JEE ASSIGNMENT 2

EE1030 : Matrix Theory
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1) A plane passes through the points $A\,(1,2,3)$, $B\,(2,3,1)$ and $C\,(2,4,2)$. If O is the origin and P is $(2,-1,))$, then the projection of vector $\overrightarrow{OP}$ on this plane is of length: (2021 - 4 Marks)

a) $\sqrt{\frac{2}{5}}$ b) $\sqrt{\frac{2}{3}}$ c) $\sqrt{\frac{2}{11}}$ d) $\sqrt{\frac{2}{7}}$

2) The contrapositive of the statement "If you will work, you will earn money" is: (2021 - 4 Marks)

a) If you will not earn money, you will not work
b) You will earn money, if you will not work
c) If you will earn money, you will work
d) To earn money, you need to work

3) If $\alpha,\beta \in \mathbb{R}$ are such that $1-2i$ (here $i^2=-1$) is a root of $z^2+\alpha z+\beta=0$, then $(\alpha-\beta)$ is equal to: (2021 - 4 Marks)

a) 7 b) -3 c) -7 d) 3

4) If $\int_{\pi/4}^{\pi/2} cot^n x dx$, then: (2021 - 4 Marks)

a) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in G.P.
b) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.
c) $I_2+I_4,\ I_3+I_5,\ I_4+I_6$ are in A.P.
d) $I_2+I_4,\ I_3+I_5,\ I_4+I_6$ are in G.P.

5) $A=\begin{pmatrix}1 & -\alpha\\ \alpha & \beta\end{pmatrix}$, $AA^\top = I_2$, then the value of $\alpha^4+\beta^4$ is: (2021 - 4 Marks)

a) 1 b) 2 c) 3 d) 4

6) Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set $A\times B$. Then: (2021 - 4 Marks)

a) $y = 273x$ b) $2y = 91x$ c) $y = 91x$ d) $2y = 273x$

7) If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is: (2021 - 4 Marks)

a) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$ b) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$ c) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$ d) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$

8) The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}}$, $x > 0$ is equal to:
(2021 - 4 Marks)

a) $\log_e |x^2 + 5x - 7| + c$
b) $\frac{1}{4}\log_e |x^2 + 5x - 7| + c$
c) $4\log_e |x^2 + 5x - 7| + c$
d) $\log_e \sqrt{x^2 + 5x - 7} + c$

9) A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is:
(2021 - 4 Marks)

a) $\frac{x^2}{9} - \frac{y^2}{4} = 1$
b) $\frac{x^2}{9} - \frac{y^2}{16} = 1$
c) $x^2 - y^2 = 9$
d) $\frac{x^2}{9} - \frac{y^2}{25} = 1$

10) $\lim_{n\to\infty}\left[\frac{1}{n} + \frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} + \ldots\ldots + \frac{n}{(2n-1)^2}\right]$ is equal to: (2021 - 4 Marks)

a) 1 b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{1}{4}$

11) In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from chest disorder. The probability that the selected person is a smoker and non-vegetarian is:
(2021 - 4 Marks)

a) $\frac{7}{45}$ b) $\frac{8}{45}$ c) $\frac{14}{45}$ d) $\frac{28}{45}$

12) The following system of equations, $2x + 3y + 2z = 9, 3x + 2y + 2z = 9, x - y + 4z = 8$
(2021 - 4 Marks)

a) does not have any solution
b) has a unique solution
c) has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$
d) has infinitely many solutions

13) The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$ where $a, x \in \mathbb{R}$ and $a > 0$, is equal to:
(2021 - 4 Marks)

a) $a + \frac{1}{a}$ b) $a + 1$ c) $2a$ d) $2\sqrt{a}$

14) The function $f(x)$ is given by $f(x) = \frac{5x}{5x+5}$, then the sum of the series $f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \ldots + f\left(\frac{39}{20}\right)$ is equal to: (2021 - 4 Marks)

a) $\frac{19}{2}$ b) $\frac{49}{2}$ c) $\frac{39}{2}$ d) $\frac{29}{2}$

15) Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is: (2021 - 4 Marks)

a) 4 b) 1 c) 2 d) 3