NCERT 11.9.3.Q10

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Question: Find the sum to indicated number of terms in the geometric progression $x^3, x^5, x^7, ...n$ terms (if $x \neq \pm 1$).

Solution: Let $S(n)$ be the sum of the first n terms in G.P starting from $x(0)$. We have

$$x(n) = x(0) \cdot r^n \tag{1}$$

$$S(n) = \sum_{k=0}^{n-1} x(k) \tag{2}$$

$$= x(0)\frac{r^n - 1}{r - 1} \text{ (for r}\neq 1) \tag{3}$$

Input Parameters	Values
$x(0)$	x^3
$x(1)$	x^5
$x(2)$	x^7
Number of terms	n

TABLE 0
GIVEN INPUTS

Hence the common ratio, r, can be calculated by

$$r = \frac{x(1)}{x(0)} = \frac{x^5}{x^3} = x^2 \tag{4}$$

Since $x \neq \pm 1$, $r \neq 1$,

$$S(n) = x(0)\frac{r^n - 1}{r - 1} \tag{5}$$

$$\therefore S(n) = x^3\frac{x^{2n} - 1}{x^2 - 1} \tag{6}$$

$$x(n) \overset{Z}{\longleftrightarrow} X(z)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{7}$$

$$= \sum_{n=0}^{\infty} x(n)z^{-n} \tag{8}$$

$$= \sum_{n=0}^{\infty} x(0)r^n z^{-n} \tag{9}$$

$$= \frac{x(0)}{1 - rz^{-1}} \tag{10}$$

$$= \frac{x^3}{1 - x^2 z^{-1}} \tag{11}$$

The z transform is defined only when $|x^2 z^{-1}| < 1$. So, ROC : $|z| > x^2$.