Signal Processing: Where Physics and Mathematics Meet

Much of the statistical signal processing developed in the 20th century falls under the umbrella of classical signal processing, which rests on two basic stochastic assumptions:

- ▲ Stationarity;
- ▲ Gaussianity.

The use of these two assumptions is justified on the following grounds:

- ▲ The first- and second-order statistics, inherent to the *full characterization* of a Gaussian model, are of fundamental interest in their own individual ways.
- ▲ The assumptions of stationarity and Gaussianity allow the development of a *frame of reference* that is mathematically elegant and, in certain cases, it can be insightful in a practical way.

Many, if not all, of the physical phenomena responsible for generating the signals that we process in practice are, however, both nonstationary and non-Gaussian. Unlike Gaussian distributions, almost all real-world probability distributions have long tails and therefore require higher order statistics in addition to second-order statistics for their characterization. Stated in another way, Gaussian distributions occupy a vanishingly small subspace in the space of all probability distributions. Moreover, the problem is compounded by the fact that the underlying statistics are often unknown.

In a related context, a critical examination of the current literature on statistical signal processing prompts the following observations:

- ▲ Abundant use of abstract mathematics.
- ▲ Reliance on computer simulations.
- ▲ Almost complete lack of experimentation with real-life data.

Computer simulations, when properly applied, provide a great deal of insight into a problem of interest, but they are no substitute for tests with real-life data. It is therefore not surprising that many algorithms fail to survive the "test of time." The failure may be attributed to one or more of the following causes:

- ▲ Unrealistic assumptions made in formulating the mathematical model, hence a "mismatch" between the mathematical basis of the algorithm and the underlying physics of the problem at hand, which, in turn, produces a "loss" of valuable information.
- ▲ Failure to acquire "prior information" from the underlying physics of the problem and embed it into the design of the algorithm; the end result may be an erratic performance.
- ▲ Sensitivity of the algorithm to deviations from the assumed mathematical model, which may lead to instability.

Without question, mathematics is a powerful tool that gives an algorithm both elegance and general applicability. By the same token, however, an algorithm that ignores physical reality may end up being of limited or no practical use.

It is in order here that we remind ourselves of what Henri Poincaré, the famous French mathematician, once said many years ago:

> ➤ **Signal processing is at its best when it successfully combines the unique ability of mathematics to generalize with both the insight and prior information gained from the underlying physics of the problem at hand.**

"The science of physics does not only give us (mathematicians) an opportunity to solve problems, but helps us also to discover the means of solving them, and it does this in two ways: it leads us to anticipate the solution and suggests suitable lines of argument."

To paraphrase this insightful quotation in the context of signal processing, I say the following:

Signal processing is at its best when it successfully combines the unique ability of mathematics to generalize with both the insight and prior information gained from the under-

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lying physics of the problem at hand; the combination should lead to reliable algorithms that make a practical difference.

A word of caution should be noted here. There is a big gap between laboratory physics and real-world physics. In laboratory physics, for example, we may be studying an idealized chaotic oscillator such as the Lorenz attractor. (This attractor is named to honor Edward Lorenz, who derived it by truncating the Navier-Stokes equation for turbulent media.) In contrast, in real-world physics, we may be examining climate data whose underlying mechanism mandates the consideration of many chaotic and quite possibly stochastic processes that are all mushed together in some unknown fashion. For obvious reasons, I advocate the real-world physical realities of the signal processing problem at hand.

The simplified mathematical models formulated by many of the pioneers in the twentieth century were the result of careful thinking about specific signal processing problems that were difficult to solve in those times; the elegant signal processing theories so developed were compelled by the thoughts of those pioneers. With nonstationarity and non-Gaussianity being the physical realities of nearly every signal processing problem of practical interest, and with the signal processing power of computing devices no longer the impediment to practical implementation that it used to be, I see the evolution of modern signal processing that emphasizes five ingredients essential for satisfactory performance:

1) *Prior information*, the extraction of which requires understanding the physical laws that govern the generation of signals of interest.

2) *Regularization*, which is achieved by embedding prior information in a computationally efficient manner into algorithmic design so as to stabilize the solution.

3) *Adaptivity*, which is made possible by learning from the operational environment so as to account for the unknown statistical structure of the environment and track its nonstationary behavior.

4) *Robustness*, which, in a deterministic sense, means that unavoidable disturbances (e.g., errors due to choice of initial conditions, model mismatch, and use of finite-precision arithmetic) are not magnified by the algorithm. In a statistical sense, robustness means that the algorithm is insensitive to small deviations of the actual probabilty distribution form the probability distribution of the assumed model.

5) *Feedback*, a powerful engineering principle, the proper application of which has many beneficial effects (e.g., improved convergence, reduced sensitivity to parameter variations, and improved robustness to the presence of unavoidable disturbances).

Naturally, the ways in which these ingredients are built into the design of a signal processing algorithm are problem specific.

To conclude, classical signal processing developed in the 20th century is an integral part of the ever-expanding discipline of signal processing and will always be so, just like classical (Newtonian) mechanics is an integral part of physics. In the 21st century, I see the evolution of modern signal processing with its own list of performance ingredients and kit of powerful tools (not discussed here) that are used to provide reliable solutions to practical signal processing problems, with emphasis on bringing mathematics and physics together and reconciling the ever-present "tension" between them, and where we do not lose sight of the need to do two things: 1) test the performance of algorithms with real-life data and 2) learn from the data.

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