

GATE: IN - 50.2023

EE23BTECH11224 - Sri Krishna Prabhas Yadla*

Question: The phase margin of the transfer function $G(s) = \frac{2(1-s)}{(1+s)^2}$ is _____ degrees. (rounded off to the nearest integer). (GATE IN 2023)

Solution:

Parameters	Description
ω_c	crossover frequency
$\angle G(j\omega)$	phase angle of the transfer function
PM	$\angle G(j\omega_c) + 180^\circ$; Phase Margin

TABLE 1
PARAMETERS

$$G(j\omega) = \frac{2(1-j\omega)}{(1+j\omega)^2} \quad (1)$$

$$= \frac{2(1-j\omega)^3}{|1+j\omega|^4} \quad (2)$$

$$= \frac{2}{(1+\omega^2)^2} (1-j\omega)^3 \quad (3)$$

$$= \frac{2}{\sqrt{1+\omega^2}} (e^{-j\omega})^3 \quad (4)$$

$$\Rightarrow |G(j\omega)| = \frac{2}{\sqrt{1+\omega^2}} \quad (5)$$

$$\Rightarrow \angle G(j\omega) = 3 \tan^{-1}(-\omega) \quad (6)$$

$$G(j\omega_c) = 1 \quad (7)$$

$$\frac{2}{\sqrt{1+\omega_c^2}} = 1 \quad (8)$$

$$1 + \omega_c^2 = 4 \quad (9)$$

$$\Rightarrow \omega_c = \sqrt{3} \quad (10)$$

$$\angle G(j\omega_c) = 3 \tan^{-1}(-\sqrt{3}) \quad (11)$$

$$= -180^\circ \quad (12)$$

From Table 1,

$$PM = 0^\circ \quad (13)$$

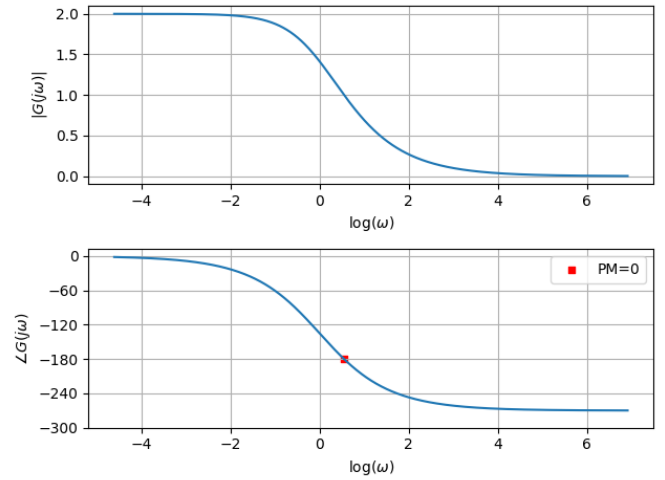


Fig. 1. Bode Plot of Transfer Function $G(s)$