ICV21_Assignment #1

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Problem 1. 2D Fourier Transform of Images

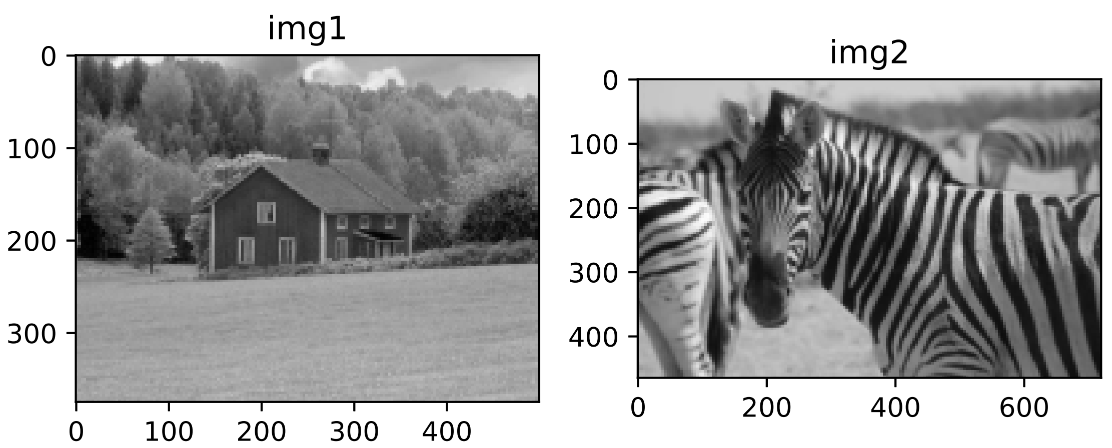
In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import cv2
```

1. load images img1, and img2 with 'imread'

In [2]:
```
img1 = cv2.imread('data/img1.jpg', cv2.IMREAD_GRAYSCALE)
img2 = cv2.imread('data/img2.jpg', cv2.IMREAD_GRAYSCALE)
```

1. Display the images

In [3]:
```
fig, ax = plt.subplots(1, 2)

ax[0].imshow(img1, cmap='gray')
ax[1].imshow(img2, cmap='gray')
ax[0].set_title("img1")
ax[1].set_title("img2")

fig.tight_layout()
plt.show()
```



1. Next resize the img2 to be the same size as the img1

In [4]:
```
img2 = cv2.resize(img2, dsize=(img1.shape[1],img1.shape[0]))
```

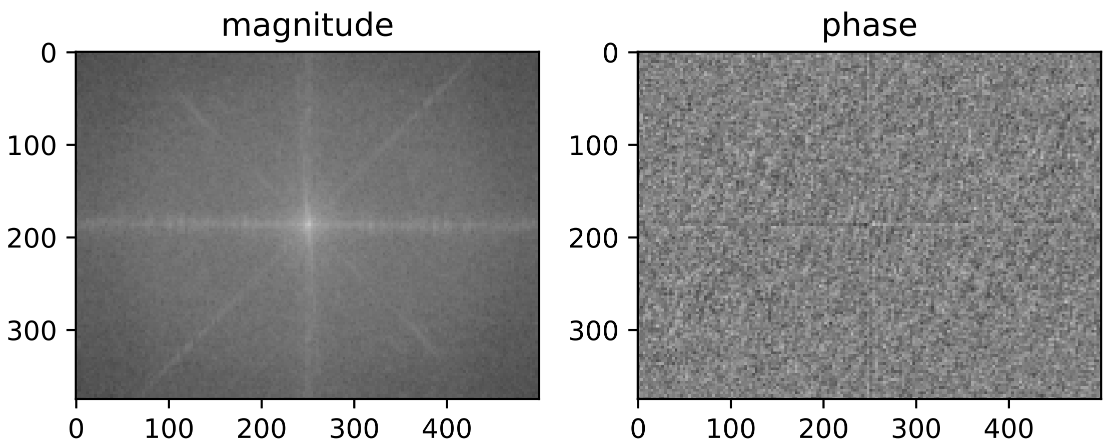
1. Perform 2D DFT on img1, and img2

In [5]:
```
img1_fft = np.fft.fftshift(np.fft.fft2(img1))
img2_fft = np.fft.fftshift(np.fft.fft2(img2))
```

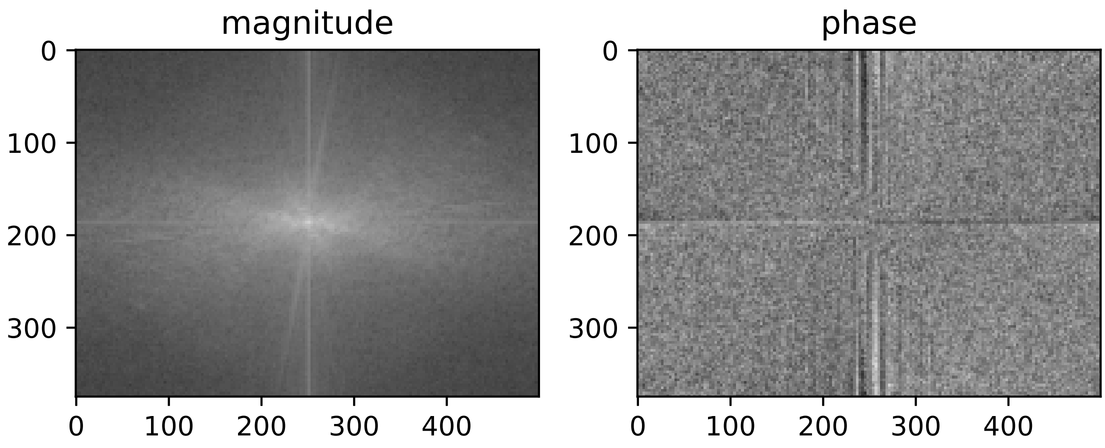
1. Display magnitude and phase of eache image

In [6]:
```
img1_mag = np.abs(img1_fft)
img1_phase = np.angle(img1_fft)

fig, ax = plt.subplots(1, 2)
ax[0].imshow(20*np.log(img1_mag), cmap='gray')
ax[1].imshow(img1_phase, cmap='gray')
ax[0].set_title("magnitude")
ax[1].set_title("phase")
fig.tight_layout()

print("img1")
plt.show()

img2_mag = np.abs(img2_fft)
img2_phase = np.angle(img2_fft)

fig, ax = plt.subplots(1, 2)
ax[0].imshow(20*np.log(img2_mag), cmap='gray')
ax[1].imshow(img2_phase, cmap='gray')
ax[0].set_title("magnitude")
ax[1].set_title("phase")
fig.tight_layout()

print("img2")
plt.show()
```

img1



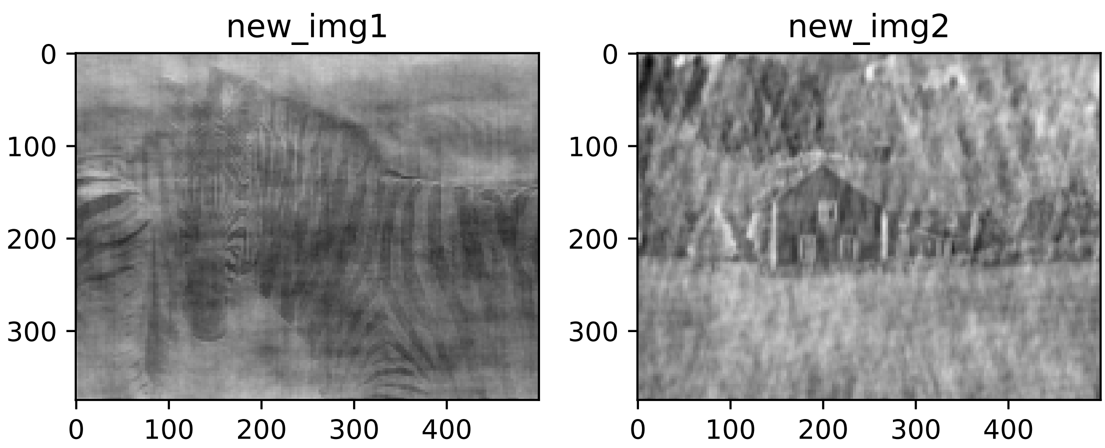
img2



1. Synthesize new images new_img1 and new_img2 by combining img1_mag + img2_phase and img2_mag + img1_phase, respectively

In [7]:

```
new_img1 = np.real(np.fft.ifft2(np.fft.ifftshift(np.multiply(img1_mag,
np.exp(1j*img2_phase)))))

new_img2 = np.real(np.fft.ifft2(np.fft.ifftshift(np.multiply(img2_mag,
np.exp(1j*img1_phase)))))

fig, ax = plt.subplots(1, 2)
ax[0].imshow(new_img1, cmap='gray')
ax[1].imshow(new_img2, cmap='gray')
ax[0].set_title("new_img1")
ax[1].set_title("new_img2")
fig.tight_layout()

plt.show()
```



Problem 1. Discussion

By looking new_img1 and new_img2, we can see that both images look similar with the original image which has the same phase.

It is because phase has the information of the contour of the images.

Problem 2. Perspective Image Transforms

In [8]:
```
import numpy as np
import matplotlib.pyplot as plt
import cv2
```

1. Perform the 2D projective transforms defined by the following 4 matrices for img1

1) Read img1

In [9]:
```
img1 = cv2.imread('data/img1.jpg')
img1 = cv2.cvtColor(img1, cv2.COLOR_BGR2GRAY)
```

2) Given M_transposes

In [10]:
```
M_T_list = [np.array([[1.6322, 0, 0],
                      [0.2120, 1.6336, 0.0013],
                      [-101.9757, -0.6322, 1.0000]]),

            np.array([[1.4219, 0.3183, 0.0013],
                      [0, 1.4206, 0],
                      [-0.4206, -101.8704, 1.0000]]),

            np.array([[0.7033, -0.2239, -0.0009],
                      [0, 0.9991, 0],
                      [0.2958, 0.2239, 1.0000]]),

            np.array([[1.1044, -0.3493, 0.0003],
                      [0.0011, 1.5066, 0.0011],
                      [-0.1041, 0.1560, 1.0000]])
            ]
```

3) function to get projection image by given image and M matrix used equation below

$[p\ q\ r]^T = \mathbf{M}\,[x\ y\ 1]^T$, and $u = p/r$ and $v = q/r$.

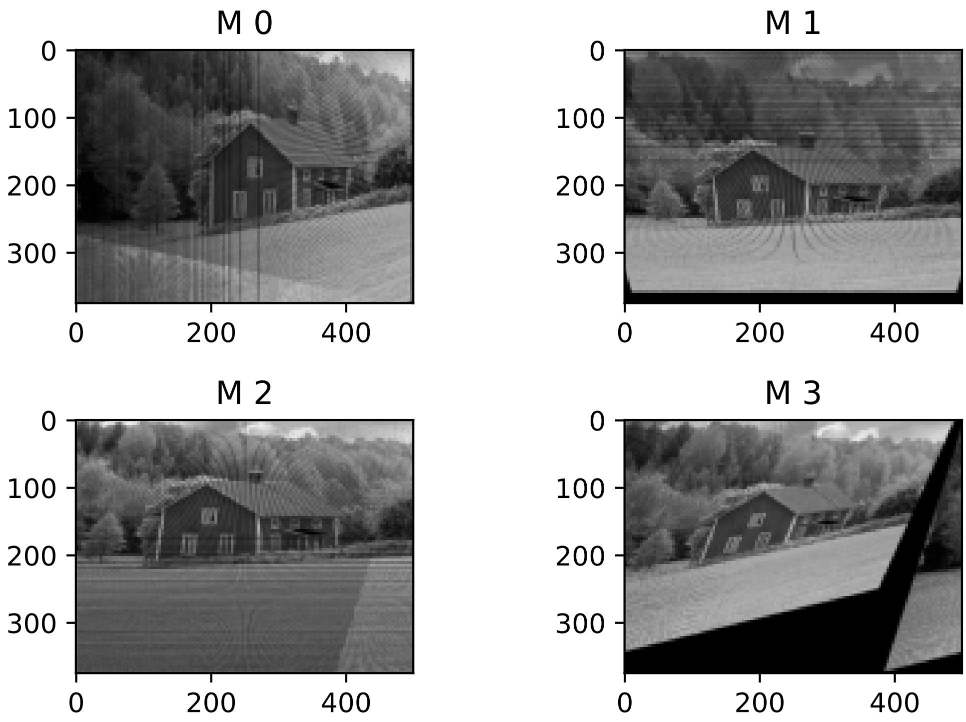
In [11]:
```
def get_proj_img(img, M):
    proj_img = np.zeros(img1.shape)
    for i in range(img.shape[0]):
        for j in range(img.shape[1]):
            p, q, r = np.matmul(M, np.array([i,j,1]))
            u = p/r
            v = q/r
            if u <= img.shape[0] and v <= img.shape[1]:
```

```
                proj_img[int(u)][int(v)] = img[i][j]

    return proj_img
```

4) get projection images and save as projected{i}.png file

(image fie will be saved in directory projected_imgs/)

In [22]:
```
fig, ax = plt.subplots(2, 2)

for i, M_T in enumerate(M_T_list):
    proj_img = get_proj_img(img1, np.transpose(M_T))
    ax[i//2, i%2].set_title('M {}'.format(i))
    ax[i//2, i%2].imshow(proj_img, cmap='gray')
    plt.imsave('projected_imgs/projected{}.png'.format(i+1), proj_img,
cmap='gray')

fig.tight_layout()
plt.show()
```



Problem 2. Discussion

using the equation, projection image seems to be well made

Problem 3. Camera Calibration

In [13]:
```
import numpy as np
import matplotlib.pyplot as plt
import cv2
```

3.a) Show that the solution p is the last column of V, corresponding to the smallest eigenvalue of A.

$$p = min_p(||Ap||),\ A = UDV^T$$

$$||Ap|| = p^TA^TAp = p^TVD^TU^TUDV^Tp = p^TVD^2V^Tp = p^T\begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}p$$

$$p\ can\ be\ decomposed\ into\ \ p = \Sigma c_i v_i$$

$$since\ ||p|| = 1, \quad following\ is\ true\ \Sigma c_i^2 = 1$$

$$p^T\begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}p = \Sigma c_i^2\lambda_i^2$$

$$min_p(||Ap||) = min_p(\Sigma c_i^2\lambda_i^2)$$

$$In\ np.linalg.svd(), \quad \lambda_i > 0\ \ and\ \ (\lambda_1 > \lambda_2 > \cdots > \lambda_n)$$

$$So,\ \lambda_n\ is\ smallest\ eigenvalue\ and\ p\ minimizing\ the\ expression\ ||Ap||\ is\ as\ below$$

$$c_n = 1, p = v_n$$

3.b) Given the following correspond points, determine the camera projection matric P using the SVD method.

1) Given (u,v) and (x,y,z)

In [14]:

```
uv = np.array([[880,214],[43,203],[270,197],[886,347],[745,302],
[943,128],
               [476,590],[419,214],[317,335],[783,521],[235,427],
[665,429],[655,362],[427,333],
               [412,415],[746,351],[434,415],[525,234],[716,308],
[602,187]],dtype=np.float32)

XYZ = np.array([[312.747,309.14,30.086],[305.796,311.649,30.356],
                    [307.694,312.358,30.418],[310.149,307.186,29.298],
[311.937,310.105,29.216],
                    [311.202,307.572,30.682],[307.106,306.876,28.66],
                    [309.317,312.49,30.23],[307.435,310.151,29.318],
                    [308.253,306.3,28.881],[306.65,309.301,28.905],
                    [308.069,306.831,29.189],[309.671,308.834,29.029],
                    [308.255,309.955,29.267],[307.546,308.613,28.963],
                    [311.036,309.206,28.913],[307.518,308.175,29.069],
                    [309.95,311.262,29.99],[312.16,310.772,29.08],
[311.988,312.709,30.514]],dtype=np.float32)
```

2) make A by Given equation below and Given (u,v) and (x,y,z)

$$m_{11}X_i + m_{12}Y_i + m_{13}Z_i + m_{14} - m_{31}u_iX_i - m_{32}u_iY_i - m_{33}u_iZ_i - m_{34}u_i = 0$$
$$m_{21}X_i + m_{22}Y_i + m_{23}Z_i + m_{24} - m_{31}v_iX_i - m_{32}v_iY_i - m_{33}v_iZ_i - m_{34}v_i = 0$$

In [15]:

```
A = np.zeros((2*len(uv),12))

for i in range(len(uv)):
    u_i, v_i = uv[i]
    X_i, Y_i, Z_i = XYZ[i]
    A[2*i] = np.array([X_i, Y_i, Z_i, 1, 0, 0, 0, 0, -u_i*X_i, -u_i*Y_i,
-u_i*Z_i, -u_i])
    A[2*i+1] = np.array([0, 0, 0, 0, X_i, Y_i, Z_i, 1, -v_i*X_i, -
v_i*Y_i, -v_i*Z_i, -v_i])
```

3) use SVD method to decompose A, then last column of V is p

In [16]:

```
u, s, vh = np.linalg.svd(A, full_matrices=True)
p = vh[-1]
p /= p[-1]
```

4) reshape vector p into matrix P

In [17]:

```
P = np.reshape(p, (3,4))
print(P)
```

```
[[-2.33261286e+00 -1.10020234e-01  3.37490343e-01  7.36686704e+02]
 [-2.31041064e-01 -4.79515779e-01  2.08721751e+00  1.53626639e+02]
 [-1.26376391e-03 -2.06774463e-03  5.14662103e-04  1.00000000e+00]]
```

3.c) In equation in b), set m_34 = 1, get equation Ap = b, then use pseudo inverse to get the least square solution p

1) make b, A with Given equation below p is size 11 (m_34 is 1 so will be added later)

$$m_{11}X_i + m_{12}Y_i + m_{13}Z_i + m_{14} - m_{31}u_iX_i - m_{32}u_iY_i - m_{33}u_iZ_i = u_i$$
$$m_{21}X_i + m_{22}Y_i + m_{23}Z_i + m_{24} - m_{31}v_iX_i - m_{32}v_iY_i - m_{33}v_iZ_i = v_i$$

In [18]:

```
b = np.zeros((2*len(uv),))

for i in range(len(uv)):
    u_i, v_i = uv[i]
    b[2*i] = u_i
    b[2*i+1] = v_i

A = np.zeros((2*len(uv),11))

for i in range(len(uv)):
```

```
    u_i, v_i = uv[i]
    X_i, Y_i, Z_i = XYZ[i]
    A[2*i] = np.array([X_i, Y_i, Z_i, 1, 0, 0, 0, 0, -u_i*X_i, -u_i*Y_i,
-u_i*Z_i])
    A[2*i+1] = np.array([0, 0, 0, 0, X_i, Y_i, Z_i, 1, -v_i*X_i, -
v_i*Y_i, -v_i*Z_i])
```

2) get pseudo inverse of A

In [19]:
```
A_T = np.transpose(A)
A_pseudo_inv = np.matmul(np.linalg.inv(np.matmul(A_T, A)), A_T)
```

3) use equation below to get p

$$\mathbf{p} = \mathbf{A}^{+}\mathbf{b}.$$

In [20]:
```
p = np.matmul(A_pseudo_inv, b)
```

4) reshape vector p into matrix P

$$(add\ m_{34} = 1\ in\ matrix\ P)$$

In [21]:
```
P = np.resize(p, (3,4))
P[-1][-1] = 1
print(P)
```

```
[[-2.33259421e+00 -1.09988242e-01  3.37390954e-01  7.36674050e+02]
 [-2.31047155e-01 -4.79506733e-01  2.08717179e+00  1.53627131e+02]
 [-1.26378942e-03 -2.06771124e-03  5.14584938e-04  1.00000000e+00]]
```

Problem 3. Discussion

If we have world coordinates and projection coordinates, we can get projection matrix.