GATE: EE - 49.2022

EE23BTECH11224 - Sri Krishna Prabhas Yadla*

Question: The discrete time Fourier series representation of a signal $x[n]$ with period N is written as $x[n] = \sum_{k=0}^{N-1} a_k e^{j(2kn\pi/N)}$. A discrete time periodic signal with period $N = 3$, has the non-zero Fourier series coefficients: $a_{-3} = 2$ and $a_4 = 1$. The signal is

(A) $2 + 2e^{-\left(j\frac{2\pi}{6}n\right)} \cos\left(\frac{2\pi}{6}n\right)$

(B) $1 + 2e^{\left(j\frac{2\pi}{6}n\right)} \cos\left(\frac{2\pi}{6}n\right)$

(C) $1 + 2e^{\left(j\frac{2\pi}{3}n\right)} \cos\left(\frac{2\pi}{6}n\right)$

(D) $2 + 2e^{\left(j\frac{2\pi}{6}n\right)} \cos\left(\frac{2\pi}{6}n\right)$

(GATE EE 2022)

Solution:

Parameters	Description	Value
$x[n]$	Signal	
N	Period	3
a_k	Fourier series coefficient	
a_{-3}	a_k at $k = -3$	2
a_4	a_k at $k = 4$	1

TABLE 1
PARAMETERS

$$x[n] = \sum_{k=-\infty}^{\infty} a_k e^{j\left(\frac{2k\pi}{3}n\right)} \tag{1}$$

$$= a_{-3}e^{j\frac{-6\pi}{3}n} + a_4 e^{j\frac{8\pi}{3}n} \tag{2}$$

$$= 2 + e^{j\frac{2\pi}{3}n} \tag{3}$$

$$= 1 + 1 + e^{j\frac{2\pi}{3}n} \tag{4}$$

$$= 1 + e^{j\frac{2\pi}{6}n}e^{-j\frac{2\pi}{6}n} + e^{j\frac{2\pi}{6}n}e^{j\frac{2\pi}{6}n} \tag{5}$$

$$= 1 + 2e^{j\frac{2\pi}{6}n}\left(\frac{e^{j\frac{2\pi}{6}n} + e^{-j\frac{2\pi}{6}n}}{2}\right) \tag{6}$$

$$= 1 + 2e^{j\frac{2\pi}{6}n}\cos\left(\frac{2\pi}{6}n\right) \tag{7}$$

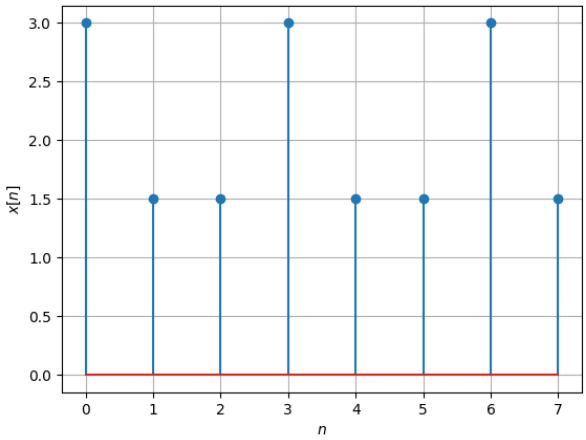

Fig. 1. Stem Plot of $x[n]$