

# Assignment 3

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April 22, 2020

1. (a) See Table 1 for the summary statistics.
- (b) Under the lognormality assumption, we have

$$\begin{aligned}\log \mathbb{E}_t[\delta(\frac{C_{t+1}}{C_t})^{-\gamma} R_{i,t+1}] &= \mathbb{E}_t[\log \delta(\frac{C_{t+1}}{C_t})^{-\gamma} R_{i,t+1}] + \frac{1}{2} \text{var}_t[\log \delta(\frac{C_{t+1}}{C_t})^{-\gamma} R_{i,t+1}] \\ &= \log 1 \\ &= 0\end{aligned}$$

Note

$$\begin{aligned}\log \delta(\frac{C_{t+1}}{C_t})^{-\gamma} R_{i,t+1} &= \log \delta - \gamma \Delta c_{t+1} + r_{i,t+1} \\ &= {}_d \mathbb{N}(\mu, \sigma^2)\end{aligned}$$

where

$$\begin{aligned}\mu &= \log \delta - \gamma \mathbb{E} \Delta c_{t+1} + \mathbb{E} r_{i,t+1} \\ \sigma^2 &= \gamma^2 \sigma_c^2 + \sigma_i^2 - 2\gamma \sigma_{ic}\end{aligned}$$

Then, for the risk-free asset, we have

$$\begin{aligned}\log \delta - \gamma \mathbb{E} \Delta c_{t+1} + r_{f,t+1} + \frac{1}{2} \gamma^2 \sigma_c^2 &= 0 \\ r_{f,t+1} &= -\log \delta + \gamma \mathbb{E} \Delta c_{t+1} - \frac{1}{2} \gamma^2 \sigma_c^2\end{aligned}\tag{1}$$

For the asset  $i$ , we have

$$\log \delta - \gamma \mathbb{E} \Delta c_{t+1} + \mathbb{E} r_{i,t+1} + \frac{1}{2} (\gamma^2 \sigma_c^2 + \sigma_i^2 - 2\gamma \sigma_{ic})\tag{2}$$

Substituting (1) into (2) yields

$$\mathbb{E}(r_{i,t+1} - r_{f,t+1}) + 0.5 \sigma_i^2 = \gamma \sigma_{ic}\tag{3}$$

- (c) See Table 2 for moments and correlations.

- (d) By rearranging (3), we obtain

$$\gamma = \frac{\mathbb{E}(r_{i,t+1} - r_{f,t+1}) + 0.5 \sigma_i^2}{\sigma_{ic}}$$

Table 1: Summary statistics

This table summarizes number of observations ( $n$ ), means ( $mean$ ), standard deviations ( $sd$ ), and first-order autocorrelation coefficients ( $ar1coef$ ) for following variables: real consumption growth ( $gc\_real$ ), real return on the aggregate stock portfolio ( $rm\_real$ ), real T-bill return ( $rf\_real$ ), the excess return on the two ( $erm$ ).

(a) Annual

	n	mean	sd	ar1coef
gc_real	86	-1.303	3.766	0.527
rm_real	87	7.982	20.372	-0.021
rf_real	87	0.680	4.138	0.632
erm	87	7.302	20.520	0.022

(b) Quarterly

	n	mean	sd	ar1coef
gc_real	275	-1.586	2.297	0.407
rm_real	276	8.263	16.448	0.079
rf_real	276	1.017	1.947	0.354
erm	276	7.246	16.329	0.081

Table 2: Moments and correlations

This table presents moments (left) and correlations (right) for following variables: log consumption growth ( $dc$ ), log return on the stock ( $r$ ), log return on the T-bill ( $rf$ ).

(a) Annual

mean		sd	dc		r	rf
dc	0.018	0.021	dc	1		
r	0.058	0.203	r	0.09	1	
rf	0.006	0.042	rf	-0.34	0.09	1

(b) Quarterly

mean		sd	dc		r	rf
dc	0.005	0.005	dc	1		
r	0.017	0.083	r	0.18	1	
rf	0.002	0.010	rf	0.12	0.11	1

The first measure is calculated as

$$\begin{aligned}\gamma_1 &= \frac{\mathbb{E}(r_{i,t+1} - r_{f,t+1}) + 0.5\sigma_i^2}{\sigma_i\sigma_c\rho_{ic}} \\ &= \begin{cases} \frac{0.058-0.006+0.5\times(0.203)^2}{0.203\times0.021\times0.09} = 189.2368 & \text{annual} \\ \frac{0.017-0.002+0.5\times(0.083)^2}{0.083\times0.005\times0.18} = 246.9143 & \text{quarterly} \end{cases}\end{aligned}$$

The second measure is calculated as

$$\begin{aligned}\gamma_2 &= \frac{\mathbb{E}(r_{i,t+1} - r_{f,t+1}) + 0.5\sigma_i^2}{\sigma_i\sigma_c} \\ &= \begin{cases} \frac{0.058-0.006+0.5\times(0.203)^2}{0.203\times0.021} = 17.0313 & \text{annual} \\ \frac{0.017-0.002+0.5\times(0.083)^2}{0.083\times0.005} = 44.4446 & \text{quarterly} \end{cases}\end{aligned}$$

An a priori justification for restricting the value of  $\gamma$  is a maximum of ten (Mehra and Prescott, 1985). Therefore,  $\gamma$  backed out from the estimates are too big to be justified. This result is a well-known equity premium puzzle.

(e) Since  $\theta$  is overidentified, we need to consider

$$\hat{\theta} = \arg \min_{\theta} m_T(\theta)' W_T m_T(\theta)$$

for some weighting matrix  $W_T$  where

$$\begin{aligned}\theta &= (\delta, \gamma) \\ h(x_t, \theta) &= \begin{bmatrix} \delta(\frac{C_{t+1}}{C_t})^{-\gamma} R_{s,t+1} z_t - z_t \\ \delta(\frac{C_{t+1}}{C_t})^{-\gamma} R_{f,t+1} z_t - z_t \end{bmatrix}_{6 \times 1} \\ m_T(\theta) &= \frac{1}{T} \sum_t h(x_t, \theta)\end{aligned}$$

Consider the optimal GMM estimator with

$$W_T = \hat{V}_T^{-1}$$

where  $\hat{V}_T$  is a consistent estimate of  $V = \mathbb{E}h(x_i, \theta)h(x_i, \theta)'$ . Define

$$\begin{aligned}M_T(\theta) &= \frac{1}{T} \sum_t \frac{\partial}{\partial \theta'} h(x_t, \theta) \\ &= \begin{bmatrix} \frac{1}{T} \sum_t (\frac{C_{t+1}}{C_t})^{-\gamma} R_{s,t+1} z_t & \frac{1}{T} \sum_t \delta (\frac{C_{t+1}}{C_t})^{-\gamma} (-\log \frac{C_{t+1}}{C_t}) R_{s,t+1} z_t \\ \frac{1}{T} \sum_t (\frac{C_{t+1}}{C_t})^{-\gamma} R_{f,t+1} z_t & \frac{1}{T} \sum_t \delta (\frac{C_{t+1}}{C_t})^{-\gamma} (-\log \frac{C_{t+1}}{C_t}) R_{f,t+1} z_t \end{bmatrix}\end{aligned}$$

Then, under the regularity conditions, we have

$$\sqrt{T}(\hat{\theta} - \theta_0) \rightarrow_d \mathbb{N}(0, [M'V^{-1}M]^{-1})$$

For a large  $T$ , we may approximate

$$\hat{\theta} \approx_d \mathbb{N}(\theta_0, \hat{\Omega})$$

Table 3: Results for the GMM two-step estimator

This table shows results for the GMM two-step estimator.

	firststage	secondstage	se	teststat	pvalue
delta	1	1	0.418	2.391	0.017
gamma	41.279	19.347	20.818	0.929	0.353
J				20.702	0.0004

where

$$\hat{\Omega} = \frac{1}{T} [M_T(\hat{\theta})' \hat{V}_T^{-1} M_T(\hat{\theta})]^{-1}$$

Moreover, since

$$\sqrt{T} m_T(\theta) \rightarrow_d \mathbb{N}(0, V)$$

we may approximate

$$\begin{aligned} J(\hat{\theta}) &= T m_T(\hat{\theta})' \hat{V}_T^{-1} m_T(\hat{\theta}) \\ &\approx_d \chi_{p-q}^2 \end{aligned}$$

where  $p$ : number of moment conditions,  $q$ : number of parameters. To estimate  $\hat{\theta}$  and  $\hat{V}_T$ , I proceed in the following steps. First, I estimate  $\hat{\theta}$  using the identity matrix  $I$  as weighting matrix. Second, I estimate a Newey-West estimator of the covariance matrix  $\hat{V}_T$  with one lag using  $\hat{\theta}$ . Lastly, I re-estimate  $\hat{\theta}$  using  $\hat{V}_T$  as a weighting matrix. For the numerical procedure, I employ L-BFGS-B method for the optimization (Byrd et. al., 1995) which is a quasi-Newton method allowing box constraints. I restrict

$$\delta \in [0, 1]$$

$$\gamma \in [0, \infty)$$

and set the value of initial parameters as  $\hat{\theta} = (0.9, 30)$ . I find that results are very sensitive not only to the value of initial parameters, but also to optimization methods. Table 3 presents the results. For  $\hat{\delta}$  and  $\hat{\gamma}$ , I compute  $t$ -ratios as

$$t(\hat{\delta}) = \frac{\hat{\delta} - \delta_0}{\sqrt{\hat{\Omega}_{11}}}$$

$$t(\hat{\gamma}) = \frac{\hat{\gamma} - \gamma_0}{\sqrt{\hat{\Omega}_{22}}}$$

and evaluate their two-sided  $p$ -values from  $\mathbb{N}(0, 1)$  under  $H_0 : \delta_0 = 0$  and  $H_0 : \gamma_0 = 0$ . For  $J(\hat{\theta})$ , I evaluate its  $p$ -value from  $\chi_4^2$ .

(f) Consider a multivariate regression model

$$Y_{T \times 10} = X_{T \times 2} B_{2 \times 10} + U_{T \times 10}$$

Assume

$$\text{var}(U) = I_T \otimes \Sigma$$

where

$$\Sigma = \mathbb{E}u_t u_t'$$

for any  $t$ . Then, the LS estimate yields

$$\hat{B} = (X'X)^{-1}XY$$

and

$$\hat{\Sigma} = \frac{1}{T}\hat{U}'\hat{U}$$

where

$$\hat{U} = Y - X\hat{B}$$

Under the regularity condition, we have

$$\sqrt{T}(\hat{B} - B) \rightarrow_d \mathbb{N}(0, M^{-1} \otimes \Sigma)$$

where

$$\frac{X'X}{T} \rightarrow_p M$$

For a large  $T$ , we may approximate

$$\hat{B} \approx_d \mathbb{N}(B, \hat{\Omega})$$

where

$$\begin{aligned} \hat{\Omega} &= \frac{1}{T}(M^{-1} \otimes \hat{\Sigma}) \\ &= (X'X)^{-1} \otimes \hat{\Sigma} \end{aligned}$$

Table 4 presents the estimates of  $\alpha_i$  and  $\beta_{i,M}$  and standard errors on them.

- (g) The results of hypothesis testing for decile portfolios from 1 to 10 is reported in Table 4. For decile portfolio  $i$ , I compute  $t$ -ratio

$$t(\hat{\alpha}_i) = \frac{\hat{\alpha}_i - \alpha_0}{\sqrt{\hat{\Omega}_{i,i}}}$$

and evaluate their two-sided  $p$ -values from  $\mathbb{N}(0,1)$  under  $H_0 : \alpha_0 = 0$ . Except the decile 1 portfolio, the null hypotheses are not rejected at 5 percent of significance.

- (h) For Fama-Macbeth regression, I employ  $\hat{\beta}_{i,M}$  estimated from part (f). A cross-sectional  $R^2$  is calculated as

$$R^2 = \frac{\sum(\hat{\lambda}_{0,FM} + \hat{\lambda}_{M,FM}\hat{\beta}_{i,M})^2}{\sum\hat{\mu}_i^2}$$

Table 5 reports the results. The CAPM predicts  $\lambda_0 = 0$ . By the rule of thumb, we cannot reject the null hypothesis  $H_0 : \lambda_0 = 0$ , suggesting the CAPM holds in the data. Figure 1 show the relationships between actual and predicted mean excess returns. The CAPM also predicts a linear relationship between beta and excess returns. While decile 1 portfolio yields higher returns than the model predicts (*size anomaly*), beta and excess returns seem to have a well-fitted linear relationship.

Table 4: Results for the multivariate regression

This table shows results for the multivariate regression.

	1	2	3	4	5	6	7	8	9	10
alpha	0.018	0.007	0.003	0.003	0.001	0.001	0.001	0.001	0.002	-0.0002
beta	1.282	1.246	1.243	1.233	1.225	1.220	1.198	1.170	1.117	0.956
se_alpha	0.007	0.006	0.005	0.004	0.004	0.003	0.003	0.002	0.002	0.001
se_beta	0.084	0.068	0.055	0.049	0.042	0.038	0.031	0.026	0.019	0.007
t_alpha	2.525	1.196	0.613	0.690	0.302	0.241	0.264	0.314	1.232	-0.368
t_beta	15.218	18.313	22.788	25.192	28.980	32.330	39.180	45.539	57.507	144.245
p_alpha	0.012	0.232	0.540	0.490	0.762	0.809	0.792	0.754	0.218	0.713
p_beta	0	0	0	0	0	0	0	0	0	0

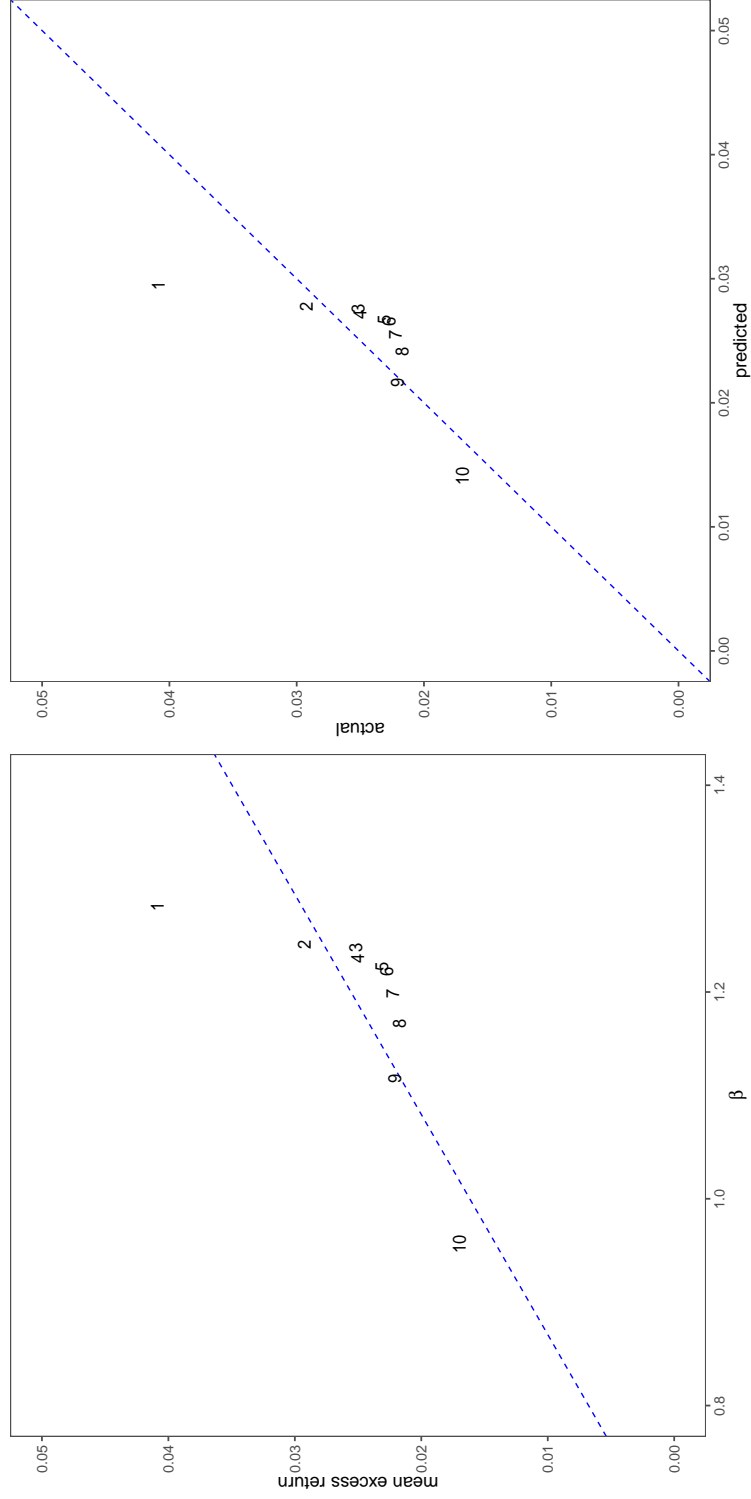


Figure 1: Cross-sectional regressions

This figure shows results of the cross-sectional regression. On the left hand side, I plot the actual mean excess returns versus predicted mean excess returns from the model. The dashed line represents a 45-degree line. Labels denote deciles of portfolios.

Table 5: Results for Fama-Macbeth estimator

This table shows results for Fama-Macbeth estimator.

	estimate	se	t
lambda0	-0.031	0.020	-1.507
lambdaM	0.047	0.020	2.311
rsq	0.971		

- (i) Table 6 report the estimates of  $\beta_{i,G}$ . Further, Table 7 report the results for Fama-Macbeth regression. Note

$$\begin{aligned}\mathbb{E}_t M_{t+1} R_{t+1} &= \mathbb{E}_t M_{t+1}^2 \text{cov}_t(R_{M,t+1}, R_{t+1}) + \frac{1}{R_{f,t+1}} \mathbb{E}_t R_{t+1} \\ &= 1\end{aligned}$$

$$\mathbb{E}_t M_{t+1}^2 \text{var}_t(R_{M,t+1}) + \frac{1}{R_{f,t+1}} \mathbb{E}_t R_{M,t+1} = 1$$

$$\begin{aligned}\mathbb{E}_t R_{t+1} - R_{f,t+1} &= -R_{f,t+1} \mathbb{E}_t M_{t+1}^2 \text{cov}_t(R_{M,t+1}, R_{t+1}) \\ &= (\mathbb{E}_t R_{M,t+1} - R_{f,t+1}) \beta_M\end{aligned}$$

where

$$\beta_M = \frac{\text{cov}_t(R_{M,t+1}, R_{t+1})}{\text{var}_t(R_{M,t+1})}$$

Motivated by the consumption-based CAPM model, suppose

$$M_t = \phi(G_t)$$

Under the regularity condition (e.g., CRRA with  $\gamma \geq 0$ ), it is natural to expect

$$G_t \uparrow \Rightarrow M_t \downarrow$$

Hence,  $\beta_{i,G}$  would be negatively priced. Table 7 shows that this is the case. Moreover, Figure 2 suggests that a fit of the model is better when we consider the consumption growth rather than the market return.

2. (a) Table 8 presents the means and the standard deviations of  $5 \times 5$  portfolios and the zero cost portfolios. When we eyeball the table, it seems that small stocks yield higher returns than large stocks for each BM decile; High BM stocks yield higher returns than Low BM stocks for each size decile. Moreover, small stocks show higher standard deviations than large stocks for each BM decile; High BM stocks show lower standard deviations than Low BM stocks for each size decile. Consistent with the above observations, means of SMB and HML portfolios show positive monthly return, which is comparable to that of the excess market return.
- (b) Consider a multivariate regression model

$$Y = XB + U$$



Table 6: Results for the multivariate regression: consumption growth

This table shows results for the multivariate regression.

	1	2	3	4	5	6	7	8	9	10
alpha	-0.289	-0.806	-0.966	-1.091	-1.209	-1.061	-1.136	-1.072	-1.090	-1.239
beta	0.332	0.839	0.995	1.121	1.237	1.088	1.163	1.099	1.117	1.262
se_alpha	0.811	0.717	0.654	0.627	0.598	0.582	0.551	0.527	0.492	0.404
se_beta	0.814	0.720	0.657	0.629	0.601	0.584	0.553	0.530	0.494	0.406
t_alpha	-0.357	-1.124	-1.476	-1.741	-2.021	-1.823	-2.060	-2.033	-2.217	-3.068
t_beta	0.408	1.165	1.515	1.782	2.060	1.863	2.101	2.075	2.263	3.111
p_alpha	0.721	0.261	0.140	0.082	0.043	0.068	0.039	0.042	0.027	0.002
p_beta	0.683	0.244	0.130	0.075	0.039	0.063	0.036	0.038	0.024	0.002

Table 7: Results for Fama-Macbeth estimator: consumption growth

This table shows results for Fama-Macbeth estimator.

	estimate	se	t
lambda0	0.048	0.012	4.074
lambdaM	-0.023	0.007	-3.312
rsq	0.996		

Table 8: The means and the standard deviations

This table reports the means and the standard deviations of  $5 \times 5$  portfolios and the zero cost portfolios.

(a) means for the  $5 \times 5$  portfolios

	Small	2	3	4	Big
LoBM	0.003	0.005	0.005	0.006	0.005
2	0.008	0.008	0.008	0.006	0.005
3	0.008	0.009	0.008	0.007	0.006
4	0.010	0.009	0.009	0.008	0.005
HiBM	0.011	0.010	0.010	0.008	0.007

(b) standard deviations for the  $5 \times 5$  portfolios

	Small	2	3	4	Big
LoBM	0.079	0.071	0.066	0.058	0.046
2	0.069	0.060	0.054	0.051	0.044
3	0.060	0.054	0.050	0.049	0.043
4	0.057	0.052	0.049	0.048	0.046
HiBM	0.060	0.060	0.056	0.057	0.054

(c) means and standard deviations of the zero cost portfolios

	Mkt-RF	SMB	HML	RMW	CMA
mean	0.005	0.002	0.003	0.003	0.003
sd	0.044	0.030	0.028	0.022	0.020

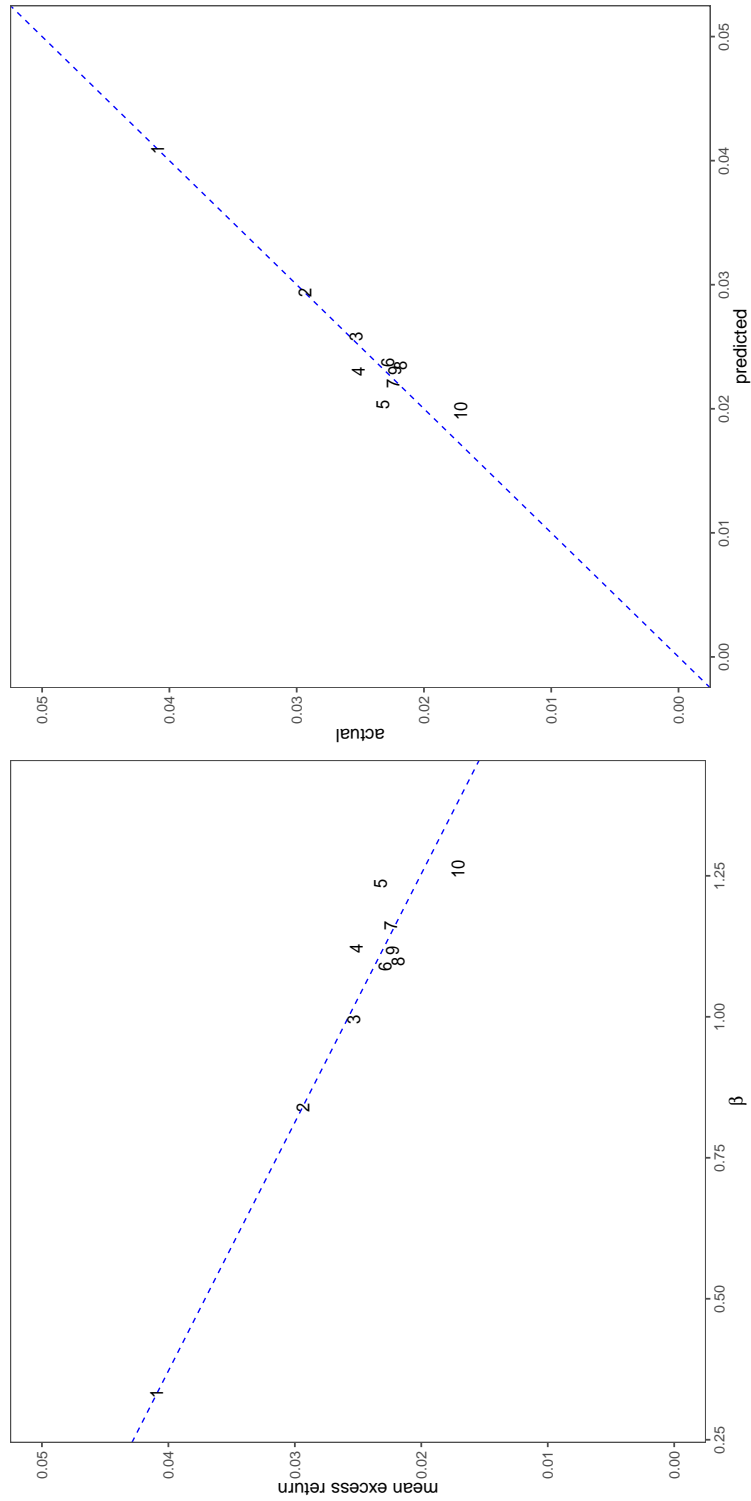


Figure 2: Cross-sectional regressions: consumption growth

This figure shows results of the cross-sectional regression. On the left hand side, I plot the observations in regression and the regression line (dashed line). On the right hand side, I plot the actual mean excess returns versus predicted mean excess returns from the model. The dashed line represents a 45-degree line. Labels denote deciles of portfolios.

where

$$\begin{aligned}
Y &= [r_1, \dots, r_N]_{T \times N} \\
X &= [\iota, F_1, \dots, F_L]_{T \times L} \\
B &= \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_N \\ \beta_{11} & \beta_{21} & & \beta_{N1} \\ \beta_{21} & \beta_{22} & & \beta_{N2} \\ \vdots & \vdots & & \vdots \\ \beta_{L1} & \beta_{L2} & & \beta_{NL} \end{bmatrix}_{L \times N} \\
U &= [u_1, \dots, u_N]_{T \times N}
\end{aligned}$$

Assume

$$\text{var}U = I \otimes \Sigma$$

The LS estimate yields

$$\hat{B} = (X'X)^{-1}X'Y$$

and

$$\hat{\Sigma} = \hat{U}'\hat{U}/T$$

where

$$\hat{U} = Y - X\hat{B}$$

Under the regularity condition, we have

$$\sqrt{T}(\hat{B} - B) \rightarrow_d \mathbb{N}(0, M^{-1} \otimes \Sigma)$$

where

$$\frac{X'X}{T} \rightarrow_p M$$

For a large  $T$ , we may approximate

$$\hat{B} \approx_d \mathbb{N}(B, (X'X)^{-1} \otimes \hat{\Sigma})$$

Therefore, I compute

$$\begin{aligned}
\hat{\alpha} &= \hat{B}_{1,\cdot} \\
\text{se}(\hat{\alpha}) &= \text{diag}((X'X)^{-1} \otimes \hat{\Sigma})_{1:25,1:25} \\
t(\hat{\alpha}_i) &= \frac{\hat{\alpha}_i}{\text{se}(\hat{\alpha}_i)}
\end{aligned}$$

(individually) evaluate their two-sided  $p$ -values from  $\mathbb{N}(0, 1)$  under  $H_0 : \alpha_{i0} = 0$ . Table 9 reports  $\hat{\alpha}$  from the market model.

(c) Consider a multivariate regression model

$$Y = XB + U$$

where

$$\begin{aligned}
Y &= [r_1, \dots, r_N]_{T \times N} \\
X &= [\iota, F_1, \dots, F_L]_{T \times L}
\end{aligned}$$

Table 9: Estimates from the time-series regression: the market model

This table reports  $\hat{\alpha}$  from the market model.

(a)  $\hat{\alpha}$

	Small	2	3	4	Big
LoBM	−0.005	−0.002	−0.002	−0.0001	−0.0001
2	0.001	0.001	0.002	0.0005	0.0004
3	0.002	0.003	0.002	0.002	0.001
4	0.005	0.004	0.004	0.003	0.0002
HiBM	0.005	0.004	0.004	0.003	0.001

(b)  $\text{se}(\alpha)$

	Small	2	3	4	Big
LoBM	0.002	0.001	0.001	0.001	0.001
2	0.002	0.001	0.001	0.001	0.001
3	0.001	0.001	0.001	0.001	0.001
4	0.001	0.001	0.001	0.001	0.001
HiBM	0.001	0.001	0.001	0.001	0.001

(c)  $t(\hat{\alpha})$

	Small	2	3	4	Big
LoBM	−2.560	−1.641	−1.592	−0.141	−0.137
2	0.799	1.311	2.326	0.682	0.711
3	1.490	2.959	2.494	1.964	1.308
4	3.373	3.496	3.833	3.818	0.252
HiBM	3.590	2.840	3.628	2.152	1.141

(d)  $p\text{-value}$

	Small	2	3	4	Big
LoBM	0.010	0.101	0.111	0.888	0.891
2	0.424	0.190	0.020	0.496	0.477
3	0.136	0.003	0.013	0.050	0.191
4	0.001	0.0005	0.0001	0.0001	0.801
HiBM	0.0003	0.005	0.0003	0.031	0.254

Table 10: GRS tests

This table reports  $F$ -statistics of Gibbons, Ross, and Shanken (1989) for the market, FF3, and FF5 models.

	Fstat	pvalue
market	4.559	0
FF3	3.849	0
FF5	3.100	0

$$B = \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_N \\ \beta_{11} & \beta_{21} & & \beta_{N1} \\ \beta_{21} & \beta_{22} & & \beta_{N2} \\ \vdots & \vdots & & \vdots \\ \beta_{L1} & \beta_{L2} & & \beta_{NL} \end{bmatrix}_{L \times N}$$

$$U = [u_1, \cdots, u_N]_{T \times N}$$

Assume

$$\text{var}U = I \otimes \Sigma$$

Under the normality assumption, Gibbons, Ross, and Shanken (1989, ETCA) show that

$$GRS \equiv \frac{T - N - L}{N} \frac{\hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha}}{1 + \hat{\mu}' \hat{\Omega}^{-1} \hat{\mu}}$$

$$=_d F_{N, T-N-L}$$

where

$T$  : time

$N$  : # of assets

$L$  : # of factors

$$\hat{B} = (X'X)^{-1}X'Y$$

$$\hat{\alpha} = \hat{B}_1,$$

$$\hat{U} = Y - X\hat{B}$$

$$\hat{\Sigma} = \hat{U}'\hat{U}/(T - 1 - L)$$

$$\hat{\mu} = [\bar{F}_1, \cdots, \bar{F}_L]'$$

$$\hat{\Omega} = [F_1, \cdots, F_L]'[F_1, \cdots, F_L]/(T - 1)$$

Table 10 reports the results for GRS test.  $F$ - statistic and  $p$ -value presented in line 1 of Table 10 show that the null hypothesis that the alpha are jointly zero is rejected.

- (d) i. FF3: Table 11 reports  $\hat{\alpha}$  from the FF3 model. The result for GRS test can be found in line 2 of Table 10. FF3 model yields smaller  $F$ -statistic than the market model, but the null hypothesis is still rejected.

Table 11: Estimates from the time-series regression: FF3

This table reports  $\hat{\alpha}$  from the FF3 model.

(a)  $\hat{\alpha}$ 

	Small	2	3	4	Big
LoBM	-0.005	-0.002	-0.001	0.001	0.002
2	-0.00002	0.0002	0.001	-0.0005	0.0002
3	-0.0002	0.001	0.00005	-0.0003	0.00001
4	0.002	0.001	0.001	0.001	-0.002
HiBM	0.001	-0.0002	0.001	-0.001	-0.002

(b)  $se(\alpha)$ 

	Small	2	3	4	Big
LoBM	0.001	0.001	0.001	0.001	0.0004
2	0.001	0.001	0.001	0.001	0.001
3	0.001	0.001	0.001	0.001	0.001
4	0.001	0.0005	0.001	0.001	0.001
HiBM	0.001	0.001	0.001	0.001	0.001

(c)  $t(\hat{\alpha})$ 

	Small	2	3	4	Big
LoBM	-5.233	-2.741	-1.300	2.004	3.779
2	-0.032	0.406	1.408	-0.721	0.427
3	-0.363	1.696	0.083	-0.417	0.009
4	3.411	1.582	1.393	1.405	-3.586
HiBM	2.656	-0.430	0.961	-1.177	-1.919

(d)  $p$ -value

	Small	2	3	4	Big
LoBM	0.00000	0.006	0.194	0.045	0.0002
2	0.975	0.685	0.159	0.471	0.669
3	0.717	0.090	0.934	0.676	0.993
4	0.001	0.114	0.164	0.160	0.0003
HiBM	0.008	0.667	0.336	0.239	0.055

Table 12: Estimates from the time-series regression: FF5

This table reports  $\hat{\alpha}$  from the FF5 model.

(a)  $\hat{\alpha}$ 

	Small	2	3	4	Big
LoBM	-0.003	-0.001	0.0002	0.002	0.001
2	0.001	-0.0002	0.0001	-0.002	-0.001
3	-0.0002	0.00004	-0.001	-0.001	-0.001
4	0.002	0.0001	-0.00003	0.0005	-0.002
HiBM	0.001	-0.0004	-0.0001	-0.001	-0.0002

(b)  $se(\alpha)$ 

	Small	2	3	4	Big
LoBM	0.001	0.001	0.001	0.001	0.0004
2	0.001	0.001	0.001	0.001	0.001
3	0.001	0.001	0.001	0.001	0.001
4	0.001	0.0005	0.001	0.001	0.001
HiBM	0.001	0.001	0.001	0.001	0.001

(c)  $t(\hat{\alpha})$ 

	Small	2	3	4	Big
LoBM	-3.289	-1.377	0.335	2.947	2.794
2	1.838	-0.403	0.250	-2.712	-1.650
3	-0.397	0.072	-1.326	-1.951	-1.096
4	3.292	0.264	-0.052	0.698	-3.919
HiBM	2.212	-0.688	-0.199	-1.327	-0.203

(d)  $p$ -value

	Small	2	3	4	Big
LoBM	0.001	0.169	0.737	0.003	0.005
2	0.066	0.687	0.803	0.007	0.099
3	0.691	0.942	0.185	0.051	0.273
4	0.001	0.791	0.959	0.485	0.0001
HiBM	0.027	0.491	0.842	0.185	0.839



- ii. FF5: Table 12 reports  $\hat{\alpha}$  from the FF5 model. The result for GRS test can be found in line 3 of Table 10. FF5 model yields smaller  $F$ -statistic than the market model and FF3 model, but the null hypothesis is still rejected.
- (e) Figure 3 plots the actual versus predicted mean excess returns. Trivially, residuals seem to be smaller as we include more factors in the model.

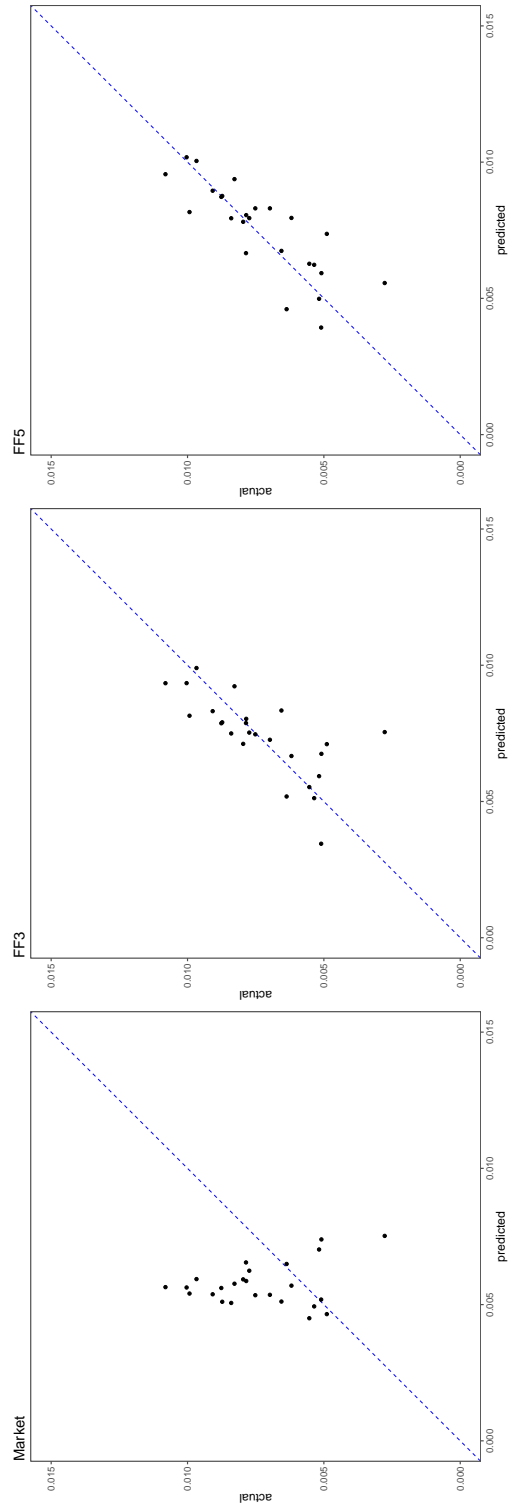


Figure 3: Time-series regressions

This figure plots the actual mean excess returns versus predicted mean excess returns from the market, FF3, and FF5 models. The dashed line represents a 45-degree line.

## Appendix

For R codes used to produce above results, see <https://github.com/ysugk/Course-FIN625/tree/master/HW/code>.