JEE ASSIGNMENT 6

EE1030 : Matrix Theory
Indian Institute of Technology Hyderabad

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1) The set of all values of a for which $\lim_{x \to a} ([x-5] - [2x+2]) = 0$, where $[\cdot]$ denotes the greatest integer less than or equal to $\cdot$, is equal to (2023 - 4 Marks)

   a) $[-7.5, -6.5)$ b) $(-7.5, -6.5]$ c) $[-7.5, -6.5]$ d) $(-7.5, -6.5)$

2) Let p and q be two statements. Then $\sim (p \wedge (p \Rightarrow \sim q)$ is equivalent to (2023 - 4 Marks)

   a) $(\sim p) \vee q$ b) $p \vee ((\sim p) \wedge q)$ c) $p \vee (p \wedge q)$ d) $p \vee (p \wedge (\sim q))$

3) The locus of the mid points of the chords of the circle $C1 : (x-4)^2 + (y-5)^2 = 4$ which subtend an angle θ_i at the centre of the circle C_1, is a circle of radius r_i. If $\theta_1 = \frac{\pi}{3}, \theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to (2023 - 4 Marks)

   a) $\frac{3\pi}{4}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

4) If $f(x) = \frac{2^{2x}}{2^{2x}+2}, x \in \mathbb{R}$, then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \text{........} + f\left(\frac{2022}{2023}\right)$ is equal to (2023 - 4 Marks)

   a) 1010 b) 2011 c) 1011 d) 2010

5) If the system of equations

$$x + 2y + 3z = 3$$

$$4x + 3y - 4z = 4$$

$$8x + 4y - \lambda z = 9 + \mu$$

   has infinitely many solutions, then the ordered pair (λ, μ) is equal to : (2023 - 4 Marks)

   a) $\left(-\frac{72}{5}, \frac{21}{5}\right)$ b) $\left(\frac{72}{5}, -\frac{21}{5}\right)$ c) $\left(\frac{72}{5}, \frac{21}{5}\right)$ d) $\left(-\frac{72}{5}, -\frac{21}{5}\right)$

6) Let the plane containing the line of intersection of the planes $P_1 : x + (\lambda + 4) y + z = 1$ and $P_2 : 2x + y + z = 2$ pass through the points $(0, 1, 0)$ and $(1, 0, 1)$. Then the distance of the point $(2, \lambda, -\lambda)$ from the plane P_2 is (2023 - 4 Marks)

a) $4\sqrt{6}$ b) $3\sqrt{6}$ c) $5\sqrt{6}$ d) $2\sqrt{6}$

7) If $\left(\binom{30}{1}\right)^2 + 2\left(\binom{30}{2}\right)^2 + 3\left(\binom{30}{3}\right)^2 + ... + 30\left(\binom{30}{30}\right)^2 = \frac{\alpha 60!}{(30!)^2}$, then α is equal to (2023 - 4 Marks)

a) 60 b) 30 c) 15 d) 10

8) If the foot of the perpendicular drawn from $(1, 9, 7)$ to the line passing through the point $(3, 2, 1)$ and parallel to the planes $x+2y+z = 0$ and $3y-z = 3$ is (α, β, γ), then $\alpha+\beta+\gamma$ is equal to (2023 - 4 Marks)

a) -1 b) 1 c) 3 d) 5

9) Let A be a 3×3 matrix such that $|\text{adj}(\text{adj}(\text{adj}A))| = 12^4$. Then $|A^{-1}\text{adj}A|$ is equal to (2023 - 4 Marks)

a) 12 b) $2\sqrt{3}$ c) $\sqrt{6}$ d) 1

10) The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is (2023 - 4 Marks)

a) $\frac{1}{2}\left(\sqrt{3}+1\right)$ c) $\frac{1}{2}\left(1-\sqrt{3}\right)$
b) $-\frac{1}{2}\left(1-i\sqrt{3}\right)$ d) $-\frac{1}{2}\left(\sqrt{3}-1\right)$

11) The number of square matrices of order 5 with entries from the set $\{0, 1\}$, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is (2023 - 4 Marks)

a) 120 c) 150
b) 225 d) 125

12) $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}dx$ is equal to (2023 - 4 Marks)

a) $\frac{\pi}{6}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) 2π

13) The number of real solutions of the equation $3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5=0$ is (2023 - 4 Marks)

a) 3 b) 0 c) 2 d) 4

14) Let $\alpha = 4\hat{i}+3\hat{j}+5\hat{k}$ and $\beta = 2\hat{i}+4\hat{j}$. Let β_1 be parallel to α and β_2 be perpendicular to α. If $\beta = \beta_1+\beta_2$, then the value of $5\beta_2 \cdot \left(\hat{i}+\hat{j}+\hat{k}\right)$ is (2023 - 4 Marks)

a) 7 b) 9 c) 6 d) 11

15) Let $f(x)$ be a function such that $f(x+y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{R}$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is. (2023 - 4 Marks)

a) 8
b) 9
c) 6
d) 7