

STAT400 Final Exam Review

1.

$$\begin{aligned}P(|X - 15| > 3) &= 1 - P(|X - 15| \leq 3) \\&= 1 - P(-3 \leq X - 15 \leq 3) \\&= 1 - P(12 \leq X \leq 18) \\&= 1 - (P(X \leq 18) - P(X < 12)) \\&= 1 - (F_X(18) - F_X(12)) \\&= 1 - F_X(18) + F_X(12)\end{aligned}$$

2. todo

3. todo

4. todo

5. todo

6. todo

7. todo

8. todo

8.a. todo

$$E(XY) =$$

8.b. todo

9. todo

10. todo

11. todo

12. todo

... todos

42. todo

42.a.

$$E(X) = \frac{1}{n} \sum_{i=1}^n X_i$$

$$\frac{\hat{\alpha}}{\hat{\alpha} + 1} = \bar{X}$$

$$\hat{\alpha} = \hat{\alpha} \bar{X} + \bar{X}$$

$$\hat{\alpha} \bar{X} - \hat{\alpha} = -\bar{X}$$

$$\hat{\alpha} = \frac{-\bar{X}}{\bar{X} - 1}$$

todo verify

42.b.

$$\begin{aligned} \ell(\alpha) &= \ln \prod_{i=1}^n f(x_i; \alpha) \\ &= \ln \prod_{i=1}^n \frac{\Gamma(\alpha + 1)}{\Gamma(\alpha)\Gamma(1)} \cdot x_i^{\alpha-1} (1 - x_i)^{1-1} \\ &= \ln \prod_{i=1}^n \alpha \cdot x_i^{\alpha-1} \\ &= \sum_{i=1}^n \ln \alpha + \ln(x_i^{\alpha-1}) \\ &= \sum_{i=1}^n \ln \alpha + (\alpha - 1) \ln x_i \end{aligned}$$

$$0 = \frac{d}{d\alpha} \ell(\alpha)$$

$$0 = \frac{d}{d\alpha} \sum_{i=1}^n (\ln \alpha + (\alpha - 1) \ln x_i)$$

$$0 = \sum_{i=1}^n \frac{d}{d\alpha} (\ln \alpha + (\alpha - 1) \ln x_i)$$

$$0 = \sum_{i=1}^n \left(\frac{1}{\alpha} + \ln x_i \frac{d}{d\alpha} (\alpha - 1) \right)$$

$$0 = \sum_{i=1}^n \left(\frac{1}{\alpha} + \ln x_i \right)$$

$$0 = \frac{n}{\alpha} + \sum_{i=1}^n \ln x_i$$

$$-\sum_{i=1}^n \ln x_i = \frac{n}{\alpha}$$

$$-\alpha \sum_{i=1}^n \ln x_i = n$$

$$\alpha = \frac{-n}{\sum_{i=1}^n \ln x_i}$$

todo verify