Constrained Geometric Approximation Approach for Robot Planning and Decentralized Formation Algorithm for Multi-Robot Systems

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Constrained Geometric Approximation

- **Goal**:
  For an extremely simple robot with:
  - computation limitations
  - moving and sensing uncertainties

  represent and reason about uncertainty in its own states efficiently.
- **Basic Idea**:
  Explicitly represent what the robot knows as an information state (*I-state*).
- **Intuition**:
  Accelerate time-consuming operations by maintaining only an overapproximation of the true *I-state*, and constraining this approximation to have a simple geometric form.

SRV-1 Surveyor Robot

Robot Model

Assume that current real state of the robot could not be observed directly. The robot could maintain an *I-state* η_k, to make its decisions.

- Robot state at stage k: $x_k \in X$,
- State transition function: $F(x_k, u_k)$.
- Robot action at stage k: u_k.
- Information state (*I-state*) at stage k: η_k is a set of possible states at stage k

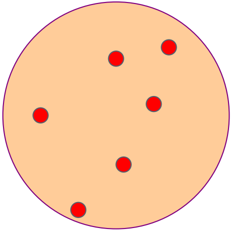


Figure: I-state η_k contains all possible states

Prior Work

- Prior research done by (B. Tovar and S. M. LaValle) and (J. van den Berg, P. Abbeel, and K. Goldberg) used probabilistic representations for planning
- Prior work by the J.O'Kane has used preliminary versions of the constrained geometric approximation method using specific, fixed range spaces.

New contributions

1. A careful formulation of the operations in the range space $\mathcal{R}$.
2. Algorithms for double-rectangle range space $\mathcal{R}_{direct}$.
3. A series of experiments for effectiveness comparison of different range spaces.

Range Space

Definition

A range space $\mathcal{R} \subseteq \mathcal{I}$ is a set of I-states, contains approximation of *I-states*, $A(\eta_k) \in \mathcal{R}$, equipped with two operations:

1. An *approximate observation update function* $O : \mathcal{R} \times Y \to \mathcal{R}$, such that if $\eta_k \subseteq A(\eta_k)$, then

$$\eta_k \cap H(y_k) \subseteq O(A(\eta_k), u_k)$$

2. An *approximate action update function* $T : \mathcal{R} \times U \to \mathcal{R}$, such that if $\eta_k \subseteq A(\eta_k)$, then

$$\bigcup_{x_k \in \eta_k} F(x_k, u_k) \subseteq T(A(\eta_k), u_k)$$

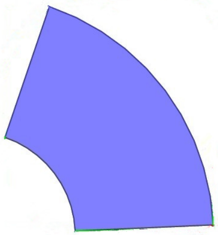
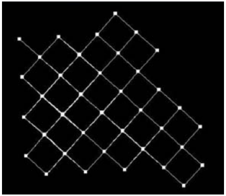


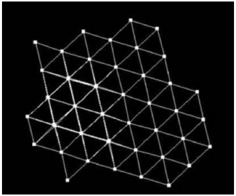
Figure: The blue region denotes the *I-state* η_k

Related Work

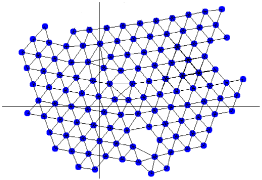
Formation using virtual force

W. Spears, D. Spears, J. Hamann and R. Heil, 2004

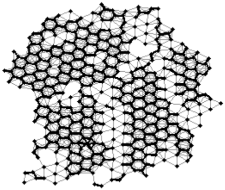




I. Navarro, J. Pugh, A. Martinoli, and F. Matia, 2008

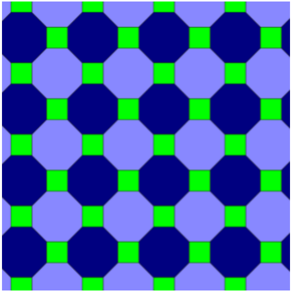


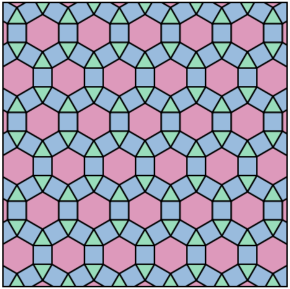
S. Prabhu, W. Li, J. McLurkin, 2012



Motivation

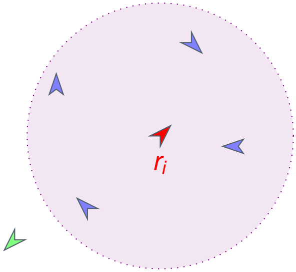
Question: How to use one algorithm to generate various (repeating) lattice pattern formations?





Robot Model

- Differential Drive robots.
- Each robot has an unique **ID**.
- Use a vector $p = [x, y, \theta]^T$ to represent robot's **pose**.
- Each robot has a **range** within which it can sense and communicate with other robots.
- Each robot gets **observation** of its neighbors' IDs and relative poses in its body frame.

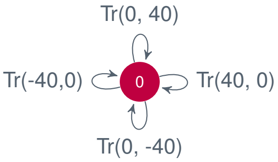


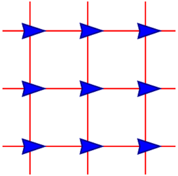
Robot r_i has 4 neighbors

Input: Lattice Graph

Definition

A **lattice graph** is a strongly connected directed multigraph in which each edge e is labeled with a rigid body transformation $T(e)$ and each $v \xrightarrow{T(e)} w$ has an inverse edge $w \xrightarrow{T(e)^{-1}} v$.



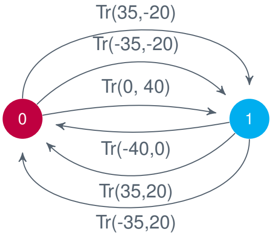


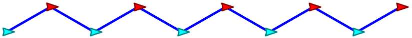
Role Function

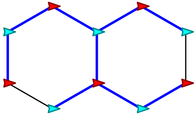
Definition

Given a lattice graph $G = (V, E)$ and a set of robots $R = \{r_1, \dots, r_n\}$, R **satisfies** G if there exists a role function $f : R \to V$ that preserves the neighborhood structure of G.

Specifically, for any i and j, if r_i and r_j are neighbors, there must exist an edge $e_{ij} : f(r_i) \longrightarrow f(r_j)$ in E, such that $T(r_j) = T(r_i)T(e_{ij})$.







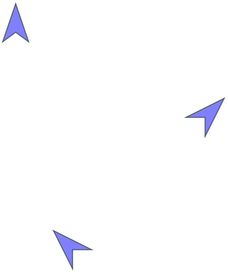
Algorithm

General Description

Robot broadcasts message containing its

- authority
- matching.

1. Form tree structure.
2. Use tree structure to computer local task assignment.
3. Make movement decision.



Define authority and comparison operator

An **authority** is an ordered list of robot IDs

$$(\mathrm{id}_1, \ldots, \mathrm{id}_k)$$

The first ID in the list, id_1 is called the **root** ID. The final ID in the list, id_k is called the **sender** ID.

Authority A_2 is **higher than** A_1 if:

- root ID of $A_2 >$ root ID of A_1, or
- length of $A_2 <$ length of A_1 if they have the same root, or
- sender ID of $A_2 >$ sender ID of A_1 if they have the same root and length.

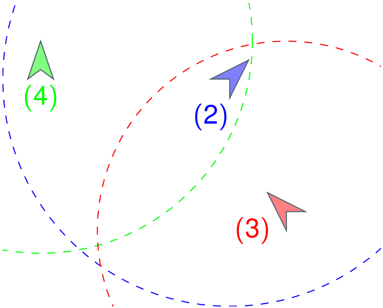
1.Construct Authority Tree

Decide to be root or descendant

The robots use these authorities to establish a collection of authority trees

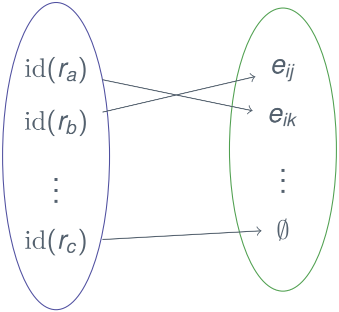
1. Discards any message in which the authority contains its own ID.
2. Forms an authority containing only its own ID, compares it with the authorities of remaining messages and selects the highest authority.

- If its authority is the highest, then it is a root;
- Otherwise, it selects the one who sends the highest authority as its parent. Append its own ID to the highest authority to create its own authority.



Matching

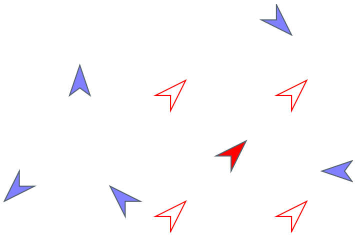
A **matching** for a robot is a function $\eta : \{\mathrm{id}(r_a), \mathrm{id}(r_b), ...\} \rightarrow \{\emptyset, e_{ij}, e_{ik}, ...\}$ that associates each neighbor ID with either a lattice graph edge from its role vertex or with the null value $\emptyset$.



2. Local Task Assignment
Hungarain Algorithm

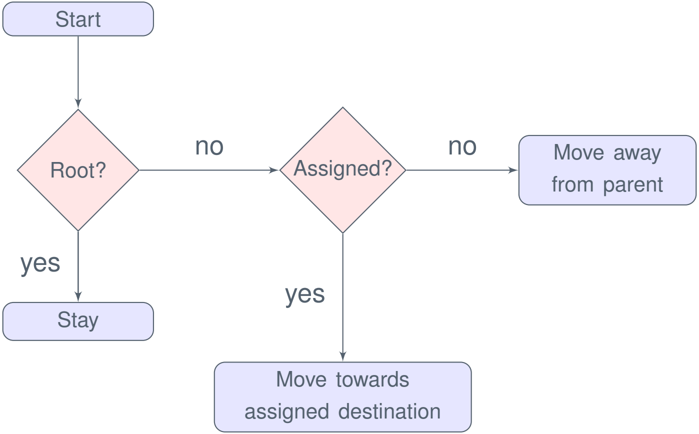
To compute an optimal matching of a robot with N neighbors and E out-going edges of its role in the lattice graph, define a weight matrix of size $N \times \max(N, E)$ and apply Hungarian Algorithm (Harold W. Kuhn, 1955).

1. Each row corresponds to a neighbor;
2. Each column corresponds to an out-going edge of robot's role or a null value $\emptyset$.
3. The entries of the matrix are the Euclidean distance between current position of each neighbor and the desired position if matched with a lattice graph edge.



5 neighbors, 4 out-going edges.

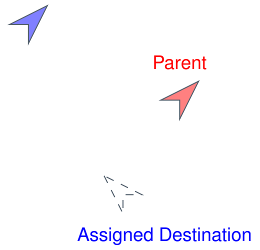
3. Robot Movement Strategy



Bounded Movement

Goal: let descendant stay in the range of its parent

- Within the set O (Red circle), the parent is guaranteed to get observation at next stage.
- Descendant can reach anywhere in set P (blue circle) at next stage.
- The real destination for descendant is the closest point on the boundary of the intersection ($O \cap P$) to the assigned destination.



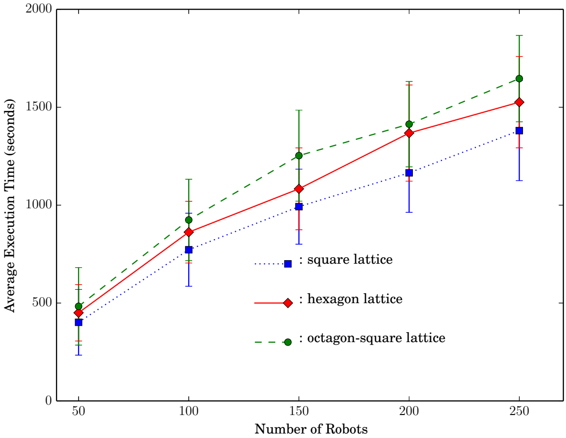
Simulation

Octagon-square formation with 100 robots

Simulation

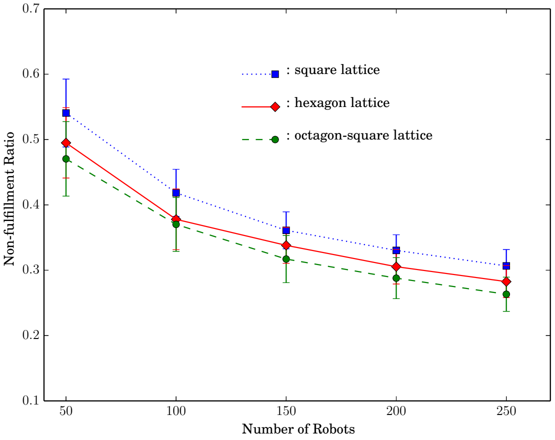
Hexagon formation with 100 robots

Experiment Results
on three kinds of repeated lattice patterns



Average time to the static position with 50 trials, uniform distribution.

Experiment Results
on three kinds of repeated lattice patterns



Average non-fulfillment ratio $\Gamma = \frac{1}{n}\sum_{i=1}^{n}\frac{E_i - N_i}{E_i}$ with 50 trials, uniform distribution.

Conclusions

Summary

- Robots can form different types of geometric formations, including the repeated lattice patterns
- Algorithm scales reasonably well with increasing numbers of robot
- Algorithm is robust to the situation when some robots are removed from or with more robots added to the system

Future Work

- Improve the motion strategy
- Prove the convergence
- Nonholonomic constraints

Questions