CFRM 541 - Quantitative Risk Management
Homework 1

Due: January 26, 2025 - 11:59 pm

Question 1 (10 pts). Consider the options data in the OptionsPivotTables.xls Excel workbook. We want to consider joint stresses of the underlying security price and implied volatility. In particular, we want to consider shifting the underlying security by -20%, -10%, -5%, 0%, +5%, +10% and +20%, and shifting the implied volatility by -5, -2, 0, +2, +5, +10 and +20 volatility points. We therefore have a total of $7 \times 7 = 49$ scenarios. Write a Python program that takes all the options data in the OptionsData worksheet, computes the P&L for each option under each scenario and outputs the results to the same spreadsheet beginning at column AR. *Hint*: To see the code behind functions such as BSPrice type *Alt-F11*.

Question 2 (10 pts).

1. Once the P&L's from the 49 scenarios have been calculated and written to the Options Data worksheet, create a pivot-table that can display the P&L from one of these scenarios as a function of strike (rows), maturity (columns) and underlying security (page or filter).
2. Create additional pivot tables for appropriate delta, gamma, vega, volga and vanna numbers. Check to see if the numbers in these pivot tables are consistent with the pivot table in (a) when the underlying security is shifted by -10% and the implied volatilities increase by +10 volatility points.
3. Suppose you want to create a pivot-table to view the P&L data as a function of the shift in the underlying (rows), the shift in the implied volatilities (columns) and the underlying security (page or filter). Is it possible to do this given the way we have organized the data in the OptionsData worksheet? If yes, do it. If not, explain briefly what design changes you would need to have made in Question 1 so that it would be possible to create such a pivot-table.

Note: There are two additional worksheets in the OptionsPivotTables.xls Excel workbook that contain pivot tables based on the data in cells A4 to AQ87 of the OptionsData worksheet. You can play with these pivot tables if you wish to get a better feeling for how pivot tables work.

Question 3 (10 pts).

1. Do you think parallel shifts in the volatility surface represent reasonable scenarios? Can you think of a better set of scenarios for representing shifts in the implied volatility surface? Put another way, suppose a crisis occurs tomorrow and all implied volatilities increase. Do you think near-term volatilities will increase more or less than long-term implied volatilities? (This question is designed to show that designing realistic scenario should be based on risk factors movements during crisis.)
2. Suppose an options portfolio is delta, gamma, vega and vanna neutral. Suppose also that the options portfolio is designed to be very short skew, that is the portfolio will benefit if the skew flattens. Do you think scenarios involving shifts in the underlying security and implied volatility surface will reveal the full risk in the portfolio? Explain your answer. What sort of additional scenarios might you want to include? What issues arise when designing these additional scenarios? (This question should impress on you the importance of considering the trading strategy / portfolio composition when designing scenarios. A more basic example occurs if you consider a market-neutral portfolio of stocks. Then just shifting the underlying securities by uniform amounts will not reveal any risk in the portfolio. Likewise, a fixed income portfolio that has a zero duration will reveal very little risk if it is only subjected to stresses based on parallel shifts in the yield curve. Such a portfolio might still be very risky if, for example, it is exposed to changes in the slope of the yield curve or other risk factors.)

Question 4 (10 pts). Compute the VaR and Expected-Shortfall (ES) for (i) the normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 0.310000/\sqrt{250}$ and (ii) the t4 distribution with the same values of μ and σ. (Note that if we assume that the horizon is $\Delta = 1$ day, then a value of $\sigma = .3/\sqrt{250}$ corresponds to an annual volatility of approx 30%. The value of 250 corresponds to the fact that there are approx. 250 trading days in a calendar year. The multiplier of 10000 is there simply to make the numbers more readable.) You should compute the VaR and ES for the following values of α:

$$0.90,\ 0.95,\ 0.975,\ 0.99,\ 0.995,\ 0.999,\ 0.9999,\ 0.99999,\ 0.999999.$$

What do you notice? Now compute the ES to VaR ratio for each value of α. Report all your results in a table.

Question 5 (10 pts). This question is based on material from Sections 19.2 and 19.3 of Ruppert and Matteson's Statistics and Data Analysis for Financial Engineering. (Feel free to look at those sections but you should try and figure out the details yourself rather than just coping the code there.) First install R's Ecdat package from CRAN and then load the package by including the line "library(Ecdat)" at the beginning of your R session, script or function. We will be using the SP500 dataframe from the Ecdat package. This dataframe contains returns on the S&P500 index between 1981 and 1991.

1. Suppose you hold a 1m position in the S&P500. Use the return data in the SP500 dataframe to estimate the 95% VaR and expected shortfall (ES) of your position.

2. Use R's fitdistr function to fit a t-distribution to the return data. Now use this fitted distribution to again estimate the 95% VaR and ES of your position. Compare your answer to your answer from part 1.