ELEN 50 Class 03 – Ohms Law ..Circuit Topology

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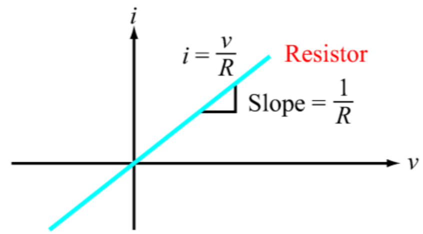
Some review :

Last time we talked about valid circuits and types of sources



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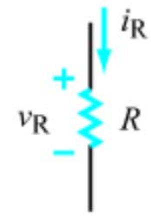
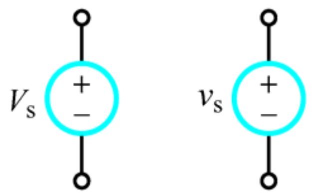


Ohm's Law

$v_R = Ri_R$

independent sources



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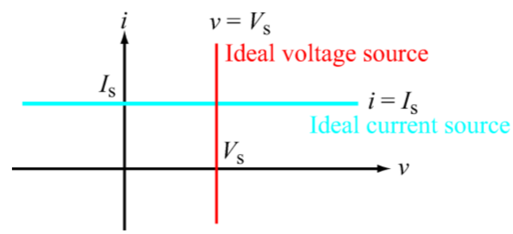

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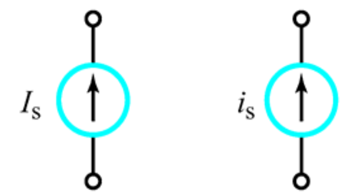

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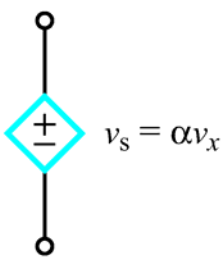
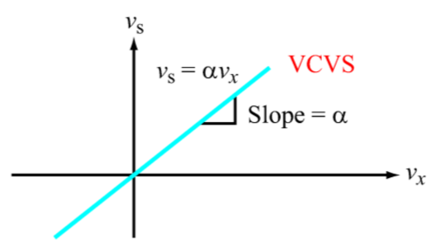
dependent sources





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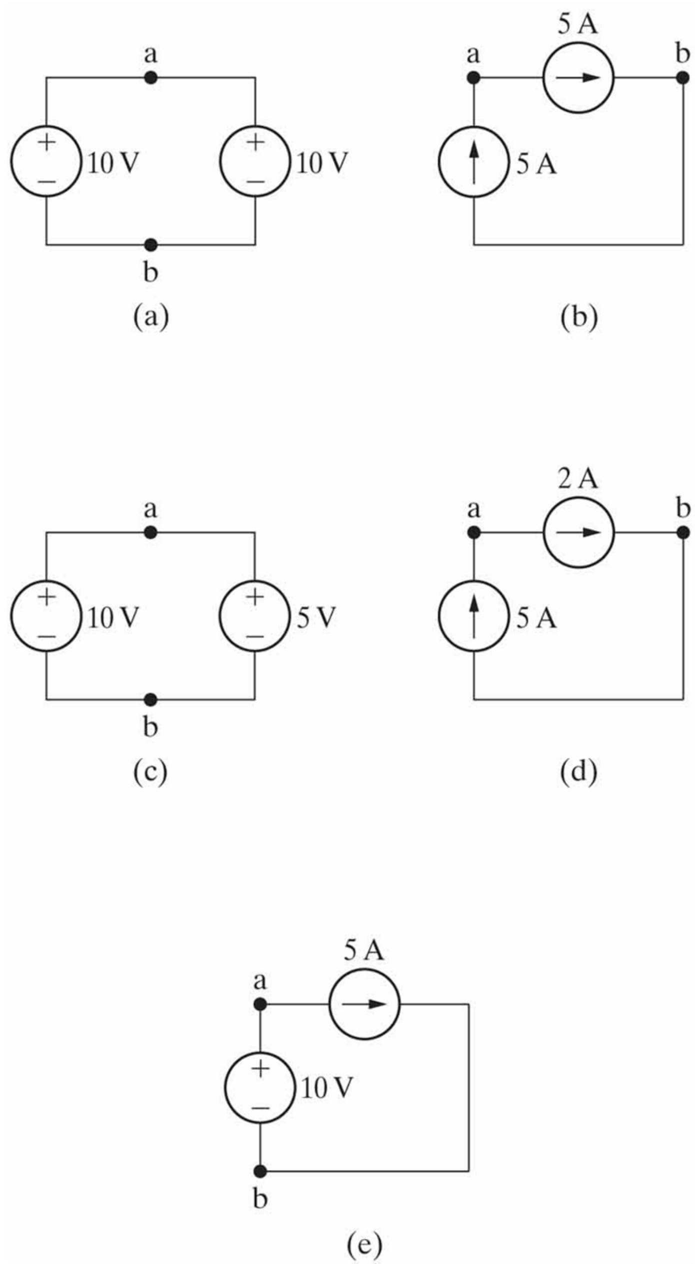

We discussed these circuits – is it clear which of them are valid given the constraints imposed by ideal sources?

Are these circuits valid or invalid …and why?

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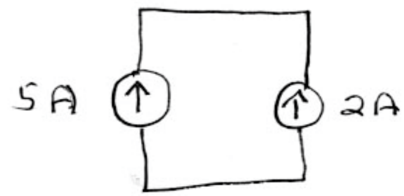

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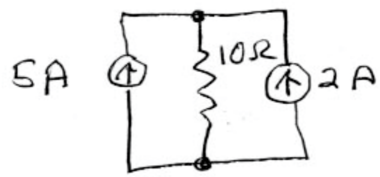
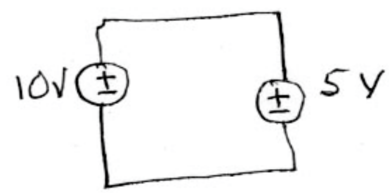


(c.)



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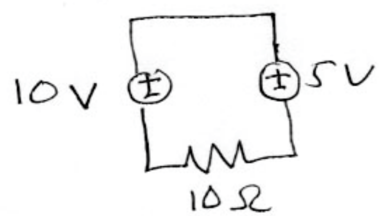
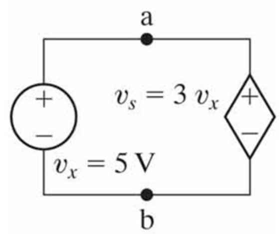
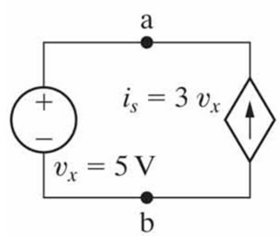




(a)



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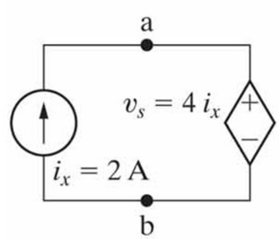

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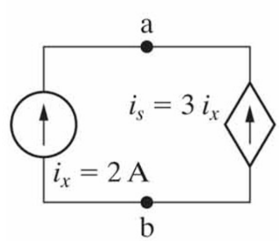
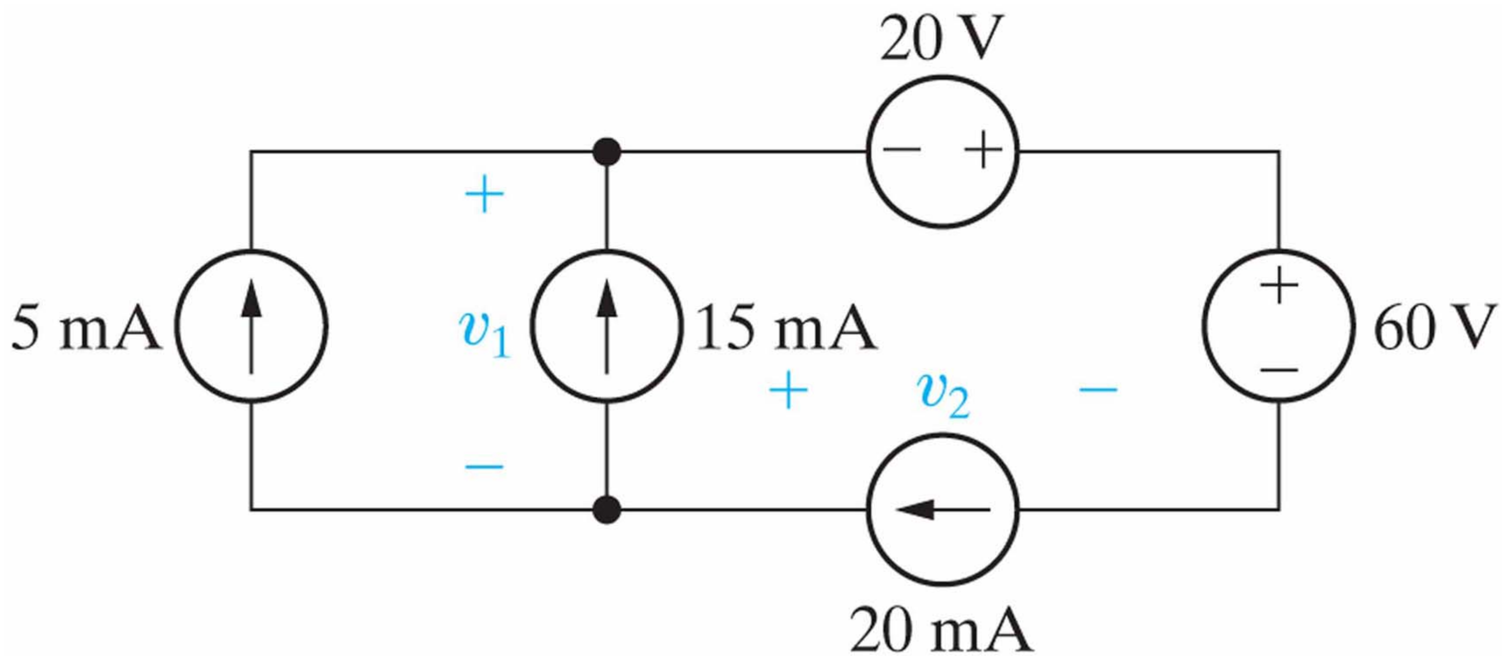


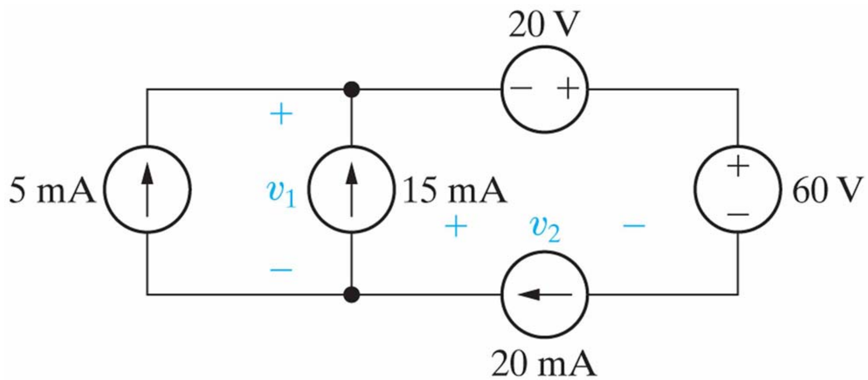
(d)

These circuits involve dependent sources. Which of them are valid given the constraints imposed by ideal sources?

We haven't introduced any formal systematic circuit analysis strategies yet …but based on what you know now, you should be able to answer these questions:

Is this circuit valid? If it is, what are the voltages v1 and v2?





The interconnection is valid, since the voltage sources can carry the 20 mA current supplied by the current source, and the current sources can support whatever voltage drop is required by the interconnection. In particular, note the the voltage drop across the three sources in the right hand branch must be the same as the voltage drop across the 15 mA current source in the middle branch, since the middle and right hand branches are connected between the same two terminals. In particular, this means that:

v1(the voltage drop across the middle branch) = −20V + 60V − v2

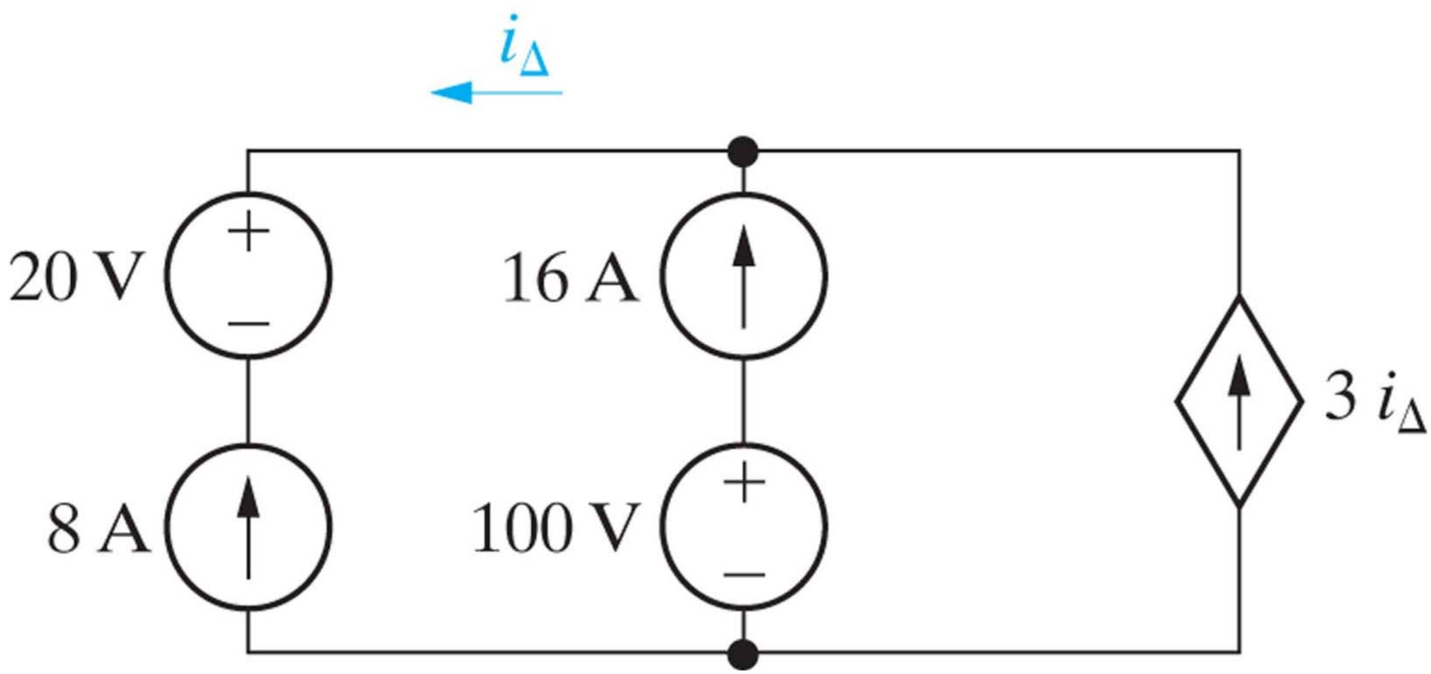
Hence <u>any combination of v1 and v2 such that v1 + v2 = 40V is a valid solution.</u> **<u>Solution is possible but not unique!</u>**

How can this be? If I built this circuit in the lab and measured voltages v1 and v2, I'd certainly get valid readings wouldn't I?

Here's another circuit:

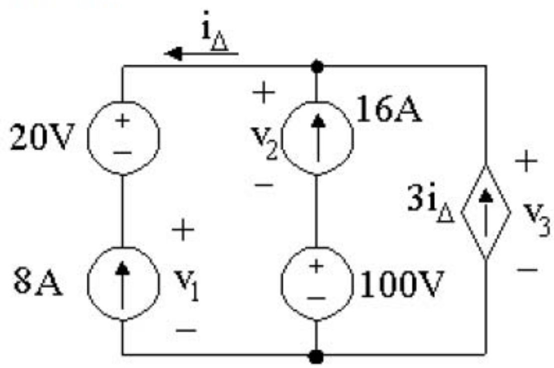
(a.) Is this circuit valid?

(b.) Can you solve for all of the voltage drops developed in the circuit?



[a] Yes, each of the voltage sources can carry the current required by the interconnection, and each of the current sources can carry the voltage drop required by the interconnection. (Note that $i_\Delta = -8$ A.)

[b] No, because the voltage drop between the top terminal and the bottom terminal cannot be determined. For example, define v_1, v_2, and v_3 as shown:



The voltage drop across the left branch, the center branch, and the right branch must be the same, since these branches are connected at the same two terminals. This requires that

$$20 + v_1 = v_2 + 100 = v_3$$

But this equation has three unknown voltages, so the individual voltages cannot be determined, and thus the power of the sources cannot be determined.

Again, we have a valid circuit with no unique solution …actually this is a rare situation….You shouldn’t conclude from seeing this twice in a row that it is a frequent occurance – in fact you’ll probably not see it again in this class!

What I wanted to show here is that current sources can develop any voltage drop necessary to satisfy other parts of the circuit ..

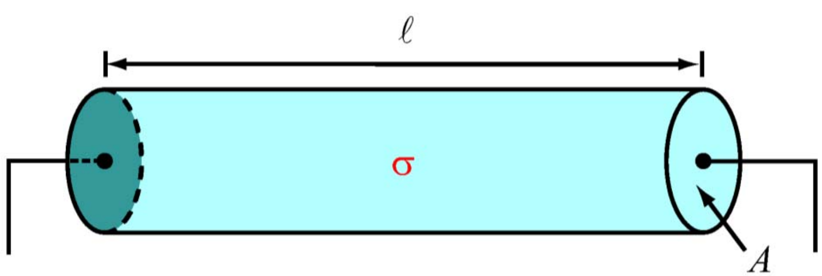
….and voltage sources and carry whatever current it required by the rest of the circuit.

as we saw in the last class

Ohms Law

$$\boxed{v = iR}$$

We mentioned this relationship several times briefly in the last class. The resistance of a resistor is dependent on the resistivity of the materials it is made out of …and the resistor's geometry. For a longitudinal resistor, you probably learned in your introductory physics class:



$$R = \frac{l}{\sigma A} = \rho \frac{l}{A}$$

For purposes of circuit theory, we don't care about the physical properties of resistors. Resistors are just ideal circuit elements that obey Ohm's law. Ohm's law describes the resistor's I-V characteristic and that's all we need to know for circuit theory.

$$\boxed{v = iR}$$

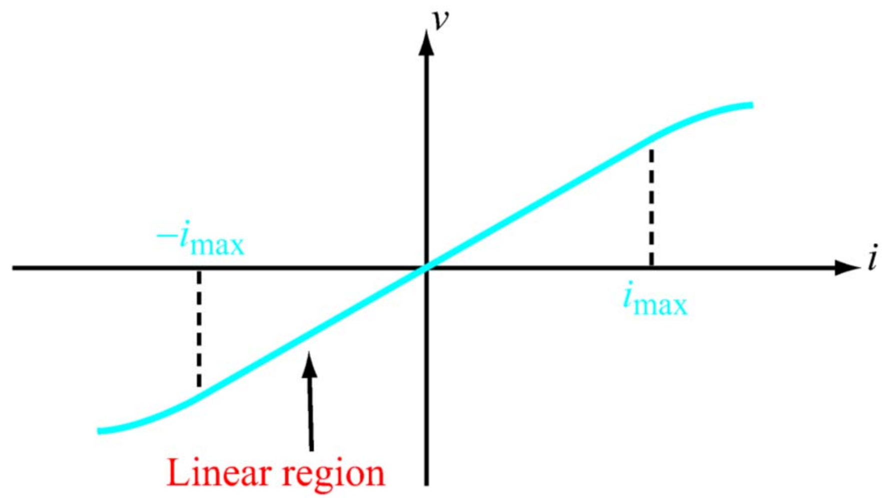
Ohms Law

$$v = iR$$

Discovered in 1826 by Georg Ohm. It seems almost intuitive now, but Ohm's work was actually denounced by the German Ministry of Education at the time. The ministry apparently proclaimed that "a professor who preached such heresies was unworthy to teach science." [*Hart, Ivor B. Makers of Science. London, Oxford University Press, 1923. p. 243*].

If you are interested, there is more on the history of Ohm's Law on Wikipedia

Ohm's Law can be theoretically derived using classical kinetic theory and the Drude model for conductors (classical electrons that bounce off of a stationary lattice of fixed ions as they flow through a material). Why is complex solid state physics required to derive such an intuitive theory as Ohm's Law?



Real resistors have a linear region in which V = IR pretty accurately. Beyond i_{max}, however, voltage increases less steeply with current in real resistors. This happens because of device heating and the fact that some (but not all) resistors have a negative temperature coefficient (they become less resistive with increasing temperature)….so a further increase in current through the device will result in a smaller increase in voltage drop.

Ideal resistors (the kind we use in circuit theory) will always be assumed to be linear for all values of current.

Some More Relationships

Because: $$v = iR$$ Ohm's Law

and

$$P = vi$$ Definition of power – we talked about this earlier

therefore:

$$\boxed{\begin{aligned} P &= i^2 R \\ P &= v^2 / R \end{aligned}}$$

Some Topology Definitions

A planar circuit is a circuit that can be drawn schematically in two-dimensional space such that no two branches have to cross one another.

A **node** is an electrical connection point for two or more devices.

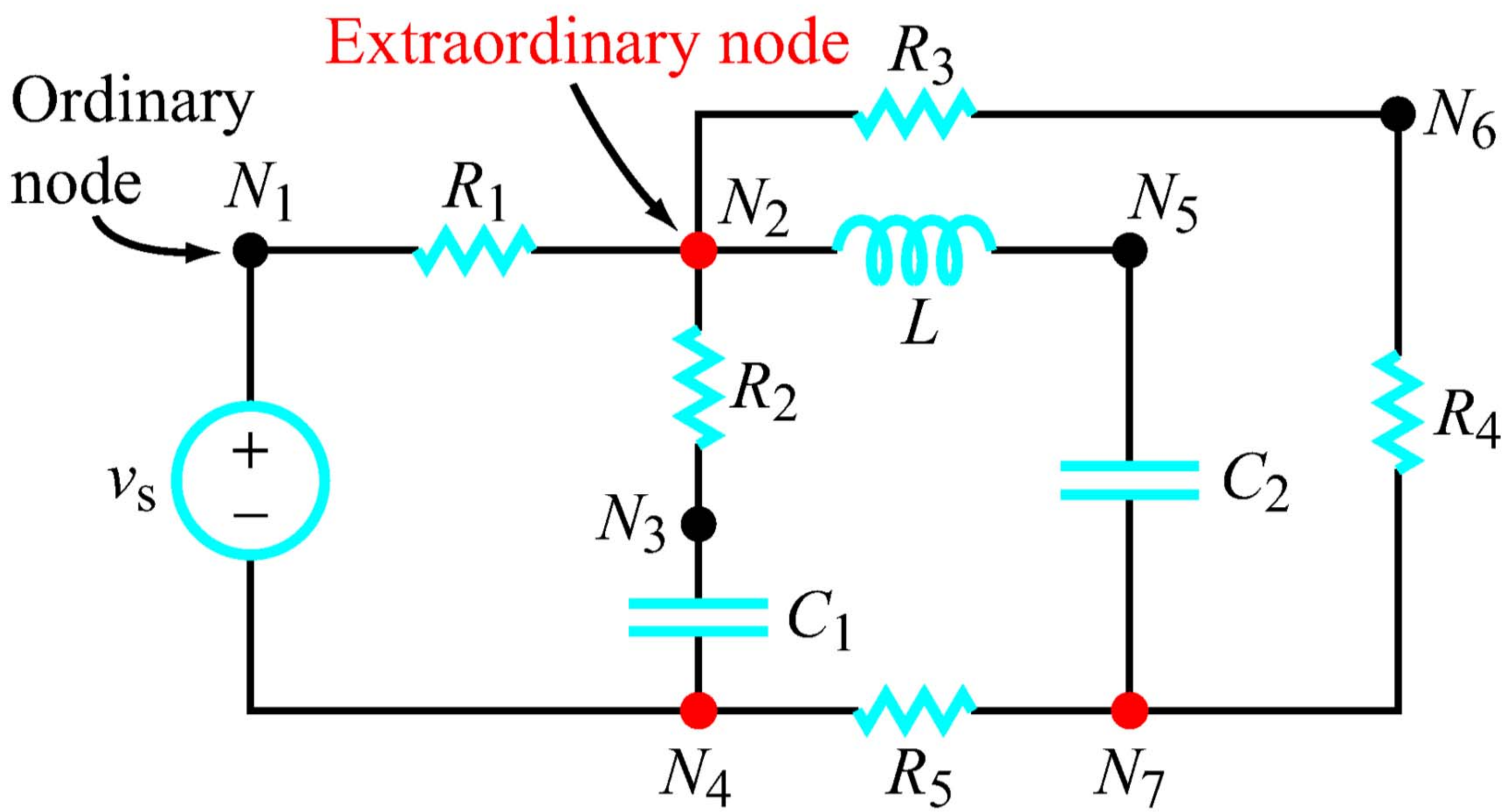
A **branch** is the trace between two consecutive nodes containing only one element between them.

Planar Circuit terminology.

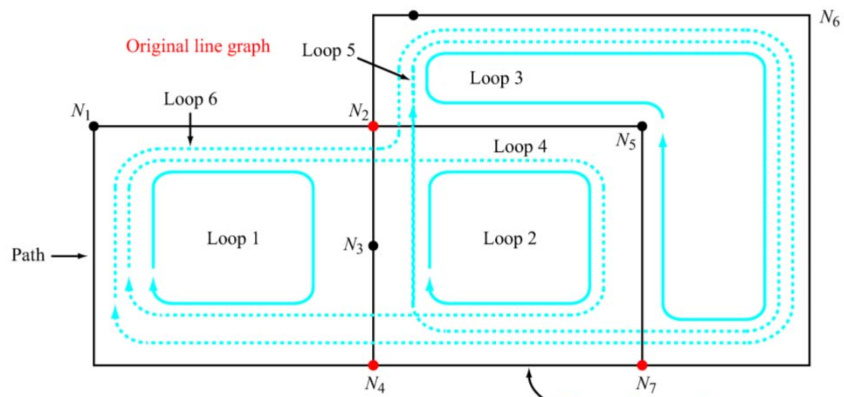
Term	Definition
Ordinary node	An electrical connection point that connects to only two elements.
Extraordinary node	An electrical connection point that connects to three or more elements.
Branch	Trace between two consecutive nodes with only one element between them.
Path	Continuous sequence of branches with no one node encountered more than once.
Extraordinary path	Path between two adjacent extraordinary nodes.
Loop	Closed path with the same start and end node.
Independent loop	Loop containing one or more branches not contained in any other independent loop.
Mesh	Loop that encloses no other loops.
In-series	Elements that share the same current.
In-parallel	Elements that share the same voltage.

Be sure you understand this!

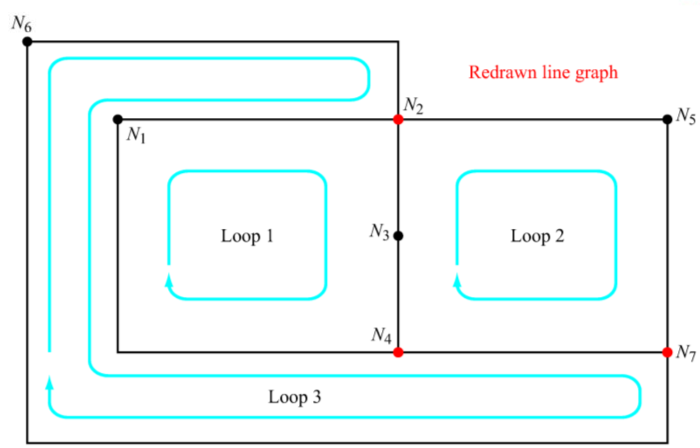
Extraordinary and Ordinary Nodes



We can redraw this circuit as a linear graph (suppressing circuit elements)



(a)



(b)

There are some important relationships among these planar circuit elments:

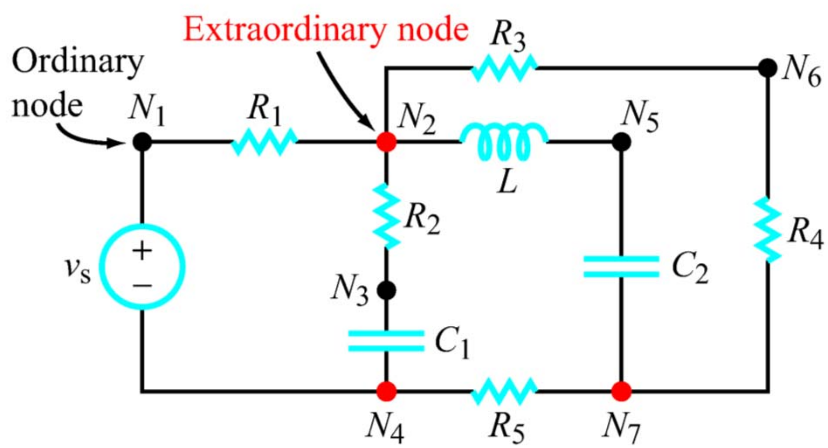
The number of branches, b is related to the number of nodes, n and the number of independent loops l_{ind} by the relationship:

$$\boxed{b = n + l_{ind} - 1}$$

Also, the number of extraordinary nodes, n_{ex}, extraordinary paths, p_{ex}, and independent loops, l_{ind}, are related by:

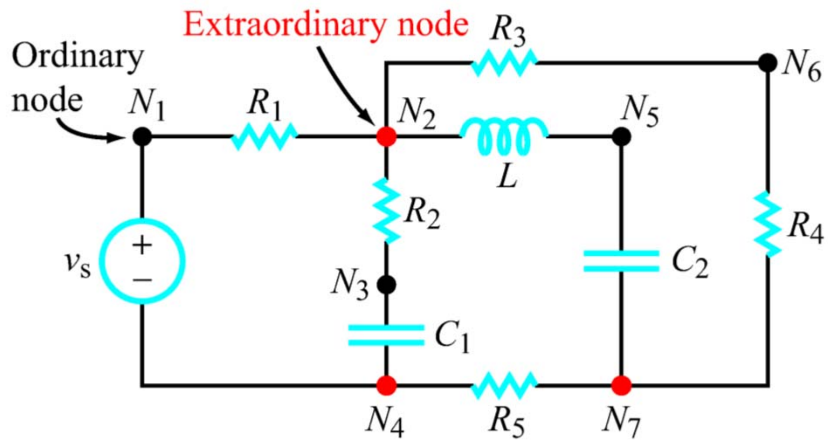
$$\boxed{p_{ex} = n_{ex} + l_{ind} - 1}$$

Let's check these relationships with our circuit:



$$\boxed{b = n + l_{ind} - 1}$$

branches nodes independent loops



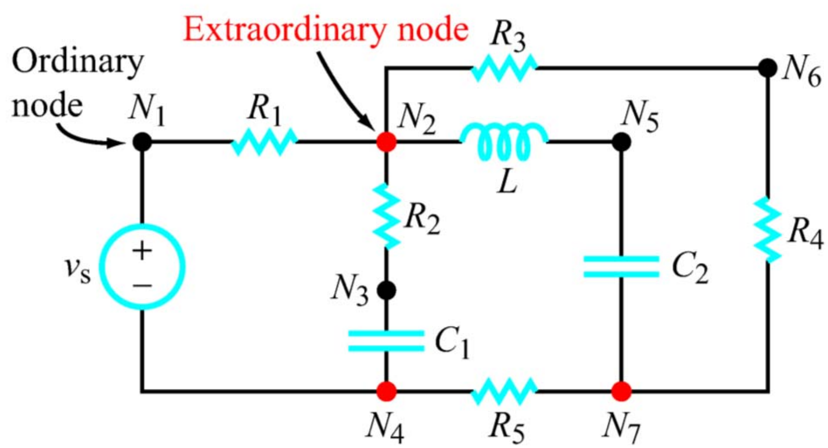
$$\boxed{b = n + l_{ind} - 1}$$

branches

nodes

independent loops

9 = 7+3 - 1

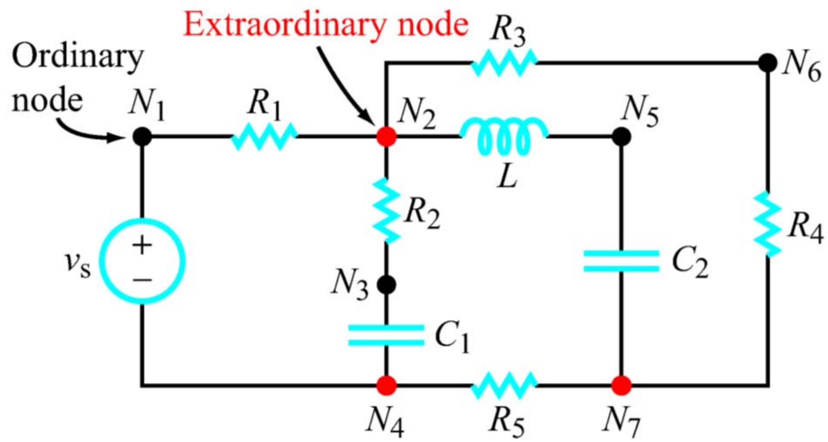


$$\boxed{p_{ex} = n_{ex} + l_{ind} - 1}$$

extraordinary paths

extraordinary nodes

independent loops



$$p_{ex} = n_{ex} + l_{ind} - 1$$

extraordinary paths

extraordinary nodes

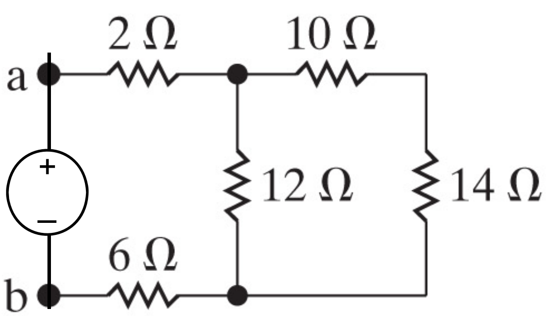
independent loops

5 = 3+3 - 1

How many nodes are in circuit (a)? How many are extraordinary nodes? How about independent loops in circuit (a).

$$\boxed{b = n + l_{ind} - 1}$$

branches nodes independent loops



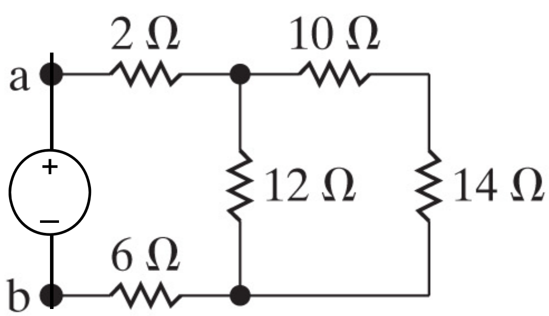
(a)

Is the relationship between branches, nodes and independent loops satisfied?

How many nodes are in circuit (a)? How many are extraordinary nodes? How about independent loops in circuit (a).

$$\boxed{b = n + l_{ind} - 1}$$

branches nodes independent loops



(a)

Is the relationship between branches, nodes and independent loops satisfied?

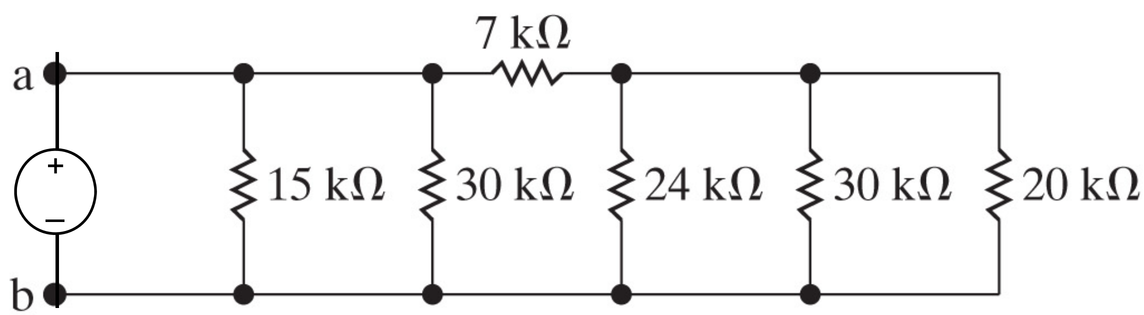
why is this 5 ?

5 = 4+2 - 1

These are important definitions

- Two or more devices are in **series** if the **same current** flows through all of them.
- Two or more devices are in **parallel** if they share the same pair of nodes, thereby having the **same voltage** across them

In this circuit, which components are in series ….which are in parallel?



(b)

Do you think it might be possible to simplify this circuit by combining circuit elements?

Combining series and parallel circuit elements is a very useful strategy in solving circuits. It's an example of the use of "equivalent circuits." We'll talk about other types of equivalent circuits later in the course.

We’ll talk about series and parallel resistors some more next week …but you should be able to figure this out right now in terms of what you know.

In a series circuit …what can we say about the current flowing in the circuit?

In a parallel circuit …what can we say about the voltage drop across the parallel circuit elements?

Using just Ohms Law …can you figure out how to combine resistors that are arranged in series?

How about a parallel arrangement of resistors?

In the next lecture we'll discuss the Kirchhoff Laws. Together with Ohm's Law these will provide us with everything needed to solve DC resistive circuits …of any complexity.

I have assigned a problem set for next week. It's in the Week 1 folder on Camino now and it's <u>due next Friday in class</u>. Several of the problems involve the Kirchhoff Laws …which we'll discuss next Monday.