ED – 775

VI Semester B.E. (CSE/ISE) Degree Examination, Dec. 2014/Jan. 2015
(2K11 Scheme)
CI 62 : PROBABILITY AND STOCHASTIC PROCESSES

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　　　Max. Marks : 100

***Instruction** : Answer **any five** questions, selecting atleast **two** from **each** Part.*

PART – A

1. a) A given lot of IC chips contain 2 percent defective chips. Each chip is tested before delivery. The tester itself is not totally reliable so that :

   P("Tester says chip is good"| "Chip is actually good") = 0.95

   P("Tester says chip is defective"| "Chip is actually defective") = 0.94

   If a tested device is indicated to be defective, what is the probability that it is actually defective ? **10**

   b) A certain firm has plants A, B and C producing, respectively, 35%, 15% and 50% of the total output. The probabilities of a non defective product are respectively, 0.75, 0.95 and 0.85. A customer receives a defective product. What is the probability that it came from plant C ? **10**

2. a) Consider tossing a coin three times. The possible outcomes are contained in Table 1 and the values of f is given in Equation 1. Determine the cumulative distribution function.

Table 1 : Tossing a coin three times.

Element of sample space	Probability	Value of the random variable X,x
HHH	1/8	3
HHT	1/8	2
HTH	1/8	2
THH	1/8	2
HTT	1/8	1
THT	1/8	1
TTH	1/8	1
TTT	1/8	0

$$P(X=0)=\frac{1}{8};P(X=1)=\frac{3}{8};P(X=2)=\frac{3}{8};P(X=3)=\frac{1}{8} \text{ equation (1)}$$

P.T.O.

	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

10

b) The ages of people in India is a continuous random variable with PDF, f(x) is given by :

$$f(x)=\begin{cases} \frac{1}{80} & 0\le x\le 60 \\ \frac{100-x}{3200} & 60\le x\le 100 \\ 0 & \text{otherwise} \end{cases}$$

i) Find the corresponding cumulative distribution function and

ii) Using the cumulative distribution function, find the probability that a Indian is between 50 and 60 years of age. **10**

3. a) In a period of time, 5 out of 20 screws produced by a manufacturing company are found to be defective. If 10 of the total screws are selected at random for inspection, what is the probability that 2 of that 10 will be defective ? **10**

b) Suppose the average number of lions seen on a 1-day safari is 5. What is the probability that tourists will see fewer than four lions on the next 1 day safari ? **10**

4. a) If jobs arrive every 15 seconds on average, λ = 4 per minute, what is the probability of waiting less than or equal to 30 seconds, i.e. 5 minutes ? **10**

b) The joint probability mass function (pm f) of X and Y is given the table below: Compute the following :

i) $P(X \le 1)$

ii) $P(Y \le 3)$

iii) $P(X\le 1, Y\le 3)$

iv) $P(X \le 1|Y \le 3)$

v) $P(Y \le 3|X \le 1)$

vi) $P(X + Y \le 4)$. **10**

PART – B

5. a) A group of telephone subscribes is observed continuously during a 80-minute busy-hour period. During this time they make 30 calls, with the total conversation time being 4,200 seconds. Compute the cell arrival rate and the traffic intensity. **10**

b) Suppose that the probability of a dry day following a rainy day is 1/3 and the probability of a rainy day following a dry day is 1/2. Given that May 1 is a dry day. Find the probability that May 3 is a dry day and also May 5 is a dry day. **10**

6. a) Consider a Markov chain with state space {0,1} and the tpm is given by

$$P = \begin{Bmatrix} 1 & 0 \\ 1/2 & 1/2 \end{Bmatrix}$$

i) Draw the transition diagram

ii) Show that state 0 is recurrent

iii) Show that state 1 is transient

iv) Is the state 1 is periodic ? If so, what is the period ?

v) Is the chain irreducible ?

vi) Is the chain ergodic ? Explain. **10**

b) Consider an M|M|1 queuing system in which the total number of jobs is limited to n owing to a limitation on queue size.

i) Find the steady -state probability that an arriving request is rejected because the queue is full.

ii) Find the steady-state probability that the processor is idle.

iii) Given that a request has to be accepted, find its average response time. **10**

7. a) Explain the M/M/1 queuing system in detail. **10**

b) Discuss the differences between open queuing networks and closed queuing networks. **10**

8. a) Derive a closed for expression for average system throughput for a closed queuing network under monoprogramming (i.e., $n = 1$). **10**

b) Consider a pure birth process with birth rate λ_j . Let $X(t)$ denote the population size at time t, and let $P_{ij}(t) = P(X(t) = j|X(0) = i)$.

i) Write down the Kolmogorov forward equations for a general pure birth process.

ii) Show that if $\lambda_j = j\lambda$, then

$$P_{1j} = \exp(-\lambda t)(1 - \exp(-\lambda t))^{j-1}, j \geq 1$$

iii) Show that for a general pure birth process with birth rate λ_j, that

$$P_{ij} = \lambda_{j-1} \exp(-\lambda_j t) \int_0^t \exp(\lambda_j s) P_{i,j-1}(s)\, ds \quad \text{for } j > i.$$

Assume $P_{ij}(0) = 0$ and $P_{ii} = 1 \ \forall$ i and j. **10**
