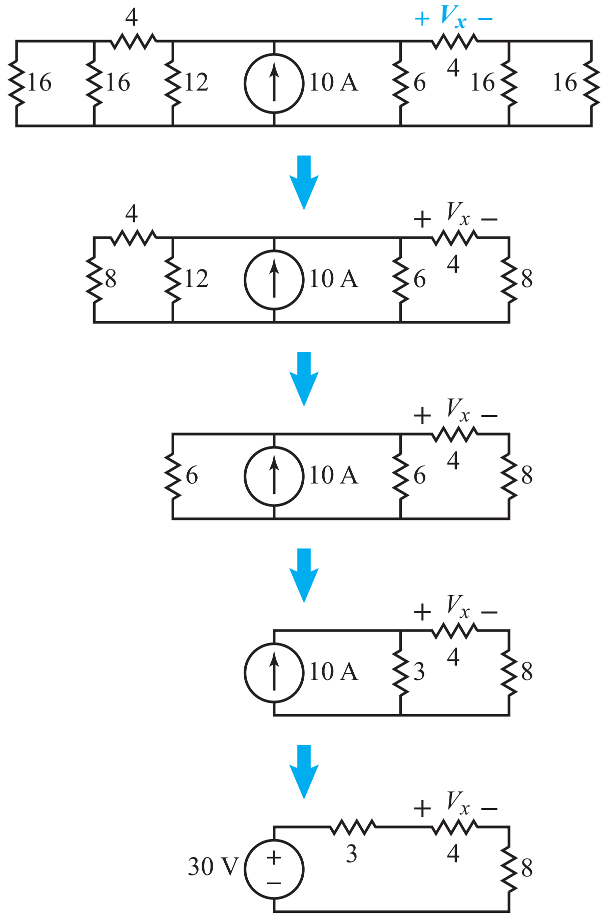
Problem 2.31 Use resistance reduction and source transformation to find V_x in the circuit of Fig. P2.31. All resistance values are in ohms.

Solution:

Figure P2.31: Circuit for Problem 2.31.



$$V_x = \frac{30 \times 4}{3+4+8} = 8 \text{ V}.$$

Problem 2.37 Find R_{eq} for the circuit in Fig. P2.37. All resistances are in ohms.

Solution:

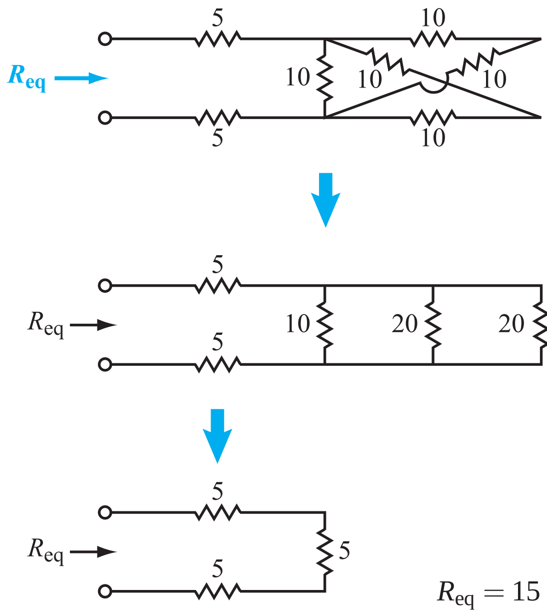


Figure P2.37: Circuit for Problem 2.37.

$R_{eq} = 15\ \Omega.$

Problem 3.25 Apply mesh analysis to find I_x in the circuit of Fig. P3.25.

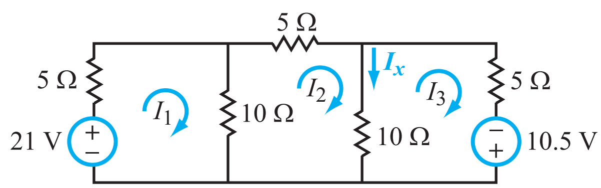


Figure P3.25: Circuit for Problem 3.25.

Solution:

$$\text{Mesh 1:} \quad -21 + 5I_1 + 10(I_1 - I_2) = 0$$

$$\text{Mesh 2:} \quad 10(I_2 - I_1) + 5I_2 + 10(I_2 - I_3) = 0$$

$$\text{Mesh 3:} \quad 10(I_3 - I_2) + 5I_3 - 10.5 = 0$$

Solution is:

$$I_1 = \frac{13}{5} \text{ A}, \qquad I_2 = \frac{9}{5} \text{ A}, \qquad I_3 = \frac{19}{10} \text{ A},$$

and

$$I_x = I_2 - I_3 = \frac{9}{5} - \frac{19}{10} = -\frac{1}{10} = -0.1 \text{ A}.$$

Problem 3.26 Apply mesh analysis to determine the amount of power supplied by the voltage source in Fig. P3.26.

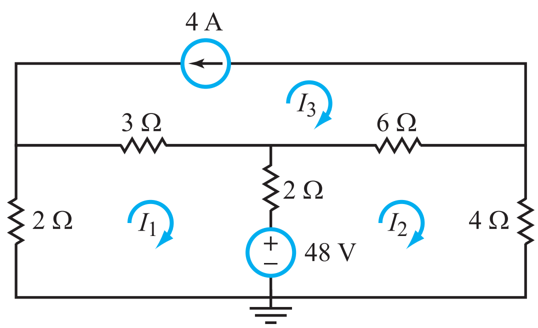


Figure P3.26: Circuit for Problem 3.26.

Solution:

Mesh 1: $2I_1 + 3(I_1 - I_3) + 2(I_1 - I_2) + 48 = 0$

Mesh 2: $-48 + 2(I_2 - I_1) + 6(I_2 - I_3) + 4I_2 = 0$

Mesh 3: $I_3 = -4$ A.

Solution is:

$$I_1 = -8.4 \text{ A}, \qquad I_2 = 0.6 \text{ A}, \qquad I_3 = -4 \text{ A}.$$

Current entering "+" terminal of voltage source is:

$$I = I_1 - I_2 = -8.4 - 0.6 = -9 \text{ A}.$$

Hence,

$$P = VI = 48 \times (-9) = -432 \text{ W}.$$

Problem 3.4 For the circuit in Fig. P3.4:

(a) Apply nodal analysis to find node voltages V_1 and V_2.

(b) Determine the voltage V_R and current I.

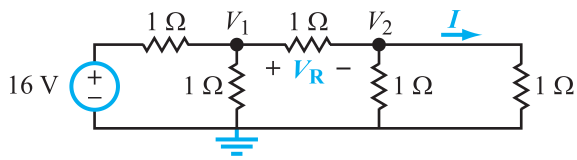


Figure P3.4: Circuit for Problem 3.4.

Solution: (a) At nodes V_1 and V_2,

$$\text{Node 1:} \qquad \frac{V_1 - 16}{1} + \frac{V_1}{1} + \frac{V_1 - V_2}{1} = 0 \qquad (1)$$

$$\text{Node 2:} \qquad \frac{V_2 - V_1}{1} + \frac{V_2}{1} + \frac{V_2}{1} = 0 \qquad (2)$$

Simplifying Eqs. (1) and (2) gives:

$$3V_1 - V_2 = 16 \qquad (3)$$

$$-V_1 + 3V_2 = 0. \qquad (4)$$

Simultaneous solution of Eqs. (3) and (4) leads to:

$$V_1 = 6\text{ V}, \qquad V_2 = 2\text{ V}.$$

(b)

$$V_R = V_1 - V_2 = 6 - 2 = 4\text{ V}$$

$$I = \frac{V_2}{1} = \frac{2}{1} = 2\text{ A}.$$

Problem 3.5 Apply nodal analysis to determine the voltage V_R in the circuit of Fig. P3.5.

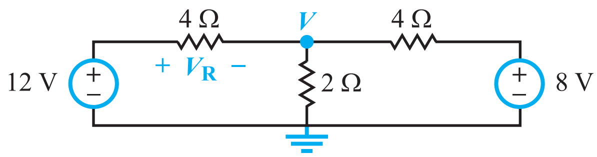


Figure P3.5: Circuit for Problem 3.5.

Solution: At node V:

$$\frac{V-12}{4} + \frac{V}{2} + \frac{V-8}{4} = 0,$$

which leads to

$$V = 5 \text{ V}.$$

Hence,

$$V_R = 12 - V = 12 - 5 = 7 \text{ V}.$$

Problem 3.55 Find the Thévenin equivalent circuit at terminals (a,b) for the circuit in Fig. P3.55.

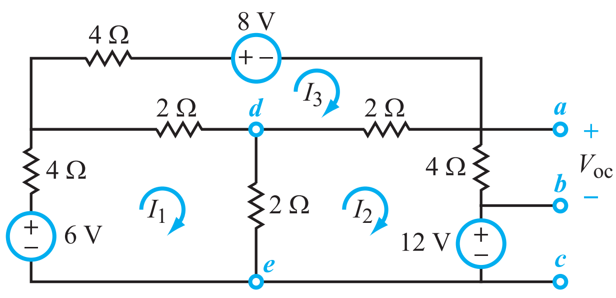


Figure P3.55: Circuit for Problem 3.55.

Solution: Since all sources in the circuit are voltage sources, the mesh current by-inspection method can be applied:

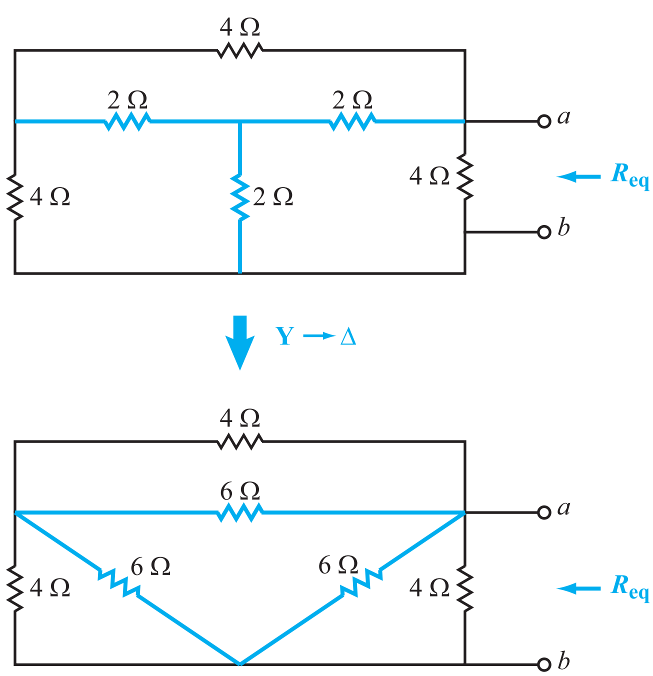
$$\begin{bmatrix} 8 & -2 & -2 \\ -2 & 8 & -2 \\ -2 & -2 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -12 \\ -8 \end{bmatrix}$$

Matrix inversion yields:

$$I_1 = -0.1 \text{ A}, \qquad I_2 = -1.9 \text{ A}, \qquad I_3 = -1.5 \text{ A}.$$

$$V_{\text{Th}} = V_{\text{oc}} = 4I_2 = 4(-1.9) = -7.6 \text{ V}.$$

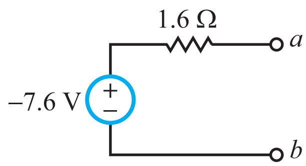
Suppressing sources:



Further simplification leads to

$$R_{\text{Th}} = 1.6\ \Omega.$$

Hence, the Thévenin circuit is:



Problem 3.71 Determine the maximum power extractable from the circuit in Fig. P3.71 by the load resistor R_L.

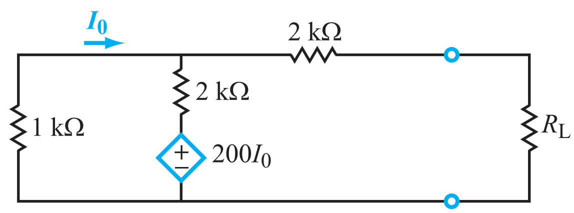


Figure P3.71: Circuit for Problem 3.71.

Solution: The circuit has no independent sources. Hence,

$$V_{Th} = 0.$$

Consequently, R_L cannot extract any power from the circuit.

Problem 4.26 Relate v_o in the circuit of Fig. P4.26 to v_s and specify the linear range of v_s. Assume $V_0 = 0$.

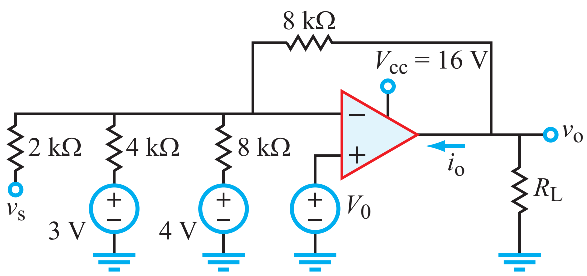


Figure P4.26: Circuit for Problem 4.26.

Solution: With $V_0 = 0$, this is a summing amplifier circuit. Hence,

$$\begin{aligned} v_o &= -\left[\left(\frac{8}{2}\right) v_s + \left(\frac{8}{4}\right) \times 3 + \left(\frac{8}{8}\right) \times 4\right] \\ &= -[4v_s + 6 + 4] \\ &= -[4v_s + 10] \qquad \text{(V)}. \end{aligned}$$

For $v_o = V_{cc} = 16$ V,

$$16 = -4v_s - 10, \qquad \text{or } v_s = -6.5 \text{ V}.$$

For $v_o = -V_{cc} = -16$ V,

$$-16 = -4v_s - 10, \qquad \text{or } v_s = 1.5 \text{ V}$$

Hence,

$$-6.5 \text{ V} \leq v_s \leq 1.5 \text{ V}.$$

Problem 4.34 The circuit in Fig. P4.34 contains two single-pole single-throw switches, S_1 and S_2. Determine the closed circuit gain $G = v_o/v_s$ for each of the four possible closed/open switch combinations.

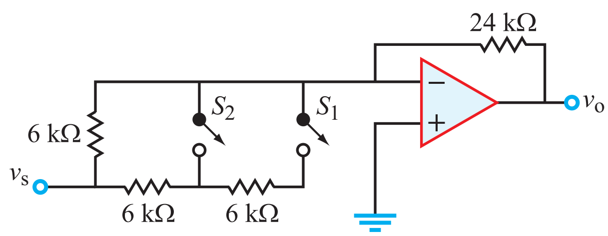


Figure P4.34: Circuit for Problem 4.34.

Solution: This is an inverting amplifier circuit.

S_1	S_2	G	
open	open	$-\frac{24}{6} = -4$	
open	closed	$-\frac{24}{3} = -8$	$(6 \parallel 6 = 3)$
closed	open	$-\frac{24}{4} = -6$	$(12 \parallel 6 = 4)$
closed	closed	$-\frac{24}{3} = -8$	$(6 \parallel 6 = 3;\ 6 \parallel 0 = 0)$

Problem 4.41 Relate v_o in the circuit of Fig. P4.41 to v_s.

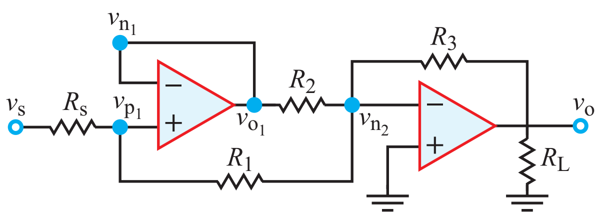


Figure P4.41: Circuit for Problem 4.41.

Solution: The second stage is a standard summing amplifier:

$$v_o = \left(-\frac{R_3}{R_2}\right) v_{o_1} - \left(\frac{R_3}{R_1}\right) v_{p_1}.$$

For the first stage,

$$v_{n_1} = v_{o_1}$$
$$v_{n_1} = v_{p_1}$$
$$\frac{v_{p_1} - v_s}{R_s} + \frac{v_{p_1} - v_{n_2}}{R_1} = 0$$

Also,

$$v_{n_2} = v_{p_2} = 0.$$

Hence,

$$v_{p_1} = \frac{v_s R_1}{R_1 + R_s}$$
$$v_{o_1} = v_{n_1} = v_{p_1}$$
$$v_o = \left(-\frac{R_3}{R_2} - \frac{R_3}{R_1}\right) v_{p_1} = -\left(\frac{v_s R_1}{R_1 + R_s}\right) \times \frac{R_3(R_1 + R_2)}{R_1 R_2} = -\left(\frac{R_3}{R_2}\right)\left(\frac{R_1 + R_2}{R_1 + R_s}\right) v_s.$$

Problem 5.17 Reduce the circuit in Fig. P5.17 into a single equivalent capacitor at terminals (a,b). Assume that all initial voltages are zero at $t = 0$.

Solution:

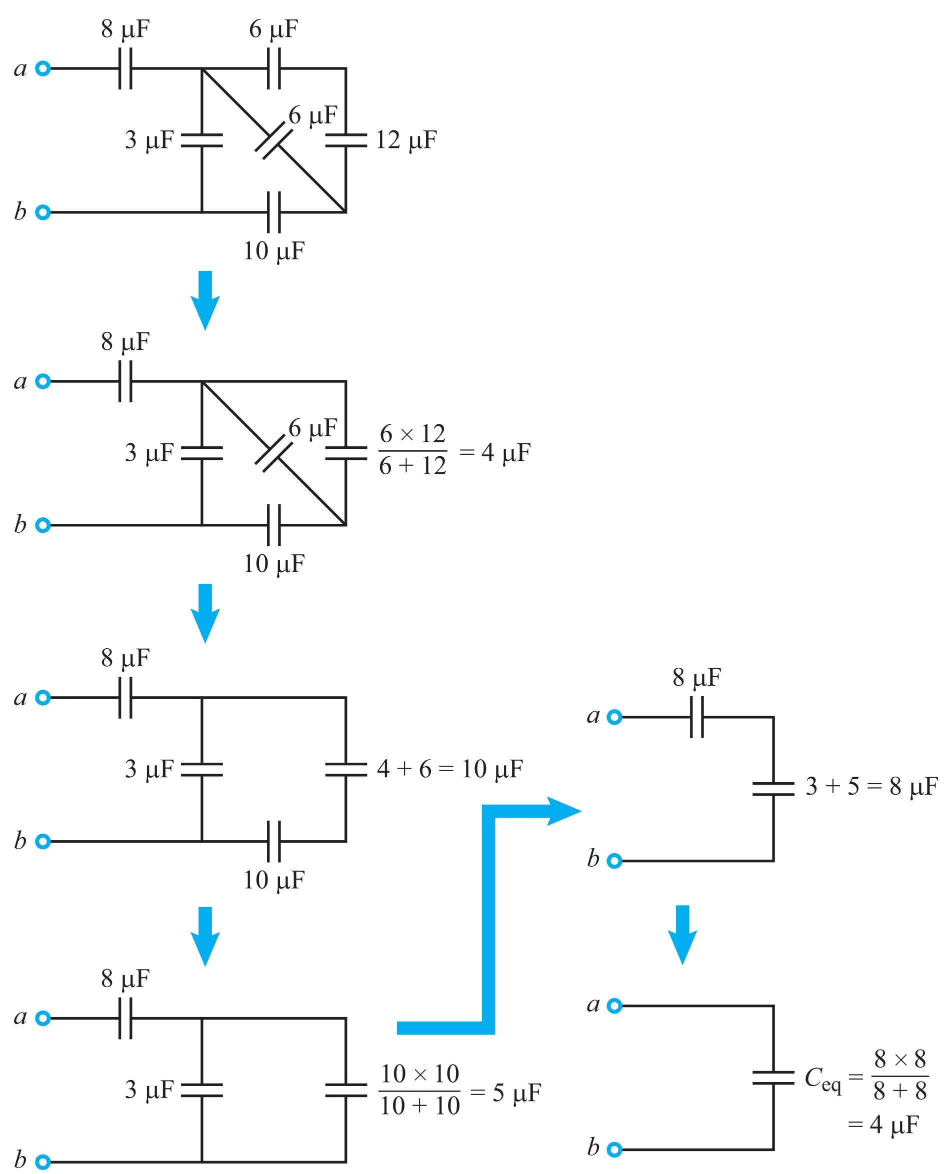


Figure P5.17

Problem 5.28 For the circuit in Fig. P5.28, determine the voltage across C and the currents through L_1 and L_2 under dc conditions.

Solution:

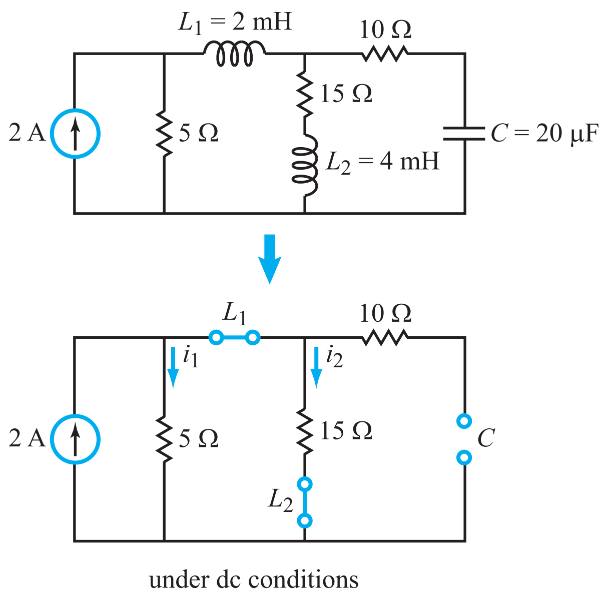


Figure P5.28

Current division gives:

$$i_1 = \frac{2 \times 15}{5 + 15} = 1.5 \text{ A}, \qquad \Longrightarrow \qquad i_2 = 0.5 \text{ A}.$$

$$v_C = i_1 \times 5 = 1.5 \times 5 = 7.5 \qquad \text{(V)},$$

Current through both L_1 and L_2 is $i_2 = 0.5$ A.

Problem 5.49 After having been in position 1 for a long time, the switch in the circuit of Fig. P5.49 was moved to position 2 at $t = 0$. Determine $i_1(t)$ and $i_2(t)$ for $t \geq 0$ given that $I_0 = 6$ mA, $R_0 = 12\ \Omega$, $R_1 = 10\ \Omega$, $R_2 = 40\ \Omega$, $L_1 = 1$ H, and $L_2 = 2$ H.

Solution:

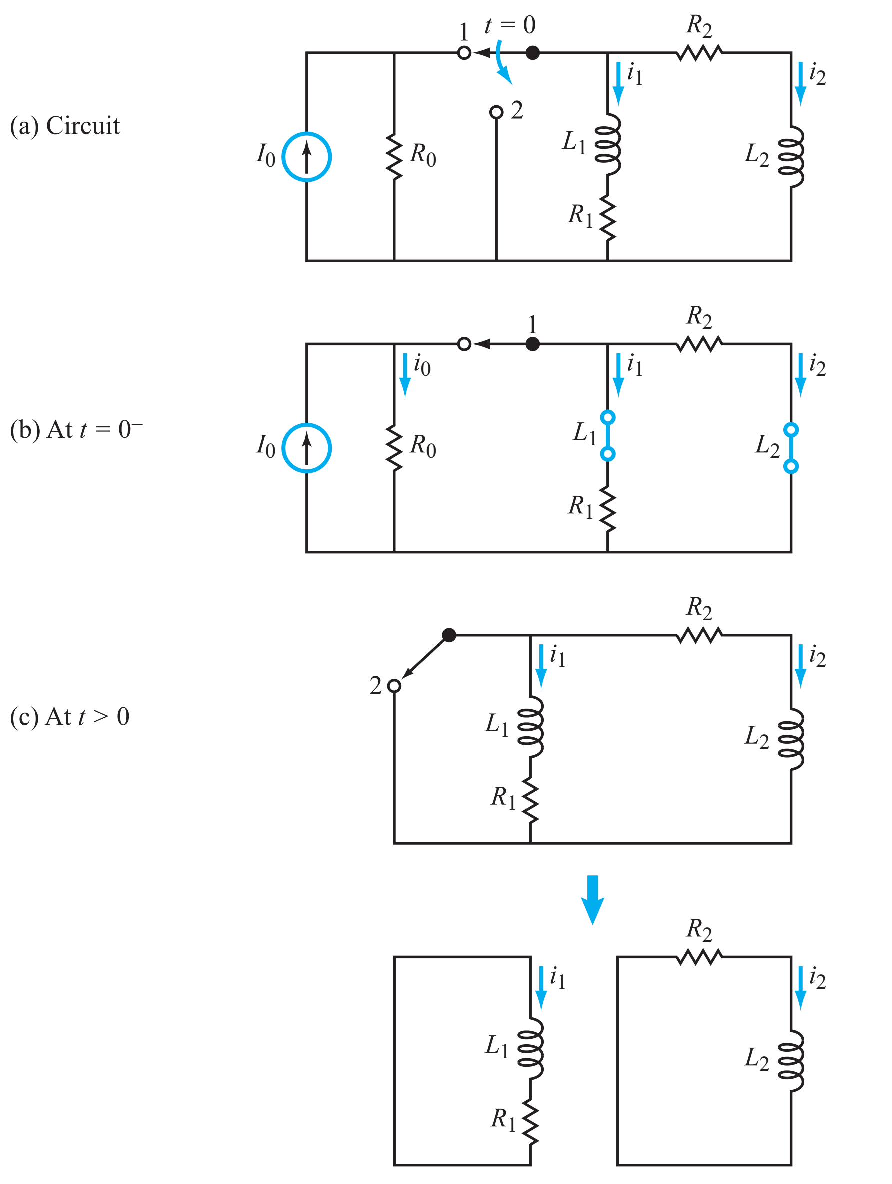


Figure P5.49

At $t = 0^-$ (Fig. P5.49(b)), I_0 will flow through the three branches such that

$$i_0 R_0 = i_1 R_1 = i_2 R_2,$$

and $I_0 = i_0 + i_1 + i_2$. Hence,

$$i_1(0^-) = \frac{R_0 R_2 I_0}{R_0 R_1 + R_0 R_2 + R_1 R_2} = 2.88 \qquad \text{(mA)},$$

$$i_2(0^-) = \frac{R_0 R_1 I_0}{R_0 R_1 + R_0 R_2 + R_1 R_2} = 0.72 \qquad \text{(mA)}.$$

At $t > 0$, we have two independent RL circuits sharing a common short circuit.

$R_1 L_1$ Circuit

$$\begin{aligned}
i_1(0) &= i_1(0^-) = 2.88 \qquad \text{(mA)} \\
i_1(\infty) &= 0 \\
\tau_1 &= \frac{L_1}{R_1} = \frac{1}{10} = 0.1 \text{ s} \\
i_1(t) &= 2.88 e^{-10t} \qquad \text{(mA)}, \qquad \text{for } t \geq 0.
\end{aligned}$$

$R_2 L_2$ Circuit

$$\begin{aligned}
i_2(0) &= i_2(0^-) = 0.72 \qquad \text{(mA)} \\
i_2(\infty) &= 0 \\
\tau_2 &= \frac{L_2}{R_2} = \frac{2}{40} = 0.05 \text{ s} \\
i_2(t) &= 0.72 e^{-20t} \qquad \text{(mA)}, \qquad \text{for } t \geq 0.
\end{aligned}$$

Problem 6.7 For the circuit of Fig. P6.7, determine $i_1(0)$ and $i_2(0)$.

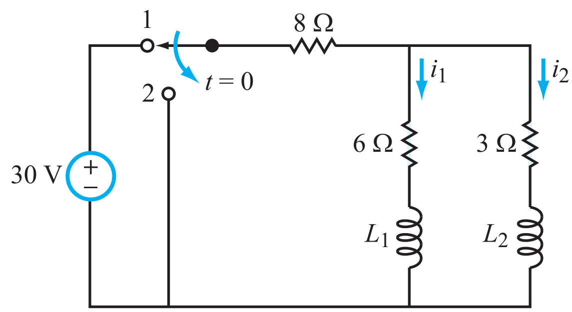
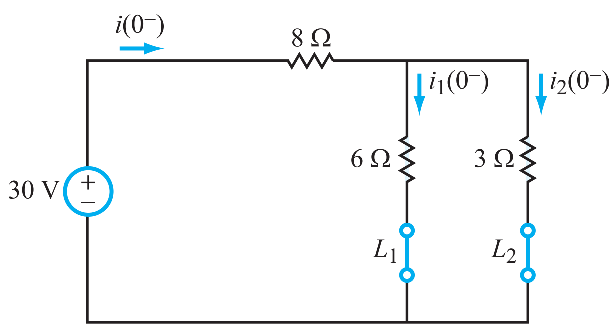


Figure P6.7: Circuit for Problem 6.7.

Solution:



At $t = 0^-$

$$3 \parallel 6 = \frac{3 \times 6}{3+6} = 2\ \Omega.$$

Hence,

$$i(0^-) = \frac{30}{8+2} = 3\ \text{A}.$$

Current division gives

$$i_1(0^-) = \frac{i(0^-) \times 3}{6+3} = \frac{3 \times 3}{9} = 1\ \text{A},$$

$$i_2(0^-) = \frac{i(0^-) \times 6}{6+3} = 2\ \text{A}.$$

Since i in an inductor cannot change instantaneously,

$$i_1(0) = 1\ \text{A}, \qquad i_2(0) = 2\ \text{A}.$$

Problem 7.27 Determine the equivalent impedance:

(a) $\mathbf{Z}_1$ at 1000 Hz (Fig. P7-27)(a))

(b) $\mathbf{Z}_2$ at 500 Hz (Fig. P7-27)(b))

(c) $\mathbf{Z}_3$ at $\omega = 10^6$ rad/s (Fig. P7-27)(c))

(d) $\mathbf{Z}_4$ at $\omega = 10^5$ rad/s (Fig. P7-27)(d))

(e) $\mathbf{Z}_5$ at $\omega = 2000$ rad/s (Fig. P7-27)(e))

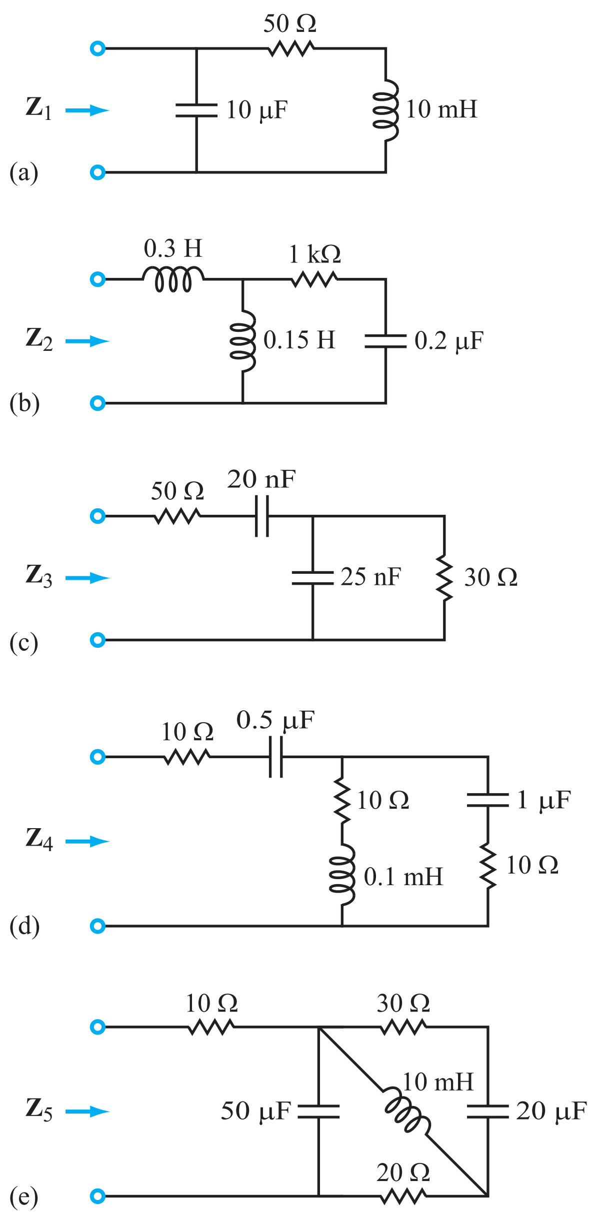
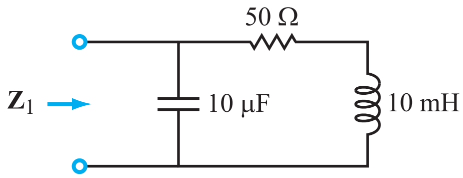
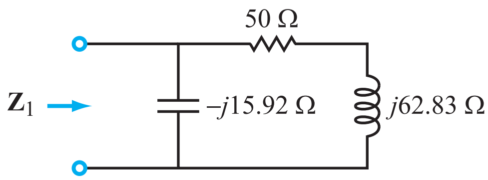


Figure P7.27: Circuit for Problem 7.27.

Solution:

(a)



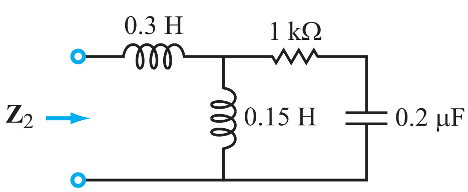


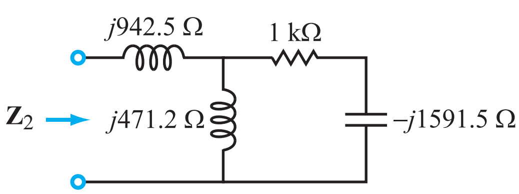
$$\mathbf{Z}_\mathrm{C} = \frac{-j}{\omega C} = \frac{-j}{2\pi \times 10^3 \times 10 \times 10^{-6}} = -j15.92\ \Omega$$

$$\mathbf{Z}_\mathrm{L} = j\omega L = j2\pi \times 10^3 \times 10^{-2} = j62.83\ \Omega$$

$$\begin{aligned}\mathbf{Z}_1 &= (50 + j62.83) \parallel (-j15.92)\\ &= \frac{(50 + j62.83)(-j15.92)}{50 + j62.83 - j15.92}\\ &= \frac{1000 - j796}{50 + j46.91} \cdot \left(\frac{50 - j46.91}{50 - j46.91}\right)\\ &= \frac{12660 - j86710}{4701} = (2.7 - j18.5)\ \Omega.\end{aligned}$$

(b)





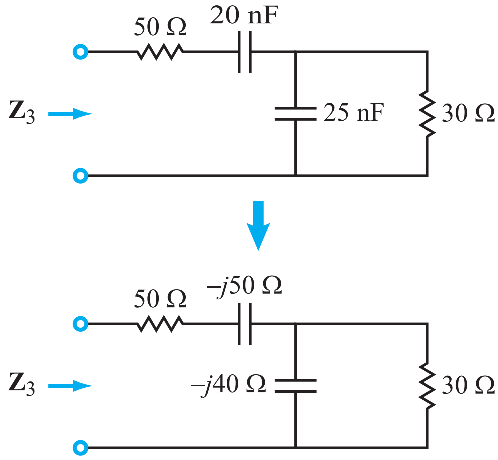
$$\mathbf{Z}_{L_1} = j\omega L_1 = j2\pi \times 500 \times 0.3 = j942.5\ \Omega$$

$$\mathbf{Z}_{L_2} = j\omega L_2 = j471.2\ \Omega$$

$$\mathbf{Z}_\mathrm{C} = \frac{-j}{\omega C} = \frac{-j}{2\pi \times 500 \times 0.2 \times 10^{-6}} = -j1591.5\ \Omega$$

$$\begin{aligned}\mathbf{Z}_2 &= j942.5 + j471.2 \parallel (1000 - j1591.5)\\ &= j942.5 + \frac{j471.2(1000 - j1591.5)}{1000 - j1591.5 + j471.2} = (98.5 + j1524.0)\ \Omega\end{aligned}$$

(c)

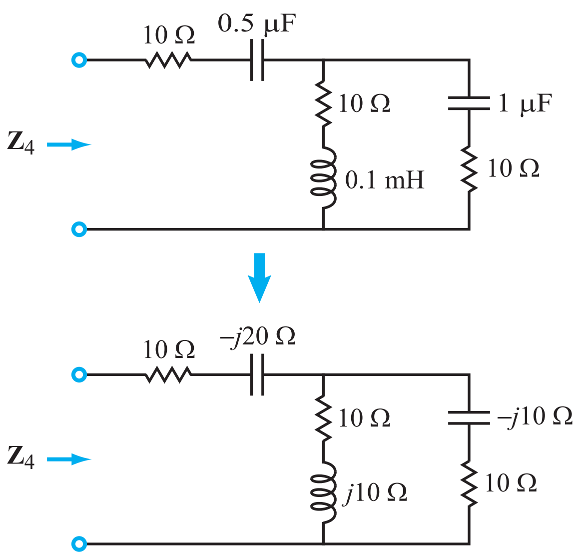


$$\mathbf{Z}_{C_1} = \frac{-j}{\omega C_1} = \frac{-j}{10^6 \times 20 \times 10^{-9}} = -j50\ \Omega$$

$$\mathbf{Z}_{C_2} = \frac{-j}{\omega C_2} = \frac{-j}{10^6 \times 25 \times 10^{-9}} = -j40\ \Omega$$

$$\mathbf{Z}_3 = 50 - j50 + \frac{30 \times (-j40)}{30 - j40} = (69.2 - j64.4)\ \Omega.$$

(d)



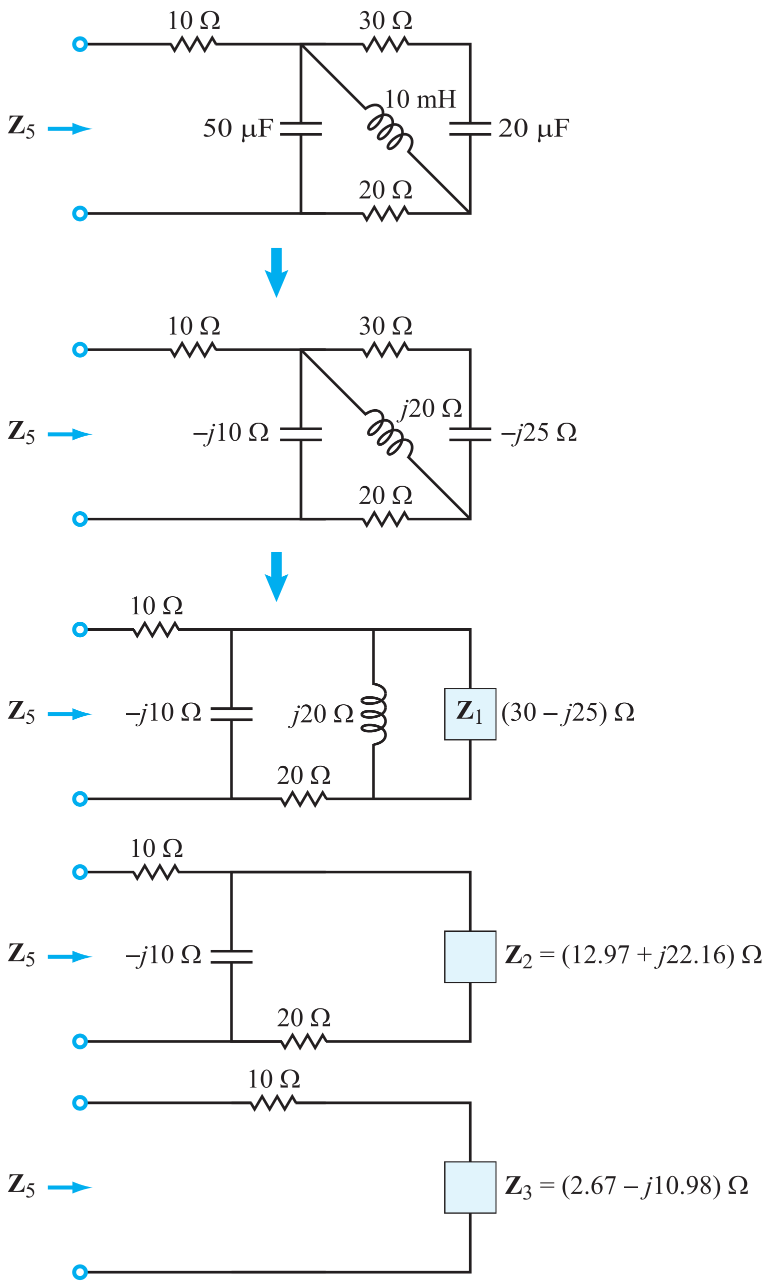
$$\mathbf{Z}_{C_1} = \frac{-j}{\omega C_1} = \frac{-j}{10^5 \times 0.5 \times 10^{-6}} = -j20\ \Omega$$

$$\mathbf{Z}_{C_2} = \frac{-j}{\omega C_2} = -j10\ \Omega$$

$$\mathbf{Z}_{\mathrm{L}} = j\omega L = j \times 10^5 \times 10^{-4} = j10\ \Omega$$

$$\mathbf{Z}_4 = (10 - j20) + \frac{(10 + j10)(10 - j10)}{10 + j10 + 10 - j10} = (10 - j20) + 10 = (20 - j20)\ \Omega.$$

(e)



$$\mathbf{Z}_{C_1} = \frac{-j}{\omega C_1} = \frac{-j}{2000 \times 50 \times 10^{-6}} = -j10\ \Omega$$

$$\mathbf{Z}_{C_2} = \frac{-j}{\omega C_2} = \frac{-j}{2000 \times 20 \times 10^{-6}} = -j25\ \Omega$$

$$\mathbf{Z}_{\mathrm{L}} = j\omega L = j2000 \times 10 \times 10^{-3} = j20\ \Omega$$

$$\begin{aligned}\mathbf{Z}_2 &= \mathbf{Z}_1 \parallel (j20)\\ &= \frac{(30 - j25)(j20)}{30 - j25 + j20}\\ &= \frac{500 + j600}{30 - j5} \cdot \frac{(30 + j5)}{(30 + j5)}\end{aligned}$$

$$
\begin{aligned}
&= \frac{12000 + j20500}{925} = (12.97 + j22.16)\ \boldsymbol{\Omega} \\
\mathbf{Z}_3 &= (20 + \mathbf{Z}_2) \parallel (-j10) \\
&= \frac{(32.97 + j22.16)(-j10)}{32.97 + j22.16 - j10} \\
&= \frac{221.6 - j329.7}{32.97 + j12.16} \cdot \frac{(32.97 - j12.16)}{(32.97 - j12.16)} \\
&= (2.67 - j10.98)\ \boldsymbol{\Omega} \\
\mathbf{Z}_5 &= 10 + \mathbf{Z}_3 = (12.67 - j10.98)\ \boldsymbol{\Omega}.
\end{aligned}
$$

Problem 7.34 At what angular frequency ω is the current $i(t)$ in the circuit of Fig. P7.34 in-phase with the source voltage $v_s(t)$?

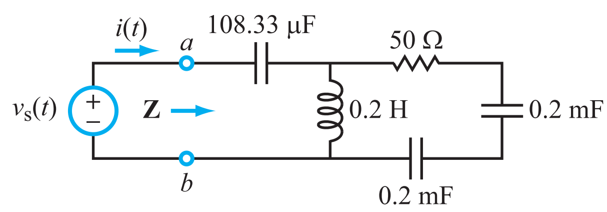


Figure P7.34: Circuit for Problem 7.34.

Solution: For $i(t)$ to be in-phase with $v_s(t)$, it is necessary that $\mathbf{Z}$ be purely real. Noting that the two in-series capacities add up to 0.1 mF,

$$\begin{aligned}
\mathbf{Z} &= \frac{-j}{108.33\omega \times 10^{-6}} + j0.2\omega \parallel \left(50 - \frac{j}{10^{-4}\omega}\right) \\
&= -j\frac{9.23\times 10^3}{\omega} + \frac{j0.2\omega\left(50 - \dfrac{j10^4}{\omega}\right)}{50 - j\dfrac{10^4}{\omega} + j0.2\omega} \\
&= -j\frac{9.23\times 10^3}{\omega} + \frac{2000 + j10\omega}{50 + j\left(0.2\omega - \dfrac{10^4}{\omega}\right)} \\
&= -j\frac{9.23\times 10^3}{\omega} + \frac{(2000 + j10\omega)\left[50 - j\left(0.2\omega - \dfrac{10^4}{\omega}\right)\right]}{2500 + \left(0.2\omega - \dfrac{10^4}{\omega}\right)^2} \\
&= -j\frac{9.23\times 10^3}{\omega} + \frac{[1\times 10^5 + (2\omega^2 - 10^5)] + j\left[500\omega - 400\omega + \dfrac{2\times 10^7}{\omega}\right]}{2500 + \left(0.2\omega - \dfrac{10^4}{\omega}\right)^2}
\end{aligned}$$

The imaginary part of $\mathbf{Z}$ has to be zero,

$$-j\frac{9.23\times 10^3}{\omega} + \frac{j\left(100\omega + \dfrac{2\times 10^7}{\omega}\right)}{2500 + \left(0.2\omega - \dfrac{10^4}{\omega}\right)^2} = 0$$

Solution of quadratic equation, obtained after cross multiplication, is

$$\omega = 200 \text{ rad/s} \qquad \text{or} \qquad 292.8 \text{ rad/s}.$$

Problem 8.13 Determine the average power dissipated in the load resistor R_L of the circuit in Fig. P8.13, given that $\mathbf{V}_s = 100$ V, $R_1 = 1$ kΩ, $R_2 = 0.5$ kΩ, $R_L = 2$ kΩ, $\mathbf{Z}_L = j0.8$ kΩ, and $\mathbf{Z}_C = -j4$ kΩ.

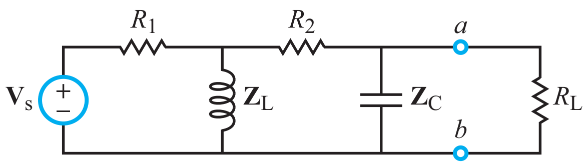
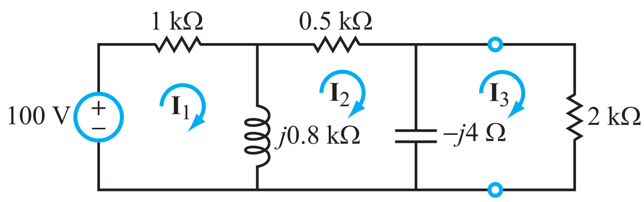


Figure P8.13: Circuit for Problem 8.13.

Solution:



Mesh-current by inspection gives:

$$\begin{bmatrix} (1+j0.8) & -j0.8 & 0 \\ -j0.8 & (0.5-j3.2) & j4 \\ 0 & j4 & (2-j4) \end{bmatrix} \times 10^3 \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}.$$

MATLAB® software solution gives:

$$\mathbf{I}_1 = 65.36\angle{-31.55^\circ} \quad \text{(mA)}$$

$$\mathbf{I}_2 = 24.87\angle{58.48^\circ} \quad \text{(mA)}$$

$$\mathbf{I}_3 = 22.28\angle{31.98^\circ} \quad \text{(mA)}$$

$$P_{av} = \frac{1}{2}|\mathbf{I}_3|^2 R_L = 496.4 \text{ mW}.$$

Section 8-4: Power Factor

Problem 8.22 The RL load in Fig. P8.22 is compensated by adding the shunt capacitance C so that the power factor of the combined (compensated) circuit is exactly unity. How is C related to R, L, and ω in that case?

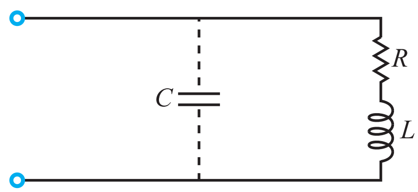


Figure P8.22: Circuit for Problem 8.22.

Solution: For the combined load, the impedance is

$$\begin{aligned}
Z &= (R + j\omega L) \parallel \left(\frac{1}{j\omega C}\right) \\
&= \frac{(R + j\omega L)\left(\frac{-j}{\omega C}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \\
&= \frac{\omega L - jR}{\omega RC - j(1 - \omega^2 LC)} \\
&= \frac{(\omega L - jR)[\omega RC + j(1 - \omega^2 LC)]}{[\omega RC - j(1 - \omega^2 LC)][\omega RC + j(1 - \omega^2 LC)]} \\
&= \frac{[\omega^2 RLC + R(1 - \omega^2 LC)]}{\omega^2 R^2 C^2 + (1 - \omega^2 LC)^2} + \frac{j[\omega L(1 - \omega^2 LC) - \omega R^2 C]}{\omega^2 R^2 C^2 + (1 - \omega^2 LC)^2}
\end{aligned}$$

For the *pf* to be unity, the imaginary component of **Z** has to be zero, which is realized if

$$\omega L(1 - \omega^2 LC) - \omega R^2 C = 0.$$

or

$$C = \frac{L}{R^2 + \omega^2 L^2}\,.$$