No. of Printed Pages : 3

GE-187

VI Semester B.E. (CSE/ISE) Examination, June/July - 2019

2K11CI62

Probability and Stochastic Processes

Time : 3 Hours Max. Marks : 100

Instruction : Answer **any five** full questions selecting atleast **TWO** from each part.

PART - A

1. (a) Consider four computer firms A, B, C and D bidding for a certain contract. A survey of past bidding success of these firms on similar contracts shows the following probabilities of winning : **10**

   $P(A) = 0.35,\ P(B) = 0.15,\ P(C) = 0.3,\ P(D) = 0.2$

   Before the decision is made to award the contract, firm B withdraws its bid. Find the new probabilities of winning the bid for A, C and D.

   (b) A box contains 10 red and 12 blue balls. Two balls are drawn at random and are discarded without their colors being seen. What is the probability that a third ball drawn is blue ? **10**

2. (a) Consider tossing a coin three times. Let X be a random variable denoting the number of heads. Determine the cumulative distribution function $F(x)$. **8**

   (b) A random variable X has the following pdf : **12**

$$f(x) = \begin{cases} Cx(1-x) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   (i) Find the value of C.

   (ii) Compute $P\left(\frac{1}{2} \le X \le \frac{3}{4}\right)$.

   (iii) Find the CDF $F(x)$.

   (iv) Sketch the plot of $F(x)$ against x.

P.T.O.

(a) Among the 120 participants for a job, only 80 are actually qualified. If 5 of the applicants are randomly selected for the final interview, find the probability that only two of the five will be qualified for the job by using : **10**

(i) Hypergeometric distribution.

(ii) Binomial distribution.

(b) Bulbs manufactured by a company have a mean life of 23 hours. A purchase agent requires 1000 bulbs. **10**

(i) What is the probability that a single bulb will last more than 28 hours ?

(ii) How many of them will last at most 20 hours ?

(a) Let X be a random variable denoting the lifetime or time to failure of a component. Derive expressions for the following : **12**

(i) Instantaneous failure rate

(ii) Cumulative failure rate

(iii) Conditional failure density

(iv) Conditional reliability

(b) Let X and Y have joint pdf : **8**

$$f(x,y) = \begin{cases} \frac{1}{\pi} & x^2+y^2 \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the marginal pdf's of X and Y. Are X and Y independent.

PART - B

(a) Consider a computer system with poisson job-arrival stream at an average rate of 60 per hour. Determine the probability that the time interval between successive job arrivals is : **12**

(i) Longer than 4 minutes.

(ii) Shorter than 8 minutes.

(iii) Between two and six minutes.

(b) For a cascade of binary communication channels, let $P(X_0=1)=\alpha$ and $P(X_0=0)=1-\alpha$, $\alpha \ge 0$, and assume that a=b. Compute the probability that a one was transmitted, given that a one was received after the n^{th} stage; i.e compute : **8**

$P(X_0=1/X_n=1)$.

6. (a) Assume that a computer system is in one of three states : busy, idle, or undergoing repair, respectively denoted by states 0, 1 and 2. Observing its state at 2 pm each day, we believe that the system approximately behaves like a homogeneous Markov chain with the transition probability matrix : **10**

$$P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{bmatrix}$$

Prove that the chain is irreducible, and determine the steady state probabilities.

(b) Suppose that customers arrive at a bank according to a poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes. **10**

(i) Exactly 4 customers arrive.

(ii) More than 4 customers arrive.

7. (a) Explain the M/M/I queuing system in detail. **10**

(b) What are pure birth and death processes ? Explain each of them for constant and linear rate. **10**

8. (a) Derive an expression for average system throughput for a closed queuing network under mono programming (i.e. n=1). **10**

(b) Discuss the differences between open queuing networks and closed queuing networks. **10**

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