ELEN 50 Class 18 – Capacitors and Inductors

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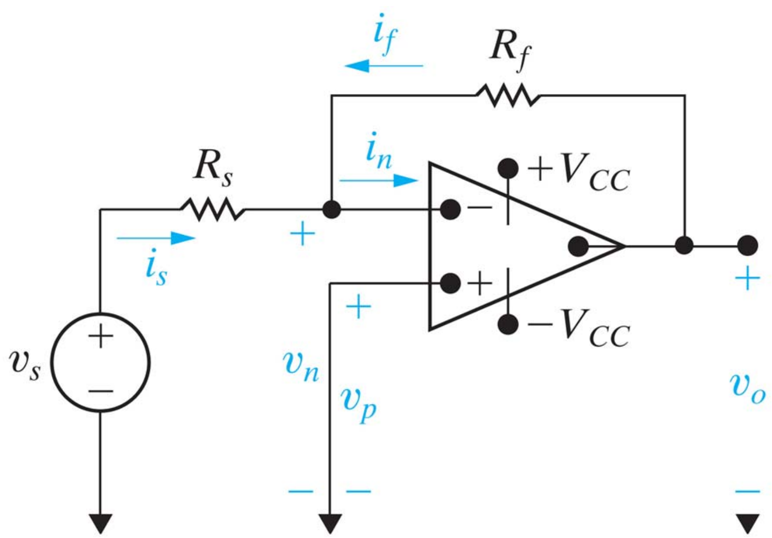
Here's a circuit we saw before:

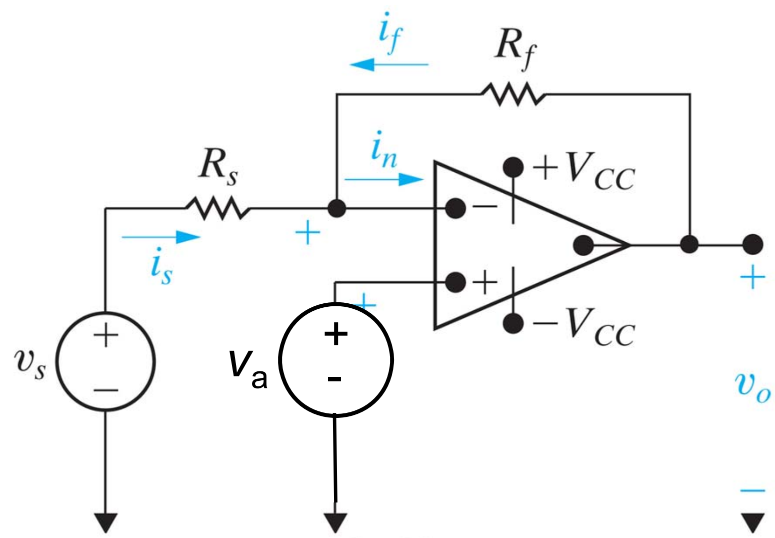
Could you figure out (without do any calculation) what would happen in the inverting amplifier circuit if the noninverting input were connected to a voltage source instead of ground:

instead of this



$$\boxed{v_0 = -(R_f / R_s)\, v_s}$$

we have this



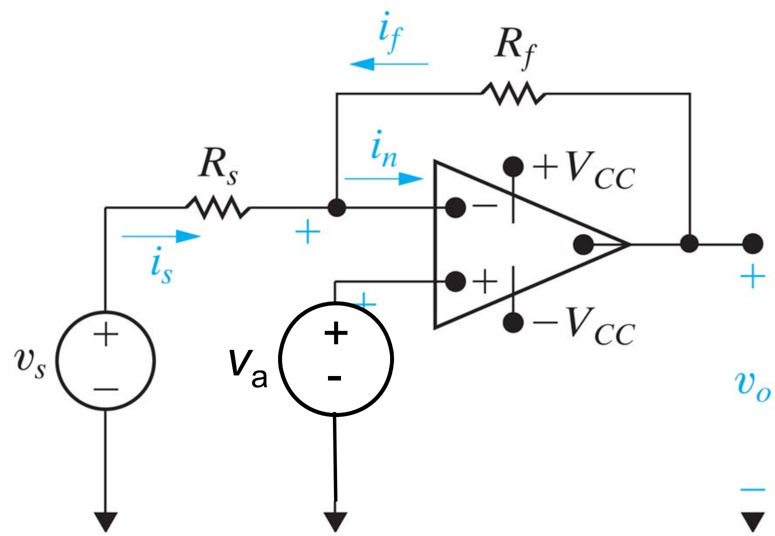
v_o = ????

What is the gain equation?

How would you solve it?

You could guess!…..You might guess that the output voltage would be the same as before …but with an "offset" voltage of v_a …..or maybe you would guess the "offset voltage" would be scaled …something like - v_a (R_f/R_s).

Well …we can just write the KCL equation at the inverting input as we usually do …and we don't have to guess. The golden rule tells us that, because of feedback, $v_n = v_p$ so the KCL equation is:



$$\frac{v_s - v_a}{R_s} + \frac{v_o - v_a}{R_f} = 0$$

so

$$v_o = -\left(\frac{R_f}{R_s}\right)\left[v_s - v_a\right] + v_a$$

So, both of those guesses were wrong! We also notice that the expression reduces to the ordinary inverting amp equation when $v_a = 0$ as we would expect.

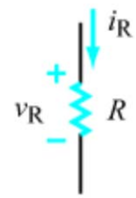
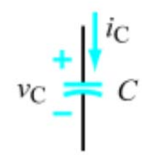
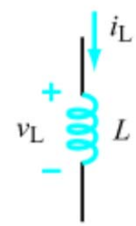
Enough about op amps. You can find lots of examples of op amp circuits online if you want to practice deriving the gain equation.

Now we're going to introduce two new circuit elements.

Capacitors and Inductors

Up to now, all of the circuits we've analyzed have been constructed from DC current and voltage sources, wires, and resistors – and as we saw last time, op amps can really be modelled as dependent voltage sources. Now we're going to introduce two new circuit elements. They are still linear circuit elements, but their i-v curves are quite different from resistors – they are not constant …they depend explicitly on time. You'll need to understand how these new elements work to do all of the remaining labs and projects in ELEN 50.

Table 1-4: Passive circuit elements and their symbols.

Element	Symbol	i–v Relationship
Resistor		$v_R = Ri_R$
Capacitor		$i_C = C\dfrac{dv_C}{dt}$
Inductor		$v_L = L\dfrac{di_L}{dt}$

The Inductor

(a)

$$v = L\frac{di}{dt}$$

The voltage across an inductor depends on the **derivative of current with time through the inductor**. A time independent current (DC) produces zero voltage, therefore the inductor behaves as a DC short circuit.

The polarity of the induced voltage in the inductor is in the direction opposing the change in current. Inductors resist changes in current through them …in this respect they sort of act like the physical property of momentum. Momentum resists changes in velocity….this is Newton's First Law of Motion.

Note: current cannot change instantaneously through an inductor since this would result in an infinite voltage across the inductor.

• So an inductor is really just a type of wire ….it has zero resistance as far as DC current is concerned.

• However, it’s a very special kind of wire --- because of the way it is coiled up, there is significant magnetic field produced when current passes through it. This causes energy to be stored in the magnetic field. As the current through the inductor is changed, the magnetic field changes ..and this changing field induces a voltage in the coils of the inductor, resulting in the relationship:

$$\boxed{v = L\frac{di}{dt}}$$

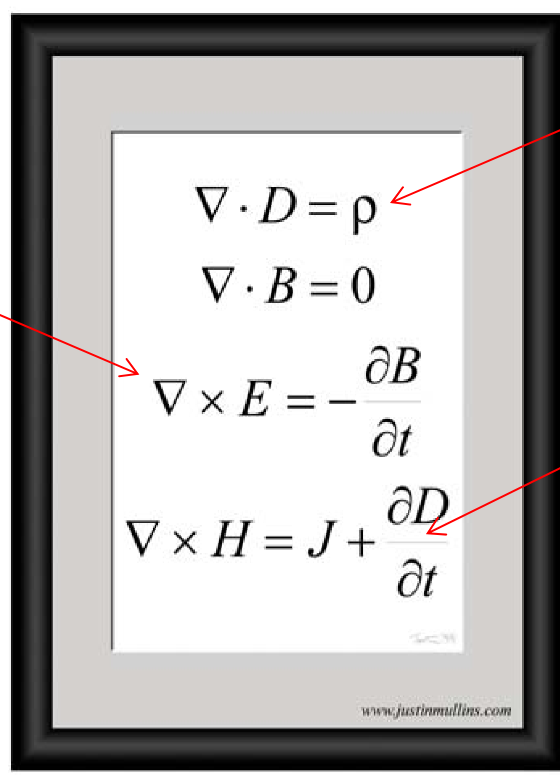
The reason for the unusual I-V characteristic of inductors is Faraday's Law ------ which you normally see in physics classes as part of Maxell's Equations:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \text{or in integral form} \qquad \oint_{\partial\Sigma} \mathbf{E} \cdot d\boldsymbol{\ell} = -\int_{\Sigma} \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{A}$$

Faraday's Law

Gauss' Law

Ampere's Law

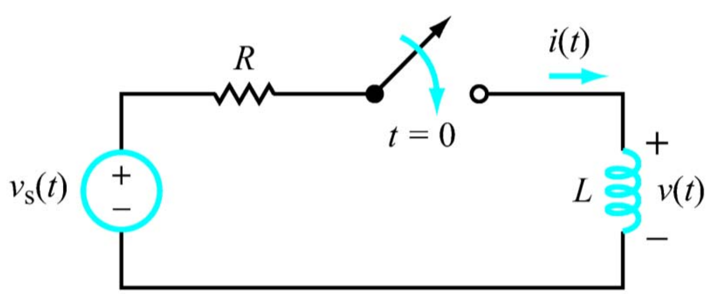


Let There Be Light--Maxwell's Equations

$$\oint_{\partial\Sigma} \mathbf{E} \cdot d\boldsymbol{\ell} = - \int_{\Sigma} \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{A}$$

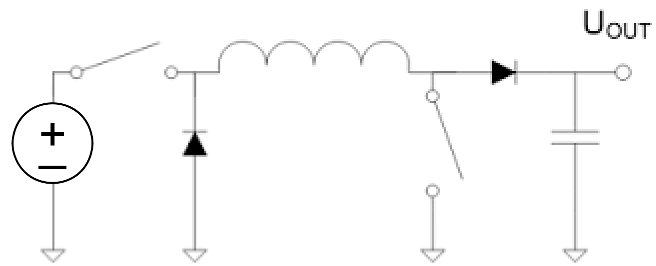
Faraday's Law says that the line integral of the electric field around a loop is equal to the negative of the time rate of change of the surface integral of the magnetic field across the area of the loop. In other words, as the current in a coil (inductor) changes it will change the magnetic field in the coil which will, in turn, induce a voltage (the line integral of the electric field) in the direction opposing the original change in current.

You can actually demonstrate this to yourself quite easily (and dramatically) with this simple circuit:

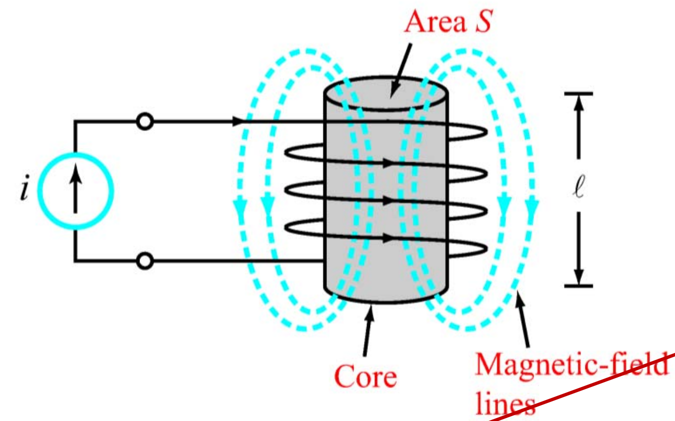


If the switch is closed for t<0, a current I = v_s/R will be flowing (since the inductor has zero DC resistance). However, when the switch is opened at t=0, the current will try to stop instantaneously …but this will be opposed by the inductor …and, if the inductor is large enough a substantial voltage (and a spark) will occur at the switch terminals.

The opposition to the change in current flow through an inductor is taken advantage of in circuits like this:

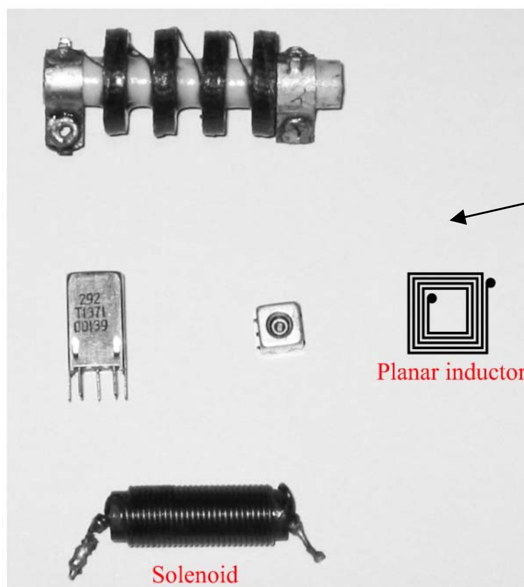


This is basically the heart of a switched inductor “boost” circuit …used to build a DC to DC convertor.



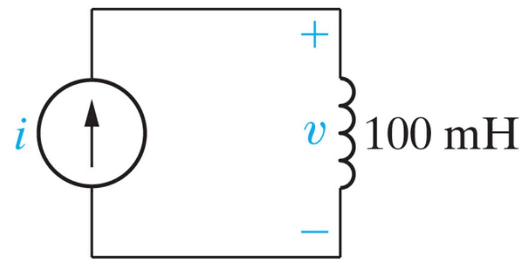
You don't need to know this!

The inductance of a solenoid of length l and cross sectional area S is $L=\mu N^2 S/l$, where N is the number of turns and μ is the magnetic permeability of the core material.



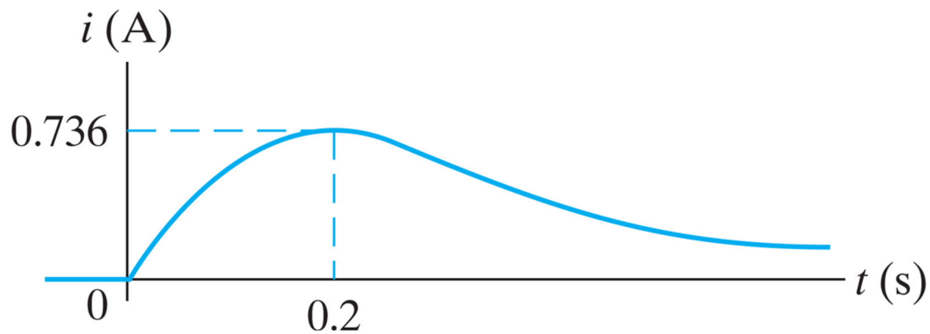
Types of inductors

If we apply a time varying current to an inductor:

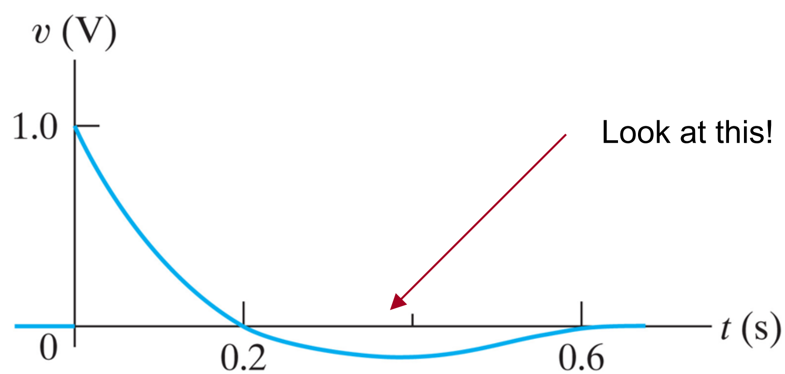


$$i = 0, \qquad t < 0$$

$$i = 10te^{-5t}\text{A}, \qquad t > 0$$



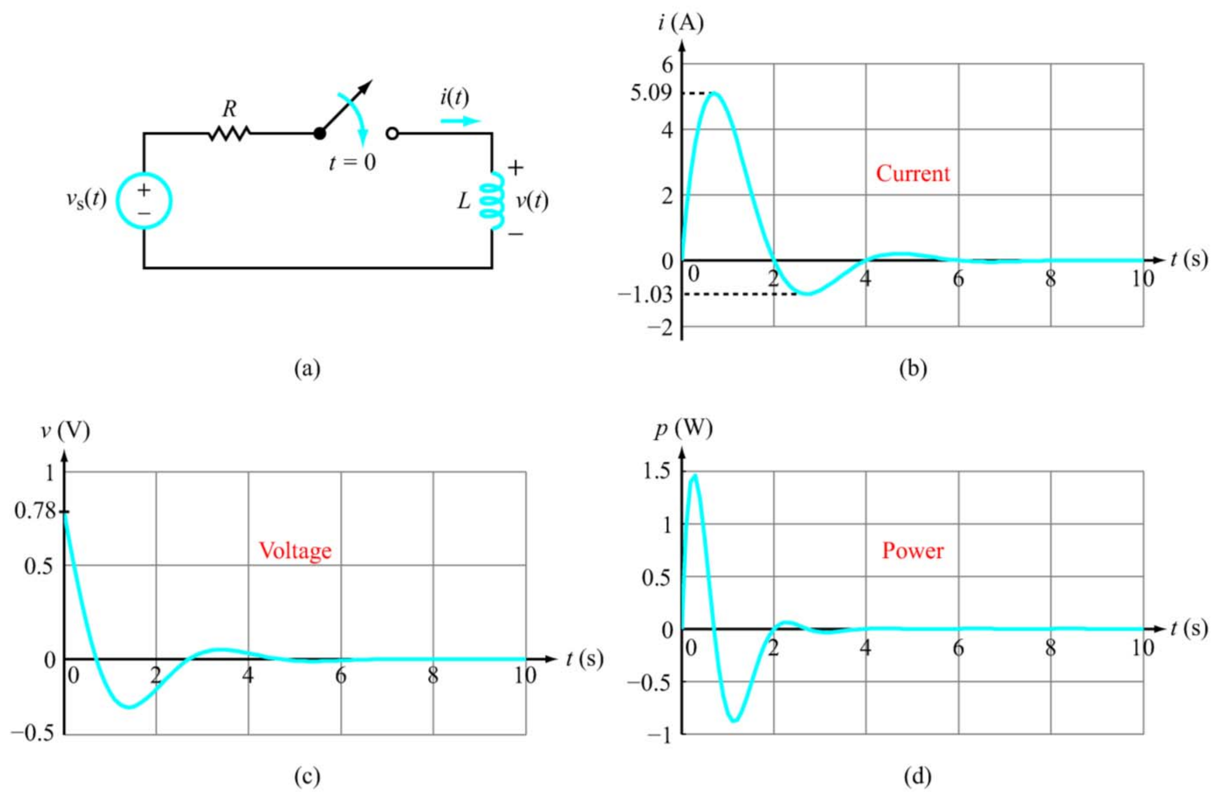
Since: $$\frac{di}{dt} = 10(-5te^{-5t} + e^{-5t}) = 10e^{-5t}(1-5t)$$



Look at this! It actually reverses polarity!

Here's another example – at t= 0 the switch closes and the voltage source, $v_s(t)$ provides a current that is:

$$i(t) = 10e^{-0.8t} \sin(\pi t / 2) \quad for\ t \geq 0$$



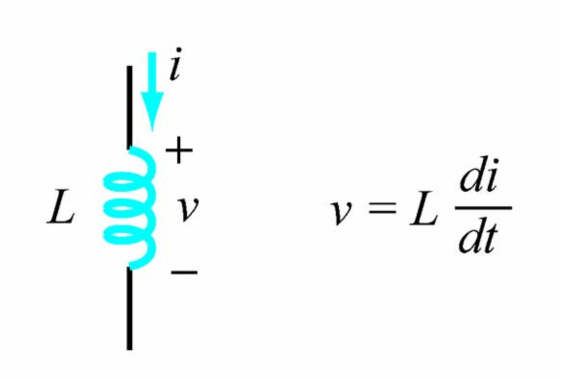
We can also derive an expression for the current through an inductor as a function of the voltage by integrating both sides of the initial equation:

Since: $$v = L\frac{di}{dt}$$

$$v\,dt = L\,di$$

$$L\int_{i(t_0)}^{i(t)} dx = \int_{t_0}^{t} v\,dt$$

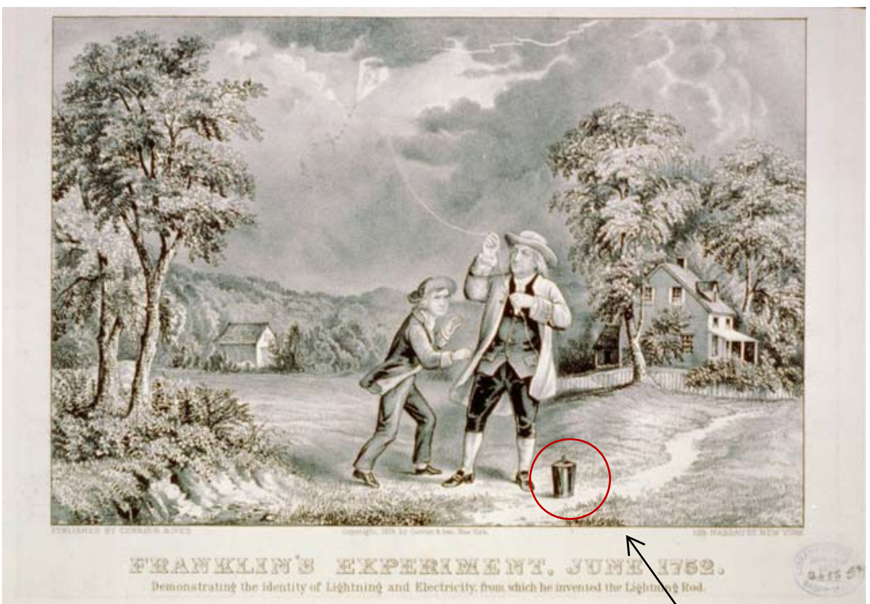
$$i(t) = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)$$



The passive sign convention for inductors

The Capacitor

Capacitors as circuit elements have been around about as long as inductors



notice

The Capacitor

(a)

$$\boxed{i = C\frac{dv}{dt}}$$

Capacitors act in a manner which resists changes in voltage across them. When the voltage across a capacitor changes, the capacitor produces a current in a direction that resists the change in voltage.

The current through a capacitor depends on the **time derivative of the voltage across the capacitor**. A time independent voltage (DC) results in zero current …so the capacitor acts like an open circuit to DC voltages.

Note: voltage cannot change instantaneously across a capacitor since this would result in an infinite current through the capacitor

Here's another equation you need to know:

since: $$i = C\frac{dv}{dt}$$

If you integrate both sides of the equation you get: $$\boxed{Q = CV}$$

remember that $$i = \frac{dQ}{dt}$$

So capacitors and inductors are complimentary components:

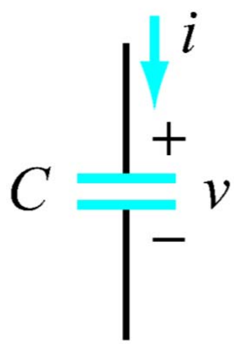
Inductors act in a manner to oppose changes in current through them – when the current changes the inductor produces a <u>voltage whose polarity will oppose the current change</u>. Under DC conditions <u>an inductor is a short circuit.</u>

Capacitors act in a manner to oppose changes in voltage across them – when voltage changes across the capacitor, it produces a <u>current in the direction that opposes the voltage change</u>. Under DC conditions <u>a capacitor is an open circuit.</u>

$$v = L\frac{di}{dt} \qquad i = C\frac{dv}{dt}$$

Using a similar argument to the one we used with the inductor, one can show:

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i\, dt + v(t_0)$$



$$i = C\,\frac{dv}{dt}$$

The passive sign convention used for capacitors

Capacitors and inductors are frequently used in electronic circuitry – particularly in filters, resonant circuits and a variety of energy storage applications. Capacitors are probably more common than inductors because of this usefulness as a energy storage system and because of their use in computer memory (both DRAM [volatile system memory] and Flash [nonvolatile storage memory]) store data capacitativly.

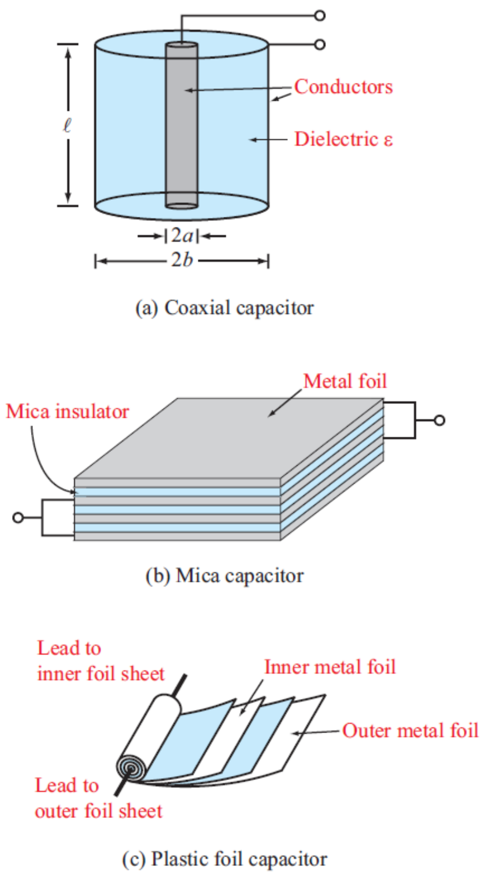
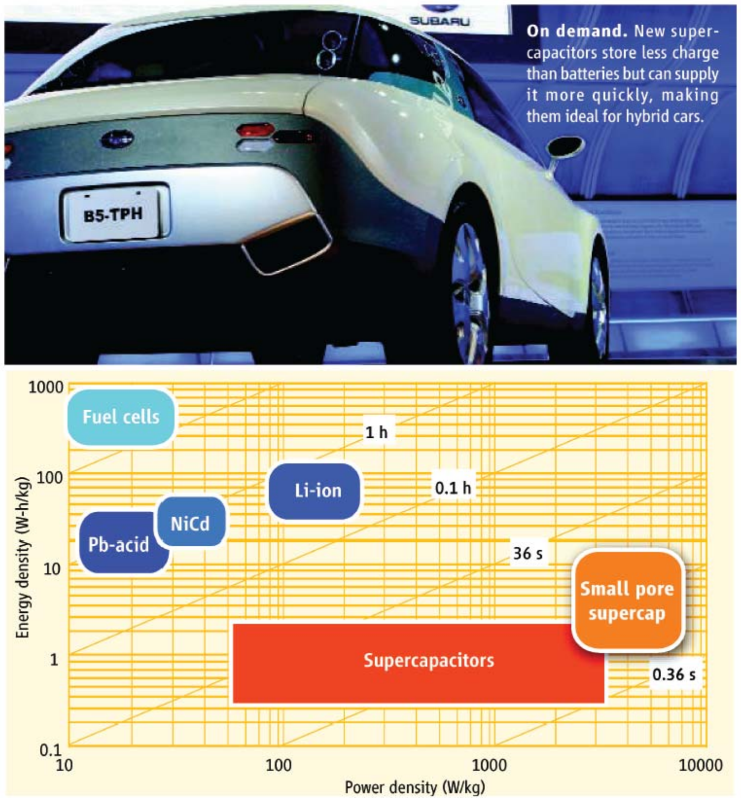
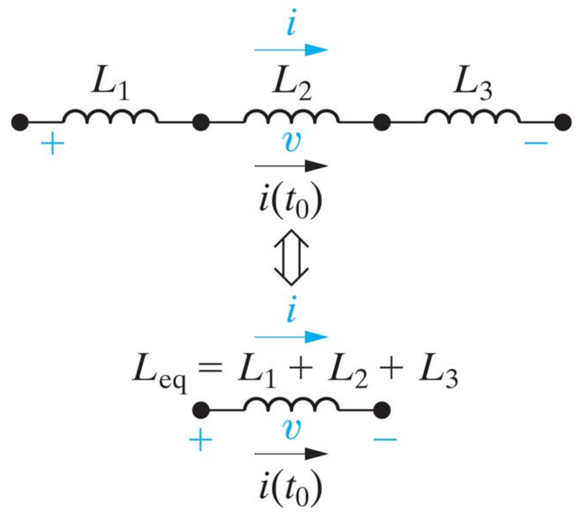


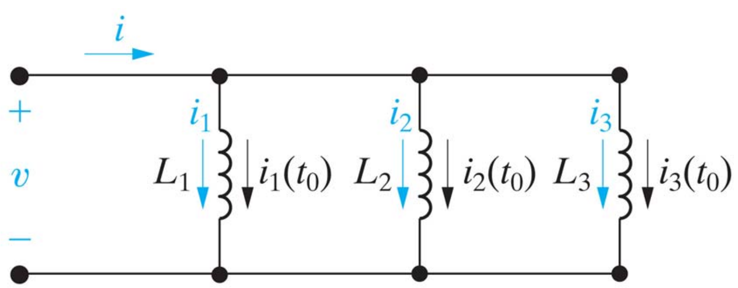
Figure 5-12: Various types of capacitors.



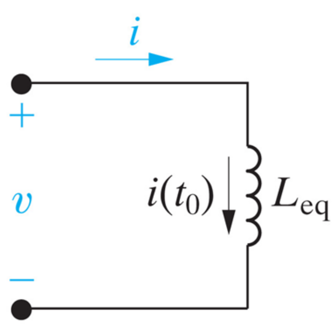
Rules for Series and Parallel Inductors:



For series connected inductors, the current is the same through every inductor, so the voltages developed across each inductor add.



For parallel connected inductors, the terminal voltages are the same for every inductor. The inverse of the equivalent inductance of the parallel combination is equal to the sum of the inverse inductances of the inductors in parallel.

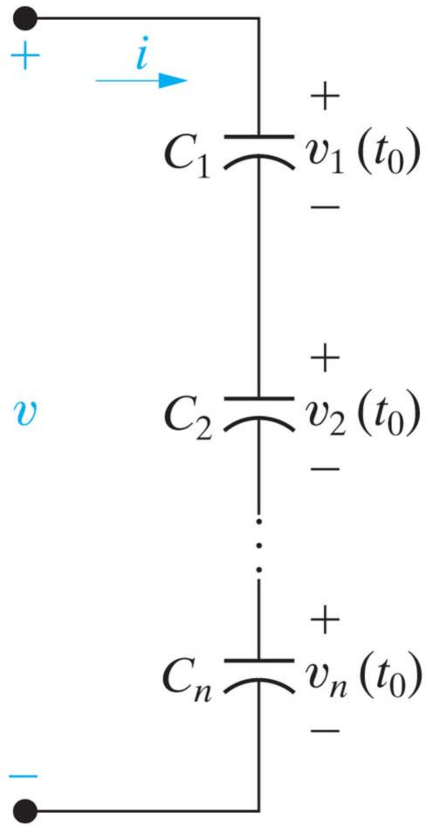


$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

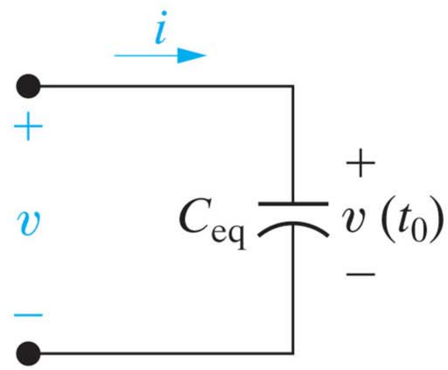
$$i(t_0) = i_1(t_0) + i_2(t_0) + i_3(t_0)$$

In other words, inductors behave in series and parallel just like resistors

Series and Parallel Capacitors:



(a)



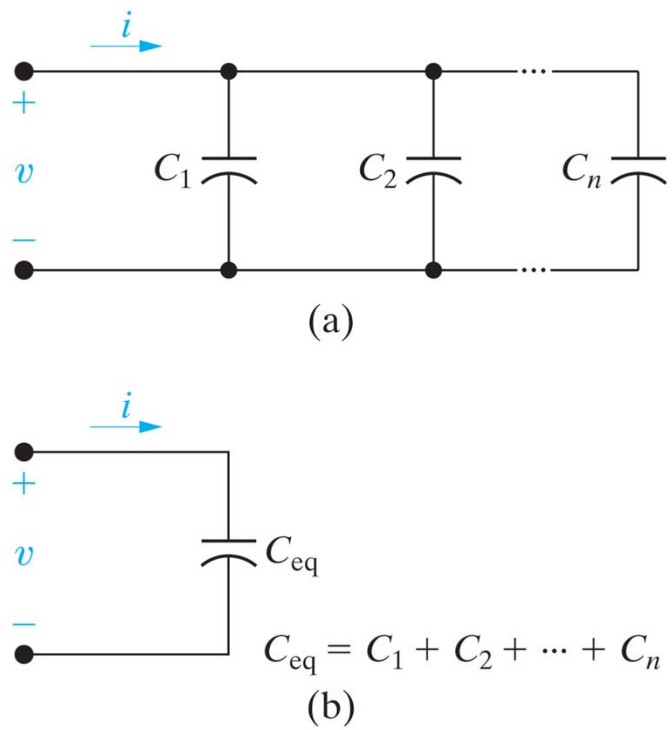
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

$$v(t_0) = v_1(t_0) + v_2(t_0) + \cdots + v_n(t_0)$$

(b)

The inverse of the equivalent capacitance of a series connection of capacitors is equal to the sum of the inverse capacitances of each of the capacitors.

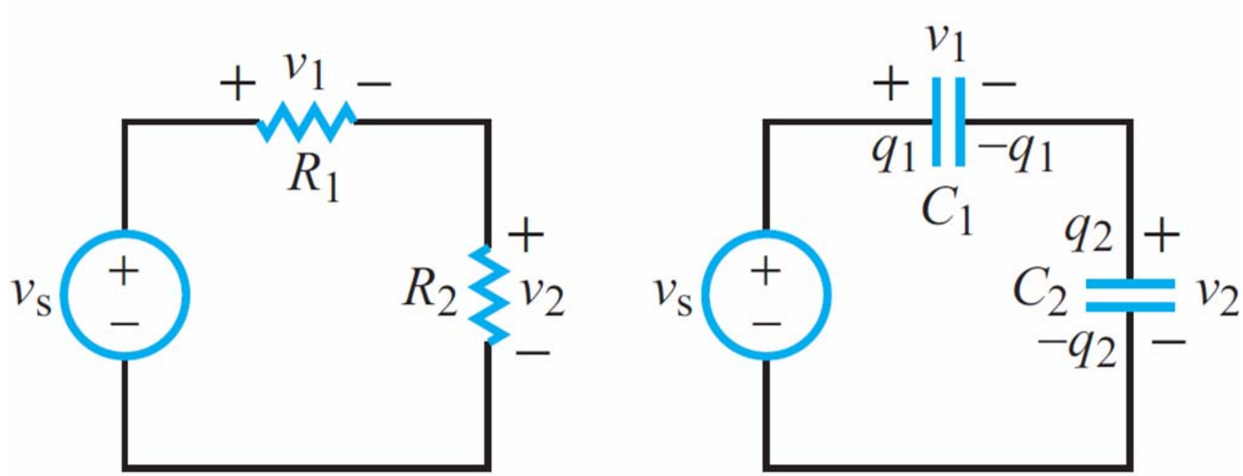
Series and Parallel Capacitors:



The equivalent capacitance of a parallel connection of capacitors is equal to the sum of the capacitances of each of the capacitors. So capacitors in series behave like resistors in parallel and capacitors in parallel behave like resistors in series.

Voltage Division with Resistors and Capacitors

Resistive voltage divider | Capacitive voltage divider



(a) $$v_1 = \left(\frac{R_1}{R_1 + R_2}\right) v_s$$

$$v_2 = \left(\frac{R_2}{R_1 + R_2}\right) v_s$$

(b) $$v_1 = \left(\frac{C_2}{C_1 + C_2}\right) v_s$$

$$v_2 = \left(\frac{C_1}{C_1 + C_2}\right) v_s$$

Do you see how the equations for the capacitive voltage divider were obtained?

As we saw, the two capacitors in series result in a combined capacitance across the voltage source of:

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

The voltage source, v_s will cause a total charge $Q = C_{eq} v_s$ to be stored on each capacitor. So

$$Q = v_s \frac{C_1 C_2}{C_1 + C_2}$$

Why is this?

This charge on the first capacitor will produce a voltage $v_1 = Q/C_1$

$$v_1 = v_s \frac{C_1 C_2}{C_1 + C_2}\left(\frac{1}{C_1}\right) = v_s \frac{C_2}{C_1 + C_2}$$ and similarly $$v_2 = v_s \frac{C_1}{C_1 + C_2}$$

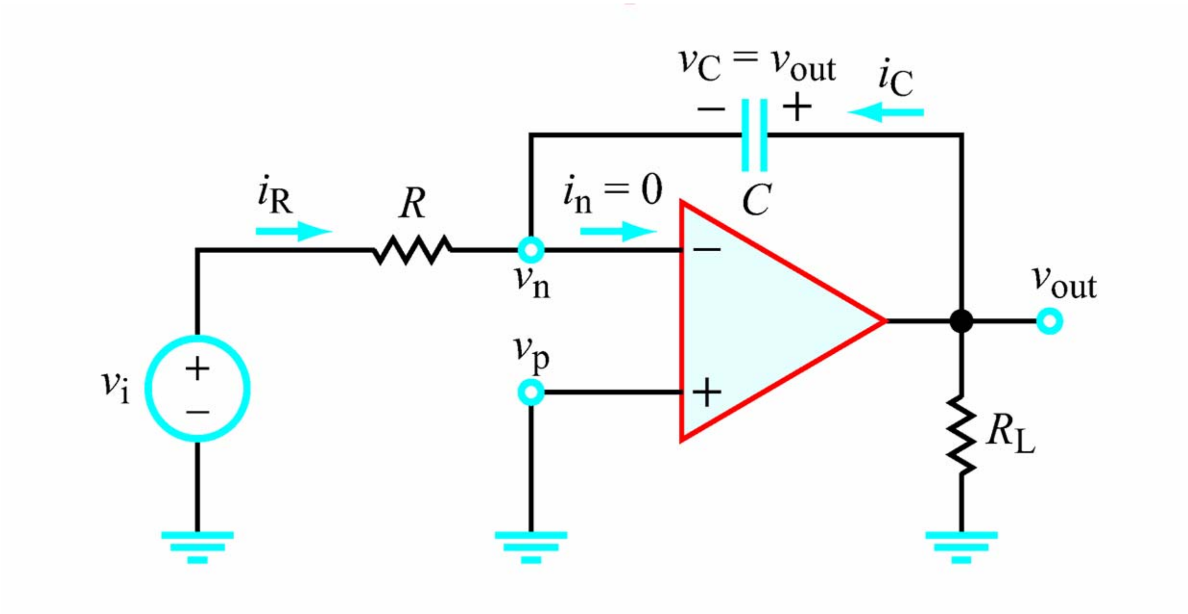
Basic Properties of R, L and C

Property	R	L	C
i–v relation	$i = \frac{v}{R}$	$i = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)$	$i = C\,\frac{dv}{dt}$
v-i relation	$v = iR$	$v = L\,\frac{di}{dt}$	$v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$
p (power transfer in)	$p = i^2 R$	$p = Li\,\frac{di}{dt}$	$p = Cv\,\frac{dv}{dt}$
w (stored energy)	0	$w = \frac{1}{2}Li^2$	$w = \frac{1}{2}Cv^2$
Series combination	$R_{eq} = R_1 + R_2$	$L_{eq} = L_1 + L_2$	$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$
Parallel combination	$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$	$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$	$C_{eq} = C_1 + C_2$
dc behavior	no change	short circuit	open circuit
Can v change instantaneously?	yes	yes	no
Can i change instantaneously?	yes	no	yes

We’re going to analyze the time response of RL and RC circuits to impulse functions later. It’s probably already apparent to you that a DC analysis of a circuit containing inductors and capacitors isn’t anything more than an analysis of the circuit with the inductors and capacitors deactivated (open circuit for the capacitors and short circuit for the inductors).

The time domain behavior as a result of time varying sources (impulse, AC, or otherwise) can be quite interesting, however.

Also Notice: we can put inductors and capacitors in the feedback loop in an operational amplifier – and this can result in very useful circuits. We can solve this circuit, as before, by <u>writing the KCL equation at the inverting input node</u> and using the op amp golden rules – just as we always do:

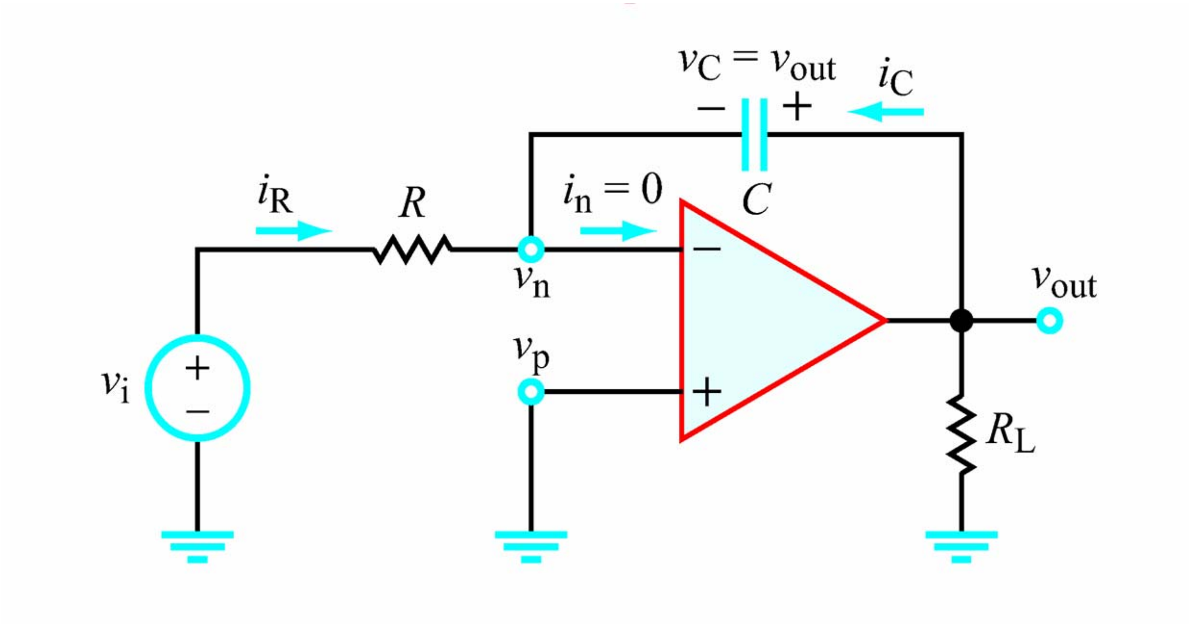


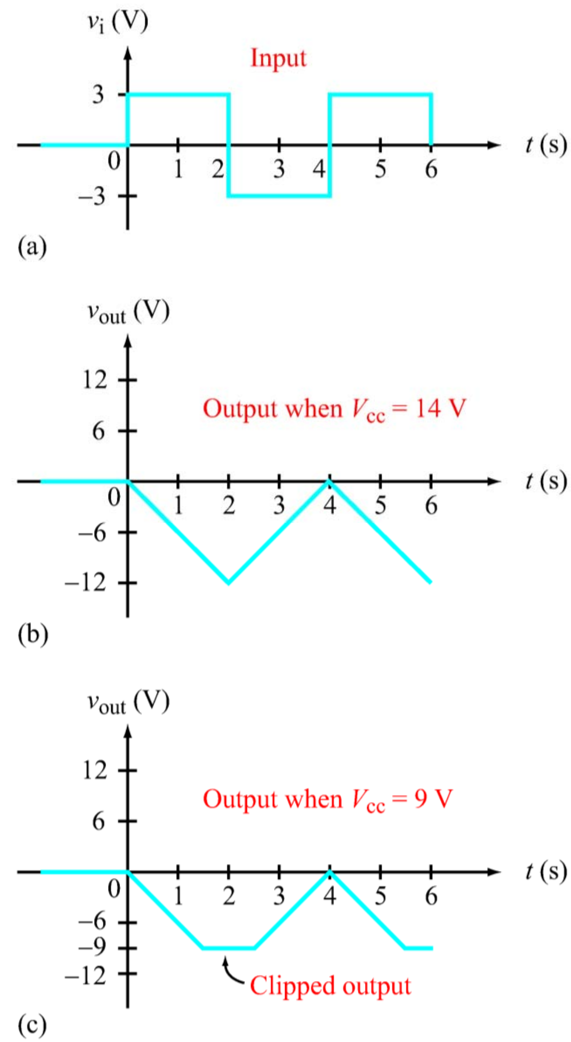
$$\frac{v_i}{R} + i_C = 0$$

$$\frac{v_i}{R} + C\frac{dv_o}{dt} = 0$$

$$\frac{dv_o}{dt} = -\frac{1}{RC}v_i$$

$$v_o = -\frac{1}{RC}\int_{t_0}^{t} v_i dt + v(t_0)$$

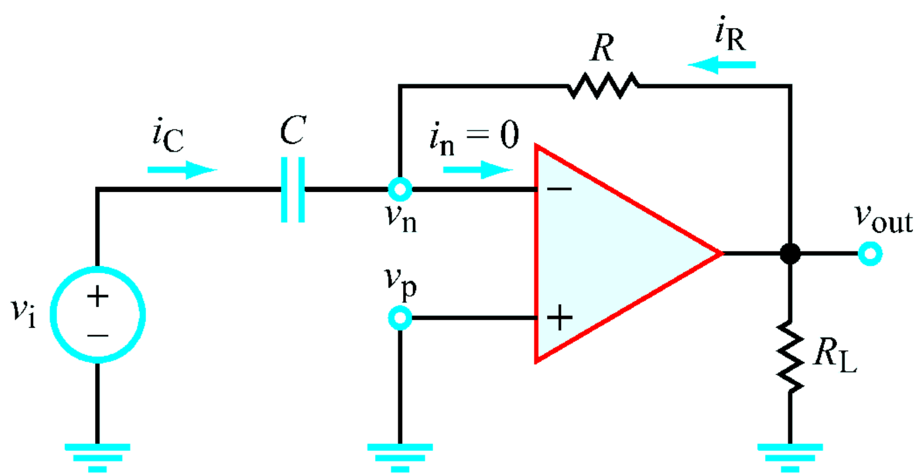




So this thing is an RC Integrator a circuit whose output is proportional to the time integral of the input signal

$$v_{out} = -\frac{1}{RC}\int_{t_0}^{t} v_i dt + v_{out(t_0)}$$

We could also put a capacitor in the input branch of an operational amplifer – and this results in another useful circuit. Will it matter very much if the capacitor is on the input side as opposed to the feedback loop? Let's use the op amp golden rules and write the KCL equation at the inverting input node again …..and solve for v_o.



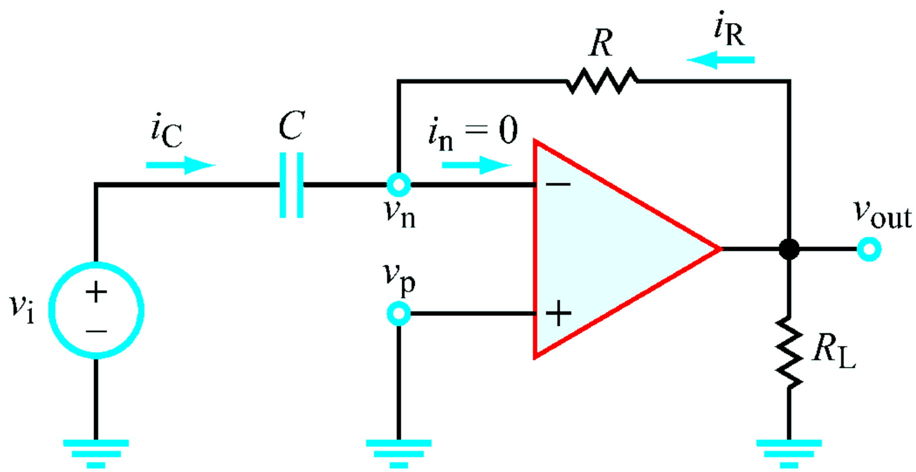
$$\frac{v_o}{R} + i_C = 0$$

$$\frac{v_o}{R} + C\frac{dv_i}{dt} = 0$$

$$\frac{dv_i}{dt} = -\frac{1}{RC}v_o$$

$$v_o = -RC\frac{dv_i}{dt}$$

So moving the capacitor to the input branch of this circuit changes it from an RC Integrator to an RC Differentiator -- a circuit whose output is proportional to the time derivative of the input signal.



$$v_{out} = -RC\frac{dv_i}{dt}$$

RC Differentiator

Capacitors and/or inductors can also be used with operational amplifiers to make frequency filters – and this will be the subject of your next lab project.