# Constrained Geometric Approximation Approach for Robot Planning and Decentralized Formation Algorithm for Multi-Robot Systems

Yang Song, Jason M. O'Kane
song24@email.sc.edu
jokane@cse.sc.edu

Dept. of Computer Science and Engineering
University of South Carolina

UNIVERSITY OF SOUTH CAROLINA

# Outlines

# Constrained Geometric Approximation




- **Goal**:
  For an extremely simple robot with:
  - computation limitations
  - moving and sensing uncertainties

  represent and reason about uncertainty in its own states efficiently.
- **Basic Idea**:
  Explicitly represent what the robot knows as an information state (*I-state*).
- **Intuition**:
  Accelerate time-consuming operations by maintaining only an overapproximation of the true *I-state*, and constraining this approximation to have a simple geometric form.

SRV-1 Surveyor Robot

# Robot Model



Assume that current real state of the robot could not be observed directly. The robot could maintain an *I-state* $\eta_k$, to make its decisions.

- Robot state at stage $k$: $x_k \in X$,
- State transition function: $F(x_k, u_k)$.
- Robot action at stage $k$: $u_k$.
- Information state (*I-state*) at stage $k$: $\eta_k$ is a set of possible states at stage $k$

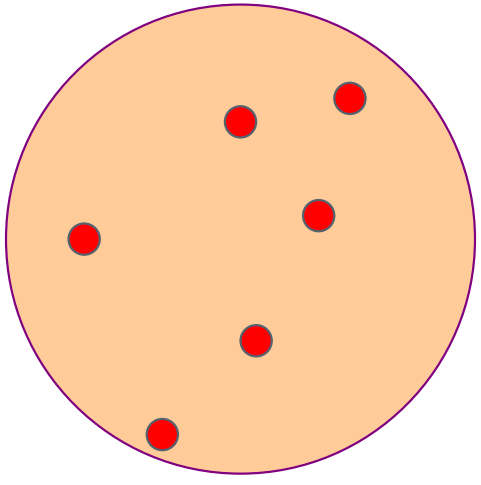


Figure: I-state $\eta_k$ contains all possible states

# Prior Work




- Prior research done by (B. Tovar and S. M. LaValle) and (J. van den Berg, P. Abbeel, and K. Goldberg) used probabilistic representations for planning
- Prior work by the J.O'Kane has used preliminary versions of the constrained geometric approximation method using specific, fixed range spaces.

New contributions

1. A careful formulation of the operations in the range space $\mathcal{R}$.
2. Algorithms for double-rectangle range space $\mathcal{R}_{drect}$.
3. A series of experiments for effectiveness comparison of different range spaces.

# Outlines

# Range Space



## Definition

**A range space** $\mathcal{R} \subseteq \mathcal{I}$ is a set of I-states, contains approximation of *I-states*, $A(\eta_k) \in \mathcal{R}$, equipped with two operations:

1. An *approximate observation update function* $O : \mathcal{R} \times Y \to \mathcal{R}$, such that if $\eta_k \subseteq A(\eta_k)$, then

$$\eta_k \cap H(y_k) \subseteq O(A(\eta_k), u_k)$$

2. An *approximate action update function* $T : \mathcal{R} \times U \to \mathcal{R}$, such that if $\eta_k \subseteq A(\eta_k)$, then

$$\bigcup_{x_k \in \eta_k} F(x_k, u_k) \subseteq T(A(\eta_k), u_k)$$

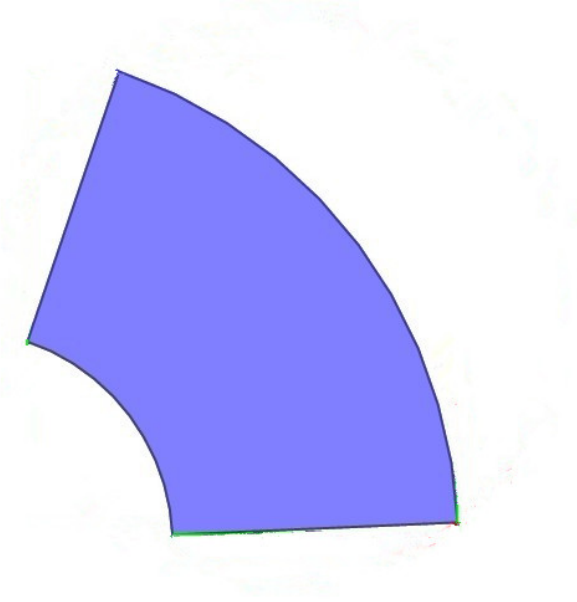


Figure: An *I-state* $\eta_k$

# Range Space



## Definition

**A range space** $\mathcal{R} \subseteq \mathcal{I}$ is a set of I-states, contains approximation of *I-states*, $A(\eta_k) \in \mathcal{R}$, equipped with two operations:

1. An *approximate observation update function* $O : \mathcal{R} \times Y \to \mathcal{R}$, such that if $\eta_k \subseteq A(\eta_k)$, then

$$\eta_k \cap H(y_k) \subseteq O(A(\eta_k), u_k)$$

2. An *approximate action update function* $T : \mathcal{R} \times U \to \mathcal{R}$, such that if $\eta_k \subseteq A(\eta_k)$, then

$$\bigcup_{x_k \in \eta_k} F(x_k, u_k) \subseteq T(A(\eta_k), u_k)$$

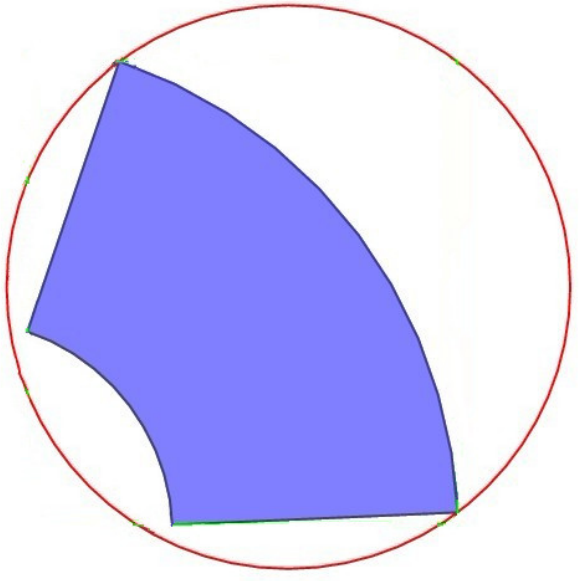


Figure: Disk overapproximation $A(\eta_k) \in \mathcal{R}_{disk}$ in red.

# Range Space



## Definition

**A range space** $\mathcal{R} \subseteq \mathcal{I}$ is a set of I-states, contains approximation of *I-states*, $A(\eta_k) \in \mathcal{R}$, equipped with two operations:

1. An *approximate observation update function* $O : \mathcal{R} \times Y \to \mathcal{R}$, such that if $\eta_k \subseteq A(\eta_k)$, then

$$\eta_k \cap H(y_k) \subseteq O(A(\eta_k), u_k)$$

2. An *approximate action update function* $T : \mathcal{R} \times U \to \mathcal{R}$, such that if $\eta_k \subseteq A(\eta_k)$, then

$$\bigcup_{x_k \in \eta_k} F(x_k, u_k) \subseteq T(A(\eta_k), u_k)$$

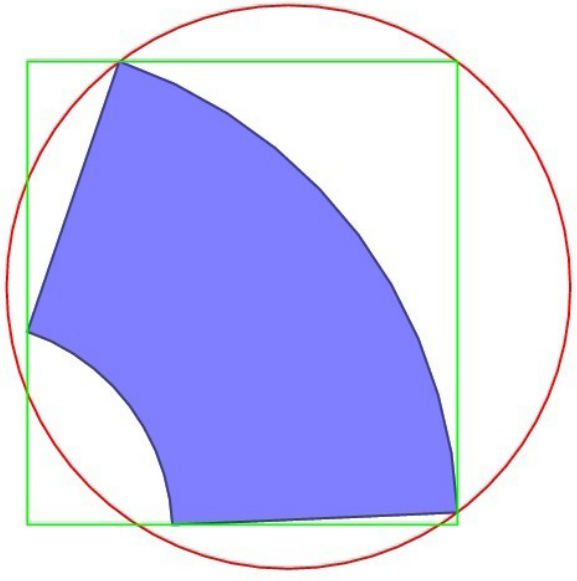
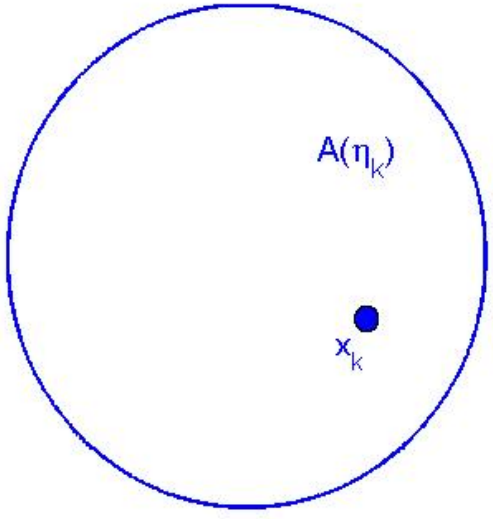


Figure: Rectangle overapproximation $A(\eta_k) \in \mathcal{R}_{rect}$.

# Observation Update in $\mathcal{R}_{disk}$



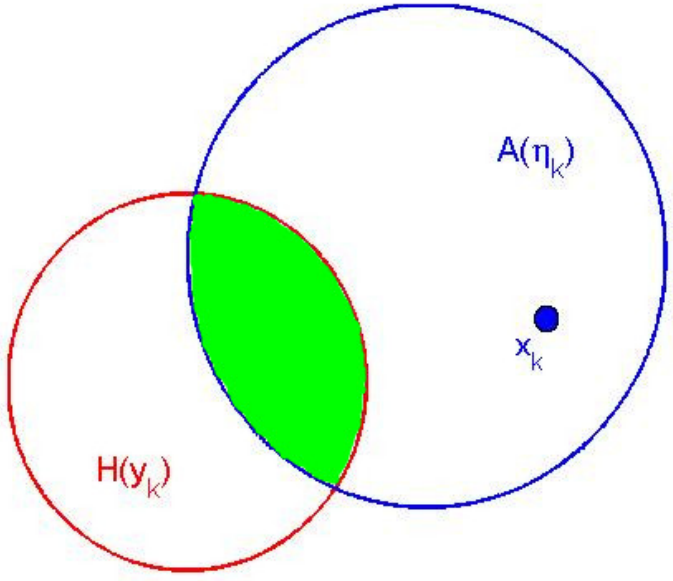
Approximated I-state $A(\eta_k) \in \mathcal{R}_{disk}$, where $x_k$ denotes the real state but unknown to robot.



# Observation Update in $\mathcal{R}_{disk}$



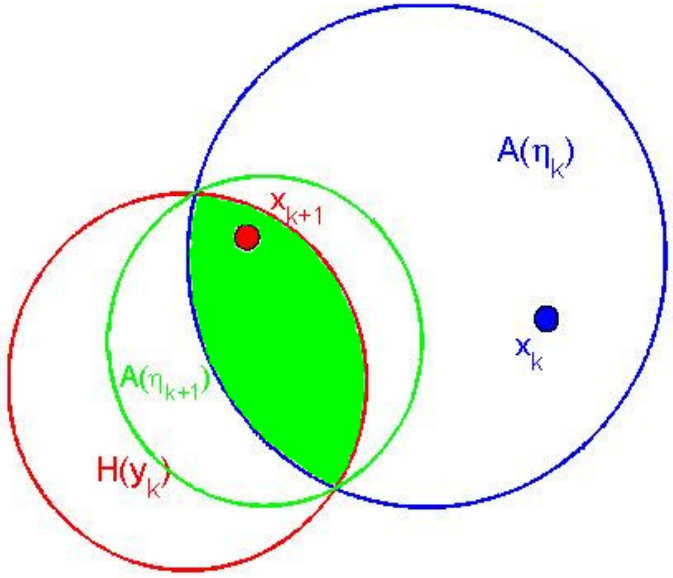
Approximated I-state $A(\eta_k) \in \mathcal{R}_{disk}$ intersects with observation preimage $H(y_k)$ .



# Observation Update in $\mathcal{R}_{disk}$



The green region of intersection is the updated I-state $\eta_{k+1}$, and the green disk is the approximation $A(\eta_{k+1})$ of $\eta_{k+1}$.



# Observation Update in $\mathcal{R}_{rect}$

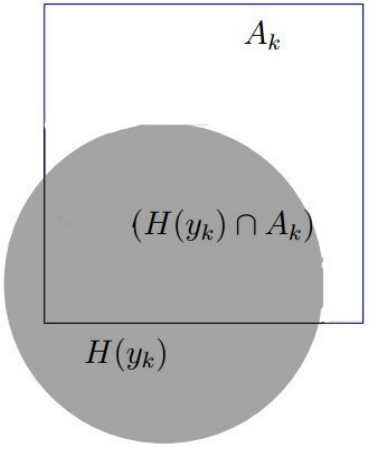



## Definition

*AABB*($S$) : For any compact set $S \subset \mathbb{R}^2$, let *AABB*($S$) denote the its smallest "axis-aligned bounding box."

In $\mathcal{R}_{rect}$, computing approximate observation update function $O_{rect}$ takes $O(1)$ time:

$$O_{rect}(A_k, y_k) = AABB(H(y_k) \cap A(\eta_k)), A_k = A(\eta_k) \in \mathcal{R}_{rect}$$



# Observation Update in $\mathcal{R}_{rect}$

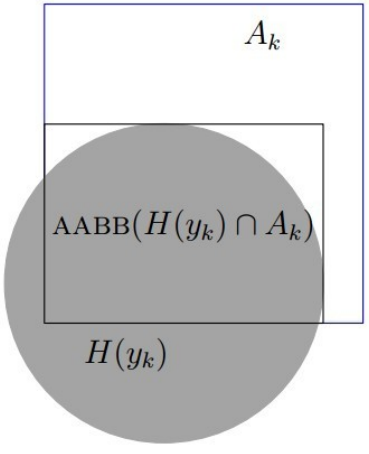



## Definition

*AABB*$(S)$ : For any compact set $S \subset \mathbb{R}^2$, let *AABB*$(S)$ denote the its smallest "axis-aligned bounding box."

In $\mathcal{R}_{rect}$, computing approximate observation update function $O_{rect}$ takes $O(1)$ time:

$$O_{rect}(A_k, y_k) = AABB(H(y_k) \cap A(\eta_k)), A_k = A(\eta_k) \in \mathcal{R}_{rect}$$



# Action Update in $\mathcal{R}_{rect}$



In $\mathcal{R}_{rect}$, computing approximate action update function $T_{rect}$ :

$$T_{rect}(A(\eta_k), u_k) = AABB(X_{free} \cap [A(\eta_k) \oplus \{u_k\} \oplus AABB(\Theta(u_k))])$$

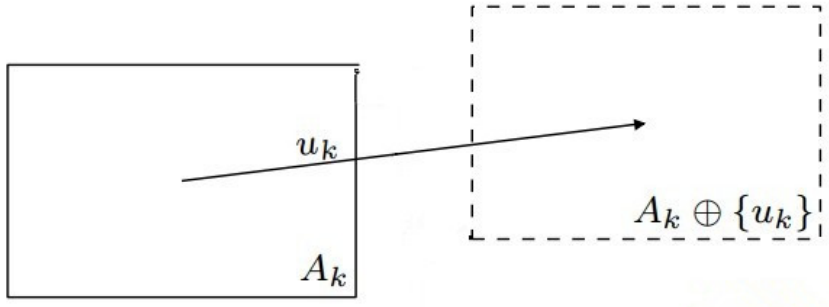



Figure: Transition of $A(\eta_k)$ given action $u_k$, $\oplus$ is Minkowski addition of two sets.

# Action Update in $\mathcal{R}_{rect}$



In $\mathcal{R}_{rect}$, computing approximate action update function $T_{rect}$ :

$$T_{rect}(A(\eta_k), u_k) = AABB(X_{free} \cap [A(\eta_k) \oplus \{u_k\} \oplus AABB(\Theta(u_k))])$$

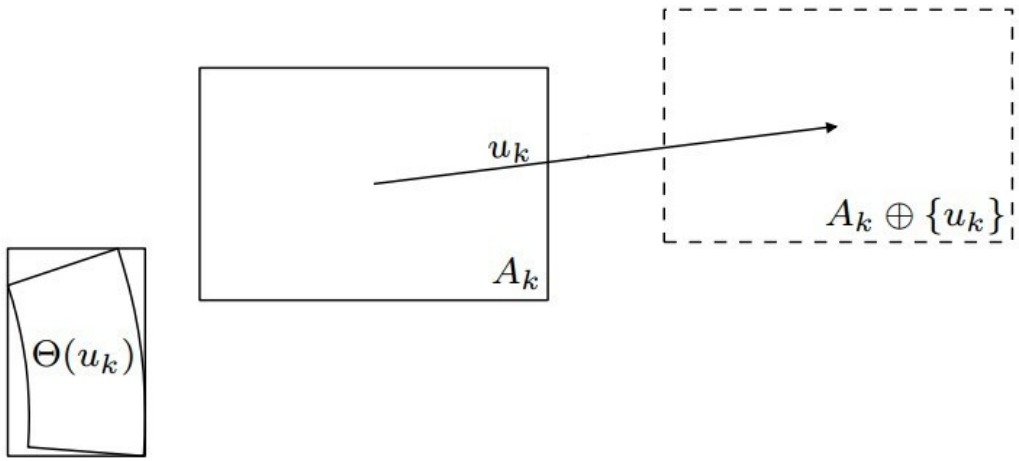



Figure: Consider approximation of the bounded motion uncertainty $\Theta(u_k)$.

# Action Update in $\mathcal{R}_{rect}$



In $\mathcal{R}_{rect}$, computing approximate action update function $T_{rect}$ :

$$T_{rect}(A(\eta_k), u_k) = AABB(X_{free} \cap [A(\eta_k) \oplus \{u_k\} \oplus AABB(\Theta(u_k))])$$

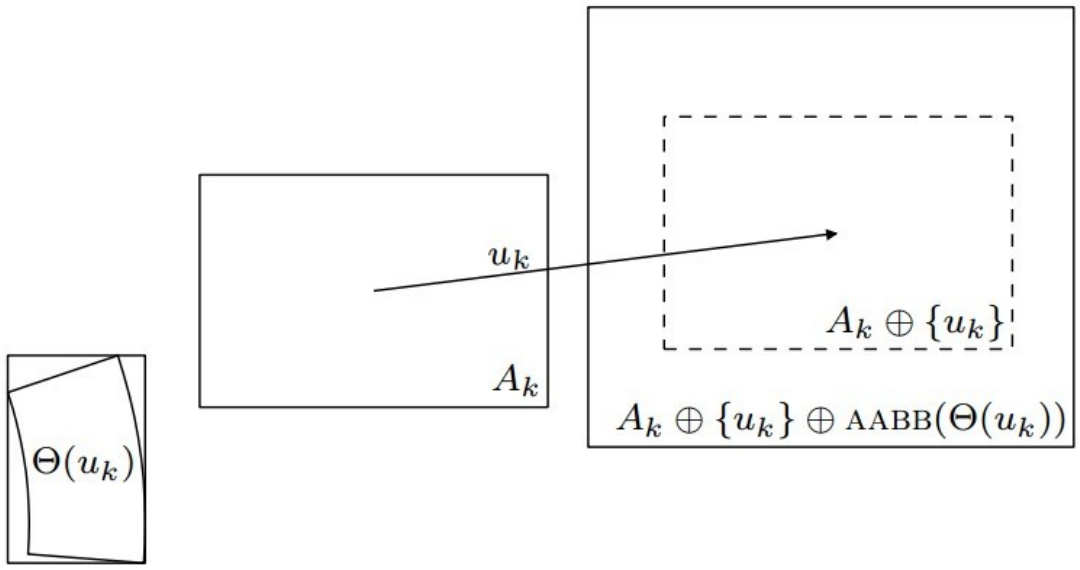



Figure: Find Minkowski sum of bounded noise and approximation transition

# Action Update in $\mathcal{R}_{rect}$



In $\mathcal{R}_{rect}$, computing approximate action update function $T_{rect}$ :

$$T_{rect}(A(\eta_k), u_k) = AABB(X_{free} \cap [A(\eta_k) \oplus \{u_k\} \oplus AABB(\Theta(u_k))])$$

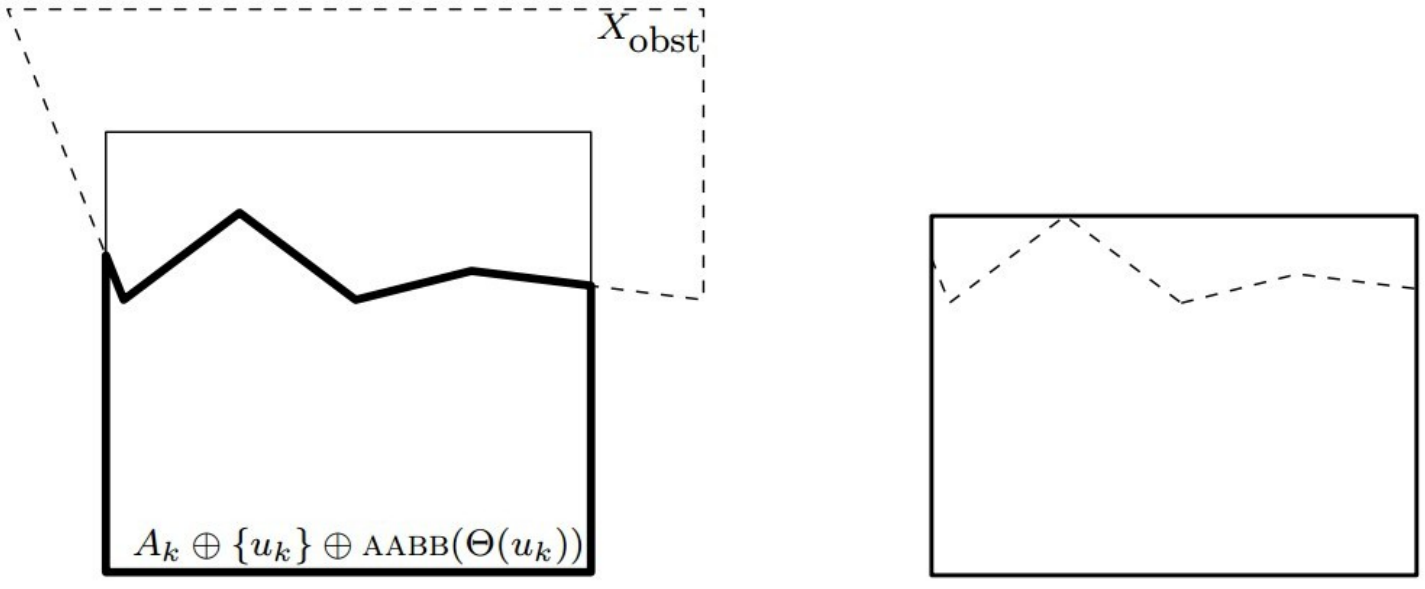



Figure: If there is obstacles, intersect with $X_{free}$ first and then find the bounding box of the intersection.

# Double-Rectangle approximated I-state



- For better overapproximation quality for non-convex *I-states*, we proposed a more expressive range space of *double rectangles*:

$$\mathcal{R}_{drect} = \{R_1 \cup R_2 \mid R_1, R_2 \in \mathcal{R}_{rect}\} \quad (1)$$

- Aims to improve the approximation quality.

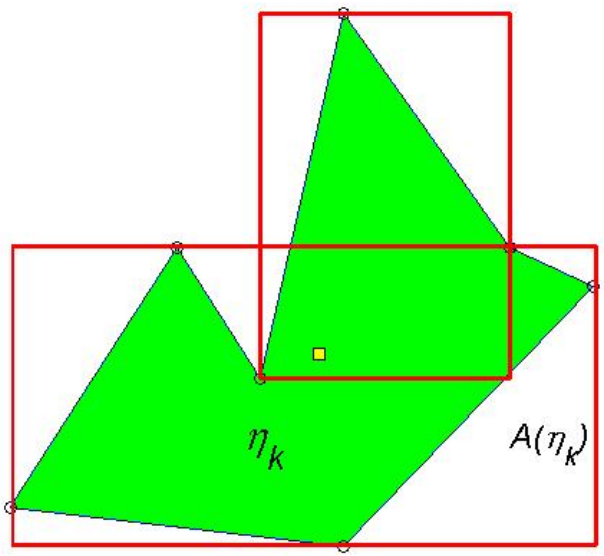


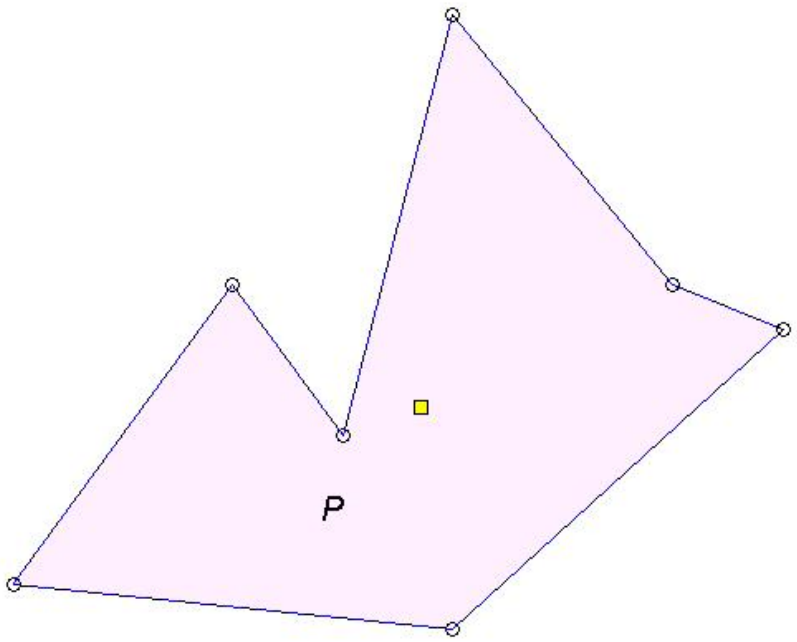

Figure: Non-convex *I-state*

# DRAP Algorithm



"Double Rectangle Around Polygon" (**DRAP**) algorithm:

- input is a $n$-edge polygonal region of the plane
- output is a small double rectangle containing that polygon
- run time: $O(n^3)$

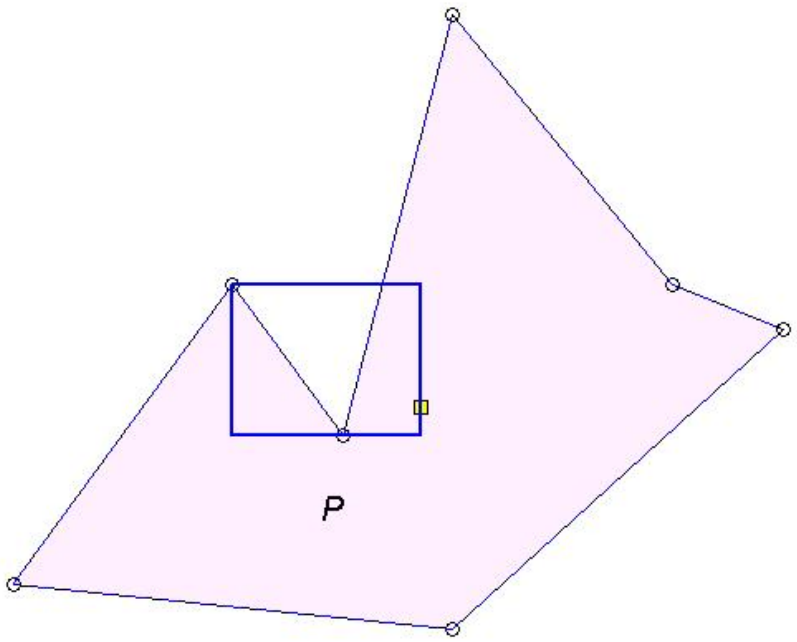


# DRAP Algorithm



"Double Rectangle Around Polygon" (**DRAP**) algorithm:

- input is a $n$-edge polygonal region of the plane
- output is a small double rectangle containing that polygon
- run time: $O(n^3)$

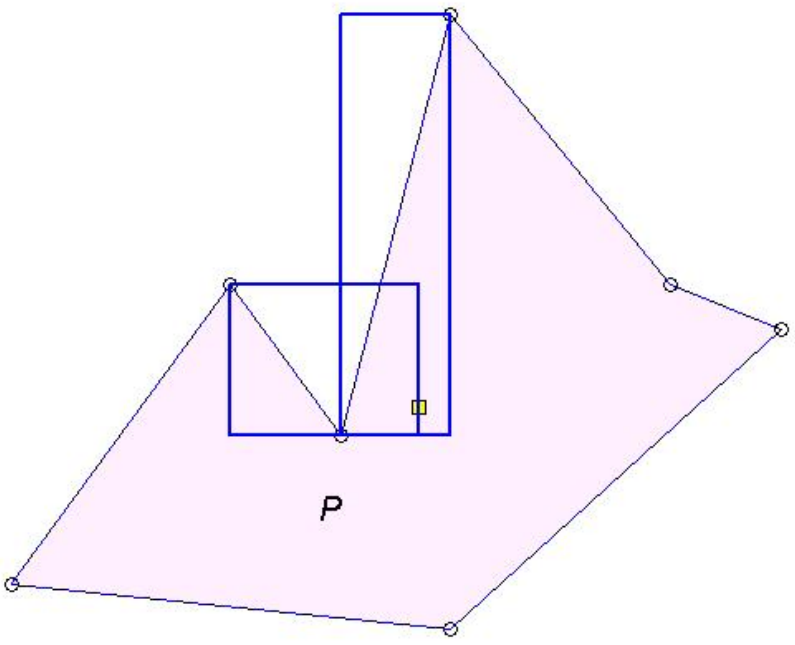


# DRAP Algorithm



"Double Rectangle Around Polygon" (**DRAP**) algorithm:

- input is a $n$-edge polygonal region of the plane
- output is a small double rectangle containing that polygon
- run time: $O(n^3)$

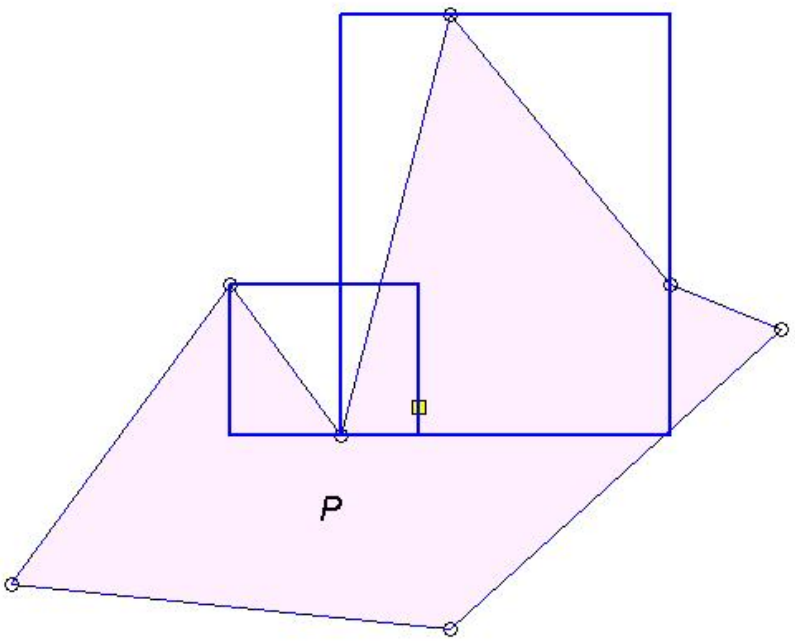


# DRAP Algorithm



"Double Rectangle Around Polygon" (**DRAP**) algorithm:

- input is a $n$-edge polygonal region of the plane
- output is a small double rectangle containing that polygon
- run time: $O(n^3)$

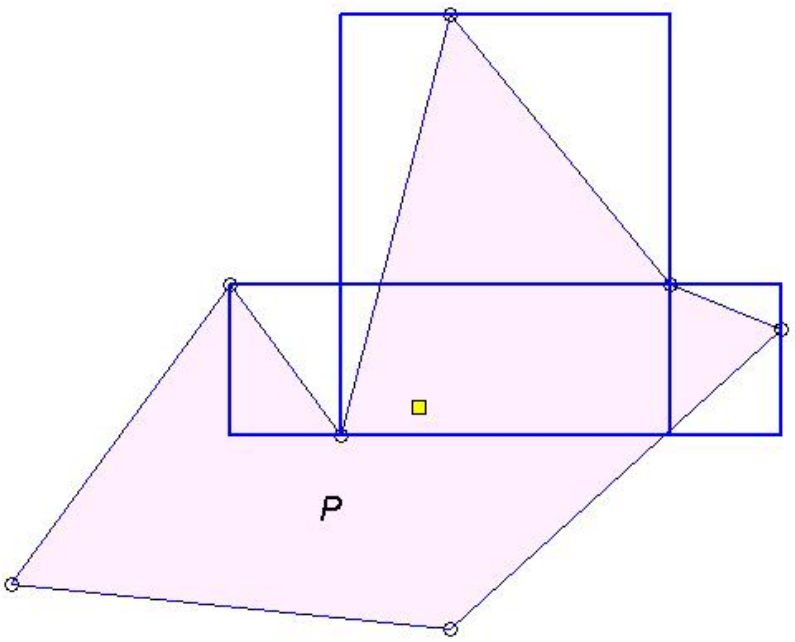


# DRAP Algorithm



"Double Rectangle Around Polygon" (**DRAP**) algorithm:

- input is a $n$-edge polygonal region of the plane
- output is a small double rectangle containing that polygon
- run time: $O(n^3)$

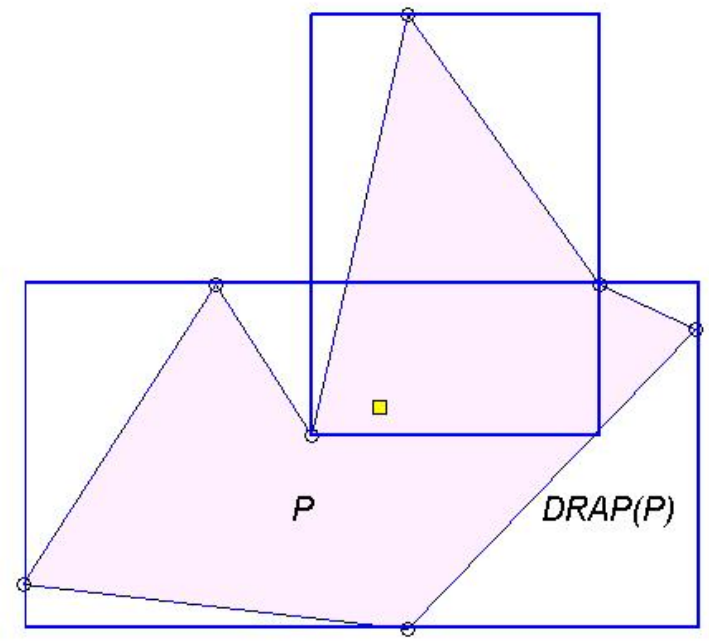


# DRAP Algorithm



"Double Rectangle Around Polygon" (**DRAP**) algorithm:

- input is a $n$-edge polygonal region of the plane
- output is a small double rectangle containing that polygon
- run time: $O(n^3)$



# Outlines

# Experiments in Various Environments




Comparison with using the true I-state, we conducted experiments using 3 environments, and 3 range spaces $\mathcal{R}_{disk}$, $\mathcal{R}_{rect}$, and $\mathcal{R}_{drect}$.
**ASSUMPTIONS:**

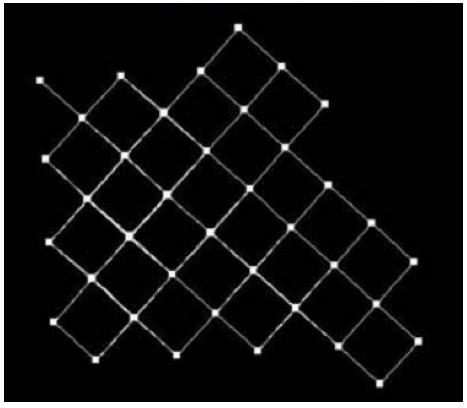
- Robot is guided by centroid point of the approximated *I-state*
- Robot can detect presence but not distance to the settled waypoints
- Landmarks are pseudo-randomly generated (Number Matters)
- Initial I-state $\eta_0$ is given.

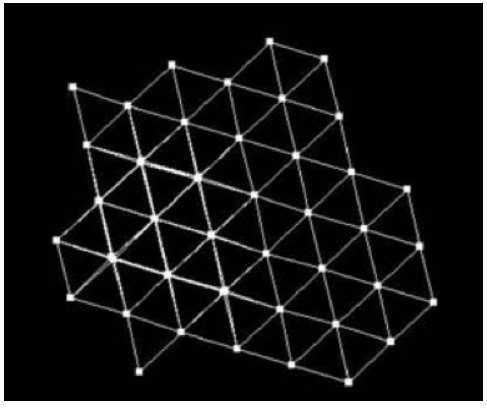
# Simulation
## Navigation in $\mathcal{R}_{dbrect}$

# Related Work
## Formation using virtual force

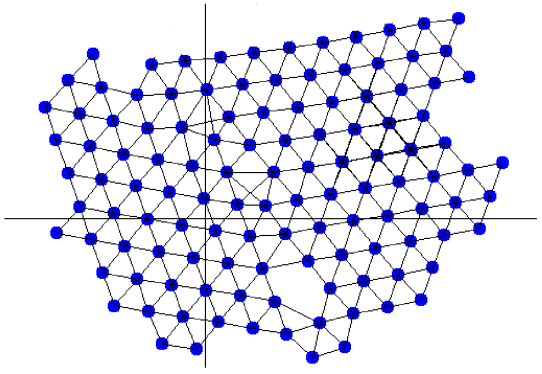


W. Spears, D. Spears, J. Hamann and R. Heil, 2004

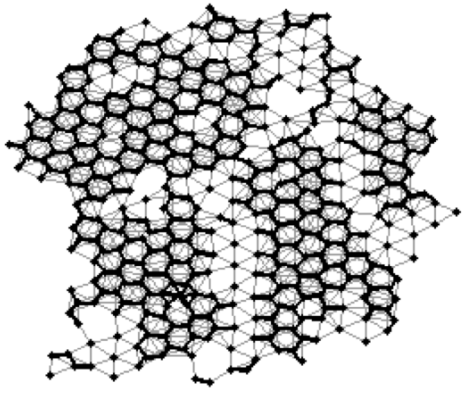




I. Navarro, J. Pugh, A. Martinoli, and F. Matia, 2008



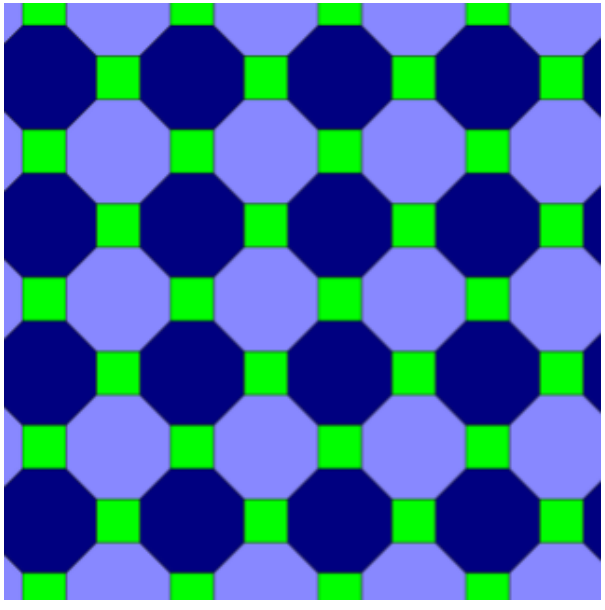
S. Prabhu, W. Li, J. McLurkin, 2012

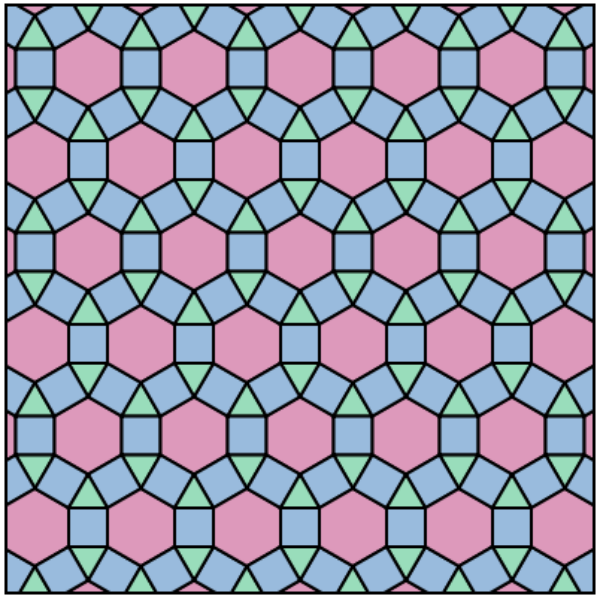


# Motivation



Question: How to use one algorithm to generate various (repeating) lattice pattern formations?

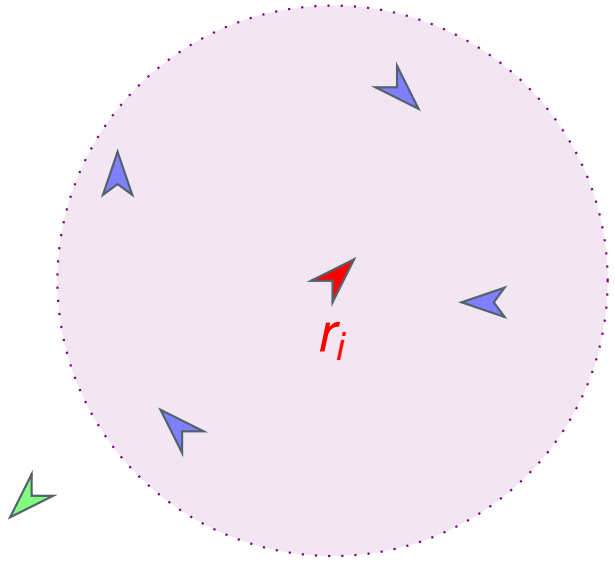




# Robot Model



- Differential Drive robots.
- Each robot has an unique **ID**.
- Use a vector $p = [x, y, \theta]^T$ to represent robot's **pose**.
- Each robot has a **range** within which it can sense and communicate with other robots.
- Each robot gets **observation** of its neighbors' IDs and relative poses in its body frame.



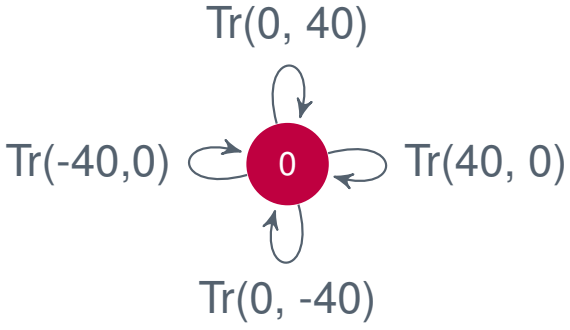

Robot $r_i$ has 4 neighbors

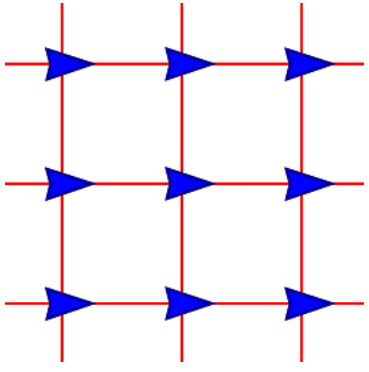
# Input: Lattice Graph



**Definition**

A **lattice graph** is a strongly connected directed multigraph in which each edge $e$ is labeled with a rigid body transformation $T(e)$ and each $v \xrightarrow{T(e)} w$ has an inverse edge $w \xrightarrow{T(e)^{-1}} v$.



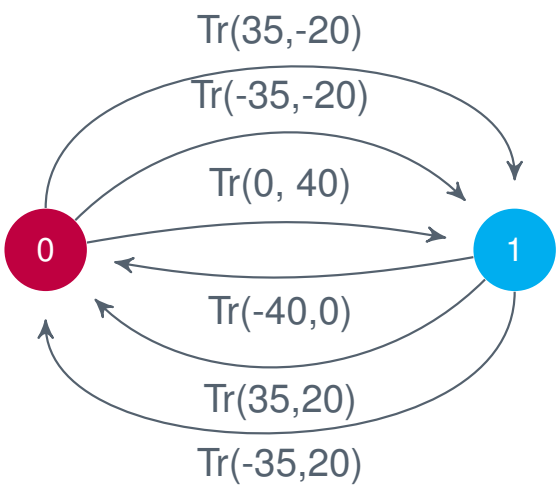



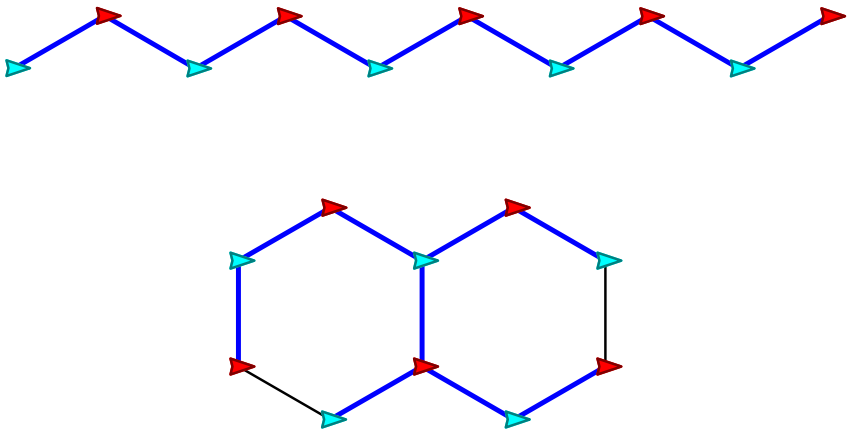
# Role Function



## Definition

Given a lattice graph $G = (V, E)$ and a set of robots $R = \{r_1, \ldots, r_n\}$, $R$ **satisfies** $G$ if there exists a role function $f : R \to V$ that preserves the neighborhood structure of $G$.
Specifically, for any $i$ and $j$, if $r_i$ and $r_j$ are neighbors, there must exist an edge $e_{ij} : f(r_i) \longrightarrow f(r_j)$ in $E$, such that $T(r_j) = T(r_i)T(e_{ij})$.



# Algorithm




## General Description

Robot broadcasts message containing its

- authority
- matching.

1. Form tree structure.
2. Use tree structure to computer local task assignment.
3. Make movement decision.

# Authority



## Define authority and comparison operator

An **authority** is an ordered list of robot IDs

$$(\mathrm{id}_1, \ldots, \mathrm{id}_k)$$

The first ID in the list, $\mathrm{id}_1$ is called the **root** ID. The final ID in the list, $\mathrm{id}_k$ is called the **sender** ID.

Authority $A_2$ is **higher than** $A_1$ if:

- root ID of $A_2 >$ root ID of $A_1$, or
- length of $A_2 <$ length of $A_1$ if they have the same root, or
- sender ID of $A_2 >$ sender ID of $A_1$ if they have the same root and length.

# 1.Construct Authority Tree

Decide to be root or descendant



The robots use these authorities to establish a collection of authority trees

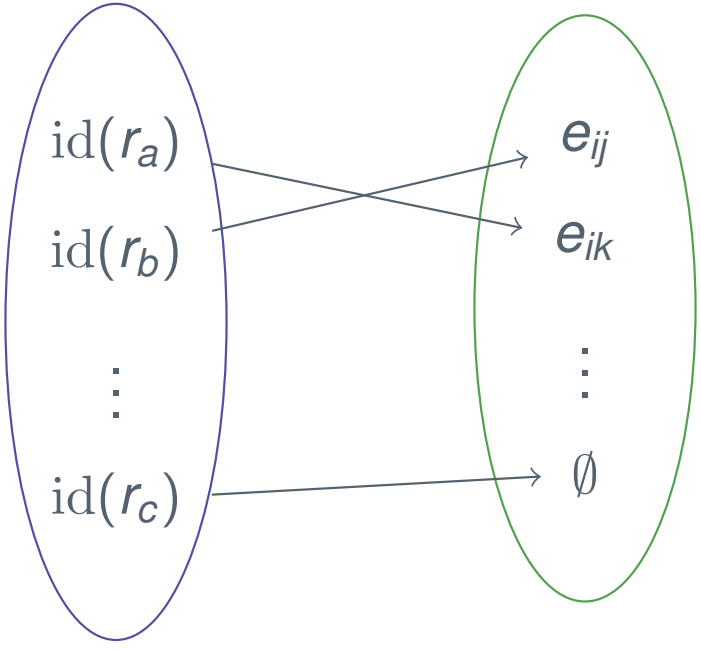
1. Discards any message in which the authority contains its own ID.
2. Forms an authority containing only its own ID, compares it with the authorities of remaining messages and selects the highest authority.

- If its authority is the highest, then it is a root;
- Otherwise, it selects the one who sends the highest authority as its parent. Append its own ID to the highest authority to create its own authority.

# Matching



A **matching** for a robot is a function $\eta : \{\mathrm{id}(r_a), \mathrm{id}(r_b), ...\} \rightarrow \{\emptyset, e_{ij}, e_{ik}, ...\}$ that associates each neighbor ID with either a lattice graph edge from its role vertex or with the null value $\emptyset$.



# 2. Local Task Assignment

## Hungarain Algorithm



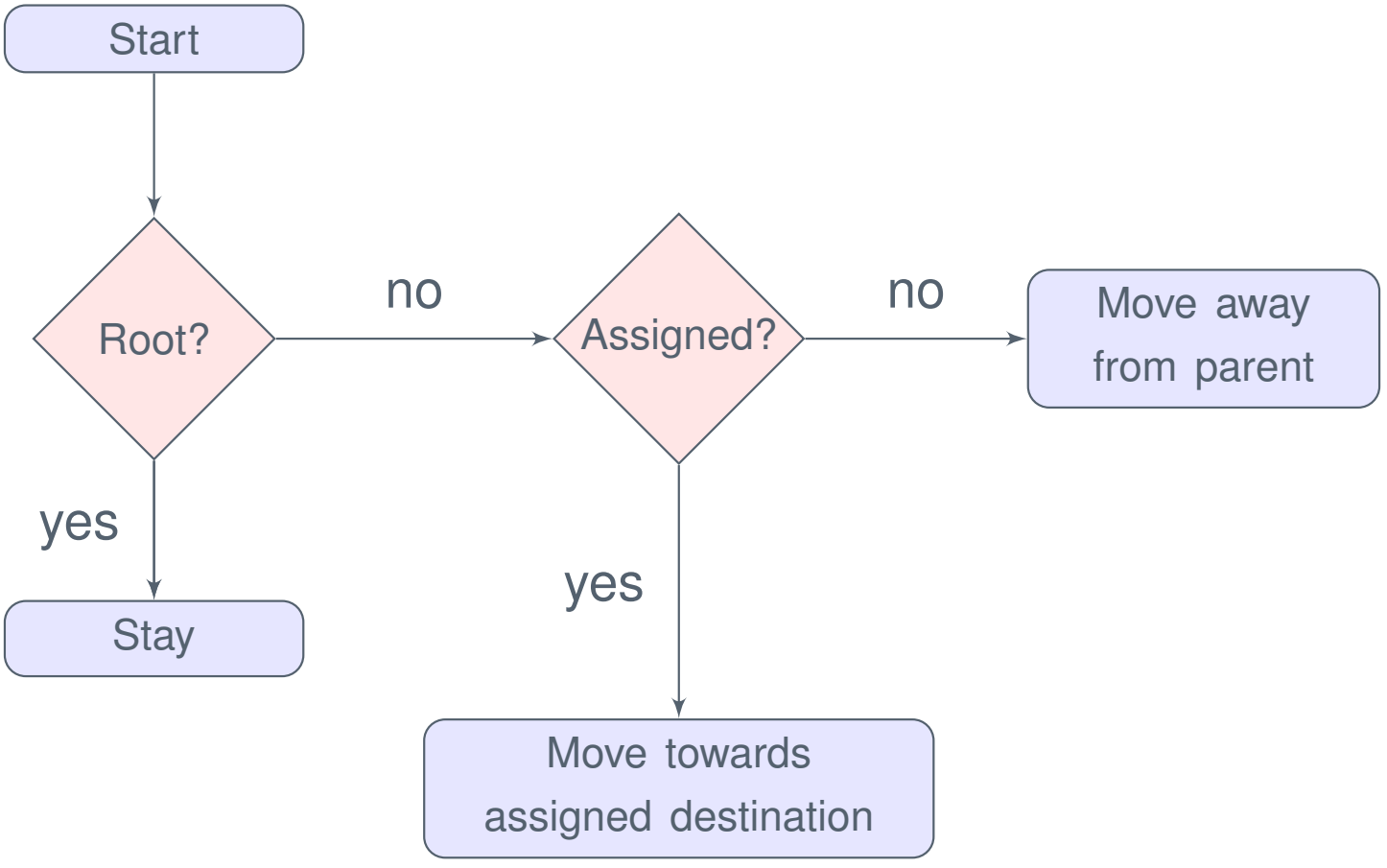

To compute an optimal matching of a robot with $N$ neighbors and $E$ out-going edges of its role in the lattice graph, define a weight matrix of size $N \times \max(N, E)$ and apply Hungarian Algorithm (Harold W. Kuhn, 1955).

1. Each row corresponds to a neighbor;
2. Each column corresponds to an out-going edge of robot's role or a null value $\emptyset$.
3. The entries of the matrix are the Euclidean distance between current position of each neighbor and the desired position if matched with a lattice graph edge.

5 neighbors, 4 out-going edges.

# 3. Robot Movement Strategy





# Bounded Movement




## Goal: let descendant stay in the range of its parent

- Within the set $O$ (Red circle), the parent is guaranteed to get observation at next stage.
- Descendant can reach anywhere in set $P$ (blue circle) at next stage.
- The real destination for descendant is the closest point on the boundary of the intersection ($O \cap P$) to the assigned destination.

# Simulation

Octagon-square formation with 100 robots

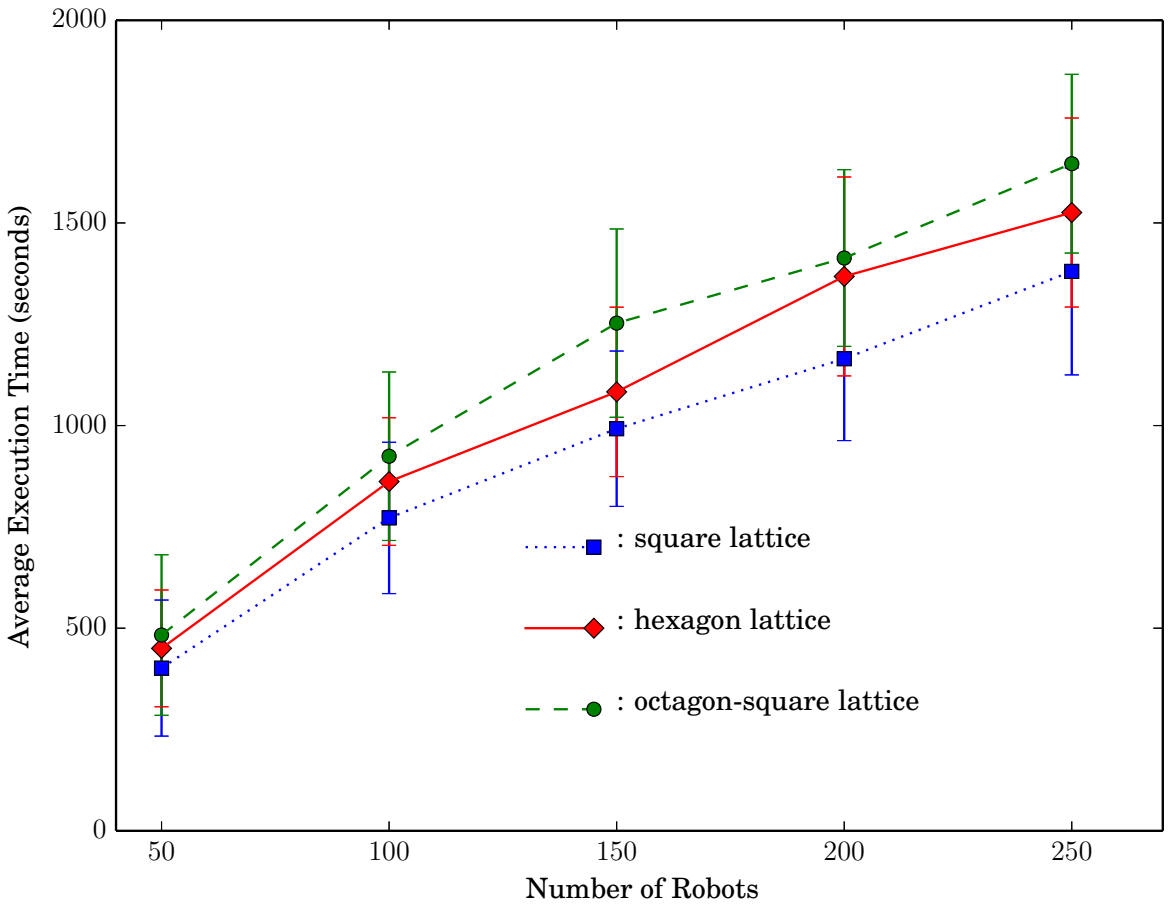
# Simulation

Hexagon formation with 100 robots

# Experiment Results

## on three kinds of repeated lattice patterns

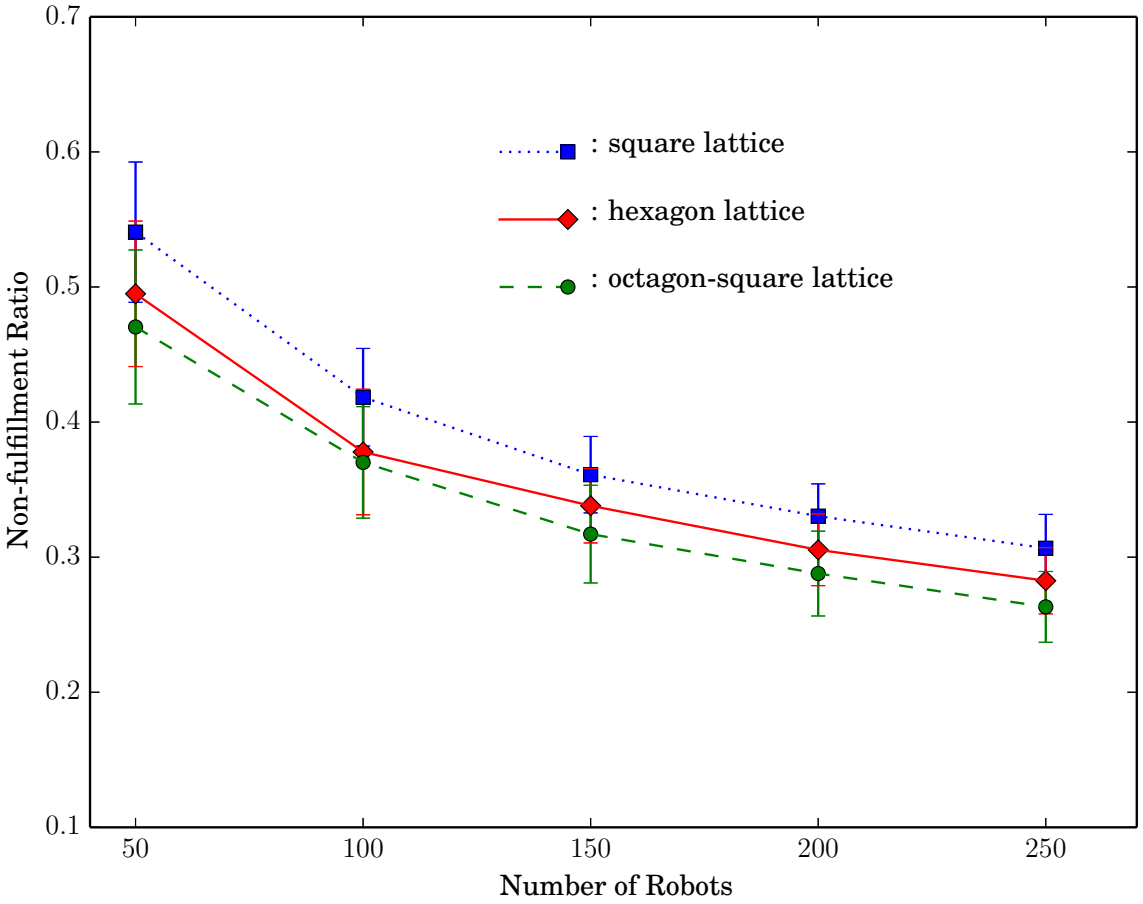





Average time to the static position with 50 trials, uniform distribution.

# Experiment Results
## on three kinds of repeated lattice patterns






Average non-fulfillment ratio $\Gamma = \frac{1}{n}\sum_{i=1}^{n}\frac{E_i - N_i}{E_i}$ with 50 trials, uniform distribution.

# Conclusions




## Summary

- Robots can form different types of geometric formations, including the repeated lattice patterns
- Algorithm scales reasonably well with increasing numbers of robot
- Algorithm is robust to the situation when some robots are removed from or with more robots added to the system

## Future Work

- Improve the motion strategy
- Prove the convergence
- Nonholonomic constraints