STAT 311: Homework 3
Due: Jul 15, in class

Name: Answer Key

The material covered includes chapter 4 as well as Lectures 5, 6 and 7. In general, rounding to 2 digits is sufficient.

1 Punxsutawney Phil: Prophet or Phraud?

Legend has it that Punxsutawney Phil, a groundhog from Punxsutawney, Pennsylvania is capable of predicting the severity of the weather. On Groundhog day each year (Feb 2), Phil rises from his burrow and if he sees his shadow, it means that is will be a long winter[1]. If Phil doesn't see his own shadow, it means that there will be a early spring. Phil has appeared on The Oprah Winfrey Show and was immortalized by the 1993 movie "Groundhog Day" starring Bill Murray. But has Phil been fooling us this whole time, or is he the real deal? Let's take a look.

For this homework, we will consider data from the National Oceanic and Atmospheric Administration on the average March monthly temperature (from Pittsburgh, the closest major city to Punxsutawney) and Phil's predictions from 1903-2016[2].

Consider the following table. Since temperature is a continuous variable, we have first converted to z-scores, then discretized the z-scores into "Hot", "Medium", "Cold." Any z-score below -.5 is considered "Cold", any z-score between -.5 and .5 is considered "Medium" and any z-score above .5 is considered "Hot." The rows indicate Phil's predictions, and the columns indicate the actual resulting March temperatures.

	Cold	Hot	Medium	Total
Early Spring	6	4	5	15
Long Winter	27	25	30	82
No Prediction	0	1	1	2
Total	33	30	36	99

1. *The average of the average March temperatures is 39.9 and the standard deviation of the average March temperatures is 4.46. What is the z-score for 1906, when the average March temperature was 33.3 degrees? What is the z-score for 1906, when the average March temperature was 33.3 degrees? What about 2012, when the temperature was 50.36?*

$$z_{1906} = \frac{33.3 - 39.9}{4.46} = -1.48$$

$$z_{2012} = \frac{50.6 - 39.9}{4.46} = 2.40$$

[1] `https://en.wikipedia.org/wiki/Punxsutawney_Phil`

[2] If you want to check out the data for yourself, you can get it up at `http://www.stat.washington.edu/~ysamwang/notes/ground_hog_day_data.csv`

2. *If 2005 had a z-score of -.9, what average March temperature does that correspond to?*

$$z_{2005} = -.9 = \frac{x_{2005} - 39.9}{4.46}$$

$$x_{2005} = 35.89$$

3. *Given that March was cold, what is the distribution of Phil's prediction?*

   - Early Spring = 6 / 33 = .18
   - Long Winter = 27 / 33 = .82
   - No Prediction = 0 / 33 = 0

4. *What is the risk ratio of a "Cold" march, given that Phil has predicted a "Long Winter" vs given that Phil has predicted an "Early Spring"?*

   - $P(\text{Cold}|\text{Long Winter}) = 27/82 = .33$
   - $P(\text{Cold}|\text{Early Spring}) = 6/15 = .4$
   - Risk Ratio = P(Cold — Long Winter) / P(Cold — Early Spring) = .83

5. *What is the risk ratio of a "Hot" march, given that Phil has predicted a "Long Winter" vs given that Phil has predicted an "Early Spring"?*

   - $P(\text{Hot}|\text{Long Winter}) = 25/82 = .30$
   - $P(\text{Hot}|\text{Early Spring}) = 4/15 = .27$
   - Risk Ratio = $P(\text{Hot}|\text{Long Winter})/P(\text{Hot}|\text{Early Spring}) = 1.11$

6. *What do the two risk ratios you calculated above say about Phil's ability to predict the weather?*

   Phil is wrong more often than not

2 Simpson's Paradox in the Florida Justice System

In a 1991 study, Radelet and Pierce [1991] examine the sentencing outcome of all murder convictions in Florida from 1976-1987. They record the race of the convicted defendant as well as whether or not the defendant received the death penalty. Unfortunately, this is real data.

	Death Penalty	No Death Penalty	Total
Black	15	176	191
White	53	430	483
Total	68	606	674

1. *What is the conditional probability of the death penalty, given that the defendant is white?*

   $P(\text{Death}|\text{White Defendant}) = 53/483 = .11$

2. *What about the conditional distribution if the defendant is black?*

   $P(\text{Death}|\text{Black Defendant}) = 15/191 = .08$

3. *Which race is more likely to receive the death penalty?*

   White

Let's dive deeper into the data now. In addition to the race of the defendant, let's also consider the race of the victim.

Victim Race	Defendent Race	Death	No Death
White	White	53	414
White	Black	11	37
Black	White	0	16
Black	Black	4	139

4. *For the cases where the victim is white, what is the conditional distribution of sentencing outcome, given that the defendant is white?*

   $P(\text{Death}|\text{White victim} \cap \text{White Defendant}) = 53/(53 + 414) = .11$

   $P(\text{No Death}|\text{White victim} \cap \text{White Defendant}) = 414/(53 + 414) = .89$

5. *For the cases where the victim is white, what is the conditional distribution of sentencing outcome, given that the defendant is black?*

   $P(\text{Death}|\text{White victim} \cap \text{Black Defendant}) = 53/(53 + 414) = .22$

   $P(\text{No Death}|\text{White victim} \cap \text{Black Defendant}) = 414/(53 + 414) = .77$

6. *For the cases where the victim is black, what is the conditional probability of the death penalty, given that the defendant is white?*

   $P(\text{Death}|\text{Black victim} \cap \text{White Defendant}) = 0/(0 + 16) = 0$

   $P(\text{No Death}|\text{Black victim} \cap \text{White Defendant}) = 16/(0 + 16) = 1$

7. *For the cases where the victim is black, what is the conditional distribution of sentencing outcome, given that the defendant is black?*

   $P(\text{Death}|\text{Black victim} \cap \text{Black Defendant}) = 4/(4 + 139) = .03$

   $P(\text{No Death}|\text{Black victim} \cap \text{Black Defendant}) = 139/(4 + 139) = .97$

8. *What proportion of all convictions result in the death penalty when the victim is white? What proportion of all convictions result in the death penalty when the victim is black?*

   $P(\text{Death}|\text{White victim}) = (53 + 11)/(53 + 11 + 414 + 37) = .12$

   $P(\text{Death}|\text{Black victim}) = (0 + 4)/(0 + 4 + 16 + 139) = .03$

9. *Which race (of the defendant) is more likely to receive the death penalty in the disaggregated data?*

   We see when splitting the data by the race of the victim, black defendants have a higher conditional probability of receiving the death penalty both when the victim is white and when the victim is black.

10. *How does this compare to the results we saw when the data was aggregated?*

    This is an example of Simpson's paradox where the aggregated data tells the opposite story of the disaggregated data. The confounding variable in this case is the race of the victim.

    We see in the data, when the victim is white, the defendant is much more likely to receive the death penalty than when the victim is black. We also see that white defendants tend to have more white victims, while black defendants tend to have more black victims. Thus, although black defendants are more likely to receive the death penalty in either case, this observation is not clear when only considering the aggregated data.

3 Testing for HIV

In a 2006 metastudy, Delaney et al. [2006] examine the efficacy of the OraQuick HIV test. OraQuick was approved by the FDA in 2012 and can be purchased over the counter. The data on OraQuick's test results from a saliva sample is reproduced below.

	HIV+	HIV-	Total
Test+	327	54	381
Test-	3	12010	12013
Total	330	12064	12394

1. *What is the Sensitivity of the OraQuick test?*

   $P(Test + |True+) = 327/330 = .99$

2. *What is the Specificity of the OraQuick test?*

   $P(Test - |True-) = 12010/12064 = .996 \approx 1$

3. *What is the Positive Predictive value of the OraQuick test?*

   $P(True + |Test+) = 327/381 = .86$

4. *What is the Negative Predictive value of the OraQuick test?*

   $P(True - |Test-) = 12010/12013 = .9998 \approx 1$

5. *Suppose the company releases a new tests called OraQuick2. If the true status of each individual remains the same, but the specificity of the OraQuick2 test decreased slightly to .99, and the sensitivity decreased to .985, what would the new table look like? Round to the nearest person.*

	HIV+	HIV-	Total
Test+	325	121	446
Test-	5	11943	11948
Total	330	12064	12394

   If there are 330 HIV+ individuals, and the sensitivity is .985, the new test would detect $.985 \times 330$ of those individuals, so the HIV+ and Test + individuals would be 325. We can calculate the other cells of the table in the same way.

6. *What are the new Positive Predictive Value and Negative Predictive Value for OraQuick2?*

   $P(True + |Test+) = 325/446 = .72$

   $P(True - |Test-) = 11943/11948 = .9995 \approx 1$

7. *Which values in the table changed the most? What does this say about the effect of a small change in the sensitivity or specificity of a test?*

   The number of false positives drastically changed (more than doubles), while the other cells did not see large relative changes. This is because there is a much larger number of individuals who are HIV-. So, for rare diseases, specificity will affect more outcomes than sensitivity.

8. *What should the company which produces OraQuick be most concerned about- false positive or false negatives? What are the implications of either error? Note that there is not a single right answer.*

   A false positive can be traumatic for the patient temporarily, but generally a follow-up test will be prescribed which will show that the patient is not actually HIV+. However, a false negative may lead to non-treatment of the HIV+ virus which can lead to AIDS as well as transmission to other individuals. Thus, it seems that a false negative could be potentially more damaging.

References

Kevin P Delaney, Bernard M Branson, Apurva Uniyal, Peter R Kerndt, Patrick A Keenan, Krishna Jafa, Ann D Gardner, Denise J Jamieson, and Marc Bulterys. Performance of an oral fluid rapid hiv-1/2 test: experience from four cdc studies. *Aids*, 20(12):1655–1660, 2006.

Michael L Radelet and Glenn L Pierce. Choosing those who will die: Race and the death penalty in florida. *Fla. L. Rev.*, 43:1, 1991.