JEE ASSIGNMENT 1

EE1030 : Matrix Theory
Indian Institute of Technology Hyderabad

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1) If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies: (2020 - 4 Marks)

    a) $e^4 + 2e^2 - 1 = 0$
    b) $e^2 + 2e - 1 = 0$
    c) $e^4 + e^2 - 1 = 0$
    d) $e^2 + e - 1 = 0$

2) The set of all real values of λ for which the function $f(x) = \left(1 - \cos^2 x\right)(\lambda + \sin x), x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, has exactly one maxima and exactly one minima, is: (2020 - 4 Marks)

    a) $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$
    b) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$
    c) $\left(-\frac{3}{2}, \frac{3}{2}\right)$
    d) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

3) The probabilities of three events A, B and C are given by $P(A) = 0.6$, $P(B) = 0.4$ and $P(C) = 0.5$. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \leq \alpha \leq 0.95$, then β lies in the interval.
(2020 - 4 Marks)

    a) $[0.36, 0.40]$
    b) $[0.25, 0.35]$
    c) $[0.35, 0.36]$
    d) $[0.20, 0.25]$

4) The common difference of the $A.P.$ $b_1, b_2, \ldots, b_m$ is 2 more than the common difference of $A.P.$ $a_1, a_2, \ldots, a_n$. If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to: (2020 - 4 Marks)

    a) -127
    b) 81
    c) 127
    d) -81

5) The integral $\int_1^2 e^x.x^x(2 + log_e x)\,dx$ equals (2020 - 4 Marks)

    a) $e(4e - 1)$
    b) $e(4e + 1)$
    c) $4e^2 - 1$
    d) $e(2e - 1)$

6) If the tangent to the curve, $y = f(x) = x\log_e x$, $(x > 0)$ at a point $(c, f(c))$ is parallel to the line-segment joining the points $(1, 0)$ and (e, e), then c is equal to:
(2020 - 4 Marks)

a) $e^{\left(\frac{1}{1-e}\right)}$ b) $\frac{(e-1)}{e}$ c) $\frac{1}{(e-1)}$ d) $e^{\left(\frac{1}{e-1}\right)}$

7) If $y = \left(\frac{2}{\pi}x - 1\right)\csc x$ is the solution of the differential equation, $\left(\frac{dy}{dx}\right) + p(x)y = \left(\frac{2}{\pi} - 1\right)\csc x$, $0 < x < \frac{\pi}{2}$, then the function $p(x)$ is equal to: (2020 - 4 Marks)

a) $\operatorname{cosec} x$ b) $\cot x$ c) $\tan x$ d) $\sec x$

8) If α and β are the roots of the equation $2x(2x+1) = 1$, then β is equal to: (2020 - 4 Marks)

a) $2\alpha(\alpha - 1)$
b) $-2\alpha(\alpha + 1)$
c) $2\alpha^2$
d) $2\alpha(\alpha + 1)$

9) For all twice differentiable functions $f : \mathbb{R} \to \mathbb{R}$, with $f(0) = f(1) = f'(0) = 0$, (2020 - 4 Marks)

a) $f''(x) = 0$, at every point $x \in (0, 1)$
b) $f''(x) \neq 0$, at every point $x \in (0, 1)$
c) $f''(x) = 0$, for some $x \in (0, 1)$
d) $f''(0) = 0$

10) The area (in sq.units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to: (2020 - 4 Marks)

a) $\frac{4}{3}$ b) $\frac{7}{2}$ c) $\frac{16}{3}$ d) $\frac{8}{3}$

11) For a suitably chosen real constant a, let a function, $f : \mathbb{R} - \{-a\} \to \mathbb{R}$ be defined by $f(x) = \frac{a-x}{a+x}$. Further suppose that for any real number $x \neq -a$ and $f(x) \neq -a$, $(f \circ f)(x) = x$. Then $f\left(\frac{-1}{2}\right)$ is equal to: (2020 - 4 Marks)

a) -3 b) 3 c) $\frac{1}{3}$ d) $-\frac{1}{3}$

12) Let $\theta = \frac{\pi}{5}$ and $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$. If $B = A + A^4$, then $\det(B)$: (2020 - 4 Marks)

a) is one
b) lies in $(1, 2)$
c) lies in $(2, 3)$
d) is zero

13) The center of the circle passing through the point $(0, 1)$ and touching the parabola $y = x^2$ at the point $(2, 4)$ is : (2020 - 4 Marks)

a) $\left(\frac{3}{10}, \frac{16}{5}\right)$ b) $\left(\frac{6}{5}, \frac{53}{10}\right)$ c) $\left(-\frac{16}{5}, \frac{53}{10}\right)$ d) $\left(-\frac{53}{10}, \frac{16}{5}\right)$

14) A plane P meets the coordinate axes at A, B and C respectively. The centroid of a triangle ABC is given to be $(1, 1, 2)$. Then the equation of the line through this centroid and perpendicular to the plane P is: (2020 - 4 Marks)

a) $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$
c) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$
b) $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$
d) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$

15) Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \max\left\{x, x^2\right\}$. Let S denote the set of all points in $\mathbb{R}$, where f is not differentiable. Then (2020 - 4 Marks)

a) $\{0, 1\}$
c) $\{1\}$
b) an empty set
d) $\{0\}$