JE – 327

V Semester B.Tech. (ISE) Degree Examination, February 2021
(CBCS Scheme)
18CIPC 502 : PROBABILITY AND STOCHASTIC PROCESSES

Time : 3 Hours Max. Marks : 100

Instruction : *Q. 1, Q. 2 and Q. 9 are* ***compulsory****. Answer Q. 3 or Q. 4, Q. 5 or Q. 6, Q. 7 or Q. 8.*

1. a) Define the term 'Median' with an example. **(15×1=15)**

b) What is 'Standard Deviation' in a distribution ?

c) What do you mean by 'Infinite Sample Space' ? Provide an example.

d) If A and B are not mutually exclusive events, then $A \cup B$ = ________

e) If A and B are independent events, then P (A|B) = ________

f) What do you mean by 'Prior' probability ?

g) List the two properties of a probability density function pdf.

h) If X is a continuous random variable, then E(X) = ________

i) Define Covariance.

j) Write the 'pmf' of hypergeometric distribution.

k) What is 'memorylessness property' in exponential distribution ?

l) Give the 'mean' and 'variance' of generalized normal distribution.

m) Define 'autocorrelation' of a stochastic process.

n) Define Discrete-Time Markov Chain.

o) Give the Chapman-Kolmogorov equation.

2. a) Suppose that five good fuses and two defective ones have been mixed up. To find the defective fuses, we test them one-by-one at random and without replacement. What is the probability that we are lucky and find both of the defective fuses in the first two tests ? **10**

b) A card is chosen at random from a deck of 52 playing cards. What is the probability that a randomly chosen card is a 'king' given the evidence that the card chosen is a 'face' card ? **7**

3. a) A random variable has the following probability distribution function. 10

x	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find :

i) The value of k.

ii) Evaluate $P(X < 6)$.

iii) If $P(X \le c) > \frac{1}{2}$, then find the minimum value of c.

b) Suppose that X is a continuous random variable with pdf given by : 7

$$f(x) = \begin{cases} 2x & x \in (0, 1) \\ 0 & \text{otherwise} \end{cases}$$

Find the expectation and variance of X.

OR

4. a) The amount of time in hours that an electric bulb functions before breaking down is a continuous random variable with pdf given by : 10

$$f(x) = \begin{cases} \lambda e^{-x/100} & \text{if } x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that

i) The bulb will function between 200 to 300 hours before breaking down and

ii) It will function for less than 250 hours.

b) If X and Y are discrete random variables, then prove that

$E[X + Y] = E[X] + E[Y]$. 7

5. a) A bulb manufacturing company is known to produce 5% defective. In a random sample of 15 bulbs, what is the probability that there are :

i) Exactly 3 defectives

ii) Not more than 3 defectives. 8

b) Messages arrive at a computer at an average rate of 15 messages per second. The number of messages that arrive in 1 second is known to be a Poisson random variable.

i) Find the probability that no messages arrive in 1 second.

ii) Find the probability that more than 10 messages arrive in a 1-second period. 9

OR

6. a) After the first 6 hours, the lifetime of a cell phone batteries are exponentially distributed with an average remaining lifetime of 18 hours. Let the random variable X measure the remaining lifetime after these initial 6 hours, and let the random variable Y measure the total lifetime.

i) Give the pdf and cdf of X.

ii) Compute E(X), σ(X), E(Y) and σ(Y). 10

b) Electric trains on a certain line run every half an hour between mid-night and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least twenty minutes ? 7

7. a) Classify the stochastic process into four different types based on the parameter space T and state space I. 7

b) Consider an example of the Gambler's Ruin. A gambler enters a casino with $50 with him to play. He decides to play the wheel roulette. At each spin, he places $25 on red. If red occurs, he wins $25. If black comes up, he loses $25. Therefore, the probability of winning is 50% and the probability of losing is 50%. He will quit playing when he either has zero money left or he has $75 in total. Derive the long run probabilities and find the probability of losing all his money or gaining an addition $25 if he starts the game with $25 and $50 in his pocket. 10

OR

8. a) Suppose that the probability of a dry day following a rainy day is $\frac{1}{3}$ and the probability of a rainy day following a dry day is $\frac{1}{2}$. Given that May 1 is a dry day. Find the probability that May 3 is a dry day and also May 5 is a dry day. 7

b) Consider a Markov chain with state space {0, 1} and the tpm :

$$p = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

i) Draw the transition diagram.

ii) Show that state 0 is recurrent.

iii) Show that state 1 is transient.

iv) Is the state 1 is periodic ? If so, what is the period ?

v) Is the chain irreducible ?

vi) Is the chain ergodic ? Explain. 10

9\. a) Consider a M|M|m queuing system and derive the Erlang's C formula. 7

b) Consider a variation of the queuing model of figure given below, where the CPU node consists of two parallel processors with a service rate of μ_0 each. Draw a state diagram for this system and proceed to solve the balance equations. Obtain an expression for the average response time E[R] as a function of μ_0, μ_1, p_0 and λ. 10

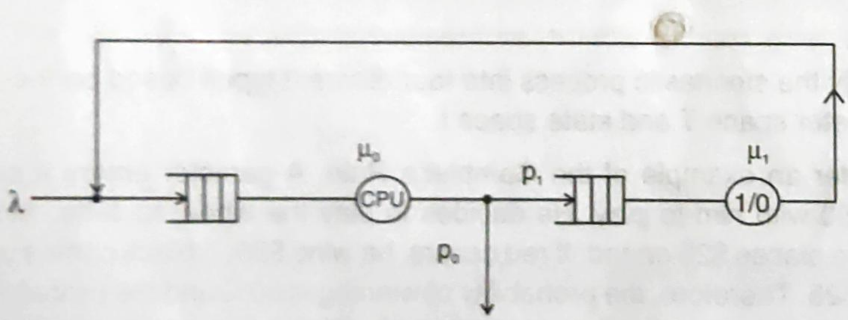

Fig. Q. No. 9.(b)