

1.

- (a) In unsigned binary under, the overflow is from 0000 (0 in decimal) to 1111 (15 in decimal).
- (b) In 2's complement integer, the overflow is from 0111 (+7 in decimal) to 1000 (-8 in decimal).

2.

- (a) An unsigned number

$$\begin{aligned} & 11001001.0101_2 \\ &= 1 * 2^7 + 1 * 2^6 + 1 * 2^3 + 1 * 2^0 + 1 * 2^{-2} + 1 * 2^{-4} \\ &= 128 + 64 + 8 + 1 + 0.25 + 0.0625 \\ &= 201.3125_{10} \end{aligned}$$

- (b) A 2's complement number

$$\begin{aligned} & 11001001.0101_2 \\ &= -128 + 64 + 8 + 1 + 0.25 + 0.0625 \\ &= -54.6875_{10} \end{aligned}$$

3.

- (a) 101101_2

$$\begin{aligned} &= 32 + 8 + 4 + 1 \\ &= 45_{10} \end{aligned}$$

- (b) $BEEF_{16}$

$$\begin{aligned} &= 11 * 16^3 + 14 * 16^2 + 14 * 16 + 15 \\ &= 48879_{10} \end{aligned}$$

4.

- (a) 150_{10} to base 2

$$\begin{array}{lll} 150/2 & Q = 75 & R = 0 \\ 75/2 & Q = 37 & R = 1 \\ 37/2 & Q = 18 & R = 1 \\ 18/2 & Q = 9 & R = 0 \\ 9/2 & Q = 4 & R = 1 \\ 4/2 & Q = 2 & R = 0 \\ 2/2 & Q = 1 & R = 0 \end{array}$$

$$\text{Thus, } 150_{10} = 10010110_2$$

- (b) 1500_{10} to base 16

$$\begin{array}{lll} 1500/16 & Q = 93 & R = 12 \\ 93/16 & Q = 5 & R = 13 \\ 5/16 & Q = 0 & R = 5 \end{array}$$

$$\text{Thus, } 1500_{10} = 5DC_{16}$$

5.

(a) 0.9_{10} to base 2

$$0.9 * 2 = 1.\underline{8}$$

$$0.8 * 2 = 1.6$$

$$0.6 * 2 = 1.2$$

$$0.2 * 2 = 0.4$$

$$0.4 * 2 = 0.\underline{8}$$

Thus, $0.9_{10} = 0.1110_2$ (= 0.875, not exactly, but close enough?)

(b) 0.9_{10} to base 16

$$0.9 * 16 = 14.\underline{4}$$

$$0.4 * 16 = 6.\underline{4}$$

Thus, $0.9_{10} = 0.E_{16}$ (= 0.8984375)

6.

(a) $ACE5_{16} = 1010\ 1100\ 1110\ 0101_2$

(b) $FA.CE_{16} = 1111\ 1010.\ 1100\ 1110_2$

(c) $101011.01101_2 = 0010\ 1011.\ 0110\ 1000_2 = 2B.68_{16}$

7.

(a) $01010101 = 64 + 16 + 4 + 1 = 85_{10}$ (because the most significant bit is 0, so this is positive, just do polynomial evaluation)

(b) $10101010 = -128 + 32 + 8 + 2 = -86_{10}$

(c) $1000.0001 = -8 + 0.0625 = -7.9375_{10}$

(d) $1001.0110 = -8 + 1 + 0.25 + 0.125 = -6.625_{10}$

(e) $0111.1110 = 4 + 2 + 1 + 0.5 + 0.25 + 0.125 = 7.875_{10}$ (positive)

8.

(a) -6.7

$$-8 + 1 + 0.25 + 0.0625 = -6.6875 \text{ (pretty close I would say)}$$

Thus, -6.7's representation in 2's complement is 1001.0101_2

(b) -37.1

$$-64 + 16 + 8 + 2 + 0.5 + 0.25 + 0.125 = 37.125$$

Thus, -37.1's representation in 2's complement is 1011010.111_2

(c) -100

$$-128 + 16 + 8 + 4 = -100$$

Thus, -100's representation in 2's complement is 10011100_2

(d) -7.7

$$-8 + 0.25 + 0.0625 = -7.6875$$

Thus, -7.7's representation in 2's complement is 1000.0101_2

9.

(a) 10101011

(b) 01010110

- (c) 0111.1111
- (d) 0110.1010
- (e) 1000.0010

10.

- (a) $2^{-(n-2)}$
- (b) When $n = 8$, the bit with 1 should be the sixth bit from the decimal point, so 00.000001
- (c) $-2^{-(n-2)}$
- (d) 11.111111

11.

Most positive: $011111 = 31_{10}$

Most negative: $100000 = -32_{10}$

12.

Minimum: $000000 = 0_{10}$

Maximum: $111111 = 64_{10}$

13.

When we are dealing with unsigned number.

14.

No, they are different.

For example, in 2's complement, $0111 = +7_{10}$, $1000 = -8_{10}$

15.

- (a) 0.324_7 to base 10
 $324_7 = 3 * 7^2 + 2 * 7 + 4 = 165_{10}$
Thus, $0.324_7 = 165/7^3 = 0.4810495627_{10}$
- (b) 400_{10} to base 7
 $400/7 \quad Q = 57 \quad R = 1$
 $57/7 \quad Q = 8 \quad R = 1$
 $8/7 \quad Q = 1 \quad R = 1$
Thus, $400_{10} = 1111_7$
- (c) 0.9_{10} to base 3
 $0.\underline{9} * 3 = 2.7$
 $0.7 * 3 = 2.1$
 $0.1 * 3 = 0.3$
 $0.3 * 3 = 0.\underline{9}$
Thus, $0.9_{10} = 0.220_3$
- (d) 12.34_5 to base 7
 $1234_5 = 1 * 5^3 + 2 * 5^2 + 3 * 5 + 4 = 194_{10}$
 $12.34_5 = 194/5^2 = 7.76_{10}$

$$7_{10} = 10_7$$

$$0.\textcolor{red}{76} * 7 = 5.32$$

$$0.32 * 7 = 2.24$$

$$0.24 * 7 = 1.68$$

$$0.68 * 7 = 4.\textcolor{red}{76}$$

$$\text{Thus, } 12.34_5 = 10.5214$$

(e) 35.2_7 to base 10

$$352_7 = 3 * 7^2 + 5 * 7 + 2 = 184_{10}$$

$$\text{Thus, } 35.2_7 = 184/7 = 26.28571429_{10}$$

(f) 35.2_{10} to base 7

$$35_{10} = 50_7$$

$$0.\textcolor{red}{2} * 7 = 1.4$$

$$0.4 * 7 = 2.8$$

$$0.8 * 7 = 5.6$$

$$0.6 * 7 = 4.\textcolor{red}{2}$$

$$\text{Thus, } 35.2_{10} = 50.1254_7$$

16.

(a) $\text{FACE}_{16} = 1111\ 1010\ 1100\ 1110_2 = 001\ 111\ 101\ 011\ 001\ 110_2 = 175316_8$

(b) $1011.0111_2 = 001\ 011.\ 011\ 100_2 = 13.34_8$

(c) $232.1_4 = 10\ 11\ 10.\ 01_2 = 101\ 110.\ 010_2 = 56.2_8$

(d) $17.6_9 = 0121.20_3$

(e) $1100011.11001_2 = 001\ 100\ 011.110\ 010_2 = 143.62_8$

(f) $71.3_8 = 111\ 001.011_2 = 11\ 10\ 01.01\ 10_2 = 321.12_4$

17.

$$\frac{1}{6} = 0.1666666667$$

$$0.125 + 0.03125 + 0.0078125 = 0.1640625 = \frac{21}{128}$$

$$\frac{1}{6} \rightarrow 0.00101010_2$$

$$\text{Error} = \left| \frac{1}{6} - \frac{21}{128} \right| = \left| \frac{128 - 21 * 6}{128 * 6} \right| = \left| \frac{2}{768} \right| = \frac{1}{384} = 0.002604167$$

18.

For example,

$$(13\text{-bit})\ 0.0001110001101_2 = 0.1109619140625_{10} \approx 0.111_{10}$$

$$0.111 * 2 = 0.222$$

$$0.222 * 2 = 0.444$$

$$0.444 * 2 = 0.888$$

$$0.888 * 2 = 1.776$$

$$0.776 * 2 = 1.552$$

$$1.552 * 2 = 1.104$$

$$0.104 * 2 = 0.208$$

$$0.208 * 2 = 0.416$$

$$0.416 * 2 = 0.832$$

$$0.832 * 2 = 1.664$$

$$0.664 * 2 = 1.328$$

$$0.328 * 2 = 0.656$$

$$0.656 * 2 = 1.312$$

$$(12\text{-bit}) 0.000111000110_2 = 0.11083984375_{10} \approx 0.111_{10}$$

$$(11\text{-bit}) 0.00011100011_2 = 0.11083984375_{10} \approx 0.111_{10}$$

$$(10\text{-bit}) 0.0001110001_2 = 0.1103515625_{10} \approx 0.110_{10}$$

Another example, just to check.

$$0.123 * 2 = 0.246$$

$$0.246 * 2 = 0.492$$

$$0.492 * 2 = 0.984$$

$$0.984 * 2 = 1.968$$

$$0.968 * 2 = 1.936$$

$$0.936 * 2 = 1.872$$

$$0.872 * 2 = 1.744$$

$$0.744 * 2 = 1.488$$

$$0.488 * 2 = 0.976$$

$$0.976 * 2 = 1.952$$

$$0.952 * 2 = 1.904$$

$$(11\text{-bit}) 0.00011111011_2 = 0.12255859375_{10} \approx 0.123_{10}$$

$$(10\text{-bit}) 0.0001111101_2 = 0.1220703125_{10} \approx 0.122_{10}$$

Thus, we should have at least 11 bits of binary numbers.

