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Production planning for a ramp-up process in a multi-stage production system with worker learning and growth in demand

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ABSTRACT

In a response to changes in customer requirements and environmental dynamics, product lifecycles have become shorter and shorter over the last decades. As a result, production ramp-ups have become more frequent in many industries, and they now often account for a significant share of the entire product lifecycle. Due to the prominent role production ramp-ups play in the lifecycle of a product, efficient production ramp-ups are now an important determinant of business success. This work proposes a mathematical model for managing production ramp-ups in a serial multi-stage production system. In the scenario considered here, both the productivity of workers and demand increase over time until a steady-state phase is reached at the end of the ramp-up. The model proposed in this paper supports the assignment of workers to the different stages of the production system and the balancing of production and demand to ensure a smooth transition from the production ramp-up to steady-state production. The model is analysed in numerical experiments to illustrate its potential for managing the production ramp-up. Our experiments show that high learning rates can have drawbacks in terms of large inventories if learning is not aligned with demand growth and across production stages.

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KEYWORDS
Production ramp-up; multi-stage production system; learning effect; demand growth

1. Introduction

The production ramp-up can be defined as the phase between the end of product development and full-capacity production, where production is scaled up from small batches to the large volumes requested by the market (e.g. Terwiesch and Bohn 2001; Haller, Peikert, and Thoma 2003; Terwiesch and Xu 2004; Doltsinis, Ratchev, and Lohse 2013). Especially during early phases of the ramp-up, the production process is often not well understood and error-prone, making process adjustments necessary that are costly to the company and that may lead to delays in the introduction of the product to the market (Doltsinis, Ratchev, and Lohse 2013). Problems encountered during the production ramp-up include disturbances in process and product quality, a lack of planning reliability, unplanned capacity losses, and low supplier performance (e.g. Almgren 1999; Almgren 2000; Surbier, Alpan, and Blanco 2013).

Prior research has hypothesised that the importance of the production ramp-up has increased in recent years (e.g. Terwiesch and Bohn 2001; Terwiesch, Bohn, and Chea 2001; Ceglarek et al. 2004; Matta, Tomasella, and Valenta 2007). First, product lifecycles have become shorter over the last decades, leading to more frequent new product introductions and associated production ramp-ups. Secondly, production has become more complex in many industries, which has made the planning and realisation of efficient production ramp-ups much more difficult. As the production ramp-up may account for a large share of the product's lifecycle today, ramp-up management is an important lever for lowering cost and for attracting customers in a market segment. Given that companies are often able to charge higher prices early in the lifecycle of a product (Haller, Peikert, and Thoma 2003), a fast and smooth introduction of the product into the market may help companies to increase profits and to strengthen and improve their market positions.

During the production ramp-up, companies can adjust the production process in various ways. Prior research has, for example, identified the adjustment of production capacities, the sequencing or changing of production processes, worker training or the assignment of

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workers to work stations as important levers in the management of the ramp-up (e.g. Gopal et al. 2013; Li et al. 2014; Glock and Grosse 2015). To support managers in controlling the ramp-up process, researchers have developed several mathematical models in the past that support different planning problems companies usually face during this stage of the product lifecycle. A recent review of the literature in this field is the work of Glock and Grosse (2015), who proposed a classification scheme for decision support models for the production ramp-up. The scheme took account both of the attributes of the ramp-up process considered in the model (e.g. the production of defective items or an increase in demand over time) as well as the managerial actions the model aims to support (e.g. worker assignment or capacity expansion).

The work at hand contributes to the literature on the production ramp-up by extending two earlier works of Glock, Jaber, and Zolfaghari (2012) and Kim (2018), who investigated a situation where both the productivity of resources (workers) and customer demand increase over time until a steady-state phase is reached at the end of the production ramp-up. As the development of resource productivity and the demand rate may follow different patterns, the production planner has to decide on the number of workers to assign to the production system to ensure a smooth ramp-up of the product at minimal cost and without encountering any shortages. While Glock, Jaber, and Zolfaghari (2012) studied the production ramp-up in a simple two-stage production system, Kim (2018) investigated a vendor-buyer supply chain. One limitation of these works is that both considered production systems with only a single producing and a single consuming stage. Acknowledging that production systems are often more complex in practice, the work at hand extends the models of Glock, Jaber, and Zolfaghari (2012) and Kim (2018) to account for multiple serial production stages. With both production and consumption increasing during the course of the ramp-up, but each stage following a different pattern, a complex planning problem arises where a smooth transition to steady-state production is not easy to obtain. The work at hand supports production planning in this scenario.

The remainder of the paper is structured as follows. The next section presents a review of related works, and Section 3 introduces the problem investigated in this paper. Section 4 proposes a mathematical model for managing the ramp-up of a serial multi-stage production system that is illustrated in numerical experiments in Section 5. Section 6 summarises the paper and presents an outlook on future research opportunities.

2. Literature review

The management of the production ramp-up has been the subject of research for several decades. During this phase of the product lifecycle, production planners have to solve different decision problems to ensure a smooth and cost-efficient ramp-up of the production system. First, to keep up with the increasing end customer demand (e.g. Terwiesch and Xu 2004; Surbier, Alpan, and Blanco 2013), companies may have to invest into machinery or decide to assign additional workers to the stages of the production system to expand the company's production capacity. Secondly, given that the quality of the product still changes during the ramp-up (e.g. Terwiesch, Bohn, and Chea 2001), the company has to manage inventories carefully to avoid that stock is built up that later needs to be reworked or scrapped. Finally, also the production workflow, which includes the determination of production and machine feed rates and the definition of production sequences (e.g. Nembhard and Birge 1998), needs to be optimised to minimise production costs. Surprisingly, especially the assignment of workers to the stages of the production system during ramp-ups has not attracted much attention in the literature so far.

One of the first works that studied the sizing of the workforce during the production ramp-up we are aware of is the one of Carrillo and Franza (2006), who considered a ramp-up scenario where a company can assign labour hours both to product design and to production. While product design influences the unit revenue the company can earn during the ramp-up, assigning labour hours to production increases the production capacity. Both the design and the production capacity were also assumed to increase over time subject to a learning effect. The model proposed by the authors supports production planners in finding an optimal worker assignment that maximises the profit generated during the ramp-up. Glock, Jaber, and Zolfaghari (2012) studied a simple two-stage inventory model during a production ramp-up. The authors assumed that demand at the second stage increases over time and that the company has to assign workers to the first stage in regular time intervals to match production and demand. Workers were assumed to learn during the course of the ramp-up, leading to an increase in their production rates. The authors showed that a frequent restructuring of the production stage helps to match the total production rate well with the demand rate, which reduces inventory in the system, albeit at the expense of high setup cost. The proposed model helps to balance these two conflicting costs. The model was extended by Kim (2018) to the case of a supply chain where production occurs at a single vendor and demand at a single buyer. In addition to the assignment of workers to the production stage, the supply chain now also has to decide about how to ship finished products from the vendor to the buyer to minimise the total supply chain costs.

Other authors did not study the assignment of workers to production stages explicitly, but instead referred to resource investments to expand the production capacity during the ramp-up. Given that some of these works considered learning effects in production that have often been used to model the accumulation of knowledge on the worker level, these works could support the sizing of the workforce during ramp-ups as well.

Hiller and Shapiro (1986), for example, studied a multi-period ramp-up scenario where learning occurs in production and where the price of the product decreases over time. For each period of the ramp-up, the company faces a set of investment opportunities it can select to further increase its production capacity. The model proposed by the authors determines an optimal investment plan for the ramp-up that maximises the discounted sum of net revenues. Gatto and Ghezzi (1992) considered a two-stage production system where demand grows over time and where the unit production cost decreases as a result of learning. The authors developed an optimal control policy that determines how much to invest into the production capacity over time. They showed that in the optimal control policy, the entire profit obtained during the ramp-up is reinvested until an optimal capacity level has been reached. Hansen and Grunow (2015) studied a company that produces a new product in a number of production facilities and that sells the product to different markets. The company can increase the production capacity of its facilities by assigning (further) production lines to the facilities, and in addition, it benefits from learning that occurs at the production lines as a function of the cumulative output produced. The model proposed by the authors determines when to open production lines at the facilities with the objective to maximise the net present value of the ramp-up phase. Becker, Stolletz, and Stäblein (2017) studied a strategic production ramp-up problem where the tasks are to allocate products to plants, to time ramp-ups and ramp-downs, to select suitable ramp-up patterns for each plant, and to decide on the production volumes for each product for each market over time. The authors proposed a mixed-integer programme to simultaneously optimise these decisions for given sets of ramp-up curves. Numerical studies indicate that a simultaneous optimisation of the strategic ramp-up decisions leads to substantially better results than a sequential optimisation. Meisel and Glock (2018) investigated a scenario where a single buyer faces two sources of supply during the production ramp-up. The authors assumed that demand increases during the ramp-up and that learning occurs at both suppliers, leading to an increase in the suppliers' production rates and a reduction in procurement costs over time. In addition, they assumed that the buyer has the option to implement supplier development projects at both suppliers, which leads to an additional improvement in the suppliers' production rates and procurement costs. The model proposed by the authors supports the supplier selection decision and assists the buyer in deciding whether to implement supplier development projects, or to rely solely on self-induced performance improvements at the supplier(s).

The works discussed so far all considered two-stage production systems where production occurs only on a single stage (with parallel production lines or facilities in some cases). While these works help gaining valuable insights into how individual production stages (or production facilities on an aggregated level) should be managed to minimise the costs of the ramp-up, they provide only limited support for the management of multi-stage production systems. Prior research on multi-stage production systems has shown that the production rates of subsequent stages need to be carefully managed to avoid unnecessary inventories and to ensure that the cost of production can be minimised (e.g. Glock 2011). We hypothesise that the management of multi-stage production systems is even more challenging during ramp-ups where demand increases over time, and where the production rates of the stages develop according to different patterns. The only work we are aware of that studied a serial multi-stage production system during a ramp-up is the one of Neumann and Medbo (2017), who developed a discrete event simulation model to compare a serial production system to a parallel production system for a production ramp-up with worker learning. The trade-off studied in the paper is that the serial system will have shorter cycles times, leading to more repetitions and hence more opportunities to learn, while the initially slower parallel system will display a higher productivity over time. The results of the simulation showed that after a certain threshold duration of the ramp-up, the parallel system outperformed the serial system. Hence, the parallel system should be preferred for ramp-up processes whose expected duration exceeds this threshold value. Despite the important insights the paper provides on the relative advantage of the two production systems, it assumed that learning at the production stages is identical and also did not consider inventory that emerges between the stages.

To further our understanding of how multi-stage production systems should be operated during production ramp-ups, this paper develops a model of a serial multi-stage production system where both the demand rate and the production rates of the stages increase over time, but where all rates may develop according to different patterns. Based on our review of the literature, we formulate the following research questions:

RQ1: What is the influence of the demand pattern on the ramp-up of the production facility?

RQ2: If demand and worker productivity develop according to different patterns, can a company be interested in withdrawing workers from the production stages over time?

RQ3: If the learning rates of the stages are non-identical, how should the stages be sequenced (or how should workers be assigned to the stages) to minimize total cost?

RQ4: Is there a theoretically optimal learning rate that ideally matches production and demand and that minimizes total cost?

RQ5: Does the number of stages of the production system have an influence on the ramp-up pattern?

The next two sections develop a mathematical model based on the earlier works of Glock, Jaber, and Zolfaghari (2012) and Kim (2018) to answer the research questions formulated above.

3. Problem description

This paper studies a serial multi-stage production system for a single type of product during production ramp-up. While moving from low-volume to large-volume production, workers are assumed to improve their knowledge of the production process that was initially not well understood (e.g. Wilhelm and Sastri 1979; Terwiesch and Bohn 2001; Surbier, Alpan, and Blanco 2013), leading to an increase in worker productivity over time. We use the term 'worker learning' in this context to refer to the productivity improvement of workers that results from the repetitive execution of tasks, from familiarising with the production process and from organisational and technological changes during the ramp-up, including both cognitive and motor learning. Worker learning is modelled using learning curves in the paper at hand, taking account of the fact that learning may follow different patterns at the various stages of the system. For an overview of how the learning curve concept has been used in the operations management domain in the past, we refer the reader to the recent review of Glock et al. (2019).

In addition to worker learning, we assume that customer demand increases as well as the new product diffuses in the market (e.g. Terwiesch and Xu 2004; Niroomand, Kuzgunkaya, and Bulgak 2012). We use an established demand growth function to model the increase in customer demand and assume that demand reaches a saturation level at the end of the ramp-up. Figure 1 illustrates the system investigated in this paper. The production system we consider is linear with a single machine/production unit on each stage. The stage producing the final product is referred to as Stage 1. If we focus on a particular Stage i, then Stages $i+1, \ldots, N$ would be referred to as the upstream part and stages $i-1, \ldots, 1$ as the downstream part of the manufacturing system (from the perspective of Stage i). The objective of the paper is to develop a model that supports managing the production ramp-up in this scenario by determining how workers should be assigned to the different stages of the system.

In developing the proposed model, the following assumptions are made:

(1) Items completed at one of the stages can immediately be consumed by the subsequent downstream stage due to short distances between the stages. Shipment times between stages are thus negligible.
(2) The storage capacity at all production stages is sufficient to accommodate items waiting at the respective stages.
(3) Demand develops according to a logistics curve, and worker productivity at the stages of the production system develops according to the time-constant model described in Hackett (1983). Glock, Jaber, and Zolfaghari (2012) have shown that these learning curves fit empirical data of a ramp-up process well. The logistics curve has been used to model the development of demand in ramp-up scenarios also in Kim (2018) and Meisel and Glock (2018). We note

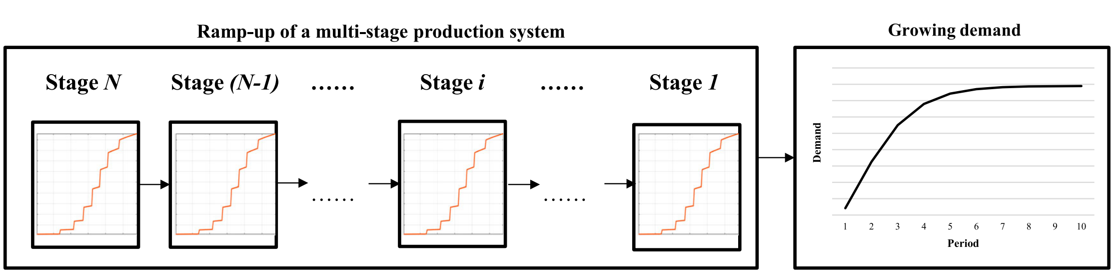


Figure 1. Multi-stage production system during the ramp-up phase.

that the proposed model could be applied as well if demand followed a different pattern, such as the demand diffusion process analyzed by Bass (1969), for example.

(4) Workers need to be assigned to the production stages. They are available without limitation (for example because they can be hired at a temporal employment agency), albeit at a unit cost. The proposed model assigns worker-time-equivalents to the production stages. If a period is one week with 40 working hours, for example, and if the model assigns 14.5 worker-time-equivalents to a particular stage, then workers work a total of $40*14.5 = 580$ h at that stage.

(5) Workers can be withdrawn from production stages. If a worker is withdrawn from a stage, this does not necessarily mean that the worker is laid off; instead, the company could assign the worker to a different department where the worker could support another ramp-up or continue working in his/her regular position (e.g. in case a foreman was temporarily assigned to a production stage to support the ramp-up with his/her experience). Withdrawing worker-time-equivalents assigned to the stages in earlier periods comes at a unit cost (e.g. due to compensation payments for workers that are laid off).

(6) We consider a finite planning horizon consisting of T periods. At the beginning of each period, the company can set up the stages to assign worker-time-equivalents to or withdraw worker-time-equivalents from the stages.

(7) All parameters are deterministic.

(8) Shortages are not permitted due to the high prices the company is able to charge during the ramp-up phase. Consequently, the company has to control production in a way that ensures that demand is met at any point in time.

In the following, when referring to the assignment or withdrawal of workers to/from the stages of the production system, we mean the assignment or withdrawal of worker-time-equivalents. Thus, if the production planner decides to withdraw a small number of worker-time-equivalents from the stages, the actual number of workers assigned to the stay may remain unchanged, but the time the workers spend at the stage is reduced.

Model development

This section develops the proposed model. The following notation will be used throughout the paper:

Model parameters:

T	Number of periods, index t
N	Number of production stages, index i
d_t	Customer demand at stage 1 in period t
h_{it}	Inventory holding cost per unit per unit of time at stage i in period t
μ_{it}	Cost for assigning one worker to stage i in period t
r	Cost for laying off one worker employed in an earlier period
s_i	Setup cost at stage i
$p_{\max,i}$	Maximum production rate at stage i
δ_i	Difference between $p_{\max,i}$ and the initial production rate at stage i
τ_i	Parameter for the learning effect at stage i
$g_i(t,\tau_i)$	Unit production rate at stage i, where $g_i(t,\tau_i) = \left(p_{\max,i} - \delta_i e^{-\frac{t}{\tau_i}}\right)$
α	Parameter for the setup policy; 1 if workers can only be either added to or be withdrawn from a stage in a certain period, and 2 if a simultaneous assignment and withdrawal of workers is possible in a certain period.
M	Sufficiently large number

Decision variables:

l_{it}	Continuous variable: inventory at stage i at the end of period t
p_{it}	Continuous variable: number of units produced at stage i in period t
$w_{it't}$	Continuous variable: number of workers at stage i committed in period t' used in period t
y_{it}	Binary variable: 1, if a setup occurs at stage i at the beginning of period t; 0 otherwise
y^a_{it}	Binary variable: 1, if a setup occurs for adding new workers to stage i at the beginning of period t; 0 otherwise
y^w_{it}	Binary variable: 1, if a setup occurs for withdrawing existing workers from stage i at the beginning of period t; 0 otherwise
$z_{it't}$	Continuous variable: number of workers at stage i employed in period t' and laid off in period t

Here, we consider two types of workforce-related cost factors, namely the cost for assigning workers to the production stages (i.e. $\sum_{i=1}^{N}\sum_{t=1}^{T}\mu_{it}\sum_{t'=1}^{t}w_{it't}$) and the cost for withdrawing workers from the stages (i.e. $r\sum_{i=1}^{N}\sum_{t=1}^{T-1}\sum_{t'=1}^{t}z_{i,t',t+1}$). Two further costs considered in the proposed model are setup costs ($\sum_{i=1}^{N}\sum_{t=1}^{T} s_i y_{it}$) and inventory carrying cost ($\sum_{i=1}^{N}\sum_{t=1}^{T} h_{it} I_{it}$). Based on the above assumptions, we propose the following model:

Minimise

$$TRC(\boldsymbol{w}, \boldsymbol{y}) = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(s_i y_{it} + h_{it} I_{it} + \mu_{it}\sum_{t'=1}^{t} w_{it't}\right) + r\sum_{i=1}^{N}\sum_{t=1}^{T-1}\sum_{t'=1}^{t} z_{i,t',t+1} \tag{1}$$

subject to

$$I_{1t} = \sum_{t'=1}^{t}(p_{1t'} - d_{t'}) \quad \forall t = 1, \ldots, T \tag{2}$$

$$I_{it} = \sum_{t'=1}^{t}(p_{it'} - p_{i-1,t'}) \quad \forall i = 2, \ldots, N; t = 1, \ldots T \tag{3}$$

$$p_{it} = \sum_{t'=1}^{t} w_{it't} g_i(t - t' + 1, \tau_i) \quad \forall i = 1, \ldots, N; t = 1, \ldots, T \tag{4}$$

$$w_{it't} - w_{i,t',t+1} \leq y_{i,t+1} M \quad \forall i = 1, \ldots, N; t = 1, \ldots, T - 1; t' = 1, \ldots, t \tag{5}$$

$$w_{i,t',t+1} - w_{it't} \leq y_{i,t+1} M \quad \forall i = 1, \ldots, N; t = 1, \ldots, T - 1; t' = 1, \ldots, t \tag{6}$$

$$y_{it}^{a} + y_{it}^{w} \leq \alpha y_{it} \quad \forall i = 1, \ldots, N; t = 1, \ldots, T \tag{7}$$

$$z_{i,t',t+1} = w_{it't} - w_{i,t',t+1} \quad \forall i = 1, \ldots, N; t = 1, \ldots, T - 1; t' = 1, \ldots, t \tag{8}$$

$$w_{itt} \leq y_{it}^{a} M \quad \forall i = 1, \ldots, N; t = 1, \ldots, T \tag{9}$$

$$z_{it't} \leq y_{it}^{w} M \forall i = 1, \ldots, N; t = 1, \ldots, T - 1; t' = 1, \ldots, t \tag{10}$$

$$z_{it't} \geq 0 \quad \forall i = 1, \ldots, N; t = 1, \ldots, T; t' = 1, \ldots, t \tag{11}$$

$$I_{it} \geq 0 \quad \forall i = 1, \ldots, N; t = 1, \ldots, T \tag{12}$$

$$w_{it't} \geq 0 \quad \forall i = 1, \ldots, N; t = 1, \ldots, T; t' = 1, \ldots, t \tag{13}$$

$$y_{it}, y_{it}^{a}, y_{it}^{w} \in \{0, 1\} \quad \forall i = 1, \ldots, N; t = 1, \ldots, T \tag{14}$$

Objective function (1) seeks to minimise the sum of setup cost, inventory carrying cost, and worker deployment cost. Inventory flow equalities (2) and (3) ensure that inventory at the end of period t equals the total production quantity in t minus the total demand in t. Note that demand at the final stage 1 equals the given customer demand d_t, whereas demand at the previous stages $2, \ldots, N$ equals the production quantity of the respective succeeding stage. Constraint (4) sets the production quantity at stage i in period t depending on the number of workers employed and the learning effect if workers were assigned to the stage in previous periods. Both constraints (5) and (6) force a setup at stage i in period t (i.e. $y_{it} = 1$) if the number of workers assigned to the stage has changed from the previous period. Constraint (7) is used to permit or prohibit the simultaneous assignment and withdrawal of workers to/from a certain stage in the same period. Constraint (8) defines the number of withdrawn workers in period t at stage i. Constraints (9) and (10) limit possible values of the decision variables according to the setup mode. Finally, constraints (11) through (14) define the domains of the variables.

4. Numerical studies

This section conducts numerical studies to answer the research questions formulated in Section 2. In Section 4.1, we first introduce a benchmark example with two production stages and a demand stage and investigate ramp-up patterns for alternative demand scenarios as well as for identical and non-identical learning rates at both stages. The experiments provided in this section help us especially in answering RQs 1–3. In Section 4.2, we conduct a sensitivity analysis in which we add further stages to the production system and systematically vary the learning rates to answer RQs 4 and 5. The developed optimisation model was solved using the LINGO Solver 18.0 on a PC with an i5-7500 CPU @3.40 GHz processor.

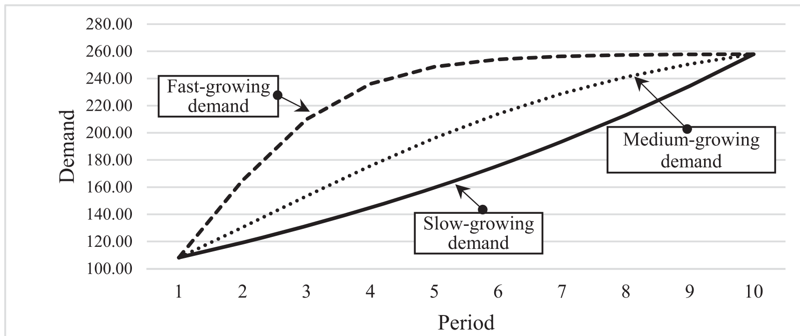
4.1. Numerical examples

Case 1) Identical learning rates

In our first experiment, we use the parameters listed in Table 1 and assume that both stages have the same learning rates, i.e. $\tau_1 = \tau_2 = 1.00$. Both stages also have the same operating characteristics except for the holding costs that are assumed to increase along the supply chain due to the value-adding concept (see, e.g. Sajadieh, Thorstenson, and Akbari Jokar 2010; Kim and Glock 2013; Zanoni et al. 2014). We also assume three different demand scenarios, namely slow-, medium- and fast-growing demand (see Table 1).

We first assume that workers can be added to and be withdrawn from a stage in the same period (i.e. $\alpha = 2$). The results obtained from solving the optimisation model for the case where laying off workers employed

Table 1. Parameters used for numerical experimentation.

Production-related parameters								
Stage(i)	S_i	h_i	μ_i	$p_{max,i}$	δ_i	τ_i	T	α
1	50.0	3.0	5.0	10.0	5.0	1.00	10	2
2	50.0	2.5	5.0	10.0	5.0	1.00		
Demand functions								
Demand Type	Slow-growing demand			Medium-growing demand		Fast-growing demand		
Function	$d(t)=\frac{5000}{1+50e^{-0.10t}}$			$d(t)=\frac{279}{1+2.2e^{-0.33t}}$		$d(t)=\frac{258}{1+3.4e^{-0.90t}}$		
Demand Patterns								

in earlier periods does not lead to costs (i.e. $r = 0.0$) are shown in Table 2(a). As can be seen, for the scenario with slow-growing demand, the workers assigned to the stages are kept constant for the first four periods. Initially, the production rates (lines d_t plus lines l_{it}) increase faster than the demand rate (lines d_t), which leads to inventory being built up during periods 1–3 (lines l_{it}). The demand rate then increases faster, which makes a setup at both stages necessary. During this setup, workers assigned in period 1 (that are more productive due to learning) are withdrawn from both stages and new workers (that have not benefitted from an increase in productivity as a result of learning yet) are added. What may sound counterintuitive at first glance can be explained by the system's intention to reduce inventory: Using a larger number of new workers makes it easier to match the production rates with the demand rate than using more productive workers with a longer tenure on the job, which lowers inventory at the expense of higher worker cost. We remind the reader that withdrawing experienced workers from the production stages does not necessarily imply that these workers are laid off. Instead, the experienced workers could have been delegated from a different department of the company to the production system during the ramp-up, and once they are no longer needed, they are sent back to their original workplace. In the system studied here, inventory only occurs between the final production stage and the demand stage; since both production stages are perfectly synchronised, no inventory occurs in-between both stages.

The results change for faster growing demand rates. In this case, the stages need to be set up earlier to ensure that production can keep up with the increase in demand. For the scenario with quickly growing demand, demand approaches a plateau around period 7 (see also Table 2(a)). This demand pattern makes it easier for the production planner to align the production capacities with customer demand, such that only a single setup for restructuring the production stages is necessary in this case (compared to two in the other two demand scenarios).

We next investigate the case where laying off workers comes at a cost. Table 2(b) shows that for the case where $r = 2.0$, the system does not lay off any workers that were employed in earlier periods in the slow-growing demand scenario anymore. Instead, the system is now set up more frequently to match production and demand. For the slow-growing demand scenario, the total inventory increases in this case (34.31 units for $r = 0.0$ vs. 44.73 units for $r = 2.0$). For the medium- and fast-growing demand scenarios, the system withdraws workers once in each case, but also to a lesser extent than in the case where $r = 0.0$. The cases where $r = 0.1$ and $r = M$ are illustrated in Tables A4 and A5 in the appendix.

Case 2) Non-identical learning rates

We now illustrate the case where the learning rates of both stages are different. We start with the case where workers at stage 1 learn faster than workers at stage 2 and assume $\tau_1 = 0.50$ and $\tau_2 = 1.00$. The results illustrated in Table 3 for the case where $r = 0.0$ show that it is much more difficult now to match the production rates both with one another and with the demand rate for all three demand scenarios. To lower inventory, the

Table 2. (a) Optimal worker profiles and stock levels ($r = 0.0$).

Demand type	Measure	Period (t')	Stage (i)	Period (t) 1	2	3	4	5	6	7	8	9	10
Slow-growing demand	$w_{it't}$	1	1,2	13.56	13.56	13.56	13.56	1.11	1.11	1.11	1.11	1.11	1.11
		5	1,2					18.20	18.20	18.20	0.64	0.64	0.64
		8	1,2								23.98	23.98	23.98
		d_t		108.13	119.23	131.44	144.86	159.61	175.81	193.58	213.07	234.43	257.81
	I_{it}		1	2.53	9.72	10.51	–	–	4.99	–	–	6.56	–
				TRC = 2296.36, Solver Time = 19.57 (seconds)									
Medium-growing demand	$w_{it't}$	1	1,2	14.40	14.40	14.40	0.19	0.19	0.19	0.0	0.0	0.0	0.0
		4	1,2				21.30	21.30	21.30	12.09	12.09	12.09	12.09
		7	1,2							13.38	13.38	13.38	13.38
		d_t.		108.87	130.55	153.51	175.73	196.13	214.00	228.99	241.14	250.70	258.06
	I_{it}		1	9.43	13.11	–	–	4.37	–	–	4.11	4.63	–
				TRC = 2502.52, Solver Time = 24.39 (seconds)									
Fast-growing demand	$w_{it't}$	1	1,2	15.64	15.64	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46
		3	1,2			25.18	25.18	25.18	25.18	25.18	25.18	25.18	25.18
		d_t		108.30	165.17	210.01	236.07	248.61	254.10	256.40	257.35	257.73	257.89
	I_{it}		1	19.34	–	–	3.30	4.85	4.89	4.09	2.88	1.48	–
				TRC = 2686.97, Solver Time = 11.08 (seconds)									

(b) Optimal worker profiles and stock levels ($r = 2.0$)

Demand type	Measure	Period (t')	Stage (i)	Period (t) 1	2	3	4	5	6	7	8	9	10
Slow-growing demand	$w_{it't}$	1	1,2	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56
		5	1,2					3.71	3.71	3.71	3.71	3.71	3.71
		7	1,2							3.58	3.58	3.58	3.58
		9	1,2									4.38	4.38
		d_t		108.13	119.23	131.44	144.86	159.61	175.81	193.58	213.07	234.43	257.81
	I_{it}		1	2.53	9.72	10.51	–	5.80	–	7.35	–	8.82	–
				TRC = 2343.61, Solver Time = 16.22 (seconds)									
Medium-growing demand	$w_{it't}$	1	1,2	14.40	14.40	14.40	14.40	14.40	14.40	6.07	6.07	6.07	6.07
		4	1,2				5.72	5.72	5.72	5.72	5.72	5.72	5.72
		7	1,2							13.68	13.68	13.68	13.68
		d_t		108.87	130.55	153.51	175.73	196.13	214.00	228.99	241.14	250.70	258.06
	I_{it}		1	9.43	13.11	–	13.65	14.38	–	–	4.11	4.63	–
				TRC = 2565.69, Solver Time = 27.24 (seconds)									
Fast-growing demand	$w_{it't}$	1	1,2	15.64	15.64	15.64	15.64	14.21	14.21	14.21	14.21	14.21	14.21
		3	1,2			7.93	7.93	7.93	7.93	7.93	7.93	7.93	7.93
		5	1,2					3.64	3.64	3.64	3.64	3.64	3.64
		d_t		108.30	165.17	210.01	236.07	248.61	254.10	256.40	257.35	257.73	257.89
	I_{it}		1	19.34	–	7.19	–	–	0.29	0.40	0.35	0.20	–
				TRC = 2719.75, Solver Time = 13.02 (seconds)									

*The orange shaded cells highlight periods where workers are withdrawn from the stages.

system schedules more setups now and more frequently removes workers from the stages. For the slow-growing demand scenario, the inventory in the system nevertheless increases as compared to the case where the stages have the same learning rates; for the other two demand scenarios where demand is ramped up earlier, the system manages to reduce inventory by means of more frequent setups. Somewhat counterintuitively, the improvement in learning at stage 1 leads to a cost increase for all three demand scenarios due to the consequent misalignment of the two production stages.

Table A1 in the appendix again illustrates the case where $r = 2.0$. The results are similar for the three demand scenarios. Given the high cost of laying off workers, the system now does not remove workers from stage 2 at all. For stage 1, one or two setups that remove some of the workers assigned to the stage in period 1 are necessary to avoid that the improvement in the production rate pushes too much inventory into the system. Not surprisingly, both the inventory levels and the total costs increase as compared to the situation where $r = 0.0$ (Table 3).

Table 3. Optimal worker profiles and stock levels ($r = 0.0$).

				Period (t)									
Demand type	Measure	Period (t')	Stage (i)	1	2	3	4	5	6	7	8	9	10
Slow-growing demand	$w_{it't}$	1	1	11.82	11.82	11.82	11.82	9.21	9.21	9.21	9.21	7.20	7.20
			2	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56
		3	1			2.08	2.08	0.0	0.0	0.0	0.0	0.0	0.0
		5	1					7.86	7.86	3.31	3.31	0.0	0.0
			2					3.71	3.71	3.71	3.71	3.71	3.71
		7	1							8.12	8.12	8.12	8.12
			2							3.58	3.58	3.58	3.58
		9	1									9.67	9.67
			2									4.38	4.38
		d_t		108.13	119.23	131.44	144.86	159.61	175.81	193.58	213.07	234.43	257.81
	I_{it}		1	2.09	–	6.04	–	5.80	–	7.35	–	8.82	–
			2	0.44	9.72	4.47	–	–	–	–	–	–	–
				TRC = 2365.92, Solver Time = 25.43 (seconds)									
Medium-growing demand	$w_{it't}$	1	1	12.41	12.41	12.41	0.0	0.0	0.0	0.0	0.0	0.0	0.0
			2	14.40	14.40	14.40	4.69	4.69	4.69	0.0	0.0	0.0	0.0
		3	1			3.17	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		4	1				19.34	19.34	0.0	0.0	0.0	0.0	0.0
			2				16.40	16.40	16.40	11.58	11.58	11.58	11.58
		6	1						23.03	23.03	11.11	11.11	11.11
		7	2							13.91	13.91	13.91	13.91
		8	1								14.26	14.26	14.26
		d_t		108.87	130.55	153.51	175.73	196.13	214.00	228.99	241.14	250.70	258.06
	I_{it}		1	7.61	–	–	4.55	–	0.76	–	2.80	4.51	–
			2	1.82	13.11	–	–	8.03	–	–	1.13	–	–
				TRC = 2569.22, Solver Time = 21.47 (seconds)									
Fast-growing demand	$w_{it't}$	1	1	11.62	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
			2	13.34	13.34	13.34	7.39	7.39	7.39	7.39	7.39	7.39	7.39
		2	1		19.51	19.51	3.69	3.69	3.69	3.69	3.69	3.69	3.69
			2		6.98	6.98	6.98	6.98	6.98	6.98	6.98	6.98	6.98
		4	1				21.37	21.37	17.39	17.39	17.39	17.39	17.39
			2				11.40	11.40	11.40	11.40	11.40	11.40	11.40
		6	1						4.67	4.67	4.67	4.67	4.67
		d_t		108.30	165.17	210.01	236.07	248.61	254.10	256.40	257.35	257.73	257.89
	I_{it}		1	–	16.71	–	–	–	–	0.62	0.69	0.42	–
			2	0.58	–	1.83	–	0.41	0.78	0.22	–	–	0.09
				TRC = 2734.04, Solver Time = 8.90 (seconds)									

*The orange shaded cells highlight periods where workers are withdrawn from the stages.

Table 4 and Table A2 in the appendix again consider the case where the learning rates of stages 1 and 2 are different, but now assumes faster learning at stage 2 (i.e. $\tau_1 = 1.00$ and $\tau_2 = 0.50$). The system now shows an opposite behaviour than in the earlier case we analyzed in Table 3 for all demand scenarios: while we see that workers are mainly added to the slower production stage over time, workers need to be laid off at stage 2 from time to time to avoid that too much inventory accumulates in the system. Table A2 (appendix) again shows that higher cost of laying off workers limits this ability of the system, resulting in higher inventory levels and higher total costs. Comparing the total costs of the cases illustrated in Table 3 and Table A1 (appendix) with those displayed in Table 4 and Table A2 (appendix), it becomes apparent that the costs are lower for cases where the higher learning rate occurs on upstream stages of the production system.

The case where a simultaneous assignment and withdrawal of workers to/from a stage is not permitted within a single period (i.e. the case where $\alpha = 1$) is illustrated in Table A3 in the appendix. The table illustrates that workers are now only withdrawn in the case of slow-growing demand and in case setup and worker withdrawal costs are low. In all other cases, the system decides not to withdraw any workers and instead balances the production rates and the demand rate just by assigning workers to the stages over time.

4.2. Sensitivity analysis

This section presents the results of a sensitivity analysis that aims on gaining insights into how the results obtained in Section 4.1 change if a third stage is added to the production system. The benchmark instance used for the purpose of our experiment is introduced in Table

Table 4. Optimal worker profiles and stock levels ($r = 0.0$).

Demand type	Measure	Period (t')	Stage (i)	Period (t) 1	2	3	4	5	6	7	8	9	10
Slow-growing demand	$w_{it't}$	1	1	13.25	13.25	13.25	13.25	9.68	9.68	9.68	9.68	9.68	9.68
			2	12.35	12.35	12.35	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		4	1				6.94	6.94	6.94	6.94	6.94	6.94	6.94
			2				16.37	16.37	16.37	11.45	11.45	6.09	6.09
		7	1							4.29	4.29	4.29	4.29
			2							9.24	9.24	9.24	9.24
		9	1									4.32	4.32
			2									9.66	9.66
	d_t			108.13	119.23	131.44	144.86	159.61	175.81	193.58	213.07	234.43	257.81
	I_{it}		1	–	4.31	2.07	9.79	11.41	–	7.03	–	8.82	–
			2	7.02	5.85	–	–	0.94	–	–	–	–	–
	TRC = 2361.98, Solver Time = 25.05 (seconds)												
Medium-growing demand	$w_{it't}$	1	1	13.65	13.65	13.65	13.65	9.45	9.45	9.45	9.45	9.45	9.45
			2	12.41	12.41	6.39	6.39	0.0	0.0	0.0	0.0	0.0	0.0
		3	1			3.48	3.48	3.48	3.48	3.48	3.48	3.48	3.48
			2			10.48	10.48	0.0	0.0	0.0	0.0	0.0	0.0
		5	1					9.28	9.28	9.28	3.54	3.54	3.54
			2					21.87	21.87	21.87	4.36	4.36	4.36
		8	1								9.41	9.41	9.41
			2								21.18	21.18	21.18
	d_t			108.87	130.55	153.51	175.73	196.13	214.00	228.99	241.14	250.70	258.06
	I_{it}		1	3.31	–	8.01	–	7.80	9.27	–	–	1.62	–
			2	4.30	–	–	–	–	1.26	–	–	1.22	–
	TRC = 2540.41, Solver Time = 21.89 (seconds)												
Fast-growing demand	$w_{it't}$	1	1	15.64	15.64	15.64	15.64	14.21	14.21	14.21	14.21	14.21	14.21
			2	14.22	14.22	3.17	3.17	0.0	0.0	0.0	0.0	0.0	0.0
		3	1			7.93	7.93	7.93	7.93	7.93	7.93	7.93	7.93
			2			19.90	19.90	13.61	13.61	13.61	13.61	13.61	13.61
		5	1					3.64	3.64	3.64	3.64	3.64	3.64
			2					12.08	12.08	12.08	12.08	12.08	12.08
	d_t			108.30	165.17	210.01	236.07	248.61	254.10	256.40	257.35	257.73	257.89
	I_{it}		1	19.34	–	7.19	–	–	0.29	0.40	0.35	0.20	–
			2	4.93	–	–	–	–	1.44	1.37	0.73	–	
	TRC = 2717.95, Solver Time = 7.12 (seconds)												

*The orange shaded cells highlight periods where workers are withdrawn from the stages.

Table 5. Parameters used for numerical experimentation.

Stage (i)	S_i	Scenario 1 h_i	Scenario 2 h_i	μ_i	$p_{max,i}$	δ_i	τ_i	T
1	50.0	3.0	2.0	5.0	10.0	5.0	1.00	10
2	50.0	2.5	2.0	5.0	10.0	5.0	1.00	
3	50.0	2.0	2.0	5.0	10.0	5.0	1.00	

5 and relies on the slow-growing demand scenario. We consider two scenarios in the following, one where the inventory carrying costs are identical for the three stages and one where the inventory carrying costs increase along the stages of the production system. For both scenarios, we study how changes in the learning rate τ_i and in the cost for laying off workers r influence the performance of the system.

Case 1) Identical learning rates ($\tau_i = \tau, \forall i$)

We again first investigate the case where the learning rates of the different stages are identical. Figure 2(a) shows how the total cost of the system change for alternative values for $\tau \in [0.01, 1.50]$ and $r \in \{0.0, 0.1, 1.0, M\}$.

In Figure 2(a), we can observe three distinctive zones that show a different behaviour of the total cost of the system in r. For very fast and very slow learning (zones 1 and 3), the total costs do not depend on r, as the system tends not to withdraw any workers from the stages. In zone 2 where the system can improve its performance by withdrawing some workers assigned earlier to the stages over time, the total costs are lower for lower values of r. In addition, we could observe an optimal learning rate for the stages depending on r for which the total system costs can be minimised ($\tau = 0.83, \tau = 0.82$ and $\tau = 0.97$ for $r = 0.0, r = 0.1$ and $r \in \{1.0, M\}$, respectively). This optimal learning rate helps to match the production rates to the demand rate and reduces the need to withdraw workers from the stages.

Figure 2(b) illustrates the total system costs for alternative values for r and τ for the case where the stages

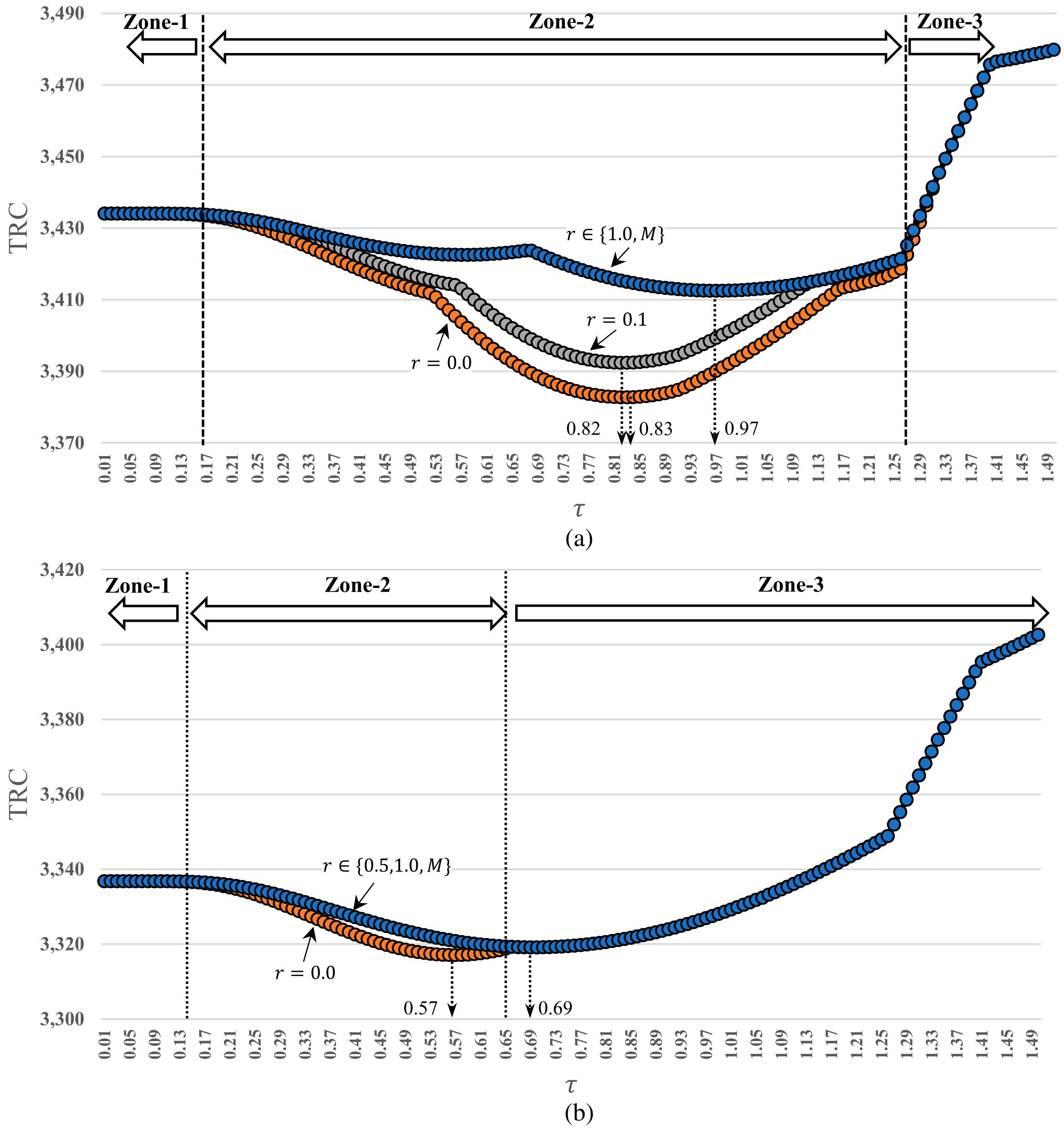


Figure 2. (a) Total costs of the system for alternative values of τ and r (Scenario 1, slow-growing demand). (b) Total costs of the system for alternative values of τ and r (Scenario 2, slow-growing demand).

have identical inventory holding costs (Scenario 2). Compared to Figure 2(a), zone 2 where the system is interested in withdrawing workers from the production stages is smaller now. Given that our earlier results have already shown that in the case where the production rates are synchronised, inventory is only built up at the final production stage, the system now benefits from the lower inventory carrying costs at this stage, which reduces the pressure on the system to match the production rates with the demand rate (that may make a withdrawal of workers necessary over time). Also in the case of Scenario 2, the total system costs have a piece-wise convex pattern with an optimal learning rate.

The case of fast-growing demand is illustrated in Figures A6 and A7 in the appendix. As can be seen in both figures, the two inventory holding cost scenarios have similar patterns for the total relevant costs (i.e. the total costs have a piece-wise convex form in τ). The learning performance of the workers is consequently a critical factor in minimising the total system cost.

Case 2) Non-identical learning rates

We now investigate the case where the learning rates of the stages may differ. Table 6 illustrates the case where the learning rate of stage i is reduced or increased from the benchmark instance with $\tau_i = 1 \forall i$. First, we can see that both an increase or a reduction of τ_i for any of the

Table 6. Total system costs for alternative values of τ_i and r (Scenario 1, slow-growing demand).

		Value of τ_i (Change from the benchmark instance with $\tau_i = 1 \forall i$)						
r	Stage	**0.01** (−0.99)	**0.05** (−0.95)	**0.1** (−0.9)	**1.0** (0.0)	**2.0** (+1.0)	**3.0** (+2.0)	**4.0** (+3.0)
0.0	1	3522.4	3522.4	3522.3	**3393.1**	3502.5	3550.1	3596.1
	2	3533.3	3533.3	3533.2	**3393.1**	3546.3	3596.2	3599.7
	3	3478.9	3478.9	3478.9	**3393.1**	3520.9	3573.2	3583.8
0.1	1	3524.3	3524.3	3524.3	**3402.1**	3503.3	3551.3	3597.4
	2	3535.8	3535.8	3535.8	**3402.1**	3547.2	3597.7	3601.4
	3	3478.9	3478.9	3478.9	**3402.1**	3522.4	3576.8	3585.3
2.0	1	3534.4	3534.4	3534.4	**3412.6**	3508.4	3558.2	3605.0
	2	3536.8	3536.8	3536.8	**3412.6**	3562.7	3624.6	3624.9
	3	3478.9	3478.9	3478.9	**3412.6**	3528.1	3585.1	3601.6

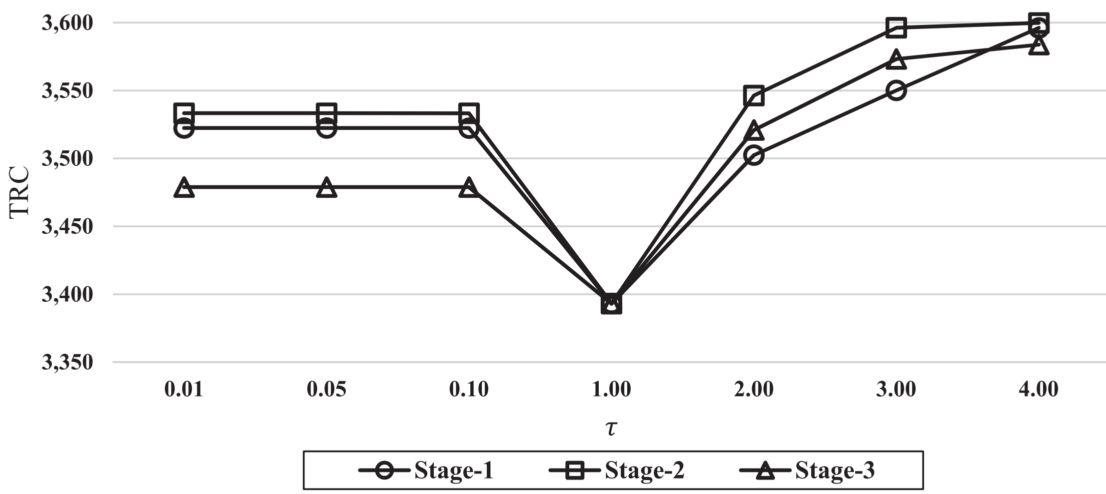


Figure 3. Total system cost for changes in the learning rate of the system's stages (Scenario 1, $r = 0.0$).

stages from $\tau_i = 1.0$ (recall from Figure 2(a) that the optimal learning rate is close to 1.0 in this case) leads to an increase in costs. We can further observe that the middle stage ($i = 2$) is most sensitive towards changes in the learning rate. This result is caused by the fact that stage 2 connects the other two stages. If the learning rate changes at stage 1 (or, similarly, at stage 3), then stages 2 and 3 (or, similarly, stages 1 and 2) can still be synchronised to avoid inventory between the stages. If stage 2 is affected by a change in τ_i, in contrast, then matching the production rates of stages 1 and 3 with the production rate of stage 2 is not that easy anymore, which leads to the build-up of inventory between the stages. This, in turn, induces a stronger increase in system costs. Consistent with the above results, the system realises lower total costs if faster workers are assigned to upstream stages. The behaviour of the total costs for changes in τ_i is also illustrated graphically in Figure 3. Table A8 in the appendix confirms these results also for Scenario 2. Table A9 and Figure A10 in the appendix further illustrate the case where $N = 4$.

5. Discussion and conclusion

This paper proposed a model for managing the ramp-up of a multi-stage production system. We assumed that learning increases the productivity of workers and that the demand rate grows over time. Both worker productivity and the demand rate develop according to different patterns. The proposed model supports the decision of how many workers (worker-time-equivalents) to assign to the production stages that influences setup, inventory carrying and worker costs. A set of research questions was formulated that guided our numerical experiments.

The first aspect we were interested in is how the demand pattern the production system faces influences the ramp-up of the production stages (RQ1). We found that demand that grows quickly and that then reaches a plateau relatively early in the ramp-up phase makes it easier to manage the ramp-up. In this case, the production planner has to scale up the production rate quickly with a few setups and can then benefit from an almost stable system. If the production rates improve further due to learning after demand has reached a plateau, this can lead to an opportunity for the production planner to withdraw workers not needed at the production line again.

Secondly, we were interested in gaining insights into when production planners may be interested in withdrawing workers from the production stages during the ramp-up (RQ2). Apart from the case where demand plateaus earlier than the production rates, we found that if withdrawing workers from the stages is not restricted and cheap, the company decides to withdraw (experienced) workers relatively frequently and to replace them by less experienced ones to better match the production rates with each other and with the demand rate, which reduces inventory in the system. If withdrawing workers is expensive, then the company prefers to set up the stages more frequently and to use (only) worker assignments as a measure for matching production and demand. Workers are only withdrawn in exceptional cases in such scenarios. Less flexibility in the management of the workforce (in this case caused by higher withdrawal costs) leads to higher inventory levels however. If the learning rates of the production stages are different, the company can benefit from removing some workers over time from the faster stage to better balance the production rates and to lower inventory levels. If withdrawing workers is too expensive, the company again balances the production rates by assigning workers more frequently to the faster learning stages. If a simultaneous assignment and withdrawal of workers is not permitted within a period, then the system generally has a tendency to withdraw fewer workers, and it instead balances the production rates and the demand rate by assigning workers more frequently to the production stages.

A third question we investigated is how the production stages should be sequenced (or how workers should be assigned to the stages) in case the learning rates of the stages are non-identical (RQ3). First, we found that a misalignment in the learning rates of the production stages creates inventory between the stages and thus increases costs. If the production stages are synchronised with identical learning rates at all stages, the company can avoid inventory in-between stages altogether, with inventory just piling up at the farthest downstream stage in this case. If managers are unable to fully balance the learning performance of the production system's stages, then costs can be reduced by assigning faster learning workers to upstream stages of the system. Given the usually lower value of products produced at upstream stages, pushing extra inventory into the system at these stages has a less severe effect on total cost than pushing high-value inventory into the system that usually occurs at downstream stages. Our results confirm earlier research on the management of multi-stage production lines (e.g. Jaber and Khan 2010), but differ from findings on the management of assembly lines with learning effects, where it was shown that an unbalanced allocation of workers to production lines worked best (e.g. Cohen, Vitner, and Sarin 2006), while in our case a balanced allocation led to the lowest total costs. We acknowledge, however, that companies may not always be fully flexible in assigning workers to the different stages of the production system, for example if the stages are under control of different companies or if not all workers have the required skills to work at all stages of the system. Still, our results show that taking account of worker learning and demand growth characteristics in assigning workers to the stages of the production system – to the extent possible – may lead to cost savings. Our results also confirm the findings of Bogaschewsky and Glock (2009), who have shown that learning effects in production (despite reflecting a free-of-cost performance improvement) may reduce the system's performance if they lead to too much inventory being pushed into the system.

Fourthly, we investigated if there is an optimal learning rate for the production system (RQ4). As already mentioned, the system benefits from identical learning rates at all production stages that help to avoid inventory in-between stages. Apart from this, an optimal learning rate exists for the stages that helps to match the production rates to the demand rate and that lowers the pressure on the system to frequently change the workers assigned to the stages over time. Even though learning rates are worker-specific and both difficult to measure exactly and to influence by management, managers could nevertheless use the theoretically optimal learning rates in assigning workers to work stations and in planning worker training activities.

Finally, we intended to gain insights into how the structure of the production system influences the ramp-up pattern (RQ5). We found that in multi-stage production systems, especially the middle stages that connect the upstream and downstream parts of the production system are sensitive towards changes in the learning rate. If middle stages have a (fundamentally different) learning rate than the other stages, it is difficult to arrive at a smooth material flow and low inventory levels along the production system, which increases total cost. Production planners therefore have to pay special attention to manage these stages carefully and to align them with the upstream and downstream parts of the production system.

This paper has several limitations that could be addressed in future research. First, we assumed that all parameters that are relevant for the management of the ramp-up are known. Earlier research has highlighted the stochastic nature of the production ramp-up that may result in machine breakdowns or product quality-related issues, however. As a result, the model proposed in this paper could be extended to take account of stochastic influences as well. Secondly, we considered a pure serial production system even though production systems often follow an assembly-type structure in practice. Therefore, extending the proposed model towards a production network and analysing how changes in learning parameters influence the performance of the system would be worthwhile. Thirdly, we assumed that worker learning is a function of time only and that it cannot be influenced by management above and beyond the assignment of workers to stages. Prior research has discussed the case where the company can schedule training activities in addition to actively influence the learning performance of workers (e.g. Glock 2016; Meisel and Glock 2018). Considering training activities in our model would further increase the flexibility of the company to manage the production ramp-up. Fourthly, we did not consider quality aspects in the proposed model. However, earlier research has shown that during early phases of the production ramp-up, products do often not meet the quality standards defined by the company, and that quality improves over time as workers gain experience. It would therefore be a natural extension of our proposed model to consider the production of imperfect items with defect rates decreasing as workers gain experience over time. If we assume that experienced workers commit fewer mistakes than less experienced ones during the ramp-up, then adding product quality to the model may also change some of the results we obtained, as withdrawing experienced workers from the production stages may become less attractive to the company. Finally, we assumed that workers employed at a production stage have the same characteristics. Taking account of heterogeneous workers and investigating how they should be assigned to the production stages over time would further increase the practical applicability of our work. We leave these and further extensions for future research.

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No potential conflict of interest was reported by the author(s).

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Appendix

Table A1. Optimal worker profiles and stock levels ($r = 2.0$).

				Period (t)									
Demand type	Measure	Period (t')	Stage (i)	1	2	3	4	5	6	7	8	9	10
Slow-growing demand	$w_{it't}$	1	1	11.82	11.82	11.82	11.82	7.28	7.28	7.28	7.28	7.28	7.28
			2	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56
		3	1			2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08
		5	1					7.70	7.70	7.70	7.70	7.70	7.70
			2					3.71	3.71	3.71	3.71	3.71	3.71
		7	1							3.40	3.40	3.40	3.40
			2							3.73	3.73	3.73	3.73
		9	1									4.31	4.31
			2									4.05	4.05
		d_t		108.13	119.23	131.44	144.86	159.61	175.81	193.58	213.07	234.43	257.81
	I_{it}		1	2.09	–	6.04	–	5.83	–	8.71	–	10.41	–
			2	0.44	9.72	4.47	–	–	0.08	–	2.82	–	–
				TRC = 2387.05, Solver Time = 18.49 (seconds)									
Medium-growing demand	$w_{it't}$	1	1	11.59	11.59	11.59	8.85	8.85	8.85	8.85	2.87	2.87	2.87
			2	14.40	14.40	14.40	14.40	14.40	14.40	14.40	14.40	14.40	14.40
		2	1		2.78	2.78	2.78	2.78	2.78	2.78	2.78	2.78	2.78
		4	1				7.25	7.25	7.25	7.25	7.25	7.25	7.25
			2				5.04	5.04	5.04	5.04	5.04	5.04	5.04
		6	1						3.41	3.41	3.41	3.41	3.41
			2						3.08	3.08	3.08	3.08	3.08
		8	1								8.92	8.92	8.92
			2								2.79	2.79	2.79
		d_t		108.87	130.55	153.51	175.73	196.13	214.00	228.99	241.14	250.70	258.06
	I_{it}		1	–	10.21	–	8.06	–	6.46	–	5.09	5.87	–
			2	9.43	2.91	–	–	2.41	–	–	0.70	–	–
				TRC = 2619.82, Solver Time = 22.43(seconds)									
Fast-growing demand	$w_{it't}$	1	1	14.22	14.22	14.22	14.22	5.10	5.10	5.10	5.10	5.10	5.10
			2	16.25	16.25	16.25	16.25	16.25	16.25	16.25	16.25	16.25	16.25
		3	1			8.42	8.42	8.42	8.42	8.42	8.42	8.42	8.42
			2			6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67
		5	1					12.18	12.18	12.18	12.18	12.18	12.18
			2					2.82	2.82	2.82	2.82	2.82	2.82
		d_t		108.30	165.17	210.01	236.07	248.61	254.10	256.40	257.35	257.73	257.89
	I_{it}		1	24.08	–	10.49	–	–	1.73	2.13	1.72	0.94	–
			2	–	10.57	2.89	0.44	1.74	0.50	–	–	0.21	0.51
				TRC = 2787.17, Solver Time = 9.95 (seconds)									

*The orange shaded cells highlight periods where workers are withdrawn from the stages.

Table A2. Optimal worker profiles and stock levels ($r = 2.0$).

				Period (t)									
Demand type	Measure	Period (t')	Stage (i)	1	2	3	4	5	6	7	8	9	10
Slow-growing demand	$w_{it't}$	1	1	13.25	13.25	13.25	13.25	13.25	13.25	13.25	13.25	13.25	13.25
			2	12.35	12.35	12.35	12.35	12.35	12.35	12.35	12.35	12.35	12.35
			4	1			3.03	3.03	3.03	3.03	3.03	3.03	3.03
			2				3.69	3.69	3.69	3.69	3.69	3.69	3.69
		7	1							4.66	4.66	4.66	4.66
			2							4.47	4.47	4.47	4.47
		9	1									4.29	4.29
			2									4.27	4.27
		d_t		108.13	119.23	131.44	144.86	159.61	175.81	193.58	213.07	234.43	257.81
	I_{it}		1	–	4.31	2.07	13.22	13.92	–	6.93	–	8.82	–
			2	7.02	5.85	–	1.84	1.56	–	1.51	–	1.60	–
				TRC = 2379.59, Solver Time = 17.64(seconds)									
Medium-growing demand	$w_{it't}$	1	1	13.65	13.65	13.65	13.65	13.65	13.65	13.65	13.65	13.65	13.65
			2	12.41	12.41	12.41	12.41	12.41	12.41	12.41	12.41	12.41	12.41
		3	1			3.48	3.48	3.48	3.48	3.48	3.48	3.48	3.48
			2			4.23	4.23	4.23	4.23	4.23	4.23	4.23	4.23
		5	1					4.67	4.67	4.67	4.67	4.67	4.67
			2					4.80	4.80	4.80	4.80	4.80	4.80
		8	1								3.55	3.55	3.55
			2								3.66	3.66	3.66
		d_t		108.87	130.55	153.51	175.73	196.13	214.00	228.99	241.14	250.70	258.06
	I_{it}		1	3.31	–	8.01	–	11.96	12.33	–	5.32	5.52	–
			2	4.30	–	1.80	–	2.91	2.42	–	1.97	1.66	–
				TRC = 2580.32, Solver Time = 22.51 (seconds)									
Fast-growing demand	$w_{it't}$	1	1	15.64	15.64	15.64	15.64	14.21	14.21	14.21	14.21	14.21	14.21
			2	14.22	14.22	14.22	14.22	5.10	5.10	5.10	5.10	5.10	5.10
		3	1			7.93	7.93	7.93	7.93	7.93	7.93	7.93	7.93
			2			8.42	8.42	8.42	8.42	8.42	8.42	8.42	8.42
		5	1					3.64	3.64	3.64	3.64	3.64	3.64
			2					12.18	12.18	12.18	12.18	12.18	12.18
		d_t		108.30	165.17	210.01	236.07	248.61	254.10	256.40	257.35	257.73	257.89
	I_{it}		1	19.34	–	7.19	–	–	0.29	0.40	0.35	0.20	–
			2	4.93	–	3.30	–	–	1.44	1.73	1.37	0.73	–
				TRC = 2742.96, Solver Time = 11.39(seconds)									

*The orange shaded cells highlight periods where workers are withdrawn from the stages.

Table A3. Effects of S_i and r on the sum of withdrawn workers, i.e. $\sum z_{it't}$, in case of $\alpha = 1$.

		r														
	S_i	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0
Slow-growing demand	0.5	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.22	0.22	0.22	0.22
	1.0	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.22	0.22	0.22	0.22	0.22
	1.5	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.22	0.22	0.22	0.22	0.22	0.22	0.22
	2.0	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22
	2.5	0.92	0.92	0.92	0.92	0.92	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22
	3.0	0.92	0.92	0.92	0.92	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.00
	3.5	0.92	0.92	0.92	0.22	0.22	0.22	0.22	0.22	0.22	0.00	0.00	0.00	0.00	0.00	0.00
	4.0	0.92	0.22	0.22	0.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Medium-& Fast-growing demand		$\sum z_{it't} = 0$, for all S_i and r														

Table A4. Optimal worker profiles and stock levels ($r = 0.1$).

				Period (t)									
Demand type	Measure	Period (t')	Stage (i)	1	2	3	4	5	6	7	8	9	10
Slow-growing demand	$w_{it't}$	1	1,2	13.56	13.56	13.56	13.56	1.11	1.11	1.11	0.0	0.0	0.0
		5	1,2					18.20	18.20	18.20	1.80	1.80	1.80
		8	1,2								23.92	23.92	23.92
		d_t		108.13	119.23	131.44	144.86	159.61	175.81	193.58	213.07	234.43	257.81
	I_{it}		1	2.53	9.72	10.51	–	–	4.99	–	–	6.56	–
				TRC $=$ 2302.35, Solver Time $=$ 19.00 (seconds)									
Medium-growing demand	$w_{it't}$	1	1,2	14.40	14.40	14.40	0.19	0.19	0.19	0.0	0.0	0.0	0.0
		4	1,2				21.30	21.30	21.30	12.09	12.09	12.09	12.09
		7	1,2							13.38	13.38	13.38	13.38
		d_t		108.87	130.55	153.51	175.73	196.13	214.00	228.99	241.14	250.70	258.06
	I_{it}		1	9.43	13.11	–	–	4.37	–	–	4.11	4.63	–
				TRC $=$ 2507.24, Solver Time $=$ 17.92 (seconds)									
Fast-growing demand	$w_{it't}$	1	1,2	15.64	15.64	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46
		3	1,2			25.18	25.18	25.18	25.18	25.18	25.18	25.18	25.18
		d_t		108.30	165.17	210.01	236.07	248.61	254.10	256.40	257.35	257.73	257.89
	I_{it}		1	19.34	–	–	3.30	4.85	4.89	4.09	2.88	1.48	–
				TRC $=$ 2690.00, Solver Time $=$ 12.26 (seconds)									

*The orange shaded cells highlight periods where workers are withdrawn from the stages.

Table A5. Optimal worker profiles and stock levels ($r = M$).

				Period (t)									
Demand type	Measure	Period (t')	Stage (i)	1	2	3	4	5	6	7	8	9	10
Slow-growing demand	$w_{it't}$	1	1,2	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56
		5	1,2					3.71	3.71	3.71	3.71	3.71	3.71
		7	1,2							3.58	3.58	3.58	3.58
		9	1,2									4.38	4.38
		d_t		108.13	119.23	131.44	144.86	159.61	175.81	193.58	213.07	234.43	257.81
	I_{it}		1	2.53	9.72	10.51	–	5.80	–	7.35	–	8.82	–
				TRC $=$ 2343.61, Solver Time $=$ 3.26 (seconds)									
Medium-growing demand	$w_{it't}$	1	1,2	14.40	14.40	14.40	14.40	14.40	14.40	14.40	14.40	14.40	14.40
		4	1,2				4.90	4.90	4.90	4.90	4.90	4.90	4.90
		6	1,2						3.37	3.37	3.37	3.37	3.37
		8	1,2								2.62	2.62	2.62
		d_t		108.87	130.55	153.51	175.73	196.13	214.00	228.99	241.14	250.70	258.06
	I_{it}		1	9.43	13.11	–	6.94	–	5.10	–	5.91	5.95	–
				TRC $=$ 2569.32, Solver Time $=$ 14.29 (seconds)									
Fast-growing demand	$w_{it't}$	1	1,2	15.64	15.64	15.64	15.64	15.64	15.64	15.64	15.64	15.64	15.64
		3	1,2			7.93	7.93	7.93	7.93	7.93	7.93	7.93	7.93
		5	1,2					2.14	2.14	2.14	2.14	2.14	2.14
		d_t		108.30	165.17	210.01	236.07	248.61	254.10	256.40	257.35	257.73	257.89
	I_{it}		1	19.34	–	7.19	–	2.00	2.57	2.34	1.71	0.90	–
				TRC $=$ 2734.56, Solver Time $=$ 9.59(seconds)									

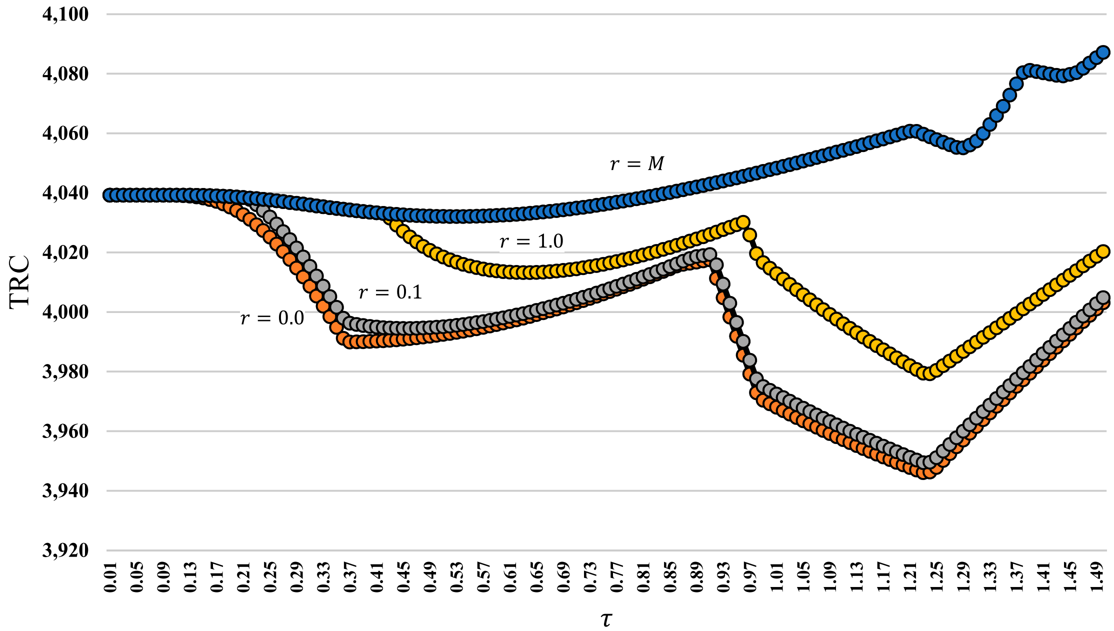

Figure A4. Total costs of the system for alternative values of τ and r (Scenario 1, Fast-growing demand).

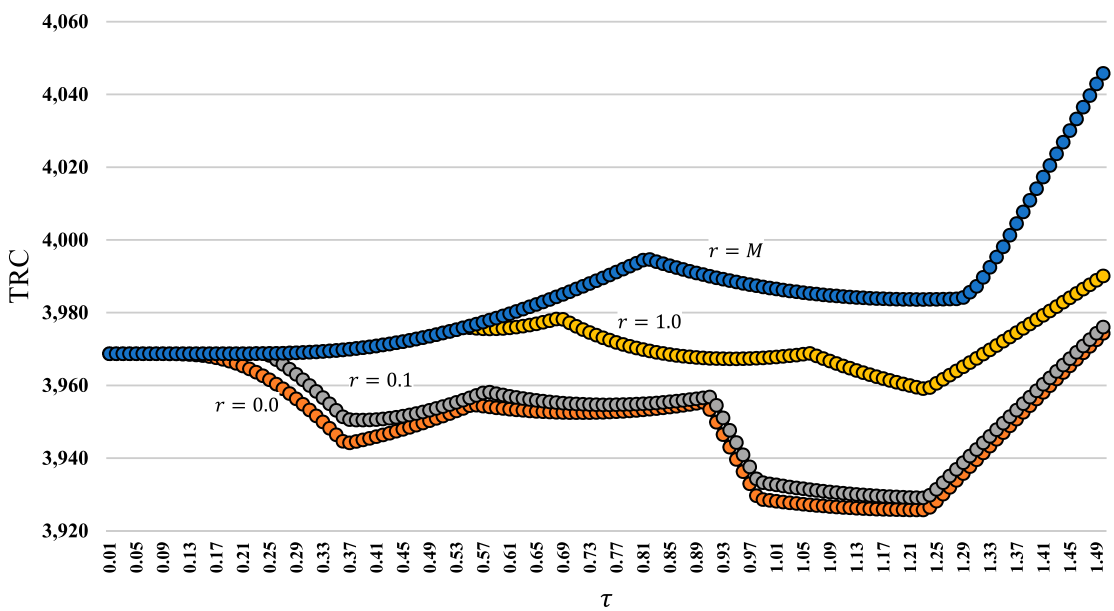

Figure A5. Total costs of the system for alternative values of τ and r(Scenario 2, Fast-growing demand).

Table A6. Total system costs for alternative values of τ_i and r (Scenario 2).

		Value of τ_i (Change from the benchmark instance with $\tau_i = 1 \forall i$)						
r	Stage	**0.01** (−0.99)	**0.05** (−0.95)	**0.1** (−0.9)	**1.0** (0.0)	**2.0** (+1.0)	**3.0** (+2.0)	**4.0** (+3.0)
0.0	1	3440.1	3440.1	3440.0	**3328.6**	3433.0	3485.4	3531.3
	2	3455.9	3455.9	3455.9	**3328.6**	3477.5	3519.4	3554.2
	3	3394.9	3394.9	3394.9	**3328.6**	3448.0	3502.4	3530.0
0.1	1	3440.5	3440.5	3440.5	**3328.6**	3433.9	3486.5	3531.8
	2	3456.4	3456.4	3456.4	**3328.6**	3479.1	3520.9	3555.9
	3	3394.9	3394.9	3394.9	**3328.6**	3449.4	3505.3	3533.6
2.0	1	3440.9	3440.9	3440.9	**3328.6**	3435.8	3488.9	3532.6
	2	3465.6	3465.6	3465.6	**3328.6**	3498.1	3547.8	3576.6
	3	3394.9	3394.9	3394.9	**3328.6**	3462.6	3523.0	3554.7

Table A7. Total system costs for alternative values of $\tau_i$and r (Scenario 1, $N = 4$, $h_4 = 1.5$).

		Value of τ_i (Change from the benchmark instance with $\tau_i = 1 \forall i$)						
r	Stage	**0.01** (−0.99)	**0.05** (−0.95)	**0.1** (−0.9)	**1.0** (0.0)	**2.0** (+1.0)	**3.0** (+2.0)	**4.0** (+3.0)
0.0	1	4588.3	4588.3	4588.3	4466.1	4560.9	4610.7	4657.9
	2	4601.3	4608.8	4601.2	4466.1	4607.8	4658.4	4693.5
	3	4615.1	4580.2	4580.1	4466.1	4607.0	4668.2	4694.8
	4	4540.2	4515.5	4515.5	4466.1	4580.8	4639.2	4667.1
0.1	1	4588.3	4588.3	4588.3	4466.1	4561.0	4610.8	4657.9
	2	4604.8	4604.8	4604.8	4466.1	4609.3	4660.5	4694.7
	3	4580.7	4580.7	4580.7	4466.1	4609.2	4654.3	4699.9
	4	4515.5	4518.5	4515.5	4466.1	4582.8	4641.4	4668.9
2.0	1	4588.3	4588.3	4588.3	4466.1	4562.0	4611.8	4658.5
	2	4613.4	4613.4	4613.4	4466.1	4624.5	4678.2	4709.4
	3	4586.7	4597.5	4586.7	4466.1	4634.5	4685.2	4730.3
	4	4515.5	4515.5	4515.5	4466.1	4619.2	4653.6	4708.5

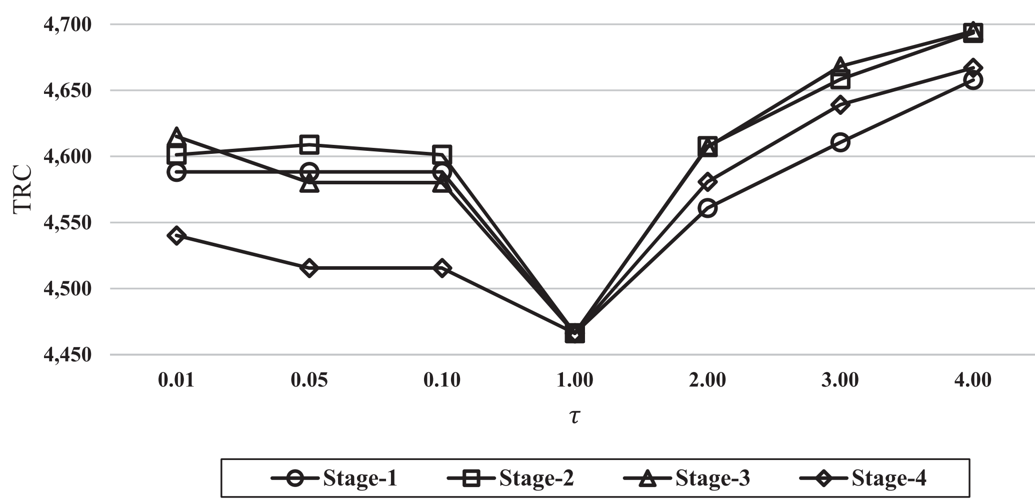

Figure A6. Total system cost for changes in the learning rate of the system's stages (Scenario 1, $r = 0.0$).