Part IV: Problem Set 1

Yongseok Kim

April 26, 2020

1. (a) See Figure 1.
   (b) See Figure 2.
   (c) See Figure 3.
2. (a) See Figure 4.
   (b) See Figure 4. Compared to $\hat{\alpha} + \hat{\beta} x_i$, $\hat{m}(x_i)$ is much more volatile.
   (c) See Figure 5.
   (d) See Table 1 for h_n^{cv}. See Figure 6 for the local linear estimator using this bandwidth. Compared to $\tilde{m}(x)$ (dashed line) obtained in Part (c), $\tilde{m}^{cv}(x)$ (solid line) is more smooth when $h_n^{cv} > h_n$.
   (e)
3. (a) Note

$$
\begin{aligned}
\mathbb{E}[\varepsilon_i | x_i] &= \mathbb{E}[\mathbb{E}[\varepsilon_i | w_i, x_i] | x_i] \\
&= 0
\end{aligned}
$$

$$\mathbb{E}[y_i | x_i] = \mathbb{E}[w_i | x_i]\beta + m(x_i)$$

   Then

$$y_i - \mathbb{E}[y_i | x_i] = (w_i - \mathbb{E}[w_i | x_i])\beta + \varepsilon_i$$

   (b) See Figure 7.
   (c) See Table 2.
   (d) See Figure 8.
   (e) See Figure 8. Compared to $\hat{m}(x)$ (dashed lines), $\hat{m}^{PL}(x)$ (solid lines) are shifted downward.

Table 1: h_n^{cv}

This figure presents h_n for each kernel estimators. h_n denotes the bandwidth set by the rule of thumb. h_cv denotes the bandwidth set by minimizing the cross-validation function.

	h_n	h_cv
Gauss	2.717	2.073
Epanechnikov	2.717	4.631
Parzen	2.717	7.042
Bartlett	2.717	5.000

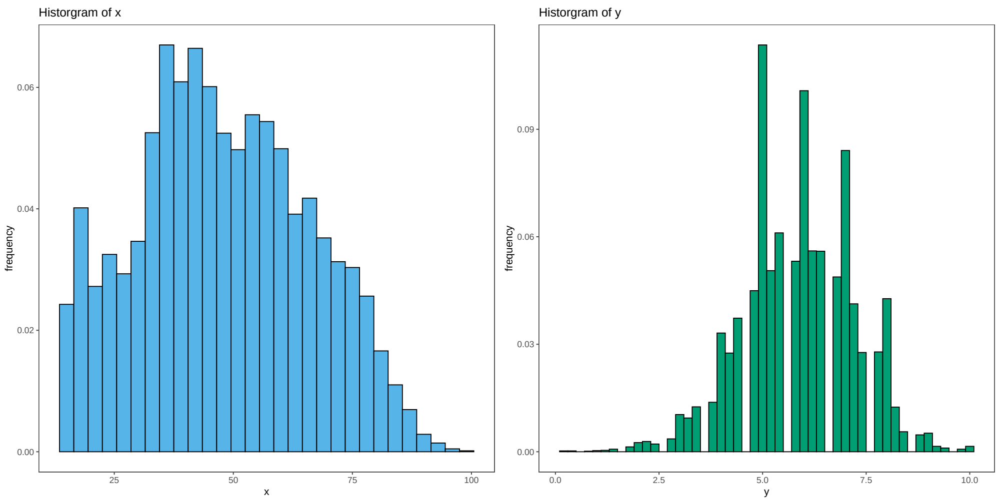


Figure 1: Histogram

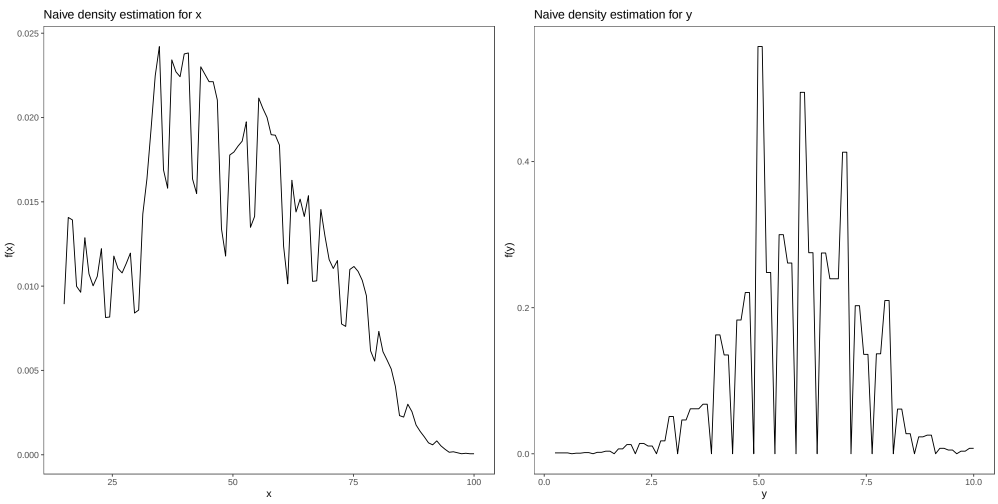
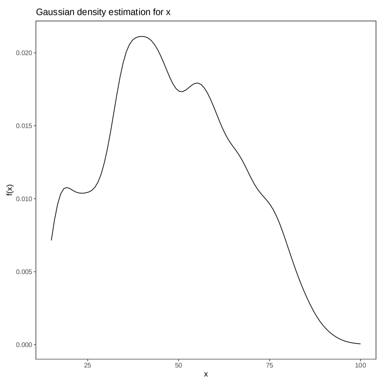
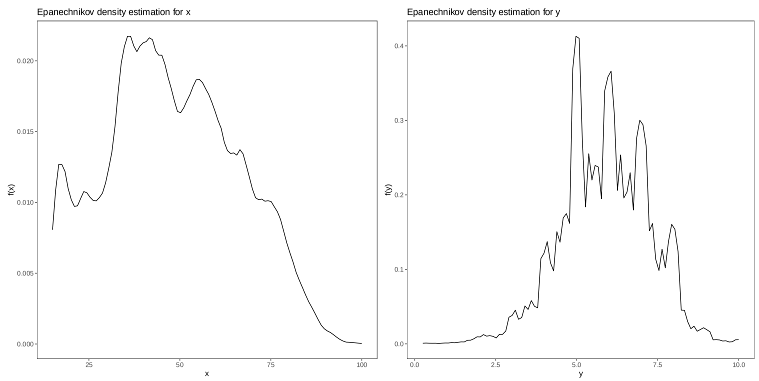


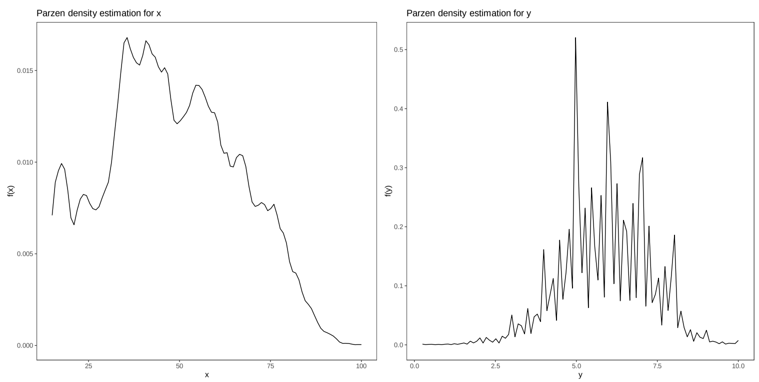
Figure 2: Naive density estimator



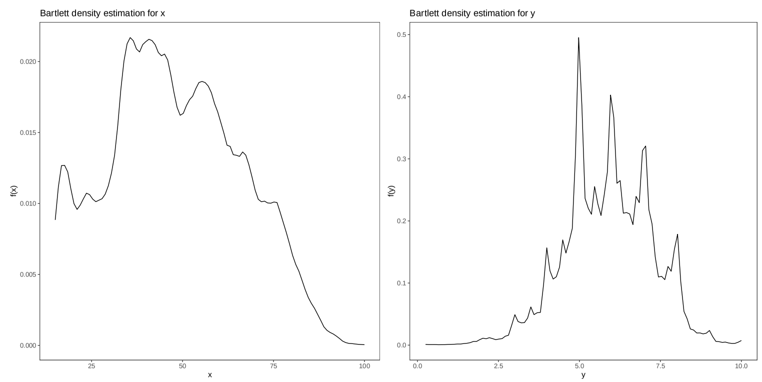
(a) Gaussian



(b) Epanechnikov

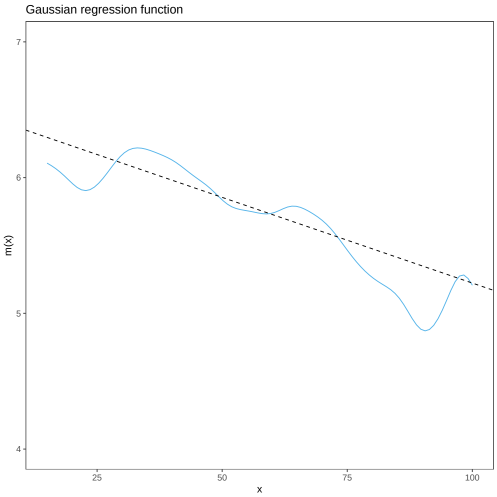


(c) Parzen

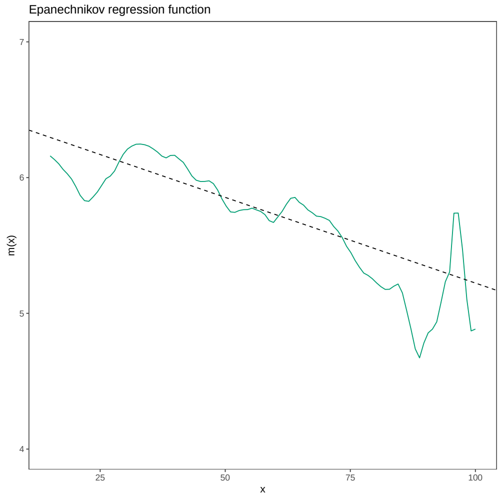


(d) Bartlett

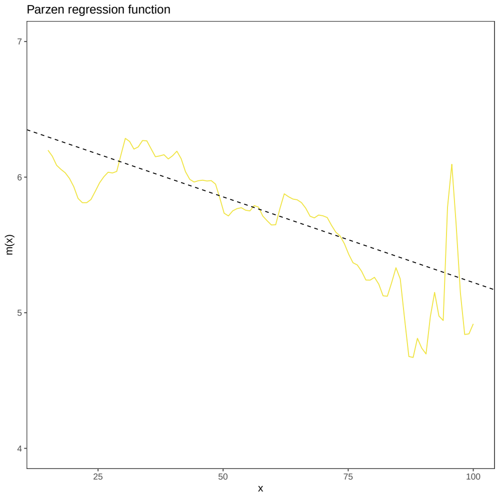
Figure 3: Kernel density estimator



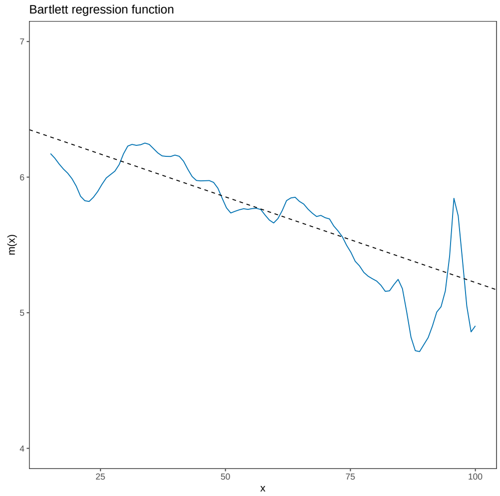
(a) Gaussian



(b) Epanechnikov



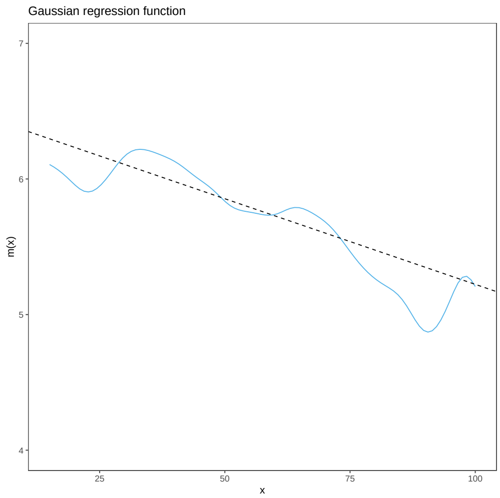
(c) Parzen



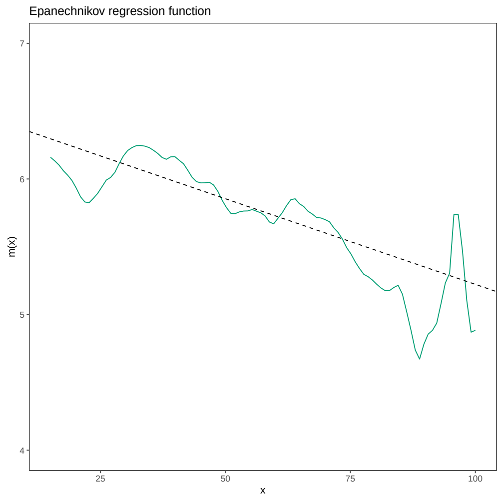
(d) Bartlett

Figure 4: The local constant estimation

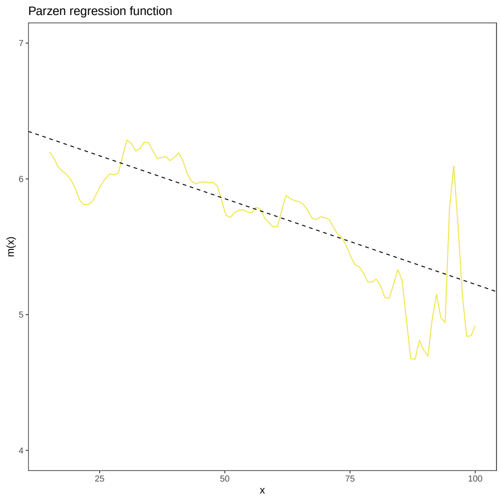
This figure presents the local consant estimation method with the four Kernal functions: Gaussian, Epanechnikov, Parzen, and Bartlett. The dashed line represents the linear regression model $\hat{y}_i = \hat{\alpha} + \hat{\beta} x_i$.



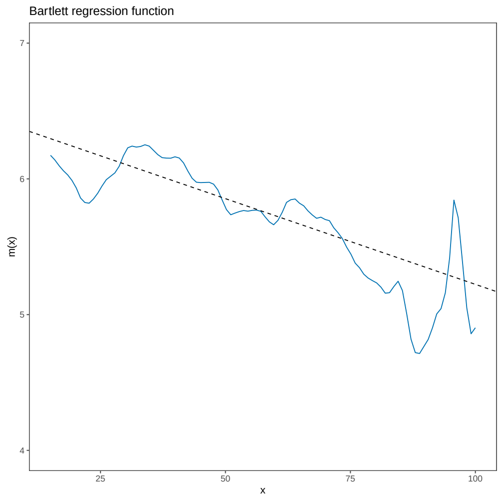
(a) Gaussian



(b) Epanechnikov



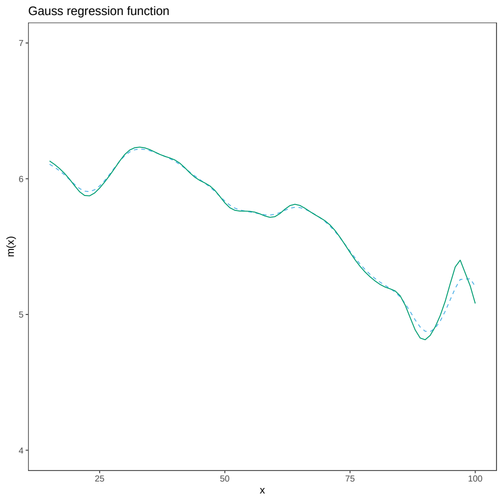
(c) Parzen



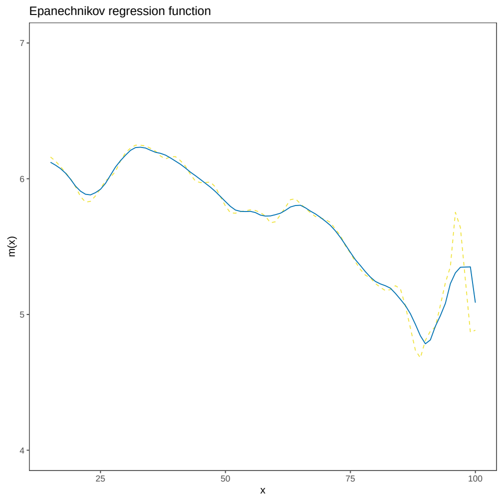
(d) Bartlett

Figure 5: The local linear estimation

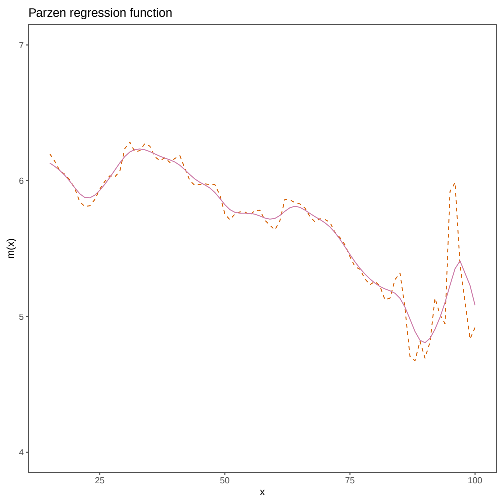
This figure presents the local linear estimation method with the four Kernal functions: Gaussian, Epanechnikov, Parzen, and Bartlett. The bandwidth h_n is set by the rule of thumb, i.e. $h_n(x) = \hat{\sigma}_n n^{-1/5}$. The dashed line represents the linear regression model $\hat{y}_i = \hat{\alpha} + \hat{\beta} x_i$.



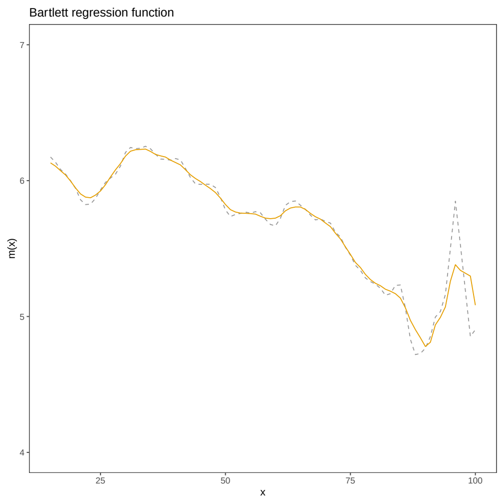
(a) Gaussian



(b) Epanechnikov



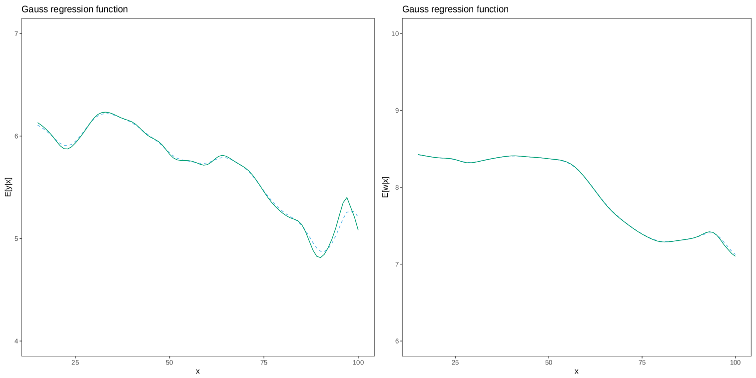
(c) Parzen



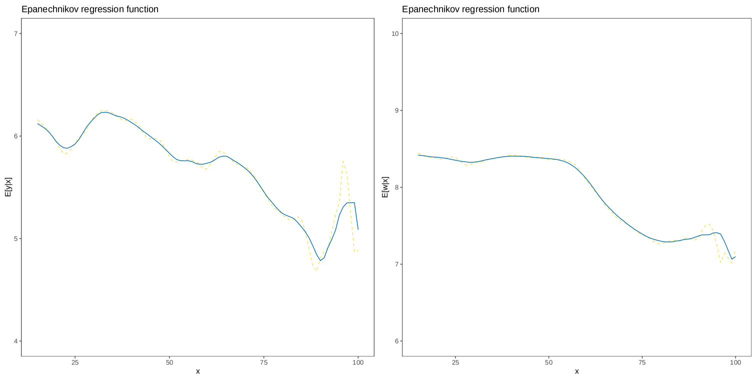
(d) Bartlett

Figure 6: The local linear estimation

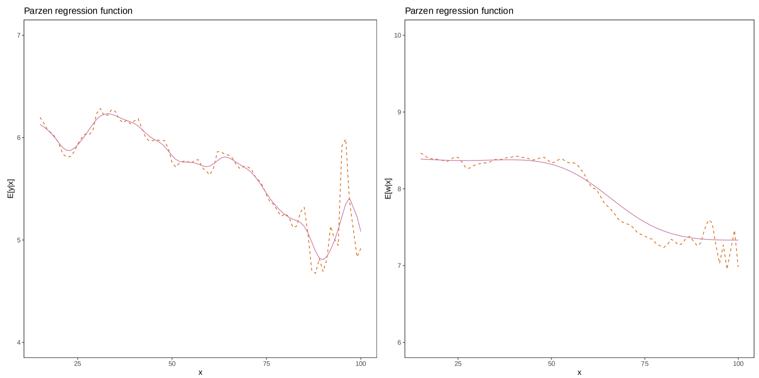
This figure presents the local linear estimation method with the four Kernal functions: Gaussian, Epanechnikov, Parzen, and Bartlett. The bandwidth h_{cv} is set by $\arg\min_h CV(h)$. The dashed line represents the local linear estimation method using the bandwidth h_n set by the rule of thumb.



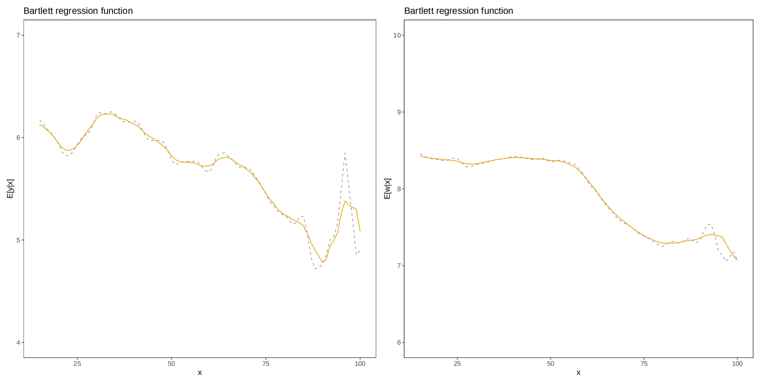
(a) Gaussian



(b) Epanechnikov



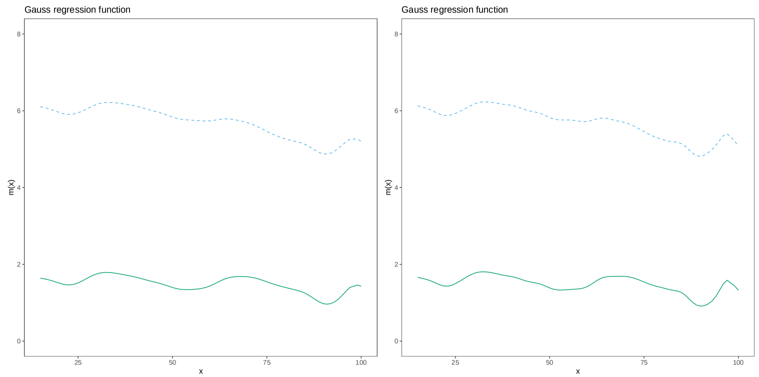
(c) Parzen



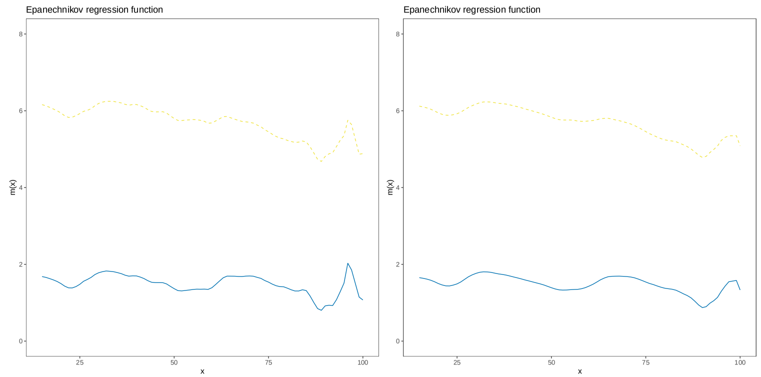
(d) Bartlett

Figure 7: $\hat{\mathbb{E}}[y_i|x_i]$ and $\hat{\mathbb{E}}[w_i|x_i]$ by the local constant estimation

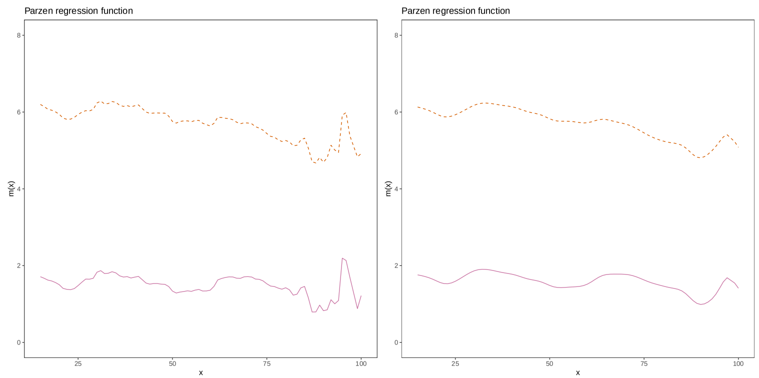
This figure shows $\hat{\mathbb{E}}[y_i|x_i]$ and $\hat{\mathbb{E}}[w_i|x_i]$ from various local constant estimators. The left figure of each figure is $\hat{\mathbb{E}}[y_i|x_i]$. The right figure of each figure is $\hat{\mathbb{E}}[w_i|x_i]$. Dashed lines represent local constant estimator using the bandwidth set by the rule of thumb while solid lines represent local constant estimator using the bandwidth set by minimizing the cross-validation function.



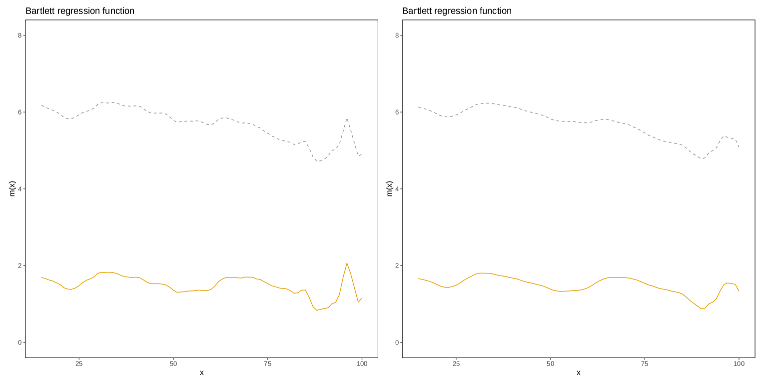
(a) Gaussian



(b) Epanechnikov



(c) Parzen



(d) Bartlett

Figure 8: $\hat{m}^{PL}(x)$ and $\hat{m}(x)$ by the local constant estimation

This figure shows $\hat{m}^{PL}(x)$ and $\hat{m}(x)$ from various local constant estimators. Dashed lines represent $\hat{m}(x)$ while solid lines represent $\hat{m}^{PL}(x)$. The left figure of each figure is $\hat{m}^{PL}(x)$ and $\hat{m}(x)$ by using $\hat{\beta}$ from the local constant estimator using the bandwidth set by the rule of thumb. The right figure of each figure is $\hat{m}^{PL}(x)$ and $\hat{m}(x)$ by using $\hat{\beta}$ from the local constant estimator using the bandwidth set by minimizing the cross-validation function.

Table 2: $\hat{\beta}$

This table presents $\hat{\beta}$ from various local constant estimators obtained in Part (b). beta_rot denotes $\hat{\beta}$ from the local constant estimator using the bandwidth set by the rule of thumb. beta_cv denotes $\hat{\beta}$ from the local constant estimator using the bandwidth set by minimizing the cross-validation function.

	beta_rot	beta_cv
gauss	0.530	0.530
epanechnikov	0.530	0.530
parzen	0.530	0.519
bartlett	0.530	0.530

A Appendix

For R codes to get the above results, see `https://github.com/ysugk/Course-ECON572/blob/master/code/EmpiricalHW-partIV.R`