F625 - Empirical Asset Pricing - Assignment 3

I Information about the assignments

The assignments are an integral part of this course. I believe that you have to "do it yourself" to really learn something. Most of the questions requires some form of programming. Feel free to choose the programming language of your choice. It is your responsibility to make sure you have access to adequate software. You most hand in your own work, but you are encouraged to talk/discuss the problems with your class mates. Explain your approach and label figures and tables carefully. In this way it is easy for me to follow what you have been doing. Also, please add the code in the appendix.

II Questions

Question 1 – Consumption based asset pricing

For this exercise you need the data in "rates.xls."

Consider the annual and quarterly data set in the excel file. The annual data set is from 1929 to 2015 and contains a real per capita consumption (of non-durable goods and services) index, returns on an aggregated stock portfolio, a T-bill portfolio, and an inflation series.

The quarterly data set covers the period 1947 to 2015 and contains a real per capita consumption (of non-durable goods and services) index, returns on an aggregated stock portfolio, a T-bill portfolio, and then size portfolios (stocks sorted into deciles based on their market capitalization), and an inflation series.

(a) Provide (for both data sets) means, standard deviations, and first-order autocorrelation coefficients for real consumption growth, the real return on the aggregated stock portfolio, the real T-bill return, and the excess return on the two. Note that all returns are simple nominal returns (expressed in percent), which need to be transformed into real returns. Annualize all the statistics under an IID assumption (that is, for quarterly data means are multiplied by 4 and standard deviations are multiplied by 2).

Consider the canonical asset pricing equation

$$E\left(M_{t+1}R_{i,t+1}|I_t\right),$$

where M_{t+1} is the stochastic discount factor, $R_{i,t+1}$ is the gross return on asset i, and $E\left(\cdot|I_t\right)$ denotes the conditional expectation given information at time t. In the case of a representative agent with (constant) time discount factor δ who maximizes a time-separable utility function defined over aggregate consumption $U\left(C_t\right)$, the stochastic discount factor equals the investors's intertemporal marginal rate of substitution (i.e., $M_{t+1} = \delta\frac{U'(C_t)}{U'(C_t)}$). The asset pricing equation is then the investor's first-oder condition. With power utility

$$U\left(C_t\right) = \frac{C_t^{1-\gamma}}{1-\gamma},$$

the first-order condition is:

$$E\left(\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{i,t+1}|I_t\right) = 1.$$

Note that by the law of iterated expectations the first-order condition also holds unconditionally.

Under the assumption that the consumption growth and returns are jointly log-normally distributed, expected excess returns can be written

$$E\left(r_{i,t+1} - r_{f,t}\right) + 0.5\sigma_i^2 = \gamma\sigma_{ic},$$

and the risk-free rate can be written as

$$r_{f,t} = -ln\left(\delta\right) + \gamma E\left(\Delta c_{t+1}\right) - \frac{\gamma^2\sigma_c^2}{2},$$

where $r_{i,t+1}$ is the log return on asset i between the data t and $t+1$, $r_{f,t}$ is the log risk-free return observed at date t for the period between t and $t+1$, σ_i is the standard deviation of the log return on asset i, σ_{ic} is the covariance between the log return on asset i and log consumption growth, and σ_c is the standard deviation of log consumption growth.

(b) Derive the above expressions for the expected excess return and the risk-free rate under the assumption of lognormality.

(c) Consider the annual and quarterly data set. Provide estimates of the moments in the equations for the excess returns and the risk-free asset above. Let the stock be the risky asset and let the T-bill return be a proxy for the risk-free asset. Also, report the sample correlation between excess returns and consumption growth.

(d) From the above estimates (from (b)) back out two measures of the risk aversion coefficient, γ. The first measure uses the sample covariance between the excess return of stocks and consumption growth as an estimate of σ_{ic}. The second measure assumes that the correlation between excess returns on stocks and consumption growth is perfect (i.e., equal to one), which means that an estimate of σ_{ic} can be derived from the product of the standard deviations of the excess return and the consumption growth. Interpret your results.

Now consider again the first-order condition for an investor with power utility

$$E\left(\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{i,t+1}|I_t\right) = 1.$$

which can be seen as a conditional moment condition. Let z_t denote a vector of instruments (information variables known at time t and part of the greater information set I_t). Then, by the law of iterated expectations, the following unconditional moment condition should hold:

$$E\left(\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{i,t+1}z_t - z_t\right) = 0.$$

(e) Use the moment conditions for the gross returns to estimate the parameters δ and γ on the annual data set. Consider as system of moment conditions with the gross returns on stocks and T-Bills (i.e., $R_{i,t+1}$ equals $R_{s,t+1}$ and $R_{f,t+1}$), and instruments given by $z_t = (1, C_t/C_{t-1}, R_{s,t} - R_{f,t})'$. Provide results for the GMM two-step estimator. Use a

Newey and West (1987) estimator of the covariance matrix; assume that there is no serial correlation beyond one lag. Report parameter estimates, standard errors, and tests for overidentifying restrictions with $p-values$ for the estimator.

We will now consider the 10 size portfolios. We will examine both the CAPM and a version of the consumption CAPM. Let $r_{i,t} = R_{i,t} - R_{f,t}$ and $r_{M,t} = R_{M,t} - R_{f,t}$ be the excess return on decile portfolio i and on the market, respectively. Let $G_t = C_t/C_{t-1}$ denote the per capita real consumption growth.

(f) Run a market model regression for the ten decile portfolios. That is, for each decile portfolio run the time-series regression

$$r_{i,t} = \alpha_i + \beta_{i,M} r_{M,t} + \varepsilon_{i,t},$$

where α_i and $\beta_{i,M}$ are regression coefficients, and $\varepsilon_{i,t}$ is an error term. Report the estimates of α_i and $\beta_{i,M}$ and standard errors on them.

(g) Test the hypothesis that each alpha is (individually) zero.

(h) Run a cross-sectional regression of the mean excess return versus the estimated market beta. That is, run

$$\hat{\mu}_i = \lambda_0 + \lambda_M \hat{\beta}_{i,M} + \eta_i,$$

where $\hat{\mu}_i = \frac{1}{T}\sum_{t=1}^{T} r_{i,t}$ and $\hat{\beta}_{i,M}$ is the estimated beta for portfolio i. Report estimates of λ_0 and λ_M, and standard error as in Fama and MacBeth (1973). Also report a cross-sectional R^2. Plot the observations in the regression and the regression line. Does the CAPM hold? Plot the actual mean excess returns versus the predicted mean excess return. Interpret.

(i) Redo the analysis above, but now with consumption growth. That is, run first the following time-series regression:

$$r_{i,t} = \alpha_i + \beta_{i,G} G_t + \varepsilon_{i,t},$$

and then the following cross-sectional regression:

$$\hat{\mu}_i = \lambda_0 + \lambda_G \hat{\beta}_{i,G} + \eta_i.$$

Report the estimates of $\beta_{i,G}$. Further, report estimates of λ_0 and λ_G together with Fama and MacBeth (1973) standard errors on them. Also, report the cross-sectional R^2. Provide two plots: (i) the mean excess returns versus the estimated betas, and (ii) the actual mean excess returns versus the predicted mean excess returns. Comment on the results. Compare to those in (h).

Question 2 – Fama and French 3 and 5 factor models

Consider the monthly data in "FF5factors.csv" and "'FF25.csv" Let $r_{SMB,t}$ denote the return on small (market capitalization) portfolio minus the return on a big (market capitalization) portfolio; let $r_{HML,t}$ denote the return on a high (book-to-market) portfolio minus the return on a low (book-to-market) portfolio, let $r_{RMW,t}$ denote the return on a robust (operating profitability) portfolio minus the return on a weak (operating profitability) portfolio; let $r_{CMA,t}$ denote the return on a conservative investment portfolio minus the return on an aggressive investment portfolio; and let $r_{M,t}$ denote the market return in excess of a T-bill return. The 5×5 portfolios are sorted on market capitalization and book-to-market ratio and are also referred to as size and value/growth portfolios; these are excess returns and denoted by $r_{i,t}$ where $i = 1, \ldots, 25$. Note that returns in the sheet are raw returns and excess returns are constructed by subtracting the risk-free rate.

(a) Provide means and standard deviations for the 5×5 portfolios. Also provide means and standard deviations of the zero cost portfolios $r_{M,t}$, $r_{SMB,t}$, $r_{HML,t}$, $r_{RMW,t}$ and $r_{CMW,t}$. Note that the time-series mean of the returns on a zero-cost portfolio is a measure of the risk premium on that investment strategy. Use the means and the standard deviations to characterize the portfolios.

(b) Run regressions of the excess returns on the 5×5 portfolios on the market excess returns; that is, run the following regressions for the 25 portfolios:

$$r_{i,t} = \alpha_i + \beta_{i,M} r_{M,t} + \varepsilon_{i,t}.$$

Given the proxy for the market portfolio and the risk-free rate, the CAPM suggest that the alpha is zero and the risk premium of asset i is given by the product of the beta and the market risk premium. Report the alphas.

(c) Use the Gibbons, Ross, and Shanken (1989) finite sample test of the null hypothesis that the alphas are jointly zero.

(d) Redo (b) and (c) when using the 3 factor model ($r_{M,t}$, $r_{SMB,t}$ and $r_{HML,t}$) and the 5 factor model.

(e) For all the models, plot the actual versus predicted mean excess returns (not including the alpha).