

GATE: EE - 49.2022

EE23BTECH11224 - Sri Krishna Prabhas Yadla*

Question: The discrete time Fourier series representation of a signal $x[n]$ with period N is written as $x[n] = \sum_{k=0}^{N-1} a_k e^{j(2k\pi n/N)}$. A discrete time periodic signal with period $N = 3$, has the non-zero Fourier series coefficients: $a_{-3} = 2$ and $a_4 = 1$. The signal is

- (A) $2 + 2e^{-j(\frac{2\pi}{6}n)} \cos\left(\frac{2\pi}{6}n\right)$
 (B) $1 + 2e^{j(\frac{2\pi}{6}n)} \cos\left(\frac{2\pi}{6}n\right)$
 (C) $1 + 2e^{j(\frac{2\pi}{3}n)} \cos\left(\frac{2\pi}{6}n\right)$
 (D) $2 + 2e^{j(\frac{2\pi}{6}n)} \cos\left(\frac{2\pi}{6}n\right)$

(GATE EE 2022)

Solution:

Parameters	Description	Value
$x[n]$	Signal	
N	Period	3
a_k	Fourier series coefficient	
a_{-3}	a_k at $k = -3$	2
a_4	a_k at $k = 4$	1

TABLE I
PARAMETERS

In discrete-time Fourier series,

$$a_k = a_{k+N} \quad (1)$$

$$\Rightarrow a_0 = a_{-3} \quad (2)$$

$$a_1 = a_4 \quad (3)$$

$$x[n] = \sum_{k=0}^2 a_k e^{j(\frac{2k\pi}{3}n)} \quad (4)$$

$$= a_0 + a_1 e^{j\frac{2\pi}{3}n} + a_2 e^{j\frac{4\pi}{3}n} \quad (5)$$

$$= 2 + e^{j\frac{2\pi}{3}n} + 0 \quad (6)$$

$$= 1 + 1 + e^{j\frac{2\pi}{3}n} \quad (7)$$

$$= 1 + e^{j\frac{2\pi}{6}n} e^{-j\frac{2\pi}{6}n} + e^{j\frac{2\pi}{6}n} e^{j\frac{2\pi}{6}n} \quad (8)$$

$$= 1 + 2e^{j\frac{2\pi}{6}n} \left(\frac{e^{j\frac{2\pi}{6}n} + e^{-j\frac{2\pi}{6}n}}{2} \right) \quad (9)$$

$$= 1 + 2e^{j\frac{2\pi}{6}n} \cos\left(\frac{2\pi}{6}n\right) \quad (10)$$

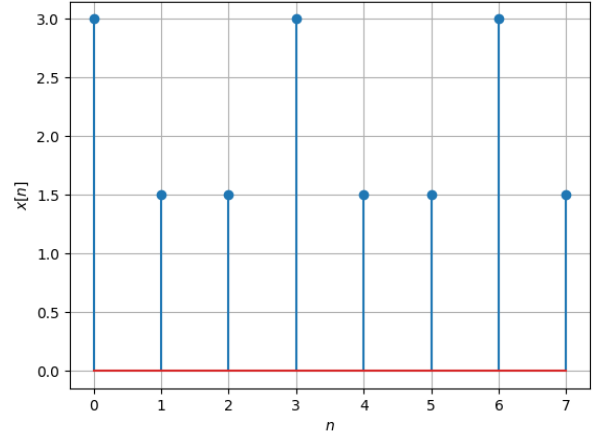


Fig. 1. Stem Plot of $x[n]$