STAT 311: Hypothesis Testing for Two Way Tables

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Logistics

- Final next Friday
- Practice final posted today
- Review practice final on Wednesday
- Thursday will be general review / Questions

Example: Ebola case outcomes

Consider the Ebola data from we've previously examined[1]

	Death	Survive	Total
Guinea	2536	1268	3804
Liberia	4806	5860	10666
Sierra Leone	3955	10167	14122
Total	11297	17295	28592

[1] Data available from World Health Organization: `http://apps.who.int/gho/data/view.ebola-sitrep.ebola-summary-latest?lang=en`. Up to date as of Dec 2015

Two Way Tables

	Col 1	Col 2	Col 3	Total
Row 1	n_{11}	n_{12}	n_{13}	n_{1+}
Row 2	n_{21}	n_{22}	n_{23}	n_{2+}
Row 3	n_{31}	n_{32}	n_{33}	n_{3+}
Total	n_{+1}	n_{+2}	n_{+3}	n_{++}

- Joint
- Marginal
- Conditional

Independence

Two events are independent if the conditional distribution is equal to the marginal distribution

$$P(A|B) = P(A)$$

which implies the joint is the product of the marginals

$$P(A \bigcap B) = P(A|B)P(B) = P(A)P(B)$$

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$$\frac{n_{i+}}{n_{++}}$$

or

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Expected Counts under independence

Under the assumption of independence,

$$P(R = r_i \bigcap C = c_j) = P(R = r_i)P(C = c_j) = \frac{n_{i+}}{n_{++}}\frac{n_{+j}}{n_{++}}$$

so the expected count is

$$n_{++}\frac{n_{i+}}{n_{++}}\frac{n_{+j}}{n_{++}} = \frac{n_{i+}n_{+j}}{n_{++}}$$

Measure of deviation

Given that there are many cells in a table, how do we measure how "different" the counts are from what we would expect if the variables are independent? To test

$$H_0 : \text{No association between row and column variables}$$

$$H_A : \text{Association between row and column variables}$$

we can use the following test statistic

$$\chi = \sum_{ij} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

where O_{ij} is the observed counts and E_{ij} is the expected counts

Under the null hypothesis, χ follows a χ^2 distribution with (Rows -1)(Columns - 1) degrees of freedom.

χ^2 Distribution

The χ^2 distribution has a single parameter k which is the degrees of freedom.

Given standard normal random variables Z_i, we can form a χ^2 variable with k degrees of freedom-

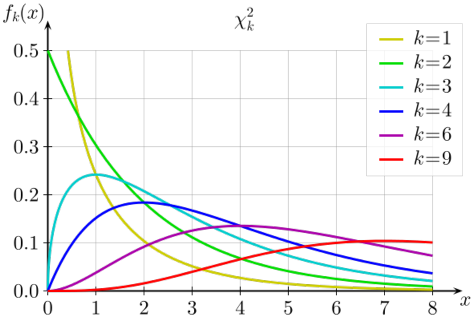
$$\chi = \sum_{i=1}^{k} Z_i^2$$

χ^2 Distribution

For $X \sim \chi^2_k$,

$$E(X) = k$$

$$Var(X) = 2k$$



Example: Ebola Case Outcomes

The observed counts are

	Death	Survive	Total
Guinea	2536	1268	3804
Liberia	4806	5860	10666
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Total	11297	17295	28592

The Expected counts are

	Death	Survive	Total
Guinea	1503.00	2301.00	3804
Liberia	4214.25	6451.75	10666
Sierra Leone	5579.75	8542.25	14122
Total	11297	17295	28592

Example: Ebola Case Outcomes

The deviations are

	Death	Survive	Total
Guinea	709.97	463.75	-
Liberia	83.09	54.27	-
Sierra Leone	473.10	309.037	-
Total	-	-	-

$$\chi = 2093$$

Under the null hypothesis, χ should be distributed as a $\chi^2(3)$
So we reject the null hypothesis that there is no association between the country and case outcome

Class Roadmap