GATE: IN - 50.2023

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Question: The phase margin of the transfer function $G(s) = \frac{2(1-s)}{(1+s)^2}$ is __ degrees. (rounded off to the nearest integer). (GATE IN 2023)

Solution:

Parameters	Description
ω_c	crossover frequency, where the $\lvert G(j\omega_c)\rvert = 1$
$\angle G(j\omega_c)$	phase angle of the transfer function at ω_c
PM	Phase Margin

TABLE 1

$$G(j\omega_c) = \frac{2(1 - j\omega_c)}{(1 + j\omega_c)^2} \tag{1}$$

$$|G(j\omega_c)| = \left|\frac{2(1 - j\omega_c)}{(1 + j\omega_c)^2}\right| \tag{2}$$

$$= \frac{2}{\sqrt{1 + \omega_c^2}} = 1 \tag{3}$$

$$\implies \omega_c = \sqrt{3} \tag{4}$$

$$\implies G(j\omega_c) = \frac{(1 - \sqrt{3}j)^3}{8} \tag{5}$$

$$= \left(\frac{1 - \sqrt{3}j}{2}\right)^3 \tag{6}$$

$$= e^{-3\frac{j\pi}{3}} \tag{7}$$

$$\implies \angle G(j\omega_c) = -180^\circ \tag{8}$$

$$PM = \angle G(j\omega_c) + 180^\circ \tag{9}$$

$$= 0^\circ \tag{10}$$