4.4.2.22

EE24BTECH11059 - Yellanki Siddhanth

Question:
Show that two lines $a_1x + b_1y + c_1 = 0$and $a_2x + b_2y + c_2 = 0$ where $b_1b_2 \neq 0$ are parallel if $\frac{a_1}{b_1} = \frac{a_2}{b_2}$ **Solution:**

Variable	Description	Formula
L_1	First Line	$a_1x + b_1y + c_1 = 0$
L_2	Second Line	$a_2x + b_2y + c_2 = 0$

TABLE 0

Rewriting both the lines in $n^\top x = c$ form,

$$L_1 \equiv n_1^\top x = -c_1 \tag{0.1}$$

$$n_1 = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} \implies \|n_1\| = \sqrt{a_1^2 + b_1^2} \tag{0.2}$$

$$L_2 \equiv n_2^\top x = -c_2 \tag{0.3}$$

$$n_2 = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} \implies \|n_2\| = \sqrt{a_2^2 + b_2^2} \tag{0.4}$$

If L_1 and L_2 are parallel lines, then their direction cosines must be the same.

$$\frac{n_2}{\|n_2\|} = \frac{n_1}{\|n_1\|} \tag{0.5}$$

$$\implies \frac{a_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2}{\sqrt{a_2^2 + b_2^2}} \tag{0.6}$$

$$\implies \frac{b_1}{\sqrt{a_1^2 + b_1^2}} = \frac{b_2}{\sqrt{a_2^2 + b_2^2}} \tag{0.7}$$

Thus proving that $\frac{a_1}{b_1} = \frac{a_2}{b_2}$.