Lecture 9/4

Probability functions

Sigma algebra

A sigma algebra is a collection of interesting events in some sample space.

A collection $\mathcal{B}$ of subsets of a sample space S is a sigma algebra if:

1. $\emptyset \in \mathcal{B}$
2. $\forall A \in \mathcal{B}, A^c \in \mathcal{B}$
3. If $\{A_i : i \in \mathbb{N}\}$ is a countable collection such that $A_i \in \mathcal{B}$ for all i, then $\bigcup_{i \in \mathbb{N}} A_i \in \mathcal{B}$

Probability functions

Consider a sample space S with a sigma algebra $\mathcal{B}$.

A probability function is a function from events to probabilities ($\mathcal{B} \to \mathbb{R}$). It must satisfy the following axioms:

1. (finite measure) $P(S) = 1$
2. (positivity) $\forall A \in \mathcal{B}, P(A) \geq 0$
3. (countable additivity) For A_1, A_2, A_3, ..., the collection of pairwise disjoint subsets of S in $\mathcal{B}$, we must have

$$P\left(\bigcup_{i \in \mathbb{N}} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$