F625 - Empirical Asset Pricing - Assignment 1

I Information about the assignments

The assignments are an integral part of this course. I believe that you have to "do it yourself" to really learn something. Most of the questions requires some form of programming. Feel free to choose the programming language of your choice. It is your responsibility to make sure you have access to adequate software. You most hand in your own work, but you are encouraged to talk/discuss the problems with your class mates. Explain your approach and label figures and tables carefully. In this way it is easy for me to follow what you have been doing. Also, please add the code in the appendix.

II Questions

Question 1:

Consider the file "FF3factorsMonthly.xlsx" on Canvas. The file contains the excess return on the market, the SMB, HML and the risk-free rate for the period July 1926 until October 2018. This data can also be found on Kenneth French's web page (Link).

a) Calculate key summary statistics for the for time series. You can choose what you want to report. Make a table in "journal style" with a caption explaining the data.

b) Consider the excess return on the market. Assume that it is normally distributed and provide an estimate of μ and σ^2 and their standard deviations.

c) The Sharpe ratio defined as $\frac{\mu}{\sigma}$ is an important quantity in finance. Use the "delta method" to provide a standard error of the Sharpe ratio. Do it both analytically and numerically.

Question 2:

Simulate 100 identical and independent normally distributed numbers with a mean of one and a standard deviation of two. With these simulated numbers, set up the sample log likelihood function for a univariate normal distribution with mean μ and standard deviation σ. Find, numerically, the values of μ and σ that maximize the log likelihood function. What is the mean log likelihood value? Show graphically that you have found a maximum (for instance, by plotting the surface and/or contour of the mean log-likelihood for a range of parameter values).

Question 3:

Consider the file "FF3factorsMonthly.xlsx" on Canvas (same file as for question 1). Let $r_{M,t}$ denote the excess return on the market, $r_{SMB,t}$ the return on the small minus the return on the big portfolio, and $r_{HML,t}$ the return on the high book to market minus low book to market portfolio.

a) Provide summary statistics on the three portfolios. Interpret them.

b) Run the following regressions:

$$r_{SMB,t} = \alpha_{SMB} + \beta_{SMB} r_{M,t} + \epsilon_{SMB,t} \qquad (1)$$

$$r_{HML,t} = \alpha_{HML} + \beta_{HML} r_{M,t} + \epsilon_{HML,t} \qquad (2)$$

Report the ordinary least squares estimates. Also report the ordinary least square standard errors. Test if $\alpha_{SMB} = 0$ and $\alpha_{HML} = 0$ separately.

b) Correct the standard errors for conditional heteroskedasticity as in White (1980). Test if $\alpha_{SMB} = 0$ and $\alpha_{HML} = 0$. Report the p-values and comment on your results.

c) Use a Wald statistics for the the following three hypothesis:

$$H_0 : \alpha_{SMB} = 0 \qquad (3)$$

$$H_0 : \alpha_{HML} = 0 \qquad (4)$$

$$H_0 : \alpha_{SMB} = \alpha_{HML} = 0 \qquad (5)$$

Report the Wald statistics and report the p-values. Relate it to b).

Question 4:

Consider the excess return on the market in "FF3factorsMonthly.xlsx."

a) Estimate the market risk premium and it's variance by the following moment conditions:

$$E\begin{bmatrix} r_{M,t}-\mu \\ (r_{M,t}-\mu)^2-\sigma^2 \end{bmatrix}=0 \tag{6}$$

Provide the estimates of μ and σ^2 and their standard deviations. You can assume that there is no autocorrelation in the moment conditions.

b) Derive the covariance matrix of the parameter estimates under the additional assumption that excess returns are normally distributed.

c) Provide Newey and West (1987) standard errors of the parameter estimates using one lag. Compare them with the standard errors calculated in a).

Question 5:

Conduct a Monte Carlo study of the properties of the OLS estimates β and ρ in the following system

$$r_{t+1} = \alpha+\beta x_t+u_{t+1} \tag{7}$$

$$x_{t+1} = \theta+\rho x_t+e_{t+1} \tag{8}$$

where it is assumed that $\rho^2<1$ and that

$$\begin{bmatrix} u_{t+1} \\ e_{t+1} \end{bmatrix} = iidN\left(0,\begin{bmatrix} \sigma_u^2 & \sigma_{ue} \\ \sigma_{ue} & \sigma_e^2 \end{bmatrix}\right). \tag{9}$$

Assume that $\alpha=\theta=0$, $\beta=0.210$, $\rho=0.972$, and that $\sigma_u^2=0.0030050$, $\sigma_e^2=0.0000108$, and $\sigma_{ue}=-0.0001621$. (These parameter values capture important aspects in the case where r_t is the continuously compounded market excess return in month t and x_t is the dividend-price ratio at the end of month t.) Simulate 840 monthly observations of the above model, conditioning on that the initial value of x_t is zero (i.e., $x_0=0$). Estimate β and ρ with ordinary least squares. Store the estimates. Repeat this 10,000 times (the number of replications). Are there any biases in the estimators? Provide the mean, standard deviation, skewness, and kurtosis for the distributions of the estimators. Interpret. Redo the analysis with 240 observations. Comment on the results.

Question 5:

In this exercise you will estimate an $MA(1)$ process. I want you to code up the Maxmium Likelihood estimation and not just run a standardized program. Consider the following model

$$y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}. \tag{10}$$

where ϵ_t is $iidN\left(0,\sigma^2\right)$. First you will create date generated by the model in equation. To do this you will simulate the model using a Monte Carlo simulation. Let $T = 1000$ (you will simulate 1000 periods, i.e., $t = 1,\ldots,1000$). Moreover, $\theta = 0.1$, $\mu = 1.5$ and $\sigma^2 = 1.9$. Assume that $\epsilon_0 = 0$. Using the simulated data, estimate the parameters (a reference to this model is Hamilton ch 5.4 p.127).

Question 6:

Consider the following deterministic system (ODE)

$$dx_t = \kappa(\bar{x} - x_t)dt \tag{11}$$

for $t \in [0,\infty)$, with $\bar{x} > 0$ and $\kappa > 0$. Assume that $x_0 = x$.

a) Provide a solution of the differential equation. What happens to x_t when $t \to \infty$.

b) Consider a discretized version of the ODE in equation 11:

$$\Delta x_t = \kappa\left(\bar{x} - x_t\right)\Delta t. \tag{12}$$

Set $\Delta t = \frac{1}{12}$ and let $\kappa = 0.1$, $\bar{x} = 0.01$ and $x_0 = 0.1$. Simulate the system forward starting at time $t = 0$ and plot the evolution.

c) Now consider the stochastic differential equation (SDE)

$$dx_t = \kappa(\bar{x} - x_t)dt + \sigma dW_t, \tag{13}$$

where $\sigma > 0$ and W_t is a standard Brownian motion. Continue to assume that $\bar{x} > 0$ and $\kappa > 0$. Provide a solution to the SDE. What is $E_t\left(x_s\right)$ and $var_t\left(x_s\right)$ for $s > t$. What happens to the these quantities when $s - t \to \infty$?

d) The process in equation 13 is a simple version of a common way of modeling the short interest rate (it is the Vasicek (1977) model). You will now simulate the SDE forward using what is called an Euler discretization. Set $\Delta t = \frac{1}{12}$ and let $\kappa = 0.1$, $\bar{x} = 0.01$, $x_0 = 0.1$, and $\sigma = 0.05$. Simulate the system forward using the following

$$\Delta x_t = \kappa \left(\bar{x} - x_t\right) \Delta t + \sigma \left(\sqrt{\Delta t}\right) \varepsilon_t, \tag{14}$$

where ε_t is $iidN(0,1)$. Provide a plot of your simulated process i) for time 0 to 5 and for ii) time 0 until 100.