1.1.6.19

EE24BTECH11059 - Yellanki Siddhanth

Question:

The vectors $\lambda\hat{i}+\hat{j}+2\hat{k}$, $\hat{i}+\lambda\hat{j}-\hat{k}$ and $2\hat{i}-\hat{j}+\lambda\hat{k}$ are coplanar if $\lambda =$
Solution:

Variable	Description	Formula
A	A vector in terms of λ	$A = \begin{pmatrix}\lambda\\1\\2\end{pmatrix}$
B	A vector in terms of λ	$B = \begin{pmatrix}1\\\lambda\\-1\end{pmatrix}$
C	A vector in terms of λ	$C = \begin{pmatrix}2\\-1\\\lambda\end{pmatrix}$
M	It is a matrix comprising of vectors A, B, C	$M = [A, B, C]$
λ	It is the variable with which A, B, C are established	$\lvert M\rvert = 0$

TABLE 0

The rank of a matrix M is the maximum number of linearly independent rows or columns in the matrix. If the rank of the matrix M is 2, it means that there are only two linearly independent vectors in the set, and hence, the three vectors are coplanar (since the third vector is a linear combination of the first two).

$$Rank(M) = 2 \tag{0.1}$$

Equivalently,

$$|M| = 0 \tag{0.2}$$

$$|M| = \begin{vmatrix}\lambda & 1 & 2\\ 1 & \lambda & -1\\ 2 & -1 & \lambda\end{vmatrix} = 0 \tag{0.3}$$

$$\lambda(\lambda^2-1) - 1(\lambda+2) + 2(-1-2\lambda) = 0 \tag{0.4}$$

$$\lambda^3 - 6\lambda - 4 = 0 \tag{0.5}$$

$$(\lambda + 2)(\lambda^2 - 2\lambda - 2) = 0 \tag{0.6}$$

Therefore,

$$\lambda = -2 \text{ or } \lambda = 1 \pm \sqrt{3} \tag{0.7}$$

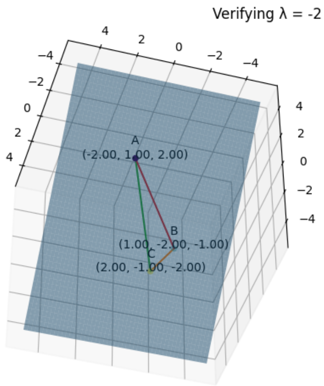

Fig. 0.1

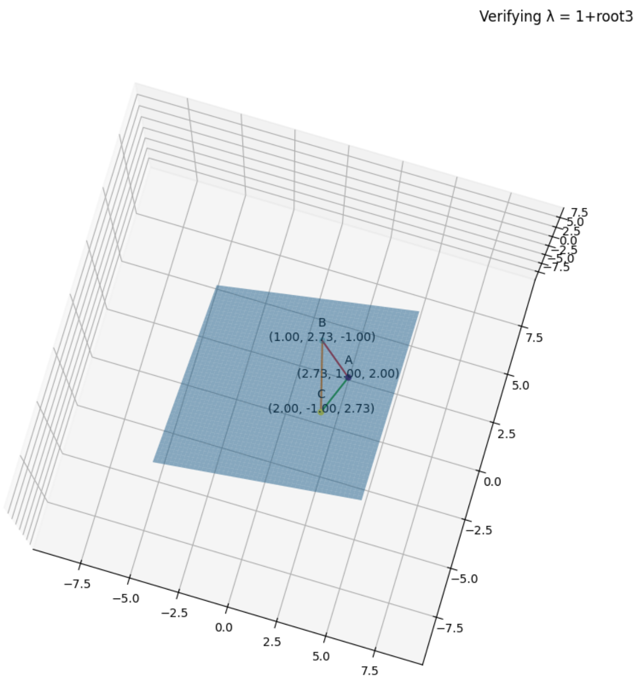
Verifying λ = 1+root3
B
(1.00, 2.73, -1.00)
A
(2.73, 1.00, 2.00)
C
(2.00, -1.00, 2.73)
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5

Fig. 0.2

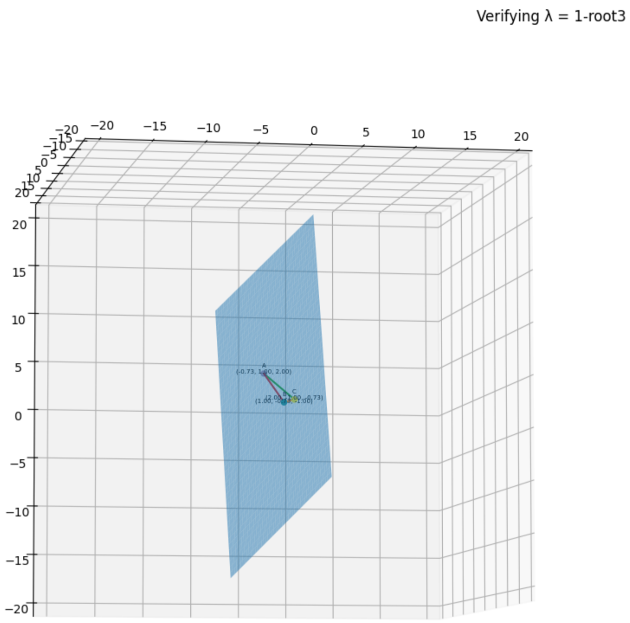

Fig. 0.3