S. No.	Date	Title	Page No.	Teacher's Sign / Remarks

03/01/19

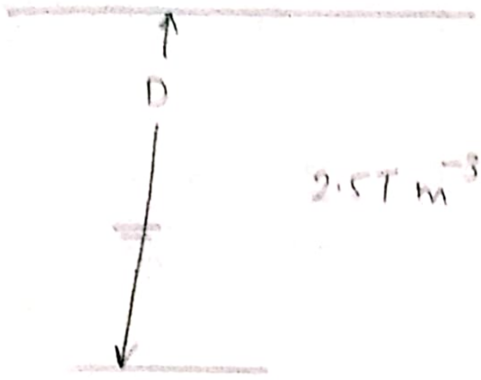
Ques 10 ton load on 54mm dia specimen

Strength $= \frac{F}{A}, \frac{N}{M^2}, \frac{MN}{m^2}, MPa, \frac{Kg}{cm^2}, \frac{kgf}{cm^2}$

$$\frac{10 \times 1000 \times 10}{\frac{\pi (54)^2}{4} \times 10^{-6}}$$

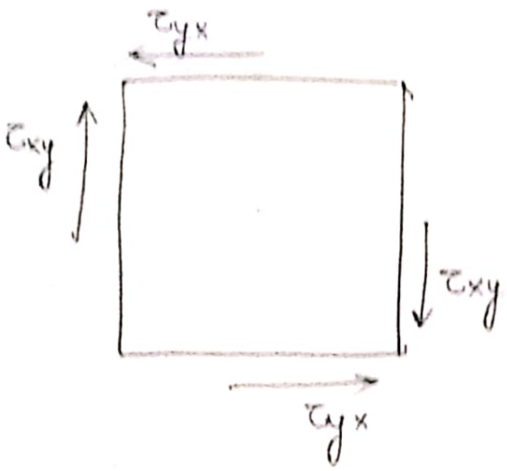
$$= 43.66 \frac{MN}{m^2} = 43.66 MPa$$

04/01/19



2.5 $T\,m^{-3}$ Rock mass density

- Stress will be discussed upon a point Force eqm. and torque eqm. should be maintained.

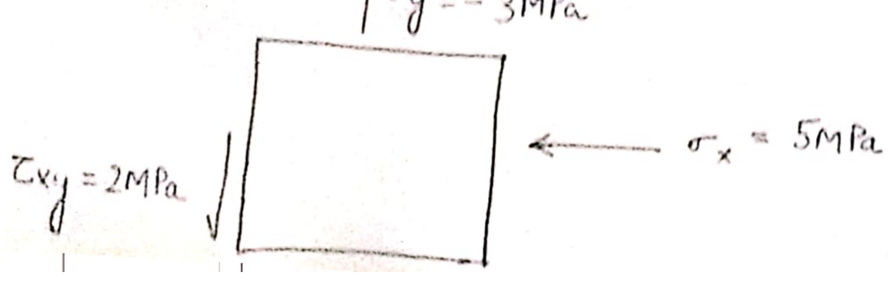


$\left.\begin{matrix}\sigma_{xx} & \sigma_x \\ \sigma_{yy} & \sigma_y\end{matrix}\right\}$ normal stress $\quad$ $\left.\sigma_{xy} = \tau_{xy}\right\}$ shear stress

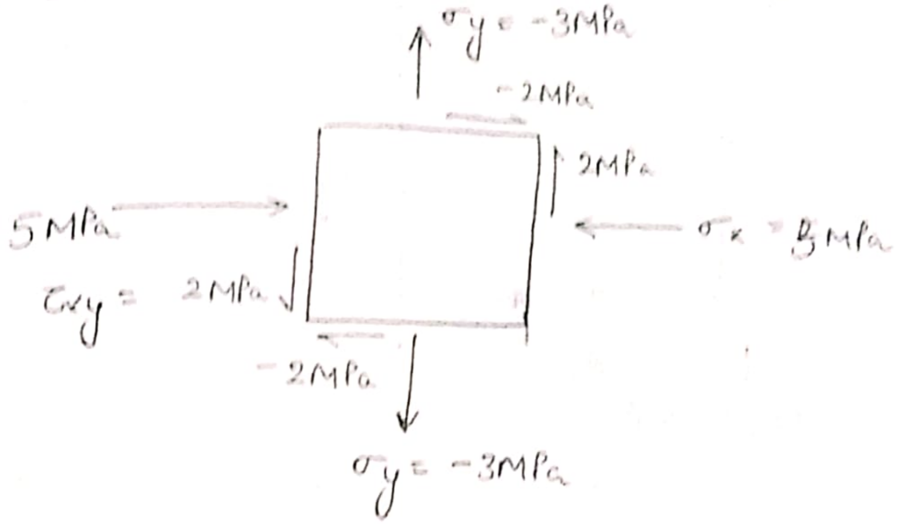
Compression +ve

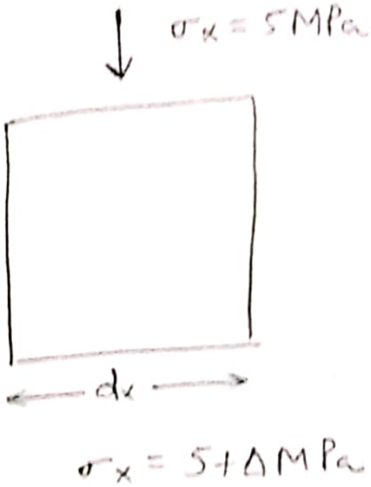
Tension −ve

Anti clockwise +ve $\quad$ Clockwise −ve.



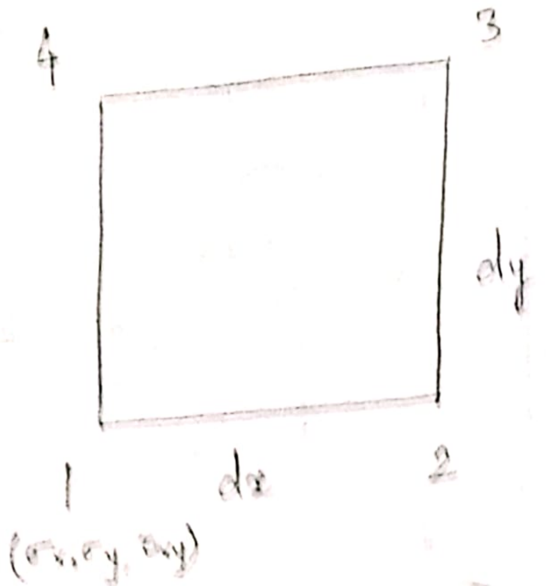
To maintain eqm. opposing and balancing forces will be there.





Δ exists due to body force

f_x has unit $\frac{N}{m^2 \text{ or } m^3}$



$$\sigma_x \longrightarrow \frac{\partial \sigma_x}{\partial x}, \quad \frac{\partial \sigma_x}{\partial y}$$

$$\sigma_y \longrightarrow \frac{\partial \sigma_y}{\partial y}, \quad \frac{\partial \sigma_y}{\partial x}$$

$$\tau_{xy} \longrightarrow \frac{\partial \tau_{xy}}{\partial x}, \quad \frac{\partial \tau_{xy}}{\partial y}$$

<u>At point 2</u>

$$\sigma_x + \frac{\partial \sigma_x}{\partial x} \cdot dx$$

$$\sigma_y + \frac{\partial \sigma_y}{\partial x} \cdot dx$$

$$\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x} \cdot dx$$

<u>At point 4</u>

$$\sigma_x + \frac{\partial \sigma_x}{\partial y} \cdot dy$$

$$\sigma_y + \frac{\partial \sigma_y}{\partial y} \cdot dy$$

$$\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y} \cdot dy$$

At point 3

$$\sigma_x + \frac{\partial \sigma_x}{\partial y} \cdot dy + \frac{\partial \sigma_x}{\partial x} \cdot dx$$

$$\sigma_y + \frac{\partial \sigma_y}{\partial y} \cdot dy + \frac{\partial \sigma_y}{\partial x} \cdot dx$$

$$\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y} \cdot dy + \frac{\partial \tau_{xy}}{\partial x} \cdot dx$$

1-2

$$\bar{\sigma}_y = \frac{(\sigma_y) + \left\{ (\sigma_y) + \left(\frac{\partial \sigma_y}{\partial x} \cdot dx \right) \right\}}{2}$$

$$\bar{\sigma}_y = \sigma_y + \frac{1}{2} \left(\frac{\partial \sigma_y}{\partial x} \cdot dx \right)$$

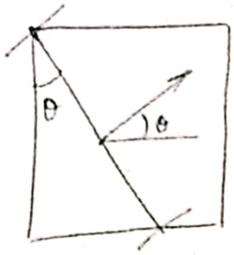
$$F_y \, (1-2) = \bar{\sigma}_y \cdot dx \uparrow$$

3-4

$$\left\{ \sigma_y + \frac{1}{2} \frac{\partial \sigma_y}{\partial x} \cdot dx + \frac{\partial \sigma_y}{\partial y} \cdot dy \right\} \cdot dx \downarrow$$

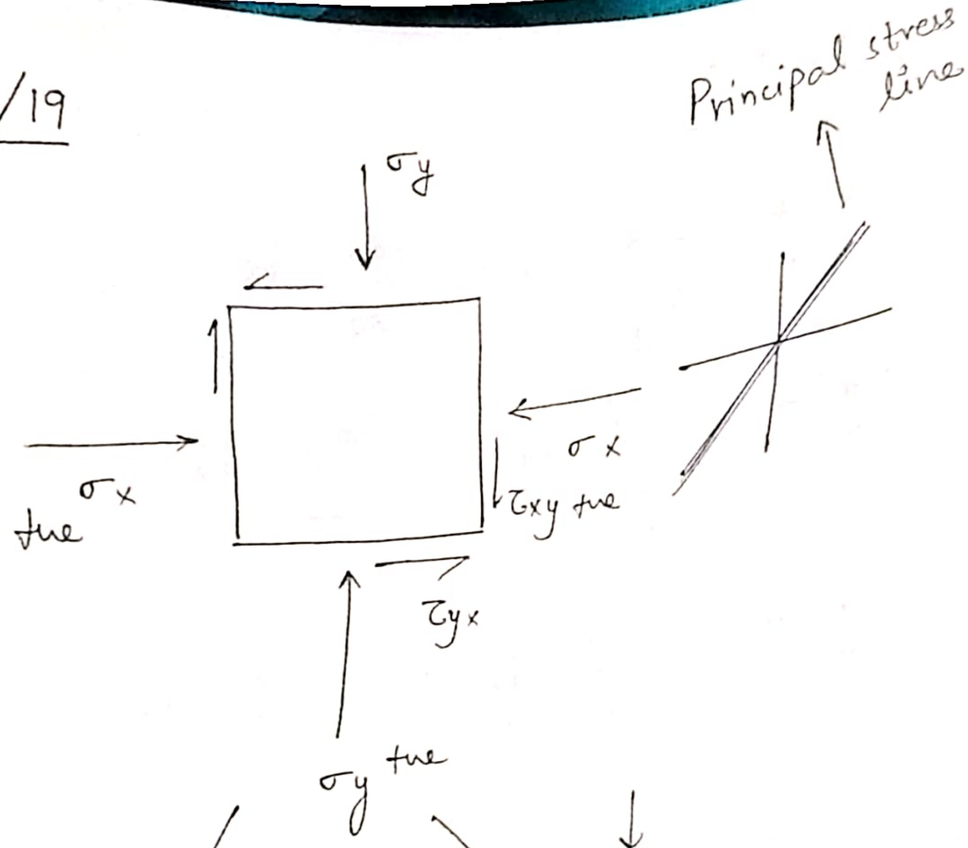
Principal stress

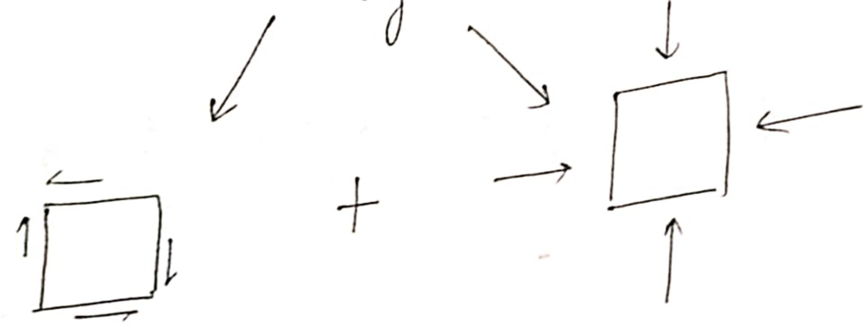
$$\begin{vmatrix} \sigma_1 \\ \sigma_3 \\ 0 \end{vmatrix}_{\theta} \begin{vmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{vmatrix} = \begin{vmatrix} \sigma_{x'} \\ \sigma_{y'} \\ \tau_{x'y'} \end{vmatrix}$$

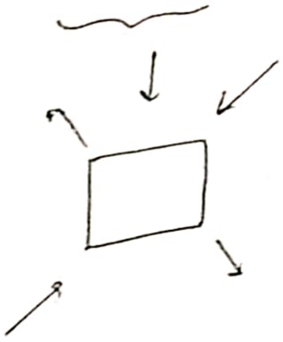


limit

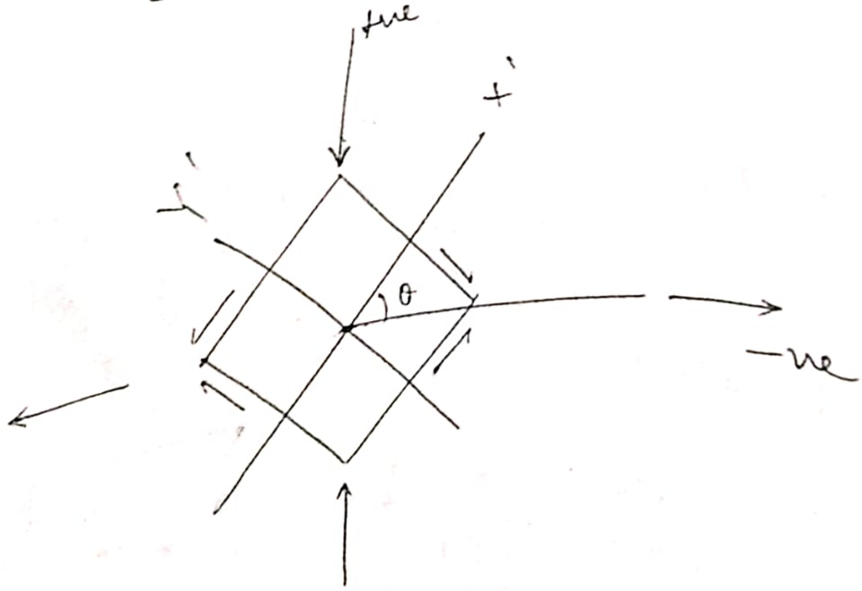
24/01/19

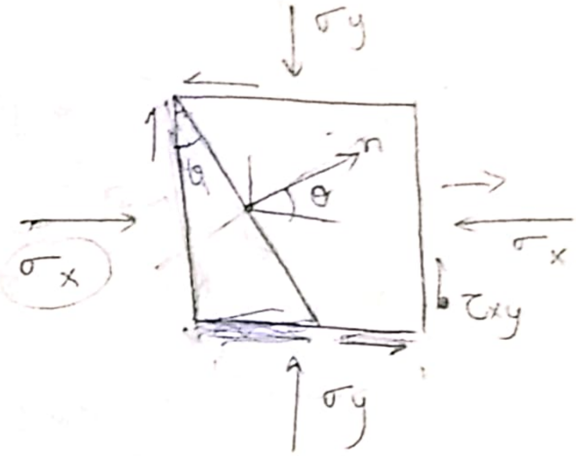






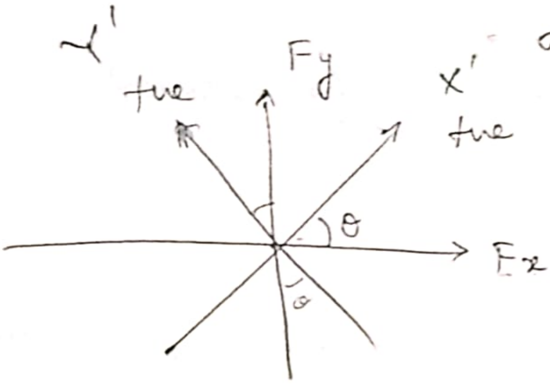
(feliure in mining occurs due to compression)





$F_x = \sigma_x \cos\theta + \tau_{xy} \sin\theta$

$F_y = \sigma_y \sin\theta + \tau_{xy} \cos\theta$



$\sigma_{x'}$

$\sigma_{x'}$ $\sigma_{y'}$

$$F_{x'} = F_x \cos\theta + F_y \sin\theta$$

$$F_{y'} = F_y \cos\theta - F_x \sin\theta$$

$$\sigma_{nn} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

if we treat σ_{nn} as $f(\theta)$ then $\sigma_{y'}$ can be found out by $f(\theta + 90^\circ)$ since $\hat{\sigma_{nn}} \perp \hat{\sigma_{tt}}$.

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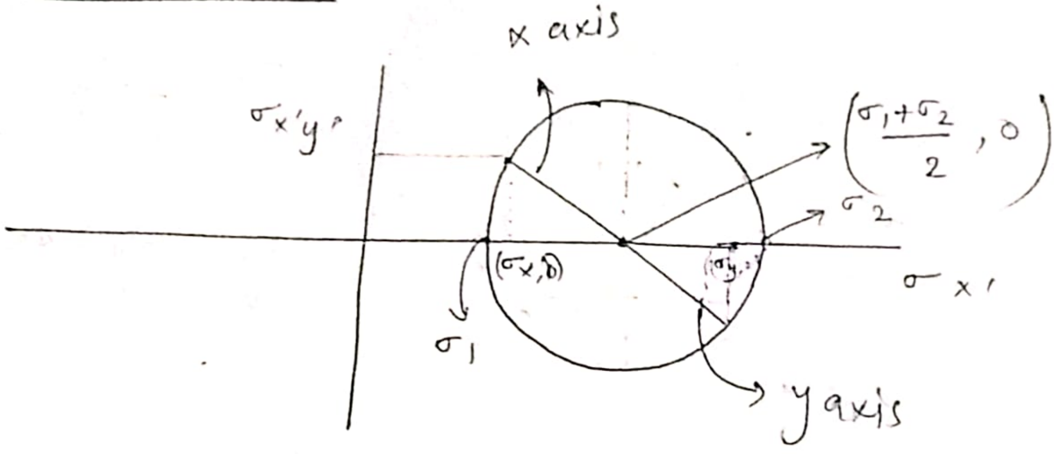
Mohr's Circle



σ_1 & σ_2 are principal stresses.

Homework

$$\sigma_{nn} = (\sigma_x \cos\theta + \tau_{xy}\sin\theta)\cos\theta + (\sigma_y \sin\theta + \tau_{xy}\cos\theta)\sin\theta$$

$$= \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + \tau_{xy}\sin 2\theta$$

$$= \sigma_x\left(\frac{1+\cos 2\theta}{2}\right) + \sigma_y\left(\frac{1-\cos 2\theta}{2}\right) + \tau_{xy}\sin 2\theta$$

$$\boxed{f(\theta) = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta}$$

$$\sigma_{y'} = f(\theta+90^\circ)$$

$$\boxed{\sigma_{y'} = \left(\frac{\sigma_x+\sigma_y}{2}\right) - \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta}$$

$$\tau_{x'y'} = F_y \cos\theta - F_x \sin\theta$$

$$= (\sigma_y \sin\theta + \tau_{xy}\cos\theta)\cos\theta - (\sigma_x \cos\theta + \tau_{xy}\sin\theta)\sin\theta$$

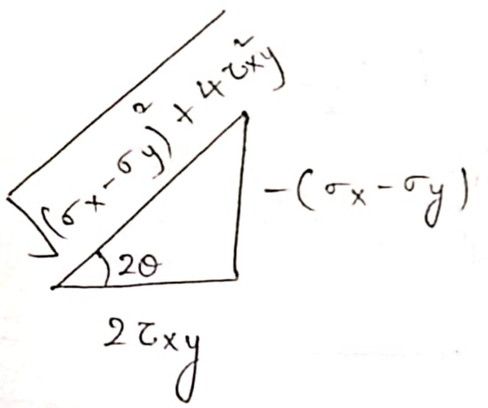
$$= (\sigma_y - \sigma_x)\cos\theta\sin\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= \left(\frac{\sigma_y - \sigma_x}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

for finding principal stresses

$$\frac{d(\tau_{x'y'})}{d\theta} = (\sigma_y - \sigma_x)\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0$$

$$\Rightarrow \tan 2\theta = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$



$$\sin 2\theta = \frac{-(\sigma_x - \sigma_y)/2}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\cos 2\theta = \frac{\cancel{2}\tau_{xy}}{\cancel{2}\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

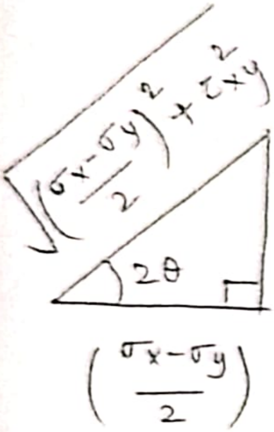
$$\Rightarrow \tau_{x'y'\,max} = \frac{\left[\frac{-(\sigma_x - \sigma_y)}{2}\right]^2}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} + \frac{\tau_{xy}^2}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\Rightarrow \quad \tau_{x'y'max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

<u>Now</u>

$$\frac{d\sigma_{x'}}{d\theta} = +(\sigma_y - \sigma_x)\sin 2\theta + 2\tau_{xy}\cos\theta = 0$$

$$\Rightarrow \quad \tan 2\theta = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)} = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}$$



$$\sin 2\theta = \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\cos 2\theta = \frac{\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\boxed{\sigma_{x'max} = \frac{\sigma_x + \sigma_y}{2} + \cancel{2\tau_{xy}}\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

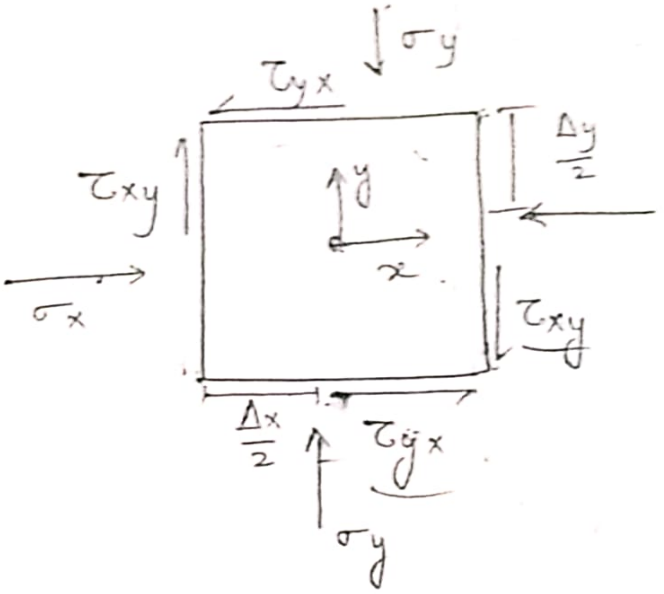
for $\sigma_{y'max}$ 2θ is same as for $\sigma_{x'max}$

$$\boxed{\sigma_{y'max} = \frac{\sigma_x + \sigma_y}{2} - \cancel{2\tau_{xy}}\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

Principal stresses

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \underbrace{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}_{\text{radius for mohr's circle}}$$

Stress eqm. Eqⁿ



cuboid

$(\Delta x, \Delta y, \Delta z)$ sides

Origin at centre

for force balance $\quad \sigma_y, \sigma_x$ shown

for torque balance

$$\left[\underbrace{(\Delta x \times \Delta z) \times \tau_{yx}}_{\text{force}} \times \frac{\Delta y}{\cancel{2}}\right] \underbrace{\times \cancel{2}}_{\text{couple}} + \left\{-\left(\tau_{yx} \times \frac{\Delta y}{2} \times \Delta z \times \Delta x\right) \times 2\right\}$$

$$= \underbrace{\rho}_{\text{density of object}} \underbrace{\Delta x \Delta y \Delta z \frac{(\Delta x^2 + \Delta z^2)}{12}}_{\text{moment of inertia}} \cdot \underset{\alpha \text{ angular accl}^n}{\omega}$$

$$\Rightarrow \tau_{xy}\cancel{(\Delta x \Delta y \Delta z)} - \tau_{yx}\cancel{(\Delta x \Delta y \Delta z)}$$

$$= b\cancel{(\Delta x \Delta y \Delta z)}\frac{(\Delta x^2 + \Delta y^2)}{12}\dot{\omega}$$

by limiting $\Delta x \rightarrow 0$, $\Delta y \rightarrow 0$

$$\Rightarrow \boxed{\tau_{xy} = \tau_{yx}}$$

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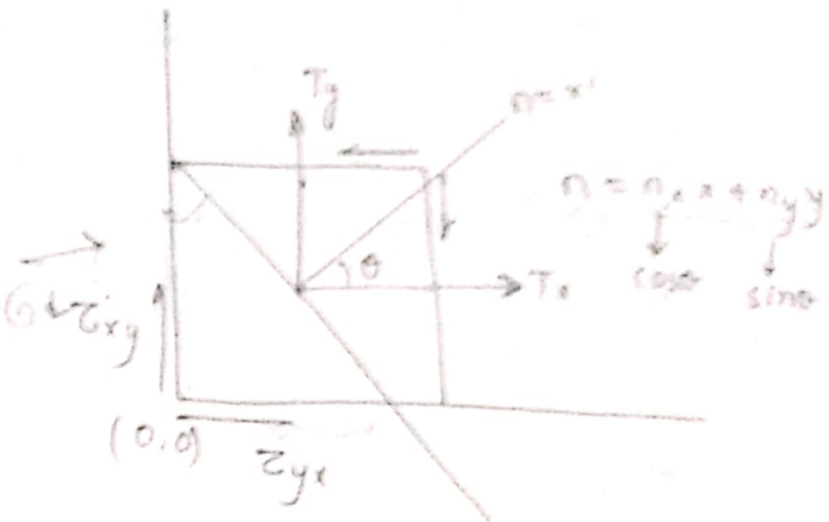
25/01/19

2D-stress

$\sigma_{nn} = \sigma_{x'}$

$\downarrow \quad \downarrow$

$\sigma_{nt} \quad \tau_{x'y'}$



$$T_x = \sigma_x n_x + \tau_{xy} n_y +$$

$$T_y = \sigma_y n_y + \tau_{xy} n_x$$

(Traction force)

$$[\sigma] = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix}_{2\times2} \begin{bmatrix} n_x \\ n_y \end{bmatrix}_{2\times1}$$

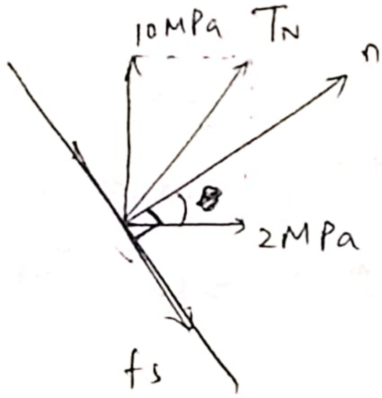
$$[n] = [n_x \quad n_y \quad n_z]$$

$$\underset{2\times1}{[T]} = \underset{2\times2}{[\sigma]}\ \underset{2\times1}{[n]^T}$$

$$\sigma_{x'} = T_x n_x + T_y n_y \begin{pmatrix} T_x & T_y \\ T_y & -T_x \end{pmatrix} \begin{pmatrix} n_x \\ n_y \end{pmatrix}$$

$$\tau_{x'y'} = T_y n_x - T_x n_y$$

31/01/19



~~$\tau_{x'y'} = 10\sin\theta - 2\cos\theta$~~

$$f_s = \underline{10 n_x - 2 n_y} \quad 5$$

$$f_n = 2n_x + 10 n_y \quad 5$$

T_x

T_y

f_n

$f_s = ?$

$$T_N = \sqrt{T_x^2 + T_y^2}$$

$$5 f_s + f_n = 52 n_x$$

$$\boxed{n_x = \frac{5 f_s + f_n}{52}}$$

$$f_s - 5 f_n = -52 n_y$$

$$\boxed{n_y = \frac{5 f_n - f_s}{52}}$$

$$\tau_{x'y'} = 10\left(\frac{5 f_s + f_n}{52}\right)$$

$$- 2\left(\frac{5 f_n - f_s}{52}\right)$$

$$=$$

$\sigma_{x'x'} = T_x n_x + T_y n_y$

$\sigma_{y'y'} = T_x(-\sin\theta) + T_y(\cos\theta)$

$\tau_{x'y'} = -T_x n_y + T_y n_x$] <u>T_x diff.</u>

$$\begin{bmatrix} \sigma_{x'x'} & \sigma_{x'y'} \\ \sigma_{y'x'} & \sigma_{y'y'} \end{bmatrix}$$

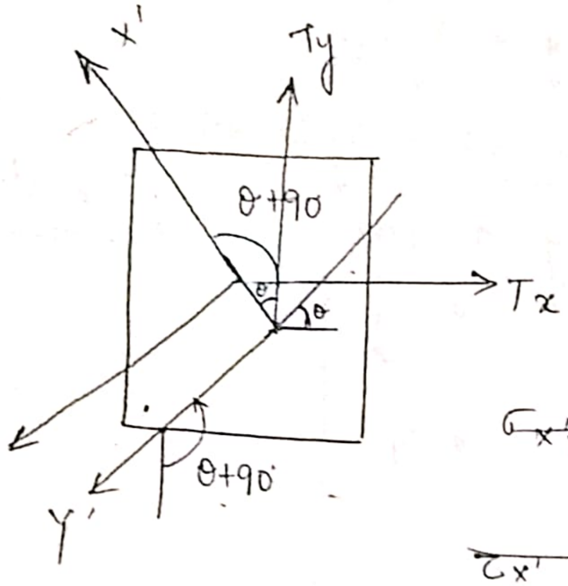
$$= \begin{bmatrix} T_x & T_y \\ T_x & T_y \end{bmatrix} \times \begin{bmatrix} n_x & -n_y \\ n_y & n_x \end{bmatrix}$$

2×2 $\qquad$ 2×2

$$\begin{bmatrix} T_x & T_y \\ T_x & T_y \end{bmatrix} \begin{bmatrix} n_x & 0 \\ 0\, n_y & n_y \end{bmatrix}$$

$T_x n_x \quad T_y n_y$

$\cancel{-T_y n_x}$



$$T_x = n_x\sigma_x + n_y\tau_{xy}$$

$$T_y = \sigma_y n_y + \tau_{xy} n_x$$

$$\cancel{\sigma_{x'x'} = T_x n_x - T_y n_y}$$

$$\cancel{\tau_{x'}}$$

$$\cancel{\sigma_{y'y'} =}$$

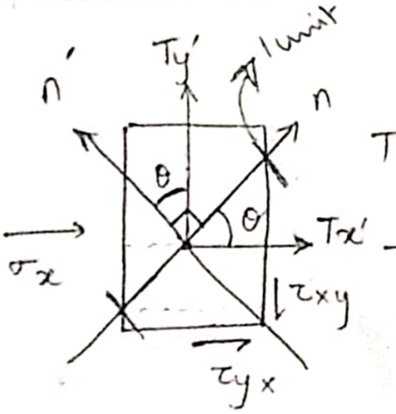
$$\sigma_{x'x'} = T_y n_x - T_x n_y$$

$$\sigma_{y'y'} = -(T_x n_x + T_y n_y)$$

Homework

finding T_x' and T_y'

Ask.



$T_x' = \sigma_x(-\sin\theta) + \tau_{yx}\cos\theta$ (Consider ⊥ plane

$T_y' = \sigma_y \cos\theta + \tau_{xy}(-\sin\theta)$ $(-\sin\theta, c$

$$T_x' = -\sigma_x \sin\theta + \tau_{yx}\cos\theta$$

$$= \sigma_x(-n_y) + \tau_{yx}(n_x)$$

$$T_y' = \sigma_y n_x + \tau_{xy}(-n_y)$$

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$$\boxed{\sigma_{y'y'} = T_{y'}\cos\theta - T_{x'}\sin\theta}$$

$-\sin\theta(\tau_{xy}) + \cos\theta(\tau_{yy})$

$\left(-\sin\theta(\tau_{xx}) + \cos\theta(\tau_{yx})\right)$

$-\sin\theta$

To form T_x' and T_y'

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sigma_x & \sigma_{xy} \\ \sigma_{yx} & \sigma_y \end{bmatrix}$$

$$\begin{array}{cc} T_x & T_y \\ \end{array}$$

$$\begin{bmatrix} \sigma_x(\cos\theta) + \sigma_{yx}(\sin\theta) & \sigma_{xy}(\cos\theta) + \sigma_y(\sin\theta) \\ \sigma_x(-\sin\theta) + \sigma_{yx}(\cos\theta) & \sigma_{xy}(-\sin\theta) + \sigma_y(\cos\theta) \end{bmatrix}$$

$$\begin{array}{cc} T_x' & T_y' \\ \end{array}$$

Since the 1st matrix gave T_x, T_y in new coordinates we can call it rotation matrix R.

$$\begin{bmatrix} T_x & T_y \\ T_x' & T_y' \end{bmatrix} \cdot R^T$$

$$= \begin{bmatrix} T_x & T_y \\ T_x' & T_y' \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{array}{cc} \sigma_{x'} & \sigma_{x'y'} \\ \end{array}$$

$$\begin{bmatrix} T_x\cos\theta + T_y\sin\theta & T_x(-\sin\theta) + T_y(\cos\theta) \\ T_x'(\cos\theta) + T_y'(\sin\theta) & T_x'(-\sin\theta) + T_y'(\cos\theta) \end{bmatrix}$$

$$\begin{array}{cc} \sigma_{y'x'} & \sigma_{y'} \\ \end{array}$$

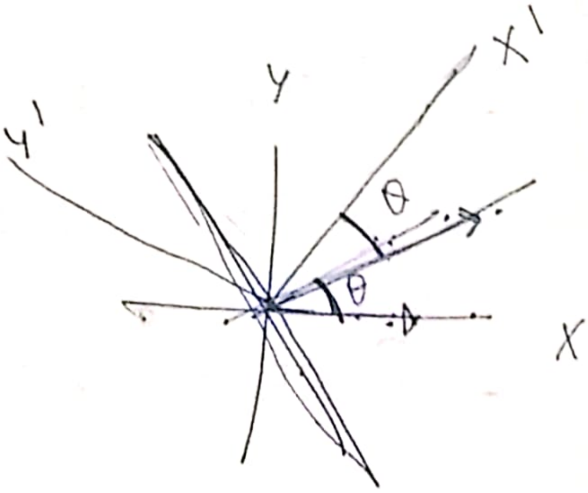
$$\Rightarrow \begin{bmatrix} \sigma_{x'} & \sigma_{x'y'} \\ \sigma_{y'x'} & \sigma_{y'} \end{bmatrix} = R \begin{bmatrix} \sigma_x & \sigma_{xy} \\ \sigma_{yx} & \sigma_y \end{bmatrix} R^T$$

where R is rotation matrix

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$|\sigma_p) = \begin{pmatrix} \sigma_p & 0 & 0 \\ 0 & \sigma_p & 0 \\ 0 & 0 & \sigma_p \end{pmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

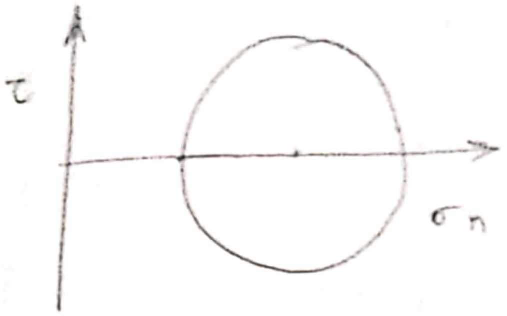
$$= \begin{matrix} T_x \\ T_y \\ T_z \end{matrix}$$



$\theta = 30^0$

$X' = 45$

01/02/19 Principal stresses



$$\frac{\sigma_1+\sigma_3}{2} = \frac{\sigma_x+\sigma_y}{2}$$

$$= \frac{\sigma_{x'}+\sigma_{y'}}{2} = \text{const.}$$

Invariants

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 8 \end{bmatrix} \qquad \begin{bmatrix} 5 & -2 & 8 \\ -2 & 6 & 2 \\ 8 & 2 & 4 \end{bmatrix}$$

$\sigma_{x'x'} \quad \sigma_{x'y'} \quad \sigma_{x'z'} \qquad \sigma_p \quad 0 \quad 0$

$$\boxed{\underset{3\times3}{[\sigma]}\,\underset{3\times1}{[n]} = [\sigma_p][n]}$$

$$[\sigma][n] = \underset{3\times1}{\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}}$$

$$\big[[\sigma] - [\sigma_p]\big][n] = 0$$

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \\ & & \sigma_{zz} \end{bmatrix} - \begin{bmatrix} \sigma_p & 0 & 0 \\ 0 & \sigma_p & 0 \\ 0 & 0 & \sigma_p \end{bmatrix}$$

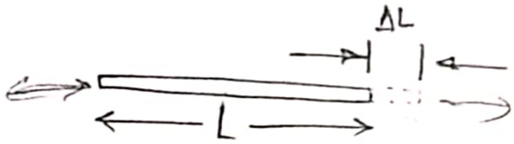
$$\det \begin{vmatrix} \sigma_{xx}-\sigma_p & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy}-\sigma_p & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz}-\sigma_p \end{vmatrix} = 0$$

$x'x$	$y'y$	$z'z$
$y'x$	$y'y$	$y'z$
$z'x$	$z'y$	$z'z$

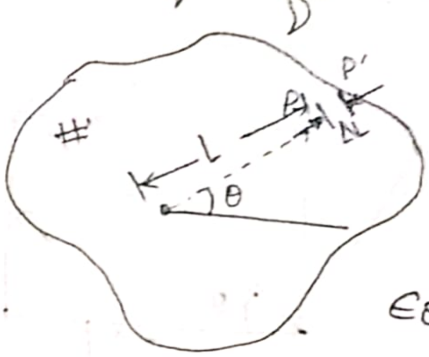
$$A\sigma_p^3 + B\sigma_p^2 + C\sigma_p^2 + D = 0$$

07/02/19

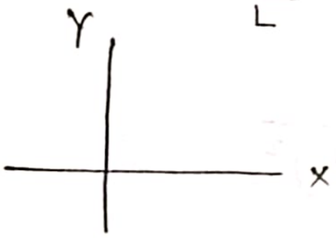
2D strain



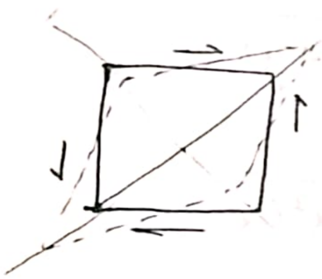
$$\epsilon_x = \frac{\Delta L}{L}$$



$$\epsilon_\theta = \frac{\Delta L}{L}$$

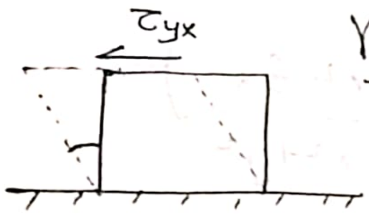


$$\sigma_{xx} \equiv \epsilon_{xx}$$



$$\theta = \gamma\,(\epsilon_{xy})$$

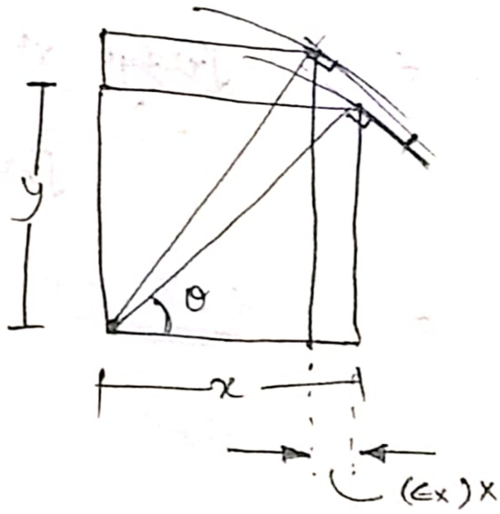
$$\epsilon_{xy} = \frac{\gamma}{2}$$



γ_{yx}

$$\boxed{\frac{\tau}{\gamma} = G}$$ shear modulus

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \theta\gamma \end{bmatrix} = \begin{bmatrix} \epsilon_\theta \\ \epsilon_{\theta+90} \\ \theta\,\gamma_\theta \end{bmatrix}$$



$$\sqrt{x^2+y^2}$$

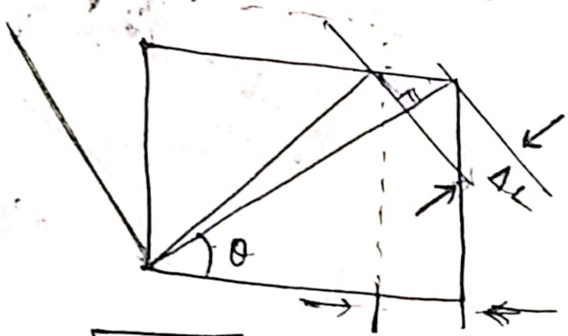
$$\sqrt{(x+\epsilon_x x)^2 + (y+\cancel{\epsilon_y y})^2}$$

$$= \sqrt{x^2(1+2\epsilon_x) + y^2(1+2\epsilon_y)}$$

$$= \sqrt{x^2+y^2+2(\epsilon_x x^2+\epsilon_y y^2)}$$

$$= \sqrt{x^2+y^2}\left\{1 + \frac{2(\epsilon_x x^2+\epsilon_y y^2)}{(x^2+y^2)}\cdot\frac{1}{2}\right\}$$

$\cancel{\sqrt{x^2+y^2}\left(\epsilon_x x^2+\epsilon_y y^2\right)}$



ϵ_x

$$\sqrt{x^2+y^2} - \sqrt{(x-\epsilon_x x)^2 + y^2}$$

$$= \sqrt{x^2+y^2} - \sqrt{x^2\left(1-\frac{2\cancel{x}\epsilon_x}{\cancel{x}}\right)+y^2}$$

$$= \sqrt{x^2+y^2} - \sqrt{x^2+y^2-2x^2\epsilon_x}$$

$$= \sqrt{x^2+y^2} - \sqrt{x^2+y^2}\left(1-\frac{x^2\epsilon_x}{x^2+y^2}\right)$$

$$= \frac{\cancel{\sqrt{x^2+y^2}}\; x^2\epsilon_x}{\sqrt{x^2+y^2}}$$

$$\tan\theta = \frac{y}{x}$$

$$\tan(\theta+d\theta) = \frac{y}{x-\epsilon_x x}$$

$$= \boxed{\epsilon_x x\cos\theta = \epsilon_\theta}$$

$$\epsilon_{\theta+90} = \epsilon_y\, y\sin\theta$$

$\cancel{\epsilon_y}\, y$

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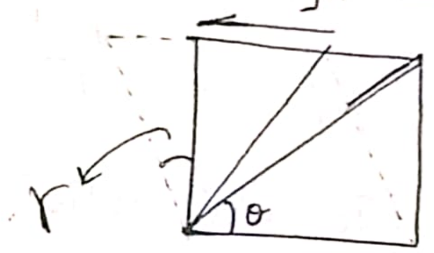
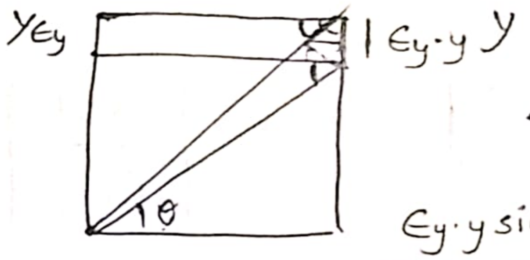
τ_{yx} $-ve$ γ



$\tan\gamma$

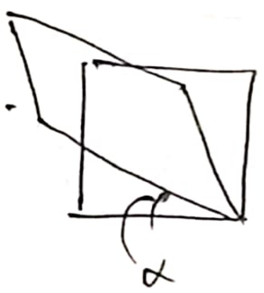
$\tan\theta - \theta \leq \frac{5}{100}$

$y\gamma\cos\theta$



$\epsilon_\theta = f(\epsilon_x \, \epsilon_y \, \gamma, \theta)$

$\epsilon_y \cdot y \sin\theta$



$(\alpha - \beta)$

To Prove

$\boxed{\text{max is } 0.5}$

$\frac{LS}{Lon.S} = \nu$

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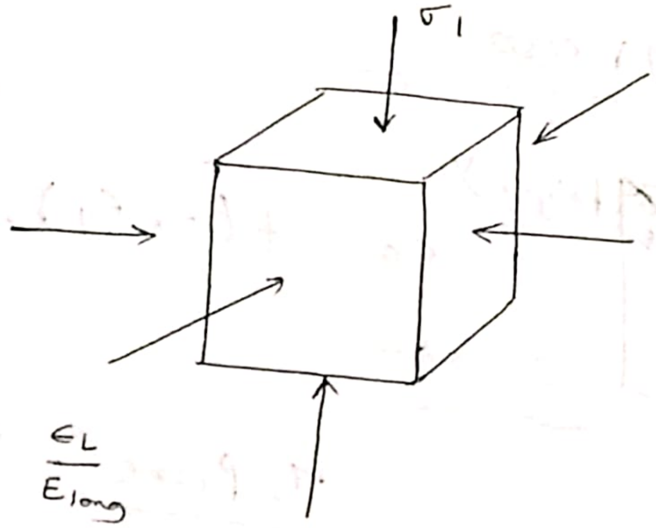
γ ~~$\gamma = \{0 - 0.5\}$~~

$\nu = \{0 - 0.5\}$ (Poission's ratio)



$V_{final} \leq V_{initial}$

$\nu = \left| \frac{\epsilon_{Lat}}{\epsilon_{long}} \right|$

$\nu = \frac{\epsilon_L}{\epsilon_{long}}$

ϵ_{long}

$\frac{a - a \cdot \epsilon_{long}}{a + a \nu \epsilon_{long}}$

$a + a \nu \epsilon_{long}$

~~$V_f = a^3 (1 - \epsilon_{long}^2)(1 + \nu \epsilon_{long})$~~

~~$V_f = a^3 (-\epsilon_{long}^3 - \epsilon_{long}^2 + \epsilon_{long} + 1)$~~

$V_f = \cancel{a^3} (1 - \epsilon_{Long})(1 + \nu \epsilon_{long})^2 \leq \cancel{a^3}$

$(1 - \epsilon_{long})(1 + 2\nu\epsilon_{long} + \nu^2 \epsilon_{long}^2) \leq 1$

$-\cancel{\nu^2 \epsilon_{long}^3} - \cancel{2\nu\epsilon_{long}^2} - \epsilon_{long} + \cancel{1} + 2\nu\epsilon_{long}$

$+ \cancel{\nu^2 \epsilon_{long}^2} \leq \cancel{1}$

$\Rightarrow \quad \boxed{\nu \leq 0.5}$

$\sigma_x \longrightarrow \left\{ \epsilon_x = \frac{\sigma_x}{E}, \quad \epsilon_y = \frac{\sigma_y}{E}, \quad \epsilon_z = \frac{\sigma_z}{E} \right.$

σ_y

$\sigma_z \qquad \nu = \frac{\epsilon_y}{\epsilon_z} \qquad \nu \epsilon_z$

$\epsilon_y = -\nu \epsilon_x = -\frac{\nu \sigma_x}{E}, \quad \epsilon_z = -\nu \epsilon_x = -\frac{\nu \sigma_x}{E}$

$\sigma_y \Rightarrow \quad \epsilon_y = \frac{\sigma_y}{E} \qquad \epsilon_x = -\frac{\nu \sigma_y}{E}, \qquad \epsilon_z = -\frac{\nu \sigma_y}{E}$

$\sigma_z \Rightarrow \quad \epsilon_z = \frac{\sigma_z}{E} \qquad \epsilon_x = -\frac{\nu \sigma_z}{E} \qquad \epsilon_z = -\frac{\nu \sigma_z}{E}$

$\Rightarrow \quad \epsilon_x = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E} - \frac{\nu \sigma_z}{E} \qquad \nu\nu$

$\gamma_{xy} = \frac{\tau_{xy}}{G}$

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G \end{bmatrix}}_{?} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yx} \\ \tau_{yz} \end{bmatrix}$$

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$[\epsilon] = [C][\sigma]$

$[\sigma] = [C]^{-1}[\epsilon]$
$\downarrow$
$[D]$

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Tutorial

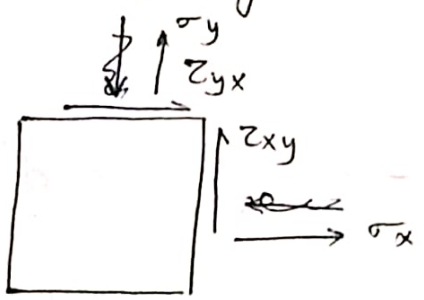
1) At a point P in a body

$$[T_{ij}] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

Find normal & shear stress at a plane having

$n_x = ?$, $n_y = ?$, $n_z = ?$

2)



Find principal stress & their angles with std. axis Also Find Maximum Shear value and plane.

σ_x	σ_y	τ_{xy}
5	−2	3

$\boxed{\sigma^2 + \tau^2 = T^2}$

for 1)

$$[T_{ij}] = \begin{bmatrix} 5 & 4 & 2 \\ 4 & -2 & 6 \\ 2 & 6 & 3 \end{bmatrix} \qquad \begin{aligned} n_x &= 1/\sqrt{3} \\ n_y &= 1/\sqrt{3} \\ n_z &= 1/\sqrt{3} \end{aligned}$$

Ah.

$$T_x = 5\left(\frac{1}{\sqrt{3}}\right) + 4\left(\frac{1}{\sqrt{3}}\right) + 2\left(\frac{1}{\sqrt{3}}\right)$$

$$T_y = 4\left(\frac{1}{\sqrt{3}}\right) +$$

$$\sigma = \frac{T_x\left(\frac{1}{\sqrt{3}}\right) + T_y + T_z}{\sqrt{3}}$$

$$\tau = T^2 - \sigma^2$$

$$T = \sqrt{T_x^2 + T_y^2 + T_z^2}$$

$$[\sigma] = [D][E]$$

$$D = \frac{1}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5-\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5-\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5-\nu \end{bmatrix}$$

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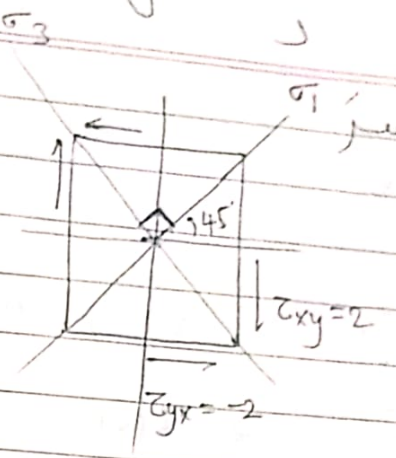

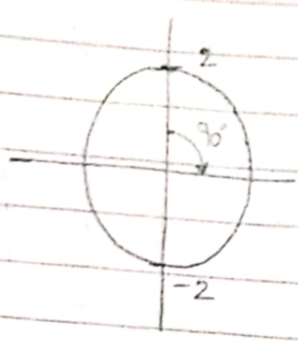

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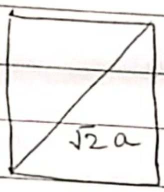
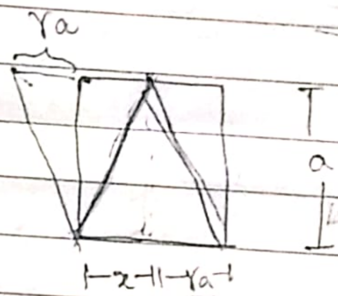
$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = \begin{bmatrix}\sigma_1\\ \sigma_3\\ \theta\end{bmatrix} = \begin{bmatrix}\sigma_x'\\ \sigma_y'\\ \tau_{x'y'}\end{bmatrix}$$





$$\cancel{\sqrt{(ra)^2 + (\underbrace{a-ra}_{x})^2}}$$

$$\cancel{= a\sqrt{r^2 + (1-r)^2}}$$

$$\sqrt{(a-ra)^2 + a^2}$$

$$a\sqrt{(1-r)^2+1}$$

$$a\sqrt{1-2r+1} = \sqrt{2}a\sqrt{1-r}$$

$$\epsilon_x' = \frac{\sqrt{2}a(1-\sqrt{1-r})}{\sqrt{2}a}$$

$$= \left|\frac{r}{2}\right| \qquad \epsilon_x' = f(G,\tau)$$

$$= \frac{\tau}{2G} \quad \text{—(1)}$$

$$\sigma_x' = \tau \qquad \sigma_y' = -\tau \qquad \tau_{x'y'} = 0$$

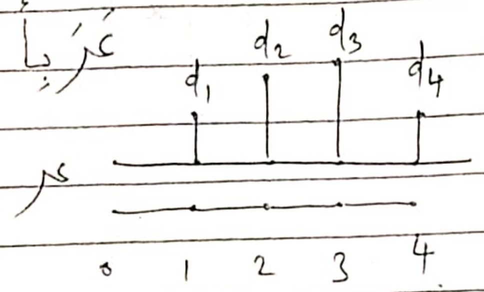
$$\epsilon_x' = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} = \frac{\tau}{E} + \frac{\nu\tau}{E} = \frac{\tau}{E}(1+\nu) \quad \text{—(2)}$$

(1) = (2)

$$\frac{\cancel{\tau}}{2G} = \frac{\cancel{\tau}}{E}(1+\nu) \qquad \boxed{G = \frac{E}{2(1+\nu)}}$$

Date
Page

Strain Compactibility equation Compact
قهمبكت



$y_2 - y_1 = \Delta y$

$d = f(x)$

أ

$\frac{\Delta y}{\Delta x} = \epsilon \quad \frac{\partial u}{\partial x} = \epsilon_x$

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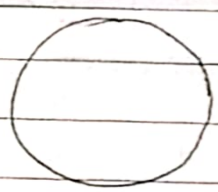
$[\sigma] = [D][E]$

$$D = \frac{1}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5-\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5-\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5-\nu \end{bmatrix}$$

$$\left.\begin{array}{l} \text{Plane Strain} \\ \text{Plane Stress} \end{array}\right\} \longrightarrow 2D$$

$L >> R$



No strain along axis of the tunnel.

γ

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\nu \sigma_y}{E} - \frac{\nu \sigma_x}{E} = 0$$

$\gamma_{xz} = \gamma_{yz} = 0$

$[\epsilon] = [C][\sigma]$

$[\sigma] = [C]^{-1}[\epsilon]$

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ f(\sigma_x, \sigma_y) \\ \tau_{xy} \\ 0 \\ 0 \end{bmatrix} = [D] \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z = 0 \\ \gamma_{xy} \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{\sigma_z = \nu(\sigma_y + \sigma_z)}$$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E} - \frac{\nu \sigma_z}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu \sigma_x}{E} - \frac{\nu \sigma_z}{E}$$

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$$\epsilon_x = \frac{\sigma_x}{E}(1-\nu^2) - \frac{\nu\sigma_y}{E}(1+\nu)$$

$\epsilon_y =$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

Idealization from 3D-2D

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 3\times 3 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

<u>Transversly isotropic</u>

2 direction has same young's modulus and 3^{rd} direction is $\perp$.

<u>Anisotropic</u> - behaviour is different in all directions.

if material is perfectly isotropic E, G., B has relationships.