1. Let $x(t) = (1 - t^2)^4$ for $\mid t \mid < 1$ and $x(t) = 0$ otherwise.

    - Use the symbolic toolkit in Matlab or Python's sympy module to find the Fourier transform $X(f)$ of $x(t)$. Plot $x(t)$ and its Fourier transform $X(f)$ on separate charts.
    - Denote by $X_F(f)$ the periodization of $X(f)$ with period F. Find a value $F > 0$ such that $|X(f) - X_F(f)| < 0.001$ for all $\mid f \mid \leq F/2$.
    - Use the numerical procedure outlined in the class notes to get a discrete approximation of $X(f)$ with error less than 0.001 for $\mid f \mid \leq F/2$. Set the number of samples N so that$\Delta f = 1/32$.
    - What value are you using for T in your calculations T ? Show $x(t)$, its periodization $x_T(t)$ of period T, and the sample values $x_T(j\,\Delta t)$ on a single graph.
    - On another graph show the exact Fourier transform $X(f)$, the periodized Fourier transform $X_F(f)$, and the approximations $X_F(n\,\Delta f)$ of $X(f)$ calculated using the discrete Fourier transform.

2. Let $x(t) = \frac{e^{-|t|}}{1+|t|}$.

    - Plot an approximation of its Fourier transform using F = 4 and N = 64
    - Repeat the above with F = 8 and N = 256.

3. The series $\sum_{n=1}^{\infty} \frac{1}{n} \cos(n\,t)$ is the Fourier series of some periodic function $h(t)$.

    - What are the period T and fundamental frequency f_o of $h(t)$?
    - Let $N = 40$. Plot the partial sum $s_N(t)$ of this series for $0 \leq t \leq T$.
    - Plot the Cesaro sum $\sigma_N(t)$ of this series.
    - What would the graph of $h(t)$ look like ? Validate your guess by calculating the cosine and sine coefficients of the "guessed" function and showing they match those in the series.

4. Consider the function $x(t)$ defined as $x(t) = e^t$ when $-1 < t < 1$ and $x(t) = 0$ otherwise. Define $x_T(t) = \sum_{-\infty}^{+\infty} x(t - 3\,n)$.

    - Plot $x_T(t)$ for $-6 \leq t \leq 6$.
    - Use the discrete Fourier transform to calculate the Fourier coefficients $\hat{x}_T(n)$ with $-32 \leq n \leq +32$.
    - With the Fourier coefficients just calculated find, and plot, the partial and Cesaro sums of the Fourier series of $x_T(t)$ for $-6 \leq t \leq 6$.