

1.1.5.33

EE24BTECH11059 - Yellanki Siddhanth

Question:

The vectors $\lambda\hat{i} + \hat{j} + 2\hat{k}$, $\hat{i} + \lambda\hat{j} - \hat{k}$ and $2\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar if $\lambda =$

Solution:

Variable	Description	Formula
A	A vector in terms of λ	$A = \begin{pmatrix} \lambda \\ 1 \\ 2 \end{pmatrix}$
B	A vector in terms of λ	$B = \begin{pmatrix} 1 \\ \lambda \\ -1 \end{pmatrix}$
C	A vector in terms of λ	$C = \begin{pmatrix} 2 \\ -1 \\ \lambda \end{pmatrix}$
M	It is a matrix comprising of vectors A, B, C	$M = [A, B, C]$
λ	It is the variable with which A, B, C are established	$ M = 0$

TABLE 0

The rank of a matrix M is less than 3, then the matrix is coplanar. For a rank 1 matrix, there is only one linearly independent column (or row), and all other columns (or rows) are linearly dependent on it.

$$\text{Rank}(M) = 2 \quad (0.1)$$

Equivalently,

$$|M| = 0 \quad (0.2)$$

$$|M| = \begin{vmatrix} \lambda & 1 & 2 \\ 1 & \lambda & -1 \\ 2 & -1 & \lambda \end{vmatrix} = 0 \quad (0.3)$$

$$\lambda(\lambda^2 - 1) - 1(\lambda + 2) + 2(-1 - 2\lambda) = 0 \quad (0.4)$$

$$\lambda^3 - 6\lambda - 4 = 0 \quad (0.5)$$

$$(\lambda + 2)(\lambda^2 - 2\lambda - 2) = 0 \quad (0.6)$$

Therefore,

$$\lambda = -2 \text{ or } \lambda = 1 \pm \sqrt{3} \quad (0.7)$$

Verifying $\lambda = -2$

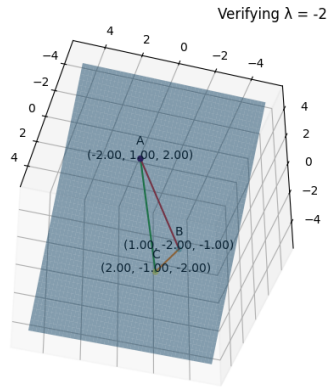


Fig. 0.1

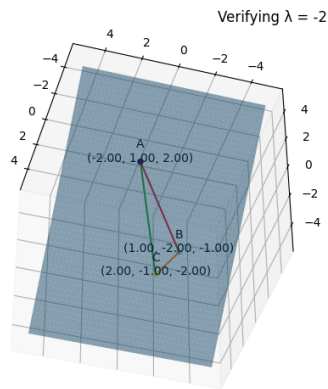


Fig. 0.2