JEE ASSIGNMENT 3

EE1030 : Matrix Theory

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2021 March 18 Shift 2 1 to 15

1) Let the system of linear equations $4x + \lambda y + 2z = 0, 2x - y + z = 0, \mu x + 2y + 3z = 0, \lambda, \mu \in \mathbb{R}$ Has a non-trivial solution. Then which of the following is true?
(2021 - 4 Marks)

a) $\mu = 6, \lambda \in \mathbb{R}$
b) $\lambda = 2, \mu \in \mathbb{R}$
c) $\lambda = 3, \mu \in \mathbb{R}$
d) $\mu = -6, \lambda \in \mathbb{R}$

2) A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle of $\triangle ABC$ is 2, then the height of the pole is equal to

a) $\frac{1}{\sqrt{3}}$
b) $\sqrt{3}$
c) $2\sqrt{3}$
d) $\frac{2\sqrt{3}}{3}$

3) Let in a series of $2n$ observations, half of them are equal to a and the remaining half are equal to $-a$. Also by adding a constant b in each of these observations, the mean and standard deviation of the new set become 5 and 20, respectively. Then the value of $a^2 + b^2$ is equal to: (2021 - 4 Marks)

a) 250
b) 925
c) 650
d) 425

4) Let $g(x) = \int_0^x f(t)\,dt$ where f is a continuous function in $[0, 3]$ such that $\frac{1}{3} \leq f(t) \leq 1$ for all $t \in [0, 1]$ and $0 \leq f(t)$ for all $t \in (1, 3]$. The largest possible interval in which $g(3)$ lies is: (2021 - 4 Marks)

a) $[1, 3]$
b) $\left[-1, -\frac{1}{2}\right]$
c) $\left[-\frac{3}{2}, -1\right]$
d) $\left[\frac{1}{3}, 2\right]$

5) If $15\sin^4\theta + 10\cos^4\theta = 6$, for some $\theta \in \mathbb{R}$, then the value of $27\sec^6\theta + 8\csc^6\theta$ is equal to: (2021 - 4 Marks)

a) 250
b) 400
c) 500
d) 350

6) Let $f : \mathbb{R} - \{3\} \to \mathbb{R} - \{1\}$ be defined by $f(x) = \frac{x-2}{x-3}$. Let $g : \mathbb{R} \to \mathbb{R}$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to
(2021 - 4 Marks)

a) 7 b) 5 c) 2 d) 3

7) Let S_1 be the sum of the first $2n$ terms of an arithmetic progression. Let S_2 be the sum of the first $4n$ terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first $6n$ terms of the arithmetic progression is equal to:
(2021 - 4 Marks)

a) 3000 b) 7000 c) 5000 d) 1000

8) Let $S_1 = x^2 + y^2 = 9$ and $S_2 = (x-2)^2 + y^2 = 1$. Then the locus of the centre of a variable circle S which touches S_1 internally and S_2 externally always passes through the points:
(2021 - 4 Marks)

a) $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$ b) $\left(2, \pm\frac{3}{2}\right)$ c) $(1, \pm 2)$ d) $\left(0, \pm\sqrt{3}\right)$

9) Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line $x + y = 3$. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then $(R + r)$ is equal to
(2021 - 4 Marks)

a) $2\sqrt{2}$ b) $3\sqrt{2}$ c) $7\sqrt{2}$ d) $\frac{9}{\sqrt{2}}$

10) In a triangle ABC, if vector $\overrightarrow{BC} = 8$, $\overrightarrow{CA} = 7$, $\overrightarrow{AB} = 10$, then the projection of the vector AB on AC is equal to: (2021 - 4 Marks)

a) $\frac{25}{4}$ b) $\frac{85}{14}$ c) $\frac{127}{20}$ d) $\frac{115}{16}$

11) Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to: (2021 - 4 Marks)

a) $\frac{80}{243}$ b) $\frac{32}{625}$ c) $\frac{128}{625}$ d) $\frac{40}{243}$

12) Let a and b be two non-zero vectors perpendicular to each other and $|a| = |b|$. If $|a \times b| = |a|$, then the angle between the vectors $(a + b + (a \times b))$ and a is equal to:
(2021 - 4 Marks)

a) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ b) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ c) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$ d) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

13) Let a complex number be $w = 1 - \sqrt{3}i$. Let another complex number z be such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to:

(2021 - 4 Marks)

a) $\frac{1}{2}$ b) 4 c) 2 d) $\frac{1}{4}$

14) The area bounded by the curve $4y^2 = x^2(4-x)(x-2)$ is equal to:
(2021 - 4 Marks)

a) $\frac{3\pi}{2}$ b) $\frac{\pi}{16}$ c) $\frac{\pi}{8}$ d) $\frac{3\pi}{8}$

15) Define a relation R over a class of $n \times n$ real matrices A and B as ARB if there exists a non-singular matrix P such that $PAP^{-1} = B$. Then which of the following is true?
(2021 - 4 Marks)

a) R is reflexive, symmetric but not transitive
b) R is symmetric, transitive but not reflexive
c) R is an equivalence relation
d) R is reflexive, transitive but not symmetric