

NCERT 12.10.Q21

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Question: In deriving the single slit diffraction pattern, it was stated that the intensity is zero at angles of $\frac{n\lambda}{a}$. Justify this by suitably dividing the slit to bring out the cancellation.

Solution:

Parameters	Description
λ	Wavelength
a	Slit width
k	Wave number
ω	Angular frequency
ds	Infinitesimally small part of slit at a distance s from top of the slit
Δx	Path difference
θ	Angle of elevation of point P on screen from slit
dE	Electric field at point P associated with light wavelet at ds
c_1, c_2	Proportionality constants
E_1	Amplitude of E_P
ϕ'	Phase difference between E_P and incident light's electric field
I	Intensity at point P
I_0	Intensity at central bright band

TABLE 1
VARIABLES USED

$$I \propto E_1^2 \quad (10)$$

$$I = c_2 \cdot \left(\frac{2c_1}{k \sin \theta} \sin \left(\frac{ka \sin \theta}{2} \right) \right)^2 \quad (11)$$

$$= c_2 c_1^2 a^2 \frac{\sin^2 \left(\frac{ka \sin \theta}{2} \right)}{\left(\frac{ka \sin \theta}{2} \right)^2} \quad (12)$$

$$I_0 = \lim_{\theta \rightarrow 0} I = c_2 c_1^2 a^2 \quad (13)$$

$$I = I_0 \frac{\sin^2 \beta}{\beta^2} \quad (14)$$

$$\Rightarrow \beta = \frac{ka \sin \theta}{2} = \frac{\pi a \sin \theta}{\lambda} \quad (15)$$

For zero intensity,

$$\beta = n\pi \quad (16)$$

$$a \sin \theta = n\lambda \quad (17)$$

$$\sin \theta \approx \theta \quad (\text{for small angles}) \quad (18)$$

$$\Rightarrow \theta = \frac{n\lambda}{a} \quad (19)$$

$$k = \frac{2\pi}{\lambda} \quad (1)$$

$$\Delta x = s \cdot \sin \theta \quad (2)$$

$$dE = (c_1 \cdot ds) \cos(k(x + \Delta x) - \omega t) \quad (3)$$

$$= c_1 \cos(ks \sin \theta + kx - \omega t) ds \quad (4)$$

$$E_P = \int_0^a c_1 \cos(ks \sin \theta + kx - \omega t) ds \quad (5)$$

$$= \frac{c_1 \sin(ka \sin \theta + kx - \omega t) - c_1 \sin(kx - \omega t)}{k \sin \theta} \quad (6)$$

$$= \frac{2c_1}{k \sin \theta} \sin \left(\frac{ka \sin \theta}{2} \right) \cos \left(kx - \omega t + \frac{ka \sin \theta}{2} \right) \quad (7)$$

$$= E_1 \cos(kx - \omega t + \phi') \quad (8)$$

$$\Rightarrow E_1 = \frac{2c_1}{k \sin \theta} \sin \left(\frac{ka \sin \theta}{2} \right) \quad (9)$$