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III Semester B.TECH. (CSE/ISE) Examination,
December - 2019/January - 2020
(CBCS Scheme)

18CIPC305 : DISCRETE MATHEMATICAL STRUCTURES

Time : 3 Hours | Max. Marks : 100

Instructions : *(i) Q.No.**1** is **compulsory**.*

*(ii) Q.No.**2** and Q.No.**3** are **compulsory**.*

*(iii) Answer to Q.No.**4** or Q.No.**5**, Q.No.**6** or Q.No.**7** and Q.No.**8** or Q.No.**9**.*

1. Answer **all** the following **15** questions. Each question carries **one** mark. **15x1=15**

(a) Write English sentence corresponding to the statement $p \vee \sim p$.

(b) A coin is tossed four times and the result of each toss is recorded. How many different sequences of heads and tails are possible ?

(c) What is the value of ${}_7C_3$?

(d) Let R be the following symmetric relation on the set A = {1, 2, 3, 4, 5}:
R = {(1, 2), (2, 1), (3, 4), (4, 3), (3, 5), (5, 3), (4, 5), (5, 4), (5, 5)}
Draw the graph of R.

(e) Let A = B = {1, 2, 3} and R = {(1, 1), (1, 2), (2, 3), (3, 1)}. Compute R^{-1}.

(f) How does a function differ from a general relation ?

(g) The degree of the recurrence relation $P_n = (1.11)P_{n-1}$ is ________.

(h) Find the roots of the characteristic equation $s^2 - 4s + 4 = 0$.

(i) What is the difference between a linear homogeneous recurrence relation and linear nonhomogeneous recurrence relation ?

(j) What is an Euler circuit ?

(k) Define the chromatic polynomial of a graph g.

(l) Give an example of a connected graph with five vertices that is planar.

(m) When do you call a relation R on a set A is a partial order ?

(n) On the set A = {a, b, c}, find all partial orders ≤ in which a ≤ b.

(o) Let A = {a, b}. Which of the following tables define a semigroup on A ? Which define a monoid on A ?

(i)

*	a	b
a	b	a
b	a	b

(ii)

*	a	b
a	a	b
b	b	a

P.T.O.

2. (a) Find the solution for the recurrence relation 8

$$a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$$

with the initial conditions $a_0 = 1$, $a_1 = -2$ and $a_2 = -1$.

(b) Solve the recurrence relation 9

$$a_{r+2} - 2a_{r+1} + a_r = 0$$

by the method of generating functions with the initial conditions $a_0 = 2$ and $a_1 = 1$.

3. (a) Let A = {1, 2, 3, 4, 12}. Consider the partial order of divisibility on A. That is, if a and b $\in$ A, a $\leq$ b if and only if a | b. Draw the Hasse diagram of the poset (A, $\leq$). 9

(b) Let G be a group and let a and b be elements of G. Then prove that 8

(i) The equation $ax = b$ has a unique solution in G.

(ii) The equation $ya = b$ has a unique solution in G.

4. (a) Let P(n) be the statement $n^2 + n$ is an odd number for $n \in \mathbf{Z}^+$. 10

(i) Prove that $P(k) \Rightarrow P(k+1)$ is a tautology.

(ii) Is P(n) true for all n ? Explain.

(b) If thirteen people are assembled in a room, show that at least two of them must have their birthday in the same month. 7

OR

5. (a) Construct the truth tables to determine whether the given statement is a tautology, a contingency, or an absurdity. 9

(i) $p \Rightarrow (q \Rightarrow p)$

(ii) $q \Rightarrow (q \Rightarrow p)$

(b) In how many ways can five balls be chosen so that 8

(i) all five are red ?

(ii) all five are black ?

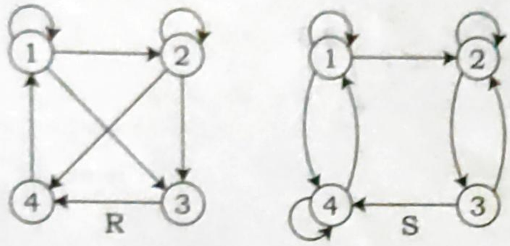
6. (a) Let R and S be two relations whose corresponding graphs are shown below. Compute 8

(i) $\overline{R}$

(ii) $R \cap S$

(iii) $R \cup S$

(iv) S^{-1}



(b) Let A = {1, 2, 3, 4} and let R = {(1, 2), (2, 3), (3, 4), (2, 1)}. Find the transitive closure of R using Warshall's algorithm. 9

OR

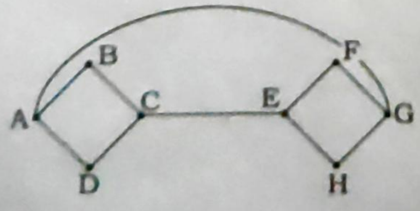
7. (a) Let $A = B = C = \mathbf{R}$, and let $f : A \to B$, $g : B \to C$ be defined by $f(a) = a - 1$ and $g(b) = b^2$. Find 8

(i) $(f \circ g)(2)$

(ii) $(g \circ f)(2)$

(b) Let A = {1, 2, 3, 4, 5, 6}. Compute (4, 1, 3, 5) ∘ (5, 6, 3) and (5, 6, 3) ∘ (4, 1, 3, 5). 9

8. (a) Use Fleury's algorithm to construct an Euler circuit for the graph shown below. 9



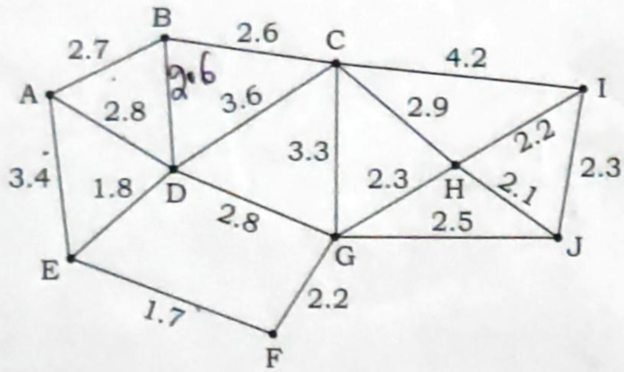
(b) Prove that : 8

(i) If a graph G has a vertex of odd degree, there can be no Euler circuit in G.

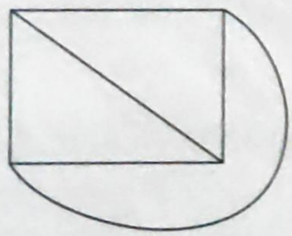
(ii) If G is a connected graph and every vertex has even degree, then there is an Euler circuit in G.

OR

9. (a) Let G be the graph shown below. Use Prims's algorithm to find a minimal spanning tree for the communication network graph. **9**



(b) Find the chromatic polynomial P_G for the graph given below and use P_G to find the chromatic number of G. **8**



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