$W_1\Omega^0 \quad W_1\Omega^1 \quad W_1\Omega^2 \oplus \overset{W_1}{\Omega^0}\{1\}$

$\Omega^0 \quad \Omega^1 \quad \Omega^2 \oplus \Omega^0\{1\}$

$\Omega^0 \quad \Omega^1 \quad \Omega^2$

$W_1\Omega^j\{i\}$

$\Omega^{j-1}\{i\}$

$\Omega^{j-1}\{i\}$

$W_1\Omega^j\{i\}$

$\Omega^j\{i\}$

$\Omega^{j-1}\{i\}$

$W_1\Omega^j\{i\}/V$

$B\Omega^{j+1}\{i\}$

$\Omega^{j+1}\{i\}$

Mnemosyne ニーモシネ

$\cancel{\Omega^j\{i\}}$

$\Omega^j\{i\}$

$B\Omega^j\{i\}$

$\mathrm{im}(F \subset \Omega^j\{i\})$

so $\mathrm{im} = V(Z^1)$

$Fd(V\omega) = d\omega$

$V\omega \xrightarrow{Fd} B\Omega^1$

$\ker = V(Z_n)$

Ω°

$B\Omega'$

$pX^{n/p} \in F\Omega^\circ$

$\boxed{\omega, \Omega^\circ}$

$pX^{n/p}$

N181A - A4 / 無地

空間認識力に応える横型、無地

- イメージを広げ、俯瞰しやすい横型タイプ
- 罫線にとらわれず、自由な表現が可能

360°スムーズに開閉するツインワイヤ製本

- フラットに折り返せるから書きやすい
- リングが上部にあり、手元がストレスフリー

ページをきれいにカットできるミシン目加工

- タイトルを明記し、切り離してファイリングできる
- 切り取りA4（正寸）サイズだから、ページをデータ化しやすい

書きやすさを追求し開発された、マルマン国産オリジナル筆記用紙

- さまざまな筆記具に対し良好な適性を備えた、『筆記』のための紙
- 文字のかすれ・にじみ・裏抜けが少ない滑らかな書き心地

$x^{1/p}\, d\log x \in \log F$

$\ker F = \operatorname{im} V$

$V(x^n\, d\log x) = px^{n/p}\, d\log x$

$\Omega^\circ \otimes \{p^2\}$

$B\Omega'$

$F\Omega^\circ$

$\ker = \langle px^n\, d\log x\rangle$

$F\langle x^n\, d\log x\rangle$

$Fd(Vx^n) = dx^n$

$W_1\langle x^n\, d\log x\rangle$

$\langle x^{1/p}\, d\log x\rangle$

$(\operatorname{im} V)$

$$\tilde{\alpha}(C_p) = 2 - \lambda_0 = (0;1)$$

$$\tilde{\alpha}(C_{p^2}) = 2 - \lambda_1 = (0,0;1)$$

$$\left.\begin{matrix} \Omega^0 u_{\lambda_0} \\ \Omega^0 u_{\lambda_0} \end{matrix}\right\} = \boxed{W_1 \Omega^0 u_{\lambda_0}}$$

invariant?

$$TR^0_2 \xrightarrow{V} TR^1_2 \xrightarrow{a_{\lambda_0}} TR^1_{2-\lambda_0} \xrightarrow{F} TR^0_0$$

$$W_1\Omega^0\sigma \longrightarrow$$

$$\begin{tikzcd} W_1\Omega^0\sigma \arrow[r, "a_{\lambda_0}"] & W_1\Omega^0 u_{\lambda_0} \arrow[d, "F"] \\ & \Omega^0 \end{tikzcd}$$

$$\sigma a_{\lambda_0} = \xi u_{\lambda_0}$$

$TR^0_2 \longrightarrow TR^1_{\mathrm{lp}\lambda}$ $\quad TC^-_{2+\lambda_0} \longrightarrow TC^-_2 \longrightarrow TR^0_2$

- Mac paper
- email transfer systems

~~ffmpeg multiple~~
get Internet
somewhere

at some point

$$\begin{array}{ccccc} TC^-_{2+\lambda_0} & \xrightarrow{a_{\lambda_0}} & TC^-_2 & \longrightarrow & \pi^e_2 T \\ \| & & \| & & \| \\ N^{\geq 2}\hat{\Delta}^{(1)}\{1\} & \longrightarrow & N^{\geq 1}\hat{\Delta}^{(1)}\{1\} & \longrightarrow & \end{array}$$

~~multiple sources~~

$\tilde{\alpha} = -\tilde{\lambda}_k$

C_p - $\quad \alpha = (-1; 0)$

$W_1 \Omega^j$

$\boxed{\Omega^j{}_{a_0}}$

C_p^2: $\tilde{\alpha} = -\lambda_1 = (-1, -1; 0)$

$W_1 \Omega^j{}_a \longleftarrow$

$\Omega^j{}_{a_0 a_1}$

$\Omega^j{}_{a_0 a_1} \longleftarrow$

$\Omega^j{}_{a_0 a_1}$

R perfectoid, $\Delta_R = (A, (\tilde{\xi}))$

$(\xi) = N^{\geq 1}\Delta_R$

$|\sigma| = 2$

$|t| = -2$

$TC^-_*(R) = \dfrac{A[\sigma, t]}{\sigma t - \xi} \xrightarrow{\text{can}} TP_*(R) = A[t^{\pm}]$

$\pi_2 THH = A/\xi\langle\sigma\rangle$

$\pi_{2p} T^{hC_p} = A/\xi\phi(\xi)\langle\sigma^p\rangle$

$\pi_{[p]_\lambda} T^{hC_p} = A/\xi\phi(\xi)\langle\sigma^p u\lambda_0^{-(p-1)}\rangle$

$[p]_\lambda = \lambda^0 + \underbrace{\lambda^1 + \cdots + \lambda^{p-1}}$

$= \lambda_\infty + \underbrace{(p-1)\lambda_0}$

$$N: TR^0_2(S) \longrightarrow \boxed{TR^1_{[p]_\lambda}(S)}$$

$$\shortparallel \qquad\qquad \shortparallel$$

$$\cancel{W_1\Omega^2_S \oplus W_1\Omega^0_S\,\sigma} \qquad \cancel{W_1\Omega^j v_0}$$

$$\Omega^2_S \oplus \Omega^0_S\,\sigma$$

S = smooth k-algebra

$TR^n_{\alpha,j} = W\Omega^j$ contribution to $TR^n_{\alpha+j}$

k perfect $\mathbb{F}_p$-alg

$$\begin{tikzcd}
TR^1_{[p]_\lambda} \arrow[r] \arrow[d] & TR^0_{([p]_\lambda)}{}^{C_p} \arrow[d] \\
\pi^{hC_p}_{[p]_\lambda} T \arrow[r, "hC_p"] & \pi^{tC_p}_{[p]_\lambda} T
\end{tikzcd}$$

C_p only sensitive to d_∞ (αx_0 invertible)

tC_p only sensitive to d_0 (v_0 invertible)

$[p]_\lambda = (p, \infty)$

$\quad d_0 \quad d_\infty$

ε^0

$TR^1_{[p]_\lambda, j}$

$$v_0^p \longmapsto p^p a_0^{-p} = 0$$

$$\Omega^j a_0^{-p} \Longleftarrow \boxed{W_1\Omega^j v_0^p}$$

hC_p

$$\Omega^j a_0^{-p}$$

$$\cancel{\Omega^j a_0^{-p}} \quad \boxed{W_1\Omega^j v_0^p} \quad \cancel{\Omega^j a_0^{-p}}$$

$F N x = x^p$

$N(x+y) = N(x) + N(y) + V s_p(x,y)$

where $(x+y)^p = x^p + y^p + p\, s_p(x,y)$

If assume

$N(dx) = 0$, $k[x_1, y_1, \ldots]$

then forces $(X = A^{2p})$

$N(\underbrace{dx_1 dy_1 + \cdots + dx_p dy_p}_{\Omega^2})$

$= V \underbrace{(p-1)!\, dx_1 dy_1 \cdots dx_p dy_p}_{\omega_1 \Omega^{2p}}$

$K \to H$

target degree is $\cancel{\quad} \underset{\lambda^0}{2} + \underset{\lambda^1}{\lambda_0} = [2]_\lambda$

$\underbrace{TR^1_{-2+\lambda_0}}, 4 = W\Omega^4$ contribution to $TR^1_{[2]\lambda}$

$\alpha = (0; -1)$

$\underbrace{\Omega^4 (\alpha_0 \, v^{-1}_{\lambda_0})}_{\in C_p} \longleftarrow \underbrace{W_1 \Omega^4 (v_0^{d_0} \, v^{-1}_{\lambda_0})}_{nC_p}$

$\boxed{0}$

E_∞: $\quad 0 \quad \mathrm{Fil}^1 W_1 \Omega^4_S$.

0

$\mathrm{Fil}^n = V^n + dV^n$

$= \ker(W_0 \longrightarrow W_{n-1} \Omega^1)$

$[C_2/e_+ \to S^0 \to S^\sigma, \underline{\mathbb{Z}}]$

$$\underline{\pi}_{\star+\sigma}\underline{\mathbb{Z}} \xrightarrow{|a_\sigma| = -\sigma} \underline{\pi}_{\star}\underline{\mathbb{Z}} \to \underline{\pi}_{\star} C_2/e_+ \wedge \underline{\mathbb{Z}} \xrightarrow{\partial}$$

$$\underline{\mathbb{Z}}^{\Sigma C_2/e_+} \to \underline{\mathbb{Z}}^{S^\sigma} \to \underline{\mathbb{Z}}$$

$$\mathrm{Hom}(S^\sigma, \underline{\mathbb{Z}})$$

$$= \Sigma^{-1}\underline{\mathbb{Z}}^{C_2/e_+}$$

$$= \Sigma^{-1} C_2/e_+ \wedge \underline{\mathbb{Z}}$$

$$\begin{array}{ccc} S^0 & \to & S^\sigma \\ & & \downarrow \\ & & \Sigma C_2/e_+ \end{array}$$

$$\begin{array}{ccc} \mathbb{Z} & \xrightarrow{1} & \mathbb{Z} \\ 1 \uparrow \downarrow 2 & & \Delta \downarrow \uparrow \nabla \\ \mathbb{Z} & \xrightarrow{\Delta} & \mathbb{Z}[C_2] \end{array}$$

$n \mapsto ?$

$n \mapsto (1+t)n$

0 . . -1

0 0

$TR^1_{2-\lambda_0} = TR^1_{2-\lambda,0} \oplus TR^1_{-\lambda,2}$

$(1,0)$ $(0,-1)$

Ω^ε

$W_1\Omega^0 \oplus \{\xi\}$

$W_1\Omega^0/2$ $W_1\Omega^1/2$ $W_1\Omega^2 \oplus W_1\Omega^0\{1\}/2$

$\cong$

$(+)\Omega^0$ $(+)\Omega^1$

$B\Omega^1$ $B\Omega^2 \oplus \Omega^0\{1\}$

$C_2/e_+ \to S^0 \to S^\sigma$ ⇝ filtration

Ch(Mack C_2)

$$S^0 \xrightarrow{a_\sigma} S^\sigma \to \Sigma C_2/e_+$$

$$\begin{matrix} \underline{\mathbb{Z}} & \mathbb{Z} \\ = & 1 \circlearrowleft \uparrow 2 \\ & \mathbb{Z} \end{matrix}$$

$$\underline{\mathbb{Z}} \to \Sigma^\sigma \underline{\mathbb{Z}} \to \Sigma \boxed{C_2/e_+ \wedge \underline{\mathbb{Z}}} = \mathrm{Ind}_e^{C_2} \mathrm{Res}_e^{C_2} \underline{\mathbb{Z}}$$

$$\underline{\pi}_0(\Sigma^\sigma \underline{\mathbb{Z}}) = \underline{\pi}_{-\sigma}(\underline{\mathbb{Z}}) = \mathrm{infl}_{C_2}(\mathbb{Z}/2)$$

$$H_\cdot(S^\sigma; \underline{\mathbb{Z}})$$

$$\boxed{\mathrm{res} \circ \mathrm{tr} = C_2}$$

$$\begin{matrix} 0 \\ \circlearrowleft \uparrow \\ \mathbb{Z} \end{matrix} \quad \circlearrowleft x \mapsto -x$$

$$= \begin{matrix} \mathbb{Z} & \xrightarrow{2} & \mathbb{Z} \\ \Delta \downarrow \uparrow \nabla & & 1 \downarrow \uparrow 2 \\ \mathbb{Z}[C_2] & \xrightarrow{\nabla} & \mathbb{Z} \end{matrix}$$

$\Delta(n) = (1+\gamma)n$

$\nabla(a+b\gamma) = a+b \cdot \gamma$

$\gamma(1-\gamma)x = (\gamma-1)x$

$= -[(1-\gamma)x]$

	1	0
	0	$\mathbb{Z}/2$
H_q	$(1-\gamma)\mathbb{Z}[C_2]$	0
	1	0

$\mathrm{infl}_{C_2} \mathbb{Z}/2$

0	$\mathbb{Z}/2$
$\mathbb{Z}^-$	0

• fix the React recording!

$$S^0 \to S^\sigma \qquad S^\sigma \to S^{2\sigma}$$

$$\downarrow \qquad \xrightarrow{\otimes S^\sigma} \qquad \downarrow$$

$$\Sigma C_2/e_+ \qquad \Sigma C_2/e_+ \wedge S^\sigma$$

$$\cong \Sigma C_{2+} \wedge S^1$$

$$= \Sigma^2 C_2/e_+$$

$$0 \to \hat{H}^{-1} \to H_0 \xrightarrow{tr} H^0 \to \hat{H}^0 \to 0$$

$$S^0 \xrightarrow{a_\sigma} S^\sigma \xrightarrow{a_\sigma} S^{2\sigma}$$

$$\downarrow \qquad \downarrow$$

$$\Sigma C_2/e_+ \qquad \Sigma^2 C_2/e_+$$

$(a + b \otimes \gamma)$

$- \cancel{b\gamma \cdot b}$

$\pi_{\star-2\sigma} M$

$\Delta H^0(M)$

$$M^e \xrightarrow{1-\gamma} M^e \xrightarrow{tr} M^{C_2}$$

$H^0(M^e) \quad \hat{H}^{-1}(M^e) \quad M^{\Phi C_2} = M^{C_2}/tr$

$\Delta \quad \nabla \quad 1 \quad 2$

$$M^e[C_2] \xrightarrow{(1-\gamma)} M^e[C_2] \xrightarrow{\nabla} M^e$$

$2 \qquad 1 \qquad 0$

$\pi_\star$

$H^0(M^e) \quad \hat{H}^{-1} \quad M^e/tr$

0

$(a + b \otimes \gamma)$

$$\frac{\ker(a + b\otimes\gamma \mapsto a + b\gamma)}{\gamma - 1 \quad \ker}$$

TITLE: DATE/NO.

M^{C_2}

M^e

$\circlearrowleft_\gamma$

$S^0 \longrightarrow S^0$

$\Sigma C_2/e_+$

$$\ker\left[H_0(M^e) \xrightarrow{tr} \underline{H^0(M^e)} \right] \longrightarrow \underline{\hat{H}^0(M^e)} = 0$$

$$\begin{array}{ccc} 0 \to M^e & \xrightarrow{tr} & M^{C_2} \\ \Delta \downarrow \uparrow \nabla & & res \downarrow \uparrow tr \\ M^e[C_2] & \xrightarrow{\nabla} & M^e \end{array} \to 0$$

$\xrightarrow{\pi_*}$

$\ker(M^e \xrightarrow{tr} M^{C_2}) \longrightarrow$

$\boxed{(a+b\otimes\gamma}\ : a+b\gamma=0 \longrightarrow M^e/tr$

$\nabla(a+b\otimes\gamma) = a+b\gamma$

$\{(a+b\overset{\otimes}{\gamma}) : a+b\gamma=0\}$

$m \xleftarrow{res}$

$\downarrow tr$

$m+m\otimes\gamma \longmapsto m+m\gamma = C_2/e \wedge m$

0

$\boxed{\mathbb{N}_\pm}$

$S^0 \simeq S^?$

$\Sigma C_2/e_+$

$\Sigma^2 C_2/e_+$

$S^0\{1\}$

$B\Omega'$

$(+)\Omega^0 = \Sigma\Omega^0$

$\ker F$

crucial recording elements

• aspect Ratio (dimension)

$TR'_{2,2} \cong TR'$

$\underline{\pi}_*^{C_p} \underline{M}^{hC_p} = H_0(M^e)$ $\quad \hat{H}^q(M^e) \quad \hat{H}^q(M^e)$

$\underline{M}_{hC_p} \to \underline{M} \to \underline{M}^{\Phi C_p}$

$\underline{M} \to \boxed{\underline{M}^{\Phi C_p}} \to \Sigma \underline{M}_{hC_p}$

$gr^1 \qquad gr^0$

$\underline{\pi}_*^{C_p} \underline{M} = M_0^{C_p} \quad 0 \quad \hat{H}^{-2} \quad 0 \quad 0$

$\underline{\pi}_* \underline{M} \to \underline{\pi}_* \underline{M}^{\Phi C_p} \to \underline{\pi}_{*-1} M^{hC_p}$

$H_1 \to 0 \to \hat{H}^{-1}$

$H_0 \to \hat{H}^0 \to \hat{H}^0$

0

$gr^1 \quad M^{C_p}_2$

gr^0

$H_0(M^e) \quad \hat{H}^0(M^e) \quad \hat{H}^{-1}(M^e)$

$0 \quad 1 \quad 2$

$\pi_0 \quad \pi_1 \quad \pi_2$

$\underline{\pi}_0 M^{\Phi C_p}$

$\cancel{M^{\Phi C_p}}$ $\quad \cancel{M}$

$M^{\Phi C_p}$

$\ker(tr) \quad \hat{H}^0(M^e) \quad \hat{H}^{-1}(M^e)$

λ = default rep^n of S^1 on $\mathbb{C}$

$$\begin{tikzcd} \underline{M}_{hC_p} \arrow[r] \arrow[d, equal] & \underline{M} \arrow[r] \arrow[d] & \boxed{\underline{M}^{\Phi C_p}} \arrow[d] \\ \underline{M}^e_{hC_p} \arrow[r] & \underline{M}^{hC_p} \arrow[r] & \underline{M}^{tC_p} \end{tikzcd}$$

$$M^{C_p} \underset{F}{\overset{V}{\rightleftarrows}} M^e$$

$H_0(M^e)$

$\pi_* M^{hC_p} =$

$H^0(M^e) \qquad H^1(M^e) \qquad H^2(M^e)$

$\cong$

~~$H^0(M^e) \qquad \hat{H}^{-1}(M^e) \qquad \hat{H}^0(M^e)$~~

$\pi_* M^{tC_p} = \hat{H}^0(M^e) \qquad \hat{H}^{-1}(M^e) \qquad \hat{H}^0(M^e)$

$\underline{\pi}_{\lambda+*}\underline{M}$

$\underline{\pi}_{\lambda+*}\underline{M} \longrightarrow \underline{\pi}_{\lambda+*}\underline{M^{\Phi C_p}}$

$= a_\lambda^{-1}\pi_* M^{\Phi C_p}$

$\pi_{\lambda+*}M^{hC_p} = \underbrace{u_\lambda^{-1}}_{\lambda-2}\pi_{*+2}M^{hC_p}$

$\pi_{\lambda+*}M^{tC_p} = a_\lambda^{-1}\pi_* M^{tC_p}$

$\boxed{M^{\Phi C_p}}$

$\boxed{M^{hC_p} \to M^{tC_p}}$

depends on d_0

$\alpha = x + \lambda$

good for cellular $S^0 \xrightarrow{a_\lambda} S^\lambda$

$d_0 = \dim_{\mathbb{C}}(\alpha)$

$d_\infty = \dim_{\mathbb{C}}(\alpha^{C_p})$ = triv part

$d_\infty = \dim_{\mathbb{C}}(\alpha^{\pi})$

(d_i)

$\alpha = x + y\lambda \doteq \boxed{(d_0 ; d_\infty)}$

$= d_\infty \lambda^{\text{triv}} + (d_0 - d_\infty)\lambda$

depends only on d_∞

$G \to X$, $g \mapsto g.x_0$, $+ n\mathbb{G}$

$\pi_n(G) \to \pi_n(X) \to \pi_n(X_{hG})$

non-canonical iso $\pi_\lambda E^{hC_p} \cong \pi_2 E^{hC_p}$ $\boxed{a_\lambda}$

$\pi_\star \underline{M}_{hC_p}$	$H_0(M^e)$				
$\underline{\pi}_\star \underline{M}^{hC_p}$	$H^0(M^e)$	$\hat{H}$			
$\underline{\pi}_\star M^{tC_p}$	$\hat{H}^0(M^e)$	$\hat{H}^{-1}(M^e)$	$\hat{H}^0(M^e)$		
$\underline{\pi}_\star M^{\Phi C_p}$	$M^{\Phi C_p}$	$\ker(tr)$	$\hat{H}^0(M^e)$		
	0	1	2	3	4

$$\begin{tikzcd} \underline{M} \arrow[r] \arrow[d] & \underline{M}^{\Phi C_p} \arrow[d] \\ \underline{M}^{hC_p} \arrow[r] & \underline{M}^{tC_p} \end{tikzcd}$$

a_λ invertible here

u_λ invertible

$\mathbb{Z}$ complex orientable

E even w/ S^1-action

$\pi_\star E^{hC_p}$ computed by $\hat{S}$ H^{*} spectral sequence

S^1 connected $\Rightarrow$ action on $\pi_\star$ trivial

$\Rightarrow H^*(BS^1, \pi_* E) \Rightarrow$ SS collapses

$a_\lambda \in \pi_{-\lambda}^{C_p} \mathbb{S}_{C_p}$ canonical

$u_\lambda \in \pi_{\cancel{}\,2-\lambda} M^{hC_p}$ invertible non-canonical

$G \times X \longrightarrow X$

(X, x_0)

$G \longrightarrow X$

$g \longmapsto g x_0$

$S^n \longrightarrow G \longrightarrow X$

$\pi_n(G \times X) \longrightarrow \pi_n(X)$

fix $x_0 \in X$

$\pi_n(G) \times \pi_n(X, x_0) \longrightarrow \pi_n(X)$

$S^n \longrightarrow G \Rightarrow G \longrightarrow X$

$g(t) \longmapsto g(t) \cdot x_0$

$S^n \xrightarrow{g(t)} G$

$x(t) \downarrow$

X

$\Rightarrow g(t) \cdot x(t) \in \pi_n(X)$

$\pi_n(G) \longrightarrow \pi_n(X)$

g

$g(t) \cdot x_0(t)$

$\pi_n(G) \quad S^n \longrightarrow G \longrightarrow X$

$g \longmapsto g x_0$

$G = \mathrm{Aut}(S^1, S^n) = \pi_1(G)$

$n \geq 2$

$\pi_n = \mathbb{Z}/2$

$\pi_1 = \mathbb{Z}/2$

$$\begin{array}{ccccc} \{g x_0\} & \xrightarrow{x_0} & \{x_0 \xrightarrow{g} g x_0\} & & x_0 \\ \downarrow & & \downarrow & & \downarrow \\ G & \longrightarrow & X & \longrightarrow & X_{hG} \\ g & \longmapsto & g x_0 & & \end{array}$$

$\pi_n(G) \to \pi_n(X)$

$g \longmapsto g x_0$

1-truncation of X_{hG} = action groupoid $X/\!/G$

$Ob(X/\!/G) = X$ $\qquad \pi_0(X/\!/G) = X/G$

$Hom_{X/\!/G}(x,y) = \{g \in G : gx = y\}$ $\qquad \pi_1(X/\!/G, x) = Stab(x) = G_x$

$Orb(x) = G/G_x$

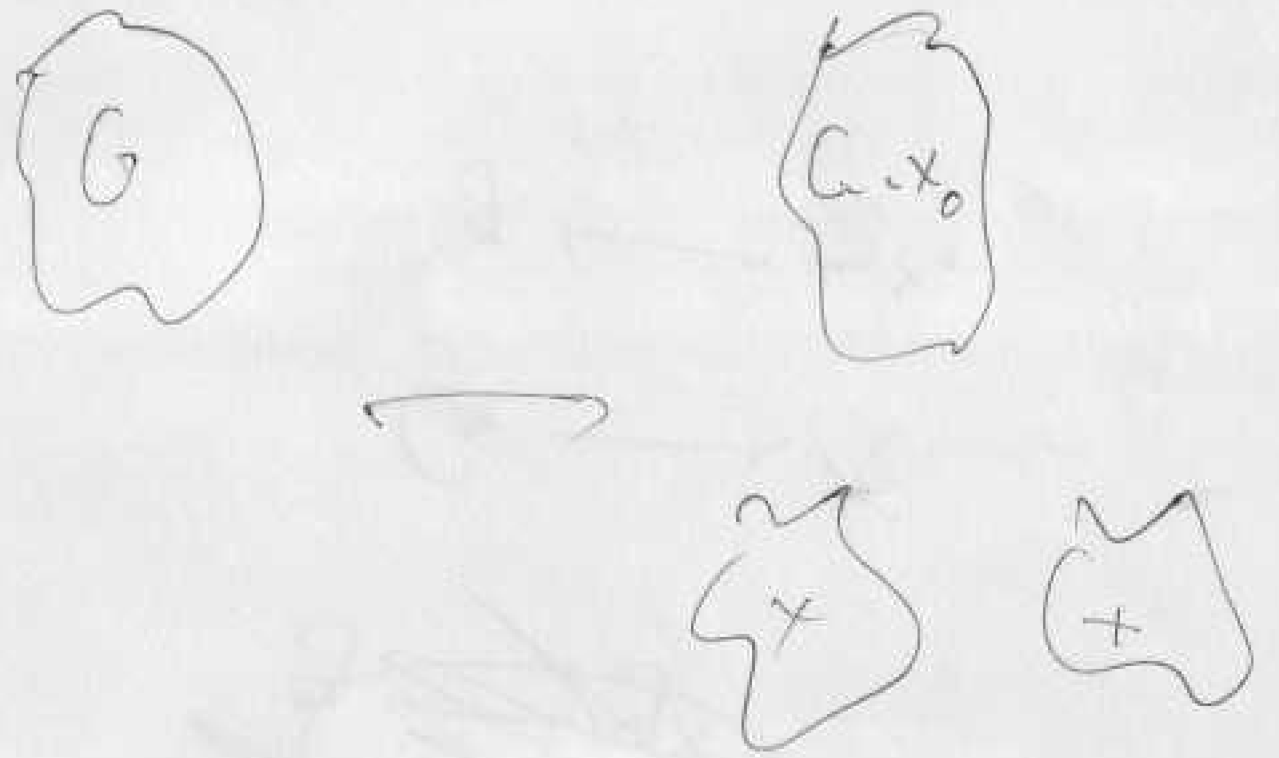
$$\begin{array}{ccc} Y & \longrightarrow & * \\ \downarrow & \lrcorner & \downarrow x_0 \\ X & \longrightarrow & X/\!/G \end{array} \qquad X_{hG}$$

$x \in X$, $x_0 \in X/\!/G$: $g x_0 = x$

$0 \to \pi_1(X/\!/G, x) \to \pi_0(fib) \to \pi_0(X, x_0) \to \pi_0(X/\!/G, x) \to 0$

$$\begin{array}{ccccccc} & & & & \| & & \| \\ G_x & \longrightarrow & G & \longrightarrow & X & \longrightarrow & X/G \end{array}$$

$g \in G$, $(x_0 \longrightarrow g \cdot x_0)$



$\pi_0 T^{hC_p} = \mathbb{Z}/p\langle t^{-1}\rangle \quad \mathbb{Z}/p^2 \quad \mathbb{Z}/p^2\sigma \quad \mathbb{Z}/p^2\sigma^2 \quad - - -$

$$= \frac{\mathbb{Z}/p^2[\sigma, t]}{pt=0,\ \sigma t = p}$$

$\alpha = (d_0 ; d_\infty)$

$= \mathbb{F}_p[t^{-1}, a_{\lambda_0}^{\pm}]$

$$\begin{array}{ccccc}
\pi_\alpha T_{hC_p} & \longrightarrow & \pi_\alpha T^{C_p} & \longrightarrow & \pi_\alpha T^{\Phi C_p} \\
\| & & \downarrow & & \downarrow \varphi \\
\pi_\alpha T_{hC_p} & \longrightarrow & \pi_\alpha T^{hC_p} & \longrightarrow & \pi_\alpha T^{tC_p}
\end{array}$$

$\pi_\star T^{\Phi C_p} = \pi_\star THH[a_\lambda^{\pm}]$

E^0

$\pi_\alpha T^{tC_p} \longleftarrow \pi_\alpha T^{hC_p}$

$\pi_\alpha T^{\Phi C_p}$

$\pi_\alpha T_{hC_p}$ $d_0 \geq 0$

$\pi_\alpha T^{\Phi C_p}$ d_∞

$\pi_{\alpha-1}$ π_α

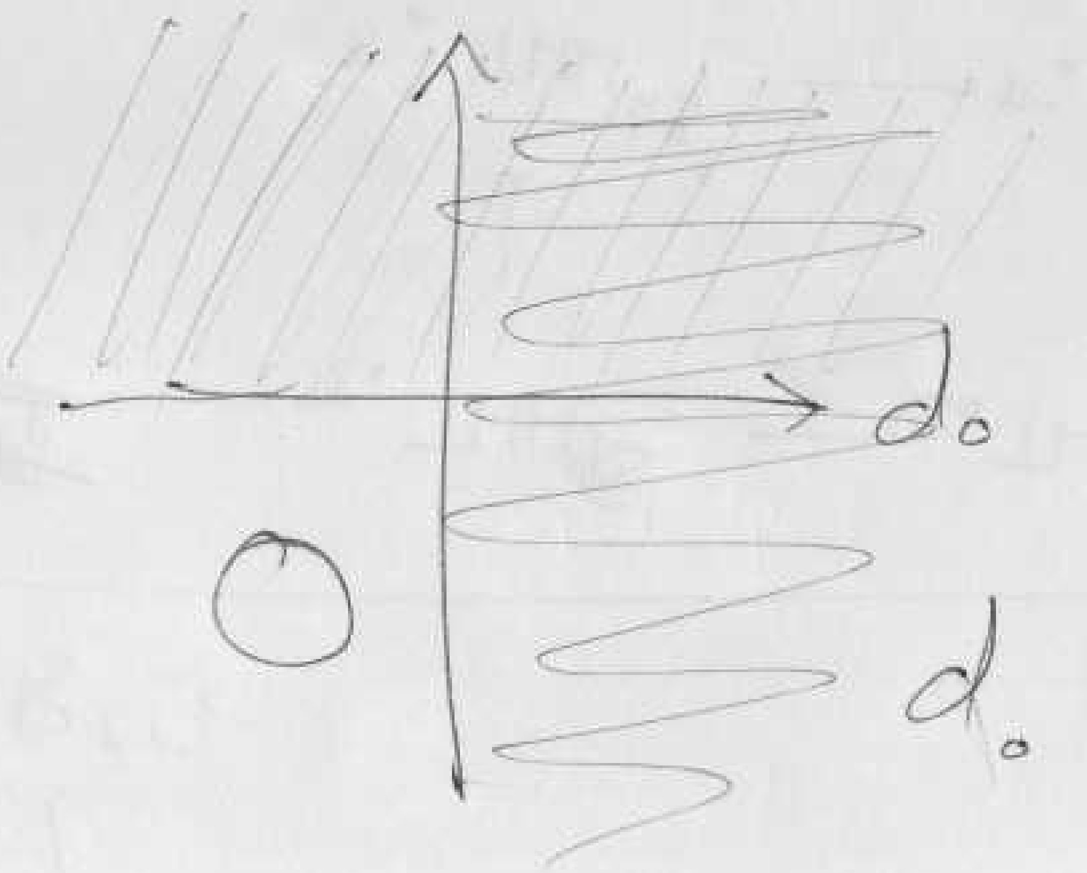
$d_0 < 0$

d_∞ $\varphi : THH \longrightarrow THH^{tC_p}$

conn. cov.

$\pi_\alpha T^{\Phi C_p}$ for $\mathbb{F}_p$

(Segal conj.)



$$\begin{array}{cccccc}
\pi_* T^{hC_p} & \mathbb{Z}/p\, t^{+2} & \mathbb{Z}/p\, t^{+1} & \mathbb{Z}/p^2 & \mathbb{Z}/p^2\sigma & \mathbb{Z}/p^2\sigma^2 \\
 & \downarrow\wr & \downarrow\wr & & & \\
\pi_* T^{tC_p} & \mathbb{Z}/p\, t^{+2} & \mathbb{Z}/p\, t^{+1} & \mathbb{Z}/p & \mathbb{Z}/p\, t^{-1} & \mathbb{Z}/p\, t^{-2}
\end{array}$$

$\mathring{I} : \mathcal{C}^{op} \to Sp$

$$\begin{tikzcd} B_{\mathring{I}}(x,x)_{hC_2} \arrow[r] \arrow[d, equal] & \mathring{I}(x) \arrow[r] \arrow[d] & \Lambda_{\mathring{I}}(x) \arrow[d] \\ B_{\mathring{I}}(x,x)_{hC_2} \arrow[r] & B_{\mathring{I}}(x,x)^{hC_2} \arrow[r] & B^{tC_2} \end{tikzcd}$$

$\pi_*^{C_p} THH(\mathbb{F}_p)$

$\pi_* THH^{hS^1} = \dfrac{\mathbb{Z}_p[\sigma, t]}{\sigma t - p}$ $\quad |\sigma| = 2,$

$\pi_* THH^{tS^1} = \mathbb{Z}_p[t^{\pm}]$ $\quad |t| = -2$

$\pi_* THH^{\Phi S^1} = \mathbb{Z}_p[t^{-1}]$

$\pi_* THH^{hC_p} = \dfrac{\pi_* T^{hS^1}}{pt = 0}$

$~~TC_*(\mathbb{F}_p)~~$

$$\begin{tikzcd} \pi_* THH^{\Phi S^1} \arrow[r] \arrow[d] & \pi_* THH^{\Phi S^1} \arrow[d] \\ \pi_* THH^{hS^1} \arrow[r] & \pi_* T^{tS^1} \end{tikzcd}$$

$\shortparallel$

$$\begin{tikzcd} \mathbb{Z}_p[\sigma] \arrow[r] \arrow[d] & \mathbb{Z}_p[t^{-1}] \arrow[d] \\ \dfrac{\mathbb{Z}_p[\sigma, t)}{\sigma t - p} \arrow[r] & \mathbb{Z}_p[t^{\pm}] \end{tikzcd}$$

$pt = 0 \implies p\sigma t = 0$

$\implies p^2 = 0$

d_∞

$\mathbb{Z}/p\, a_0^{-d_0} u_{\lambda_0}^{d_\infty} \xleftarrow{\sim} \mathbb{Z}/p\, a_0^{-d_0} u_{\lambda_0}^{d_\infty}$

$\mathbb{Z}/p\, a_0^{-d_0} u_{\lambda_0}^{d_\infty}$

$\mathbb{Z}/p^2\, u_0^{d_0} u_{\lambda_0}^{d_\infty}$

$\mathbb{Z}/p\, a_0^{-d_0} u_{\lambda_0}^{d_\infty}$

$\mathbb{Z}/p\, a_0^{-d_0} u_{\lambda_0}^{d_\infty}$

$\pi_\star T^{C_p}$

$\pi_\star T^{\Phi C_p}$

d_0

$\mathbb{Z}/p\, a_0^{-d_0} u_{\lambda_0}^{d_\infty} \leftarrow \mathbb{Z}/p^2\, u_0^{-d_0} u_{\lambda_0}^{d_\infty}$

O

$\pi_\star T_{hC_p}$

$(p\mathbb{Z}/p^2)\, u_0^{-d_0} (u_{\lambda_0}^{d_\infty})$

$\ker(F: W_1(\mathbb{F}_p) \to \mathbb{F}_p)$

$d_0 = 0$: $\mathbb{Z}/p\, u_{\lambda_0}^{d_\infty} \xleftarrow{p} \mathbb{Z}/p^2\, u_{\lambda_0}^{d_\infty}$

$\|$

$\mathbb{Z}/p\, u_{\lambda_0}^{d_\infty} \to \mathbb{Z}/p^2\, u_{\lambda_0}^{d_\infty}$

TITLE: DATE/NO.

$|v_0| = (1;0)$

$|a_0| = (-1;0)$

$|\sigma| = 2$

d_∞

$\mathbb{Z}/p[t^{-1}] \longrightarrow \mathbb{Z}/p[t^{\pm}]$

$\pi_\alpha T^{\Phi C_p}$ $\pi_\cdot T^{C_p}$ d_0

$\pi_\alpha T_{hC_p}$

$\alpha = (2;1)$ $\Rightarrow$ $\lambda^0 + \lambda^1$

$d_\infty = 1$ $\quad$ $\mathbb{C}^{triv}$

$|v_\lambda| = 2 - \lambda$

$|a_{\lambda_0}| = -\lambda$

$\pi_4 \xrightarrow{\cong} \pi_{2+\lambda}$ $|t| = -2$

T^{tC_p} $\quad$ πT^{hC_p} $\quad$ $\pi T^{\Phi C_p}$

$\mathbb{Z}/p\langle t^{-2}\rangle v_{\lambda_0}^{-1}$ $\mathbb{Z}/p^2\langle\sigma^2\rangle$ $v_{\lambda_0}^{-1}$

$\mathbb{Z}/p\langle t^{-1}\rangle a_{\lambda_0}^{-1}$

$\boxed{v_0 = \sigma v_\lambda^{-1}}$

$|v_0| = \lambda$

$a_0 = a_{\lambda_0}$

$t = a_0 v_{\lambda_0}^{-1}$

$(d_0; d_\infty)$

$v_0^{d_0}$ $\quad$ $v_{\lambda_0}^{d_\infty}$

or $a_0^{-d_0}$

$\mathbb{Z}/p\ a_0^{-2} v_{\lambda_0}$ $\quad$ $\mathbb{Z}/p^2\ v_0^2 v_{\lambda_0}$

$t^{-2} v_{\lambda_0}^{-2}$

$\boxed{v_0 a_0 = p}$

$\mathbb{Z}/p\ a_0^{-2} v_{\lambda_0}$

$= (a_{\lambda_0} v_{\lambda_0}^{-1})^{-2} v_{\lambda_0}^{-2}$

$v_0^2 = (\sigma^2 v_{\lambda_0}^{-2}) v_{\lambda_0} = \sigma^2$

$-a_{\lambda_0}^{-2}$

$v^{-1} a_0 = (a_0 v_{\lambda_0}^{-1})^{-1} \ldots^{-2}$

$$\begin{tikzcd} \arrow[d, "N"'] & \arrow[u, "1/p"'] \\ \mathbb{G}_m(A/I_1) \arrow[r, "d\log_1"'] & A/I_1\{1\}[1] \end{tikzcd}$$

$$\begin{tikzcd} P \arrow[r] \arrow[d] & \mathbb{G}_m(A/I_1^2) \arrow[r] \arrow[d] & 0 \arrow[d] \\ \mathbb{G}_m(A/I) \arrow[r, "N"'] & \mathbb{G}_m(A/I_1) \arrow[r] & A/I_1\{1\}[1] \end{tikzcd}$$

$P = \{ x \in \mathbb{G}_m(A/I)$

$\underline{y} \in \mathbb{G}_m(A/I_1^2)$

$y = N(x) \bmod I_1$.

$}$

$Fd(px^n) = F(pn\, x^n\, d\log x) = 0$

$Fd(px^{j/p}) = F(j\, x^{j/p}\, d\log x)$

$\boxed{px^n}$

$px^n = \Omega^0 \otimes \{\xi\}$

	x^n	px^n	$px^{j/p}$
F		0	0
Fd		0	$-$

$\}$

$d_0 \geq 1, \; d_\infty \geq 0$

$v_0 d_0 = p$

$v_0^{d_0} \longmapsto p^{d_0} a_0^{-d_0}$

$\Longrightarrow \cancel{v_0 d}$

E_0

$\mathbb{Z}/p \; a_0^{-d_0} \xleftarrow{\cancel{2}\; d_0} \mathbb{Z}/p^2 \; v_0^{d_0}$

$\mathbb{Z}/p \; a_0^{-d_0}$

torsion

E^1

$\mathbb{Z}/p \; a_0^{-d_0} \qquad \mathbb{Z}/p^2 \; v_0^{d_0}$

$\sim \; \mathbb{Z}/p \; a_0^{-d_0}$

E_∞

$\mathbb{Z}/p^2 \; v_0^{d_0} \; v_{\lambda_0}^{d_\infty}$

plan(A/I, B/J)

$$\begin{tikzcd} I \arrow[r] \arrow[d] & A \arrow[d] \\ 0 \arrow[r] & B/J \end{tikzcd} \qquad \begin{tikzcd} A \arrow[r] \arrow[d] & 0 \arrow[d] \\ J[1] \arrow[r] & B[1] \end{tikzcd}$$

$\Longleftrightarrow$ extension of

$$\begin{tikzcd} I \arrow[r] \arrow[d] & A \arrow[r] \arrow[d] & 0 \arrow[d] \\ 0 \arrow[r] & J[1] \arrow[r] & B[1] \end{tikzcd}$$

$$\begin{tikzcd} I \arrow[r] \arrow[d] & A \arrow[r] \arrow[d] & 0 \arrow[d] \\ 0 \arrow[r] & J[1] \arrow[r] & B[1] \end{tikzcd}$$

extension of A by I

$$0 \to J \to A \times J \to A \to 0$$

$$0 \to I \to A \times I \to A \to 0$$

$$0 \to A \to A \times A \to A \to 0$$

$$\begin{tikzcd} & 0 \arrow[d] & \\ A \times B \arrow[r] \arrow[d] & B/I \arrow[r] \arrow[d] & 0 \arrow[d] \\ A \arrow[r] & I[1] \arrow[r] & B[I] \end{tikzcd}$$

$\varphi^r(I) = I^{p^r}$ mod p by δ-stuff!

$= J_r$

$J_r = I I_r^{p-1}$

$A|B,\ C|D$

$F^{C_p}\underline{W} = (\ker F) \to (\operatorname{coker} F)$

group action on M/I in terms of I.

$\operatorname{Hom}(M/I, M/I)$

$\mathbb{Z}$

$\Delta \left(\int \right) \triangleright$

$\mathbb{Z}C_p$

$\cancel{\mathbb{Z}C_p[-1]}$

$\Longleftrightarrow \mathbb{Z}C_p$

$$\mathbb{Z}C_p \xrightarrow{\gamma} \mathbb{Z}C_p$$

$\mathrm{Hom}(M[-1], N)$

$= \mathrm{Hom}(M, N[1])$

= extensions of M by N

$$(*)\quad \begin{tikzcd} \widetilde{\mathbb{Z}[C_p]}[-1] \arrow[r] \arrow[d] & \mathbb{Z} \arrow[d] \\ 0 \arrow[r] & \mathbb{Z}[C_p] \end{tikzcd}$$

$$0 \to \mathbb{Z} \to \mathbb{Z}[C_p] \to \widetilde{\mathbb{Z}[C_p]} \to 0$$

$\cancel{0 \to M^{C_p} \to \ ? \to \mathrm{coker}\, M^e \to 0}$

gives extension of $\mathbb{Z}[C_p]$

$$\begin{tikzcd} & \boxed{} \arrow[r] \arrow[d] & 0 \arrow[d] \\ A \arrow[r] & M[1] \arrow[r] & N[1] \end{tikzcd}$$

$$\begin{tikzcd} 0 \arrow[r] & \mathbb{Z} \arrow[r] \arrow[d] & \mathbb{Z}[C_p] \arrow[r] \arrow[d] & \widetilde{\mathbb{Z}[C_p]} \arrow[r] \arrow[d] & 0 \\ 0 \arrow[r] & \mathbb{Z}[C_p] \arrow[r] & \arrow[r] & \widetilde{\mathbb{Z}[C_p]} \arrow[r] & 0 \end{tikzcd}$$

$$\begin{tikzcd} & \Sigma^\infty : \Sigma \arrow[l] & 0 \arrow[l] \\ & N : \Sigma' M \arrow[u] & 0 \arrow[l] \end{tikzcd}$$

$\cancel{A \to M[1] \to N[1]}$

$$\widetilde{\mathbb{Z}_p[i]}(k[x]/x^e) = \frac{W_e(k)}{V W_e(k)}$$

$$\begin{tikzcd}
0 \arrow[r] & J \arrow[r] \arrow[d, equal] & I \times J \arrow[r] \arrow[d] & I \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & J \arrow[r] \arrow[d] & A \times J \arrow[r] \arrow[d] & A \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & B \arrow[r] & A \times B \arrow[r] & A \arrow[r] & 0
\end{tikzcd}$$

$$\begin{tikzcd}
I \arrow[r] \arrow[d] & A \arrow[r] \arrow[d] & 0 \arrow[d] \\
0 \arrow[r] & J[1] \arrow[r] & B[1]
\end{tikzcd}$$

$$\begin{tikzcd}
\mathbb{Z} \\
\mathbb{Z}[C_p] \arrow[u, bend left] \arrow[u, phantom, "\Delta"]
\end{tikzcd}$$

$$\Rightarrow F^{C_p}\,\mathbb{Z}[C_p] = \widetilde{\mathbb{Z}[C_p]}[-1]$$

$$\begin{tikzcd}
\widetilde{\mathbb{Z}[C_p]}[-1] \arrow[r] \arrow[d] & \mathbb{Z} \arrow[r] \arrow[d] & 0 \arrow[d] \\
0 \arrow[r] & \mathbb{Z}[C_p] \arrow[r] & \mathbb{Z}[1]
\end{tikzcd}$$

$$M^e/M^G[-1] \to M^G \to M^e$$

M^G

~~splitting of~~

$$\begin{tikzcd}
M^e/M^G[-1] \arrow[r] \arrow[d] & M^G \arrow[d] \\
0 \arrow[r] & M^e
\end{tikzcd}$$

~~splitting of~~

extensions

$$\begin{tikzcd}
0 \arrow[r] & M^G \arrow[r] \arrow[d] & M^e \arrow[r] \arrow[d] \arrow[dl, "\sigma ?"'] & M^e/M^G \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & M^e \arrow[r] & \boxed{} \arrow[r] & M^e/M^G \arrow[r] & 0
\end{tikzcd}$$

$M^e \oplus_G M^e$?

$M^e \xrightarrow{\sigma} M^e$

$\sigma(x) = x$ for all things in M^G

$$\boxed{I \to A \to B}$$

$\mathbb{Z}[C_p][-1] \to \mathbb{Z}$

$N \to A[1] \to B[1]$

$N \times A \to \quad 0$

$N \times B \to I[1] \to 0$

$N \to A[1] \to B[1]$

split?

so $A^{G} \to A^{e}$

$i \oplus {}_2 U \quad {}_1 U \quad {}_0 U$

$A/(p^2)_q$:

$A/(p^2)_q\langle x^{p^{2n}}\rangle$

$A/(p)_q\langle (p)_q^p\, x^{pj}\rangle$

ϕ^{-2}

$A/\xi_1\langle x^{2n}\rangle$

$A/\phi^{-1}(\xi)\langle \xi\, x^{i/p}\rangle$

$A/(p^2)_q\langle x^{p^{2n}}\, d\log x\rangle$

$A/(p)_q\langle x^{pj}\, d\log x\rangle$

$A/\phi^{-1}(\xi)\langle \xi\, x^{i/p}\rangle$

$A/\phi^{-1}(\xi)\langle x^{j}\, d\log x\rangle$

$F(W\Omega^{\bullet})$

$F(a x^n) = \phi(a) x^{np}$

$F(\xi x^{i/p}) = \xi^{p} x^{i}$

$\nabla(x^j\, d\log x)$

$= \xi\, x^{i/p}\, d\log x$

$$\begin{tikzcd}
0 \arrow[r] & M^G \arrow[r] \arrow[d] & M^e \arrow[r] \arrow[d] \arrow[dl, dashed, "\sigma"'] & M^e/M^G \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & M^e \arrow[r] & M^e \oplus_{M^G} M^e \arrow[r] & M^e/M^G \arrow[r] & 0
\end{tikzcd}$$

$$\begin{tikzcd}
M^e \arrow[r] \arrow[d] & 0 \arrow[d] \\
M^e/M^G \arrow[r] & M^G[1]
\end{tikzcd}$$

$$\begin{tikzcd}
0 \arrow[r] & M^G \arrow[r] \arrow[d, equal] & M^e \times_{M^e/M^G} M^e \arrow[r] \arrow[d] & M^e \arrow[r] \arrow[d] \arrow[dl, dashed, "\sigma"'] & 0 \\
0 \arrow[r] & M^G \arrow[r] \arrow[d] & M^e \arrow[r] \arrow[d] \arrow[dl, dashed, "\sigma"'] & M^e/M^G \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & M^e \arrow[r] & M^e \oplus_{M^G} M^e \arrow[r] & M^e/M^G \arrow[r] & 0
\end{tikzcd}$$

$$A/I[-1] \to I \to A$$

$$B/J[-1] \to J \to B$$

$$\begin{tikzcd}
0 \arrow[r] & I \arrow[r] \arrow[d] & A \arrow[r] \arrow[d] & A/I \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & B \arrow[r] & A \oplus_I B \arrow[r] & A/I \arrow[r] & 0
\end{tikzcd}$$

$$B \to B/J \to J[1]$$

MG

$$\begin{tikzcd}
I \arrow[r] & B & \iff & I \arrow[r] \arrow[d, "N^e/NG"] & 0 \\
 & & & B/J \arrow[r] & J[1]
\end{tikzcd}$$

Floral

extension of

$$\begin{tikzcd}
0 \arrow[r] & J \arrow[r] \arrow[d] & I \times_{B/J} B \arrow[r] \arrow[d] & I \arrow[r] \arrow[d] \arrow[dl, dashed] & 0 \\
0 \arrow[r] & J \arrow[r] & B \arrow[r] & B/J \arrow[r] & 0
\end{tikzcd}$$

$\mathcal{M}^G$
$\cup$
$I \longrightarrow A \quad '' \ \mathcal{M}^c$

~~maps between~~ $\mathcal{M}^G \longrightarrow \mathcal{M}^c$

maps $A \longrightarrow B$ in terms of

$\boxed{A/I[-1] \longrightarrow I}$ $\iff$

$\boxed{B/J[-1] \longrightarrow B}$

$A/I[-1] \longrightarrow I$

given $\boxed{I, A/I}$

$$\begin{tikzcd}
0 \arrow[r] & I \arrow[r] \arrow[d] & A \arrow[r] \arrow[d] \arrow[dl, dashed] & A/I \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & B \arrow[r] & A \otimes_I B \arrow[r] & A/I \arrow[r] & 0
\end{tikzcd}$$

$A \longrightarrow B$

$\iff$
$$\begin{tikzcd}
A/I[-1] \arrow[r] \arrow[d] & I \arrow[d] \\
0 \arrow[r] & B
\end{tikzcd}$$

Λ ~~injective~~

$\ker f = \phi^{-1}(\Sigma)$

Ω^0

$\Omega^0 \otimes \{\Sigma \ \phi^{-1}(\Sigma)\}$

Λ injective here ???

~~injective~~

$\boxed{M, \Omega^0 \otimes \{\Sigma\}}$

$$\begin{tikzcd}
\cancel{(A/I, J)} \arrow[r] \arrow[d] & \cancel{(A, J)} \arrow[r] \arrow[d] & \cancel{(I, J)} \arrow[d] & \\
{} \arrow[r] \arrow[d] & (A, B) \arrow[r] \arrow[d] & (I, B) \arrow[r] \arrow[d] & (A/I[-1], B) \arrow[d] \\
{} \arrow[r] \arrow[d] & (A, B/J) \arrow[r] \arrow[d] & (I, B/J) \arrow[r] \arrow[d] & (A/I[-1], B/J) \arrow[d] \\
{} \arrow[r] & (A[-1], J) \arrow[r] & (I[-1], J) \arrow[r] & (A/I[-1], J[1])
\end{tikzcd}$$

$$\begin{tikzcd}
I \arrow[r] \arrow[d] & J \arrow[d] \\
A \arrow[r] & B
\end{tikzcd}$$

$$\begin{tikzcd}
0 \arrow[r] & I \arrow[r] \arrow[d] & A \arrow[r] \arrow[d] & A/I \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & B/J \arrow[r] & A \oplus B/J \arrow[r] & A/I &
\end{tikzcd}$$

$$0 \to J \to I \times J \to I \to 0$$

$$\boxed{\begin{array}{ccc} \mathrm{Hom}(X, I) & \longrightarrow & \mathrm{Hom}(X, A) \\ \downarrow & & \downarrow \\ 0 & \longrightarrow & \mathrm{Hom}(X, A/I_\xi) \end{array}}$$

~~Hom(B/Y~~

$I = M^{cp}$

$A = M^{e}$

injective

~~Hom(B/Y~~

$$\begin{array}{ccccccc} \mathrm{Hom}(A/I, J) & \longrightarrow & \cancel{\mathrm{Hom}(A, J)} & \longrightarrow & \mathrm{Hom}(I, J) & \longrightarrow & \mathrm{Hom}(A/I[1], J) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \mathrm{Hom}(A/I, B) & \longrightarrow & \boxed{\mathrm{Hom}(A, B)} & \longrightarrow & \mathrm{Hom}(I, B) & \longrightarrow & \mathrm{Hom}(A/I[1], B) \\ \downarrow & & \downarrow & & \downarrow & & \\ \mathrm{Hom}(A/I, B/J) & \longrightarrow & \mathrm{Hom}(A, B/J) & \longrightarrow & \mathrm{Hom}(I, B/J) & \longrightarrow & \mathrm{Hom}(A/I[1], B/J) \\ & & \downarrow & & \downarrow & & \\ & & \mathrm{Hom}(A, J[1]) & \longrightarrow & \mathrm{Hom}(I, J[1]) & \longrightarrow & \mathrm{Hom} \end{array}$$

$\boxed{\mathrm{Hom}(A,B)}$ $\mathrm{Hom}(A,B) \longrightarrow \mathrm{Hom}($

in terms of extensions

$(A,B) \longrightarrow (I,B) \longrightarrow ($

Hom

$O \to J \to X \to I \to O$

$J \to X \to I$

$A/I[-1] \to I$

~~$O \to J \to X \to A \to A/I$~~

$A/I[-1]$

$J \to X \to I$

Trivial!

$O \to J \to X \xrightarrow{A/I[-1]} A/I[-1]$

$O \to J \to X \to I \to O$

$O \to X$

$O \to I \to A \to A/I \to O$

$(M, F^{cp} N[1]) \longrightarrow (F^{cp} M, F^{cp} N)[1]$

$\downarrow \qquad\qquad \downarrow$

$(M, N[1]) \longrightarrow (F^{cp} M, N)[1]$

heuristically

$\mathrm{Ext}^2(\longrightarrow)$

$\cancel{0} \to F^{cp} N \to$

$\mathrm{Ext}^2(\longrightarrow)$

$0 \to F^{cp} N \to X \to M \to 0$

$A^{cp} \to A^{lc}$

$\mathrm{Hom}(I, B[1])$

$A \otimes C_p \longrightarrow \hat{H}_{cp}(A)$

$\cancel{cp(x) = x^{cp}}$

Sunday

coffee or beer

$$\begin{tikzcd}
(A,B) \arrow[r] \arrow[d] & (I,B) \arrow[r] \arrow[d] & (A/I[-1],B) \arrow[d] \\
(A,B/J) \arrow[r] \arrow[d] & (I,B/J) \arrow[r] \arrow[d] & (A/I[-1],B/J) \arrow[d] \\
(A,J[1]) \arrow[r, dash] & (I,J[1]) \arrow[r] & (A/I[-1], \cancel{B/J}\ J[1])
\end{tikzcd}$$

figure out what Ext^2 actually is

$\cancel{I = M^{cp}}$

$$\begin{tikzcd}
(M, F^{cp}N[1]) \arrow[r] \arrow[d] & (F^{cp}M, F^{cp}N[1]) \arrow[d] \\
(M, N[1]) \arrow[r] & (F^{cp}M, N[1])
\end{tikzcd}$$

$\cancel{Ext^2}$

$\cancel{I = M^{cp}}$ $I = M$

$A/I[-1] = \cancel{F^{cp}M}$

$F^{cp}M$

$\cancel{(M, F^{cp}N}$

$$\begin{tikzcd}
\cancel{(M, F^{cp}N)^{[1]}} \arrow[r] \arrow[d] & \cancel{(F^{cp}M, F^{cp}N)^{[1]}} \arrow[d] \\
(M, N[1]) \arrow[r, dash] & \cancel{(F^{cp}M, N[1])}
\end{tikzcd}$$

$\{v_{H/K} \in A\}$.

$$
\begin{tikzcd}
0 \arrow[r] & I_K/I_K^2 \arrow[r] \arrow[d, "J_{H/K} = |H/K|"] & \mathrm{Gm}(A/I_K^2) \arrow[r] \arrow[d] & \mathrm{Gm}(A^K) \arrow[r] \arrow[d, "N_K^H"] & 0 \\
0 \arrow[r] & \frac{I_H}{I_K I_H} \arrow[r] & \mathrm{Gm}(A/I_K I_H) \arrow[r] & \mathrm{Gm}(A^H) \arrow[r] & 0
\end{tikzcd}
$$

$$\frac{I_H}{I_K I_H} \qquad \cancel{\frac{I_K J_{H/K}}{I_K I_H}}$$

$|H|$

$$\frac{I_H}{I_K I_H} = \frac{I_K}{I_K^2}$$

$\cancel{N^{H/K}(1+t) = (1+t)^{H/K} + \mathrm{tr}}$

$\cancel{(1+t)^{H/K}}$ $N_K^H($

• quenching thumbnails

(→ server)

• Liquid, try to update old content

• vuuii

• Michael

• Lean guy

• ~~Liquid something~~

need for $A/I_H I_K \hookrightarrow A/I_K^2 \times A/J_{H/K}$

$\varphi^{H/K}(I_K) \subset J_{H/K}$

$$\begin{array}{ccc} A^H & \longrightarrow & A^{H/K} \\ | & \overset{\phi^{H/K}}{\nearrow} & \\ A^K & & \end{array}$$

$\psi^{H/K}(f) = f^{H/K} + \text{—}$.

spoke representatives

$\boxed{I_H I_K}$

$N_H^K(\mathfrak{p}+1)$ pow

~~$\varphi^r(f)$~~ ~~Frobenius?~~ ? ~~$\varphi(d) = d^p +$~~

~~$\varphi(f) \equiv p \mod I_1^2$~~ ~~$N_e^{cp}(f) =$~~

~~$\varphi(d) = d^p + pu$~~

$\varphi^r(\mathcal{I}) =$

im ψ?

assume primary!

$$\begin{array}{ccc} TR^1_{2-\lambda} & \xrightarrow{R} & TR^0_2 \\ \downarrow F & \searrow a_\lambda & TR^1_2 \\ TR^0_0 & & \end{array}$$

$\cong W_0\Omega^0$

give the ~~associated~~ tensored at line bundles so that "~~the~~ Δ ~~comp~~ $\mathbb{F}_p$" $\frac{\Delta_R}{\mathbb{Z}_p}$

all maps are

$F(\sigma_n) = \sigma_{n-1} \circ$ unit

$R(\sigma_n) = \phi$ ~~~ σ_{n-1}

$\varphi^p: A^e \to A^{\varphi}$

$W_1 \otimes \{\xi \phi^{-1}(\xi)\}$

$F \downarrow F(f) =$

$+ \Omega_0 \otimes \{\xi\}$

$\mathbb{Z}_p^{cyc}$

$\mathcal{O}_{\mathbb{C}_p}$

$\mathbb{F}_p[x^{1/p^\infty}]/(x)$

$\mathbb{Z}_p^{cyc}[x^{1/p^\infty}]^\wedge_p$

$\widetilde{W}\mathbb{Z}_p[[q-1]][q^{1/p^\infty}]^\wedge_{(p,q-1)}$

$\otimes$ "$A_{inf}(\mathcal{O}_{\mathbb{C}_p})$"

$\mathbb{Z}_p\left\langle \frac{x^n}{[n]_q!} \right\rangle$

$\Delta_R \otimes \{(p^\infty)_q\}$

~~$W_n\Omega^R$~~

$\otimes \}$ ~~$W_n\Omega^i_S$~~ $\otimes \{(p^n)_q\}$

$N(f) = \phi(f) - VS(f)$

S-structure

$\varphi^{ce}(f) = N^{ce}_{e}(f) + V^{ce}_{e} S(f)$

$\varphi^p(f) = N^p$

~~$A^e \xrightarrow{\varphi^p} A^{\varphi}$~~

$$\varphi^2(I) = I^{p^2} + p\delta_1(I)^p + p^2\delta_2(I)$$

$$\Rightarrow \varphi^2(I) \equiv I^{p^2} \mod p$$

$$= I^{p^2-p} I^p$$

$$\Rightarrow$$

$$\varphi(I) = I^p \mod p$$

$$= I^p[I\varphi(I)]^{p-1}$$

TITLE: DATE / NO.

δ which orbits?

ring map $\delta(xy) = x^{\ell p}\,\delta(y) + \cdots +$

$\delta : A \longrightarrow A_{\oplus}$

$A \otimes_{c_p}^{e} \longrightarrow A^{C_p}$

φ^{c_p}

$W_{C_p}(A)$

$\varphi^{c_p}(f) = f^{C_p} + C_p k \cdot \delta(f)$

$\varphi^{c_p}(f) = \begin{cases} f^{C_p} \end{cases}$

$\gamma \nearrow L_R$

$A \longrightarrow A_{\otimes}$

$\mathrm{Fil}^{\circ - n}\, W_k \Omega^i_S$

$\mathrm{TR}_*(\mathcal{O}_{C_i})$

$RO(G)$-graded norms for prismatic & de Rham–Witt theory?

$$\boxed{N(\omega) = V\frac{\omega^p}{p} + (p - V(1))\frac{\psi^p(\omega)}{p}}$$

vectors to Rede Rham–Witt complex!

$$\begin{array}{ccc} \mathcal{H}^{cp} & \longrightarrow & 0 \\ \downarrow & & \downarrow \\ M^{cp} & \longrightarrow & M^{cp} \end{array} \Rightarrow M^c \longrightarrow M^{cp}$$

(assuming ~~nes injective~~

~~surjective~~

~~ker(nes) in degs 0~~

~~coker (F)~~

surjective!

~~ker(F) → W(R) →~~

$$\begin{array}{ccc} A/I[-n] & \longrightarrow & 0 \\ \downarrow & & \downarrow \\ A & \xrightarrow[\ tr\ ]{} & A \end{array}$$

$0 \longrightarrow$

$M^e \to M^{cp}$

$F^{cp}M \to M^{cp} \to M^e$

$$\begin{tikzcd} F^{cp}M \arrow[r] \arrow[d] & 0 \arrow[d] \\ M^{cp} \arrow[r, "tr"'] & M^{cp} \end{tikzcd}$$

$M^{cp} \to M^{cp}$

is the map. $\boxed{VF(1)}$?

$$\begin{tikzcd} 0 \arrow[r] & M^{cp} \arrow[r] \arrow[d, "tr"'] & M^e \arrow[r] \arrow[d] \arrow[dl, dashed] & M^e/M^{cp} \arrow[r] \arrow[d] & 0 \\ 0 \arrow[r] & M^{cp} \arrow[r] & M^e \otimes_{M^{cp}} M^{cp} \arrow[r] & \boxed{} \arrow[r] & 0 \end{tikzcd}$$

transfer

If H/K is prime, then

A^e A^{c_2} A^{c_3} A^{c_4}

$u = 1 + t, \quad t \in I$

$u - 1 = t$ | p^t? $\qquad \phi(t)$

I/I^2 | $I/I^2 \qquad \phi(I)/\phi(I)^2$

$u^p - 1$

$\boxed{N(t)???}$

$u^p - 1 = \phi(u-1) = \phi(t) \qquad (p)_u = p \mod I$

$u^p - 1 = (p)_u \underbrace{(u-1)}_{t} \qquad (p+I)t = pt$?

(A, I) should be refractive, not ΔI

$\mathrm{im}(tr) = p \bmod (\ker F)^p$

A ring w/ C_p-action

$\varphi(-) = -^p$

$u \in 1 + I$,

$u^p \equiv 1 \mod I\phi(I)$

$u^p - 1 = \phi(u-1) \in \phi(I)$ ✓

mod I, $u \equiv 1 \Rightarrow u^p = 1$

$u - 1 \in I_0$

$u^p - 1 \in I_1$

$u^{p^2} - 1 \in I_2$

$\vdots$

$\frac{u^p - 1}{p} \equiv u - 1 \mod I$

~~$\xi \equiv p \bmod I$~~

$u = 1 + \xi$

$\frac{u^p - 1}{p} = \underbrace{(p)_u}_{\equiv p \bmod I} (u - 1)$

$\log_\Delta(u)$

$\frac{I\varphi(I)}{I^2\varphi(I)^2} \hookrightarrow \frac{I}{I^2} \times \frac{\varphi(I)}{\varphi(I)^2}$?

~~transversal~~

~~Suppose $xI\varphi(I)$~~

$\boxed{x}\, d\varphi(d) = y\, d$

$u = 1+t,\ t \in I$

$u - 1 = t \in A/I\{1\}$

$u^p - 1 = pt \in A/I\{1\}$

$= \phi(t) \in A/\phi(I)\{1\}$

$u^{p^2} - 1 = p^2 t \in A/I\{1\}$

$= p\phi(t) \in A/\phi(I)\{1\}$

$= \phi^2(t) \in A/\phi^2(I)\{1\}$

$\log_G$?

$u^{p^2} - 1 = (p^2)_u (u-1)$

$= (p)_{u^p} (u^p - 1)$

looks very similar to norm!

$u^{p^r} - 1 = p^{r-i} \phi^i(t) \in A/\phi^i(I)\{1\}$

Q. Does $A\{1\} \hookrightarrow$???

roughly looks like $(t, \phi(t), \phi^2(t), \dots)$

$(u-1, u^p-1, u^{p^2}-1, \dots)$ = image under ghost map?

What is his norm map doing?

q-logarithm characterized by Δ?

Liquid

- captions
- TypeScript λ
- FCK-NAT
- post in Discord

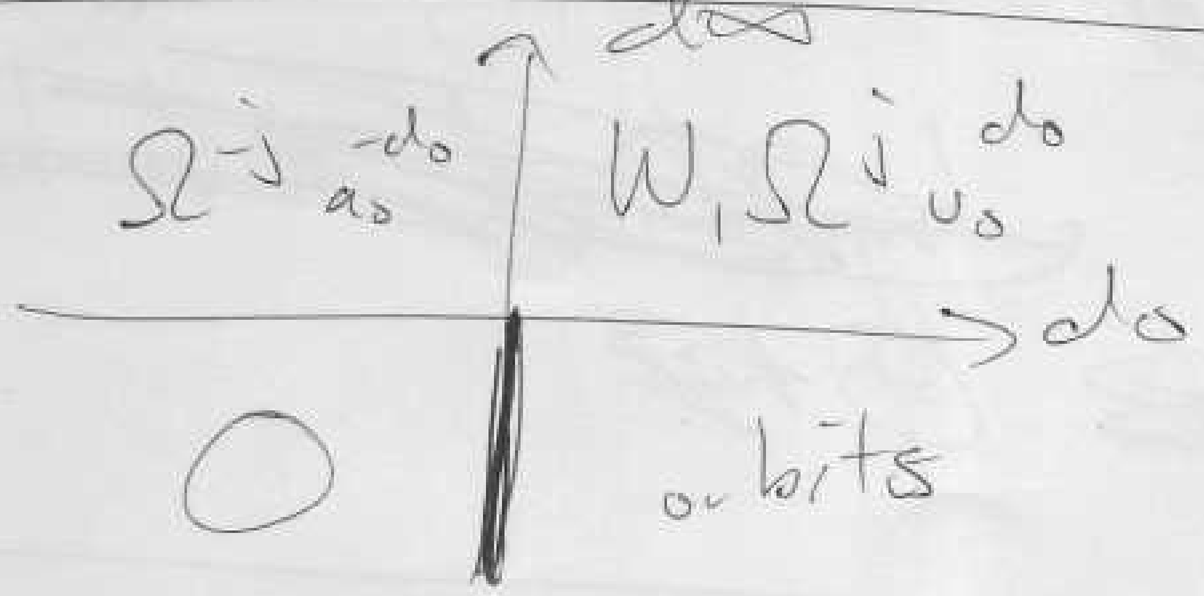
Epiplexis

- fix the slugs you dummy!
- content???

~~N~~ $\cancel{\nabla x^2}$

$TC^-_*(R) = \mathrm{Rees}(N^{\geq *}\Delta\{*\}) = \mathrm{Fil}^{\geq i} t^{-i}$

$\alpha = (d_0 ; d_\infty)$



~~$\Omega^j{}^{-d_0}_{a_0}$~~ ~~$\Phi_{\ldots}\mathrm{THH}(R;\mathbb{Z}_p) = \Sigma^{[-1]}\,\mathrm{Fil}^1\,\mathrm{THH}(R;\mathbb{Z}_p)$~~

$\Omega^j{}^{-d_0}_{a_0}{}^{d_\infty}_{u_{\lambda_0}} \longleftarrow W_1\Omega^j{}^{d_0}_{u_0}{}^{-d_\infty}_{u_{\lambda_0}}$

literally is fiber of

$W_1\hat{\Omega}^j\{i\} \longrightarrow \Omega^j\{i\}$

$$\begin{cases} \mathrm{Fil}^1 W_1\Omega^j\, u_{\lambda_0}^{d_\infty} \\ W_1\Omega^j\, u_0^{d_0}\, u_{\lambda_0}^{d_\infty} \\ \quad\oplus\ \Omega^j{}^{-d_0}_{a_0}[-1] \end{cases}$$

~~$\mathrm{TC}^-(R;\mathbb{Z}_p)$~~

$d_0 = 0$ ~~$\mathrm{Fil}^{1-d_0}\, u_0^{d_0}$~~

$d_0 > 0$ $\mathrm{Fil}^{d_0}\, a_0^{-d_0}$

$\mathrm{Fil}^0 W_1\Omega^\cdot = 0$ ~~Fil^1 $W_1\Omega$ $W_1\Omega$ $W_1\Omega$~~

2 1 0 -1 -2

$d_0 \geq 0$ shifted Rees?

$\mathrm{Fil}^{1-d_0} W_1\hat{\Omega}^j u_0^{d_0}$

$\mathrm{Fil}^{d_0}\, \Omega^j{}^{-d_0}_{a_0}$

d_0 negative!

Mod(R)-graded

~~$\alpha \in RO(C_p)$, $TR_\alpha = ?$~~ ~~diffuse~~

$$\pi_0^e \xrightarrow{N} \pi_0^{C_p} \xrightarrow{F} \pi_0^e$$

---

moving restriction past norm

~~$(A,\alpha) \leftarrow (D,\delta) \longrightarrow (X,x) = (X,x)$~~

$(n,\tilde{n})$: $(A,\alpha) \longrightarrow (B,\beta)$ in $RO(G)^N$

$(r,\tilde{r})$: $\underbrace{(X,x)}_{=A} \longrightarrow (B,\beta)$ in $RO(G)^F$

---

n: $A \to B$ map of G-sets

$\tilde{n}$: $\beta \Rightarrow n_\oplus \alpha$ nat trafo

r: $X \to B$ map of G-sets

$\tilde{r}$: $x \Rightarrow r^* \beta$

$(r,\tilde{r}) \circ (n,\tilde{n})$ = bispan

$(A,\alpha) \xleftarrow{(r',\tilde{r}')} (D,\delta) \xrightarrow{(n',\tilde{n}')} (X,x) = (X,x)$

$$\begin{tikzcd} D \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner"] & X \arrow[d] \\ A \arrow[r] & B \end{tikzcd}$$

δ: $B_G D \longrightarrow \hat{\mathcal{I}}^{op}$

$d = (a,x) \in D$, set $\delta(a,x) = \alpha(a)$

on g: $(a,x) \to (ga, gx)$

set $\delta(g) = \alpha(g)$: $\alpha(a) \to \alpha(ga)$

$\tilde{r}'$: $\delta \Rightarrow (r')^{*}\alpha$ identity on objects

$\tilde{n}'$: $\chi \Rightarrow (n')^{\oplus}_{n} \delta$ in composite

$$\chi(x) \xrightarrow{\tilde{r}} \beta(r(x)) \xrightarrow{\tilde{n}} \bigoplus_{n(a)=r(x)} \alpha(a) = \bigoplus_{d \in D,\ n'(d)=x} \delta(d)$$

??????

$$\boxed{\begin{array}{l} \pi_0^e \Rightarrow A = Gle \\ \pi_0^G \Rightarrow B = * \end{array}}$$

$(0,0)$ $\quad Gle_+ \wedge S^0$

α: $B_G(Gle) \longrightarrow \hat{\mathcal{I}}$

β: $B_{G^*} \longrightarrow \hat{\mathcal{I}}$ $\quad S^0 \cap G$

$\tilde{n}^*_0 \beta \longrightarrow n^{\oplus}_0 \alpha$

$\Rightarrow$ ~~$S^0 \longrightarrow Gle$~~ $\wedge S^0$

$$\begin{tikzcd} D \arrow[r] \arrow[d] & Gle \arrow[d] \\ Gle \arrow[r] & * \end{tikzcd}$$

$$\begin{array}{ccc} D & \longrightarrow & X \\ \big\downarrow & & \big\downarrow n \\ A & \xrightarrow{\;n\;} & B \end{array}$$

$F^B_X \, N^B_A = \prod$

$Gle \longmapsto S^0$

$S^0 \longrightarrow S^0 \longrightarrow \oplus$

$n(a) = r(x)$

multiply along fibers.

Cartier iso?

$(\rho)_{q^{1/p}} = \xi$.

$\Delta^{(1)} = q^{\xi_c}$

$A[x] \longrightarrow A[x]\, d\log x$

$A[\tilde{\xi}][x] \xrightarrow{\nabla_q} A[\tilde{\xi}][x]\, d\log x$

$A[\tilde{\xi}][x^{\pm p}] \longrightarrow A[\tilde{\xi}][x^{\pm p}]\, d\log x$

$A[\tilde{\xi}][x^{pn}]$

$\bigoplus$

$H^0(q\Omega_{\mathbb{Z}[x]}/(p)_q)$

$\mathbb{Z}[\zeta_p]\langle x^{\pm p}\rangle$

$\mathbb{Z}[\zeta_p]\langle x^{\pm p}\, d\log x\rangle$

$H^0(q\Omega_{\mathbb{Z}[x]}/(p)_q) = q\Omega_{\mathbb{Z}[x^p]}$

$H^0(q\Omega_{\mathbb{Z}[x]}/(p^2)_q) \longrightarrow$

$$q\Omega_{(p^2)_q} : \left.\begin{matrix} x^{p^2} \\ (p)_{q^p}\, x^p \end{matrix}\right]$$

$$\cancel{q\Omega_{(p^2)}} \quad \cancel{q\Omega_R/(p^2)_q \simeq W_1\Omega_R^{\cdot}}$$

$$q\Omega_R/(6)_q : \quad \begin{matrix} x^6 \\ (2)_{q^3}\, x^3 \end{matrix} \quad \begin{matrix} (3)_{q^2}\, x^2 \\ (6)_q\, x \end{matrix}$$

$$\Rightarrow \cancel{W_6(R[x^6])}$$

$$H^0(q\Omega_{\mathbb{Z}[x]}/(n)_q) = W_{\langle n\rangle}\Omega^{\cdot}_{\mathbb{Z}[x^n]}$$

~~canonical identification~~...

$TP_0(S) = W\Omega^{\cdot}_S$ [17]

$\mathbb{P}$. $TC^{-}_{\cdot} = W\Omega^{\cdot}[\sigma, t]/(\sigma t - p)$???

$W\Omega^{\cdot}$

$\cancel{V}W\Omega^{0} \longrightarrow W\Omega^{\cdot}$

$pVW\Omega^{0} \longrightarrow VW\Omega^{1} \longrightarrow W\Omega^{\cdot}$

$\Omega^{1} \longrightarrow TR^{1}_{1+\lambda}$

$TR^{1}_{\lambda,1}$. $\cancel{\lambda = (1,0)}$

$\cancel{TR^{1}_{\lambda-2,3}}$

$T_\lambda^{hc_p} \;?\; \frac{N^{\geq 1}\Delta}{pN^{\geq 2}\Delta}$

~~$T_\lambda \simeq N^{\geq 1}\Delta$~~

$N^{\geq 1}: \; VW\Omega^0 \to W\Omega^1 \to W\Omega^2 \to \cdots$

$N^{\geq 2}: \; pVW\Omega^0 \to VW\Omega^1 \to W\Omega^2 \to \cdots$

$$\frac{VW\Omega^0}{p^2VW\Omega^0} \to \frac{W\Omega^1}{pVW\Omega^1} \to \frac{W\Omega^2}{p} \to \frac{W\Omega^3}{p} \to \cdots$$

$$d\left(p\alpha x^{n/p}\right) = (\alpha n)\, x^{n/p}\, d\log x$$

$$V(x^n\, d\log x) = p\, x^{n/p}\, d\log x$$

$n \geq 1$

$$\begin{array}{ccc} \mathbb{Q} & \xrightarrow{a^{(-)}} & \mathbb{R} \\ {\scriptstyle i}\downarrow & & \\ \mathbb{R} & & \end{array}$$

$$\begin{array}{ccc} \mathbb{Q}^{op} & \xrightarrow{a^{-}} & \mathbb{R} \\ {\scriptstyle i}\downarrow & \nearrow & \\ \mathbb{R}^{op} & & \end{array}$$

$\Leftarrow$?

$p^v \Leftrightarrow p^2 \mid (\alpha n)$?

$p^2 \mid n$

$p \mid \alpha,\ p \mid n$

~~$(\mathrm{Lan}_i f)($~~

~~$\tilde f(a) \leq x$~~

$$\begin{array}{ccc} A & \xrightarrow{f} & Y \\ {\scriptstyle i}\downarrow & \nearrow_{\tilde f} & \\ X & & \end{array}$$

$\tilde f \circ i = f$

A, X, Y posets
i, f monotonic

$TR^0_1 = \Omega^1$

$TR^1_{\lambda,1}$: $(1;0) \Longrightarrow \boxed{W_1\Omega^1 \sigma v_{\lambda_0}^{-1}}$

$TR^1_{\lambda-2,3}$: $(0;-1) \Longrightarrow Fil^1 W_1 \Omega^3 v_{\lambda_0}^{-1}$

$TP_0(S) = W\Omega^\bullet_S[t^\pm]$,

$pW\Omega^1$

$\cancel{W\Omega^1_0 \longrightarrow W\Omega^1 t^{-1}}$

$\Omega^1 \longrightarrow$ "$N^{-1}W\Omega^1$"

sup

left Kan extension

right Kan extension $Fil^1 W_1\Omega^3 v_{\lambda_0}^{-1}$

$(\mathrm{Lan}_i f)(x) = \sup\{f(a) \mid i(a) \leq x\}$

$(\mathrm{Ran}_i f)(x) = \inf_{i(a) \leq x} \{f(a)\}$ $f(a) \leq \tilde{f}(i(a)) \leq \tilde{f}(x)$

$\Longrightarrow f(a) \leq \tilde{f}(x)$

$$\Omega^1 \cong W_1\Omega^1 \sigma \cup \lambda_0^{-1} \longrightarrow \Omega^1$$

$\oplus$

$\mathrm{Fil}^1 W_1\Omega^3 \cup \lambda_0^{-1}$ p

x^p $\Big\downarrow F$

$$\Omega^1\sigma \oplus \Omega^3$$

$dx\,dy\,dz$

$dy\,dx\,dz$

$(p=3)$

~~dx should go to~~

~~$(dx+dy+dz)^p$~~ $(dx+dy+dz)^3 = 0$ yes but --

~~$N(dx+dy+dz)$~~

$\oplus$

$F(dx) = F(x\, d\log x) = x^p\, d\log x$

$= x^{p-1}\, dx$

perfectoid case

$TR^1_\lambda(R) = A[\xi \phi(\xi)\langle \sigma \cup \partial_0^{-1}\rangle \quad \mapsto \phi^{-1}(\xi)$

$R \downarrow \phi^{-1}$

$= A[\xi\langle t^{-1}\rangle$

$\Rightarrow 0$ except

$\xi^{p-1}\, dx \mapsto$

$dx \mapsto$

x^2

$C_2/e_+ \to S^0 \to S^\sigma$

$S^{-\sigma} \to S^0 \to C_2/e_+$

$\Sigma^{-1} C_2/e_+ \to S^{-\sigma} \to S^0$

Ω^0/F

-1 0 1 2 3

Ω^0 Ω^1 Ω^2

$\oplus$ $\Omega^1\{1\}$

F $\Omega^0\{1\}$

F

$W_1\Omega^0$ $W_1\Omega^1$ $W_1\Omega^0\{1\}$

$\oplus$

$W_1\Omega^2$

$FdV =$

Ω^0/F Ω^1/F $\ker F$ Ω^2/F

$\oplus$

$\Omega^0/F\{1\}$

$(\ker F)$ $(\ker F)W_1\Omega^1$

$F\,dV(x^n) = Fd(px^{n/p})$

$= F(n\, x^{n/p}\, d\log x)$

$= n\, x^n\, d\log x$

$W_1\Omega^0$	$W_1\Omega^1$	$W_1\Omega^2$	$W_1\Omega^0/V$	$W_1\Omega^1/V$	$W_1\Omega^2/V$
Ω^0	Ω^1	Ω^2	0	$B\Omega^1$	$B\Omega^2$
	Ω^0	Ω^1		Ω^0	Ω^1

dV

Ω^0 Ω^1 Ω^2

$\Omega^{0,2,3}$

$\Omega^{i+1}\{-1\}$

$\Omega^{i}\{i-1\}$

$\Omega^{i}\{-i\}$

Tomorrow, schedule.

$W\Omega^0 \quad W\Omega^1 \quad W\Omega^2$

V^k

$W\Omega^0 \quad W\Omega^1$

dV^k

$x^\alpha y^\beta \, d\log x \qquad W\Omega^0$

$dV^k x = V^k y$

$dx = p^k y$

$\Rightarrow \quad \Rightarrow \; \in F^k ?$

$W\Omega^* \qquad W\Omega^0 / V^k$

$W\Omega^{\cdot -1} \xrightarrow{dV^k} 0$

$W\Omega^{\cdot -2} \qquad W\Omega^{\cdot -2}$

$W_1\Omega^0 \quad W_1\Omega^1 \quad W_1\Omega^2 \qquad W_1\Omega^0/V \quad W_1\Omega^1/V \quad W_1\Omega^2/V$

$\Omega^0 \quad \Omega^1 \qquad 0$

Ω^0

poly-representation, negative range

(what is the invariant answer?)

$\Rightarrow$ infinite product / calculus on $\mathcal{O}_{\mathbb{C}_p}$

$\Omega^0/F \quad \Omega^1/F \quad \Omega^2/F$

$\oplus$

$\Omega^0/F\{1\}$

$W_1\Omega^0[F] \quad W_1\Omega^1[F] \quad W_1\Omega^2[F]$

$\pi_0 \cancel{(Th(p)) = Fil^1}$

$(Th(p)) \cancel{\Omega} ?$

$Fil^1 W\Omega^1 \qquad W\Omega^1\{1\}$

$0 \qquad 2$

$-1 \quad 0 \quad 1? \quad 2 \quad 3$

$\Omega^1 \leftarrow \boxed{\cancel{Fil^1 W\Omega^1}}$; $\mathrm{ev} + R$? $\boxed{\Omega^1\{1\}}$ should be R $\quad W_1\Omega^1\{1\} \quad \Omega^1\{2\} \leftarrow W_1\Omega^1\{2\}$

Ω^1

$TR^1_{0,1} = Fil^1 W\Omega^1_{0}{}^{-1}$???

$W_1(R) \otimes_{A_{inf}} (\xi, A_{inf})$

$= W_1(R) \otimes_{W(R)} \quad \xi, W$

tensor product with Ω !

$W_1(R)\{n\} = W_1(R)\{1\}^{\otimes n} \xrightarrow{\sim} TR^1_{2n}$

(ξ, A_{inf})

$W_1(R)\{1\} = TR^1_2(R)$

$\Omega^0\{1\}_{/F} \xrightarrow{dV} \mathrm{Fil}^1 W_1\Omega^1 \longrightarrow \cancel{W_1\Omega}$

$\cancel{VW\Omega}$?

$V\Omega^0$

$V+dV$

$V(x^n\, d\log x) = p\, x^{n/p}\, d\log x$

$dV(x^n) = n\, x^{n/p}\, d\log x$

$$\begin{array}{ccc} A/I\,\phi(I) & \longrightarrow & A/\phi(I) \\ \downarrow & & | \\ A/I & \longrightarrow & A/I,\phi(I) \end{array}$$

$$\begin{array}{ccc} \dfrac{I\,\phi(I)}{I^2\,\phi(I)^2} & \longrightarrow & \dfrac{\phi(I)}{\phi(I)^2} \\ \downarrow{\scriptstyle 1/p} & & | \\ I/I^2 & \longrightarrow & \end{array}$$

$I \to A \Rightarrow A \to I^{-1}$ (not surjective)

$W\Omega^1$ $\quad$ $W\Omega^0$

$W\Omega^2$ $\quad$ $W\Omega^1$ $\quad$ $W\Omega^0$

$W\Omega^3$ $\quad$ $W\Omega^2$ $\quad$ $W\Omega^1$ $\quad$ $W\Omega^0$

$E/\mathbb{Q}$ ell curve, $\rho_E : G_{\mathbb{Q}} = \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \longrightarrow GL_2(\mathbb{Q}_p)$

$I/I^2 \otimes_A (A \longrightarrow I^{-1})$

good reduction $\Longleftrightarrow \rho_E$ crystalline

HT wts $0, 1$

$I = (\xi)$, $\xi^{-1}A \subset A[\xi^{\pm 1}]$

prismatic F-gauges $\Rightarrow$ family of crystalline repns

$I \longrightarrow A \longrightarrow A/I \xrightarrow{\otimes A/I} I/I^2 \longrightarrow A/I \longrightarrow A/I, I$

$I/I^2 = A/I\{1\}$

$\phi(I) \equiv p \mod I^p$

$\otimes I^{-1}$

mod p crystalline repn must be extra data

$A \longrightarrow I^{-1} \longrightarrow \dfrac{I^{-1}A}{A} = A/I\{-1\}$

$\dfrac{I + \phi(I)}{I\phi(I), I^2} \simeq \dfrac{I}{I^2, \cancel{I\phi(I)}}$ pI

• Adams operations on THH

• $p=2$ case

• cell structures method

$$\begin{array}{ccc}
\dfrac{I\phi(I)}{I^2\phi(I)^2} & \longrightarrow & \dfrac{\phi(I)}{\phi(I)^2} \\
\Big\downarrow{\scriptstyle 1/p} & & \Big\downarrow \\
I/I^2 & \longrightarrow & \dfrac{I+\phi(I)}{I\phi(I),\ I\phi(I)}
\end{array}$$

$A/(p)_q = \mathbb{Z}[\zeta_p]$; $A/(p)_{q^p} = \mathbb{Z}[\zeta_{p^2}]$???

$\underbrace{1+q}_{q=-1}$; $\underbrace{1+q^2}_{q=\sqrt{-1}}$

$(p^2)_q \longmapsto$

$F(f) = \phi(f)$

$\cancel{(1+q^2)^2 = 1+2q^2+q^4}$ $\qquad \cancel{\phi(I) = p + I}$

$\cancel{\phi(I) \equiv p \bmod}$

$\cancel{I\phi(I) = pI + I^2}$

I/I^2 p-torsionfree

$\phi(I) \equiv p \mod I$

$I \equiv ? \mod \phi(I)$

Suppose oriented e.g. $(p)_q$

$$\frac{(p^2)_q A}{(p^2)_q^2 A} \longrightarrow$$

$$\frac{(p)_q A}{(p)_q^2 A} \longrightarrow$$

~~$(p^{n+1})_{q^{1/i}}$~~

$A/(p)_{q^{1/p}}$ $\{(p^{n+1})_{q^{1/p^{n+1}}}^{i}\}$

$A/(p^2)_{q^{1/p^2}}$ ~~free~~

$(p^{n+1})_{q^{1/p^{n+1}}}$

$q^{1/p^{n+1}}$

$(p)_q A/(p)_q^2 \cong A/(p)_q \cong \mathbb{Z}[\zeta_p]$ p-torsion free.

~~$\frac{(p)_{q^p} A}{(p)_{q^p}^2 A}$~~

but, this is not surjective

natural map

in dir. by I

$$\frac{I\phi(I)}{I^2\phi(I)^2} \longrightarrow \frac{\phi(I)}{I\phi(I)^2}$$

$$\Big\downarrow$$

$$\frac{\phi(I)}{\phi(I)^2}$$

$$\frac{I\phi(I)}{I^2\phi(I)^2} \longrightarrow \frac{\phi(I)}{\phi(I)^2}$$ div. by I

$\mathbb{Z}_p^{syn}$ compatible ...

$(p)_q$ mod $(p)_{q^p}$. I-torsionfree but what is the image???

$\mathbb{Z}[\sqrt{-1}]$, $(1+i)$ ideal???

so not ... is true

$(1+i)(1-i) = 2$

~~but I think~~

$$\boxed{\delta(1+q) = -q}$$

$$\phi(\xi) = \xi^p + p\delta(\xi)$$

$$\boxed{1+q^2 = (1+q)^2 + 2\delta(1+q)}$$

$$\Rightarrow 0 = (1+i)^2 \cancel{+} - 2\cancel{q}i$$

$$\Rightarrow 2\cancel{\delta} = \frac{(1+i)^2}{i}$$

$\frac{(1+i)^2}{i} = \frac{2i}{i}$ | 2 does not have literal $\sqrt{\ }$ here...

no control whatsoever of $\delta(A^*)$...

~~$\phi(I) = I^2$ +~~

$A\{1\}$ transversal description needs

~~canonical image?~~

$$\left(\begin{array}{ccc} A/I_1 & \longrightarrow & A/\phi(I) \\ \downarrow & & | \\ A/I & \longrightarrow & \frac{A}{I_1\phi(I)} \end{array}\right) \otimes I_1 = \begin{array}{ccc} I_1/I_1^2 & \twoheadrightarrow & \frac{I\phi(I)}{I\phi(I)^2} \\ \downarrow & & \downarrow \\ \frac{I\phi(I)}{I^2\phi(I)} & \longrightarrow & \boxed{} \end{array}$$

$$\left(I \longrightarrow A \longrightarrow A/I\right) \otimes_A (A/I) = \left(A/I\{1\} \longrightarrow A/I \longrightarrow A/I, I\right)$$

$$\underline{\underline{\text{not}}} \quad \begin{array}{ccccc} A/I\{1\} & \longrightarrow & A/I^2 & \longrightarrow & A/I \\ \| & & | & & | \\ A/I\{1\} & \longrightarrow & A/I & \longrightarrow & A/I, I \end{array}$$

A

$\underline{\mathbb{Z}[\sqrt{-1}]}$ $\times$ $\mathbb{F}_2[x]/(x^2+1)$ $\phi^i(\mu)$

I maps to p-nilpotent thing in $A/\phi(I)$, i σ_1

$d = 1+t$, $\boxed{t = d-1}$ $TR_\circ(S; \mathbb{Z}_p)$

$= W\Omega^\circ_S[\beta]$

~~$\phi(d) = (1+t)^p + p\delta(1+t)$~~

~~characterize~~ log $A/I\phi(I) \longrightarrow A/I \times A/\phi(I)$

$$\frac{I\phi(I)}{I^2\phi(I)^2} \longrightarrow \frac{I}{I^2} \times \frac{\phi(I)}{\phi(I)^2} \ ???$$

$\beta = \{\mu\}$

$$\frac{I\phi(I)}{I^2\phi(I)^2}$$ $\phi^{-(i+1)}(\mu)\,\sigma_0$

$\xi = \dfrac{\mu}{\phi^{-1}(\mu)}$

$1/p$

$$\frac{I}{I^2}$$ $\Rightarrow \mu \longmapsto \phi^{-1}(\mu) \cdot \xi$

ξ^m

$$\begin{tikzcd} A/I_1 \arrow[d, bend right] \\ A/I \arrow[u, hook, bend right] \end{tikzcd} \qquad \begin{tikzcd} A/I_1\{1\} \arrow[d, bend right] \\ A/I\{1\} \arrow[u, hook, bend right] \end{tikzcd}$$

$$A\{1\} \otimes_A A/I_n \xrightarrow{\sim} I_n/I_n^2 \qquad \varprojlim \left(\cdots \to A/I_1\{1\} \to A/I\{1\} \right)$$

$$\boxed{A\{1\} \otimes_A (A/\phi(I)) \ ???}$$

$$I_r \equiv I^{(r)p} \mod p$$

$$A\{1\} \longrightarrow I_r/I_r^2 \quad \text{induces} \quad \frac{A\{1\}}{I_r} \xrightarrow{\sim} I_r/I_r^2$$

$$\boxed{\frac{A\{1\}}{\phi(I)} = ???}$$

p-complete & p-torsionfree

$$\cancel{A\{1\} \otimes_A A/\phi(I)}$$

$$A\{1\} \otimes_A (A/\phi(I)) = \left(A\{1\} \otimes_A A/I_1 \right) \otimes_{A/I_1} A/\phi(I)$$

$$\boxed{= \frac{I_1}{I_1^2} \otimes_{A/I_1} A/\phi(I)}$$

$$(p^{n+1})_{q^{1/p^{n+1}}}$$

$$TR^n_r(S;\mathbb{Z}_p) = \widehat{\bigoplus_{i=0}} W_r\Omega_S^{*-2i}\{i\}$$

$$\begin{array}{ccccc} \frac{I_1}{I_1^2} & \longrightarrow & A/I_1^2 & \longrightarrow & A/I_1 \\ \| & & \downarrow & & \\ \frac{I_1}{I_1^2} & \longrightarrow & A/I_1 & \longrightarrow & A/(I_1, I_1) \end{array}$$

$$\frac{I\,\phi(I)}{I^2\phi(I)^2} \underset{A/I_1}{\otimes} (A/I)$$

$$\frac{I\,\phi(I)}{I^2\phi(I)^2,\ I^2\phi(I)} \simeq \boxed{\frac{I}{I^2}}$$

$$\frac{I\,\phi(I)}{I^2\phi(I)} = \frac{I}{I^2}$$

versus $\dfrac{I\,\phi(I)}{I^2\phi(I)^2,\ I\,\phi(I)^2} = \boxed{\dfrac{I\,\phi(I)}{I\,\phi(I)^2}}$

no canonical way to get rid of extra I factor.

$$\begin{array}{ccc} I_1/I_1^2 & \longrightarrow & \dfrac{I\,\phi(I)}{I\,\phi(I)^2} \\ \downarrow & & \downarrow \\ I/I^2 & \longrightarrow\!\!\!\!\to & I/(I^2, p) \end{array}$$

$\left[\simeq \dfrac{\phi(I)}{\phi(I)^2}\right]$

$\Rightarrow$ prismatic logarithm!

$\cancel{\lambda}$ default

$\alpha = a + b\lambda$

$(d_0 ; d_\infty) \Rightarrow \cancel{\alpha = d_\infty}$ $\quad \alpha = d_\infty \lambda_\infty + (d_0 - d_\infty)\lambda_\infty$

$T^e \quad d_\infty$

$\Omega^j \qquad W_1\Omega^j \leftarrow T^{hC_p}$

$\to d_0$

$\Omega^j[-1] \qquad W_1\Omega^j$ or $\mathrm{Fil}^1 W_1\Omega^j$

$\Omega^j/p^{d_0}[-1] \qquad \mathrm{Fil}^{1-d_0} W_1\Omega^j$

T_{hC_p}

$u \in (1+I)_{rk=0}$ $\qquad A/I,\ A/\phi(I)$

$\cancel{u^p - 1 = (u)_p(u-1)}$

$u^p - 1 = \underbrace{(p)_u}_{= p \bmod} \underbrace{(u-1)}_{\in I}$

$\cancel{u - 1 \in I \Rightarrow \phi(u-1) \in \phi(I)}$

$\cancel{\text{p-adic lo}}$ $\quad \log_{\Delta}(u) = \varprojlim \left(u^{p^n} - 1 \in I_n / I_n^2 \right)$

group $\varphi_{A\{1\}}(\log_{\Delta}(u)) = \log_{\Delta}(u) \quad \Big| \quad \log_{\Delta}(q^p) \in A\{1\}$

$q^p - 1 = (p)_q (q-1) \Rightarrow q^p \equiv 1 \bmod 1$ $\qquad A\{1\}$

$\cancel{\Rightarrow \log_{\Delta}(q^p) = \log_q(q)?}$ $\qquad e_A$ generates $\cancel{[p^n]_q}$

$(q-1)(p^n)_q = q^{p^n} - 1$

$\Rightarrow (q-1)e_A = \log_{\Delta}(q^p)$

$$e_A = \frac{\log_\Delta(q^p)}{q-1}, \quad \phi(e_A) = \frac{\log_\Delta(q^p)}{q^p-1}$$

q-logarithm.

$$= \frac{e_A}{(p)_q}$$

$\mathbb{Z}_{p,q}$

$$A = \widehat{\bigoplus} \mathbb{Z}_p[[q-1]] \left\langle \frac{(u-1)_q^n}{(n)_q!} \right\rangle$$

$$\log_q(u) = \sum_{n=1}^{\infty} (-1)^{n-1} q^{-\binom{n}{2}} \frac{(u-1)_q^n}{(n)_q}$$

$\log_q(1) = 0$, $\quad \nabla_q \log_q(u) = \frac{1}{u}$.

$\log_\Delta(u^p) = \log_q(u)\, e_A$.

$\log_\Delta(1) = 0$,

$$\cancel{\frac{\log_\Delta(q^p u^p) - \log_\Delta(u^p)}{qu}} = \log_\Delta(q^p) = (q-1)e_A$$

$u_{\lambda_0} u_{\lambda_1}^{-1}$

$$\begin{tikzcd}
\Sigma^2 E \arrow[r, "\sigma"] \arrow[dr, "u_{\lambda_0}"'] & E \\
 & \Sigma^{\lambda_0} E \arrow[u]
\end{tikzcd}$$

$$\begin{tikzcd}
S^0 \arrow[d, "a_{\lambda_0} = a_0"'] \arrow[dr, "a_{\lambda_1}"] \arrow[drr, bend left, "a_{\lambda_2}"] & & \\
S^{\lambda_0} \arrow[r, "a_1"'] & S^{\lambda_1} \arrow[r, "a_2"'] & S^{\lambda_2}
\end{tikzcd}$$

$\sigma u_{\lambda_0}^{-1}$

$u_{\lambda_0} u_{\lambda_1}^{-1}$

$(u_{\lambda_0}$

$$\begin{tikzcd}
 & \Sigma^2 R \arrow[dl, "\sigma"'] \arrow[d, "u_{\lambda_0}"] \arrow[dr, "u_{\lambda_1}"] & & \\
R & \Sigma^{\lambda_0} R \arrow[l, "u_0"] & \Sigma^{\lambda_1} R \arrow[l, "u_1"] & \cdots \arrow[l]
\end{tikzcd}$$

~~R invertible~~

~~$TC_2(R)$~~

$$\begin{tikzcd}
THH^{C_p} \arrow[r] \arrow[d, dash] & THH^{\Phi C_p} \arrow[d] \\
T^{hC_p} \arrow[r] & T^{tC_p}
\end{tikzcd}$$

a_λ invertible

u_λ invertible

$TR^1_\star(S) = W_1 \Omega^*_S \otimes_{W_1(k)} TR^1_{\alpha_1}(k)$

~~$TC_2(R;\mathbb{Z}_p) = N^{21} \Delta_{R|A}\{1\}$~~

$TR^1_{\alpha,j}(S) = W_1 \Omega^j_S$ contribution to $TR^1_{\alpha+j}(S)$

$\xi = (p)_{q^{1/p}}$ $\quad \xi_n = (p^n)_{q^{1/p^n}}$

$\tilde{\xi} = (p)_q$ $\quad A/\xi, \langle x^n \, d\log x \rangle \oplus A/\phi^{-1}(\xi) \langle x^{i/p} d\log x \rangle$

$\tilde{\xi}_n = (p^n)_q$

~~Universal case~~

$$\sum_{n=1}^{\infty} (-1)^{n-1} q^{-\binom{n}{2}} \frac{(u-1)_q^n}{(n)_q}$$

$$\nabla_{q,u} \Longrightarrow \sum_{n=1}^{\infty} (-1)^{n-1} q^{-\binom{n}{2}} (u-1)_q^{n-1}$$

$$\frac{\log_q(qu) - \log_q(u)}{qu - u} = \frac{\log_q(q)}{(q-1)u} = \frac{1}{u}$$

prismatic logarithm. $u = 1$ mod I

$$\frac{I_1}{I_1^2} \longrightarrow \frac{I\phi(I)}{I\phi(I)^2} \quad \frac{I\phi(I)}{I\phi(I)^2,\ I\phi(I)^2}$$

$$\downarrow$$

$$\frac{I}{I^2}$$

$u^p - 1$

$$\begin{array}{ccc} u^p-1 \quad \dfrac{I\,\phi(I)}{I^2\,\phi(I)^2} & \longrightarrow & \dfrac{I\,\phi(I)}{I\,\phi(I)^2} \\ \downarrow & & \downarrow \\ \dfrac{I\,\phi(I)}{I^2\,\phi(I)} & \longrightarrow & \boxed{} \\ (1/p)\ \downarrow\ \cong & & \\ \dfrac{I}{I^2} & & \end{array}$$

$u-1$

$d(p^{-1})$,

$u-1$,

probably just

$u^p - 1 = \underbrace{(p)_u}_{\in\phi(I)}\underbrace{(u-1)}_{I}$ mod $\phi(I)^2$

mod

$$T^e = \Omega^j \qquad T^{hC_p} = W_1\Omega^j \qquad d_0$$

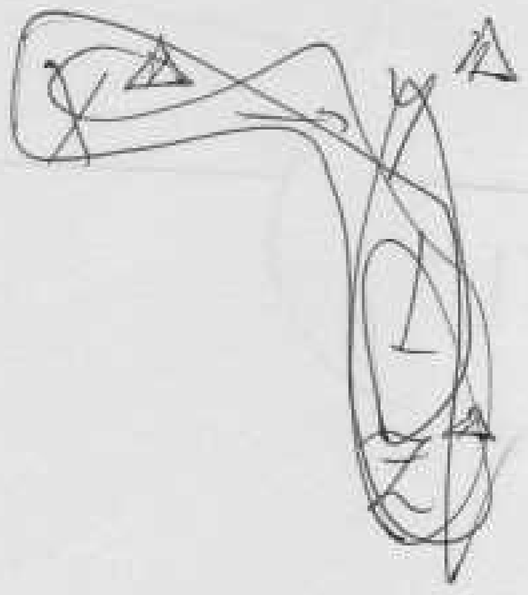
$\deg$

$$T_{hC_p} = \begin{array}{cc} \Omega^j[-1] \quad W_1\Omega^j & d_0 > 0 \\ \text{or} \quad \mathrm{Fil}^1 W_1\Omega^j & d_0 = 0 \end{array}$$

O

$\Omega^j/p^{d_0}[-1] \qquad \mathrm{Fil}^{1-d_0} W_1\Omega^j$

$$\begin{tikzcd} \mathbb{G}_m(A/I) \arrow[r, "d\log"] \arrow[d, "N"'] & I/I^2[1] \arrow[d, "p", "\sim"'] \\ \mathbb{G}_m(A/I_1) \arrow[r] & I_1/II_1[1] \end{tikzcd}$$



$$\begin{tikzcd} 0 \arrow[r] & I/I^2 \arrow[r, "\exp"] \arrow[d, "p", "\simeq"'] & \mathbb{G}_m(A/I^2) \arrow[r] \arrow[d, "\tilde{N}"] & \mathbb{G}_m(A/I) \arrow[r] \arrow[d, "N"] & 0 \\ 0 \arrow[r] & \dfrac{I\phi(I)}{I^2\phi(I)} \arrow[r, "\exp"] & \mathbb{G}_m(A/I^2\phi(I)) \arrow[r] & \mathbb{G}_m(A/I_1) \arrow[r] & 0 \end{tikzcd}$$

$\hookrightarrow A/I^2 \times A/\phi(I)$

retraction?

$\phi(p) \equiv p \mod I^p \quad \mod \phi\, I^2$

$I(I_1^{p-1})$

$$I_{c_p} = I_e I_{\phi c_p}$$

$$A \longrightarrow A/I_{c_p} \longrightarrow A/I_e$$

$$I_{\phi c_p} = c_i \text{ mod}$$

$$\begin{array}{ccc} G_m(A^e) & \longrightarrow & A^e\{1\}[1] * I_e/I_e^2[1] \\ \downarrow \wr & & \uparrow \\ G_m(A^{c_p}) & \longrightarrow & I_{c_p}/I_{c_p}^2[1] \end{array}$$

$\boxed{\nabla c_p}$

$$I_{c_p}/I_{c_p}^2 = \frac{I_e I_{\phi c_p}}{I_e^2 I_{\phi c_p}^2} \longrightarrow \frac{I_e I_{\phi c_p}}{I_e^2 I_{\phi c_p}} \overset{c_p^{-1}}{\cong} \frac{I_e}{I_e^2} \#$$

$$\begin{array}{ccc} G_m(A^e) & \longrightarrow & I_e/I_e^2[1] \\ \downarrow \wr & & \uparrow \\ G_m(A^{c_p}) & \longrightarrow & \dfrac{I_e I_{\phi c_p}}{I_e^2 I_{\phi c_p}}[1] \end{array}$$

$$\begin{array}{ccccc} I_e/I_e^2 & \xrightarrow{\exp} & G_m(A/I_e^2) & \longrightarrow & G_m(A^e) \\ {\scriptstyle c_p}\downarrow \wr & & \downarrow \widetilde{N} & & \downarrow N \\ \dfrac{I_e I_{\phi c_p}}{I_e^2 I_{\phi c_p}} & \longrightarrow & G_m(A/I_e^2 I_{\phi c_p}) & \longrightarrow & G_m(A^{c_p}) \end{array}$$

~~suppose it's a retraction?~~

~~$I_{\phi c_p} = I_{c_p}$ mod I_e^p~~

$$A/I_e^2 I_{\phi C_p} \longrightarrow A/I_e^2 \times A/I_{\phi C_p}$$

~~log refractor~~

Q. make refined prismatic logarithm ~~precise~~ "concrete".

$$\mathbb{G}_m(\bar{A}) \xrightarrow{\widetilde{\log}_\Delta} A\{1\}(?) \qquad H_1(\mathbb{G}_m(\bar{A})^\wedge_p \to A\{1\})$$

recovers $\log_\Delta$.

$\log_\Delta(\ )$

$$\begin{array}{ccccc} I/I^2 & \longrightarrow & \mathbb{G}_m(A/I^2) & \longrightarrow & \mathbb{G}_m(A/I) \\ \big\lceil & & \big\lceil & & \big\lceil N \\ & \longrightarrow & \mathbb{G}_m(\ \ ) & \longrightarrow & \mathbb{G}_m(A/I_{cp}) \end{array}$$

de Rham-Witt complex

$$\boxed{N(\omega) = V\frac{\omega^p}{p} + (p - V(1))\frac{\varphi^P(\omega)}{p}}$$

p-adic numbers

~~$\mathbb{G}_m(A/I) \xrightarrow{N} \mathbb{G}_m$~~

$$\begin{tikzcd}
\mathbb{G}_m(A/I) \arrow[rr] \arrow[d, "N"'] & & I/I^2[1] \arrow[d, leftrightarrow, "\cong"] \\
\mathbb{G}_m(A/I_1) \arrow[r] & I_1/I_1^2[1] \arrow[r, two heads] & \dfrac{I\phi(I)}{I^2\phi(I)}[1]
\end{tikzcd}$$

$$\begin{tikzcd}
I/I^2 \arrow[r] & \boxed{\cancel{\mathbb{G}_m(A/I^2)}} \arrow[r] \arrow[dd, dash] & \mathbb{G}_m(A/I) \arrow[dd, dash, "N"] \\
& & \\
I_1/I_1^2 \arrow[r] \arrow[d] & \mathbb{G}_m(A/I_1^2) \arrow[r] \arrow[d, dash] & \mathbb{G}_m(A/I_1) \arrow[d, equal] \\
\dfrac{I\phi(I)}{I^2\phi(I)} \arrow[r] & \mathbb{G}_m(A/I^2\phi(I)) \arrow[r] & \mathbb{G}_m(A/I_1)
\end{tikzcd}$$

~~I_1/I_1^2~~ $\mathbb{G}_m(A/I_1^2) \times_{\square} \mathbb{G}_m(A/I)$

<u>Prop</u> S smooth k-algebra,

$$\pi_{2j}\left[\pi_i^{cyc}\, THH(S)\right]^{h\mathbb{T}} = N^{\geq i+j}\, W\Omega^i$$

$$\pi_{2j}\left[\pi_i^{cyc}\, THH(S)\right]^{t\mathbb{T}} = W\Omega^i_A$$

$W\Omega^i[t^{\pm}]$, Tate spectral sequence $W\Omega^i/V$

$$\pi_{2j}\left[\pi_i^{cyc}THH(S)\right]^{h\mathbb{T}} = N^{\geq i+j}W\Omega^i\{j\}\ ?$$

(upper limit)

$x = \sup(S)$ means: • $s \leq x$ for all $s \in S$

• If $s \leq y$ for all $s \in S$, then $x \leq y$

e.g. $\sup_{x\in\mathbb{Q}}\{x \mid x^2 \leq 2\} = \sqrt{2}$

$\sup\{x\in\mathbb{Q} \mid x^2 \leq 2\} = \sqrt{2} \notin \mathbb{Q}$

$W\Omega^\bullet[\beta]$

$W\Omega^i/p$

$W\Omega^i/p$

$W\Omega^i/V$

$$\pi_{2i}\left[\pi_j^{cyc}THH(S)\right]^{h\mathbb{T}} = N^{\geq i+j}W\Omega^j\{i\}$$

$$\pi_{2i}\left[\pi_j^{cyc}THH(S)\right]^{t\mathbb{T}} = W\Omega^j\{i\}$$

$\mathrm{Cond}(\mathrm{Ab})$

$W\Omega^i/p$ $\quad W\Omega^i/p$ $\quad W\Omega^i/p$ ~~actually $VW\Omega^i$~~

secretly

$W\Omega^i/\cancel{p}$ $\quad W\Omega^i/\cancel{p}$ $\quad W\Omega^i/\cancel{p}$

$\mathrm{gr}^1 W\Omega^{\geq 13}$

so $\frac{V}{pV}$

$W\Omega^i$

$VW\Omega^i$

$pVW\Omega^i$

$p^2VW\Omega^i$

$\vdots$

(ok to trivialize over here...)

~~$F^1 W\Omega^i \subseteq VW\Omega^i$~~

~~$F^j W\Omega^{i+j} \quad N^{\geq i+j} W\Omega^{\geq i}$~~

HH ... II

(S) HH ...

$TC^-_\bullet(S) = ?$ $S = A^2$ ~~π^{cyc}~~

THH(S) has cyclotomic gr^i, gr^1, gr^j

$TC^-_{2i}(\pi^{cyc}_j \, THH(S))$

$= N^{\geq i+j} W\Omega^j\{i\}$

$N^{\geq 2} W\Omega^2$ $\quad N^{\geq 3} W\Omega^2\{1\}$ $\quad N^{\geq 4} W\Omega^2\{2\}$

$N^{\geq 1} W\Omega^1$ $\quad N^{\geq 2} W\Omega^1\{1\}$ $\quad N^{\geq 3} W\Omega^1\{2\}$

$N^{\geq 0} W\Omega^0$ $\quad N^{\geq 1} W\Omega^0\{1\}$ $\quad N^{\geq 2} W\Omega^0\{2\}$

$\Rightarrow TC^-_\star(\pi^{cyc}_j \, THH(S))$

$W\Omega^j$

0 $\quad$ 2 $\quad$ 4

(should split) $\quad$ only focus on $W\Omega^j$ contribution.

$TR^1_\star(\pi^{cyc}_j \, THH(S))$.

$\cancel{TC_{2,j}^{-} = N^{\geq j+1} W\Omega^j\{1\}}$

$\cancel{TC}$

$TC_{2,j}^{-} = N^{\geq j+1} W\Omega^j\{1\}$

$TC_0^{-} = N^{\geq j} W\Omega^j \quad (= W\Omega^j)$

$\downarrow$

$THH_0 = W\Omega^j/V$

$TC_{\star,j}^{-}(S)$

$= TC_\star^{-} \, \mathrm{gr}_j^{\pi \, cyc} \, THH(S)$

$TC_{\lambda,j}^{-}(S) = \cancel{N^{\geq j+1} W\Omega^j}$

$TC_2^{-} = \cancel{N^{\geq j} W} \quad N^{\geq j+1} W\Omega^j\{1\}$

~~probably~~ should

$\downarrow$

$THH_2 = W\Omega^j/p$

$TC_2^{-} = N^{\geq j+1} W\Omega^j\{1\}$

$\downarrow$

$THH_2 = W\Omega^j/p\{1\}$

$= \frac{N^{\geq j+1}}{N^{\geq j+2}}$

$TC_{2+\lambda}^{-} = N^{\geq j+2} W\Omega^j\{1\}$

$\ker(VW\Omega^j \longrightarrow W\Omega^j/p)$

$\uparrow$

$TR^1_{1+\lambda} \leftarrow TR^1_0$

$N^{\geq 1} W\Omega^2\{-1\}$ $N^{\geq 2} W\Omega^2$ $N^{\geq 3} W\Omega^2\{1\}$

$N^{\geq 1} W\Omega^1$ $N^{\geq 2} W\Omega^1\{1\}$

$W\Omega^0$ $N^{\geq 1} W\Omega^0\{1\}$ $N^{\geq 2} W\Omega^0\{2\}$

0 1 2 3 4

collapse?

could be d

most likely d
weird differential

Want $TR^1_{?,2}$

$p=2$

$TR^0_2 \xrightarrow{N} TR^1_{2+?} \xrightarrow{R} TR^0_2$

$TR^1_{2+?} \xrightarrow{F} TR^0_2$

x^2

$N^{\geq 3} W\Omega^2\{1\}$

$N^{\geq 3} W\Omega^2$

$N^{\geq 2} W\Omega^2\{2\}$

$N^{\geq 1} W\Omega^1$

$N^{\geq 2} W\Omega^0\{2\}$

$N^{\geq 1} W\Omega^0\{1\}$

$W\Omega^0$

$\boxed{\sigma}$

$N^{\geq j-1} W\Omega^j\{-1\}$ $\xrightarrow{12}$ $W\Omega^j\{-1\}$

$N^{\geq j} W\Omega^j$ $\xrightarrow{12}$ $W\Omega^j$

$N^{\geq j+1} W\Omega^j\{1\}$

$W\Omega^j\{1\}$

$N^{\geq j+2} W\Omega^j\{2\}$

$W\Omega^j\{2\}$

$N^{\geq i+j} W\Omega^j\{i\}$

$\Longrightarrow W\Omega^j[t^{\pm}]$, $N^{\geq j+1} W\Omega^j = V W\Omega^j$

$(W\Omega^j)^{tC_p}$, $(W\Omega^j)^{hC_p}$

THH^{hC_p} "canonically".

more likely ~~$T^{hC_p} = T$~~

$W\Omega^j/p$ $\square$ $W\Omega^j/p$ $\square$ $W\Omega^j/p$ $\square$ $W\Omega^j/p$ $\square$ $W\Omega^j/p$

$W\Omega^j/V$ $\square$ $W\Omega^j/V$ $\square$ $W\Omega^j/V$ $\square$ $\sim$ $\square$ $\sim$

-4 $\quad -2 \quad 0 \quad 2 \quad 4$

$W\Omega^j/V$ [p-torsion]

= all?

$(p=2)$

$TR^1_{2+\lambda} = TR^1_{\lambda,2} \oplus TR^1_{\lambda-2,0}$

$(1;0)$ $\quad$ $(0;-1)$

$W\Omega N$

werses $/p$ $\quad \dfrac{N^{\geq j+1}W\Omega^j}{pN^{\geq j+2}W\Omega^j}$

Ω^2

$$\boxed{T^{tC_p}_{\lambda,2}} \xleftarrow{i} T^{hC_p}_{\lambda,2}$$

$T^{e}_{\lambda,2}$

$$\frac{N^{\geq j+1}W\Omega^j}{pVW\Omega^j} = \frac{VW\Omega^j}{p^2VW\Omega^j}$$

$$\Omega^j \xleftarrow{0} \frac{N^{\geq j+1}W\Omega^j}{pN^{\geq j+2}W\Omega^j} + \frac{VW\Omega^j}{p^2VW\Omega^j}$$

Ω^j

no ...

$$\Omega^2 \longrightarrow \underbrace{\frac{VW\Omega^2}{p^2VW\Omega^2}}_{gr^{j+1}_N W\Omega^2} \oplus$$

$TR^1_{\lambda-2}$

$T^{tC_p}_{-2}$

$$\frac{N^{\geq j}W\Omega^j\{-1\}}{pN^{\geq j+1}W\Omega^j\{-1\}}$$

T^{tC_p}

$x^{1/p}\, d\log x$

$x^{1/p^2}\, d\log x$

$\sigma^2 t^2 \qquad \sigma^2 t \qquad \sigma^2$

$W\Omega^i / V \qquad \sigma t^2 \qquad \sigma t \qquad \sigma$

versus t^2 p $\quad t \quad 1 \quad e \quad t^{-1} \quad t^{-2}$

$W\Omega^1 / V$:

$W\Omega^1 / p$: $\{x^{n/p^n}\, d\log x\}$

$\dfrac{W\Omega^1}{V + dV}$

$$V(a x^n\, d\log x) = ap\, x^{n/p}\, d\log x$$

$$dV(x^n) = n\, x^{n/p}\, d\log x$$

$$x^{1/p}\, d\log x = dV(x)$$

$$x^{1/p^2}\, d\log x \in W\Omega^1$$

~~$V(x^{1/p}\, d\log x)$~~

~~$V(x^{1/p}\, d\log x)$~~

~~mod~~ dV

$dV(x^2)$

$W\Omega$ $\quad W\Omega \quad W\Omega$

$W\Omega \quad W\Omega \quad W\Omega$

Δ_R

$N^{\geq d_0} \hat{\Delta}_R \otimes \{ I^{d_1} \phi(I)^{d_2}$

$\Delta_R \otimes \{ I^{d_{\alpha'}} \}$

$N^{\geq d_0} \hat{\Delta}_R \otimes I^{d_{\alpha'}}$

$TC^-_\alpha(R;\mathbb{Z}_p) \simeq N^{\geq d_0} \hat{\Delta}_R \otimes I^{\alpha'}$

$TP(R;\mathbb{Z}_p) \simeq \hat{\Delta}_R \otimes I^{\alpha'}$

$\otimes I^{\alpha'}$

$\otimes \prod_{i=0} \phi^{i*}(I)^{d_i}$

$d_i(\alpha') = \alpha_{i+1}$

$d_i(\alpha^{(n)}) = \alpha_{i+n}$

$\boxed{W\Omega^j/p\,?}$

$M/p \cdot t^2$ $\quad M/p\cdot t \quad M/p\cdot \quad M/p\cdot t^{-1}$

$M/V\cdot t \quad M/V\cdot t \quad M/V \quad \varepsilon \quad M/V\cdot t^{-1} \quad M/V\cdot t^{-2}$

$M/V[p\text{-torsion}]$ b.c. M/V

map should be F?

kernel of $M/V \to M/p$?

$\boxed{\text{map???}}$

$M/V^{u} \longrightarrow M/p \cdot t^2$

~~$M/p \to M/V$~~

$\boxed{W\Omega^1/p}$ $\quad W\Omega^1$ if $p \mid n$ $\to$ killed

~~$dV(x^n) = n\, x^{n/p}\, d\log x$~~

so ...

$THH^{tC_p} = TP/(C_p t = 0)$.

$M/F \longrightarrow M/p \xrightarrow{V} M/V$

$M/V \longrightarrow M/p \xrightarrow{F} M/F$

$W\Omega^j/p$ F

$W\Omega^j/V$ $W\Omega^j/V$

$W\Omega^j/F$

$W\Omega^j/p$

$W\Omega^j/V$

THH^{hC_p} : $W\Omega^j/pV$

THH^{tC_p} : $W\Omega^j/p$

$(= pW\Omega^j)$

$W\Omega^j/p \longleftarrow W\Omega^j/pV$

~~possibly unti...~~

F $W\Omega^j/V$

$pW\Omega^j/pV$

$pW\Omega^j/pV$

Ω^j/V

~~$W\Omega^j \longrightarrow \Omega^j$~~

$\frac{W\Omega^j}{V^2 + dV^2}$?

$\frac{pW\Omega^j}{pV} \longrightarrow \boxed{\frac{W\Omega^j}{pV}} \xrightarrow{R} W\Omega^j/V$

$pW\Omega^j$

$\frac{pW\Omega^j}{pV} \longrightarrow ? \longrightarrow \frac{W\Omega^j}{V}$

hexis Liquesad)

$0 \longleftarrow \frac{\Lambda}{W} \longleftarrow \iota \longleftarrow \frac{W\Lambda^d}{W^d}$

$\cancel{WS\Omega^{3}p}$ $WS\Omega^{3}p$ ~~should get~~

~~should~~ ~~actually~~ ~~should~~

$()^{Mp}$ $M/p\,t$ M/p $M/p\,t^{-1}$ $M/p\,t^{-2}$

$()^{Mp}$ M/pt M/pV $\cancel{N^{\ni 1}M/pV}$ $N^{\ni 2}M/p$ $N^{\ni 1}$

$\frac{N^{\ni 1}M}{pN^{\ni 2}M}$

-1 0 1 2

$\cancel{M/p\ni -1\ni}$ M/p $M/p\ni 1\ni$ $M/p\ni 2\ni$

$\cancel{M/p\ni -1\ni}$ $\frac{M}{pN^{\ni 1}}$ $\frac{N^{\ni 1}M\ni 1\ni}{pN^{\ni 2}}$ $\frac{N^{\ni 2}M\ni 2\ni}{pN^{\ni 3}}$

instead of M/pV, M/V $\cancel{M/p\ni 1\ni}$ $M/p \doteq \frac{N^{\ni 1}M^{\ni 0}}{AN^{\ni 2}M}\ni 1\ni$ $N^{\ni 2}M\ni 2\ni$

0 1 2 3 4 5 6

$\frac{pM}{pN^{\ni 1}}$ $\cancel{M/V}$ $= \cancel{\frac{V(FM)}{p^2VM}}$

M/V $\cancel{\frac{N^{\ni 1}M}{N^{\ni 2}M}\ni 1\ni}$ $\frac{pM}{p^2VM} = \frac{M}{pV}$

$\cancel{M/pt}$ $\frac{M}{p}t$ $\cancel{\frac{M}{p}}$ $\frac{M}{p}t^{-1}$ $\frac{M}{p}t^{-2}$

$\frac{M}{pV}$ $\frac{VM}{p^2V}$ $\frac{pVM}{p^3VM}$

M/pV

M/V M/V

~~$H^i(W\Omega^\bullet/p)$~~

$W\Omega^\bullet/p \simeq \Omega^\bullet$

$\pi_* \mathrm{TMH}(S)^{t\mathbb{C}_p} = \Omega^\bullet_S[t^{\pm}]$???

~~$W\Omega^1/p$~~

$W\Omega^0/p$ $\quad$ $W\Omega^1/p \cdot t^{-1}$ $\quad$ $W\Omega^0/p \cdot t^{-2}$

$\Rightarrow$

Σ

$W\Omega^2/p\{-1\}$ $\quad$ $W\Omega^2/p$ $\quad$ $W\Omega^2/p\{1\}$

$W\Omega^1/p$ $\quad$ $W\Omega^1/p\{1\}$ $\quad$ $W\Omega$

$W\Omega^0/p$ $\quad$ $W\Omega^0/p\{1\}$ $\quad$ $W\Omega^0/p\{2\}$

$\Rightarrow$

Ω^4

Ω

Ω^2

Ω^1

Ω^0 $\quad$ Ω^0

$\boxed{W\Omega^\bullet/p\ ?}$

~~$A(\ )_x^{1,0}$~~

$d(p\, x^{1/p}) = x^{1/p}\, d\log x$ so acyclic anyway

$\boxed{H^0(W\Omega^{\circ}/p^2)}$ $d\left[p^2\, x^{1/p}\right] = p\, x^{1/p}\, d\log x$

$\{x^{p^2 n}\} \in H^0(W\Omega/p^2)$

$W/p^2 \langle x^{p^2 n}\rangle \in H^0(W\Omega/p^2)$

$W/p \langle x^p \rangle$

$d\left[p^2\, x^{1/p}\right] = p\, x^{1/p}\, d\log x$

$d(x^p) = p\, x^p\, d\log x \rightsquigarrow W/p \langle x^p\, d\log x\rangle$

$\pi_* \mathrm{THH}^{hC_p}(S)$

only up to $W\Omega^{?}$?

$\dfrac{N^{\geq 2} W\Omega^2}{pN^{\geq 3} W\Omega^2}\{-1\}$

$\dfrac{N^{\geq 1} W\Omega^1}{pN^{\geq 2} W\Omega^1}$ $\qquad$ $\dfrac{N^{\geq 2} W\Omega^1}{pN^{\geq 3} W\Omega^1}\{1\}$

$\dfrac{N^{\geq 0} W\Omega^0}{pN^{\geq 1} W\Omega^0}$ $\qquad$ $\dfrac{N^{\geq 1} W\Omega^0}{pN^{\geq 2} W\Omega^0}\{1\}$ $\boxed{???}$ $\qquad$ $\dfrac{N^{\geq 2} W\Omega^0}{pN^{\geq 3} W\Omega^0}\{2\}$

$\dfrac{W\Omega^0}{pV} \to \dfrac{W\Omega^1}{pV} \to \dfrac{W\Omega^2}{pV}$

$H^*\left[\dfrac{VW\Omega^0}{p^2 V} \xrightarrow{d} \dfrac{W\Omega^1}{pV} \xrightarrow{d} \dfrac{W\Omega^2}{pV}\right]$

$$dV(\alpha x^n) = d\left[p\alpha\, x^{n/p}\right] = (n\alpha)\, x^{n/p}\, d\log x$$

$\Rightarrow p \mid (n \text{ or } \alpha)$ $\quad$ Im F

$$pV\left[\frac{n\alpha}{p}\, x^{n}\, d\log x\right]$$

$$dV\left(\alpha p^k x^{n/p^k}\right) = d\left[\alpha p^{k+1} x^{n/p^{k+1}}\right]$$

$$= (\alpha n)\, x^{n/p^{k+1}}\, d\log x$$

$FW\Omega^0$ should be everything

$$\overset{?}{=} pV\left[\frac{\alpha n}{p}\, x^{n/p^k}\, d\log x\right]$$

$$dVx = pVy \Rightarrow dx = p^2 y$$

$$\Rightarrow x \in F^2$$

$$\cancel{dV(x^p) = d[px] = p\,x\,d\log x}$$

$$\cancel{= pV\left[x^p\, d\log x\right]} \;?$$

$$\frac{VF^2W\Omega^0}{p^2V} = \frac{pFW\Omega^0}{p^2V} = \frac{FW\Omega^0}{pV} \;?$$

$\cancel{dx = pVy}$ $\quad$ $H^*(W\Omega^\cdot/pV)$

$\cancel{H^*(\ldots)}$???

$N^{\geq *} W\Omega^2\{-1\}$ $\quad$ $N^{\geq ?} W\Omega^2$ $\quad$ $\boxed{TC_\bullet^-(S)\ ???}$

$N^{\geq 1} W\Omega^1 \xleftarrow{d}$ $\quad$ $N^{\geq 2} W\Omega^1\{1\}$ $\quad$ $\cancel{H^i(N^{\geq i} W\Omega^\bullet)}$

$N^{\geq 0} W\Omega^0$ $\quad$ $N^{\geq 1} W\Omega^0\{1\}$ $\quad$ $N^{\geq 2} W\Omega^0\{2\}$

BMS.

$N^{\geq 1} W\Omega^1 = N^{\geq 1} W\Omega^\bullet$

$$W\Omega^1 \xleftarrow{d} W\Omega^0$$

-2 $\quad$ -1 $\quad$ 0 $\quad$ 1 $\quad$ 2

$W\Omega^1 \leftarrow W\Omega^0$

$\boxed{TC^-_{*}(A^1) = ???}$ $\quad H^0(N^{\geq i} W\Omega^0 \to N^{\geq i} W\Omega^1)$

$\Rightarrow H^0(W\Omega^0 \to W\Omega^1) \Rightarrow H^0_{crys}(A^1_{k}) \oplus H^1_{crys}(A^1_{k})$

$d(p x^{1/p}) = x^{1/p}\, d\log x$

avoid cell structures

$H^0(VW\Omega^0 \to W\Omega^1) \Rightarrow pW(k) \oplus$

$\cancel{d(x^k) = k x^k d}$

$\cancel{d(N^k) = k x^k\, d\log x}$

$\cancel{d[p x^{1/p}] = x^{1/p}\, d\log x}$

$\cancel{d(Vx) = d[p x]}$

$d\,V(x^p) = [p x] \Rightarrow$

filtration?

$W_1\Omega^*_S$

$\oplus W/n\langle x^n \sim\rangle$

$\oplus W/pn \langle x^n d\log x\rangle$

$H^i(N^{\geq r}_n W\Omega) \subset \varphi^{-n}(p^{nr} H^i_{crys}(X/W))$

$H^*(VW\Omega^0 \to W\Omega^1)$

$\cancel{H^0(W\Omega^0}$

$\cancel{H^0(W\Omega^0} \to W\Omega^1)$

$p x^n \longmapsto n p x^n\, d\log x$

$p x^{n/p} \longmapsto x^{n/p}\, d\log x$

φ

back to $\log_\Delta$ ... | $\cancel{\log_\Delta(q^p)}$

$$\begin{array}{ccc} \mathbb{G}_m(A/I) & \longrightarrow & I/I^2[1] \\ \downarrow & & \\ \mathbb{G}_m(A/I\phi(I)) & \longrightarrow & \dfrac{I\phi(I)}{I^2\phi(I)^2}[1] \end{array}$$

$\cong$ $\dfrac{I\phi(I)}{I^2\phi(I)}[1]$

$\cancel{0 \to I/I^2 \to \mathbb{G}_m(A/I^2) \to \mathbb{G}_m(A/I) \to 1}$

$$\begin{array}{ccccccc} 0 \to \dfrac{I_1}{I_1^2} & \longrightarrow & ? & \twoheadrightarrow & \mathbb{G}_m(A/I) & \to & 1 \\ \| & & \downarrow & & \downarrow \wr & & \\ 0 \to \dfrac{I\phi(I)}{I^2\phi(I)^2} & \longrightarrow & \mathbb{G}_m(A/I_1^2) & \longrightarrow & \mathbb{G}_m(A/I_1) & \to & 1 \end{array}$$

x y

$\cancel{\$ \; y = \phi(x) \; \nabla\delta(x) \; \mathrm{mod}\ I}$ $\boxed{\mathrm{mod}\ I}$

$$A/I^2\phi(I)^2 \xrightarrow{???} A/I^2 \times A/\phi(I)^2$$

$\boxed{A/I^2 \text{ p-torsionfree!}}$

$\mathrm{mod}\ I^2$, ...

$(A/I,\ A/\phi(I))$

$(x^{p^2},\ \phi(x)^p,\ \phi^2(x))$

$x^p,\ \phi(x)$

$H^0(N^{\geq i}$

log-refractive???

$\mathbb{G}_m(A/I) \xrightarrow{N^2} \mathbb{G}_m(A/I_2)$ $\zeta_K(s)$

$\zeta_K(s)$

$\mathbb{G}_m(A/I) \xrightarrow{N} \mathbb{G}_m(A/I_1)$ given by

~~$N(f) = \phi(f) - V$~~

$\tilde{\xi} = p$ and I^p

$N(f) = \phi(f) - \xi\,\delta(f)$

(???)

~~$VS(f)$~~

$= p$ and I_1 ???

$TP_0(A^1_k) = H^0_{crys}(A^1_{k})[t^{\pm}]$

$H^0_{crys}(A^1_k) = W(k)\{0\} \oplus \bigoplus_n W/n\langle x^n\, d\log x\rangle$

$\{\mathbb{Z}_p^\sim\}$

H^i_{crys}

~~Th THH~~ $V(x^{np^2}) = V(y^{np})$

$W/p^2 \langle \underbrace{p\, x^{np}}\rangle$ py^n

$W/p \langle \underbrace{p^2\, x^j}_{pV(y^j)} \rangle$

y^n

$W/p^2 \langle x^{np} \rangle$

$W/p \langle p\, x^j \rangle$

$= V(x^{pj}) = V(y^j)$

$\boxed{T^{hC_p}_\lambda = W\Omega^\circ_1 \{1\}}$???

~~except~~ ---

over perfectoid base???

ok, actually is ok relative to $\mathbb{F}_p$.

Ω°

$u \longmapsto$ starts in ——

$\Omega^2\sigma^2$

$\Omega^2\sigma t^3$ $\quad \cancel{\Omega^2\sigma t^2}$ $\quad \Omega^2\sigma t$ $\quad \Omega^2\sigma$

$\Omega^2 t^2$ $\quad \cancel{\Omega^2 u}$ $\quad \Omega^2 t$ $\quad \cancel{\Omega^2 u}$ $\quad \Omega^2$ $\quad \Omega^2 t^{-1}$

$-4 \quad -2 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$

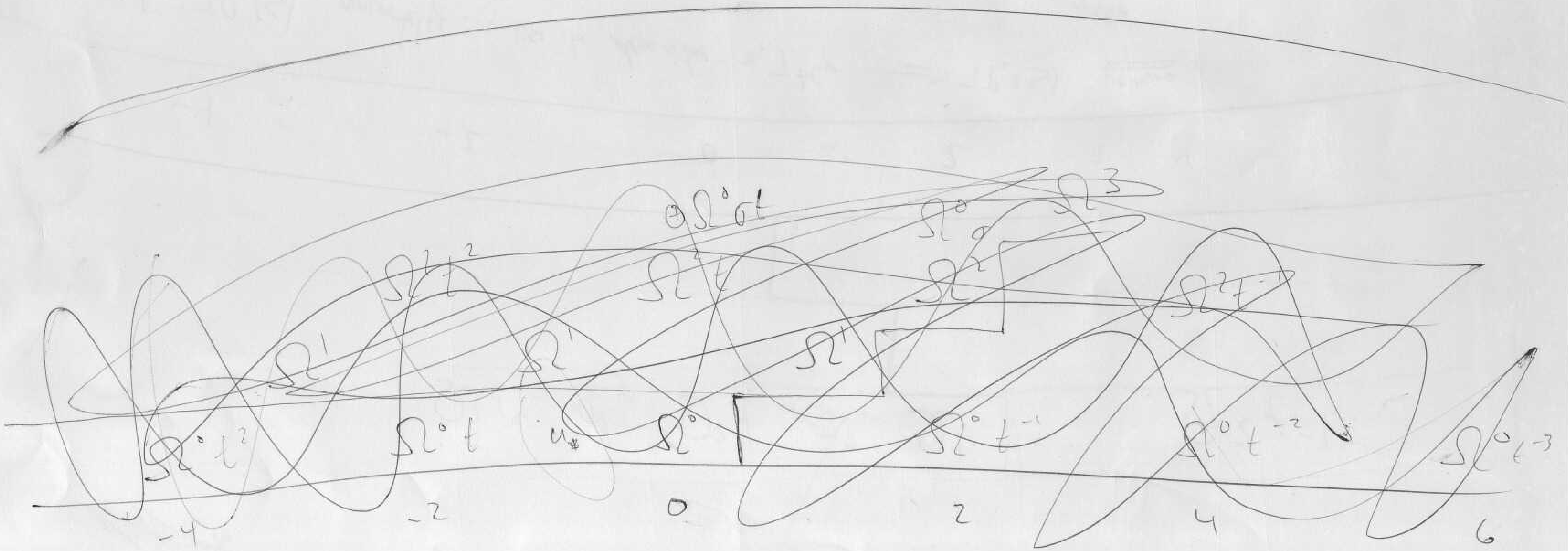
identify $TP_*(S)$ concretely ... use to describe $\pi_* T^{tC_p} \Longrightarrow TP_*(S)$ versus

$\pi_* T^{tC_p}$

$H^*_{\mathrm{cris}}(S) \longmapsto$ $\quad W\Omega^1 \quad W\Omega^0$ $\quad \Omega^0$

$W\Omega^1 \leftarrow W\Omega^0$

p=2, $\boxed{\Omega^1 \longrightarrow W_1\Omega^2}$???



TITLE: DATE / NO.

$\pi_{0,j} T^{hC_p} = W_1 \Omega^j$

$\pi_{2,j} T^{hC_p} =$

$H^*_{cryp}(S) \equiv \varprojlim_{n,F} W_n \Omega^*_S$?

p=0 n tC_p

$\Omega^2 \sigma t^2$ $\Omega^2 \sigma t$ $\Omega^2 \sigma$

$\Omega^2 t$ $\Omega^2 u$ Ω^2 $\Omega^2 t^{-1}$ $\Omega^2 t^{-2}$

-4 -2 0 2 4 6 8

hmm. bad example.

$\sigma^2 t^2$ $\sigma^2 t$ σ^2

σt^2 σt σ σt^{-1}

t^2 t u 1 ut^{-1} t^{-1} t^{-2}

$(\mathcal{H}^\cdot(W_n\Omega), \pi)$?

$(H^\cdot(W_n\Omega), R) \simeq (W_n\Omega^\cdot, F)$

ker $F = V^n$?

$\boxed{\pi_2 T^{h(p} = ???}$

$W_2\Omega^j\{i\} \xrightarrow{p^i ?} W_1\Omega^j\{i\}$

R map

$TR^n_\cdot(S) = \bigoplus W_n\Omega^i_S$???

$$\begin{array}{c} W_2\Omega \\ F\downarrow \uparrow V \quad (R) \\ W_1\Omega \\ F\downarrow \uparrow V \\ W_0\Omega \end{array}$$

$(H^\cdot(W_n\Omega^\cdot), \pi)$

?

$(H^\cdot(W_n\Omega), \pi)$

$\simeq (W_n\Omega^\cdot, F)$

?

$\boxed{???}$

$N(dx) = dx$, $\frac{1}{F}(dx) = V(x\, d\log x) = p\, x^{1/p}\, d\log x$ $(=0)$

actually empty.

$H^i(W\Omega^1)$

(acyclic)

$H^i(W\Omega/p)$ ~~ignored~~ mostly acyclic.

$\{V^k x^{n/p^k}\}$, $\{dV^k x^{n/p^k}\}$

$\to x^n\, d\log x \rightsquigarrow W/n$

new words need p^r.

$\cong W_r\Omega_{A^{\mathrm{act}}}$

$W_n\Omega_S[\sigma_n]$

$\frac{\Delta}{p^n}$, $\frac{N^{\geq i}\Delta}{pN^{\geq i+1}}$

(d_0, d_1, d_2)

$d_\cdot(\alpha)$

$W_n\Omega^*_S[\sigma_n]$

(new word moratorium)

$\underline{\alpha} = (d_0, d_1, d_2, \dots)$

$N^{\geq 1}A_{\inf}(K)$

$TF^{\cdot}_p(K; \mathbb{Z}_p)$

$TC(K; \mathbb{Z}_p)$

$H^{\cdot}\left(\frac{VW\Omega^0}{pVW\Omega^0} \to \frac{W\Omega^1}{V}\right)$

Δ / p

$H^{\cdot}\left(N^{2i}\ \Delta / pN^{2i+1}\right)$

$V(x^{-p} d\log x)$

$\{_p x^{n\emptyset} d\log x\}$

$VW\Omega^1 \leftarrow pW\Omega^0$

$W\Omega^1 \leftarrow VW\Omega^0$

$V(x^{np}) = p^2 x^n$

$\cancel{W\Omega^0}$ $W\Omega^1 \leftarrow W\Omega^0$

$W\Omega^0 \to W\Omega^1$

$VW\Omega^0 \to W\Omega^1$

$pVW\Omega^0 \to VW\Omega^1$

$p^2VW\Omega^0 \to pVW\Omega^1$

$H^{\cdot}\left(\frac{W\Omega^0}{pV} \xrightarrow{d} \frac{W\Omega^1}{p}\right)$

$\{V^k x^n\} \xrightarrow{\sim} \{dV^k x^n\}$ $p \nmid n$?

$\{x^n\}$ $\{x^n d\log x\}$

$\frac{W(k)}{p^V} \langle x^{np} \rangle$

$W/_{n,p} \langle x^n d\log x \rangle$

$Vd = p^d$

$\Rightarrow Edx = py$

Ω

$dx = Vy$

$Vdx = V^2y$ $W_n \oplus p^{ni}$

$\Rightarrow pdVx = V^2 y$ p^{2i} $\to p^i$

$dx = Vy$

W, Ω

$$H^{\cdot}\left(\frac{VW\Omega^0}{p^2 VW\Omega^0} \longrightarrow \frac{W\Omega^1}{pV}\right)$$

$\{V^k x^n\} \longrightarrow \{dV^k x^n\}$

$W_1\Omega^0 \qquad W_1\Omega^1$

$\{x^n\} \qquad \{x^n \, d\log x\}$

$V(x^{p^2} \, d\log x) = p \, x^p \, d\log x$

$pV(x^{np} \, d\log x) = p^2 x^n \, d\log x$

$dV(x^{np}) = d[p x^n] = p(n) x^n \, d\log x$

$V(x^{np}) =$

$T_2^{h c_p} = H^0\left(\frac{N^{\geq 1}\Delta}{pN^{\geq 2}\Delta}\{1\}\right) = H^0\left(\frac{VW\Omega^0}{p^2 V}\right)$

$\mathbb{Z}_p^{syn,tr}$ on CAlg

$H^0\left(\frac{V}{p^2 V}W\Omega^0 \longrightarrow \frac{1}{pV}W\Omega^0\right)$

$\mathbb{Z}_p^{syn}(R)$
$= \mathbb{Z}_p^{syn,tr}(HR)$

~~remove~~ remove BK twists

Tannaka duality

$\mathbb{Z}_p^{syn}$

$T_2^{hc_p} \longrightarrow T_0^{hc_p}$

(WHHS)

Buchweitz–Auslander X

$\mathbb{Z}_p^{\Delta} \rightsquigarrow TP^{(1)}$ (decompleted)

formal completion @ closed pt TC^-

MRT, SHIFT

THH

HT division THH

MRT

MHC, HP, HCE

HP

$TP(R[\zeta_p]/\mathbb{S}[q])$ Fuchs closed

$VW\Omega^0$

(Bruno – Lipson) Hopkins–Mahowald is known. of perfectoids

Ω have, blargh

$px^{np} = py^n = V(y^{np})$

$p^2 x^j = pV(x^{pj}) = pV(y^j)$

y^n

$V(y^j)$

$W_1\Omega^0_{k[y]}$

$TC^-_{2i}(R) = \hat{\Delta}^{(1)}_R$

$N^{2i}\hat{\Delta}^{(1)}_R\{i\}$ $T^{h C_p}_{\lambda++}$

$p\cancel{x} = V(x^p) \longmapsto$

$p^2 x = V(x^p) \longmapsto$

$\longmapsto p \, x \, d\log x$

$\cancel{pV_x} \longmapsto 0$

0 2 4

-4 -2

$TC^-_\lambda = N^{21}\Delta$ (no twist)

$\boxed{y = x^p}$

$TC^-_{2\lambda} = N^{22}\Delta$ $K^{\cdot}$ X

$W_1\Omega^1_{\mathbb{Z}[y]}$

$$H^\cdot\left(\frac{VW\Omega^0}{p^2 V} \longrightarrow \frac{W\Omega^1}{pV}\right)$$

$\{x^n\} \longrightarrow \{x^n d\log x\}$

$W_0\langle x^j d\log x\rangle$ $W_1\langle x^n\rangle$

$$H^\cdot\left(\frac{W\Omega^0}{pV} \longrightarrow \frac{W\Omega^1}{p}\right)$$

$W/p^2\langle y^n\rangle$ $W/p\langle y^n\rangle$

$W/p\langle$

???

$\Omega^1_{\mathbb{Z}[y]}$

$W_1\Omega^0_y$ $px \longmapsto 0.$ $\boxed{pV}$

$W/p^2\langle y^n\rangle$ $\frac{W\Omega^1}{pV}$ on top line

W/p

$\hat{p x}$

$p x^{i} \longmapsto THH_{G}, HH_{G}$

$p^2 x^j \longmapsto p^2 x^j \, d\log x$

$V(x^{p^2 j})$

$p^2 x^j \, d\log x \quad = p x^{i}$

$= p V(x^{pj} \, d\log x)$

$p^2 x^{n} \, d\log x$

$= p V(x^{-p} \, d\log x)$

$p^2 x^j \longmapsto p$ —

$= \boxed{p V(x^{pj})} \quad p^2_j x^i \, d\log x$

$= {}_j p V(\text{—})$

$p V(x^{pj})$

p

$$H^{\cdot}\left(\frac{VW\Omega^0}{p^2 V} \longrightarrow \frac{W\Omega^1}{pV}\right) \qquad W_1\Omega^1_{k[y]}$$

$$W/p^2\langle p\, x^{np}\rangle \;\; V(x^{np^2}) \qquad W_1\Omega^0_{k[y]}\{1\}\ ??? \qquad W/p^2\langle x^{pn}\, d\log x\rangle$$

$$W/p\langle p^2 x^{j}\rangle \;\; pV(x^{j}) \qquad p\,y^{n} \qquad W/p\langle x^{j}\, d\log x\rangle$$

$$H^{\cdot}\left(\frac{W\Omega^0}{pV} \longrightarrow \underbrace{\frac{W\Omega^1}{p}}\right)$$

$$\left.\begin{array}{l} W/p^2\langle x^{np}\rangle \\ W/p\langle \underbrace{p\,x^{j}}_{V x^{pj}}\rangle \end{array}\right] W_1\Omega^0_{k[y]} \qquad \left. W/p\langle x^{np}\, d\log x\rangle \right\} \Omega^1_{k[y]}\{-1\}$$

$\uparrow !\ 1/p$

$V(x^{np^2})$

$x^{n}\, d\log x$

$x = y^{1/p}$

$\cancel{VW\Omega^0}$

$$(x+y)^2 = 2xy \qquad e_1 = x+y \qquad e_2 = xy \qquad e_2 = \frac{e_1^2}{2}$$

$$(W^{\otimes n})^{\Sigma_n} = N[x^k/k!]/gr^{>n}$$

$$W^{\otimes 2} \qquad \frac{N[x,y]}{x^2, y^2}$$

$$\underbrace{(x+y)}^2 = 2xy \qquad x+y = t_1 \qquad xy = \frac{t^2}{2}$$

$$\cancel{(x+y+z)^3 = 3!\, xyz}$$

$$\cancel{(x+y+z)^2 = 2}$$

$$e_1^n = n!\, e_n$$

$\boxed{W_r\Omega_S = ???}$

$$\boxed{\begin{aligned} y &= x^p, \\ x &= y^{1/p} \end{aligned}}$$

$k[x^p] = k[x]^{(1)}$

Ω_q

$d\log$

$A/(p)_q\langle x^n\rangle$

$y = \phi(x)$

$$A/(p)_q\langle x^n\rangle \longrightarrow A/(p)_q\langle x^n \, d\log x\rangle$$

$$A \quad \{A/(p)_q \, x^n \, d\log x$$

ϕ

A-action is non-lin? ϕ

$x^{np} = \phi(x^n)$

$$\underline{A/(p)_q \otimes_A A^{(1)} = ????}$$

$$A/(p)_q \xrightarrow{\phi} A/(p)_{q^p}$$

$$\begin{tikzcd} A \arrow[r, "\phi"] \arrow[d] & A \arrow[d] \\ A/(p)_q \arrow[r, "\phi"'] & A/(p)_{q^p} \end{tikzcd}$$

$A^{(1)}/(p)_{q^{1/p}}$

$\mathrm{Lan}_\ell f \cong f \cong \mathrm{Ran}_\ell f ??$

$\mathrm{Ran}_\ell f : X \to Y$

$\mathrm{Lan}_\ell f : X \to Y$

left Kan extension of f along ℓ is

$\mathrm{Lan}_\ell f \cong f_! \cong \mathrm{Ran}_\ell f$

$A/(p)_q\langle x^p\rangle$ $\quad y = x^p \Rightarrow x = y^{1/p}$

$A/(p)_{q^{1/p}} \xrightarrow{\phi} A/(p)_{q^{1/p}} \longrightarrow A$

$\cancel{A/(p)_{q^{1/p}}}$ $\quad A[x] \otimes_A A^{(1)}$ $\quad A[x] \otimes_A A$ ~~mixed~~ characteristic

$A/(p)_{q^{1/p}}\langle x^n\rangle \xrightarrow{\phi} A/(p)_q\langle x^{np}\rangle$

$H^\bullet(\Delta^{(1)}/(p)_q) \simeq \Omega_{A/A|\xi}$? $\quad$ [concrete goal: mixed characteristic $W\Omega^\bullet$]

$A/(p^2)_q\langle x^n\rangle \longrightarrow A/(p^2)_q\langle x^n \, d\log x\rangle$

$\Rightarrow A/(p^2)_q\langle x^{np^2}\rangle \oplus A/(p)_q\langle \underbrace{(p)_{q^p}\, x^{pj}}_{= V(x^j)}\rangle$

$\cancel{\oplus A/(p)_{q^p}\langle x^{jp}\rangle}$

$A/\xi\phi^{-1}(\xi)$

$\Rightarrow W_{A/\xi\phi^{-1}(\xi)}\Omega^\bullet$

$\underbrace{A/(p^2)_{q^{1/p^2}}}_{A/\xi\phi^{-1}(\xi)?}$

$A/(p^2)_q\langle x^{np^2}\, d\log x\rangle$

$\cancel{x^{np^2}\, d\log x}$

$\underbrace{A/(p)_q\langle x^{pj}\; d\log x\rangle}$ $\quad$! $\quad \boxed{A/\phi^{-2}(\xi)}$

$$\underbrace{\frac{q-1}{q^{1/p}-1}}_{(p)_{q^{1/p}}\ \xi} \qquad \underbrace{\frac{q^{1/p}-1}{q^{1/p^2}-1}}_{\underbrace{(p)_{q^{1/p^2}}}_{\phi^{-1}(\xi)}}$$

$$\cancel{A/\tilde\xi\,\phi(\tilde\xi)\langle x^{np^2}\rangle} \oplus \cancel{A/\phi(\xi)\langle}$$

$$P_{2n}\mathrm{THH}(R;\mathbb{Z}_p) = \Sigma^{[?]}\lambda\,\mathrm{THH}(R;\mathbb{Z}_p)$$

$$A/\tilde\xi\,\phi(\tilde\xi)\langle x^{np^2}\rangle \oplus A/\tilde\xi\langle\phi(\tilde\xi)\,x^{pj}\rangle \quad ; \quad A/\tilde\xi\,\phi(\tilde\xi)\langle x^{np^2}\,d\log x\rangle \oplus A/\tilde\xi\langle x^{pj}\,d\log x\rangle$$

$$A/\xi\,\phi^{-1}(\xi)\langle y^n\rangle \oplus A/\phi^{-1}(\xi)\langle \underbrace{\xi\, y^{j/p}}_{V(y^j)}\rangle$$

$$A/\xi\,\phi^{-1}(\xi)\langle y^n \xrightarrow{d\log x}\rangle \ \frac{1}{(p^2\cdot)_{2k}}$$

$$\cancel{A}\ A/\phi^{-1}(\xi)\langle y^{j/p}\,d\log x\rangle$$

crazy twisting

$$\cancel{W_{}R}\ \cancel{W_{}S_{}}$$

$$\begin{CD} A/\xi\,\phi^{-1}(\xi) @>{\phi^2}>> A/\tilde\xi\,\phi(\tilde\xi) \\ @A{\phi}AA @AA{V}A \\ A/\xi @>>{\phi}> A/\tilde\xi \end{CD}$$

$f \mapsto \phi^{-1}(f)\,\tilde\xi$ $\quad F$

$\cancel{f \mapsto \phi(f)}$ $\quad \cancel{\phi(f)}$

$$\phi^{-2}\left[\phi(f)\,\frac{\phi(\tilde\xi)}{\delta(\tilde\xi)}\right] = \phi^{-1}(f)\ \xi$$

$y = x^{p^2}$, $d(y^k)$

$\nabla_{q,y}[y^k] = (k)_q \, y^k \, d\log y$

$\nabla_{q,x}[y^k] = \nabla_{q,x}[x^{p^2 k}]$

$= (p^2 k)_q \, x^{p^2 k} \, d\log x$

$= (k)_q \, (p^2)_{q^k} \, x^{p^2 k} \, d\log x$

$y^k \, d\log y = (p^2)_{q^k} \, x^{p^2 k} \, d\log y$

$W, \Omega^1_{R(?)} \otimes \frac{1}{(p^2)_{q^k}}$

$\nabla_{q,y}[y^{j/p} \, d\log y] =$

[look up action of ϕ on $A\Omega_{\mathcal{X}}$]

$\boxed{f = i \circ \tilde{f}}$

$\sup\{f(a) \mid i(a) \le x\} \le \tilde{f}(x) \le \inf\{f(a) \mid x \le i(a)\}$

~~sup-closed upper bound~~

sup(S).

$f : i(a_1) = i(a_2)$ but $f(a_1) \neq f(a_2)$, impossible

ff : is injective

be possible

$y = x^p$

$A/(p)_q\langle x^{np}\rangle$ $\qquad$ $A/(p)_q\langle x^{np}\, d\log x\rangle$ $\qquad$ $\cancel{N^G_H(f) = f^{G/H}}$

$\cancel{\phi(dx) = (p)_q\, dx}$ $\qquad$ $d(x^p) =$

$\cancel{\phi(x) = \phi}$

$$\phi(x^n\, d\log x) = \phi(x^{n-1}\, dx) = x^{pn-p}\, d(x^p)$$

$$= (p)_q\, x^{np}\, d\log x \qquad K_{2i-1}(R;\mathbb{Z}_p)$$

$$\phi(d\log x) = \frac{d(x^p)}{\phi(x)} = (p)_q\, d\log x$$

$N^H_K(f) = f^{H/K}$ $\qquad$ $\phi(f) = f^p + p\delta(f)$

$$\begin{array}{ccc}
A/(p)_{q^{1/p}}\langle y^n\rangle & & A/(p)_{q^{1/p}}\langle y^n\, d\log y\rangle\,\frac{1}{(p)_{q^n}} \\
\phi\big\downarrow\wr & & \phi\big\downarrow \\
A/(p)_q\langle x^{np}\rangle & & A/(p)_q\langle x^{np}\, d\log x\rangle
\end{array}$$

$\phi(y) = x^p$

$$\cancel{d\log x^p} \quad d\log y = \frac{d(x^p)}{x^p}$$

$$\cancel{d\log y = d\log} \quad \cancel{= (p)_q\, d\log x} \quad (p)_q\, d\log x$$

~~In fact,~~ Suppose $y = x^e$

$$\cancel{\nabla_{q,y}[y^n] = d\log y] = (n)_q\, y^n\, d\log y}$$

$$\nabla_{q,y}(y^n) = (n)_q\, y^n\, d\log y$$

$$\nabla_{q,x}(y^n) = \nabla_{q,x}(x^{ne}) = (ne)_q\, x^{ne}\, d\log x$$

$$\Rightarrow y^n\, d\log y = (e)_{q^n}\, x^{ne}\, d\log x$$

$$T_2^{hlp} = W_1\Omega^2 \oplus W_1\Omega^0\{1\}$$

$$\uparrow$$

$$T_{2+\lambda}^{hlp}$$

$$\uparrow$$

$$T_{2+2\lambda}^{hlp}$$

for $p=2$ yes

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "i"'] & Y \\ X \arrow[ur, dashed, "\tilde f"'] & \end{tikzcd}$$

? $\tilde f \circ i = f$

$\cancel{(\tilde f \circ i)(a) \le f(a)}$

$\tilde f(i(a)) \le \tilde f(x)$

$f(a) \le \tilde f(x)$

$$\Rightarrow \overbrace{\sup\{f(a) \mid i(a) \le x\}}^{\mathrm{Lan}_i(f)} \le \tilde f(x)$$

$$\le \overbrace{\inf\{f(a) \mid x \le i(a)\}}^{\mathrm{Ran}_i(f)}$$

~~adjunction~~

$2+2\lambda$

$N^{\geq 3}\Delta\{1\}$

$$\frac{\xi^3 A\Omega^0}{\xi^4\tilde{\xi}}\{1\} \longrightarrow \frac{\xi^2 A\Omega^1}{\xi^3\tilde{\xi}}$$

$V(f) = \xi\phi^{-1}(f)$

$2+\lambda$

$= N^{\geq 2}\Delta\{1\}$

$$\frac{\xi^2 A\Omega^0}{\xi^3\tilde{\xi}}\{1\} \longrightarrow \frac{\xi A\Omega^1}{\xi^2\tilde{\xi}}\{1\}$$

$N^{\geq 1}\Delta\{1\}$

$(N^{\geq 1}\Delta)\{1\}$

or $N^{\geq 1}(\Delta\{1\})$

2

$= N^{\geq 1}\Delta\{1\}$

$$\frac{\xi A\Omega^0}{\xi^2\tilde{\xi}}\{1\} \longrightarrow \frac{N^{\geq 1} A\Omega^1}{\xi\tilde{\xi}}\{1\}$$

$\cancel{\frac{\phi}{\xi}}$ $\frac{\phi}{\xi}$

$$A/\xi\tilde{\xi}\langle\xi^3 x^{p^n}\rangle\{1\} \oplus A/\xi\langle\xi^3\tilde{\xi}\, x^j\rangle\{1\} \xleftarrow[\sim]{\phi} A/\xi\phi^{-1}(\xi)\langle\phi^{-1}(\xi)^3 x^n\rangle\{1\} \oplus A/\phi^{-1}(\xi)\langle\xi\phi^{-1}(\xi)^3 x^{j/p}\rangle\{1\}$$

$\downarrow$

$$A/\xi\tilde{\xi}\langle\xi^2 x^{p^n}\rangle\{1\} \oplus A/\xi\langle\xi^2\tilde{\xi}\, x^j\rangle\{1\} \xleftarrow{\phi} A/\xi\phi^{-1}(\xi)\langle\phi^{-1}(\xi)^2 x^n\rangle\{1\} \oplus A/\phi^{-1}(\xi)\langle\phi^{-1}(\xi)^2\xi\, x^{j/p}\rangle$$

$\downarrow$

$$A/\xi\tilde{\xi}\langle\xi\, x^{p^n}\rangle\{1\} \oplus A/\xi\langle\xi\tilde{\xi}\, x^j\rangle\{1\} \xleftarrow{\phi} A/\xi\phi^{-1}(\xi)\langle\phi^{-1}(\xi)\, x^n\rangle\{1\} \oplus A/\phi^{-1}(\xi)\langle\xi\phi^{-1}(\xi)\, x^{j/p}\rangle\{1\}$$

$\xi \leftarrow$

$TH_2^{\mathrm{tc}} = W_1\Omega^0\{1\}$ $A/\xi\Omega \oplus \ldots \leftarrow d/\xi\Omega \leftarrow d/\xi\Omega$ $W\Omega^\cdot$?

looks like just $\phi^{-1}(\xi)$-adic filtration on $\cancel{W}$

$\leftarrow d/\xi\Omega \leftarrow d/\xi\Omega$ $\cancel{\Omega}$ $A/\xi\Omega$

$0 \leftarrow d/\xi\Omega \leftarrow d/\xi\Omega \leftarrow \Omega/\xi\Omega \leftarrow 0$ $A/\xi\Omega$

$V(x\, d\log x) = p\, x^{1/p}\, d\log x = 0$

noyau de V ⇝ Illusie, Dev.-Mond.

$TR_\star(\mathcal{O}_C; \mathbb{Z}_p)$, $TR_\star(S; \mathbb{Z}_p)$,

$RO(G)$-graded norms & the poly-de Rham-Witt complex

Φ_p $\qquad \Phi^{b}_p$

$N^{\geq d_0} N^{\geq d_1} \hat{\Delta}_R$

$\underline{W\Omega}^\star_S \longrightarrow \underline{TC}^-_\star$

$$\boxed{N(\omega) = V\frac{\omega^p}{p} + (p - V(1))\frac{\varphi^p(\omega)}{p}}$$

$$N^{\geq d_0} N^{\geq d_1} \hat{\Delta}_R \begin{Bmatrix} I^{d_2}\, \phi(I)^{d_3}\, \phi^2(I)^{d_4} \cdots \\ \cdots \end{Bmatrix}$$

- test step function!
- add Bisue to Epiphanies - Next

- ~~Liquid~~ Liquid 2.2 ???
- business bank acct
- write articles {EMN}
- revise + submit

Goals for this week

{S[kit]} TE, ~~TE~~ {A}

EMN?

Rotation article probably exists in a notebook somewhere.

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "i"'] & Y \\ X \arrow[ur, "\mathrm{Lan}_i(f)"] \arrow[ur, bend right, "\mathrm{Ran}_i(f)"'] & \end{tikzcd}$$

$A\Omega^1 \longleftarrow A\Omega^0$

$A\Omega^1\{1\} \longrightarrow A\Omega^0\{1\}$

TP

$\mathcal{C}_T^{t\epsilon_p} = \cancel{TP}$

$$\begin{tikzcd} S^0 \arrow[d, "a_{\lambda_0} = a_0"'] \arrow[dr, "a_{\lambda_1}"] \arrow[drr, bend left, "a_{\lambda_2}"] & & \\ S^{\lambda_0} \arrow[r, "a_1"] & S^{\lambda_1} \arrow[l, bend left, "a_1"] \arrow[r, "a_2"] & S^{\lambda_2} \\ S^2 \arrow[u, "u_{\lambda_0}"] \arrow[ur, "u_{\lambda_0}"'] & & \end{tikzcd}$$

$u_{\lambda_0}\ u_{\lambda_1}^{-1}$

(x^p) $a_{\lambda_{-1}}^{-1}$

$$\begin{tikzcd} S^0 \arrow[r, shift left, "a_0"] & S^{\lambda_0} \arrow[l, shift left, "u_0"] \arrow[r, shift left, "a_1"] & S^{\lambda_1} \arrow[l, shift left, "u_1"] \arrow[r, shift left, "a_2"] & S^{\lambda_2} \arrow[l, shift left, "u_2"] \\ & S^2 \arrow[ul, "u"] \arrow[u, "u_{\lambda_0}"] \arrow[ur, "u_{\lambda_1}"'] & & \end{tikzcd}$$

$$\begin{tikzcd} S^0 \arrow[d, "a_0 = a_{\lambda_0}"'] \arrow[dr, "a_{\lambda_1}"] \arrow[drr, bend left, "a_{\lambda_2}"] & & \\ S^{\lambda_0} \arrow[r, shift left, "a_1"] & S^{\lambda_1} \arrow[l, shift left, "u_1"] \arrow[r, shift left, "a_2"] & S^{\lambda_2} \arrow[l, shift left, "u_2"] \\ S^2 \arrow[u, "u_{\lambda_0}"] \arrow[ur, "u_{\lambda_1}"'] & & \end{tikzcd}$$

~~$0 \to d(\Omega^0) \to ? \to F(W, \Omega^0) \to 0$~~

$\Omega^0\{1\}$, $B\Omega^1$, $F(W, \Omega^0)$

im d ker d

ker V

$\to R\{1\} \to W_1(R)\{1\} \to R\{1\} \to 0$

im F $\to R\{1\} \to$ ker d

$\Omega^0 \xrightarrow{d} \Omega^1$

$d = FdV$

$F(W, \Omega^0)$

ker(F) = im V+

Ω^i, $B\Omega^i$, $F\Omega^i$ in char p,

$$\Omega^0 \qquad \Omega^1 \qquad \begin{matrix}\Omega^2 \\ \oplus \\ \Omega^0\{1\}\end{matrix}$$

im d $\quad B\Omega^1$

ker d $\quad (\mathrm{im}\, F)\Omega^0$

$$TR^1_{(1,0),0} = W_1\Omega^0 \otimes \{\xi\}$$

$$\Omega^0 \xrightarrow{V} W_1\Omega^0 \xrightarrow{\phi^{-1}(\xi)} W_1\Omega^0 \xrightarrow{F} \Omega^0$$

$\Omega^0\{1\}, \quad B\Omega^1, \quad (\mathrm{im}\, F)\Omega^0$

surjects onto im F

$$\ker F = \phi^{-1}(\xi)$$

$$dV(x^n) = d(\xi x^{n/p}) = \frac{n\xi}{p} x^{n/p}\, d\log x$$