An Identifiability Perspective on Representation Learning

Yash Sharma

github.com/bethgelab/slow_disentanglement
arxiv.org/abs/2007.10930

github.com/brendel-group/cl-ica
arxiv.org/abs/2102.08850

Outline

1. Towards Nonlinear Disentanglement in Natural Data with Temporal Sparse Coding
    a. Identifiability w/ assumptions derived from natural video

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    a. Identifiability w/ assumptions derived from natural video
2. Contrastive Learning Inverts the Data Generating Process
    a. Identifiability & InfoNCE
3. Self-Supervised Learning with Data Augmentations Provably Isolates Content from Style
    a. Identifiability when augmentations leave factors invariant

Towards Nonlinear Disentanglement in Natural Data with Temporal Sparse Coding

Overview

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    a. What is disentanglement?

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3. Empirical Contributions
    a. Qualitative and quantitative results on existing and contributed datasets demonstrate outperformance in aggregate

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State of the world, **z**:

- shape
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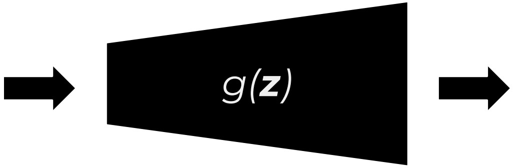

data generating process

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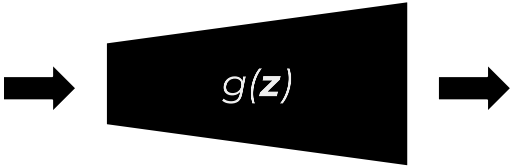

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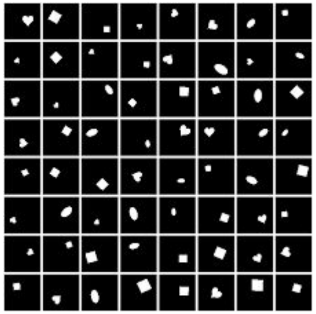

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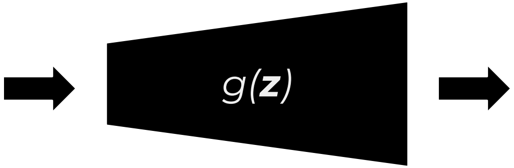

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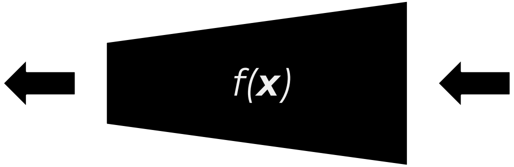

representation learning

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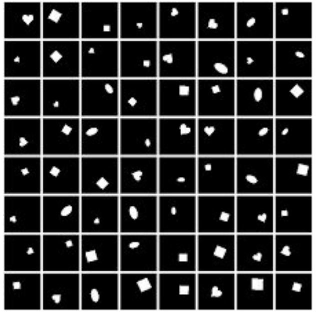

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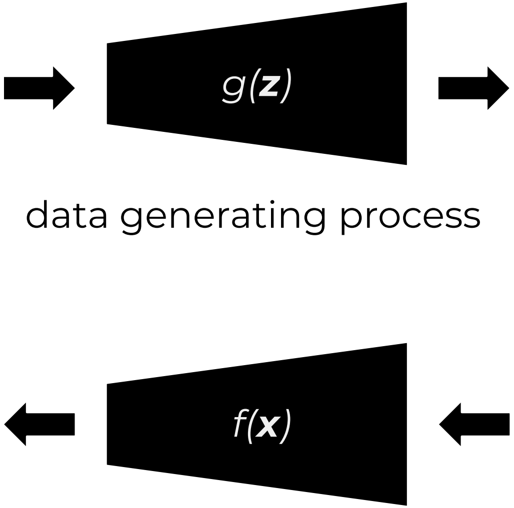

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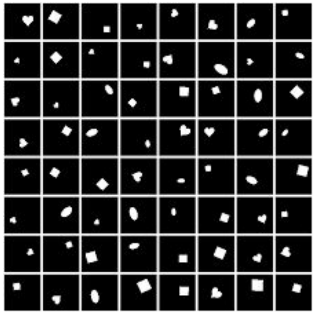

Too simplistic

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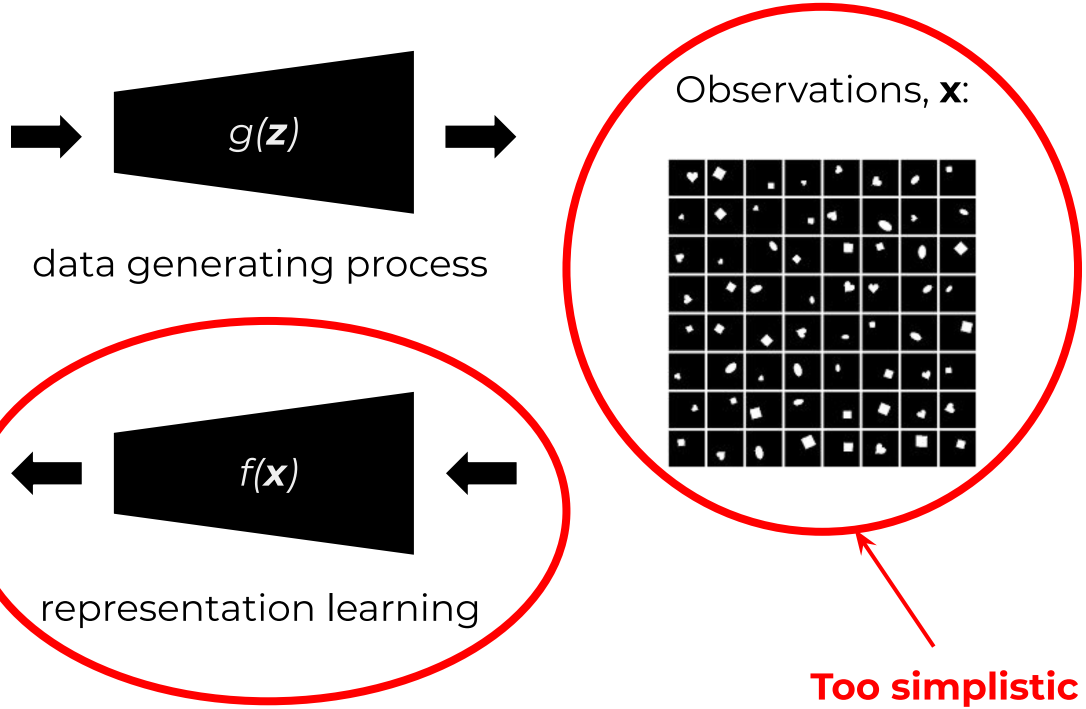
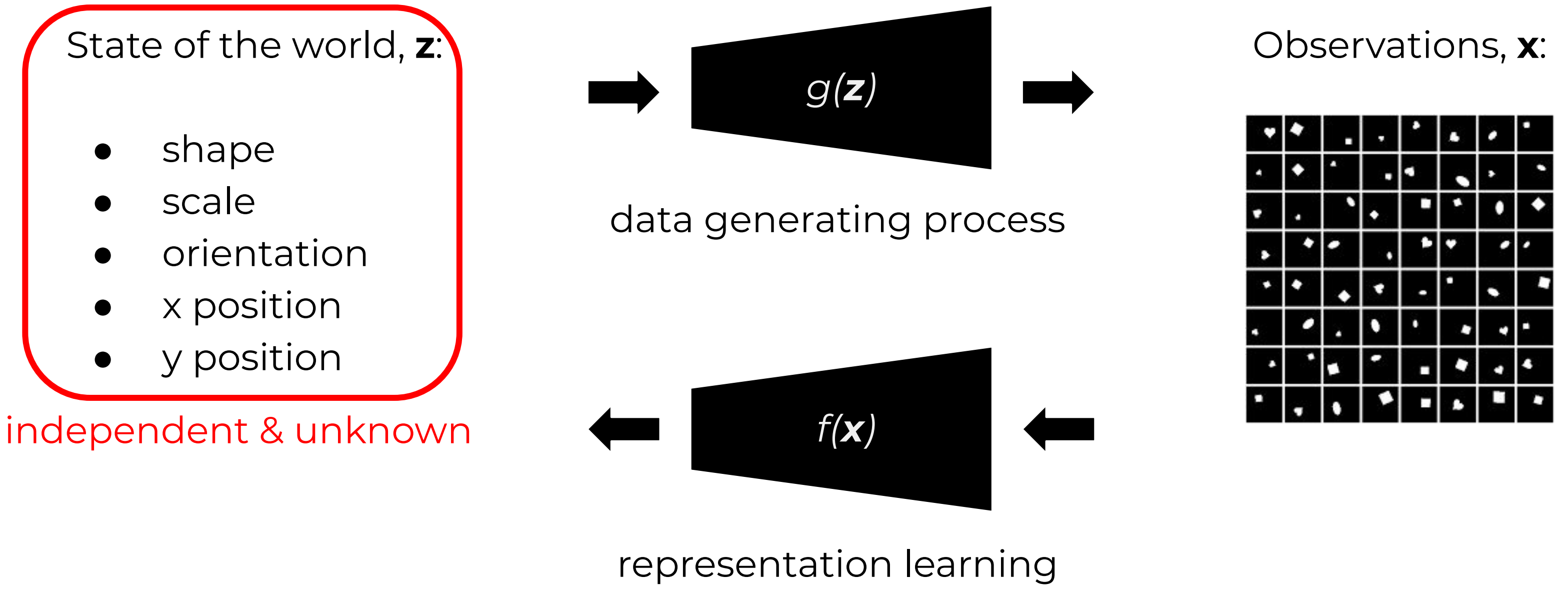


prev solutions do not account for observed natural scene statistics

Too simplistic

Non-identifiability



Hyvärinen & Pajunen (1999) *Nonlinear independent component analysis: Existence and uniqueness results*

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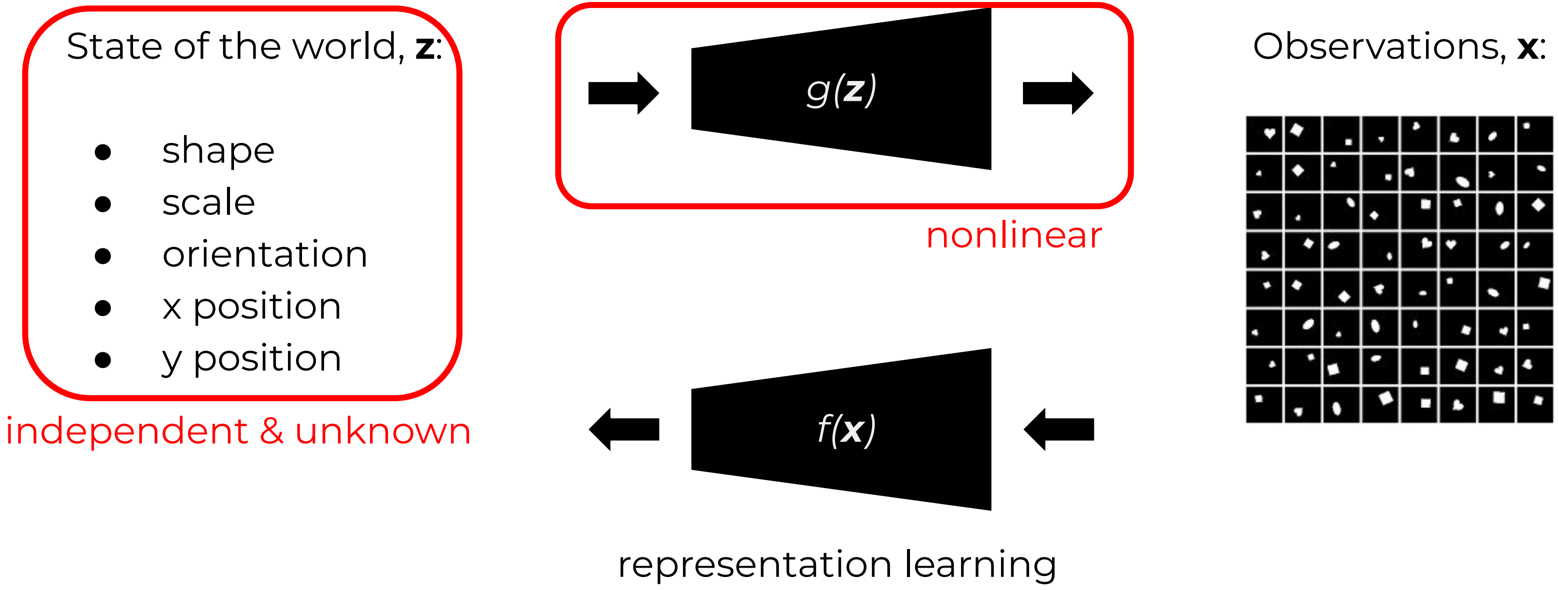

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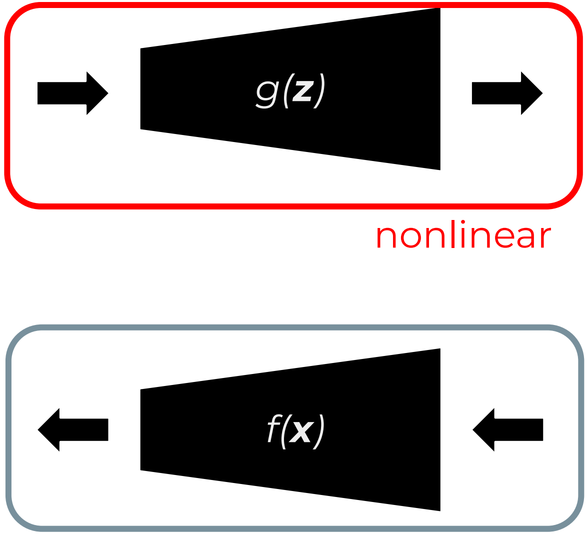
independent & unknown



nonlinear

Non-identifiable
🥺

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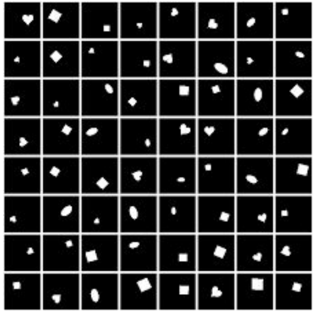
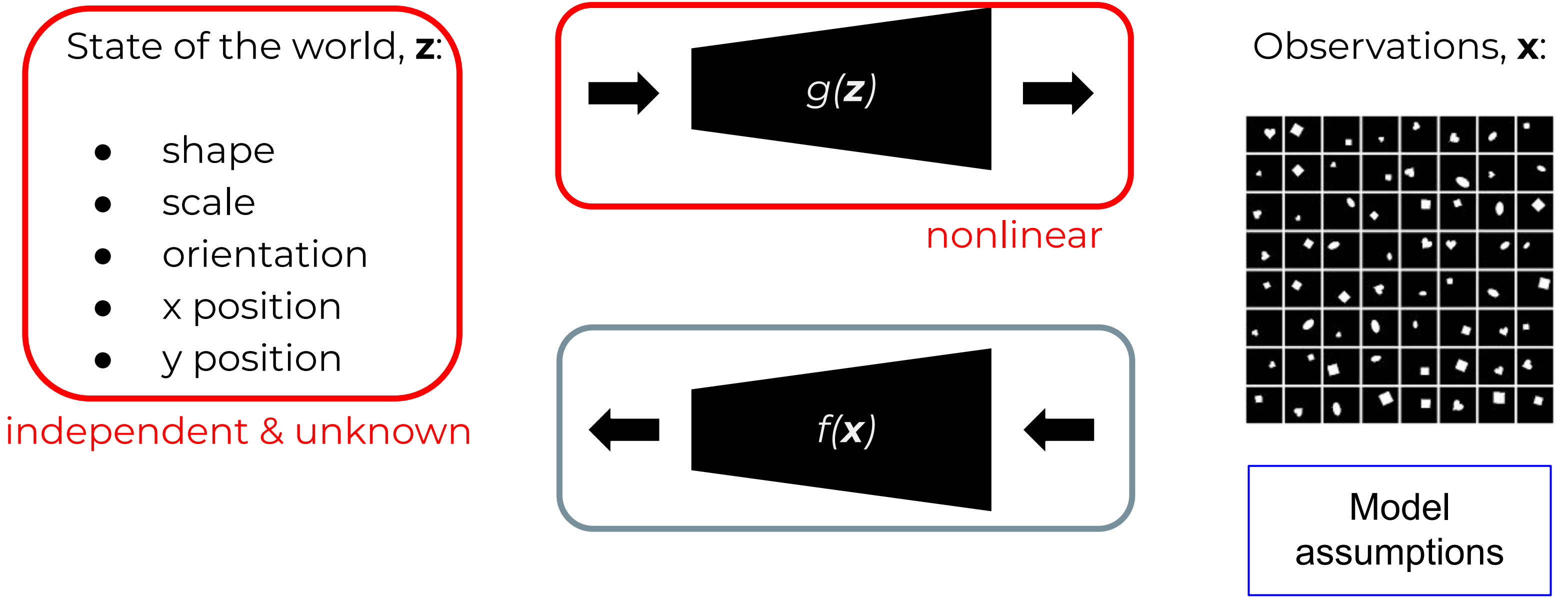


independent identically distributed (i.i.d.)

Locatello et al. (2018) *Challenging Common Assumptions in the Unsupervised Learning of Disentangled Representations*

Hyvärinen & Pajunen (1999) *Nonlinear independent component analysis: Existence and uniqueness results*

Nonlinear Disentanglement



Locatello et al. (2020) *Weakly-Supervised Disentanglement Without Compromises*

Hyvärinen & Morioka (2017) *Nonlinear ICA of Temporally Dependent Stationary Sources*

Hyvärinen & Morioka (2016) *Unsupervised Feature Extraction by Time-Contrastive Learning and Nonlinear ICA*

Natural Video

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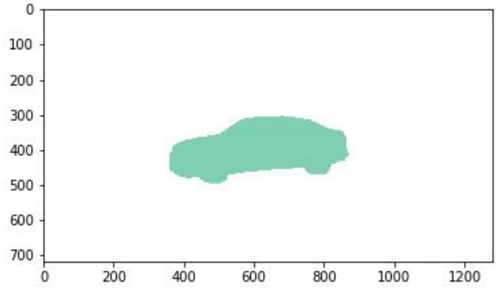

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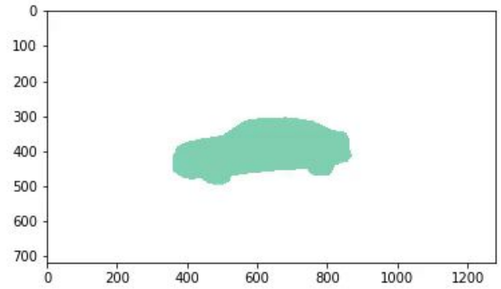

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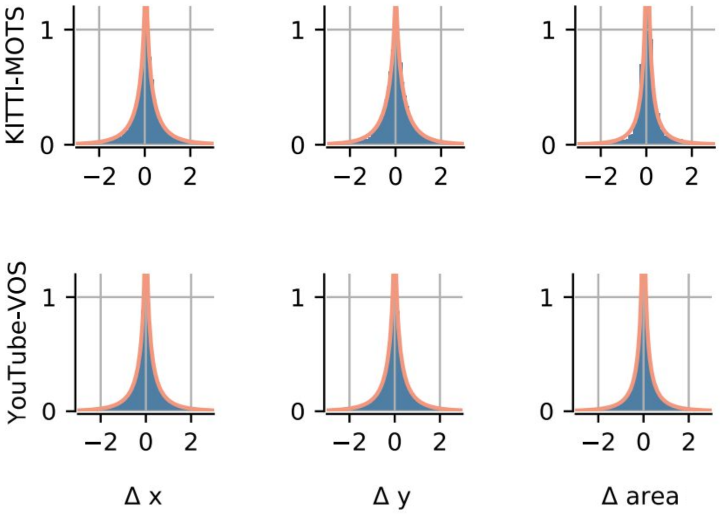
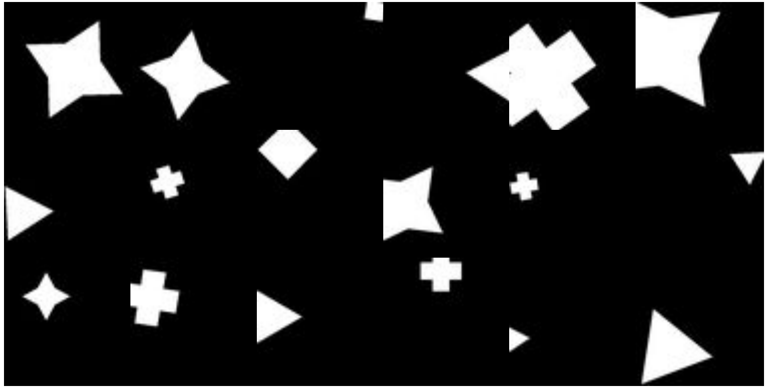
Natural Data Analysis



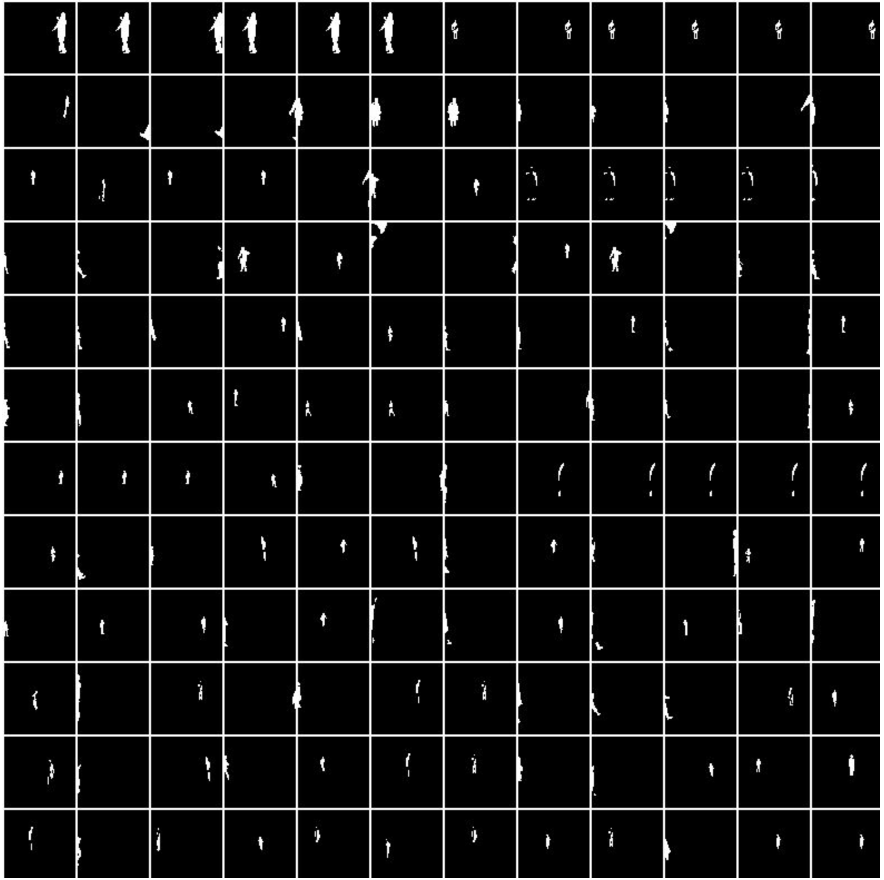
Natural Sprites

- Images generated online using renderer
- Simple, well-controlled objects
- Transitions sampled from YouTube-VOS



KITTI Masks

- Pedestrian masks extracted directly from autonomous recorded videos
- Realistic objects & transitions



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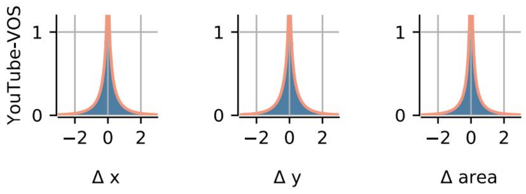

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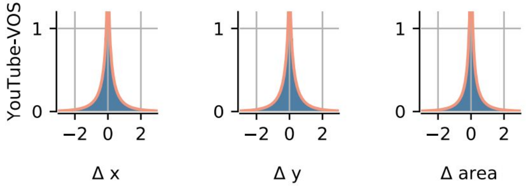

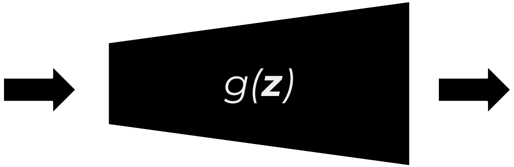

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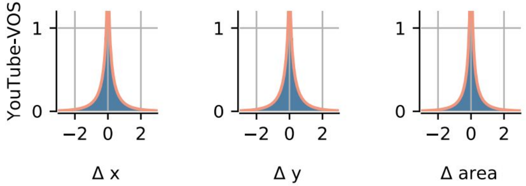

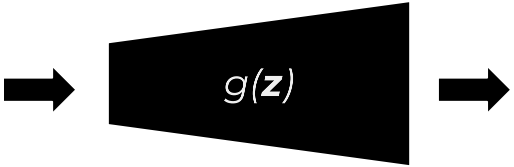

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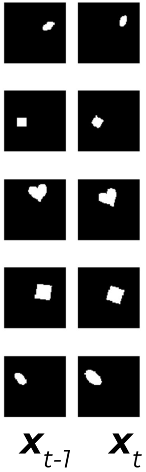

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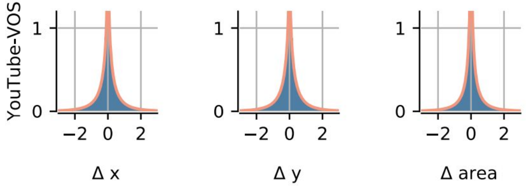

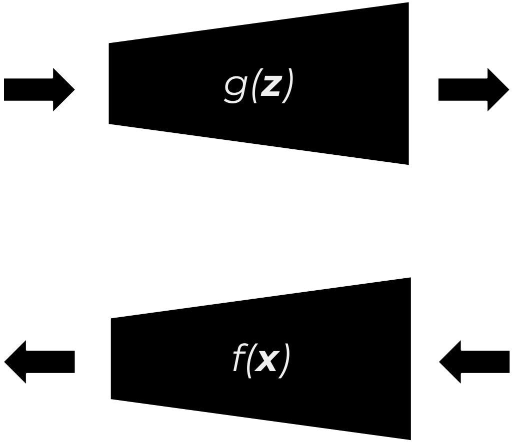

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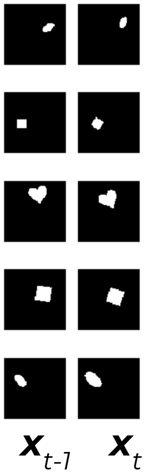

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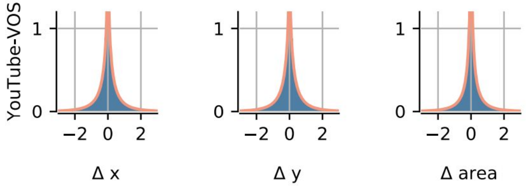

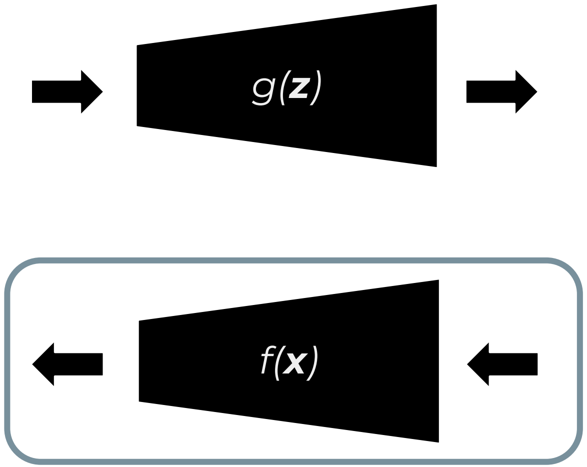

identifiable 🤩

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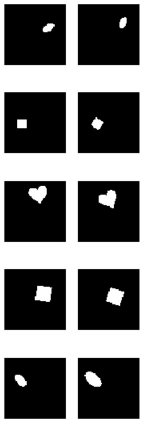
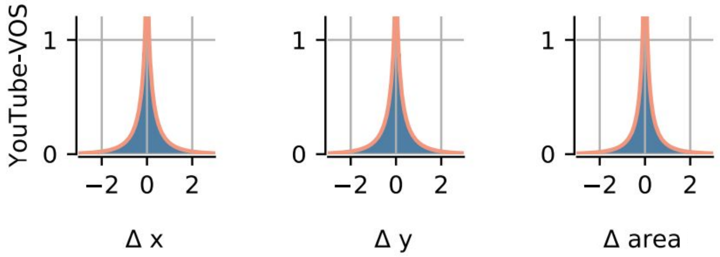


$\boldsymbol{x}_{t-1}$ $\boldsymbol{x}_t$

Identifiability Proof Intuition



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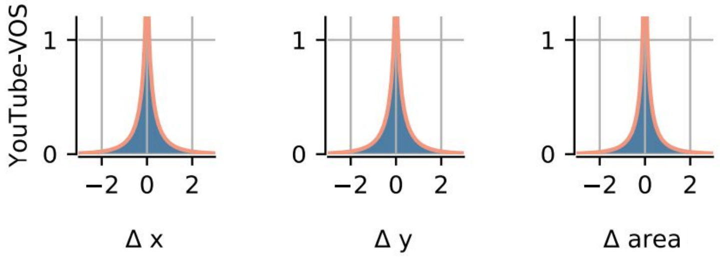

Prior:
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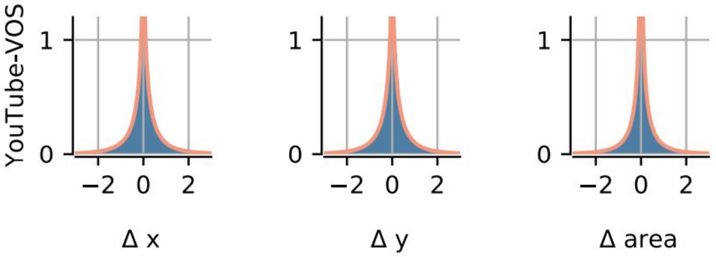

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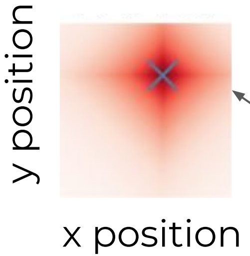

Probability of next position given previous position

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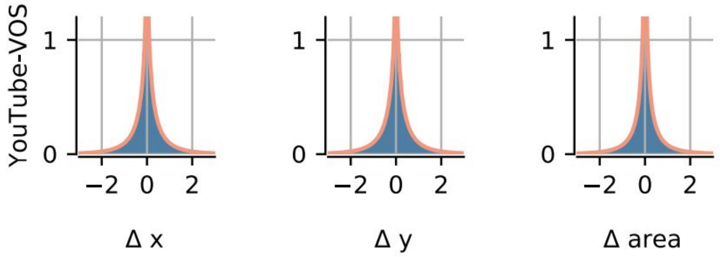

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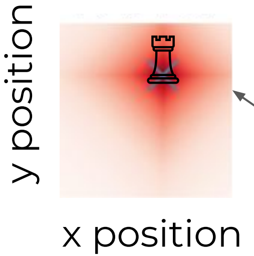

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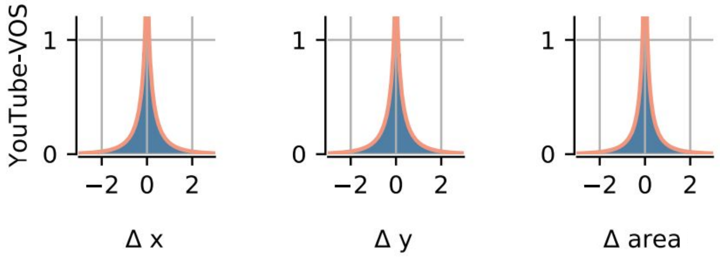

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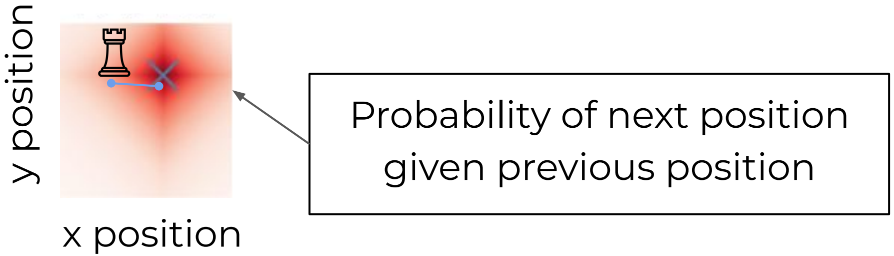

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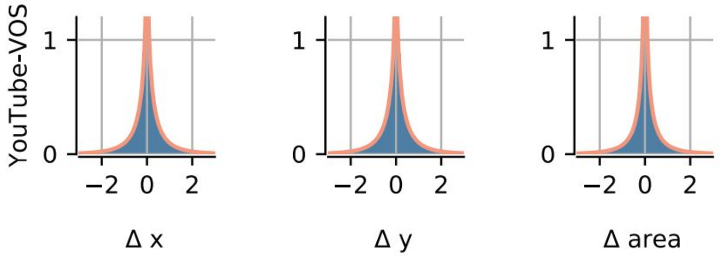
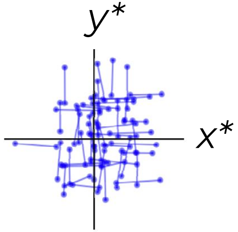


True Model



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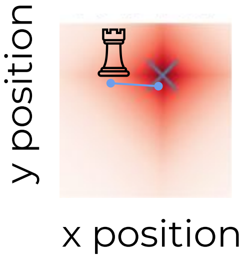

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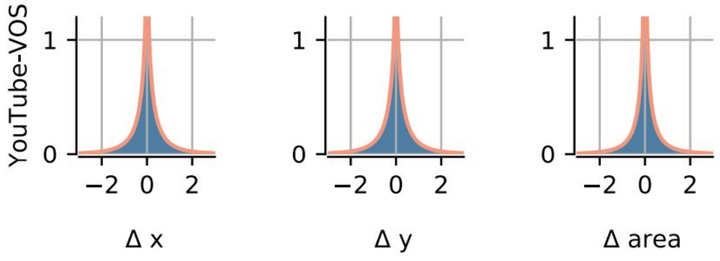

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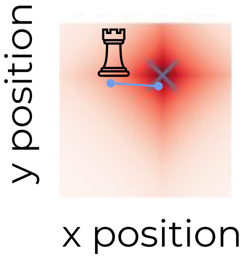

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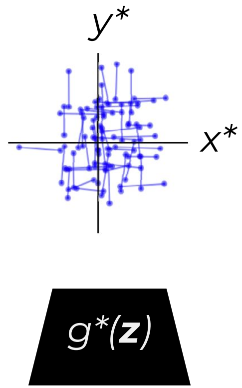

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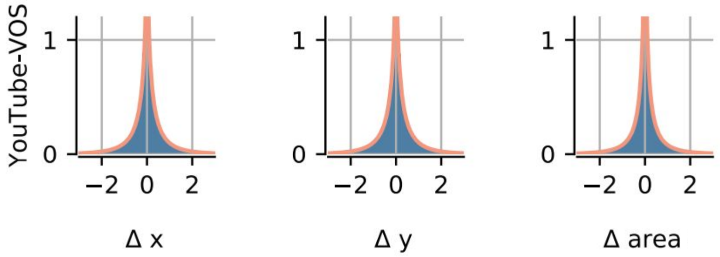

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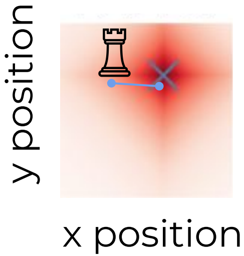

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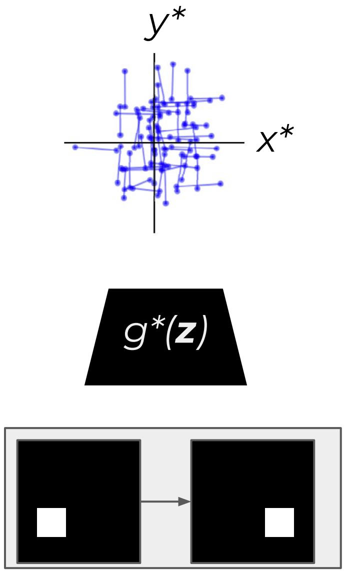

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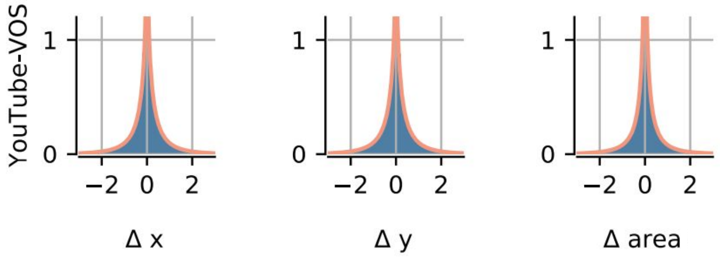

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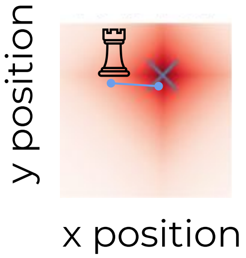

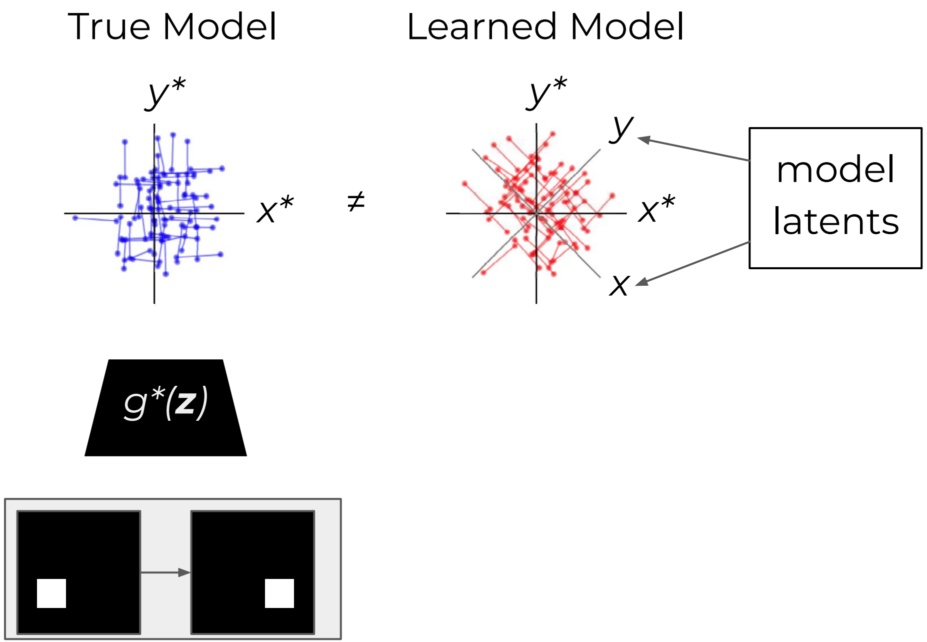

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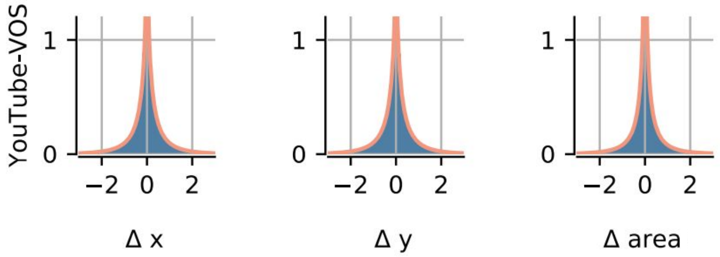

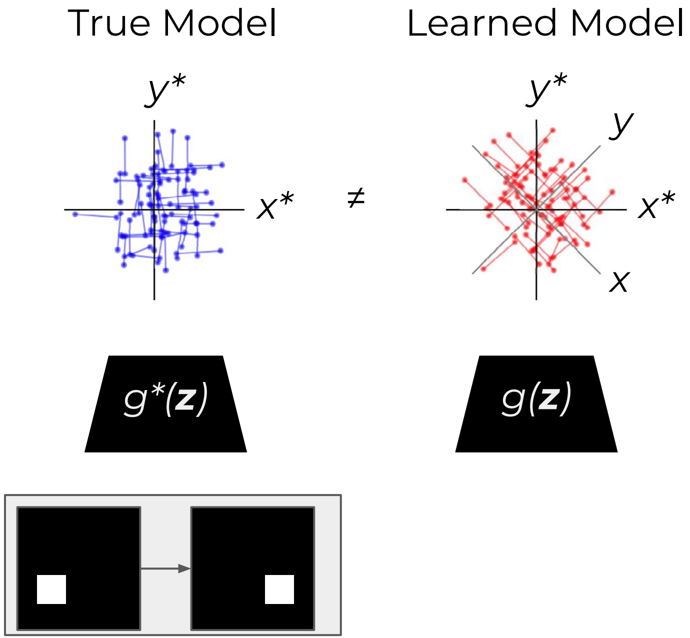

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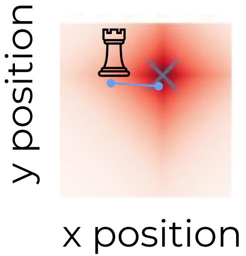

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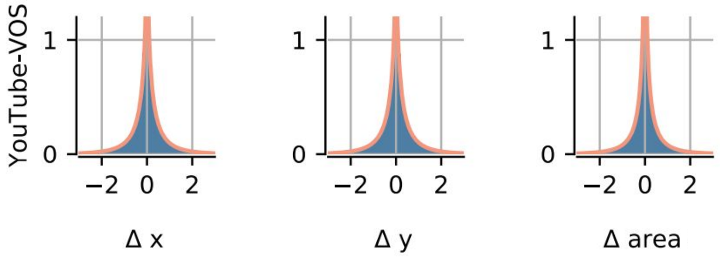

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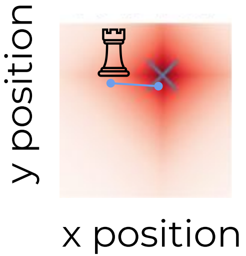

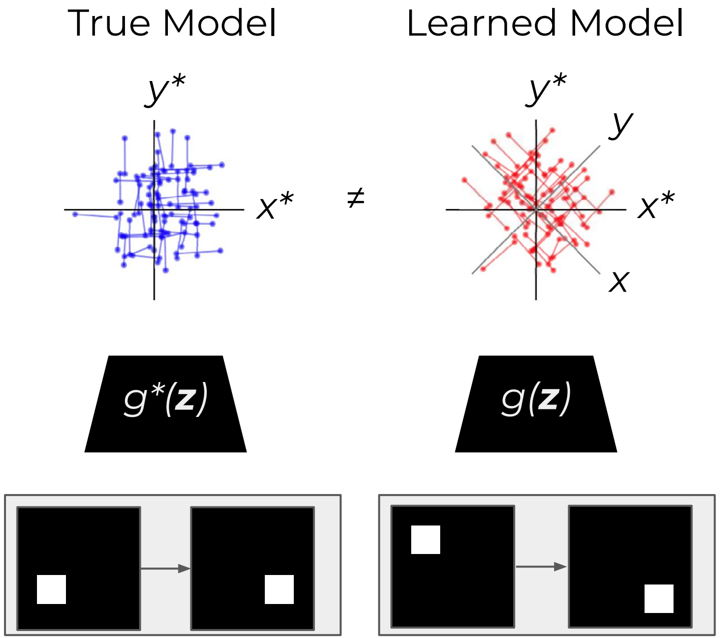

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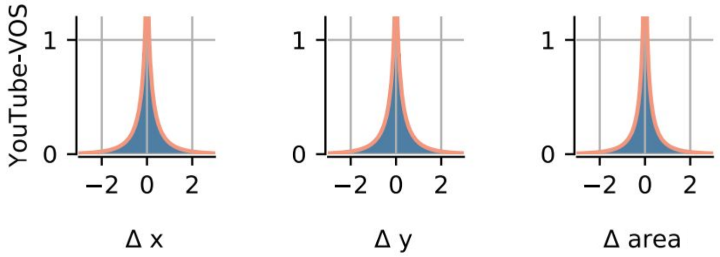

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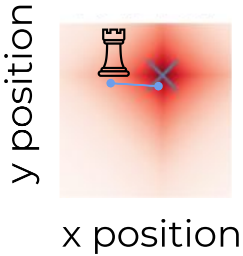
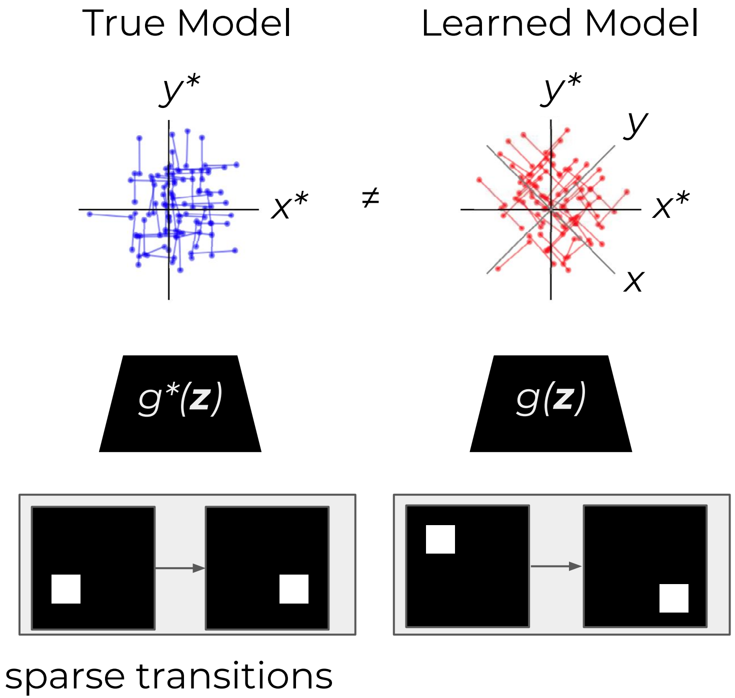
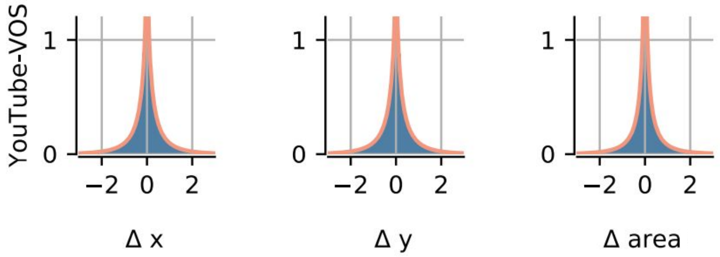
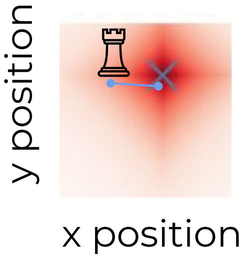


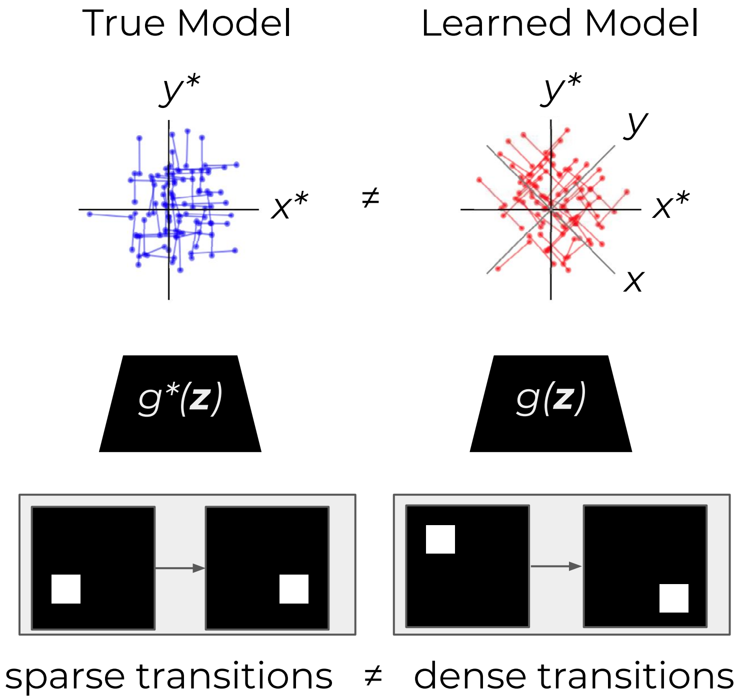


Identifiability Proof Intuition

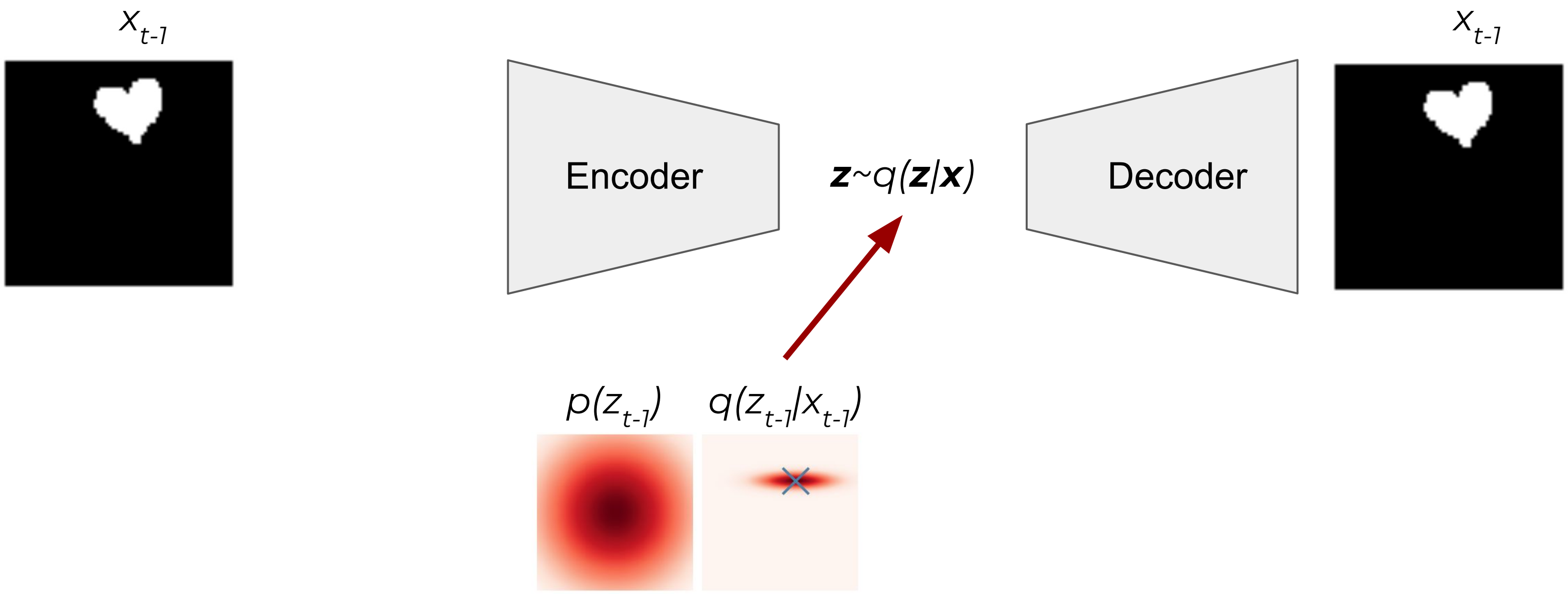


Prior:
objects in nature change sparsely

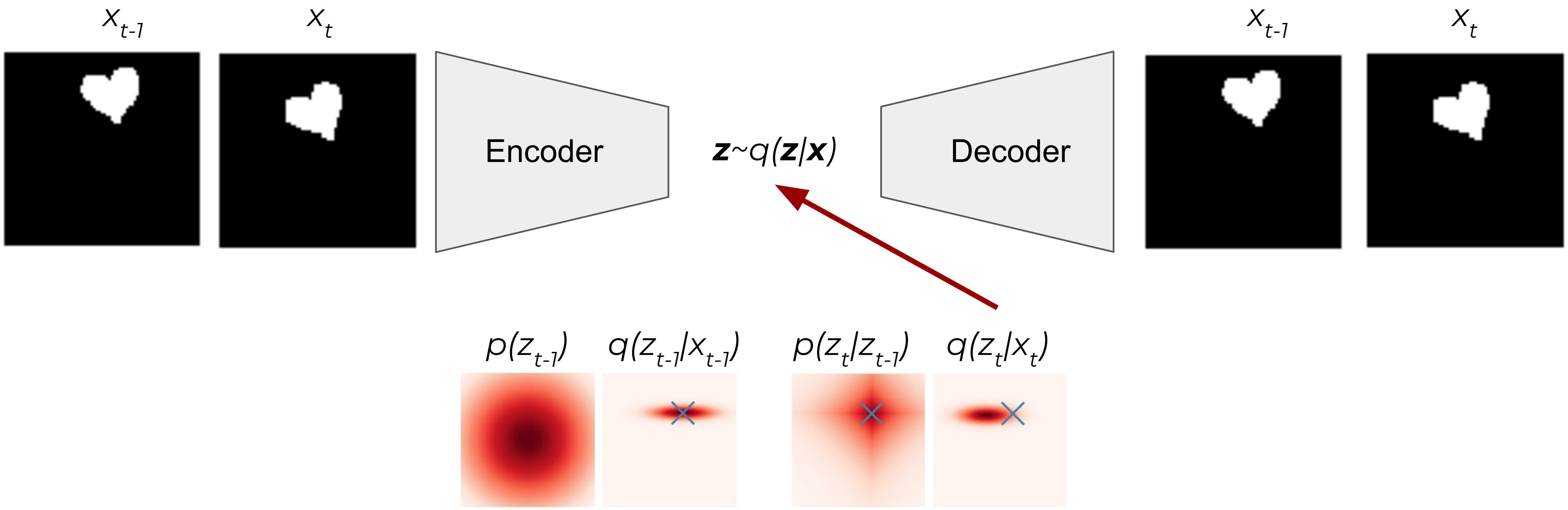




Slow Variational Autoencoder at time *t-1*



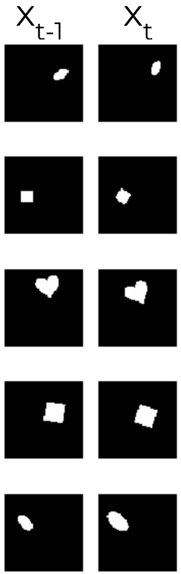
Slow Variational Autoencoder at time *t-1*



Quantitative Results - dSprites

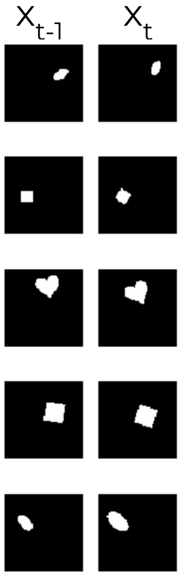
Quantitative Results - dSprites

Data



Quantitative Results - dSprites

Data



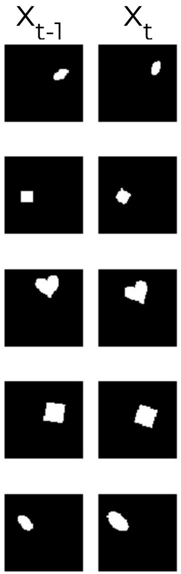
Ada-GVAE [Locatello et al. 19]

PCL [Hyvärinen et al. 17]

SlowVAE (ours)

Quantitative Results - dSprites

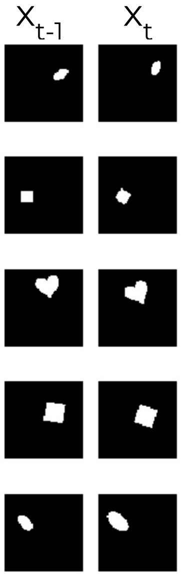
Data



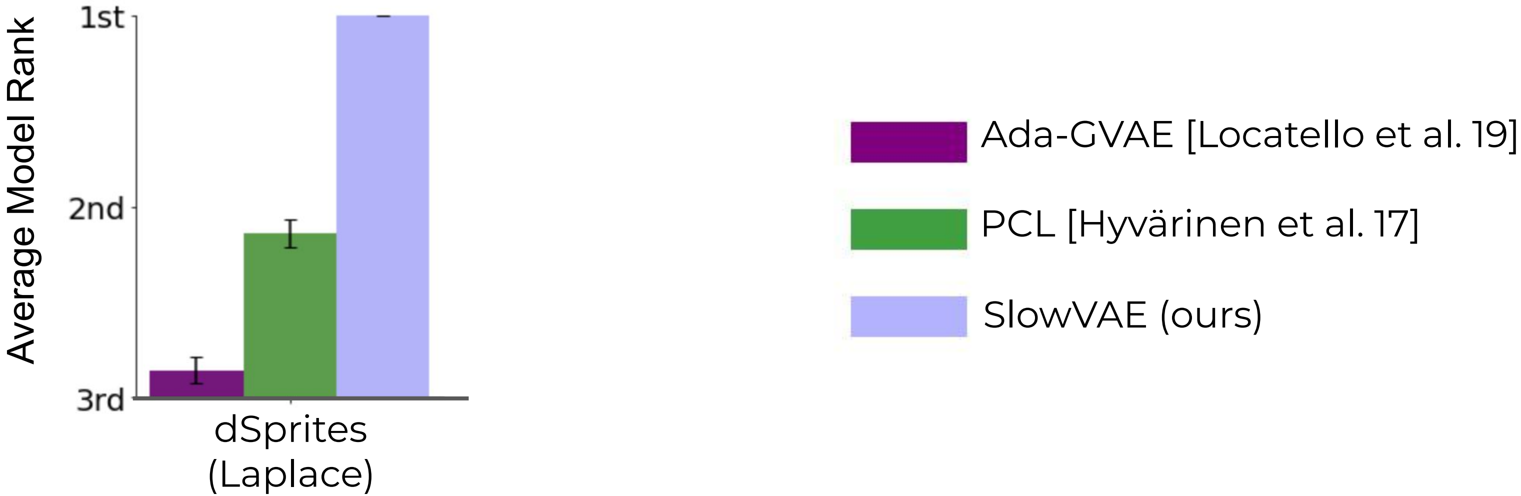
Disentanglement Performance

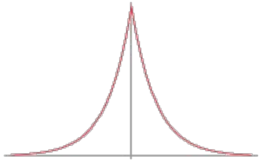
Quantitative Results - dSprites

Data



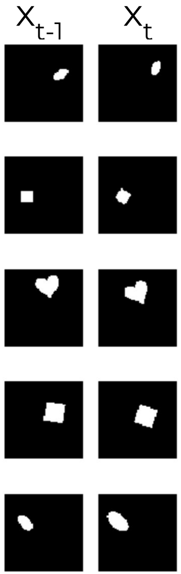
Disentanglement Performance



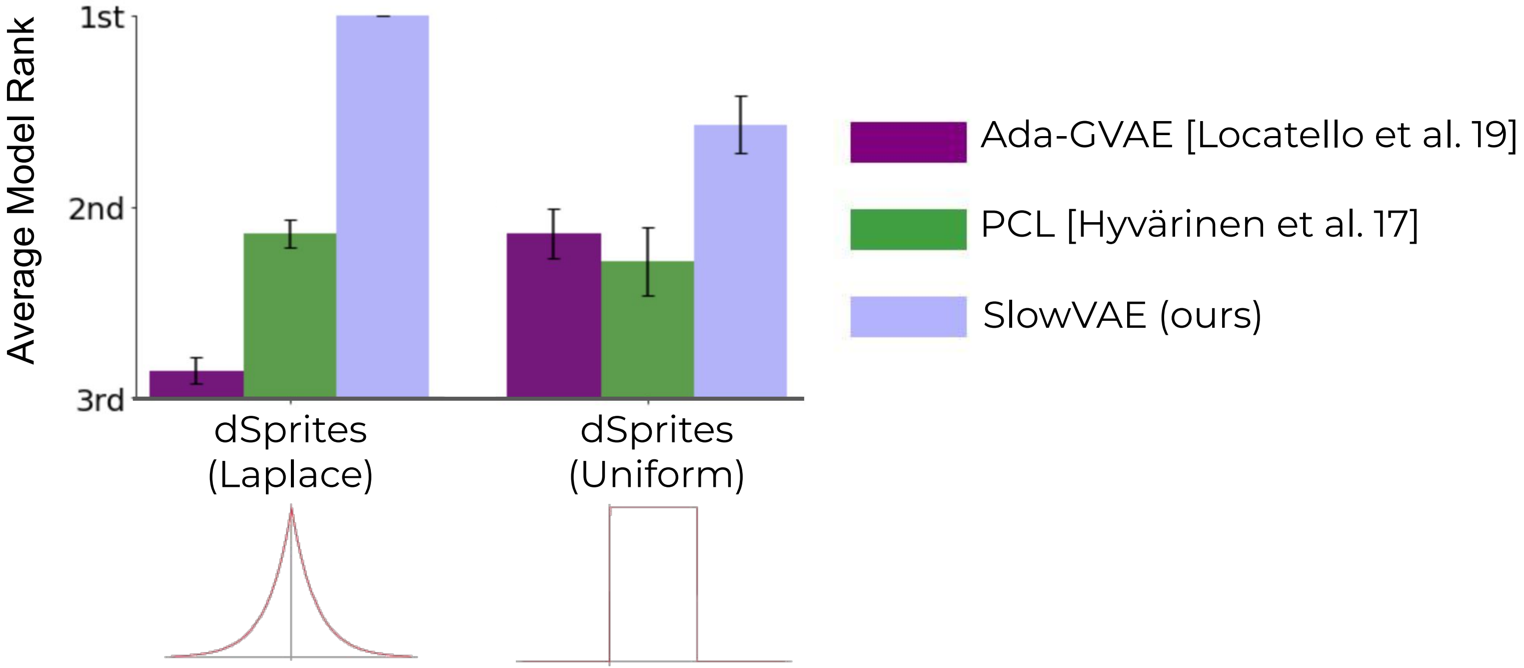


Quantitative Results - dSprites

Data

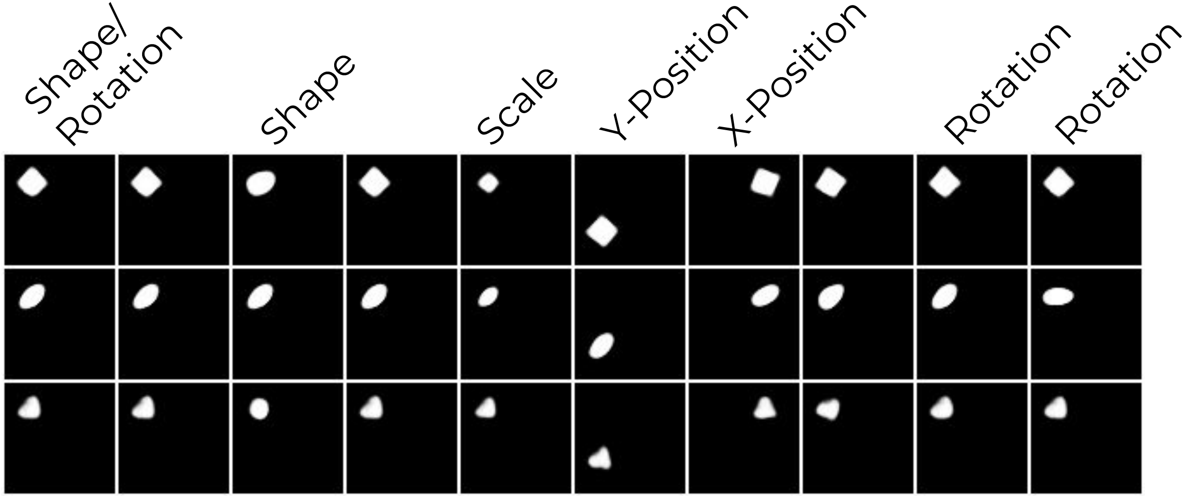


Disentanglement Performance

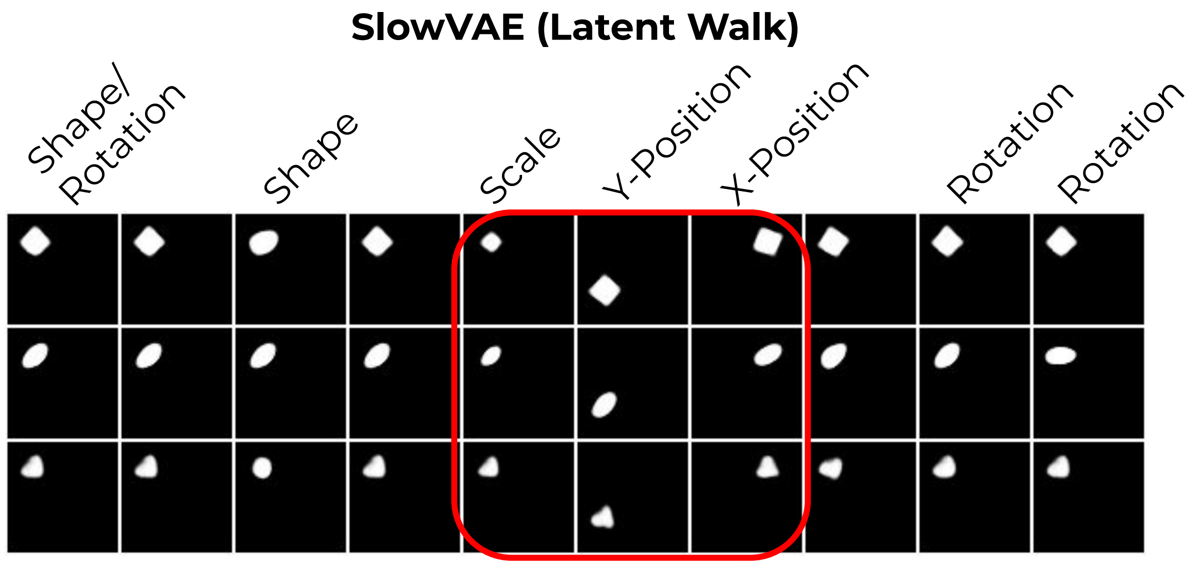


Qualitative Results - dSprites

SlowVAE (Latent Walk)

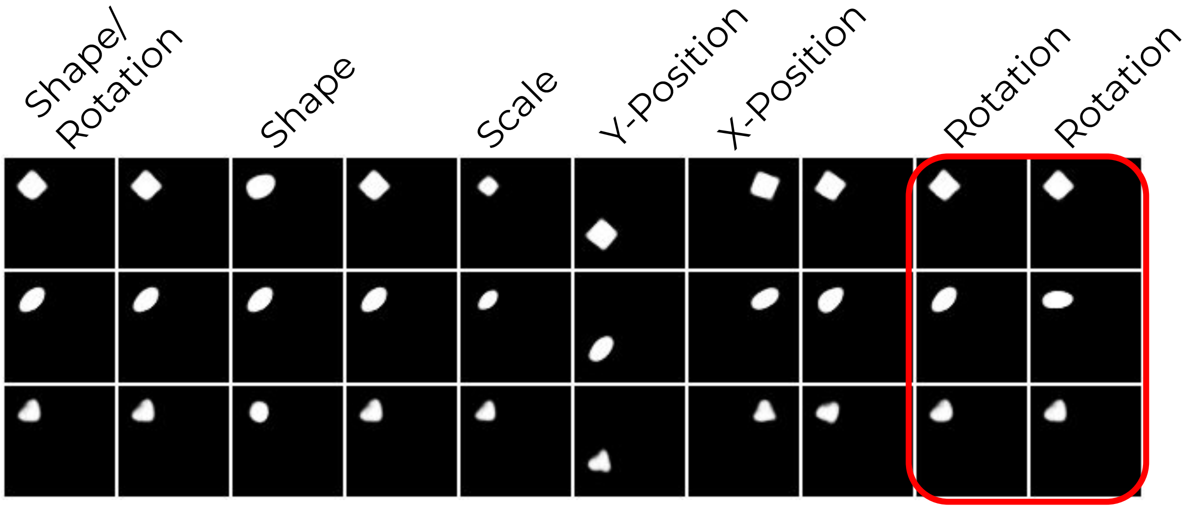


Qualitative Results - dSprites

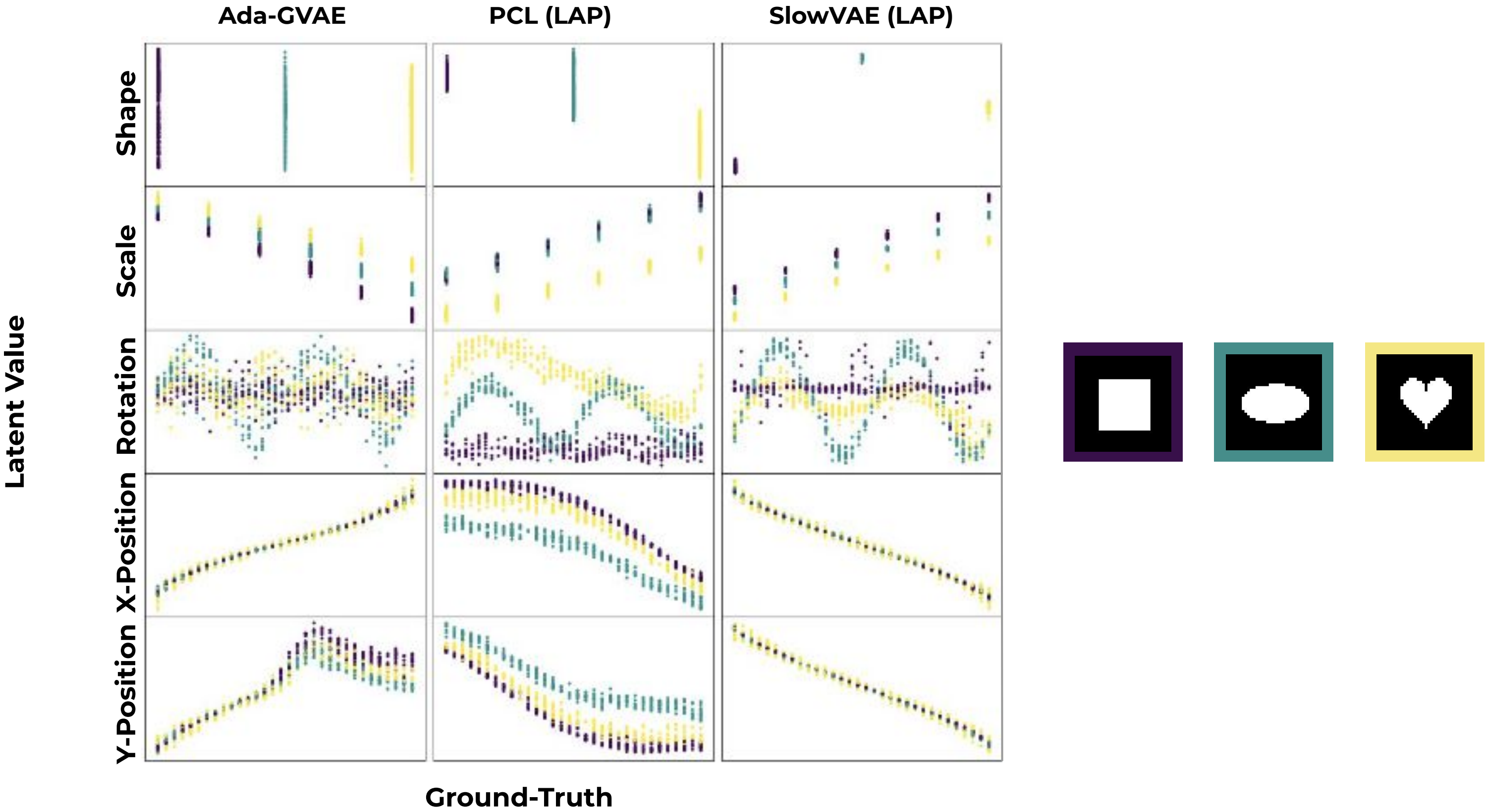


Qualitative Results - dSprites

SlowVAE (Latent Walk)



Failure Cases - dSprites



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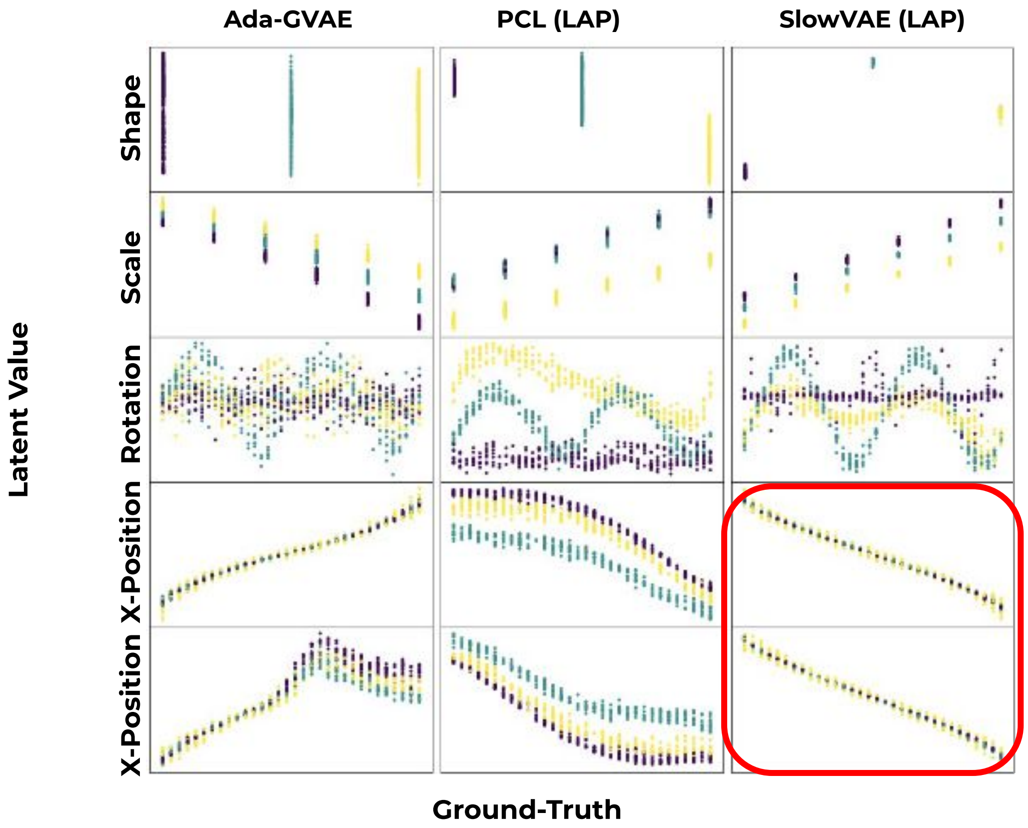
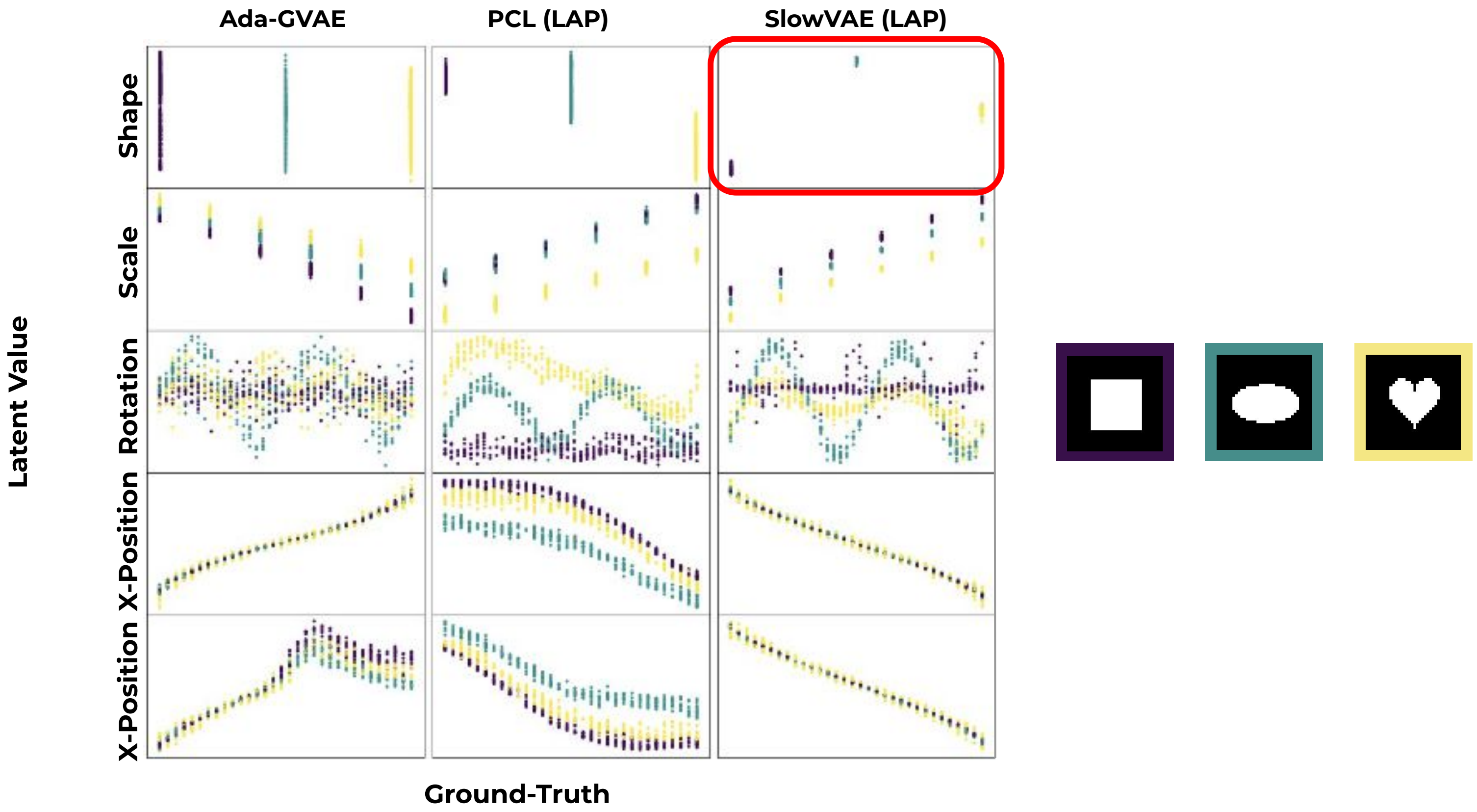
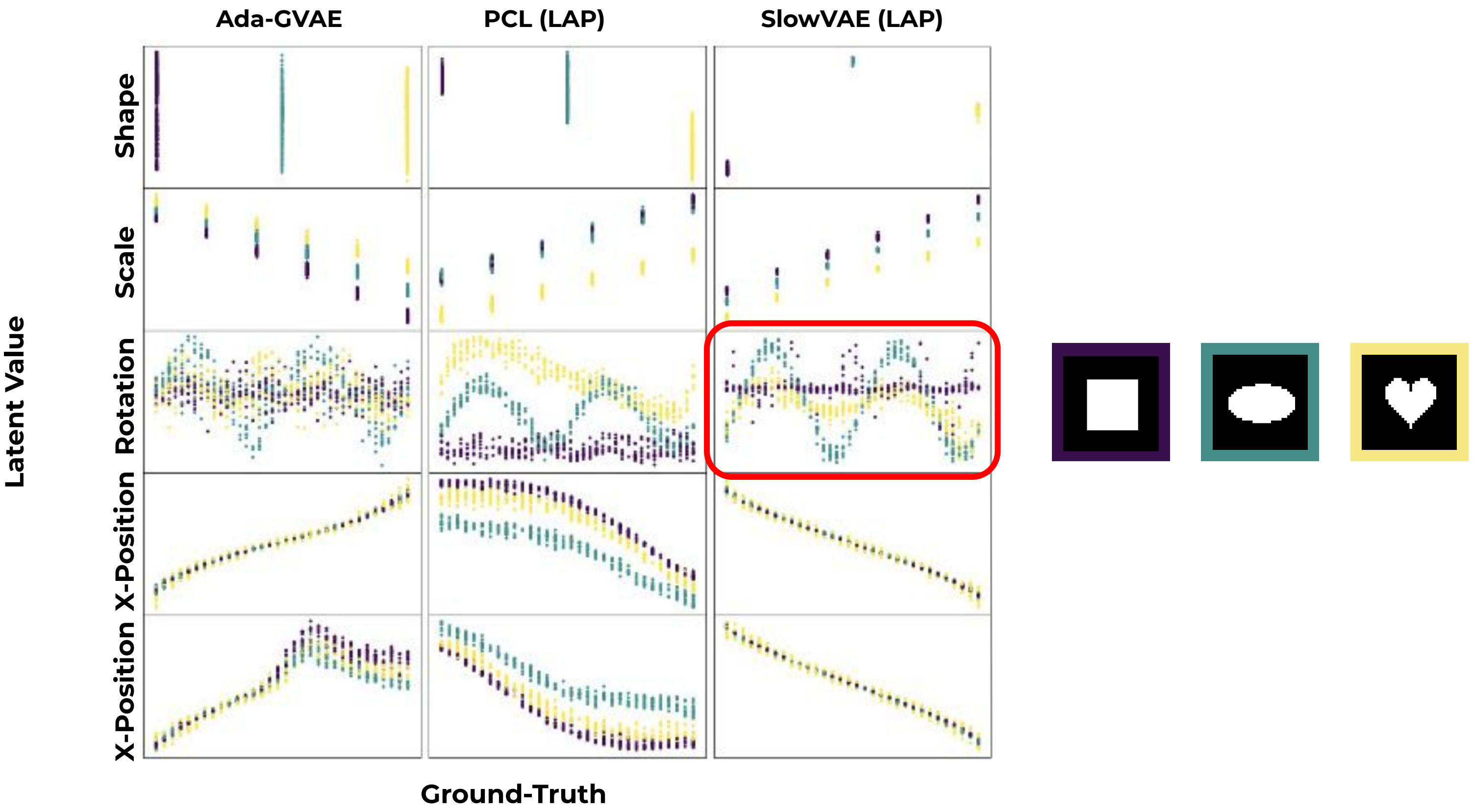


Failure Cases - dSprites

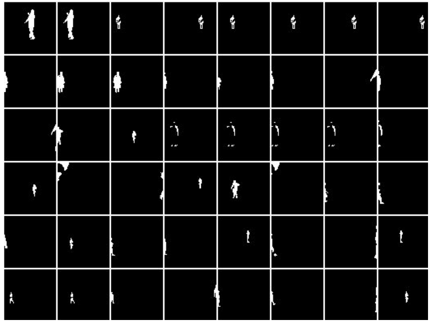


Failure Cases - dSprites



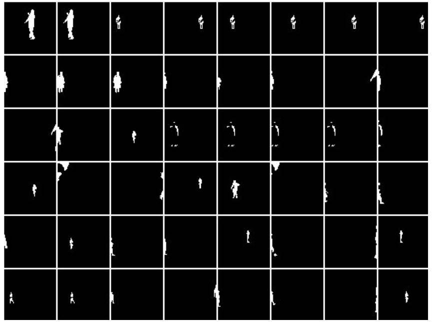
Results on Natural Data

KITTI-Masks

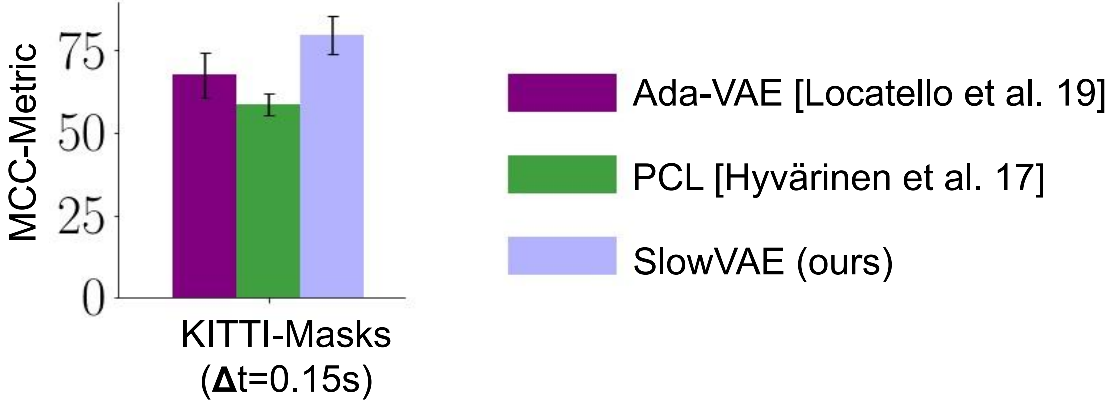


Results on Natural Data

KITTI-Masks

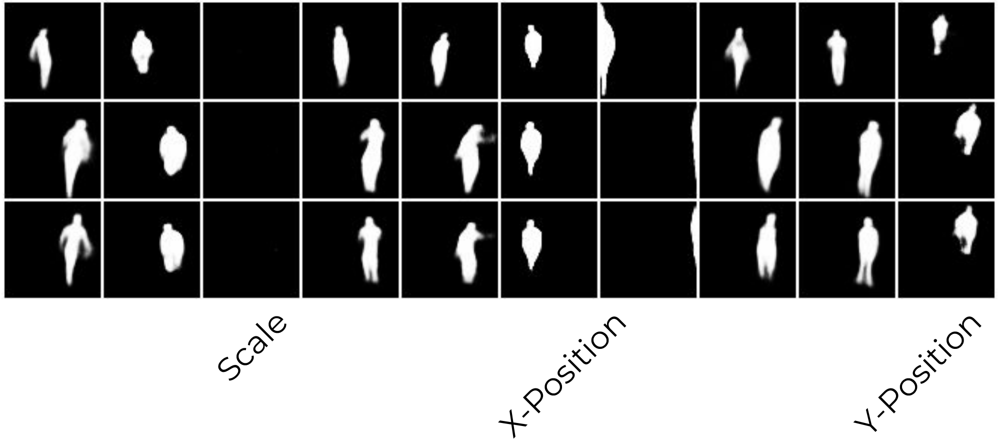


Disentanglement Performance



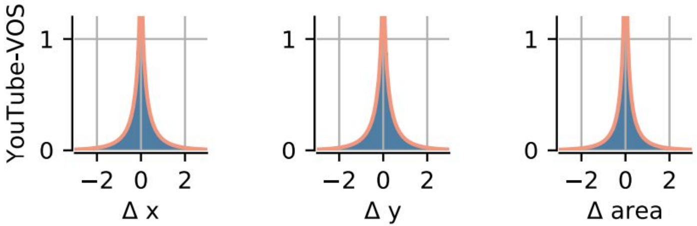
Results - KITTI-Masks

SlowVAE Latent Walk 🕺



Paper Contributions

Objects in natural scenes have **sparse** marginal transition statistics



Paper Contributions

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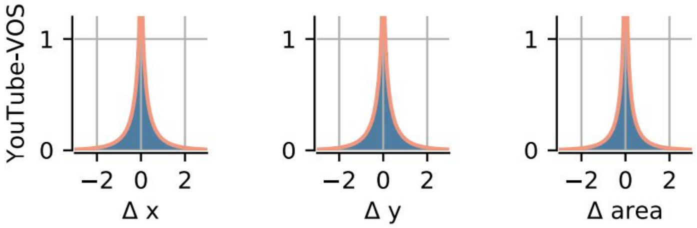
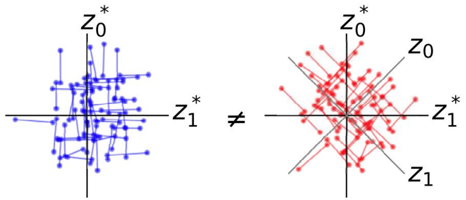


Intuitive proof for **identifiability** in Nonlinear ICA & Disentanglement



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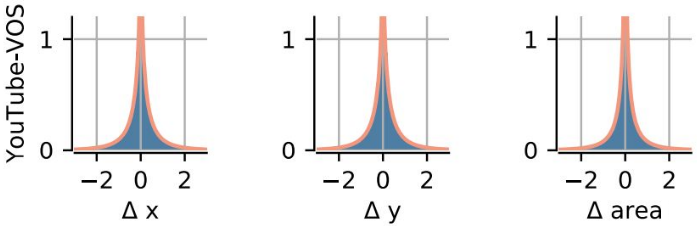

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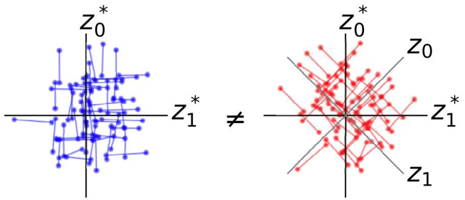
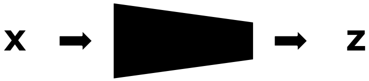


Empirical results using a **Flow** and a **VAE** based implementation of the theoretical model



Paper Contributions

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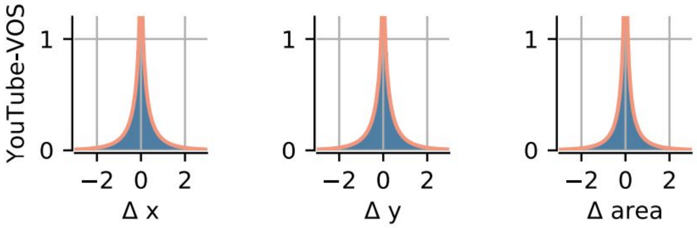
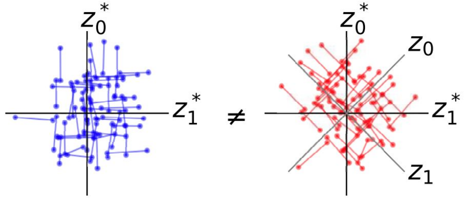
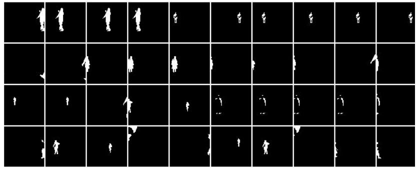
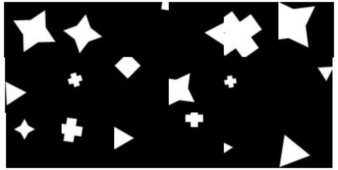


Intuitive proof for **identifiability** in Nonlinear ICA & Disentanglement



Two challenging new **datasets** to push disentanglement towards **natural** video





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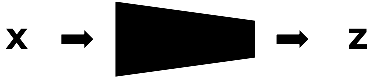

Contrastive Learning Inverts the Data Generating Process

Overview

1. Theoretical Connection between InfoNCE & Nonlinear ICA

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2. Empirical Test on robustness to mismatch (in assumptions)

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3. Identifiability on 3DIdent

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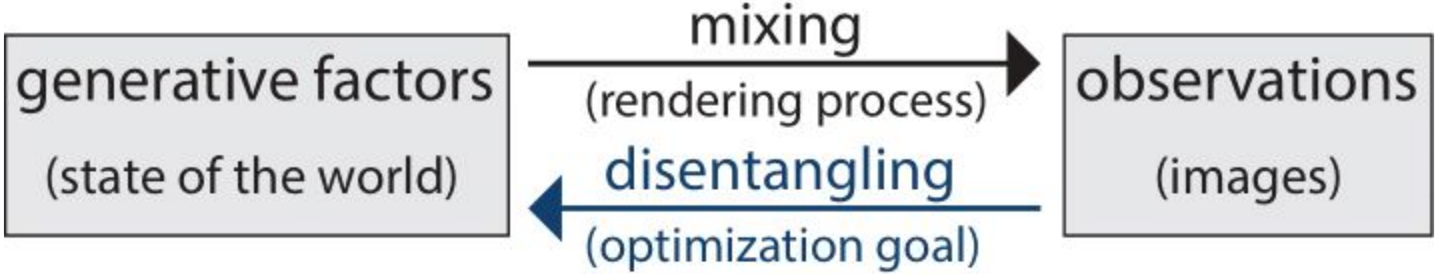
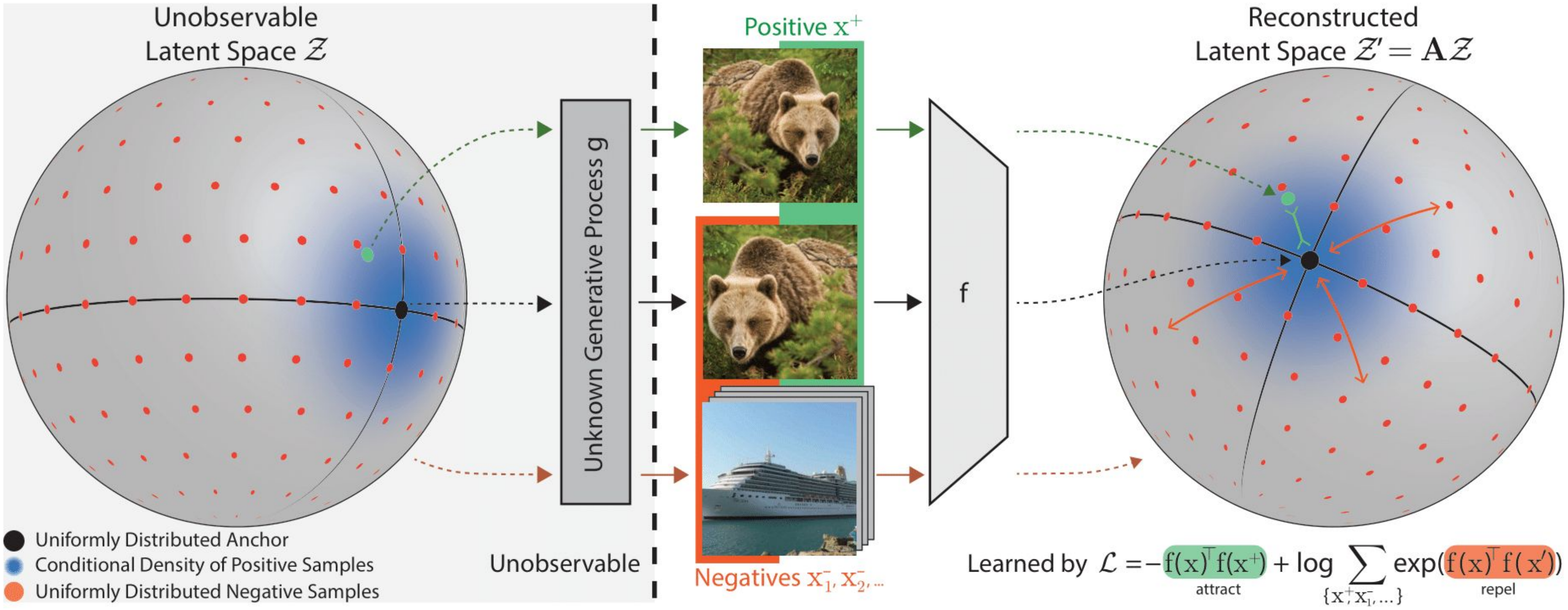
3. Identifiability on 3DIdent
    a. complex, high-resolution images

Nonlinear ICA



Theoretical Framework



Theoretical Framework

Theorem 2. *Let $\mathcal{Z} = \mathbb{S}^{N-1}$, the ground-truth marginal be uniform, and the conditional a vMF distribution (cf. Eq. 2). Let the mixing function g be differentiable and injective. If the assumed form of q_{h}, as defined above, matches that of p, and if f is differentiable and minimizes the CL loss (1), then for fixed $\tau > 0$ and $M \to \infty$, $h = f \circ g$ is linear, i.e., f recovers the latent sources up to orthogonal linear transformations.*

$$\mathcal{L}_{\mathsf{contr}}(f;\tau,M) := \underset{\substack{(\mathbf{x},\tilde{\mathbf{x}})\sim p_{\mathsf{pos}} \\ \{\mathbf{x}_i^-\}_{i=1}^M \overset{\text{i.i.d.}}{\sim} p_{\mathsf{data}}}}{\mathbb{E}} \left[-\log \frac{e^{f(\mathbf{x})^\top f(\tilde{\mathbf{x}})/\tau}}{e^{f(\mathbf{x})^\top f(\tilde{\mathbf{x}})/\tau} + \sum_{i=1}^{M} e^{f(\mathbf{x}_i^-)^\top f(\tilde{\mathbf{x}})/\tau}} \right]. \tag{1}$$

$$p(\mathbf{z}) = |\mathcal{Z}|^{-1}, \qquad p(\mathbf{z}|\tilde{\mathbf{z}}) = C_p^{-1} e^{\kappa \mathbf{z}^\top \tilde{\mathbf{z}}}$$
$$\text{with} \quad C_p := \int e^{\kappa \mathbf{z}^\top \tilde{\mathbf{z}}} \, d\tilde{\mathbf{z}} = \text{const.}, \quad \mathbf{x} = g(\mathbf{z}). \tag{2}$$

$$q_{\mathrm{h}}(\tilde{\mathbf{z}}|\mathbf{z}) = C_h(\tilde{\mathbf{z}})^{-1} e^{h(\tilde{\mathbf{z}})^\top h(\mathbf{z})/\tau}$$
$$\textit{with} \quad C_h(\mathbf{z}) := \int e^{h(\tilde{\mathbf{z}})^\top h(\mathbf{z})/\tau} \, d\mathbf{z},$$

Theoretical Framework

Theorem 1 ($\mathcal{L}_{\mathsf{contr}}$ converges to the cross-entropy between latent distributions)**.** *If the ground-truth marginal distribution p is uniform, then for fixed $\tau > 0$, as the number of negative samples $M \to \infty$, the (normalized) contrastive loss converges to*

$$\lim_{M \to \infty} \mathcal{L}_{\mathsf{contr}}(f; \tau, M) - \log M + \log |\mathcal{Z}| = \mathop{\mathbb{E}}_{\mathbf{z} \sim p(\mathbf{z})} \left[H(p(\cdot|\mathbf{z}), q_h(\cdot|\mathbf{z})) \right] \tag{14}$$

where H is the cross-entropy between the ground-truth conditional distribution p over positive pairs and a conditional distribution q_{h} parameterized by the model f, and $C_h(\mathbf{z}) \in \mathbb{R}^+$ is the partition function of q_{h} (see Appendix A.1.1):

$$q_{\mathrm{h}}(\tilde{\mathbf{z}}|\mathbf{z}) = C_h(\mathbf{z})^{-1} e^{h(\tilde{\mathbf{z}})^\top h(\mathbf{z})/\tau}$$
$$\textit{with} \quad C_h(\mathbf{z}) := \int e^{h(\tilde{\mathbf{z}})^\top h(\mathbf{z})/\tau} \, \mathrm{d}\tilde{\mathbf{z}}. \tag{15}$$

Theoretical Framework

Proposition 1 (Minimizers of the cross-entropy maintain the dot product)**.** *Let* $\mathcal{Z} = \mathbb{S}^{N-1}$, $\tau > 0$ *and consider the ground-truth conditional distribution of the form* $p(\tilde{\mathbf{z}}|\mathbf{z}) = C_p^{-1}\exp(\kappa\tilde{\mathbf{z}}^\top\mathbf{z})$. *Let* h *map onto a hypersphere with radius* $\sqrt{\tau\kappa}$.[3] *Consider the conditional distribution* q_h *parameterized by the model, as defined above in Theorem 1, where the hypothesis class for* h *is assumed to be sufficiently flexible such that* $p(\tilde{\mathbf{z}}|\mathbf{z})$ *and* $q_{\mathrm{h}}(\tilde{\mathbf{z}}|\mathbf{z})$ *can match. If* h *is a minimizer of the cross-entropy* $\mathbb{E}_{p(\tilde{\mathbf{z}}|\mathbf{z})}[-\log q_{\mathrm{h}}(\tilde{\mathbf{z}}|\mathbf{z})]$, *then* $p(\tilde{\mathbf{z}}|\mathbf{z}) = q_{\mathrm{h}}(\tilde{\mathbf{z}}|\mathbf{z})$ *and* $\forall \mathbf{z}, \tilde{\mathbf{z}} : \kappa\mathbf{z}^\top\tilde{\mathbf{z}} = h(\mathbf{z})^\top h(\tilde{\mathbf{z}})$.

Proposition 2 (Extension of the Mazur-Ulam theorem to hyperspheres and the dot product)**.** *Let* $\mathcal{Z} = \mathbb{S}^{N-1}$. *If* $h : \mathcal{Z} \to \mathcal{Z}$ *maintains the dot product up to a constant factor, i.e.,* $\forall \mathbf{z}, \tilde{\mathbf{z}} \in \mathcal{Z} : \kappa\mathbf{z}^\top\tilde{\mathbf{z}} = h(\mathbf{z})^\top h(\tilde{\mathbf{z}})$, *then* h *is an orthogonal linear transformation.*

Theoretical Framework

Theorem 5. *Let $\mathcal{Z}$ be a convex body in $\mathbb{R}^N$, $h = f \circ g : \mathcal{Z} \to \mathcal{Z}$, and δ be a metric. Further, let the ground-truth marginal distribution be uniform and the conditional distribution be as (5). Let the mixing function g be differentiable and injective. If the assumed form of q_{h} matches that of p, i.e.,*

$$q_{\mathrm{h}}(\tilde{\mathbf{z}}|\mathbf{z}) = C_q^{-1}(\mathbf{z}) e^{-\delta(h(\tilde{\mathbf{z}}), h(\mathbf{z}))/\tau}$$
$$\textit{with} \quad C_q(\mathbf{z}) := \int e^{-\delta(h(\tilde{\mathbf{z}}), h(\mathbf{z}))/\tau} \, \mathrm{d}\tilde{\mathbf{z}}, \tag{7}$$

and if f is differentiable and minimizes the $\mathcal{L}_{\delta\text{-contr}}$ objective in (6) for $M \to \infty$, we find that $h = f \circ g$ is invertible and affine, i.e., we recover the latent sources up to affine transformations.

$$p(\mathbf{z}) = |\mathcal{Z}|^{-1}, \qquad p(\mathbf{z}|\tilde{\mathbf{z}}) = C_p^{-1} e^{-\delta(\mathbf{z}, \tilde{\mathbf{z}})}$$
$$\text{with} \quad C_p(\mathbf{z}) := \int e^{-\delta(\mathbf{z}, \tilde{\mathbf{z}})} \, \mathrm{d}\tilde{\mathbf{z}}, \quad \mathbf{x} = g(\mathbf{z}), \tag{5}$$

$$\mathcal{L}_{\delta\text{-contr}}(f; \tau, M) \quad := \tag{6}$$
$$\underset{\substack{(\mathbf{x}, \tilde{\mathbf{x}}) \sim p_{\mathrm{pos}} \\ \{\mathbf{x}_i^-\}_{i=1}^M \overset{\text{i.i.d.}}{\sim} p_{\mathrm{data}}}}{\mathbb{E}} \left[-\log \frac{e^{-\delta(f(\mathbf{x}), f(\tilde{\mathbf{x}}))/\tau}}{e^{-\delta(f(\mathbf{x}), f(\tilde{\mathbf{x}}))/\tau} + \sum_{i=1}^{M} e^{-\delta(f(\mathbf{x}_i^-), f(\tilde{\mathbf{x}}))/\tau}} \right].$$

Theoretical Framework

Theorem 3. *Let δ be a semi-metric and $\tau, \lambda > 0$ and let the ground-truth marginal distribution p be uniform. Consider a ground-truth conditional distribution $p(\tilde{\mathbf{z}}|\mathbf{z}) = C_p^{-1}(\mathbf{z}) \exp(-\lambda\delta(\tilde{\mathbf{z}}, \mathbf{z}))$ and the model conditional distribution*

$$q_{\mathrm{h}}(\tilde{\mathbf{z}}|\mathbf{z}) = C_h^{-1}(\mathbf{z}) e^{-\delta(h(\tilde{\mathbf{z}}), h(\mathbf{z}))/\tau}$$
$$\text{with} \quad C_h(\mathbf{z}) := \int_{\mathcal{Z}} e^{-\delta(h(\tilde{\mathbf{z}}), h(\mathbf{z}))/\tau} \mathrm{d}\tilde{\mathbf{z}}. \tag{61}$$

Then the cross-entropy between p and q_{h} is given by

$$\lim_{M\to\infty} \mathcal{L}_{\delta\text{-contr}}(f; \tau, M) - \log M + \log |\mathcal{Z}| = \underset{\mathbf{z}\sim p(\mathbf{z})}{\mathbb{E}} \left[H(p(\cdot|\mathbf{z}), q_{\mathrm{h}}(\cdot|\mathbf{z})\right], \tag{62}$$

which can be implemented by sampling data from the accessible distributions.

Theorem 4. *Let $\mathcal{Z} = \mathcal{Z}'$ be a convex body in $\mathbb{R}^N$. Let the mixing function g be differentiable and invertible. If the assumed form of q_{h} as defined in (4) matches that of p, and if f is differentiable and minimizes the cross-entropy between p and q_{h}, then we find that $h = f \circ g$ is affine, i.e., we recover the latent sources up to affine transformations.*

Theoretical Framework

Theorem 6. *Let $\mathcal{Z}$ be a convex body in $\mathbb{R}^N$, $h : \mathcal{Z} \to \mathcal{Z}$, and δ be an L^α metric for $\alpha \geq 1, \alpha \neq 2$. Further, let the ground-truth marginal distribution be uniform and the conditional distribution be as (5), and let the mixing function g be differentiable and invertible. If the assumed form of $q_{\mathrm{h}}(\cdot|\mathbf{z})$ matches that of $p(\cdot|\mathbf{z})$, i.e., both use the same metric δ up to a constant scaling factor, and if f is differentiable and minimizes the $\mathcal{L}_{\delta\text{-contr}}$ objective in (6) for $M \to \infty$, we find that $h = f \circ g$ is a composition of input independent permutations, sign flips and rescaling.*

Theorem D. *Suppose $1 \leq \alpha \leq \infty$ and $\alpha \neq 2$. An $n \times n$ matrix $\mathbf{A}$ is an isometry of L^α-norm if and only if $\mathbf{A}$ is a generalized permutation matrix, i.e., $\forall \mathbf{z} : (\mathbf{A}\mathbf{z})_{\mathbf{i}} = \alpha_{\mathbf{i}} \mathbf{z}_{\sigma(\mathbf{i})}$, with $\alpha_i = \pm 2$ and σ being a permutation.*

Proof. See Li & So (1994). Note that this can also be concluded from the Banach-Lamperti Theorem (Lamperti et al., 1958). □

$$p(\mathbf{z}) = |\mathcal{Z}|^{-1}, \qquad p(\mathbf{z}|\tilde{\mathbf{z}}) = C_p^{-1} e^{-\delta(\mathbf{z},\tilde{\mathbf{z}})}$$

$$\text{with} \quad C_p(\mathbf{z}) := \int e^{-\delta(\mathbf{z},\tilde{\mathbf{z}})} \, \mathrm{d}\tilde{\mathbf{z}}, \quad \mathbf{x} = g(\mathbf{z}), \tag{5}$$

$$\mathcal{L}_{\delta\text{-contr}}(f; \tau, M) := \tag{6}$$

$$\mathop{\mathbb{E}}_{\substack{(\mathbf{x},\tilde{\mathbf{x}}) \sim p_{\text{pos}} \\ \{\mathbf{x}_i^-\}_{i=1}^M \overset{\text{i.i.d.}}{\sim} p_{\text{data}}}} \left[-\log \frac{e^{-\delta(f(\mathbf{x}),f(\tilde{\mathbf{x}}))/\tau}}{e^{-\delta(f(\mathbf{x}),f(\tilde{\mathbf{x}}))/\tau} + \sum_{i=1}^{M} e^{-\delta(f(\mathbf{x}_i^-),f(\tilde{\mathbf{x}}))/\tau}} \right].$$

Different Assumptions, Different Losses

$\bigcirc$ + vMF $\longrightarrow \exp(f(x)^\top f(x'))$

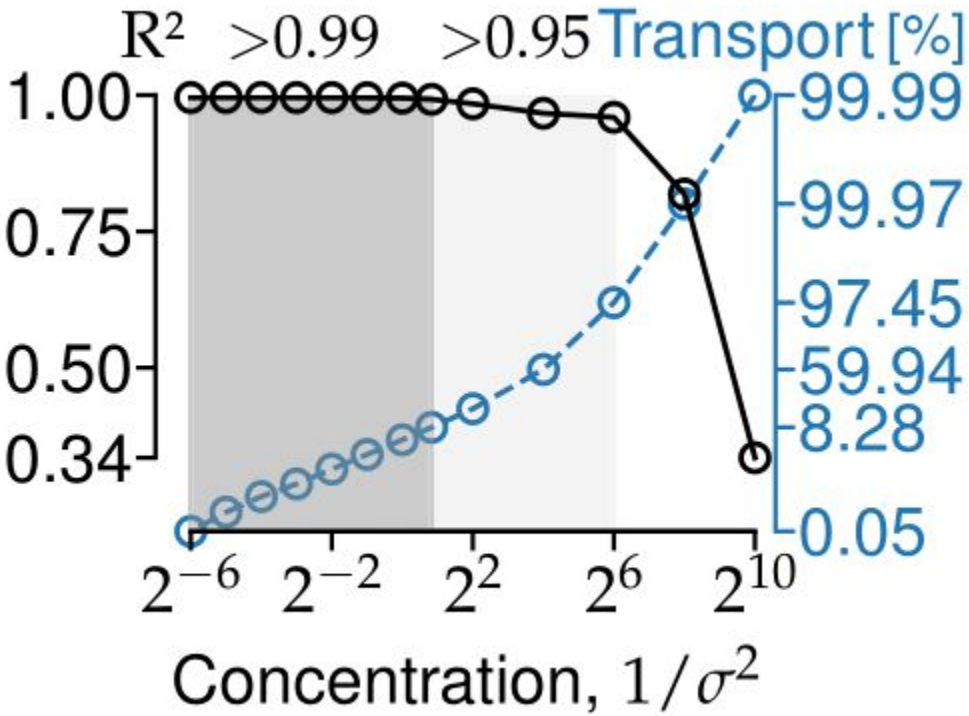
$\square$ + Normal $\longrightarrow \exp(-\|f(x) - f(x')\|_2)$

$\square$ + Laplace $\longrightarrow \exp(-\|f(x) - f(x')\|_1)$

Empirical Results

Generative process g			Model f			R^2 Score [%]		
Space	$p(\cdot)$	$p(\cdot\|\cdot)$	Space	$q_{\mathrm{h}}(\cdot\|\cdot)$	M.	Identity	Supervised	Unsupervised
Sphere	Uniform	vMF(κ=1)	Sphere	vMF(κ=1)	✓	66.98 ± 2.79	99.71 ± 0.05	99.42 ± 0.05
Sphere	Uniform	vMF(κ=10)	Sphere	vMF(κ=1)	✗	—〃—	—〃—	99.86 ± 0.01
Sphere	Uniform	Laplace(λ=0.05)	Sphere	vMF(κ=1)	✗	—〃—	—〃—	99.91 ± 0.01
Sphere	Uniform	Normal(σ=0.05)	Sphere	vMF(κ=1)	✗	—〃—	—〃—	99.86 ± 0.00
Box	Uniform	Normal(σ=0.05)	Unbounded	Normal	✗	67.93 ± 7.40	99.78 ± 0.06	99.60 ± 0.02
Box	Uniform	Laplace(λ=0.05)	Unbounded	Normal	✗	—〃—	—〃—	99.64 ± 0.02
Box	Uniform	Laplace(λ=0.05)	Unbounded	GenNorm(β=3)	✗	—〃—	—〃—	99.70 ± 0.02
Box	Uniform	Normal(σ=0.05)	Unbounded	GenNorm(β=3)	✗	—〃—	—〃—	99.69 ± 0.02
Sphere	Normal(σ=1)	Laplace(λ=0.05)	Sphere	vMF(κ=1)	✗	63.37 ± 2.41	99.70 ± 0.07	99.02 ± 0.01
Sphere	Normal(σ=1)	Normal(σ=0.05)	Sphere	vMF(κ=1)	✗	—〃—	—〃—	99.02 ± 0.02
Unbounded	Laplace(λ=1)	Normal(σ=1)	Unbounded	Normal	✗	62.49 ± 1.65	99.65 ± 0.04	98.13 ± 0.14
Unbounded	Normal(σ=1)	Normal(σ=1)	Unbounded	Normal	✗	63.57 ± 2.30	99.61 ± 0.17	98.76 ± 0.03

Empirical Results



Empirical Results

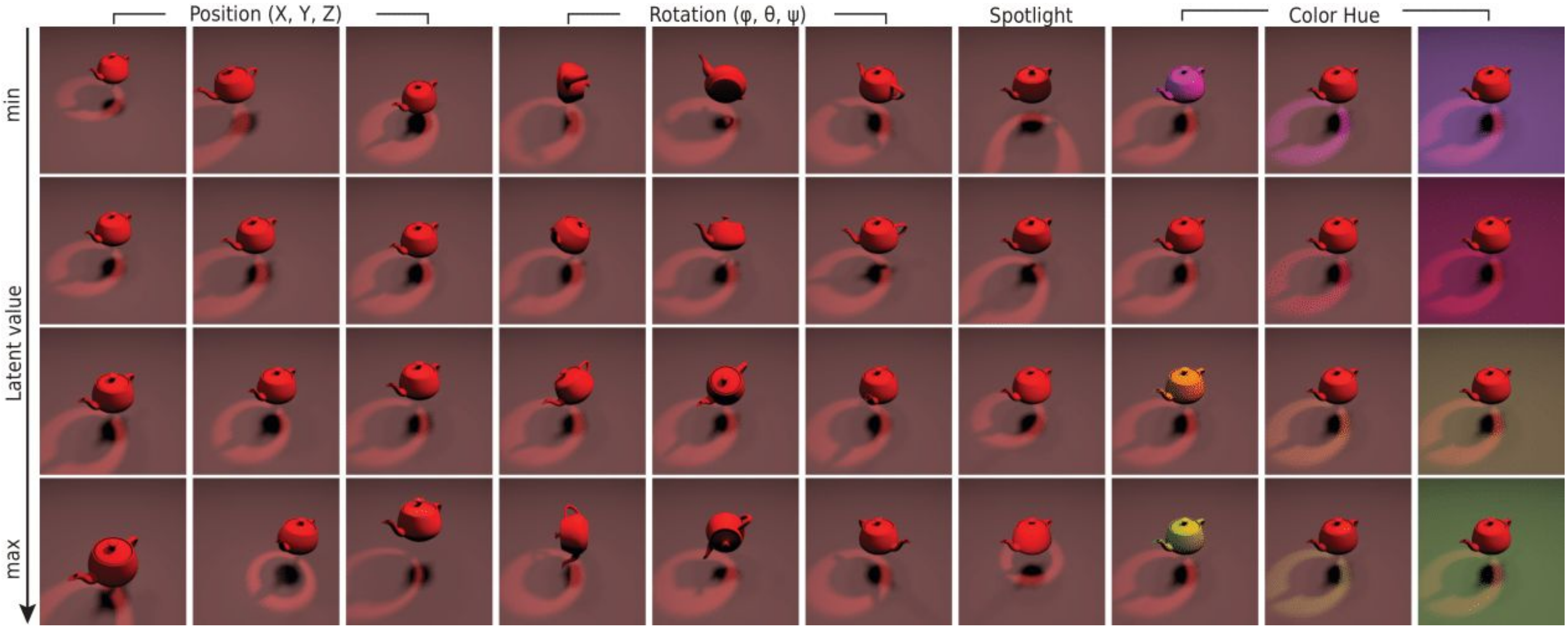
Generative process g			Model f			MCC Score [%]		
Space	$p(\cdot)$	$p(\cdot\|\cdot)$	Space	$q_{\mathrm{h}}(\cdot\|\cdot)$	M.	Identity	Supervised	Unsupervised
Box	Uniform	Laplace(λ=0.05)	Box	Laplace	✓	46.55 ± 1.34	99.93 ± 0.03	98.62 ± 0.05
Box	Uniform	GenNorm(β=3; λ=0.05)	Box	GenNorm(β=3)	✓	—"—	—"—	99.90 ± 0.06
Box	Uniform	Normal(σ=0.05)	Box	Normal	✗	—"—	—"—	99.77 ± 0.01
Box	Uniform	Laplace(λ=0.05)	Box	Normal	✗	—"—	—"—	99.76 ± 0.02
Box	Uniform	GenNorm(β=3; λ=0.05)	Box	Laplace	✗	—"—	—"—	98.80 ± 0.02
Box	Uniform	Laplace(λ=0.05)	Unbounded	Laplace	✗	—"—	99.97 ± 0.03	98.57 ± 0.02
Box	Uniform	GenNorm(β=3; λ=0.05)	Unbounded	GenNorm(β=3)	✗	—"—	—"—	99.85 ± 0.01
Box	Uniform	Normal(σ=0.05)	Unbounded	Normal	✗	—"—	—"—	58.26 ± 3.00
Box	Uniform	Laplace(λ=0.05)	Unbounded	Normal	✗	—"—	—"—	59.67 ± 2.33
Box	Uniform	Normal(σ=0.05)	Unbounded	GenNorm(β=3)	✗	—"—	—"—	43.80 ± 2.15

KITTI Masks

Table 3. **KITTI Masks**. Mean $\pm$ standard deviation over 10 random seeds. $\overline{\Delta t}$ indicates the average temporal distance of frames used.

	Model	Model Space	MCC [%]
$\overline{\Delta t} = 0.05s$	SlowVAE	Unbounded	66.1 ± 4.5
	Laplace	Unbounded	77.1 ± 1.0
	Laplace	Box	74.1 ± 4.4
	Normal	Unbounded	58.3 ± 5.4
	Normal	Box	59.9 ± 5.5
$\overline{\Delta t} = 0.15s$	SlowVAE	Unbounded	79.6 ± 5.8
	Laplace	Unbounded	79.4 ± 1.9
	Laplace	Box	80.9 ± 3.8
	Normal	Unbounded	60.2 ± 8.7
	Normal	Box	68.4 ± 6.7

3DIdent


Position (X, Y, Z)
Rotation (φ, θ, ψ)
Spotlight
Color Hue
min
Latent value
max

3DIdent

Dataset	Model f			Identity [%]	Unsupervised [%]	
$p(\cdot\|\cdot)$	Space	$q_{\mathrm{h}}(\cdot\|\cdot)$	M.	R^2	R^2	MCC
Normal	Box	Normal	✓	5.25 ± 1.20	96.73 ± 0.10	98.31 ± 0.04
Normal	Unbounded	Normal	✗	—″—	96.43 ± 0.03	54.94 ± 0.02
Laplace	Box	Normal	✗	—″—	96.87 ± 0.08	98.38 ± 0.03
Normal	Sphere	vMF	✗	—″—	65.74 ± 0.01	42.44 ± 3.27

Ongoing Work

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1. Extend framework to object-centric methods
    a. MONet, IODINE, Slot Attention etc.

Ongoing Work

1. Extend framework to object-centric methods
    a. MONet, IODINE, Slot Attention etc.
2. Extend framework to data augmentations
    a. Content & Style Disambiguation
    b. Invariant factors == delta conditional

Self-supervised learning with data augmentations provably isolates content from style

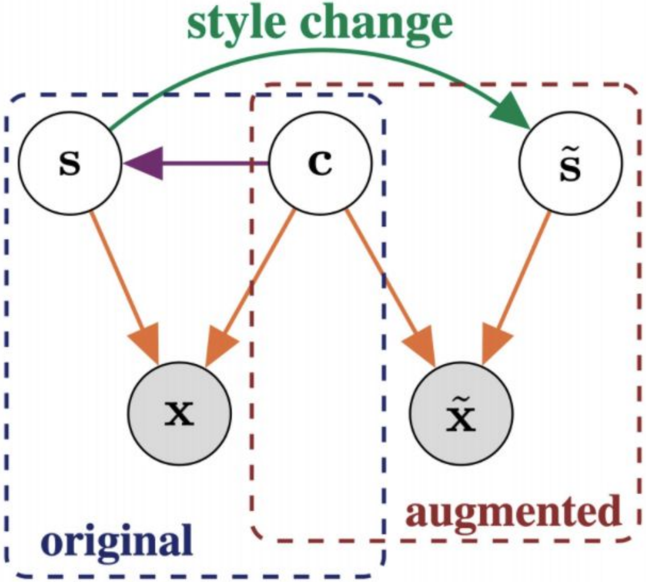
with **Julius von Kügelgen*, Yash Sharma*, Luigi Gresele***, Wieland Brendel, Michel Besserve, Francesco Locatello

Formalise generation $x = f(z)$ and augmentation $\tilde{x} = f(\tilde{z})$ processes as latent variable model with a content-style partition $z = (c, s)$:

- *invariant content* c*:* always shared between pairs $(x, \tilde{x})$ of views;
- *varying style* s*:* may change across pairs $(x, \tilde{x})$ of views.

Allow causal dependence of style on content (*Causal3DIdent* dataset):

augmented view $\tilde{x}$ = counterfactual under soft style intervention on x.



Theory: Can identify* invariant content partition in generative and discriminative learning with entropy maximisation (e.g., SimCLR).

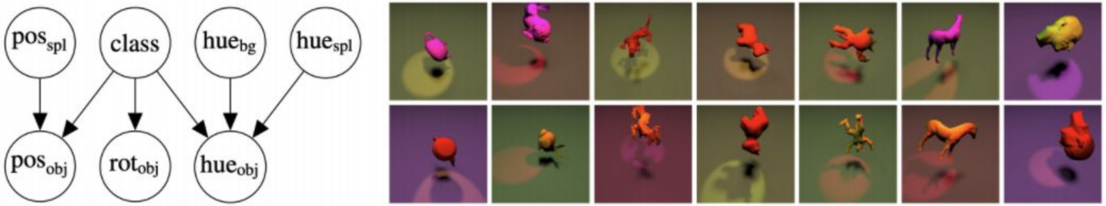


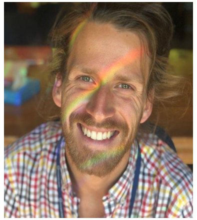
Figure 2: *(Left)* Causal graph for the *Causal3DIdent* dataset. *(Right)* Two samples from each object class.

*up to invertible transformation

Ongoing Work

1. Extend framework to object-centric methods
    a. MONet, IODINE, Slot Attention etc.
2. Extend framework to data augmentations
    a. Content & Style Disambiguation
    b. Invariant factors == delta conditional
3. Extend framework for causal discovery
    a. Robustness in downstream tasks?

Thank you for your attention!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