1. Using Matlab's or Python's built in FFT routines for vectors of length 8, write a script for calculating Discrete Fourier Transforms of length 32. Validate your script by using as input the vector of 32 leading digits of π and comparing results with the Matlab or Python built-in FFTs.

2. Consider a homogeneous circular plate of radius=1. If r and θ represent polar coordinates, find the steady-state temperature $u(r, \theta)$ of the plate if the temperature on the boundary is given by $u(1, \theta) = \cos(\theta) + \frac{1}{2}\cos(2\theta)$. Display your answer as a density plot.

3. Let

$$x(t) = \left(\frac{\sin \pi t}{\pi t}\right)^2.$$

What is the Nyquist frequency of $x(t)$? Define

$$y(t) = \sum_{-N}^{+N} x(k/F)\ \mathrm{sinc}(F(t - k/F)).$$

Plot $y(t)$ for $-2 \leq t \leq 2$ with the following choices of F and N: (a) $F = 3$ and $N = 12$, (b) $F = 2$ and $N = 12$, and (c) $F = 1$ and $N = 12$. Compare to $x(t)$. Explain the results.

4. Write a Matlab or Python script that uses FFTs to multiply two 10,000-decimal digit integers a and b where the digits of a are all 9 and the digits of b are all 6. Print the 5 leading digits and 5 trailing digits of your result.