Computer Graphics

Unit 2 – Part1

–By Manjula. S

2D Transformation

Transformation:

Changing position, shape, size or orientation of an object on display is known as transformation.

Basic Geometric Transformations are:

- Translation
- Rotation
- Scaling

Other Transformations:

- Reflection
- Shear

2D Linear Transformations

➢ We can use a 2 × 2 matrix to change, or transform, a 2D vector:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{11}x & + & a_{12}y \\ a_{21}x & + & a_{22}y \end{bmatrix}$$

- This kind of operation, which takes in a 2-vector and produces another 2-vector by a simple matrix multiplication, is a linear transformation.

- By this simple formula we can achieve a variety of useful transformations, depending on what we put in the entries of the matrix.

2-D Geometric Transformations

In order to manipulate an object in 2-D space, we must apply various transformation functions to the object. This allows us to change the position, size, and orientation of the objects.

There are two complementary points of view for describing object movement.

1.) **Geometric Transformation :** The object itself is moved relative to a stationary coordinate system or background.

2.) **Coordinate transformation :** The object is held stationary while the coordinate system is moved relative to the object.

The Basic geometric transformations are:

- **Translation**
- **Rotation**
- **Scaling**
- **Reflection**
- **Shear**

2D Translation

We can *translate* points in the (x, y) plane to new positions by adding translation amounts to the coordinates of the points. For each point P(x, y) to be moved by d_x units parallel to the x-axis and by d_y units parallel to the y-axis to the new point P′(x′, y′), we can write

$$x' = x + d_x \qquad y' = y + d_y \qquad \text{---- (1)}$$

If we define the column vectors

$$\boldsymbol{P} = \begin{bmatrix} x \\ y \end{bmatrix}, \qquad \boldsymbol{P}' = \begin{bmatrix} x' \\ y' \end{bmatrix}, \qquad \boldsymbol{T} = \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

Then equation (1) can be expressed more concisely as

$$P' = P + T$$

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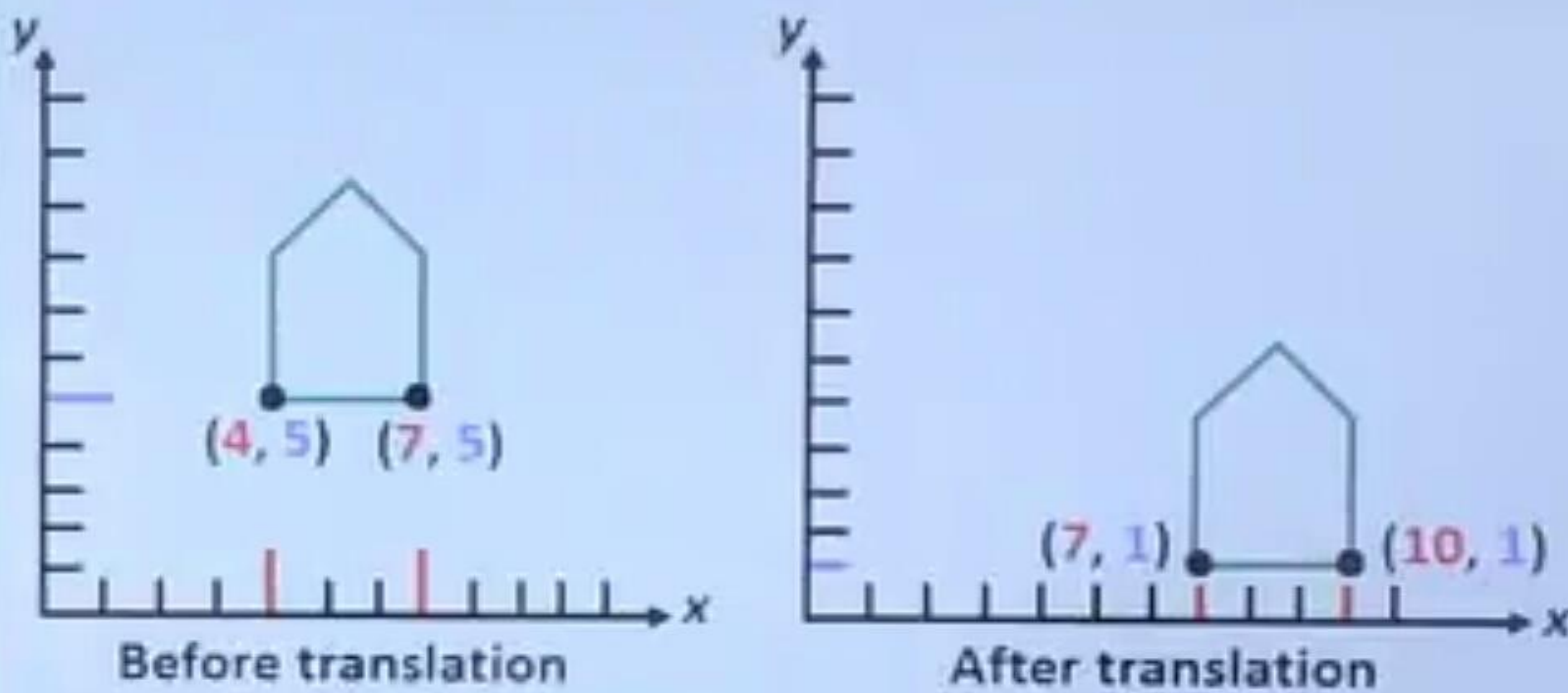
2D Translation

Let us consider the following example –



Translation of a house

Here $(X_1, Y_1) = (4, 5)$, $(X_2, Y_2) = (7, 5)$ and $d_x = 3$, $d_y = -4$

So, $(X_1', Y_1') = (4 + 3, 5 - 4) = (7, 1)$ and

$(X_2', Y_2') = (7 + 3, 5 - 4) = (10, 1)$

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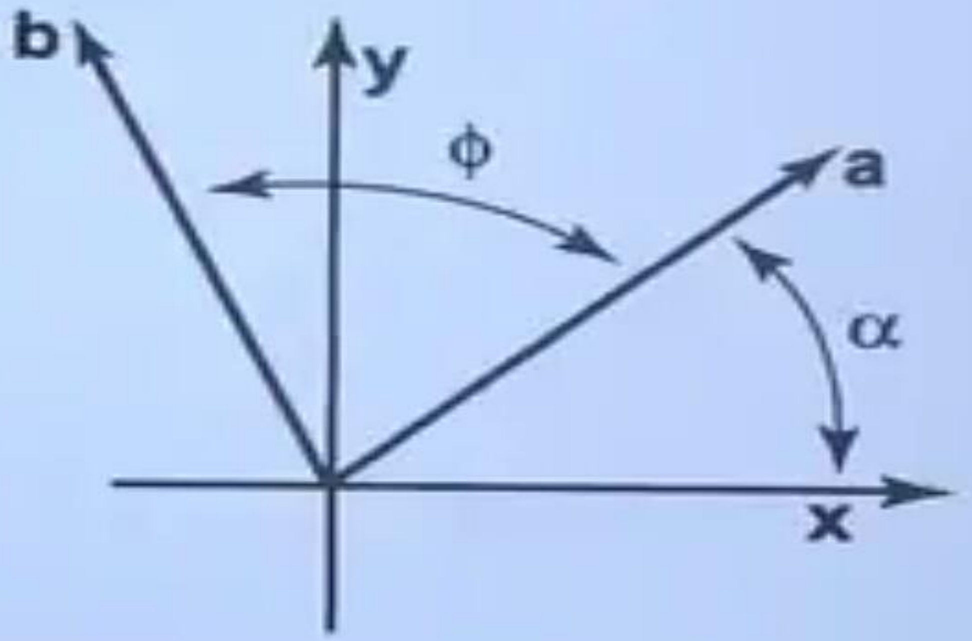
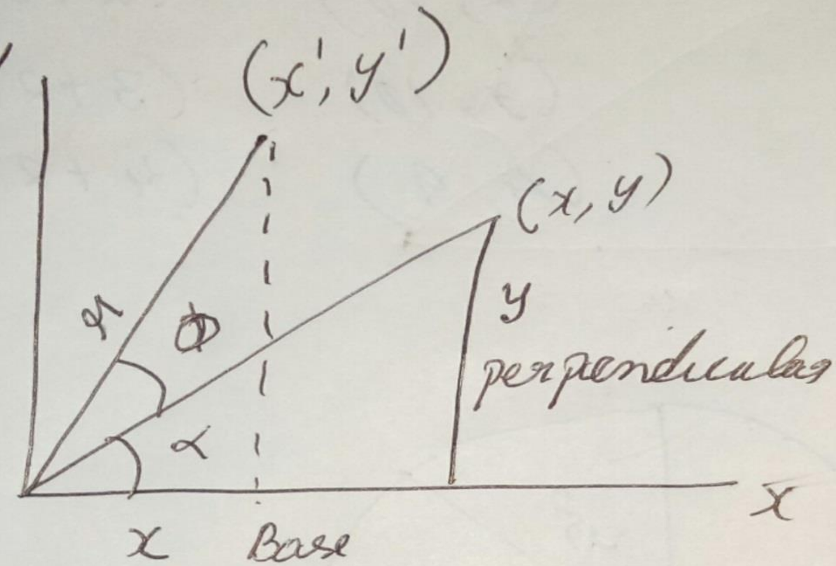
2D Rotation

- Suppose we want to rotate a vector **a** by an angle φ counter-clockwise to get vector **b**. If **a** makes an angle α with the x-axis, and its length is $r = x^2_a + y^2_a$, then we know that

$$x_a = r\cos\alpha,$$
$$y_a = r\sin\alpha.$$



2D Rotation



$$x' = r\cos(\alpha+\phi) \quad \text{——} \quad (1)$$

$$\therefore \cos(\alpha+\phi) = \frac{x'}{r} \qquad \left[\cos\theta = \frac{\text{Base}}{\text{Hypotenus}}\right]$$

$$y' = r\sin(\alpha+\phi) \quad \text{——} \quad (2)$$

$$\therefore \sin(\alpha+\phi) = \frac{y'}{r} \qquad \left[\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenus}}\right]$$

2D Rotation

$$x' = r\cos(\alpha+\phi)$$

$$= r[\cos\alpha\cos\phi - \sin\alpha\sin\phi]$$

$$x' = r\cos\alpha\cos\phi - r\sin\alpha\sin\phi \quad \text{—— (3)}$$

$$y' = r\sin(\alpha+\phi)$$

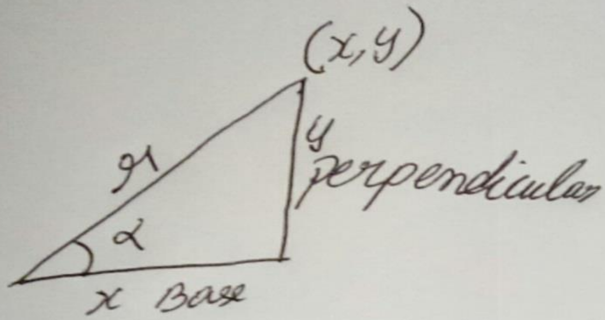
$$= r[\sin\alpha\cos\phi + \cos\alpha\sin\phi]$$

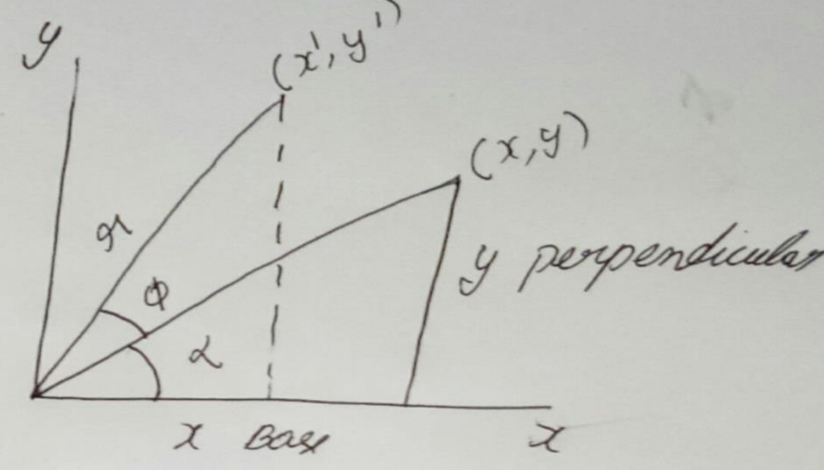
$$y' = r\sin\alpha\cos\phi + r\cos\alpha\sin\phi \quad \text{—— (4)}$$

[Because $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$]

2D Rotation





$$\cos\alpha = \frac{Base}{Hy}$$

$$\cos\alpha = \frac{x}{r}$$

$$x = r\cos\alpha \quad \text{——} \quad (5)$$

$$\sin\alpha = \frac{Per}{Hy}$$

$$\sin\alpha = \frac{y}{r}$$

$$y = r\sin\alpha \quad \text{——} \quad (6)$$

$$x' = r\cos(\alpha+\phi) \quad \text{——} \quad (1)$$

$$x' = \underline{r\cos\alpha}\cos\phi - \underline{r\sin\alpha}\sin\phi \quad \text{——} \quad (3)$$

$$x' = x\cos\phi - y\sin\phi \quad \text{——} \quad (7)$$

$$y' = r\sin(\alpha+\phi) \quad \text{——} \quad (2)$$

$$y' = \underline{r\sin\alpha}\cos\phi + \underline{r\cos\alpha}\sin\phi \quad \text{——} \quad (4)$$

$$y' = y\cos\phi + x\sin\phi$$

$$y' = x\sin\phi + y\cos\phi \quad \text{——} \quad (8)$$

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2D Rotation

$$x' = x\cos\phi - y\sin\phi \quad \text{—} \; (7)$$

$$y' = x\sin\phi + y\cos\phi \quad \text{—} \; (8)$$

$$R = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

$$p' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$p = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$p' = RP$$

$$\boxed{\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}}$$

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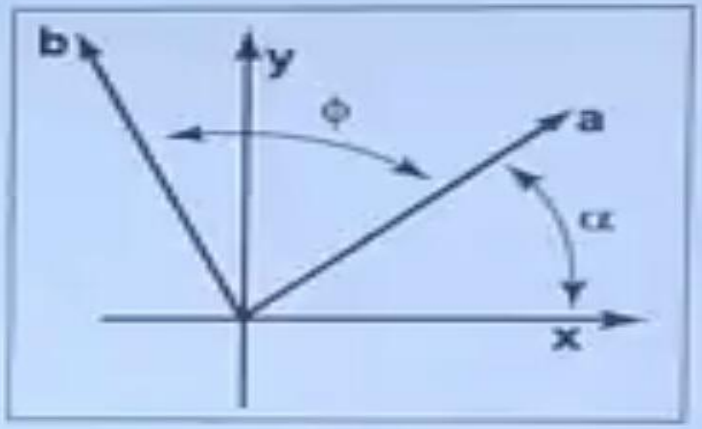
2D Rotation



The geometry for Equation

$$x_b = r\,cos(\alpha + \emptyset) = r \cos \alpha \cos \emptyset - r \sin \alpha \sin \emptyset$$

$$y_b = r\,sin(\alpha + \emptyset) = r \sin \alpha \cos \emptyset - r \cos \alpha \sin \emptyset$$

Substituting $x_a = r \cos \alpha$ and $y_a = r \sin \alpha$ gives

$$x_b = x_a \cos \emptyset - y_a \sin \emptyset,$$

$$y_b = y_a \cos \emptyset + x_a \sin \emptyset.$$

In matrix form, the transformation that takes **a** to **b** is then

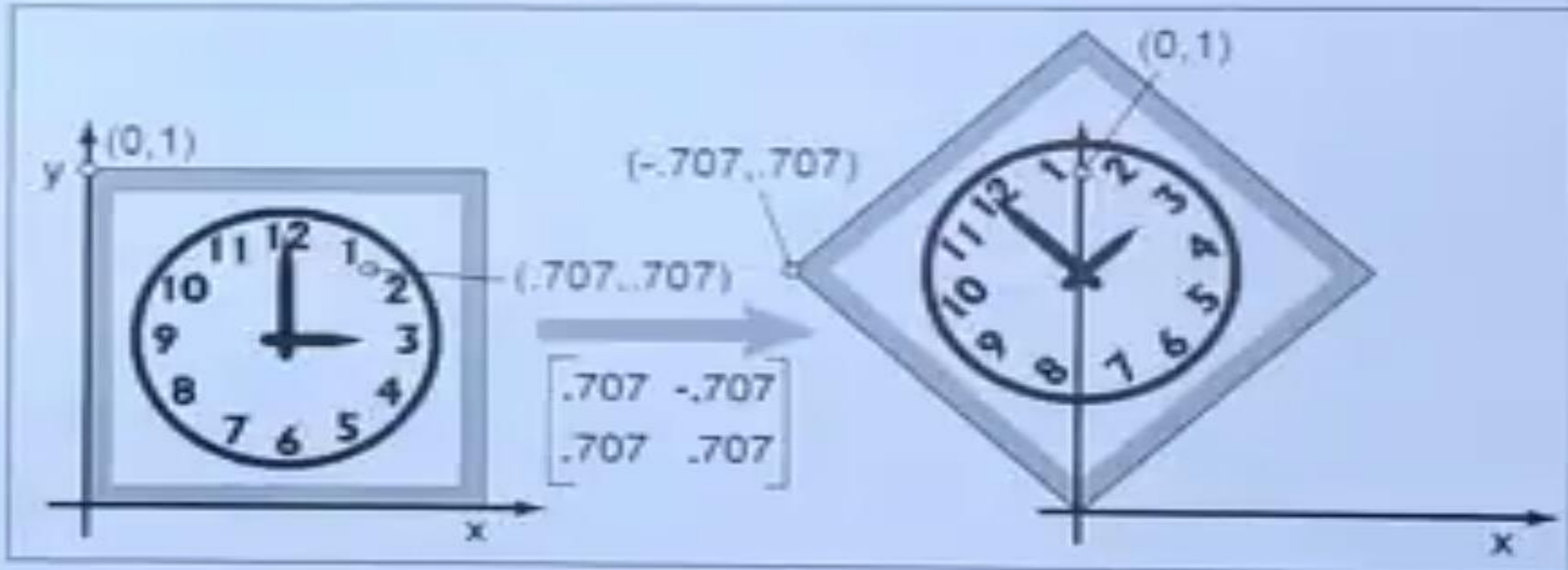
$$\text{Rotate}(\emptyset) = \begin{bmatrix} cos\emptyset & -sin\emptyset \\ sin\emptyset & cos\emptyset \end{bmatrix}$$

2D Rotation

Example-1:

A matrix that rotates vectors by $\pi/4$ radians (45 degrees) is

$$\begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} = \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix}$$



A rotation by 45 degrees. Note that the rotation is counter-clockwise and that $\cos(45^\circ) = \sin(45^\circ) = .707$.

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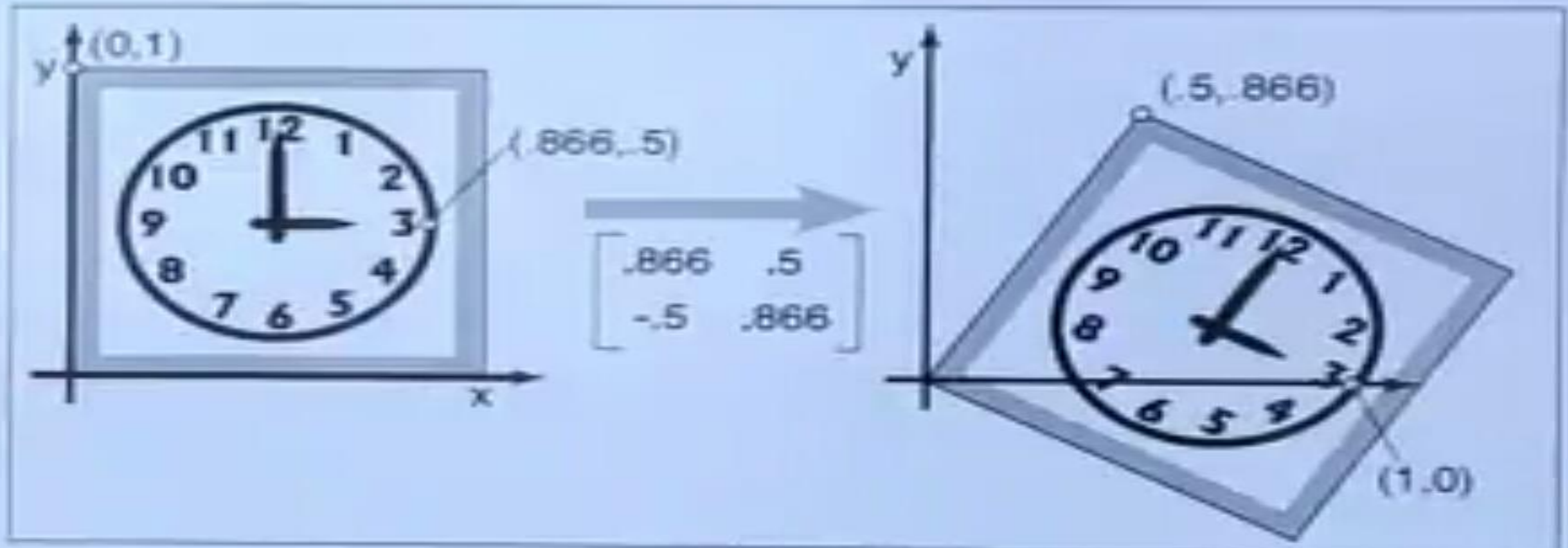
2D Rotation

Example-2:

A matrix that rotates by $\pi/6$ radians (30 degrees) in the *clockwise* direction is a rotation by $-\pi/6$ radians in our framework

$$\begin{bmatrix} \cos\frac{-\pi}{6} & -\sin\frac{-\pi}{6} \\ \sin\frac{-\pi}{6} & \cos\frac{-\pi}{6} \end{bmatrix} = \begin{bmatrix} 0.866 & 0.5 \\ -0.5 & 0.866 \end{bmatrix}$$



A rotation by minus thirty degrees. Note that the rotation is clockwise and that $\cos(-30^\circ) = .866$ and $\sin(-30^\circ) = -.5$.

2D Scaling

The most basic transform is a *scale* along the coordinate axes. This transform can change length and possibly direction:

$$\mathbf{scale}(s_x, s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

Note what this matrix does to a vector with Cartesian components (x, y):

$$\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$$

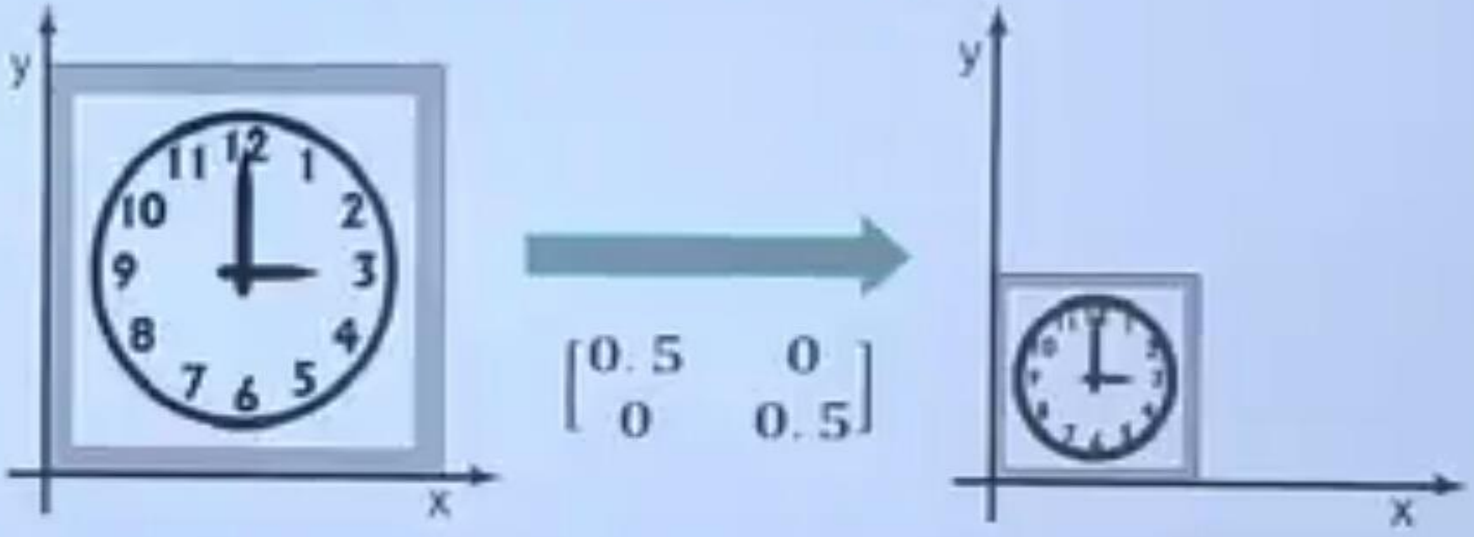
So just by looking at the matrix of an axis-aligned scale we can read off the two scale factors.

2D Scaling

Example-1:

The matrix that shrinks *x* and *y* uniformly by a factor of two is

$$\textbf{scale}(0.5, 0.5) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$



Scaling uniformly by half for each axis: The axis-aligned scale matrix has the proportion of change in each of the diagonal elements and zeroes in the off-diagonal elements.

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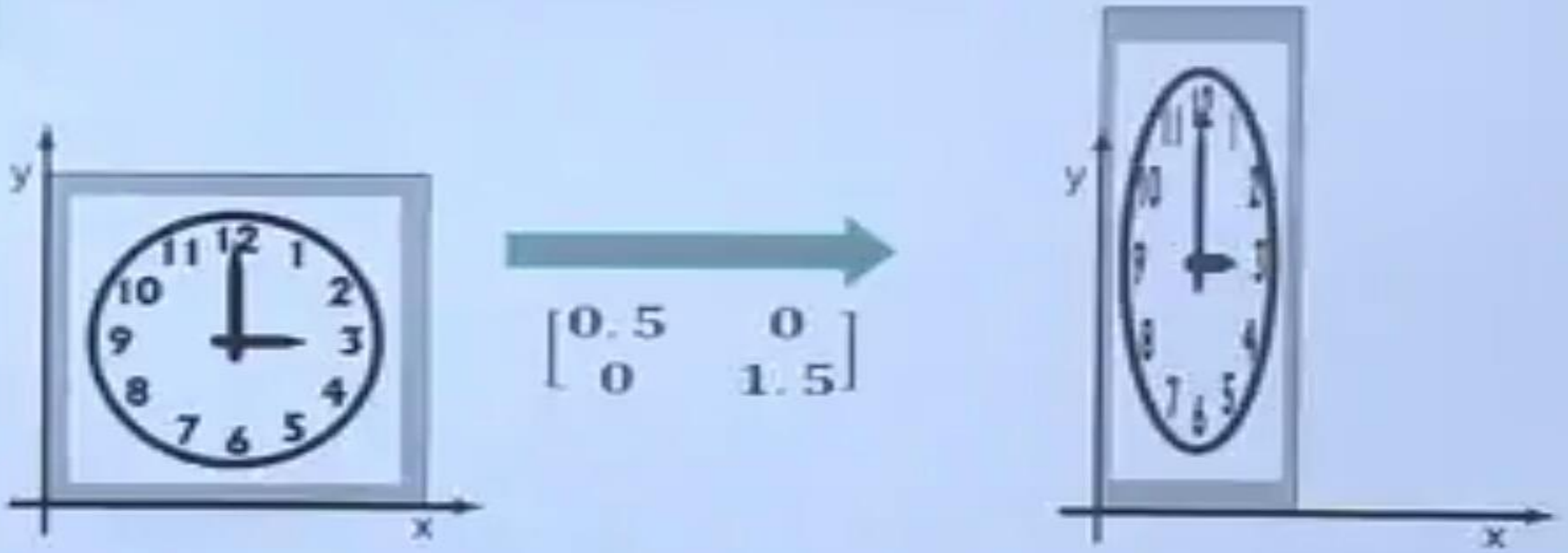
2D Scaling

Example-2:
A matrix which halves in the horizontal and increases by three-halves in the vertical is

$$\textbf{scale}(0.5, 1.5) = \begin{bmatrix} 0.5 & 0 \\ 0 & 1.5 \end{bmatrix}$$



$$\begin{bmatrix} 0.5 & 0 \\ 0 & 1.5 \end{bmatrix}$$

Scaling non-uniformly in x and y: The scaling matrix is diagonal with non-equal elements. Note that the square outline of the clock becomes a rectangle and the circular face becomes an ellipse.

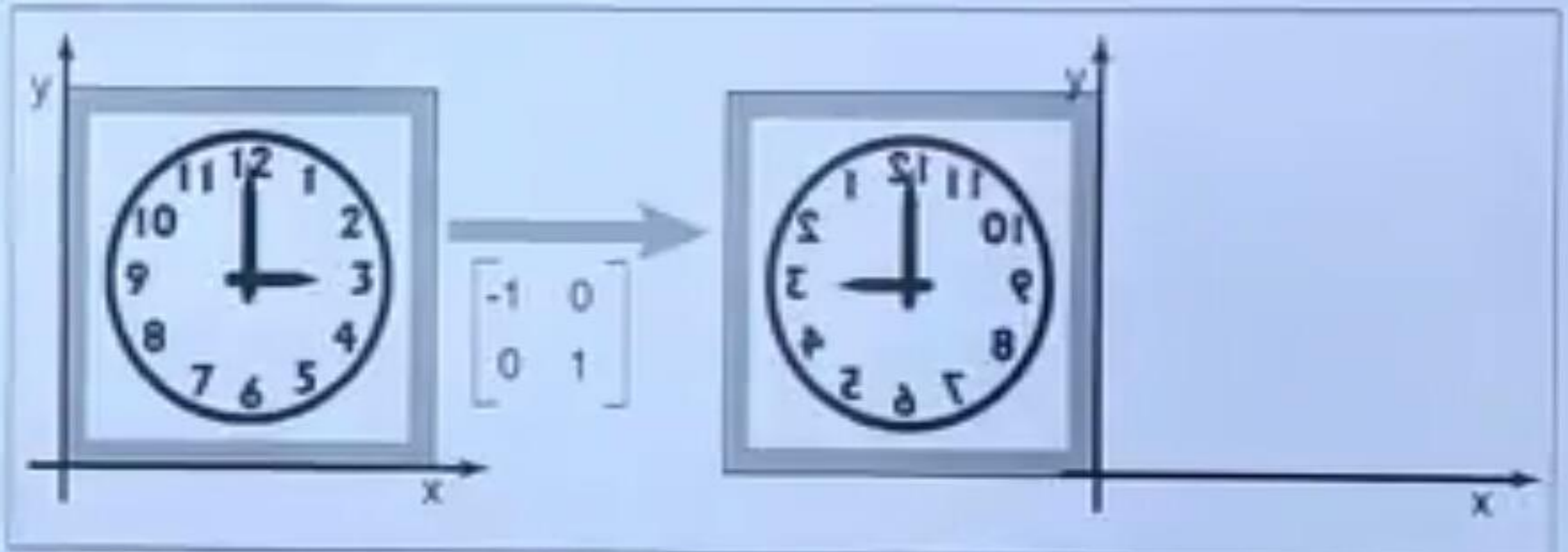
2D Reflection

We can reflect a vector across either of the coordinate axes by using a scale with one negative scale factor

$$\textbf{reflect-y} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \textbf{reflect-x} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Example-1:

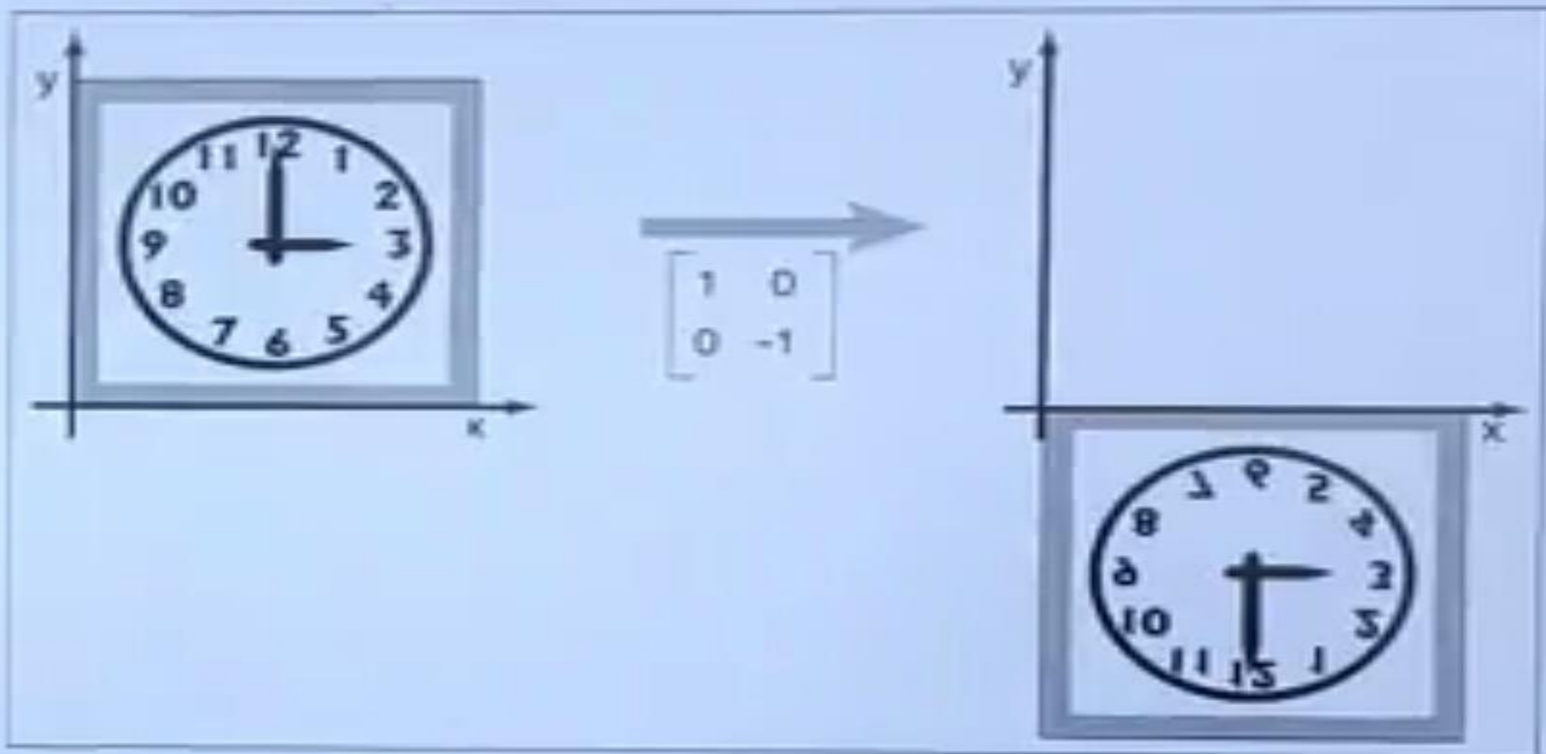
Reflection about the y-axis



A reflection about the y-axis is achieved by multiplying all x-coordinates by -1.

2D Reflection

Example-2:
Reflection about the x-axis



A reflection about the x-axis is achieved by multiplying all y-coordinates by -1.

Note: While one might expect that the matrix with −1 in both elements of the diagonal is also a reflection, in fact it is just a rotation by π radians.

2D Shearing

A shear is something that pushes things sideways, producing something like a deck of cards across which you push your hand; the bottom card stays as it is and cards move more the closer they are to the top of the deck.

The horizontal and vertical shear matrices are

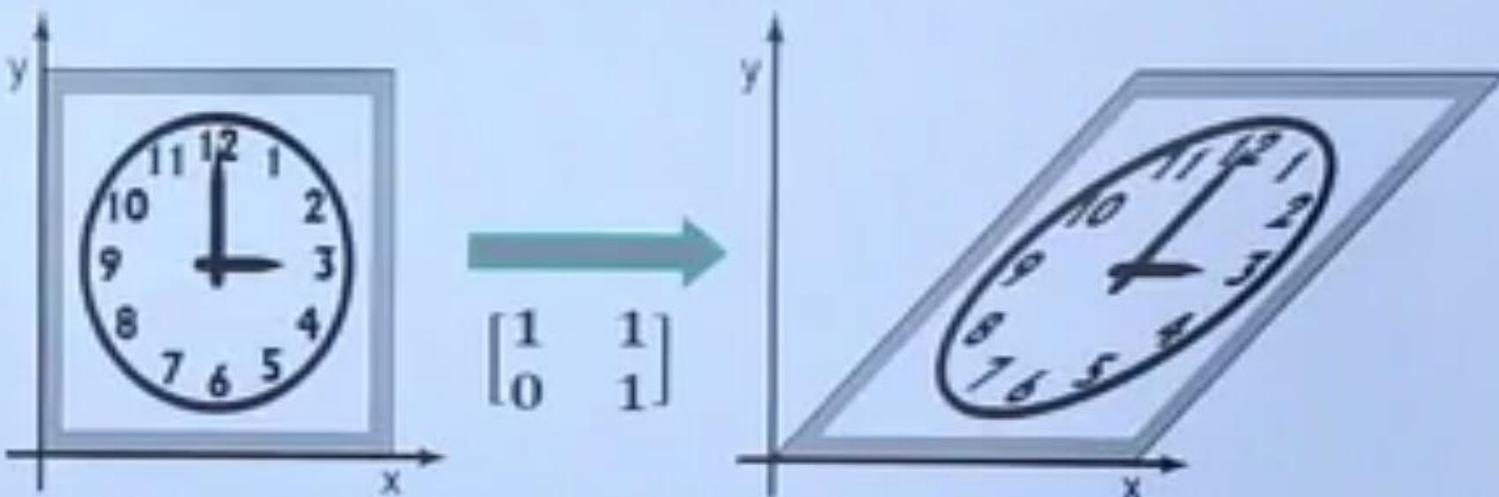
$$\textbf{shear-x}(s) = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}, \quad \textbf{shear-y}(s) = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$

2D Shearing

Example-1:

The transform that shears horizontally so that vertical lines become 45° lines leaning towards the right is

$$\textbf{shear-x(1)} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

An x-shear matrix moves points to the right in proportion to their y-coordinate. Now the square outline of the clock becomes a parallelogram and, as with scaling, the circular face of the clock becomes an ellipse.

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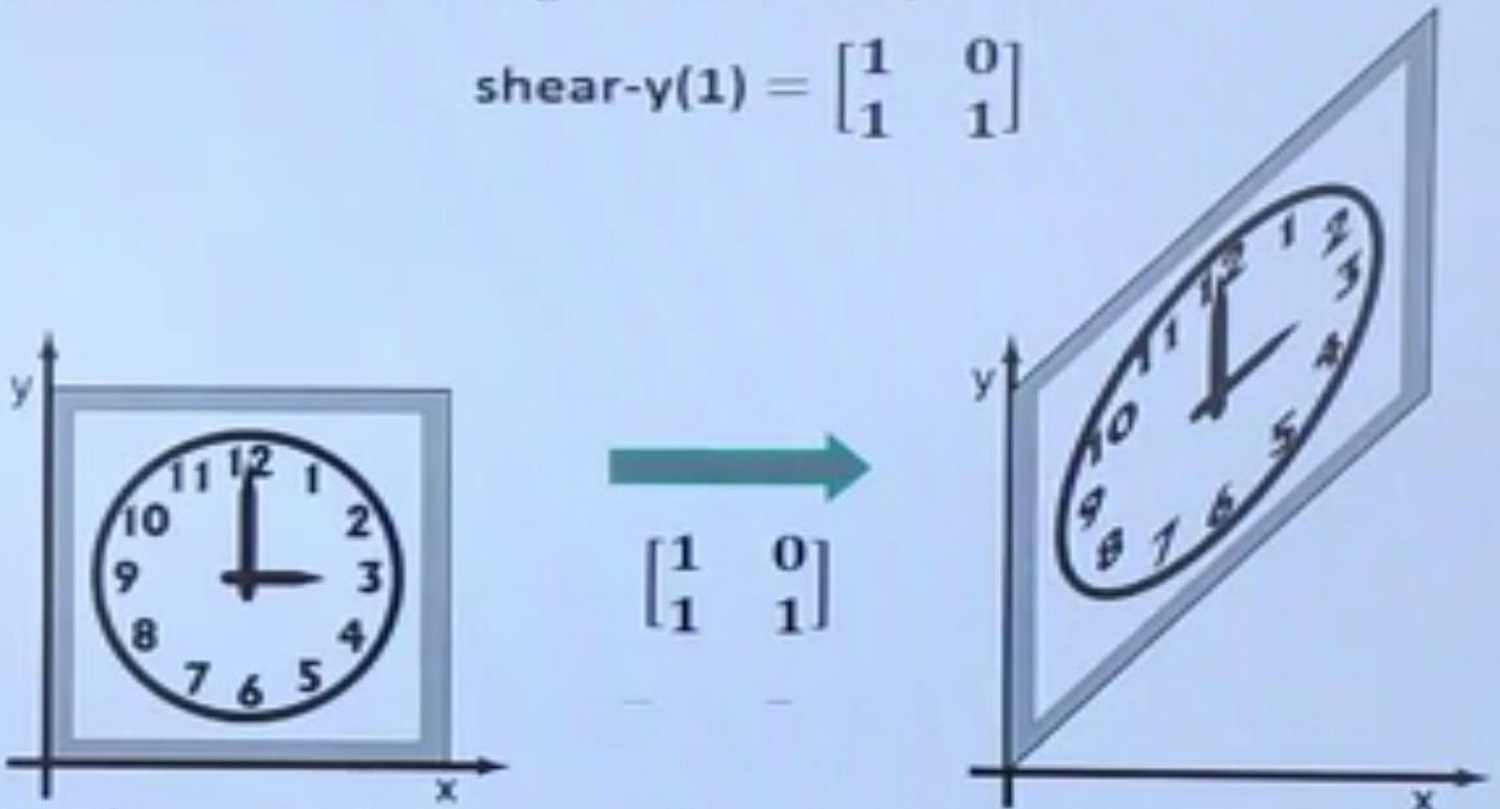
2D Shearing

Example-2:

The transform that shears vertically so that horizontal lines become 45° lines leaning towards the upward is

$$\textbf{shear-y(1)} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

A y-shear matrix moves points up in proportion to their x-coordinate.

2D Shearing

In both cases the square outline of the sheared clock becomes a parallelogram, and the circular face of the sheared clock becomes an ellipse.

Another way to think of a shear is in terms of rotation of only the vertical (or horizontal) axes. The shear transform that takes a vertical axis and tilts it clockwise by an angle ϕ is

$$\begin{bmatrix} 1 & \tan\phi \\ 0 & 1 \end{bmatrix}$$

Similarly, the shear matrix which rotates the horizontal axis counter-clockwise by angle ϕ is

$$\begin{bmatrix} 1 & 0 \\ \tan\phi & 1 \end{bmatrix}$$

Composition of 2D Transformations

It is common for graphics programs to apply more than one transformation to an object. For example, we might want to first apply a scale **S**, and then a rotation **R**. This would be done in two steps on a 2D vector **v1**:

$$\text{first, } \mathbf{v}_2 = \mathbf{S}\mathbf{v}_1, \text{ then, } \mathbf{v}_3 = \mathbf{R}\mathbf{v}_2 .$$

Another way to write this is

$$\mathbf{v}_3 = \mathbf{R}\,(\mathbf{S}\mathbf{v}_1) .$$

Because matrix multiplication is associative, we can also write

$$\mathbf{v}_3 = (\mathbf{RS})\,\mathbf{v}_1 .$$

In other words, we can represent the effects of transforming a vector by two matrices in sequence using a single matrix of the same size, which we can compute by multiplying the two matrices: **M** = **RS** .

It is *very important* to remember that these transforms are applied from the *right side first*. So the matrix **M** = **RS** first applies **S** and then **R**.

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Composition of 2D Transformations

Example-1:

Suppose we want to scale by one-half in the vertical direction and then rotate by $\pi/4$ radians (45 degrees). The resulting matrix is

$$\begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.707 & -0.353 \\ 0.707 & 0.353 \end{bmatrix}$$

It is important to always remember that matrix multiplication is not commutative. So the order of transforms *does* matter. In this example, rotating first, and then scaling, results in a different matrix.

$$\begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix} = \begin{bmatrix} 0.707 & -0.707 \\ 0.353 & 0.353 \end{bmatrix}$$

Composition of 2D Transformations

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Composition of 2D Transformations

Example-1 (Contd.):
Suppose we want to scale by one-half in the vertical direction and then rotate by $\pi/4$ radians (45 degrees). The resulting matrix is

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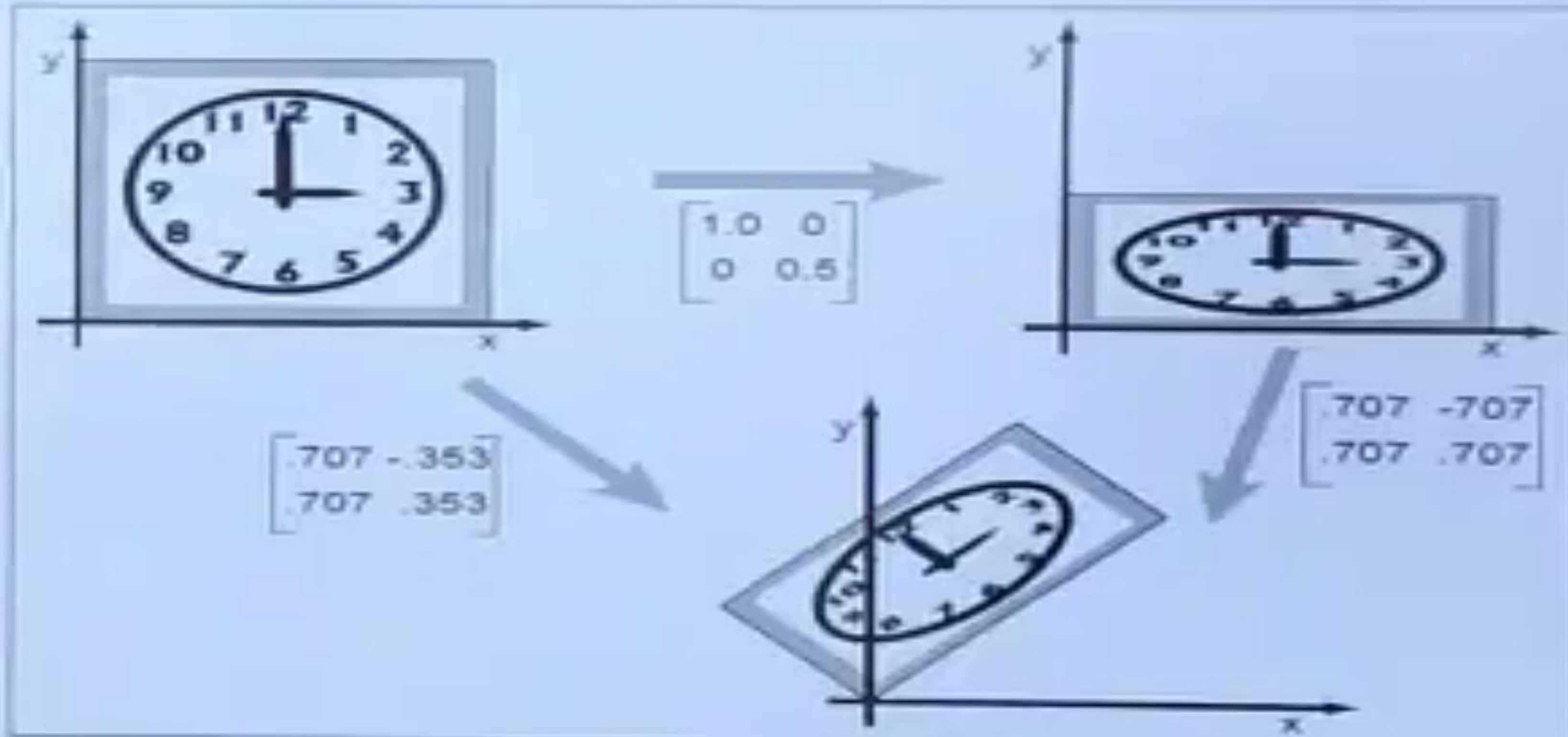

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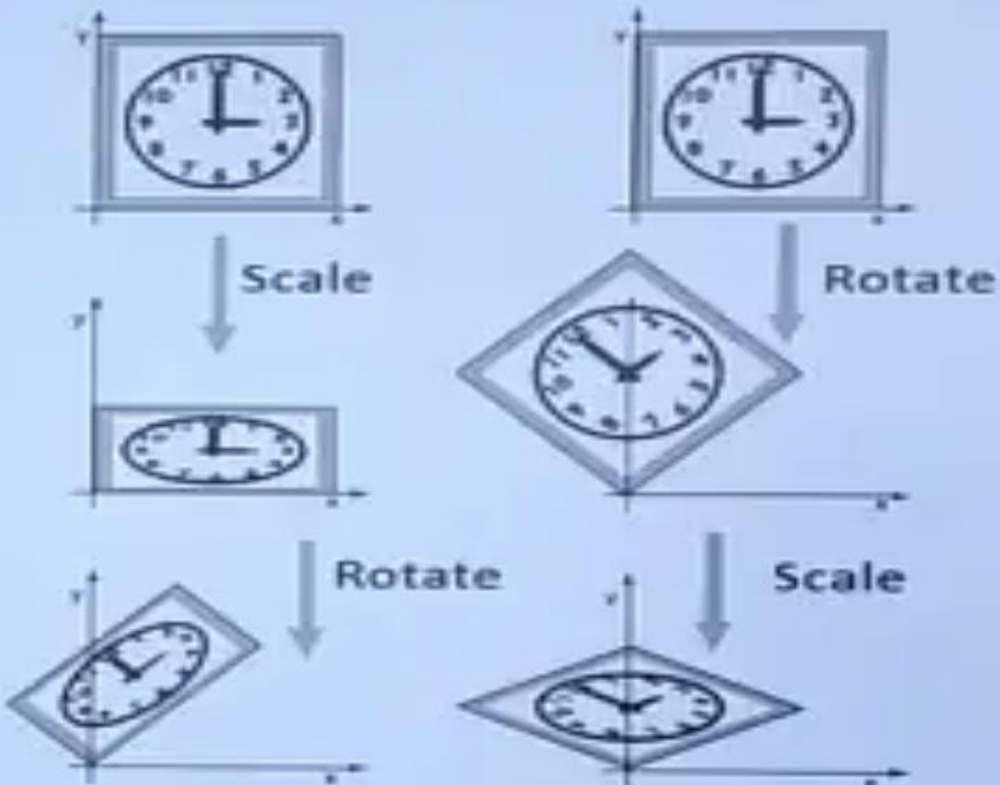
Composition of 2D Transformations

Example-2:



The order in which two transforms are applied is usually important. In this example, we do a scale by one-half in y and then rotate by 45°. Reversing the order in which these two transforms are applied yields a different result.

Homogeneous Coordinates

- The matrix representations for translation, scaling, and rotation are, respectively,

$$P' = T + P,$$
$$P' = S \cdot P,$$
$$P' = R \cdot P.$$

- Unfortunately, translation is treated differently (as an addition) from scaling and rotation (as multiplications). We would like to be able to treat all three transformations in a consistent way, so that they can be combined easily.

- If points are expressed in *homogeneous coordinates*, all three transformations can be treated as multiplications.

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Homogeneous Coordinates

- In homogeneous coordinates, we add a third coordinate to a point. Instead of being represented by a pair of numbers *(x, y)*, each point is represented by a triple (x, *y, W)*.
- At the same time, we say that two sets of homogeneous coordinates (x, *y, W)* and *(x', y', W')* represent the same point if and only if one is a multiple of the other. Thus, (2, 3 , 6) and (4, 6, 12) are the same points represented by different coordinate triples.
- That is, each point has many different homogeneous coordinate representations. Also, at least one of the homogeneous coordinates must be nonzero: (0, 0, 0) is not allowed.
- If the W coordinate is nonzero, we can divide through by it: *(x, y, W)* represents the same point as *(x/W, y/W, 1)*. Where W ≠ 0.

2D Translation in Homogeneous Coordinates

The Because points are now three-element column vectors, transformation matrices, which multiply a point vector to produce another point vector, must be 3 x 3. In the 3 x 3 matrix form for homogeneous coordinates, the translation equations are

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2D Translation in Homogeneous Coordinates

Example:

What happens if a point P is translated by $T(d_{x1}, d_{y1})$ *to* P' and then translated by $T(d_{x2}, d_{y2})$ to P''?

The result we expect intuitively is a net translation $T(d_{x1} + d_{x2}, d_{y1} + d_{y2})$. To confirm this intuition, we start with the given:

$$P' = T(d_{x1}, d_{y1}) \cdot P,$$

$$P'' = T(d_{x2}, d_{y2}) \cdot P'.$$

Now, substituting, we obtain

$$P'' = T(d_{x2}, d_{y2}) \cdot (T(d_{x1}, d_{y1}) \cdot P) = (T(d_{x2}, d_{y2}) \cdot T(d_{x1}, d_{y1})) \cdot P$$

The matrix product $T(d_{x2}, d_{y2}) \cdot T(d_{x1}, d_{y1})$ is

$$\begin{bmatrix} 1 & 0 & d_{x2} \\ 0 & 1 & d_{y2} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & d_{x1} \\ 0 & 1 & d_{y1} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & d_{x1} + d_{x2} \\ 0 & 1 & d_{y1} + d_{y2} \\ 0 & 0 & 1 \end{bmatrix}$$

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2D Scaling in Homogeneous Coordinates

The scaling equations are represented in matrix form in homogeneous coordinates as

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Thus defining

$$S(s_x, s_y) = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2D Scaling in Homogeneous Coordinates

Just as successive translations are additive, we expect that successive scaling should be multiplicative. Given

$$P' = S(s_{x1}, s_{y1}) \cdot P,$$
$$P'' = S(s_{x2}, s_{y2}) \cdot P',$$

then, substituting we get

$$P'' = S(s_{x2}, s_{y2}) \cdot (S(s_{x1}, s_{y1}) \cdot P) = (S(s_{x2}, s_{y2}) \cdot S(s_{x1}, s_{y1})) \cdot P$$

The matrix product $S(s_{x2}, s_{y2}) \cdot S(s_{x1}, s_{y1})$ is

$$\begin{bmatrix} s_{x2} & 0 & 0 \\ 0 & s_{y2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_{x1} & 0 & 0 \\ 0 & s_{y1} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_{x1} \cdot s_{x2} & 0 & 0 \\ 0 & s_{y1} \cdot s_{y2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2D Rotation in Homogeneous Coordinates

The rotation equations are represented in matrix form in homogeneous coordinates as

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Thus we have,

$$\boldsymbol{R}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So we have,

$$P' = R(\theta) \cdot P$$

2D Rotation in Homogeneous Coordinates

If we apply two successive rotations of angles α and Θ, then the rotation equations can be represented in matrix form in homogeneous coordinates as

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\alpha + (-\sin\theta)\cdot\sin\alpha + 0\cdot 0 & \cos\theta\cdot(-\sin\alpha) + (-\sin\theta)\cdot\cos\alpha + 0\cdot 0 & \cos\theta\cdot 0 + (-\sin\theta)\cdot 0 + 0\cdot 1 \\ \sin\theta\cos\alpha + \cos\theta\cdot\sin\alpha + 0\cdot 0 & \sin\theta\cdot(-\sin\alpha) + \cos\theta\cdot\cos\alpha + 0\cdot 0 & \sin\theta\cdot 0 + \cos\theta\cdot 0 + 0\cdot 1 \\ 0\cdot\cos\alpha + 0\cdot\sin\alpha + 1\cdot 0 & 0\cdot(-\sin\alpha) + 0\cdot\cos\alpha + 1\cdot 0 & 0\cdot 0 + 0\cdot 0 + 1\cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\alpha - \sin\theta\sin\alpha & -(\cos\theta\sin\alpha + \sin\theta\cos\alpha) & 0 \\ \sin\theta\cos\alpha + \cos\theta\sin\alpha & -\sin\theta\sin\alpha + \cos\theta\cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta+\alpha) & -\sin(\theta+\alpha) & 0 \\ \sin(\theta+\alpha) & \cos(\theta+\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This demonstrates that 2 successive rotations are additive.

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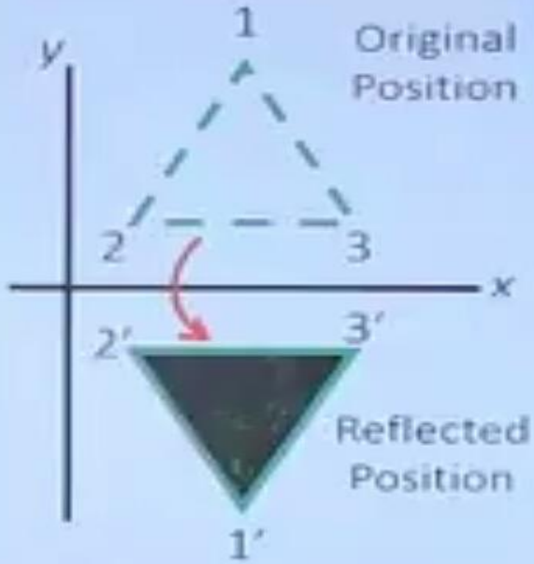
2D Reflection in Homogeneous Coordinates

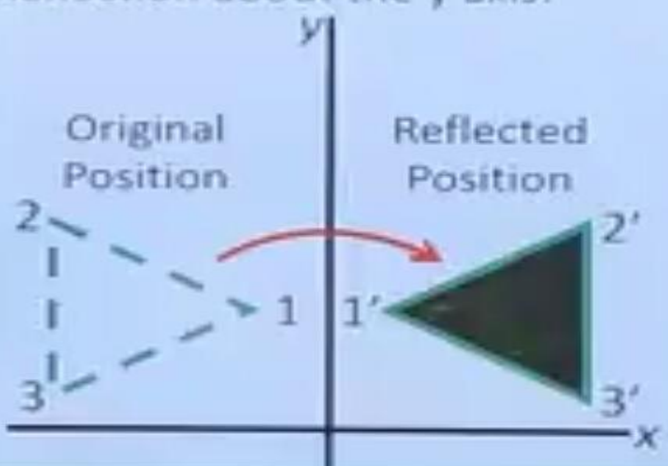
Reflection about the x axis:



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

i.e. $x' = x$; $y' = -y$

Reflection about the y axis:

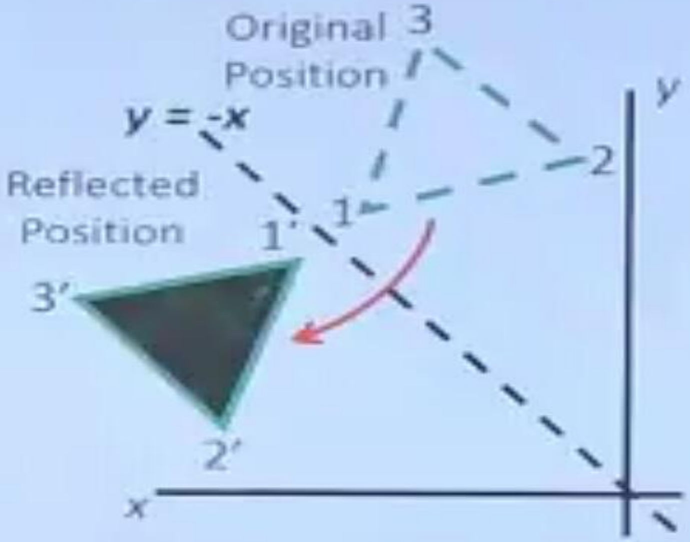


$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

i.e. $x' = -x$; $y' = y$

2D Reflection in Homogeneous Coordinates

Flipping both x and y coordinates of a point relative to the origin:



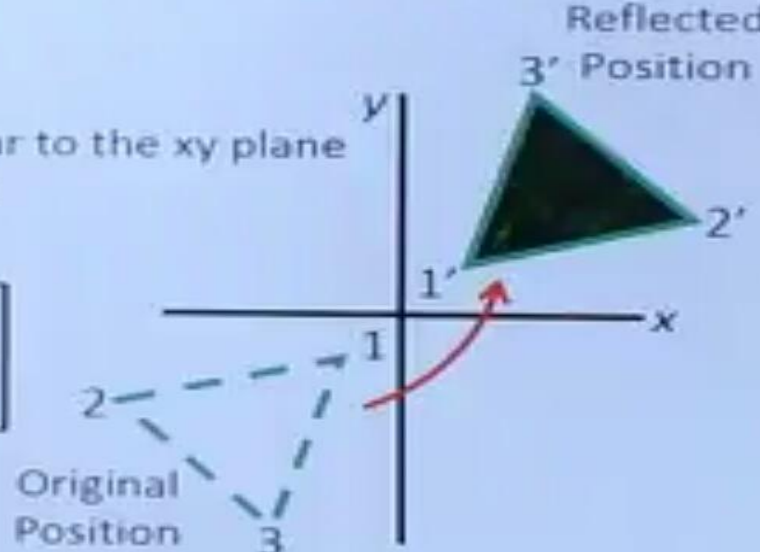
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

i.e. $x' = y$; $y' = x$

Reflection about the axis perpendicular to the xy plane Passing through the coordinate origin:

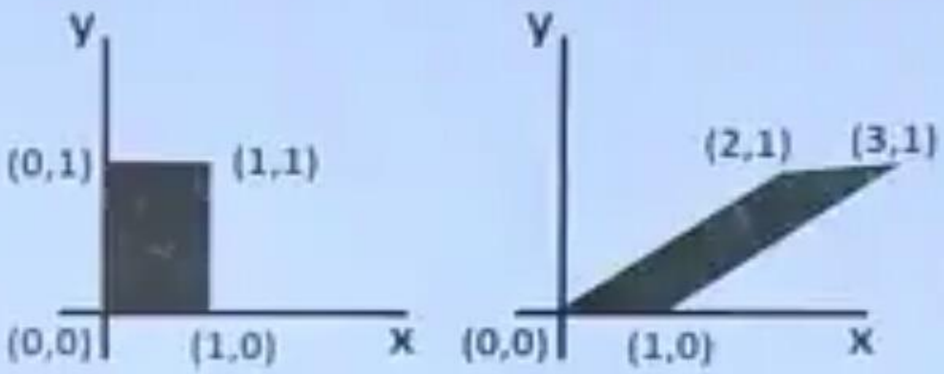
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

i.e. $x' = -x$; $y' = -y$



2D Shear in Homogeneous Coordinates

X-direction shear, with a shearing parameter sh_x, relative to the x-axis:



(a) (b)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

i.e. $x' = x + y * sh_x$; $y' = y$

Y-direction shear, with a shearing parameter sh_y, relative to the y-axis:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

i.e. $x' = x$; $y' = y + x * sh_y$

-By Manjula. S

Inverse Transformations

- Inverse translation matrix
  - Translate in the opposite direction

$$T^{-1} = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

- Inverse rotation matrix
  - Rotate in the clockwise direction

$$R^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Inverse scaling matrix

$$S^{-1} = \begin{bmatrix} \frac{1}{s_x} & 0 & 0 \\ 0 & \frac{1}{s_y} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thank You

-By Manjula. S