The expected utility hypothesis

- A choice under risk is made by maximizing the expected utility,

$$a^* = \underset{a \in \{a_1, a_2, \dots\}}{\operatorname{argmax}} \ \mathbb{E}[U(a)]$$

- The objective function $\mathbb{E}[U(a)]$ consists of two types of quantities:
  - $Pr(a)$: probability distribution
  - $U(a)$: utility function
- n.b. each possible choice $a_1, a_2, \dots$ is a distribution; i.e. we choose one of the random variables. $U(\cdot)$ is a deterministic function, and $U(a)$ may be seen as a random variable as well.

The existence of probability distribution

- Under the assumptions As1-As5, for any event A, there exists a unique probability function $Pr(A)$ satisfying $A \sim_L G[0, Pr(A)]$ where $G[a, b]$ is the event that a uniformly distributed random variable lies in the interval (a, b).

Elicitation of probabilities

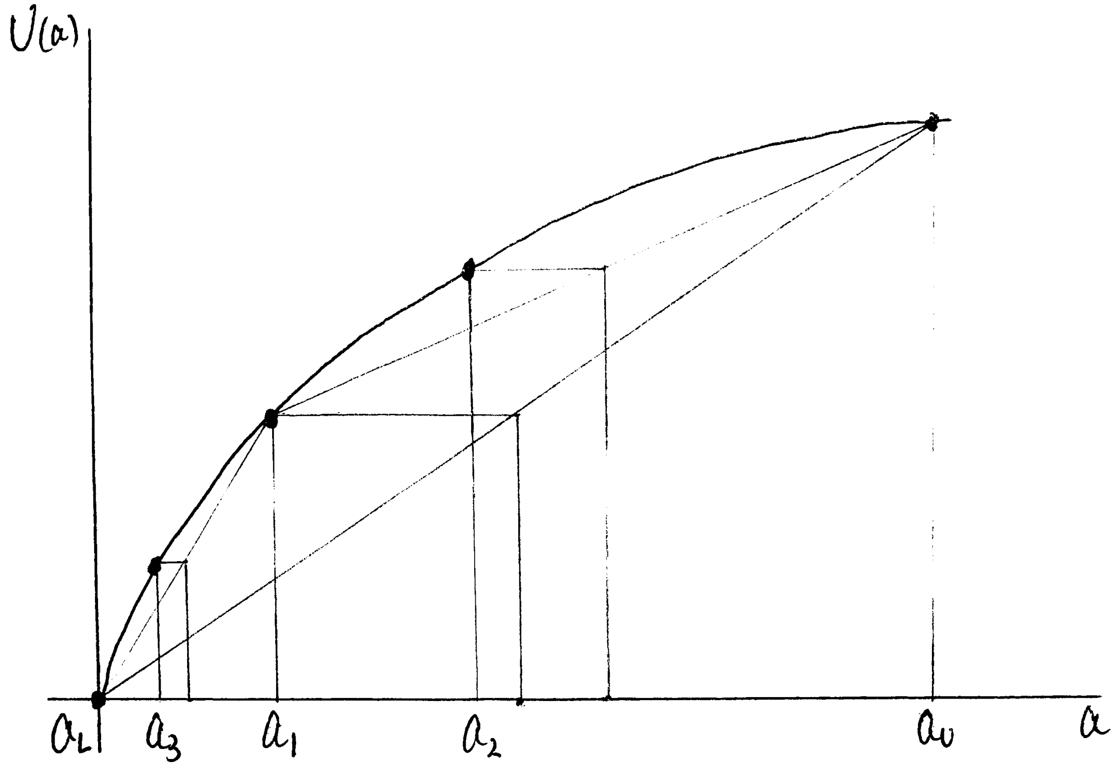
- Repeatable events
  - nonparametric statistics (e.g. quantile regression)
  - parametric statistics (e.g. MLE)
  - sample moments (e.g. mean, variance, etc.)
- Non-repeatable events
  - relying on the relative likelihood
  - reference lotteries
  - the fractile method

The existence of utility function

- A2 (independence), e.g. Does pizza (p) $\succ$ burrito (b) get reversed if ordered with horchata (h), i.e. $\beta p + (1 - \beta) h \prec \beta b + (1 - \beta) h$?
- "$U(a)$ is unique up to an affine transformation." Technically, the implication goes in both directions. That is, $U(a)$ and $V(a)$ represents the same preferences if and only if $V(a) = \alpha + \beta U(a), (\beta > 0)$. For a proof, "if" part is straightforward, while "only if" part requires some work. See some graduate textbook, e.g. Jehle & Reny (2011, p.108).

Elicitation of preferences

- We may estimate $U(\cdot)$ by using a questionnaire and tracing certainty equivalents, a concept you will learn next week.
- We sequentially determine the certainty equivalent of $\mathbb{E}[U(a)]$ where a takes only two possible values with the equal probability. We start with an lower bound (a_L) and an upper bound (a_U). Note that since an affine transformation of $U(a)$ is allowed, we may set $U(a_L) = 0$ and $U(a_U) = 1$. For example,
  - $U(a_1) = \mathbb{E}[U(a)]$ where $a \in \{a_L, a_U\}$
  - $U(a_2) = \mathbb{E}[U(a)]$ where $a \in \{a_1, a_U\}$
  - $U(a_3) = \mathbb{E}[U(a)]$ where $a \in \{a_L, a_1\}$



Solver in Excel

- A way to solve an equation or a constrained optimization problem in Excel
- If not found "Data" tab, see the link