MATHEMATICS-III (MATH F211) TEST- I (Open Book)

Date: 24-10-2020   Max. Marks: 50   Weightage: 25%   Max.time:50 min

_____

Answer all the questions

1. a) Solve $ydx - xdy - y^2 dx = y^2 \cos x \, dx$ [5+5]

    b) Solve $y^{(4)} - 2y''' + 5y'' - 8y' + 4y = 0$

2. Solve the following   [8]

    $$2x^2 y'' + 4xy' + 3y = 0$$

3. By using reduction of order solve   [8]
    $$xy'' = y' + 2x^2 y'$$

4. If one solution of $x^2 y'' + y' - \dfrac{1}{x} y = 0$ is given $y_1 = x$, then find second solution y_2

    and also the general solution.   [8]

5. Find the integrating factor and solve   [8]

    $$\left(\frac{1}{e^{\sin x}} + y^2 \cos x \right) dx + 2y dy = 0$$

6. Solve the following   [8]

    $$3y' + \frac{y}{x} = \frac{e^{2x}}{y^2}$$

_____Best of Luck! _____

MATHEMATICS-III (MATH F211) TEST- I (Open Book)

Date: 24-10-2020   Max. Marks: 50   Weightage: 25%   Max.time:50 min

---

Answer all the questions

1. a) Solve $\ ydx - xdy - x^2 dx = x^2 \sin x\, dx$ [5+5]

   b) Solve $y^{(4)} - 4y''' + 8y'' - 16y' + 16y = 0$

2. Solve $3x^2 y'' + 5xy' + 2y = 0$ [8]

3. If one solution of $4xy'' + 2y' + y = 0$ is given $y_1 = \sin \sqrt{x}$, then find second

   Solution y_2 and also the general solution.   [8]

4. Solve $x^2 y' + y = \dfrac{1}{y}$ [8]

5. By using reduction of order solve

   $$xy'' = y' - xy'$$ [8]

6. Find the integrating factor and solve   [8]

   $$\left(y + \frac{y^3}{3} + e^x \right) dx + \left(\frac{x + xy^2}{4} \right) dy = 0$$

MATHEMATICS-III (MATH F211) TEST- I (Open Book)

Date: 24-10-2020   Max. Marks: 50   Weightage: 25%   Max.time:50 min

Answer all the questions

1. a) Solve $xdy + ydx = x^2 y^3 dy + x^2 ydy$ [5+5]

   b) Solve $y^{(4)} + 2y''' + 2y'' + 2y' + y = 0$

2. Solve the following   [8]
   $$4x^2 y'' + 2xy' + y = 0$$

3. If one solution of $xy'' - (x+2)y' + 2y = 0$ is given $y_1 = e^x$, then find second

   solution y_2 and also the general solution.   [8]

4. Solve $xy' + y = \dfrac{1}{y}$ [8]

5. Find the integrating factor and solve   [8]

   $$\left(y^4 + 2y \right) dx + \left(xy^3 - 4x + y^4 e^y \right) dy = 0$$

6. By using reduction of order solve   [8]

   $$yy'' = y'^2 + y'^3$$

MATHEMATICS-III (MATH F211) TEST- 2 (Open Book)

Date: 25-11-2020   Max. Marks: 50   Weightage: 25%
Max.time:50 min

_____

Answer all the questions

1. By using the method of undetermined coefficient find the particular solution of

$$y'' - 2y' = 3e^{2x} + 4x$$ [8M]

2. By using the method of undetermined coefficient find the particular solution of

$$y'' - 4y = \cos 2x + \sin x$$ [8M]

3. By using the method of variation of parameters find the particular solution of

$$y'' - 2y' + y = \frac{e^x}{1 - x^2}$$ [8M]

4. By using operator method find the particular solution of

$$y'' - 5y' + 6y = (x^2 + x)e^{2x}$$ [8M]

5. Solve by Frobenius series method and find one solution for the following differential equation

$$xy'' + y' + 5y = 0$$ [8M]

6. Solve following differential equation by Power series method

$$(x^2 + 1)y'' - 6y = 0$$ [10M]

_____

Answer all the questions

1. By using the method of undetermined coefficient find the particular solution of

$$y'' - 3y' = e^{3x} - 6x$$ [8M]

2. By using the method of undetermined coefficient find the particular solution of

$$y'' - y = 2\sin x - \cos 2x$$ [8M]

3. By using the method of variation of parameters find the particular solution of

$$y'' - 2y' + y = \frac{xe^x}{1 + x^2}$$ [8M]

4. By using operator method find the particular solution of

$$y'' - 5y' + 6y = (x^2 - 3)e^{2x}$$ [8M]

5. Solve by Frobenius series method and find one solution for the following differential equation

$$xy'' + y' + 7y = 0$$ [8M]

6. Solve following differential equation by Power series method

$$(1 - x^2)y'' + 6y = 0$$ [10M]

Answer all the questions

1. By using the method of undetermined coefficient find the particular solution of

$$y'' - 4y' = e^{4x} - 8x$$ [8M]

2. By using the method of undetermined coefficient find the particular solution of

$$y'' - y = 2\cos x - \sin 2x$$ [8M]

3. By using the method of variation of parameters find the particular solution of

$$y'' - 2y' + y = (x+1)e^{2x}$$ [8M]

4. By using operator method find the particular solution of

$$y'' - 5y' + 6y = x^2 e^{2x}$$ [8M]

5. Solve by Frobenius series method and find one solution for the following differential equation

$$xy'' + y' + 11y = 0$$ [8M]

6. Solve following differential equation by Power series method

$$(x^2 + 1)y'' - 2y = 0$$ [10M]

MATHEMATICS-III (MATH F211)
COMPREHENSIVE EXAMINATION (Open Book) PART A

Date: 31-12-2020   **Max. Marks: 30**   **Weightage: 15%**   **Max. time: 70 min**
_____

Answer all the questions

1. Find the value of the expression $F(-1,2,3,x) + \frac{d}{dx}F(-2,1,2,x)$ where $F(a,b,c,x)$

    represents Hypergeometric series.   4M

2. Find the value of $\int_{-1}^{1} P_2(x)[P_1(x) + P_3(x)]dx$

    where $P_n(x)$ represents Legendre polynomials.   3M

3. Solve the eigen value problem and find the eigen values and functions.

    $y'' + \lambda^2 y = 0$; $y'(0) = 0 = y'(L)$.   4M

4. Find the general solution of

    $x^2 y'' - 5xy' + 8y = 0$; $y(2) = 32$, $y'(2) = 0$.   4M

    Find Laplace transformation of

5. $f(x) = \begin{cases} 3 & ; \ 0 \le x \le \pi \\ e^{3x} & ; \pi \le x < \infty \end{cases}$ 4M

    Find Laplace inverse

6. $L^{-1}\left(\dfrac{p+4}{(p^2 + 2p + 10)p} \right)$ 4M

7. Evaluate $\int_{0}^{\pi} x^{\frac{3}{2}} J_{1/2}(x)dx$ where $J_{1/2}(x)$ is Bessel's function.   4M

    Solve the following differential equation

8. $\dfrac{ydx - xdy}{xdx + ydy} = \dfrac{y^2}{x^2 + y^2}$ 3M

MATHEMATICS-III (MATH F211)
COMPREHENSIVE EXAMINATION (Open Book) PART B

Date: 31-12-2020   **Max. Marks: 30**   **Weightage: 15%**   **Max. time: 70 min**
_____

Answer all the questions

1. Find the value of the expression $\frac{d}{dx}F(3,-2,6,x) + F(4,-1,7,x)$

   where $F(a,b,c,x)$ represents Hypergeometric series.   4M

2. Find the value of $\int_{-1}^{1}\left[\left(P_1(x)\right)^2 + \left(P_3(x)\right)^2\right]dx$ 3M

   where $P_n(x)$ represents Legendre polynomials.

3. Solve the eigen value problem and find the eigen values and functions.   4M

   $$y'' + 4\lambda^2 y = 0 \; ; \; y'(0) = 0 = y'(L).$$

4. Find the general solution of

   $$x^2 y'' + xy' + y = 0 \; ; y(1) = 1, \; y'(1) = 2.$$ 4M

   Find Laplace transformation of

5. $f(x) = \begin{cases} e^x & ; \; 0 \le x \le \pi \\ x & ; \; \pi \le x < \infty \end{cases}$ 4M

   Find Laplace inverse

6. $L^{-1}\left(\dfrac{p+2}{p(p^2 - 2p +10)}\right)$ 4M

7. Evaluate $\displaystyle\int_{0}^{\pi} x^{\frac{3}{2}} J_{-1/2}(x)dx$ where $J_{-1/2}(x)$ is Bessel's function.   4M

   Solve the following differential equation

8. $\dfrac{ydx - xdy}{xdy + ydx} = \dfrac{x^2 + y^2}{x^2 y^2}$ 3M

MATHEMATICS-III (MATH F211)
COMPREHENSIVE EXAMINATION (Open Book) PART C

Date: 31-12-2020   **Max. Marks: 30**   **Weightage: 15%**   **Max. time: 70 min**

_____

Answer all the questions

1. Find the value of the expression $\frac{d}{dx}F(3,-3,9,x) + F(4,-2,10,x)$ 4M

   where $F(a,b,c,x)$ represents Hypergeometric series.

2. Find the value of $\int_{-1}^{1}\left[\left(P_2(x)\right)^2 + \left(P_1(x)\right)^2\right]dx$ 3M

   where $P_n(x)$ represent Legendre polynomials.

3. Solve the eigen value problem and find the eigen values and functions.

   $y'' + 9\lambda^2 y = 0 ; \ y'(0) = 0 = y'(L).$ 4M

4. Find the general solution of

   $x^2 y'' - 3xy' + 4y = 0 ; y(1) = 5, \ y'(1) = 3.$ 4M

   Find Laplace transformation of

5. $f(x) = \begin{cases} 1 & ; \ 0 \le x \le \pi \\ 1-x^2 & ; \pi \le x < \infty \end{cases}$ 4M

   Find Laplace inverse

6. $L^{-1}\left(\dfrac{p+4}{p(p^2+4p+8)}\right)$ 4M

7. Evaluate $\displaystyle\int_0^\pi x^2 J_{1/2}(x)\, J_{-1/2}(x)\, dx$ where $J_{1/2}(x)$ and $J_{-1/2}(x)$ are Bessel's function.   4M

   Solve the following differential equation

8. $\dfrac{ydx - xdy}{xdx + ydy} = \dfrac{xy}{x^2 + y^2}$ 3M

MATHEMATICS-III (MATH F211)
COMPREHENSIVE EXAMINATION (Open Book) PART B

Date: 31-12-2020   Max. Marks: 50   Weightage: 25%   Max.time:100 min
_____
Answer all the questions

1. By using method of variation of parameters find particular solution of
$y'' + 9y = \sec 3x + \operatorname{cosec} 3x$ 5M

2. Solve the following differential equation
$\dfrac{1}{x}\dfrac{dy}{dx} + \dfrac{y}{x^2} = y^2 \log x$ 4M

3. Find the fourier series of

$$f(x) = \begin{cases} 0 & ; \quad -\pi < x < 0 \\ \pi + x & ; \quad 0 < x < \pi \end{cases}$$

sketch the graph in $(-3\pi, 3\pi)$. Deduce the series at $x = 0$ 6M

$$1 + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \dfrac{1}{7^2} + ...$$

4. Solve the following system of differential equations
$\dfrac{dx}{dt} = -6x + 2y$ 5M
$\dfrac{dy}{dt} = -3x - 2y$

5. Solve the following differential equations by using
Frobenius series method $x^2 y'' - x(1+x)y' + y = 0$ 5M

6. By using operator method find the particular solution of   5M

$\left(D^2 - \frac{1}{4}\right) y = e^{\frac{3}{2}x}(10x^3 - 10x).$

7. Solve by Power series method: $y'' - x^2 y' - y = 0.$ 5M

8. Solve by reduction of order technique: $y^3 y'' - 2(y')^3 = 0$ 5M

9. Using Laplace transform method solve

$y'' - 6y' + 15y = 2\sin(3t); \ \ y(0) = -1, \ y'(0) = -4$ 5M

10. A tightly stretched elastic string of length π unit is set to vibrate by an initial

deformation $f(x) = \begin{cases} 100\pi x & ; \ 0 \le x < \frac{\pi}{2} \\ 100\pi(\pi - x) & ; \ \frac{\pi}{2} \le x < \pi \end{cases}$

Find the displacement in the string at any time t.   5M

MATHEMATICS-III (MATH F211)
COMPREHENSIVE EXAMINATION (Open Book) PART C

Date: 31-12-2020   Max. Marks: 50   Weightage: 25%   Max.time:100 min
_____
Answer all the questions

1. By using method of variation of parameters find particular solution of

   $y'' + 4y = \sec^2 2x + \tan 2x$ 5M

2. Solve the following differential equation

   $\cos x \dfrac{dy}{dx} + y \sin x = (x+1)^2 y^3$ 4M

3. Find Fourier series of

   $$f(x) = \begin{cases} \pi - x ; & -\pi < x < 0 \\ 0 \; ; & 0 < x < \pi \end{cases}$$

   sketch the graph in $(-3\pi, 3\pi)$. Deduce the series at $x = 0$ 6M

   $$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + ...$$

4. Solve the following system of differential equations

   $\dfrac{dx}{dt} = x - y$

   $\dfrac{dy}{dt} = 3x + y$ 5M

5. Solve the following differential equations by using

   Frobenius series method $x^2 y'' + x(3+x) y' + y = 0$ 5M

6. Using Operator method find the particular solution of

$$\left(D^2 - \frac{1}{4}\right)y = e^{-\frac{3}{2}x}(10x^3 + 10x).$$ 5M

7. Solve by Power series method:

$$y'' + x^2 y' - y = 0.$$ 5M

8. Solve by Reduction of order technique:

$$y^3 y'' - (y')^3 = 0$$ 5M

9. Using Laplace transform method solve

$$y'' - 6y' + 15y = 2\cos 3t; \quad y(0) = 0, \ y'(0) = 1$$ 5M

10. A tightly stretched elastic string of length π unit is set to vibrate by an initial

deformation $$f(x) = \begin{cases} \frac{100\pi}{k}x & ; \ 0 \le x < \frac{\pi}{2} \\ \frac{100\pi}{k}(\pi - x) & ; \ \frac{\pi}{2} \le x < \pi \end{cases}$$

Find the displacement in the string at any time t. 5M