JEE ASSIGNMENT 5

EE1030 : Matrix Theory

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1) The total number of functions, $f : \{1,2,3,4\} \to \{1,2,3,4,5,6\}$ such that $f(1)+f(2) = f(3)$, is equal to (2022 - 4 Marks)

   a) 60 b) 90 c) 108 d) 126

2) If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4+x^3+x^2+x+1=0$, then $\alpha^{2022}+\beta^{2022}+\gamma^{2022}+\delta^{2022}$ is equal to (2022 - 4 Marks)

   a) -4 b) -1 c) 1 d) 4

3) For $n \in \mathbb{N}$ $S_n = \left\{z \in C : |z-3+2i| = \frac{n}{4}\right\}$ and $T_n = \left\{z \in C : |z-2+3i| = \frac{1}{n}\right\}$. Then the number of elements in the set $\{n \in N : S_n \cap T_n = \phi\}$ is (2022 - 4 Marks)

   a) 0 b) 2 c) 3 d) 4

4) The number of $q \in (0, 4\pi)$ for which the system of linear equations

$$\begin{aligned} 3(\sin 3\theta)x - y + z &= 2 \\ 3(\cos 2\theta)x + 4y + 3z &= 3 \\ 6x + 7y + 7z &= 9 \end{aligned}$$

   has no solution, is (2022 - 4 Marks)

   a) 6 b) 7 c) 8 d) 9

5) If $\lim_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$ then $8(\alpha+\beta)$ is equal to (2022 - 4 Marks)

   a) 4 b) -8 c) -4 d) 8

6) If the absolute maximum value of the function $f(x) = \left(x^2-2x+7\right)e^{\left(4x^3-12x^2-180x+31\right)}$ in the interval $[-3,0]$ is $f(\alpha)$, then (2022 - 4 Marks)

a) $\alpha = 0$ b) $\alpha = -3$ c) $\alpha \in (-1, 0)$ d) $\alpha \in (-3, -1]$

7) The curve $y(x) = ax^3 + bx^2 + cx + 5$ touches the x-axis at the point P(–2, 0) and cuts the y-axis at the point Q, where y' is equal to 3. Then the local maximum value of $y(x)$ is (2022 - 4 Marks)

a) $\frac{27}{4}$ b) $\frac{29}{4}$ c) $\frac{37}{4}$ d) $\frac{9}{2}$

8) The area of the region given by $A = \left\{(x, y)\,;\, x^2 \le y \le \min\{x + 2, 4 - 3x\}\right\}$ is text (2022 - 4 Marks)

a) $\frac{31}{8}$
b) $\frac{17}{6}$
c) $\frac{19}{6}$
d) $\frac{27}{8}$

9) For any real number x, let $[x]$ denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval $[-10, 10]$ by

$$f(x) = \begin{cases} x - [x], & \text{if } [x] \text{ is odd} \\ 1 + [x] - x, & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\frac{\pi^2}{10} \int_{-10}^{10} f(x) \cos \pi x \, dx$ is (2022 - 4 Marks)

a) 4 b) 2 c) 1 d) 0

10) The slope of the tangent to a curve $C : y = y(x)$ at any point (x, y) on it is $\frac{2e^{2x} - 6e^{-x} + 9}{2 + 9e^{-2x}}$. If C passes through the points $\left(0, \frac{1}{2} + \frac{\pi}{2\sqrt{2}}\right)$ and $\left(\alpha, \frac{1}{2}e^{2\alpha}\right)$, then e^α is equal to (2022 - 4 Marks)

a) $\frac{3+\sqrt{2}}{3-\sqrt{2}}$
b) $\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$
c) $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$
d) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$

11) The general solution of the differential equation $(x - y^2)dx + y(5x + y^2)dy = 0$ is : (2022 - 4 Marks)

a) $\left(y^2 + x\right)^4 = C\left|\left(y^2 + 2x\right)^3\right|$
b) $\left(y^2 + 2x\right)^4 = C\left|\left(y^2 + x\right)^3\right|$
c) $\left|\left(y^2 + x\right)^3\right| = C\left(2y^2 + x\right)^4$
d) $\left|\left(y^2 + 2x\right)^3\right| = C\left(2y^2 + x\right)^4$

12) A line, with the slope greater than one, passes through the point $A(4, 3)$ and intersects the line $x - y - 2 = 0$ at the point B. If the length of the line segment AB is $\frac{\sqrt{29}}{3}$, then B also lies on the line (2022 - 4 Marks)

a) $2x + y = 9$ b) $3x - 2y = 7$ c) $x + 2y = 6$ d) $2x - 3y = 3$

13) Let the locus of the centre (α,β), $\beta > 0$, of the circle which touches the circle $x^2 + (y-1)^2 = 1$ externally and also touches the x-axis be L. Then the area bounded by L and the line $y = 4$ is: (2022 - 4 Marks)

a) $\frac{32\sqrt{2}}{3}$ b) $\frac{40\sqrt{2}}{3}$ c) $\frac{64}{3}$ d) $\frac{32}{3}$

14) Let P be the plane containing the straight line $\frac{x-3}{9} = \frac{y+4}{-1} = \frac{z-7}{-5}$ and perpendicular to the plane containing the straight lines $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ and $\frac{x}{3} = \frac{y}{7} = \frac{z}{8}$. If d is the distance P from the point $(2, -5, 11)$, then d^2 is equal to: (2022 - 4 Marks)

a) $\frac{147}{2}$ b) 96 c) $\frac{32}{3}$ d) 54

15) Let ABC be a triangle such that $\overrightarrow{BC} = \vec{a}, \overrightarrow{CA} = \vec{b}, \overrightarrow{AB} = \vec{c}, \left|\vec{a}\right| = 6\sqrt{2}, \left|\vec{b}\right| = 2\sqrt{3}$ and $\vec{b} \cdot \vec{c} = 1$. Consider the statements :

$$(S1) : \left|\left(\vec{a} \times \vec{b}\right) + \left(\vec{c} \times \vec{b}\right)\right| - \left|\vec{c}\right| = 6\left(2\sqrt{2} - 1\right)$$

$$(S2) : \angle ACB = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

, then (2022 - 4 Marks)

a) Both (S1) and (S2) are true
b) Only (S1) is true
c) Only (S2) is true
d) Both (S1) and (S2) are false