a) The best fit parameters are

```
Fit params using Newton are [1.42281068e+00 1.92358649e-04 1.79236908e-05]
```

where the parameters are ordered as a, t_0, w.

b) The noise is estimated by taking the standard deviation of the differences between fitted values and data. The error in the fitted parameters are estimated using the covariance matrix,

```
Estimated noise in data is 0.025044391788604244
Fit param errors using Newton are [1.68617122e-02 2.12350953e-07 3.00715894e-07]
```

where the parameters are ordered as a, t_0, w.

c) Using numeric differentiation with the same number of step as a), we get

```
Fit params using Newton and numeric diff are [1.42281068e+00 1.92358649e-04 1.79236908e-05]
with error [1.68617122e-02 2.12350953e-07 3.00715894e-07]
```

where the parameters are ordered as a, t_0, w. The difference is statistically insignificant as the they are well below the uncertainties.

d) We take the value from 1 a) for a, t_0, w. We estimate the b, c to be around one order of magnitude smaller than a, and $\mathrm{d}t$ to be around one order of magnitude smaller than t_0. The fit is performed and we get

```
Fit params using Newton and numeric diff and three Lorentzians are
 [1.44204652e+00 1.01234553e-01 5.97087340e-02 1.92591564e-04
 4.50740613e-05 1.61618899e-05]
with error [1.84257662e-02 1.84964450e-02 1.80116489e-02 2.21089912e-07
 2.72288801e-06 4.09233511e-07]
```

where the parameters are ordered as $a, b, c, t_0, \mathrm{d}t, w$.

e) The residues are shown in Figure 1. The residues are showing clear structures. This indicate that the error bars are not independent given the three-Lorentzians model used. This means the three-Lorentzians model may not be a complete description of the data.

f) The plot is shown in Figure 2. We can see that the error is generally larger at the two small bumps. The typical χ^2 for perturbed and best fit parameters are

```
Average of the perturbed chi2 is 27.63233127414122
Newton best fit chi2 is 21.374163797484997
Difference is 6.258167476656222
```

The perturbed χ^2 is generally larger than the best fit one as we expected. Note the differences is around $\sqrt{2k}$ where k is the best fit χ^2, i.e. the perturbed χ^2 is off by around one standard deviation.

g) The fit results using MCMC is

```
Fit params using MCMC and three Lorentzians are
 [1.44281071e+00 8.72898094e-02 6.02726339e-02 1.92426691e-04
 4.26812411e-05 1.63578543e-05]
with error [4.58566614e-02 1.91667835e-02 2.07841084e-02 6.58571982e-07
 9.47444449e-06 8.40912277e-07]
MCMC fit chi2 is 24.224894541731246
```

This is mostly the same result as we obtained in (d). To determine whether the MCMC converges, we first plot the chi^2 as a function of steps in Figure 3. We can see from the figure that chi^2 converges to a constant at large steps. We also plot the chain for each parameters as shown in Figure 4 and their respective FFT transformed results in Figure 5. All the parameters converges and there is no low frequency peaks in their FFT. Based on these observations, we believe the MCMC chain converges.

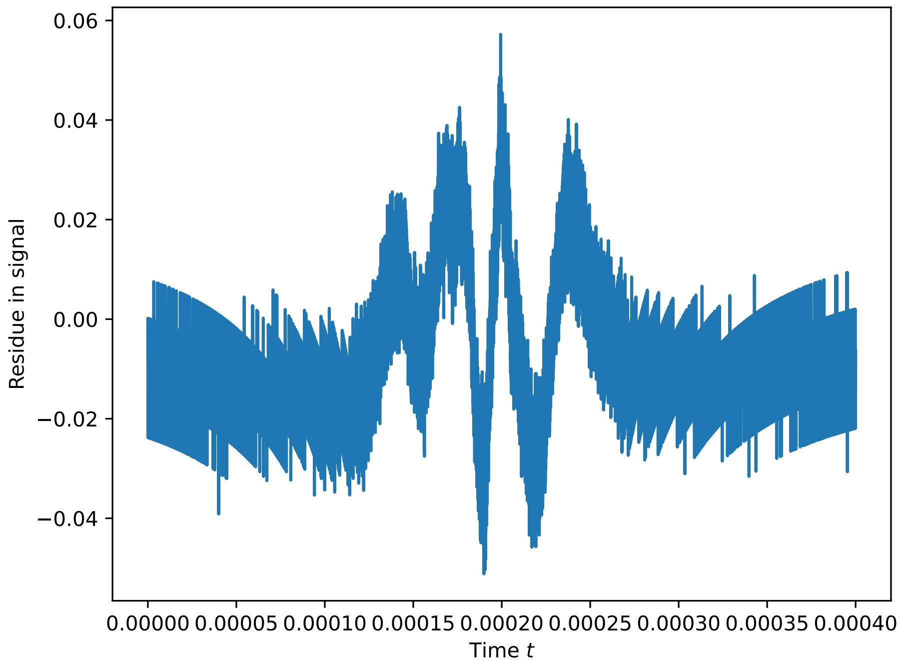

Figure 1: Plot of fit residues using three-Lorentzians. There is clear structure in the residue, indicating possible correlation of error bars.

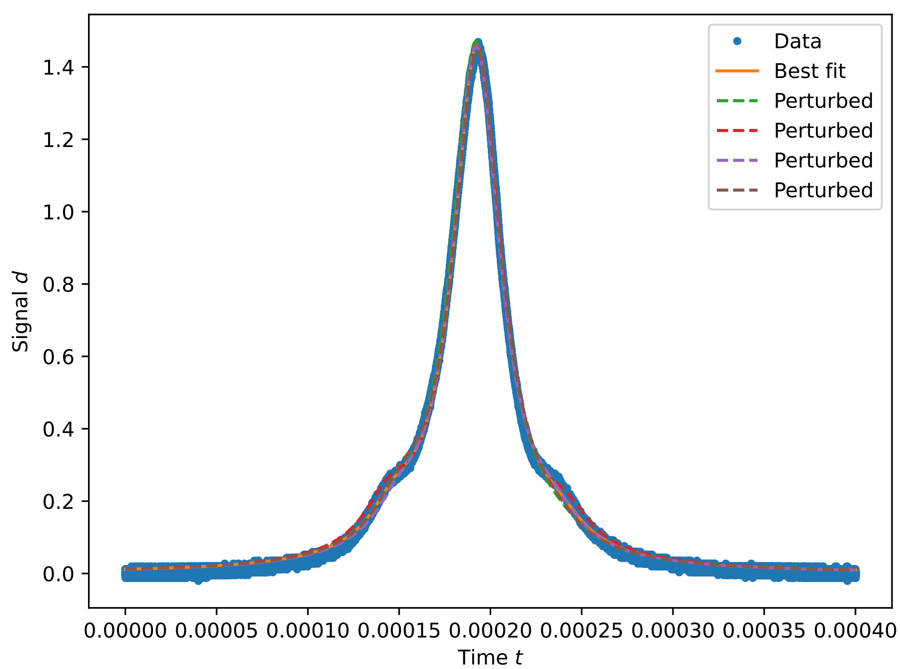

Figure 2: Plot of best fit line to the data and a three-Lorentzians using the parameters with realized parameter errors.

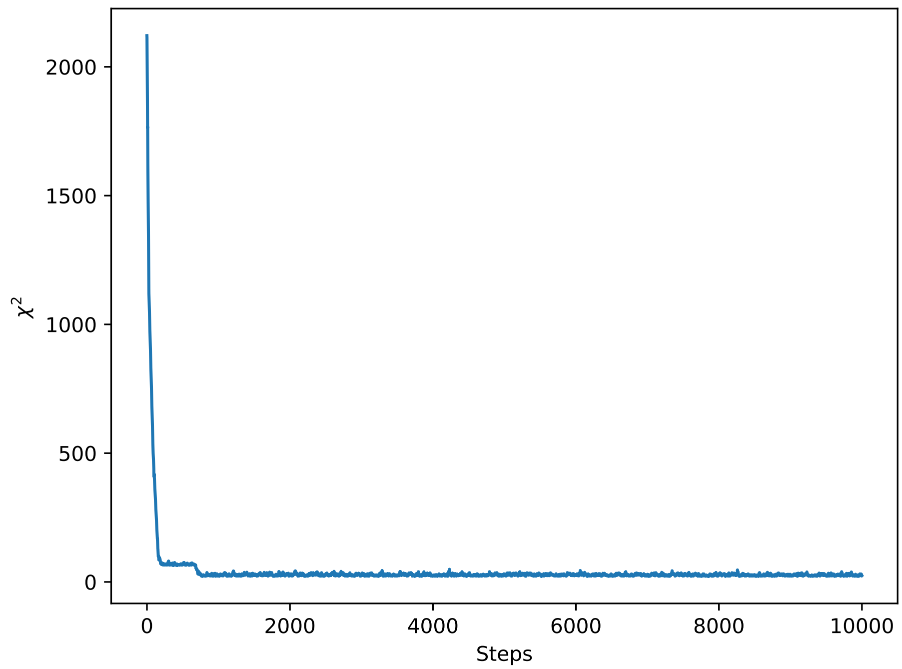

Figure 3: Plot of χ^2 as a function of steps.

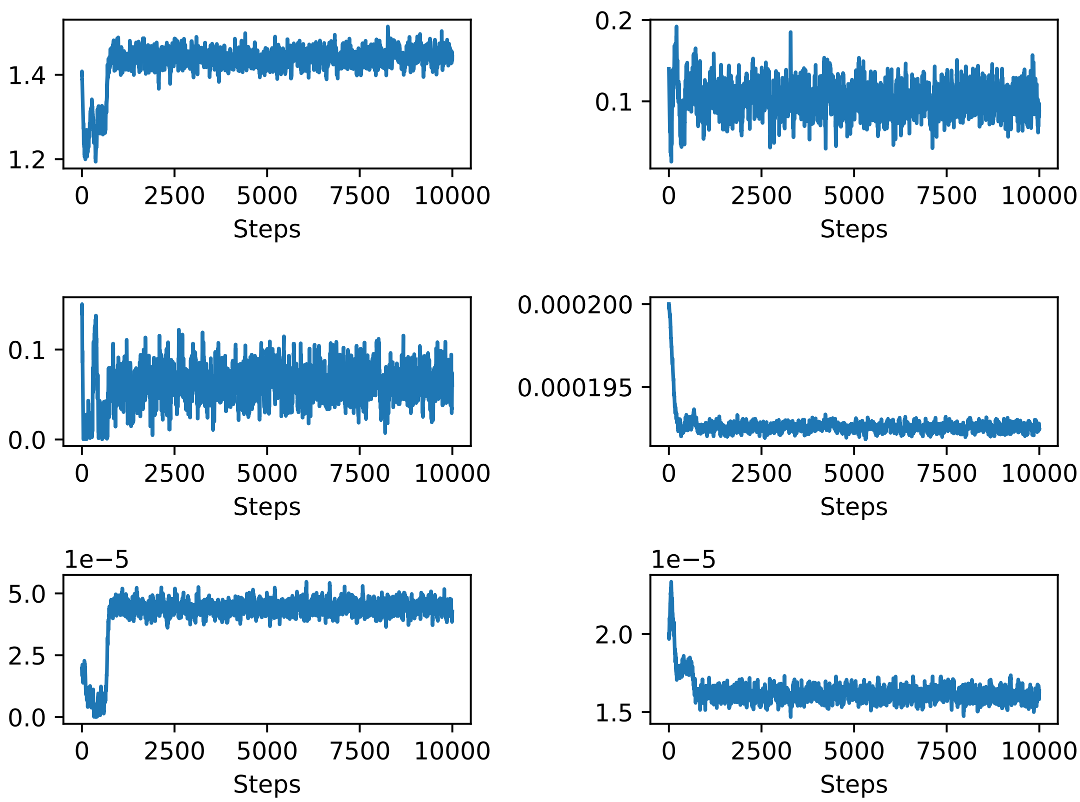

Figure 4: Plot of the chain of MCMC for each parameters. The sub-plots shows the chain of $a, b, c, t_0, \mathrm{d}t, w$ respectively.

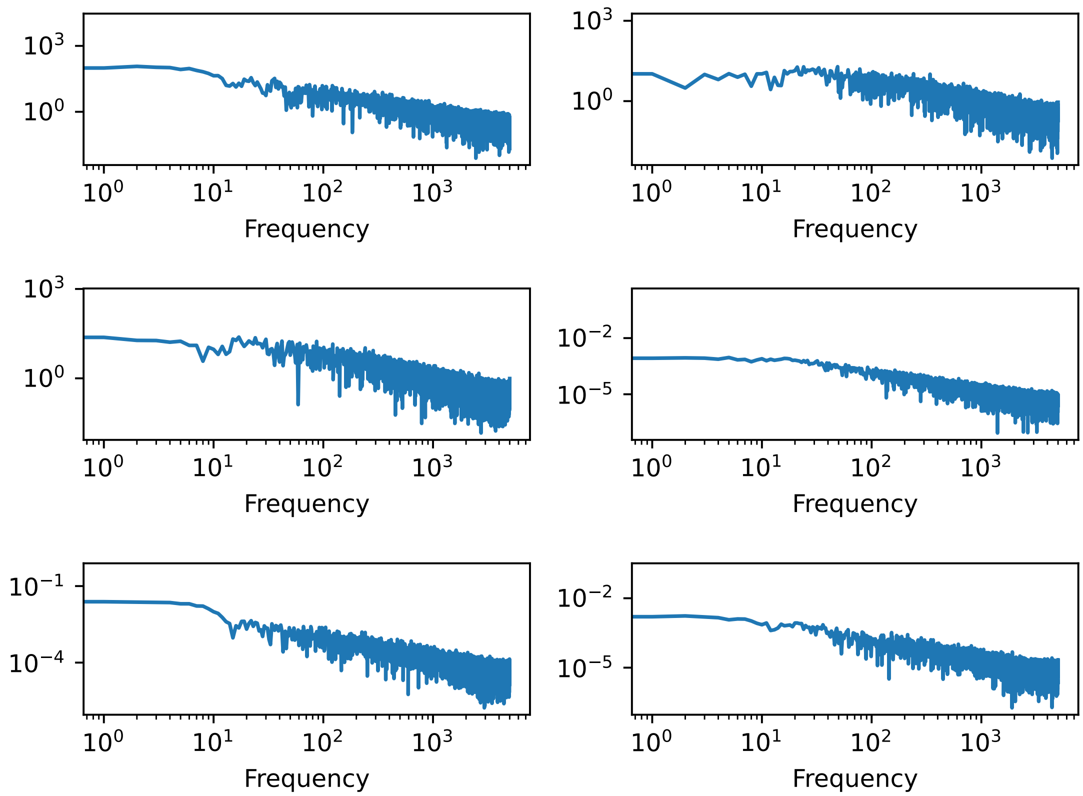

Figure 5: Plot of the FFT transformed chain of MCMC for each parameters. The sub-plots shows the FFT of $a, b, c, t_0, \mathrm{d}t\,, w$ respectively.

h) Based on the given 9 Ghz separation, we have

$$\frac{\mathrm{d}t}{9\,\mathrm{GHz}} = \frac{w}{w_{\mathrm{real}}}$$

then

$$w_{\mathrm{real}} = \frac{w \cdot 9\,\mathrm{GHz}}{\mathrm{d}t}.$$

and its uncertainty can be obtained via error propagation

$$\sigma_{w_{\mathrm{real}}} = w_{\mathrm{real}}\sqrt{\left(\frac{\sigma_w}{w}\right)^2 + \left(\frac{\sigma_{\mathrm{d}t}}{\mathrm{d}t}\right)^2}$$

assuming no uncertainties in the given 9 GHz. The result computed using Python is

```
Real width is 3.449306642459153
with error 0.7859461228037559
```

in units of GHz.