

**BITS PILANI, DUBAI CAMPUS
SECOND SEMESTER 2020- 2021**

MATHEMATICS-III (MATH F211) TEST- I (Open Book).

Date: 09-03-2021

Max. Marks: 75

Weightage: 25%

Max.time:50 min.

Answer all the questions

1. Solve $ydx - xdy + (1 + x^2)dx + x^2 \sin y dy = 0.$ [10]

2. Solve $(2x^2y - y - ax^3)dx + x(1 - x^2)dy = 0.$ [15]

3. Solve $\frac{dy}{dx} + y \cot x = xy^3 \tan^2 x$ [13]

4. Find particular solution of the following differential equation by using variation of parameters

$$x^2 y'' + xy' - 4y = x^2 e^{x^2} \quad [12]$$

5. Solve the differential equation by reduction of order method:

$$y'' - y' \cot(x) = (y')^2 \quad [12]$$

6. Solve the differential equation by the method undetermined coefficients:

$$y'' + 6y' + 9y = -11e^{-3x} + 5 \quad [13]$$

Best of Luck!

**BITS PILANI, DUBAI CAMPUS
SECOND SEMESTER 2020- 2021**

MATHEMATICS-III (MATH F211) TEST- II (Open Book).

Date: 18-04-2021

Max. Marks: 75

Weightage: 25%

Max.time:50 min.

Answer all the questions

1) Find only the particular solution using operator's method:

$$y''' - 3y'' + 3y' - y = x^2 e^x + 2x e^x + e^x \quad [10]$$

2) Find the function g such that $f(t) = \int_0^t \sin(4\tau) g(t - \tau) d\tau$ has the Laplace transform

$$L(f(t)) = \frac{p}{(p^2+16)((p-1)^2+9)} \quad [15]$$

3) By using Laplace transformation solve following differential equation

$$y'' + 2y' + y = x + e^{-x}$$

$$\text{with } y(0) = 0 \text{ and } y(-1) = \frac{3e}{2} \quad [15]$$

4) Find the Sine series of $f(x) = 1 - x$ in $0 \leq x \leq 1$

Also find the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$ [10]

5) Find the Laplace transform of $(x+1)e^{-3x} \cos 2x$ [10]

6) Find the Fourier series of the function $f(x) = \begin{cases} -\pi, & -\pi \leq x < 0 \\ x^2, & 0 \leq x \leq \pi \end{cases}$ [15]

MATHEMATICS-III (MATH F211) COMPREHENSIVE EXAMINATION (Open Book).

Date: 30-05-2021

Marks: 120

Time: 3 hours

Answer all the questions

Write the answers for each section in separate answer books.

SECTION-A

1. Solve the following differential equation [8M]

$$x\sqrt{x} \frac{dy}{dx} - (1+x)y\sqrt{x} = y^2 \left(x + \frac{1}{2} \right)$$

2. Solve the following differential equation [8M]

$$\frac{\ln y}{x+2} dx + \left(e^{-\ln y} \ln(4x+8) - \frac{y^2}{e^y} \right) dy = 0$$

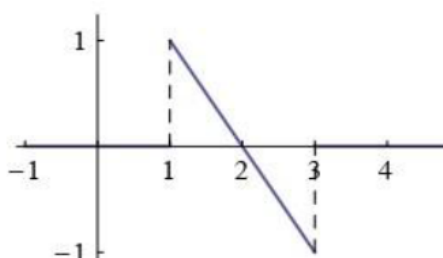
3. Solve by Power series method [10M]

$$\frac{d^2 y}{dx^2} - x \frac{dy}{dx} - xy = 0 \quad \text{with } y(0) = y'(0) = 1.$$

4. Find eigen values and eigen functions for [6M]

$$\frac{d^2 y}{dx^2} + \frac{\lambda}{4} y = 0 \quad \text{with } y'(0) = y'(\pi) = 0$$

5. Find the Laplace transform of following graph from $x = 1$ to $x = 3$. [8M]



SECTION-B

1. Find the particular solution of the following differential equation by reduction of order:

$$x^2 y'' + 2xy' + 2 = 0, \quad y(1) = 2, \quad y'(1) = -2. \quad [8]$$

2. Solve the following initial value problem using Laplace transform:

$$y'' + 2y' + 5y = 6\cos 3x - 4\sin 3x; \quad y(0) = 0, \quad y'(0) = 5. \quad [10]$$

3. Determine the solution to the heat equation $4\frac{\partial^2 w}{\partial x^2} = \frac{\partial w}{\partial t}$ with

$$w(0, t) = w(\pi, t) = 0 \quad \text{and} \quad w(x, 0) = \begin{cases} x, & 0 < x \leq \pi/2 \\ \pi - x, & \pi/2 < x \leq \pi \end{cases} \quad [8]$$

4. Find the indicial roots and the recurrence relation of the following differential equation:

$$2xy'' + y' - x^2y = 0. \quad [8]$$

5. Given $f(x) = x^2 J_1(x) + 4J_0(x)$. Determine $f'(x)$, $f''(x)$ and reduce the expressions to functions of $J_0(x)$, $J_1(x)$. [6]

SECTION-C

1) Solve by method of Variation of Parameters and find the complete solution:

$$y'' + 3y' + 2y = \sin(e^x) \quad [8]$$

2) Solve by Operator method and find only the particular solution:

$$(D^3 - D^2 - D + 1)y = x \cosh(x) \quad [8]$$

3) Solve the following System of differential equation:

$$\begin{aligned} \frac{dx}{dt} + \frac{dy}{dt} + 10x + y &= 0; \\ \frac{dy}{dt} + 17x + 3y &= 0 \end{aligned} \quad [8]$$

4) Find the general solution of the following Hypergeometric differential equation at the regular singular point $x = 0$

$$12x(1-x)y'' + (4-24x)y' + 9y = 0 \quad [6]$$

5) Find the Fourier series of the following function:

$$f(x) = 2x^2 - 3x; \quad -2 \leq x < 2$$

Hence, find the sum of the series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$ [10]
