8.2.5

EE24BTECH11059 - Y Siddhanth

Question:
Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$, and $x = 4$.

Solution:
Theoretical Solution:
First, we find the intersection points, clearly x = 4 is the intersection point for 2 sides.

$$-3x + y = 1 \tag{0.1}$$
$$-2x + y = 1 \tag{0.2}$$

Taking an augmented matrix to solve the above equations,

$$\begin{pmatrix} -3 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} \tag{0.3}$$

Using row operations to reduce it,

$$\begin{pmatrix} -3 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} \xrightarrow[R_2 \to R_2 - \frac{2}{3}R_1]{} \begin{pmatrix} -3 & 1 & 1 \\ 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \xrightarrow[R_1 \to -\frac{1}{3}R_1]{R_2 \to 3R_2} \begin{pmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow{R_1 \to R_1 + \frac{1}{3}R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \tag{0.4}$$

Thus, the intersection of the lines $(-3x + y = 1, -2x + y = 1)$ is $(0, 1)$
To find the area, we integrate the difference between the two lines from their intersection point at $x = 0$ to $x = 4$:

$$Area = \int_0^4 ((3x + 1) - (2x + 1))dx \tag{0.5}$$
$$= \int_0^4 xdx \tag{0.6}$$
$$= \left[\frac{1}{2}x^2\right]_0^4 \tag{0.7}$$
$$= \frac{1}{2}(4^2) - \frac{1}{2}(0^2) \tag{0.8}$$
$$= 8 \tag{0.9}$$

Thus, the area of the triangular region is 8 square units.

Numerical Solution:

The numerical solution for this question can be found using the trapezoidal rule (0.10).

$$J = \int_a^b f(x)\, dx \approx h\left(\frac{1}{2}f(x) + f(x_1) + f(x_2) \cdots + f(x_{n-1}) + \frac{1}{2}f(b)\right) \tag{0.10}$$

$$h = \frac{b-a}{n} \tag{0.11}$$

$$f(x) = (3x+1) - (2x+1) = x \tag{0.12}$$

$$J = j_n, \text{ where, } j_{i+1} = j_i + h\frac{f(x_{n+1}) + f(x_n)}{2} \tag{0.13}$$

$$\rightarrow j_{i+1} = j_i + \frac{bh}{a}(x_{n+1} + x_n) \tag{0.14}$$

$$x_{n+1} = x_n + k \tag{0.15}$$

Taking n =100000 and applying (0.14) iteratively, we get

$$J = 7.99992 \tag{0.16}$$

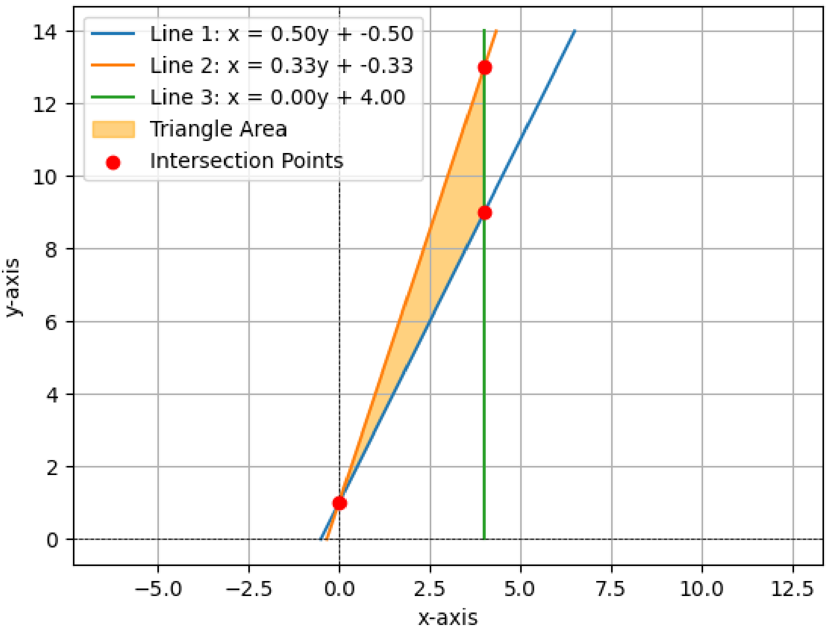

Fig. 0.1: Plot of the given triangle