1.1.8.28

EE24BTECH11059 - Yellanki Siddhanth

Question:
Find the equation of the set of the points P such that its distances from the points $A\,(3,4,-5)$ and $B\,(-2,1,4)$ are equal.
Solution:

Variable	Description	Formula
A	A Point to be plotted	$A = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$
B	A Point to be plotted	$B = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$
C	Midpoint of A and B	$C = \left(\frac{A+B}{2}\right)$
O	The set of points which are equidistant from A and B	$(A-B)^\top O = \frac{\|\|A\|\|^2 - \|\|B\|\|^2}{2}$

TABLE 0

If **O** is equidistant from the points **A** and **B**

$$\|\boldsymbol{O}-\boldsymbol{A}\| = \|\boldsymbol{O}-\boldsymbol{B}\| \tag{0.1}$$

(0.2)

$$\|\boldsymbol{O}-\boldsymbol{A}\|^2 = \|\boldsymbol{O}-\boldsymbol{B}\|^2 \tag{0.3}$$

(0.4)

$$\|\boldsymbol{O}\|^2 - 2\boldsymbol{O}^\top\boldsymbol{A} + \|\boldsymbol{A}\|^2 = \|\boldsymbol{O}\|^2 - 2\boldsymbol{O}^\top\boldsymbol{B} + \|\boldsymbol{B}\|^2 \tag{0.5}$$

By simplifying further,

$$(\boldsymbol{A}-\boldsymbol{B})^\top O = \frac{\|\boldsymbol{A}\|^2 - \|\boldsymbol{B}\|^2}{2} \tag{0.6}$$

The above equation is the general expression for the perpendicular bisecting plane between any points $\boldsymbol{A}$ and $\boldsymbol{B}$.
Substituting the $\boldsymbol{A}$ and $\boldsymbol{B}$ values in the derived equation.

$$\begin{pmatrix} 5 \\ 3 \\ -9 \end{pmatrix}^{\top} O = \frac{\left\| \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} \right\|^2 - \left\| \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} \right\|^2}{2} = \frac{29}{2} \tag{0.7}$$

Comparing with $n^{\top} x = c$

$$\boldsymbol{n} = \begin{pmatrix} 5 \\ 3 \\ -9 \end{pmatrix} \tag{0.8}$$

$$\boldsymbol{c} = \frac{29}{2} \tag{0.9}$$

$$\tag{0.10}$$

Final Plane equation is:

$$Plane \equiv 5x + 3y - 9z = 14.5 \tag{0.11}$$

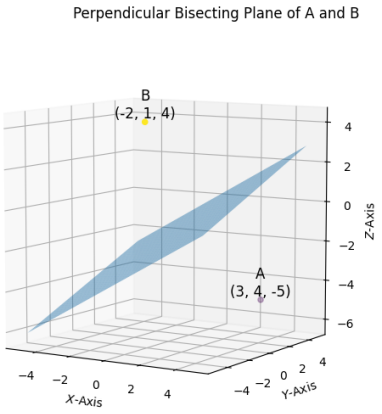

Fig. 0.1