

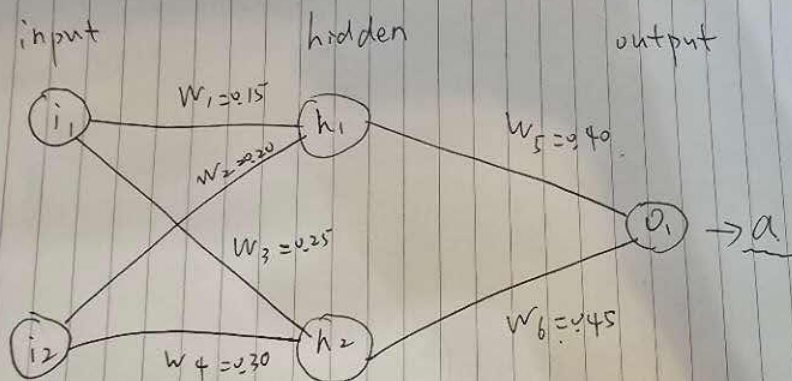
神经网络结构: $\begin{cases} 2 \text{ 个输入层} \\ 1 \text{ 个隐藏层} \\ 1 \text{ 个输出层} \end{cases}$

激活函数 $\rightarrow \text{sigmoid } f(x) = \frac{1}{1+e^{-x}}$
损失函数 $\rightarrow E = \frac{1}{2} (\text{target} - \text{output})^2$

参数: 隐藏层: $W^H = \begin{pmatrix} w_1 & w_2 \\ w_3 & w_4 \end{pmatrix} = \begin{pmatrix} 0.15 & 0.20 \\ 0.25 & 0.30 \end{pmatrix}$ $b_1 = 0.35$
输出层: $W^O = (w_5, w_6) = (0.40, 0.45)$ $b_2 = 0.60$

训练数据 (batch size = 2) sample 1: $X_1 = [0.05, 0.10]$ $y_1 = 0.01$
sample 2: $X_2 = [0.80, 0.20]$ $y_2 = 0.99$

超参数: 学习率: ~~0.5~~ $\eta = 0.5$



step 1: 前向传播 $x_1 = [0.05, 0.10] \leftarrow$ 样本 1

$$z_{h1} = w_1 \cdot i_1 + w_2 \cdot i_2 + b_1 = 0.15 \times 0.05 + 0.2 \times 0.10 + 0.35 = 0.375$$

接着计算 sigmoid $\rightarrow a_{h1} = \text{sigmoid}(z_{h1}) = \frac{1}{1+e^{-0.375}} = 0.59326$

$$z_{h2} = w_3 \cdot i_1 + w_4 \cdot i_2 + b_1 = (0.25 \times 0.05) + (0.30 \times 0.10) + 0.35 = 0.3925$$

sigmoid $\rightarrow a_{h2} = \text{sigmoid}(z_{h2}) = \frac{1}{1+e^{-0.3925}} = 0.59688$

↑
样本 1 隐藏层

输出层: $z_o = w_5 \cdot a_{h1} + w_6 \cdot a_{h2} + b_2 = (0.40 \times 0.59326) + (0.45 \times 0.59688) + 0.60 = 1.10590$

sigmoid: $\frac{1}{1+e^{-1.1059}} = 0.75136$

所以样本一误差是 $\rightarrow \frac{1}{2}(0.01 - 0.75136)^2 = 0.27483$

我们还要计算出样本二的误差, 然后相加算出总误差

样本二误差算出来是 0.02692 \rightarrow 省略计算

总误差: $\frac{0.27483 + 0.02692}{2} = 0.150875$

step 2: 反向传播

回顾 $w_5 \rightarrow z_o \rightarrow a_o \rightarrow E$

根据链式法则 $\frac{\partial E}{\partial w_5} = \frac{\partial E}{\partial a_o} \times \frac{\partial a_o}{\partial z_o} \times \frac{\partial z_o}{\partial w_5}$

核心!!!

对 $\frac{1}{2}(y_1 - a_o)^2$ 求导 $E_1 = \frac{1}{2}(y_1 - a_o)^2 \rightarrow (0.75136 - 0.01)$

$\frac{\partial E_1}{\partial w_5} = 0.74136 \times \frac{\partial z_o}{\partial w_5}$

$= 0.74136$

$$\frac{\partial E_1}{\partial z_{01}} = \frac{\partial E_1}{\partial a_{01}} \cdot \frac{z_{001}}{\partial z_{01}} = 0.74136 \cdot 0.75136 (1 - 0.75136) = 0.13849$$

样本2 同理 $\rightarrow -0.04267$

\downarrow

$$\begin{aligned} \frac{\partial E_{total}}{\partial w_5} &= \frac{1}{2} (\delta_{01} \cdot a_{11} + \delta_{01} \cdot a_{11}) \\ &= \frac{1}{2} (0.13849 \cdot 0.593626 + (-0.04267) \cdot 0.62983) \\ &= 0.02777 \end{aligned}$$

w_6 同理 $\rightarrow 0.02755$

\downarrow

$$\frac{\partial E_{total}}{\partial b_2} = \frac{1}{2} (0.13849 - 0.04267) = 0.04791$$

一直往前

直到算出更新

的权重

最终算出

更新的 w_1, w_2, w_3, w_4 和 b_1