# 1.1.6.19

EE24BTECH11059 - Yellanki Siddhanth

**Question**:

The vectors $\lambda\hat{i} + \hat{j} + 2\hat{k}$, $\hat{i} + \lambda\hat{j} - \hat{k}$ and $2\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar if $\lambda =$

**Solution:**

| Variable | Description | Formula |
|---|---|---|
| $A$ | A vector in terms of $\lambda$ | $A = \begin{pmatrix} \lambda \\ 1 \\ 2 \end{pmatrix}$ |
| $B$ | A vector in terms of $\lambda$ | $B = \begin{pmatrix} 1 \\ \lambda \\ -1 \end{pmatrix}$ |
| $C$ | A vector in terms of $\lambda$ | $C = \begin{pmatrix} 2 \\ -1 \\ \lambda \end{pmatrix}$ |
| $M$ | It is a matrix comprising of vectors $A, B, C$ | $M = [A, B, C]$ |
| $\lambda$ | It is the variable with which $A, B, C$ are established | $\lvert M \rvert = 0$ |

TABLE 0

The rank of a matrix $M$ is less than 3, then the matrix is coplanar.

$$Rank\,(M) = 2 \tag{0.1}$$

Equivalently,

$$|M| = 0 \tag{0.2}$$

$$|M| = \begin{pmatrix} \lambda & 1 & 2 \\ 1 & \lambda & -1 \\ 2 & -1 & \lambda \end{pmatrix} = 0 \tag{0.3}$$

$$\lambda\left(\lambda^2 - 1\right) - 1\,(\lambda + 2) + 2\,(-1 - 2\lambda) = 0 \tag{0.4}$$

$$\lambda^3 - 6\lambda - 4 = 0 \tag{0.5}$$

$$(\lambda + 2)\left(\lambda^2 - 2\lambda - 2\right) = 0 \tag{0.6}$$

$$\lambda = -2 \text{ or } \lambda = 1 \pm \sqrt{3} \tag{0.7}$$

$\therefore$ Verifying $\lambda$ values:

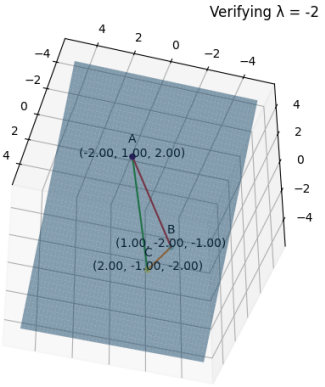


Fig. 0.1

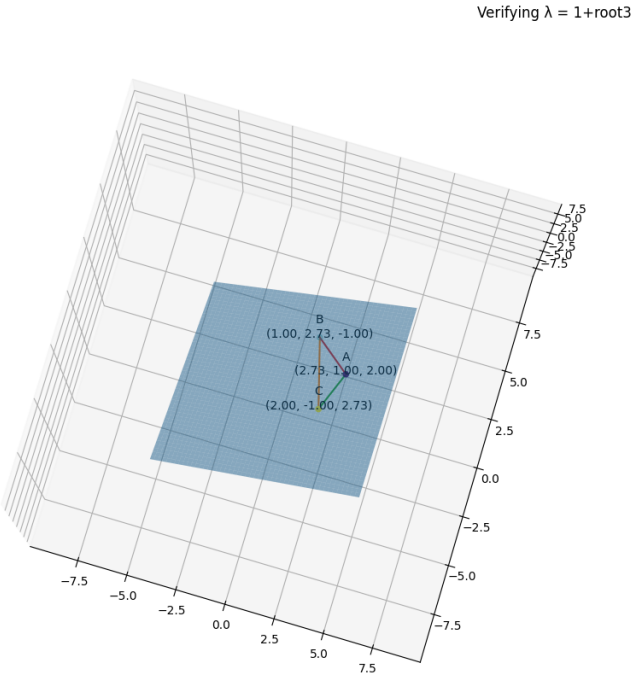
Verifying λ = 1+root3
B
(1.00, 2.73, -1.00)
A
(2.73, 1.00, 2.00)
C
(2.00, -1.00, 2.73)
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5

Fig. 0.2

Verifying λ = 1-root3

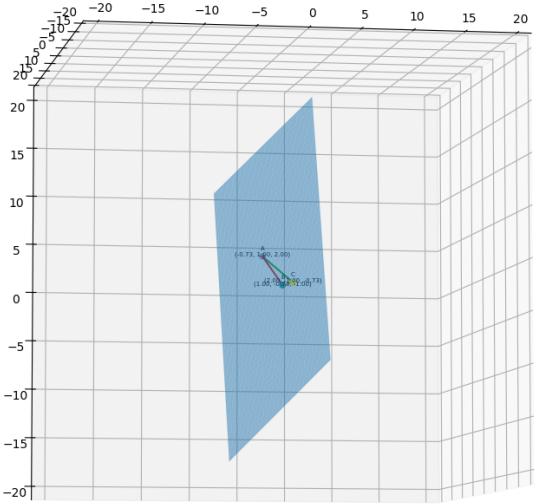


Fig. 0.3