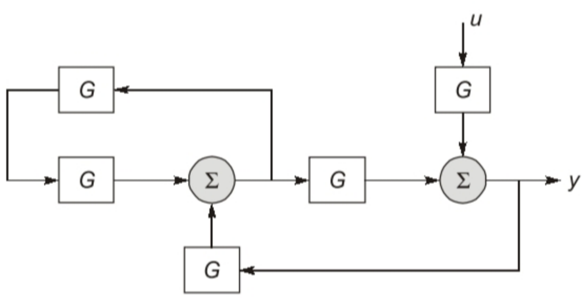
GATE: CH - 58.2022

EE23BTECH11224 - Sri Krishna Prabhas Yadla*

Question: In the block diagram shown in the figure, the transfer function $G = \frac{K}{\tau s+1}$ with $K > 0$ and $\tau > 0$. The maximum value of K below which the system remains stable is _____(rounded off to two decimal places) (GATE CH 2022)



Solution:

Parameter	Value	Description
G	$\frac{K}{\tau s+1}$	Transfer function shown in blocks
Y		Laplace transform of y(output)
U		Laplace transform of u(input)
X,Z		Laplace transform of x and z
T	$\frac{Y}{U}$	Transfer function of complete system

TABLE 1
PARAMETERS

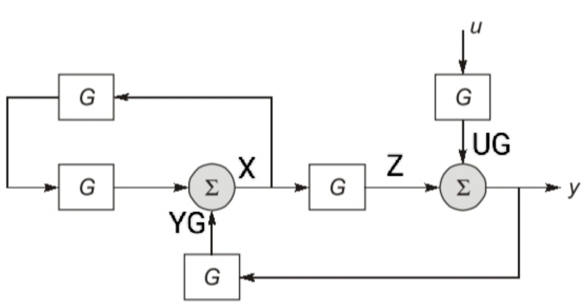


Fig. 1. Block Diagram

$$X = XG^2 + YG \tag{1}$$

$$\implies X = \frac{YG}{1-G^2} \tag{2}$$

$$Z = XG \tag{3}$$

$$Y = Z + UG \tag{4}$$

$$Y = XG + UG \tag{5}$$

$$Y = \frac{YG^2}{1-G^2} + UG \tag{6}$$

$$\implies Y = \frac{UG(1-G^2)}{1-2G^2} \tag{7}$$

From Table 1,

$$T = \frac{G(1-G^2)}{1-2G^2} \tag{8}$$

$$= \frac{K\left(1-\frac{K^2}{(\tau s+1)^2}\right)}{\left(1-\frac{2K^2}{(\tau s+1)^2}\right)(\tau s+1)} \tag{9}$$

$$= \frac{K(\tau^2 s^2 + 2\tau s + 1 - K^2)}{\tau^3 s^3 + 3\tau^2 s^2 + (3\tau - 2K^2\tau)s + 1 - 2K^2} \tag{10}$$

So, Characteristic equation : $1 - 2G^2 = 0$

$$1 - 2\frac{K^2}{(\tau s+1)^2} = 0 \tag{11}$$

$$\implies \tau^2 s^2 + 2\tau s + 1 - 2K^2 = 0 \tag{12}$$

For a characteristic equation $a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + ... a_n = 0$,

s^n	a_0	a_2	a_4	...
s^{n-1}	a_1	a_3	a_5	...
s^{n-2}	$b_1 = \frac{a_1a_2 - a_3a_0}{a_1}$	$b_2 = \frac{a_1a_4 - a_5a_0}{a_1}$	...	..
s^{n-3}	$c_1 = \frac{b_1a_3 - b_2a_1}{b_1}$	$\vdots$		
$\vdots$	$\vdots$	$\vdots$		
s^1	$\vdots$	$\vdots$		
s^0	a_n			

TABLE 2
ROUTH ARRAY

From Table 2:

s^3	τ^3	$3\tau - 2K^2\tau$
s^2	$3\tau^2$	$1 - 2K^2$
s^1	$\frac{8}{3}\tau(1 - K^2)$	0
s^0	$1 - 2K^2$	

TABLE 3

Given $\tau > 0$ and $K > 0$, for system to be stable,

$$1 - K^2 > 0 \tag{13}$$

$$1 - 2K^2 > 0 \tag{14}$$

$$\implies 0 < K < \frac{1}{\sqrt{2}} \tag{15}$$

$$K_{max} \approx 0.71 \tag{16}$$