GATE: AE - 37.2022

EE23BTECH11224 - Sri Krishna Prabhas Yadla*

Question: Consider the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$. The boundary conditions are $y = 0$ and $\frac{dy}{dx} = 1$ at $x = 0$. Then the value of y at $x = \frac{1}{2}$ (GATE AE 2022)

Solution:

Parameters	Values	Description
$y(0)$	0	y at $x = 0$
$y'(0)$	1	$\frac{dy}{dx}$ at $x = 0$

TABLE 1
PARAMETERS

$$\frac{d^2y}{dx^2} \overset{\mathcal{L}}{\longleftrightarrow} s^2Y(s) - sy(0) - y'(0) \tag{1}$$

$$\frac{dy}{dx} \overset{\mathcal{L}}{\longleftrightarrow} sY(s) - y(0) \tag{2}$$

Applying Laplace Transform, using (1) and (2),

$$s^2Y(s) - sy(0) - y'(0) - 2(sY(s) - y(0)) + Y(s) = 0 \tag{3}$$

From Table 1,

$$(s^2 - 2s + 1)Y(s) - 1 = 0 \tag{4}$$

$$Y(s) = \frac{1}{(s-1)^2} \tag{5}$$

$$t^n \overset{\mathcal{L}}{\longleftrightarrow} \frac{n!}{s^{n+1}} \tag{6}$$

$$e^a t x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s-a) \tag{7}$$

Taking Inverse Laplace Transform for $Y(s)$, using (6) and (7),

$$y(x) = xe^x \tag{8}$$

$$\implies y\left(\frac{1}{2}\right) = \frac{\sqrt{e}}{2} \tag{9}$$

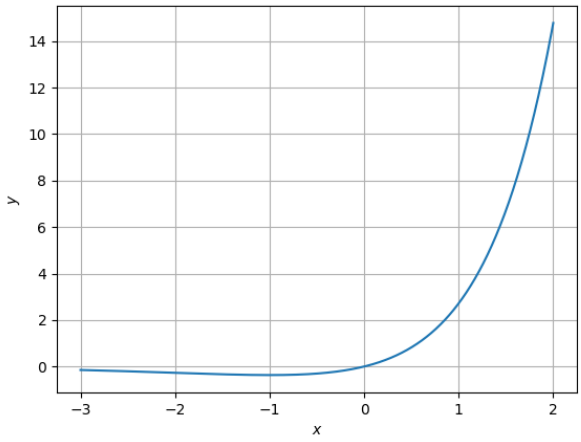

Fig. 1. Plot of $y(x)$