# STAT 311: Introduction to Probability (cont)

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# Logistics

- Midterm on Friday
- Will include material up to today
- Bring practice midterm to lab on Thursday

# Dependant vs Independent

Intuitively, we think about independent events as having no effect on each other. We define independence vs dependence formally through conditional probabilities.

Two events $A$ and $B$ are independent if

$$P(A|B) = P(A) \text{ and } P(B|A) = P(B)$$

Two events $A$ and $B$ are dependent if

$$P(A|B) \neq P(A) \text{ and } P(B|A) \neq P(B)$$

# Probability Rules

We can calculate the probability of events using the following basic probability rules

- Complement (not): $P(A^c) = 1 - P(A)$
- Union (or): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Intersection (and): $P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$
- Total Probability (marginal from joint): $P(A) = \sum_i P(A \cap B_i)$
- Bayes Rule: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

# Example: Probability Terminology

Suppose you are playing Pokemon Go, and each time you go to the park, one (or more) of the following events could happen

- Catch nothing
- Catch a Pikachu
- Catch a Clefairy
- Catch a Weedle
- Catch a Nidoran

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- Catch a Weedle
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Questions:

- What is the sample space for a single trial?
- What is a compound event?
- Are any of the outcomes mutually exclusive?
- What is complement of catching nothing?

# Example: Probability Rules

Suppose the following events have the probabilities-

- P(Catch nothing) = .3
- P(Catch a Pikachu) = .4
- P(Catch a Clefairy) = .2
- P(Catch a Weedle) = .6
- P(Catch a Nidoran) = .2

Questions:

- What is the probability of catching at least something?

# Example: Probability Rules

Suppose the following events have the probabilities-

- P(Catch nothing) = .3
- P(Catch a Pikachu) = .4
- P(Catch a Clefairy) = .2
- P(Catch a Weedle) = .6
- P(Catch a Nidoran) = .2

Questions:

- What is the probability of $P(Pikachu \cup Nothing)$? What assumption did we make?

# Example: Probability Rules

Suppose the following events have the probabilities-

- P(Catch nothing) = .3
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- P(Catch a Clefairy) = .2
- P(Catch a Weedle) = .6
- P(Catch a Nidoran) = .2

Questions:

- If $P(Pikachu \cap Clefairy) = .1$, what is the probability of $P(Pikachu \cup Clefairy)$?

# Example: Probability Rules

Suppose the following events have the probabilities-

- P(Catch nothing) = .3
- P(Catch a Pikachu) = .4
- P(Catch a Clefairy) = .2
- P(Catch a Weedle) = .6
- P(Catch a Nidoran) = .2

Questions:

- Given the previous answer, what is the conditional probability of catching a Pikachu, given that you have caught a Clefairy? Does that mean that catching a Clefairy and a Pikachu are independent events?

# Example: Probability Rules

Suppose the following events have the probabilities-

- P(Catch nothing) = .3
- P(Catch a Pikachu) = .4
- P(Catch a Clefairy) = .2
- P(Catch a Weedle) = .6
- P(Catch a Nidoran) = .2

Questions:

- Assuming you can only catch a maximum of two Pokemon, and the probability of $P(Pikachu \cap Clefairy) = .1$, $P(Pikachu \cap Weedle) = .05$, $P(Pikachu \cap Nidoran) = .1$ and $P(Pikachu \cap Pikachu) = .05$ what is the probability of catching only a Pikachu?

# Example: Probability Rules

Suppose the following events have the probabilities-

- P(Catch nothing) = .3
- P(Catch a Pikachu) = .4
- P(Catch a Clefairy) = .2
- P(Catch a Weedle) = .6
- P(Catch a Nidoran) = .2

Questions:

- Assuming you can only catch a maximum of two Pokemon, and the probability of $P(Nidoran|Weedle) = .1$, $P(Clefairy|Weedle) = .4$, $P(Pikachu|Weedle) = .1$, $P(Nothing|Weedle) = .4$, draw a probability tree to and calculate the joint probability of each outcome.