

$$\begin{array}{ccccccc}
0 & \longrightarrow & \mathrm{TF}_\alpha & \longrightarrow & \mathrm{TF}_\alpha^\Phi & \longrightarrow & \pi_\alpha \Sigma^2 \mathrm{TC}^+ \longrightarrow \mathrm{TF}_{\alpha-1} \longrightarrow \mathrm{TF}_{\alpha-1}^\Phi \longrightarrow 0 \\
& & \downarrow & & \downarrow & & \parallel \\
0 & \longrightarrow & \mathrm{TC}_\alpha^- & \longrightarrow & \mathrm{TP}_\alpha & \longrightarrow & \pi_\alpha \Sigma^2 \mathrm{TC}^+ \longrightarrow 0
\end{array}$$

$$G = C_p$$

Case	Subcase	TF_α	$\mathrm{T}\Phi_\alpha$	$\pi_\alpha \Sigma^2 \mathrm{TC}^+$	$\mathrm{TF}_{\alpha-1}$	$\mathrm{T}\Phi_{\alpha-1}$
$d_0 > 0$	$d_\infty \geq 0$	$A\langle v_0^{d_0} u_{\lambda_0}^{d_\infty} \rangle$	$A\langle a_{\lambda_0}^{-d_0} u_{\lambda_0}^{d_\infty} \rangle$	$A/\xi^{d_0} \langle a_{\lambda_0}^{-d_0} u_{\lambda_0}^{d_\infty} \rangle$	0	0
	$d_\infty < 0$	0	0	$A/\xi^{d_0} \langle a_{\lambda_0}^{-d_0} u_{\lambda_0}^{d_\infty} \rangle$	$A/\xi^{d_0} \langle \Sigma^{-1} a_{\lambda_0}^{-d_0} u_{\lambda_0}^{d_\infty} \rangle$	0
$d_0 \leq 0$	$d_\infty \geq 0$	$A\langle a_{\lambda_0}^{-d_0} u_{\lambda_0}^{d_\infty} \rangle$	$A\langle a_{\lambda_0}^{-d_0} u_{\lambda_0}^{d_\infty} \rangle$	0	0	0
	$d_\infty < 0$	0	0	0	0	0

$$G = C_{p^2}$$

Case-1	Case-2	Case-3	TF_α	$\text{T}\Phi_\alpha$	$\pi_\alpha \Sigma^2 \text{TC}^+$	$\text{TF}_{\alpha-1}$	$\text{T}\Phi_{\alpha-1}$
$d_0 > 0$	$d_\infty \geq 0$	$d_1 > 0$	$A \langle v_0^{d_0} v_1^{d_1} u_{\lambda_1}^{d_\infty} \rangle$	$A \langle a_{\lambda_0}^{-d_0} v_1^{d_1} u_{\lambda_1}^{d_\infty} \rangle$	$A/\xi^{d_0} \langle a_{\lambda_0}^{-d_0} v_1^{d_1} u_{\lambda_1}^{d_\infty} \rangle$	0	0
		$d_1 \leq 0$	$A \langle v_0^{d_0} b_1^{-d_1} u_{\lambda_1}^{d_\infty} \rangle$	$A \langle a_{\lambda_0}^{-d_0} b_1^{-d_1} u_{\lambda_1}^{d_\infty} \rangle$	$A/\xi^{d_0} \langle a_{\lambda_0}^{-d_0} v_1^{d_1} u_{\lambda_1}^{d_\infty} \rangle$	$\frac{A}{\xi^{d_0}, \phi(\xi)^{-d_1}} \langle \Sigma^{-1} a_{\lambda_0}^{-d_0} v_1^{d_1} u_{\lambda_1}^{d_\infty} \rangle$	0
	$d_\infty < 0$	$d_1 > 0$	0	0	$A/\xi^{d_0} \langle a_{\lambda_0}^{-d_0} v_1^{d_1} u_{\lambda_1}^{d_\infty} \rangle$	$\frac{A}{\xi^{d_0} \phi(\xi)^{d_1}} \langle \Sigma^{-1} a_{\lambda_0}^{-d_0} b_1^{-d_1} u_{\lambda_1}^{d_\infty} \rangle$	$A/\xi^{d_1} \langle \Sigma^{-1} a_{\lambda_0}^{-d_0} b_1^{-d_1} u_{\lambda_1}^{d_\infty} \rangle$
		$d_1 \leq 0$	0	0	$A/\xi^{d_0} \langle a_{\lambda_0}^{-d_0} v_1^{d_1} u_{\lambda_1}^{d_\infty} \rangle$	$A/\xi^{d_0} \langle \Sigma^{-1} a_{\lambda_0}^{-d_0} v_1^{d_1} u_{\lambda_1}^{d_\infty} \rangle$	0
$d_0 \leq 0$	$d_\infty \geq 0$	$d_1 > 0$	$A \langle a_{\lambda_0}^{-d_0} v_1^{d_1} u_{\lambda_1}^{d_\infty} \rangle$	$A \langle a_{\lambda_0}^{-d_0} v_1^{d_1} u_{\lambda_1}^{d_\infty} \rangle$	0	0	0
		$d_1 \leq 0$	$A \langle a_{\lambda_0}^{-d_0} b_1^{-d_1} u_{\lambda_1}^{d_\infty} \rangle$	$A \langle a_{\lambda_0}^{-d_0} b_1^{-d_1} u_{\lambda_1}^{d_\infty} \rangle$	0	0	0
	$d_\infty < 0$	$d_1 > 0$	0	0	0	$A/\phi(\xi)^{d_1} \langle \Sigma^{-1} a_{\lambda_0}^{-d_0} b_1^{-d_1} u_{\lambda_1}^{d_\infty} \rangle$	$A/\xi^{d_1} \langle \Sigma^{-1} a_{\lambda_0}^{-d_0} b_1^{-d_1} u_{\lambda_1}^{d_\infty} \rangle$
		$d_1 \leq 0$	0	0	0	0	0

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$$G = C_{p^3}$$

$$\begin{aligned} a_1 u_0 &= \phi(\xi) a_0 u_1 \\ b_1 &= \phi(\xi) v_1^{-1} \end{aligned}$$

Case-1	Case-2	Case-3	Case 4	TF_α	$\text{T}\Phi_\alpha$	$\pi_\alpha \Sigma^2 \text{TC}^+$	$\text{TF}_{\alpha-1}$	$\text{T}\Phi_{\alpha-1}$
$d_0 > 0$	$d_1 > 0$	$d_2 > 0$	$d_\infty \geq 0$	—	$A \langle b_0^{-d_0} v_1^{d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	$A/\xi^{d_0} \langle b_0^{-d_0} v_1^{d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	0	0
			$d_\infty < 0$	0	0	$A/\xi^{d_0} \langle b_0^{-d_0} v_1^{d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	$\frac{A}{\xi^{d_0} \phi(\xi)^{d_1} \phi^2(\xi)^{d_2}} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} b_2^{-d_2} u_{\lambda_2}^{d_\infty} \rangle$	$\frac{A}{\xi^{d_1} \phi(\xi)^{d_2}} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} b_2^{-d_2} u_{\lambda_2}^{d_\infty} \rangle$
		$d_2 \leq 0$	$d_\infty \geq 0$	—	$A \langle b_0^{-d_0} v_1^{d_1} b_2^{-d_2} u_{\lambda_2}^{d_\infty} \rangle$	$A/\xi^{d_0} \langle b_0^{-d_0} v_1^{d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	$\frac{A}{\xi^{d_0} \phi(\xi)^{d_1}} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	$\frac{A}{\xi^{d_1, \phi(\xi)^{-d_2}} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$
			$d_\infty < 0$	0	0	$A/\xi^{d_0} \langle b_0^{-d_0} v_1^{d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	$\frac{A}{\xi^{d_0} \phi(\xi)^{d_1}} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	$A/\xi^{d_1} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$
	$d_1 \leq 0$	$d_2 > 0$	$d_\infty \geq 0$	—	$A \langle b_0^{-d_0} b_1^{-d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	$A/\xi^{d_0} \langle b_0^{-d_0} v_1^{d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	$\frac{A}{\xi^{d_0, \phi(\xi)^{d_1}} \langle b_0^{-d_0} v_1^{d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	0
			$d_\infty < 0$	0	0	$A/\xi^{d_0} \langle b_0^{-d_0} v_1^{d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	$\frac{A}{\xi^{d_0} \phi^2(\xi)^{d_2}} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} b_2^{-d_2} u_{\lambda_2}^{d_\infty} \rangle$	$A/\phi(\xi)^{d_2} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} b_2^{-d_2} u_{\lambda_2}^{d_\infty} \rangle$
		$d_2 \leq 0$	$d_\infty \geq 0$	—	$A \langle b_0^{-d_0} b_1^{-d_1} b_2^{-d_2} u_{\lambda_2}^{d_\infty} \rangle$	$A/\xi^{d_0} \langle b_0^{-d_0} v_1^{d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	$\frac{A}{\xi^{d_0, \phi(\xi)^{d_1}} \langle b_0^{-d_0} v_1^{d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	0
			$d_\infty < 0$	0	0	$A/\xi^{d_0} \langle b_0^{-d_0} v_1^{d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	$A/\xi^{d_0} \langle \Sigma^{-1} b_0^{-d_0} v_1^{d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	0
	$d_0 \leq 0$	$d_1 > 0$	$d_\infty \geq 0$	—	$A \langle b_0^{-d_0} v_1^{d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	0	0	0
			$d_\infty < 0$	0	0	0	$\frac{A}{\phi(\xi)^{d_1} \phi^2(\xi)^{d_2}} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} b_2^{-d_2} u_{\lambda_2}^{d_\infty} \rangle$	$\frac{A}{\xi^{d_1} \phi(\xi)^{d_2}} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} b_2^{-d_2} u_{\lambda_2}^{d_\infty} \rangle$
		$d_2 \leq 0$	$d_\infty \geq 0$	—	$A \langle b_0^{-d_0} v_1^{d_1} b_2^{-d_2} u_{\lambda_2}^{d_\infty} \rangle$	0	$\frac{A}{\phi(\xi)^{d_1, \phi^2(\xi)^{d_2}} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	$\frac{A}{\xi^{d_1, \phi(\xi)^{-d_2}} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$
			$d_\infty < 0$	0	0	0	$A/\phi(\xi)^{d_1} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} v_2^{d_2} u_{\lambda_1}^{d_\infty} \rangle$	$A/\xi^{d_1} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} v_2^{d_2} u_{\lambda_1}^{d_\infty} \rangle$
	$d_1 \leq 0$	$d_2 > 0$	$d_\infty \geq 0$	—	$A \langle b_0^{-d_0} b_1^{-d_1} v_2^{d_2} u_{\lambda_2}^{d_\infty} \rangle$	0	0	0
			$d_\infty < 0$	0	0	0	$A/\phi^2(\xi)^{d_2} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} b_2^{-d_2} u_{\lambda_2}^{d_\infty} \rangle$	$A/\phi(\xi)^{d_2} \langle \Sigma^{-1} b_0^{-d_0} b_1^{-d_1} b_2^{-d_2} u_{\lambda_2}^{d_\infty} \rangle$
	$d_2 \leq 0$	$d_\infty \geq 0$	$d_\infty \geq 0$	—	$A \langle b_0^{-d_0} b_1^{-d_1} b_2^{-d_2} u_{\lambda_2}^{d_\infty} \rangle$	0	0	0
			$d_\infty < 0$	0	0	0	0	0