I. PROBLEM FORMULATION

In this paper, two coordinate systems are used: the body coordinate system and the world coordinate system. The three axes of the body coordinate system coincide with three axes of the inertial/magnetic sensor unit (accelerometers, gyroscopes and magnetic sensors). The world coordinate system is a local geographic frame with north-west-up (x-y-z) convention.

A rotation matrix $C_w^b \in SO(3)$ is used to represent the rotation relationship between the body coordinate system and the world coordinate system. Let $r_b \in R^3$ and $r_w \in R^3$ denote the same position where r_b is expressed in the body coordinate system and r_w is expressed in the world coordinate system. The two vectors are related by the rotation matrix C_w^b when two coordinate frames share the same origin:

$$r_b = C_w^b r_w.$$

The column vectors of C_w^b are denoted by $C_i \in R^{3\times 1}$:

$$C_w^b = \begin{bmatrix} C_1 & C_2 & C_3 \end{bmatrix}.$$

The kinematics of a rotation matrix is given by

$$\dot{C}_w^b = [\omega \times] C_w^b \tag{1}$$

where $\omega \in R^3$ is the body angular velocity.

Three sensors are used in this paper: accelerometer $y_a \in R^3$, gyroscope $y_g \in R^3$ and magnetic sensor $y_m \in R^3$. The sensor outputs are modeled as follows:

$$\begin{aligned} y_{a,k} &= C_w^b \tilde{g} + v_{a,k} \\ y_{m,k} &= C_w^b \tilde{m} + v_{m,k} \\ y_g(t) &= \omega(t) + v_g(t) \end{aligned} \tag{2}$$

where $v_a \in R^3$, $v_m \in R^3$ and $v_g \in R^3$ are sensor noises. It is assumed that the sensor noises are white Gaussian and their covariances are given by

$$\mathrm{E}\{v_{a,k} v'_{a,l}\} = r_a \delta_{k,l} I_3, \mathrm{E}\{v_g(t) v_g(s)'\} = r_g \delta(t-s) I_3$$
$$\mathrm{E}\{v_{m,k} v'_{m,l}\} = r_m \delta_{k,l} I_3.$$

where $r_a \in R$, $r_g \in R$ and $r_m \in R$ are positive scalars. $\delta(t)$ and $\delta_{k,l}$ are continuous and discrete time Dirac delta functions, respectively. Local gravitation vector $\tilde{g} \in R^3$ and magnetic field vector $\tilde{m} \in R^3$ are given by

$$\tilde{g} \triangleq \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}, \quad \tilde{m} \triangleq \begin{bmatrix} \cos\mu \\ 0 \\ \sin\mu \end{bmatrix} \tag{3}$$

where $g \in R$ is the magnitude of the gravitation and $\mu \in R$ is the magnetic dip angle.

Inserting (3) into (2), we have the following:

$$\begin{aligned} y_a &= gC_3 + v_a \\ y_m &= \cos\mu C_1 + \sin\mu C_3 + v_m. \end{aligned} \tag{4}$$

In (4), we note that C_2 term is not used. Thus the optimization problem is formulated, in which C_1 and C_3 are estimated. Once C_1 and C_3 are estimated, $C_2 \in R^3$ can be computed by

$$C_2 = C_3 \times C_1. \tag{5}$$

II. Attitude estimation problem as a convex optimization problem

In this section, the attitude estimation problem is formulated as a convex optimization problem. We assume that the initial discrete time index is 1 and the final discrete time index is N: for example, accelerometer output $y_{a,k}$ is available for $1 \leq k \leq N$.

$$\begin{gathered} \min_{C_{1,k},C_{3,k}} \sum_{k=1}^{N} \Big(\tfrac{1}{r_a} \|\tilde{y}_{a,k} - C_{3,k}g - d_{a,k}\|_2^2 + \\ \tfrac{1}{r_m} \|\tilde{y}_{m,k} - C_{1,k}\cos\mu - C_{3,k}\sin\mu - d_{m,k}\|_2^2 \Big) \\ + \sum_{k=1}^{N-1} \Big(\tfrac{1}{r_g T} \|C_{i,k+1} - \exp([y_g \times]T)C_{i,k}\|_2^2 \Big) \\ + \alpha \sum_{k=1}^{N} \left(\|d_{a,k}\|_1 + \|d_{m,k}\|_1 \right) \end{gathered} \tag{6}$$

subject to

$$\begin{bmatrix} C'_{1,k} \\ C'_{3,k} \end{bmatrix} \begin{bmatrix} C_{1,k} & C_{3,k} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The parameter α determines how large $d_{a,k}$ and $d_{m,k}$ could be. If α is large, large $d_{a,k}$ and $d_{m,k}$ values are discouraged.

An article containing the detailed algorithm is under submission to an academic journal and will be uploaded in `https://uouisl.wordpress.com/research/` upon acceptance.