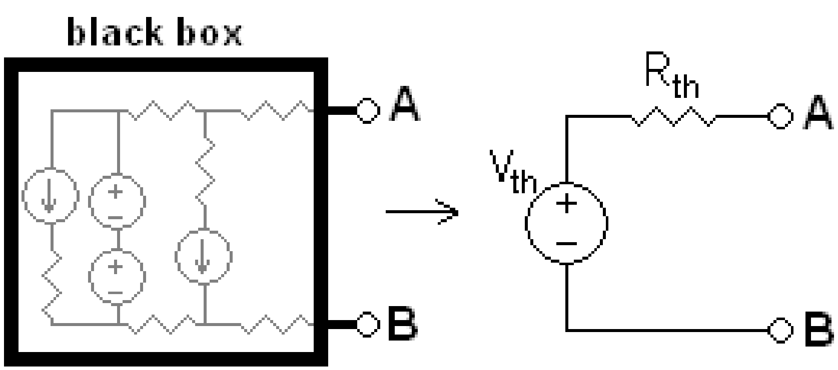
ELEN 50 Class 11 – Thevenin and Norton Equivalent Circuits

S. Hudgens

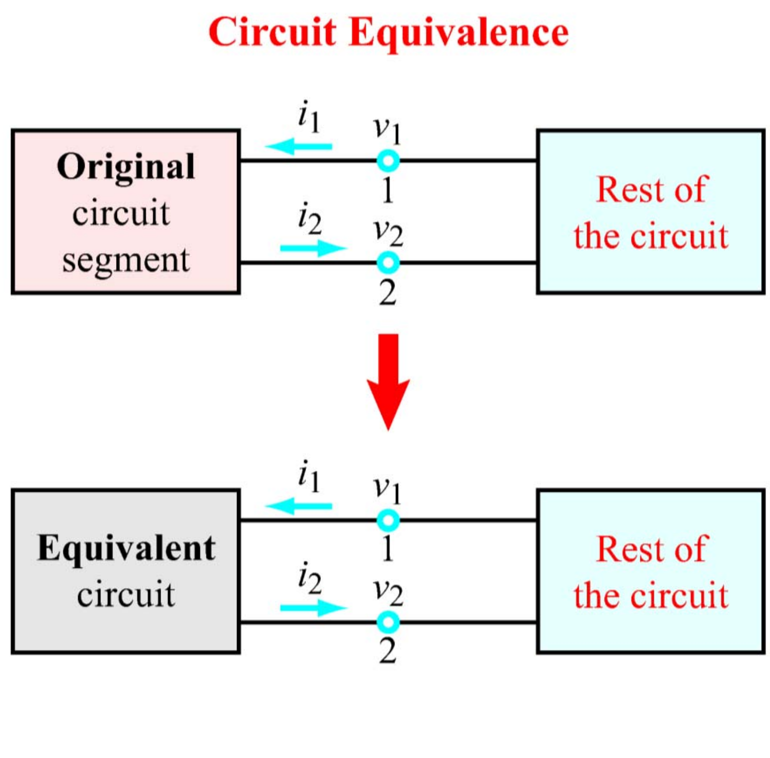
In 1883 a French engineer named Leon Thevenin published a technical paper that contained what is now known as Thevenin's Theorem. The theorem states that a linear circuit can be represented at its output terminals by an equivalent circuit consisting of a voltage source, vth in series with a resistor, Rth. The voltage source, vth, is the open circuit voltage (no load) at the terminals and Rth is the equivalent resistance between the terminals when all independent sources have been deactivated. Thevenin derived his theorem from a consideration of the Kirchoff Laws and Ohms Law. He didn't realize that his theorem had been generally accepted until 1926 (43 years later) – and shortly before his death!



Thanks to Wikipedia for the graphic

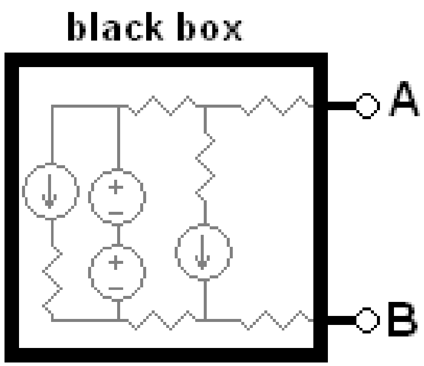
Leon Charles Thevenin

Remember in Class 05 when we introduced the principle of circuit equivalence:



We said that this is an equivalent circuit if the <u>i-v characteristics at nodes 1 and 2 are identical to the i-v characteristics at these nodes in the original circuit.</u> Thevenin's Theorem simply says that <u>the equivalent circuit for any linear circuit (at any given pair of nodes) is a voltage source in series with a resistor</u>.

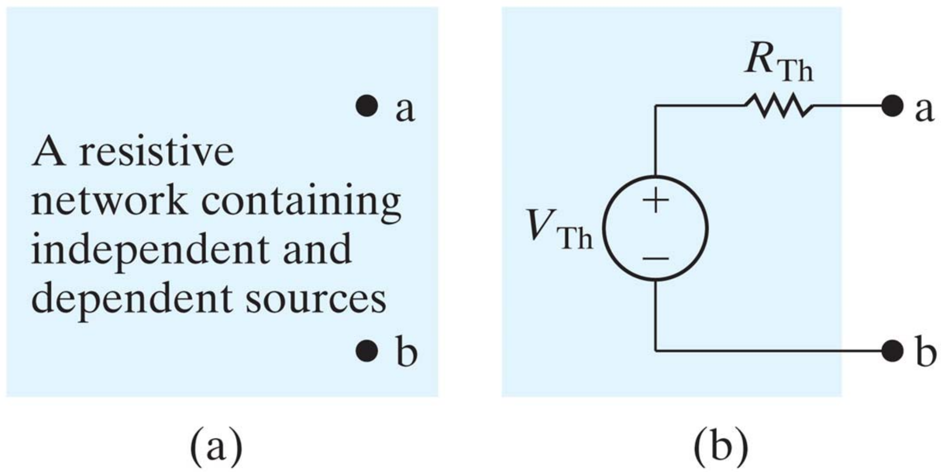
I'm not going to give you a formal proof of Thevenin's theorem (you can find it online if you are interested). I'm just going to make a heuristic argument.



If the components inside the black box are all linear (i.e. they all have linear I-V characteristics), then the I-V characteristic between A and B will also be linear. Furthermore, I know that if there are no current or voltage sources present in the black box, the open circuit voltage across A and B will be zero. This means I can just replace the components with a single resistor. If voltage and current sources are present in the box, then there will be an open circuit voltage present as well …so I can replace the circuit in the black box with a voltage source in series with the resistor….this is Thevenin's Theorem

Thevenin Equivalent Circuit

Thevenin (and Norton) equivalent circuits are ways of replacing complicated and/or irrelevant parts of a circuit with a simple circuit based on the behavior of the circuit at a particular pair of terminals using source transformations. They are <u>critical to calculating maximum power transfer.</u>



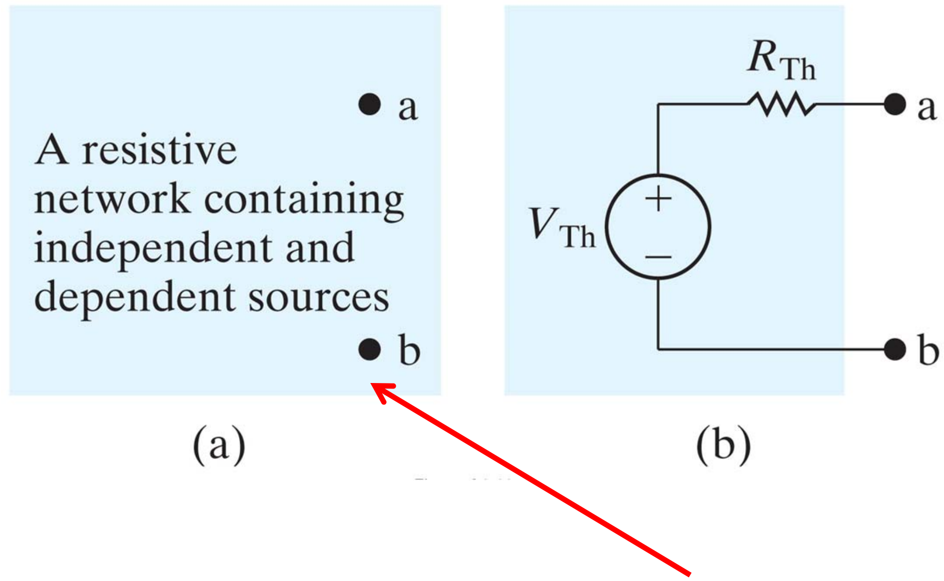
The Thevenin equivalent circuit for a resistive network is a voltage source, V_{th} in series with a resistor, R_{th}. We chose these values by measuring (or calculating) the open circuit voltage for the network …this is V_{th} …then we measure (or calculate) the short circuit current out of the network. Since this will be V_{th}/R_{th}, the value of R_{th} is: $R_{th} = V_{th}/i_{sc}$.

Norton Equivalent Circuit

Once we've got values for Vth and Rth …we can transform this into a current source with a parallel resistor <u>using a source transform</u>….and this current source in parallel with a resistor is called a Norton Equivalent Circuit. As far as the behavior at the terminals is concerned, it doesn't matter if you use a Thevenin equivalent circuit or a Norton equivalent circuit. It may be more convenient to use one or they other …but they are absolutely identical in terms of describing the behavior at the terminals as we saw earlier.

You can generate Thevenin or Norton equivalent circuits for real circuits by making open circuit voltage and short circuit current measurements, or you can calculate these equivalent circuits using circuit analysis.

Thevenin Equivalent Circuit – Measurement Strategy for a Physical Circuit



To obtain the value of Vth and Rth for this circuit, measure the open circuit voltage across terminals (a.) and (b.). In the equivalent circuit, Vth will be equal to the measured open circuit voltage. Now, measure the current that flows between (a.) and (b.) through a short circuit. In the equivalent circuit, the short circuit current will be equal to Vth/Rth ….so this measurement will give you Rth.

Thevenin Equivalent Circuit – General Calculation Strategy (this is important)

- **Obtain Vth** by calculating the voltage across the two specified terminals when no load is present (open circuit voltage) – sometimes you can do this by using source transforms.

- **Obtain Rth** by:

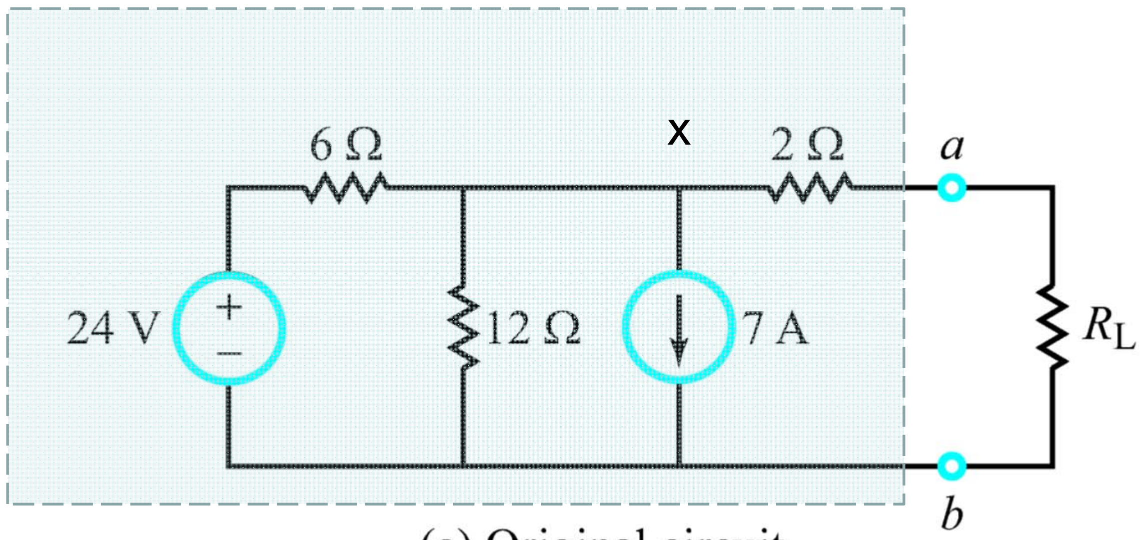
  1. Calculating the current that will flow between the specified terminals in a short circuit. Rth is obtained from Rth = Vth/Isc

     …….or

  2. If the circuit doesn't contain dependent sources, you can calculate the equivalent resistance between the specified terminals after all independent voltage sources are replaced with short circuits and all independent current sources are replaced with open circuits. This in effect is "deactivating" the independent sources and and equivalent resistance in this case is Rth, the Thevenin resistance. ….or

  3. If the circuit contains independent and dependent sources, Rth can be determined by deactivating independent sources, and adding an external source (vex)…then solve the circuit to determine the current iex supplied by the external source. Rth = vex/iex

We want to replace all of the circuit left of terminals (a.) and (b.) with a Thevenin equivalent.

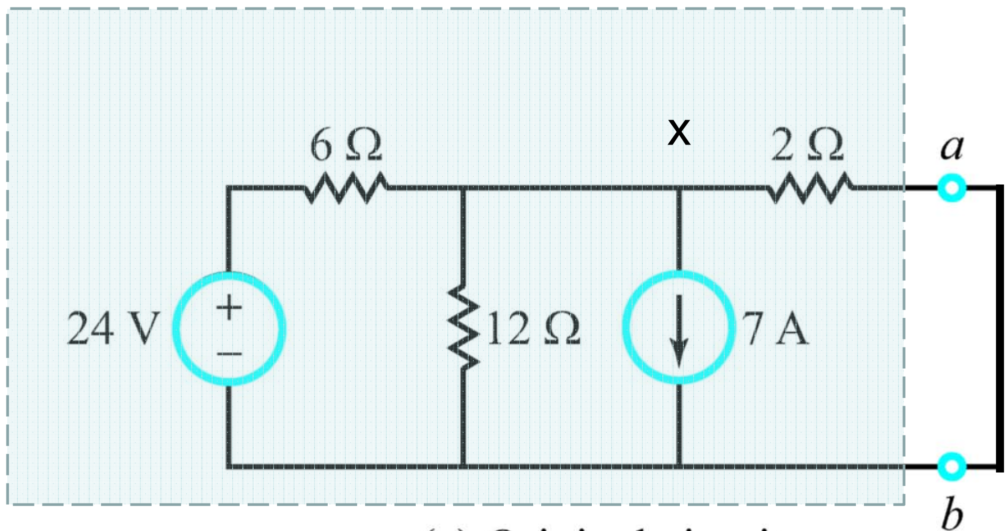


(a) Original circuit

It contains only independent sources, so we can use method 1 or method 2. Let's try method 1. First, we'll calculate Vth, the voltage between (a.) and (b.) <u>when R_L is removed (open circuit)</u>. We can do that by a Node Voltage analysis at node (x.) ….since the voltage at (a.) will be the same as the voltage at (x.) when R_L is removed.

KCL at this node is: $(v_x-24)/6 + v_x/12 + 7 = 0$ therefore, $v_x = -12 = Vth$

Now we want to do the Node Voltage analysis again with R_L replaced by a short circuit – with the voltage at x, we can calculate the current through the short circuit.

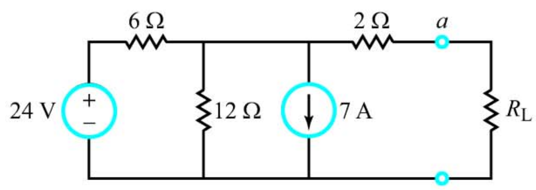


(a) Original circuit

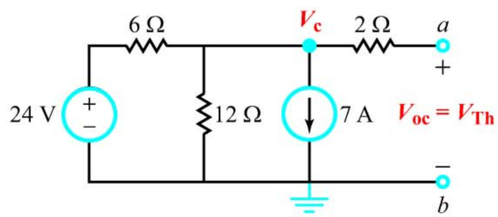
KCL at node x is now: $(v_x-24)/6 + v_x/12 + 7 + v_x/2 = 0$ therefore, $v_x = -4$ and the current through the short circuit , $I_{sc} = 4/2 = -2A$

so Rth = Vth/Isc = -12/-2 = 6 Ohms

Open Circuit – Short Circuit Method (Method 1) Summary

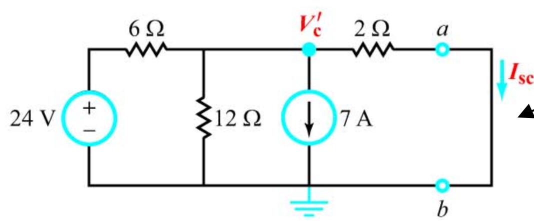


(a) Original circuit



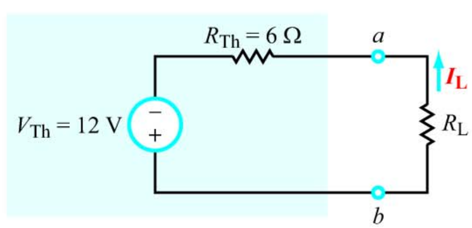
(b) Replacing R_L with open circuit

- replace Rl with open ckt. - solve node voltage equation to get Voc = Vth = -12V



(c) Replacing R_L with short circuit

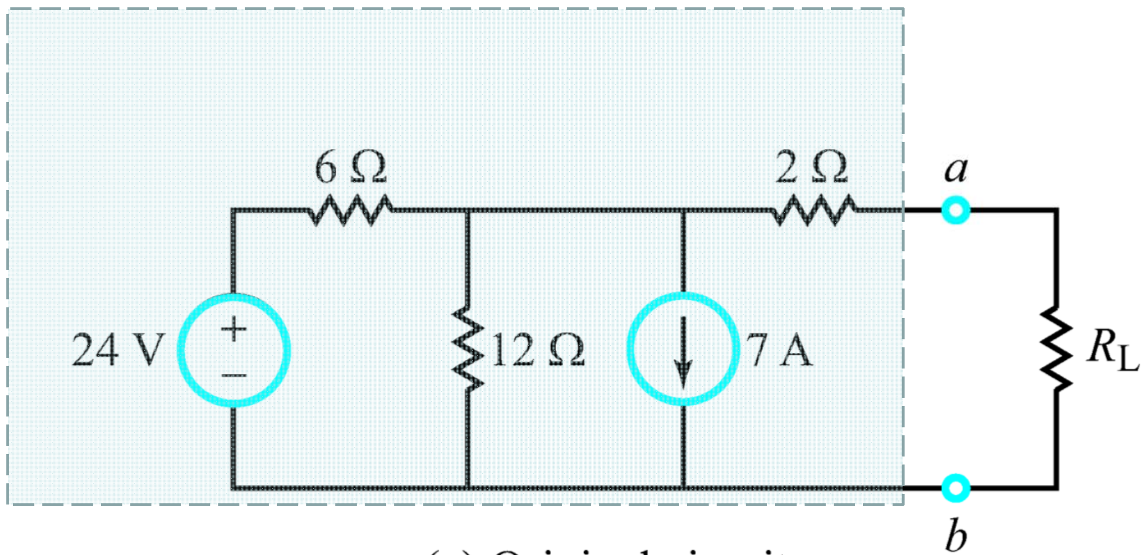
- replace Rl with short ckt.
- solve node voltage equation again to get Vc' = - 4V …so Isc = -2A



(d) Thévenin equivalent circuit

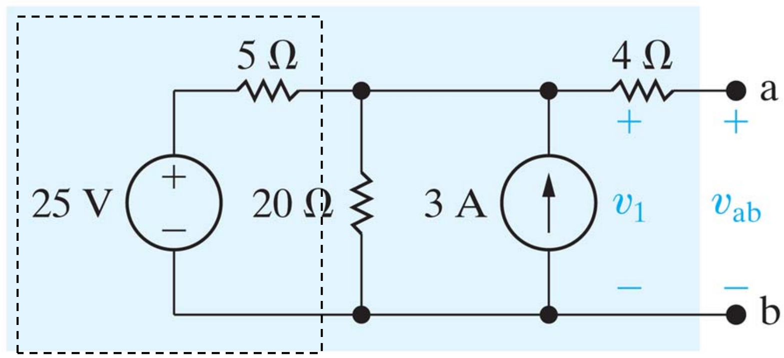
Rth = Vth/Isc = 6 Ω

We could also have gotten Rth by Method #2 – the “deactivating sources” method:



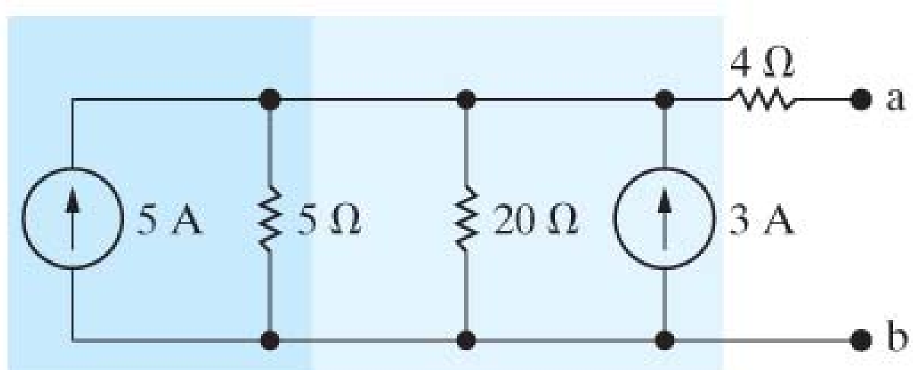
(a) Original circuit

If we deactivate the voltage source (replace with a short) and deactivate the current source (replace with an open) we have a 6Ω and a 12Ω resistor in parallel and then this is in series with a 2Ω resistor. This combination is Rth = 6Ω …as we saw earlier. Of course, Vth is still given by the open circuit voltage as before.

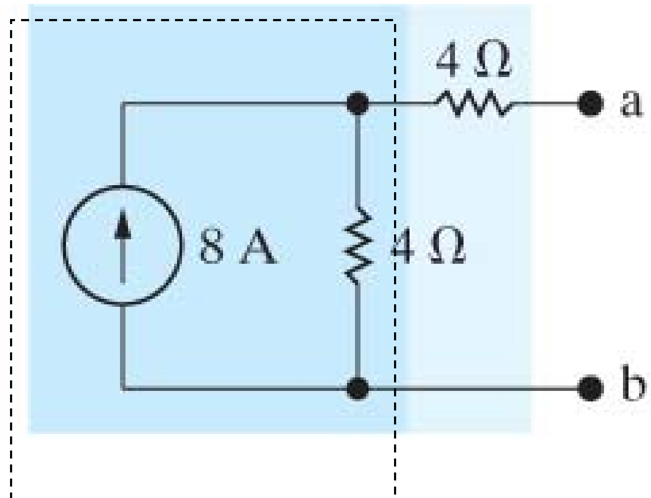


Calculate the Thevenin and Norton equivalent circuits for terminals a and b.

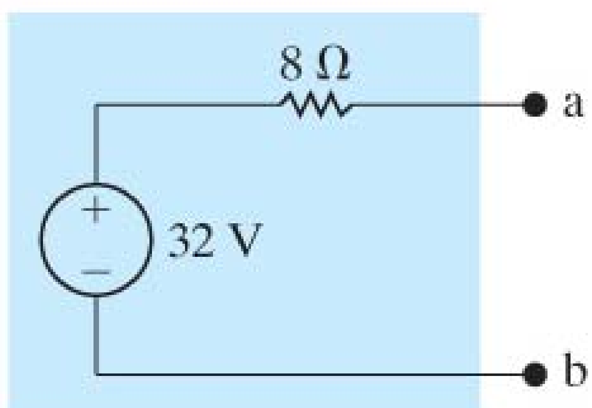
We can't eliminate any resistors – there are none in series with current sources or in parallel across voltage sources. However, we can transform this part to get:



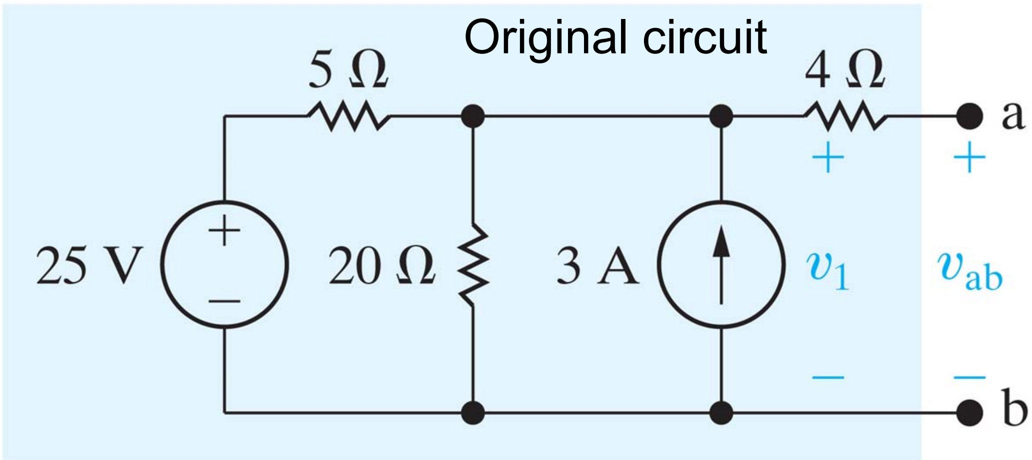
Then, combining the parallel resistors and current sources, we get:



One more source transform of this part, and we get:

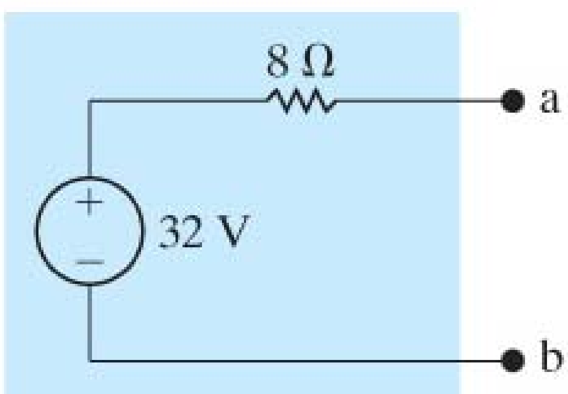


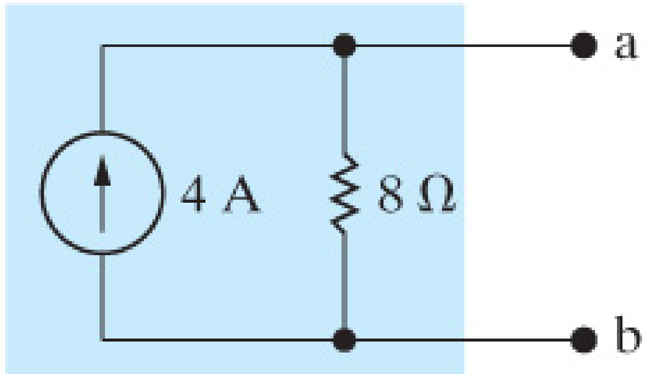
Which, of course is the Thevenin equiv. ckt. – without any Node Voltage analysis.



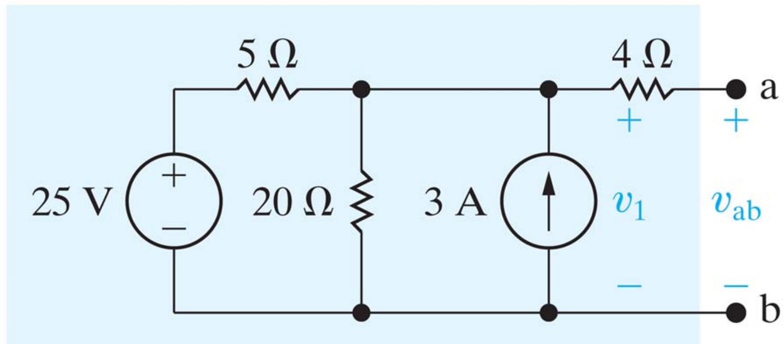
And, of course, we can get the Norton Equivalent by a source transform of the Thevenin Equivalent circuit.

Thevenin Equivalent ← → Norton Equivalent





We could also have gotten the Thevenin equivalent by method #2

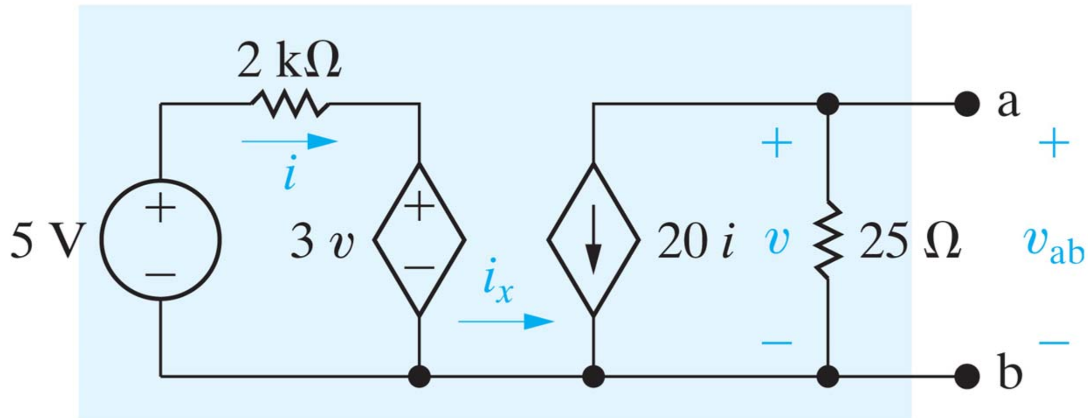


Vth is just the voltage at the one essential node in the circuit and:

$$\frac{(v-25)}{5}+\frac{v}{20}-3=0$$

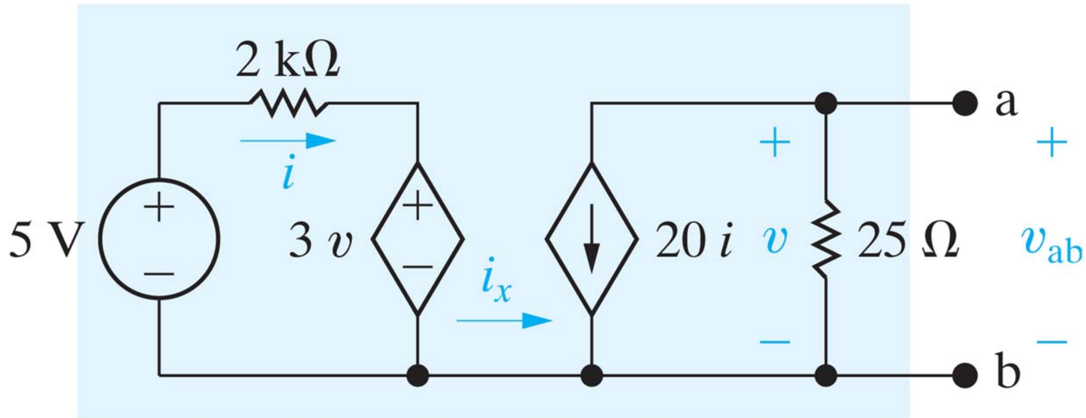
So Vth =32. Now deactivating the voltage source (replacing the voltage source with a short) and deactivating the current source (replacing the current source with an open) we can easily calculate the equivalent resistance; 5Ω || 20Ω ..and this in series with a 4Ω resistor = 8Ω

Thevenin Equivalent Circuits with Dependent Source



Find the Thevenin equivalent circuit at terminals a and b.

First, we notice that this is a sort of bogus circuit – the left hand side doesn't really have very much to do with the right hand side – other than determining i (which controls the current source on the right hand side) and being connected through a common ground connection….i.e. i_x is always zero.



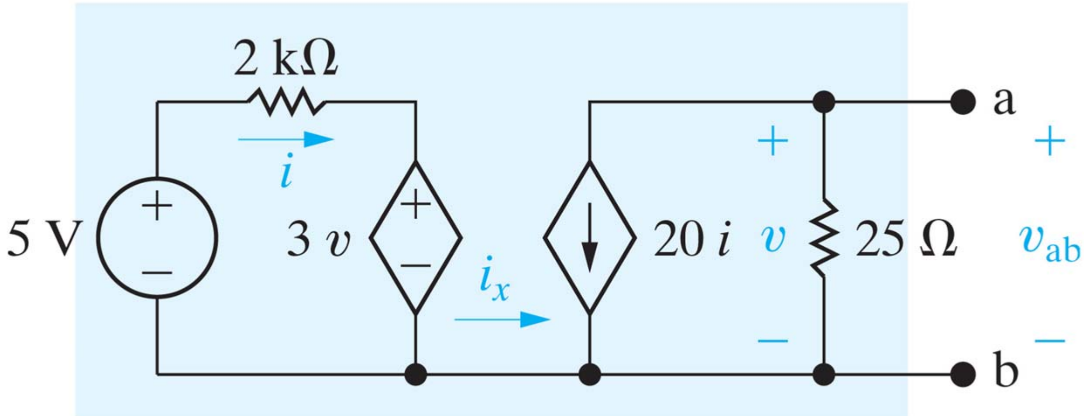
The open circuit voltage across terminals a and b is just the voltage drop across the 25Ω resistor, so:

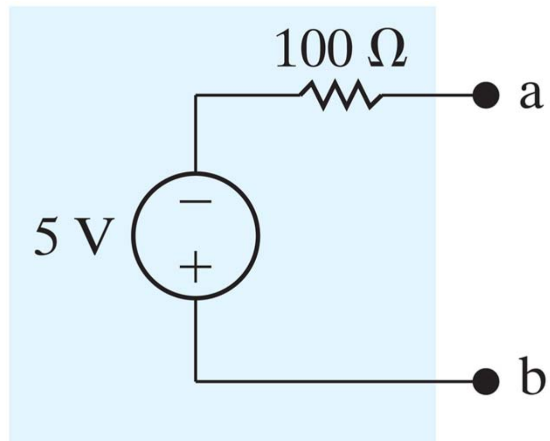
Vth = v_{ab} = v = (- 20i) (25) = - 500i

i = (5 – 3v)/2000 = (5 – 3Vth)/2000

So Vth = - 500 ([5 – 3 Vth)/2000]) → Vth = -5V

We need the short circuit current to define a resistance for the Thevenin equivalent circuit …so if we placed a short circuit across a and b, this would cause v to be zero, and i (in the left hand side of the circuit) would be 5V/2kΩ = 2.5 mA. With i = 2.5 mA, the current source on the right hand side would supply – 50 mA to the short , so the series resistor we need for the Thevenin equivalent circuit will be Vth/i_{sc} = 100Ω



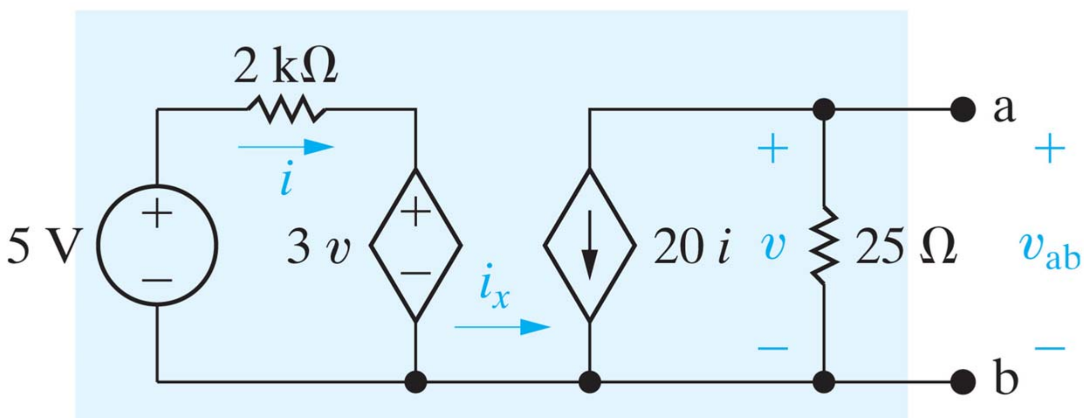


Thevenin Equivalent

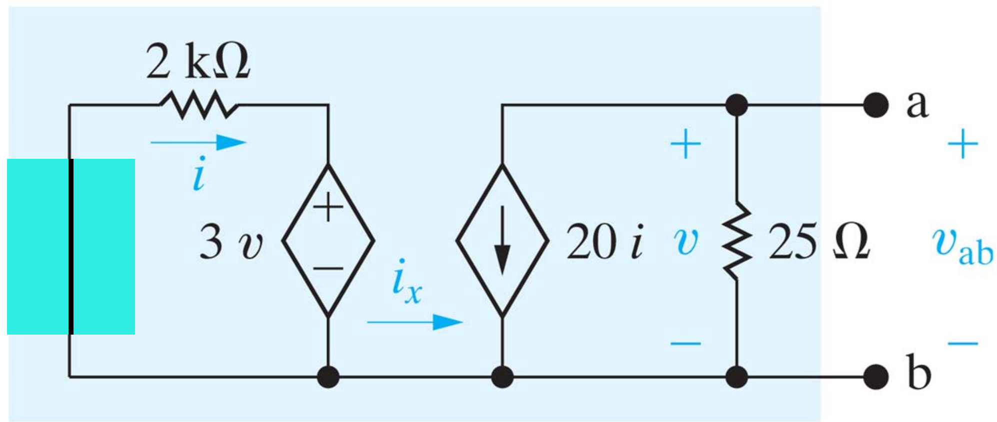
eliminates two dependent sources and one resistor

This probably wasn't a good example of calculating a Thevenin Equivalent for a circuit with independent and dependent sources. It was a kind of a trick! We saw on slide 7 that, in general, the typical way to do get Rth is Method #3:

If the circuit contains <u>independent and dependent sources</u>, Rth can be determined by <u>deactivating independent sources, and adding an external source (vex or iex)</u>…then solve the circuit to determine the unknown current or voltage required by the external source. Rth = vex/iex. How would this work on the circuit we just considered?



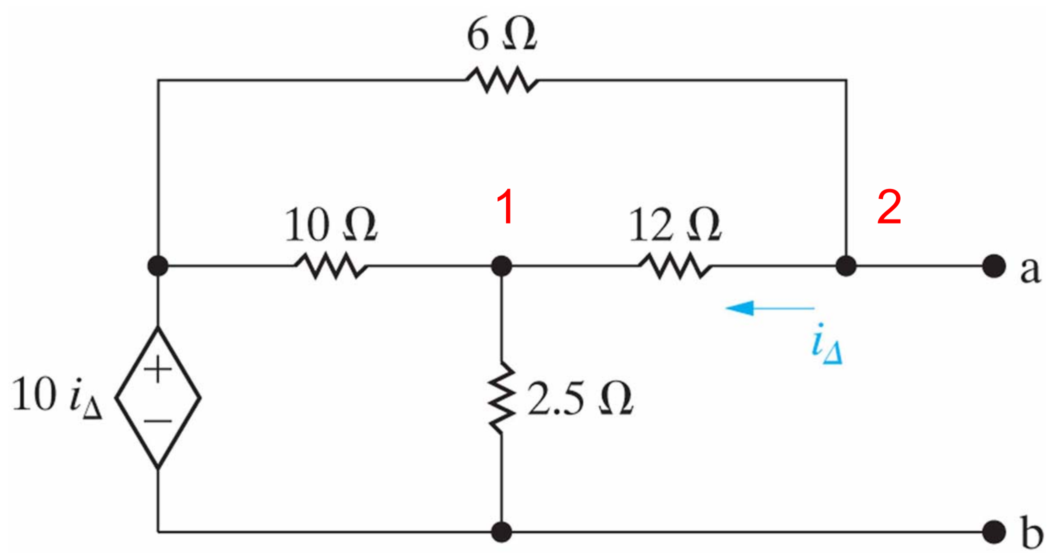
As we saw, deactivating a voltage source means replacing it with a short. If we do that and put an external vex =1V source across (a.) and (b.) , a current i = -3v/2kΩ will flow in the left hand side circuit.



Since i = -3v/2kΩ, and v = vext = 1V, this means that i= 0.0015 A and the dependent current source is providing 0.03A of current. This current alone would produce a -0.75V voltage drop on the 25 Ω resistor, so the external voltage source must provide an additional current of iex = 0.01 A to maintain v = 1 V.

Therefore vex/iex = 1/0.01 = 100 Ω as we saw before

OK here's another circuit with <u>only</u> dependent sources:

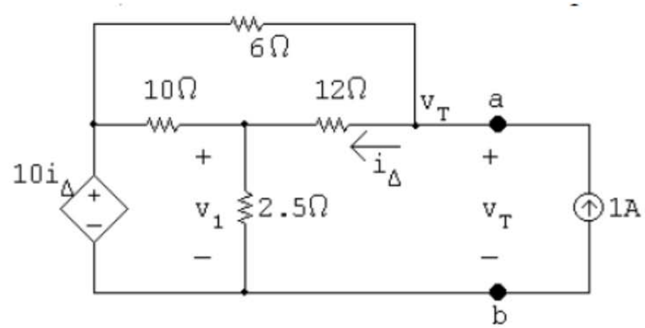


Find the Thevenin equivalent with respect to terminals (a.) and (b.)

The first step is to calculate the open circuit voltage at (a.) and (b.) – but there's something funny happening here! The only voltage source in the problem is the dependent source v =10 i_Δ but there's no source to create i_Δ . You can do a Node Voltage analysis on the two nodes shown above and you'll find the only solution that satisfies KCL is for both voltages to be zero. So **circuits with only dependent sources have Thevenin voltages of zero! This is always true….and it's important to remember.**

The circuit will still have a Thevenin resistance however, and we can calculate it by the external source method.

We'll apply an external current source of 1A to terminals (a.) and (b.) and write the Node Voltage equations for v1 and vt



$$\frac{v_1 - 10i_\Delta}{10} + \frac{v_1}{2.5} + \frac{v_1 - v_\mathrm{T}}{12} = 0$$

$$\frac{v_\mathrm{T} - v_1}{12} + \frac{v_\mathrm{T} - 10i_\Delta}{6} - 1 = 0$$

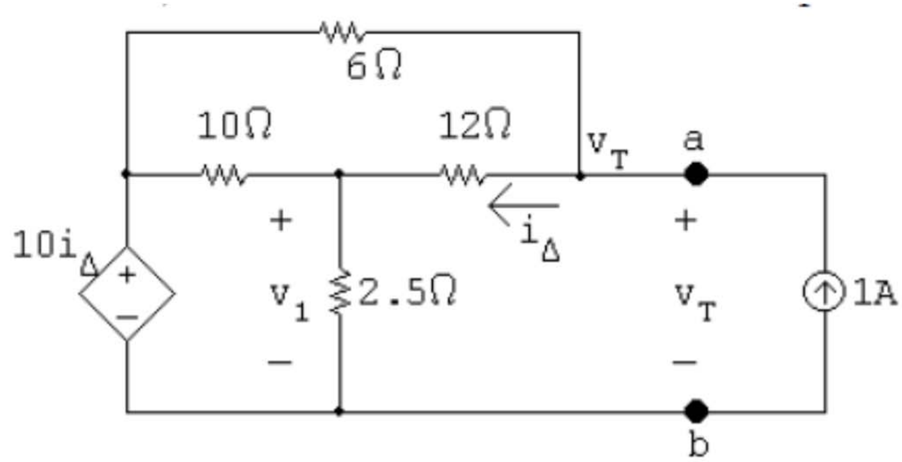
$$i_\Delta = \frac{v_\mathrm{T} - v_1}{12}$$

In standard form:

$$v_1\left(\frac{1}{10} + \frac{1}{2.5} + \frac{1}{12}\right) + v_\mathrm{T}\left(-\frac{1}{12}\right) + i_\Delta(-1) = 0$$

$$v_1\left(-\frac{1}{12}\right) + v_\mathrm{T}\left(\frac{1}{12} + \frac{1}{6}\right) + i_\Delta\left(-\frac{10}{6}\right) = 1$$

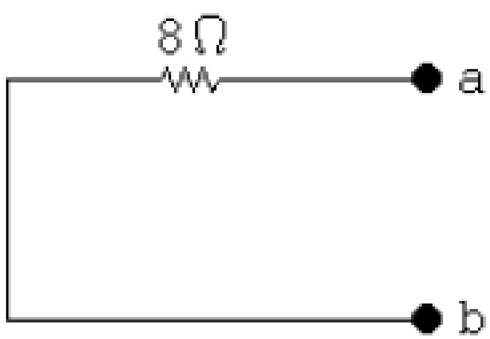
$$v_1(1) + v_\mathrm{T}(-1) + i_\Delta(12) = 0$$



Solving: v1 = 2V , vt = 8V, and i_Δ = 0.5A

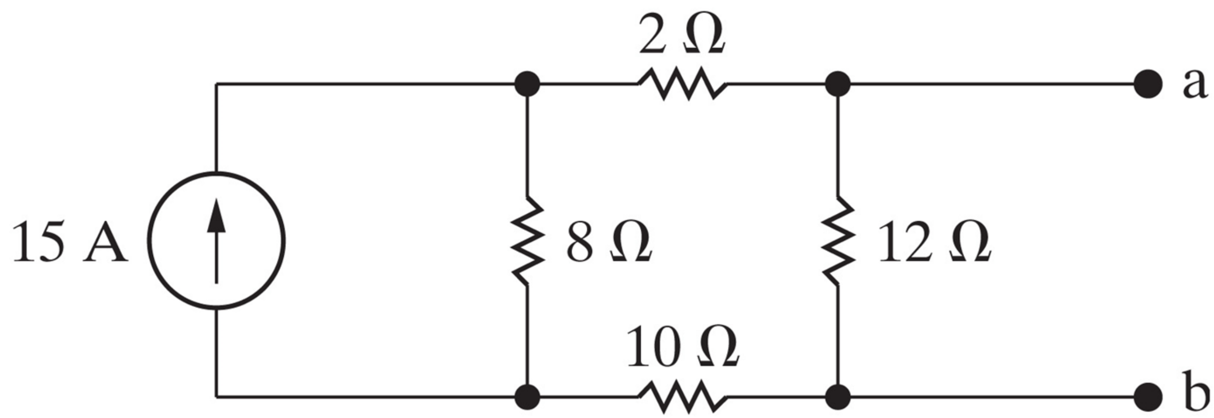
So Rth = vt/1A = 8Ω

The Thevenin equivalent circuit is therefore:



It is always true that circuits with no independent sources have zero Thevenin voltages

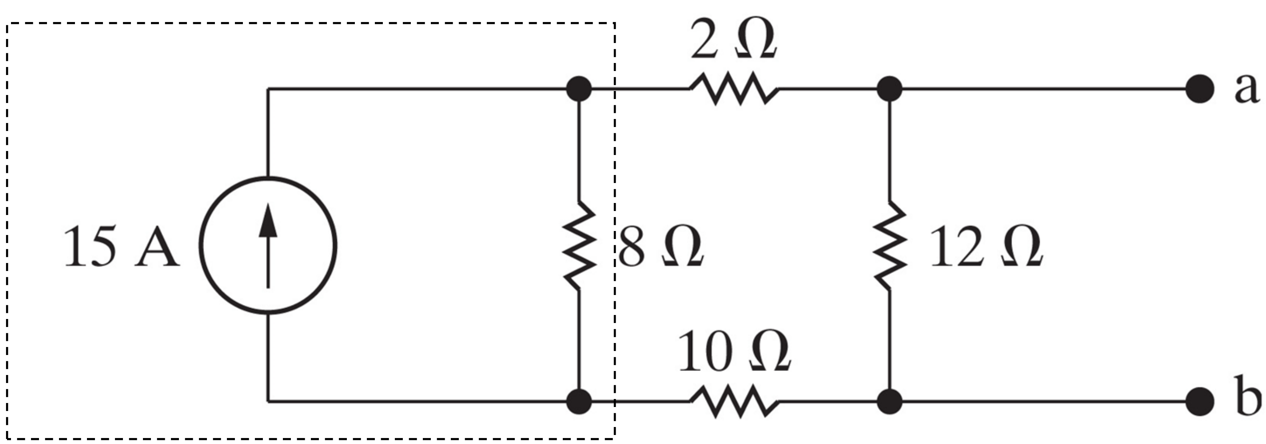
Let's do another one – find the Norton equivalent ckt. at a and b.

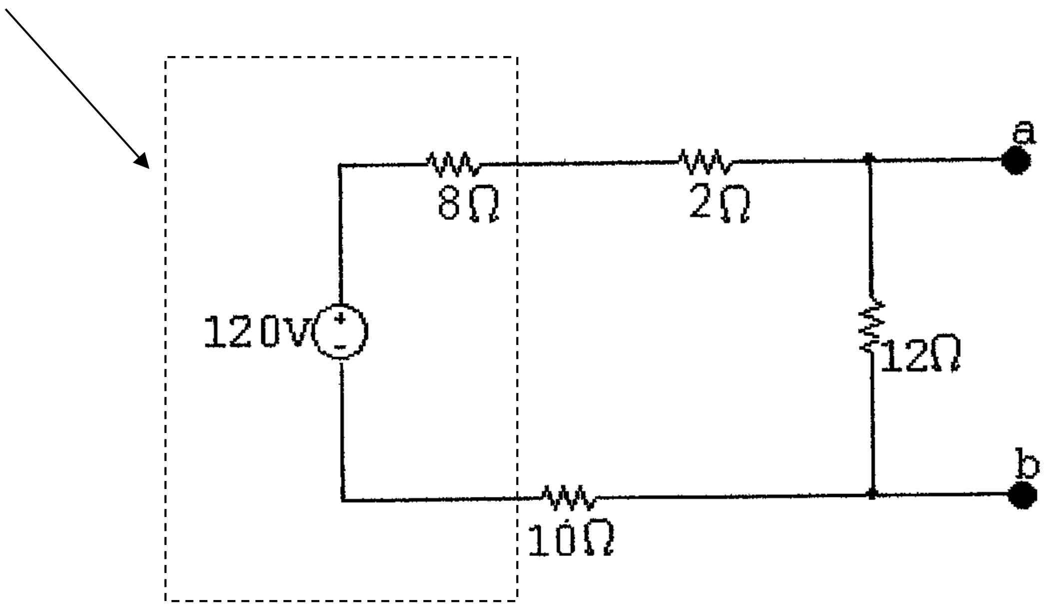


Since the question asks about a Norton equivalent we're looking for a current source in parallel with a resistor to replace this circuit as far as terminals a and b are concerned.

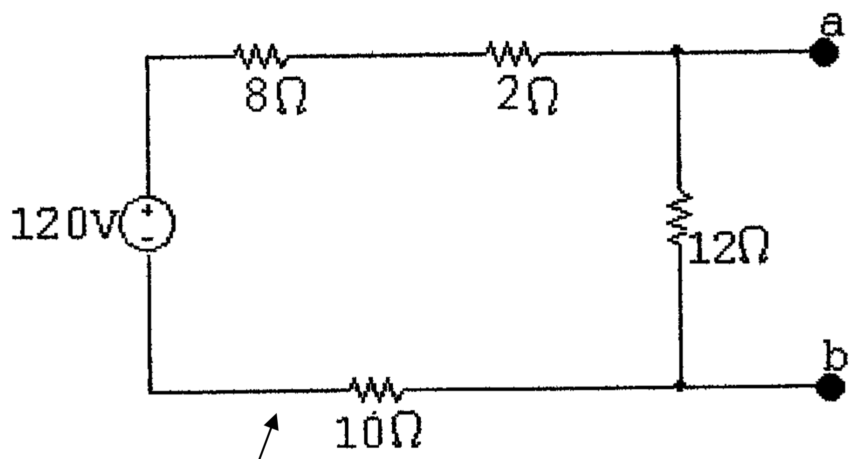
What method should we use – no dependent sources so either #1 or #2 will work …but maybe there's an even easier way – using source transforms!

We can start by transforming the 15A source and parallel 8Ω resistor into a voltage source in series with a resistor, using is = Vs/R

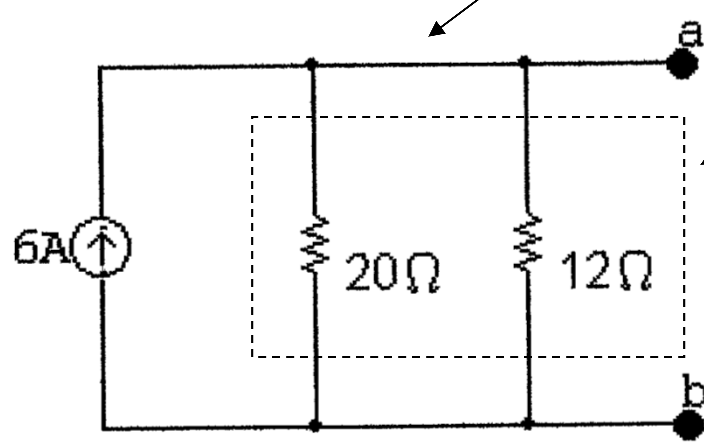




Now combine the 2Ω, 8Ω, and 10Ω resistors into a 20Ω resistor in series with the 120V source and transform this into a current source and a parallel resistor.



<u>This</u> transforms to <u>this</u>

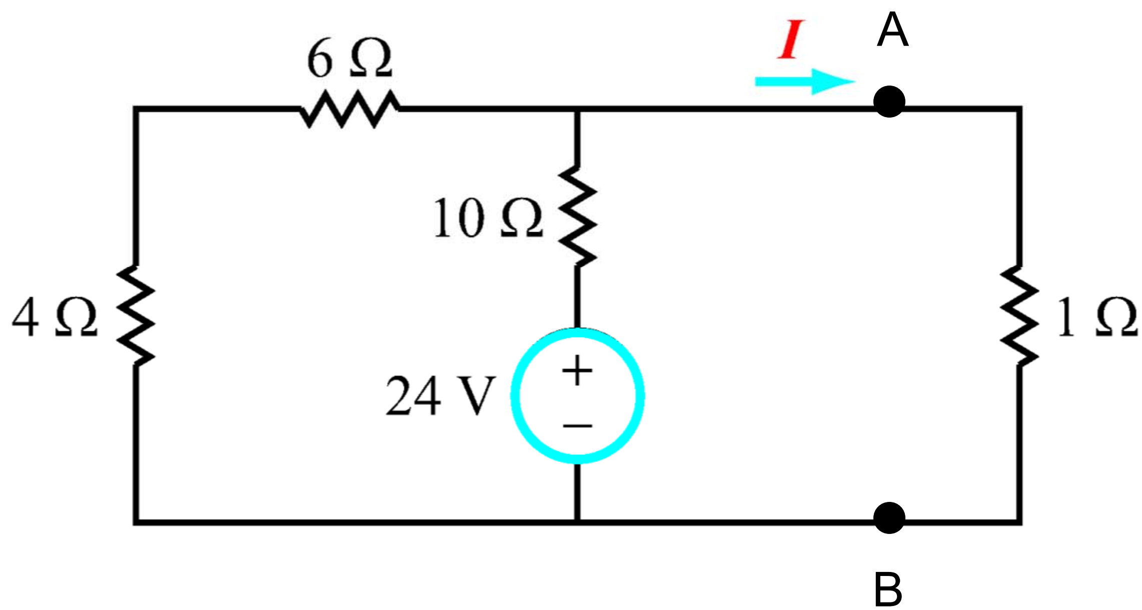


$20\Omega \parallel 12\Omega$

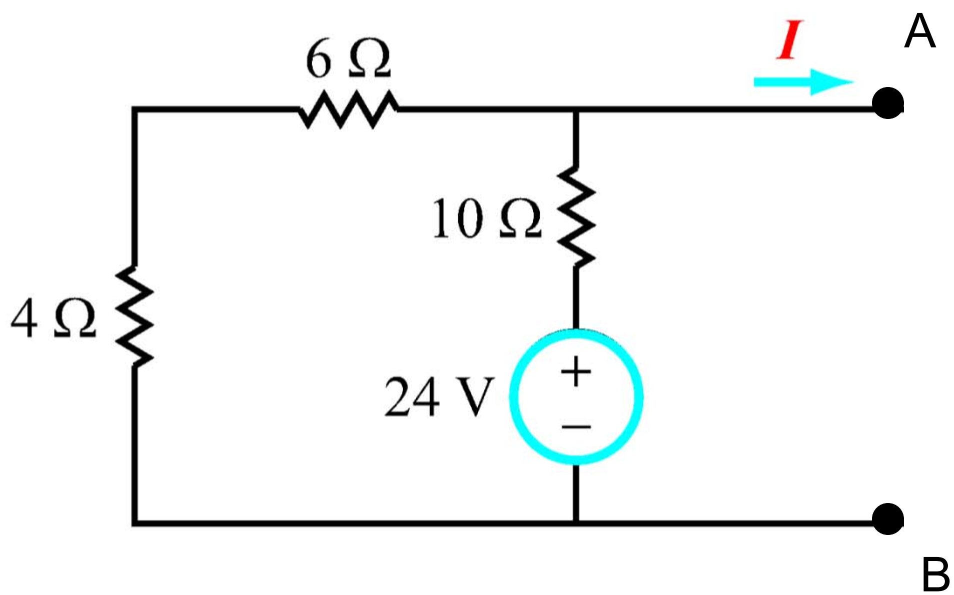
$= 7.5\Omega$

So the Norton equiv. ckt is a 6A source in parallel with a 7.5 Ω resistor

Here's a simple circuit we solved by the Node Voltage method in class 8. The question then was, what is the current, I?



What is the Thevenin equivalent of the circuit at terminals A and B with the 1Ω resistor removed?



Well, there are no dependent sources so we can use method #1 or method #2. Let's use #2. Choosing terminal B as ground, we can write the node voltage equation for terminal A as:

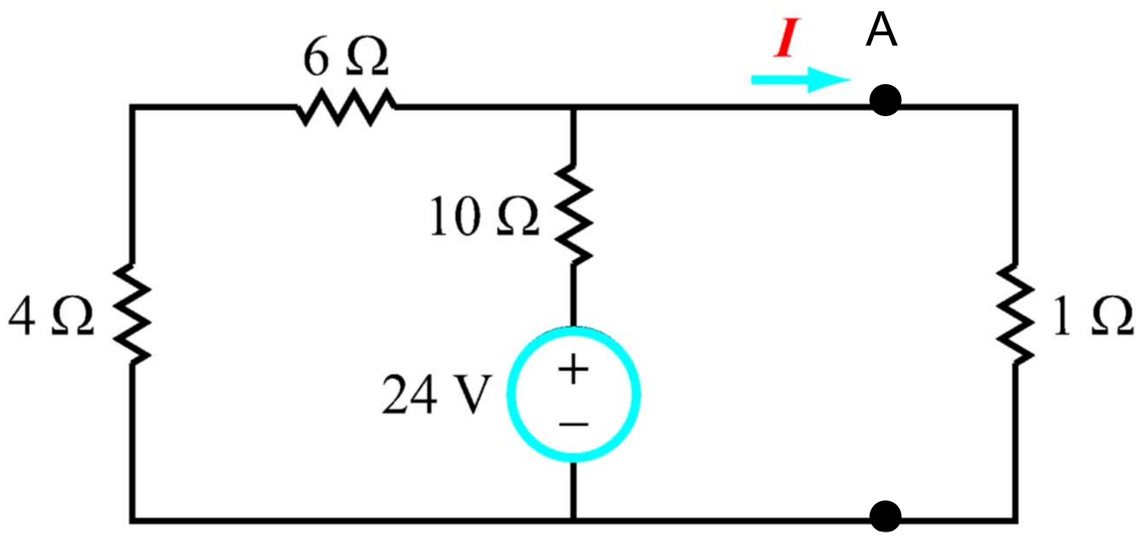
$$\frac{V_A}{10} + \frac{(V_A - 24)}{10} = 0$$

$$2V_A = 24$$

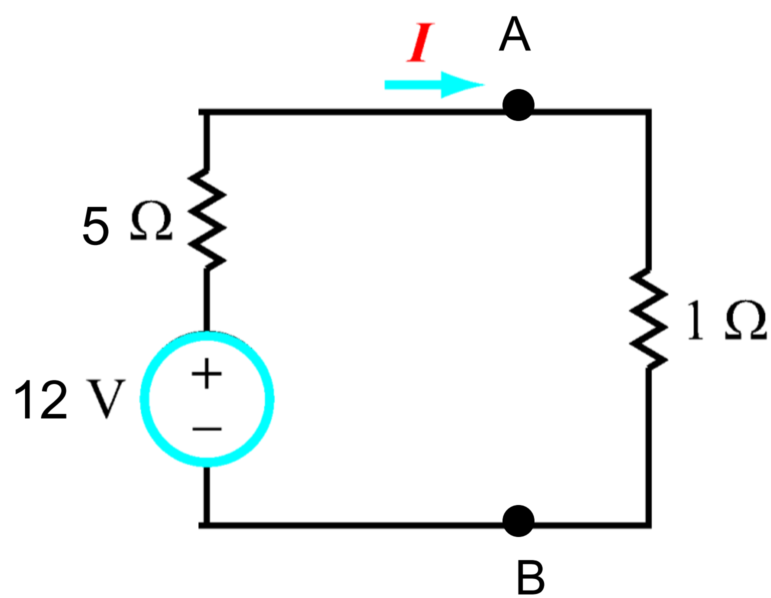
$$V_A = V_{Th} = 12$$

And we can get R_{th} by deactivating the 24V source (shorting it) and calculating the resistance from A to B as R_{th} = 10 Ω || 10 Ω = 5 Ω

So we can replace the original circuit with its Thevenin equivalent plus a 1Ω resistor across A and B.



and, of course, this Thevenin equivalent circuit will produce a current I = 12V/6Ω = 2A the same answer we got in class 08.



Now we are ready to use Thevenin Equivalent circuits to address the issue of maximum power transfer. We'll discuss this in the next class …and it is also the subject of Project I in lab.