

GATE: IN - 50.2023

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Question: The phase margin of the transfer function $G(s) = \frac{2(1-s)}{(1+s)^2}$ is ___ degrees. (rounded off to the nearest integer). (GATE IN 2023)

Solution:

Parameters	Description
ω_c	crossover frequency, where the $ G(j\omega_c) = 1$
$\angle G(j\omega_c)$	phase angle of the transfer function at ω_c
PM	Phase Margin

TABLE 1

$$G(j\omega_c) = \frac{2(1 - j\omega_c)}{(1 + j\omega_c)^2} \quad (1)$$

$$|G(j\omega_c)| = \left| \frac{2(1 - j\omega_c)}{(1 + j\omega_c)^2} \right| \quad (2)$$

$$= \frac{2}{\sqrt{1 + \omega_c^2}} = 1 \quad (3)$$

$$\Rightarrow \omega_c = \sqrt{3} \quad (4)$$

$$\Rightarrow G(j\omega_c) = \frac{(1 - \sqrt{3}j)^3}{8} \quad (5)$$

$$= \left(\frac{1 - \sqrt{3}j}{2} \right)^3 \quad (6)$$

$$= e^{-3\frac{j\pi}{3}} \quad (7)$$

$$\Rightarrow \angle G(j\omega_c) = -\pi \text{ rad} = -180^\circ \quad (8)$$

$$PM = \angle G(j\omega_c) + 180^\circ = 0^\circ \quad (9)$$