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III Semester B.Tech. Examination, December - 2019/January - 2020
(CBCS) (ME/EE/EC/CSE/ISE All Branches Except Civil)

MATHEMATICS

18BSEM301 : Engineering Mathematics - III

Time : 3 Hours | Max. Marks : 100

Instruction : *Answer **all** the questions.*

A. Answer **all** the questions. **1x15=15**

1. Sine function is periodic with period :

(a) $\frac{\pi}{2}$ (b) $-\pi$ (c) 2π (d) π

2. The value of a_0 in the Fourier series expansion of $f(x)=x-x^2$ in $(-\pi, \pi)$ is :

(a) $-\frac{2\pi^3}{3}$ (b) $\frac{2\pi^3}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2\pi^3}{4}$

3. The mean value of $f(x) \sin nx$ in $(0, 2\pi)$ is :

(a) a_0 (b) $\frac{a_n}{2}$ (c) $\frac{a_n}{4}$ (d) $\frac{b_n}{2}$

4. Fourier cosine transform of e^{-x^2} is :

(a) $\frac{\sqrt{\pi} e^{\frac{s^2}{4}}}{2}$ (b) $\frac{-\sqrt{\pi} e^{\frac{s^2}{4}}}{4}$ (c) $\frac{\sqrt{\pi} e^{\frac{3}{4}}}{2}$ (d) $\frac{-\sqrt{\pi} e^{\frac{s^2}{4}}}{2}$

5. If $F\{f(x)\}=F(s)$, then $F\{f(x-a)\}$ is :

(a) $e^{sa}f(s)$ (b) $e^{-isa}f(s)$ (c) $e^{isa}f(s)$ (d) $f(x)$

6. $Z\left(\frac{1}{n!}\right)$ is :

(a) $e^{\frac{1}{z}}$ (b) $e^{-\frac{1}{z}}$ (c) e^{Z} (d) e^{-Z}

P.T.O.

7. Which of the following curves gives shortance distance between two points on a plane ?

(a) $y=mx+c$ (b) $y^2+x^2=c$

(c) $y^2=4ax$ (d) $y^2=-mx+c$

8. The geodesic on a sphere of radius 'a' is :

(a) $z=Ax+By$ (b) $z=Ax^2+By^2$

(c) $z=Ax-By$ (d) $z=Ax^2-By^2$

9. For the functional $I[y(x)]=\int_a^b (x-y)^2\,dx$, the extremal is a :

(a) Straight line (b) Parabola

(c) Circle (d) Catenary

10. For the Lagrange's Linear PDE $Pp+Qq=R$ the subsidiary(auxiliary) equation is :

(a) $\frac{dx}{P}=-\frac{dy}{Q}=-\frac{dz}{R}$ (b) $-\frac{dx}{P}=-\frac{dy}{Q}=\frac{dz}{R}$

(c) $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$ (d) $\frac{dx}{P^2}=-\frac{dy}{Q^2}=\frac{dz}{R^2}$

11. The PDE obtained from $Z=e^y f(x+y)$ is :

(a) $p+z=q$ (b) $z-q=p$ (c) $p-q=z$ (d) pq

12. The order and degree of differential equation $\frac{\partial^2 z}{\partial x^2}+2xy\left(\frac{\partial z}{\partial x}\right)^2+\frac{\partial z}{\partial y}=5$ are respectively :

(a) 2 and 1 (b) 1 and 2 (c) 1 and 1 (d) 2 and 2

13. If A and B are two events such that $P(A)>0$ and $P(B)\neq 1$ then $P\left(\bar{A}/\bar{B}\right)$ is equal to :

(a) $1-P\left(A/B\right)$ (b) $1-P\left(\bar{A}/B\right)$ (c) $\frac{1-P(A\cup B)}{P(\bar{B})}$ (d) $\frac{P(\bar{A})}{P(\bar{B})}$

14. A bag contains 3 red balls, 4 white balls and 7 black balls. The probability of drawing a red or a black ball is :

(a) $\frac{2}{7}$ (b) $\frac{5}{7}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

15. A random variable X takes the values 0, 1, 2, 3 and its mean is 1.3. If $P(X=3)=2$, $P(X=1)=P(X=2)=0.3$, then $P(X=0)$ is :
(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

B. Answer **all** the questions. **17x5=85**

1. (a) Find the Fourier expansion of $f(x)=x^2$ over the interval $(-\pi, \pi)$ and hence prove that $\frac{\pi^2}{8}=\Sigma\frac{1}{(2n-1)^2}$. **7+6+4**

(b) Find the half-range cosine series for the following function over the interval $(0, \pi)$, $f(x)=\begin{cases}\cos x, & 0<x<\pi/2 \\ 0, & \pi/2<x<\pi\end{cases}$

(c) Obtain the Fourier series neglecting the terms higher than first harmonics :

x	0	60	120	180	240	300
y	7.9	7.2	3.6	0.5	0.9	6.8

2. (a) Find the Fourier transform of the function $f(x)=\begin{cases}x, & \lvert x\rvert\le a \\ 0, & \lvert x\rvert>a\end{cases}$ **5+4+8**

(b) Find the inverse Fourier transform of e^{-a^2}

(c) Define Z-transform of a sequence U_n.

Prove that $Z(a_n)=\frac{z}{z-a}$ and $Z(n^p)=-z\frac{d}{dz}Z(n^{p-1})$, p being a positive integer.

3. (a) State and prove Euler's equation. **6+4+7**

(b) Show that the extremal of $I=\int_{x_1}^{x_2}\sqrt{y(1+y'^2)}\,dx$ in a parabola.

(c) Find the geodesic on a right circular cylinder.

OR

4. (a) Find the extremal of the functional $I=\int_4^5\sqrt{x(1+y'^2)}\,dx$, $y(4)=0$, $y(5)=4$. **5+5+7**

(b) Show that an extremal of $I=\int_{x_1}^{x_2}\left(\frac{y'}{y}\right)^2 dx$ is expressible in the form $y=ae^{bx}$.

(c) Find the geodesic on a sphere of radius 'a'.

5. (a) Form the PDE by eliminating arbitrary functions $Z = f\left(\frac{xy}{z}\right)$. **4+5+8**

(b) Solve by the method of separation of variables $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$.

(c) Solve $q = (z+px)^2$ by Charpit's method.

OR

6. (a) Solve $\frac{\partial^2 z}{\partial x \partial y} = \frac{x}{y}$ subject to the conditions $\frac{\partial z}{\partial x} = \log x$ when $y = 1$ and $z = 0$ when $x = 1$. **5+6+6**

(b) Verify that $z = ae^{b(ax+by)}$ is a complete solution of the equation $p^2 = qz$ and prove that $z = 0$ in the singular solution and particular solution.

(c) Solve $p\cot x + q\cot y = \cot z$.

7. (a) Fit a curve of the form $y = ae^{bx}$ for the data : **5+6+6**

x	0	2	4
y	8.12	10	31.82

(b) When a coin is tossed 4 times, find the probability of getting
(i) exactly one head (ii) atleast 2 heads.

(c) Obtain the lines of regression and hence find the coefficient of correlation for the following data :

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

OR

8. (a) Derive the normal equations for second degree parabola $y = ax^2 + bx + c$. **5+6+6**

(b) Find the correlation coefficient for the groups :

A	92	89	87	86	83	77	71	63	53	50
B	86	83	91	77	68	85	52	82	37	57

(c) 2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains
(i) no defective fuses
(ii) 3 or more defective fuses.

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