BITS PILANI, DUBAI CAMPUS

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SECOND SEMESTER 2021- 2022

MATHEMATICS-III (MATH F211)

QUIZ- I

Date: 25-02-2022 Max. Marks: 30; Weightage: 15% Max.Time:25 min

NAME:	ID:	SECTION:

Answer all the **five** questions

1. Solve using suitable exact differential formulae or otherwise: $(\boldsymbol{y - xy^2})\boldsymbol{dx} - (\boldsymbol{x + x^2y})\boldsymbol{dy} = \boldsymbol{0}$ **(**6M)

2. Solve $\frac{\boldsymbol{1}}{\boldsymbol{x}}\frac{\boldsymbol{dy}}{\boldsymbol{dx}} + \boldsymbol{4y} = \boldsymbol{2}$ (6M)

3. Solve and find the general solution of the following differential equation, given one solution $\mathbf{y_1 = x^{-1}}$

$$\boldsymbol{xy'' + (2 + 2x)y' + 2y = 0}$$ (6M)

4. Solve

$$y'' - 2y' + 10y = 0,\ y(0) = 4,\ y'(0) = 1$$

(6M)

5. Solve by Reduction of order method:

$$xy'' = \sqrt{1 + (y')^2}$$

(6M)

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MATHEMATICS-III (MATH F211) MID-TERM (Open Book).

Date: 25-03-2022 Max. Marks: 60 Weightage: 30% Max.time:90 min.

Answer all the questions

1. Solve $\left(3x^2+2xy^2\right)dx+2x^2ydy=0$, given $y(2)=-3.$ [6]

2. Solve using exact differential formula (or) by any other method:
$y(1+xy)dx+x(1+xy+x^2y^2)dy=0$ [6]

3. Solve: $2x^2y'+6xy-3e^{-2x}=0.$ [6]

4. Solve: $y\sin x-\frac{dy}{dx}\cos x=y^2$ [6]

5. Solve $xy''+x\left(y'\right)^2-y'=0$ using the reduction of order method. [6]

6. Solve $y'''-3y''+4y=0,\ y(0)=1,\ y'(0)=0,\ y''(0)=-1.$ [6]

7. Solve $(1+x)^2y''+(1+x)y'+y=0$ given one solution as $y_1=\cos(\log(1+x))$ [6]

8. Solve $y''+3y'+2y=\sin(e^x)$ using the method of variation of parameters. [6]

9. Solve $y''+4y=2\cos^2(x)$ using the method of undetermined coefficients. [6]

10. Solve $2y''-3y'-y=x^2$ using the operator method. [6]

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MATHEMATICS-III (MATH F211) COMPREHENSIVE EXAMINATION **(CLOSED BOOK).**

Date: 06-06-2022 Marks: 80 Weightage: 40% Time: 3 hours

Answer all the questions

1. Solve $\left(y^4+2y\right)dx+(xy^3+2y^4-4x)dy=0$ **[4]**
2. Find the general solution of $yy''+\left(y'\right)^2=y'$ by reduction of order method. **[4]**
3. Find the general solution of $y''+y=x-\cot(x)$ by variation of parameter method. **[4]**
4. Find the general solution of $(D^2+2D)y=e^x\sin(x)$ by undetermined coefficient method. **[4]**
5. Find the power series solution of $(1-x^2)y''+2xy'-y=0.$ about the ordinary point $x=0$ **[6]**
6. Solve the following hypergeometric differential equation in terms of hypergeometric function, near the regular singular point x=0
   $8x(1-x)y''+(4-14x)y'-y=0.$ **[4]**
7. Using the generating function for Legendre polynomials prove that $P_n'(1)=\frac{n(n+1)}{2}$
   where $P_n(x)$ represent Legendre polynomial **[4]**
8. Find the eigen values and eigen functions for the equation $y''+\lambda y=0$ where $y(0)=0, y(2b)=0$ when $b>0$ **[4]**
9. Find the Fourier series of the function $f(x)=3x+2x^2, -\pi\le x<\pi$. Hence deduce the sum of the series $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+...$ **[6]**
10. Solve $x\left(x-1\right)y'-y=x^2\left(x-1\right)^2$ **[4]**
11. Verify that one solution of $xy''-(2x+1)y'+(x+1)y=0$ is given by $y_1=e^x,$ and find the general solution. **[4]**
12. Solve $y''-2y'+y=\frac{e^x}{x^3}$ by operator method. **[4]**
13. Find the general solution of the following system:
    $$\frac{dx}{dt}=4x-2y,\ \frac{dy}{dt}=5x+2y$$ **[4]**
14. Find the Frobenius series solution about $x=0$ of $4xy''+2y'+2y=0.$ **[6]**

15. Given that $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x;\ J_{-1/2}(x)=\sqrt{\frac{2}{\pi x}}\cos x$, express $J_{3/2}(x)$ and $J_{-3/2}(x)$ in terms of trigonometric functions. **[4]**

16. Find $L\left[e^{4t}\left(t^2+3t+5\right)\right]$ **[4]**

17. Solve $y''-10y'+9y=5t,\ y(0)=-1,\ y'(0)=2$ using Laplace transform. **[6]**

18. Suppose that a string of length π is stretched and fixed on both ends and is plucked in the middle such that it has the initial shape $f(x)=\begin{cases}\frac{x}{10}, & \text{if } 0\le x\le\frac{\pi}{2}\\ \frac{\pi-x}{10}, & \text{if } \frac{\pi}{2}<x\le\pi\end{cases}$

Find the deflection of the string at any point ***x*** at any time ***t.*** **[4]**
