

計量経済 II：復習テスト 4

学籍番号_____氏名_____

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注意：すべての質問に解答しなければ提出とは認めない。正答に修正した上で、復習テスト 1～8 を（左上で）ホチキス止めし、中間試験実施日（11 月 22 日の予定）にまとめて提出すること。

1. $\{y_t\}$ を共分散定常な AR(1) とする。すなわち任意の t について

$$y_t = c + \phi y_{t-1} + w_t$$
$$\{w_t\} \sim \text{WN}(\sigma^2)$$

ただし $|\phi| < 1$. $\{y_t\}$ の自己共分散関数を $\gamma(\cdot)$ とする.

(a) $E(y_t)$ を求めなさい.

(b) y_t を w_t, w_{t-1}, \dots で表しなさい.

(c) $\text{cov}(y_{t-1}, w_t)$ を求めなさい.

(d) $\text{cov}(y_t, w_t)$ を求めなさい.

(e) $\gamma(0)$ を $\gamma(1), \phi, \sigma^2$ で表しなさい.

2. $\{y_t\}$ を MA(1) とする. すなわち任意の t について

$$y_t = \mu + w_t - \theta w_{t-1}$$
$$\{w_t\} \sim \text{WN}(\sigma^2)$$

$\{y_t\}$ の自己共分散関数を $\gamma(\cdot)$, 自己相関関数を $\rho(\cdot)$ とする.

(a) $E(y_t)$ を求めなさい.

(b) $\gamma(0)$ を θ, σ^2 で表しなさい.

(c) $\gamma(1)$ を θ, σ^2 で表しなさい.

(d) $\rho(1)$ を θ で表しなさい.

(e) $\gamma(2)$ を求めなさい.

解答例

1. (a)

$$\begin{aligned} E(y_t) &= E(c + \phi y_{t-1} + w_t) \\ &= c + \phi E(y_{t-1}) + E(w_t) \\ &= c + \phi E(y_t) \\ &= \frac{c}{1 - \phi} \end{aligned}$$

(b) 逐次代入により

$$\begin{aligned} y_t &= c + \phi y_{t-1} + w_t \\ &= c + w_t + \phi y_{t-1} \\ &= c + w_t + \phi(c + w_{t-1} + \phi y_{t-2}) \\ &= \dots \\ &= \sum_{s=0}^{\infty} \phi^s (c + w_{t-s}) \\ &= c \sum_{s=0}^{\infty} \phi^s + \sum_{s=0}^{\infty} \phi^s w_{t-s} \\ &= \frac{c}{1 - \phi} + \sum_{s=0}^{\infty} \phi^s w_{t-s} \end{aligned}$$

(c) 前問より

$$\begin{aligned} \text{cov}(y_{t-1}, w_t) &= \text{cov}\left(\frac{c}{1 - \phi} + w_{t-1} + \phi w_{t-2} + \dots, w_t\right) \\ &= \text{cov}(w_{t-1} + \phi w_{t-2} + \dots, w_t) \\ &= \text{cov}(w_{t-1}, w_t) + \phi \text{cov}(w_{t-2}, w_t) + \dots \\ &= 0 \end{aligned}$$

(d) 前問より

$$\begin{aligned} \text{cov}(y_t, w_t) &= \text{cov}(c + \phi y_{t-1} + w_t, w_t) \\ &= \text{cov}(\phi y_{t-1} + w_t, w_t) \\ &= \phi \text{cov}(y_{t-1}, w_t) + \text{cov}(w_t, w_t) \\ &= \text{var}(w_t) \\ &= \sigma^2 \end{aligned}$$

(e) 前問より

$$\begin{aligned} \gamma(0) &:= \text{var}(y_t) \\ &= \text{cov}(y_t, y_t) \\ &= \text{cov}(y_t, c + \phi y_{t-1} + w_t) \\ &= \text{cov}(y_t, \phi y_{t-1} + w_t) \\ &= \phi \text{cov}(y_t, y_{t-1}) + \text{cov}(y_t, w_t) \\ &= \phi \gamma(1) + \sigma^2 \end{aligned}$$

2. (a)

$$\begin{aligned} E(y_t) &= E(\mu + w_t - \theta w_{t-1}) \\ &= \mu + E(w_t) - \theta E(w_{t-1}) \\ &= \mu \end{aligned}$$

(b)

$$\begin{aligned} \gamma(0) &:= \text{var}(y_t) \\ &= \text{var}(\mu + w_t - \theta w_{t-1}) \\ &= \text{var}(w_t - \theta w_{t-1}) \\ &= \text{var}(w_t) - 2 \text{cov}(w_t, \theta w_{t-1}) + \text{var}(\theta w_{t-1}) \\ &= \text{var}(w_t) + \theta^2 \text{var}(w_{t-1}) \\ &= \text{var}(w_t) + \theta^2 \text{var}(w_t) \\ &= (1 + \theta^2) \sigma^2 \end{aligned}$$

(c)

$$\begin{aligned} \gamma(1) &:= \text{cov}(y_t, y_{t-1}) \\ &= \text{cov}(\mu + w_t - \theta w_{t-1}, \mu + w_{t-1} - \theta w_{t-2}) \\ &= \text{cov}(w_t - \theta w_{t-1}, w_{t-1} - \theta w_{t-2}) \\ &= \text{cov}(w_t - \theta w_{t-1}, w_{t-1}) - \text{cov}(w_t - \theta w_{t-1}, \theta w_{t-2}) \\ &= \text{cov}(w_t, w_{t-1}) - \text{cov}(\theta w_{t-1}, w_{t-1}) - \text{cov}(w_t, \theta w_{t-2}) + \text{cov}(\theta w_{t-1}, \theta w_{t-2}) \\ &= -\theta \text{cov}(w_{t-1}, w_{t-1}) \\ &= -\theta \text{var}(w_{t-1}) \\ &= -\theta \sigma^2 \end{aligned}$$

(d) 前2問より

$$\begin{aligned} \rho(1) &= \frac{\gamma(1)}{\gamma(0)} \\ &= \frac{-\theta \sigma^2}{(1 + \theta^2) \sigma^2} \\ &= -\frac{\theta}{1 + \theta^2} \end{aligned}$$

(e)

$$\begin{aligned} \gamma(2) &:= \text{cov}(y_t, y_{t-2}) \\ &= \text{cov}(\mu + w_t - \theta w_{t-1}, \mu + w_{t-2} - \theta w_{t-3}) \\ &= \text{cov}(w_t - \theta w_{t-1}, w_{t-2} - \theta w_{t-3}) \\ &= \text{cov}(w_t - \theta w_{t-1}, w_{t-2}) - \text{cov}(w_t - \theta w_{t-1}, \theta w_{t-3}) \\ &= \text{cov}(w_t, w_{t-2}) - \text{cov}(\theta w_{t-1}, w_{t-2}) - \text{cov}(w_t, \theta w_{t-3}) + \text{cov}(\theta w_{t-1}, \theta w_{t-3}) \\ &= 0 \end{aligned}$$