



Machine Learning CS F464

Dr. Pranav M. Pawar

Contents

- Bayesian Classifier
- Gaussian Classifier
- Decision Tree
- Classification Accuracy Metrics

Probabilistic Classification

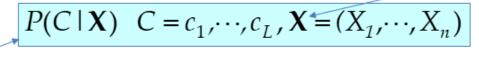
- Establishing a probabilistic model for classification
 - Discriminative model

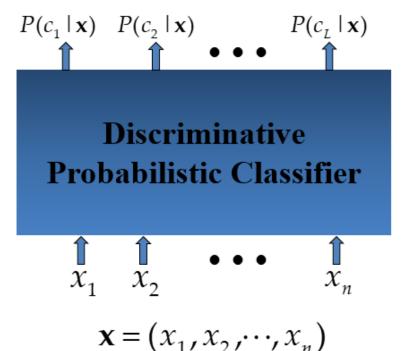
Vectors for teaching

Probability of seeing a member of this class

What is a discriminative Probabilistic Classifier?

Probability that when they show me a fruit it will be an apple



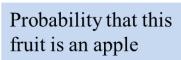


We want to know probabilities of classes for events x1

We know events x1, ... xn

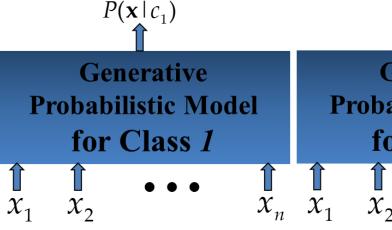
Probabilistic Classification

- Establishing a probabilistic model for classification (cont.)
 - Generative model

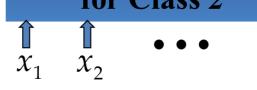


$$P(\mathbf{X} \mid C) \quad C = c_1, \dots, c_L, \mathbf{X} = (X_1, \dots, X_n)$$

Probability that this fruit is an orange



 $P(\mathbf{x} \mid c_2)$



$$P(\mathbf{x} \mid c_L)$$

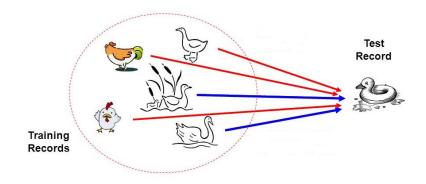
$$\bigcirc$$
Generative
Probabilistic Model
$$\mathbf{for} \ \mathbf{Class} \ L$$

$$\Diamond \quad \chi_1 \quad \chi_2 \quad \chi_n$$

$$\mathbf{x} = (x_1, x_2, \dots, x_n)$$

Vectors of random variables

- Principal
 - If it walks like a duck, quacks like a duck, then it is probably a duck.



- A statistical classifier
 - Performs *probabilistic prediction*, *i.e.*, predicts class membership probabilities.
- Assumptions
 - The classes are mutually exclusive and exhaustive.
 - The attributes are independent given the class.
- Incremental
 - Each training example can incrementally increase/decrease the probability that a hypothesis is correct — prior knowledge can be combined with observed data
- Called "Naïve" classifier because of these assumptions.
 - · Empirically proven to be useful.
 - · Scales very well.

Basis



- We defined prior, conditional and joint probability for random variables
 - Prior probability: P(X)
 - Conditional probability: $P(X_1 | X_2), P(X_2 | X_1)$
 - Joint probability: $\mathbf{X} = (X_1, X_2), P(\mathbf{X}) = P(X_1, X_2)$
 - Relationship: $P(X_1, X_2) = P(X_2 | X_1)P(X_1) = P(X_1 | X_2)P(X_2)$
 - Independence: $P(X_2 \mid X_1) = P(X_2), P(X_1 \mid X_2) = P(X_1), P(X_1, X_2) = P(X_1)P(X_2)$
- Bayesian Rule

$$P(C \mid \mathbf{X}) = \frac{P(\mathbf{X} \mid C)P(C)}{P(\mathbf{X})} \implies Posterior = \frac{Likelihood \times Prior}{Evidence}$$

- MAP classification rule
 - MAP: Maximum A Posterior
 - Assign x to c^* if

$$P(C = c^* | \mathbf{X} = \mathbf{x}) > P(C = c | \mathbf{X} = \mathbf{x}) \quad c \neq c^*, c = c_1, \dots, c_L$$

Method of Generative classification with the MAP rule

2. Then apply the MAP rule

Navie Bayes Classification

Bayes classification

$$P(C \mid \mathbf{X}) \propto P(\mathbf{X} \mid C)P(C) = P(X_1, \dots, X_n \mid C)P(C)$$

Difficulty: learning the joint probability $P(X_1,...,X_m \mid C)$

Naïve Bayes classification

Assumption that all input attributes are conditionally independent!

$$P(X_{1}, X_{2}, \dots, X_{n} | C) = P(X_{1} | X_{2}, \dots, X_{n}, C) P(X_{2}, \dots, X_{n} | C)$$

$$= P(X_{1} | C) P(X_{2}, \dots, X_{n} | C)$$

$$= P(X_{1} | C) P(X_{2} | C) \dots P(X_{n} | C)$$
in proper to the second second

Product of individual probabilities

- MAP classification rule: for $\mathbf{x} = (x_1, x_2, \dots, x_n)$

$$\mathbf{x} = (x_1, x_2, \dots, x_n)$$

$$[P(x_1 | c^*) \cdots P(x_n | c^*)]P(c^*) > [P(x_1 | c) \cdots P(x_n | c)]P(c), c \neq c^*, c = c_1, \cdots, c_L$$

Navie Bayes Classification

- Naïve Bayes Algorithm (for discrete input attributes) has two phases
 - 1. Learning Phase: Given a training set S,

For each target value of c_i ($c_i = c_1, \dots, c_L$) $\hat{P}(C = c_i) \leftarrow \text{estimate } P(C = c_i) \text{ with examples in } S; \longrightarrow \text{Prior}$ For every attribute value x_{ik} of each attribute X_i $(j = 1, \dots, n; k = 1, \dots, N_i)$

Liklihood
$$\leftarrow \hat{P}(X_j = x_{jk} \mid C = c_i) \leftarrow \text{estimate } P(X_j = x_{jk} \mid C = c_i) \text{ with examples in } \mathbf{S};$$

Output: conditional probability tables; for X_i , $N_i \times L$ elements

- **2. Test Phase**: Given an unknown instance $X' = (a'_1, \dots, a'_n)$, Look up tables to assign the label c^* to X' if

$$[\hat{P}(a'_1 \mid c^*) \cdots \hat{P}(a'_n \mid c^*)] \hat{P}(c^*) > [\hat{P}(a'_1 \mid c) \cdots \hat{P}(a'_n \mid c)] \hat{P}(c), c \neq c^*, c = c_1, \dots, c_L$$

PlayTennis: training examples

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

PlayTennis: training examples

$\overline{}$	3 8 1				
Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
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D5	Rain	Cool	Normal	Weak	Yes
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D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No C

$$P(\text{Play=Yes}) = 9/14$$

$$P(\text{Play}=No) = 5/14$$

We have four variables, we calculate for each

Outlook	Play=Yes	Play=No
Sunny	2/9	3/5
Overcast	4/9	0/5
Rain	3/9	2/5

Humidity	Play=Yes	Play=No
High	3/9	4/5
Normal	6/9	1/5

Temperature	Play=Yes	Play=No
Hot	2/9	2/5
Mild	4/9	2/5
Cool	3/9	1/5

Wind	Play=Yes Play=1	
Strong	3/9	3/5
Weak	6/9	2/5

Example: Testing Phase

Test Phase

Given a new instance of variable values,

x'=(Outlook=*Sunny*, Temperature=*Cool*, Humidity=*High*, Wind=*Strong*)

Given calculated Look up tables

$$P(Outlook=Sunny | Play=No) = 3/5$$

 $P(Temperature=Cool | Play==No) = 1/5$
 $P(Huminity=High | Play=No) = 4/5$
 $P(Wind=Strong | Play=No) = 3/5$
 $P(Play=No) = 5/14$

Use the MAP rule to calculate Yes or No

 $\begin{array}{l} \textbf{P(Yes | x')} : [P(Sunny | Yes)P(Cool | Yes)P(High | Yes)P(Strong | Yes)]P(Play=Yes) = 0.0053 \\ \textbf{P(No | x')} : [P(Sunny | No) P(Cool | No)P(High | No)P(Strong | No)]P(Play=No) = 0.0206 \\ \end{array}$

Given the fact P(Yes | x') < P(No | x'), we label x' to be "No".

1. Violation of Independence Assumption

Events are correlated

- For many real world tasks, $P(X_1,\dots,X_n|C) \neq P(X_1|C)\dots P(X_n|C)$
- Nevertheless, naïve Bayes works surprisingly well anyway!

2. Zero conditional probability Problem

- Such problem exists when no example contains the attribute value $X_i = a_{ik}$, $\hat{P}(X_i = a_{ik} | C = c_i) = 0$
- In this circumstance, $\hat{P}(x_1 | c_i) \cdots \hat{P}(a_{jk} | c_i) \cdots \hat{P}(x_n | c_i) = 0$ during test
- For a remedy, conditional probabilities are estimated with

$$\hat{P}(X_j = a_{jk} \mid C = c_i) = \frac{n_c + mp}{n + m}$$
 n_c : number of training examples for which $X_j = a_{jk}$ and $C = c_i$
 n : number of training examples for which $C = c_i$
 p : prior estimate (usually, $p = 1/t$ for t possible values of X_j)
 m : weight to prior (number of "virtual" examples, $m \ge 1$)

- What to do in the case of Continuous Valued Inputs?
 - Numberless values for an attribute
 - Conditional probability is then modeled with the normal distribution

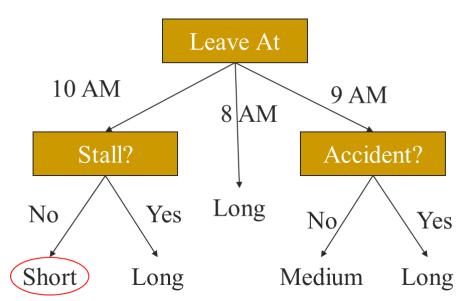
$$\hat{P}(X_j \mid C = c_i) = \frac{1}{\sqrt{2\pi} \sigma_{ji}} \exp\left(-\frac{(X_j - \mu_{ji})^2}{2\sigma_{ji}^2}\right)$$

 μ_{ji} : mean (avearage) of attribute values X_j of examples for which $C = c_i$ σ_{ji} : standard deviation of attribute values X_j of examples for which $C = c_i$

- Learning Phase: for $\mathbf{X} = (X_1, \dots, X_n)$, $C = c_1, \dots, c_L$ Output: $n \times L$ normal distributions and $P(C = c_i)$ $i = 1, \dots, L$
- Test Phase: for $X' = (X'_1, \dots, X'_n)$
 - 1. Calculate conditional probabilities with all the normal distributions
 - 2. Apply the MAP rule to make a decision

Decision Tree

- An inductive learning task
 - Use particular facts to make more generalized conclusions
- A predictive model based on a branching series of Boolean tests
 - These smaller Boolean tests are less complex than a one-stage classifier
- In this decision tree, do a series of Boolean decisions and follow the corresponding branch
 - Did we leave at 10 AM?
 - Did a car stall on the road?
 - Is there an accident on the road?
- By answering each of these yes/no questions, we then came to a conclusion on how long our commute might take



Decision Tree Classification Task

innovate

achieve

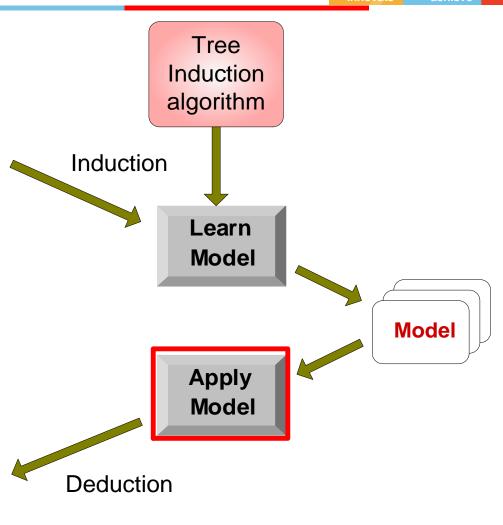
lead

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

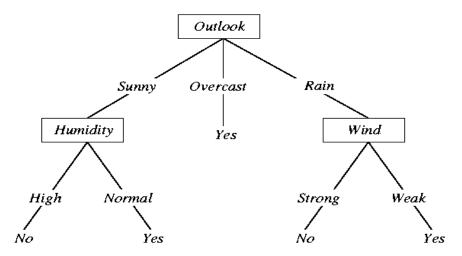
Training Set

Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set



Decision tree for Play Tennis Dataset



- Classifies instances into one of a discrete set of possible categories
- Learned function represented by tree
- Each node in tree is test on some attribute of an instance
- Branches represent values of attributes
- Follow the tree from root to leaves to find the output value.

Decision Tree Learning Steps

- 1. Choose an attribute from our dataset.
- 2. Calculate the significance of the attribute in the splitting of the data.
- 3. Split the data based on the value of the best attribute.
- 4. Go to step 1

How we can determine attribute for classification?

Information Gain



- Information gain is our metric for how well one attribute A^{i} classifies the training data.
- Information gain for a particular attribute =

 Information about target function,
 given the value of that attribute.

 (conditional entropy)
- Mathematical expression for information gain:

$$Gain(S, A_i) = H(S) - \sum_{v \in Values(A_i)} P(A_i = v)H(S_v)$$
entropy

entropy

entropy

- Measure of randomness and uncertainty.
- For an ensemble of random events: $\{A_1, A_2, ..., A_n\}$, occurring with probabilities: $\mathbf{z} = \{P(A_1), P(A_2), ..., P(A_n)\}$

$$H = -\sum_{i=1}^{n} P(A_i) \log_2(P(A_i))$$

(Note:
$$1 = \sum_{i=1}^{n} P(A_i)$$
 and $0 \le P(A_i) \le 1$)

- If you consider the self-information of event, i, to be: -log2(P(Ai))
- Entropy is weighted average of information carried by each event.
- For two states: Positive examples and Negative examples from set

$$H(S) = -p_{+}log_{2}(p_{+}) - p_{-}log_{2}(p_{-})$$

• If an event <u>always occurs</u>, $P(A_i)=1$, then it carries no information.

$$-log_{2}(1) = 0$$

• If an event <u>rarely occurs</u> (e.g. $P(A_i)=0.001$), it carries a lot of info.

$$-log_2(0.001) = 9.97$$

 The less likely the event, the more the information it carries.

- Calculate the <u>entropy</u> for <u>all training examples</u>
 - positive and negative cases
 - $-p_{+} = \text{\#pos/Tot}$ $p_{-} = \text{\#neg/Tot}$
 - $H(S) = -p_{+}log_{2}(p_{+}) p_{-}log_{2}(p_{-})$
- Determine which <u>single attribute</u> best classifies the training examples using information gain.
 - For each attribute find:
 - Use <u>attribute with greatest information gain</u> as a root

$$Gain(S, A_i) = H(S) - \sum_{v \in Values(A_i)} P(A_i = v)H(S_v)$$

Entropy before split

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	\mathbf{High}	Strong	No
D3	Overcast	Hot	\mathbf{High}	Weak	Yes
D4	\mathbf{Rain}	Mild	\mathbf{High}	\mathbf{Weak}	Yes
D5	\mathbf{Rain}	Cool	Normal	\mathbf{Weak}	Yes
D6	\mathbf{Rain}	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	\mathbf{High}	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	\mathbf{Rain}	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	\mathbf{High}	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	\mathbf{Rain}	Mild	\mathbf{High}	Strong	No

14 cases

9 positive cases

Step 1: <u>Calculate entropy</u> for all cases:

$$N_{Pos} = 9$$

$$N_{Neg} = 5$$

$$N_{Tot} = 14$$

entropy $H(S) = -(9/14)*log_2(9/14) - (5/14)*log_2(5/14) = 0.940$

- Step 2: Loop over all attributes, calculate gain:
 - Attribute = Outlook
 - Loop over values of Outlook

Outlook = Sunny

$$N_{Pos} = 2$$
 $N_{Neg} = 3$ $N_{Tot} = 5$ $H(Sunny) = -(2/5)*log_2(2/5) - (3/5)*log_2(3/5) = 0.971$ $Outlook = Overcast$ $N_{Pos} = 4$ $N_{Neg} = 0$ $N_{Tot} = 4$ $H(Overcast) = -(4/4)*log_24/4) - (0/4)*log_2(0/4) = 0.00$

	Day	Outlook	Temperature	Humidity	Wind	PlayTennis
	D1	Sunny	Hot	High	Weak	No
	D2	\mathbf{Sunny}	Hot	\mathbf{High}	Strong	No
	$\mathbf{D3}$	Overcast	Hot	\mathbf{High}	Weak	Yes
	D4	\mathbf{Rain}	Mild	\mathbf{High}	Weak	Yes
	D5	\mathbf{Rain}	Cool	Normal	Weak	Yes
	D6	\mathbf{Rain}	Cool	Normal	Strong	No
	D7	Overcast	Cool	Normal	Strong	Yes
	D8	Sunny	Mild	\mathbf{High}	Weak	No
	D9	Sunny	Cool	Normal	Weak	Yes
	D10	\mathbf{Rain}	Mild	Normal	Weak	Yes
	D11	\mathbf{Sunny}	Mild	Normal	Strong	Yes
	D12	Overcast	Mild	\mathbf{High}	Strong	Yes
	D13	Overcast	Hot	Normal	Weak	Yes
	D14	Rain	Mild	\mathbf{High}	Strong	No

Want to select best

selected attributes.

Approximate this by

separation of values for all

selecting an attribute with

the highest information gain.

Outlook = Rain

$$N_{Pos} = 3$$
 $N_{Neg} = 2$ $N_{Tot} = 5$
H(Rain) = -(3/5)*log₂(3/5) - (2/5)*log₂(2/5) = 0.971

Calculate Information Gain for attribute Outlook

$$\begin{aligned} \text{Gain}(S,Outlook) &= \text{H(S)} - \text{N}_{\text{Sunny}}/\text{N}_{\text{Tot}}^*\text{H}(\text{Sunny}) \\ &- \text{N}_{\text{Over}}/\text{N}_{\text{Tot}}^*\text{H}(\text{Overcast}) \\ &- \text{N}_{\text{Rain}}/\text{N}_{\text{Tot}}^*\text{H}(\text{Rainy}) \\ \text{Gain}(S,Outlook) &= 0.940 - (5/14)^*0.971 - (4/14)^*0 - (5/14)^*0.971 \ \textbf{Gain}(S,Outlook) &= 0.246 \end{aligned}$$

– Attribute = Temperature

(Repeat process looping over {Hot, Mild, Cool})
 Gain(S, Temperature) = 0.029

– Attribute = Humidity

- (Repeat process looping over {High, Normal})
 Gain(S, Humidity) = 0.029
- Attribute = Wind
 - (Repeat process looping over {Weak, Strong})
 Gain(S, Wind) = 0.048

Find attribute with greatest information

gain:

Gain(S,Outlook) = 0.246,= 0.029 Gain(S, Temperature)

Gain(S, Humidity) = 0.029,

Gain(S, Wind) = 0.048

.: Outlook is root node of tree

Iterate algorithm to find attributes
 which best classify training examples
 under the values of the root node

Example continued

Take three subsets:

```
- Outlook = Sunny (N_{Tot} = 5)
```

-
$$Outlook$$
 = Overcast $(N_{Tot} = 4)$

$$-Outlook = Rainy (N_{Tot} = 5)$$

 For each subset, repeat the above calculation looping over all attributes other than *Outlook*

– For example:

```
• Outlook = Sunny (N_{Pos} = 2, N_{Neg} = 3, N_{Tot} = 5) H=0.971 

- Temp = Hot (N_{Pos} = 0, N_{Neg} = 2, N_{Tot} = 2) H = 0.0 

- Temp = Mild (N_{Pos} = 1, N_{Neg} = 1, N_{Tot} = 2) H = 1.0 

- Temp = Cool (N_{Pos} = 1, N_{Neg} = 0, N_{Tot} = 1) H = 0.0 

Gain(S_{Sunny}, Temperature) = 0.971 - (2/5)*0 - (2/5)*1 - (1/5)*0 

Gain(S_{Sunny}, Temperature) = 0.571
```

Similarly:

```
Gain(S_{Sunny}, Humidity) = 0.971
Gain(S_{Sunny}, Wind) = 0.020
```

- : Humiditý classifies Outlook=Sunny instances best and is placed as the node under Sunny outcome.
- Repeat this process for Outlook = Overcast &Rainy

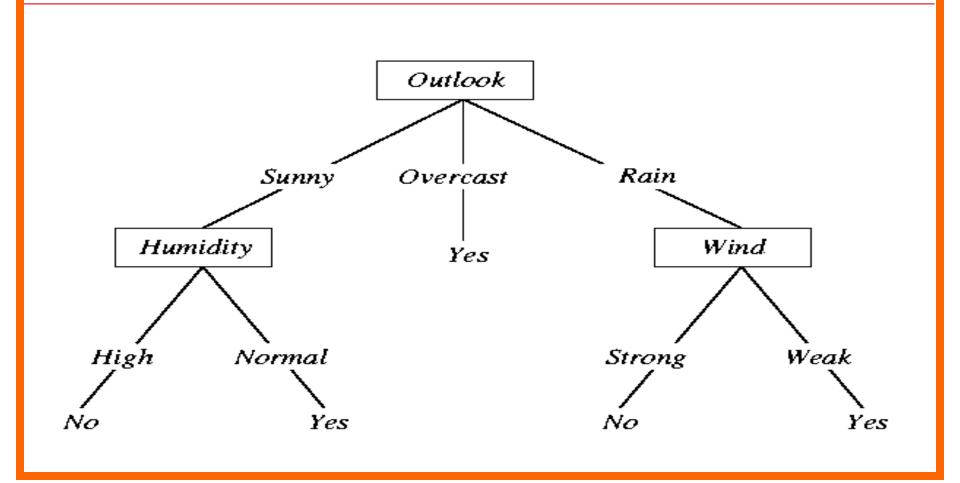
-Important:

- Attributes are excluded from consideration if they appear higher in the tree
- -Process continues for each new leaf node until:
 - Every attribute <u>has already been</u> <u>included</u> along path through the tree

or

 Training examples associated with this leaf <u>all have same target attribute value</u>. – End up with tree:

Decision Tree for PlayTennis

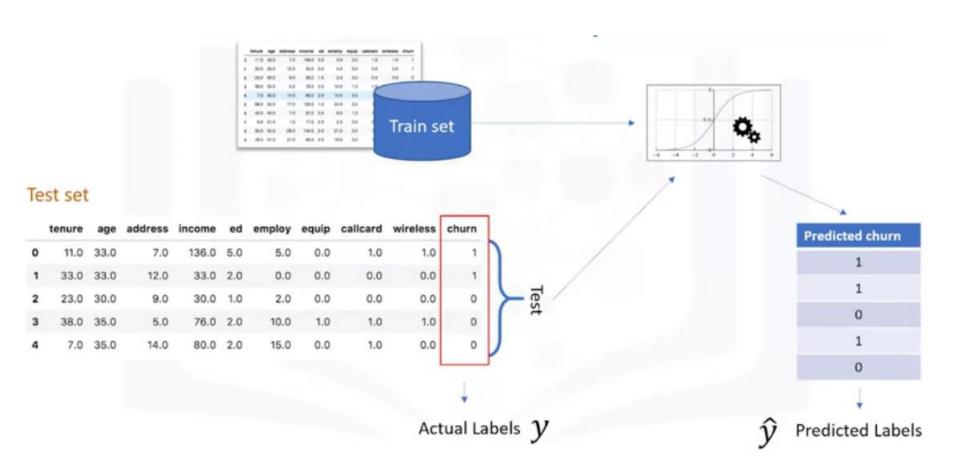


Other Decision Tree Learning Algorithm

- CART (Classification and Regression Tree)
- C 4.5

Classification Accuracy





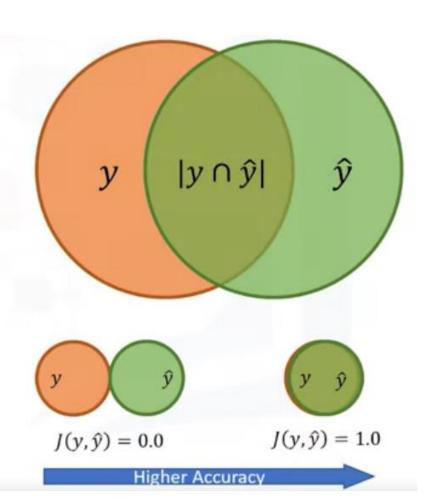
Jacquard Index

y: Actual labels

 \hat{y} : Predicted labels

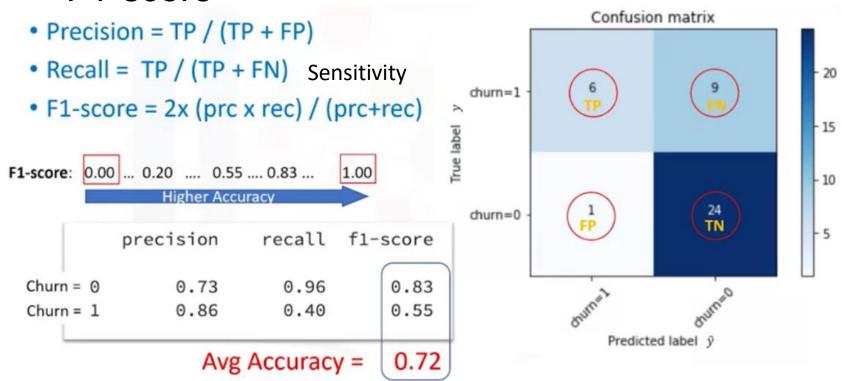
$$J(y,\hat{y}) = \frac{|y \cap \hat{y}|}{|y \cup \hat{y}|} = \frac{|y \cap \hat{y}|}{|y| + |\hat{y}| - |y \cap \hat{y}|}$$

$$J(y, \hat{y}) = \frac{8}{10+10-8} = 0.66$$



Accuracy Measures (2)

F1-score



true positive (TP): A test result that correctly indicates the presence of a condition or characteristic

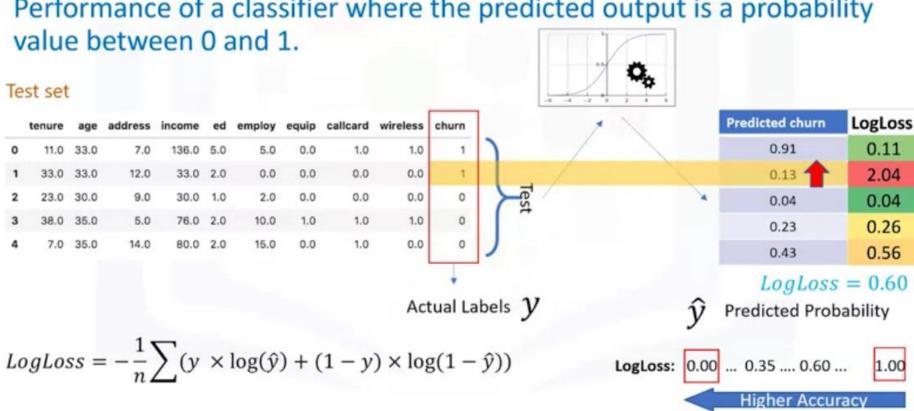
true negative (TN): A test result that correctly indicates the absence of a condition or characteristic **false positive (FP):** A test result which wrongly indicates that a particular condition or attribute is present

false negative (FN): A test result which wrongly indicates that a particular condition or attribute is absent.

Accuracy Measures (3)

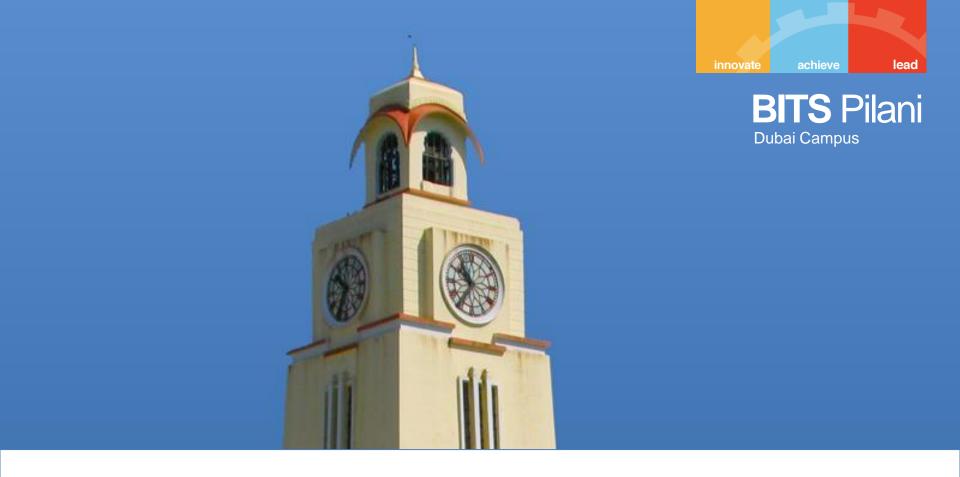
Log Loss

Performance of a classifier where the predicted output is a probability



Sources

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- Chapter 4, Tom M. Mitchell, Machine Learning, The McGraw-Hill Companies, 1st edition 2013.
- "Machine learning with Python course", IBM



Thank You!