Lab 4

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Intro

This lab

Distributions in R

In R, there are many functions which allow us to sample from or compute with probability distributions. Today we'll look at the normal, T, and χ^2 (pronounced `chi squared'' where`chi’’ is pronounced like the first part of “cayanne pepper’’) but the structure applies to many more types of distributions.

The function names have the same type of structure: the letter ‘d’, ‘p’, ‘q’, or ‘r’ followed by a name of the distribution.

- ‘r’: samples **r**andom values from the distribution
- ‘d’: computes the value of the **d**ensity at specific value
- ‘p’: computes the value of the cumulative distribution function at a specific value; i.e., the **p**robability $P(X < x)$
- ‘q’: computes the value of the **q**uantile function; i.e., given some value $0 \leq \alpha \leq 1$, what is the value of x such that $P(X < x) = \alpha$.

Random draws: rnorm

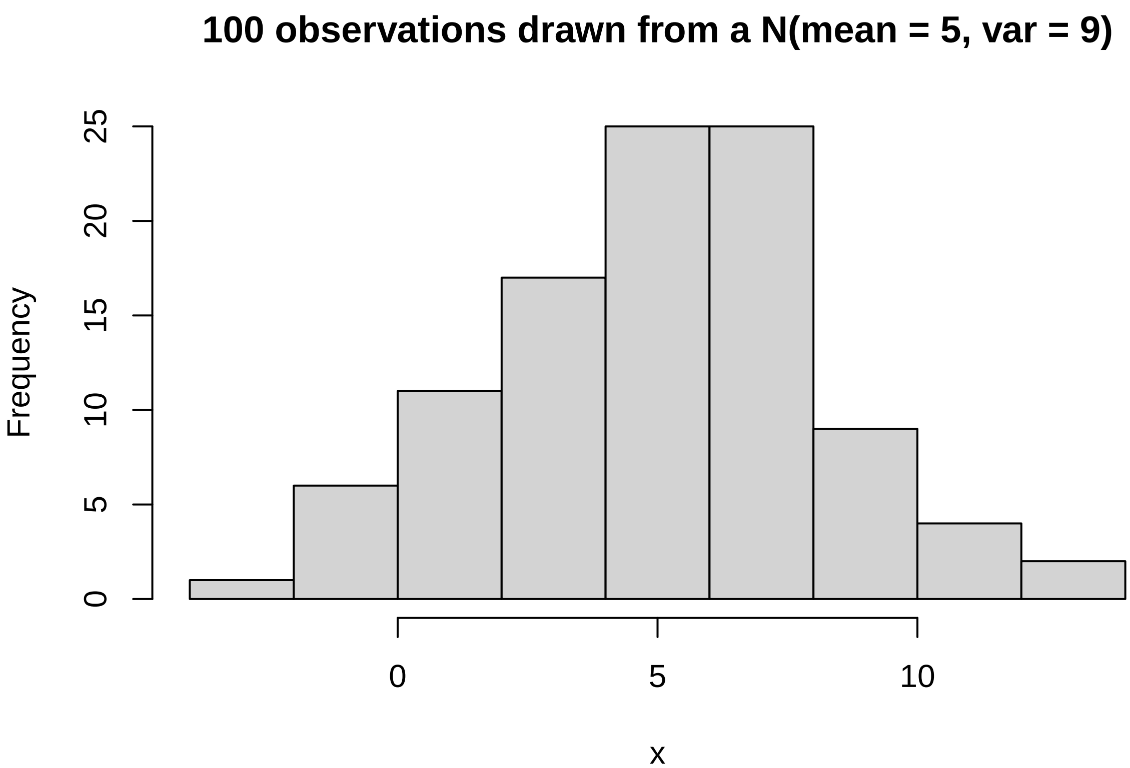
We used the `rnorm` function last week to draw random values from a normal distribution. The `rnorm` function takes 3 arguments:

- n: the number of random observations to draw
- mean: the mean of the normal distribution that the observatios will be drawn from
- sd: the standard deviation of the normal distribution that the observatios will be drawn from

If you don't enter in values, R will use the default values of mean = 0, and sd = 1. Often when we write out a normal distribution in mathematical notation, we use something like $N(3, 4)$ to indiciate a normal distribution with mean 3 and variance 4. Specifying the variance is different than the way that R specifies the normal distribution with the standard deviation so be careful when you are coding!

```
# Draw 100 observations from a normal distirbution with mean = 5, and sd = 3
x <- rnorm(100, mean = 5, sd = 3)

# plot observations
hist(x, main = "100 observations drawn from a N(mean = 5, var = 9)")
```




```
## the mean and variance of the sample aren't exactly the same as the population values
## and the values that you get when you run this will change each time
mean(x)
```

```
## [1] 5.000591
```

```
var(x)
```

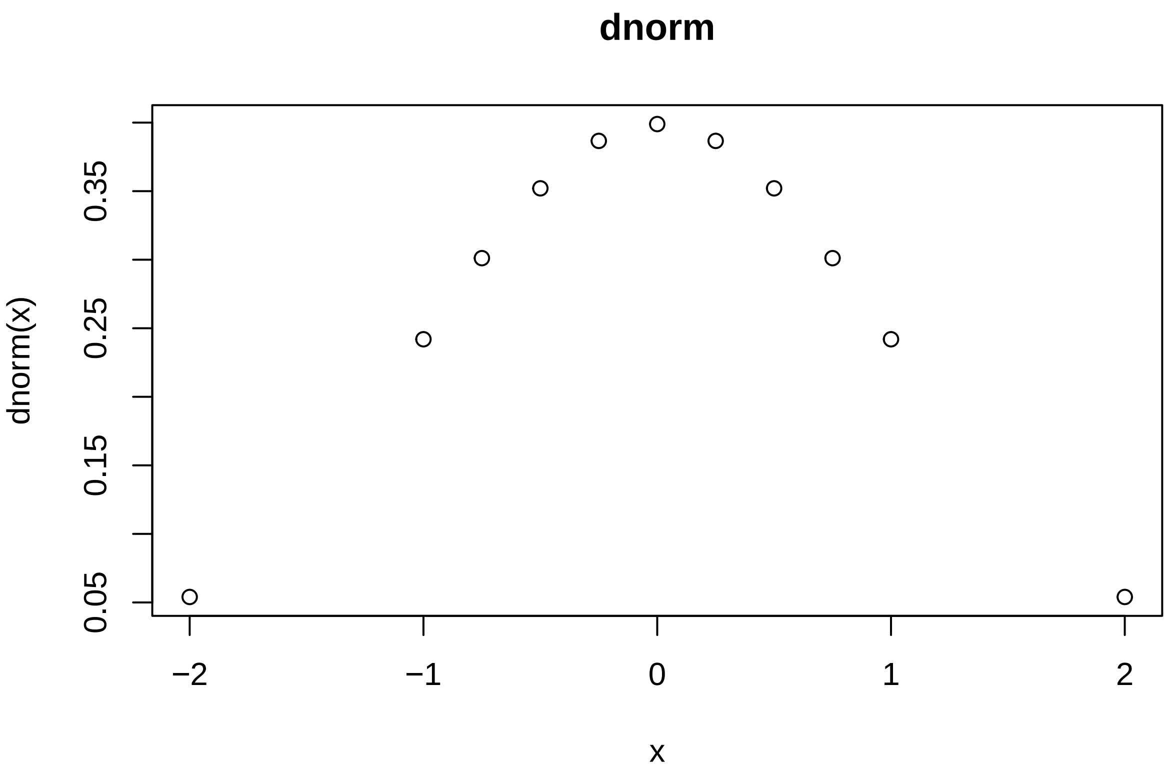
```
## [1] 10.1625
```

Density functions: dnorm

The `dnorm` takes a point (or vector of points) and evalutes the density function at that point. We will also use the `seq` function which creates a sequence of points

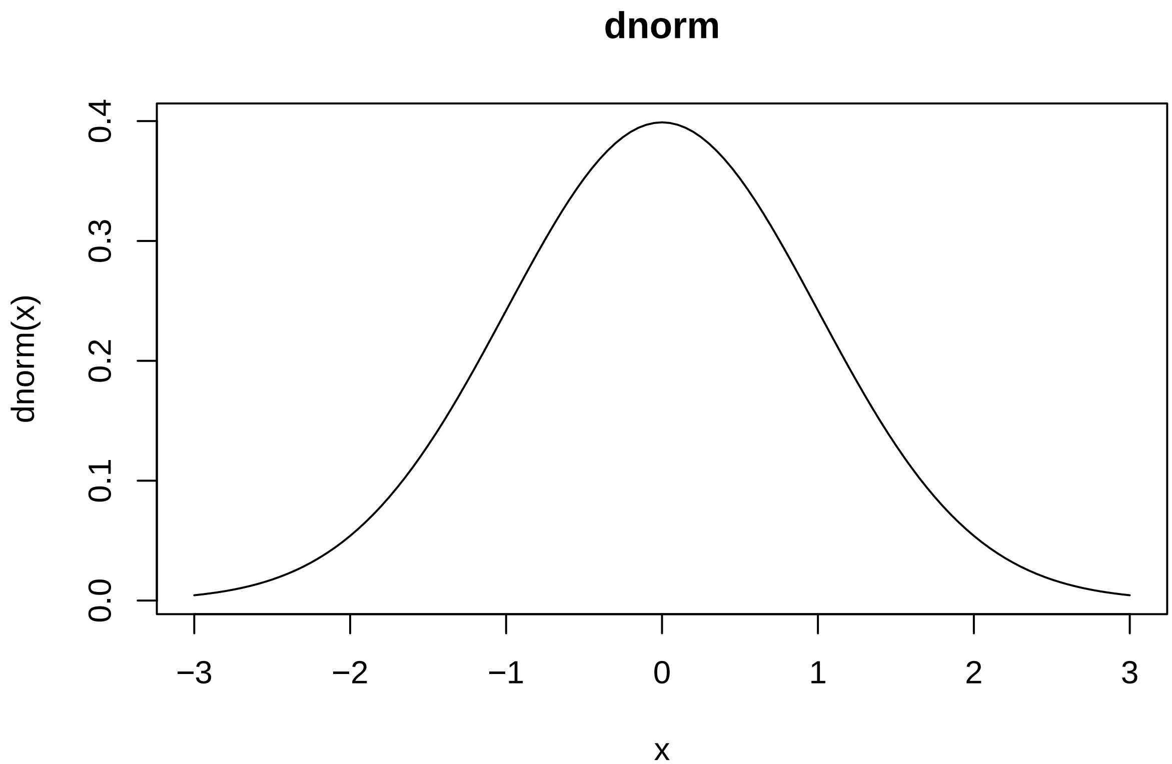
```
x <- c(-2, -1, -.75, -.5, - .25, 0, .25, .5, .75, 1, 2)

# plot the density evaluated at certain points
# since we don't specify a mean and sd, R uses the defaults
# using 'type=l' makes it a line plot
plot(x, dnorm(x), main = "dnorm")
```




```
# seq creates of numbers which starts at 'from' and goes to 'to'
# each number is spaced apart by the 'by' argument
x <- seq(from = -3, to= 3, by = .05)

# we use the 'type = l' argument to use a line plot instead of points
plot(x, dnorm(x), main = "dnorm", type = "l")
```

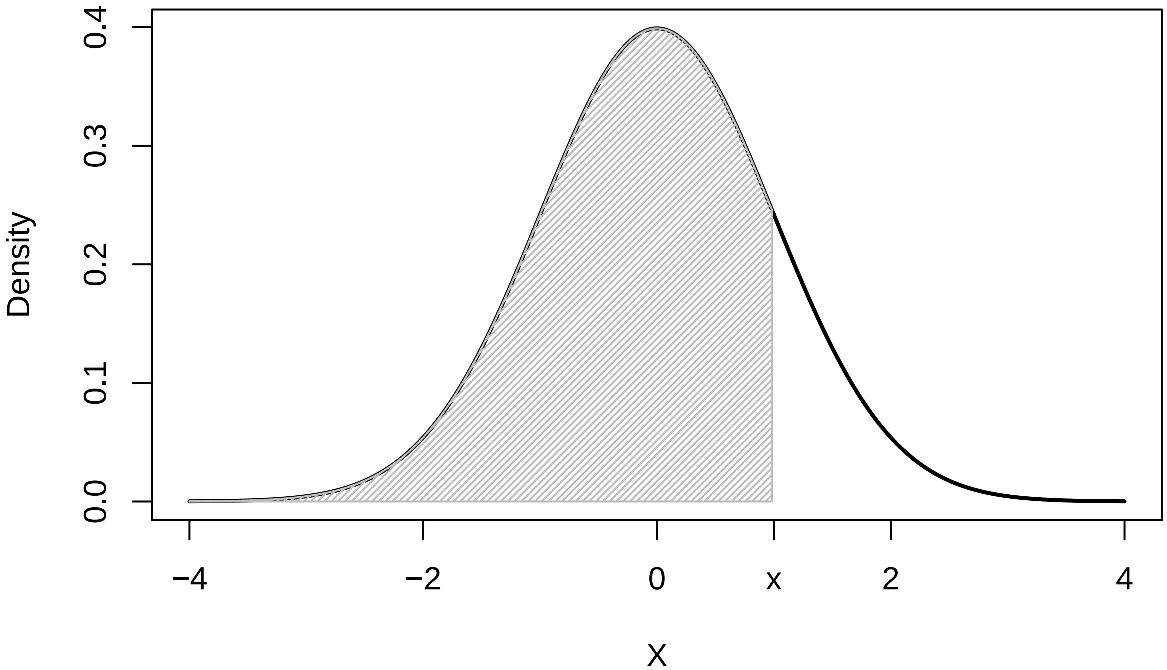

dnorm
dnorm(x)
0.0
0.1
0.2
0.3
0.4
−3
−2
−1
0
1
2
3
x

Cumulative distribution functions: pnorm

The `pnorm` takes a point (or vector of points) and evalutes the cumulative distribution function at that point. This means, given some point x, what is the probability that a random draw from a given distribution is less than x. Written out in mathematical notation, this is:

$$P(X < x).$$

In the plot below, `pnorm(x, 0, 1)` would return the area of the shaded region where the density plotted corresponds to a normal distribution with mean 0 and standard deviation 1.



For example, we can see that the probability that an observation less than 0 is drawn from a normal distribution with mean = 0 and sd = 1 is .5 because the median of the normal distribution is also the mean.

```
pnorm(0, mean = 0, sd = 1)
```

```
## [1] 0.5
```

The probability that an observation from $N(0, 1)$ is less than 1—i.e., $P(X < 1)$—is the value:

```
pnorm(1, mean = 0, sd = 1)
```

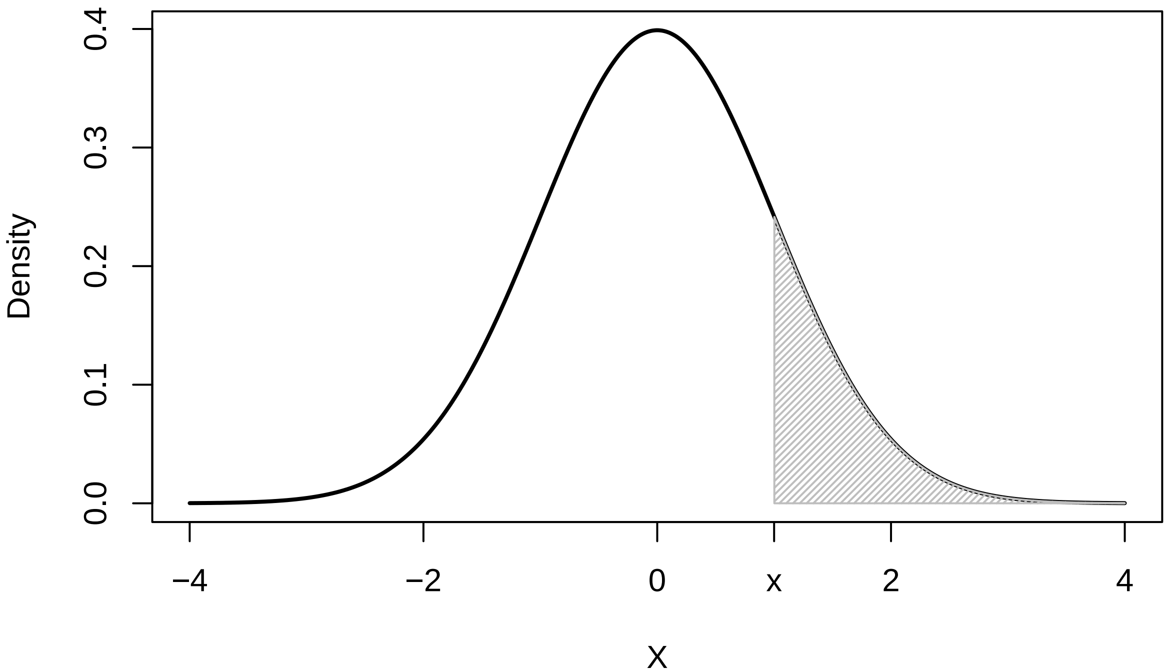
```
## [1] 0.8413447
```

Questions

- Suppose X is a normal distribution with mean .5 instead of 0 but the standard deviation is still 1—i.e., $N(.5, 1)$—do you think the probability $P(X < 1)$ increases or decrease compared to when X is $N(0, 1)$? Why?
- Suppose X is a normal distribution with mean 0 but the standard deviation is 2 instead of 1—i.e., $N(0, 2)$—do you think the probability $P(X < 1)$ increases or decreases compared to when X is $N(0, 1)$? Why?
- Using R, check whether you are correct
- Evaluate the following probabilities
  - If X is drawn from $N(2, 2)$ what is $P(X < 3)$?
  - If X is drawn from $N(2, 2)$ what is $P(X > 3)$?

- If X is drawn from $N(-2, 1)$ what is $P(-3 < X > -1)$?

By default, the `lower.tail` argument is TRUE, so we calculate the area under the density function that is in the lower tail; i.e., $P(X < x)$. We could set `lower.tail = FALSE` to calculate the area under the density function that is in the upper tail. This would give us $P(X > x)$



Since the total area under the density is always equal to 1, we know that the area to the left of a value is always equal to 1 minus the area to the right of a value.

```
# area to the left of 1
pnorm(1, mean = 0, sd = 1)
```

```
## [1] 0.8413447
```

```
# area to the right of 1
pnorm(1, mean = 0, sd = 1, lower.tail = F)
```

```
## [1] 0.1586553
```

```
# 1 minus area to the right of 1
1 - pnorm(1, mean = 0, sd = 1, lower.tail = F)
```

```
## [1] 0.8413447
```

To get the probability that a random observation, X, is between two different numbers, x_1 and x_0,

$$P(x_0 < X < x_1),$$

we can simply subtract the area to the left of x_0 from the area to the left of x_1

```
pnorm(1, mean = 0, sd = 1)
```

```
## [1] 0.8413447
```

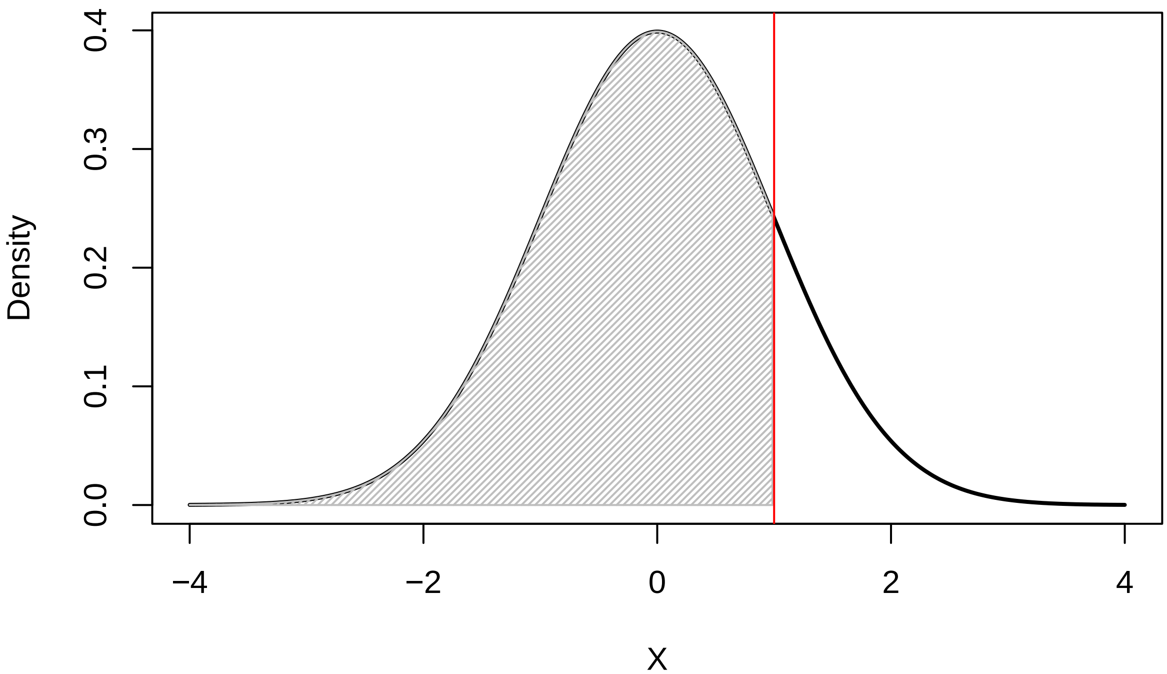
```
pnorm(0, mean = 0, sd = 1)
```

```
## [1] 0.5
```

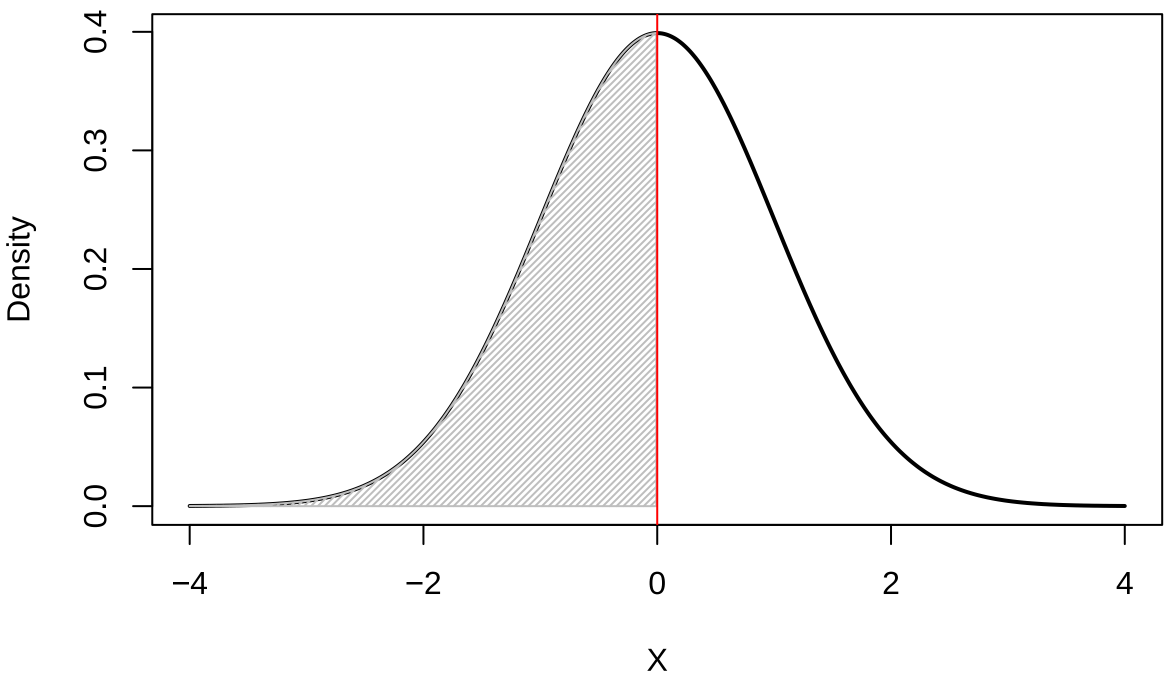
```r
pnorm(1, mean = 0, sd = 1) - pnorm(0, mean = 0, sd = 1)
```

```
## [1] 0.3413447
```

```r
shadenorm(below = 1, justbelow = T)
abline(v = 1, col = "red")
```

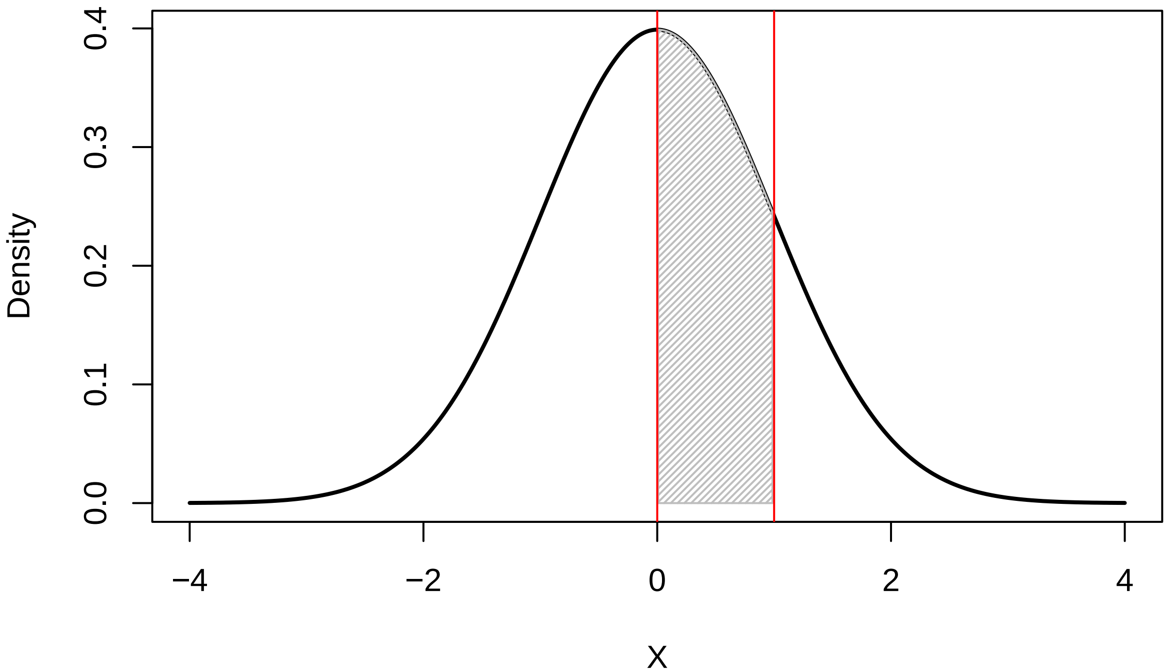



```r
shadenorm(below = 0, justbelow = T)
abline(v = 0, col = "red")
```




```r
shadenorm(between = c(0, 1))
abline(v = 1, col = "red")
```

```r
abline(v = 0, col = "red")
```



Quantile function: qnorm

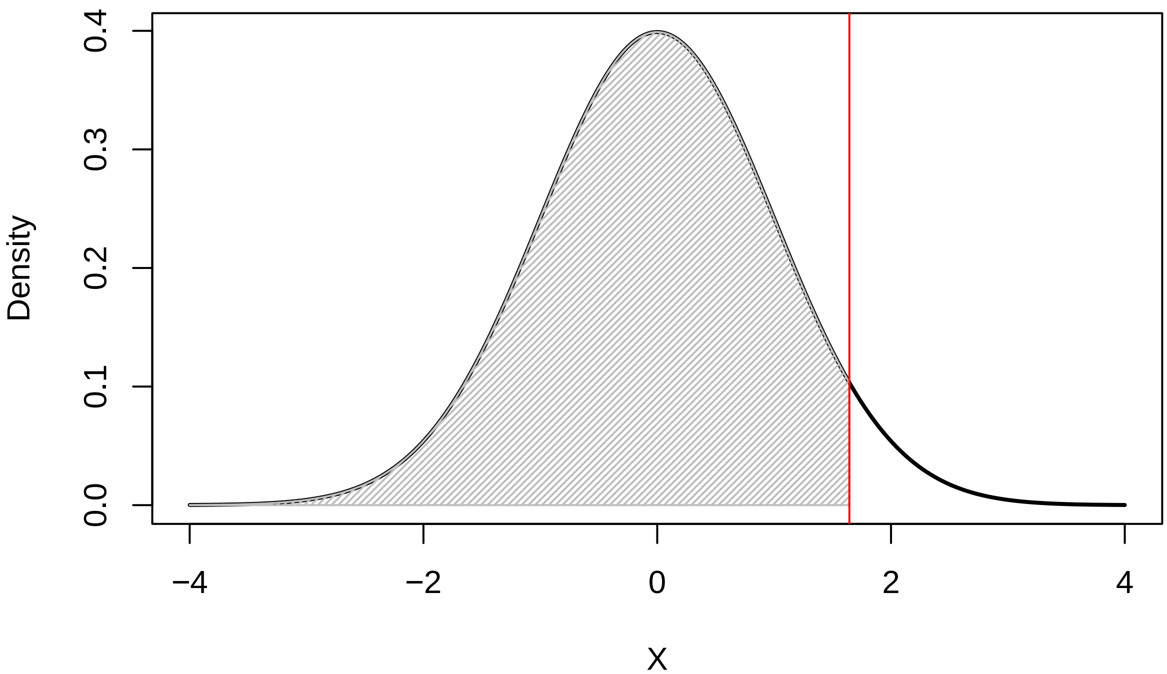
The `qnorm` function is the inverse of the pnorm function. Specifically, given a proportion, α it returns the value x such that $P(X < x) = \alpha$. If we want to know what value is larger than .95 of draws from a normal distribution with mean 0 and sd = 1:

```
qnorm(.95, mean = 0, sd = 1)
```

```
## [1] 1.644854
```

So `qnorm(.95, mean = 0, sd = 1)` finds the value such that the shaded portion has an area equal to .95

```
shadenorm(below = 1.644, justbelow = T)
abline(v = 1.644, col = "red")
```

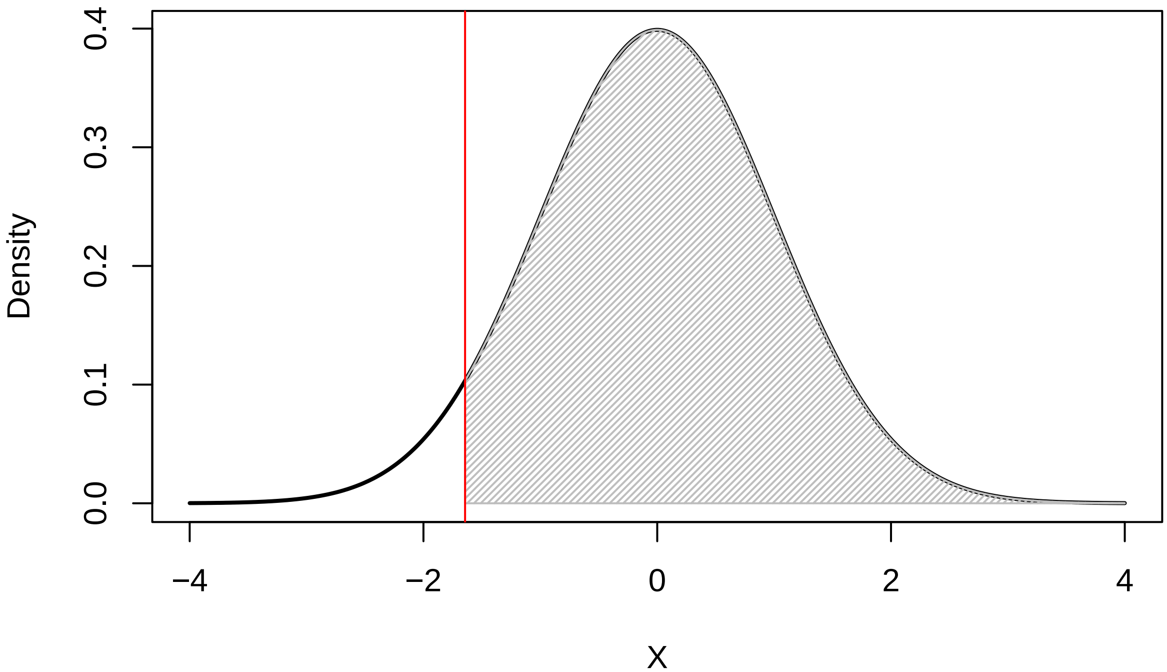


Similar to `pnorm`, the `lower.tail` option by default is set to `TRUE` so it assumes the area which is equal to α is in the lower.tail. If we set `lower.tail` to false, `qnorm` will calculate what value of x we'd need such that $P(X > x) = \alpha$.

```
qnorm(.95, mean = 0, sd = 1, lower.tail = F)
```

```
## [1] -1.644854
```

```
shadenorm(above = -1.644, justabove = T)
abline(v = -1.644, col = "red")
```



Questions

- Suppose X is a normal distribution with mean .5 instead of 0 but the standard deviation is still 1—i.e., $N(.5, 1)$—do you think `qnorm(.95, mean = .5, sd = 1)` increases or decrease compared to when X is $N(0, 1)$? Why?
- Suppose X is a normal distribution with mean 0 but the standard deviation is 2 instead of 1—i.e., $N(0, 2)$—do you think `qnorm(.95, mean = .5, sd = 1)` increases or decreases compared to when X is $N(0, 1)$? Why?
- Using R, check whether you are correct
- Evaluate the following probabilities
  - If X is drawn from $N(2, 2)$ what is x such that for $\alpha = .025$ we have $P(X < x) = \alpha$?
  - If X is drawn from $N(2, 2)$ what is x such that for $\alpha = .025$ we have $P(X > x) = \alpha$?

Look at sampling distribution of b, σ^2_ε, and t Under non-normality Under increasing df

Trade-off between Type I and Type II error