*

x_1	x_2	Y
10	12	1
8	9	0
10	15	1
10	10	0
7	4	0
13	15	1
11	13	1

Gaussian Classifier

Q. Train the model using Gaussian classifier & decide the class of data-point ($x_1 = 10$, $x_2 = 11$)

⇒ Step 1 ⇒ Calculate Prior Probabilities & mean & variance.

$$P(1) = \frac{4}{7} = 0.57, \quad P(0) = \frac{3}{7} = 0.42 \Rightarrow \text{Prior Probabilities}$$

$$\mu_1 \text{ (Mean for 1)} = \frac{1}{N_1} \sum_{x \in 1} x$$

$$= \frac{1}{4}\left[\begin{pmatrix}10\\12\end{pmatrix} + \begin{pmatrix}10\\15\end{pmatrix} + \begin{pmatrix}13\\15\end{pmatrix} + \begin{pmatrix}11\\13\end{pmatrix}\right]$$

$$= \frac{1}{4}\left[\begin{pmatrix}44\\55\end{pmatrix}\right]$$

$$= \begin{pmatrix}11\\13.75\end{pmatrix} \begin{matrix}\Rightarrow x_1\\ \Rightarrow x_2\end{matrix}$$

$$\mu_0 \text{ (Mean for 0)} = \frac{1}{N_0} \sum_{x \in 0} x$$

$$= \frac{1}{3}\left[\begin{pmatrix} 8 \\ 9 \end{pmatrix} + \begin{pmatrix} 10 \\ 10 \end{pmatrix} + \begin{pmatrix} 7 \\ 4 \end{pmatrix}\right]$$

$$= \frac{1}{3}\left[\begin{pmatrix} 25 \\ 23 \end{pmatrix}\right]$$

$$= \begin{pmatrix} 8.33 \\ 7.66 \end{pmatrix} \begin{matrix} \Rightarrow x_1 \\ \Rightarrow x_2 \end{matrix}$$

$$\sigma_1^2 = \text{Variance}_1 = \frac{1}{N_1} \sum_{x \in 1} (x_i - \mu_1)^2$$

$$= \frac{1}{4}\left[(10-11)(12-13.75) + (10-11)(15-13.75) + (13-11)(15-13.75) + (11-13)(13-13.75)\right]$$

$$= \frac{1}{4}\left[(-1)(-1.75) + (-1)(1.25) + (2)(1.25) + (-1)(-0.75)\right]$$

$$= \frac{1}{4}\left[1.75 - 1.25 + 2.5 + 0.75\right]$$

$$= \frac{1}{4}\left[0.75\right] = \underline{\underline{0.1875}} \Rightarrow \underline{\underline{\sigma_1^2}}$$

$$\sigma_0^2 = \text{Variance}_0 = \frac{1}{N_0} \sum_{x \in 0} (x_i - \mu_0)^2$$

$$= \frac{1}{3}\left[\cancel{(8-8)}(8-8.33)(9-7.66) + (10-8.33)(10-7.66) + (7-8.33)(4-7.66)\right]$$

$$= \frac{1}{3}\left[(-0.33)(1.34) + (1.67)(2.34) + (-1.33)(-3.66)\right]$$

$$\frac{1}{3}\left[-0.4422 + 3.9078 + 4.878\right] = \frac{1}{3} \times 8.3436 \quad 3$$

$$\sigma_0^2 = 2.7812$$

Step2 ⇒ Calculate log posterior probability ratio.

$$\ln\left[\frac{P(1/X=x)}{P(0/X=x)}\right] = -\frac{1}{2}\left(\frac{(x_1-\mu_1)(x_2-\mu_1)}{\sigma_1^2} - \frac{(x_1-\mu_0)(x_2-\mu_0)}{\sigma_0^2} + \ln\sigma_1^2 - \ln\sigma_0^2\right) + \ln P(1) - \ln P(0)$$

$$= -\frac{1}{2}\left(\frac{(x_1-11)(x_2-13.75)}{0.1875} - \frac{(x_1-8.33)(x_2-7.66)}{2.7812} + \ln(0.1875) - \ln(2.7812)\right) + \ln(0.57) - \ln(0.42)$$

$$= -\frac{1}{2}\left(\frac{(x_1-11)(x_2-13.75)}{0.1875} - \frac{(x_1-8.33)(x_2-7.66)}{2.7812} + \ln\left(\frac{0.1875}{2.7812}\right)\right) + \ln\left(\frac{57}{42}\right)$$

$$= -\frac{1}{2}\left(\frac{(x_1-11)(x_2-13.75)}{0.1875} - \frac{(x_1-8.33)(x_2-7.66)}{2.7812} + \ln(0.067)\right) + \ln(1.35)$$

$$\ln\left[\frac{P(1/X=x)}{P(0/X=x)}\right] = -\frac{1}{2}\left(\frac{(x_1-11)(x_2-13.75)}{0.1875} - \frac{(x_1-8.33)(x_2-7.66)}{2.7812} + \ln(0.067)\right) + \ln(1.35) \quad (4)$$

Step 3 ⇒

For ($x_1 = 10$, $x_2 = 11$),

$$\ln\left[\frac{P(1/X=x)}{P(0/X=x}\right] = -\frac{1}{2}\left(\frac{(10-11)(11-13.75)}{0.1875} - \frac{(10-8.33)(11-7.66)}{2.7812} + (-2.7030)\right) + 0.3001$$

$$= -\frac{1}{2}\left(\frac{(-1)(-2.75)}{0.1875} - \frac{(1.67)(3.34)}{2.7812} + (-2.7030)\right) + 0.3001$$

$$= -\frac{1}{2}\left(14.66 - 2.0055 - 2.7030\right) + 0.3001$$

$$= -\frac{1}{2}(9.9515) + 0.3001 = -4.975 + 0.3001$$

$$\ln\left[\frac{P(1/X=x)}{P(0/X=x)}\right] = \underline{\underline{-4.6749}} \quad (\text{Class } 0)$$