NCERT 12.10.Q21

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Question: In deriving the single slit diffraction pattern, it was stated that the intensity is zero at angles of $\frac{n\lambda}{a}$. Justify this by suitably dividing the slit to bring out the cancellation.
Solution:

Parameters	Description
λ	Wavelength
a	Slit width
k	Wave number
ω	Angular frequency
ds	Infinitesimally small part of slit at a distance s from top of the slit
Δx	Path difference
θ	Angle of elevation of point P on screeen from slit
dE	Electric field at point P associated with light wavelet at ds
c_1, c_2	Proportionality constants
E_1	Amplitude of E_P
ϕ'	Phase difference between E_P and incident light's electric field
I	Intensity at point P
I_0	Intensity at central bright band

TABLE 1
VARIABLES USED

$$k = \frac{2\pi}{\lambda} \tag{1}$$

$$\Delta x = s \cdot \sin\theta \tag{2}$$

$$dE = (c_1 \cdot ds)\cos\left(k(x + \Delta x) - \omega t\right) \tag{3}$$

$$= c_1 \cos\left(ks\sin\theta + kx - \omega t\right)ds \tag{4}$$

$$E_P = \int_0^a c_1 \cos\left(ks\sin\theta + kx - \omega t\right)ds \tag{5}$$

$$= \frac{c_1 \sin\left(ka\sin\theta + kx - \omega t\right) - c_1 \sin\left(kx - \omega t\right)}{k\sin\theta} \tag{6}$$

$$= \frac{2c_1}{k\sin\theta}\sin\left(\frac{ka\sin\theta}{2}\right)\cos\left(kx - \omega t + \frac{ka\sin\theta}{2}\right) \tag{7}$$

$$= E_1 \cos\left(kx - \omega t + \phi'\right) \tag{8}$$

$$\implies E_1 = \frac{2c_1}{k\sin\theta}\sin\left(\frac{ka\sin\theta}{2}\right) \tag{9}$$

$$I \propto E_1^2 \tag{10}$$

$$I = c_2 \cdot \left(\frac{2c_1}{k\sin\theta}\sin\left(\frac{ka\sin\theta}{2}\right)\right)^2 \tag{11}$$

$$= c_2 c_1^2 a^2 \frac{\sin^2\left(\frac{ka\sin\theta}{2}\right)}{\left(\frac{ka\sin\theta}{2}\right)^2} \tag{12}$$

$$I_0 = \lim_{\theta \to 0} I \tag{13}$$

$$= c_2 c_1^2 a^2 \tag{14}$$

$$I = I_0 \frac{\sin^2\beta}{\beta^2} \tag{15}$$

From (12),

$$\beta = \frac{ka\sin\theta}{2} \tag{16}$$

$$= \frac{\pi a\sin\theta}{\lambda} \tag{17}$$

For zero intensity,

$$\beta = n\pi \tag{18}$$

$$a\sin\theta = n\lambda \tag{19}$$

$$\sin\theta \approx \theta \quad \text{(for small angles)} \tag{20}$$

$$\implies \theta = \frac{n\lambda}{a} \tag{21}$$