

1.1.8.28

EE24BTECH11059 - Yellanki Siddhanth

Question:

Find the equation of the set of the points P such that its distances from the points $A(3, 4, -5)$ and $B(-2, 1, 4)$ are equal.

Solution:

Variable	Description	Formula
A	A Point to be plotted	$A = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$
B	A Point to be plotted	$B = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$
C	Midpoint of A and B	$C = \left(\frac{A+B}{2} \right)$
O	The set of points which are equidistant from A and B	$(A - B)^T O = \frac{\ A\ ^2 - \ B\ ^2}{2}$

TABLE 0

If \mathbf{O} is equidistant from the points \mathbf{A} and \mathbf{B}

$$\|\mathbf{O} - \mathbf{A}\| = \|\mathbf{O} - \mathbf{B}\| \quad (0.1)$$

$$(0.2)$$

$$\|\mathbf{O} - \mathbf{A}\|^2 = \|\mathbf{O} - \mathbf{B}\|^2 \quad (0.3)$$

$$(0.4)$$

$$\|\mathbf{O}\|^2 - 2\mathbf{O}^T \mathbf{A} + \|\mathbf{A}\|^2 = \|\mathbf{O}\|^2 - 2\mathbf{O}^T \mathbf{B} + \|\mathbf{B}\|^2 \quad (0.5)$$

By simplifying further,

$$(\mathbf{A} - \mathbf{B})^T \mathbf{O} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \quad (0.6)$$

The above equation is the general expression for the perpendicular bisecting plane between any points \mathbf{A} and \mathbf{B} .

Substituting the \mathbf{A} and \mathbf{B} values in the derived equation.

$$\begin{pmatrix} 5 \\ 3 \\ -9 \end{pmatrix}^T O = \frac{\left\| \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} \right\|^2 - \left\| \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} \right\|^2}{2} = \frac{29}{2} \quad (0.7)$$

Comparing with $n^T x = c$

$$\mathbf{n} = \begin{pmatrix} 5 \\ 3 \\ -9 \end{pmatrix} \quad (0.8)$$

$$\mathbf{c} = \frac{29}{2} \quad (0.9)$$

$$(0.10)$$

Final Plane equation is:

$$Plane \equiv \begin{pmatrix} 5 \\ 3 \\ -9 \end{pmatrix}^T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = (14.5) \quad (0.11)$$

Perpendicular Bisecting Plane of A and B

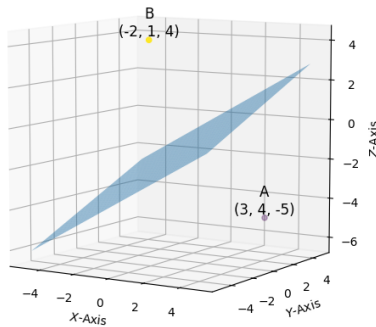


Fig. 0.1