GATE: EE - 49.2022

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Results and Derivations:

For a signal $x[n] = \sum_{k=0}^{N-1} a_k e^{j\left(\frac{2kn\pi}{N}\right)}$, defining $X[k] = Na_k$.

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\left(\frac{2kn\pi}{N}\right)} \tag{1}$$

$$\sum_{n=0}^{N-1} x[n]e^{-j\left(\frac{2rn\pi}{N}\right)} = \sum_{n=0}^{N-1}\frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\left(\frac{2kn\pi}{N}\right)}e^{-j\left(\frac{2rn\pi}{N}\right)} \tag{2}$$

$$= \sum_{k=0}^{N-1} X[k]\sum_{n=0}^{N-1}\frac{1}{N}e^{j\left(\frac{2(k-r)n\pi}{N}\right)} \tag{3}$$

$$\sum_{n=0}^{N-1}\frac{1}{N}e^{j\left(\frac{2(k-r)n\pi}{N}\right)} = \begin{cases}1 & k-r = mN, m\in\mathbb{Z}\\ 0 & \text{otherwise}\end{cases} \tag{4}$$

$$\implies X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\left(\frac{2kn\pi}{N}\right)} \tag{5}$$

If $x[n] = x[n+rN], r\in\mathbb{Z}$, from (5),

$$X[k+rN] = \sum_{n=0}^{N-1} x[n]e^{-j\left(\frac{2(k+rN)n\pi}{N}\right)} \tag{6}$$

$$= \sum_{n=0}^{N-1} x[n]e^{-j\left(\frac{2kn\pi}{N}\right)}e^{-j(2rn\pi)} \tag{7}$$

$$= \sum_{n=0}^{N-1} x[n]e^{-j\left(\frac{2kn\pi}{N}\right)} \tag{8}$$

$$= X[k] \tag{9}$$

$$\implies a_k = a_{k+rN}, r\in\mathbb{Z} \tag{10}$$

Question: The discrete time Fourier series representation of a signal $x[n]$ with period N is written as $x[n] = \sum_{k=0}^{N-1} a_k e^{j(2kn\pi/N)}$. A discrete time periodic signal with period $N = 3$, has the non-zero Fourier series coefficients: $a_{-3} = 2$ and $a_4 = 1$. The signal is

(A) $2 + 2e^{-\left(j\frac{2\pi}{6}n\right)}\cos\left(\frac{2\pi}{6}n\right)$

(B) $1 + 2e^{\left(j\frac{2\pi}{6}n\right)}\cos\left(\frac{2\pi}{6}n\right)$

(C) $1 + 2e^{\left(j\frac{2\pi}{3}n\right)}\cos\left(\frac{2\pi}{6}n\right)$

(D) $2 + 2e^{\left(j\frac{2\pi}{6}n\right)}\cos\left(\frac{2\pi}{6}n\right)$

(GATE EE 2022)

Solution:

Parameters	Description	Value
$x[n]$	Signal	
N	Period	3
a_k	Fourier series coefficient	
a_{-3}	a_k at $k=-3$	2
a_4	a_k at $k=4$	1

TABLE 1
PARAMETERS

$$x[n] = \sum_{k=0}^{N-1} a_k e^{j(2kn\pi/N)} \tag{11}$$

$$= a_0 + a_1 e^{j\frac{2\pi}{3}n} \tag{12}$$

From (10),

$$a_0 = a_{-3} \tag{13}$$

$$a_1 = a_4 \tag{14}$$

$$x[n] = 2 + e^{j\frac{2\pi}{3}n} \tag{15}$$

$$= 1 + 1 + e^{j\frac{2\pi}{3}n} \tag{16}$$

$$= 1 + e^{j\frac{2\pi}{6}n}e^{-j\frac{2\pi}{6}n} + e^{j\frac{2\pi}{6}n}e^{j\frac{2\pi}{6}n} \tag{17}$$

$$= 1 + 2e^{j\frac{2\pi}{6}n}\left(\frac{e^{j\frac{2\pi}{6}n} + e^{-j\frac{2\pi}{6}n}}{2}\right) \tag{18}$$

$$= 1 + 2e^{j\frac{2\pi}{6}n}\cos\left(\frac{2\pi}{6}n\right) \tag{19}$$

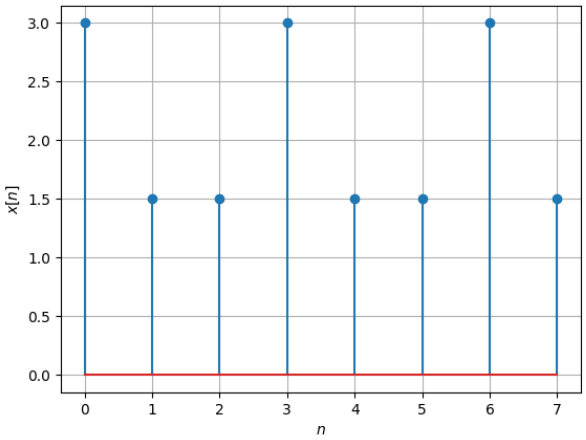

Fig. 1. Stem Plot of $x[n]$