ASSIGNMENT 1: D16(7 TO 22)

EE1030 : Matrix Theory
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D: MCQs with One or more than One correct.

1) If $f(x) = \frac{x^2-1}{x^2+1}$, for every real number x, then the minimum value of f

(1998 - 2 Marks)

a) does not exist because f is unbounded
b) is not attained even though f is bounded
c) is equal to 1
d) is equal to -1

2) The number of values of the x where the function $f(x) = \cos x + \cos(\sqrt{2}x)$ attains its maximum is

(1998 - 2 Marks)

a) 0
b) 1
c) 2
d) infinite

3) The function $f(x) = \int_{-1}^{x} t(e^t-1)(t-1)(t-2)^3(t-3)^5 dt$ has a local minimum at $x =$

(1999 - 3 Marks)

a) 0
b) 1
c) 2
d) 3

4) $f(x)$ is a cubic polynomial with $f(2) = 18$ and $f(1) = -1$. Also $f(x)$ has a local maxima at $x = -1$ and $f'(x)$ has a local minima at $x = 0$, then

(2006 - 5M, -1)

a) the distance between (-1,2) and $(a, f(a))$, where $x = a$ is the point of local minima is $2\sqrt{5}$
b) $f(x)$ is increasing for $x \in [1, 2\sqrt{5}]$
c) $f(x)$ has a local minima at x=1
d) the value of $f(0) = 15$

5) $f(x) = \begin{cases} e^x, & 0 \le x \le 1 \\ 2 - e^{x-1}, & 1 < x \le 2 \\ x - e & 2 < x \le 3 \end{cases}$ and $g(x) = \int_0^x f(t)dt, x \in [1, 3]$ then g(x) has

(2006 - 5M, -1)

a) has local maxima at $x = 1 + \ln 2$ and local minima at $x = e$
b) has local maxima at $x = 1$ and local minima at $x = 2$
c) no local maxima
d) no local minima

6) For the function

$$f(x) = x\cos\frac{1}{x}, x \ge 1$$

(2009)

a) for at least one x in the interval [1,∞), $f(x+2) - f(x) < 2$
b) $\lim_{x\to\infty} f'(x) = 1$
c) for all x in the interval [1,∞], $f(x+2)-f(x) > 2$
d) $f'(x)$ is strictly decreasing in the interval [1,∞)

7) If $f(x) = \int_0^x e^{t^2}(t-2)(t-3)dt$ for all $x \in (0, \infty)$, then

(2012)

a) f has a local maximum at $x = 2$
b) f is decreasing on (2,3)
c) there exists some c∈ $(0, \infty)$, such that $f''(c) = 0$
d) f has a local minimum at $x = 3$

8) A rectangular sheet of fixed perimeter with sides having their lengths in the ratio 8: 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the lengths of the sides of the rectangular sheet are

(JEE Adv. 2013)

a) 24
b) 32
c) 45
d) 60

9) Let $f : (0, \infty) \to R$ be given by $f(x) = \int_{\frac{1}{x}}^{x} e^{-(t+\frac{1}{t})} \frac{dt}{t}$. Then

(JEE Adv. 2014)

a) $f(x)$ is monotonically increasing on [1,∞)
b) $f(x)$ is monotonically decreasing on (0,1)
c) $f(x) + f(\frac{1}{x}) = 0$, for all $x \in (0, \infty)$
d) $f(2^x)$ is an odd function of x on R

10) Let f, g: [-1,2]→ R be continuous functions which are twice differentiable on the interval (-1,2). Let the values of f and g at the points -1,0 and 2 be as given in the following table:

	$x = -1$	$x = 0$	$x = 2$
$f(x)$	3	6	0
$g(x)$	0	1	-1

In each of the intervals (-1,0) and (0,2) the function $(f - 3g)''$ never vanishes. Then the correct statement(s) is(are)

(JEE Adv. 2015)

a) $f'(x) - 3g'(x) = 0$ has exactly three solutions in (-1,0)∪(0,2)
b) $f'(x) - 3g'(x) = 0$ has exactly one solution in (-1,0)
c) $f'(x) - 3g'(x) = 0$ has exactly one solution in (0,2)
d) $f'(x) - 3g'(x) = 0$ has exactly two solutions in (-1,0)

11) Let $f : R \to (0, \infty)$ and $g : R \to R$ be twice differentiable functions such that f'' and g'' are continuous functions on R. Suppose $f'(2) = g(2) = 0, f''(2) \neq 0$ and $g'(2) \neq 0$.

If $\lim_{x\to 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1$, then

(JEE Adv. 2016)

a) f has a local minimum at $x = 2$
b) f has a local maximum at $x = 2$
c) $f''(2) > f(2)$
d) $f(x) - f''(x) = 0$ for at least one $x \in R$

12) If $f : R \to R$ a differentiable function such that $f'(x) > 2f(x)$ for all $x \in R$ and $f(0) = 1$, then

(JEE Adv. 2017)

a) $f(x)$ is increasing in $(0, \infty)$
b) $f(x)$ is decreasing in $(0, \infty)$
c) $f(x) > e^{2x}$ in $(0, \infty)$
d) $f'(x) < e^{2x}$ in $(0, \infty)$

13) If $f(x) = \begin{vmatrix} \cos 2x & \cos 2x & \sin 2x \\ -\cos x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then

(JEE Adv. 2017)

a) $f'(x) = 0$ at exactly three points in (π, π)
b) $f'(x) = 0$ at more than three points in (π, π)
c) $f(x)$ attains its maximum at $x = 0$
d) $f(x)$ attains its minimum at $x = 0$

14) Define the collection $\{E_1, E_2, E_3, ...\}$ of ellipses and $\{R_1, R_2, R_3, ...\}$ of rectangles as follows:

$E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$;

R_1: rectangle of largest area, with sides parallel to the axes, inscribes in E_1;

E_n : ellipse $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$ of largest area inscribed in $R_{n-1}, n > 1$;

R_n: rectangle of largest area, with sides parallel to the axes, inscribes in E_n;
Then which of the following options is/are correct?

(JEE Adv. 2019)

a) The eccentricities of E_{18} and E_{19} are not equal
b) Length of the latus rectum of E_9 is $\frac{1}{6}$
c) $\sum_{n=1}^{N}$ (area of R_n) < 24, for each positive integer N
d) The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

15) Let $f : R \to R$ be given by $f(x) = (x-1)(x-2)(x-5)$.

Define $F(x) = \int_0^x f(t)dt, x > 0$.

Then which of the following options is/are correct?

(JEE Adv. 2019)

a) F has a local maximum at $x = 2$
b) F has a local minimum at $x = 1$
c) F has two local maxima and one local minimum in $(0, \infty)$
d) $F(x) = 0$ for all $x \in (0, \infty)$

16) Let $f(x) = \frac{\sin(\pi x)}{x^2}, x > 0$.

Let $x_1 < x_2 < x_3 <$ ..... $< x_n <$ ..... be all the points of local maximum of f and $y_1 < y_2 < y_3 <$ ..... $< y_n <$ ..... be all the points of local minimum of f.

Then which of the following options is/are correct?

(JEE Adv. 2019)

a) $x_{n+1} - x_n > 2$
b) $x_n \in (2n, 2n + \frac{1}{2})$ for every n
c) $|x_n - y_n| > 1$ for every n
d) $x_1 < y_1$