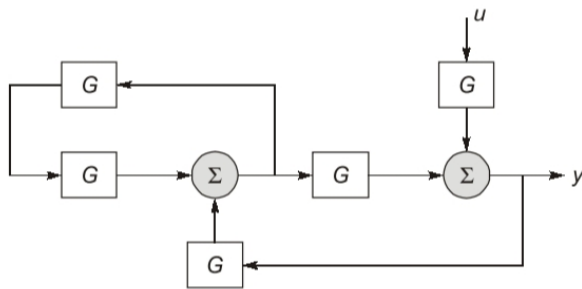


GATE: CH - 58.2022

EE23BTECH11224 - Sri Krishna Prabhas Yadla*

Question: In the block diagram shown in the figure, the transfer function $G = \frac{K}{\tau s + 1}$ with $K > 0$ and $\tau > 0$. The maximum value of K below which the system remains stable is ____ (rounded off to two decimal places) (GATE CH 2022)



$$X = XG^2 + YG \quad (1)$$

$$\Rightarrow X = \frac{YG}{1 - G^2} \quad (2)$$

$$Z = XG \quad (3)$$

$$Y = Z + UG \quad (4)$$

$$Y = XG + UG \quad (5)$$

$$Y = \frac{YG^2}{1 - G^2} + UG \quad (6)$$

$$\Rightarrow Y = \frac{UG(1 - G^2)}{1 - 2G^2} \quad (7)$$

From Table 1,

$$T = \frac{G(1 - G^2)}{1 - 2G^2} \quad (8)$$

So, Characteristic equation : $1 - 2G^2 = 0$

$$1 - 2\frac{K^2}{(\tau s + 1)^2} = 0 \quad (9)$$

$$\Rightarrow \tau^2 s^2 + 2\tau s + 1 - 2K^2 = 0 \quad (10)$$

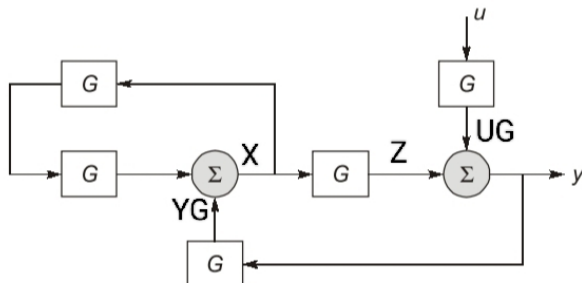
Solution:

Parameter	Value	Description
G	$\frac{K}{\tau s + 1}$	Transfer function shown in blocks
Y		Laplace transform of y(output)
U		Laplace transform of u(input)
X,Z		Laplace transform of x and z
T	$\frac{Y}{U}$	Transfer function of complete system

TABLE 1
PARAMETERS

s^n	a_0	a_2	a_4	...
s^{n-1}	a_1	a_3	a_5	...
s^{n-2}	$b_1 = \frac{a_1 a_2 - a_3 a_0}{a_1}$	$b_2 = \frac{a_1 a_4 - a_5 a_0}{a_1}$
s^{n-3}	$c_1 = \frac{b_1 a_3 - b_2 a_1}{b_1}$	\vdots		
\vdots	\vdots	\vdots		
s^1	\vdots	\vdots		
s^0	a_n			

TABLE 2
ROUTH ARRAY



From Table 2:

s^2	τ^2	$1 - 2K^2$
s^1	2τ	0
s^0	$1 - 2K^2$	

TABLE 3

Fig. 1. Block Diagram

Given $\tau > 0$ and $K > 0$, for system to be stable,

$$1 - 2K^2 \geq 0 \quad (11)$$

$$\Rightarrow 0 < K \leq \frac{1}{\sqrt{2}} \quad (12)$$

$$K_{max} \approx 0.71 \quad (13)$$