JEE ASSIGNMENT 7

EE1030 : Matrix Theory

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1) For $\lambda > 0$, let θ be the angle between the vectors $\mathbf{a} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\mathbf{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. If the vectors $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ are mutually perpendicular, then the value of $(14\cos\theta)^2$ is equal to (2024 - 4 Marks)

a) 20 b) 25 c) 40 d) 50

2) Let $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $B = I + \text{adj}(A) + (\text{adj}\,A)^2 + \ldots + (\text{adj}\,A)^{10}$ Then, the sum of all the elements of the matrix B is: (2024 - 4 Marks)

a) -88 b) -124 c) 22 d) -110

3) Let $y = y(x)$ be the solution of the differential equation $\left(x^2+4\right)^2 dy + \left(2x^3y + 8xy - 2\right)dx = 0$. If $y(0) = 0$, then $y(2)$ is equal to (2024 - 4 Marks)

a) $\frac{\pi}{32}$ b) 2π c) $\frac{\pi}{16}$ d) $\frac{\pi}{8}$

4) Let C be a circle with radius $\sqrt{10}$ units and centre at the origin. Let the line $x+y = 2$ intersects the circle C at the points P and Q. Let MN be a chord of C of length 2 unit and slope -1. Then, a distance (in units) between the chord PQ and the chord MN is (2024 - 4 Marks)

a) $\sqrt{2}+1$ b) $3-\sqrt{2}$ c) $2-\sqrt{3}$ d) $\sqrt{2}-1$

5) Consider a hyperbola H having centre at the origin and foci on the x -axis. Let C_1 be the circle touching the hyperbola H and having the centre at the origin. Let C_2 be the circle touching the hyperbola H at its vertex and having the centre at one of its foci. If areas (in sq units) of C_1 and C_2 are 36π and 4π, respectively, then the length (in units) of latus rectum of H is (2024 - 4 Marks)

a) $\frac{14}{3}$ b) $\frac{28}{3}$ c) $\frac{11}{3}$ d) $\frac{10}{3}$

6) Let $f(x) = 3\sqrt{x-2} + \sqrt{4-x}$ be a real valued function. If α and β are respectively the minimum and the maximum values of f, then $\alpha^2 + 2\beta^2$ is equal to (2024 - 4 Marks)

a) 24 b) 44 c) 38 d) 42

7) If the mean of the following probability distribution of a radam variable X :

X	0	2	4	6	8
P(X)	a	$2a$	$a+b$	$2b$	$3b$

is $\frac{46}{9}$, then the variance of the distribution is (2024 - 4 Marks)

a) $\frac{173}{27}$ b) $\frac{151}{27}$ c) $\frac{581}{81}$ d) $\frac{566}{81}$

8) Let P be the point of intersection of the lines $\frac{x-2}{1} = \frac{y-4}{5} = \frac{z-2}{1}$ and $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-3}{2}$. Then, the shortest distance of P from the line $4x = 2y = z$ is (2024 - 4 Marks)

a) $\frac{\sqrt{14}}{7}$ b) $\frac{3\sqrt{14}}{7}$ c) $\frac{6\sqrt{14}}{7}$ d) $\frac{5\sqrt{14}}{7}$

9) If the value of the integral $\int_{-1}^{1} \frac{\cos \alpha x}{1+3^x} dx$ is $\frac{2}{\pi}$.Then, a value of α is (2024 - 4 Marks)

a) $\frac{\pi}{6}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$

10) Let a relation R on $\mathrm{N} \times \mathrm{N}$ be defined as: $(x_1, y_1) R (x_2, y_2)$ if and only if $x_1 \leq x_2$ or $y_1 \leq y_2$. Consider the two statements:
(I) R is reflexive but not symmetric.
(II) R is transitive Then which one of the following is true? (2024 - 4 Marks)

a) Both (I) and (II) are correct.
b) Only (I) is correct.
c) Only (II) is correct.
d) Neither (I) nor (II) is correct.

11) Let PQ be a chord of the parabola $y^2 = 12x$ and the midpoint of PQ be at $(4, 1)$. Then, which of the following point lies on the line passing through the points P and Q? (2024 - 4 Marks)

a) $\left(\frac{3}{2}, -16\right)$ b) $(3, -3)$ c) $(2, -9)$ d) $\left(\frac{1}{2}, -20\right)$

12) The area (in sq. units) of the region $S = \{z \in \mathbb{C} : |z-1| \leq 2; (z+\bar{z}) + i(z-\bar{z}) \leq 2, \mathrm{Im}(z) \geq 0\}$ is (2024 - 4 Marks)

a) $\frac{7\pi}{4}$ b) $\frac{7\pi}{3}$ c) $\frac{17\pi}{8}$ d) $\frac{3\pi}{2}$

13) Let $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}, \mathbf{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\mathbf{c} = x\hat{i} + 2\hat{j} + 3\hat{k}, x \in \mathbb{R}$.
If $\mathbf{d}$ is the unit vector in the direction of $\mathbf{b} + \mathbf{c}$ such that $\mathbf{a} \cdot \mathbf{d} = 1$, then $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ is equal to (2024 - 4 Marks)

a) 11 b) 6 c) 9 d) 3

14) Given that the inverse trigonometric function assumes principal values only. Let x, y be any two real numbers in $[-1, 1]$ such that $\cos^{-1} x - \sin^{-1} y = \alpha, \frac{-\pi}{2} \leq \alpha \leq \pi$.
Then, the minimum value of $x^2 + y^2 + 2xy \sin \alpha$ is (2024 - 4 Marks)

a) $\frac{1}{2}$ b) 0 c) -1 d) $\frac{-1}{2}$

15) Let $f(x) = \int_0^x (t + \sin(1 - e^t))\, dt, x \in \mathbb{R}$. Then, $\lim_{x \to 0} \frac{f(x)}{x^3}$ is equal to (2024 - 4 Marks)

a) $-\frac{2}{3}$
b) $\frac{1}{6}$
c) $-\frac{1}{6}$
d) $\frac{2}{3}$