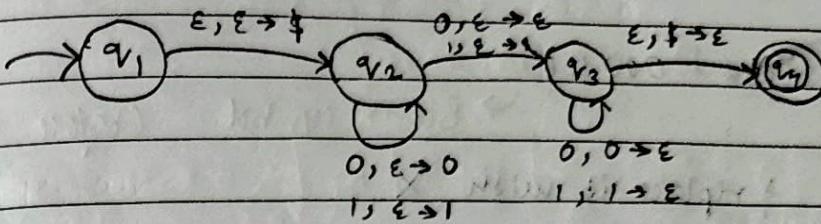
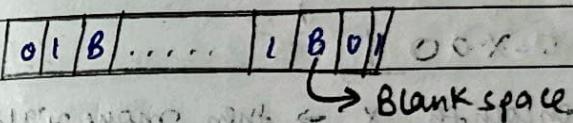


(b) $B = \{ w \in \{0,1\}^* \mid w \in w^R \text{ and length of } w \text{ is odd} \}$



1-12-23

Turing Machine \rightarrow can move in any direction in input tape
 Turing Machine \rightarrow can read & write to the tape



Formal Definition of TM

A \rightarrow Turing Machine is a 7-tuple $(Q, \Sigma, \Gamma, f, q_0, q_{acc}, q_{rej})$
 where,

- ↳ Q is a finite set of states.
- ↳ Σ is finite input alphabets.
- ↳ Γ is finite tape alphabets. (where $\square \in \Gamma \not\subseteq \Sigma \subseteq \Gamma$)
- ↳ q_0 is the start state.
- ↳ q_{acc} is the accept state.
- ↳ q_{rej} is the reject state.
- ↳ $f: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ is transition function
 head movement $\Rightarrow L = \text{Left}$
 $R = \text{Right}$

for

$\forall q \in Q, a \in \Gamma, \text{ if } f(q, a) = (q', b, L)$, then

- ↳ q' \Rightarrow New state of the machine
- ↳ b \Rightarrow letter which replaces a on the tape
- ↳ $L \Rightarrow$ next direction of head movement after reading a symbol \Rightarrow replacing with b

e.g. TM for $0^n 1^n \rightarrow \text{CFL}$

Let $n=8$: $\square 000111\square$

Blank symbol (before & after string)

- 1) Read 1st 0 & replace it with X

$\square X 00111\square$

- 2) For every input which is 0 → move right till you get a 1

- 3) When you encounter first 1 → replace with Y

↓

$\square X 00 Y 11\square$

- 4) Then, move left till you encounter X → then move right

- 5) If use a 0 again → replace with X

$\square X X 0 Y 11\square$

* Rep repeating till (keeping 0/1 intact) → from here, if input 0/Y

- 6) Then move right again.

- 7) If again 1 encountered → replace with Y

then move left till X

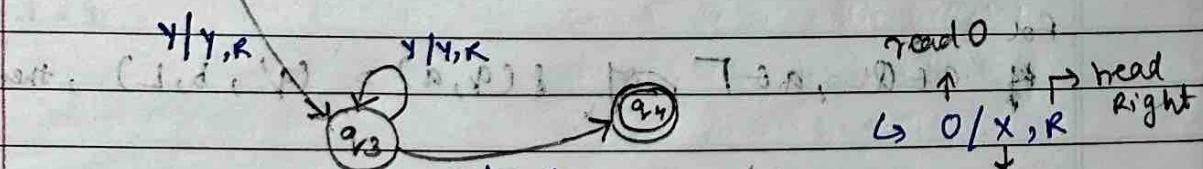
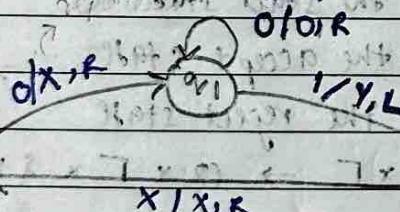
Keep repeating till all 0 → X & all 1 → Y

Y/Y, R

0/0, R

Y/Y, L

0/0, L



Y/Y, R & X/X, R

read 0
read 1
write 0
write 1
right
left

replace with X

0/0/R & 0/0/L

→ do nothing,

read 1

replace with Y

just move in the direction of head

eg:

$$\begin{array}{c} \cancel{a^n b^n c^n} \quad n \geq 1 \\ L = \{ a^n b^n c^n \mid n \geq 1 \} \\ \times \quad Y \quad Z \end{array}$$

5-12-23

Construct a) Turing Machine $w w R$ where $w \in \{a, b\}^*$

(\hookrightarrow Replace $a \rightarrow x$)

a. $a \rightarrow b \rightarrow y$

Logic:

Do all a's then

do all b's

First time

a, x, R

b, y, R

Let string: $\boxed{\square} \boxed{a} \boxed{a} \boxed{b} \boxed{b} \boxed{a} \boxed{b} \boxed{b} \boxed{a} \boxed{b} \boxed{a} \boxed{a} \boxed{\square}$

$L \rightarrow R$

w

w

x

y

z

R $\rightarrow L$

Read the

next 'a' or 'b'

do nothing

{ move R }

a, a, R

b, b, R

\square, \square, R

a, a, L

b, b, L

\square, \square, L

a, x, L

b, y, L

\square, \square, L

x, x, L

y, y, L

\square, \square, L

x, x, R

y, y, R

\square, \square, R

x, x, R

y, y, R

\square, \square, R

x, x, L

y, y, L

\square, \square, L

x, x, L

y, y, L

\square, \square, L

x, x, R

y, y, R

\square, \square, R

x, x, R

y, y, R

\square, \square, R

x, x, L

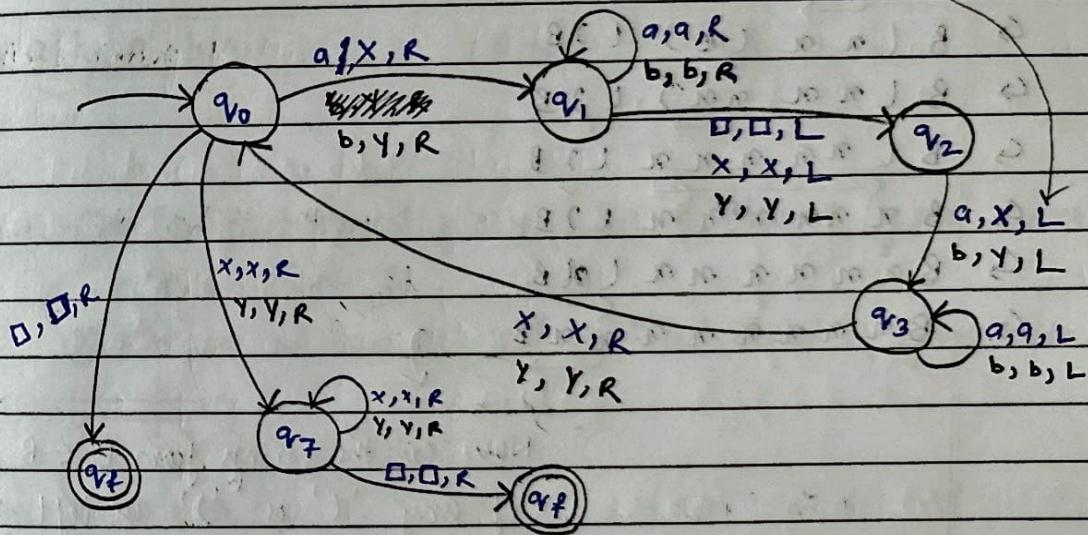
y, y, L

\square, \square, L

x, x, L

y, y, L

\square, \square, L



① a, a, R

B, b, b, R $\left. \begin{matrix} \text{P} \\ \text{a} \\ \text{p} \end{matrix} \right\}$ after changing first a into 'm' \rightarrow then move R till \square, \square, L

② Then move L { replace $a \rightarrow x$ } \rightarrow keep going till you reach an x, then 'X, X, R'.

③ Keep continuing till we transform the entire string { reach blank. Complete same for b with y. }

$$z_j \in \{a^n b^n c^n \mid n \geq 1\}$$

£1201 77 "d'0? -1

T M for well formedness of parenthesis

The diagram illustrates a stack frame structure. At the top, the word "Blank" is written above a horizontal line consisting of several short segments. Below this line, the word "Symbol" is written above a second horizontal line. The two lines are connected by a bracket spanning their entire length. Below the second line, there is a sequence of characters: B, (, (,), (,),), (,), B. Each pair of parentheses is enclosed in a rectangular box. Arrows point from the first three pairs of parentheses to the first three boxes below them, indicating they correspond to the same memory locations.

() ()) → valid string

Exception)))) C/C++ Invalid String

(C) B (-t) ()) () B

$$\left\{ \begin{array}{l} C, C, R \\ \downarrow n, L \end{array} \right\}$$

the first

first, replace closing

bracketed, with 'n' } more left;

$\mathcal{L} \circ B(\log(1))$, C_B

卷之三

$\hookrightarrow B(x_n, r) \hookrightarrow B$

$$\hookrightarrow B \xrightarrow{\text{inclusion}} (B) \hookrightarrow B$$

$\hookrightarrow B(\alpha \alpha \alpha \alpha)$ () B

\hookrightarrow B L n n n n x L) B

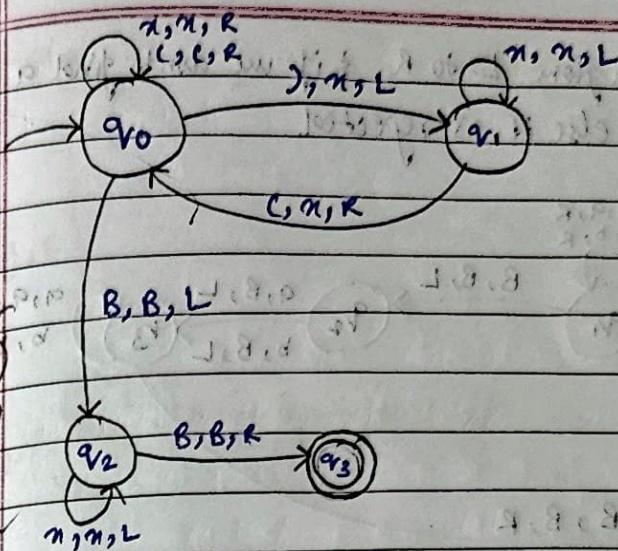
Бананы и яблоки

В а а а а а а а

QUESTION ANSWER

Now scan the strings from $B \leftarrow B$,
if any 'l' or 'r' are left \rightarrow string
is rejected

else accepted. ①



a. Design a TM for odd palindrome over $\{a, b\}^*$

↳ waw^r | $w \in \{a, b\}^*$

$B \underset{w}{a} \underset{b}{b} \underset{a}{b} \underset{b}{a} B$

$B \underset{w}{a} \underset{b}{b} \underset{b}{b} \underset{a}{b} B$

↳ Logic: ① Read first element & check last element, if they match, replace with B

② Do the same for 2nd & 2nd last element & replace with B.

Blank character ↳

$B \underset{a}{a} \underset{b}{b} \underset{a}{b} \underset{a}{b} a B$

$B \underset{B}{B} \underset{b}{b} \underset{a}{a} \underset{b}{b} a B$

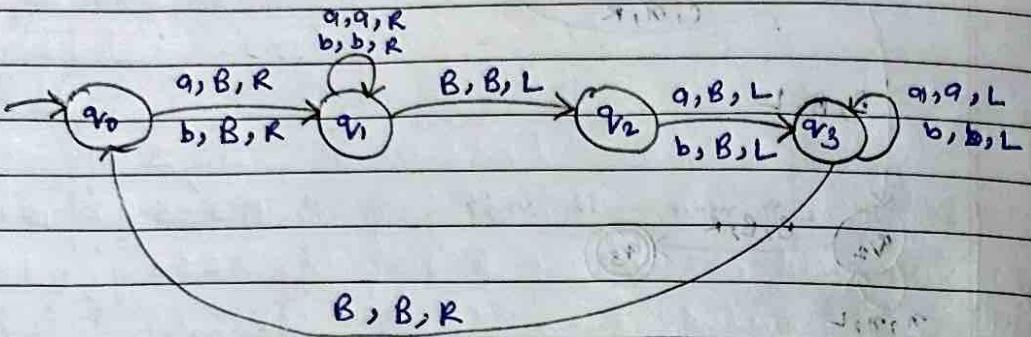
$B \underset{B}{B} \underset{b}{b} \underset{a}{a} \underset{b}{b} B B$

$B B \underset{B}{B} \underset{a}{a} \underset{b}{b} B B$

$B B \underset{B}{B} \underset{B}{B} \underset{B}{B} B$

$B B \underset{B}{B} \underset{B}{B} \underset{B}{B} B$

↳ We then do an entire scan from L to R. If we don't find a or b, then string is accepted, else it is rejected.



7-18-23

$$\underline{\text{Ex. 6}} \quad L = \{a^i b^j \mid i < j\}$$

Apprendimento

↳ Logic: connect till $x = y$,

$i=1, j=2$: $\{abb\} \rightarrow \{baab\}$ if any middle element (a, b)

i=1 , j=3 abbb

remaining → connect } now

$i=2, j=3$ $aabb$

$$i=2, j=4 \quad 99bbbbb$$

Let string \equiv aabbba

- ↳ В а а б б б В В
- ↳ В а а б б б В
- ↳ В а а у ю ю ю
- ↳ В а а у ю ю ю
- ↳ В а а ў ў ў ў
- ↳ В а а ў ў ў ў

- ↪ **а а в в в** 1 88 ⚡
- ↪ **х а в в в** 1 88 ⚡
- ↪ **н а в в у** 1 88 ⚡
- ↪ **х а в в у** 1
- ↪ **н а в в уу** 1
- ↪ **х н у у у** 1

Possible
Questions:

$i = j$

$i > j$

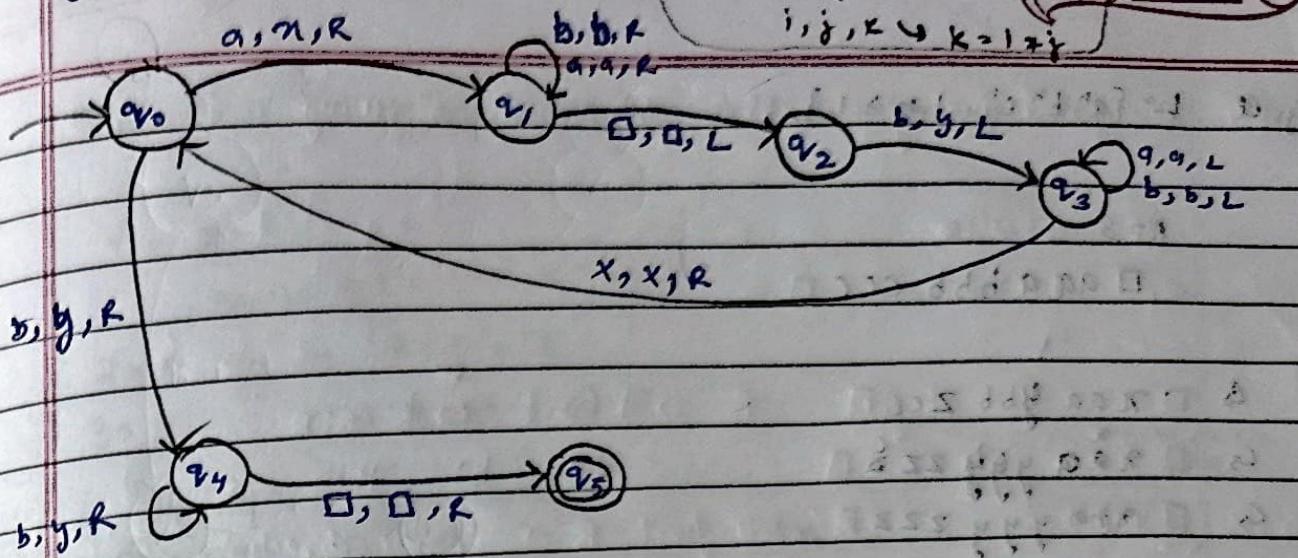
$i, j, k \in \{1, 2, 3\}$

$i > j > k$

classmate

DG

PG



$$\alpha: a^i b^j \quad 0 \leq j \leq i$$

$$i=1, j=0 \quad a$$

$$i=2, j=1 \quad aab$$

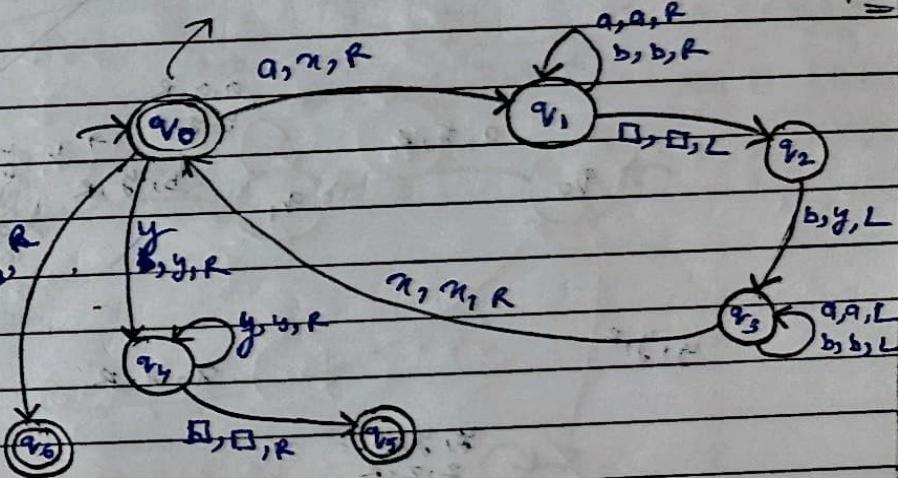
$$i=2, j=2 \quad aabb$$

$\{ \text{at } q_0 \rightarrow \text{we read } a \}$

q_0 also a final state, as only 'a' also accepted

Let string: aaa bb

- ↳ $\square a a a b b \square$
- ↳ $\square a a a b b \square$
- ↳ $\square a a a b y y \square$
- ↳ $\square a a a b y y \square$
- ↳ $\square n n a y y \square$
- ↳ $\square n n a y y \square$



eg. $L = \{a^n b^n c^n \mid n \geq 1\}$

$n=3$

□ aaabbcc □

↳ □ aaaa bbb ccc □

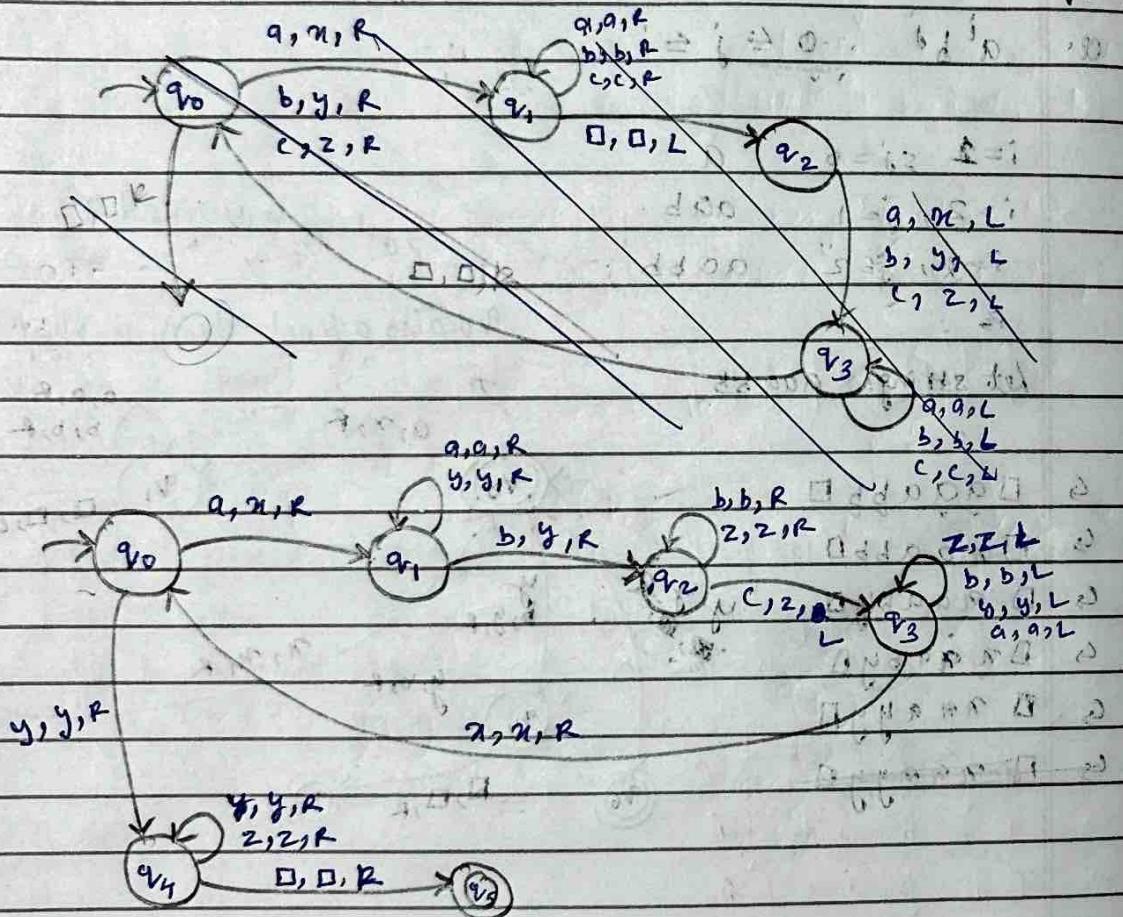
First read a's

↳ □ aaaa bbb ccc □

then b's then c's

↳ □ aaaa bbb ccc □

once change c
go left



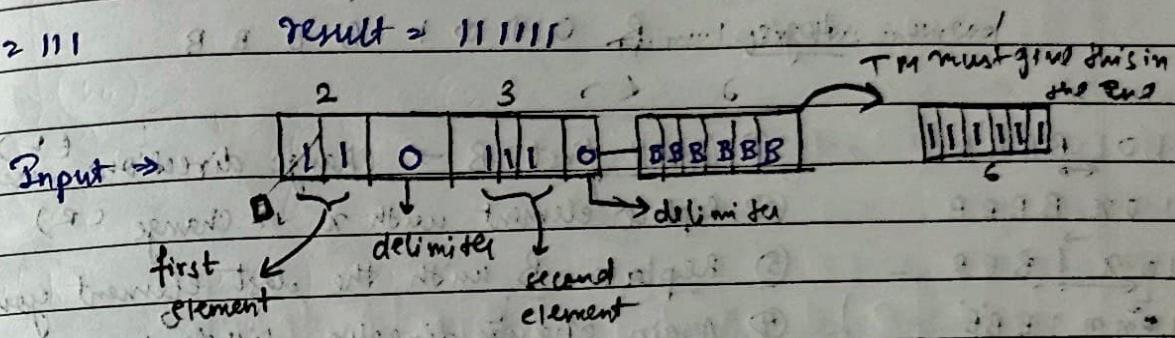
Eg:

Construct a Turing machine for multiplication operation

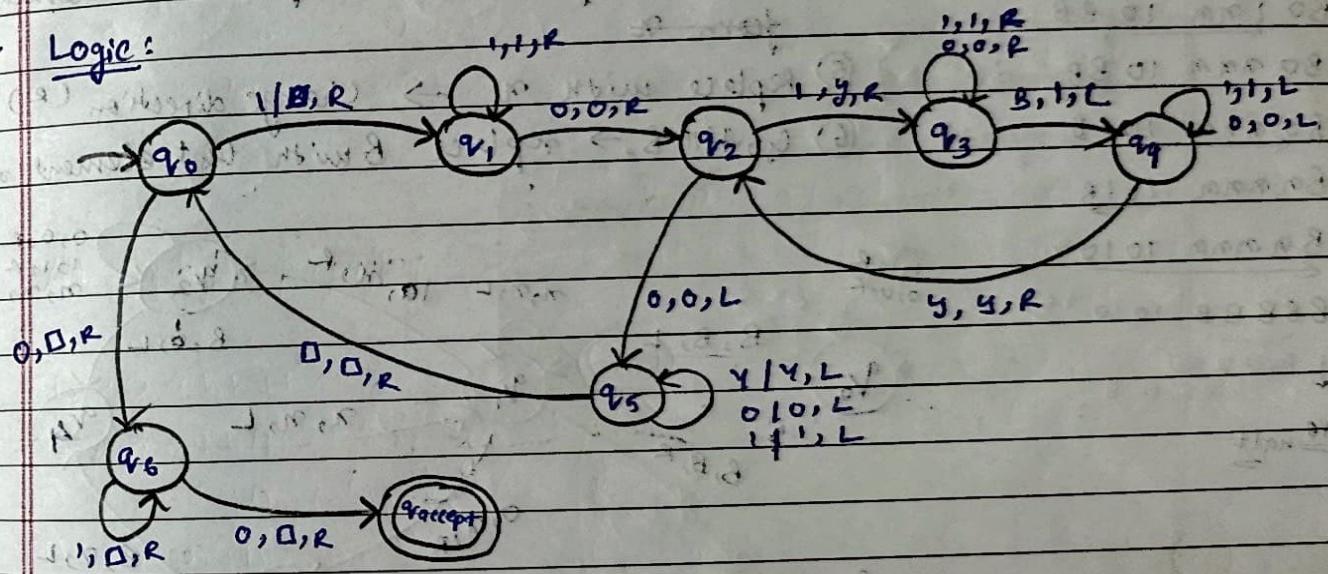
$$f = 2 \times 3$$

$$\begin{array}{l} \text{number of } 1\text{'s (not binary)} \\ 2 = 11 \quad \rightarrow \quad 2 \times 3 = 6 \end{array}$$

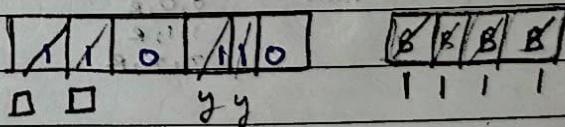
$$3 = 111 \quad \rightarrow \quad \text{result} = 11111$$



* Logic:



$\Rightarrow 2 \times 2$



Q.1 Design a Turing machine to reverse a string

Input = 01010

Output = 10101

Logic: B 0 1 0 1 → B B B B B

B 0 1 0 1 B B B B

B 0 1 0 n B B B B

B 0 1 0 n 1 B B B B

B 0 1 n m 1 B B B B

B 0 1 n m 1 0 B B B

B 0 n m 1 0 B B B

B 0 n m 1 0 1 B

Banana 1 0 1 B

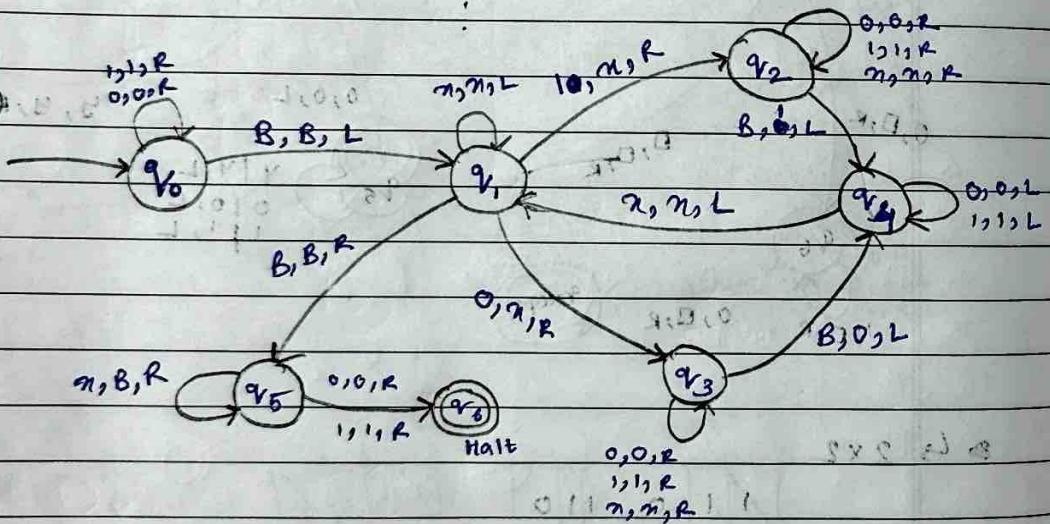
Banana 1 0 1 0

B B B B B 1 0 1 0

↓ Replace all n with B

ave halt

- ① Go till first B → Change direction (L)
- ② Replace element with n → Change (R)
- ③ Replace B with the last element you replaced
- ④ Again change direction till next element from R
- ⑤ Replace with n → Change direction (R)
- ⑥ Go till B → replace B with last element replaced



Q.2 Add b's to match the no. of a's

Input: $a^i b^j$ $0 \leq j \leq i$
 If: $\square a a b \square \square$
 Or: $\square a a b b \square$

Logic: ① Read a, replace with x

② keep going right till b \rightarrow replace with y
 { change direction (L)}

③ read a \rightarrow Go till x \rightarrow change direction (R)

④ Then change a to y and go right

⑤ If all b's are exhausted \rightarrow Read till
 $\square \rightarrow$ replace with y { go L}

⑥ If still replace any remaining

⑦ Go till blank and change direction (R)

⑧ Replace all ~~x's~~ with a's

{ all y's with b's}

Q.1 Construct a Turing machine for multiplication (start diagram drawn previously)

$$2 \times 3 = 6$$

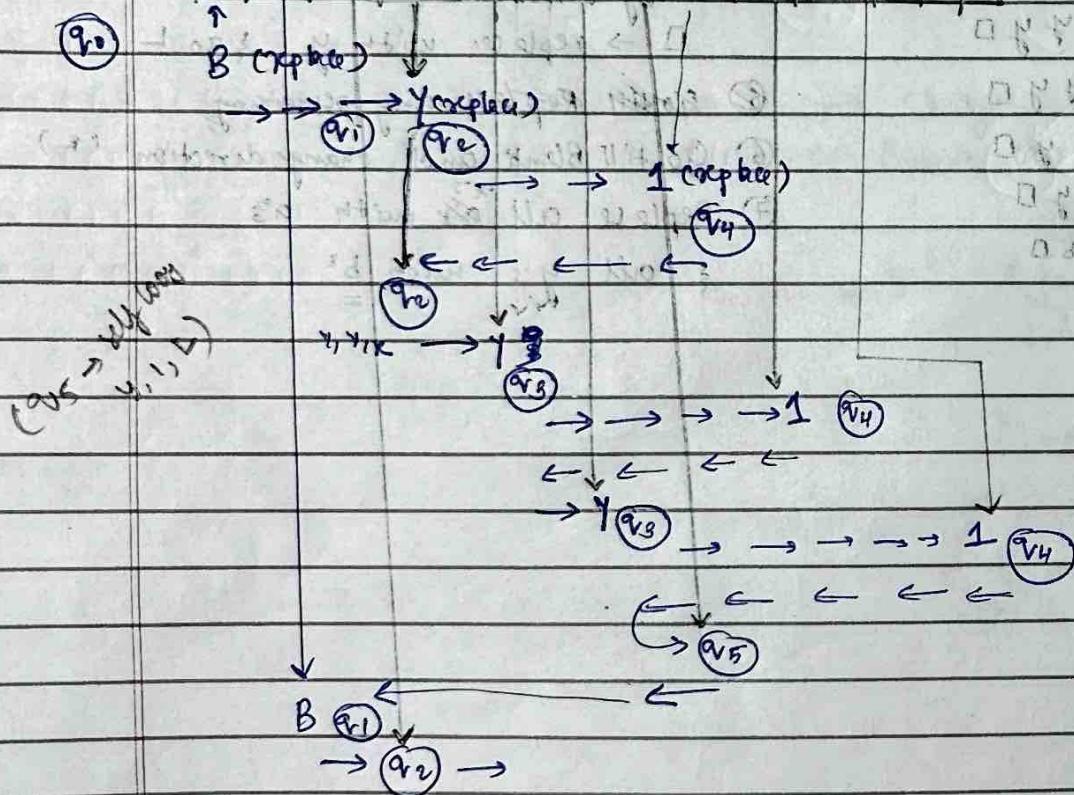
repeated addition

$$I/P \{ \begin{matrix} 2 & 2 & 1 \\ 3 & 2 & 1 \end{matrix}$$

$$\begin{matrix} 2+2+2=6 \\ 2 \\ 2 \\ 2 \end{matrix}$$

$$O/P \quad 6 = 110 \quad 110 \rightarrow 3 + 3 = 6$$

1 1 0 1 1 0 B B B B | B



(subtraction also)

Q.2

Unary addition

$2 \rightarrow 11$

$2+3 = 5$

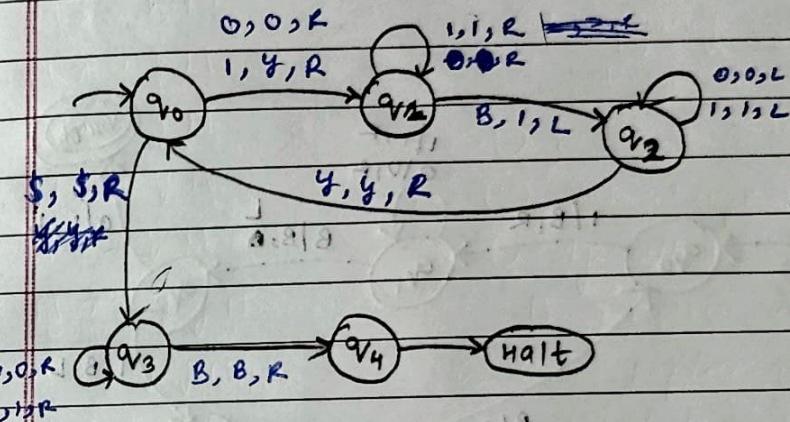
$3 \rightarrow 111$

1111

y	y	y	y	y	y	y	y	y	y	y	y
1	1	$\cancel{1}$	1	1	$\cancel{1}$	1	$\cancel{1}$	1	$\cancel{1}$	1	$\cancel{1}$

1 \hookrightarrow Logic (in form)
(of algo)2 \hookrightarrow State Diagram3 \hookrightarrow Detailed

Tracing (L)



$11 = 11 - 111$

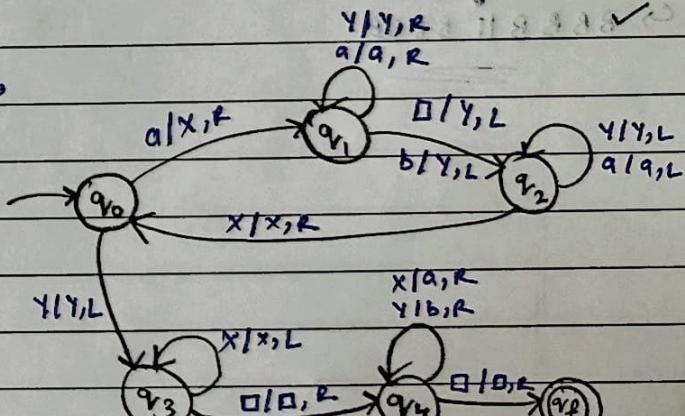
$11 = 0111110$

$11 = 0111110$

20-12-23

Q.3 Add b's to match a's $\rightarrow a^i b^j$

- $\square a a b \square \square$
- $\hookrightarrow \square x a b \square \square$
- $\square x a y \square \square$
- $\square x x y \square \square$
- $\square x x y y \square$
- $\square a x b y y \square$
- $\square a a y y \square$
- $\square a a b y \square$
- $\square a a b b \square$



input string

- $q_0 a a b \vdash X q_1, a b \vdash x a q_1, b$ ✓
 $\vdash x \vdash q_2 a y \vdash q_2 x a y \vdash x q_0 a y$
 $\vdash x x q_1, y \vdash x x q_1, y \square \vdash x x q_2 y y$
 $\vdash x x q_0 y y \vdash x q_3 x y y \vdash q_3 x x y y$
 $\vdash q_3 \square x x y y \vdash q_4 x x y y \vdash q_4 x y y$
 $\vdash a a q_4 y y \vdash a a b q_4 y y \vdash a a b b q_4 \square$
 $\vdash a a b b \square q_f$

important
to write all
step by
step

Slide 20 \hookrightarrow understand with animation

Q. 4 Unary Subtraction

$$f(m, n) = \begin{cases} m-n & , m > n \\ 0 & , m \leq n \end{cases}$$

$$\S \quad m=3, n=2$$

III - II = 1 8 8 8 8 9 0 0 0 1 1 1 + 1 1

$$\hookrightarrow m=4, n=2$$

$$1111 - 11 = 11$$

B 1111 0113 BB

GBB111011 BE

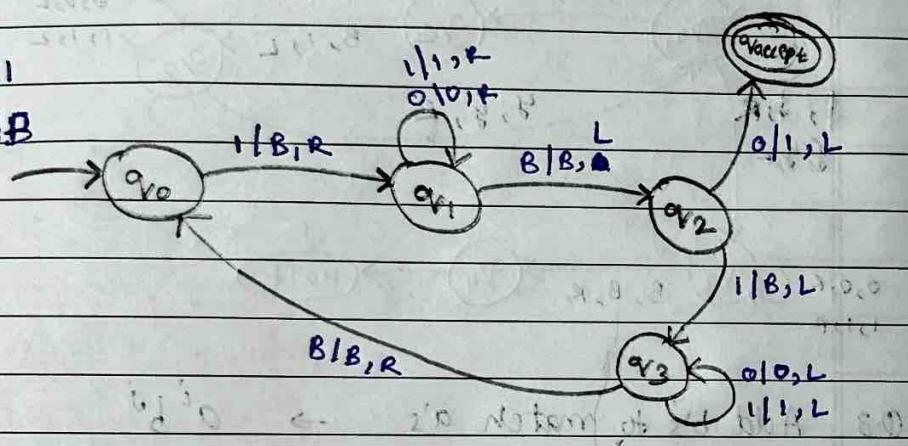
GBB B11101 BB

↪ BBBB1101BBB

$\hookrightarrow BBB_{110} \xrightarrow{\downarrow} B BBC$

↪ BBBB ↵ 10 BBBB

→ B B B B B ↓ I I R R R R



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