Stochastic Processes

A family of random variables that is indexed by a parameter such as time is known as a stochastic process (SP) (or chance or random process).

Defn. A stochastic process is a family of r.vs $\{X(t) \mid t \in T\}$, defined on a given probability space, indexed by the parameter t, where t varies over an index set T.

The values assumed by the r.v $X(t)$ are called states, and the set of all possible values forms the state space of the process denoted by I.

Recall, that a r.v. is a fn. defined on the sample space S of the experiment.

Thus, the family of r.Vs is a family of functions $\{X(t,s) \mid s \in S, t \in T\}$. For a fixed $t = t_1$, $X_{t_1}(s) = X(t_1, s)$ is a r.V denoted by $X(t_1)$ as s varies over a sample space S. At some other fixed instant of time t_2, we have another r.v $X_{t_2}(s) = X(t_2, s)$, denoted by $X(t_2)$.

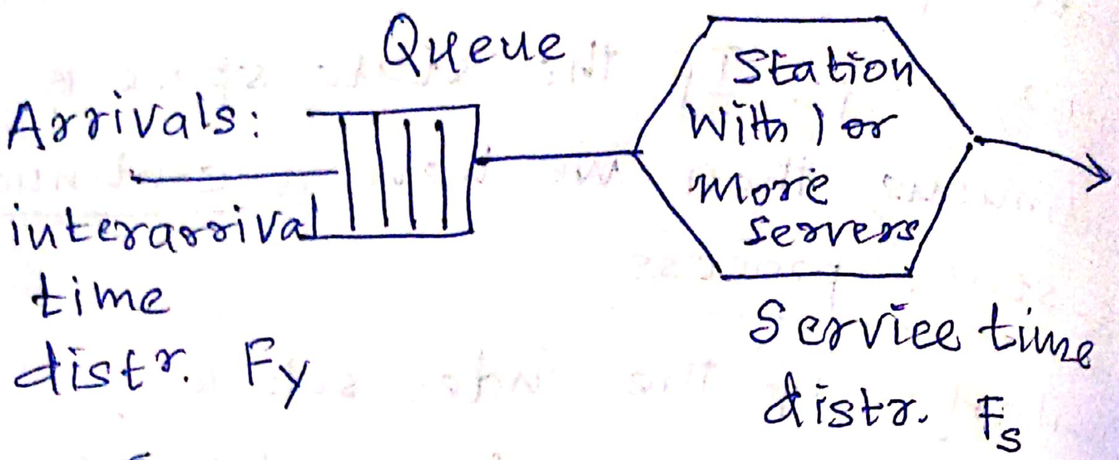
For a fixed sample point $s_1 \in S$, the expression $X_{s_1}(t) = X(t, s_1)$ is a single fn. of time t, called a sample fn. or a realization of the process. When both s and t are varied, we have the family of r.Vs constituting a stochastic process.

If the state space of a S.P is discrete, then it is a discrete space process, often refered as a "chain". In this case, the state space is assumed to be $\{0, 1, 2, \ldots\}$. If the state space is continuous, then we have a continuo-us-state process.

Similarly, if the index set is discrete, then we have discrete-time (parameter) process; otherwise we have a continuous-time (parameter) process. A discrete-time process is also called a stochastic sequence and is denoted by $\{X_n \mid n \in T\}$.

Classification of SP:

		Index set T	
		Discrete	Continuous
State space I		Discrete-time stochasic chain	Continuous-time stochastic chain
		Discrete-time continuous-state process	Continuous-time continuous-state process.



1 Fig. A queuing system.

The theory of queues (or Waiting Lines) provides many examples of SP. Assume that successive inter-arrival times $Y_1, Y_2, \ldots,$ between jobs are independent identically distributed (i.i.d) r.Vs having a distr. F_Y. Similarly, the service times $S_1, S_2, \ldots,$ are assumed to be i.i.d r.vs with a distr. F_S.

Let m = no. of servers in the system. We use Kendal's notation $F_Y|F_S|m$ to describe the queuing system.

M Expo. Distr. (Memoryless)

D Deterministic or Constant interarrival or service time

E_K K-stage Erlang Distr.

H_K K-stage hyperexponential Distr.

G General Distr.

GI General ~~inter~~dependent interarrival times.

Thus, M|G|1 denotes a single-server queue with exponential interarrival times and an arbitrary service time distr.

Based on F_Y and F_S, we need to specify a scheduling discipline that decides how the server is to be allocated to the jobs waiting for the service. otherwise by default, it is FCFS.

Example

Consider a computer server with jobs arriving at random points in time, queuing for service, and departing from the system after service completion.

① Let N_K = no. of jobs in the system @ the time of the departure of K^{th} customer (after service completion). The $\{N_K \mid K=1,2,\dots\}$ is a discrete SP time discrete process with state space $I=\{0,1,2,\dots\}$ and the index set $T=\{1,2,3,\dots\}$. A reliazation of this process is shown in Fig. ②.

② Let $X(t)$ be the no. of jobs in the system @ time t. Then, $\{X(t) \mid t \in T\}$ is a continuous-time discrete-state process with

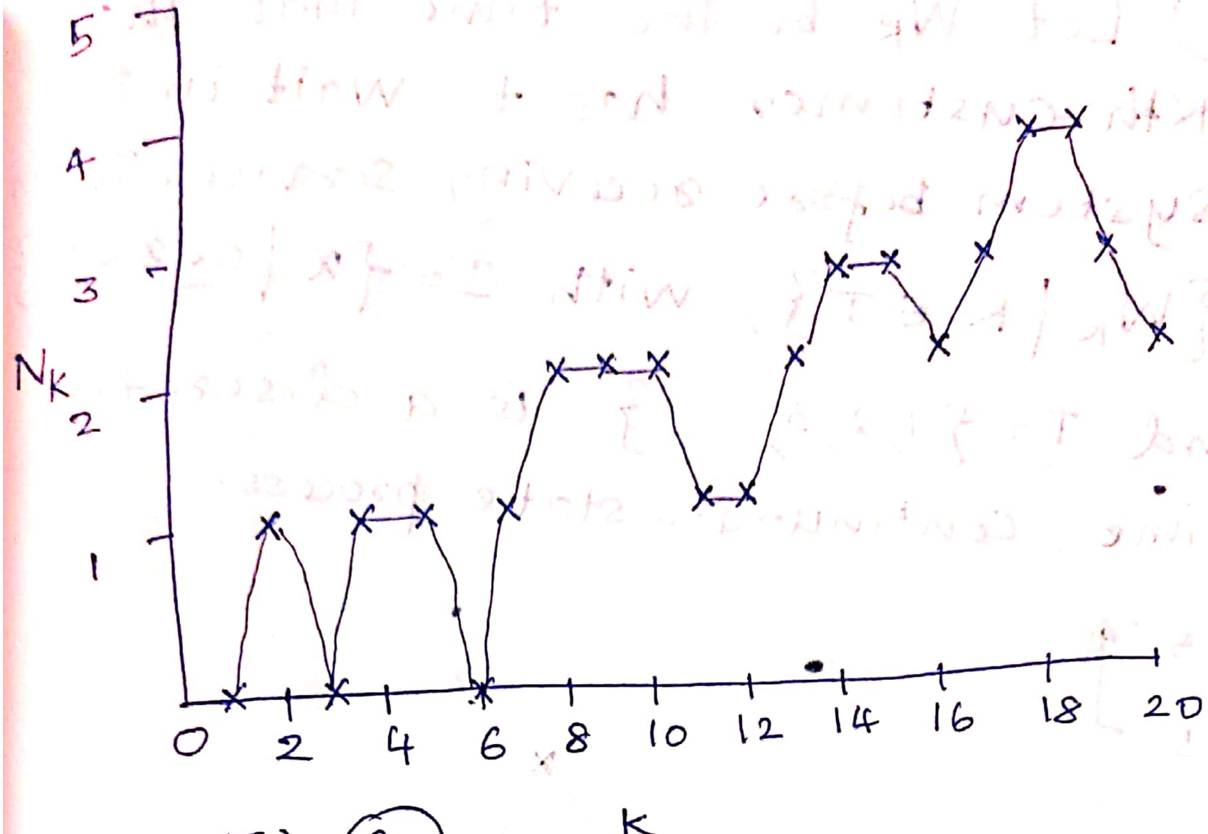


Fig. 2

$I = \{0, 1, 2, \dots\}$ and $T = \{t \mid 0 \le t < \infty\}$.

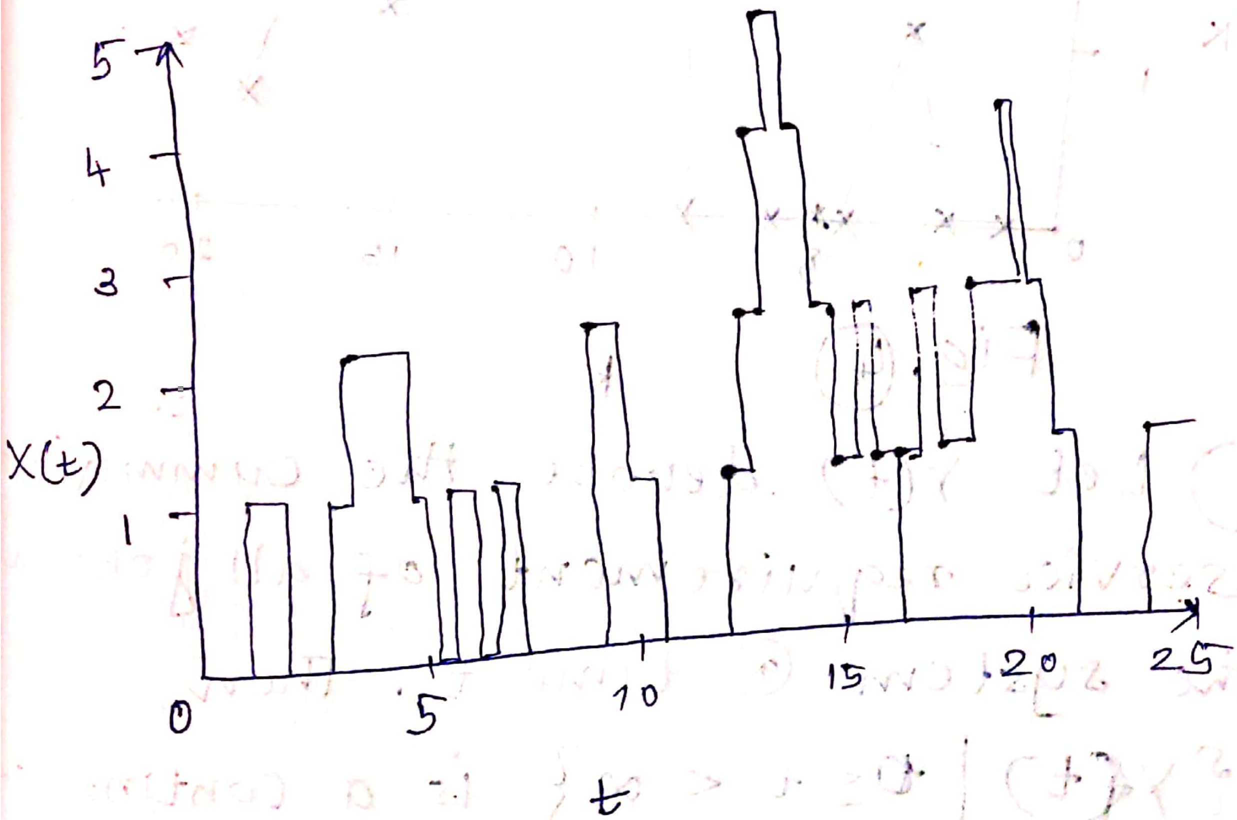


Fig. 3

③ Let W_K be the time that the Kth-customer has to wait in the system before receving service. Then, $\{W_K \mid K \in T\}$, with $I = \{x \mid 0 \leq x < \infty\}$ and $T = \{1, 2, 3, \ldots\}$ is a discrete-time, continuous-state process.

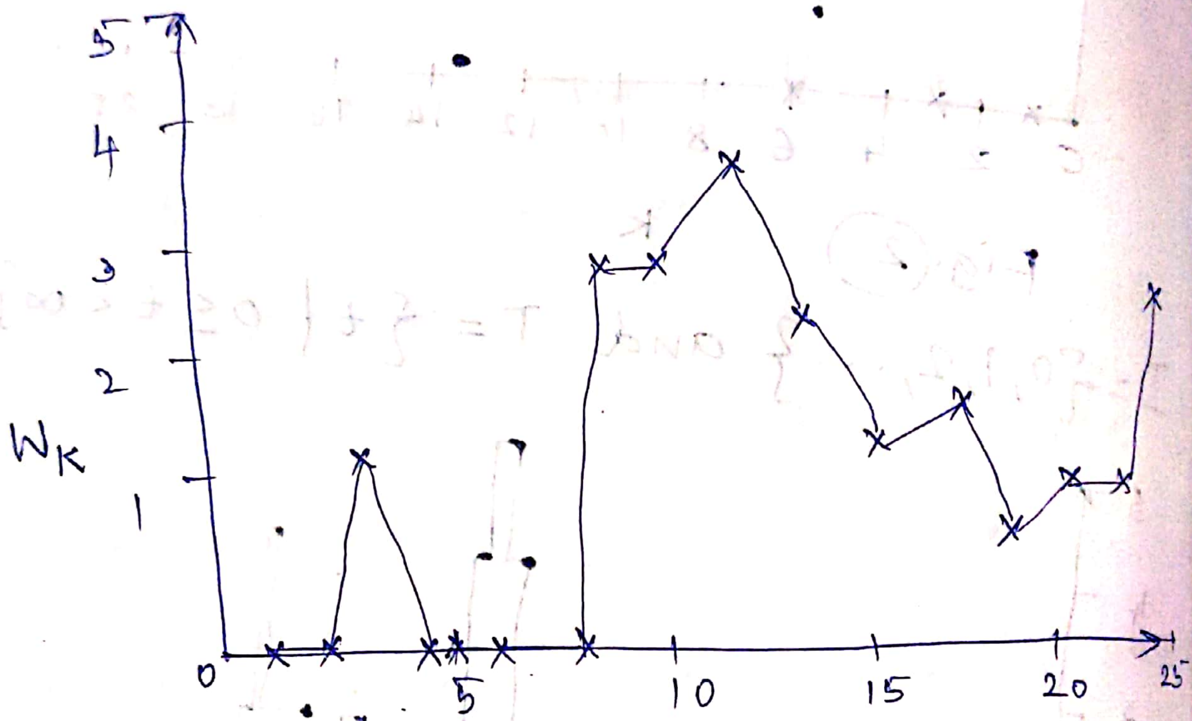


Fig. ④

④ Let $Y(t)$ denote the cummulative service requirement of all jobs in the system @ time t. Then, $\{Y(t) \mid 0 \leq t < \infty\}$ is a continous-time, continuous-state process with $I = [0, \infty)$.

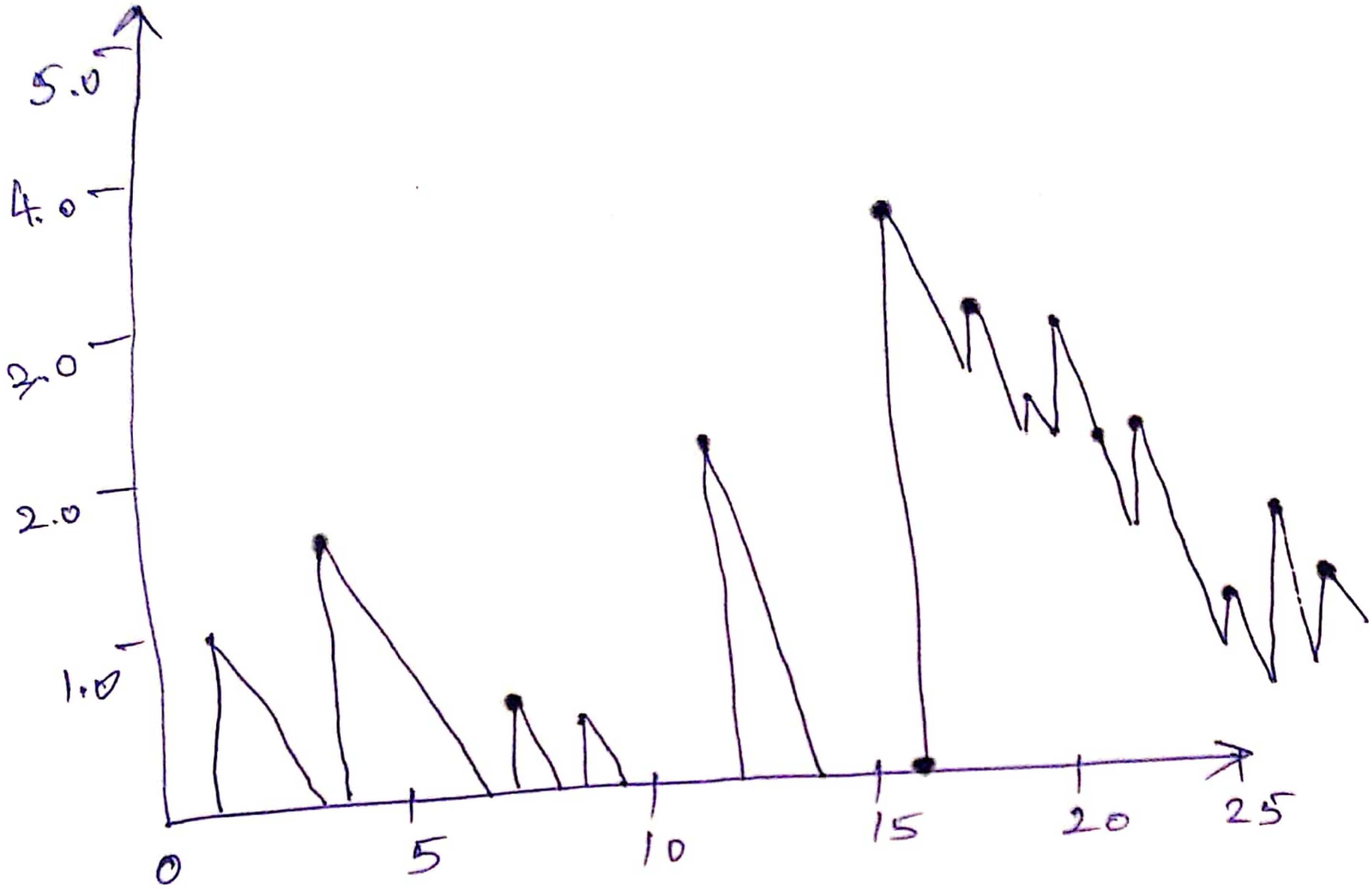


Fig. 5