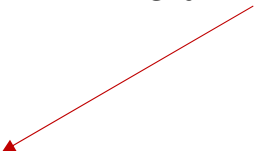ELEN 50: Mid-term #1 Review

S. Hudgens

now……. …a quick review before the mid-term

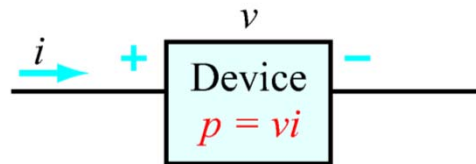Topics that will appear in mid-term problems may include:

1. Kirchoff's Laws and Ohm's Law
2. Equivalent circuits (series, parallel, and wye to delta)
3. Source transforms
4. Node Voltage Analysis (including supernodes and quasi-supernodes)
5. Thevenin and Norton Equivalent Circuits
6. Max Power Transfer

The exam will be 1 hour long and have 5 problems – about the same difficulty or slightly less difficult than the homework problems.

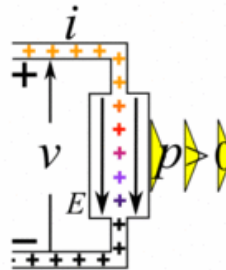The exam is open book, open notes, including access to Camino or the Design Center if you need it.

Passive Sign Convention

$p > 0$ power delivered to device
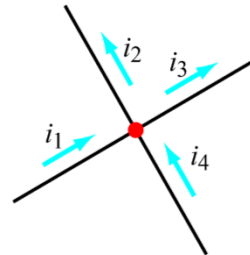$p < 0$ power supplied by device

*Note that i direction is defined as entering (+) side of v.
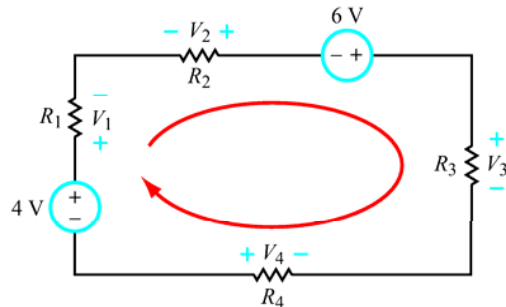
Another way to say it:

If the device is a voltage source, then current comes out of the positive side …if it's a resistor, then current goes into the positive side.

Review of KCL and KVL
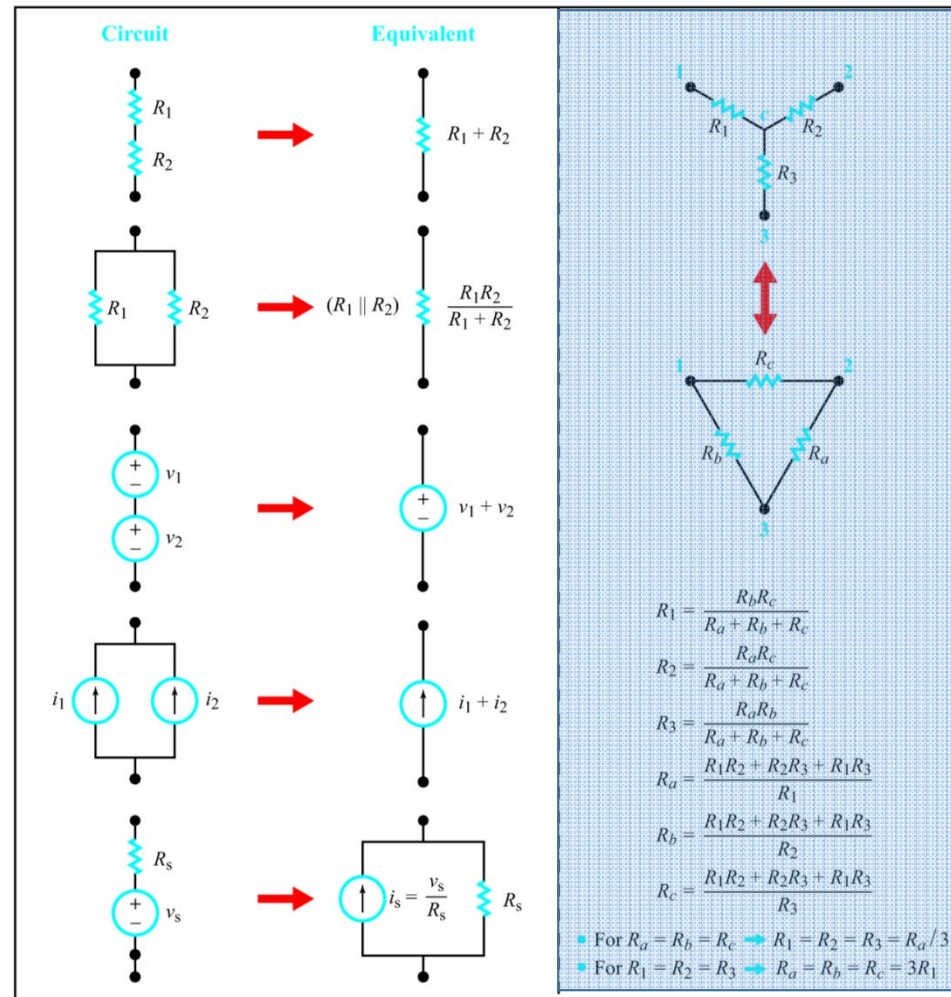

$$I_1 + i_4 = i_2 + i_3$$

Kirchhoff Current Law (KCL) – the total current entering <u>a node</u> must be equal to the total current leaving <u>a node</u> …i.e. charge doesn't "build up" at a node. The sign convention is that current leaving a node is considered positive.
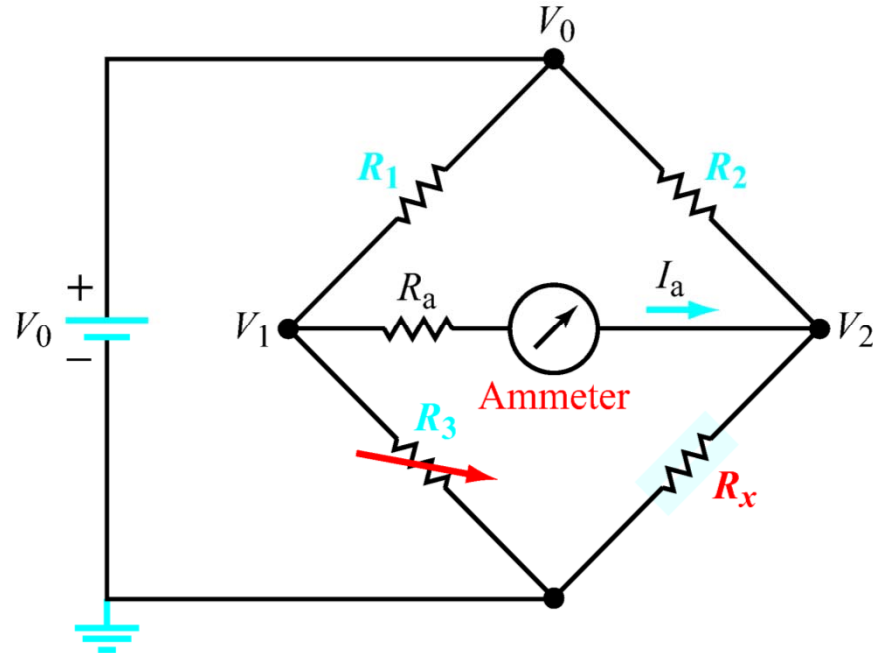

$$-4 + V1 - V2 - 6 + V3 - V4 = 0$$

Kirchhoff Voltage Law (KVL) – the algebraic sum of the voltage drops around a closed <u>loop</u> is zero. The sign convention is that the voltage drop on an element appears with the sign first encountered by the arrow on the loop.
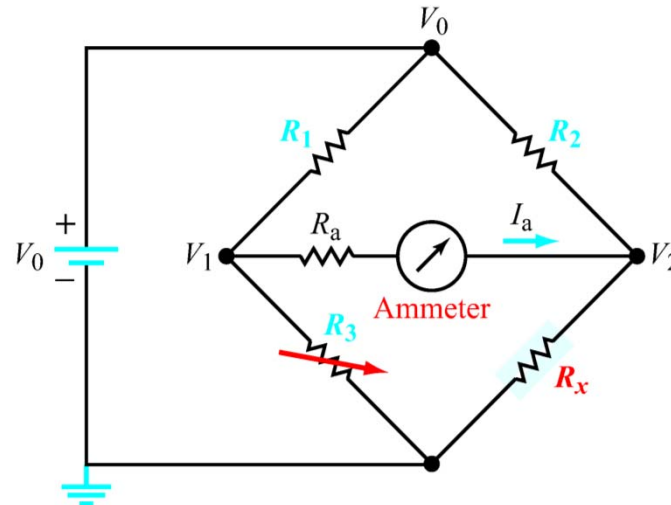
Equivalent Circuits

Won't be on exam

The Wheatstone Bridge

R1 and R2 are known, fixed resistors, R3 is a (known) adjustable resistor, and Rx is the unknown resistor. V0 is any convenient voltage source. R3 is adjusted until Ia is zero – the bridge is in balance. You can solve this without using a Wye to Delta transform …do you see why?

Balanced Bridge

In balance, V1 = V2, so, since:

$$V_1 = R_3 V_0 / (R_1 + R_3)$$

$$V_2 = R_x V_0 / (R_2 + R_x)$$

Both sides are voltage dividers!

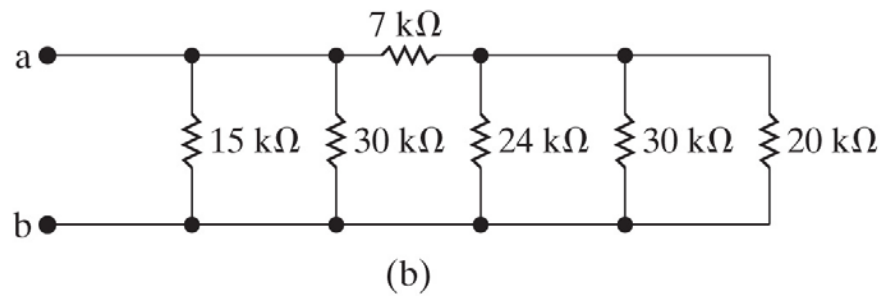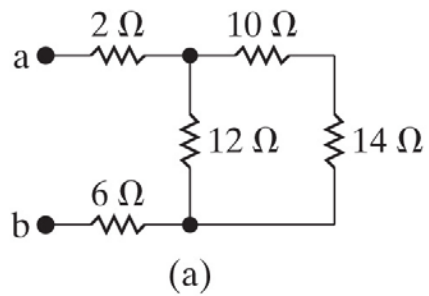$$\frac{R_3 V_0}{R_1 + R_3} = \frac{R_x V_0}{R_2 + R_x}$$

Also, since the voltages across R1 and R2 are equal at balance:
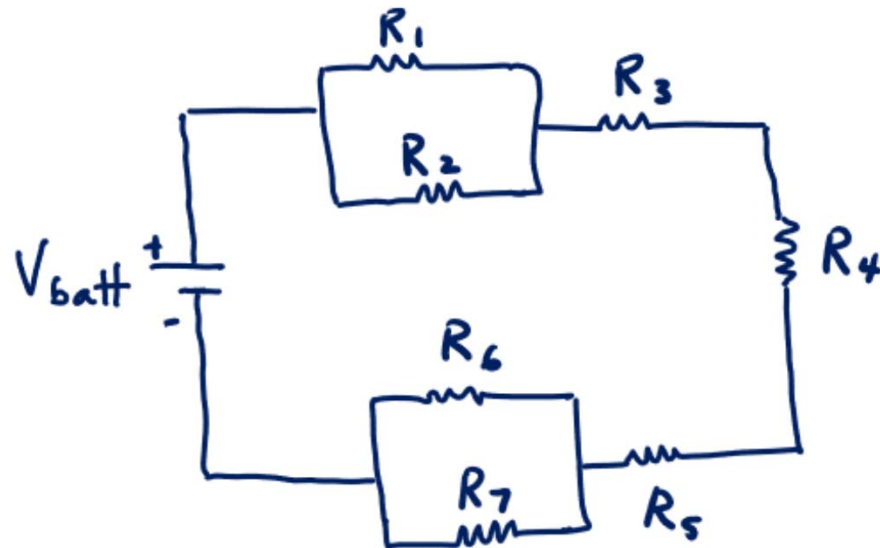
$$\frac{R_1 V_0}{R_1 + R_3} = \frac{R_2 V_0}{R_2 + R_x}$$

So, dividing these equations:

$$\frac{R_3}{R_1} = \frac{R_x}{R_2}$$

7

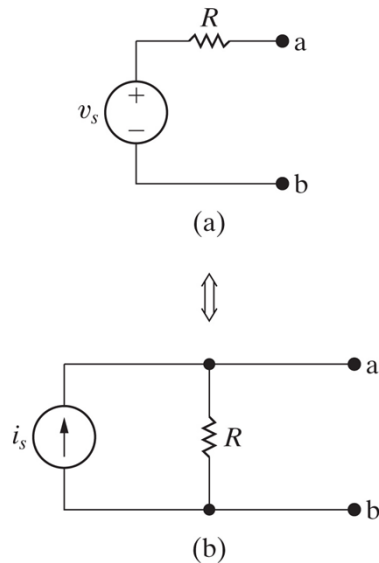Find the equivalent resistance R_{ab} for each of these circuits:

(a)

(b)

Is it clear which of these circuit elements are in series and which are in parallel?

How would you replace all these resistors with a single, equivalent resistor? Which resistors are in parallel and which are in series?

Another important kind of circuit simplification is a Source Transform

A source transform allows a voltage source in series with a resistor to be transformed into a current source in parallel with a resistor. This can be very useful in simplifying a circuit prior to doing a circuit analysis.
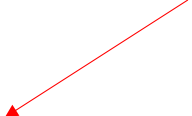

(a)

(b)

these two circuits are equivalent, if:

$$i_s = v_s/R$$

You can show this relationship by attaching a load resistor to a and b and calculating the current flowing

10

Formal Methods of Circuit Analysis
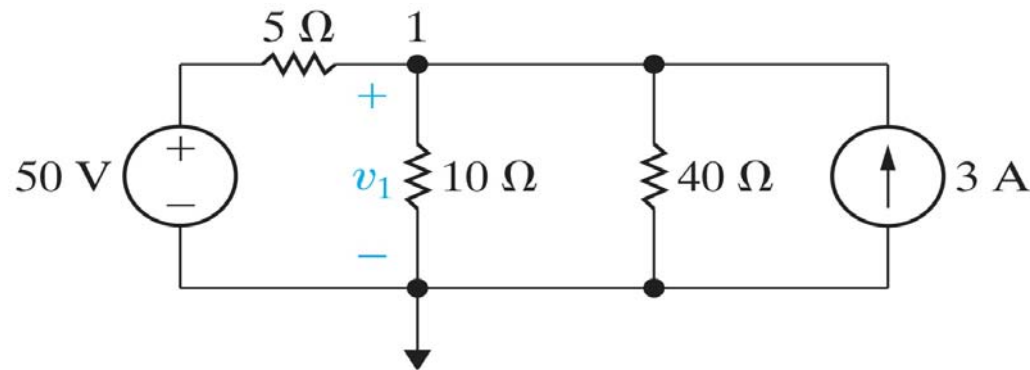
Not on this mid-term

Node Voltage Method

- Identify essential nodes

- Select reference node

- Label voltages at remaining essential nodes (v1, v2, ….vn)

- Write equations for KCL <u>at these nodes</u> in terms of node voltages referenced to reference node.

- Solve n equations in n unknowns

Mesh Current Method

- Identify mesh currents

- no reference node needed since we're explicitly calculating currents and not voltages.

- Label mesh currents (ia, ib, …in)

- Write equations for KVL <u>around the mesh current paths.</u>

- Solve n equations in n unknowns
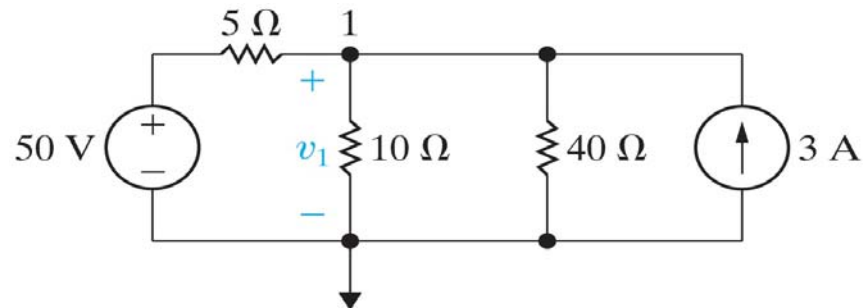
Circuit Analysis with the Node Voltage Method

- Choose a reference node and label the other essential nodes

- Well…there is only one other essential node so we can solve this circuit with a single equation! How many terms will the equation have?
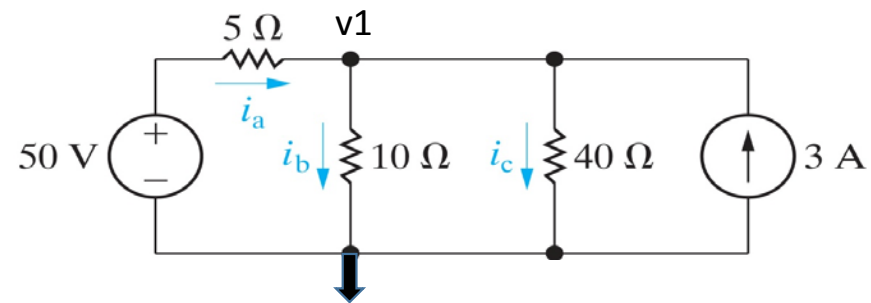
- Now we write the KCL for currents leaving the single essential node:

•If we assume that current is leaving the node through all the branches except the one with the current source:

$$(v1 - 50)/5 + v1/10 + v1/40 - 3 = 0$$

$$\rightarrow v1 = 40V$$

If, instead, we had assumed that the branch currents were flowing as shown below:
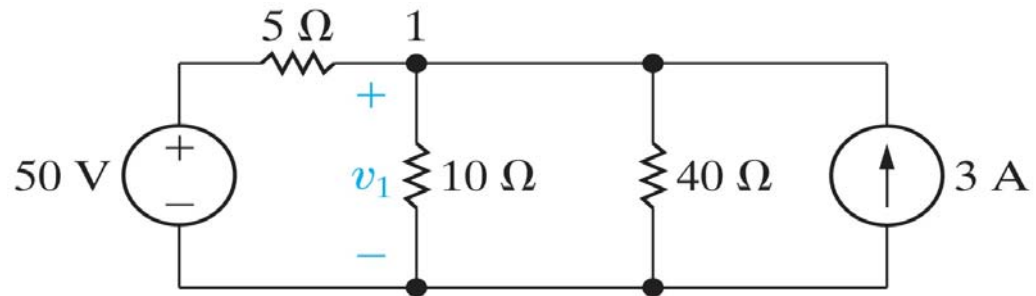

The KCL equation would be:

$-(50 - v1)/5 + v1/10 + v1/40 - 3 = 0$

$(v1 - 50)/5 + v1/10 + v1/40 - 3 = 0$ the same as before!
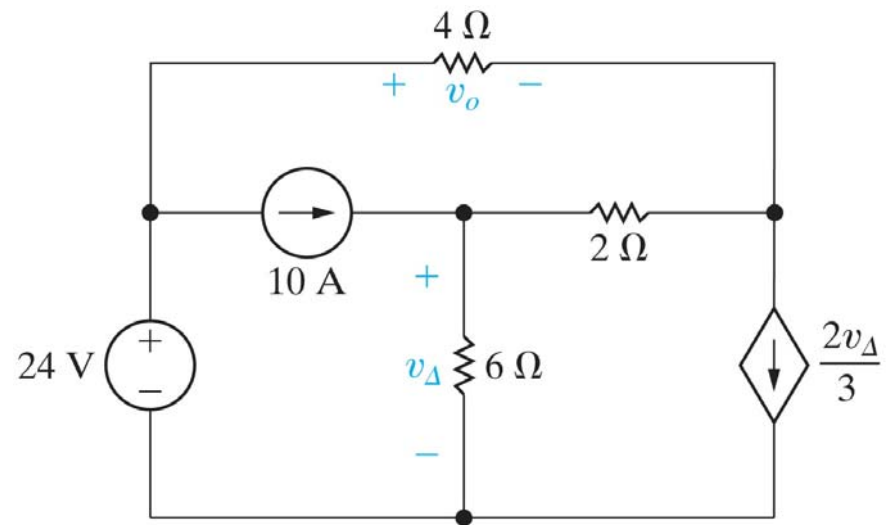
...and we get the same answer:

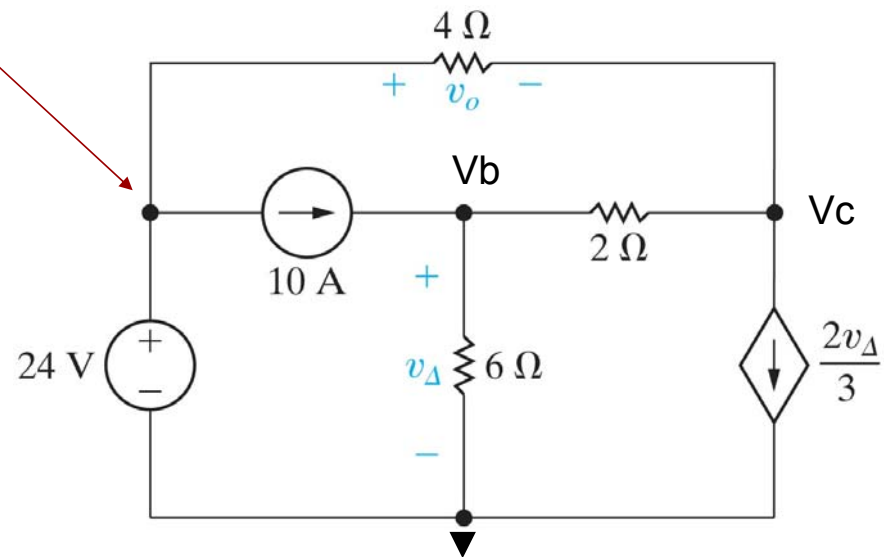→ v1 = 40V

What if you wanted to solve for v1 using source transforms?

Would this be easier than a node voltage solution? Why?

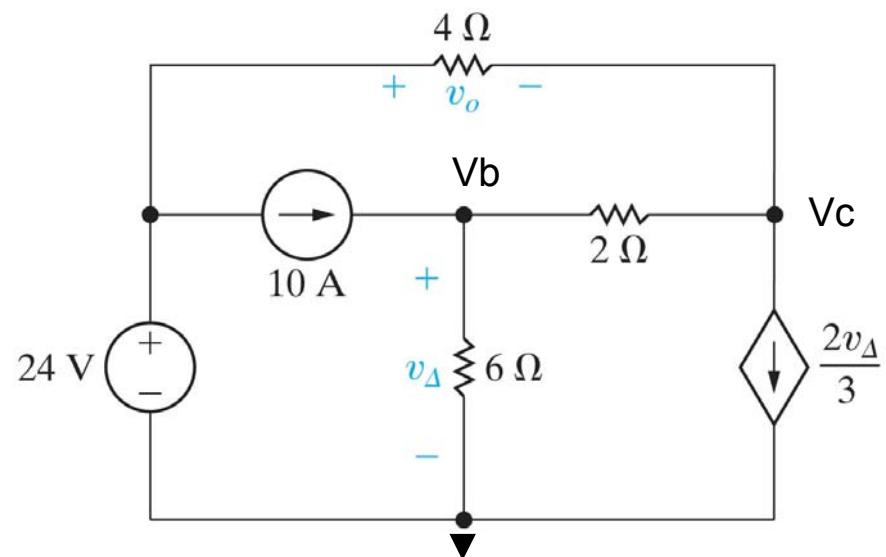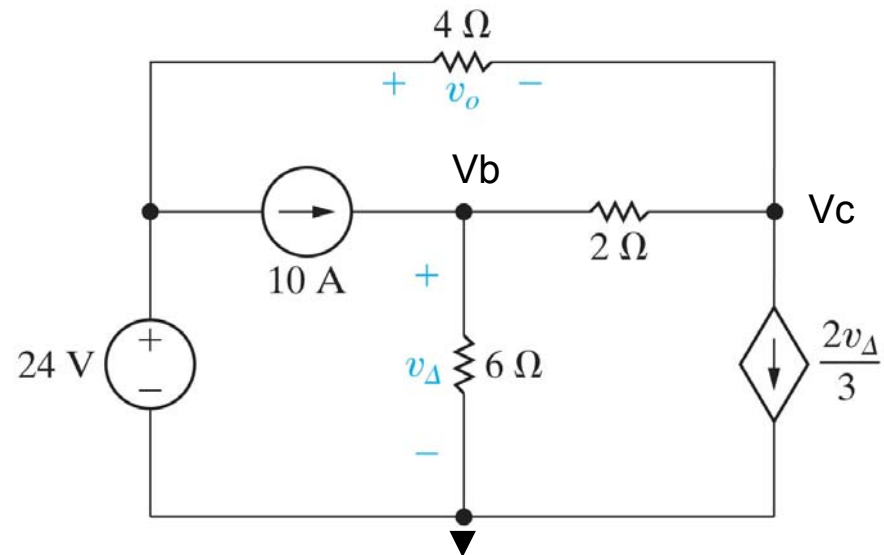Dependent Sources: -- we want to know v_0 using the node voltage method

$$4\ \Omega$$

$$+ \quad v_o \quad -$$

10 A

$$+$$

24 V

$$v_\Delta \lessgtr 6\ \Omega$$

$$2\ \Omega$$

$$\frac{2v_\Delta}{3}$$

We label the essential nodes and chose a reference node

We know
the voltage
at this
quasi-
supernode
by
inspection

$4\ \Omega$

$+ \quad v_o \quad -$

Vb

Vc

$2\ \Omega$

10 A

$+$

24 V

$v_\Delta \gtrless 6\ \Omega$

$\dfrac{2v_\Delta}{3}$

$-$
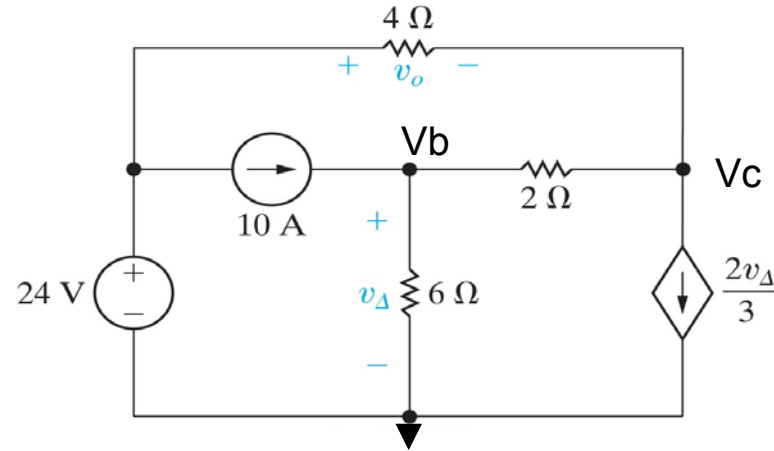
Write the node voltage equations at the two remaining essential nodes -- node b and node c

$$-10 + \frac{v_b}{6} + \frac{v_b - v_c}{2} = 0 \qquad \text{at node b}$$

$$\frac{2v_\Delta}{3} + \frac{v_c - v_b}{2} + \frac{v_c - 24}{4} = 0 \qquad \text{at node c}$$

We need another equation involving v_Δ because we've got two equations in three unknowns – we always need constraint equations when using the node voltage method with dependent sources -- what is the third equation involving v_Δ, v_c and v_b?

Well …the constraint equation here is trivial: $v_\Delta = V_b$

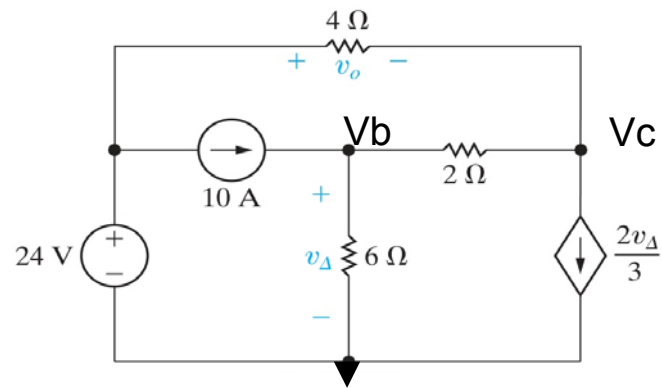Putting the equations in standard form we could write (including the constraint equation):

$$v_b\left(\frac{1}{6}+\frac{1}{2}\right) \; + \; v_c\left(-\frac{1}{2}\right) \qquad + \quad v_\Delta(0) \quad = \quad 10$$

$$v_b\left(-\frac{1}{2}\right) \qquad + \quad v_c\left(\frac{1}{2}+\frac{1}{4}\right) \; + \; v_\Delta\left(\frac{2}{3}\right) \quad = \quad \frac{24}{4}$$

$$v_b(1) \qquad\qquad + \quad v_c(0) \qquad\quad + \quad v_\Delta(-1) \quad = \quad 0$$

and solve a 3 X 3 matrix using MATLAB :

or ……..we can use the constraint equation to eliminate v_Δ immediately by substitution (giving us two KCL equations at two nodes)

$$-10 + \frac{V_b}{6} + \frac{V_b - V_c}{2} = 0$$

$$\frac{2V_b}{3} + \frac{V_c - V_b}{2} + \frac{V_c - 24}{4} = 0$$

In standard form:

$$\begin{bmatrix} \left(\dfrac{1}{6}+\dfrac{1}{2}\right) & \left(-\dfrac{1}{2}\right) \\ \left(\dfrac{2}{3}-\dfrac{1}{2}\right) & \left(\dfrac{1}{2}+\dfrac{1}{4}\right) \end{bmatrix} \begin{bmatrix} v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

And we can use MATLAB to solve this ……or we can use substitution and just write:
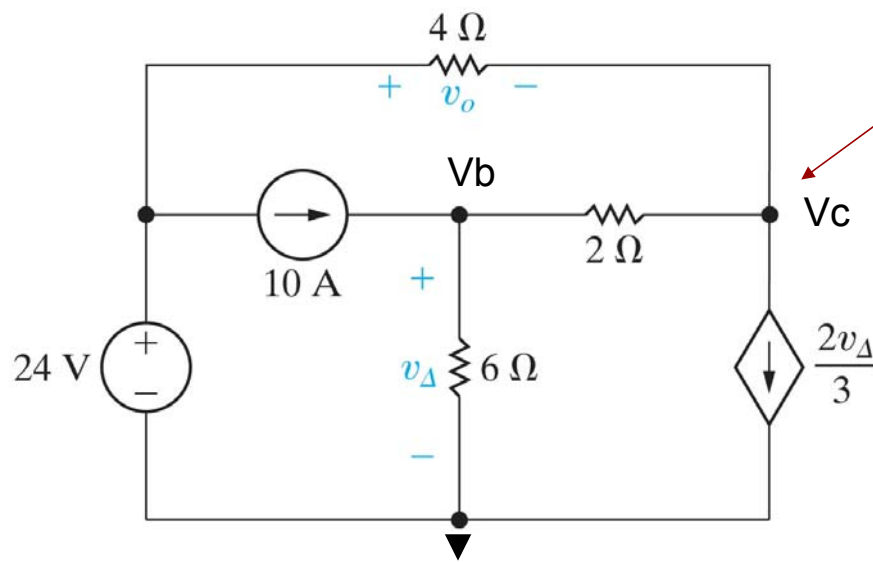
$$4V_b - 3V_c = 60$$

$$2V_b + 9V_c = 72$$

multiplying the second eq. by 2 and subtracting
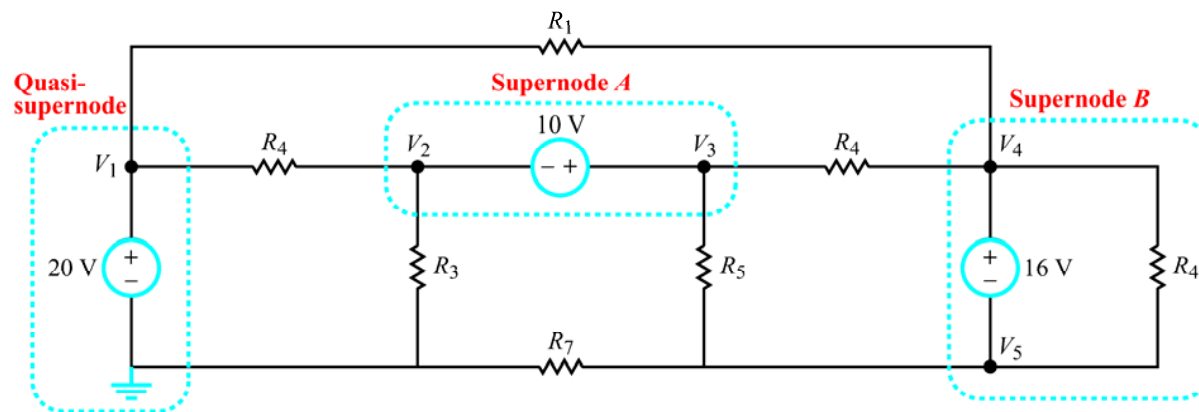
$$-21V_c = -84 \quad so \ V_c = 4, V_b = 18$$

22

We can solve:

Vb = 18V; Vc = 4V; v_Δ = 18, so v0 = 24 – Vc = 20V and we're done!

why isn't this a quasi-supernode??

4 Ω

$+$ v_o $-$

Vb

Vc

2 Ω

10 A

$+$

24 V

v_Δ 6 Ω

$\dfrac{2v_\Delta}{3}$

$-$

23

Nodes, Supernodes, and Quasi-Supernodes

Here's an exercise from another textbook that we saw before …apply the supernode concept to determine I in the circuit.
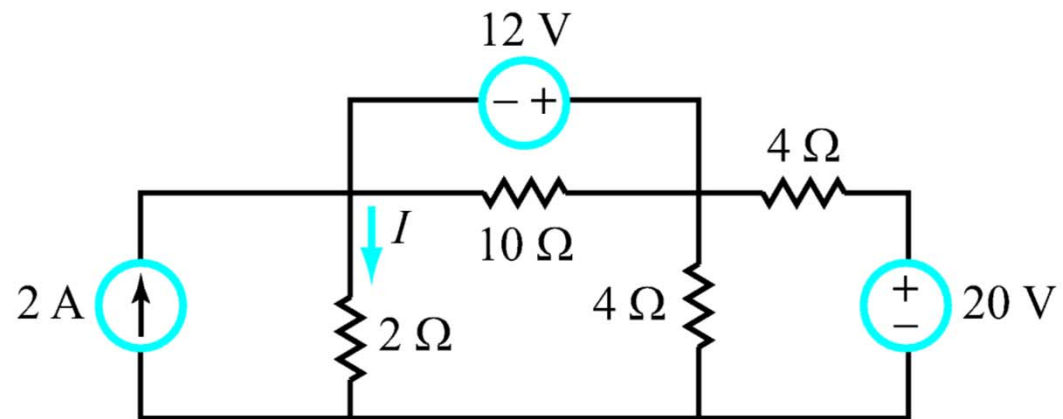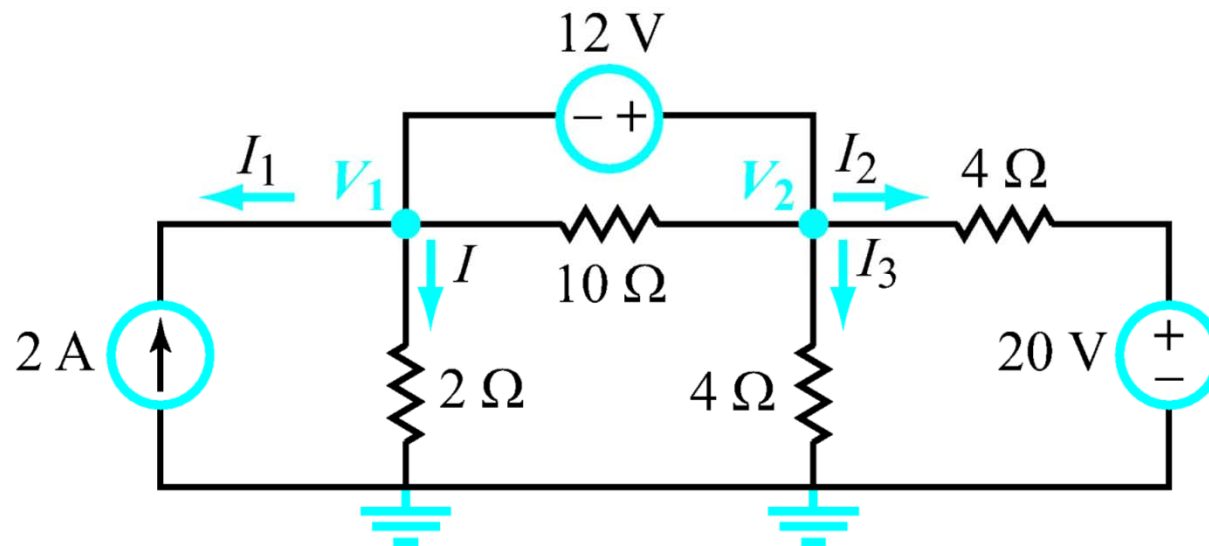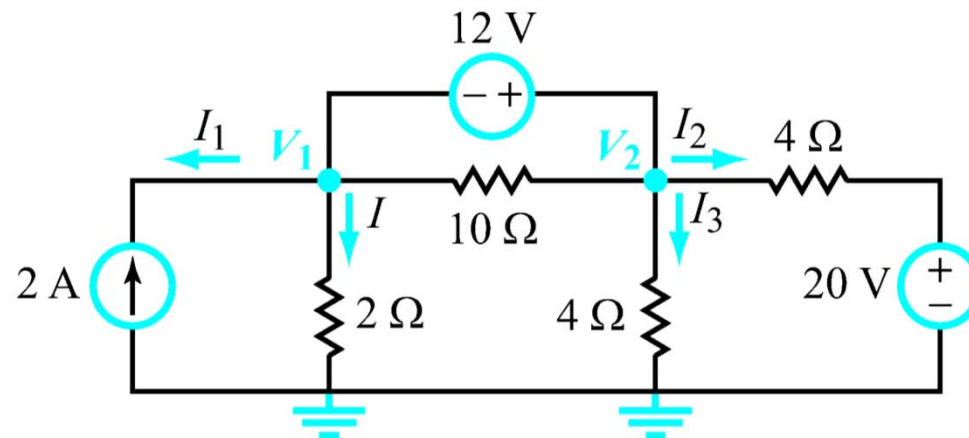

Figure E3.3

Number essential nodes and chose a reference node. Is there a supernode in this circuit?
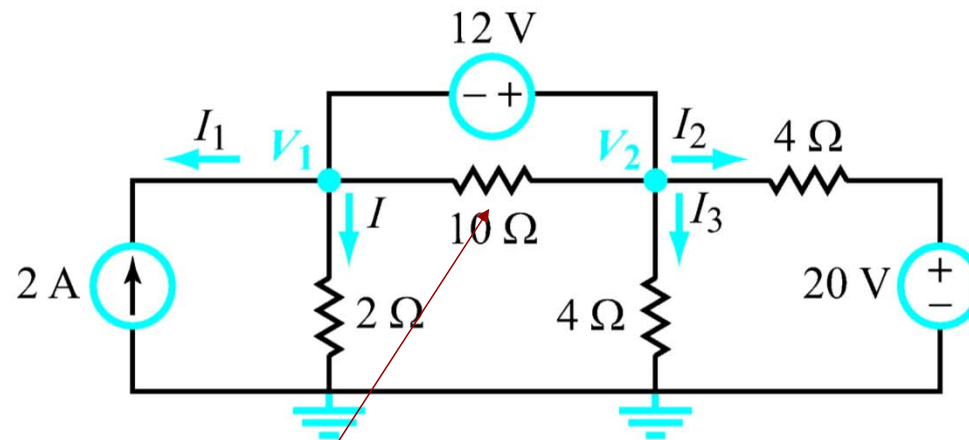
V1 and V2 form a supernode so we can write a single KCL equation for it:
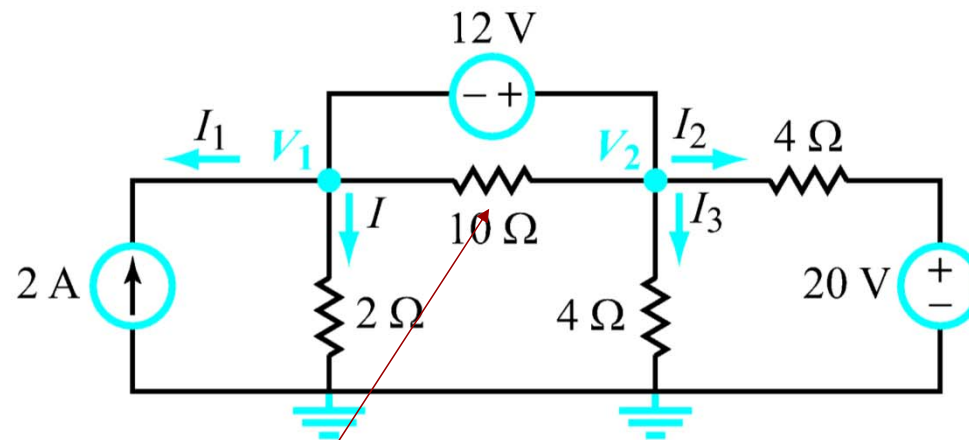
$I_1 + I + I_2 + I_3 = 0$

$I_1 = 2$, $I = V_1/2$, $I_3 = V_2/4$, $I_2 = (V_2-20)/4$ and KVL gives us $V_2-V_1=12$

So, solving for V_1 and substituting:

$I = 0.5A$, $V_1=1$, $V_2 = 13$

12 V

I_1 V_1

I_2 4 Ω
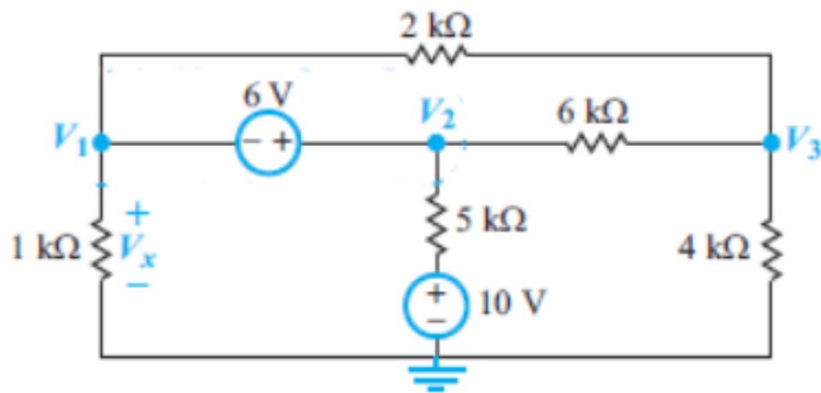
V_2

I 10 Ω

I_3

2 A

2 Ω 4 Ω

20 V

How about the current through this resistor – we didn't account for it in any of the calculations? Does the behavior of this circuit depend on the value of this resistor?
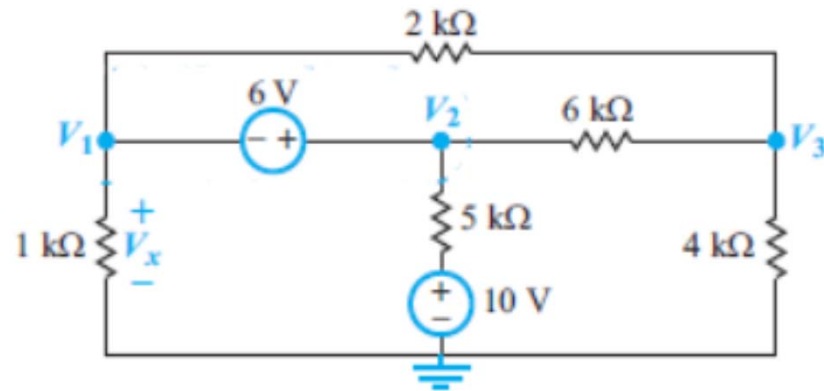
28

12 V

I_1 V_1 I_2 4 Ω

V_2

I 10 Ω

2 A 2 Ω I_3 4 Ω 20 V

This is one of these "superfluous resistors" we've talked about before… it's a resistor in parallel with a voltage source that can be removed from a circuit without affecting anything. Another way to look at it -- if we had included it in the supernode equation it would have added one current term to the v1 part of the equation and the negative of the same current term to the v2 part of the equation ….cancelling each other out.

29

Here's another node voltage problem

What is Vx …the voltage drop across the 1K resistor?

For V1 and V2 of the supernode:
$$\frac{V_1}{10^3} + \frac{V_1 - V_3}{2 \times 10^3} + \frac{V_2 - 10}{5 \times 10^3} + \frac{V_2 - V_3}{6 \times 10^3} = 0$$

And for V3:
$$\frac{V_3 - V_1}{2 \times 10^3} + \frac{V_3 - V_2}{6 \times 10^3} + \frac{V_3}{4 \times 10^3} = 0.$$

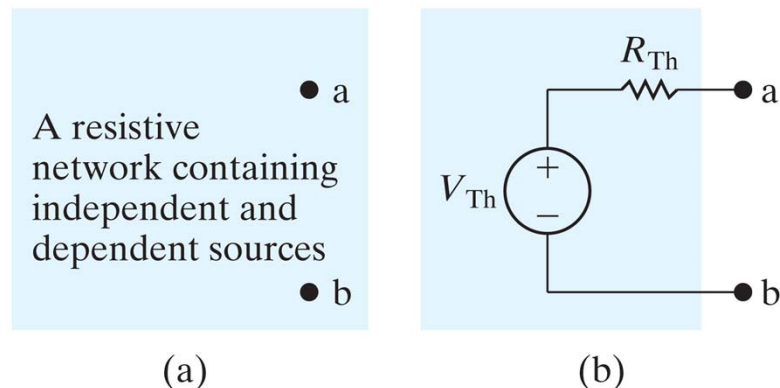Also ...there is the auxiliary equation: V2-V1 =6

MATLAB gives:

V1= 0.38V
V2= 6.38V
V3= 1.37V

So Vx = V1 = 0.38V

Thevenin Equivalent Circuit

Thevenin (and Norton) equivalent circuits are ways of replacing complicated and/or irrelevant parts of a circuit with a simple circuit based on the behavior of the circuit at a particular pair of terminals using source transformations.
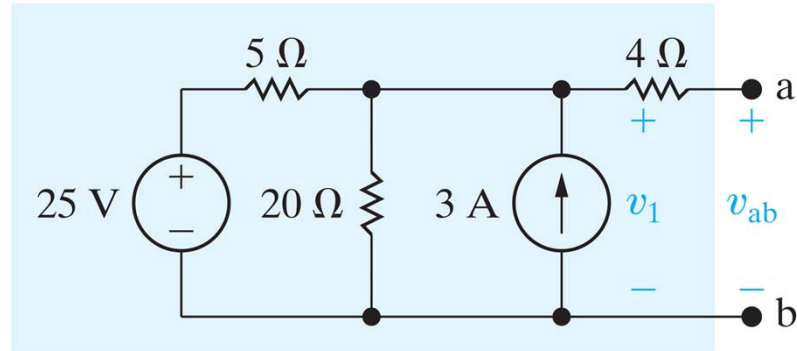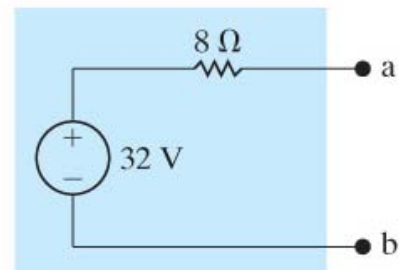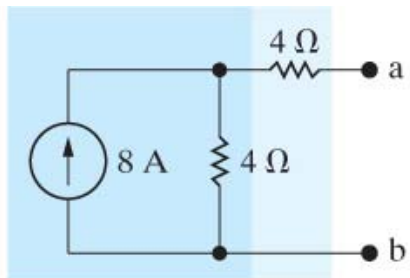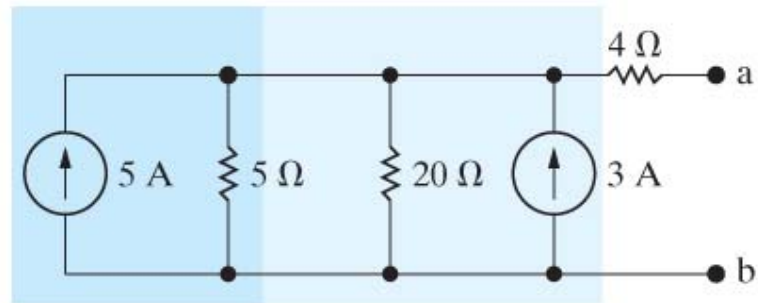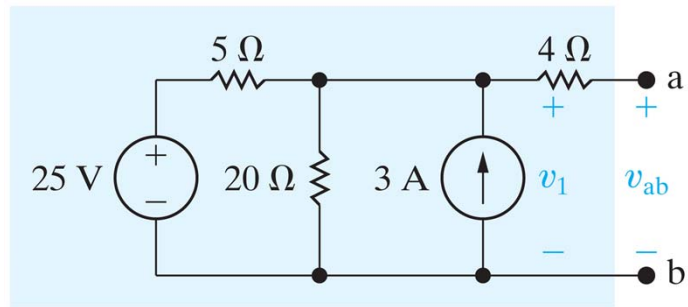

(a)   (b)

The Thevenin equivalent circuit for a resistive network is a <u>voltage source, V_{th} in series with a resistor, R_{th}.</u> We chose these values by measuring the open circuit voltage for the network …this is V_{th} …then we measure the short circuit current out of the network. Since this will be V_{th}/R_{th}, the value of R_{th} is: $R_{th} = V_{th}/i_{sc}$. The <u>Norton equivalent is just the source transform of the Thevenin circuit</u> into a current source in parallel with a resistor.

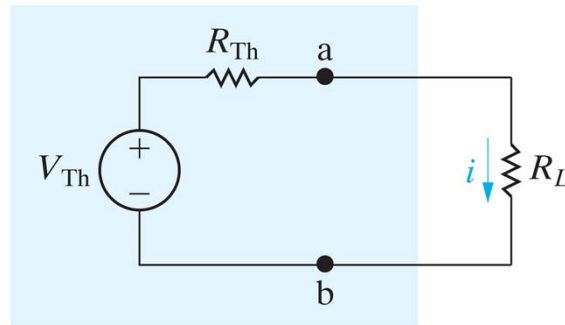Thevenin Equivalent Circuit – General Calculation Strategy

- **Obtain vth** by calculating the voltage across the two specified terminals when no load is present (open circuit voltage)

- **Obtain Rth** by either:

  - Calculating the current that will flow between the specified terminals in a short circuit. Rth is obtained from Rth = vth/Isc OR

  - If the circuit doesn't contain dependent sources, you can calculate the equivalent resistance between the specified terminals after all independent voltage sources are deactivated (replaced with short circuits)and all independent current sources are deactivated (replaced with open circuits). The equivalent resistance is Rth, the Thevenin resistance. OR

  - If the circuit contains independent and dependent sources, Rth can be determined by deactivating independent sources, and adding an external source (vex or iex)…then solve the circuit to determine the current iex or voltage vex supplied by the external source. Rth = vex/iex

Sometimes it will be possible to obtain a Thevenin equivalent circuit simply by doing a series of source transforms. For example – what is the Thevenin equivalent circuit at terminals a and b?
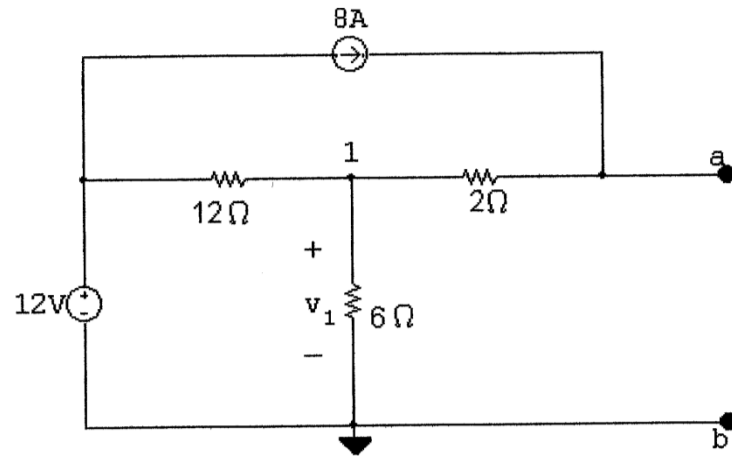
The
Thevenin
equivalent!

power transfer is maximum when the load resistance, $R_L = R_{Th}$

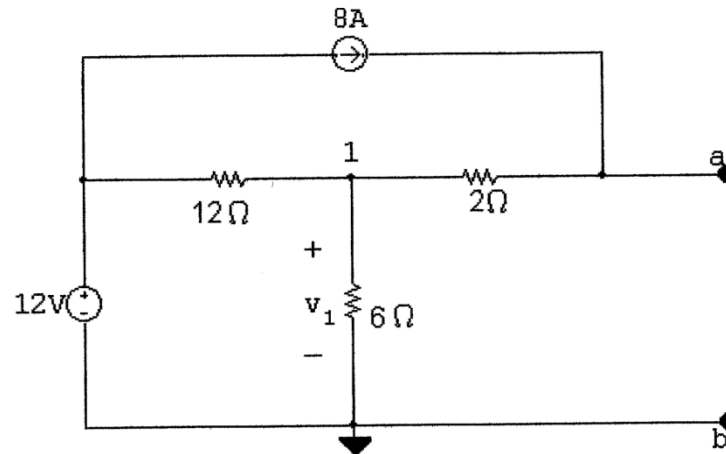and we can determine the value of the maximum power by substituting into:

$p = i2 \ R_L \ = (Vth / [R_{Th} + R_L])^2 \ R_L$

$$p_{max} = V_{Th}^2 / 4R_L$$

Notice: if the problem asks only for the load resistance that will transfer maximum power – you need to calculate Rth only – not Vth also.

What load resistor at terminals a and b will result in maximum power transfer to the load? What will that power be?

We can get Rth by method # 2 – deactivating the sources and combining resistances.

$$R_{Th} = 2 + \frac{(12)(6)}{18} = 6\,\Omega$$

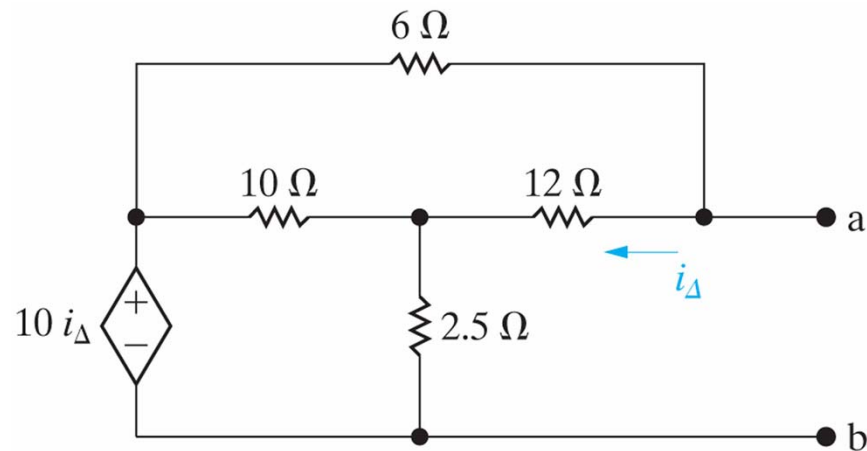We can calculate v1 using node voltage analysis….and Vth is v1 plus the voltage drop across the 2Ω resistor.

$$\frac{v_1 - 12}{12} + \frac{v_1}{6} - 8 = 0$$

$$v_1 = 36 \text{ V}$$

So p_{max}=(52)²/4*6 =112.67 W

$$v_{Th} = v_1 + (2)(8) = 52 \text{ V}$$

Same question ……what load resistor at terminals a and b will result in maximum power transfer to the load? What will that power be? Notice – this circuit has only a dependent source! What does this tell you about Vth? What does this tell you about methods for calculating Rth?

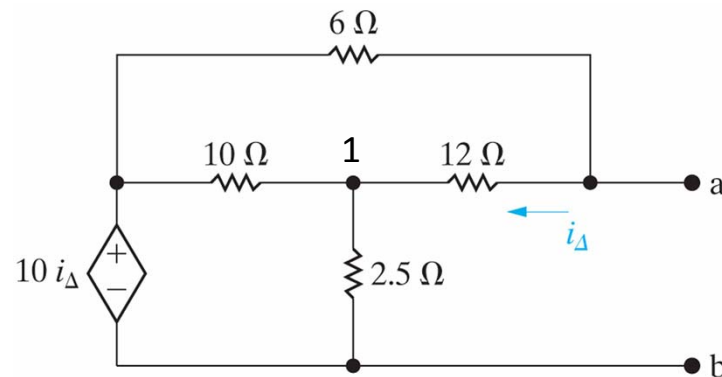Well…..if you remembered that <u>circuits with only dependent sources have Vth = 0</u> ..
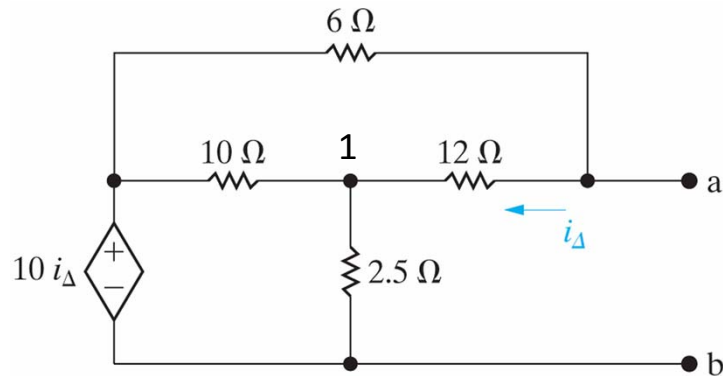..and you also remembered that:

Pmax = $(V_{th})^2/4R_{th}$

then you'd know that the maximum power transferred to the load resistor would have to be zero!

But, if you had forgotten this …you can always solve for Vth and Rth. To get V_{th} we calculate the open circuit voltage at terminals a and b. We get this by solving the single node voltage equation at node 1.

$$\frac{v_1 - 10i_\Delta}{10} + \frac{v_1}{2.5} - i_\Delta = 0$$

We also see there is another relationship between v_1 and i_Δ because of the voltage divider created by the 6Ω, 12Ω and 25Ω resistor across the dependent voltage source.
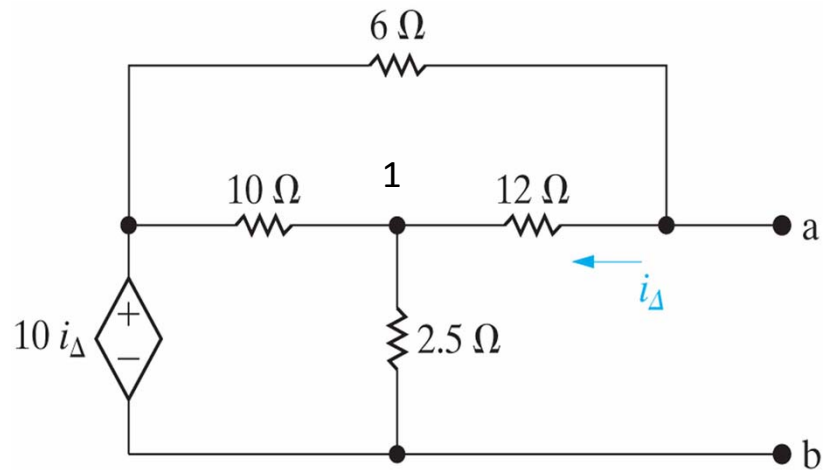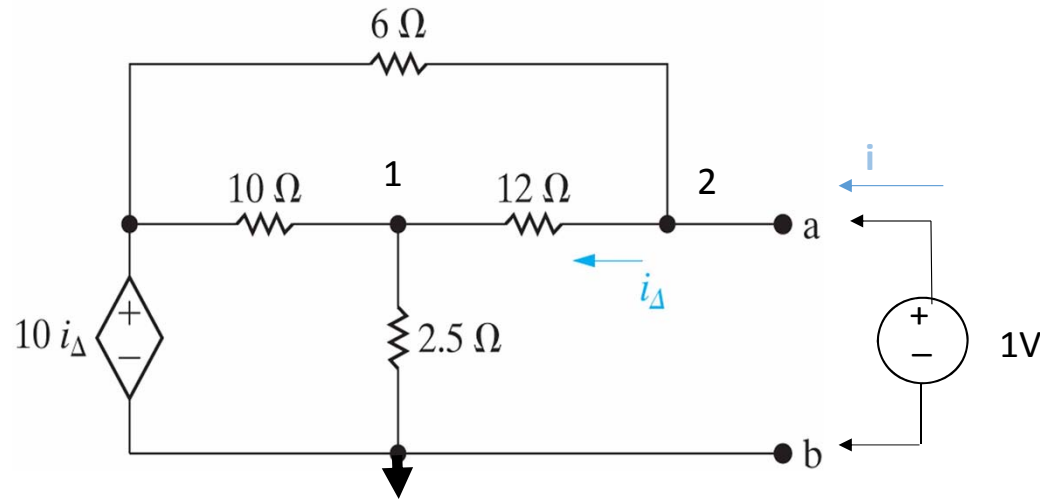

So:
$$v_1 = 10i_\Delta \frac{25}{6+12+25} = 12.195i_\Delta$$

From the first KCL equation:
$$\frac{v_1 - 10i_\Delta}{10} + \frac{v_1}{2.5} - i_\Delta = 0$$
so $v_1 = 3.64i_\Delta$

The value of i_Δ that satisfies both equations is zero...so 10 i_Δ = 0 and Vth =0 and since Pmax = $(V_{th})^2/4R_{th}$ the maximum power transferred is also zero.

What if, instead of maximum power, the problem asked you to determine the Thevenin equivalent circuit for this thing. You already know Vth = 0 ...how would you calculate Rth for the Thevenin equivalent? Which method would you use?
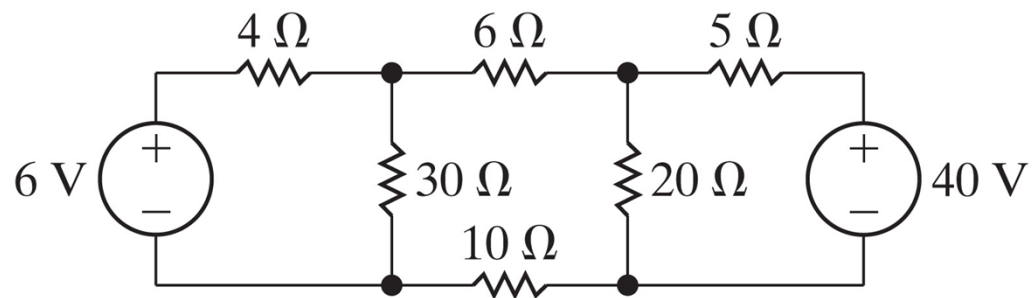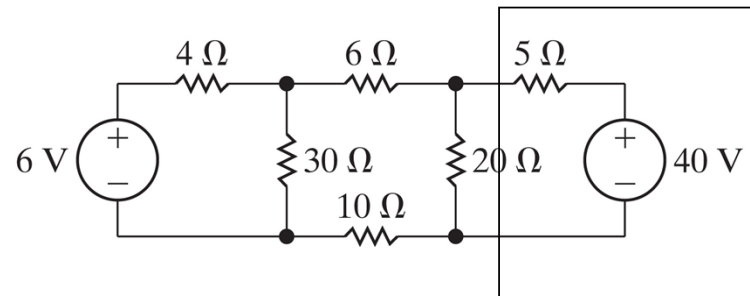
We'll use method #3 …and attach a test voltage source – here we've chosen it to be 1V. We want to calculate the current, i, flowing out of the test source …since we know from Ohm's law that the resistance from a to b will be the test voltage, 1V, divided by the current, i. If there were any independent sources in the circuit, we would deactivate them …but we leave dependent sources alone.

I'll leave the algebra to you. You'll have to solve one KCL equation at node 1 since node 2 will be a quasi-supernode (because of the 1V source) and the other node is also a quasi-supernode.

We want to find the power (absorbed or supplied) by the 6V source
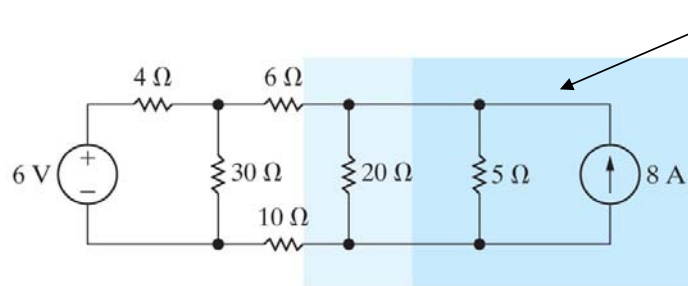

Since we only care about the current flowing into or out of the 6V source, probably the easiest way to solve this circuit is through source transforms on the 40V source – leaving the 6V source alone.
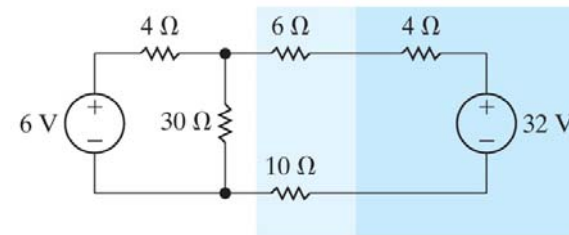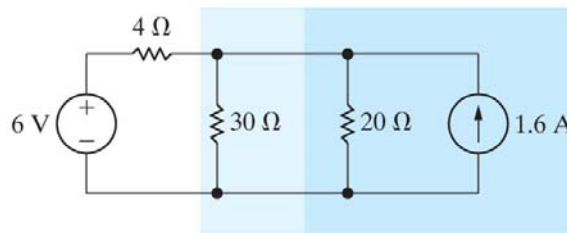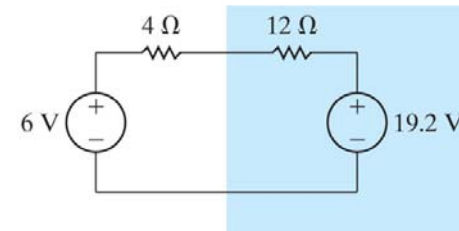
Remember

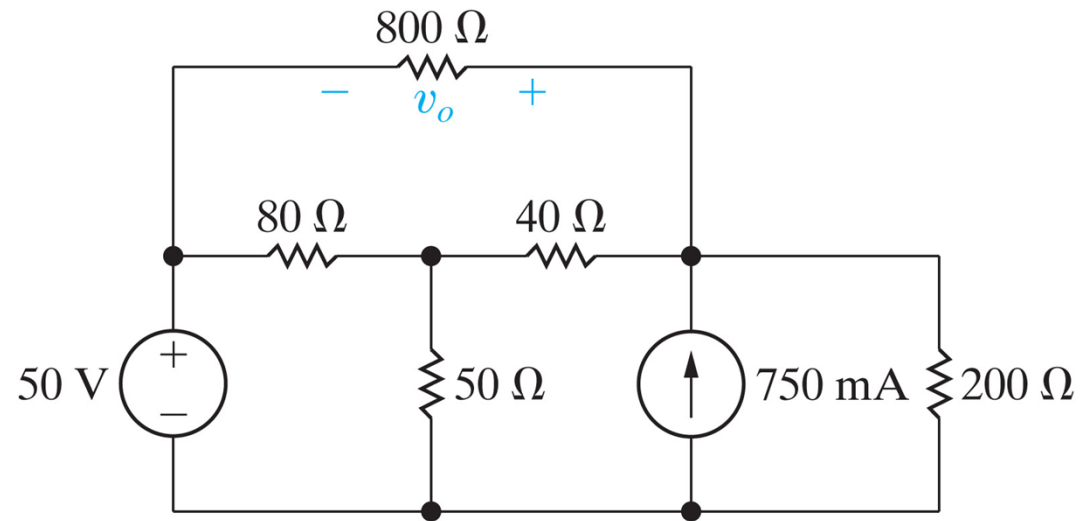$i_s = v_s/R$

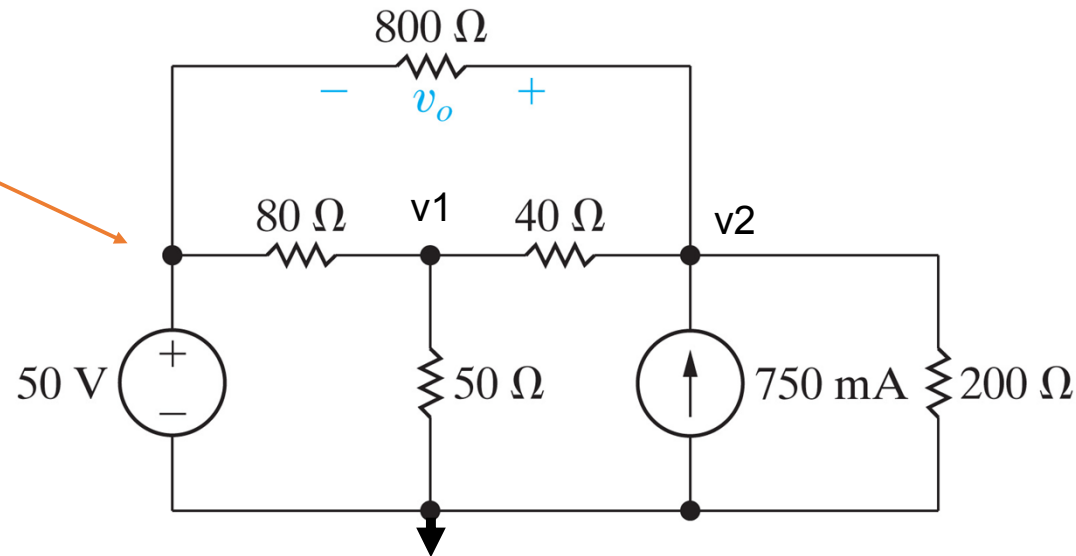(a) First step

(b) Second step

(c) Third step

(d) Fourth step

Now it's trivially easy to determine the current through the 6V source – it's (19.2 -6)/16 = 0.825A

flowing <u>backwards</u> through the voltage source so the source <u>absorbs</u> 4.95 W

Find v_0 using Node Voltage Method

Number the nodes, pick a reference node, and write the KCL equations at the two remaining essential nodes.

$(v1 - 50)/80 + v1/50 + (v1-v2)/40 = 0$ at node v1

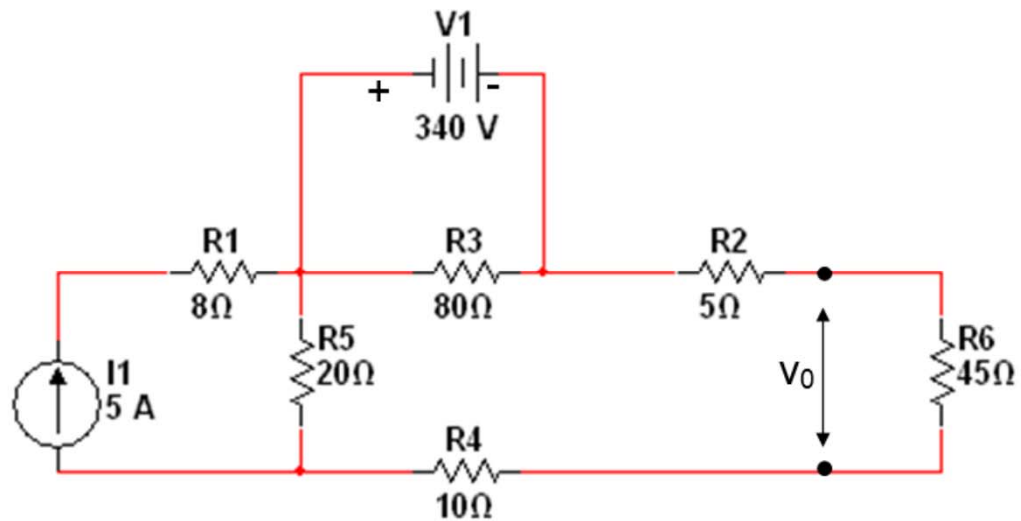$v2/200 - 0.75 + (v2-v1)/40 + (v2-50)/800 = 0$ at node v2

In standard form:

$$v_1 \left(\frac{1}{80} + \frac{1}{50} + \frac{1}{40} \right) + v_2 \left(-\frac{1}{40} \right) = \frac{50}{80}$$

$$v_1 \left(-\frac{1}{40} \right) + v_2 \left(\frac{1}{40} + \frac{1}{200} + \frac{1}{800} \right) = 0.75 + \frac{50}{800}$$
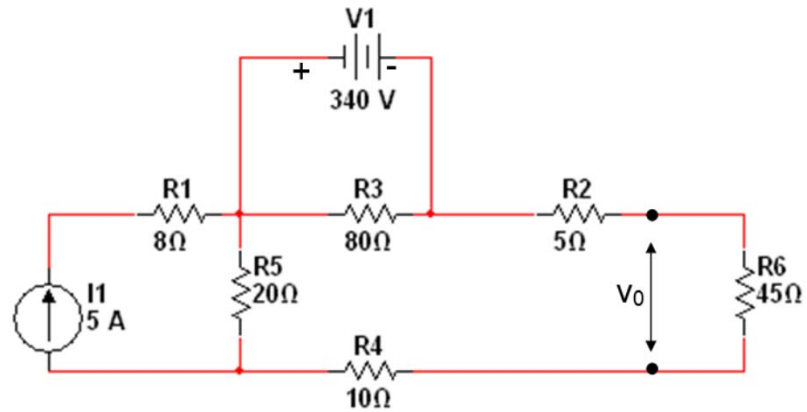
v1 = 34V;

v2 = 53.2V
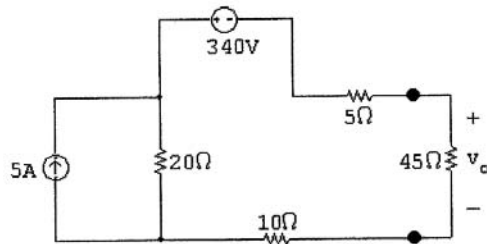
So …… v0 = v2 – 50 = 3.2V

Solve for V_0 Are there any simplifications that can be done before starting the solution?

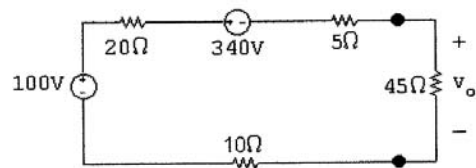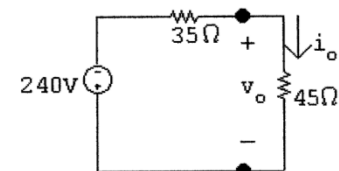P 4.61 [a] First remove the $8\,\Omega$ and $80\,\Omega$ resistors:

Next use a source transformation to convert the 5 A current source and $20\,\Omega$ resistor:
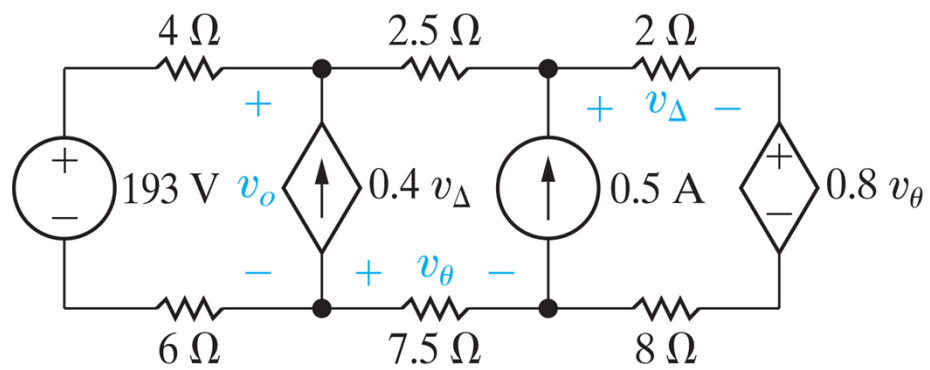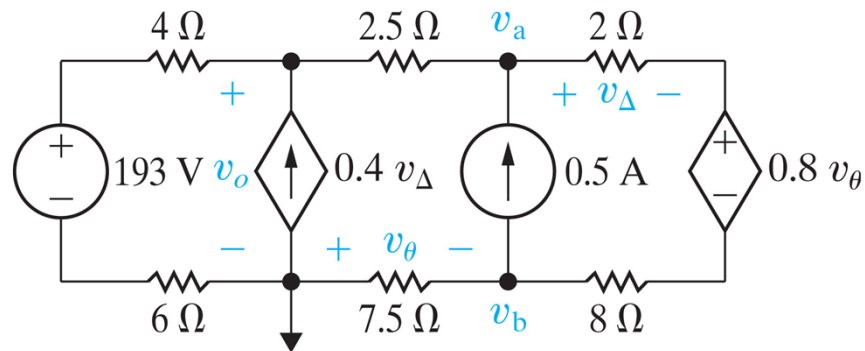

which simplifies to

$$\therefore \quad v_o = \frac{45}{80}(-240) = -135 \text{ V};$$

Can you do a node voltage analysis of this circuit to determine v0?

node voltage analysis

$(v0 - 193)/10 - 0.4 \, v_\Delta + (v0\text{-}va)/2.5 \qquad = 0$
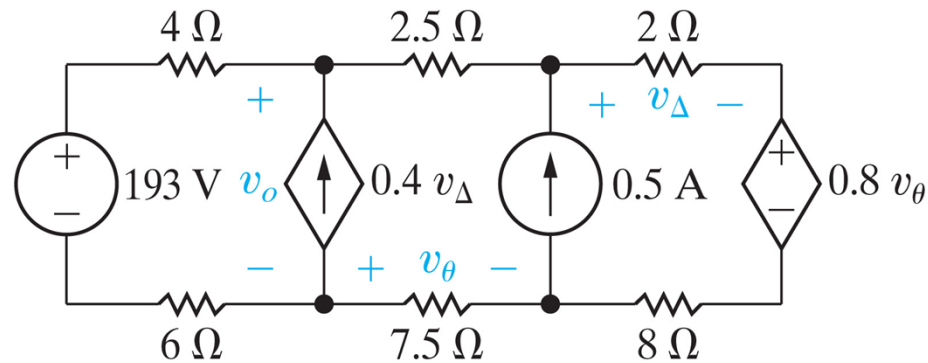
$(va\text{-}v0)/2.5 + 0.5 + [va - (vb + 0.8v\theta)]/10 = 0$

$vb/7.5 + 0.5 + (vb + 0.8 \, v\theta - va)/10 \qquad = 0$

And the constraint equations are:

$v\theta = \text{-}vb$

$v_\Delta = [va - (vb + 0.8 \, v\theta)]/10 \ * 2$

Could you have done some simplifications ? It's clear that if the two dependent sources had been independent sources, source transforms would have allowed solution of the circuit using only Ohms Law. However, we need to preserve the 7.5 Ohm and 2 Ohm resistor (not combine them) because they define the values of the dependent current and voltage sources. Probably the best strategy would be to do the ordinary node voltage solution and solve the 3 x 3 matrix.

If this all made sense to you and you understand all of the homework problems so far you'll do fine on the mid-term exam.

I'll see you Friday