* Multiple Linear Regression ⇒

Problem 3:

	Y	X_1	X_2
1	-3.7	3	8
2	3.5	4	5
3	2.5	5	7
4	11.5	6	3
5	5.7	2	1
6	?	3	2

X_1, X_2 ⇒ Independent variables.

Y ⇒ Dependent variable.

Regression equation or model is,

$$Y = b_1 X_1 + b_2 X_2 + a$$

$$b_1 = \frac{(\Sigma x_2^2)(\Sigma x y) - (\Sigma x_1 x_2)(\Sigma x_2 y)}{(\Sigma x_1^2)(\Sigma x_2^2) - (\Sigma x_1 x_2)^2}$$

$$b_2 = \frac{(\Sigma x_1^2)(\Sigma x_2 y) - (\Sigma x_1 x_2)(\Sigma x_1 y)}{(\Sigma x_1^2)(\Sigma x_2^2) - (\Sigma x_1 x_2)^2}$$

$$\Sigma x_1^2 = \Sigma X_1 X_1 - \frac{(\Sigma X_1)(\Sigma X_1)}{N}$$

$$\Sigma x_2^2 = \Sigma X_2 X_2 - \frac{(\Sigma X_2)(\Sigma X_2)}{N}$$

$$\Sigma x_1 y = \Sigma X_1 Y - \frac{(\Sigma X_1)(\Sigma Y_1)}{N}$$

$$\Sigma x_2 y = \Sigma X_2 Y - \frac{(\Sigma X_2)(\Sigma Y)}{N}$$

$$\Sigma x_1 x_2 = \Sigma X_1 X_2 - \frac{(\Sigma X_1)(\Sigma X_2)}{N.}$$

	Y	X_1	X_2	X_1X_1	X_2X_2	X_1X_2	X_1Y	X_2Y
1	-3.7	3	8	9	64	24	-11.1	-29.6
2	3.5	4	5	16	25	20	14	17.5
3	2.5	5	7	25	49	35	12.5	17.5
4	11.5	6	3	36	9	18	69	34.5
5	5.7	2	1	4	1	2	11.4	5.7
Σ	19.5	20	24	90	148	99	95.8	45.6

$$\Sigma x_1^2 = \Sigma X_1X_1 - \frac{(\Sigma X_1)(\Sigma X_1)}{N}$$

$$= 90 - \frac{\cancel{20}^{4} \times 20}{\cancel{5}}$$

$$= 90 - 80$$

$$\Sigma x_1^2 = 10$$

$$\Sigma x_2^2 = \Sigma X_2X_2 - \frac{(\Sigma X_2)(\Sigma X_2)}{N}$$

$$= 148 - \frac{(24)(24)}{5}$$

$$\Sigma x_2^2 = 32.8$$

Similary, by putting respective values.

$$\Sigma x_1 y = \Sigma X_1 Y - \frac{(\Sigma X_1)(\Sigma Y)}{N} = 17.8$$

$$\sum x_2 y = \sum X_2 Y - \frac{(\sum X_2)(\sum Y)}{N} = -48$$

$$\sum x_1 x_2 = \sum X_1 X_2 - \frac{(\sum X_1)(\sum X_2)}{N} = 3$$

~~$b_1 = 32.8 * 17.8$~~

$$b_1 = \frac{(\sum x_2^2)(\sum x_1 y) - (\sum x_1 x_2)(\sum x_2 y)}{(\sum x_1^2)(\sum x_2^2) - (\sum x_1 x_2)^2}$$

$$= \frac{32.8 * 17.8 - 3 * (-48)}{10 * 32.8 - 3 * 3} = \underline{\underline{2.28}}$$

$$b_2 = \frac{(\sum x_1^2)(\sum x_2 y) - (\sum x_1 x_2)(\sum x_1 y)}{(\sum x_1^2)(\sum x_2^2) - (\sum x_1 x_2)^2}$$

$$= \frac{10 * (-48) - 3 * 17.8}{10 * 32.8 - 3 * 3} = \underline{\underline{-1.67}}$$

$$a = \overline{Y} - b_1 \overline{X_1} - b_2 \overline{X_2} = \frac{19.5}{5} - \frac{2.28 * 20}{5} - \frac{-1.67 * 24}{5} = 2.796$$

Final regression equation or model is.

$$Y = b_1 X_1 + b_2 X_2 + a$$

$$Y = 2.28 X_1 - 1.67 X_2 + 2.796$$

$\therefore$ For $X_1 = 3$, $X_2 = 2$, $Y = ?$

$$Y = 2.28(3) - 1.67(2) + 2.796 = \underline{\underline{6.296}}$$