GATE: IN - 50.2023

EE23BTECH11224 - Sri Krishna Prabhas Yadla*

Question: The phase margin of the transfer function $G(s) = \frac{2(1-s)}{(1+s)^2}$ is _____ degrees. (rounded off to the nearest integer). (GATE IN 2023)

Solution:

Parameters	Description
ω_c	crossover frequency
$\angle G(j\omega)$	phase angle of the transfer function
PM	$\angle G(j\omega_c) + 180°$; Phase Margin

TABLE 1
PARAMETERS

Considering $s = j\omega$,

$$G(j\omega) = \frac{2(1-j\omega)}{(1+j\omega)^2} \tag{1}$$

$$= \frac{2(1-j\omega)^3}{|1+j\omega|^4} \tag{2}$$

$$= \frac{2}{(1+\omega^2)^2}(1-j\omega)^3 \tag{3}$$

$$= \frac{2}{\sqrt{1+\omega^2}}(e^{-j\omega})^3 \tag{4}$$

$$\implies |G(j\omega)| = \frac{2}{\sqrt{1+\omega^2}} \tag{5}$$

$$\implies \angle G(j\omega) = 3\tan^{-1}(-\omega) \tag{6}$$

At $\omega = \omega_c$, Gain = 0

$$\implies |G(j\omega_c)| = 1 \tag{7}$$

$$\frac{2}{\sqrt{1+\omega_c^2}} = 1 \tag{8}$$

$$\implies \omega_c = \sqrt{3} \tag{9}$$

$$\angle G(j\omega_c) = 3\tan^{-1}(-\sqrt{3}) \tag{10}$$

$$= -180° \tag{11}$$

From Table 1,

$$PM = 0° \tag{12}$$

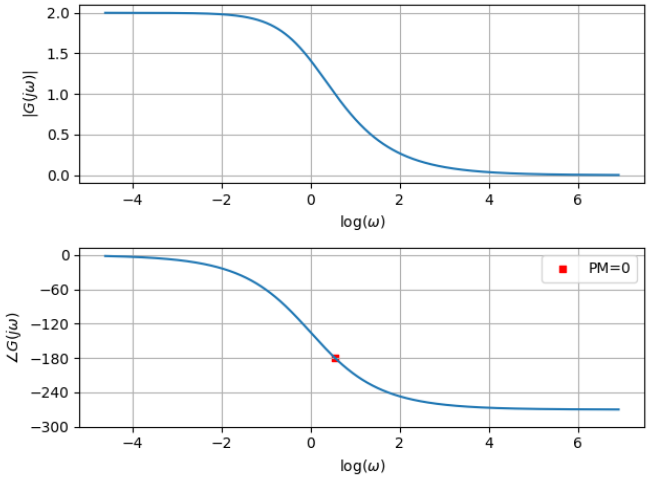

Fig. 1. Bode Plot of Transfer Function $G(s)$