F625 - Empirical Asset Pricing - Assignment 2

I Information about the assignments

The assignments are an integral part of this course. I believe that you have to "do it yourself" to really learn something. Most of the questions requires some form of programming. Feel free to choose the programming language of your choice. It is your responsibility to make sure you have access to adequate software. You most hand in your own work, but you are encouraged to talk/discuss the problems with your class mates. Explain your approach and label figures and tables carefully. In this way it is easy for me to follow what you have been doing. Also, please add the code in the appendix.

II Questions

Question 1:

This question is taken from John Campbell's book (see the syllabus), Chapter 5, Problem 5.2. It is repeated here:

In this exercise we study empirically whether the out-of-sample stock market return predictability of well-known valuation rations can be improved by imposing simple theoretical restrictions on the predictive regressions. Consider the regression:

$$R_{t+1} - R_{f,t+1} = \alpha + \beta x_t + u_{t+1}, \tag{1}$$

where R_{t+1} denotes the one-quarter-ahead return on the S&P500 index and x_t is a predictor variable. Motivated by the claim of Welch and Goyal (2008) that the historical average excess stock return forecasts future excess stock returns out of sample better than regressions of excess returns on predictor variables, we evaluate the out-of-sample R^2 statistic computed as

$$R_{OS}^2 = 1 - \frac{\sum_{t=0}^{T}\left(R_{t+1} - \hat{R}_{t+1}\right)^2}{\sum_{t=0}^{T}\left(R_{t+1} - \bar{R}_{t+1}\right)^2}, \tag{2}$$

where $\hat{R}_{t+1}$ is the fitted value from equation 1 estimated from the start date $-T_{IE}$ of the initial estimation sample through date t and $\bar{R}_{t+1}$ is the historical average arithmetic average return estimated from $-T_{IE}$ through t. Here T_{IE} is the length of the initial estimation period, and T is the length of the out-of-sample forecast evaluation period. A positive value for R_{OS}^2 means that the predictive regression has lower average mean-squared prediction error than the historical average return.

a) Calculate the in-sample R^2 statistics for the dividend yield, $x_t = D_t/P_t$, and the smoothed earnings yield, $x_t = X_t/P_t$, when regression 1 is estimated by standard ordinary least square (OLS) over the full sample from 1872 to 2016.

b) Calculate the out-of-sample R^2 statistics for the two valuation ratios when regression 1 is estimated with standard OLS, with 1872-1926 as the initial estimation period and 1927-2016 as the out-of-sample forecast period. Compare the values you obtained for the in-sample and out-of-sample R^2 statistics. Are your results consitent with Welch and Goyal's (2008) claim?

c) Repeat the calculations of the previous part for the out-of-sample R^2 statistics but now impose the (rather weak) theoretical restrictions that the slope β in the predictive regression and the forecast for the excess returns are both nonnegative. That is, calculate the return forecast as

$$\hat{R}_{t+1} = R_{f,t+1} + max\left\{0, \hat{\alpha}_{t+1} + max\left\{0, \hat{\beta}_{t+1}\right\} x_t\right\}, \tag{3}$$

where $\hat{\alpha}_{t+1}$ and $\hat{\beta}_{t+1}$ denote the intercept and slope estimates from the standard OLS regression.

Is there a significant improvement in the out-of-sample explanatory power of the two valuation rations?

In the remaining parts of the exercise, we examine whether the forecasting performance of the dividend yield improves once we impose the theoretical restrictions of the drifting steady-state valuation model (see section 5.5.2 in Campbell (2018)). From the drifting steady-state valuation model we have have[1]

$$E_t\left(1+R_{t+1}\right) \approx \frac{D_{t+1}}{P_t} + e^{E_t g_{t+1}} + 0.5Var_t\left(r_{t+1}\right). \tag{4}$$

Following the above equation, we use a version of the dividend yield adjusted for dividend growth and the real rate as our predictor variable:

$$x_t = \frac{D_t}{P_t}\left(1+G_t\right) + e^{E_t g_{t+1}} + 0.5Var_t\left(r_{t+1}\right), \tag{5}$$

where $E_t g_{t+1}$ and $Var_t\left(r_{t+1}\right)$ denote market participants' conditional expectations of future log dividend growth and the conditional variance of log returns.

d) Construct an estimate of 5 using the historical sample mean of dividend growth and the historical sample variance of log stock returns up to date t. Even though the model assumes that market participants know the value of D_{t+1} at date t, to avoid any look-ahead bias construct a real-time estimate of x_t assuming that D_{t+1} is not in the econometrician's information set at date t.

Discuss alternative procedures that you could use to construct real-time estimates of $E_t g_{t+1}$ and $Var\left(r_{t+1}\right)$.

e) Repeat the calculations of part b) and c) for x_t given by 5. Compare the forecasting performance of this adjusted version of the dividend yield with that of the (unadjusted) dividend yield.

f) Finally, fully impose the theoretical restrictions of equation 5 by calculating the predicted return as

$$\hat{R}_{t+1} = x_t - 1, \tag{6}$$

[1] It is assumed that the dividends, D_{t+1}, is known one period in advance.

where x_t is as in 5. What is the out-of-sample R^2 statistics now? Discuss your conclusions from this exercise. (The exercise in based on Campbell and Thompson (2008) as pointed out in Campbell's book.)

Question 2: *(Optional exercise)*

This exercises is also from Campbell's book (Problem 5.4) and deal with Ian Martin's paper on the SVIX and the lower bound the equity premium (discussed in class).

Martin (2017) introduces a volatility index, $SVIX_{i,t,T}$ defined in terms of the time t prices of put and call options on an underlying asset or portfolio i, usually a market index, with different strike pries and at the same maturity T. It turns out that the square of the SVIX equals the (normalized) risk-neutral conditional variance of the gross simple returns $R_{i,t,T}$ on the underlying asset i,

$$SVIX_{i,t,T}^2 = \frac{1}{T-t} Var^* \left(\frac{R_{i,t,T}}{R_{f,t,T}} \right). \tag{7}$$

The asterisk denotes moments of the risk-neutral distribution of returns. The fact that SVIX is directly observable at time t from the cross section of option prices implies that the lower bound on the equity premium that we will derive below is directly observable. In general, moments of the risk-neutral (conditional) distribution of asset returns can be backed out from option prices.

a) Show that

$$\frac{Var^* \left(R_{i,t+1} \right)}{R_{f,t+1}} = E_t R_{i,t+1} - R_{f,t+1} + Cov_t \left(M_{t+1} R_{i,t+1}, R_{i,t+1} \right), \tag{8}$$

where M_{t+1} denotes the stochastic discount factor.

Conclude that $Var^* \left(R_{i,t+1} \right) / R_{t,t+1} = R_{f,t+1} SVIX_{i,t+1}^2$ is a lower bound on the equity premium, $E_t R_{i,t+1} - R_{f,t+1}$, provided that the negative correlation condition,

$$Cov_t \left(M_{t+1} R_{i,t+1}, R_{i,t+1} \right) \leq 0 \tag{9}$$

holds.

b) Although SVIX has not yet been put to practical use, a well-known volatility index that is closely related to the SVIX is the VIX. It turns out that the VIX satisfies

$$VIX_{i,t,T}^2 = \frac{2}{T-t} L_t^* \left(\frac{R_{i,t,t+1}}{R_{f,t,T}} \right), \tag{10}$$

where $L^*()$ is the risk-neutral entropy of returns.

i) What risk-neutral distribution of returns would approximately equate the VIX and the SVIX (for short maturities T)?

ii) Assume that returns and the stochastic discount factor are jointly conditional lognormal (with respect to the objective probability measure). Show that

$$SVIX_{i,t+1}^2 = e^{\sigma_{i,t}^2} - 1 \tag{11}$$

$$VIX_{i,t+1}^2 = \sigma_{i,t}^2, \tag{12}$$

where $\sigma_{i,t}^2$ is the conditional variance of $r_{i,t+1} = logR_{i,t+1}$ To derive the expression for the VIX you will need to use of Stein's lemma: if X and Y are two jointly normally distributed variables, $COV(h(X), Y) = E(h'(X)) Cov(X, Y)$, for any differentiable function h such that $E(h'(X))$ and $Cov(X, Y)$ exist.

iii) In recent data the VIX is always higher than the SVIX. What does this tell us about the usual assumption of conditional lognormal returns?

c) We now investigate the theoretical relevance of the negative correlation condition. Take asset i to be the market portfolio of risky assets. Show whether or not (NCC) hholds in each of the following settings or whether additional assumptions need to be imposed:

1. An economy where there exists an unconstrained risk-neutral investor.

2. An economy where there exists a CRRA agent who lives only for period t and $t + 1$ and who holds the market.

3. An economy where the stochastic discount factor and the market return are condi-

tionally jointly lognormal and where the market's conditional Sharpe ratio

$$logE_t\left(R_{m,t+1}/R_{f,t+1}\right)/\sigma_t\left(logR_{m,t+1}\right), \tag{13}$$

exceeds the conditional volatility of the market, $\sigma_t\left(logR_{m,t+1}\right)$.

In light of these results, do you think that (NCC) imposes rather strong or weak conditions or weak restrictions on preferences?

d) Empirical estiamtes of $Cov_t\left(M_{t+1}R_{m,t+1}, R_{m,t+1}\right)$ in linear factor models are negative and close to zero. Indeed, the time-series average of the lower bound in recent data is around 5%, quite close to the typical estimates of the unconditional equity premium. If the lower bound is approximately satisfied with equality, what would this imply about the relative risk aversion of the marginal investor who holds the market? Consider, for example the setting in c) 2.

e) The risk-neutral variance (SVIX) approach presents an alternative to the large literature seeking to estimate the equity premium via predictive regression based on valuation ratios. Compare and discuss the advantages and disadvantages of each approach.

f) A notable divergence of forecasts between the risk-neutral variance approach and the valuation-ratios approach occurred in the late 1990s. The monthly and quarterly forecasts for the SVIX bound were high from late 1998 until the end of 1999, consistent with the high equity premia subsequently realized during that period. In contrast, valuation ratio were so low that predictive regressions forecasted a negative equity premium. Is it possible to reconcile these contrasting forecasts? Hint: Use the Campbell-Shiller approximation to the dividend yield to think about what the dividend yield forecasts.