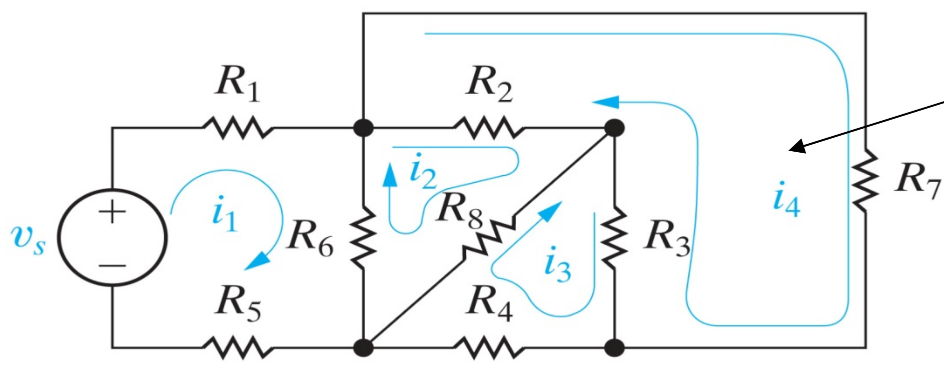
ELEN 50 Class 14 – Mesh Current Method Examples and Supermeshes

S. Hudgens

To Review

- The mesh current method is another approach to circuit analysis.
- We've seen that the node voltage method allows analysis of a circuit using $n_e - 1$ equations where n_e is the number of essential nodes in the circuit.
- The mesh current technique allows analysis of a circuit with $b_e - (n_e - 1)$ equations ….where b_e is the number of essential branches in the circuit. Remember, an <u>essential branch is a path that connects two essential nodes without passing through an essential node.</u>
- $b_e - (n_e - 1)$ is the number of meshes in the circuit.
- Remember, <u>a mesh is a loop drawn in the circuit so that there are no other loops inside of it</u> – and also remember that the mesh current approach <u>only works on planar circuits</u>.



The mesh currents are shown here

- The mesh current method is completely complementary to the node voltage method …but it uses mesh currents instead of node voltages….and it explicitly uses KVL instead of KCL.

Node Voltage Method

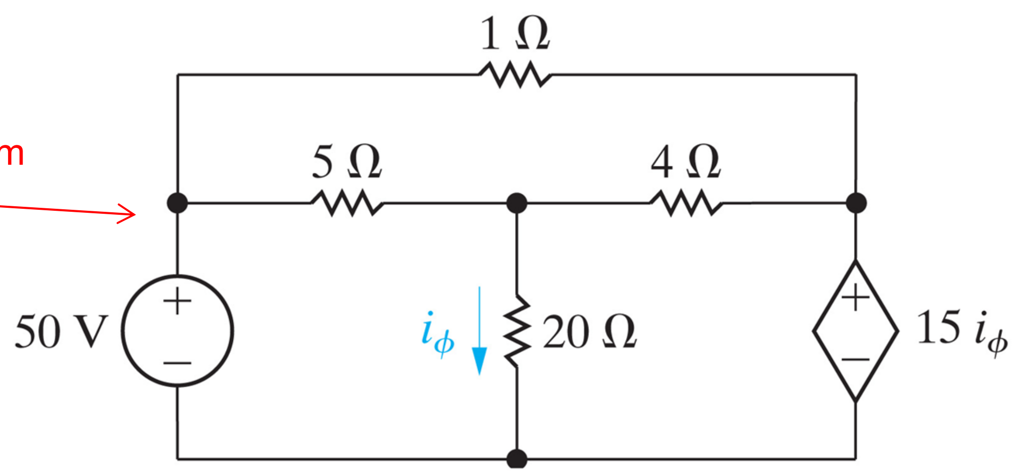
- Identify essential nodes
- Select reference node
- Label voltages at remaining essential nodes (v1, v2, ….vn)
- Write equations for KCL <u>at these nodes</u> in terms of node voltages referenced to reference node.
- Solve n equations in n unknowns

Mesh Current Method

- Identify mesh currents
- Reference node is not needed
- Label mesh currents (ia, ib, …in)
- Write equations for KVL <u>around the mesh current paths.</u>
- Solve n equations in n unknowns

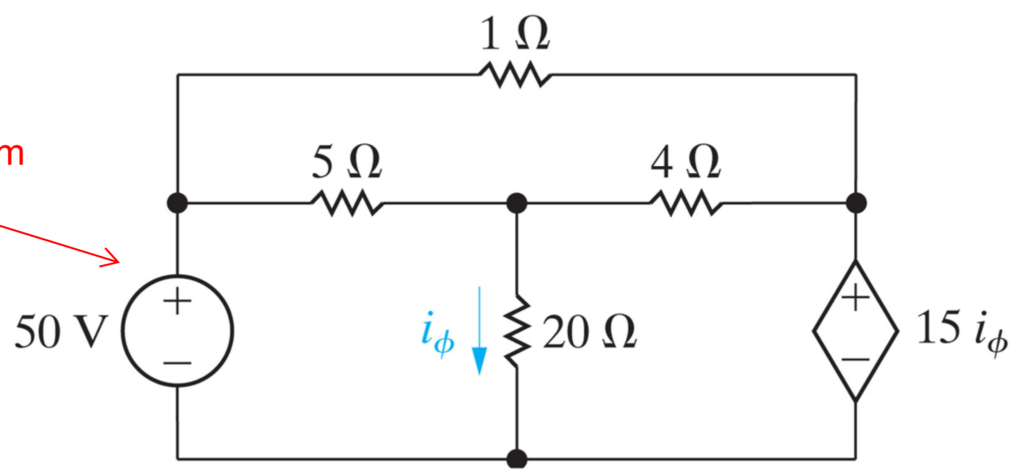
Here's another mesh current example problem – this one has a dependent voltage source …we looked at a mesh current solution with a dependent current source last time:

Source transform here?



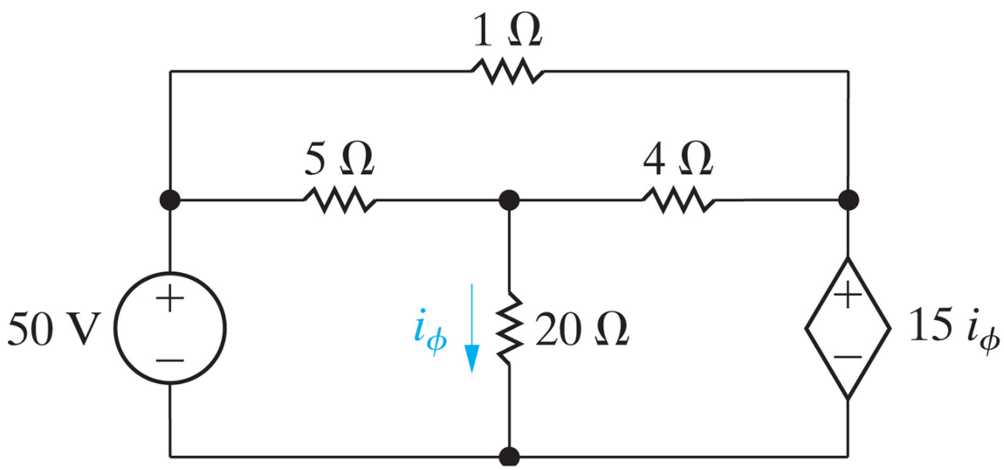
The problem asks for the power dissipated in the 4Ω resistor. Do you see any possible circuit simplifications?

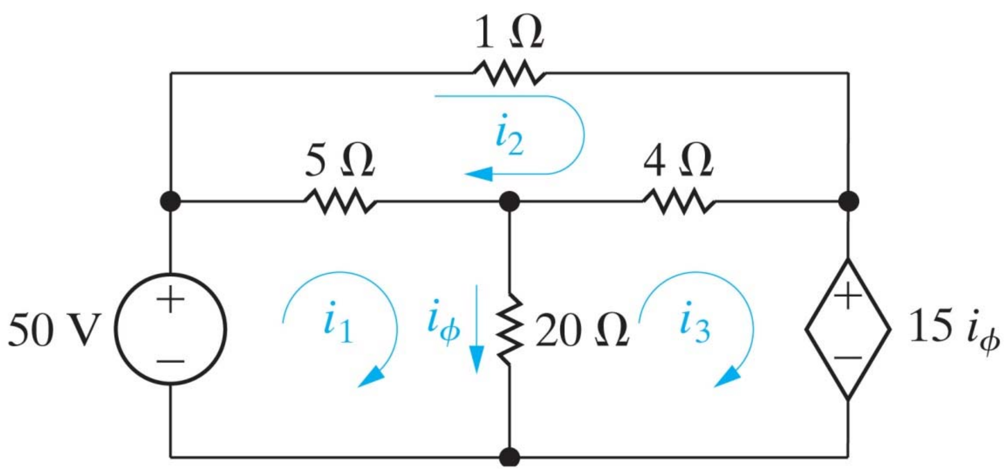
Source transform here?



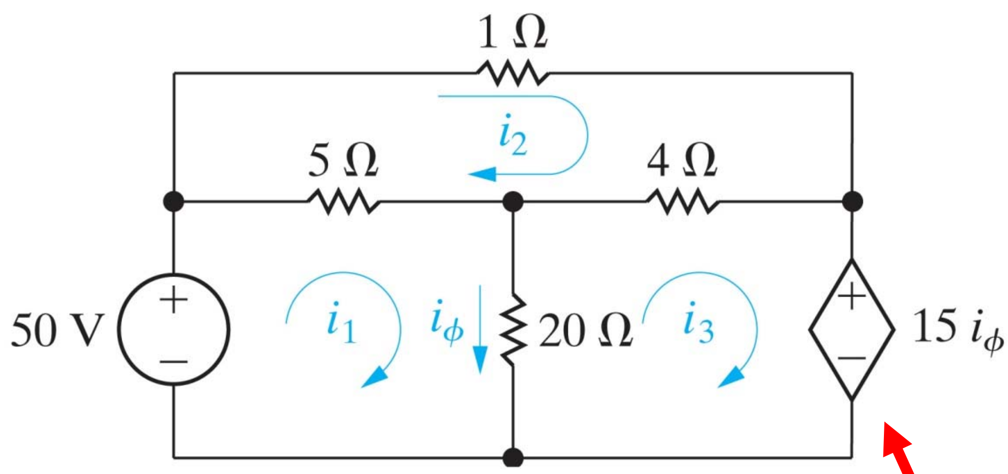
No! Don't do it! Trying to transform the 50V source and the 5Ω resistor into a current source in parallel with a resistor won't take into account current flowing through the 1Ω resistor. We saw an example of this kind of mistake earlier!

No simplifications are possible.





The first step, as always, is to identify and label the mesh currents – there are three of them, i1, i2, and i3. Notice -- none of these are the branch current, iϕ, that the problem asks for.



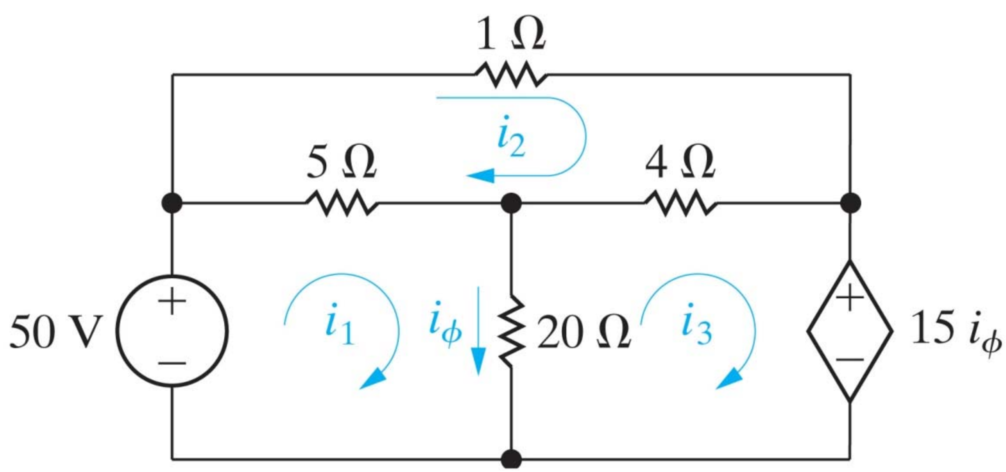
Now we write the mesh current equations using KVL.....<u>we can't do a solution by inspection because a dependent source is present</u>

- 50 + (i1 – i2) 5 + (i1 –i3) 20 = 0

(i2 –i1) 5 + i2 1 + (i2 – i3) 4 = 0

(i3 –i1) 20 + (i3 –i2) 4 + 15 iϕ = 0

and, of course, the supplemental equation: iϕ = i1 – i3 we always need this when working with dependent sources or we don't have enough equations for all of the unknowns!



Using the supplemental equation to eliminate iϕ in the mesh equations:

50 = 25 i1 – 5 i2 – 20 i3

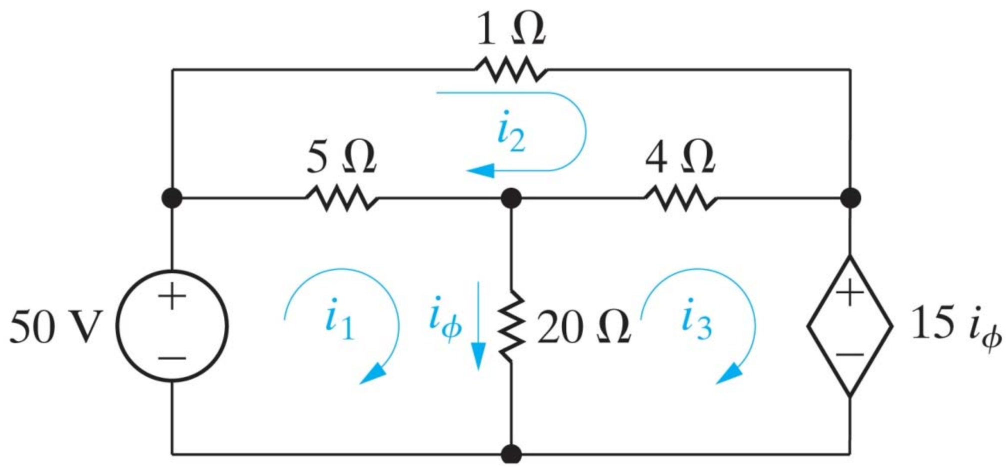
0 = -5 i1 + 10 i2 – 4 i3

0 = -5 i1 – 4 i2 + 9 i3

$$\begin{bmatrix} 25 & -5 & -20 \\ -5 & 10 & -4 \\ -5 & -4 & 9 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix}$$

Notice that the matrix isn’t symmetric -- because of the dependent voltage source.

Since the problem is to calculate the power in the 4Ω resistor, we need i2 and i3



Solving the three equations in three unknowns we can get:

I1 = 29.6A, I2 = 26A, and i3 = 28A …

So the power in the 4Ω resistor is 4 $(i3 - i2)^2$ = 16W

Again, we see that the matrix equations for this circuit didn't have that nice symmetrical appearance we had noted before when we solved other circuits by the mesh current method….i.e. .R13 is not equal to R31 !

50 = 25 i1 – 5 i2 – 20 i3

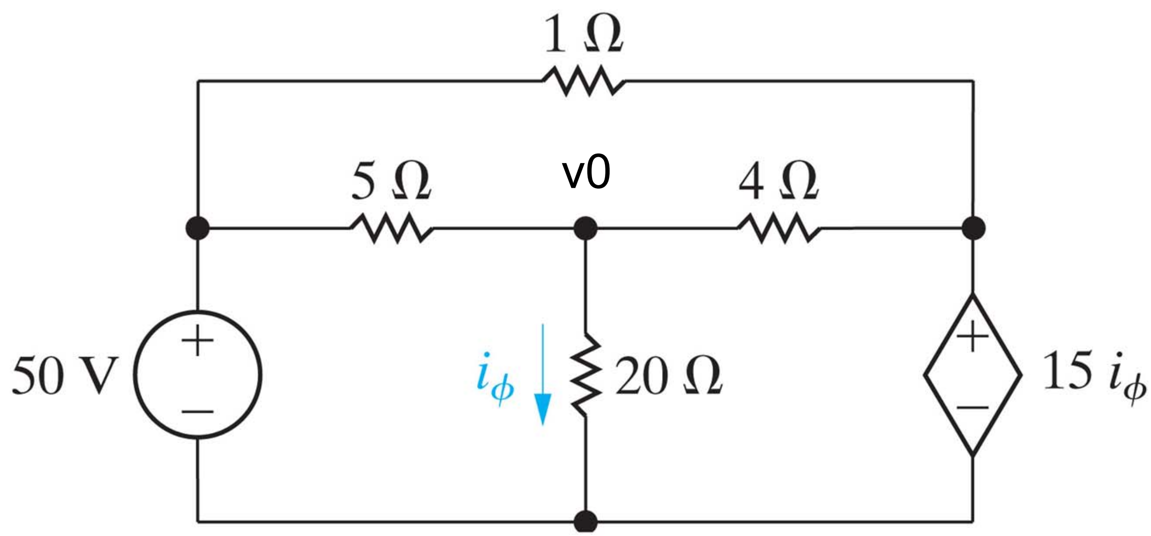
0 = -5 i1 + 10 i2 – 4 i3

0 = -5 i1 – 4 i2 + 9 i3

$$\mathbf{V} = \mathbf{IR}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

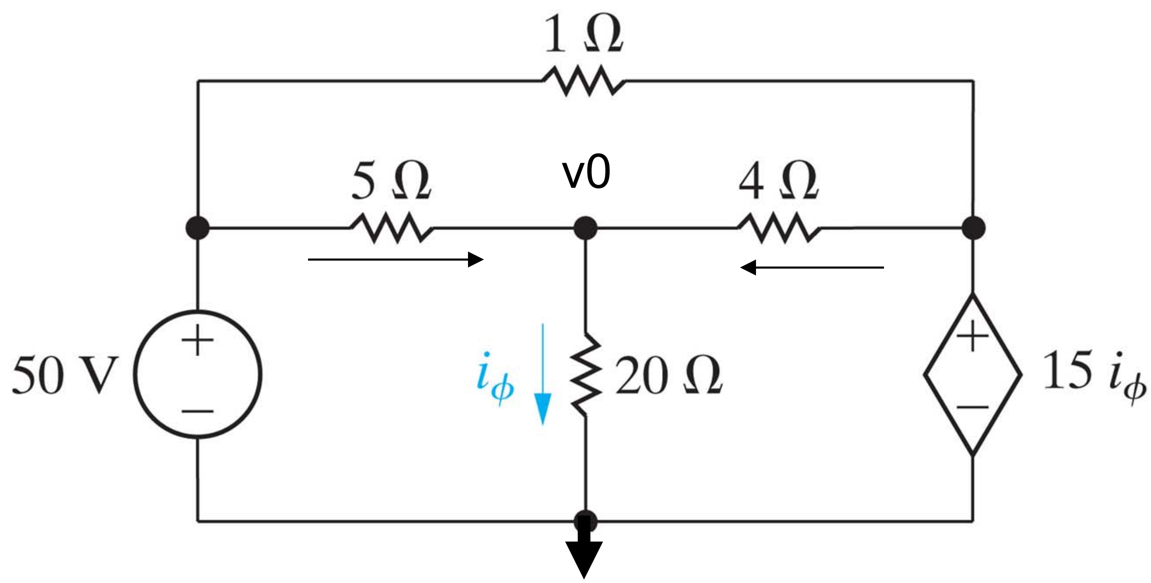
The reason for this is the dependent voltage source. This was a voltage source that got replaced by two current terms when we wrote down the KVL equations.



Let’s look at this circuit again – what if we had analyzed it using the node voltage method?

How many essential nodes are present? -- 4

If we chose the bottom node as a reference node, we can see that the two side nodes are separated in voltage by two voltage sources (quasi-supernodes) , so the only unknown voltage in the circuit is the one labeled v0. <u>We can solve this circuit with a single equation (plus the supplementary equation)!</u>



Here's the node equation:

$- (50 – v0)/5 + v0/20 – (15\ i\phi – vo)/4 = 0$

And $i\phi = vo/20$ is the supplemental equation

Substituting in the node equation:

$- (50 – v0)/5 + v0/20 – (15v0/20 – v0)/4 = 0$

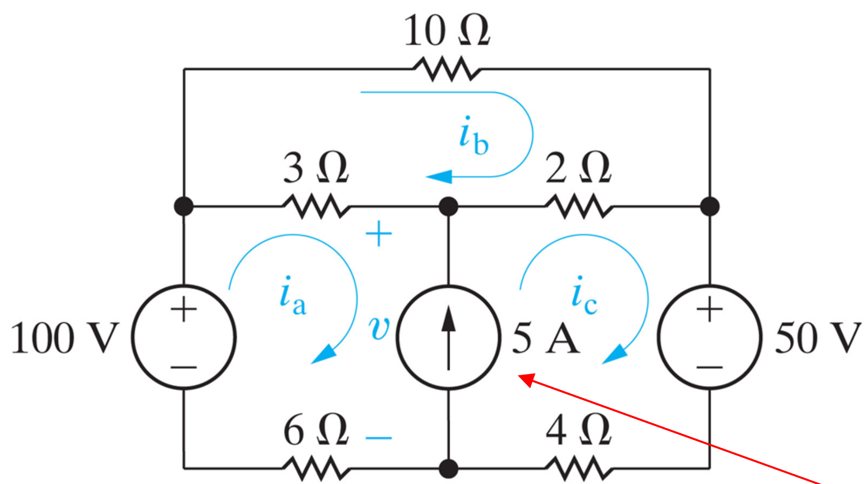
v0 = 32V …so $i\phi$ = 1.6A and the power in the 4Ω resistor is:

$P = (v0 – 15\ i\phi)^2\ 4\Omega = (32 - 24)^2 /4 = 16W$ …the answer we got before but with many fewer calculations.

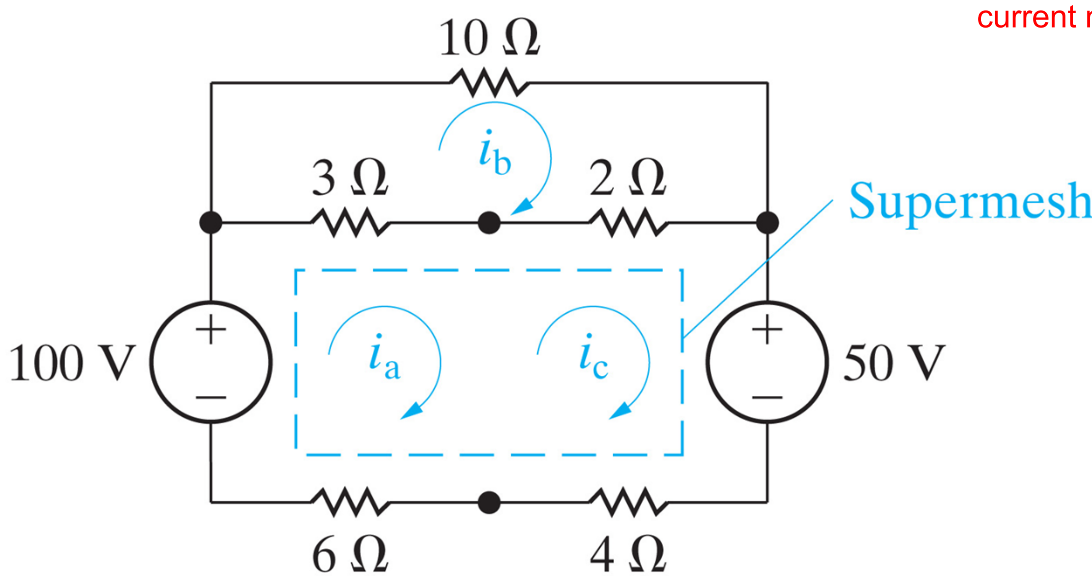
Are there any circuits where the mesh current method is preferable?

The concept of a supermesh can eliminate one of the mesh current equations …just like a supernode eliminated one of the node voltage equations.

Remember, a supernode occurred when two essential nodes were <u>connected by a voltage source</u> (dependent or independent). In a complementary fashion, a supermesh occurs when two mesh currents are <u>separated by a current source</u>.



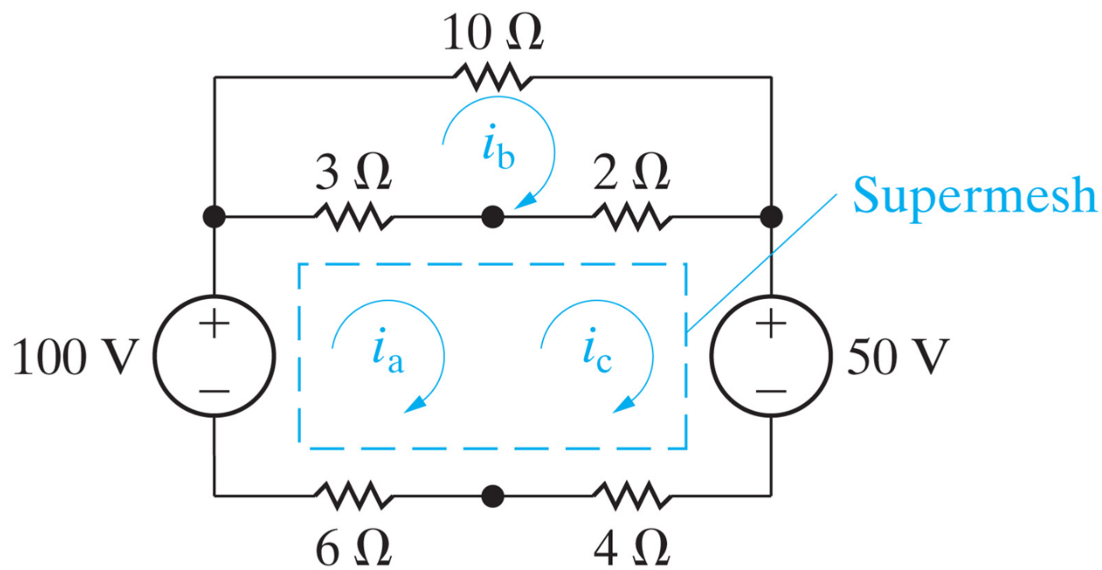
Why does this shared current source cause a problem for the mesh current method?



The motivation to use a supermesh is exactly the same as the motivation to use a supernode when doing a node voltage solution.

In the node voltage solution method we need to use KCL – to express the current into and out of a node in terms of current sources and the current through resistors connected to other nodes (with unknown voltages). When two nodes are connected by a voltage source, we don’t know how to write the current between the two nodes …so we use the supernode concept.

In the mesh current solution we need to use KVL to express the voltage drops encountered while going around a mesh. If we have two adjacent meshes containing a current source, we don’t know how to write the voltage drop across the shared current source …so we use the supermesh concept.



We write a combined mesh equation for both loops of the supermesh as:

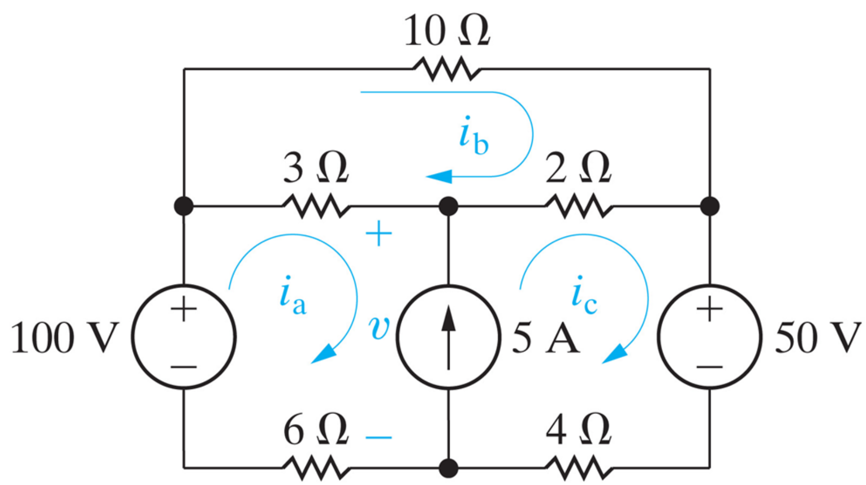
- 100 + 3 (ia – ib) + 2 (ic –ib) + 50 + 4 ic + 6 ia = 0

Notice, in writing the supermesh equation, ic is "flowing" only on the right side of the supermesh and ia is "flowing" on the left side of the supermesh.

For the upper mesh,

3 (ib-ia) + 10 ib + 2 (ib –ic) = 0

<u>This is completely analogous to the idea of a supernode where we wrote a combined KCL equation for both nodes of the supernode.</u>



What happened to the 5A source????

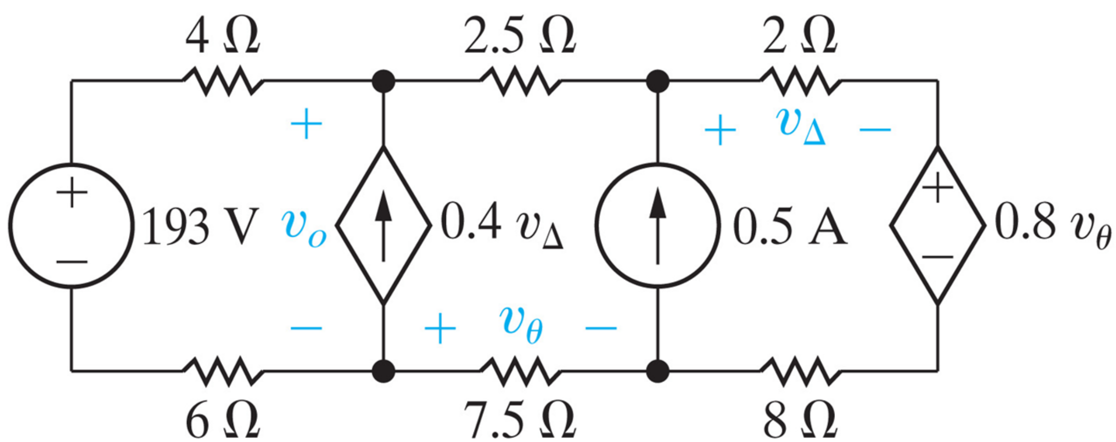
- 100 + 3 (ia – ib) + 2 (ic –ib) + 50 + 4 ic + 6 ia = 0

3 (ib-ia) + 10 ib + 2 (ib –ic) = 0

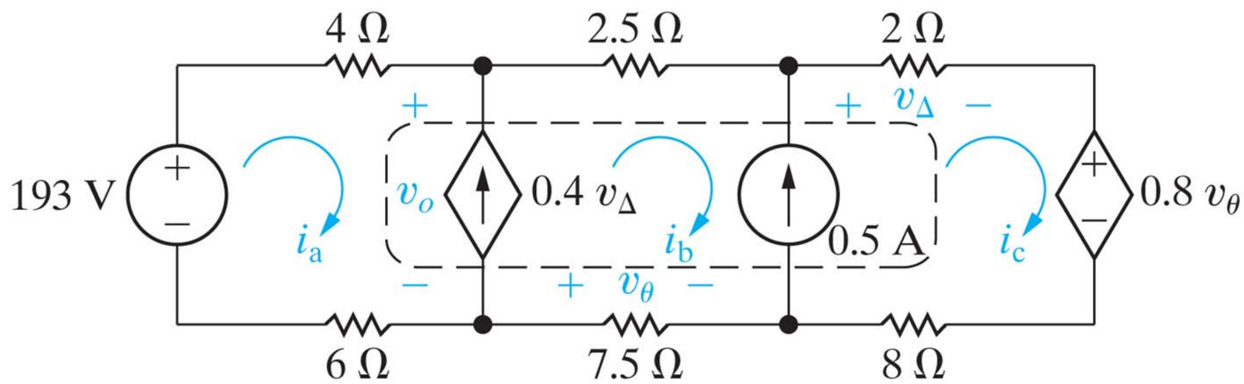
Now we know from the original circuit that ic – ia = 5A, so we can eliminate ic in the two mesh equations and solve two equations in two unknowns.

Notice, if we'd done this by the node voltage method, we would have needed three equations – here the mesh current approach wins. (finally!!!)

Here's another circuit – which should we use – node voltage or mesh current? We want to find v0. The circuit has 2 voltage sources and 2 current sources.



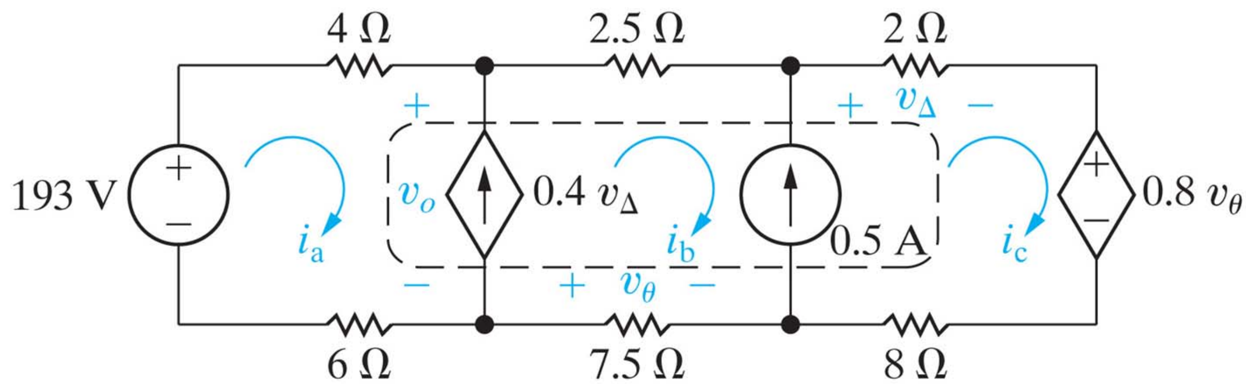
For a node voltage analysis, the circuit has four essential nodes and two dependent sources → 3 node voltage equations and 2 constraint equations.



For a mesh current analysis, there are three mesh currents (as shown), but the presence of two current sources can allow us to create a supermesh….so the analysis requires one supermesh equation and two constraint equations. Here, the current mesh approach is a clear winner!

We'll do the analysis both ways:

First, the mesh current analysis



For the supermesh:

$- 193 + (4 + 6)\ ia + (2.5 + 7.5)\ ib + (2 + 8)\ ic + 0.8\ v\theta = 0$

And the constraint equations:

$ib - ia = 0.4\ v_\Delta$

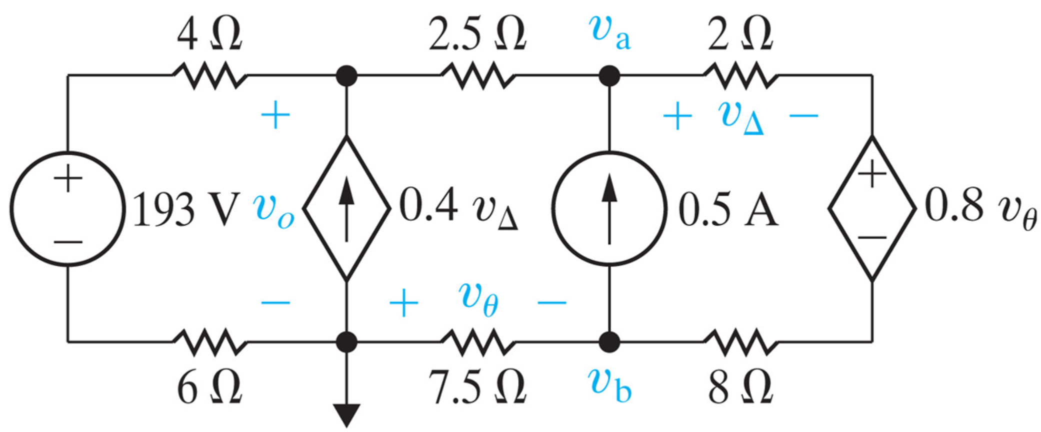
$V\theta = 7.5\ ib$

$ic - ib = 0.5$

Now ib and ic can be eliminated in the supermesh equation:

So 160 = ia → ia = 2A so v0 = 193 – 2(4) – 2(6) = 173V

Now, the node voltage analysis



$$(v0 - 193)/10 - 0.4\, v_\Delta + (v0 - va)/2.5 = 0$$

$$(va - v0)/2.5 + 0.5 + [va - (vb + 0.8v\theta)]/10 = 0$$

$$vb/7.5 + 0.5 + (vb + 0.8\, v\theta - va)/10 = 0$$

And the constraint equations are:

$$v\theta = -vb$$

$$v_\Delta = [va - (vb + 0.8\, v\theta)]/10 \;\; * \; 2$$

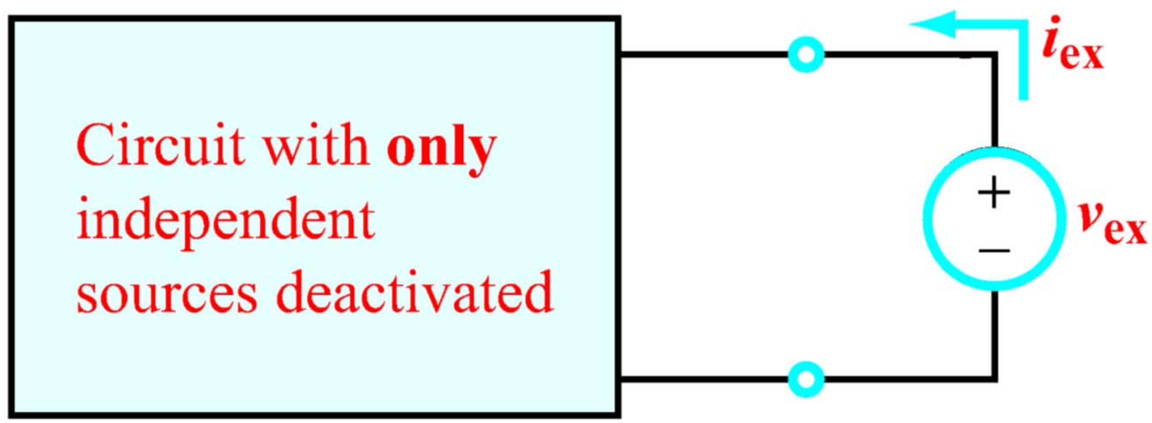
The constraint equations can be used to eliminate $v\theta$ and $v\Delta$ from the node equations so we have three equations in v0, va, and vb. Solving these gives us the value for v0 we got earlier.

So is there a general rule about node voltage versus mesh current? Probably not. Just look at the circuit from both perspectives …and pay particular attention to whether or not you can use supernodes or supermeshes to eliminate equations.

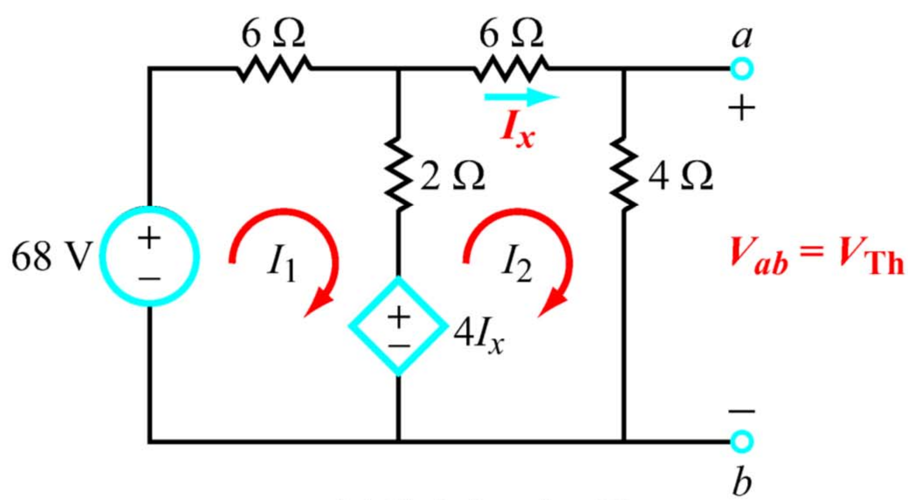
By the way ….we talked about strategies for creating Thevenin equivalent circuits last week, and, in particular, how we would approach a <u>circuit with dependent sources</u>. The technique is called the "external source method" and all of the examples in the textbook were easy to do with the mesh current method.

Now that we know how to use the mesh current approach, let's go back and talk about Thevenin equivalent circuits for systems with dependent sources.

External-Source Method



We want to find the Thevenin equivalent circuit at terminals **a** and **b** for this circuit that contains a dependent voltage source.

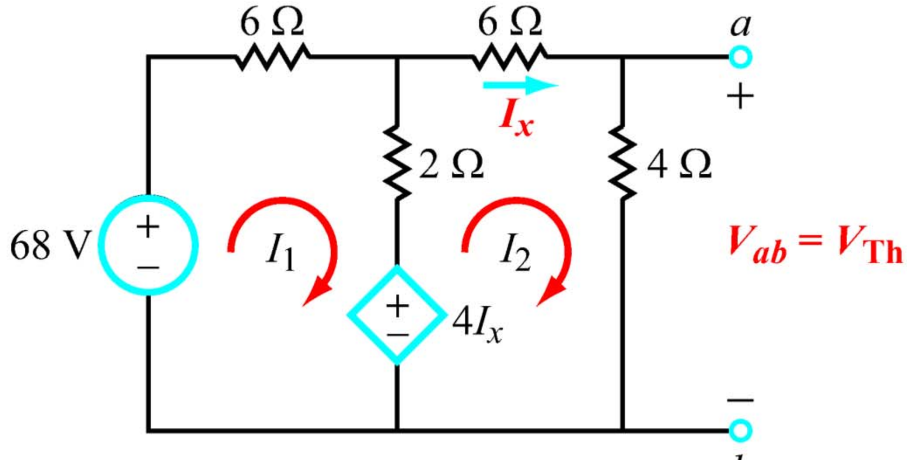


(a) Solving for V_{Th}

First we find Vth = Vab by writing the mesh current equations for I1 and I2

$$-68 + 6I_1 + 2(I_1 - I_2) + 4I_x = 0$$

$$-4I_x + 2(I_2 - I_1) + 6I_2 + 4I_2 = 0$$

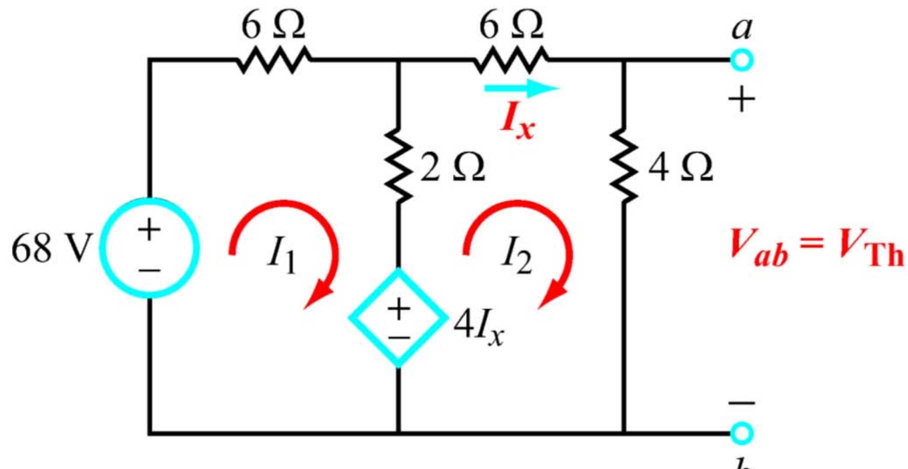


(a) Solving for V_{Th}

Since Ix = I2, we have two equations in two unknowns which we can solve

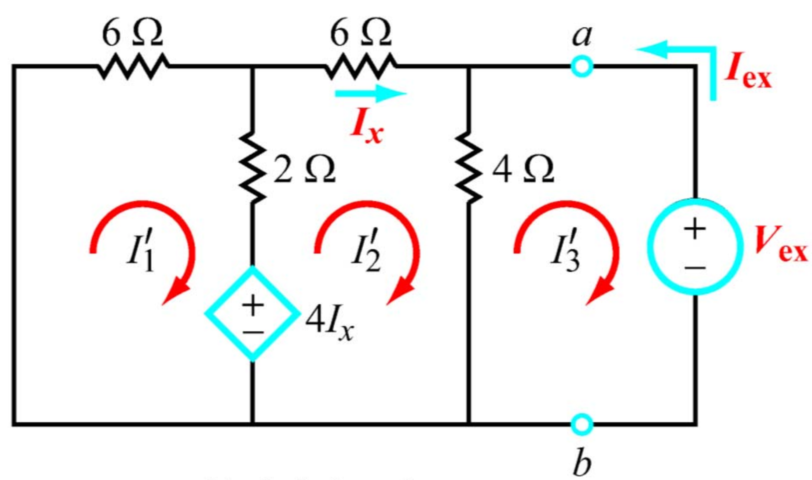
$$-68 + 6I_1 + 2(I_1 - I_2) + 4I_2 = 0$$
$$-4I_2 + 2(I_2 - I_1) + 6I_2 + 4I_2 = 0$$

I_1 = 8A and I_2 = 2A…so Vab = $4I_2$ = 8V = Vth

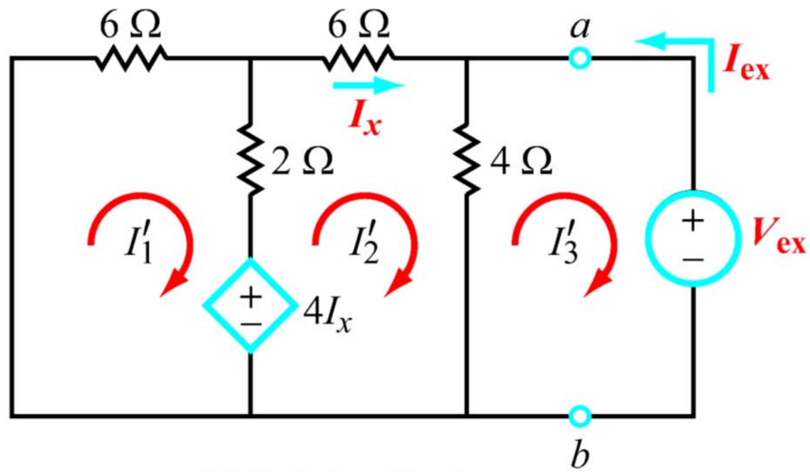


(a) Solving for V_{Th}

Now to find Rth, we deactivate the independent voltage source and attach an external voltage source, Vex. We want an expression for Iex in terms of Vex. Notice that the dependent source is still there – we can't deactivate it and use method #2 to get Rth!



(b) Solving for I_{ex}



(b) Solving for I_{ex}

We write three mesh current equations for I1', I2', and I3'

$$6I_1^{'} + 2(I_1^{'} - I_2^{l}) + 4I_x = 0$$

$$-4I_x + 2(I_2^{'} - I_1^{'}) + 6I_2^{'} + 4(I_2^{'} - I_3^{'}) = 0$$

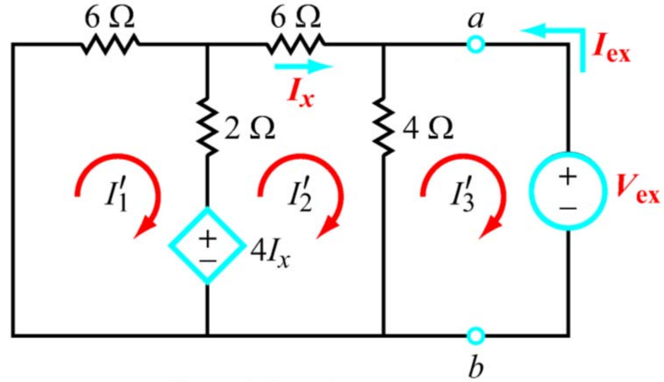
$$4(I_3^{'} - I_2^{'}) + V_{ex} = 0$$

Now, $I_x = I_2'$ so we have three equations in three unknowns. We can solve these in terms of the external voltage, V_{ex}.

$$I_1' = \frac{1}{18} V_{ex}$$

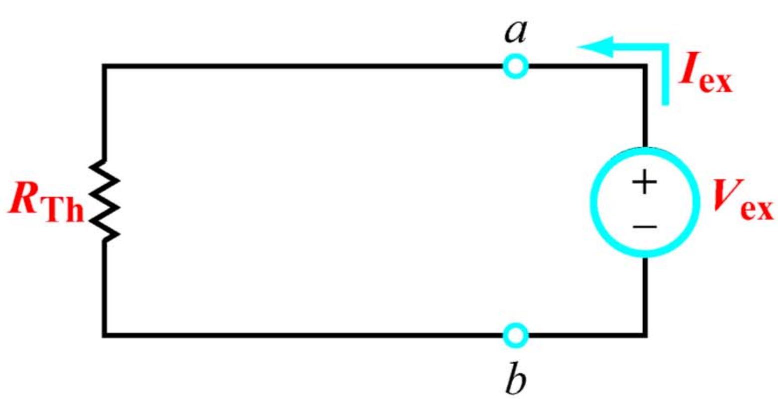
$$I_2' = -\frac{2}{9} V_{ex}$$

$$I_3' = -\frac{17}{36} V_{ex}$$



(b) Solving for I_{ex}

Rth = V_{ex}/I_{ex} and since $I_{ex} = -I_3'$ Rth = $-V_{ex}/I_3' = 36/17\ \Omega$ and Vth = 8V as we calculated earlier.



(c) Equivalent circuit for calculating R_{Th}