SANTA CLARA UNIVERSITY
Electrical Engineering Department

Homework 1

2.1. The proof is as follows:

$$\begin{aligned}(x+y)\cdot(x+z) &= xx+xz+xy+yz\\ &= x+xz+xy+yz\\ &= x(1+z+y)+yz\\ &= x\cdot 1+yz\\ &= x+yz\end{aligned}$$

2.2. The proof is as follows:

$$\begin{aligned}(x+y)\cdot(x+\overline{y}) &= xx+xy+x\overline{y}+y\overline{y}\\ &= x+xy+x\overline{y}+0\\ &= x(1+y+\overline{y})\\ &= x\cdot 1\\ &= x\end{aligned}$$

2.3. Manipulate the left hand side as follows:

$$\begin{aligned}xy+yz+\overline{x}z &= xy+(x+\overline{x})yz+\overline{x}z\\ &= xy+xyz+\overline{x}yz+\overline{x}z\\ &= xy(1+z)+\overline{x}(y+1)z\\ &= xy\cdot 1+\overline{x}\cdot 1\cdot z\\ &= xy+\overline{x}z\end{aligned}$$

2.10. Starting with the canonical sum-of-products for f get

$$\begin{aligned}f &= \overline{x}_1\overline{x}_2x_3+\overline{x}_1x_2\overline{x}_3+\overline{x}_1x_2x_3+x_1\overline{x}_2\overline{x}_3+x_1\overline{x}_2x_3+x_1x_2\overline{x}_3+x_1x_2x_3\\ &= x_1(\overline{x}_2\overline{x}_3+\overline{x}_2x_3+x_2\overline{x}_3+x_2x_3)+x_2(\overline{x}_1\overline{x}_3+\overline{x}_1x_3+x_1\overline{x}_3+x_1x_3)\\ &\quad +x_3(\overline{x}_1\overline{x}_2+\overline{x}_1x_2+x_1\overline{x}_2+x_1x_2)\\ &= x_1(\overline{x}_2(\overline{x}_3+x_3)+x_2(\overline{x}_3+x_3))+x_2(\overline{x}_1(\overline{x}_3+x_3)+x_1(\overline{x}_3+x_3))\\ &\quad +x_3(\overline{x}_1(\overline{x}_2+x_2)+x_1(\overline{x}_2+x_2))\\ &= x_1(\overline{x}_2\cdot 1+x_2\cdot 1)+x_2(\overline{x}_1\cdot 1+x_1\cdot 1)+x_3(\overline{x}_1\cdot 1+x_1\cdot 1)\\ &= x_1(\overline{x}_2+x_2)+x_2(\overline{x}_1+x_1)+x_3(\overline{x}_1+x_1)\\ &= x_1\cdot 1+x_2\cdot 1+x_3\cdot 1\\ &= x_1+x_2+x_3\end{aligned}$$

2.11. Starting with the canonical product-of-sums for f can derive:

$$\begin{aligned}
f &= (x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + \overline{x}_3) \cdot \\
&\quad (\overline{x}_1 + x_2 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3)(\overline{x}_1 + \overline{x}_2 + x_3) \\
&= ((x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3))((x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + \overline{x}_3)) \cdot \\
&\quad ((\overline{x}_1 + x_2 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3))((\overline{x}_1 + \overline{x}_2 + x_3)(\overline{x}_1 + x_2 + x_3)) \\
&= (x_1 + x_2 + x_3\overline{x}_3)(x_1 + \overline{x}_2 + x_3\overline{x}_3) \cdot \\
&\quad (\overline{x}_1 + x_2 + x_3\overline{x}_3)(\overline{x}_1 + \overline{x}_2x_2 + x_3) \\
&= (x_1 + x_2)(x_1 + \overline{x}_2)(\overline{x}_1 + x_2)(\overline{x}_1 + x_3)
\end{aligned}$$

$$\begin{aligned}
&= (x_1 + x_2\overline{x}_2)(\overline{x}_1 + x_2x_3) \\
&= x_1(\overline{x}_1 + x_2x_3) \\
&= x_1\overline{x}_1 + x_1x_2x_3 \\
&= x_1x_2x_3
\end{aligned}$$

2.25. The simplest SOP expression for the function is

$$\begin{aligned}
f &= \overline{x}_1\overline{x}_3\overline{x}_5 + \overline{x}_1\overline{x}_3\overline{x}_4 + \overline{x}_1x_4x_5 + x_1\overline{x}_2\overline{x}_3x_5 \\
&= \overline{x}_1\overline{x}_3\overline{x}_5 + \overline{x}_1\overline{x}_3\overline{x}_4 + \overline{x}_1x_4x_5 + \overline{x}_1\overline{x}_3x_5 + x_1\overline{x}_2\overline{x}_3x_5 \\
&= \overline{x}_1\overline{x}_3 + \overline{x}_1\overline{x}_3\overline{x}_4 + \overline{x}_1x_4x_5 + x_1\overline{x}_2\overline{x}_3x_5 \\
&= \overline{x}_1\overline{x}_3 + \overline{x}_1x_4x_5 + x_1\overline{x}_2\overline{x}_3x_5 \\
&= \overline{x}_1\overline{x}_3 + \overline{x}_1x_4x_5 + \overline{x}_2\overline{x}_3x_5
\end{aligned}$$

2.26. The simplest POS expression for the function is

$$\begin{aligned}
f &= (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + \overline{x}_3 + x_4)(x_1 + \overline{x}_2 + \overline{x}_3) \\
&= (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + \overline{x}_3 + x_4)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)(x_1 + \overline{x}_2 + \overline{x}_3) \\
&= (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + \overline{x}_3 + x_4)(\overline{x}_2 + \overline{x}_3) \\
&= (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + \overline{x}_3)
\end{aligned}$$