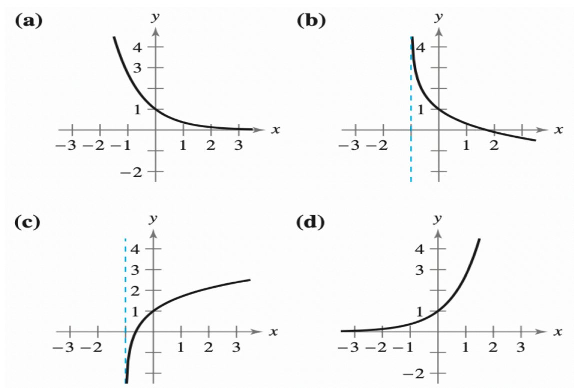
Exercises 1

1. Find equations of the lines passing through $(3, 5)$ and having the following characteristics.
   (a) Slope of $\frac{7}{16}$ (b) Parallel to the line $5x - 3y = 3$
   (c) Perpendicular to the line $3x + 4y = 8$ (d) Parallel to the y-axis.

2. Find the exact value of the given expression.
   (a) $\tan^{-1} 1$ (b) $\sec^{-1} 2$ (c) $\cos^{-1}(\frac{-\sqrt{3}}{2})$ (d) $\sin^{-1}(\sin \frac{5\pi}{6})$

3. In (1)-(4), match the function with its graph. [The graphs are labeled (a), (b), (c), and (d).]



   (1) $f(x) = e^x$ (2) $f(x) = e^{-x}$ (3) $f(x) = \ln(x + 1) + 1$ (4) $f(x) = -\ln(x + 1) + 1$.

4. Let $f(x) = \dfrac{x^2 - 4}{|x - 2|}$. Find each limit (if it exists).
   (a) $\lim\limits_{x \to 2^-} f(x)$ (b) $\lim\limits_{x \to 2^+} f(x)$ (c) $\lim\limits_{x \to 2} f(x)$

5. Evaluate the following limit:
   (a) $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{\sin x}$ (b) $\lim\limits_{x \to 0} x \sin \dfrac{1}{x}$ (c) $\lim\limits_{x \to 1} \dfrac{\sin(\pi x)}{x - 1}$ (d) $\lim\limits_{x \to 0} x^2 \sin \dfrac{1}{x}$

6. Suppose $f(x) > 0$ and $f(x + y) = f(x)f(y)$ for all x, $y \in \mathbb{R}$. Prove that if f is continuous at $x = 0$, then $f(0) = 1$ and f is continuous on $\mathbb{R} = (-\infty, \infty)$.

7. Let $f(x) = \begin{cases} -1, & \text{if } x \leq 0 \\ ax + b, & \text{if } 0 < x < 1 \\ 1, & \text{if } x \geq 1 \end{cases}$ If f is continuous on $\mathbb{R}$, then find the values of a and b.

8. Prove that, if $f : [a, b] \to \mathbb{R}$ is continuous and has no zeros on $[a, b]$, then either $f(x) > 0$ for all $x \in [a, b]$ or $f(x) < 0$ for all $x \in [a, b]$.

9. Prove that if f is continuous on $[0, 1]$ and $0 \leq f(x) \leq 1$ for $x \in [0, 1]$, then there is $c \in [0, 1]$ such that $f(c) = c$.

10. Prove that $x^5 - 4x^3 - 3x + 1 = 0$ has a root between 2 and 3.

11. Prove that the trigonometric polynomial

$$a_0 + a_1 \cos x + \cdots + a_n \cos nx,$$

where the coefficients are all real numbers and $|a_0| + |a_1| + \cdots + |a_{n-1}| < a_n$, has at least $2n$ zeros in the interval $[0, 2\pi)$.