

Spring School on Cluster Algebras (SSCA) 2025

Date February 24(Mon)--26(Wed), 2025

Venue Nagoya University, Graduate School of Mathematics Building A, Room A207

Organizers Osamu Iyama, Yoshiyuki Kimura, Tomoki Nakanishi (chief), Hironori Oya

2/24(Mon)

13:00-14:30 Stella, Cluster scattering diagrams of affine type (1)

15:00-16:30 Lee, Scattering diagrams, tight gradings, and generalized positivity (1)

2/25(Tue)

9:00-10:30 Stella, Cluster scattering diagrams of affine type (2)

10:45-11:45 Martello, Painlevé VI, symmetries, and clusters

13:30-15:00 Lee, Scattering diagrams, tight gradings, and generalized positivity (2)

15:15-16:15 Karuo, Azumaya representations of generalized skein algebras

2/26(Wed)

10:00-11:30 Stella, Cluster scattering diagrams of affine type (3)

13:00-14:30 Lee, Scattering diagrams, tight gradings, and generalized positivity (3)

14:45-15:45 Chen, A cluster theory approach from mutation invariants to Diophantine equations

Salvatore Stella (Università Degli Studi Della'Aquila)

Title Cluster scattering diagrams of affine type

Abstract Cluster scattering diagrams, since their introduction, played a central role in shaping the structure theory of cluster algebras. They consist of a combinatorial datum, a fan, together with the assignment of a formal power series for each of its codimension-1 cones. While their recursive definition is in theory explicit, constructing cluster scattering diagrams is usually a difficult task. In this lecture series we will address this problem in the special case of acyclic cluster algebras of affine type where we can leverage previous constructions built using the machinery of root systems, Coxeter groups, lattice theory to relate cluster scattering diagrams to two other fans: the mutation fan and the fan of almost-positive roots.

Kyungyong Lee (The University of Alabama)

Title Scattering diagrams, tight gradings, and generalized positivity

Abstract These lectures will be based on joint work with Amanda Burcroff and Lang Mou. I plan to

explain a recent result showing that the coefficients of the wall-functions on a generalized cluster scattering diagram of any rank are positive, which implies the Laurent positivity for generalized cluster algebras and the strong positivity of their theta bases. Several known arguments allow us to reduce the general case to the rank 2 case. We will introduce generalized rank 2 cluster algebras and their greedy bases/theta bases. A number of formulas for these bases will be presented. Every statement will be accompanied by very explicit examples. After we discuss bases for generalized rank 2 cluster algebras, we will introduce a new class of combinatorial objects which we call tight gradings. Using this, we give a directly computable, manifestly positive, and elementary but highly nontrivial formula describing rank 2 consistent scattering diagrams. In trying to make my lectures completely elementary, pictorial descriptions will be given whenever possible.

Zhichao Chen (University of Science and Technology of China/Nagoya University)

Title A cluster theory approach from mutation invariants to Diophantine equations

Abstract In this talk, we define and classify the sign-equivalent exchange matrices. We give a Diophantine explanation for the differences between rank 2 cluster algebras of finite type and infinite type. Then, we classify the positive integer points of the Markov mutation invariant and its variant. As an application, several classes of Diophantine equations with cluster algebraic structures are exhibited. This is a joint work with Zixu Li.

Hiroaki Karuo (Gakushuin University)

Title Azumaya representations of generalized skein algebras

Abstract Some generalizations of skein algebras relate to (quantum) cluster algebras of surface type. To study their algebraic structures, understanding their representations is basic and important. For non-commutative algebras, the unicity theorem claims that we can access to the representation theory using their centers. In this talk, I will explain some properties of generalized skein algebras related to the unicity theorem and share some future works.

Davide Dal Martello (Rikkyo University)

Title Painlevé VI, symmetries, and clusters

Abstract The sixth Painlevé equation (PVI) admits a native $\mathfrak{sl}_2(\mathbb{C})$ -Fuchsian representation. Taking advantage of a higher Teichmüller coordinatization for the corresponding monodromy group, we give Okamoto's symmetry of PVI a realization on the representation space in the language of cluster \mathcal{X} -mutations. The explicit mutation formula admits dual characterizations in geometric terms of colored associahedra and star-shaped fat graphs, expanding the cluster state of the art for PVI.