PHYS121 Notes

Yash Thakur

Contents

1. Chapter 1

1.1. Distance vs Displacement

- Distance is an absolute value, how much you travelled
- Displacement is $\Delta x = \text{end} - \text{start}$
  - Path doesn't matter
  - Vector quantity

1.2. Significant figures

- When multiplying or dividing, whichever operand had fewer significant figures, that's how many significant figures the result has
  - e.g. $3.73 \cdot 5.7 = 21$ (5.7 has 2 sig figs, so 21 does too)
- When adding or subtracting, whichever operand had the fewest decimal places, that's how many decimal places the result will have
  - e.g. $16.7 + 5.24 = 21.94 = 21.9$ (16.7 has 1 decimal place, so the result does too)

2. Chapter 2

2.1. Uniform Motion

Uniform motion (aka constant-velocity motion) is straight-line motion with equal displacements during anys uccessive equal-time intervals

An object's motion is uniform iff **the position vs time graph is a straight line.**

Some equations:

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$$x_f = x_i + v_x \Delta t$$

$$\Delta x = v_x \Delta t$$

2.2. Instantaneous Velocity

Speed and direction at a specific instant of time t

2.3. Acceleration

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{x_f} - v_{x_i}}{\Delta t}$$

$$v_{x_f} = v_{x_i} + a_x \Delta t$$

$$x_f = x_i + v_{x_i} \Delta t + \frac{1}{2} a_x \Delta t^2$$

$$\left(v_{x_f}\right)^2 = \left(v_{x_i}\right)^2 + 2 a_x \Delta x$$

3. Chapter 3

3.1. Vectors

- $A_x = A\cos(\theta)$ and $A_y = A\sin(\theta)$

$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

3.2. Projectile Motion

Projectile An object that moves in 2 dimensions under the influence of gravity and nothing else
Launch angle Angle of the initial velocity above the horizontal (x-axis)
Range The range of àprojectile is the horizontal distance travelled

- Horizontal direction has uniform motion
- Vertical direction has free-fall motion
- For smaller objects, air resistance is critical
  - Maximum range comes at an angle less than 45°

3.3. Circular Motion

Centripetal acceleration Acceleration pointing towards the center of a circle

$\frac{\Delta v}{v} = \frac{d}{r}$, where d is displacement

Centripetal acceleration is $a = \frac{v^2}{r}$, towards the center of the circle

3.4. Relative Velocity

???

4. Chapter 4

4.1. Motion and Forces

Newton's first law An object willre main at rest or continue moving in a straight line at the same speed unless it has forces acting on it

Contact forces Forces that act on an object by touching it

Long-range forces Forces that act on an object without physical contact

Examples of forces:

- Weight ($\vec{w}$)

5. Chapter 5

6. Chapter 6

6.1. Uniform Circular Motion

Speed is constant but not velocity, because direction is constantly changing.

Centripetal acceleration for uniform circular motion: $a = \frac{v^2}{r} = \left(\frac{2\pi}{T}\right)^2 r$

Period: Time taken to go around circle one time.

Frequency: Number of revolutions per second: $f = \frac{1}{T}$ (unit is s^{-1})

Angular velocity: $\omega = 2\pi f$

Speed: Time taken to make one revolution: $v = \frac{2\pi r}{T} = (2\pi f r) = r\omega$

6.2. Dynamics of Uniform Circular Motion

Net force producing the centripetal acceleration of uniform circular motion: $\hat{F}_{\text{net}} = m\hat{a}$ (towards the center of the circle)

When a car turns in an *unbanked* (horizontal/level) circle, static friction is the force that causes centripetal acceleration.

When a car turns in a *banked* circle, normal force is the force that causes centripetal acceleration instead.

6.3. Apparent Forces in Circular Motion}

Apparent weight equals normal force.

Critical speed: v_c is the speed for which $\hat{n} = 0$.
For roller coaster, $v_c = \sqrt{gr}$
The critical speed is the slowest speed at which the car can complete a roller coaster circle.

6.4. Circular Orbits and Weightlessness

Orbit: If the launch speed of a projectile is sufficiently large, there comes a point at which the curve of the trajectory and the curve of the Earth are parallel. Such a **closed trajectory** is called an orbit.

An orbiting projectile is in free fall.

$v_{\text{orbit}} = \sqrt{gr}$ - satellites need to maintain this speed to avoid falling into the planet.

6.5. Newton's Law of Gravity

$F_g = \frac{G m_1 m_2}{r^2}$

Gravitational constant: $G = 6.67 \times 10^{-11} \frac{\mathrm{Nm}^2}{\mathrm{kg}^2}$

$g_{\text{planet}} = \frac{G M_{\text{planet}}}{R_{\text{planet}}^2}$

6.6. Gravity and Orbits

7. Chapter 7

7.1. Describing Circular and Rotational Motion

Rotational motion Motion of objects that spin about an axis.

Angular position θ is angular position of particle when measured counterclockwise from positive x-axis. Uses radians.

Arc length The arc length $s = r\theta$ is the distance a particle has traveled along its circular path.

Angular velocity $\omega = \frac{\Delta\theta}{t}$

Every point on a rotating body has the same angular velocity.
Relationship between speed and angular speed: $v = \omega r$

Angular acceleration $\alpha = \frac{\Delta\omega}{t}$ (units are $\frac{\text{rad}}{\text{s}^2}$)

$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, just like with linear motion with constant acceleration.

Distance: $\text{S} = r\Theta$

7.2. The Rotation of a Rigid Body

Rigid body An extended object whose shape and size do not change as it moves.

7.3. Torque

The ability of a force to cause rotation depends on:

- The magnitude F of the force
- The distance r from the pivot to the point at which force is applied
- The angle at which the force is applied

Equation for torque $\tau = rF_\perp = rF\sin\varphi$ (units: $\text{N} \cdot \text{m}$).

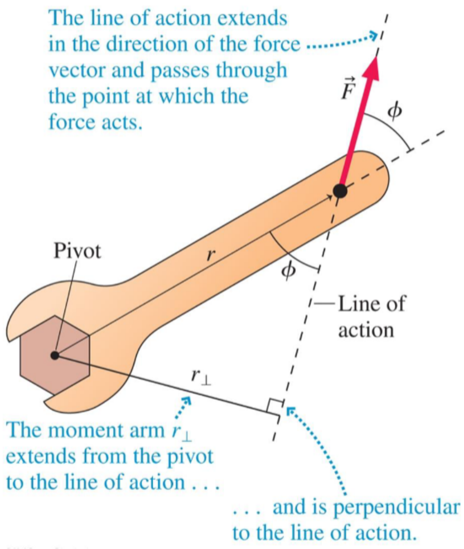
φ is measured from the radial line to the direction of the force.

Radial line Line starting at the pivot and going through the point where force is applied.

Line of action Line that is in the direction of the force and passes through the point at which the force acts.

Moment arm/lever arm Perpendicular distance from line of action to pivot.

Alternative equation for torque $\tau = r_\perp F$



7.4. Gravitational Torque and the Center of Gravity

Every particle in an object experiences torque due to the force of gravity. The gravitational torque can be calculated by assuming that the net force of gravity (the object's weight) acts as a single point. This single point is the **center of gravity**.

7.5. Rotational Dynamics and Moment of Inertia

7.5.1. Relationship between torque and angular acceleration

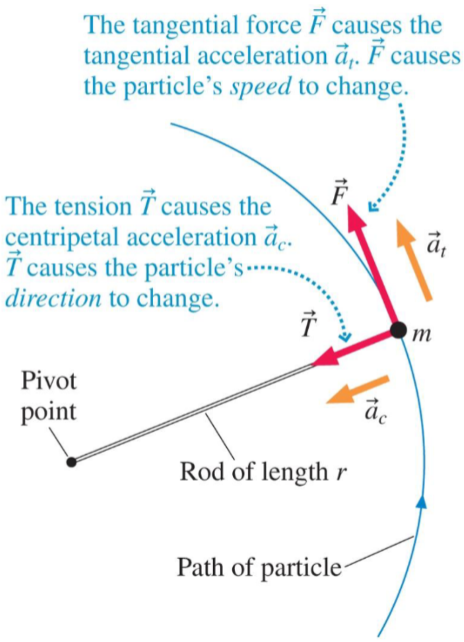
Torque causes angular acceleration.

The tangential acceleration is $a_t = \frac{F}{m}$

Tangential and angular acceleration are related by $a_t = \alpha r$, so we can rewrite equation as $\alpha = \frac{F}{mr}$

We can connect this angular acceleration to torque: $\tau = rF$

Relationship between torque and angular acceleration: $\alpha = \frac{\tau}{mr^2}$



7.5.2. Newton's Second Law for Rotational Motion

For a rigid body rotating about fixed axis, can think of object as consisting of multiple particles.
Can calculate torque on each particle.

Each particle has the same angular acceleration because the object rotates together.

Net torque:

$$\tau_{\text{net}} = \tau_1 + \tau_2 + \ldots = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + \ldots = \alpha \sum m_i r_i^2$$

Moment of Inertia (I) The proportionality constant between angular acceleration and net torque.

Units are $\text{kg} \cdot \text{m}^2$

$$I = \sum m_i r_i^2$$

Moment of inertia **depends on axis of rotation**. It depends on how the mass is distributed around rotation axis, not just how much mass there is.

The moment of inertia is the rotational equivalent of mass, i.e., $F_{\text{net}} = ma$, $\tau_{\text{net}} = I\alpha$

Newton's second law for rotation An object that experiences a net torque τ_{net} about the axis of rotation undergoes an angular acceleration of:

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

8. Chapter 8: Equilibrium and Elasticity

8.1. Torque and Static Equilibrium

An object at rest is in **static equilibrium**.

As long as the object can be modeled as a particle, static equilibrium is achieved when the net force on the particle is 0.

However, for extended objects that can rotate, we need to also consider torque: the object is only in static equilibrium if **both net force *and* net torque** are 0.

8.1.1. Choosing the Pivot Point

For an object in static equilibrium, the net torque about **every point** must be 0.

You can choose *any* point as a pivot point for calculating torque.

Natural axis: Axis about which rotation **would** occur if the object were not in static equilibrium.

8.2. Forces and Torques in the Body

8.2.1. Mechanical Advantage

If a nutcracker applies 3 times as much force to the nut as the force applied by the hand, it has a mechanical advantage of 3.

8.3. Stability and Balance

An extended object has a **base of support** on which it rests when in static equilibrium.

A wider base of support and/or a lower center of gravity improves stability.

As long as the object's center of gravity remains over the base of support, torque due to gravity will rotate the object back towards its stable equilibrium position. The object is **stable**.

If the object's center of gravity moves outside the base of support, it is **unstable**.

Critical angle Angle where center of gravity is directly above pivot.

$$\theta_c = \tan^{-1}\left(\frac{\frac{1}{2}t}{h}\right)$$

8.4. Springs and Hooke's Law

Restoring force Force that restores system to an equilibrium position
Elastic systems Systems that exhibit restoring forces, e.g. springs and rubber bands
Spring constant k, units: N/m

Spring force: $F_{\text{spring}} = -k\Delta x$ (Hooke's Law)

8.5. Stretching and Compressing Materials

Most solid materials can be modeled as being made up of particle-like atoms connected by spring-like bonds.

Pulling on a steel rod will slightly stretch the bonds between particles and the rod will stretch.

Rigid Rigid materials only experience small changes in dimension under normal forces (e.g., steel)
Pliant Pliant materials can be stretched easily or show large deformations with small forces (e.g., rubber bands)

For a rod, the spring constant depends on the cross-sectional area A, the length of the rod L, and the material from which it is made:

$$k = \frac{YA}{L}$$

Young's modulus (Y) The constant Y is a property of the material from which the rod is made (higher for stronger materials). Units are $\mathrm{N/m^2}$

The restoring force can be written in terms of the change in length ΔL:

$F = \frac{YA}{L}\Delta L$

Useful to rearrange this to put in terms of **stress** and **strain**:

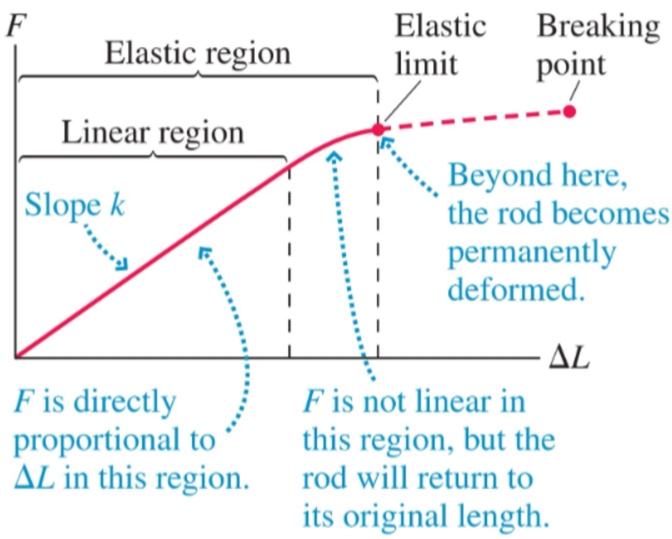
$$\frac{F}{A} = Y\left(\frac{\Delta L}{L}\right)$$

Strain Strain is $\frac{\Delta L}{L}$ (unitless)

Stress Stress is $\frac{F}{A}$, units are $\mathrm{N/m^2}$

Tensile stress Stress due to stretching

8.5.1. Beyond the Elastic Limit



As long as the stretch stays within the **linear region**, a solid rod acts like a spring and obeys Hooke's law.

Elastic limit The end of the **elastic region**.

As long as the stretch is less than the elastic limit, the rod returns to its initial length when force is removed.

Ultimate stress/tensile strength The ultimate stress/tensile strength of a rod of a rod or cable of a particular material is the largest stress the material can take before breaking (A is cross-section area):

$$\text{Tensile strength} = \frac{F_{\max}}{A}$$

9. Chapter 9: Momentum

9.1. Impulse and Momentum

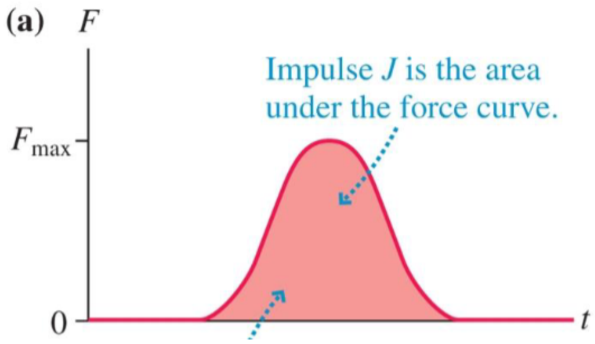
Collision Short-duration interaction between two objects.

During a collision, it takes time to compress the object, and it takes time for the object to re-expand.

The duration of a collision depends on the materials.

Impulse force A large force exerted during a short interval of time

The effect of an impulsive force is proportional to the area under the force vs time curve



Impulse The area under a force vs time curve (integral of force with respect to time?)

It's a vector quantity pointing in the same direction as the average force (units of $\mathrm{N} \cdot \mathrm{s}$):

$$\vec{J} = \vec{F}_{\text{avg}} \Delta t$$

Momentum Product of mass and velocity: $\vec{p} = m\vec{v}$

🔥 **Impulse-momentum theorem**

Impulse is change in momentum:

$$\vec{J} = \Delta\vec{p}$$

Total momentum ($\vec{P}$) Sum of momenta of all particles in system

The impulse approximation states that we can ignore the small forces that act during the brief time of the impulsive force (only consider momenta and velocities immediately before and immediately after collision).

9.2. Conservation of Momentum

i **Law of conservation of momentum**

The total momentum of the system is conserved *as long as* there are no external forces.

$\vec{F}_{\text{net}}$ is the net force due to external forces.

If $\vec{F}_{\text{net}} = \vec{0}$, the total momentum does not change.

Isolated system System with no net external force acting on it, leaving the momentum unchanged.

9.3. Explosions

Explosion When the particles of a system move apart after a brief, intense interaction (opposite of collision)

The forces in an explosion are **internal** forces, so if the system is isolated, the total momentum is 0

9.4. Inelastic Collisions

Perfectly inelastic collision Two objects stick together and move with common final velocity (e.g. clay hitting the floor)

Perfectly elastic collision Mechanical energy is conserved

> **Info**
>
> Although momentum is conserved in all collisions, mechanical energy is only conserved in a perfectly elastic collision.

In an inelastic collision, some mechanical energy is converted to thermal energy.

9.5. Angular Momentum

Angular momentum (L) Analogue of linear momentum for circular motion, since linear momentum is not conserved for spinning objects ($\mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}$)

$$L = I\omega$$

Can be written like the linear impulse-momentum equation:

$$\tau_{\mathrm{net}} \Delta t = \Delta L$$

9.5.1. Varying Moment of Inertia

Unlike linear momentum, **an isolated, rotating object can change its angular velocity**

Moment of inertia can change because the distribution of mass can change

10. Chapter 10: Energy and Work

10.1. The Basic Energy Model

Every system has a total energy E

10.1.1. Energy Transfers and Work

Energy can be transferred between a system and its environment through work and heat

Work Mechanical transfer of energy to or from a system by pushing or pulling on it
Heat Nonmechanical transfer of energy between system and environment due to temperature difference between the two

Work is change in total energy:

$$W = \Delta E$$

Isolated system No energy transferred into or out of system

10.1.2. Law of Conservation of Energy

The total energy of an isolated system remains constant

10.2. Work

Work is done on a system by **external forces**

$$W = Fd$$

Work is a **scalar** even though force and displacement are vectors

Units of work are J (joules), same as energy

A force does no work on an object if the object undergoes no displacement (or the force is perpendicular to the displacement)

10.2.1. Kinetic Energy

Translational kinetic energy Energy of motion in a line

$$K = \frac{1}{2}mv^2$$

Rotational kinetic energy Way of expressing sum of kinetic energy of all parts of a rotating object. Moment of inertia takes place of mass and angular velocity takes place of linear velocity.

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

10.3. Potential Energy

Conservative forces Forces that can store useful energy, e.g. gravity, elastic forces
Nonconservative forces Forces that can't store useful energy, e.g. friction

10.3.1. Gravitational Potential Energy

$$U_{\text{g}} = mgh$$

Only the height (h) matters, not the path the object took to get there

10.3.2. Elastic Potential Energy

$$U_{\text{s}} = \frac{1}{2}kx^2$$

10.4. Conservation of Energy

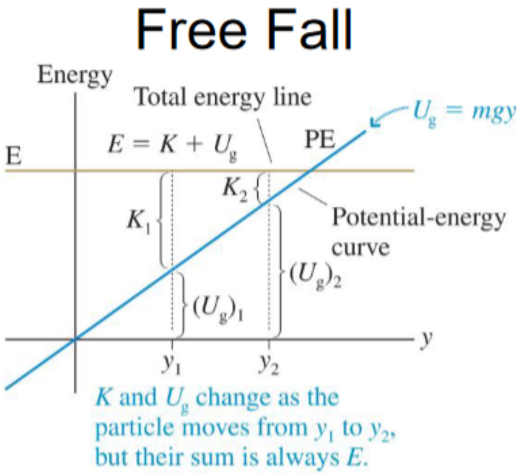
In an isolated system, $W = 0$

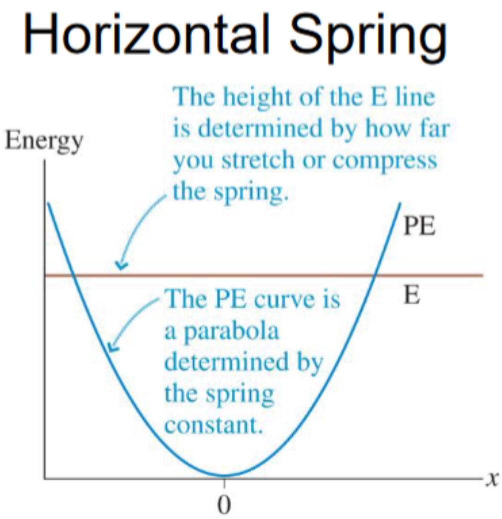
Carefully choose an isolated system to solve problems in

10.5. Energy Diagrams

Energy diagrams graph potential energy as a function of position

- Free fall energy diagrams are linear graphs of gravitational potential energy
- A spring's energy diagram is a parabola showing spring potential energy





- The distance from the axis to the PE line is the potential energy
- The distance from the PE line to the energy line E is the kinetic energy
- The object cannot be at a position where the PE curve is above the E line (because E is total energy)
- A position where the E line crosses the PE curve is a turning point where the object reverses direction
- If the E line crosses the PE curve at two positions, the object will oscillate between those two positions
- Speed will be at a maximum where the PE curve is a minimum (because that's where KE is at a maximum)

10.6. Power

Power Rate at which energy is transformed or transferred (scalar), measured in watts (W)

$$P = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t}$$

Output power A force doing work transfers energy, and the rate at which the force transfers energy is called output power

$$P = Fv$$

where P is F's output power, and v is the velocity of the object that F is acting on