PHYS121 Notes

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Contents

1. Chapter 1

2. Chapter 2

3. Chapter 3

4. Chapter 4

5. Chapter 5

6. Chapter 6

6.1. Uniform Circular Motion

Speed is constant but not velocity, because direction is constantly changing.

Centripetal acceleration for uniform circular motion: $a = \frac{v^2}{r} = \left(\frac{2\pi}{T}\right)^2 r$

Period: Time taken to go around circle one time.

Frequency: Number of revolutions per second: $f = \frac{1}{T}$ (unit is s^{-1})

Angular velocity: $\omega = 2\pi f$

Speed: Time taken to make one revolution: $v = \frac{2\pi r}{T} = (2\pi f r) = r\omega$

6.2. Dynamics of Uniform Circular Motion

Net force producing the centripetal acceleration of uniform circular motion: $\hat{F}_{\text{net}} = m\hat{a}$ (towards the center of the circle)

When a car turns in an *unbanked* (horizontal/level) circle, static friction is the force that causes centripetal acceleration.

When a car turns in a *banked* circle, normal force is the force that causes centripetal acceleration instead.

6.3. Apparent Forces in Circular Motion}

Apparent weight equals normal force.

Critical speed: v_c is the speed for which $\hat{n} = 0$.
For roller coaster, $v_c = \sqrt{gr}$
The critical speed is the slowest speed at which the car can complete a roller coaster circle.

6.4. Circular Orbits and Weightlessness

Orbit: If the launch speed of a projectile is sufficiently large, there comes a point at which the curve of the trajectory and the curve of the Earth are parallel. Such a **closed trajectory** is called an orbit.

An orbiting projectile is in free fall.

$v_{\text{orbit}} = \sqrt{gr}$ - satellites need to maintain this speed to avoid falling into the planet.

6.5. Newton's Law of Gravity

$F_g = \frac{G m_1 m_2}{r^2}$

Gravitational constant: $G = 6.67 \times 10^{-11} \frac{\mathrm{Nm}^2}{\mathrm{kg}^2}$

$g_{\text{planet}} = \frac{G M_{\text{planet}}}{R_{\text{planet}}^2}$

6.6. Gravity and Orbits

7. Chapter 7

7.1. Describing Circular and Rotational Motion

Rotational motion Motion of objects that spin about an axis.

Angular position θ is angular position of particle when measured counterclockwise from positive x-axis. Uses radians.

Arc length The arc length $s = r\theta$ is the distance a particle has traveled along its circular path.

Angular velocity $\omega = \frac{\Delta\theta}{t}$

Every point on a rotating body has the same angular velocity.
Relationship between speed and angular speed: $v = \omega r$

Angular acceleration $\alpha = \frac{\Delta\omega}{t}$ (units are $\frac{\text{rad}}{\text{s}^2}$)

$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, just like with linear motion with constant acceleration.

Distance: $\mathrm{S} = r\Theta$

7.2. The Rotation of a Rigid Body

Rigid body An extended object whose shape and size do not change as it moves.

7.3. Torque

The ability of a force to cause rotation depends on:

- The magnitude F of the force
- The distance r from the pivot to the point at which force is applied
- The angle at which the force is applied

Equation for torque $\tau = rF_{\perp} = rF \sin\varphi$ (units: $\mathrm{N} \cdot \mathrm{m}$).

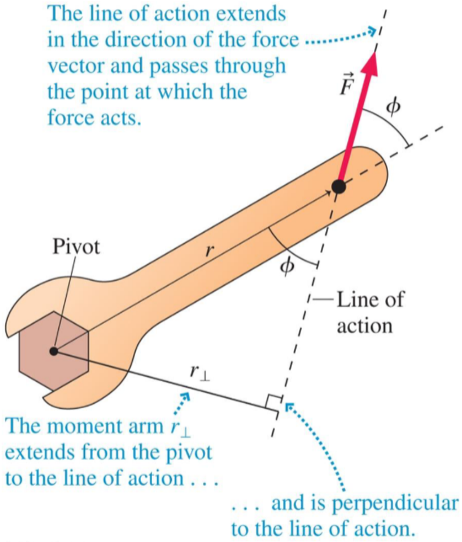
φ is measured from the radial line to the direction of the force.

Radial line Line starting at the pivot and going through the point where force is applied.

Line of action Line that is in the direction of the force and passes through the point at which the force acts.

Moment arm/lever arm Perpendicular distance from line of action to pivot.

Alternative equation for torque $\tau = r_{\perp}F$



7.4. Gravitational Torque and the Center of Gravity

Every particle in an object experiences torque due to the force of gravity. The gravitational torque can be calculated by assuming that the net force of gravity (the object's weight) acts as a single point. This single point is the **center of gravity**.

7.5. Rotational Dynamics and Moment of Inertia

7.5.1. Relationship between torque and angular acceleration

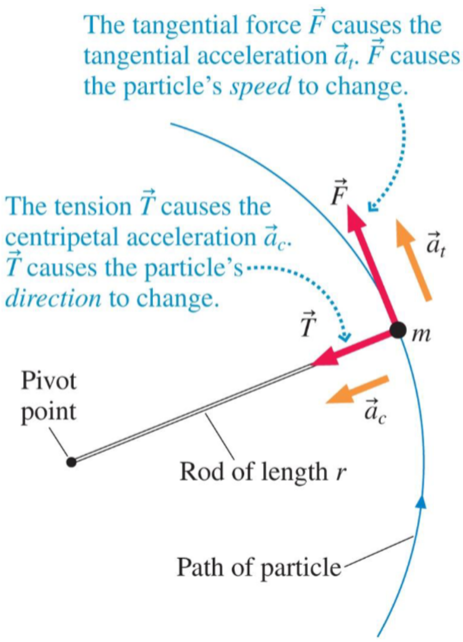
Torque causes angular acceleration.

The tangential acceleration is $a_t = \frac{F}{m}$

Tangential and angular acceleration are related by $a_t = \alpha r$, so we can rewrite equation as $\alpha = \frac{F}{mr}$

We can connect this angular acceleration to torque: $\tau = rF$

Relationship between torque and angular acceleration: $\alpha = \frac{\tau}{mr^2}$



7.5.2. Newton's Second Law for Rotational Motion

For a rigid body rotating about fixed axis, can think of object as consisting of multiple particles.
Can calculate torque on each particle.

Each particle has the same angular acceleration because the object rotates together.

Net torque:

$$\tau_{\text{net}} = \tau_1 + \tau_2 + ... = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + ... = \alpha \sum m_i r_i^2$$

Moment of Inertia (I) The proportionality constant between angular acceleration and net torque.

Units are $\text{kg} \cdot \text{m}^2$

$$I = \sum m_i r_i^2$$

Moment of inertia **depends on axis of rotation**. It depends on how the mass is distributed around rotation axis, not just how much mass there is.

The moment of inertia is the rotational equivalent of mass, i.e., $F_{\text{net}} = ma$, $\tau_{\text{net}} = I\alpha$

Newton's second law for rotation An object that experiences a net torque τ_{net} about the axis of rotation undergoes an angular acceleration of:

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

8. Chapter 8: Equilibrium and Elasticity

8.1. Torque and Static Equilibrium

An object at rest is in **static equilibrium**.

As long as the object can be modeled as a particle, static equilibrium is achieved when the net force on the particle is 0.

However, for extended objects that can rotate, we need to also consider torque: the object is only in static equilibrium if **both net force *and* net torque** are 0.

8.1.1. Choosing the Pivot Point

For an object in static equilibrium, the net torque about **every point** must be 0.

You can choose *any* point as a pivot point for calculating torque.

Natural axis: Axis about which rotation **would** occur if the object were not in static equilibrium.

8.2. Forces and Torques in the Body

8.2.1. Mechanical Advantage

If a nutcracker applies 3 times as much force to the nut as the force applied by the hand, it has a mechanical advantage of 3.

8.3. Stability and Balance

An extended object has a **base of support** on which it rests when in static equilibrium.

A wider base of support and/or a lower center of gravity improves stability.

As long as the object's center of gravity remains over the base of support, torque due to gravity will rotate the object back towards its stable equilibrium position. The object is **stable**.

If the object's center of gravity moves outside the base of support, it is **unstable**.

Critical angle Angle where center of gravity is directly above pivot.

$$\theta_c = \tan^{-1}\left(\frac{\frac{1}{2}t}{h}\right)$$

8.4. Springs and Hooke's Law

Restoring force Force that restores system to an equilibrium position
Elastic systems Systems that exhibit restoring forces, e.g. springs and rubber bands
Spring constant k, units: N/m

Spring force: $F_{\text{spring}} = -k\Delta x$ (Hooke's Law)

8.5. Stretching and Compressing Materials

Most solid materials can be modeled as being made up of particle-like atoms connected by spring-like bonds.

Pulling on a steel rod will slightly stretch the bonds between particles and the rod will stretch.

Rigid Rigid materials only experience small changes in dimension under normal forces (e.g., steel)
Pliant Pliant materials can be stretched easily or show large deformations with small forces (e.g., rubber bands)

For a rod, the spring constant depends on the cross-sectional area A, the length of the rod L, and the material from which it is made:

$$k = \frac{YA}{L}$$

Young's modulus (Y) The constant Y is a property of the material from which the rod is made (higher for stronger materials). Units are $\mathrm{N/m^2}$

The restoring force can be written in terms of the change in length ΔL:

$F = \frac{YA}{L}\Delta L$

Useful to rearrange this to put in terms of **stress** and **strain**:

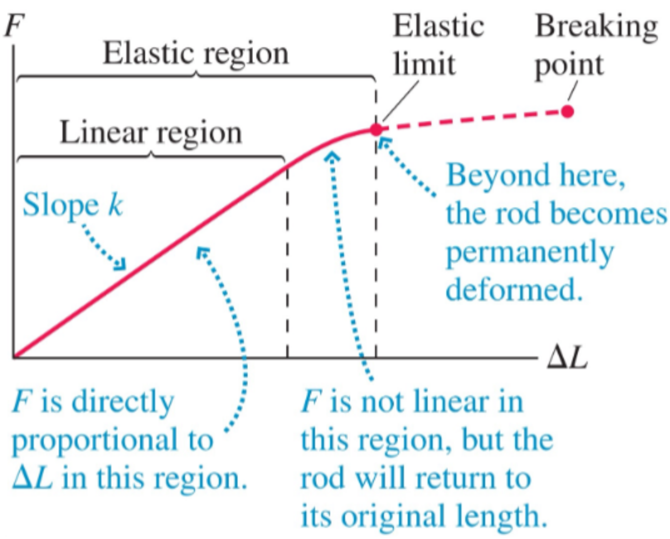
$$\frac{F}{A} = Y\left(\frac{\Delta L}{L}\right)$$

Strain Strain is $\frac{\Delta L}{L}$ (unitless)

Stress Stress is $\frac{F}{A}$, units are $\mathrm{N/m^2}$

Tensile stress Stress due to stretching

8.5.1. Beyond the Elastic Limit



As long as the stretch stays within the **linear region**, a solid rod acts like a spring and obeys Hooke's law.

Elastic limit The end of the **elastic region**.

As long as the stretch is less than the elastic limit, the rod returns to its initial length when force is removed.

Ultimate stress/tensile strength The ultimate stress/tensile strength of a rod of a rod or cable of a particular material is the largest stress the material can take before breaking (A is cross-section area):

$$\text{Tensile strength} = \frac{F_{\max}}{A}$$