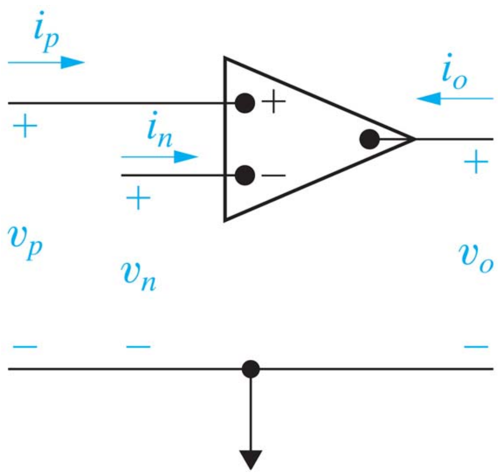
ELEN 50 Class 17 – Op. Amp. Examples

S. Hudgens

In the last class we introduced the operational amplifier ….and we discussed the assumptions that are used to solve negative feedback op amp circuits within the ideal op amp approximation. (assumes infinite open loop gain and infinite input impedance)

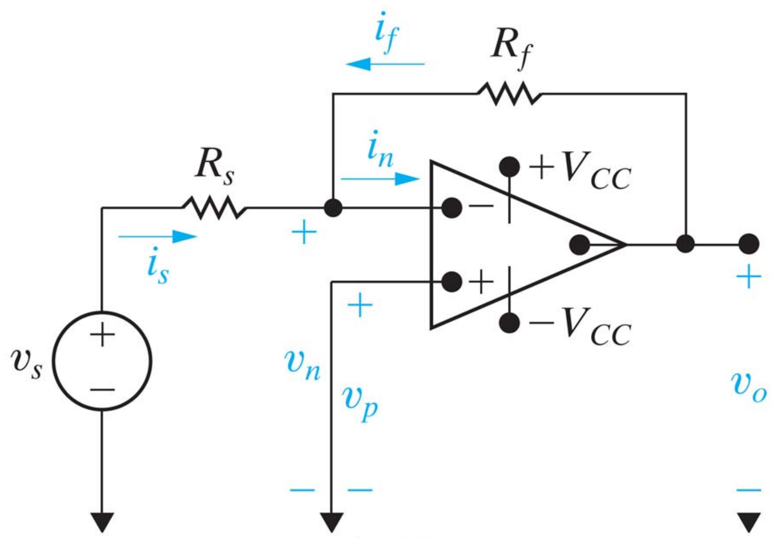
$$\boxed{v_p = v_n \quad \text{and} \quad i_p = i_n = 0}$$

The op amp
"Golden Rules"



These assumptions plus whatever else you've learned about circuit theory (KCL, KVL, etc.) will allow you to solve every op amp circuit!

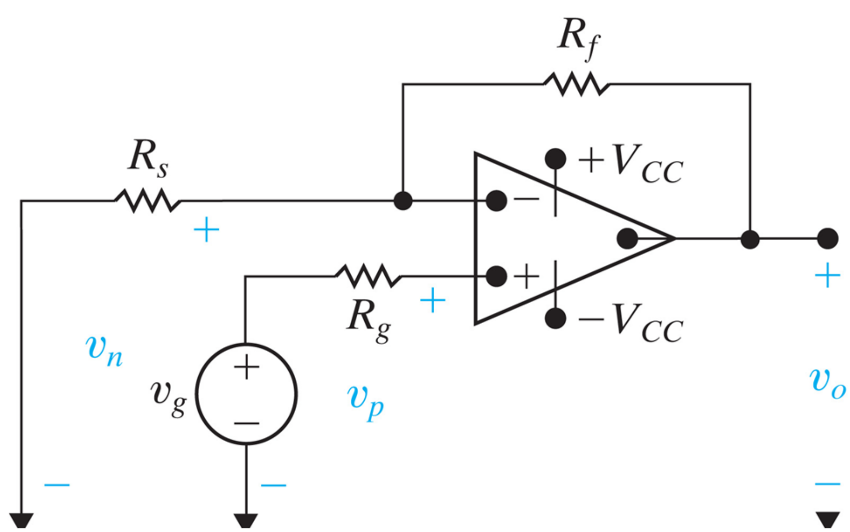
Inverting Amplifier



$$\boxed{v_0 = -(R_f / R_s)\ v_s}$$

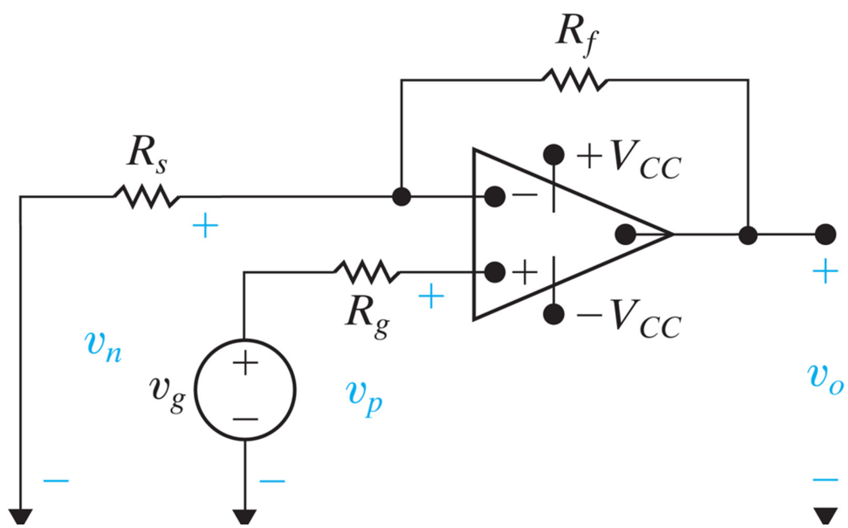
Notice: output is inverted from input

Non-inverting Amplifier



$$v_0 = v_g(R_s + R_f) / R_s$$
$$= v_g(1 + R_f/R_s)$$

Notice: (1+Rf/Rs) ..not (Rf/Rs) as with the inverting amplifier



We introduced the noninverting amplifier configuration in the last lecture. We can derive the formula for the amplifier gain easily. Because we're assuming <u>ideal op amp operation</u>, $v_p = v_n$ and since $i_n = i_p = 0$ (infinite input impedance) this means:

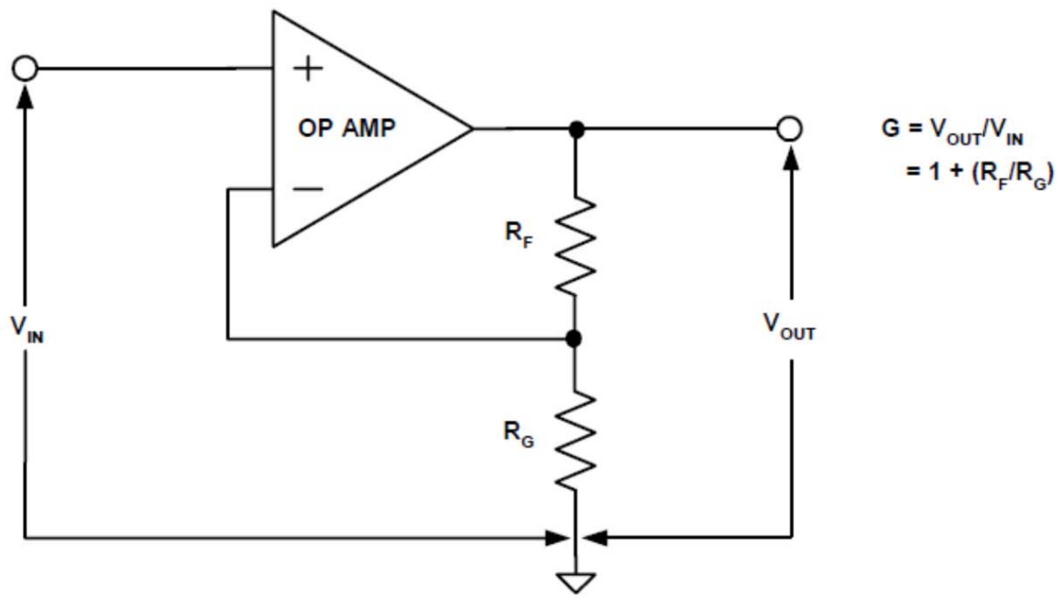
$$v_n = v_g = \frac{v_0 R_s}{R_s + R_f}$$

R_s and R_f make a voltage divider for the output voltage – solving for v_0

$$v_0 = \frac{R_s + R_f}{R_s} v_g = v_g \left[1 + \frac{R_f}{R_s} \right]$$

<u>Why doesn't this circuit depend on the choice of R_g?</u> Hint …think about the golden rules!

Here's another way of drawing the non-inverting op amp configuration – which might make the equation for its gain more intuitive:

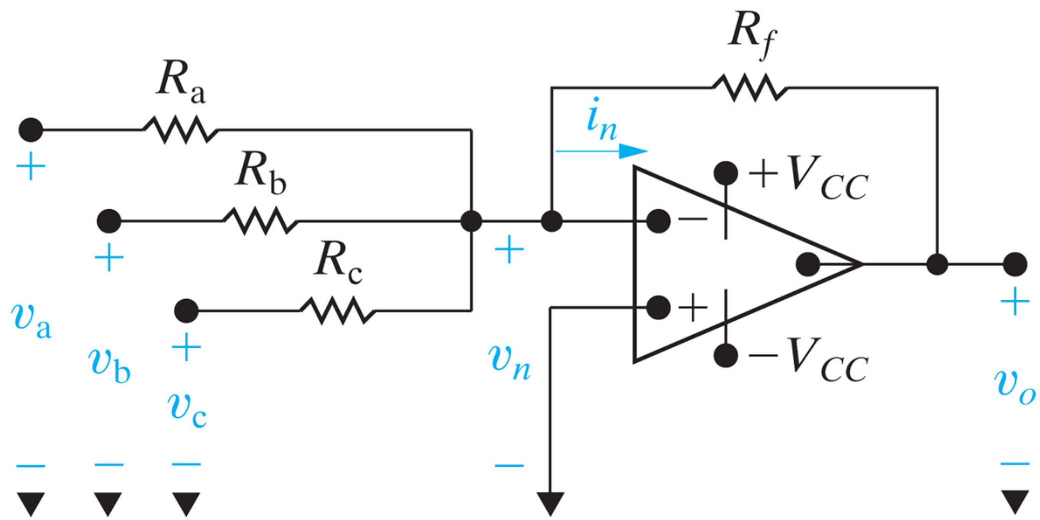


since the op amp golden rule says that $v_p = v_n$ -- because of negative feedback, this means that the voltage across R_G has to be equal to V_{in} ..and the voltage across R_G is just a voltage divider for the output voltage V_{out}

$$so \quad V_{in} = V_{out}\frac{R_G}{R_F + R_G}$$

$$V_{out} = V_{in}\left[1 + \frac{R_F}{R_G}\right]$$

The Summing Inverting Amplifier Circuit



We can analyze this circuit by writing KCL at the inverting input terminal of the op amp. This is, in general, a good strategy for solving op amp circuits.

$(v_n - v_a) / R_a + (v_n - v_b) / R_b + (v_n - v_c) / R_c + (v_n - v_0) / R_f + i_n = 0$

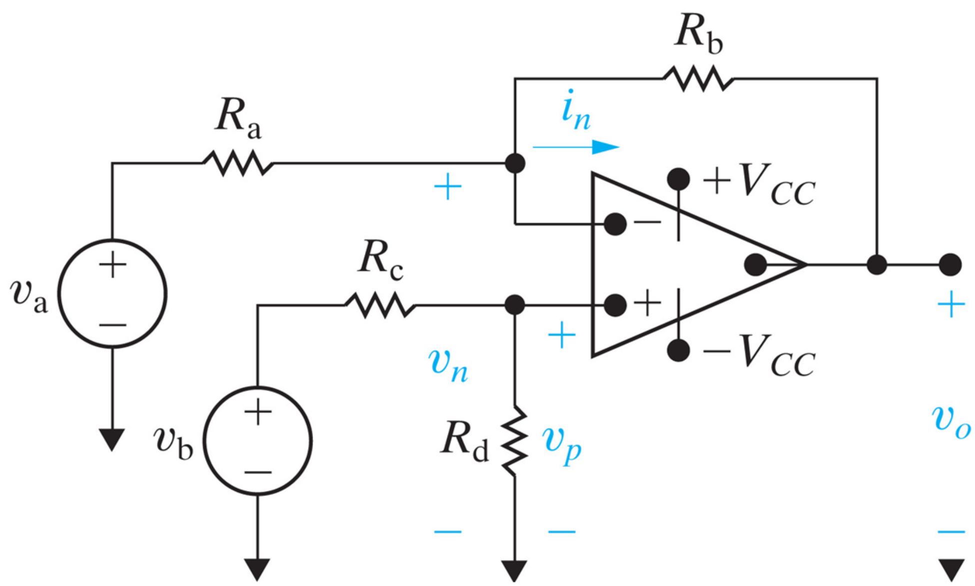
and assuming the ideal op amp Golden Rules: $i_n = 0$, and $v_n = v_p = 0$, so:

$$v_0 = -\left(\frac{R_f}{R_a} v_a + \frac{R_f}{R_b} v_b + \frac{R_f}{R_c} v_c \right)$$

We could have also gotten this answer by superposition…..right?

What could you do with this circuit? Well….if you needed to add together a number of inputs, it could be useful…..for example…

The Difference Amplifier Circuit



To solve this circuit, we can write KCL for the inverting input node as usual:

$(v_n - v_a)/ R_a + (v_n - v_0)/ R_b + i_n = 0$

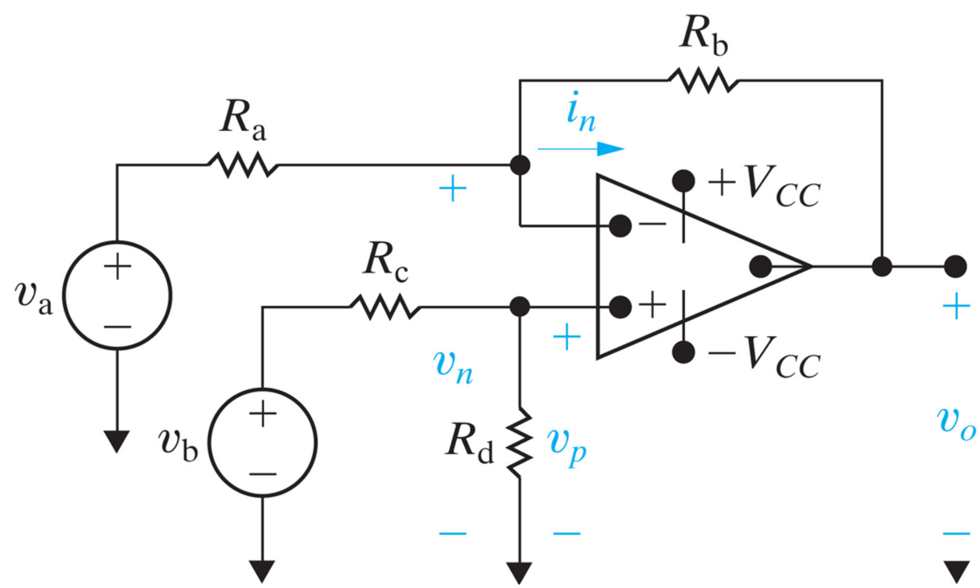
Making the <u>ideal op amp approximations</u>: $i_n = i_p = 0$ and $v_n = v_p = v_b\,[R_d/ (R_c + R_d)]$

And substituting into the first equation:

$$v_0 = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)} v_b - \frac{R_b}{R_a} v_a$$

The output is proportional to the difference between v_a and v_b with (different) scaling factors, determined by R_a, R_b, R_c, and R_d.

The Difference Amplifier Circuit



Note: if we choose the special case: Ra/Rb = Rc/Rd

$$v_0 = \frac{R_d\left(R_a + R_b\right)}{R_a\left(R_c + R_d\right)} v_b - \frac{R_b}{R_a} v_a \qquad \text{becomes} \qquad v_0 = \frac{R_b}{R_a}\left(v_b - v_a\right)$$

Why are difference amplifiers important?

Difference amplifiers can be used to amplify weak signals in the presence of much larger-amplitude, so called “common mode” interfering signals …such as electromagnetic interference from AC power lines, RF signals, etc. Because of this, they are widely used in medical instrumentation used to obtain, electrocardiograms or electroencephalograms, where the signal source (the patient) is in an electrically noisy environment.

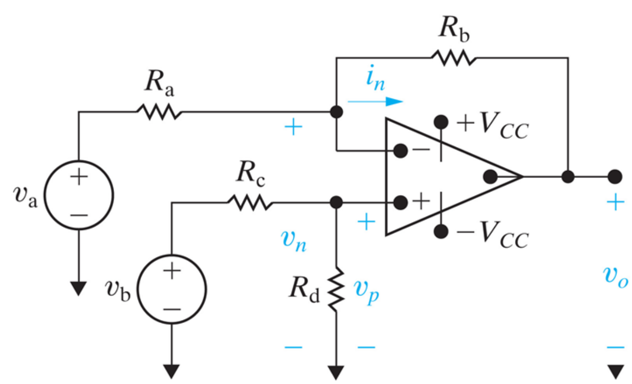
To understand this feature, we need to understand the idea of common mode rejection. We'll define two new signal amplitudes:

The differential mode input to a difference amplifier is:

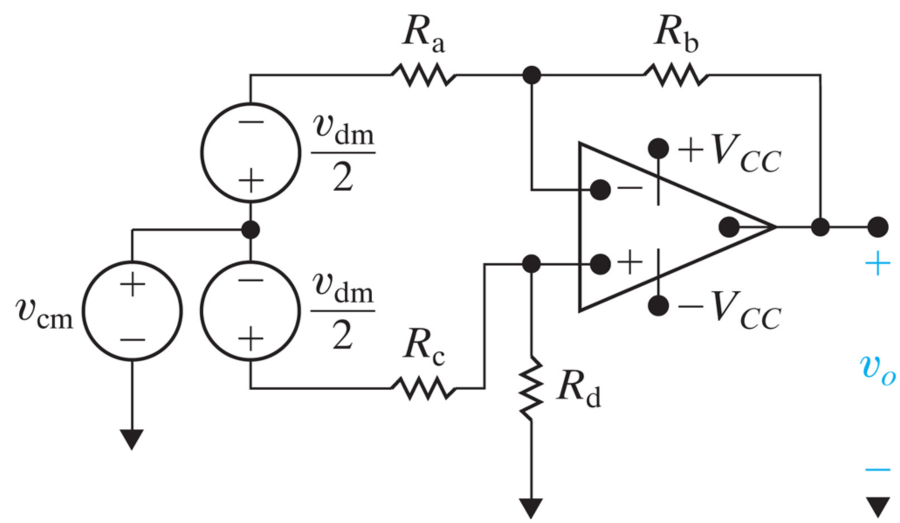
$$v_{dm} = v_a - v_b$$

The common mode input is:

$$v_{cm} = \frac{(v_a + v_b)}{2}$$



We can redraw the circuit and rewrite the gain equation for the difference amplifier in terms of the two, new signals we've defined…the common mode signal, v_{cm}, and the differential mode signal, v_{dm}



If we choose Ra/Rb = Rc/Rd, and assume ideal op amp characteristics, as we did before:

$$v_0 = (0)v_{cm} + \left(\frac{R_b}{R_a}\right)v_{dm}$$

Only the differential mode signal is amplified – the common mode signal is <u>completely rejected.</u> No real difference amplifier can accomplish this and the <u>common mode rejection ratio</u> (the ratio of differential mode gain to common mode gain) is an important performance metric for difference amplifiers.

The effect of resistance mismatch on common mode rejection ratio (CMRR)

If we haven't precisely chosen the feedback and input resistors so that Ra/Rb = Rc/Rd, CMRR is degraded. How sensitive is CMRR to mismatch?

We can describe a resistance mismatch by writing: $$\frac{R_a}{R_b} = (1-\varepsilon)\frac{R_c}{R_d}$$

Now, substituting into the full expression for the common mode gain:

$$A_{cm} = \frac{R_a(1-\varepsilon)R_b - R_a R_b}{R_a\left[R_a + (1-\varepsilon)R_b\right]}$$

$$A_{cm} = \frac{-\varepsilon R_b}{R_a + (1-\varepsilon)R_b}$$

and because ε is much smaller than 1,

$$A_{cm} \approx \frac{-\varepsilon R_b}{R_a + R_b}$$

Doing the same thing to the full expression for the differential gain

$$A_{dm} = \frac{(1-\varepsilon)R_b(R_a + R_b) + R_b[R_a + (1-\varepsilon)R_b]}{2R_a[R_a + (1-\varepsilon)R_b]}$$

$$A_{dm} = \frac{R_a}{R_b}\left[1 - \frac{\left(\varepsilon/2\right)R_a}{R_a + (1-\varepsilon)R_b}\right]$$

$$A_{dm} \approx \frac{R_b}{R_a}\left[1 - \frac{\left(\varepsilon/2\right)}{R_a + R_b}\right]$$

So, $$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| \qquad CMRR \approx \left|\frac{1 + \frac{R_b}{R_a}}{-\varepsilon}\right|$$

$$CMRR \approx \left| \frac{1+\frac{R_b}{R_a}}{-\varepsilon} \right|$$

Remember, the differential mode gain is ~ R_b/R_a, so if we make this large, e.g. 100, the CMRR will be quite large (e.g. 1000), even if <u>ε is as big as 10% mismatch.</u>

For this analysis, we are assuming that the op amp itself has identical gain for the inverting and noninverting inputs….and that all degradation to the CMRR is due to external resistors This isn't the case in the real world. Actually, the intrinsic CMRR of an op amp is one of the stated performance characteristics. For example, the inexpensive 741 op amp has a CMRR spec of 90 db…where the CMRR is expressed as:

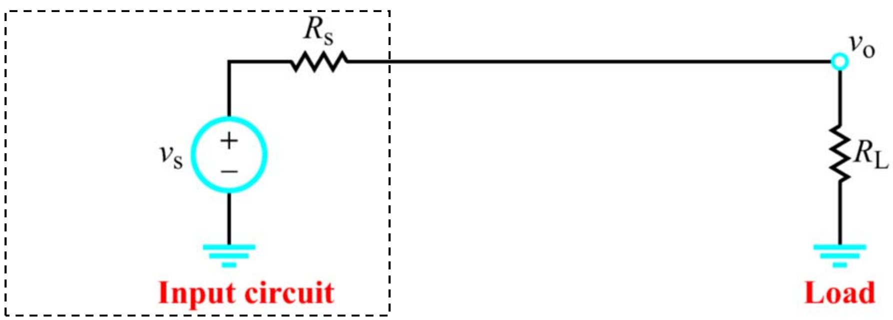
$$\mathrm{CMRR} = 10\log_{10}\left(\frac{A_\mathrm{d}}{A_\mathrm{cm}}\right)^2 = 20\log_{10}\left(\frac{A_\mathrm{d}}{|A_\mathrm{cm}|}\right)$$

So the intrinsic CMRR ~ 30,000 …we could never use this op amp in a difference amplifier circuit to get a CMRR > 30k ..but that's already a huge value!

The Voltage Follower Circuit – also very useful
we talked about this before

Thevenin equivalent of input circuit

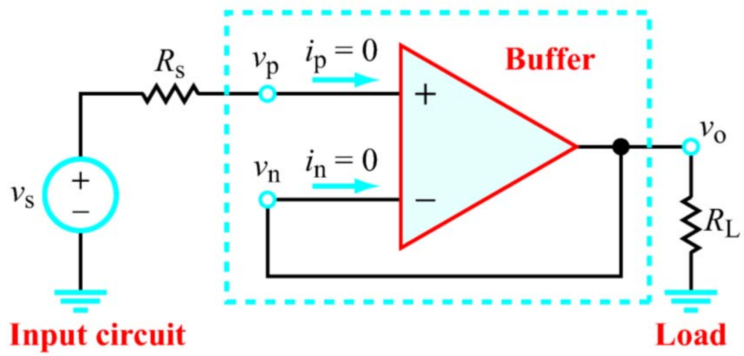
Buffer (Voltage Follower)



(a) Input circuit connected directly to a load

$$v_0 = \frac{v_s R_L}{R_s + R_L}$$

Signal amplitude is degraded by the presence of a load



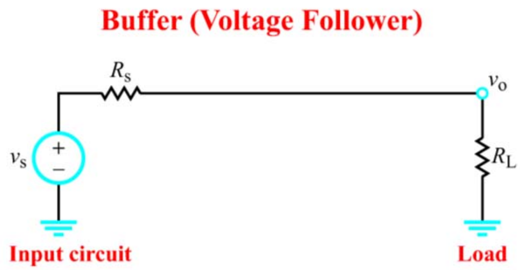
(b) Input circuit separated by a buffer

$$v_0 = v_p = v_s$$

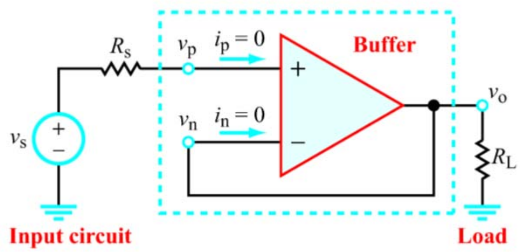
No amplitude degradation!

Output voltage follows the input regardless of load resistance or input resistance variations - this circuit is used a lot in analog designs

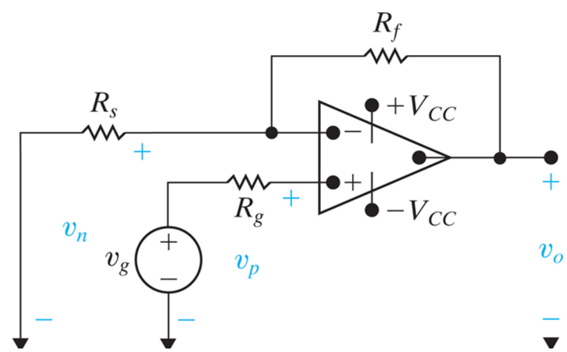
Isn’t the voltage follower just a normal non-inverting amplifier with infinite Rs and zero Rf?



(a) Input circuit connected directly to a load



(b) Input circuit separated by a buffer



Yes! So the gain is

$$\boxed{\begin{aligned} v_0 &= v_g(R_s + R_f) / R_s \\ &= v_g(1 + R_f/R_s) \end{aligned}}$$

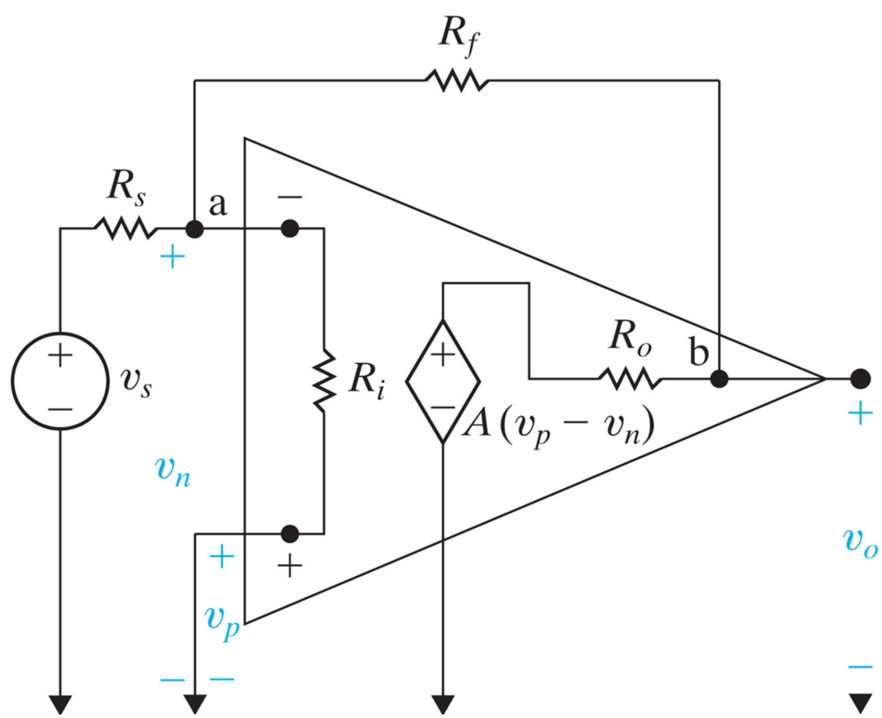
= unity!

A More Realistic Model for a Non-ideal Op Amp

Real op amps don't have infinite open loop gain, or infinite input impedance, or zero output impedance as we have assumed in the previous discussion.

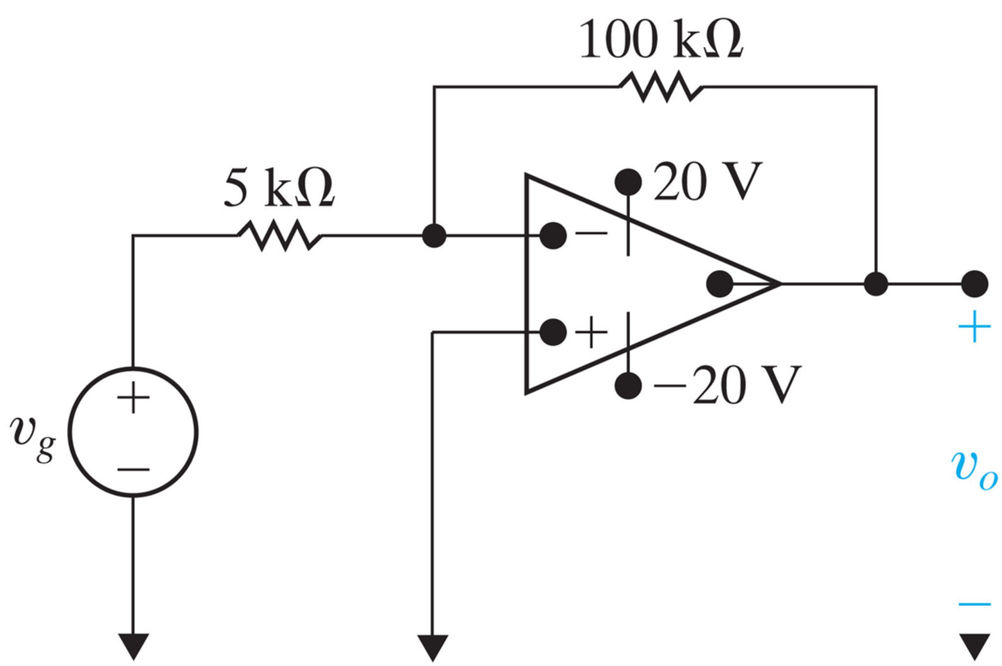
A more realistic model for an op amp is:

Here, we're showing the op amp in an inverting amplifier configuration. It has a finite input impedance of R_i, and a non-zero output impedance of R_0, and a finite open loop gain of A.

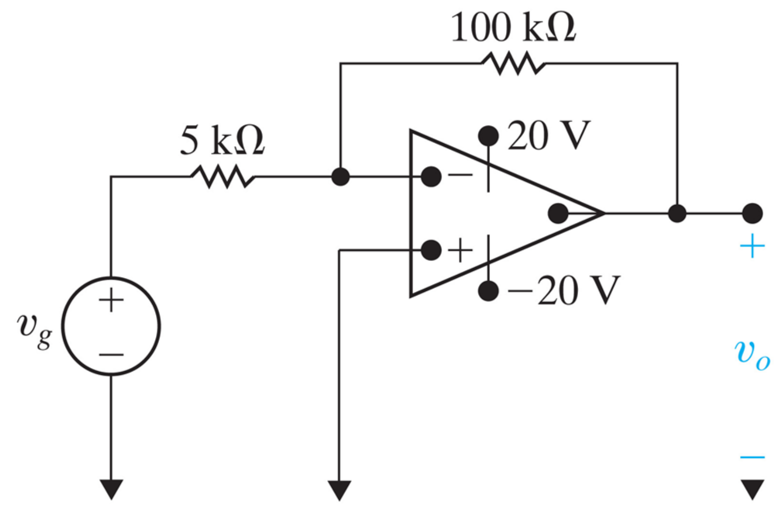


A More Realistic Model for a Non-ideal Op Amp

The derivation for v_0/v_g for this non-ideal model is in most circuits textbooks and it's a big complicated thing – I'm not going to write it down here. However, in one of the self-assessment problems in Nilsson, you are asked to calculate the actual gain, input impedance, and input voltage difference for a noninverting amplifier using the non-ideal model. For this calculation we are to assume the open loop gain is 300,000 (not infinity), the input impedance is 500kΩ (not infinity), and the output impedance is 5kΩ (not zero). What is the impact on the circuit of the nonideal behavior?



So…how good of an approximation are the "op amp golden rules?"



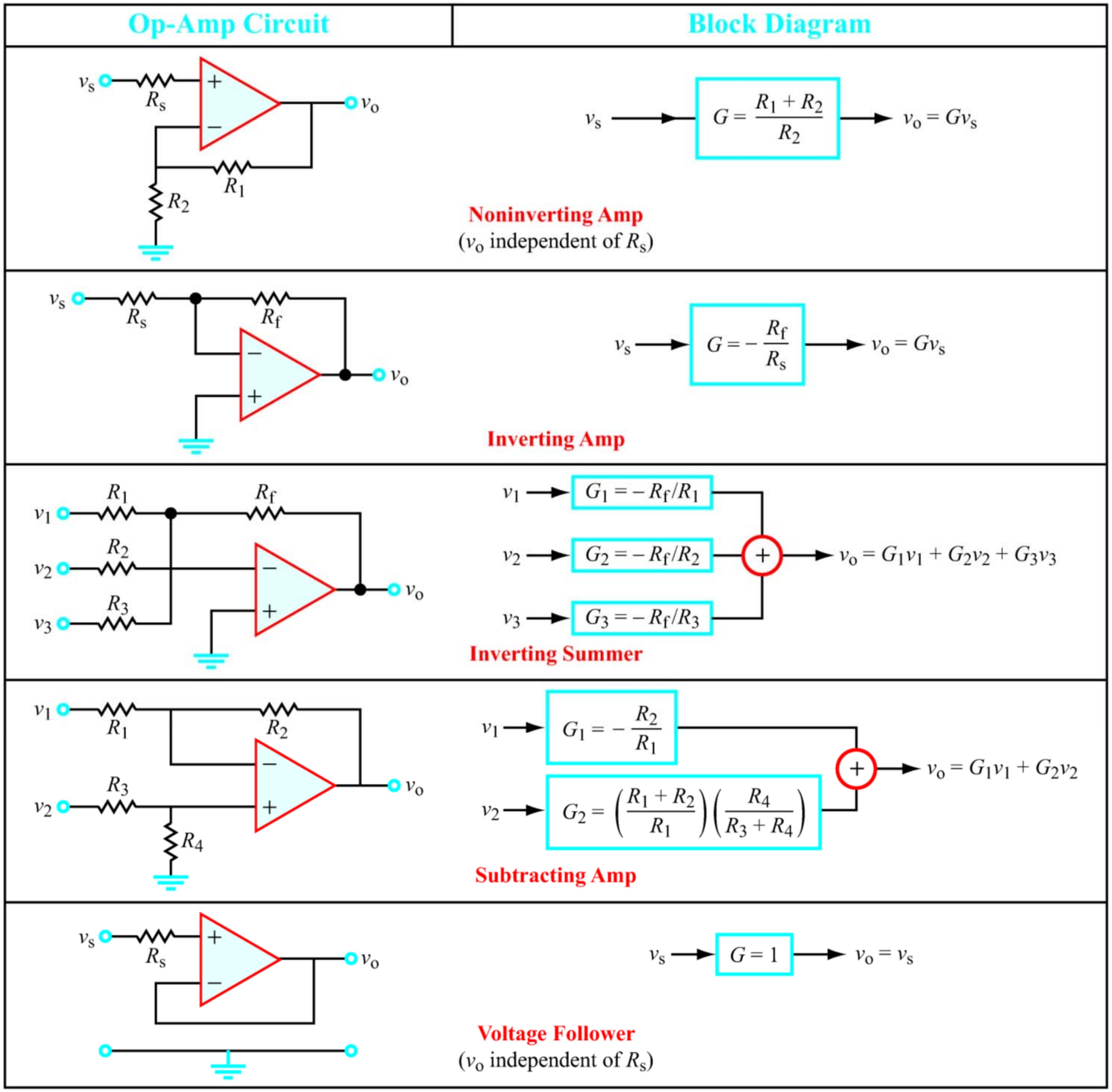
The gain for an ideal op amp in this circuit would be - 20 …for the nonideal op amp it's -19.9985 …about a 75 ppm error.

The voltage v_n at the inverting input is zero when v_g = 1V in the ideal case, and ~ 70 μV in the nonideal case. We know it can't be zero as we discussed earlier – because if it were zero the output would be zero! However, you can see that it is certainly very close to zero!

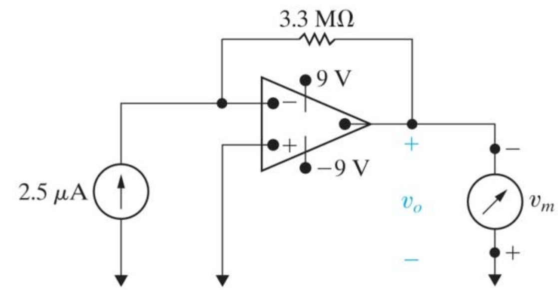
The input impedance seen by v_g is 5kΩ in the ideal case (since the inverting input is a virtual ground) and it's 5.00035 kΩ for the nonideal case….an error of 70 ppb.

Clearly, the ideal op amp approximation is a very good one and probably suitable for many applications.

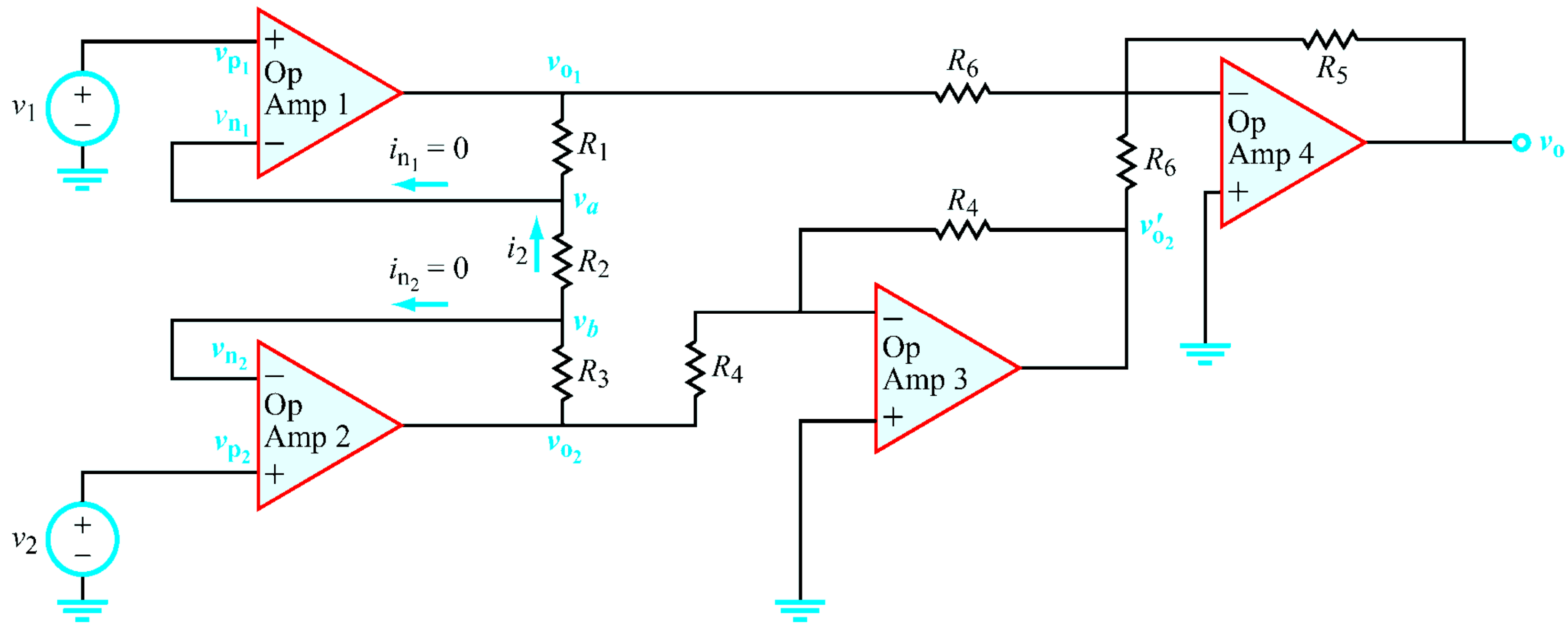
To Review: Some Simple Op Amp Circuits



also remember the current to voltage convertor we saw earlier

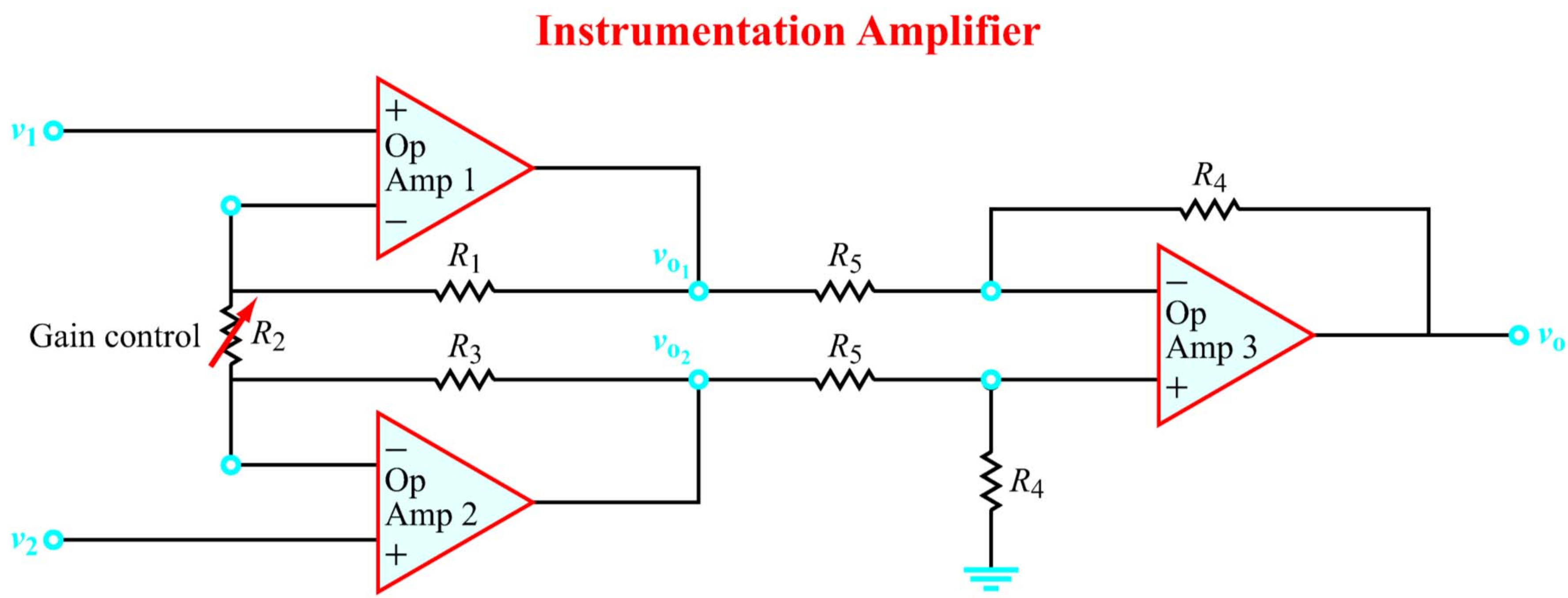


Multiple Op Amp Circuits

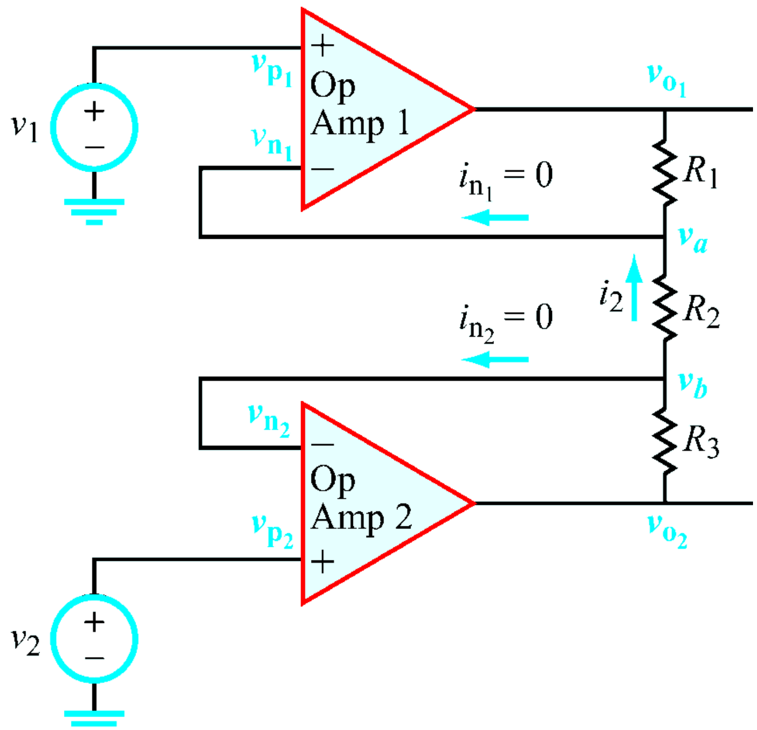


Can you quickly figure out what this circuit does? We want an expression for vo in terms of v1 and v2.

That thing was a kind of instrumentation amplifier. Instrumentation amplifiers are high-sensitivity, high-gain, deviation sensors for use in temperature sensors, strain gauges, etc. You could use one in a bridge circuit for example.



Let's look at the first part of this circuit:



Since $v_p = v_n$ for an ideal op amp

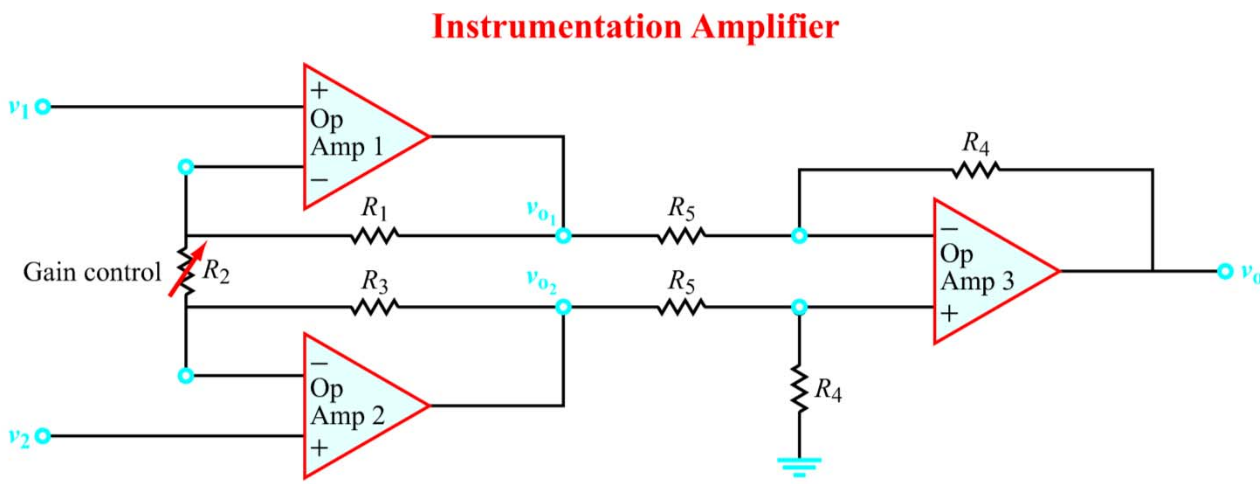
$$v_a = v_1 \quad and \quad v_b = v_2$$

and since $i_{n1} = i_{n2} = 0$ for ideal op amp

$$i_2 = \frac{v_b - v_a}{R_2} = \frac{v_2 - v_1}{R_2}$$

$$v_{02} - v_{01} = i_2(R_1 + R_2 + R_3) = \left[\frac{R_1 + R_2 + R_3}{R_2}\right](v_2 - v_1)$$

Now, for the whole circuit:



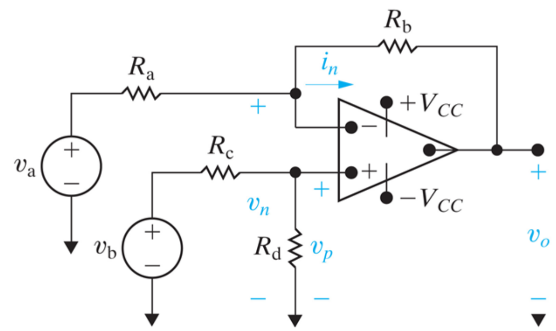
The first two amplifiers provide a differential output:

$$v_{02} - v_{01} = \left(\frac{R_1 + R_2 + R_3}{R_2} \right)(v_2 - v_1)$$

And the third amplifier is a difference amplifier that amplifies ($v_{02} - v_{01}$) by a gain factor = R_4/R_5. <u>If all of the resistors are chosen with the same value, R:</u>

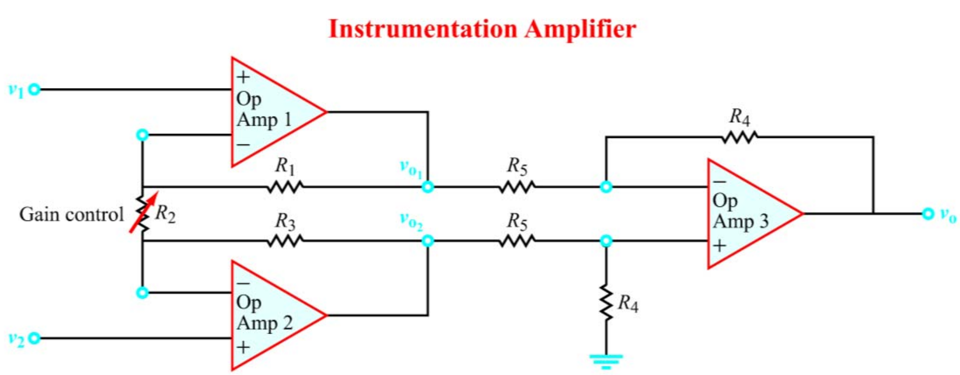
$$v_0 = \left(1 + \frac{2R}{R_2} \right)(v_2 - v_1)$$

So…..the instrumentation amplifier is essentially an ordinary differential amplifier:



$$v_0 = \frac{R_b}{R_a}(v_b - v_a)$$

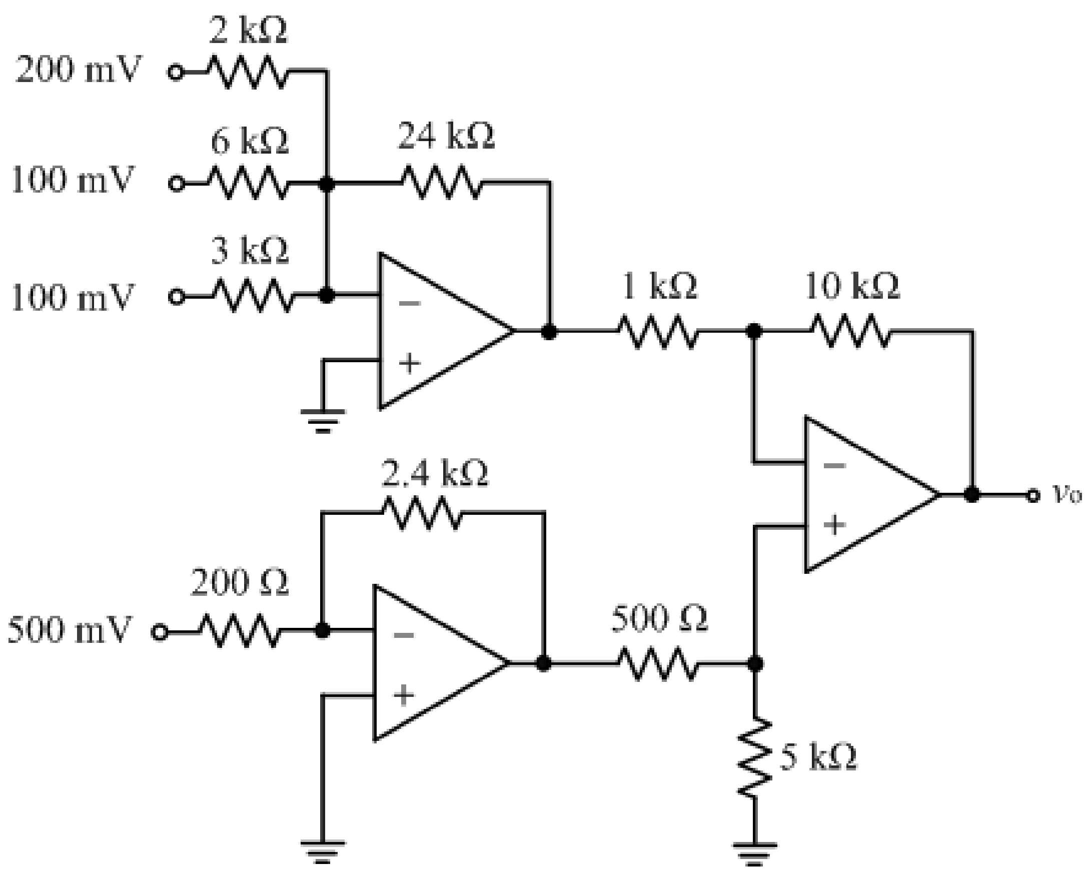
With a pair of high impedance buffer amplifiers at the input.



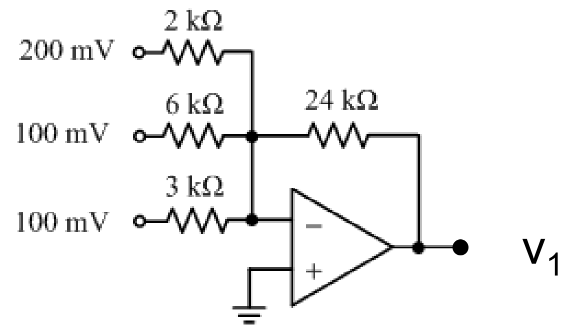
$$v_0 = \left(1 + \frac{2R}{R_2}\right)(v_2 - v_1)$$

Where $R_1 = R_2 = \ldots R_n = R$

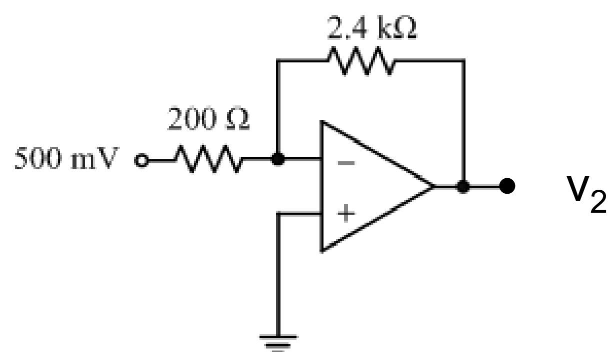
What does this circuit do? Can you systematically “decompose it” into sub-circuits so you can come up with an expression for v_0?



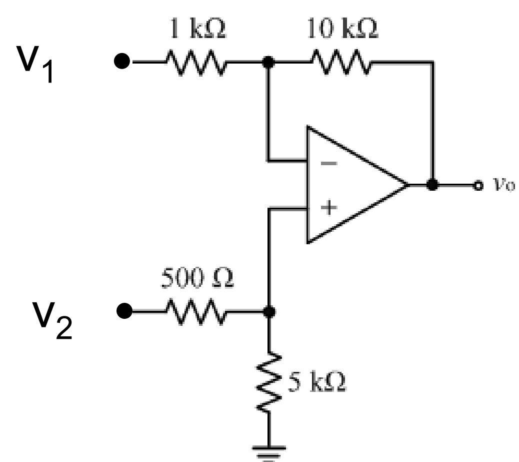
This part is an inverting summing amplifier



This part is just an inverting amplifier



And this part is a differential amplifier



So ……………. V_0= - 24V

Next time we will introduce the final two circuit components that we'll study in ELEN 50 ….capacitors and inductors. These elements are truly boring in DC circuits …since they act like either short circuits or open circuits. However, in circuits with time varying voltages and currents, capacitors and inductors can result in some very interesting and useful circuits – filters, oscillators, phase shifters, etc.

Just for Fun

We talked earlier about the situation where the output voltage is large enough to exceed the power supply voltage. In that situation we saw that the output saturated at the supply voltage value. If the signal being amplified is an AC signal, we call this situation, “clipping.” It results in distortion of the AC signal …and introduces harmonic frequencies.

Sometimes this is actually desirable ….