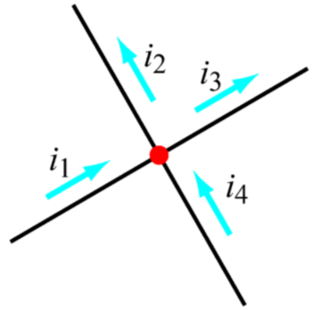
# ELEN 50 Class 05 – Resistive Circuits: Series and Parallel

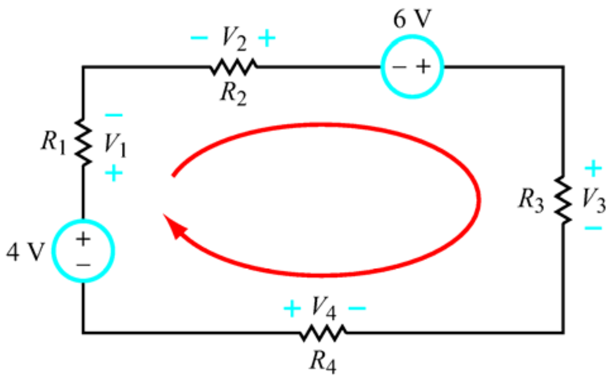
S. Hudgens

## Review of KCL and KVL




$I_1 + i_4 = i_2 + i_3$

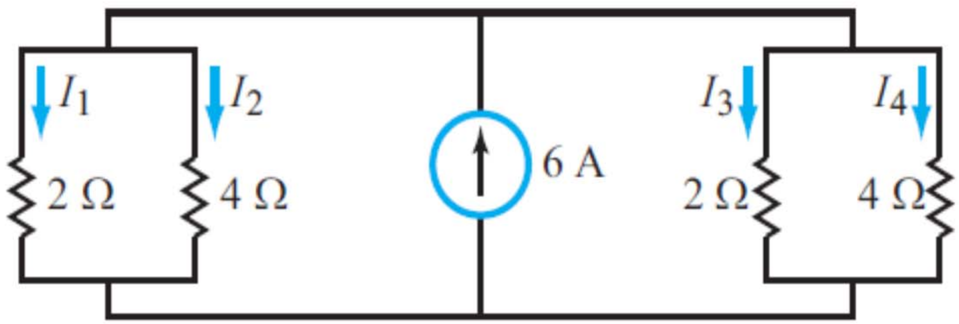
Kirchhoff Current Law (KCL) – the total current entering a node must be equal to the total current leaving a node …i.e. charge doesn't "build up" at a node. The sign convention is that current leaving a node is considered positive.




-4 + V1 – V2 -6 +V3 –V4 = 0

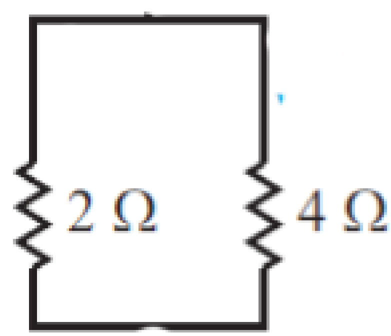
Kirchhoff Voltage Law (KVL) – the algebraic sum of the voltage drops around a closed loop is zero. The sign convention is that the voltage drop on an element appears with the sign first encountered by the arrow on the loop.

Determine currents $I_1$ to $I_4$ in the circuit



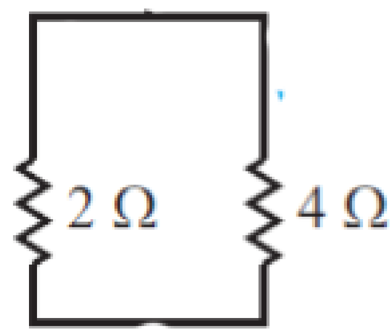

We could use Ohm's law and KCL – is this the best approach ?

First ….can you tell me if these two resistors are in series or in parallel?



What is the criterion for this choice?

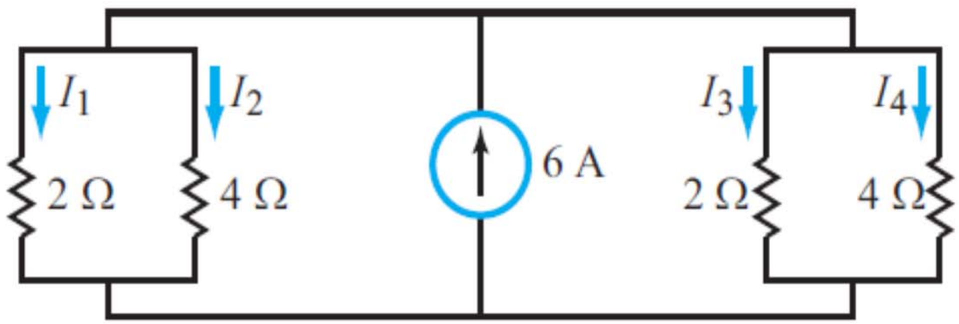
Circuit elements with the same current flowing through them are in series and circuit elements with the same voltage across them are in parallel.




Where are the currents and voltages in this circuit? There aren't any …because the circuit doesn't have any sources …explicit or implicit! Series and parallel elements are defined in terms of sources …either explicitly or implicitly (where terminals are indicated where sources would be attached).

OK…back to the problem

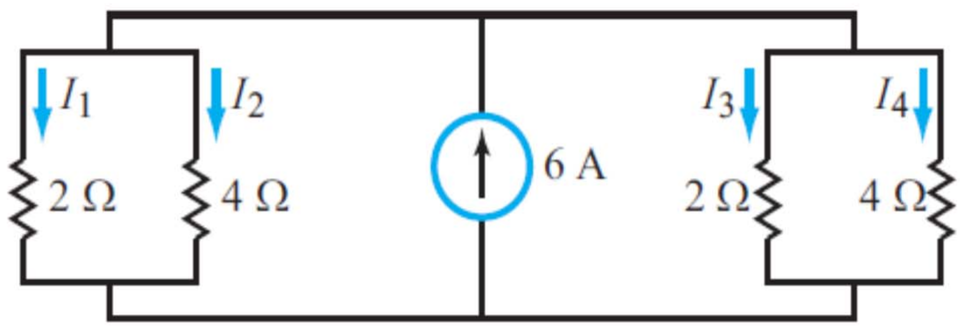
Determine currents $I_1$ to $I_4$ in the circuit




We could use Ohm's law and KCL – is this the best approach ?

Well..this will certainly work …but the easiest approach is the following:

Determine currents $I_1$ to $I_4$ in the circuit




**Solution:** The same voltage exists across all four resistors. Hence,

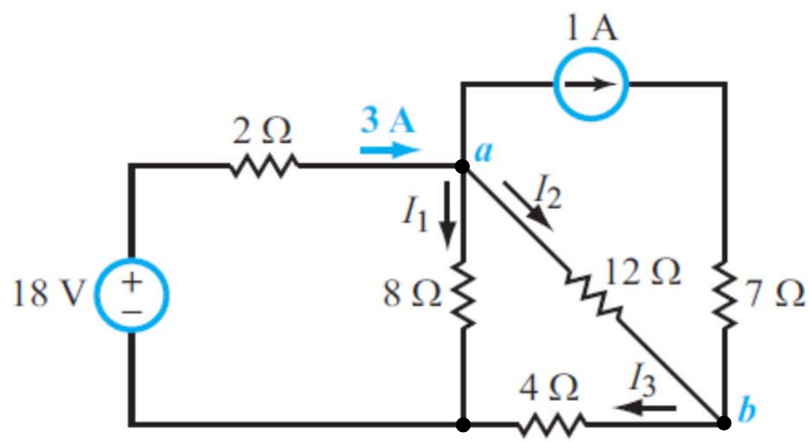
$$2I_1 = 4I_2 = 2I_3 = 4I_4.$$

Also, KCL mandates that

$$I_1 + I_2 + I_3 + I_4 = 6$$

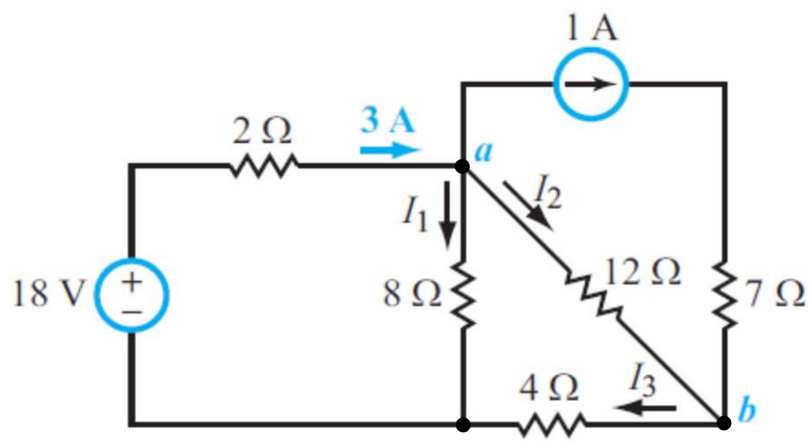
It follows that $I_1 = 2$ A, $I_2 = 1$ A, $I_3 = 2$ A, and $I_4 = 1$ A.

Can you think of a good strategy to solve this circuit?




We want to solve for three currents – and there are 3 extraordinary nodes (two plus a reference node) ..and there are three independent loops. In principle this can give us 2 KCL equations and 3 KVL equations.

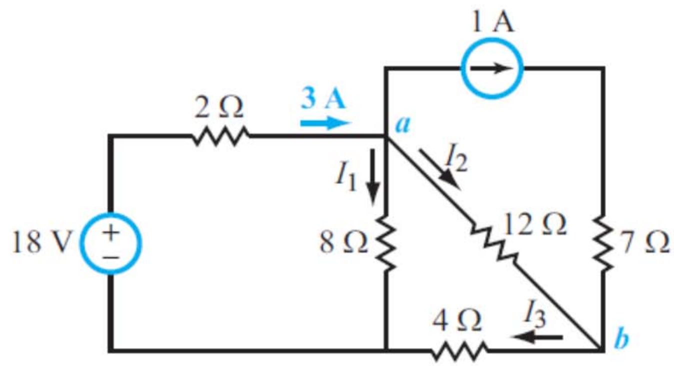
We only need 3 equations …how shall we choose?




KVL on the loop containing the 18V source and the 8Ω resistor will give $I_1$ in terms of known quantities.

Then KCL at node a will give $I_2$ in terms of $I_1$ and known quantities

Then KCL at node b will give $I_3$ in terms of $I_2$ and known quantities




**Solution:** For the loop containing the 18-V source,

$$-18 + 3 \times 2 + 8I_1 = 0.$$

Hence, $I_1 = 1.5$ A.

KCL at node $a$ gives

$$3 - 1 - I_1 - I_2 = 0$$

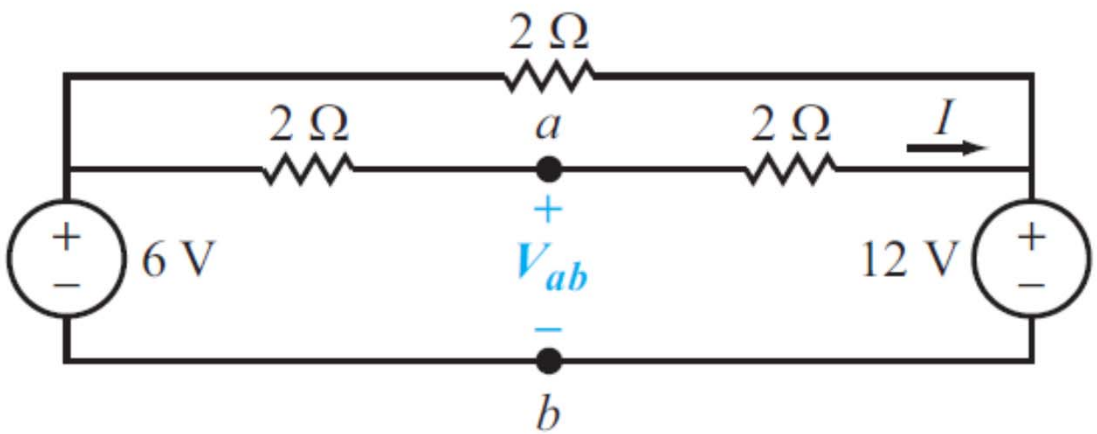
$$I_2 = 2 - I_1 = 2 - 1.5 = 0.5 \text{ A}.$$

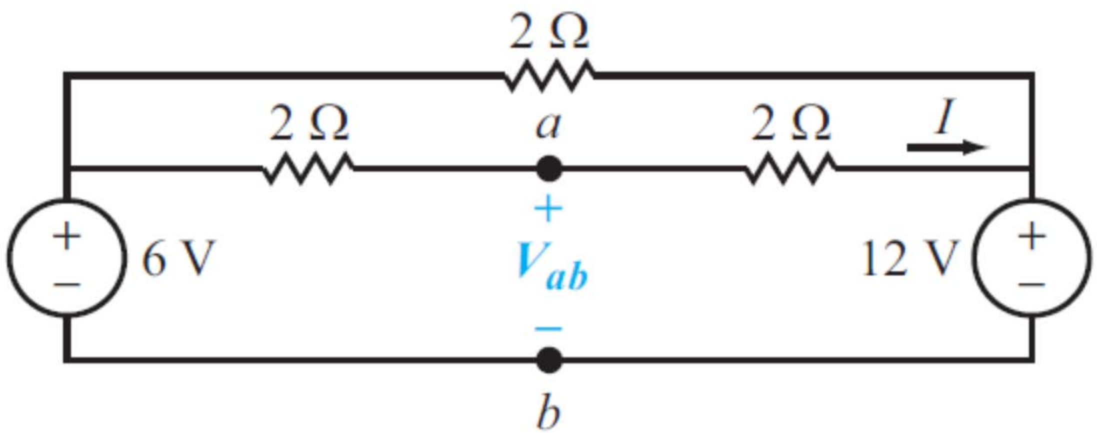
KCL at node $b$ gives

$$1 + I_2 - I_3 = 0$$

$$I_3 = 1 + I_2 = 1 + 0.5 = 1.5 \text{ A}.$$

Here's one more – what is the voltage between a and b?






For the lower loop, KVL gives

$$-6 + 4I + 12 = 0,$$

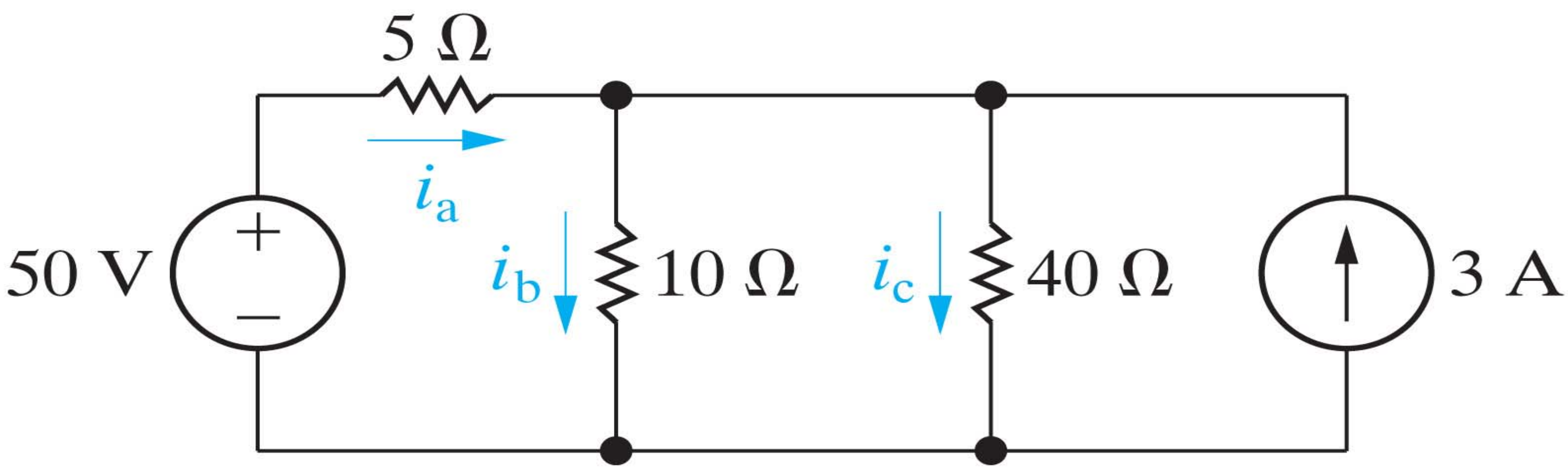
or

$$I = -1.5 \text{ A}.$$

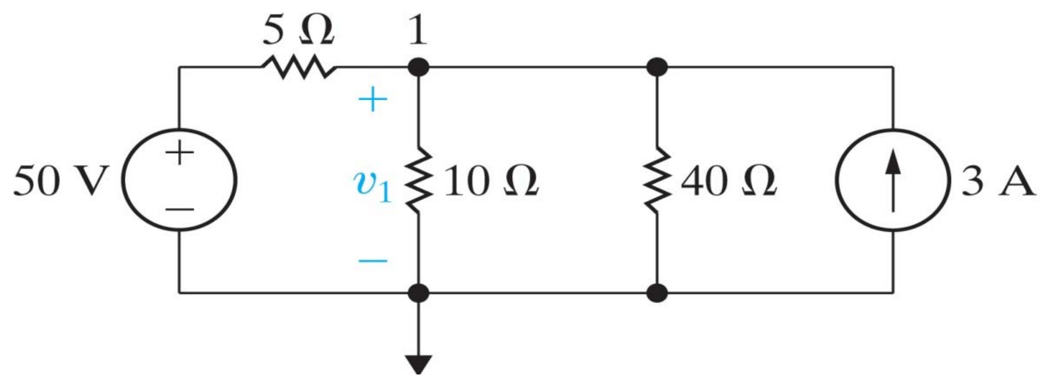
Moving from $a$ to $b$ via the 12-V supply,

$$V_{ab} = (-1.5) \times 2 + 12 = 9 \text{ V}.$$

Can you come up with a strategy to determine $i_a$, $i_b$, and $i_c$?




How many loops are there in the circuit …how many essential nodes?



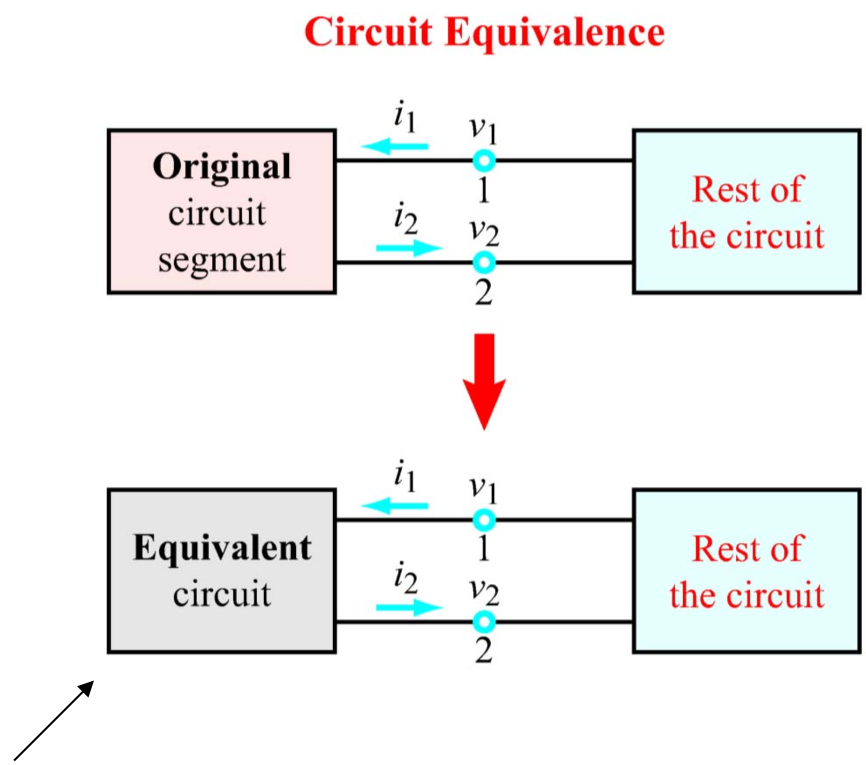

• we can write the KCL for currents leaving the single essential node:

(v1 – 50)/5 + v1/10 + v1/40 – 3 = 0

→ v1 = 40V

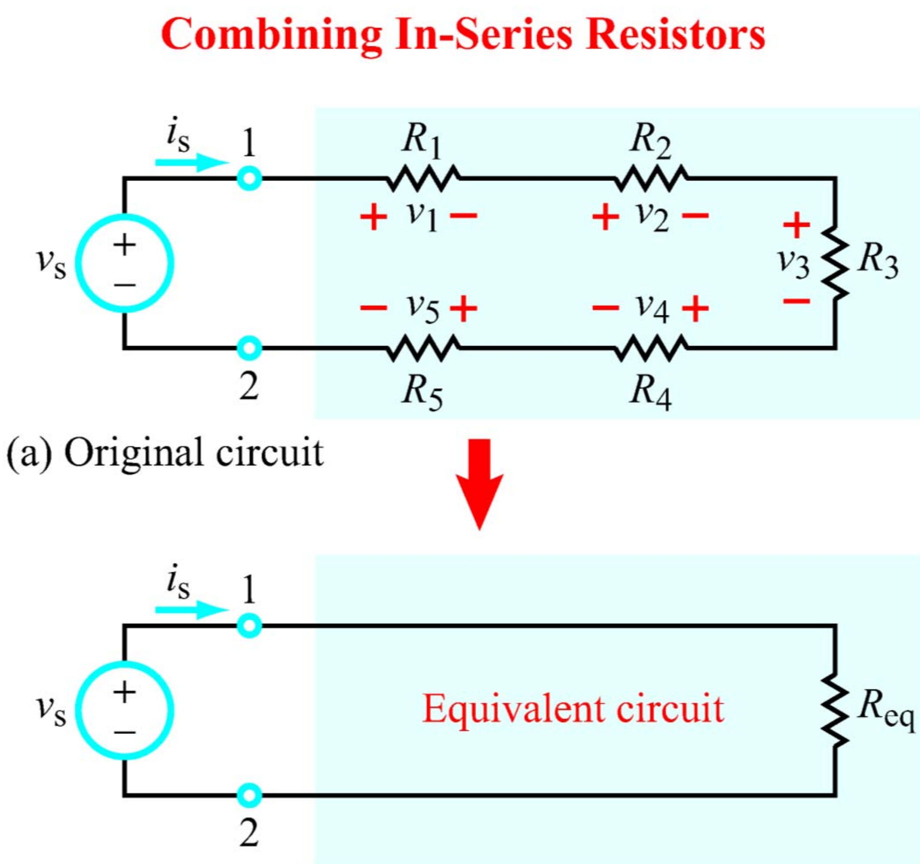
so 1a = (50-40)/5 = 2A, ib = 40/10 = 4A, and ic = 40/40 = 1A

We discussed this earlier -- solving a circuit can often be made easier by first simplifying the circuit using the principle of circuit equivalence:



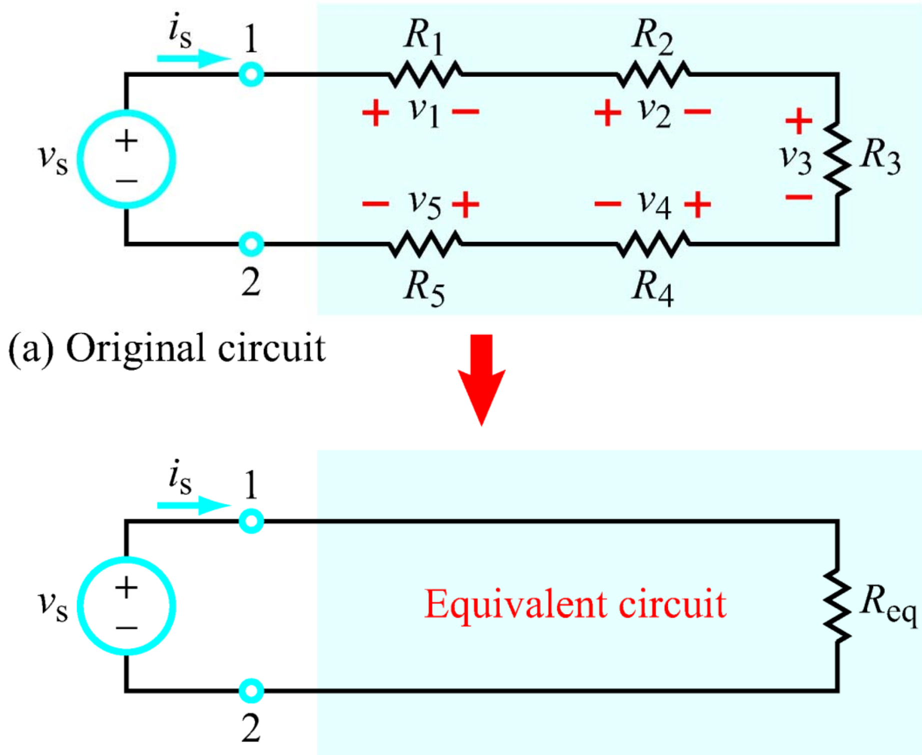

This is an equivalent circuit if the <u>i-v characteristics at nodes 1 and 2 are identical to the i-v characteristics at these nodes in the original circuit.</u>

One simple kind of circuit equivalence is accomplished by combining in-series resistors …forming a single, equivalent resistor:




$$R_{eq} = R1 + R2 + R3 + R4 + R5$$

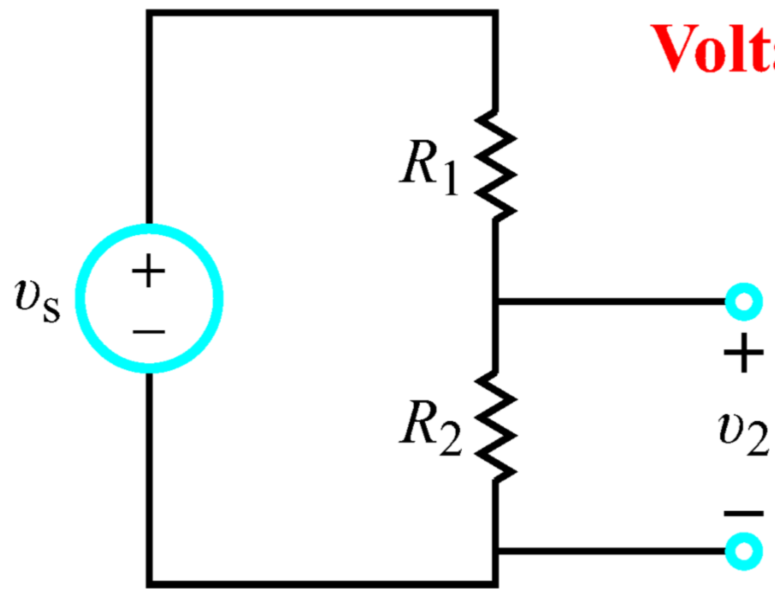
## Combining In-Series Resistors



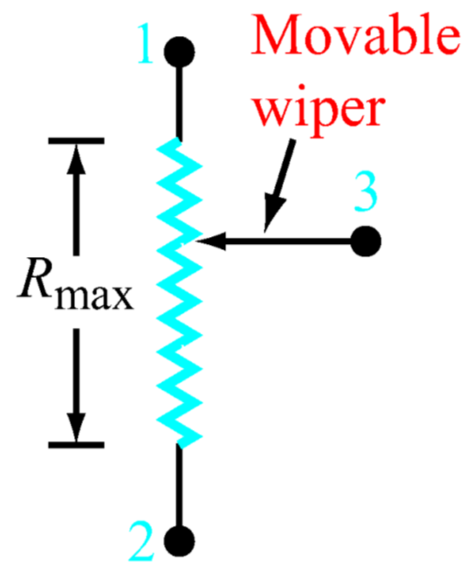

This meets the criterion for an equivalent circuit since $i_s = v_s/R_{eq}$ so the i-v characteristics at nodes 1 and 2 are the same.

The Voltage Divider – you will frequently encounter this configuration.  You should learn to recognize it when you see it.

**Voltage Divider**



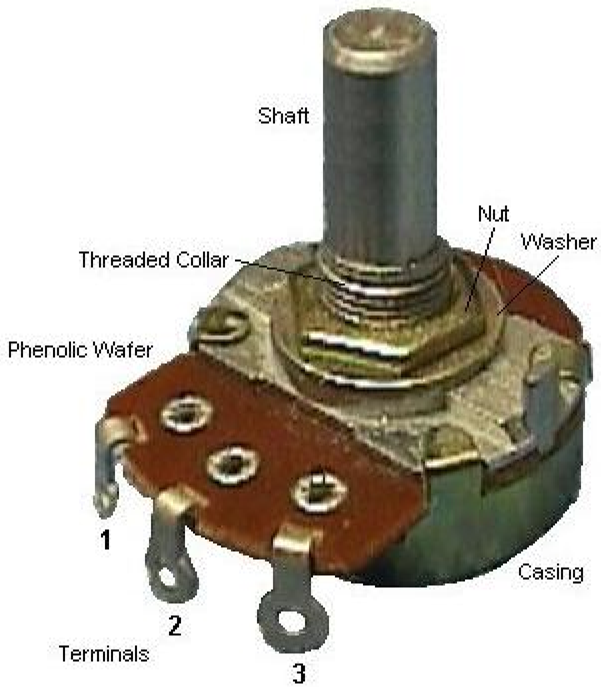

(fixed)

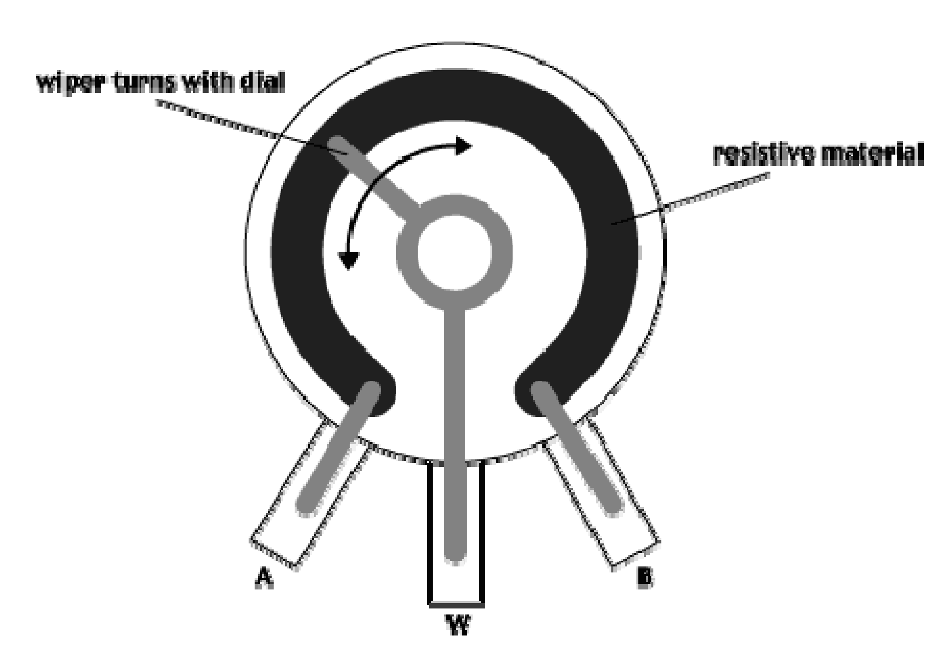



(adjustable)

$$v_2 = \left[ \frac{R_2}{R_1 + R_2} \right] v_s$$

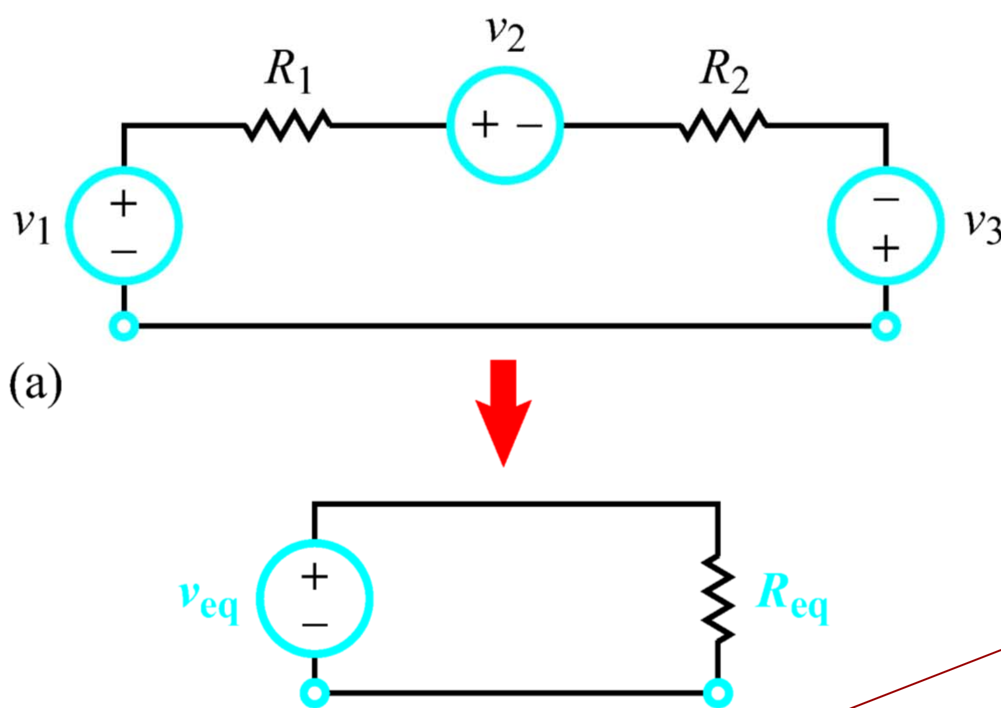
Adjustable voltage dividers (also called potentiometers) are used in many kinds of analog circuits – as gain controls and other kinds of controls where it is desirable to adjust the amplitude of a signal.



Sources in Series

We can also combine sources and resistors in series – like for example:



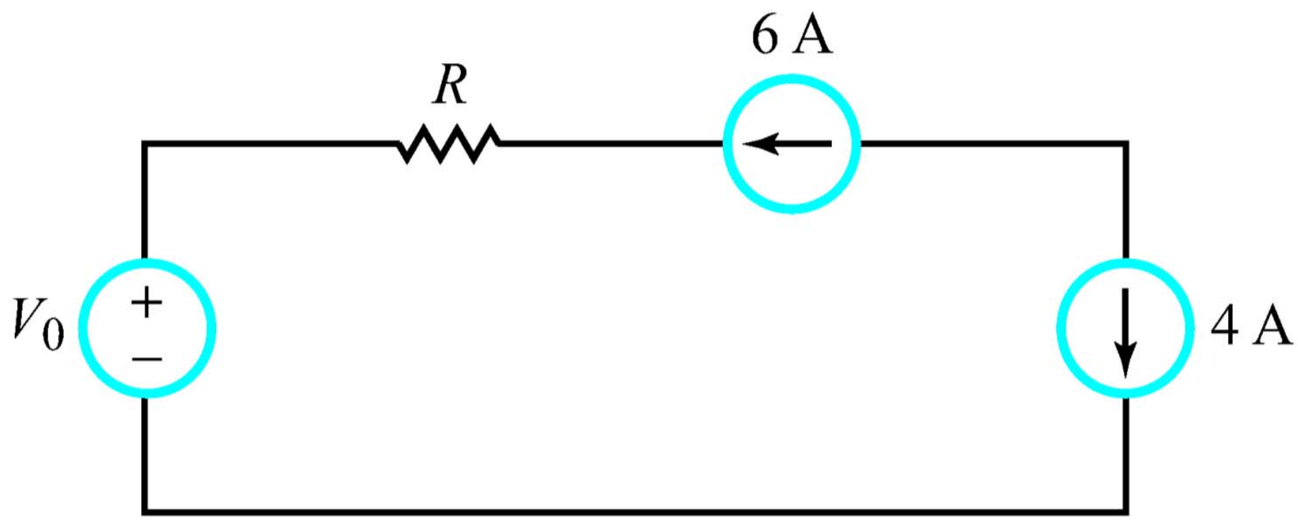

notice

$$v_{eq} = v_1 - v_2 + v_3$$

$$R_{eq} = R_1 + R_2$$

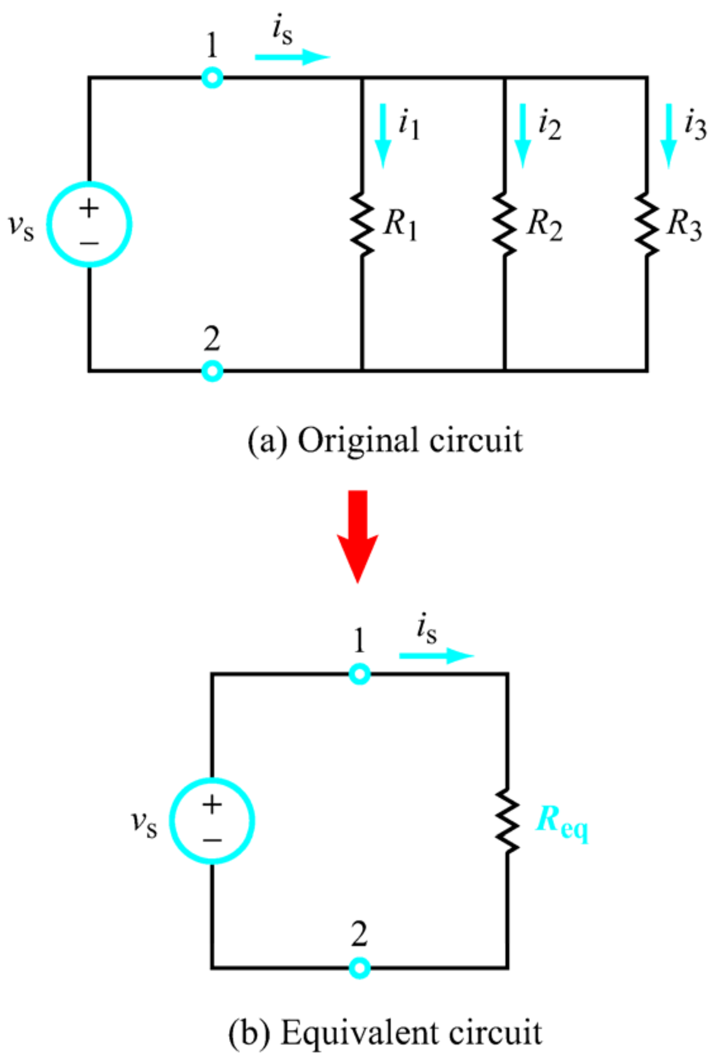
However,

We can only do this in general with voltage sources. We can't put current sources in series if they have different magnitudes or current directions as we saw earlier.




This circuit is unrealizable – is it clear to you why that is true?

## Combining In-Parallel Resistors




$$R_{eq} = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

$$or \quad \frac{1}{R_{eq}} = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

$$i_n = \left( \frac{R_{eq}}{R_n} \right) i_s$$

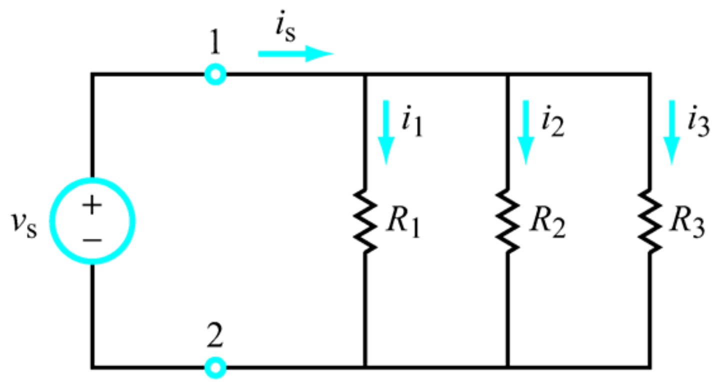
Multiple resistors connected in parallel divide the input current among them

Does it make sense to you that resistors in parallel combine in this way? We talked about this before -- it's actually intuitive if you realize that the conductance of a resistor is just the inverse of its resistance. The lower the value of the resistance …the higher the value of the conductance. If the resistance is infinite the conductance is zero.

Now, if you have several "conductances" in parallel …they will add. The conductance of the combination will just be the sum of the individual conductances.

so
$$\frac{1}{R_{eq}} = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

Basically, each parallel resistor you add provides another conductance channel for the current – so they all add up to give the equivalent conductance. We have derived this expression before using only Ohm's law and the definition of a parallel circuit. Do you remember how to do it?




Resistors in parallel all have to have the same voltage drop across them – that's the definition of a parallel connection.

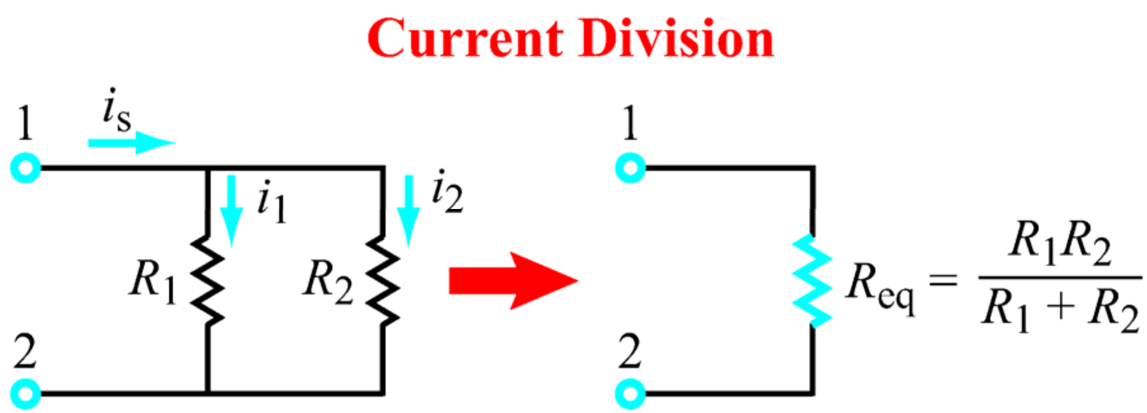
This means $$i_1 R_1 = i_2 R_2 = i_3 R_3 = v_s$$

also $$i_1 + i_2 + i_3 = i_s$$

Ohm's Law says $$v_s = i_s R_{eq} = (i_1 + i_2 + i_3) R_{eq}$$

so $$\frac{1}{R_{eq}} = \left( \frac{i_1}{v_s} + \frac{i_2}{v_s} + \frac{i_3}{v_s} \right) = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

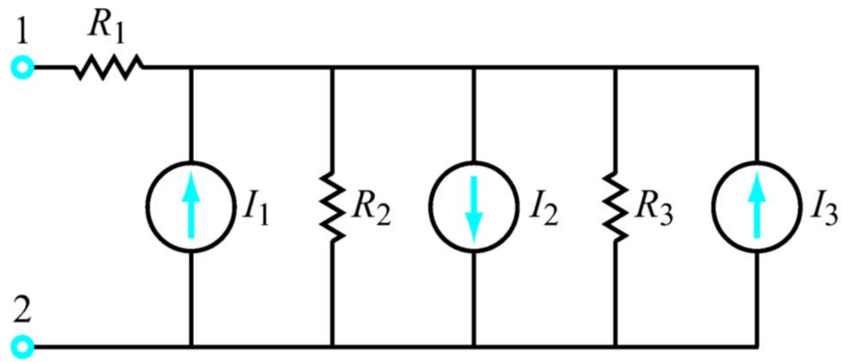
You saw how a voltage divider works – so here is a current divider circuit

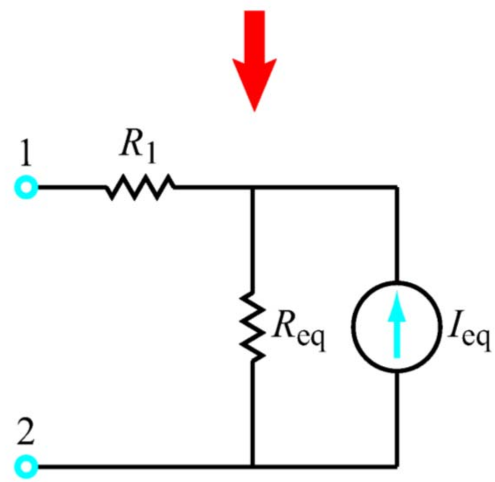



$$i_1 = \left(\frac{R_{eq}}{R_1}\right) i_s \qquad i_2 = \left(\frac{R_{eq}}{R_2}\right) i_s$$

$$= \left(\frac{R_2}{R_1 + R_2}\right) i_s \qquad = \left(\frac{R_1}{R_1 + R_2}\right) i_s$$

We can also combine current sources and resistors in parallel – like for example:

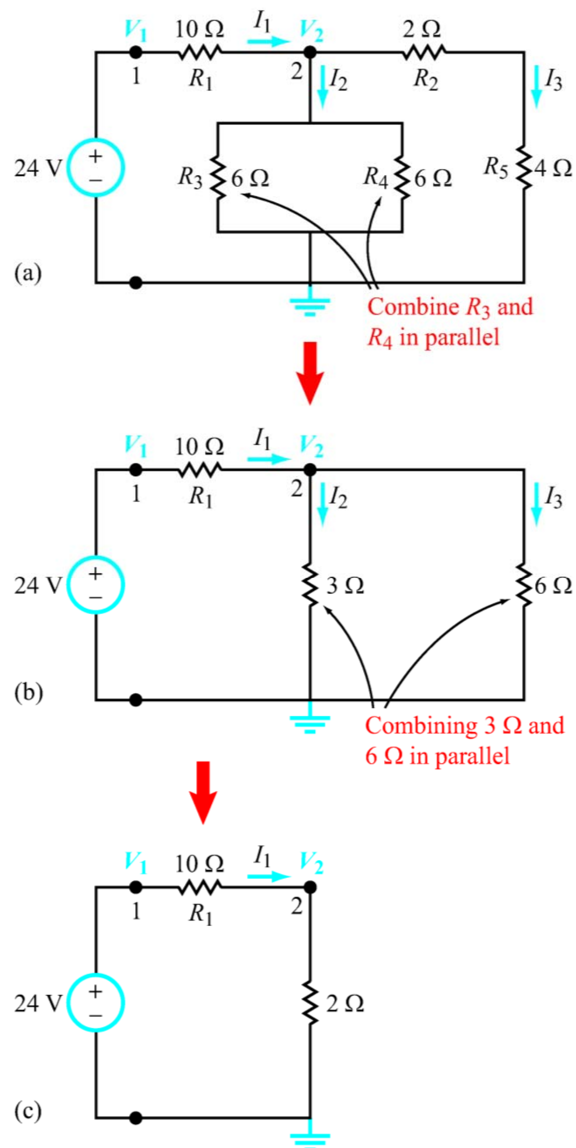



$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3}$$

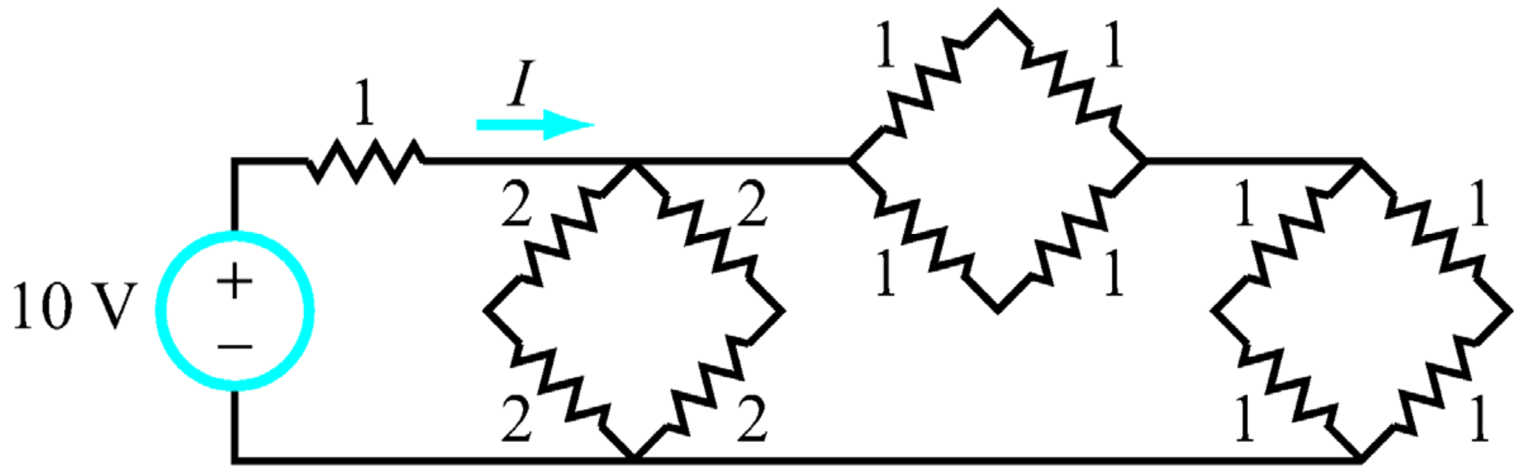
$$I_{eq} = I_1 - I_2 + I_3$$

As we discussed earlier, in general, voltage sources in parallel are unrealizable (aka nonsense).

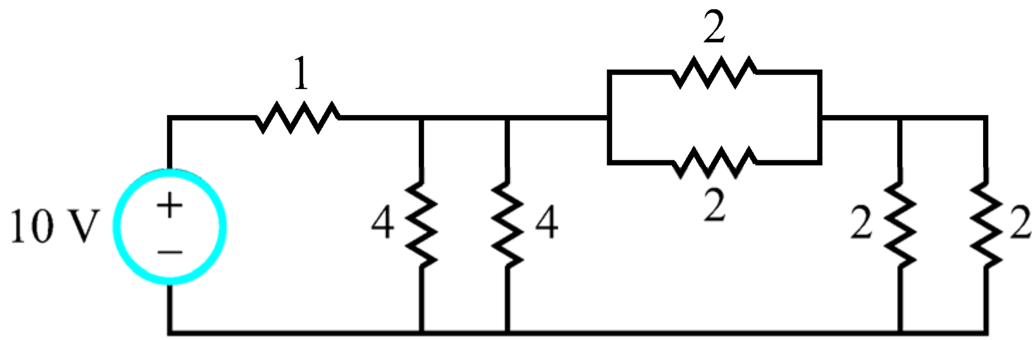



Circuit simplification by series and parallel combinations.

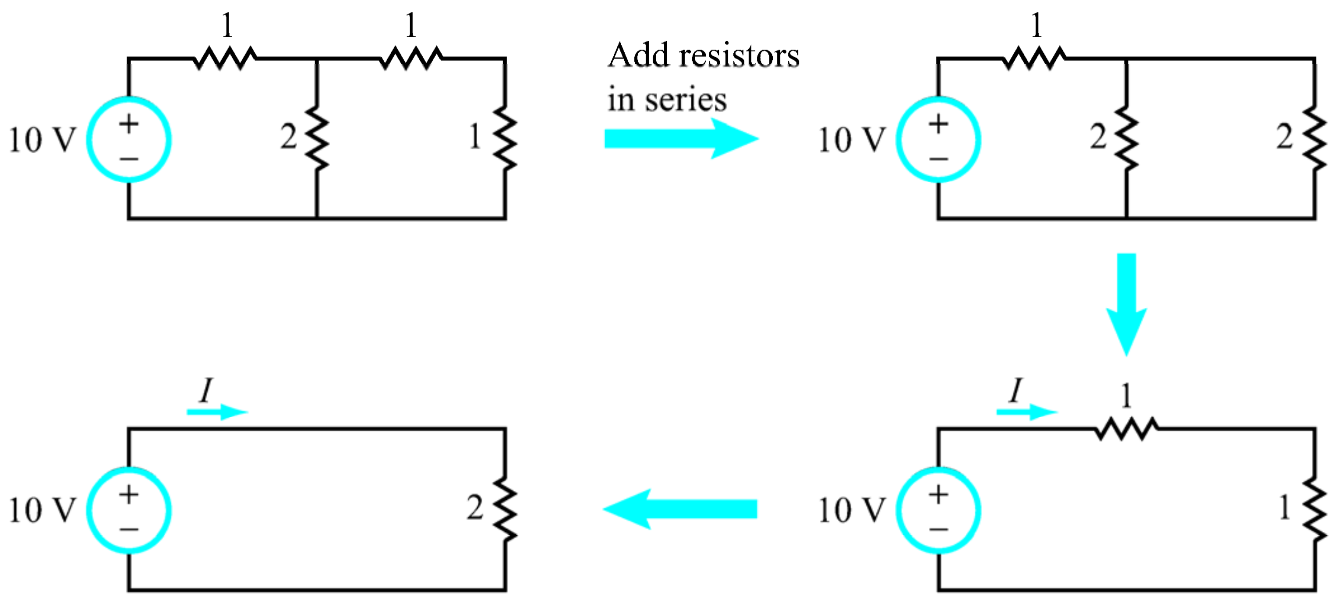
Use series and parallel combinations to simplify this circuit in order to find I



First combine resistors in series

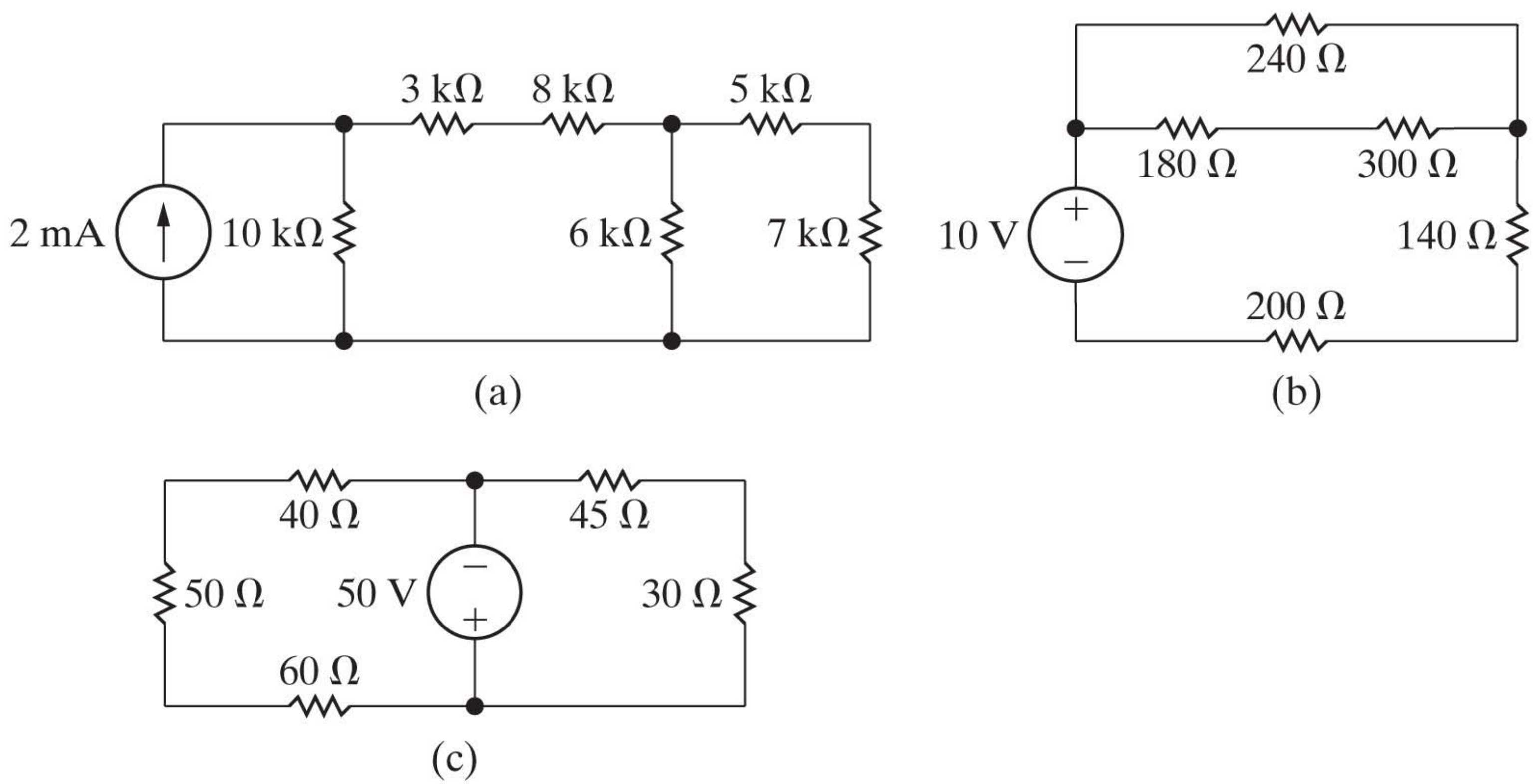



Then combine resistors in parallel

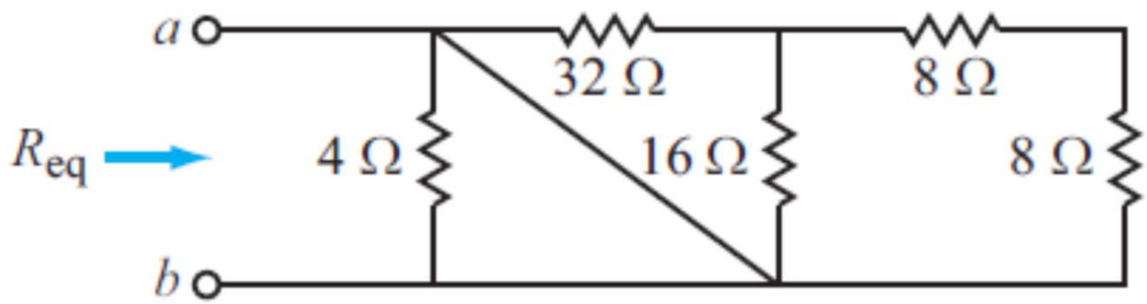



So I = 5A

How about simplifying these three circuits by series and parallel combinations:

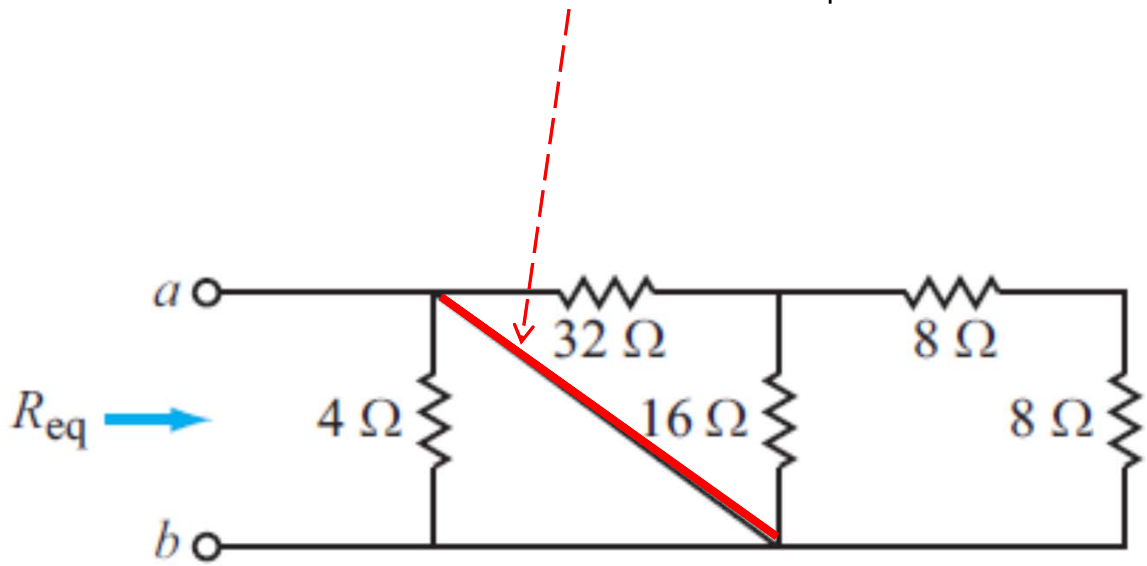


Can this circuit be simplified by series and parallel combinations?




If so, what is $R_{eq}$?

What is this????
It's a short circuit! $R_{eq}$ =0



What is $R_{eq}$ for this circuit with the short removed?