EJ – 1230

VI Semester B.E. (CSE/ISE) Degree Examination, June/July 2015
(2K11 Scheme)
CI – 62 : PROBABILITY AND STOCHASTIC PROCESSES

Time : 3 Hours | Max. Marks : 100

Instruction : *Answer **any five full** questions, selecting atleast **two** from **each** Part.*

PART – A

1. a) A 6-faced fair dice is tossed. What is the probability (conditional) of getting 4, given that an even number has occurred ? **10**

   b) State and prove Baye's Theorem. **10**

2. a) Let S be a sample space when the pair of two dice is tossed. Let X and Y be two random variables on S where X = maximum of two numbers, i.e X(a, b) = max(a, b) and Y = sum of two numbers, i.e Y(a, b) = a + b. Find

   i) The distribution f of X and

   ii) The distribution g of Y. **10**

   b) The amount of time in hours that an electric bulb functions before breaking down is a continuous random variable with pdf given by

$$f(x) = \begin{cases} \lambda e^{-x/100} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

   What is the probability that

   i) The bulb will function between 200 to 300 hours before breaking down and

   ii) It will function less than 250 hours. **10**

3. a) The number of hardware failures of a computer system in a weak of operation has the following pmf.

No. of failures	0	1	2	3	4	5	6
Probability	0.18	0.28	0.25	0.18	0.06	0.04	0.01

P.T.O.

Find :

i) The expected number of failures in a week.

ii) The variance of the number of failures in a weak. **10**

b) If X and Y are independent poisson variables such that $P(X = 1) = P(X = 2)$ and $P(Y = 2) = P(Y = 3)$. Find the variance of $X - 2Y$. **10**

4. a) The failure rate of a device is given by

$$h(t) = \begin{cases} at & \text{if} \quad 0 < t < 1000 \text{ hours} \\ b & \text{if} \quad t \geq 1000 \text{ hours} \end{cases}$$

Choose 'b' so that h(t) is continuous and find an expression for device reliability. **10**

b) State Central limit theorem. The resistors R_1, R_2, R_3, R_4 are independent random variables and each is uniform in the interval [450, 500]. Use the Central limit theorem to find $p\,\{1900 \leq R_1 + R_2 + R_3 + R_4 \leq 2100\}$. **10**

PART – B

5. a) Customers enter a stare according to a poisson process of rate $\lambda = 6$ per hour. Suppose it is known that 2 customers entered during the first hour. What is the probability that these two persons entered during the first half and the other in the second half ? **10**

b) Suppose that the probability of a dry day following a rainy day is $1/3$ and the probability of a rainy day following a dry day is $1/2$. Given that May 1 is a dry day, find the probability that May 3 is a dry day and May 5 is a dry day. **10**

6. a) A group of telephone subscribers is observed continuously during a 80-minute busy-hour period. During this time they make 30 calls, with a total conversation time being 4200 seconds. Compute the call-arrival rate and the traffic intensity. **10**

b) Assume that the weather in a certain location can be modeled as a homogeneous Markov chain whose transition probability matrix is given below.

	Tomorrow's weather		
Today's weather	Fair	Cloudy	Rain
Fair	0.8	0.15	0.05
Cloudy	0.5	0.3	0.2
Rain	0.6	0.3	0.1

i) Draw the state transition diagram and

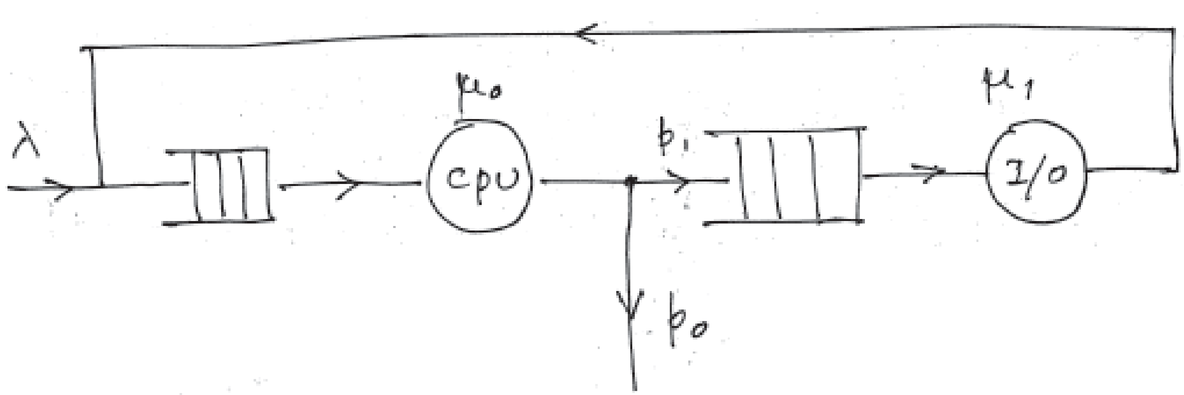
ii) If $P^T(0) = [0.4, 0.2, 0.4]$, find $P(1)$, $P(2)$ and $P(4)$. **10**

7. a) Explain the M|M|1 queuing system in detail. **10**

b) Discuss the differences between open queuing networks and closed queuing networks. **10**

8. a) What are pure birth and death processes ? Explain each one of them for constant and linear rate. **10**

b) Consider the simple model of a computer system shown the following figure.



Derive an expression for average response time at the two nodes. Assume that the two nodes are independent M|M|1 queues. **10**