ELEN 50 Class 13 – Mesh Current Method, Solution by Inspection

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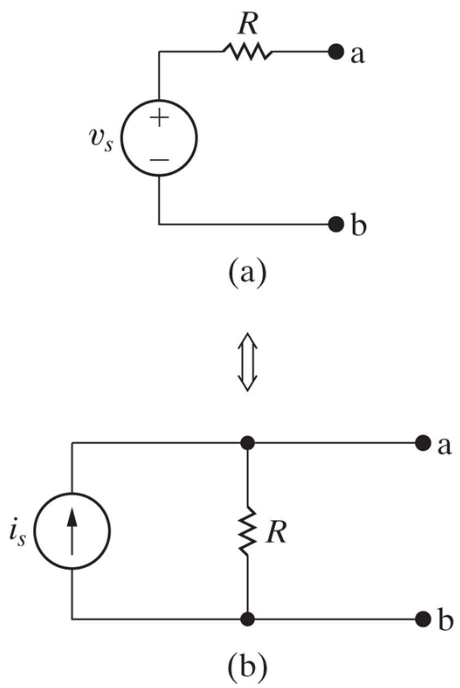
A Quick Review of Thevenin Equivalent Circuits and Max Power Transfer– sorry for the repetition, but I really want you to know this!

Source transformations are another important circuit simplification scheme like parallel and series combinations and Wye ← → Delta transformations.

A source transformation allows a voltage source in series with a resistor to be transformed into a current source in parallel with a resistor.



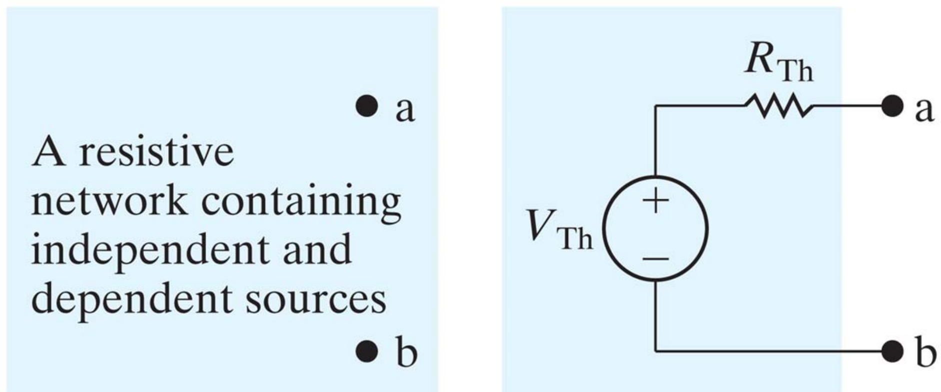
If these two circuits are equivalent, then:

$$\boxed{i_s = v_s/R}$$

You can show this by attaching a load resistor to a and b and calculating the current flowing

Thevenin Equivalent Circuit

Thevenin (and Norton) equivalent circuits are ways of replacing complicated and/or irrelevant parts of a circuit with a simple circuit based on the behavior of the circuit at a particular pair of terminals using source transformations.



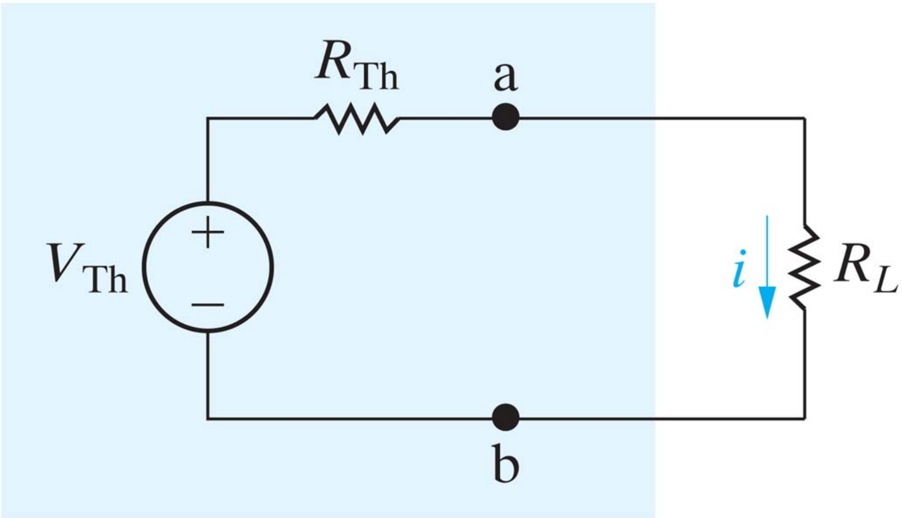
(a) (b)

The Thevenin equivalent circuit for a resistive network is a <u>voltage source, V_{th} in series with a resistor, R_{th}.</u> We chose these values by measuring the open circuit voltage for the network …this is V_{th} …then we measure the short circuit current out of the network. Since this will be V_{th}/R_{th}, the value of R_{th} is: $R_{th} = V_{th}/i_{sc}$. The Norton equivalent is just the source transform of the Thevenin circuit into a current source in parallel with a resistor. We'll discuss Thevenin equivalent circuits more later in the quarter.

Thevenin Equivalent Circuit – General Calculation Strategy

- **Obtain vth** by calculating the voltage across the two specified terminals when no load is present (open circuit voltage)
- **Obtain Rth** by either:
  - Calculating the current that will flow between the specified terminals in a short circuit. Rth is obtained from Rth = vth/Isc OR
  - If the circuit doesn't contain dependent sources, you can calculate the equivalent resistance between the specified terminals after all independent voltage sources are deactivated (replaced with short circuits)and all independent current sources are deactivated (replaced with open circuits). The equivalent resistance is Rth, the Thevenin resistance. OR
  - If the circuit contains independent and dependent sources, Rth can be determined by deactivating independent sources, and adding an external source (vex or iex)…then solve the circuit to determine the current iex or voltage vex supplied by the external source. Rth = vex/iex

For a circuit with a Thevenin equivalent resistance of R_{Th} , maximum power will be transferred to a load resistor, R_L, if $R_L = R_{Th}$



When this is the case, the power transferred will be: $p_{max} = V_{Th}^2 / 4R_L$

ELEN 50 Class 13 – Mesh Current Method

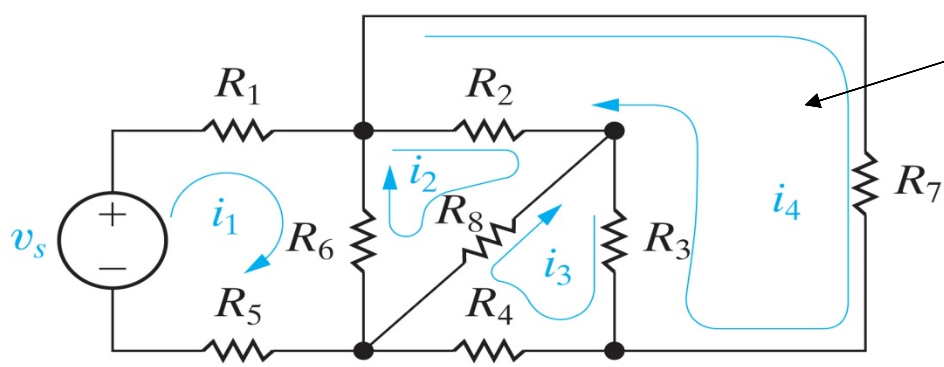
- The mesh current method is another approach to circuit analysis….complementary to the node voltage method.

- We've seen that the node voltage method allows analysis of a circuit using $n_e - 1$ equations where n_e is the number of essential nodes in the circuit.

- The mesh current technique allows analysis of a circuit with $b_e - (n_e - 1)$ equations ….where b_e is the number of essential branches in the circuit. Remember, an <u>essential branch is a path that connects two essential nodes without passing through an essential node.</u>

- $b_e - (n_e - 1)$ is the number of <u>meshes</u> in the circuit.

- Remember, <u>a mesh is a loop drawn in the circuit so that there are no other loops inside of it</u> – and also remember that the mesh current approach <u>only works on planar circuits</u>.



The mesh currents are shown here

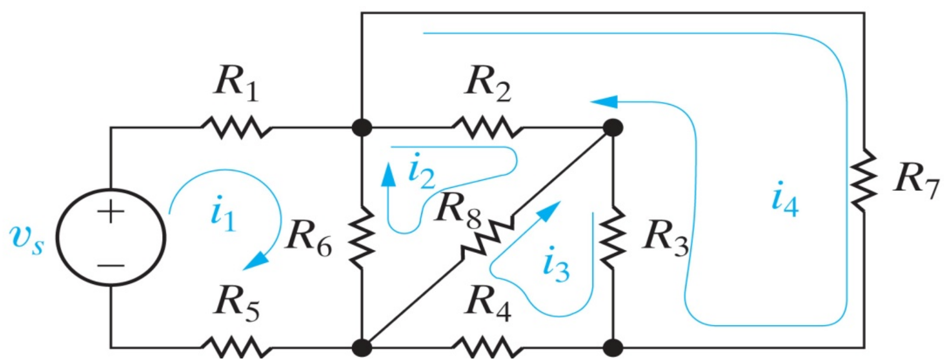
- The mesh current method is completely complementary to the node voltage method …but it uses mesh currents instead of node voltages….and it explicitly uses KVL instead of KCL.

Node Voltage Method	**Mesh Current Method**
• Identify essential nodes	• Identify mesh currents
• Select reference node	• no reference node needed since we're explicitly calculating currents and not voltages.
• Label voltages at remaining essential nodes (v1, v2, ….vn)	• Label mesh currents (ia, ib, …in)
• Write equations for KCL <u>at these nodes</u> in terms of node voltages referenced to reference node.	• Write equations for KVL <u>around the mesh current paths.</u>
• Solve n equations in n unknowns	• Solve n equations in n unknowns

- Are there fewer equations required using the mesh current method than the node voltage method? If not, why would you ever do it?

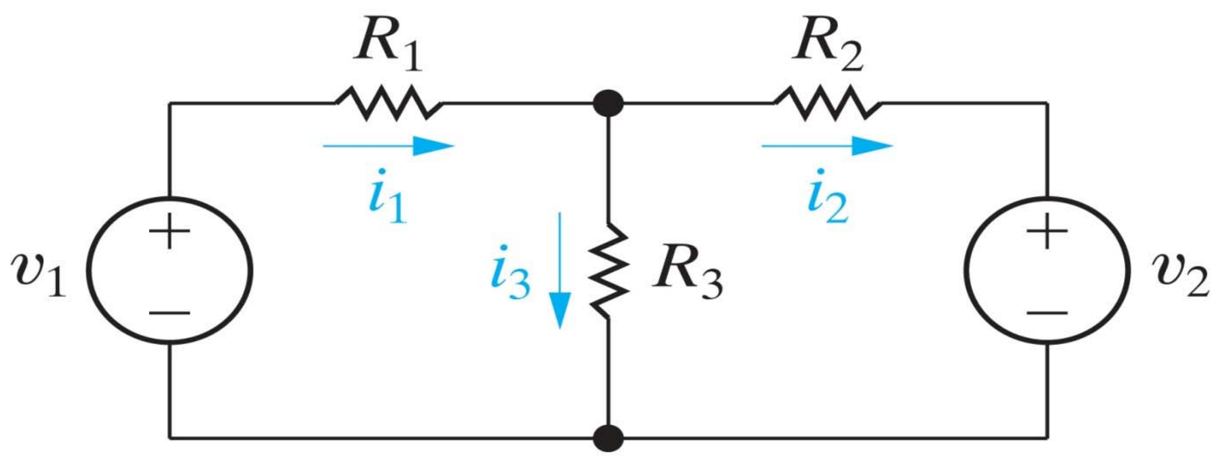
- This depends on the circuit -- we'll do a detailed comparison later and talk about how to choose between these two techniques.

- For now….we'll just learn how to use the mesh current method.

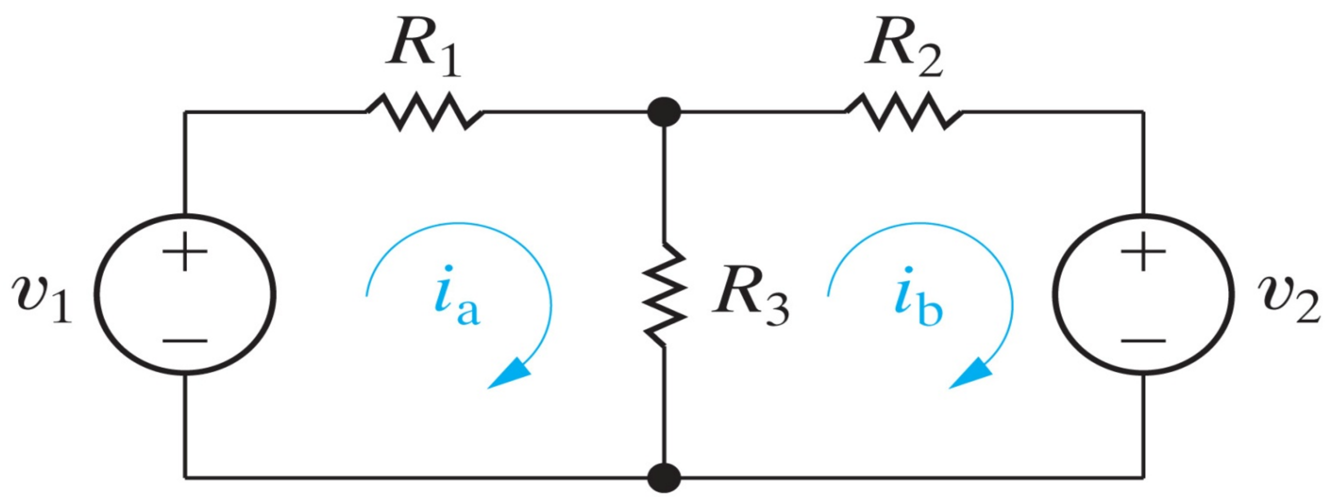


- For this circuit, there are 4 meshes and 3 essential nodes plus the reference node – so the node voltage method would actually use 1 fewer equation than the mesh current method!

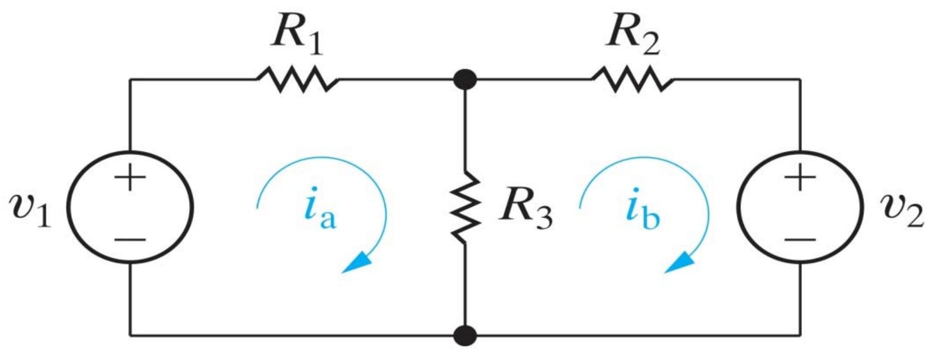
Let's look at a simpler circuit and solve it both ways:



These are the branch currents associated with the essential node in this circuit – but , for the mesh current technique, we're not interested in branch currents ….we're looking for <u>mesh currents</u>. What is the difference?



These are the mesh currents for this circuit. Notice that <u>mesh currents aren’t necessarily real branch currents</u>, but they are complete loops … drawn so no other mesh currents are inside them. In this circuit, the branch current through the R_3 resistor is actually the algebraic sum of the mesh currents i_a and i_b.



Now we write the KVL for each of the mesh currents:

(remember KVL?? The sum of the voltage drops around a closed loop is zero) as a convention we will assign polarity to a voltage drop depending on the sign that is first encountered by the mesh current as we move around the loop.

This is why this term is negative

For the ia mesh: ia R1 + (ia – ib) R3 – v1 = 0

…and this term is positive

For the ib mesh: (ib – ia) R3 + ib R2 + v2 = 0

So, in standard form:

ia (R1+ R3) – ib (R3) = v1

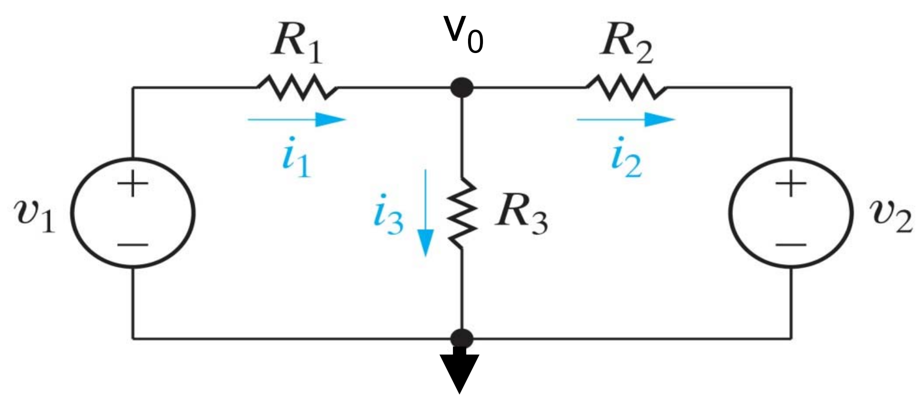
ia (-R3) + ib (R2 + R3) = -v2

And we can solve the circuit.

Notice that there is something interesting about the matrix equation for the mesh current analysis. The off-diagonal terms of coefficient matrix are symmetric around the diagonal!

$$
\begin{aligned}
ia\,(R1 + R3) + ib\,(-R3) &= v1 \\
ia\,(-R3) + ib\,(R2 + R3) &= -v2
\end{aligned}
$$

It turns out that this will always be the case for circuits that don't have dependent sources – and this forms the basis for a technique called "mesh current solution by inspection." Using the "by inspection" technique …you can basically just write down the matrix equation for a circuit directly. We'll talk about this more in a bit.



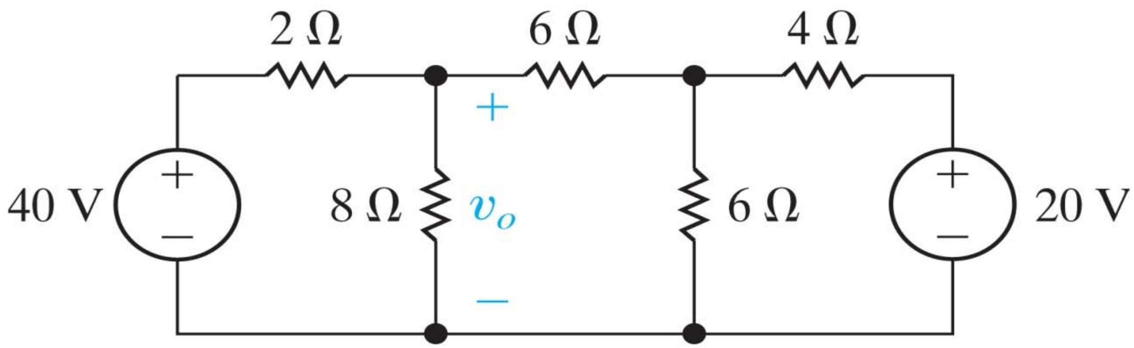
Notice, if we had done the analysis by the node voltage method, we'd only need one equation (one essential node other than the reference):

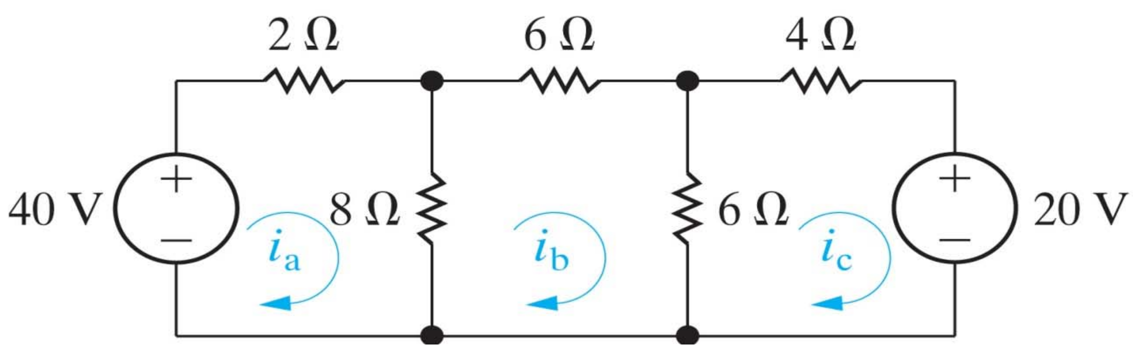
$$-(v0 - v1)/R1 + v0/R3 + (v0 - v2)/R2 = 0$$

Once we know v0, we can calculate any of the branch currents from Ohms Law.

Clearly, for this circuit, the node voltage method wins in terms of simplicity.

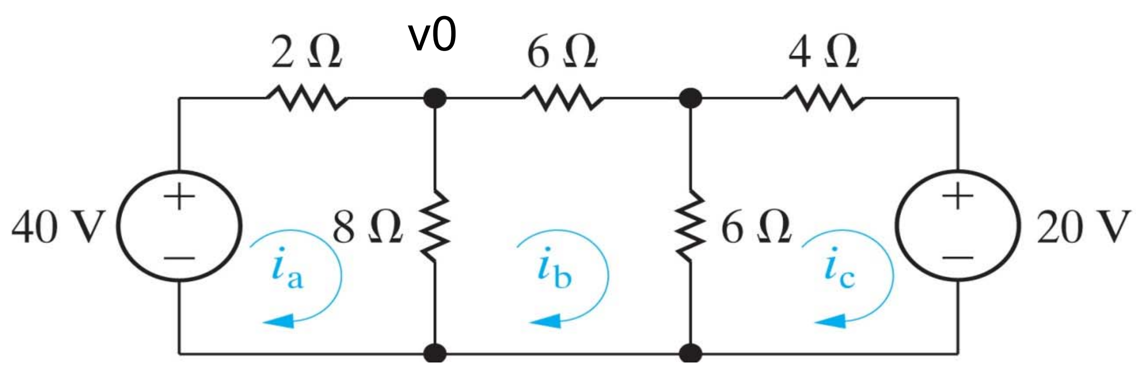
How about this circuit? We want to calculate v0





There are three mesh loops, so we'll need three KVL equations. Notice, if we'd done it by the node voltage method, even though there are 4 essential nodes, the two at the bottom get combined as a reference node, so we'd only need two KCL equations – here node voltage wins also …but we'll do the mesh current analysis anyway.

Here's the circuit:



For the three meshes, the KVL equations are:

$- 40 + 2\, i_a + 8\, (i_a - i_b) = 0$

$8\, (i_b - i_a) + 6\, i_b + 6\, (i_b - i_c) = 0$

$6\, (i_c - i_b) + 4\, i_c + 20 = 0$

In standard form:

10 ia – 8 ib + 0 ic = 40

-8 ia + 20 ib - 6 ic = 0

0 ia - 6 ib + 10 ic = -20

Here's that symmetric coefficient matrix again!

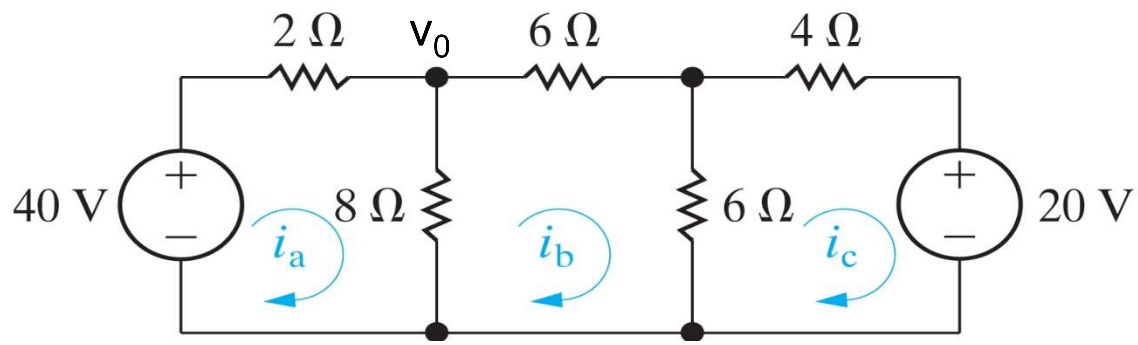
$$\begin{bmatrix} 10 & -8 & 0 \\ -8 & 20 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 40 \\ 0 \\ -20 \end{bmatrix}$$

This set of equations can be solved by, for example, Cramer's Method, or with MATLAB and we get:

i_a = 5.6 A

i_b = 2.0 A

i_c = -0.8 A

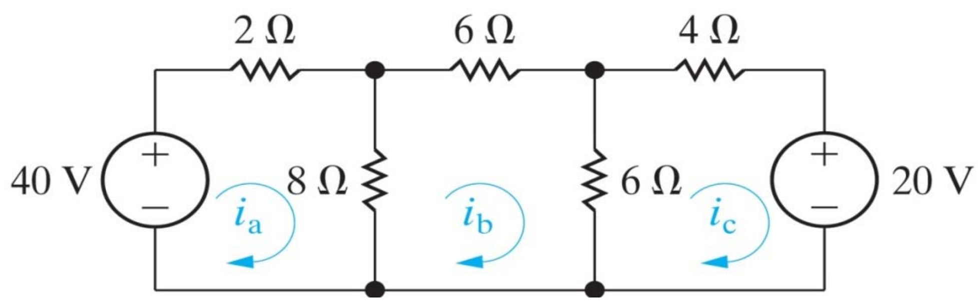


To calculate v0, we notice that v0 = 8(ia –ib), so

vo = 8 * 3.6 = 28.8 V

We can actually talk about the “by inspection” technique now since you might have discovered it for yourself looking at the circuit and the matrix equation we obtained.

Here’s the circuit:

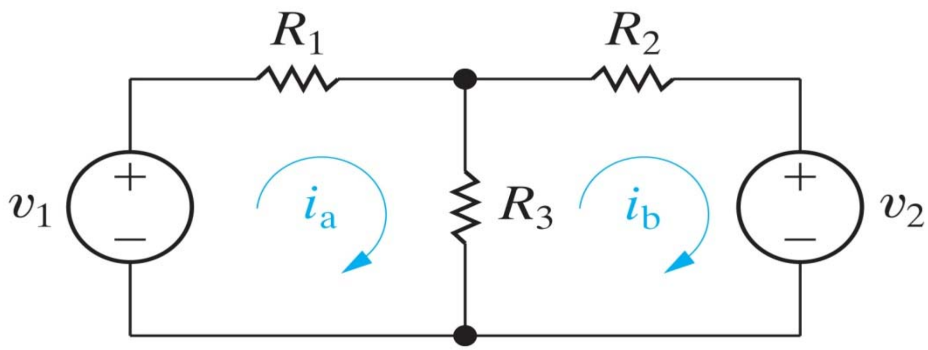


Here’s the equation:

$$\begin{bmatrix} 10 & -8 & 0 \\ -8 & 20 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 40 \\ 0 \\ -20 \end{bmatrix}$$

The diagonal terms are just the sum of all the resistances encountered in going around each of the loops. The off diagonal terms are the negative of the resistances that are shared by the loops! The voltage vector is just the negative of the sum of all of the voltage sources encountered going around the loops.

If we go back to the first circuit we did by mesh current:



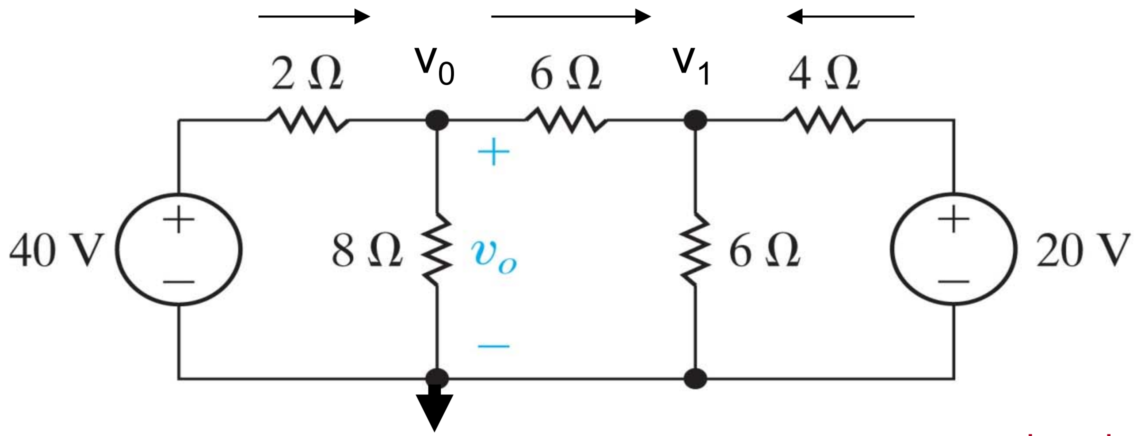
We can also write the matrix equation immediately by inspection.

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix}$$

Here was the standard form equation we got by applying KVL

ia (R1+ R3) + ib (-R3) = v1

ia (-R3) + ib (R2 + R3) = -v2



arrows show how I'm assuming current is flowing

Lets do this same analysis by the node voltage method:

At the v0 node, the KCL equation is:

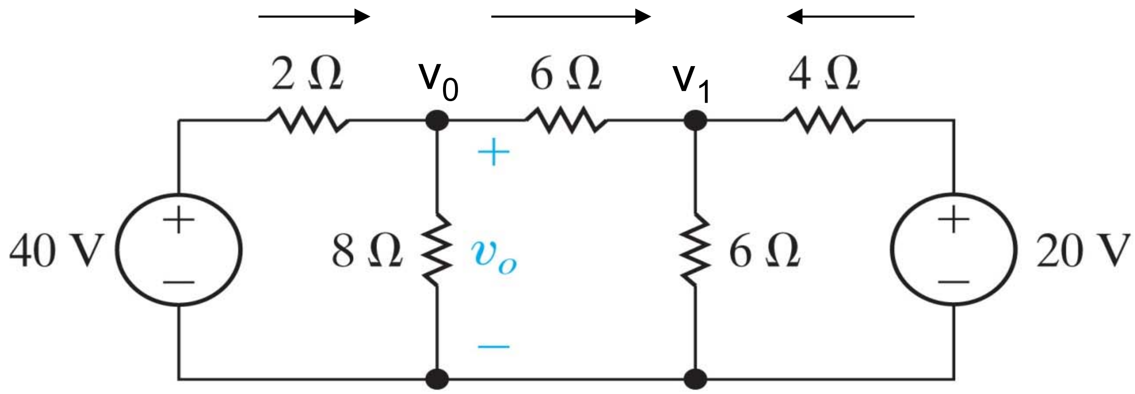
$-(40 - v0)/2 + v0/8 + (v0-v1)/6 = 0$

At the v1 node, the equation is:

$-(v0-v1)/6 + v1/6 - (20 - v1)/4 = 0$

So: $v0\,(1/2 + 1/8 + 1/6) + v1\,(-1/6) = 20$

$v0\,(-1/6) + v1\,(1/6 + 1/6 + 1/4) = 5$



$$v0\,(1/2 + 1/8 + 1/6) + v1\,(-1/6) = 20$$
$$v0\,(-1/6) + v1\,(1/6 + 1/6 + 1/4) = 5$$

$$v0\,(38) + v1\,(-8) = 960$$
$$v0\,(-4) + v1\,(14) = 120$$

Solving for v1 in the 2nd eq.: $v1 = (120 + 4\,v0)/14 = 8.571 + 0.285\,v0$

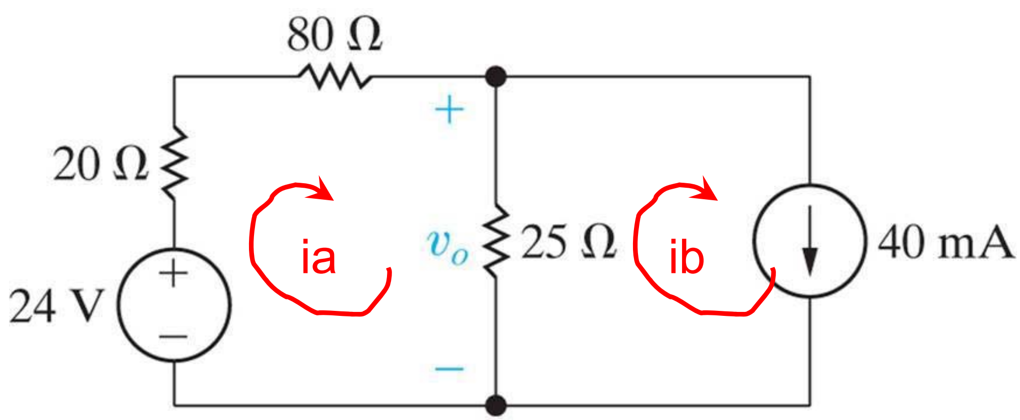
And substituting in 1st eq. $v0\,(38) - 70.00 - 2.28\,v0 = 960$

$$v0 = 28.8\text{ V}$$
$$v1 = 16.8\text{ V}$$

The same answer we got with the mesh current technique (of course!)

So which technique was easier? – the node voltage method where we had only two equations to solve …or the mesh current method where there were three equations …but where we could write the matrix equation by inspection and let MATLAB do all the algebra?

Here's a circuit similar to one you solved by node voltage on the last problem set – let's do a mesh current analysis.



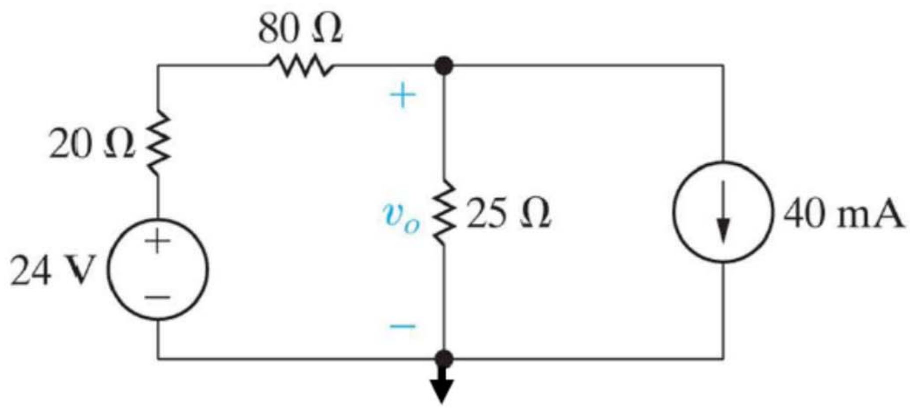
What would a mesh current solution look like? Well, there are two loop currents – but we already know one of them – so there's only one loop equation we have to solve.

ib = 0.04A

125 ia – 25 ib -24=0
125 ia -25 (0.04) =24
ia = 25/125 = 0.2 A

→ v_o= 25(0.2A – 0.04A) = 4V

The node voltage solution was:



Solution:

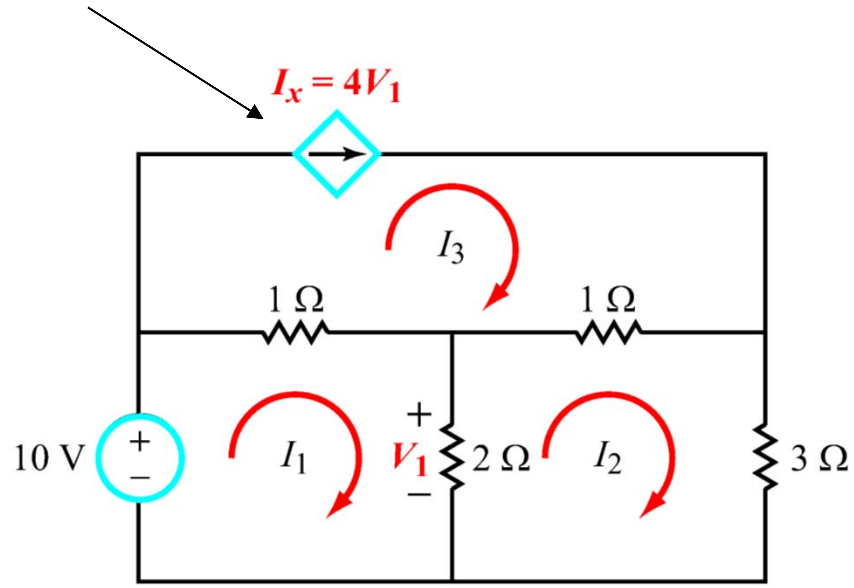
Use the lower terminal of the 25Ω resistor as the reference node:

$(v_o - 24)/ (20 + 80) + v_o/25 + 0.04 = 0$

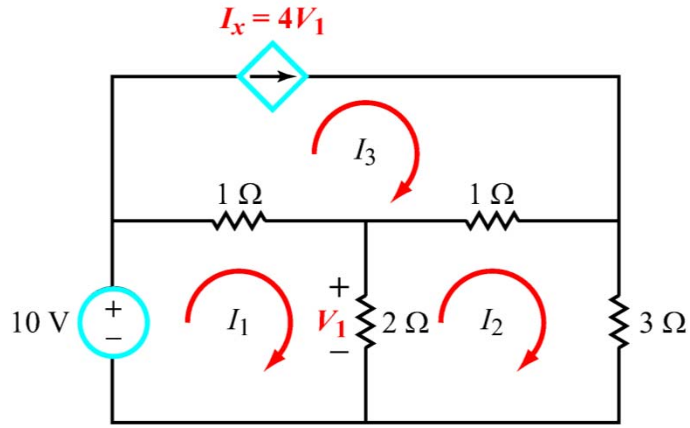
Solving, $v_o = 4$ V

Seems like both techniques were equally easy.

Here's another circuit for mesh current analysis – this one contains a dependent source.



What are the KVL equations for the mesh currents we've identified?



$$(1+2)I_1 - 2I_2 - I_3 = 10 \qquad mesh\,1$$

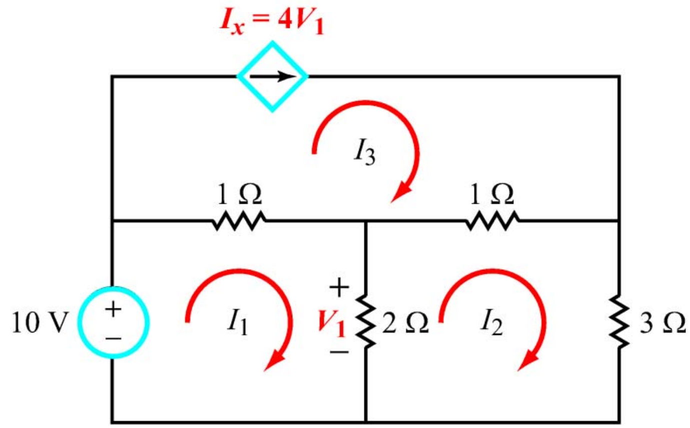
$$-2I_1 + (2+1+3)I_2 - I_3 = 0 \quad mesh\,2$$

Just like in the previous circuit, we already know the mesh current in mesh 3 (it's $4V_1$) and $V_1 = 2(I_1 - I_2)$…so $I_3 = 4V_1 = 8(I_1 - I_2)$
Substituting into the mesh 1 and mesh 2 equations:

$$-5I_1 + 6I_2 = 10$$

$$-10I_1 + 14I_2 = 0$$

$I_1 = -14A$, $I_2 = -10A$ and $V_1 = -8V$ …I_x = -32A



What if we had tried to solve this by inspection? We know that the "by inspection" technique is restricted to circuits with independent voltage sources …but let's try anyway and see what happens.

$$\begin{bmatrix} 3 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ v_3 \end{bmatrix}$$

Here were the KVL equations we got earlier for mesh 1 and mesh 2 – so these parts are OK!

$$(1+2)I_1 - 2I_2 - I_3 = 10 \qquad mesh\,1$$

$$-2I_1 + (2+1+3)I_2 - I_3 = 0 \quad mesh\,2$$

But we can't get the mesh 3 KVL equation …because v3 is some (unknown) voltage drop across the dependent current source!!!!!!

Next lecture, we'll look at a few more mesh current solutions and introduce the concept of a supermesh (analogous to a supernode for the node voltage method). Just like with the supernode, the supermesh can reduce the number of required equations.

We'll also develop some criteria for choosing between using node voltage or mesh current analysis.