T2

$$\begin{aligned}E[N(t)N(t+s)] &= E[N(t)(N(t+s)+N(t)-N(t))]\\ &= E[N(t)(N(t+s)-N(t))]+E[N(t)^2]\\ &= E[N(t)-N(0)]E[N(t+s)-N(t)]+Var[N(t)]+(E[N(t)])^2\\ &= \lambda t\times\lambda s+\lambda t+(\lambda t)^2\\ &= \lambda t(\lambda(t+s)+1)\end{aligned}$$

T4

(i)

$$P\{N(1)\leq 2\}=\sum_{k=0}^{2}P(N(1)-N(0)=k)=e^{-2}(\frac{2^0}{0!}+\frac{2^1}{1!}+\frac{2^2}{2!})$$

(ii)

$$P(N(1)=1,\ N(2)=3)=P(N(1)-N(0)=1,N(2)-N(1)=2)=P(N(1)-N(0)=1)P(N(2)-N(1)=2)=\frac{2^1}{1!}e^{-2}\times\frac{2^2}{2!}e^{-2}$$

(iii)

$$P(N(1)\geq 1,\ N(1)\geq 2)=P(N(1)\geq 2)=1-P(N(1)-N(0)=0)-P(N(1)-N(0)=1)$$

$$P(N(1)\geq 1)=1-P(N(1)-N(0)=0)$$

$$P(N(1)\geq 1\mid N(1)\geq 2)=\frac{P(N(1)\geq 1,\ N(1)\geq 2)}{P(N(1)\geq 1}=1-\frac{P(N(1)-N(0)=1)}{1-P(N(1)-N(0)=0)}=1-\frac{\frac{2^1}{1!}e^{-2}}{1-\frac{2^0}{0!}e^{-2}}$$