ELEN 50 Class 25 – Steady State AC Power – Power Factor

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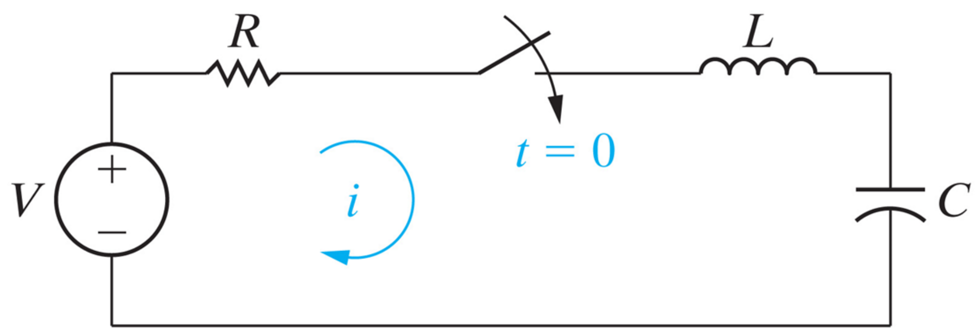
First ……let’s talk briefly about series and parallel RLC circuits

we avoided discussion the transient response of this circuit before …mostly because it is complicated and involves a second order diff. eq.



And after solving the second order diff. eq.

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} = 0$$

there are three possible cases for the series LRC circuit:

Natural Response of a Series RLC Circuit

overdamped

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

underdamped

$$i(t) = B_1 e^{-\alpha t}\cos(\omega_d t) + B_2 e^{-\alpha t}\sin(\omega_d t)$$

critically damped

$$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

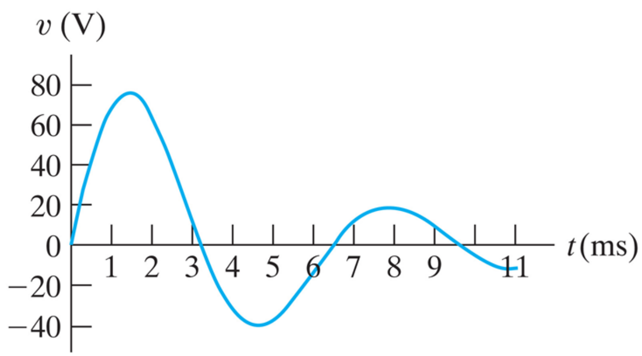
where:

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

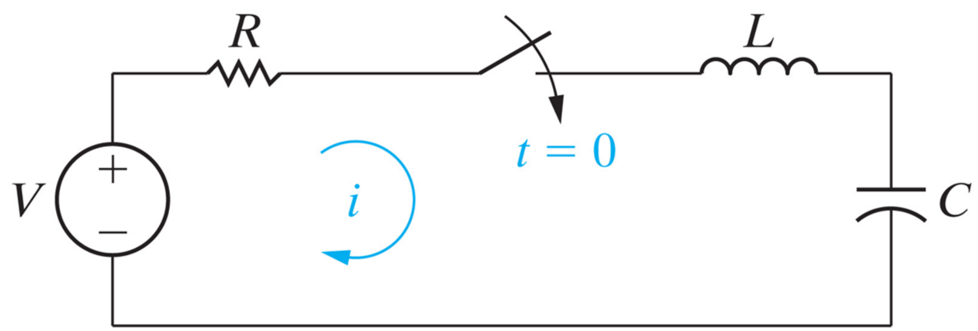
$$s_1 = -\alpha + j\omega_d$$

$$s_2 = -\alpha - j\omega_d \quad where\ \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

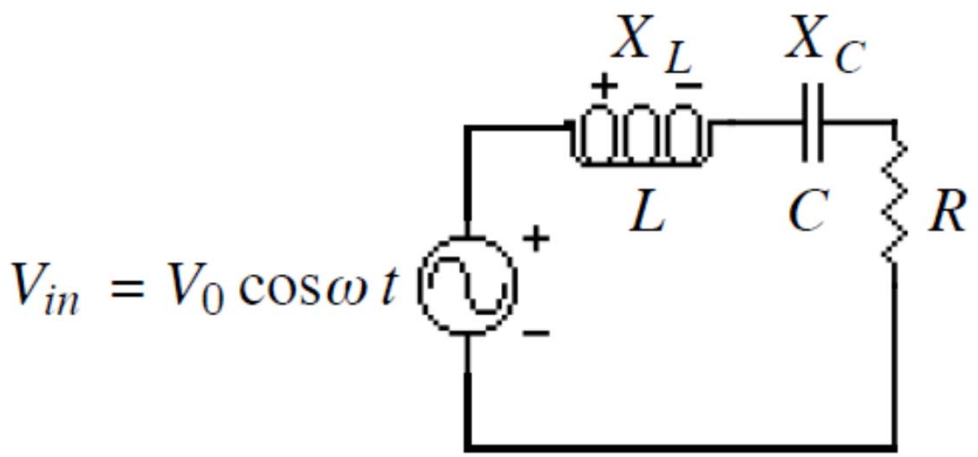
The underdamped solution is particularly interesting because the circuit is showing the property of <u>resonance</u> – a situation where a system is able easily to transfer energy between two or more storage modes – in this case the electric field in the capacitor and the magnetic field in the inductor.



What if, instead of looking at the transient response of this circuit in the time domain, we were interested only in the much simpler, steady state ac response: what would it look like? -- well, we know that the steady state response can be calculated easily using phasors!

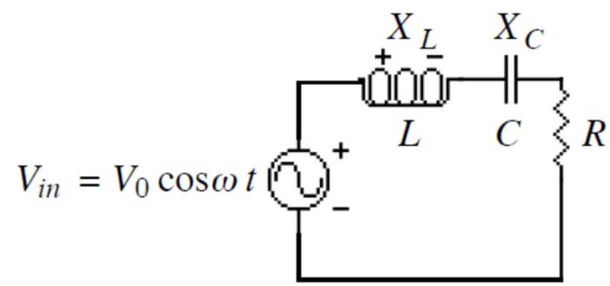


The task is to calculate the voltage across the resistor, R , in steady state with ac excitation as shown:



Without going through a formal analysis of the circuit using phasors, we can <u>use the even easier impedance model (that we talked about last time)</u>. We see that the voltage across the resistor, R can be calculated quite easily by using the voltage divider equation and the complex impedances for the reactive elements.

$$V_R = V_0 e^{j\omega t} \frac{Z_R}{Z_R + Z_C + Z_L} = \frac{V_0 e^{j\omega t} R}{R + j\left(\omega L - \dfrac{1}{\omega C}\right)}$$

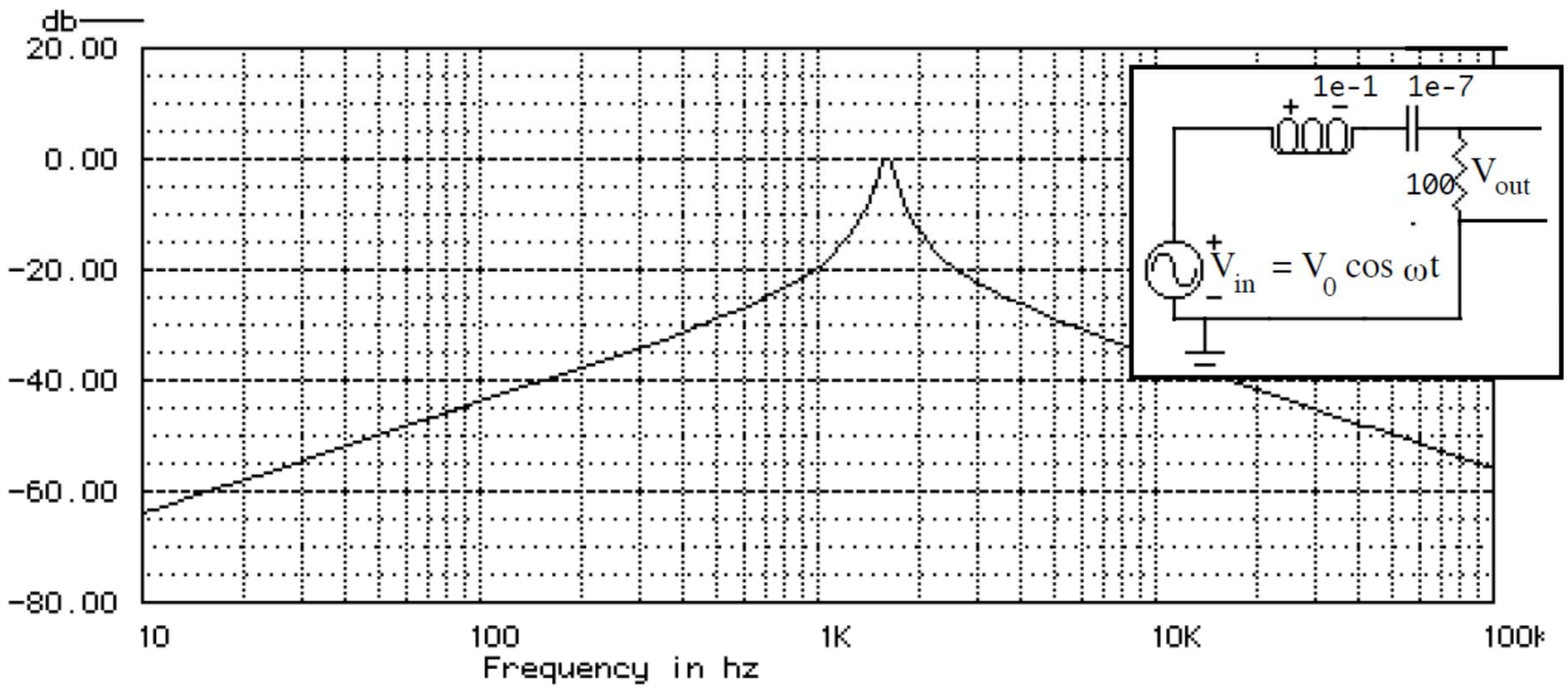


The voltage is complex …containing a magnitude and a phase. We can calculate the magnitude:

$$|V_R| = \frac{\left|V_0 e^{j\omega t}\right| |R|}{\left|R + j\left(\omega L - \dfrac{1}{\omega C}\right)\right|} = \frac{V_0 R}{\sqrt{R^2 + j\left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$

How does the magnitude of the voltage across R depend on ω – for a given choice of L, C, and R? For R= 100Ω, L= 0.1H, and C=0.1 μF: (this result – where we plot V_{out}/V_{in} in decibels is called a Bode Plot

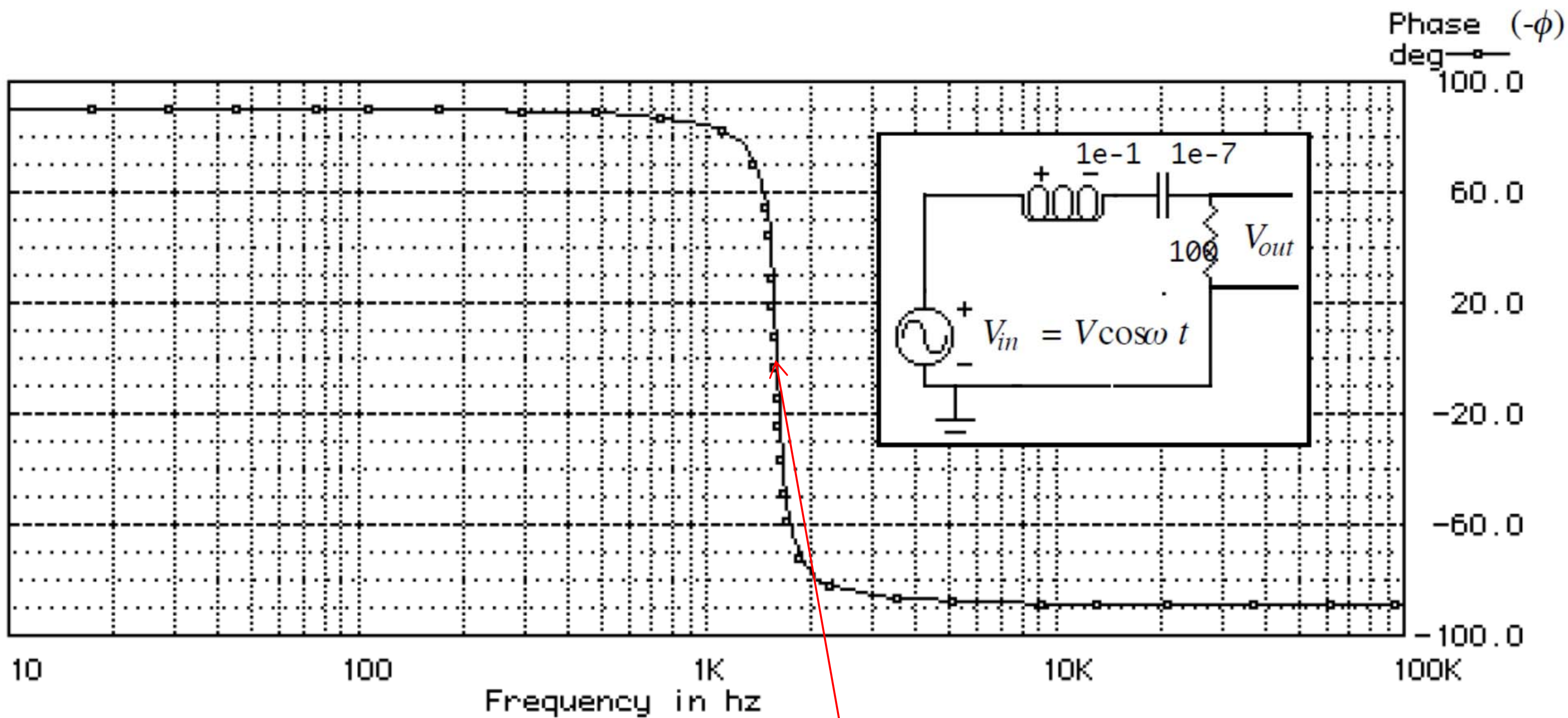
The following Bode plot shows the magnitude of V_R/V_{in} vs. frequency.



This is a band pass filter with a resonant frequency of $\omega_0 = \dfrac{1}{\sqrt{LC}} = 10000\ rad/\sec$

since $\alpha = \dfrac{R}{2L} = 500$ $\qquad \omega_0{}^2 \geq \alpha^2$ $\qquad$ so its underdamped

We can also plot the phase of the sinusoidal voltage signal across the resistor in a Bode phase plot:



So…right at the resonant frequency, the phase shift is zero – the capacitor and inductor phase shifts cancel each other out and the impedance is entirely real….and equal to the value of the resistor.

So …this is an important result for series (or parallel) LRC resonant circuits. At the resonance frequency, the magnitude of the inductive and capacitive impedances are equal …and their phase shifts are (of course) 180 degrees apart:

Since:

$$X_c = \frac{1}{j\omega C} = -\frac{j}{\omega C}$$

$$X_L = j\omega L$$

So the reactive impedances cancel one another out and the impedance of the circuit is entirely real …i.e. zero phase shift.

So ….is the resonant frequency (the frequency at which the impedance is entirely real) given by:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

for every kind of LRC circuit?

No! It's correct for a simple series or parallel LRC circuit

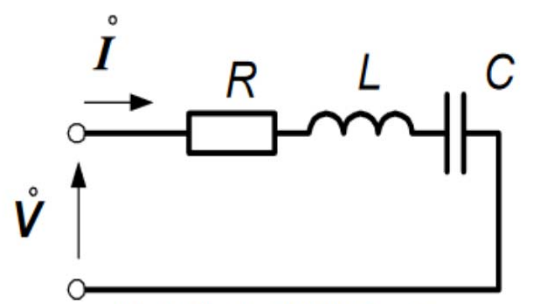


Fig.1. Series RLC circuit

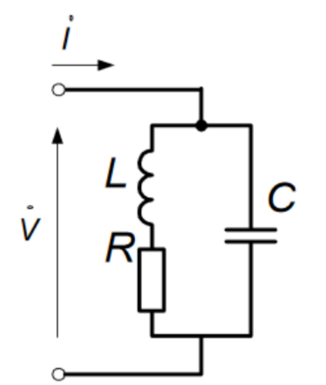


Fig. 5. Parallel RLC resonant circuit

$$\omega_0 L - \frac{1}{\omega_0 C} = 0; \quad \omega_0 = \frac{1}{\sqrt{LC}};$$

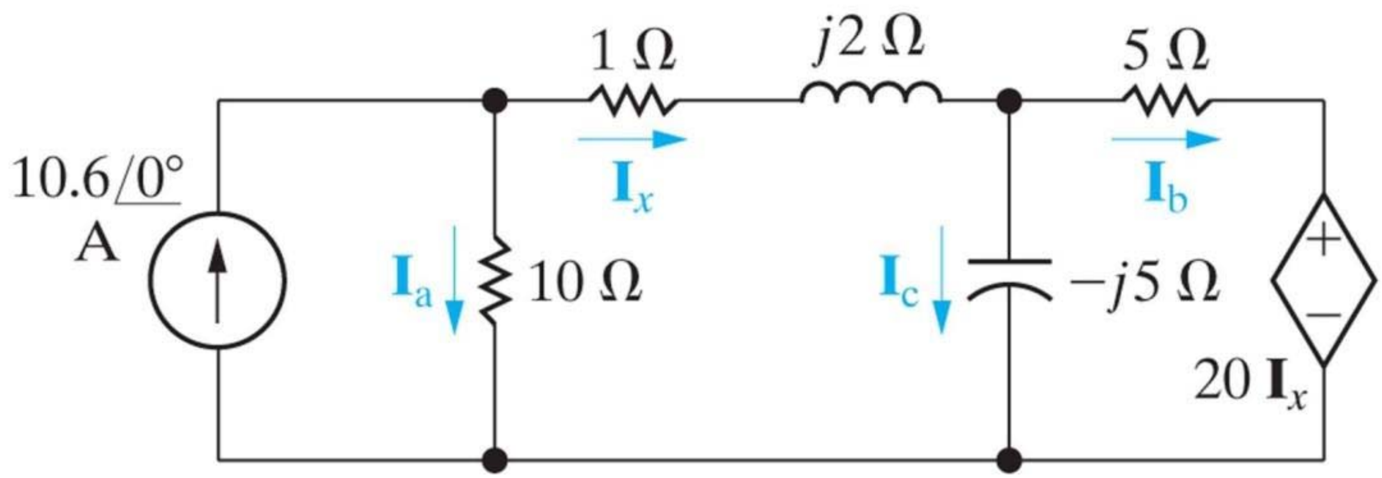
$$\omega_0 L - \frac{1}{\omega_0 C} = 0; \quad \omega_0 = \frac{1}{\sqrt{LC}};$$

but…..if a circuit has multiple capacitors and inductors, the impedances for these circuit elements will combine differently if they are in series or parallel with one another.

Consequently, the best way to determine the equivalent impedance of a circuit is to combine the impedances correctly and then determine the frequency at which the equivalent impedance is real by setting the imaginary part equal to zero and solving for the frequency

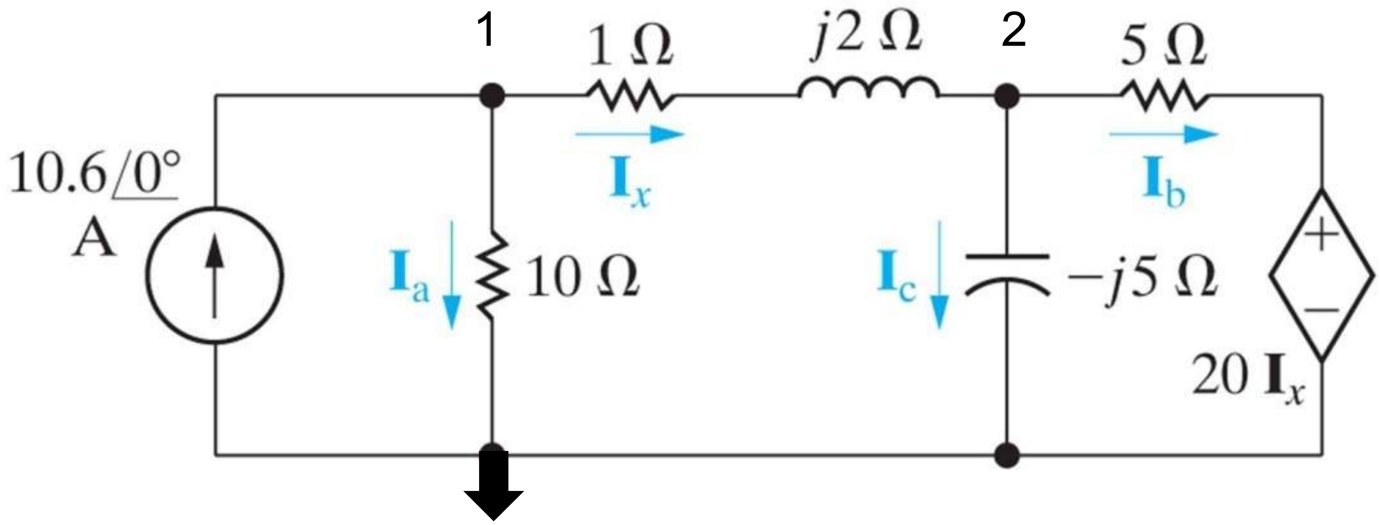
like you did for problem 3 on HW#4

Let’s also take another (slower) look at that circuit we saw last time …where we were going to use phasor analysis on an AC stead-state circuit with a dependant and an independent source



This circuit has already been transformed into the phasor domain – and we don’t know what the original AC source frequency was – so the final solution will have to be simply a phasor voltage.

We are going to do a node voltage solution in the phasor domain.

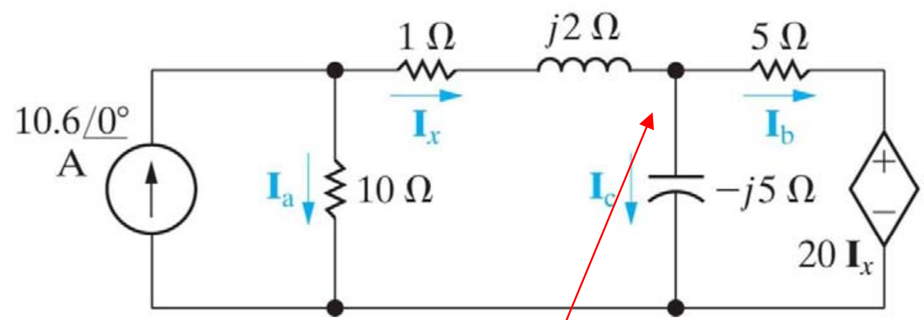


We define voltages at the two extraordinary nodes and define a reference node as shown. Now, we write the KCL equations at the two nodes using phasor voltages.

At node 1:

$$-10.6+\frac{\mathbf{V}_1}{10}+\frac{\mathbf{V}_1-\mathbf{V}_2}{1+j2}=0$$

This looks just like an ordinary node-voltage equation ….except the voltages are phasors ..and the equation has constants that are complex numbers.



$$-10.6+\frac{\mathbf{V}_1}{10}+\frac{\mathbf{V}_1-\mathbf{V}_2}{1+j2}=0$$

Multiplying by (1+j2) and collecting coefficients of the $\mathbf{V}_1$ and $\mathbf{V}_2$ phasors:

$$\mathbf{V}_1(1.1+j0.2)-\mathbf{V}_2=10.6+j21.2$$

Now we write the KCL equation at the second node:

$$\frac{\mathbf{V}_2-\mathbf{V}_1}{1+j2}+\frac{\mathbf{V}_2}{-j5}+\frac{\mathbf{V}_2-20\mathbf{I}_x}{5}=0$$

$$and \quad \mathbf{I}_x=\frac{\mathbf{V}_1-\mathbf{V}_2}{1+j2}$$ the constraint equation

Substituting the expression for $\mathbf{I}_x$ into the 2nd node equation, and multiplying through by (1+j2) …and collecting coefficients for the two phasor voltages:

$$-5\mathbf{V}_1 + (4.8 + j0.6)\mathbf{V}_2 = 0$$

So this equation along with the 1st equation:

$$\mathbf{V}_1(1.1 + j0.2) - \mathbf{V}_2 = 10.6 + j21.2$$

has the simultaneous solution for the phasor voltage: (by substitution)

$$\mathbf{V}_1 = 68.4 - j16.8V$$

$$\mathbf{V}_2 = 68 - j26V$$

from which the branch current phasors can be calculated:

Here are the 2 KCL equations in the phasor domain

In MATLAB ..the code looks like this:

$$\mathbf{V}_1(1.1 + j0.2) - \mathbf{V}_2 = 10.6 + j21.2$$

$$-5\mathbf{V}_1 + (4.8 + j0.6)\mathbf{V}_2 = 0$$

```
>>  A=[1.1+0.2*j, -1; -5, 4.8+0.6*j]

A =

   1.1000 + 0.2000i  -1.0000
  -5.0000             4.8000 + 0.6000i

> B=[10.6+21.2*j;0]

B =

  10.6000 +21.2000i
       0

>> x=B/A

x =

  68.4000 -16.8000i  13.6000 - 5.2000i
```

Notice: MATLAB easily does matrix algebra with complex numbers

Solve using the forward slash command

which are the solutions we got by substitution

Steady State AC Power

Steady-state AC power calculations are, obviously, of great importance to engineers designing electrical distribution systems. These calculations are also important to anyone focusing on efficient distribution and use of electrical energy whether from conventional or alternative energy sources – since even solar and wind generated electrical energy is usually converted to AC power before it is used.

Many electrical devices are purely resistive loads. Examples are heaters, ovens, incandescent lights, etc. Other loads can have significant reactance – electric motors and transformers are two examples. We will develop the concepts of real, reactive, and complex power to deal with these situations.

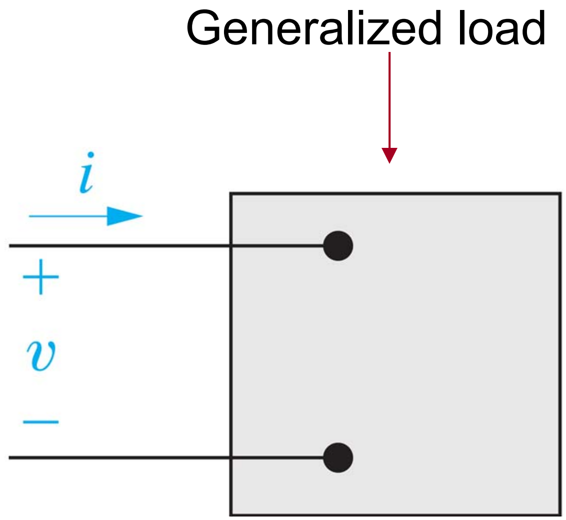
First – here are some useful trigonometric identities (that you probably have learned in high school) – we'll need some of them later.

$$
\begin{aligned}
&\sin x = \pm\cos(x \mp 90^\circ)\\
&\cos x = \pm\sin(x \pm 90^\circ)\\
&\sin x = -\sin(x \pm 180^\circ)\\
&\cos x = -\cos(x \pm 180^\circ)\\
&\sin(-x) = -\sin x\\
&\cos(-x) = \cos x
\end{aligned}
$$

$$
\begin{aligned}
&\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y\\
&\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y
\end{aligned}
$$

$$
\begin{aligned}
&2\sin x \sin y = \cos(x-y) - \cos(x+y)\\
&2\sin x \cos y = \sin(x+y) + \sin(x-y)\\
&2\cos x \cos y = \cos(x+y) + \cos(x-y)
\end{aligned}
$$

Instantaneous Power



The power at any instant of time is given by:

$$p = vi$$

Assume that the current and voltage from the source is of the form:

$$v = V_m \cos(\omega t + \theta_v)$$

$$i = I_m \cos(\omega t + \theta_i)$$

Since the reference for zero time is arbitrary, we can rewrite, without loss of generality, these two expressions as:

$$v = V_m \cos(\omega t + \theta_v - \theta_i)$$

$$i = I_m \cos \omega t$$

Using these expressions

A product of cosines

(1.) $$p = vi$$

(2.) $$p = V_m I_m \cos(\omega t + \theta_v - \theta_i)\cos \omega t$$

now, we use the trigonometric identity for the product of cosines:

$$\cos\alpha \cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$$

notice

To rewrite (2.) as:

(3.) $$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2\omega t + \theta_v - \theta_i)$$

(3.) $$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2\omega t + \theta_v - \theta_i)$$

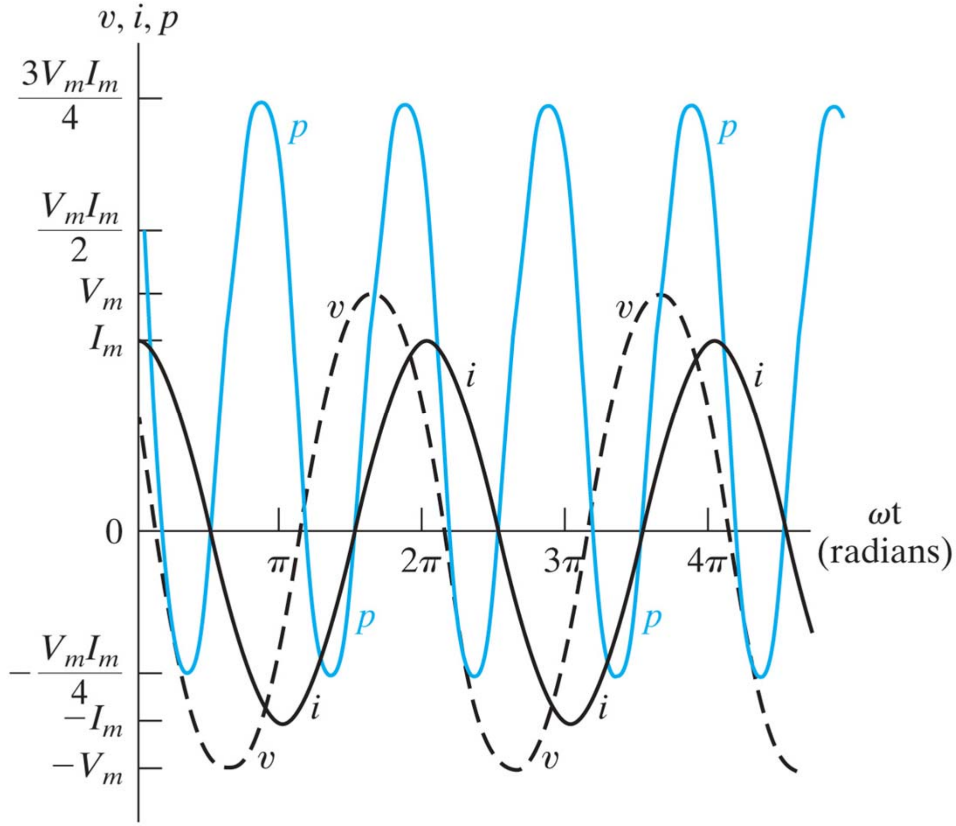
Now, using another identity: $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\,\sin\beta$

We can expand the second term in (3.) to write for the instantaneous power:

(4.) $$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

……let’s look at this result

Plotting Eq. (4.) for θ_v=60°and θ_i=0°, for example:



Notice: the power oscillates at 2ω …and sometimes it's negative – power is being sourced by the load for a portion of the cycle -- because of energy stored in reactive elements in the load. In general, the instantaneous power will sometimes be negative unless the current and voltage are perfectly in phase – i.e. $\theta_v = \theta_i$

(4.) $$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

Average and Reactive Power

We can rewrite (4.) as:

$$p = P + P\cos 2\omega t - Q\sin 2\omega t$$

$$where:$$

(5.)

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$

Average Power or Real Power (measured in Watts)

(6.)

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$

Reactive Power (measured in VAR – volt amps reactive)

The expression in (5.) is called the "Real Power" because it is the power transformed from electrical energy to nonelectrical energy (heat, motion, light, etc.) …we might have called it useful power. It is clearly the average power because:

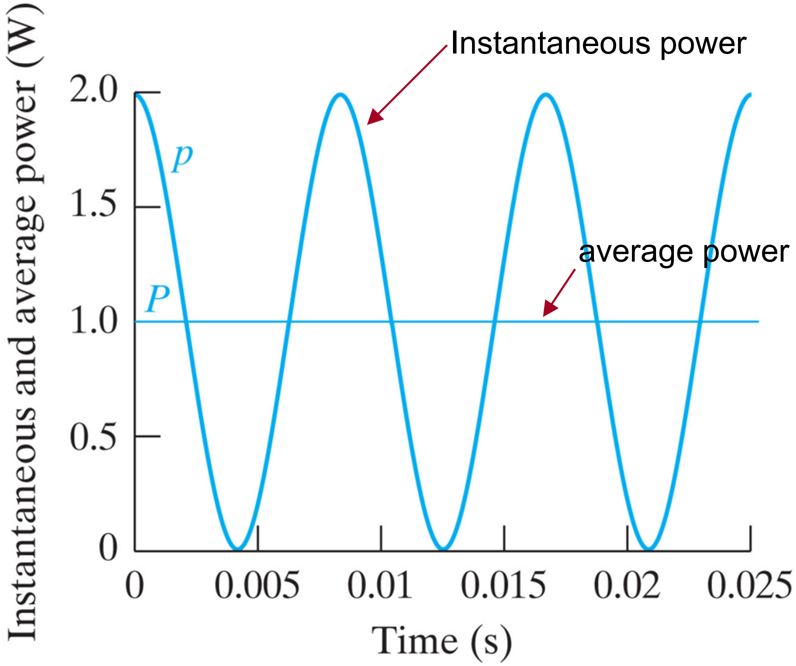
$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p\ dt \quad \ldots\ldots\ldots \int_{t_0}^{t_0+T} \cos 2\omega t\ dt = \int_{t_0}^{t_0+T} \sin 2\omega t\ dt = 0$$

Some examples will make this distinction between real and reactive power clearer:

If the load is <u>purely resistive</u>, we know that the <u>voltage and current are in phase</u>, so $\theta_v = \theta_i$.

$$\text{and} \quad p = P + P\cos 2\omega t = \frac{V_m I_m}{2}\left(1 + \cos 2\omega t\right)$$

$$\text{because} \quad \sin(0) = 0 \quad \text{and} \ \cos(0) = 1$$



$$p = P + P\cos 2\omega t - Q\sin 2\omega t$$

$$\text{where:}$$

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$

If the load is purely inductive, the current and voltage are phase shifted, the current lags the voltage by 90°, so $\theta_i = \theta_v - 90°$, so $(\theta_v - \theta_i) = +90°$.

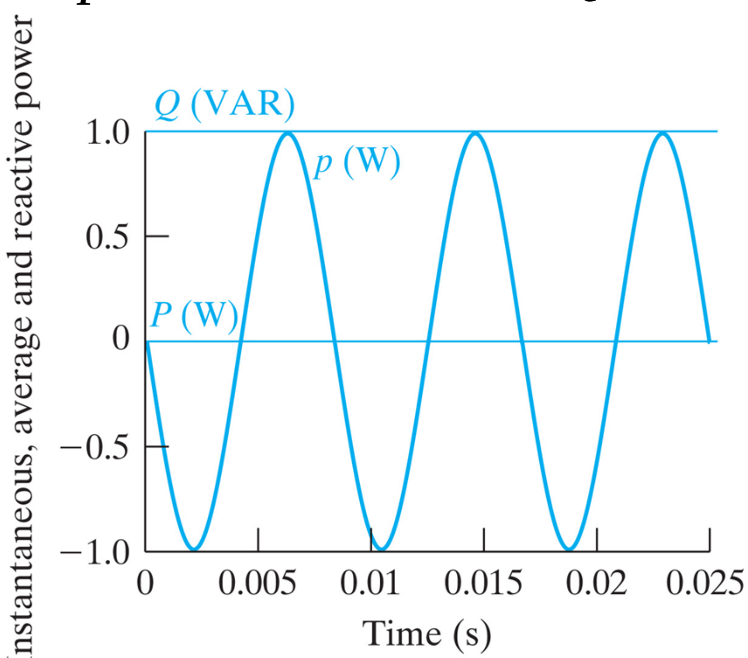
Since cos(90°) = 0 and sin(90°) = 1,

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = 0$$

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = \frac{V_m I_m}{2}$$

So the instantaneous power, p is:

$$p = P + P\cos 2\omega t - Q\sin 2\omega t = -\frac{V_m I_m}{2}\sin 2\omega t$$



And the average power (measured in Watts) is zero – all of the energy delivered to the inductor in one quarter cycle flows back out during the next quarter cycle. Q(VAR) is the peak reactive power …where VAR stands for volt-amp reactive.

Similarly, If the load is purely capacitative, the current leads the voltage by 90°, so $\theta_i=\theta_v+90°$, so $(\theta_v-\theta_i) = -90°$.

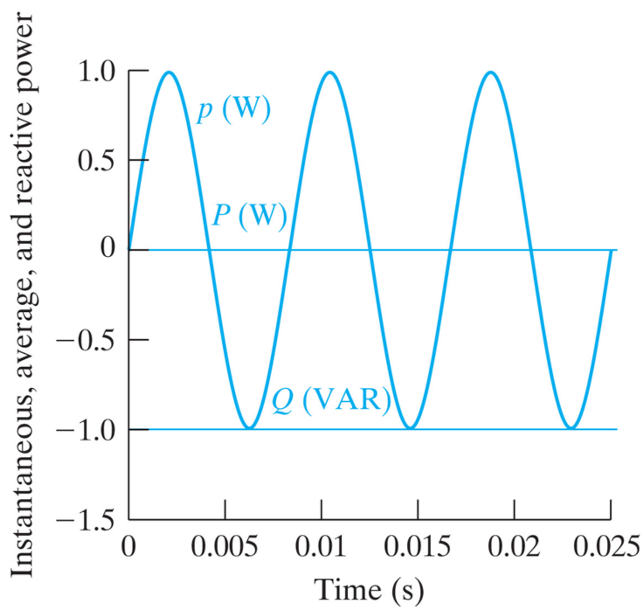
Since cos(-90°) = 0 and sin(-90°) = -1,

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = 0$$

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = -\frac{V_m I_m}{2}$$

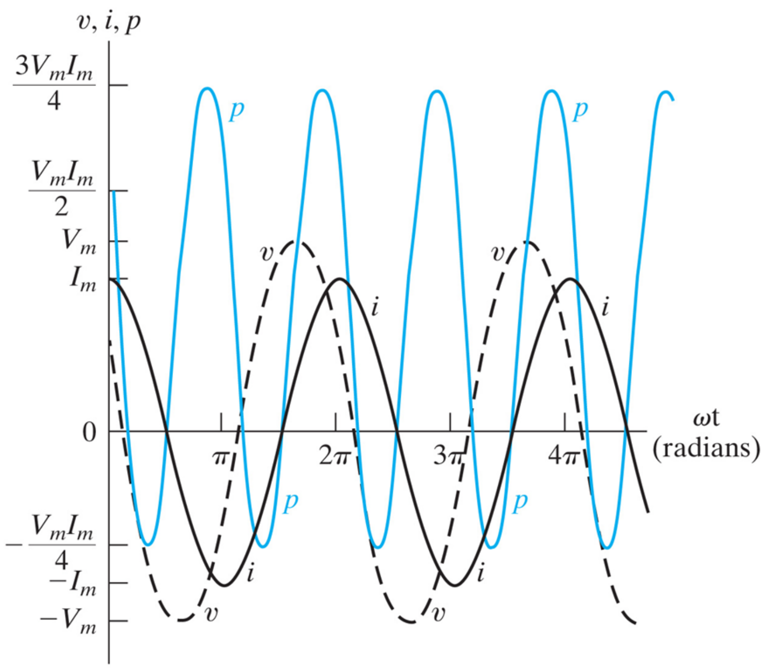
So the instantaneous power, p is:

$$p = P + P\cos 2\omega t - Q\sin 2\omega t = \frac{V_m I_m}{2}\sin 2\omega t$$



And, like the purely inductive load, the average power is zero – all of the energy delivered to the capacitor in one quarter cycle flows back out during the next quarter cycle.

The angle, $\theta_v - \theta_i$, the relative phase shift between the voltage and current, plays an important role in computation of the average and reactive power. The cosine of this angle is called the power factor and the sine of the angle is the reactive factor. Since $\cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v)$, to uniquely define the phase shift angle in terms of power factor, we have to specify a lagging power factor (current lags voltage) or a leading power factor (current leads voltage).



In the example we considered earlier, where $\theta_v = 60^o$ and $\theta_i = 0^o$, the power factor = $\cos(\theta_v - \theta_i) = \cos(60^o) = 0.5$ ..and the current lags the voltage so this load has a lagging power factor of 0.5.

The average power for this load is:

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) = \frac{V_m I_m}{4}$$

And the peak reactive power, Q (VAR) is:

$$3\frac{V_m I_m}{4}$$

Having a load with a power factor of 0.5 is definitely not a good thing for a power engineer …since the overall system has to be designed to deliver an instantaneous VAR three times larger than the actual, useful power used by the load! Even for the commercial end user of electrical power this is not a good thing, since most electrical utilities charge a premium for electricity delivered to commercial loads with power factors significantly different from unity. They have to do this because their <u>line losses increase as the peak reactive power increases.</u>

Typically, residential electric power users are charged only for the real kilowatt hours of electric power used, so residential "power factor correction devices" (often just big capacitors to correct for inductive loads from electric motors) don't save the user any money….although there are many companies offering to sell such "money saving" devices on the Internet and TV!

By the way, a typical home at peak load has a power factor of 0.9 to 0.95 lagging – which means that the load has a slight inductive component….which makes sense given that residential loads are usually either resistors (electric ovens, heaters, hair dryers, etc.) or inductors (mostly electric motors or transformers in power supplies)

The KVAR Energy Controller (KVAR EC™)

A 'Now Solution' for energy efficiency and smart grid

Although global warming has been a long-standing, universal issue, only within the past decade has it become a widespread concern for governments and the marketplace.These days its all about being cleaner and greener, sustainable and responsible, energy wise.

Recognizing a need for lowering electric consumption as a market niche, in 1995 KVAR Energy Savings, Inc. (KVAR Corp.), a Florida State corporation, began manufacturing and distributing the KVAR Energy Controller (KVAR EC™).

The KVAR EC™ is a cost effective product that reduces electric consumption and spans through the residential, commercial and industrial markets, specifically all electrical service from 600 volts and below. The tangible results of using the KVAR ECs™ are real savings on energy bills, reduction in demand for electric utilities, decreased line losses and carbon emissions.

The KVAR Corp. holds a U.S. Patent for the method and apparatus to determine the specific amount of capacitance to bring power factor to unity, thus optimizing all inductive loads. To prove its reliability and savings, KVAR ECs™ have successfully been tested and received endorsements from NASA, Honeywell and SMT Engineering LLC., an independent third party. The KVAR EC™ is UL and CSA listed, CE certified and meets RoHS (Restriction on Hazardous Substances Directives). The RoHS compliance qualifies KVAR ECs™ as Green Products since all its electric and electronic components are compliant.

NIST has analyzed residential “power factor corrector” units, and has concluded:

“If the air conditioning unit runs for 12 hours each day, the energy savings will be about 52.8 watt-hours per day. At 20 cents per kilowatt-hour, the money saved by the utility would be approximately 1 cent per day. Since in most parts of the United States air conditioners only operate for less than six months of the year, the utility’s annual savings would be about $1.80 for a single residence.”

The smallest KVAR unit costs about $100 …so it will pay for itself in saved electric utility costs (based on the air conditioner analysis) in about 56 years! If you don’t use your central air conditioner 12 hours a day for six months out of the year …it will take longer than 56 years.

Just for Fun --- I Measured the Power Factor of Some Household Appliances

Remember …power factor is the cosine of the phase shift between current and voltage

PF=1.00

Power = 424 W

Power Factor of Some Household Appliances
Paper Shredder

PF = 0.89

Power = 51.2 W

Power Factor of Some Household Appliances
120V Tungsten Halogen Light and Dimmer Control

dimmer set on half power

PF = 0.57

Power = 114 W

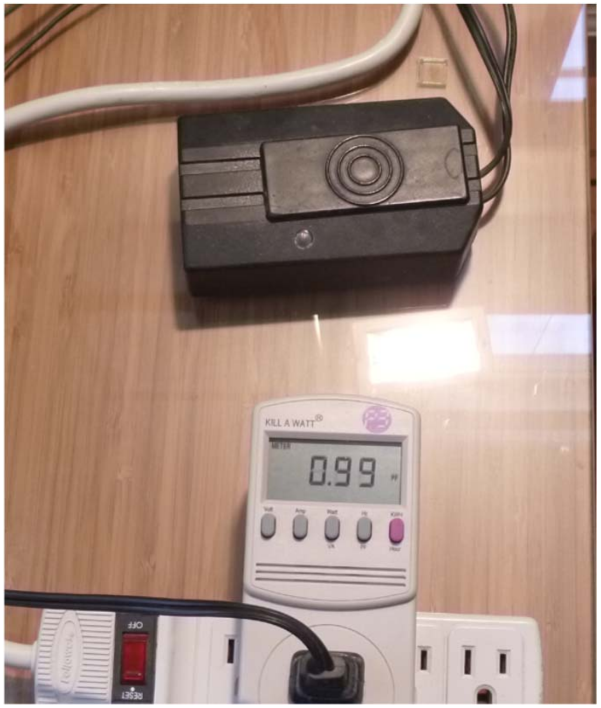
Wow!

Power Factor of Some Household Appliances
120V Tungsten Halogen Light and Dimmer Control

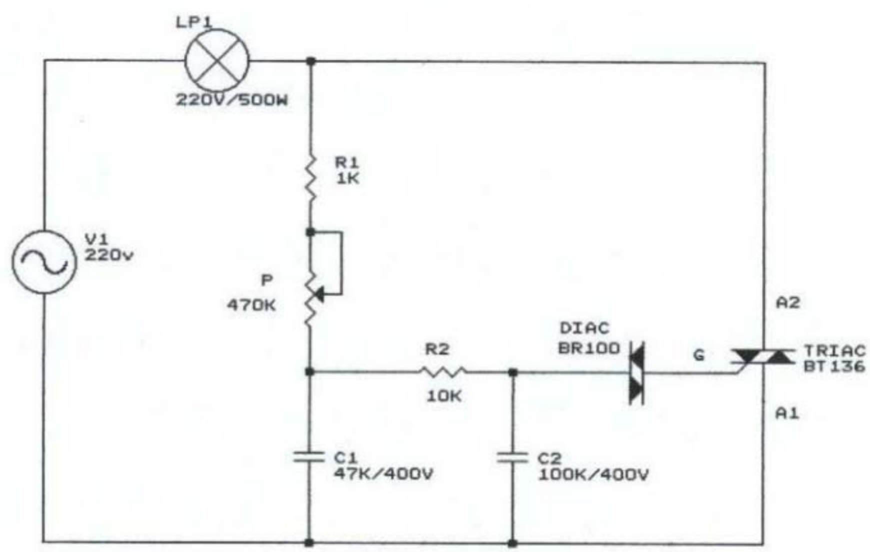
dimmer set on full power

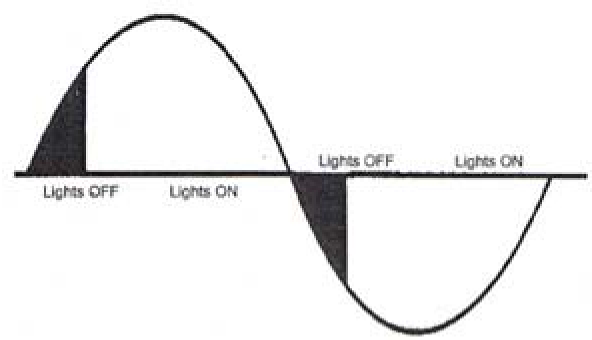
PF = 0.99

Power = 266 W



Why was the power factor so lousy when the light dimmer was working? I'm not sure. It's possible that the way the meter uses to calculate the power factor assumes the voltage and current are pure sine waves …with just a phase shift between them. The light dimmer circuit actually really messes up the waveform of the sine wave as you can see below. When the dimmer is fully on (light is brightest) the sine wave is perfect …and the power factor is measured at 0.99 …what we expect for a purely resistive load.





25% Dimmed