

DK responses in surveys on inflation expectations*

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First draft: November 2024
This version: January 6, 2025

Abstract

Empirical work on survey inflation expectations often discards ‘don’t know’ (DK) responses from the analysis, which may cause sample selection bias. A possible excuse for ignoring DK responses is that the standard sample selection model involves strong assumptions, and the classical ML and Heckit estimators are not robust. This paper proposes to use a robust Heckit estimator to check if (i) DK responses are ignorable and (ii) the classical ML and Heckit estimators are reliable. Reexamination of a recent work finds that the ML, Heckit, and robust Heckit estimates somewhat differ, suggesting that the classical estimates are not reliable.

JEL classification: C24, D84, E31

Keywords: nonresponse bias, sample selection model, robust Heckit

*This work was supported by JSPS KAKENHI Grant Number JP21K01434.

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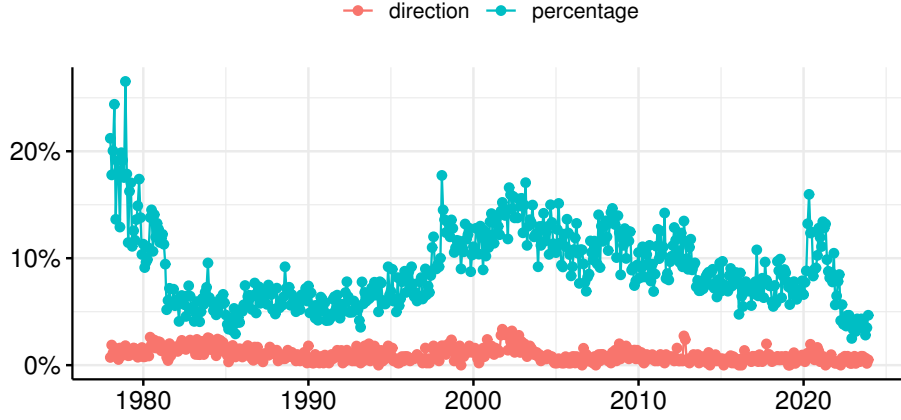


Figure 1: Missing response rates in the MSC for the direction and percentage of inflation during the next 12 months, 1978M01–2023M12

1 Introduction

Many, if not all, economists nowadays seem to believe that subjective inflation expectations matter for economic decisions and inflation.¹ Weber, D’Acunto, Gorodnichenko, and Coibion (2022, p. 158) explain,

...the key reason why subjective inflation expectations matter is that they affect the prices and wages firms set as well as the consumption–saving decisions of households.

As a consequence, the literature on the formation of subjective inflation expectations is growing. See D’Acunto, Malmendier, and Weber (2023) for a recent survey, which focuses on household inflation expectations.

Empirical work on household inflation expectations relies on surveys. Since not all consumers can always articulate their inflation expectations quantitatively, surveys that allow for ‘don’t know’ (DK) answer often have many missing responses (DK responses and item nonresponses). Figure 1 plots the missing response rates in the Michigan Survey of Consumers (MSC) for the direction and percentage of inflation during the next 12 months.² The missing response rate is much higher for percentage (around 10%) than for direction (often less than 1%).³

In practice, empirical work on survey inflation expectations often discards DK responses from the analysis. Recent works on quantitative inflation expectations ignoring DK responses include Sheen and Wang (2023), Wang, Sheen,

¹See Rudd (2022) for a criticism against the belief that subjective inflation expectations matter.

²See Curtin (1996) for the exact question wording and the actual questionnaire form of the MSC.

³The missing response rates in the New York Fed’s Survey of Consumer Expectations (SCE) are much lower, since the SCE does not allow for DK answer; see Armantier, Topa, van der Klaauw, and Zafar (2017) for an overview of the SCE. Comerford (2024) criticizes, however, that density forecasts collected by the SCE suffer from selective nonresponse and biased response.

Trück, Chao, and Härdle (2020), and Ehrmann, Pfajfar, and Santoro (2017), who use the MSC, and Tsiaplias (2020, 2021), who uses the Consumer Attitudes, Sentiments and Expectations in Australia Survey (CASIe), among many others.⁴ Ignoring DK responses may cause severe sample selection bias, unless the missingness mechanism is *ignorable* in the sense of Little and Rubin (2019, p. 133), which may not hold in practice.

A possible excuse for ignoring DK responses is that the standard sample selection model involves strong assumptions, e.g., normality and an exclusion restriction, and the classical ML and Heckit estimators are not robust to model misspecification. One can address such a concern by a robustness check in the true sense using a robust statistical method.⁵ This paper uses a robust Heckit estimator proposed by Zhelonkin, Genton, and Ronchetti (2016) to check if (i) DK responses are ignorable and (ii) the classical ML and Heckit estimators are reliable. An R package `ssmrob` developed by Zhelonkin and Ronchetti (2021) helps to apply a robust Heckit estimator.

To highlight the issue, we reexamine an analysis in Sheen and Wang (2023), who study the influence of monetary condition news on short- and medium-run household inflation expectations using data from the MSC during 2008M12–2015M12.⁶ They estimate a regression equation for the percentage of inflation by OLS, ignoring nonresponses.⁷ We assume a sample selection model instead, and compare the OLS, ML, Heckit, and robust Heckit estimates of the same equation. We find the following:

1. For both short- and medium-run expectations, the ML estimates are almost identical to the OLS estimates, with almost no correlation between the errors in the selection and outcome equations.
2. The ML and Heckit estimates somewhat differ. In particular, for medium-run expectations, the Heckit estimate of the coefficient on the bias correction term significantly differs from 0, implying that DK responses are not ignorable.
3. The classical and robust Heckit estimates somewhat differ, suggesting that the classical estimates are not reliable.

For the standard sample selection model, the ML estimator assumes a bivariate normal distribution, whereas the Heckit estimator is consistent under weaker assumptions; see Olsen (1980). Hence the difference between the two estimates may invalidate the assumption of bivariate normality, implying that the Heckit

⁴One exception is Pfajfar and Santoro (2013), who use rotating panel data from the MSC to study the effect of news on updating of inflation expectations using a binary probit model with Heckman correction to control for attrition bias; see Pfajfar and Santoro (2013, p. 1060).

⁵Existing empirical works rarely use a robust statistical method for their robustness checks.

⁶The focus of Sheen and Wang (2023) is not inflation expectations per se, but how monetary condition news affected households' readiness to spend on durables via their interest rate and inflation expectations during the 'zero lower bound' period. We focus only on one analysis in their work for our purpose of illustration.

⁷In fact, nonresponses in their sample are not DK responses but correspond to respondents who skipped the question on the percentage of inflation because they answered in the previous question that prices 'stay the same'; hence we must treat these nonresponses as '0 percent' instead of missing. We fix this error and construct our own sample with DK responses in section 4.

estimate is more reliable.⁸ Moreover, since the weaker assumptions for consistency of the Heckit estimator may not strictly hold in practice, the difference between the classical and robust Heckit estimates suggests that the robust estimate is more reliable.

Note that our robustness checks do not change the conclusion of Sheen and Wang (2023, p. 12) that ‘households do not adjust their inflation expectations upon hearing monetary condition news, neither in the short-term (1 year) nor long-term (5–10 years),’ since the estimates of the corresponding regression coefficients remain insignificant. Though acceptance of the null hypothesis is not a strong evidence in general, our robustness checks support their conclusion.

The paper proceeds as follows. Section 2 specifies a regression model with DK responses as a sample selection model. Section 3 reviews robust estimation of a sample selection model. Section 4 highlights the issue by reexamining an analysis in Sheen and Wang (2023). Section 5 discusses remaining issues.

2 Regression model with DK responses

Let (y, \mathbf{x}') be a $(1 + k)$ -variate random vector, where y is either a numerical or DK response. Let y^* be the latent numerical response underlying y , and d be the numerical response dummy so that $y = y^*$ if and only if $d = 1$. Assume a sample selection model for y given \mathbf{x} such that

$$\begin{aligned} y &= \begin{cases} y^* & \text{if } d = 1 \\ \text{NA} & \text{if } d = 0 \end{cases} \\ d &= [U > 0] \\ U &= \mathbf{x}'\boldsymbol{\alpha} + z \\ y^* &= \mathbf{x}'\boldsymbol{\beta} + u \\ \begin{pmatrix} z \\ u \end{pmatrix} | \mathbf{x} &\sim N\left(\mathbf{0}, \begin{bmatrix} 1 & \sigma_{zu} \\ \sigma_{uz} & \sigma_u^2 \end{bmatrix}\right) \end{aligned}$$

where U is a latent variable determining d , and $(z, u)'$ is an error vector with $\text{var}(z) = 1$ by rescaling U . Consider estimation of $\boldsymbol{\beta}$ given a random sample of $(d, y, \mathbf{x})'$.

By simple algebra, the outcome equation for the selected sample is

$$E(y|d = 1, \mathbf{x}) = \mathbf{x}'\boldsymbol{\beta} + E(u|z > -\mathbf{x}'\boldsymbol{\alpha}, \mathbf{x}) \quad (1)$$

If z and u are independent, then the second term is zero, so one can ignore DK responses and apply OLS to obtain a consistent estimator of $\boldsymbol{\beta}$. Otherwise the second term remains, and the OLS estimator is inconsistent (sample selection bias). One can avoid the bias using the ML or Heckit estimator, but they are not widely used in the context of DK responses, perhaps because they are not robust to model misspecification.

⁸One source of nonnormality of the percentage of inflation is that some respondents round the percentage to multiples of 5 while others do not, resulting in a multi-modal distribution. It is unclear, however, if the ML estimator becomes inconsistent while the Heckit estimator remains consistent when such rounding exists.

3 Robust Heckit estimator

3.1 Heckit estimator

Let $\Phi(\cdot)$ be the cdf of $N(0, 1)$, $\phi(\cdot) := \Phi'(\cdot)$, and $h(\cdot) := \phi(\cdot)/\Phi(\cdot)$. One can write the outcome equation (1) as

$$E(y|d = 1, \mathbf{x}) = \mathbf{x}'\boldsymbol{\beta} + \sigma_{uz}h(\mathbf{x}'\boldsymbol{\alpha})$$

See, e.g., Hansen (2022, p. 883). One obtains the Heckit estimator of $\boldsymbol{\beta}$ in two steps.

The selection equation for d is a binary probit model, so we apply the ML method to estimate $\boldsymbol{\alpha}$. Let $s := 2d - 1$. The moment restriction that defines $\boldsymbol{\alpha}$ is

$$E(s\mathbf{x}h(s\mathbf{x}'\boldsymbol{\alpha})) = \mathbf{0} \quad (2)$$

See, e.g., Hansen (2022, p. 834).

Given the bias correction term $h(\mathbf{x}'\boldsymbol{\alpha})$, one applies OLS using the selected sample to estimate $(\boldsymbol{\beta}', \sigma_{uz})'$. The moment restriction that defines $(\boldsymbol{\beta}', \sigma_{uz})'$ is

$$\begin{aligned} E(\mathbf{x}(y - \mathbf{x}'\boldsymbol{\beta} - \sigma_{uz}h(\mathbf{x}'\boldsymbol{\alpha}))|d = 1) &= \mathbf{0} \\ E(h(\mathbf{x}'\boldsymbol{\alpha})(y - \mathbf{x}'\boldsymbol{\beta} - \sigma_{uz}h(\mathbf{x}'\boldsymbol{\alpha}))|d = 1) &= 0 \end{aligned}$$

which implies

$$E(\mathbf{x}(y - \mathbf{x}'\boldsymbol{\beta} - \sigma_{uz}h(\mathbf{x}'\boldsymbol{\alpha}))d) = \mathbf{0} \quad (3)$$

$$E(h(\mathbf{x}'\boldsymbol{\alpha})(y - \mathbf{x}'\boldsymbol{\beta} - \sigma_{uz}h(\mathbf{x}'\boldsymbol{\alpha}))d) = 0 \quad (4)$$

3.2 M-estimator

The sample analogs of the moment restrictions (2)–(4) give the estimating equation that defines the Heckit estimator as an M-estimator. Let $\mathbf{z} := (d, s, y, \mathbf{x}')'$. Let $\boldsymbol{\theta} := (\boldsymbol{\alpha}', \boldsymbol{\beta}', \sigma_{uz})'$. Let

$$\begin{aligned} \psi_1(\mathbf{z}; \boldsymbol{\theta}) &:= s\mathbf{x}h(s\mathbf{x}'\boldsymbol{\alpha}) \\ \psi_2(\mathbf{z}; \boldsymbol{\theta}) &:= \begin{pmatrix} \mathbf{x} \\ h(\mathbf{x}'\boldsymbol{\alpha}) \end{pmatrix} (y - \mathbf{x}'\boldsymbol{\beta} - \sigma_{uz}h(\mathbf{x}'\boldsymbol{\alpha}))d \\ \boldsymbol{\psi}(\mathbf{z}; \boldsymbol{\theta}) &:= \begin{pmatrix} \psi_1(\mathbf{z}; \boldsymbol{\theta}) \\ \psi_2(\mathbf{z}; \boldsymbol{\theta}) \end{pmatrix} \end{aligned}$$

Let $F(\cdot)$ be the joint cdf of \mathbf{z} . Let $T(\cdot)$ be a statistical functional that defines $\boldsymbol{\theta}$, so that $\boldsymbol{\theta} := T(F(\cdot))$. Then

$$E(\boldsymbol{\psi}(\mathbf{z}; \boldsymbol{\theta})) = \mathbf{0}$$

or

$$\int \boldsymbol{\psi}(\mathbf{z}; T(F(\cdot))) dF(\mathbf{z}) = \mathbf{0}$$

Let \mathbf{Z} be a random sample of size n from $F(\cdot)$. Let $F_n(\cdot)$ be the empirical cdf of \mathbf{Z} . The M-estimator of $\boldsymbol{\theta}$ given \mathbf{Z} is $\hat{\boldsymbol{\theta}} := T(F_n(\cdot))$ such that

$$\frac{1}{n} \sum_{i=1}^n \boldsymbol{\psi}(\mathbf{z}_i; \hat{\boldsymbol{\theta}}) = \mathbf{0}$$

or

$$\int \boldsymbol{\psi}(\mathbf{z}; T(F_n(.))) dF_n(\mathbf{z}) = \mathbf{0}$$

3.3 Robustness

The influence function of $T(\cdot)$ at $F(\cdot)$ is the functional (Gâteaux) derivative of $T(\cdot)$ with respect to the cdf of a point mass (outlier) evaluated at $F(\cdot)$. If the influence function of $T(\cdot)$ is bounded, then any outlier has a bounded influence on $T(\cdot)$, so $T(\cdot)$ is robust; see Wilcox (2022, pp. 29–30). The influence function of an M-estimator is proportional to its estimating function. In our case, the influence function of $T(\cdot)$ at $F(\cdot)$ is $\forall \mathbf{z}$,

$$\text{IF}_{T(\cdot), F(\cdot)}(\mathbf{z}) = - \left(\int \boldsymbol{\psi}_{\theta}(\mathbf{z}; T(F(.))) dF(\mathbf{z}) \right)^{-1} \boldsymbol{\psi}(\mathbf{z}; T(F(.)))$$

See Hampel, Ronchetti, Rousseeuw, and Stahel (1986, p. 230). One can show by L'Hôpital's rule that $h(z) \rightarrow \infty$ as $z \rightarrow -\infty$, implying that $\boldsymbol{\psi}(\cdot; T(F(.)))$ is unbounded. Since $\text{IF}_{T(\cdot), F(\cdot)}(\cdot)$ is unbounded, an extreme outlier has a huge influence on $T(\cdot)$, so the Heckit estimator is not robust.

3.4 Bounded-influence estimator

One can obtain a robust Heckit estimator by bounding $\boldsymbol{\psi}(\cdot; \boldsymbol{\theta})$. Consider bounding $\boldsymbol{\psi}_1(\cdot; \boldsymbol{\theta})$ and $\boldsymbol{\psi}_2(\cdot; \boldsymbol{\theta})$ in turn.

Since $\boldsymbol{\psi}_1(\cdot; \boldsymbol{\theta})$ is the estimating function for a binary probit model, we can rewrite $\boldsymbol{\psi}_1(\mathbf{z}; \boldsymbol{\theta})$ as

$$\boldsymbol{\psi}_1(\mathbf{z}; \boldsymbol{\theta}) = \frac{\mathbf{x}\phi(\mathbf{x}'\boldsymbol{\alpha})(d - \Phi(\mathbf{x}'\boldsymbol{\alpha}))}{\Phi(\mathbf{x}'\boldsymbol{\alpha})\Phi(-\mathbf{x}'\boldsymbol{\alpha})}$$

See, e.g., Wooldridge (2010, p. 478). Write the binary probit model for d as a regression model:

$$\begin{aligned} E(d|\mathbf{x}) &= \Phi(\mathbf{x}'\boldsymbol{\alpha}) \\ \text{var}(d|\mathbf{x}) &= \Phi(\mathbf{x}'\boldsymbol{\alpha})\Phi(-\mathbf{x}'\boldsymbol{\alpha}) \end{aligned}$$

Let r_1 be the standardized prediction error of d given \mathbf{x} , i.e.,

$$r_1 := \frac{d - \Phi(\mathbf{x}'\boldsymbol{\alpha})}{\sqrt{\Phi(\mathbf{x}'\boldsymbol{\alpha})\Phi(-\mathbf{x}'\boldsymbol{\alpha})}}$$

which need not be symmetric around 0. We can write

$$\begin{aligned} \boldsymbol{\psi}_1(\mathbf{z}; \boldsymbol{\theta}) &= \frac{\mathbf{x}\phi(\mathbf{x}'\boldsymbol{\alpha})r_1}{\sqrt{\Phi(\mathbf{x}'\boldsymbol{\alpha})\Phi(-\mathbf{x}'\boldsymbol{\alpha})}} \\ &= \mathbf{x}\sqrt{h(\mathbf{x}'\boldsymbol{\alpha})h(-\mathbf{x}'\boldsymbol{\alpha})}r_1 \end{aligned}$$

Let $\Psi(\cdot)$ be a Huber function with bound $K > 0$, i.e., $\forall z \in \mathbb{R}$,

$$\Psi(z) := \begin{cases} z & \text{for } |z| \leq K \\ \text{sgn}(z)K & \text{for } |z| > K \end{cases}$$

Let

$$\psi_1^*(z; \theta) := w_1(\mathbf{x}) \mathbf{x} \sqrt{h(\mathbf{x}'\boldsymbol{\alpha})h(-\mathbf{x}'\boldsymbol{\alpha})} (\Psi(r_1) - E(\Psi(r_1)|\mathbf{x})) \quad (5)$$

where $w_1(\cdot)$ is a weight function to downweight extreme \mathbf{x} . Since r_1 need not be symmetric around 0, $E(\Psi(r_1)|\mathbf{x}) \neq 0$ in general, so we need an adjustment term to guarantee that $E(\psi_1^*(z; \theta)) = 0$. One can show by L'Hôpital's rule that $z^2 h(z)h(-z) \rightarrow 0$ as $z \rightarrow \infty$, implying that $\psi_1^*(\cdot; \theta)$ is bounded if $w_1(\cdot)$ is bounded, e.g., $w_1(\cdot) := 1$.

For the outcome equation, we have

$$\begin{aligned} E(y|d=1, \mathbf{x}) &= \mathbf{x}'\boldsymbol{\beta} + \sigma_{uz}h(\mathbf{x}'\boldsymbol{\alpha}) \\ \text{var}(y|d=1, \mathbf{x}) &= \sigma_w^2 \end{aligned}$$

where $\sigma_w^2 := \sigma_u^2 - \sigma_{uz}^2$. Let r_2 be the standardized prediction error of y given \mathbf{x} , i.e.,

$$r_2 := \frac{y - \mathbf{x}'\boldsymbol{\beta} - \sigma_{uz}h(\mathbf{x}'\boldsymbol{\alpha})}{\sigma_w}$$

which follows $N(0, 1)$ given $d=1$ and \mathbf{x} . We can write

$$\psi_2(z; \theta) = \begin{pmatrix} \mathbf{x} \\ h(\mathbf{x}'\boldsymbol{\alpha}) \end{pmatrix} \sigma_w r_2 d$$

Let

$$\psi_2^*(z; \theta) := w_2 \left(\begin{pmatrix} \mathbf{x} \\ h(\mathbf{x}'\boldsymbol{\alpha}) \end{pmatrix} \right) \begin{pmatrix} \mathbf{x} \\ h(\mathbf{x}'\boldsymbol{\alpha}) \end{pmatrix} \Psi(r_2) d \quad (6)$$

where $w_2(\cdot)$ is a weight function to downweight extreme $(\mathbf{x}', h(\mathbf{x}'\boldsymbol{\alpha}))'$. Since r_2 is symmetric around 0, $E(\Psi(r_1)|\mathbf{x}) = 0$, so $E(\psi_2^*(z; \theta)) = 0$ with no adjustment term. For $\psi_2^*(\cdot; \theta)$ to be bounded, however, $w_2((\mathbf{x}', h(\mathbf{x}'\boldsymbol{\alpha}))'(\mathbf{x}', h(\mathbf{x}'\boldsymbol{\alpha}))')$ must be bounded. Let

$$w_2 \left(\begin{pmatrix} \mathbf{x} \\ h(\mathbf{x}'\boldsymbol{\alpha}) \end{pmatrix} \right) := \begin{cases} 1 & \text{for } d((\mathbf{x}', h(\mathbf{x}'\boldsymbol{\alpha}))') \leq c \\ c/d((\mathbf{x}', h(\mathbf{x}'\boldsymbol{\alpha}))') & \text{for } d((\mathbf{x}', h(\mathbf{x}'\boldsymbol{\alpha}))') > c \end{cases}$$

where $c > 0$ and $d(\cdot)$ gives a distance from $\mathbf{0}$. Then $\psi_2^*(\cdot; \theta)$ is bounded.

The estimating functions (5) and (6) give the estimating equation that defines a robust M-estimator of θ , hence a robust Heckit estimator of β .

4 Illustration

4.1 Data

Sheen and Wang (2023, sec. 5) study the influence of monetary condition news on short- and medium-run household inflation expectations using consumer survey data from the MSC and macroeconomic data from FRED (Federal Reserve Economic Data) during 2008M12–2015M12. We use this work for our illustration because it is an interesting recent empirical work on household expectations published in a top journal, and because the replication data and code are available at the journal's website. Table 1 lists the variables used in the their analysis and ours. See Sheen and Wang (2023, sec. 2) for data description.

We replicated their results using R 4.4.1 developed by R Core Team (2024) and found two errors:

Table 1: Variables used to replicate Sheen and Wang (2023, p. 12)

Variable	Description
px1q1	prices up/down next year
px5q1	prices up/down next 5 years
px1q2	prices % up/down next year
px5q2	prices % up/down next 5 years
px1	price expectations 1yr recoded
px5	price expectations 5yr recoded
MPN	news: monetary condition
IN	news: inflation
yt1	income quartiles
age	age of respondent
female	female dummy
hsize	household size
edu	education of respondent
IP	industrial production (growth rate at $t - 1$)
UR	unemployment rate (at $t - 1$)
CPI	consumer price index (growth rate at $t - 1$)

1. They mistakenly use **px1q2/px5q2** as respondents' numerical inflation expectations. The questions are only for respondents expecting prices to go up/down, asking about the size of the change; hence these variables are missing if a respondent expects prices to stay the same, and positive even if a respondent expects prices to go down. We must use **px1/px5** instead.
2. For medium-run expectations, they mistakenly use **px1q2** instead of **px5q2** to construct a cross term, which makes even the sample size incorrect; see line 114 of their Stata code for variable definitions.

Table 2 replicates and corrects the two results in Sheen and Wang (2023, p. 12). The result for **px1q2** is identical to that in Sheen and Wang (2023, p. 12), but incorrect because of the first error. The result for **px1** is free from that error. The result for **px5q2** differs from that in Sheen and Wang (2023, p. 12) because of the second error, and still incorrect because of the first error. The result for **px5** is free from the two errors. Sheen and Wang (2023, p. 12) writes their main findings as follows:

The results show households do not adjust their inflation expectations upon hearing monetary condition news, neither in the short-term (1 year) nor long-term (5–10 years).

We see that fixing the two errors does not change their conclusion. For short-run expectations, the sign and significance of the effects of IP, UR, and CPI become more intuitive and consistent with their results for the direction of inflation; see Sheen and Wang (2023, p. 12). The corrected results still ignore DK responses, which may cause sample selection bias.

The replication data are not the full sample from the MSC during 2008M12–2015M12 but a subsample that excludes respondents with missing responses in variables of their interest; i.e., interest rate expectations, inflation expectations, and readiness to spend on houses, cars, and durable goods. Since a subsample

Table 2: Replication and correction of Sheen and Wang (2023, p. 12)

	px1q2	px1	px5q2	px5
MPN	0.13	0.23	0.18	-0.08
IN	0.35	0.45*	0.17	0.45**
Lpx1q2	0.32***			
MPN:Lpx1q2	0.05			
IN:Lpx1q2	0.01			
Lpx1		0.23***		
MPN:Lpx1		0.03		
IN:Lpx1		0.11**		
Lpx5q2			0.34***	
MPN:Lpx5q2			0.01	
IN:Lpx5q2			-0.04	
Lpx5				0.29***
MPN:Lpx5				0.09
IN:Lpx5				-0.07
ytl42	-0.59***	-0.41***	-0.28***	-0.23**
ytl43	-0.80***	-0.67***	-0.37***	-0.19*
ytl44	-1.05***	-0.91***	-0.46***	-0.27**
age	0.01**	0.01***	0.00	0.00
female	0.17*	0.33***	0.15***	0.19***
hsize	0.06*	0.09**	0.04*	0.05*
edu2	-0.29	-0.16	-0.25	-0.29
edu3	-0.31	-0.35	-0.36**	-0.28
edu4	-0.49*	-0.58*	-0.44**	-0.34*
IP	-0.19***	0.47***	-0.06	-0.01
UR	0.15***	-0.06*	0.10***	0.05**
CPI	0.16	1.17***	0.10	0.06
(Intercept)	2.03***	2.74***	1.69***	1.95***
Adj. R ²	0.18	0.11	0.16	0.11
Num. obs.	7785	10566	9956	10566

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$

Table 3: Summary statistics

Variable	N	Mean	SD	Min	Max	NA
px1	14386	3.45	4.07	-25	25	1151
px5	14231	3.17	2.91	-15	25	1306
MPN	15537	0.00071	0.19	-1	1	0
IN	15537	0.0077	0.23	-1	1	0
age	15537	56.70	16.15	18	97	0
hsize	15537	2.40	1.31	1	10	0
female	15537					
... No	7503	0.48				
... Yes	8034	0.52				
yt14	15537					
... 1	3343	0.22				
... 2	3804	0.24				
... 3	4124	0.27				
... 4	4266	0.27				
edu	15537					
... 1	687	0.044				
... 2	3418	0.22				
... 3	8298	0.53				
... 4	3134	0.20				

with no DK responses is not useful for our purpose, we obtain the original data from the websites of the MSC, FRED, and ALFRED (Archival FRED), and construct the full sample by ourselves. We reconstructed the replication data for double check, and successfully recovered all variables in Table 1 with correct values.

The MSC re-interviews respondents six months after their first interviews; hence one can construct a rotating panel with two waves. Following Sheen and Wang (2023), we use only wave 2 data to include a lagged dependent variable (wave 1 inflation expectations) as an explanatory variable, and exclude respondents with missing news or demographic variables. However, contrary to Sheen and Wang (2023), we keep respondents with missing responses in interest rate expectations, inflation expectations, and readiness to spend on durables; thus our sample includes DK responses in numerical inflation expectations. Table 3 shows summary statistics of our sample.

Following Sheen and Wang (2023), we further drop respondents with missing wave 1 inflation expectations, since they appear on the right-hand side of the regression equation. Table 4 is a cross table of the counts of observed/missing responses in the two waves for the percentage of inflation. Our sample sizes are 14160 (=13426+734) for short-run expectations and 14023 (=13234+789) for medium-run expectations.

4.2 Exclusion restriction

Though not necessary for identification, for precise estimation of a sample selection model, it is useful to have a variable that affects selection but not outcome directly (exclusion restriction); see, e.g., Wooldridge (2010, p. 806). Assuming that higher inflation uncertainty increases the likelihood of DK responses but

Table 4: Missing responses for the percentage of inflation

horizon	wave 2	wave 1	
		observed	missing
1 year	observed	13426	960
	missing	734	417
5 year	observed	13234	997
	missing	789	517

not the level of inflation expectations, we use the absolute difference of the CPI inflation rate in the previous month as our exclusion restriction.

Table 5 shows the ML estimates of the probit selection equations for **px1** and **px5**. We see that **px1** and **px5** tend to be observable for respondents with higher income and education, and tend to be missing for those with higher wave 1 inflation expectations, old, and female. We find that **px1** and **px5** tend to be missing when the absolute difference of the CPI inflation rate in the previous month is large, but the effects are insignificant. We still use this variable as our exclusion restriction, since it is better to have one than nothing.

4.3 Classical estimation

We use our full sample and reestimate the linear regression models for **px1** and **px5** in Table 2 by OLS as benchmarks. Then we estimate the sample selection models with the previous exclusion restriction for **px1** and **px5** by the ML and Heckit estimators, using an R package **sampleSelection** developed by Toomet and Henningsen (2008).

Table 6 compares the alternative estimates of the outcome equation for **px1**. We find that OLS and ML give almost identical results, whereas ML and Heckit give somewhat different results. The ML estimate of the correlation coefficient ρ between the errors in the selection and outcome equations is $\hat{\rho} = -0.01$, which suggests that the two equations are almost independent and hence DK responses are ignorable. The Heckit estimate of ρ is $\hat{\rho} = -0.72$, though the coefficient on the inverse Mills ratio term does not significantly differ from 0.

Table 7 compares the alternative estimates of the outcome equation for **px5**. We find here again that OLS and ML give almost identical results, whereas ML and Heckit give somewhat different results. The ML estimate of ρ is $\hat{\rho} = -0.01$, whereas the Heckit estimate of ρ is $\hat{\rho} = -1.30$, which is not in $[-1, 1]$ but now the coefficient on the inverse Mills ratio term significantly differs from 0.

The difference between the ML and Heckit estimates raises a concern about which is a better estimate. The ML estimator is asymptotically efficient under the correct specification of a bivariate normal distribution, whereas the Heckit estimator is consistent under weaker assumptions, requiring only a univariate normal distribution for the selection equation; see Olsen (1980). Thus one may conclude that the Heckit estimator is more reliable. However, OLS, ML, and Heckit are all not robust to outliers. Hence we need further analyses.

Table 5: ML estimates of the probit selection equations

	px1	px5
MPN	−0.13 (0.13)	−0.15 (0.16)
IN	−0.03 (0.12)	−0.10 (0.12)
Lpx1	−0.02 (0.00)***	
MPN:Lpx1	0.01 (0.02)	
IN:Lpx1	−0.02 (0.02)	
Lpx5		−0.03 (0.01)***
MPN:Lpx5		0.02 (0.03)
IN:Lpx5		0.01 (0.02)
ytl42	0.19 (0.05)***	0.20 (0.05)***
ytl43	0.39 (0.06)***	0.34 (0.05)***
ytl44	0.44 (0.06)***	0.40 (0.06)***
age	−0.01 (0.00)***	−0.01 (0.00)***
femaleTRUE	−0.37 (0.04)***	−0.28 (0.04)***
hsize	−0.03 (0.02)*	−0.03 (0.02)
edu2	0.44 (0.08)***	0.33 (0.08)***
edu3	0.47 (0.08)***	0.34 (0.08)***
edu4	0.45 (0.09)***	0.33 (0.09)***
IP	0.04 (0.03)	0.02 (0.03)
UR	−0.03 (0.01)	−0.03 (0.01)*
CPI	−0.10 (0.06)	−0.04 (0.06)
abs.dCPI	−0.12 (0.09)	−0.12 (0.08)
(Intercept)	1.91 (0.17)***	2.26 (0.17)***
Num. obs.	14160	14023

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$

Table 6: Alternative estimates of the outcome equation for `px1`

	OLS	ML	Heckit
MPN	0.17 (0.20)	0.17 (0.20)	0.22 (0.21)
IN	0.65 (0.18)***	0.65 (0.18)***	0.64 (0.19)***
Lpx1	0.24 (0.01)***	0.24 (0.01)***	0.25 (0.01)***
MPN:Lpx1	0.04 (0.04)	0.04 (0.04)	0.04 (0.04)
IN:Lpx1	0.08 (0.03)*	0.08 (0.03)*	0.09 (0.03)**
ytl42	-0.43 (0.10)***	-0.43 (0.10)***	-0.56 (0.14)***
ytl43	-0.65 (0.10)***	-0.65 (0.10)***	-0.87 (0.19)***
ytl44	-0.85 (0.11)***	-0.85 (0.11)***	-1.09 (0.20)***
age	0.01 (0.00)***	0.01 (0.00)***	0.01 (0.00)***
femaleTRUE	0.31 (0.06)***	0.31 (0.07)***	0.49 (0.15)***
hsize	0.08 (0.03)**	0.08 (0.03)**	0.10 (0.03)**
edu2	-0.08 (0.19)	-0.08 (0.19)	-0.44 (0.33)
edu3	-0.25 (0.18)	-0.26 (0.19)	-0.64 (0.34)
edu4	-0.51 (0.20)*	-0.51 (0.20)*	-0.88 (0.34)**
IP	0.42 (0.05)***	0.41 (0.05)***	0.39 (0.05)***
UR	-0.05 (0.02)*	-0.05 (0.02)*	-0.03 (0.03)
CPI	1.15 (0.11)***	1.15 (0.11)***	1.19 (0.11)***
(Intercept)	2.52 (0.31)***	2.53 (0.31)***	2.90 (0.42)***
sigma		3.68 (0.02)***	3.86
rho		-0.01 (0.05)	-0.72
invMillsRatio			-2.77 (2.00)
Adj. R ²	0.11		0.11
Num. obs.	13426	14160	14160
Censored		734	734
Observed		13426	13426

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$

Table 7: Alternative estimates of the outcome equation for px5

	OLS	ML	Heckit
MPN	−0.13 (0.19)	−0.13 (0.19)	−0.03 (0.22)
IN	0.53 (0.15)***	0.53 (0.15)***	0.58 (0.18)**
Lpx5	0.29 (0.01)***	0.29 (0.01)***	0.32 (0.01)***
MPN:Lpx5	0.06 (0.05)	0.06 (0.05)	0.05 (0.05)
IN:Lpx5	−0.07 (0.03)	−0.07 (0.03)	−0.06 (0.04)
ytl42	−0.19 (0.07)**	−0.20 (0.07)**	−0.41 (0.11)***
ytl43	−0.17 (0.07)*	−0.17 (0.07)*	−0.48 (0.14)***
ytl44	−0.22 (0.08)**	−0.22 (0.08)**	−0.57 (0.15)***
age	−0.00 (0.00)	−0.00 (0.00)	0.01 (0.00)*
femaleTRUE	0.20 (0.05)***	0.20 (0.05)***	0.42 (0.09)***
hsize	0.03 (0.02)	0.03 (0.02)	0.05 (0.03)*
edu2	−0.17 (0.14)	−0.18 (0.14)	−0.58 (0.21)**
edu3	−0.23 (0.13)	−0.23 (0.13)	−0.65 (0.21)**
edu4	−0.31 (0.14)*	−0.32 (0.14)*	−0.74 (0.22)***
IP	−0.02 (0.03)	−0.02 (0.03)	−0.05 (0.04)
UR	0.05 (0.02)**	0.05 (0.02)**	0.07 (0.02)***
CPI	0.15 (0.08)*	0.15 (0.08)*	0.16 (0.09)
(Intercept)	2.00 (0.22)***	2.00 (0.22)***	2.22 (0.27)***
sigma		2.62 (0.02)***	3.17
rho		−0.01 (0.05)	−1.30
invMillsRatio			−4.13 (1.42)**
Adj. R ²	0.11		0.11
Num. obs.	13234	14023	14023
Censored		789	789
Observed		13234	13234

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$

Table 8: Robust ML estimates of the probit selection equations

	px1	px5
MPN	−0.03 (0.16)	−0.17 (0.18)
IN	−0.02 (0.13)	−0.12 (0.14)
Lpx1	−0.02 (0.00)***	
MPN:Lpx1	−0.01 (0.03)	
IN:Lpx1	−0.02 (0.02)	
Lpx5		−0.03 (0.01)***
MPN:Lpx5		0.01 (0.04)
IN:Lpx5		0.01 (0.03)
yt142	0.21 (0.05)***	0.21 (0.05)***
yt143	0.37 (0.06)***	0.32 (0.06)***
yt144	0.42 (0.07)***	0.41 (0.07)***
age	−0.01 (0.00)***	−0.01 (0.00)***
femaleTRUE	−0.36 (0.04)***	−0.29 (0.04)***
hsize	−0.04 (0.02)*	−0.03 (0.02)
edu2	0.40 (0.08)***	0.30 (0.08)***
edu3	0.42 (0.08)***	0.30 (0.08)***
edu4	0.42 (0.09)***	0.30 (0.10)**
IP	0.03 (0.03)	0.02 (0.03)
UR	−0.02 (0.02)	−0.03 (0.01)*
CPI	−0.12 (0.07)	−0.03 (0.07)
abs.dCPI	−0.09 (0.10)	−0.10 (0.09)
(Intercept)	1.95 (0.19)***	2.29 (0.19)***
Num. obs.	14160	14023

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$

4.4 Robust estimation

We reestimate the sample selection models for **px1** and **px5** by a robust Heckit estimator, using an R package **ssmrob** developed by Zhelonkin and Ronchetti (2021). For both the selection and outcome equations, we set the bound for the Huber function as $K := 1.345$, which is the default value in **ssmrob** package and also a common choice in the literature. The resulting robust estimator has 95% asymptotic efficiency relative to the ML estimator when the true distribution is normal; see de Menezes, Prata, Secchi, and Pinto (2021, p. 10) and references there.

We set $w_1(\cdot) := 1$ since we need no weight for the selection equation. Let \mathbf{X} be the $n \times k$ matrix of regressors in the outcome equation including the inverse Mill's ratio term. Let $\mathbf{H} := \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}$ be the hat matrix. For the outcome equation, we set for $i = 1, \dots, n$,

$$w_2(\mathbf{x}_i) := \sqrt{1 - h_{ii}}$$

where $h_{ii} := \mathbf{x}_i'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_i$ is the i th diagonal entry of \mathbf{H} . Naghi, Váradi, and Zhelonkin (2024, p. 4) note that this weight function is simple and stable when regressors include dummy variables.

Table 8 shows the robust ML estimates of the probit selection equations for **px1** and **px5**. The results are very close to the classical ML estimates in Table 5.

Table 9 compares the classical and robust Heckit estimates of the outcome

Table 9: Robust Heckit estimates of the outcome equations

	px1		px5	
	$K = 100$	$K = 1.345$	$K = 100$	$K = 1.345$
MPN	0.22 (0.25)	0.12 (0.19)	-0.03 (0.30)	0.15 (0.22)
IN	0.64 (0.19)***	0.60 (0.14)***	0.58 (0.21)**	0.43 (0.19)*
Lpx1	0.25 (0.01)***	0.24 (0.02)***		
MPN:Lpx1	0.04 (0.06)	0.04 (0.06)		
IN:Lpx1	0.09 (0.05)	0.04 (0.05)		
Lpx5			0.32 (0.02)***	0.31 (0.02)***
MPN:Lpx5			0.05 (0.10)	-0.01 (0.06)
IN:Lpx5			-0.06 (0.06)	-0.04 (0.06)
yt142	-0.56 (0.17)***	-0.31 (0.32)	-0.41 (0.14)**	-0.38 (0.20)
yt143	-0.88 (0.23)***	-0.41 (0.48)	-0.48 (0.17)**	-0.44 (0.27)
yt144	-1.09 (0.24)***	-0.55 (0.51)	-0.57 (0.19)**	-0.48 (0.31)
age	0.01 (0.00)***	0.01 (0.01)	0.01 (0.00)*	0.01 (0.01)
femaleTRUE	0.49 (0.17)**	0.13 (0.39)	0.42 (0.11)***	0.26 (0.20)
hsize	0.10 (0.03)**	0.02 (0.05)	0.05 (0.03)*	0.04 (0.03)
edu2	-0.45 (0.40)	0.11 (0.73)	-0.58 (0.27)*	-0.41 (0.35)
edu3	-0.64 (0.40)	0.04 (0.75)	-0.65 (0.27)*	-0.41 (0.36)
edu4	-0.88 (0.40)*	-0.12 (0.75)	-0.74 (0.28)**	-0.44 (0.37)
IP	0.39 (0.07)***	0.29 (0.06)***	-0.05 (0.05)	-0.04 (0.04)
UR	-0.03 (0.02)	-0.05 (0.03)	0.07 (0.02)***	0.08 (0.02)***
CPI	1.19 (0.15)***	0.89 (0.15)***	0.16 (0.09)	0.13 (0.07)
(Intercept)	2.90 (0.49)***	2.16 (0.80)**	2.22 (0.32)***	1.77 (0.32)***
sigma	3.86	3.70	3.17	3.13
IMR1	-2.78 (2.49)	0.61 (6.23)	-4.13 (1.92)*	-3.90 (3.54)
Num. obs.	14160	14160	14023	14023
Censored	734	734	789	789
Observed	13426	13426	13234	13234

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$

equations for **px1** and **px5**. When $K := 100$, the Huber function does not bind; hence the estimates are identical to the classical Heckit estimates in Tables 6 and 7. The standard errors differ, however, since **ssmrob** package uses White's heteroskedasticity-consistent standard errors. Comparing the classical ($K := 100$) and robust ($K := 1.345$) estimates, we find the following:

1. For both **px1** and **px5**, the estimates of the coefficients on the news variables, wave 1 inflation expectations, cross terms, and macroeconomic variables do not change qualitatively.
2. For both **px1** and **px5**, the estimates of the coefficients on the demographic variables become insignificant; hence the classical Heckit estimates may not be robust.
3. The estimates of the coefficients on the inverse Mill's ratio terms become insignificant not only for **px1** but also for **px5**; thus sample selection bias may not be a problem in this application.

To summarize, we find some evidences that the classical and robust Heckit estimates differ, but no evidence that DK responses in the MSC cause sample

selection bias during 2008M12–2015M12. Throughout our analyses, we find no evidence that monetary condition news directly influences household inflation expectations. Hence our analyses supports the conclusion by Sheen and Wang (2023) that households did not adjust their inflation expectations upon hearing monetary condition news during the ‘zero lower bound’ period.

5 Discussion

DK responses are common in surveys on inflation expectations. Ignoring them in a regression analysis may cause sample selection bias. One can use a sample selection model to avoid the bias. If robustness is a concern, then one can use a robust Heckit estimator.

This paper assumes that the specification of the standard sample selection model is approximately correct (local misspecification). If this model specification is incorrect even approximately, then one may prefer a semi/non-parametric approach; e.g., Das, Newey, and Vella (2003) and Newey (2009). A semi/non-parametric estimator is not robust, however, if it has an unbounded influence function. Robust semi/non-parametric estimation of a sample selection model is an interesting and important topic for future work.

This paper still ignores two types of missing responses: (1) DK responses in the explanatory variables and (2) unit nonresponses. Ignoring the former does not cause sample selection bias, but efficiency improves by including them using DK dummies. Many empirical works in economics ignore the latter and treat the data as simple random samples. One can use the design weights to adjust for various factors including unit nonresponses. Though such weights are useful, sample selection bias remains if unit nonresponses are nonignorable. Correcting for the bias requires some information on nonrespondents; e.g, Korinek, Mistiaen, and Ravallion (2007) use the geographic structure of nonresponse rates to estimate the response probability function for reweighting, and Akande, Madson, Hillygus, and Reiter (2021) use the population counts of some demographic variables to impute missing values. Addressing the issue of nonignorable unit nonresponses may further change the results of Sheen and Wang (2023), and of many other empirical works as well.

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