BE – 170

VI Semester B.E. (CSE/ISE) Degree Examination, December 2016
(2K11 Scheme)
CI62 : PROBABILITY AND STOCHASTIC PROCESSES

Time : 3 Hours Max. Marks : 100

***Instruction** : Answer **any five** questions selecting atleast **two** from **each** Part.*

PART – A

1. a) A box contains 10 red and 12 blue balls. Two balls are drawn at random and are discarded without their colors being seen. What is the probability that a third ball drawn is blue ? **8**

   b) A random variable X has the following probability distribution function (Table 1) :

Table 1 : Probability distribution of X

x	0	1	2	3	4	5	6	7
P(X = x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find :

i) The value of k

ii) Evaluate $P\{X < 6\}$

iii) Evaluate $P\{X \geq 6\}$

iv) If $P[X \leq c] > \frac{1}{2}$, then find the minimum value of c. **12**

2. a) A random variable X has the following probability density function

$$f(x) = \begin{cases} cx(1-x) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

i) Find the value of c.

ii) Compute $P\left[\frac{1}{2} \leq X \leq \frac{3}{4}\right]$.

P.T.O.

iii) Find the cumulative distribution function F(x).

iv) Sketch the plot of F(x) against x. **12**

b) The random variable X has probability density function f(x) where

$$f(x) = \begin{cases} \dfrac{4}{x^5} & x \geq 1 \\ 0 & x < 1 \end{cases}$$

Find the median, mean, variance and standard deviation of the random variable X. **8**

3. a) Bits are sent over a communications channel in packets of 12.

i) If the probability of a bit being corrupted over this channel is 0.1 and such errors are independent, what is the probability that no more than 2 bits in a packet are corrupted ?

ii) If 6 packets are sent over the channel, what is the probability that at least one packet will contain 3 or more corrupted bits ?

iii) Let X denote the number of packets containing 3 or more corrupted bits. What is the probability that X will exceed its mean by more than 2 standard deviations ? **12**

b) Electric trains on a certain line run every half an hour between mid-night and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least twenty minutes ? **8**

4. a) 1,000 RAM IC chips are purchased from two different semiconductor houses. Let X and Y denote the time to failure of the chips purchased from two suppliers. The joint probability density of X and Y is estimated by :

$$f_{X,Y}(x,y) = \begin{cases} \lambda\mu e^{-(\lambda x+\mu y)} & x \geq 0, y > 0 \\ 0 & \text{Otherwise} \end{cases}$$

Assume $\lambda = 10^{-5}$ and $\mu = 10^{-6}$.

Determine the probability that the time to failure is greater for chips characterized by X than it is for chips characterized by Y. **10**

b) Let X and Y be two random variables. Then the expectation of their sum is the sum of their expectations; that is, if Z = X + Y, then E[Z] = E[X + Y] = E[X] + E[Y]. **10**

PART – B

5. a) Define the following types of stochastic processes with an example for each.
   i) Strict-sense stationary
   ii) Independent
   iii) Renewal
   iv) Markov processes. **12**

   b) Suppose that people arrive at a service counter in ac-cordance with a Poisson process with rate λ = 10 per hour. If customers are male with probability p = 1/2, given that 20 males arrived between 10 am and 11 am, how many females would we expect to have arrived in that time ? **8**

6. a) Discuss the superimposition and decomposition of the Poisson process. **10**

   b) Explain the $M|M|1$ queuing system in detail. **10**

7. a) Derive the expressions for the average number of jobs in the system and the average response time for a typical open queuing network. **10**

   b) A group of telephone subscribers is observed continuously during a 80-minute busy-hour period. During this time they make 30 calls, with the total conversation time being 4,200 seconds. Compute the call arrival rate and the traffic intensity. **10**

8. Write short notes on the following :
   a) Pure birth and death process
   b) Markov chains with absorbing states
   c) Closed queuing networks
   d) Random walks. **(5×4=20)**