Decentralized Formation of Arbitrary Multi-Robot Lattices

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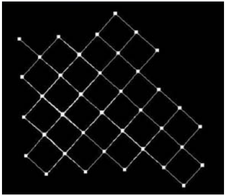
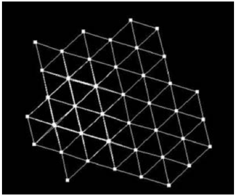
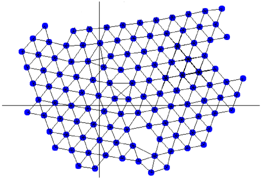
Related Work
Formation using virtual force

W. Spears, D. Spears, J. Hamann and R. Heil, 2004

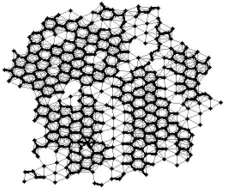




I. Navarro, J. Pugh, A. Martinoli, and F. Matia, 2008

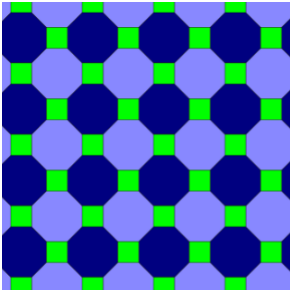


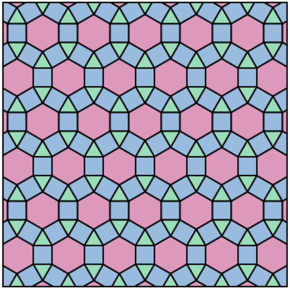
S. Prabhu, W. Li, J. McLurkin, 2012



Motivation

Question: How to use one algorithm to generate various (repeating) lattice pattern formations?





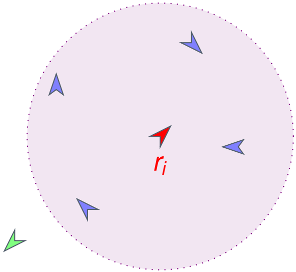
Objective

INPUT: A representation of a (repeating) lattice pattern.

OUTPUT: Move to this pattern using only local information.

Robot Model

- Differential Drive robots.
- Each robot has an unique **ID**.
- Use a vector $p = [x, y, \theta]^T$ to represent robot's **pose**.
- Each robot has a **range** within which it can sense and communicate with other robots.
- Each robot gets **observation** of its neighbors' IDs and relative poses in its body frame.

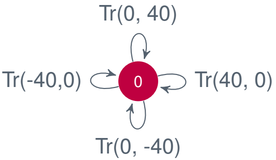


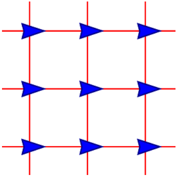
Robot r_i has 4 neighbors

Input: Lattice Graph

Definition

A **lattice graph** is a strongly connected directed multigraph in which each edge e is labeled with a rigid body transformation $T(e)$ and each $v \xrightarrow{T(e)} w$ has an inverse edge $w \xrightarrow{T(e)^{-1}} v$.



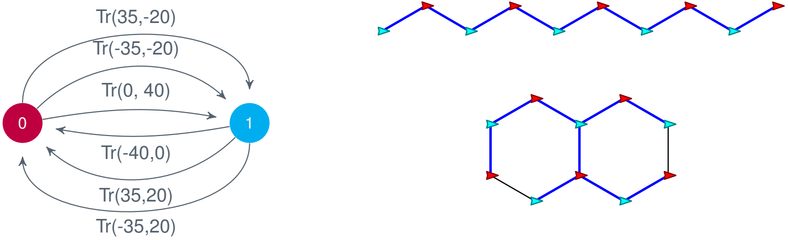


Role Function

Definition

Given a lattice graph $G = (V, E)$ and a set of robots $R = \{r_1, \ldots, r_n\}$, R **satisfies** G if there exists a role function $f : R \to V$ that preserves the neighborhood structure of G.

Specifically, for any i and j, if r_i and r_j are neighbors, there must exist an edge $e_{ij} : f(r_i) \longrightarrow f(r_j)$ in E, such that $T(r_j) = T(r_i)T(e_{ij})$.



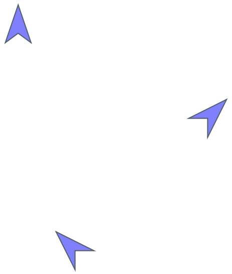
Algorithm

General Description

Robot broadcasts message containing its

- authority
- matching.

1. Form tree structure.
2. Use tree structure to computer local task assignment.
3. Make movement decision.



Authority

Define authority and comparison operator

An **authority** is an ordered list of robot IDs

$$(\mathrm{id}_1, \ldots, \mathrm{id}_k)$$

The first ID in the list, id_1 is called the **root** ID. The final ID in the list, id_k is called the **sender** ID.

Authority A_2 is **higher than** A_1 if:

- root ID of $A_2 >$ root ID of A_1, or
- length of $A_2 <$ length of A_1 if they have the same root, or
- sender ID of $A_2 >$ sender ID of A_1 if they have the same root and length.

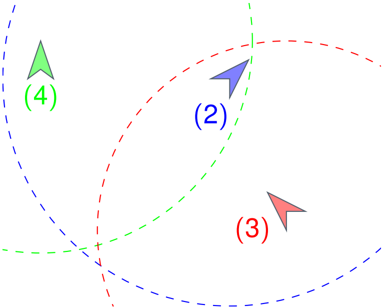
1.Construct Authority Tree

Decide to be root or descendant

The robots use these authorities to establish a collection of authority trees

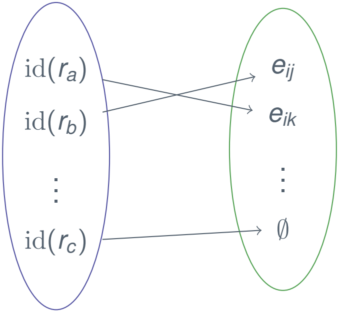
1. Discards any message in which the authority contains its own ID.
2. Forms an authority containing only its own ID, compares it with the authorities of remaining messages and selects the highest authority.

- If its authority is the highest, then it is a root;
- Otherwise, it selects the one who sends the highest authority as its parent. Append its own ID to the highest authority to create its own authority.



Matching

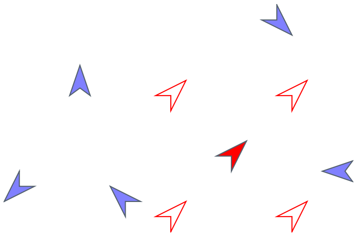
A **matching** for a robot is a function $\eta : \{\mathrm{id}(r_a), \mathrm{id}(r_b), \ldots\} \to \{\emptyset, e_{ij}, e_{ik}, \ldots\}$ that associates each neighbor ID with either a lattice graph edge from its role vertex or with the null value $\emptyset$.



2. Local Task Assignment
Hungarain Algorithm

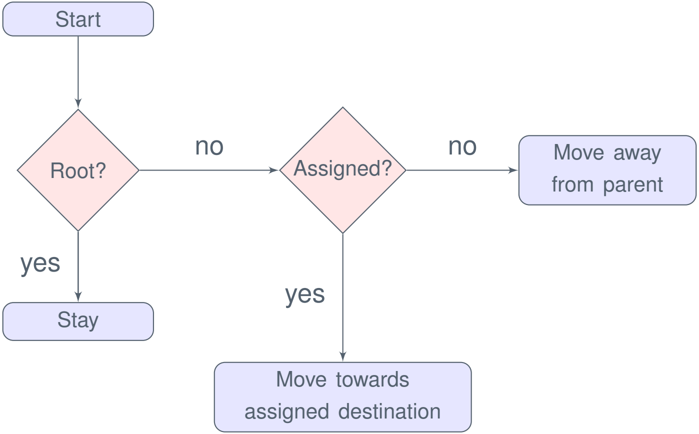
To compute an optimal matching of a robot with N neighbors and E out-going edges of its role in the lattice graph, define a weight matrix of size $N \times \max(N, E)$ and apply Hungarian Algorithm (Harold W. Kuhn, 1955).

1. Each row corresponds to a neighbor;
2. Each column corresponds to an out-going edge of robot's role or a null value $\emptyset$.
3. The entries of the matrix are the Euclidean distance between current position of each neighbor and the desired position if matched with a lattice graph edge.



5 neighbors, 4 out-going edges.

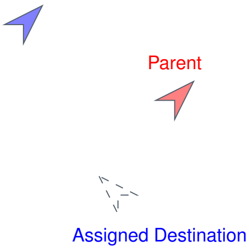
3. Robot Movement Strategy



Bounded Movement

Goal: let descendant stay in the range of its parent

- Within the set O (Red circle), the parent is guaranteed to get observation at next stage.
- Descendant can reach anywhere in set P (blue circle) at next stage.
- The real destination for descendant is the closest point on the boundary of the intersection ($O \cap P$) to the assigned destination.



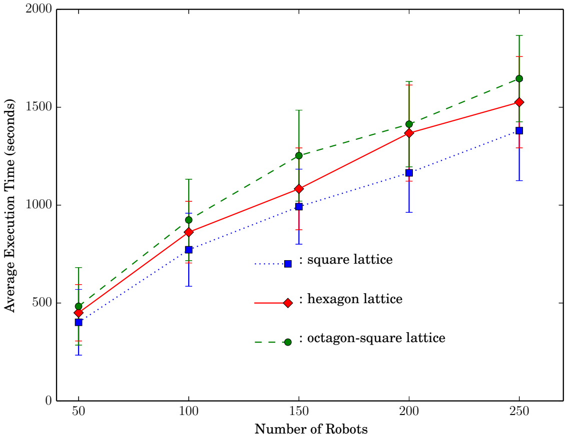
Simulation

Octagon-square formation with 100 robots

Simulation
Hexagon formation with 100 robots

Experiment Results

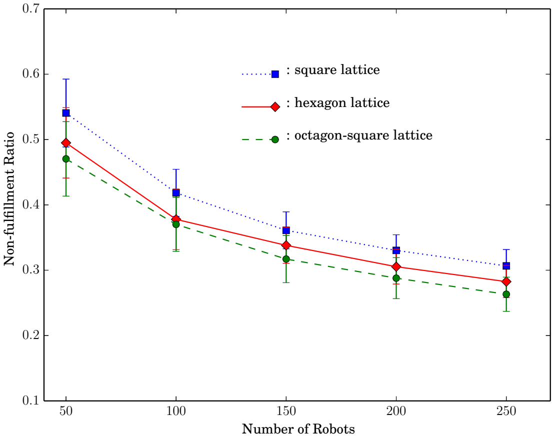
on three kinds of repeated lattice patterns



Average time to the static position with 50 trials, uniform distribution.

Experiment Results

on three kinds of repeated lattice patterns



Average non-fulfillment ratio $\Gamma = \frac{1}{n}\sum_{i=1}^{n}\frac{E_i - N_i}{E_i}$ with 50 trials, uniform distribution.

Conclusions

Summary

- Robots can form different types of geometric formations, including the repeated lattice patterns
- Algorithm scales reasonably well with increasing numbers of robot
- Algorithm is robust to the situation when some robots are removed from or with more robots added to the system

Future Work

- Improve the motion strategy
- Prove the convergence
- Nonholonomic constraints

Questions

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