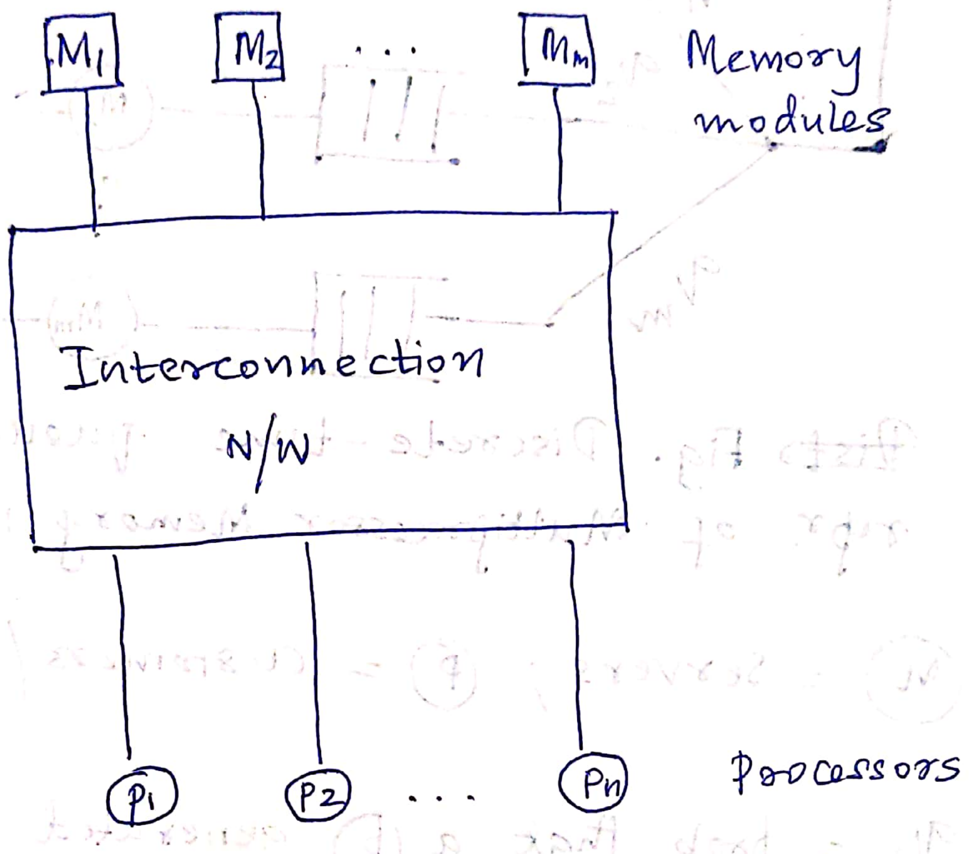
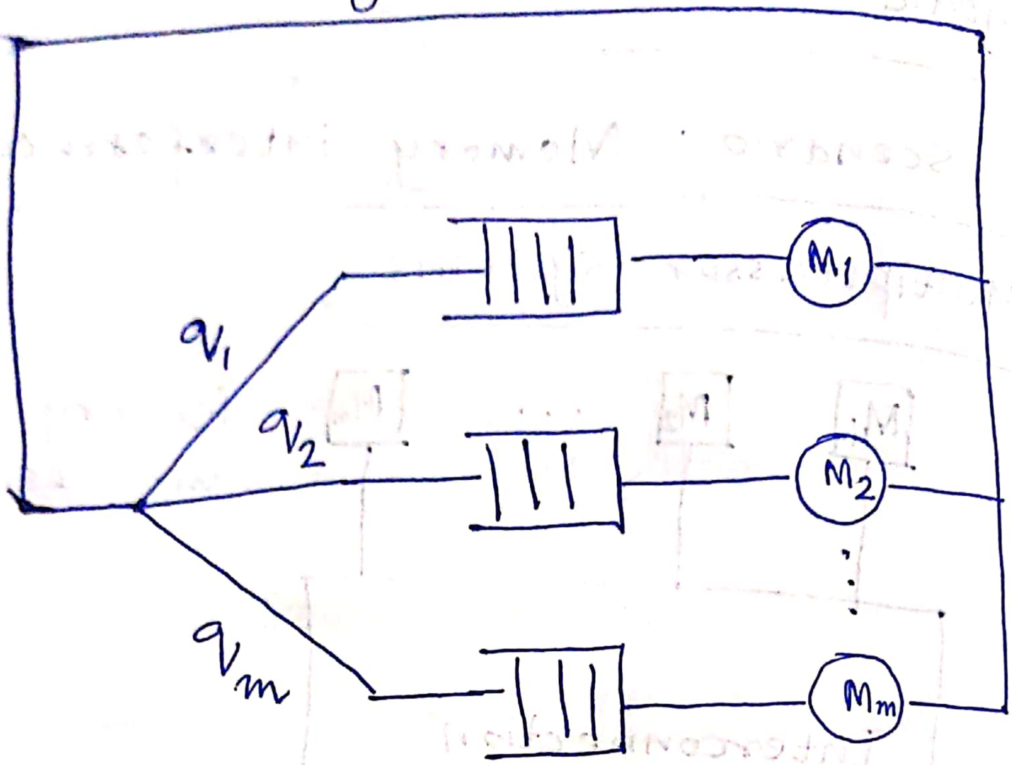
Irreducible Finite Chains with Aperiodic states

Case Scenario : Memory interference in Multiprocessor Systems



The memory (M) is accessed independently and concurrently with other modules. When more than one processor attempts to access the same module, only one processor (p) is granted access, while other (p) must wait their turn in a queue. Assume the time to complete a (M) access is a constant and that all modules are synchronized.



~~Distr~~ Fig. Discrete-time queueing n/w repr. of Multiprocessor Memory interference

(M) = Servers ; (P) = customers / jobs.

q_i = prob. that a (P) generated request is directed @ Memory module i $i = 1, 2, \ldots, m$. Thus, $\sum_{i=1}^{m} q_i = 1$.

Let $m = n = 2$.

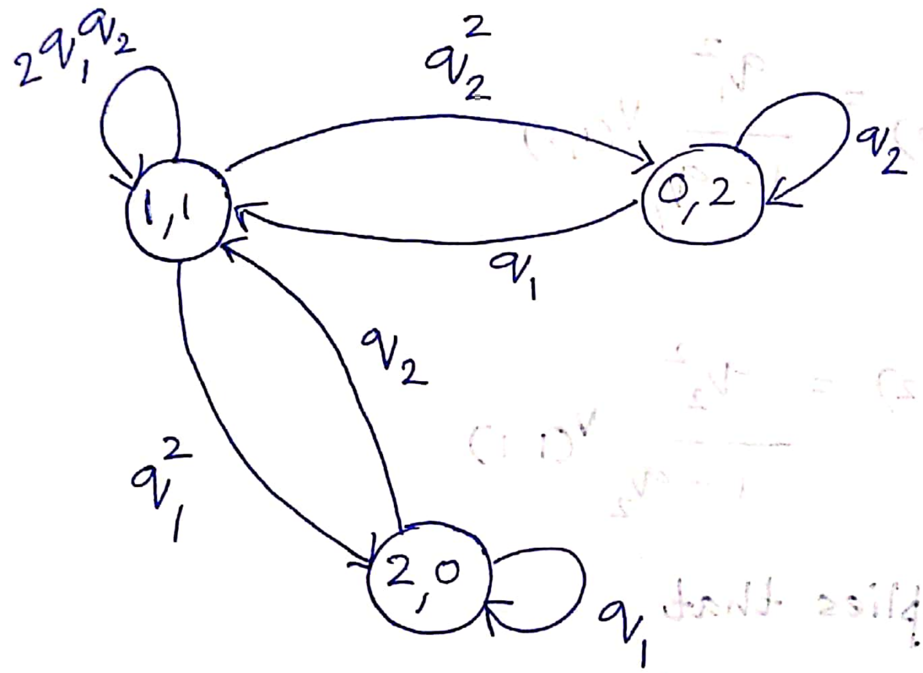
N_i = no. of (P) waiting or being served at (M) i $(i = 1, 2)$, $N_i \geq 0$, and $N_1 + N_2 = 2$.

$(N_1; N_2)$ = state of the system and

the state space $I = \{(1,1), (0,2), (2,0)\}$.

The tpm of DTMC is:

$$P = \begin{array}{c|ccc} & (1,1) & (0,2) & (2,0) \\ \hline (1,1) & 2q_1q_2 & q_2^2 & q_1^2 \\ (0,2) & q_1 & q_2 & 0 \\ (2,0) & q_2 & 0 & q_1 \end{array}$$



~~Ex: Assume~~

To obtain the steady-state ~~probabilities~~ prob. vector $v = [v_{(1,1)}, v_{(0,2)}, v_{(2,0)}]$

we use

$$v = vP \quad \text{and} \quad \sum_{(i,j) \in I} v_{(i,j)} = 1$$

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or

$$V(1,1) = 2q_1q_2 v_{(1,1)} + q_1 v_{(0,2)} + q_2 v_{(2,0)}$$

$$V_{(0,2)} = q_2^2 v(1,1) + q_2 v_{(0,2)}$$

$$V_{(2,0)} = q_1^2 v_{(1,1)} + q_1 v_{(2,0)}$$

$$V_{(1,1)} + V_{(0,2)} + V_{(2,0)} = 1$$

Thus,

$$V_{(2,0)} = \frac{q_1^2}{1-q_1} V_{(1,1)}$$

$$V_{(0,2)} = \frac{q_2^2}{1-q_2} V_{(1,1)}$$

Which implies that

$$V_{(1,1)} = \frac{1}{1 + \frac{q_1^2}{1-q_1} + \frac{q_2^2}{1-q_2}} = \frac{q_1 q_2}{1-2q_1q_2}$$

Let B = r.v. no. of memory requests completed per memory cycle in the steady state. Compute $E[B]$ = avg. memory requests completed per memory cycle.

$E[B \mid \text{system is in state } (1,1)] = 2$

$E[B \mid \text{———— '' ————} (0,2)] = 1$

$E[B \mid \text{———— '' ————} (2,0)] = 1$

We assign rewards to the three states of DTMC as follows:

$r_{(1,1)} = 2$, $r_{(2,0)} = 1$, and $r_{(0,2)} = 1$.

Then, the steady-state reward is

$$E[Z] = E[B] = 2v_{(1,1)} + v_{(0,2)} + v_{(2,0)}$$

$$= \left(2 + \frac{q_1^2}{1-q_1} + \frac{q_2^2}{1-q_2}\right) v_{(1,1)}$$

$$= \frac{1 - q_1 q_2}{1 - 2q_1 q_2}.$$

The quantity $E[B]$ achieves its maximum value $\frac{3}{2}$ when $q_1 = q_2 = \frac{1}{2}$. This is smaller than the capacity of the memory system, which is two requests per cycle.