BITS Pilani
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Machine Learning

CS F464

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Contents

- Hidden Markov Model
  - Markov model
  - Examples
  - Hidden Markov model
  - Examples

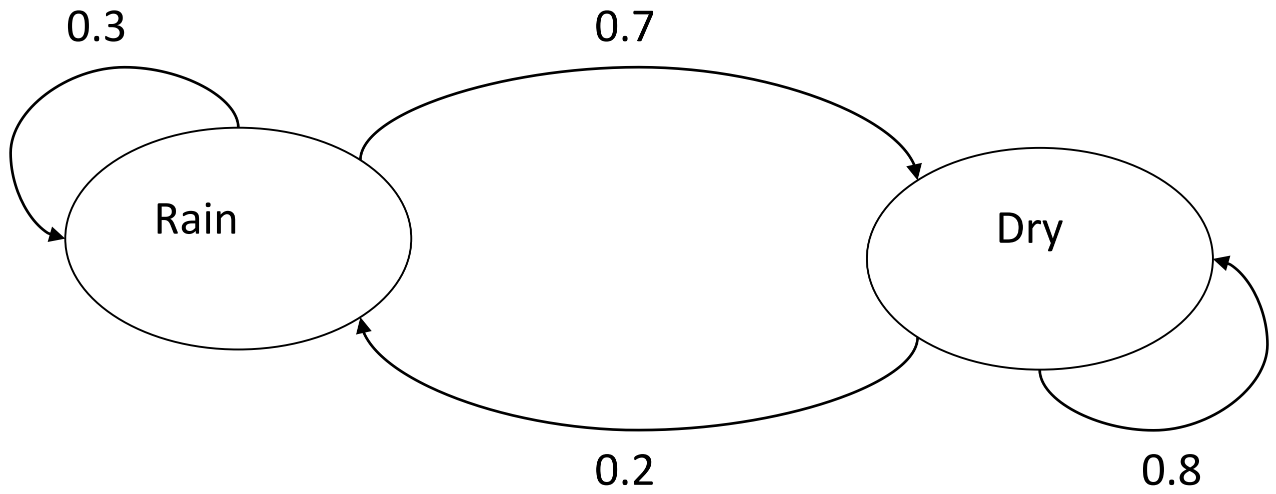
Markov Model

- Set of states: $\{s_1, s_2, \ldots, s_N\}$
- Process moves from one state to another generating a sequence of states : $s_{i1}, s_{i2}, \ldots, s_{ik}, \ldots$
- Markov chain property: probability of each subsequent state depends only on what was the previous state:

$$P(s_{ik} \mid s_{i1}, s_{i2}, \ldots, s_{ik-1}) = P(s_{ik} \mid s_{ik-1})$$

- To define Markov model, the following probabilities have to be specified:
  - transition probabilities $a_{ij} = P(s_i \mid s_j)$
  - and initial probabilities $\pi_i = P(s_i)$

Markov Model Example 1



- Two states
  'Rain' and 'Dry'.
- Transition probabilities:
  P('Rain'|'Rain')=0.3 , P('Dry'|'Rain')=0.7 ,
  P('Rain'|'Dry')=0.2, P('Dry'|'Dry')=0.8
- Initial probabilities:
  P('Rain')=0.4 , P('Dry')=0.6 .

Sequence Probability

- By Markov chain property, probability of state sequence can be found by the formula:

$$P(s_{i1}, s_{i2}, \ldots, s_{ik}) = P(s_{ik} \mid s_{i1}, s_{i2}, \ldots, s_{ik-1}) P(s_{i1}, s_{i2}, \ldots, s_{ik-1})$$

$$= P(s_{ik} \mid s_{ik-1}) P(s_{i1}, s_{i2}, \ldots, s_{ik-1}) = \ldots$$

$$= P(s_{ik} \mid s_{ik-1}) P(s_{ik-1} \mid s_{ik-2}) \ldots P(s_{i2} \mid s_{i1}) P(s_{i1})$$

- For example

P({‘Dry’,’Dry’,’Rain’,Rain’})

= P(‘Rain’|’Rain’) P(‘Rain’|’Dry’) P(‘Dry’|’Dry’) P(‘Dry’)

= 0.3*0.2*0.8*0.6

P({Dry (s1), Rain(s2), Rain(s3), Dry(s4)}) = P(Dry|Rain) P(Rain|Rain) P(Rain|Dry) P(Dry) =

Example 2

- In analysing switching by Business Class customers between airlines the following data has been obtained by XYZ airways:

Next flight by			
Last flight by		XYZ	Competition
	XYZ	0.85	0.15
	Competition	0.10	0.90

- For example if the last flight by a Business Class customer was by XYZ the probability that their next flight is by XYZ is 0.85. Business Class customers make 2 flights a year on average.
- Currently XYZ have 30% of the Business Class market. What would you forecast XYZ's share of the Business Class market to be after two years?

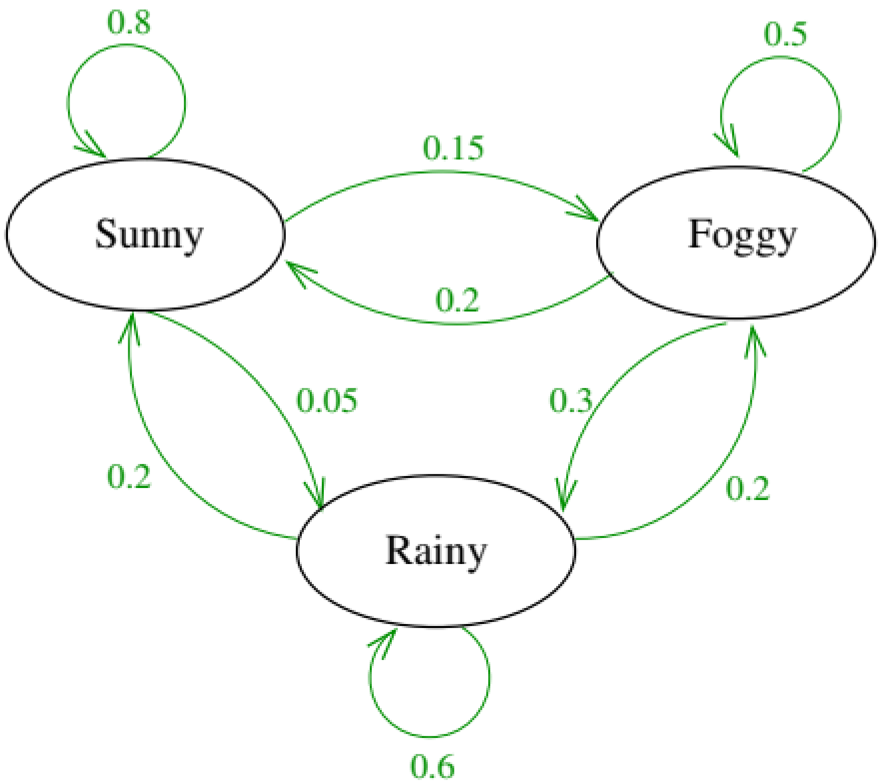
Example 2

- We have the initial system state s_1 given by s_1 = [0.30, 0.70] and the transition matrix P is given by

$$P = \begin{bmatrix} 0.85 & 0.15 \\ 0.10 & 0.90 \end{bmatrix}^2 = \begin{bmatrix} 0.7375 & 0.2625 \\ 0.1750 & 0.8250 \end{bmatrix}$$

  - Here we take square because customer make 2 flights a year on a average.
- After one year, $s_2 = s_1 P = [0.34375 \;\; 0.65625]$
- After two years, $s_3 = s_2 P = [0.368 \;\; 0.632]$
- So, after two years have elapses XYZ share of business class market is 36.8%

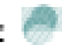
Example 3

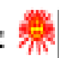
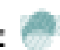


1. Given that today the weather is ☀, what's the probability that tomorrow is ☀ and the day after is 🌧?
   Using the Markov assumption and the probabilities in table 1, this translates into:

$$
\begin{aligned}
P(q_2 = ☀, q_3 = 🌧 | q_1 = ☀) &= P(q_3 = 🌧 | q_2 = ☀, q_1 = ☀) \cdot P(q_2 = ☀ | q_1 = ☀) \\
&= P(q_3 = 🌧 | q_2 = ☀) \cdot P(q_2 = ☀ | q_1 = ☀) \qquad \text{(Markov assumption)} \\
&= 0.05 \cdot 0.8 \\
&= 0.04
\end{aligned}
$$

Example 3

2. Assume, the weather yesterday was $q_1 =$ , and today it is $q_2 =$ , what is the probability that tomorrow it will be $q_3 =$ ?

$P(q_3 =$  $| q_2 =$ $, q_1 =$ $) = P(q_3 =$  $| q_2 =$ $)$ (Markov assumption)

$= 0.2.$

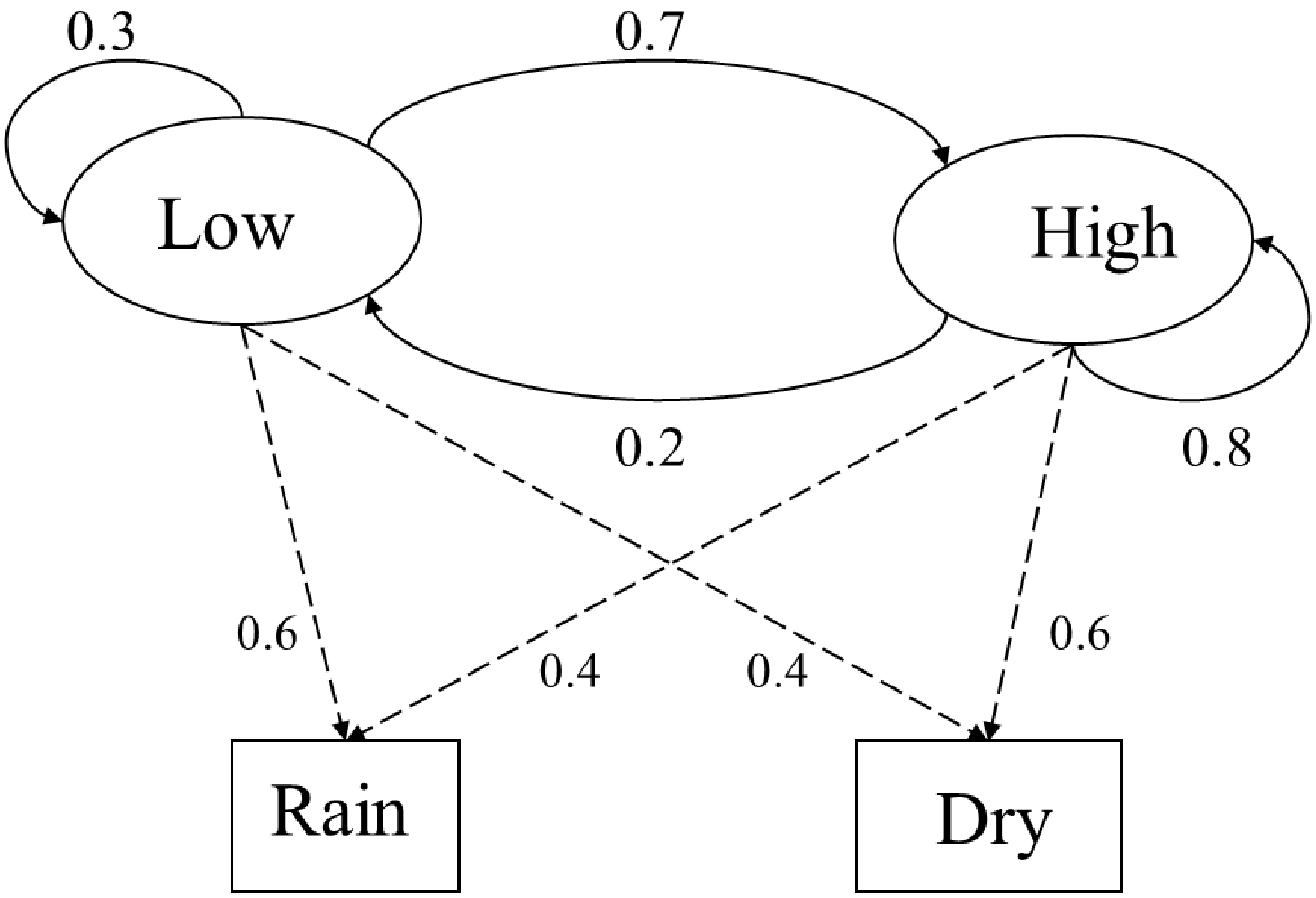
Hidden Markov Model

- Set of states: $\{s_1, s_2, \ldots, s_N\}$
- Process moves from one state to another generating a sequence of states : $s_{i1}, s_{i2}, \ldots, s_{ik}, \ldots$
- Markov chain property: probability of each subsequent state depends only on what was the previous state:

$$P(s_{ik} \mid s_{i1}, s_{i2}, \ldots, s_{ik-1}) = P(s_{ik} \mid s_{ik-1})$$

- States are not visible, but each state randomly generates one of M observations (or visible states) $\{v_1, v_2, \ldots, v_M\}$
- To define hidden Markov model, the following probabilities have to be specified:
  - matrix of transition probabilities $A=(a_{ij})$, $a_{ij}= P(s_i \mid s_j)$
  - matrix of observation probabilities $B=(b_i(v_m))$, $b_i(v_m) = P(v_m \mid s_i)$
  - vector of initial probabilities $\pi=(\pi_i)$, $\pi_i = P(s_i)$
- Model is represented as, $M=(A, B, \pi)$

Example 1 HMM



Example 1 HMM

- Two states : ‘Low’ and ‘High’ atmospheric pressure.
- Two observations : ‘Rain’ and ‘Dry’.
- Transition probabilities: P(‘Low’|‘Low’)=0.3 , P(‘High’|‘Low’)=0.7 , P(‘Low’|‘High’)=0.2, P(‘High’|‘High’)=0.8
- Observation probabilities : P(‘Rain’|‘Low’)=0.6 , P(‘Dry’|‘Low’)=0.4 , P(‘Rain’|‘High’)=0.4 , P(‘Dry’|‘High’)=0.3 .
- Initial probabilities: say P(‘Low’)=0.4 , P(‘High’)=0.6 .

Example 1 HMM

- Suppose we want to calculate a probability of a sequence of observations in our example, {'Dry','Rain'}.
- Consider all possible hidden state sequences:

P({'Dry','Rain'}) = P({'Dry','Rain'} , {'Low','Low'}) + P({'Dry','Rain'} , {'Low','High'}) + P({'Dry','Rain'} , {'High','Low'}) + P({'Dry','Rain'} , {'High','High'})

First term => P({'Dry','Rain'} , {'Low','Low'})=

P({'Dry','Rain'} | {'Low','Low'}) P({'Low','Low'}) =

P('Dry'|'Low')P('Rain'|'Low') P('Low')P('Low'|'Low)

= 0.4 * 0.6 * 0.4 * 0.3 = 0.0288

Second term => P({'Dry','Rain'} , {'Low','High'}) =

P({'Dry','Rain'} | {'Low','High'}) P({'Low','High'}) =

P('Dry'|'Low')P('Rain'|'High') P('High')P('Low'|'High')

= 0.4 * 0.4 * 0.6 * 0.2 = 0.0192

Example 1 HMM

Third term =>

P({‘Dry’,’Rain’} , {‘High’,’Low’}) =

P({‘Dry’,’Rain’} | {‘High’,’Low’}) P({‘High’,’Low’}) =

P(‘Dry’|’High’)P(‘Rain’|’Low’) P(‘Low’)P(‘High’|’Low’)

= 0.3 * 0.6 * 0.4 * 0.7 = 0.0504

Fourth term => P({‘Dry’,’Rain’} , {‘High’,’High’}) =

P({‘Dry’,’Rain’} | {‘High’,’High’}) P({‘High’,’High’}) =

P(‘Dry’|’ High’)P(‘Rain’|’ High’) P(‘High’)P(‘High’|’ High’)

= 0.3 * 0.4 * 0.6 * 0.8 = 0.0576

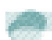
Therefore,

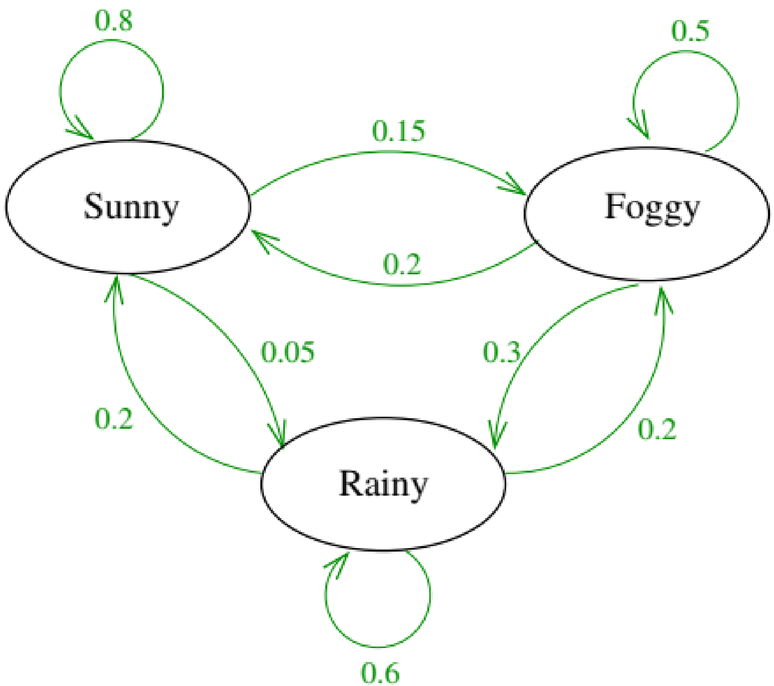
P({‘Dry’,’Rain’}) = 0.0288 + 0.0192 + 0.0504 + 0.0576

= 0.156

Example 2 HMM

Given,

Today's weather	Tomorrow's weather		
			
	0.8	0.05	0.15
	0.2	0.6	0.2
	0.2	0.3	0.5



Observed Probabilities,

Weather	Probability of umbrella
Sunny	0.1
Rainy	0.8
Foggy	0.3

Example 2 HMM

. Suppose the day you were locked in it was sunny. The next day, the caretaker carried an umbrella into the room. You would like to know, what the weather was like on this second day.

First we calculate the likelihood for the second day to be sunny:

$$L(q_2 = \text{☀} | q_1 = \text{☀}, x_2 = \text{☂}) = P(x_2 = \text{☂} | q_2 = \text{☀}) \cdot P(q_2 = \text{☀} | q_1 = \text{☀})$$
$$= 0.1 \cdot 0.8 = 0.08,$$

then for the second day to be rainy:

$$L(q_2 = \text{🌧} | q_1 = \text{☀}, x_2 = \text{☂}) = P(x_2 = \text{☂} | q_2 = \text{🌧}) \cdot P(q_2 = \text{🌧} | q_1 = \text{☀})$$
$$= 0.8 \cdot 0.05 = 0.04,$$

and finally for the second day to be foggy:

$$L(q_2 = \text{🌫} | q_1 = \text{☀}, x_2 = \text{☂}) = P(x_2 = \text{☂} | q_2 = \text{🌫}) \cdot P(q_2 = \text{🌫} | q_1 = \text{☀})$$
$$= 0.3 \cdot 0.15 = 0.045.$$

Thus, although the caretaker did carry an umbrella, it is most likely that on the second day the weather was sunny.

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Thank You!