

SANTA CLARA UNIVERSITY  
Electrical Engineering Department

ELEN 21/COEN 21 Introduction to Logic Design – Spring 2014

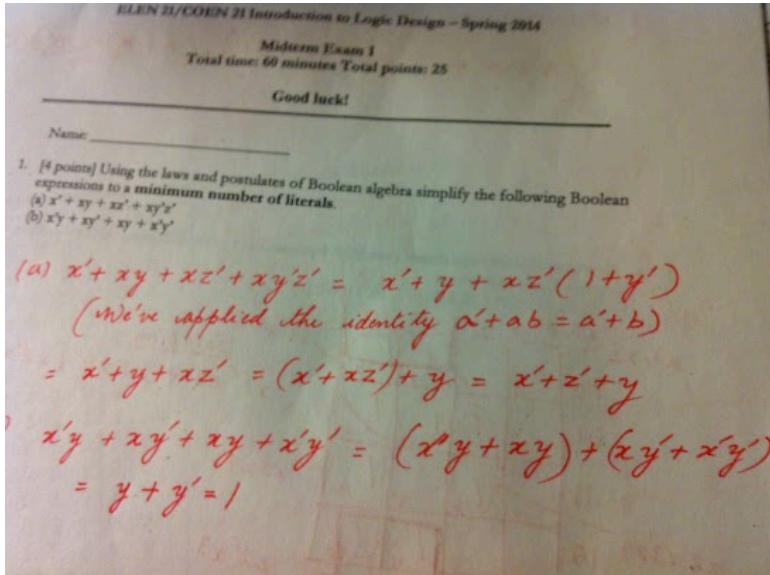
Midterm Exam 1 solution  
Total time: 60 minutes Total points: 25

Good luck!

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Name: \_\_\_\_\_

1. [4 points] Using the laws and postulates of Boolean algebra simplify the following Boolean expressions to a **minimum number of literals**.  
(a)  $x' + xy + xz' + xy'z'$   
(b)  $x'y + xy' + xy + x'y'$



2. [4 points] Express the following function in **sum of minterms** and **product of maxterms**.

$$F(x, y, z) = (xy + z)(xz + y)$$

$$\begin{aligned}
 &= x' + y + xz = (x + xz) \cdot y + xz = y + xz \\
 (b) \quad &x'y + xy' + xy + x'y' = (x'y + xy) + (xy' + x'y') \\
 &= y + y' = 1
 \end{aligned}$$

2. [4 points] Express the following function in **sum of minterms** and **product of maxterms**.  
 $F(x, y, z) = (xy + z)(xz + y)$

x	y	z	F	Minterms	Maxterms
0	0	0	0		$(x+y+z)$
0	0	1	0		$(x+y+z')$
0	1	0	0	$x'yz$	$(x'+y+z)$
0	1	1	1	$xy'z$	$(x'+y+z')$
1	0	0	0	$x'y'z$	$(x'+y+z)$
1	0	1	1	$xyz'$	$(x'+y+z')$
1	1	0	1	$xyz$	$(x'+y+z')$
1	1	1	1		

Solution:

$$\begin{aligned}
 F &= x'y'z + xy'z + xyz' + xyz \\
 &= (x+y+z)(x+y+z') \\
 &\quad \cdot (x'+y+z)(x'+y+z')
 \end{aligned}$$

3. [2 points] Find the **complement** of the following expression using DeMorgan's theorem:

$$AB(C'D + CD') + A'B'(C' + D)(C + D')$$

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$$\begin{aligned}
 &[AB(C'D + CD') + A'B'(C' + D)(C + D')]' \\
 &= (A' + B' + (C + D')(C' + D)) \cdot (A + B)(C'D)(C'D')
 \end{aligned}$$

4. [4 points] Find the **minimum cost SOP** representation of the following Boolean function using a four-variable map:  
 $F(x_1, x_2, x_3, x_4) = \sum(2, 3, 5, 6, 7, 10, 11, 13, 14)$

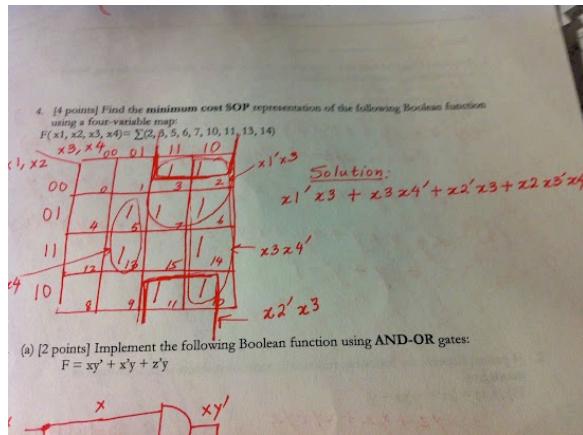
$x_3, x_4$	00	01	11	10
$x_2$	00	01	11	10
00	0	1	3	2
01	4	5	7	6

Solution:

$$x_1'x_3 + x_3x_4' + x_2'x_3 + x_2x_3'$$

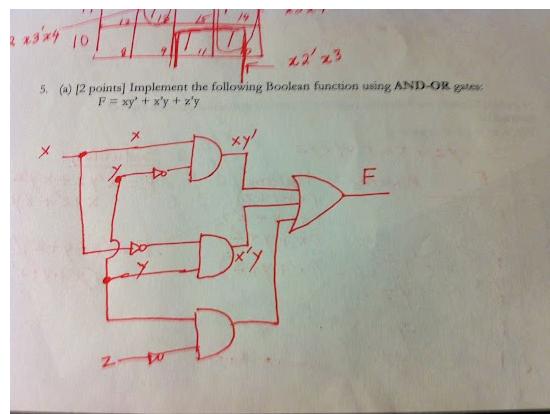
4. [4 points] Find the **minimum cost SOP** representation of the following Boolean function using a four-variable map:

$$F(x_1, x_2, x_3, x_4) = \sum(2, 3, 5, 6, 7, 10, 11, 13, 14)$$



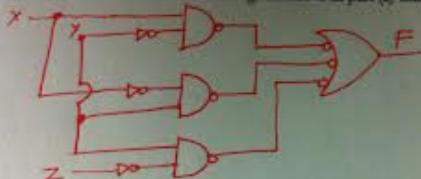
5. (a) [2 points] Implement the following Boolean function using **AND-OR** gates:

$$F = xy' + x'y + z'y$$

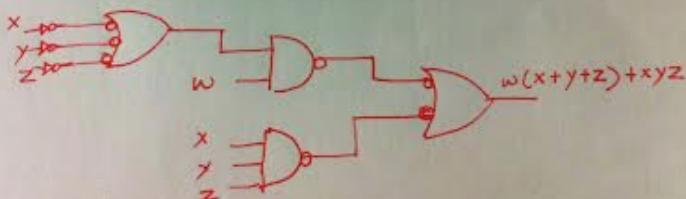


- (b) [2 points] Implement the same function F given above in part (a) using **NAND** gates only.

(b) [2 points] Implement the same function F given above in part (a) using NAND gates only.



6. [3 points] Draw the multi-level NAND circuit for the following expression:  
 $w(x + y + z) + xyz$



6. [4 points] Implement the functions F and G shown in this table on a PLA:

A	B	C	D	F	G
0	0	0	0	1	0
0	0	0	1	0	0
0	0	1	0	0	1
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	1	0
0	1	1	0	1	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	0	1	0	0
1	0	1	0	X	X
1	0	1	1	X	X
1	1	0	0	X	X
1	1	0	1	X	X
1	1	1	0	X	X
1	1	1	1	X	X

7.

[4 points] Implement the functions F and G shown in this table on a PLA:

A	B	C	D	F	G
0	0	0	0	1	0
0	0	0	1	0	0
0	0	1	0	0	1
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	1	0
0	1	1	0	1	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	0	1	0	0
1	0	1	0	X	X
1	0	1	1	X	X
1	1	0	0	X	X
1	1	0	1	X	X
1	1	1	0	X	X
1	1	1	1	X	X

$F = A'B'C'D' + BD + BC$      $G = BC + CD'$

PLA Implementation: