ELEN 50 Class 27 – Review

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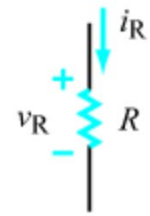
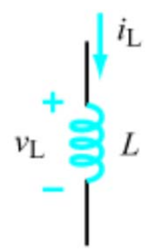
ELEN 50. Electric Circuits I

Physical basis and mathematical models of circuit components and energy sources. Circuit theorems and methods of analysis applied to DC and AC circuits. Laboratory. (Undergraduate core course.) Prerequisite: PHYS 33. (5 units)

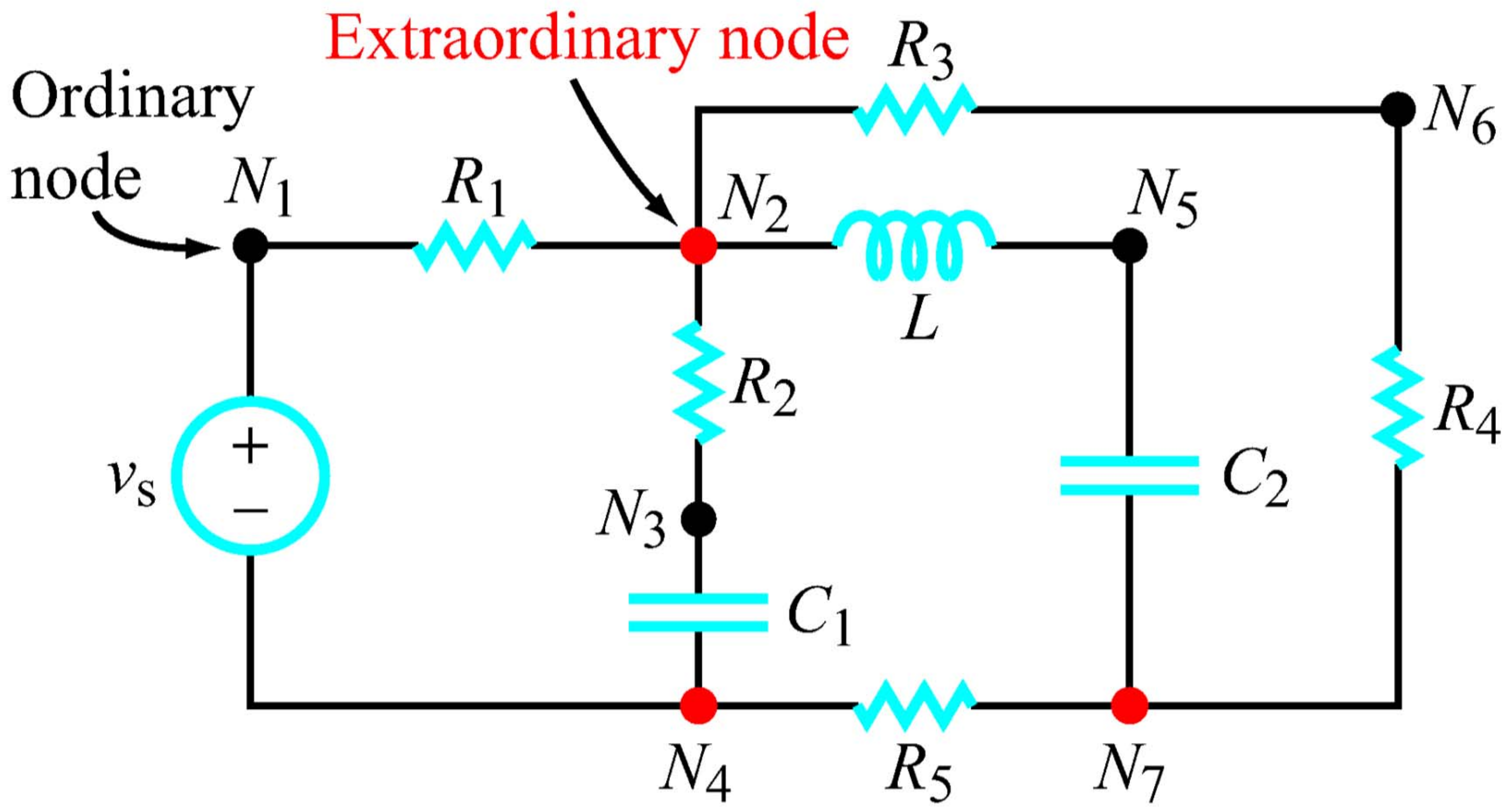
Expected learning outcomes:

1. Formulate Kirchoff current and voltage law equations in a systematic manner.
2. Formulate and solve node voltage and loop current equations
3. Compute Thevenin equivalent circuits and apply them in circuit analysis
4. Analyze circuits using operational amplifiers
5. Use phasor techniques to compute sinusoidal steady state solutions in linear circuits.
6. Design and test circuits that meet a given set of specifications.

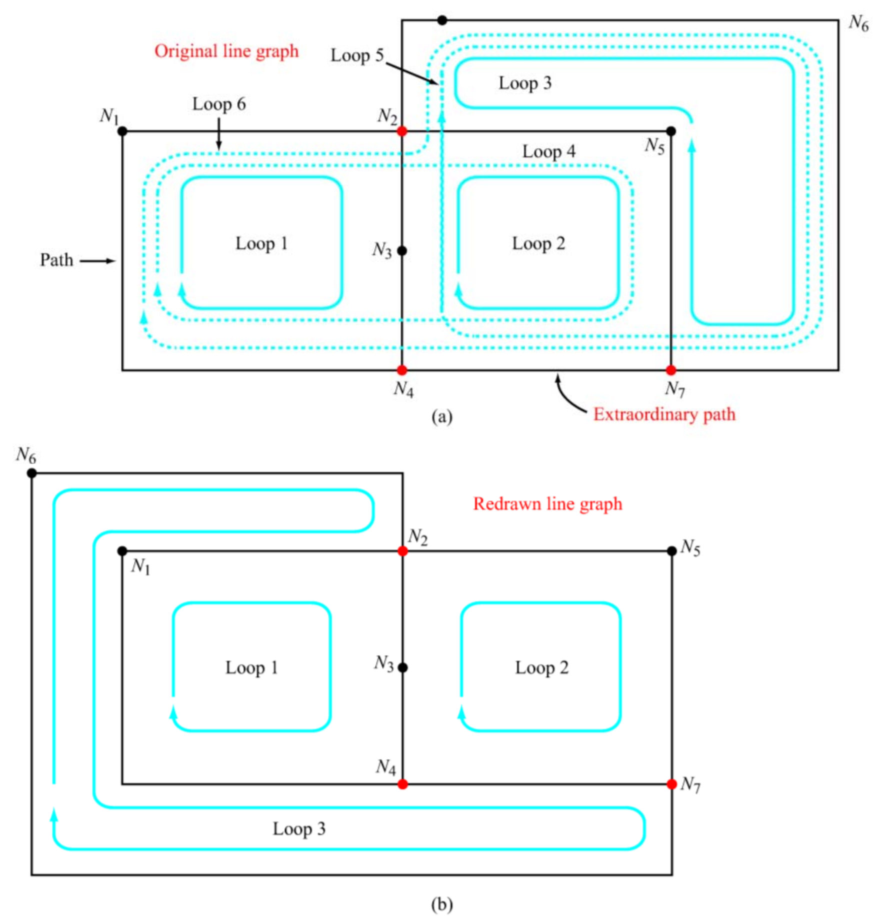
Table 1-4: Passive circuit elements and their symbols.

Element	Symbol	i–v Relationship
Resistor		$v_R = Ri_R$
Capacitor		$i_C = C\dfrac{dv_C}{dt}$
Inductor		$v_L = L\dfrac{di_L}{dt}$

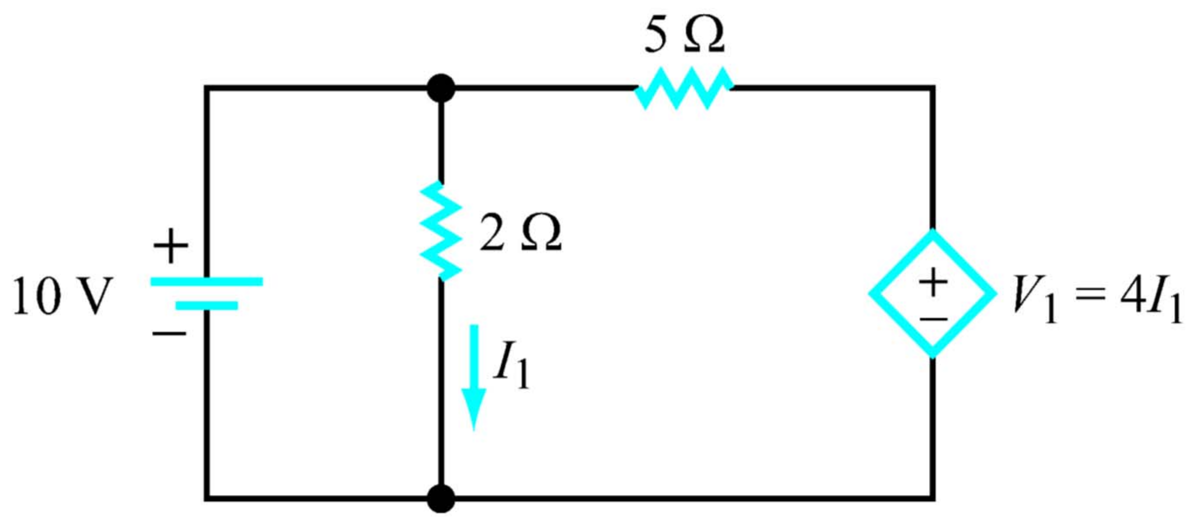
Extraordinary and Ordinary Nodes



We can redraw this circuit as a linear graph (suppressing circuit elements) and identify meshes and loops.



How do Sources Work?



The book says," The 10V dc voltage is connected across the 2 Ω resistor. Hence, the current, I_1 along the designated direction is :

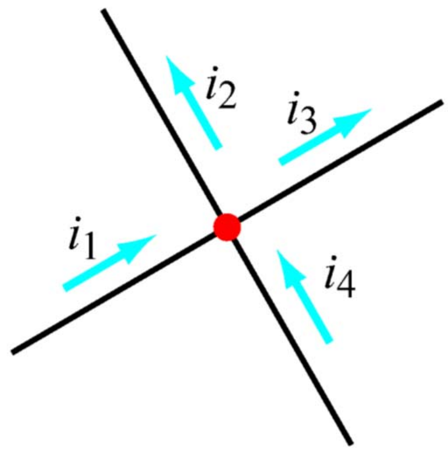
$$I_i = \frac{10}{2} = 5A$$

$$Therefore$$

$$V_1 = 4I_1 = 20V$$

Is this right? Wouldn't there also be current from the dependent source flowing through the 2Ω resistor – increasing I_1?

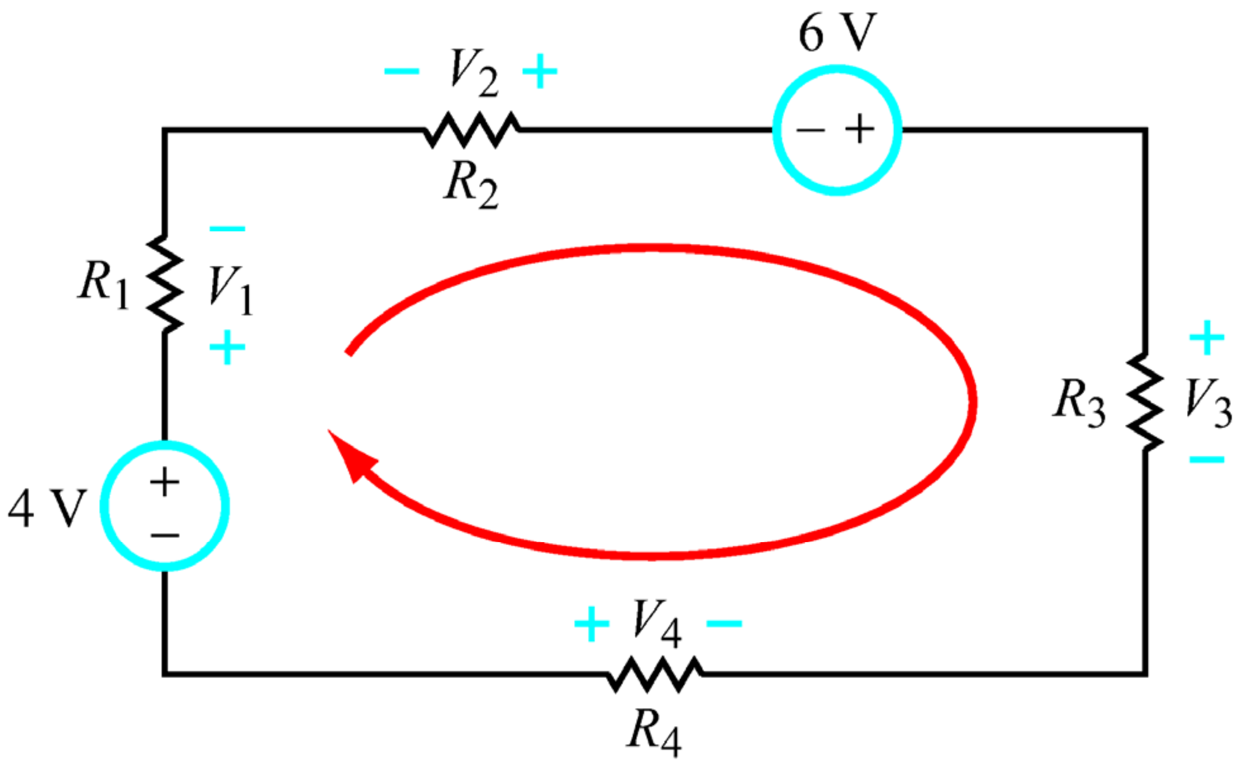
<u>Kirchhoff Current Law</u> (KCL) – the total current entering <u>a node</u> must be equal to the total current leaving <u>a node</u> …i.e. charge doesn't "build up" at a node.



$$I_1 + i_4 = i_2 + i_3$$

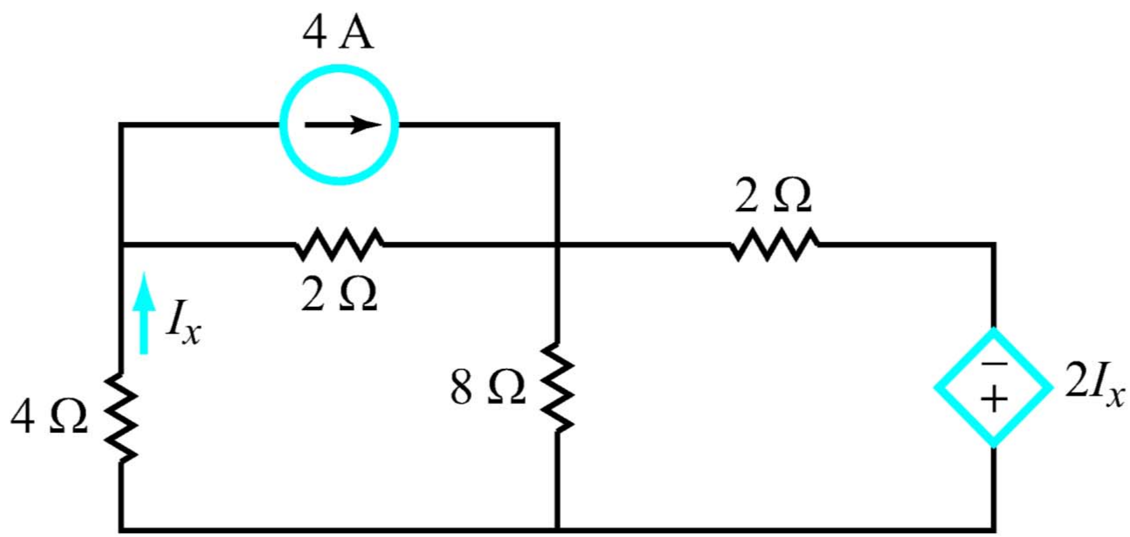
Note: a common convention is to assign a + value to current if it is entering a node and a – value if it's leaving a node although we frequently do it backwards when writing the node-voltage equations.

<u>Kirchhoff Voltage Law</u> (KVL) – the algebraic sum of the voltages around a closed <u>loop</u> is zero.



-4 + V1 – V2 -6 +V3 –V4 = 0

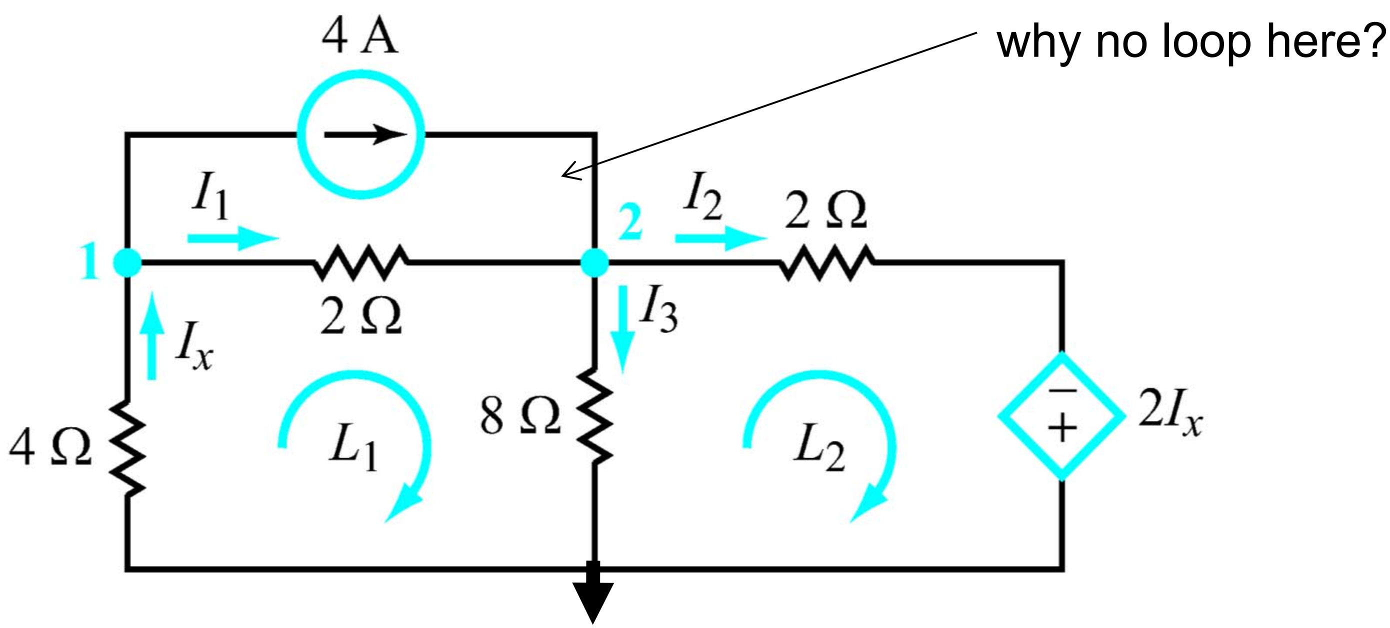
Note: a common convention is to assign a + sign to a voltage across an element if the (+) sign is the voltage encountered first as you go around the loop.



Determine I_x

….this is a more complicated circuit!

Labeling two nodes and two loops, we can write KCL and KVL



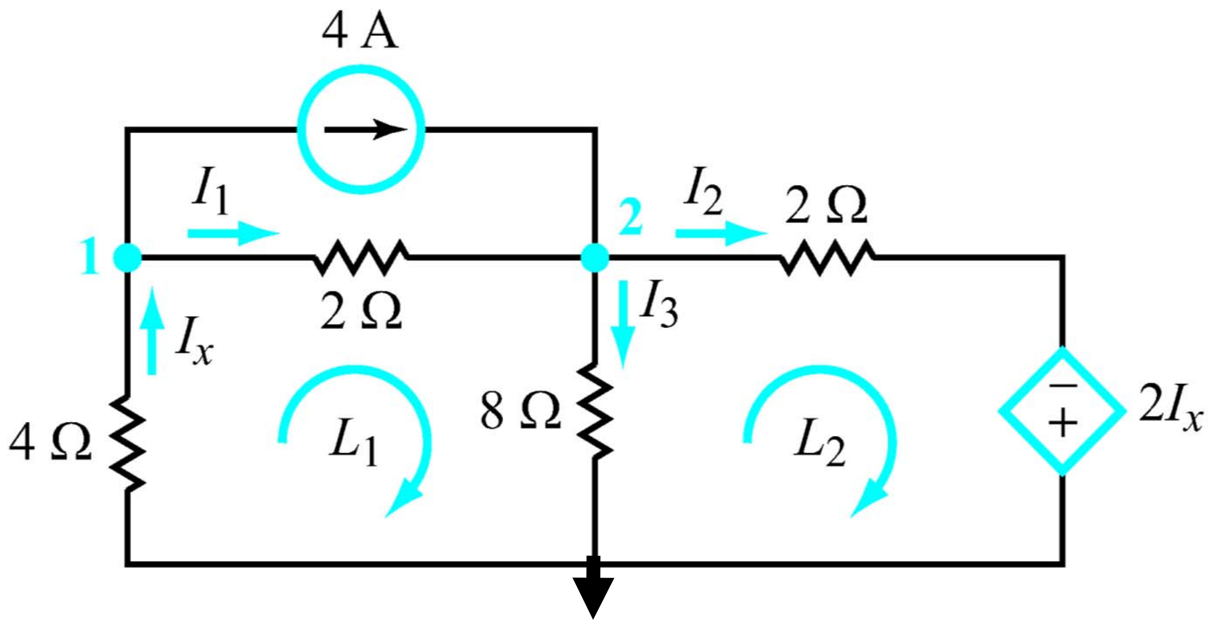
KCL @ node 1: $I_x = I_1 + 4$

KCL @ node 2: $I_1 + 4 = I_2 + I_3$

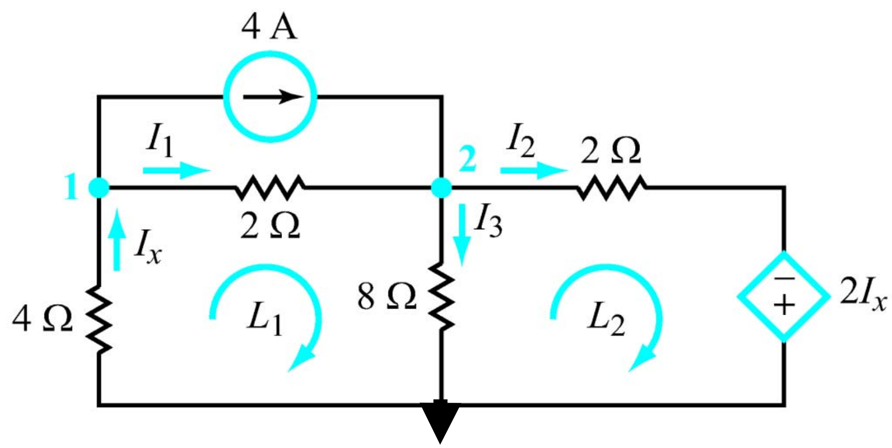
KVL Loop 1: $4I_x + 2I_1 + 8I_3 = 0$

KVL Loop 2: $-8I_3 + 2I_2 - 2I_x = 0$

Four equations in four unknowns – probably should use matrix methods to solve this – or a more systematic node voltage or mesh current approach!!!



If we did a node voltage solution, how many equations would there be?



paying attention to the current directions shown with the circuit – and writing current leaving a node as positive

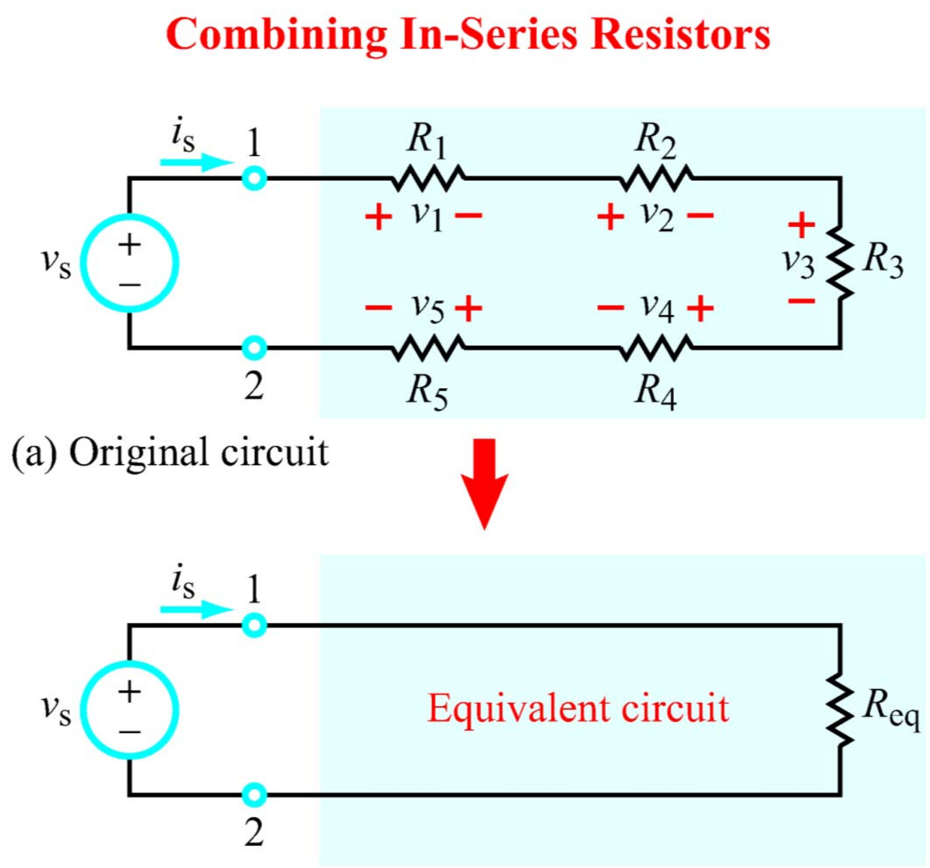
KCL at 1 $$\frac{-V_1}{4} + 4 + \frac{V_2 - V_1}{2} = 0$$ 3 terms in equation

KCL at 2 $$\frac{V_2}{8} + \frac{V_1 - V_2}{2} - 4 - \frac{2I_x - V_2}{2} = 0$$ 4 terms in equation

$$\frac{V_2}{8} + \frac{V_1 - V_2}{2} - 4 - \frac{2(-V_1/4) - V_2}{2} = 0$$

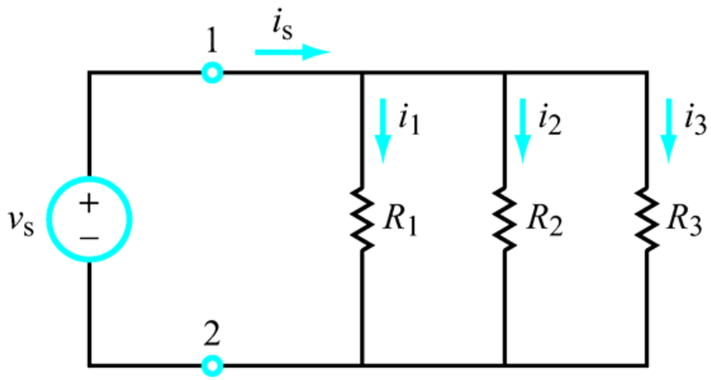
2 equations in 2 unknowns …and we get I_1, I_2, and I_3 from Ohms Law after we have obtained V_1 and V_2.

One simple kind of circuit equivalence is accomplished by combining in-series resistors …forming a single, equivalent resistor:

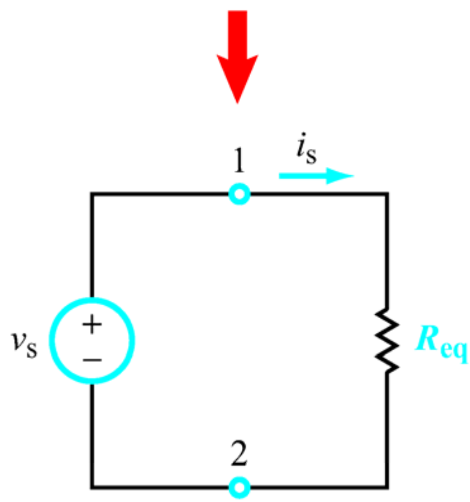


R_{eq} = R1 + R2 + R3 + R4 + R5

Combining In-Parallel Resistors



(a) Original circuit



(b) Equivalent circuit

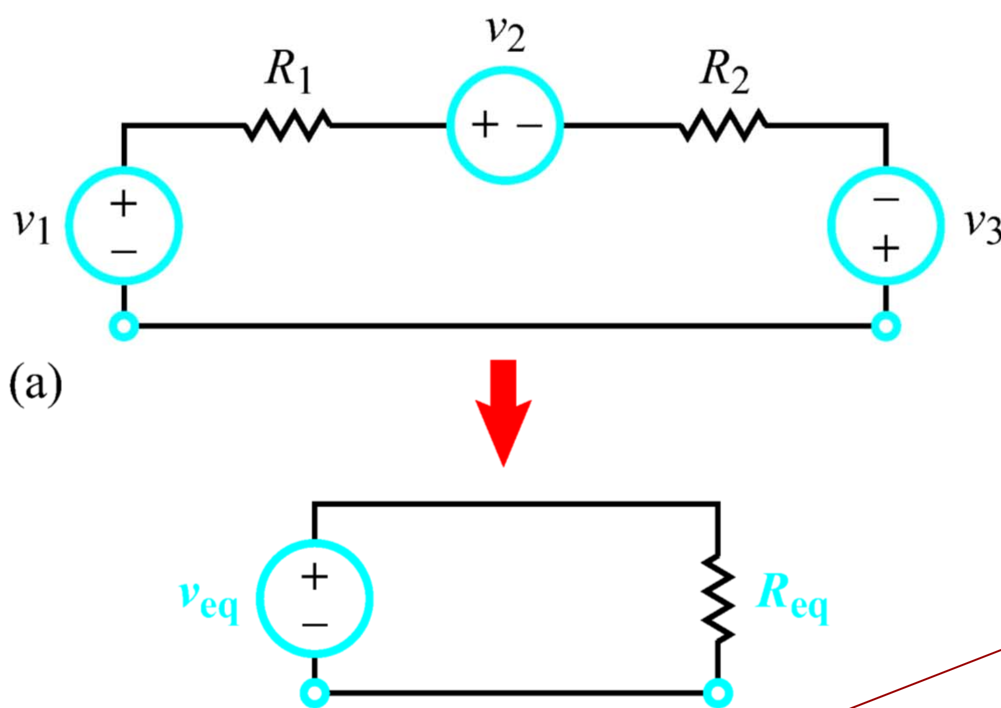
$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

$$i_n = \left(\frac{R_{eq}}{R_n} \right) i_s$$

Multiple resistors connected in parallel divide the input current among them – this is a current divider.

Sources in Series

We can also combine voltage sources and resistors in series – like for example:

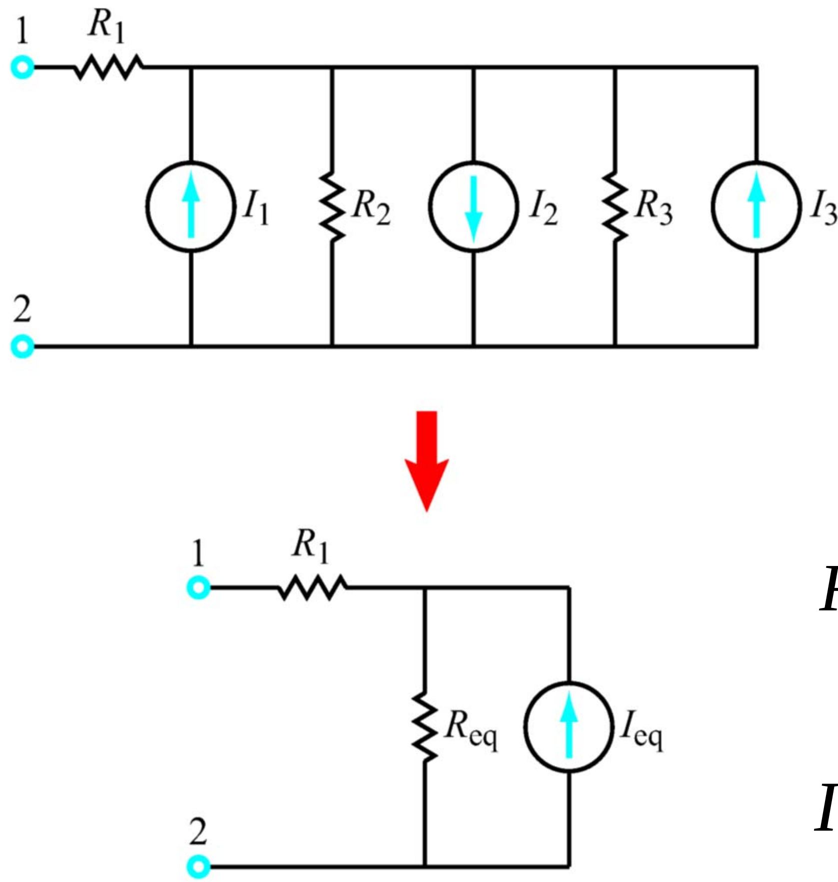


notice

$$v_{eq} = v_1 - v_2 + v_3$$

$$R_{eq} = R_1 + R_2$$

We can also combine current sources and resistors in parallel – like for example:



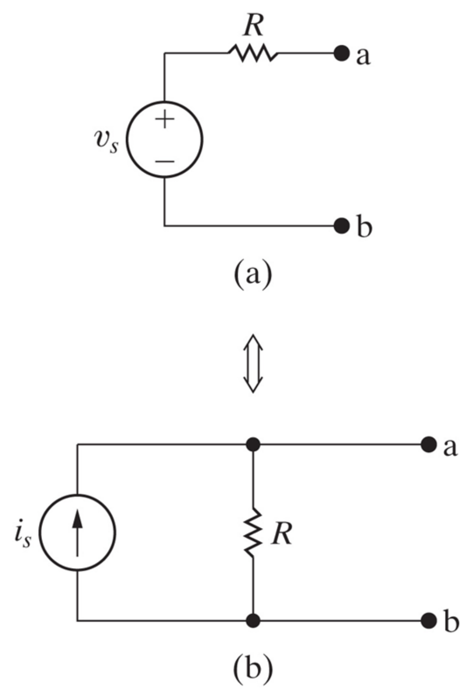
$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3}$$

$$I_{eq} = I_1 - I_2 + I_3$$

In general, voltage sources in parallel are unrealizable (aka nonsense).
Current sources in series are, similarly unrealizable in general.

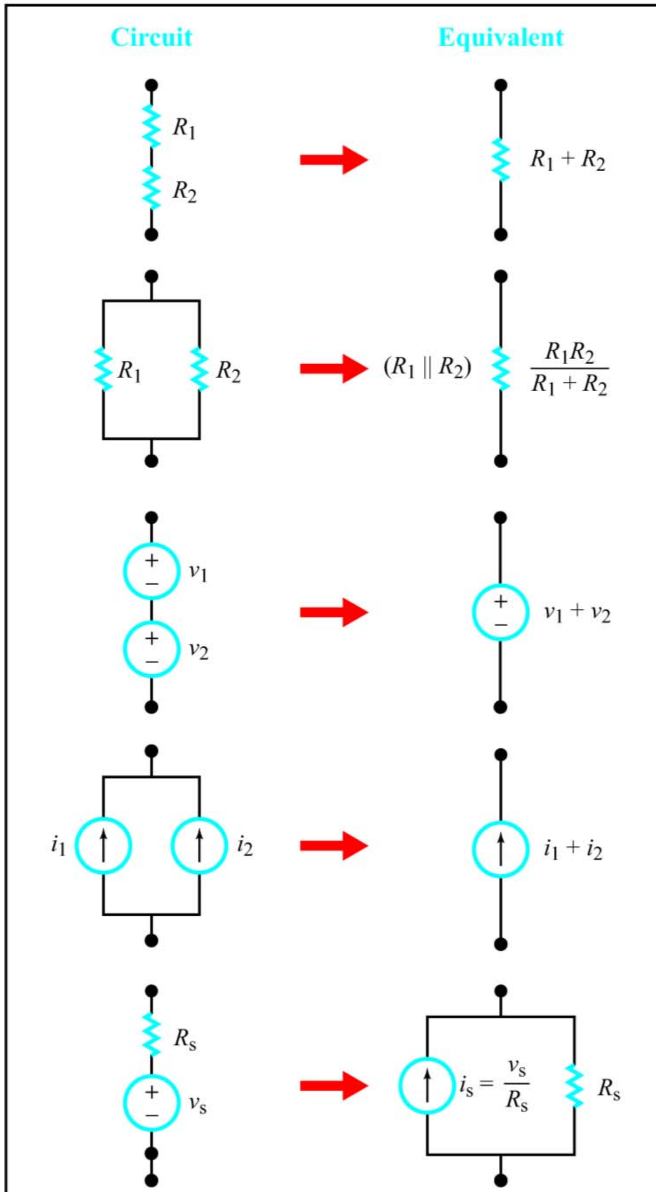
Source Transforms – we can always replace a circuit connected to two terminals with an <u>equivalent circuit that has the same I-V characteristics</u>. A series connection of a voltage source and a resistor has the same I-V curve as a current source in parallel with the same resistor …if the current source has a value of V_s/R.

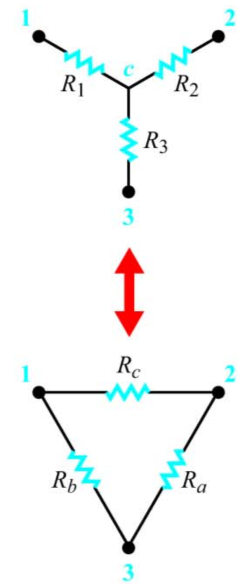
Another way of saying it is a voltage source in series with a source resistance, R_s is equivalent to the combination of a current source, $i_s = v_s/R_s$ in parallel with a shunt resistance, R_s.



$i_s = v_s/R_s$

Equivalent Circuits





$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

- For $R_a = R_b = R_c$ → $R_1 = R_2 = R_3 = R_a / 3$
- For $R_1 = R_2 = R_3$ → $R_a = R_b = R_c = 3R_1$

the Node Voltage Method:

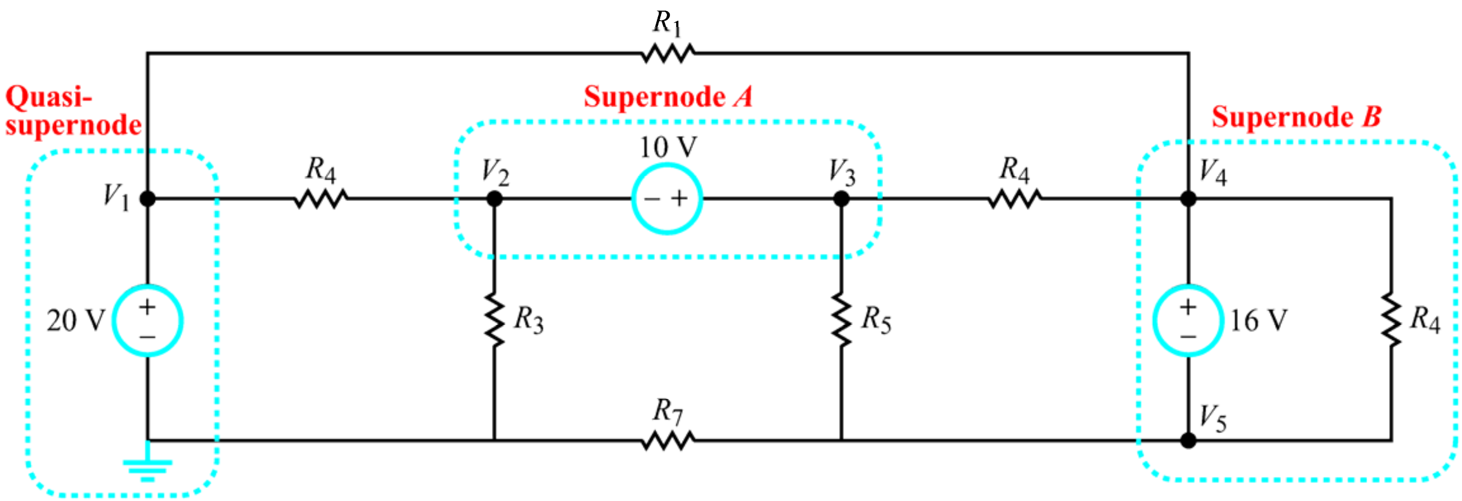
• Identify all extraordinary nodes, set one as the reference (ground) node, and assign node voltages (v1, v2, v3, etc.) to the $n_{ex} - 1$ remaining nodes.

• At each of the n_{ex}-1 nodes, write the KCL equation requiring the sum of all currents leaving the node (as a convention) to be zero.

• Solve the n_{ex}-1 independent simultaneous equations to determine the unknown node voltages.

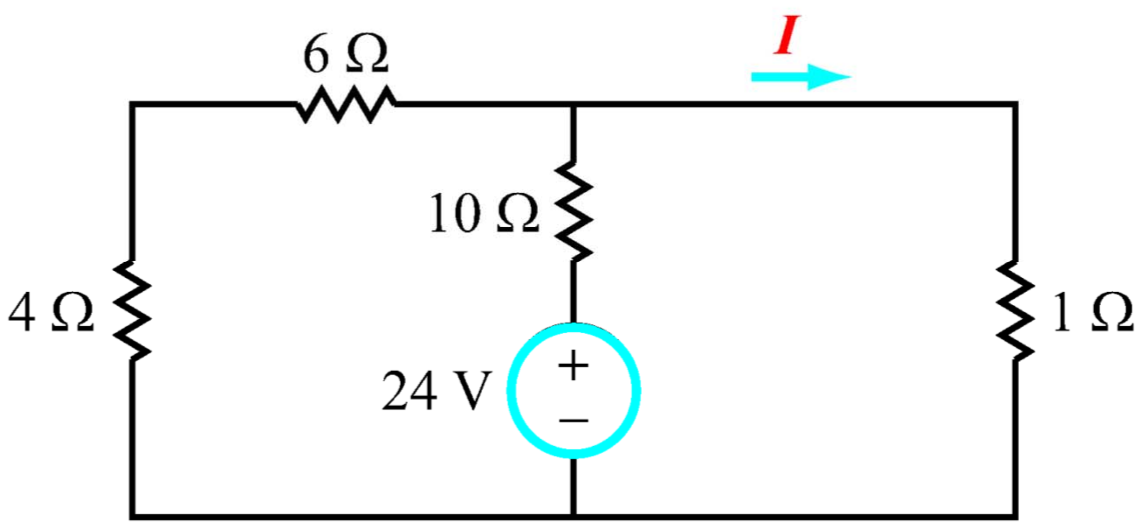
•Using the node voltage method with dependent sources is no big deal …the constraint equations for the dependent sources are just added to the KCL equations.

Nodes, Supernodes, and Quasi-Supernodes



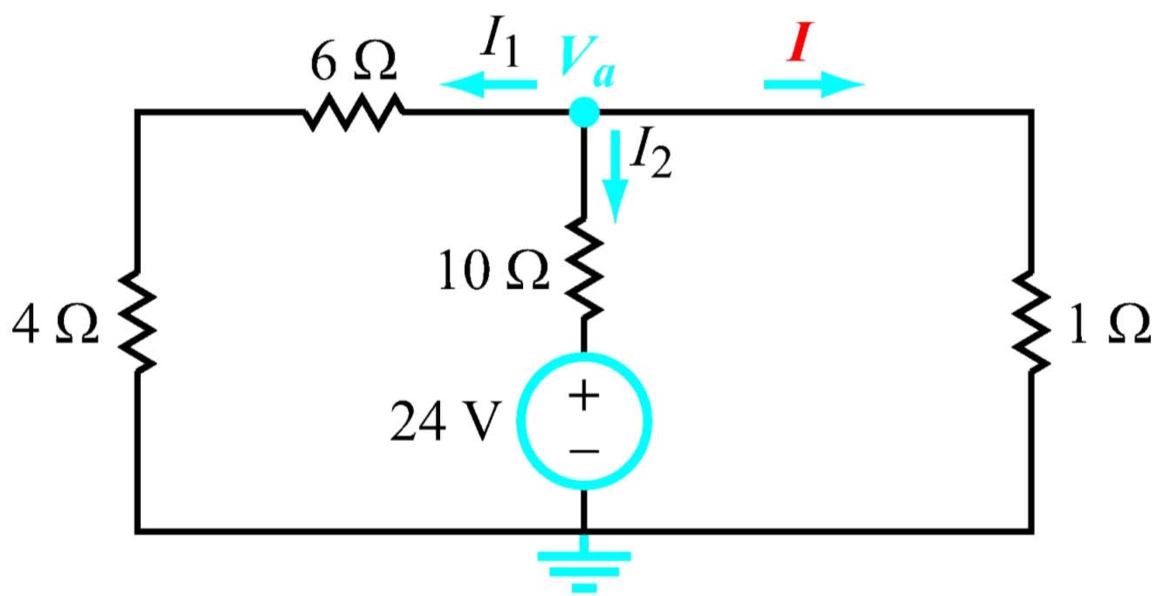
Node Voltage Example

Here's a circuit from the textbook. We want to determine the current, I. How many essential nodes are there and what is the first step in the Node Voltage Method?

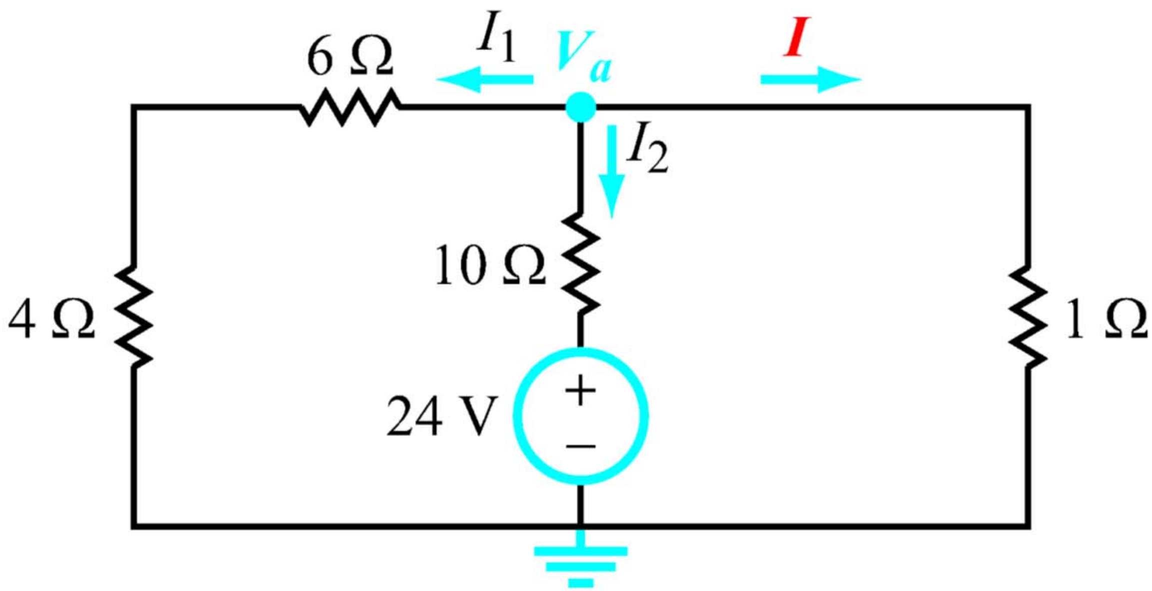


Node Voltage Example

There are two essential nodes …so we chose a reference node and label the unknown voltage (and branch currents) at the remaining essential node.



Node Voltage Example



The single node-voltage equation

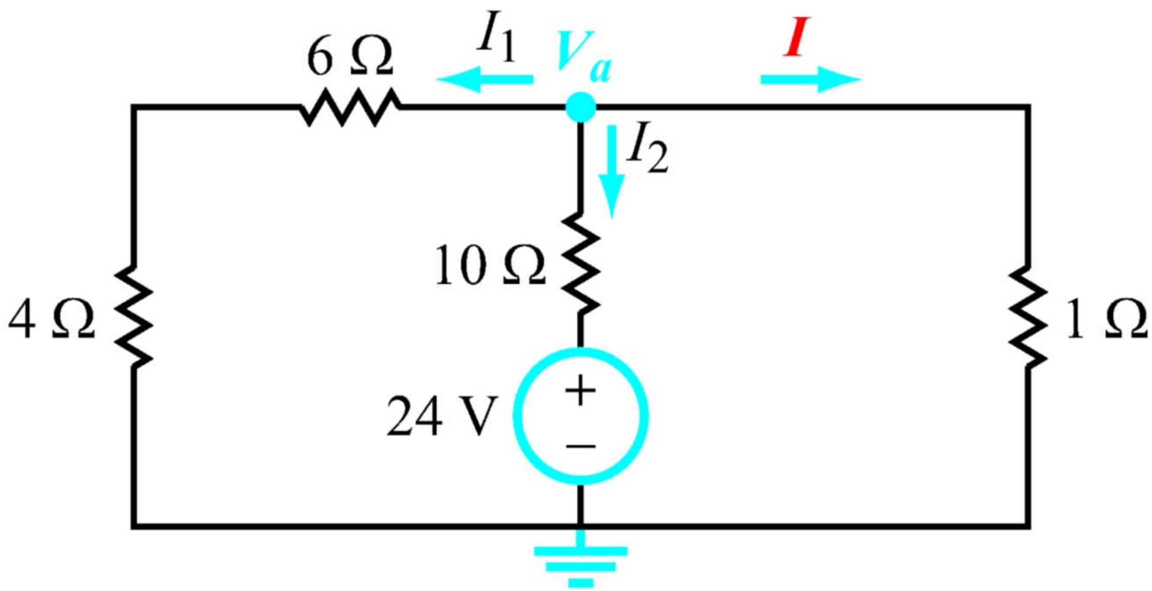
$$I_1 + I_2 + I = 0$$

Note: we're writing current leaving the node as positive

$$I_1 = \frac{V_a}{10}, \quad I_2 = \frac{(V_a - 24)}{10}, \quad I = \frac{V_a}{1}$$

$$so.... \quad \frac{V_a}{10} + \frac{(V_a - 24)}{10} + \frac{V_a}{1} = 0$$

Node Voltage Example



$$V_a\left(\frac{1}{10}+\frac{1}{10}+1\right)=\frac{24}{10}$$

$$V_a = 2V \qquad I=\frac{V_a}{1}=2A$$

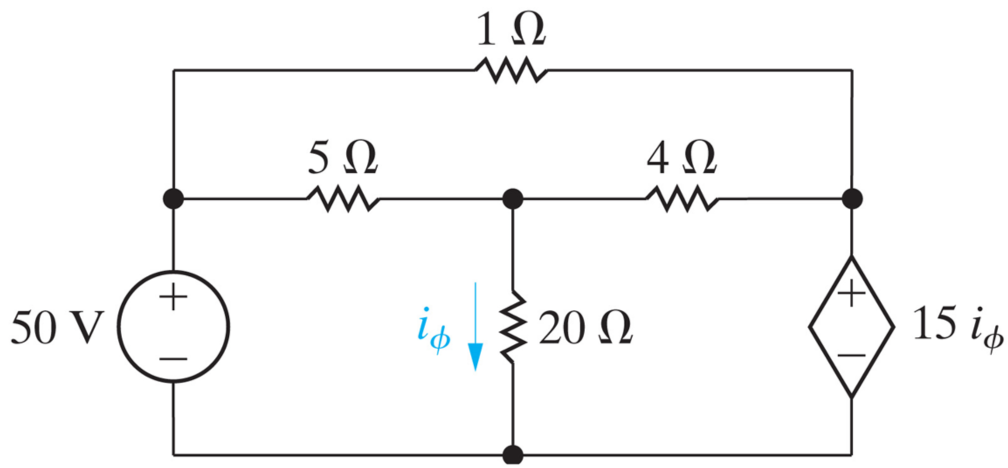
- The mesh current method is completely complementary to the node voltage method …but it uses mesh currents instead of node voltages….and it explicitly uses KVL instead of KCL.

Node Voltage Method	**Mesh Current Method**
• Identify essential nodes	• Identify mesh currents
• Select reference node	
• Label voltages at remaining essential nodes (v1, v2, ….vn)	• Label mesh currents (ia, ib, …in)
• Write equations for KCL <u>at these nodes</u> in terms of node voltages referenced to reference node.	• Write equations for KVL <u>around the mesh current paths.</u>
• Solve n equations in n unknowns	• Solve n equations in n unknowns

- In both the node voltage and mesh current method, dependent sources introduce additional constraint equations that must be used along with the KCL or KVL equations.

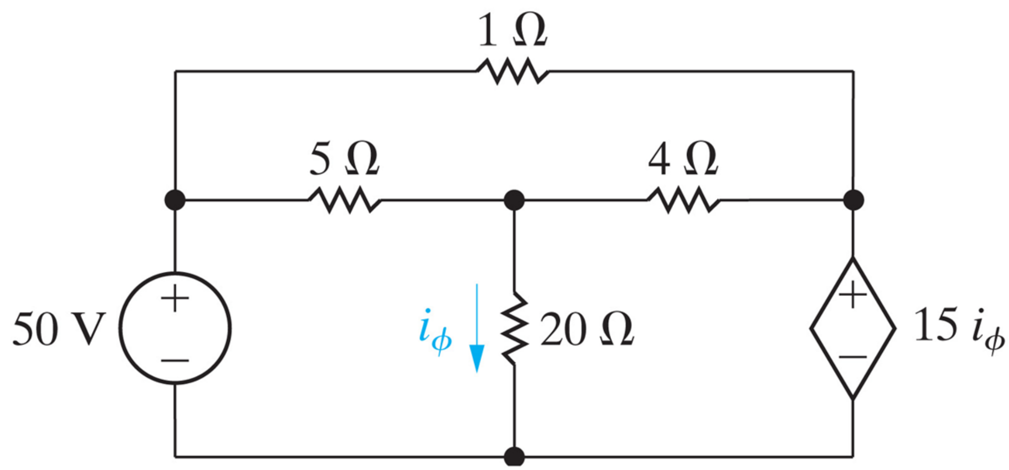
Mesh Current Example

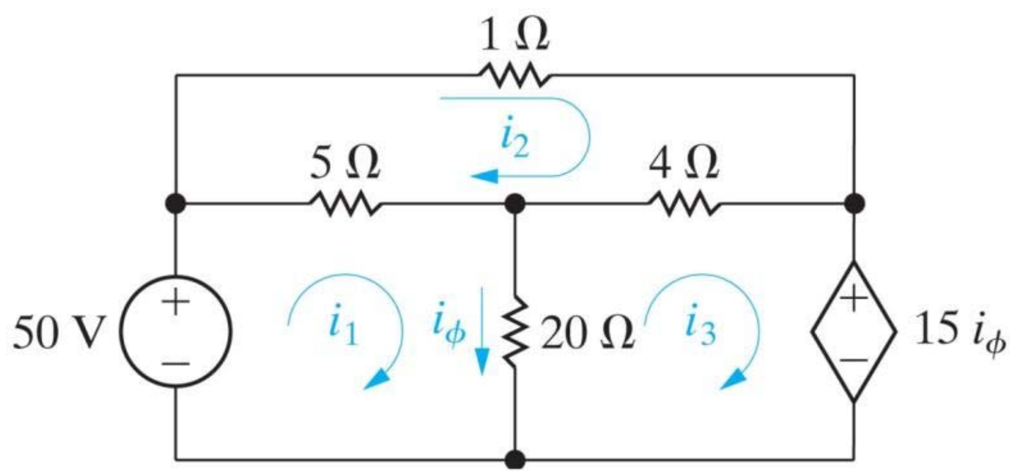
Here's another mesh current example problem – this one has a dependent voltage source:



The problem asks for the power dissipated in the 4Ω resistor.

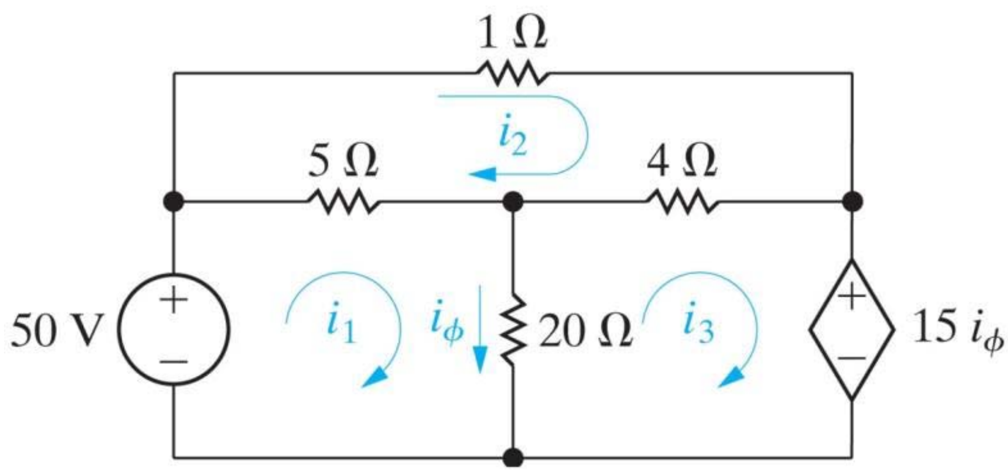
Mesh Current Example





The first step, as always, is to identify and label the mesh currents – there are three of them, i1, i2, and i3.

Mesh Current Example



Now we write the mesh current equations using KVL.

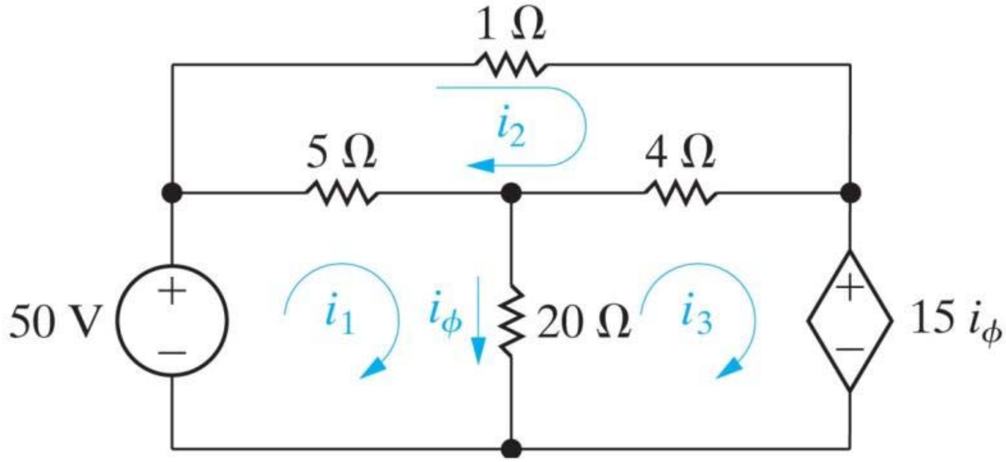
- 50 + (i1 – i2) 5 + (i1 –i3) 20 = 0

(i2 –i1) 5 + i2 1 + (i2 – i3) 4 = 0

(i3 –i1) 20 + (i3 –i2) 4 + 15 iϕ = 0

and, of course, the supplemental equation: iϕ = i1 – i3 we always need this when working with dependent sources!

Mesh Current Example



Solving the three equations in three unknowns we can get:

I2 = 26A and i3 = 28A …

So the power in the 4Ω resistor is 4 (i3 – i2)^2 = 16W

In the mesh current “analysis by inspection” technique one solves matrix equations of the form:

$$\mathbf{V} = \mathbf{IR}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & R_{22} & R_{23} & R_{24} \\ R_{31} & R_{32} & R_{33} & R_{34} \\ R_{41} & R_{42} & R_{43} & R_{44} \end{bmatrix}$$

by writing the matrix down directly from inspection of the circuit.

Mesh Current Analysis by Inspection

The circuit must contain only independent sources (the matrix for the last problem could not have been written down by inspection)

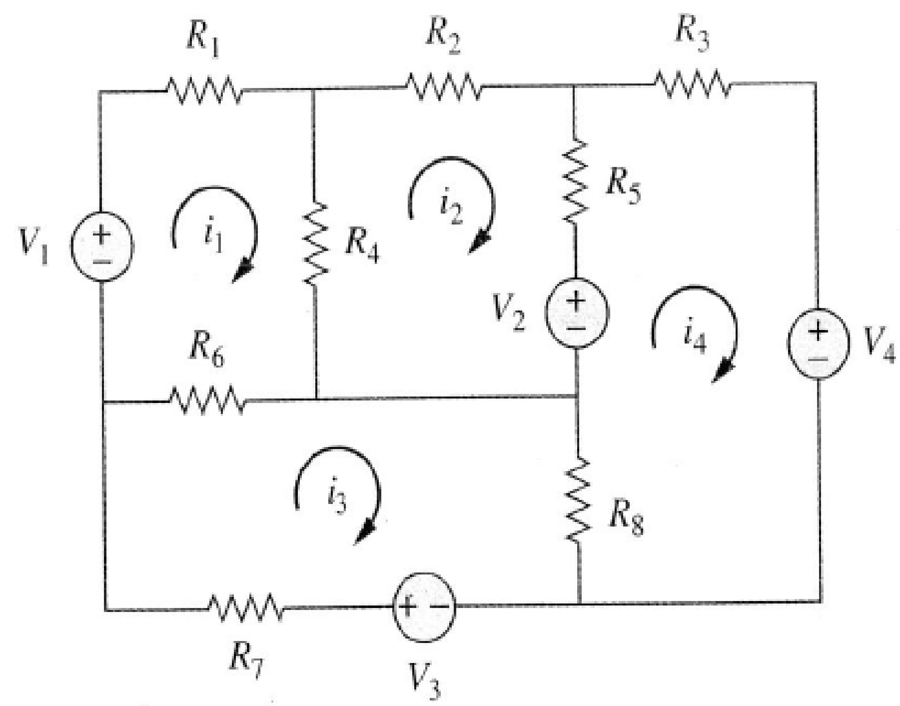
Using mesh current analysis, one identifies all of the meshes in a circuit as usual.

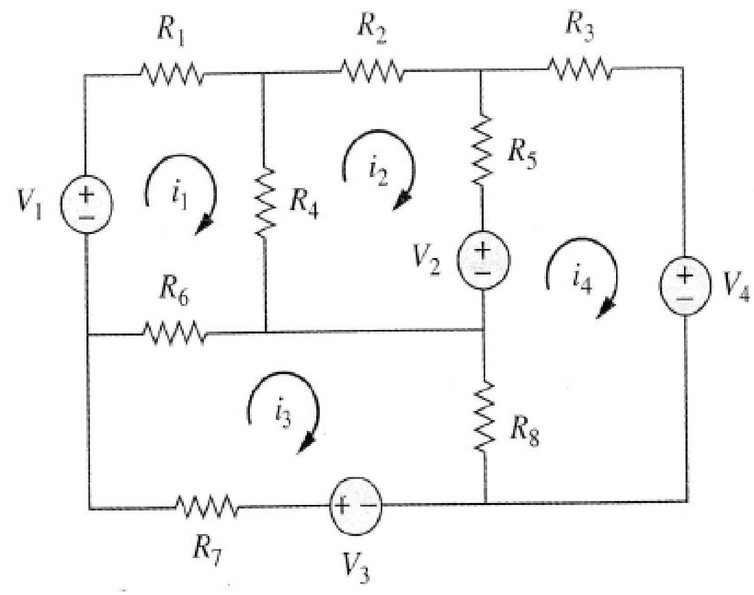
Next, one writes down the resistance matrix by inspection – noticing that the **R** matrix is symmetric ($r_{jk} = r_{kj}$) and that all off-diagonal terms are either negative or zero.

- the R_{kk} terms are the sum of all of the resistances in the k mesh
- the R_{jk} terms are the negative sum of all of the resistances common to the k and the j mesh
- the v_k term (k^{th} component of the vector **V**) is equal to the algebraic sum of all of the independent voltages in the k^{th} mesh.

One then solves the matrix equation for the **I** vector and these are the various mesh currents for the circuit.

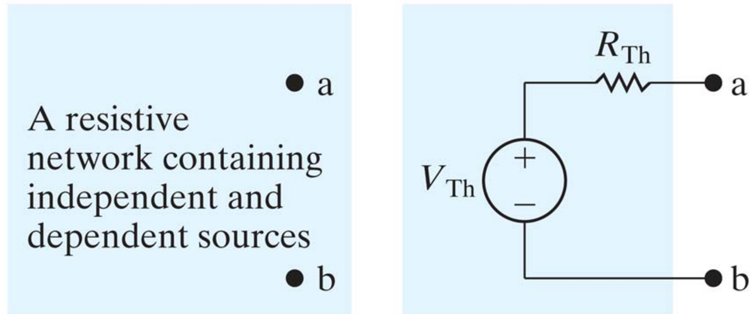
Write the matrix equation for the mesh currents in this circuit by inspection





$$\begin{bmatrix} V_1 \\ -V_2 \\ V_3 \\ -V_4 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \begin{bmatrix} R_1 + R_4 + R_6 & -R_4 & -R_6 & 0 \\ -R_4 & R_4 + R_2 + R_5 & 0 & -R_5 \\ -R_6 & 0 & R_6 + R_7 + R_8 & -R_8 \\ 0 & -R_5 & -R_8 & R_3 + R_5 + R_8 \end{bmatrix}$$

Thevenin and Norton Equivalent Circuits



(a) (b)

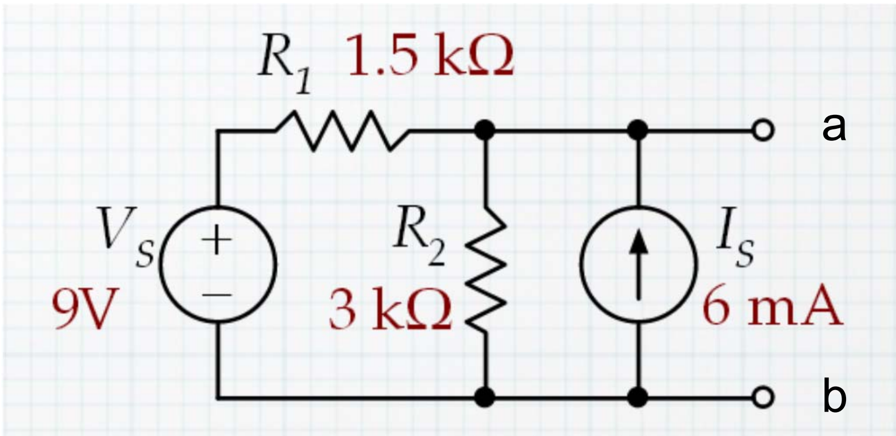
Once we've got values for Vth and Rth …we can transform this into a current source with a parallel resistor using a source transform….and this current source in parallel with a resistor is called a Norton Equivalent Circuit. As far as the behavior at the terminals is concerned, it doesn't matter if you use a Thevenin equivalent circuit or a Norton equivalent circuit. It may be more convenient to use one or they other …but they are absolutely identical in terms of describing the behavior at the terminals as we saw earlier.

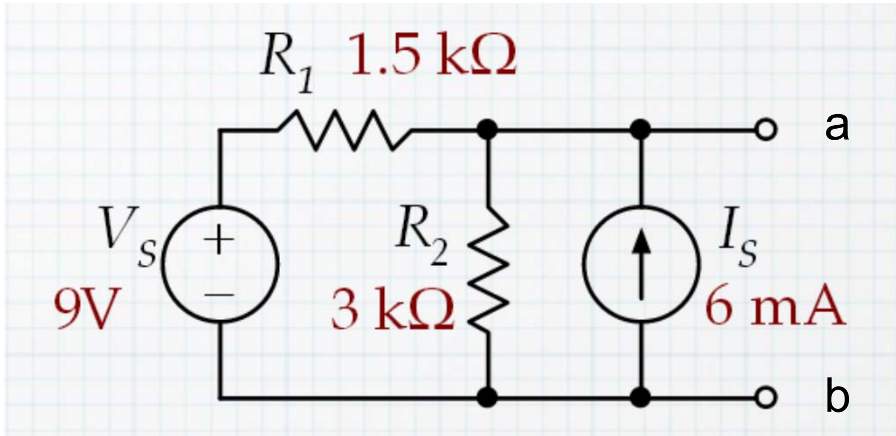
You can generate Thevenin or Norton equivalent circuits for real circuits by making open circuit voltage and short circuit current measurements, or you can calculate these equivalent circuits using circuit analysis.

Thevenin Equivalent Circuit – General Calculation Strategy

- Obtain vth by calculating the voltage across the two specified terminals when no load is present (open circuit voltage)
- Obtain Rth by:
  - Calculating the current that will flow between the specified terminals in a short circuit. Rth is obtained from Rth = vth/Isc
  - If the circuit doesn't contain dependent sources, you can calculate the equivalent resistance between the specified terminals after all independent voltage sources are replaced with short circuits and all independent current sources are replaced with open circuits. This in effect is "deactivating" the independent sources and and equivalent resistance in this case is Rth, the Thevenin resistance.
  - If the circuit contains independent and dependent sources, Rth can be determined by deactivating independent sources, and adding an external source (vex)…then solve the circuit to determine the current iex supplied by the external source. Rth = vex/iex

What is the Thevenin equivalent circuit at terminals a and b?

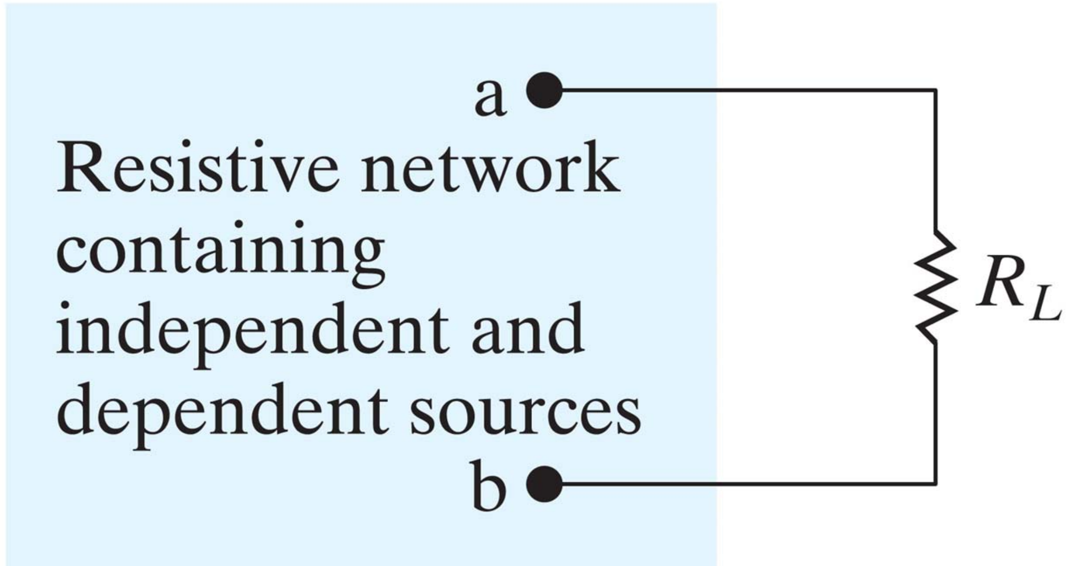




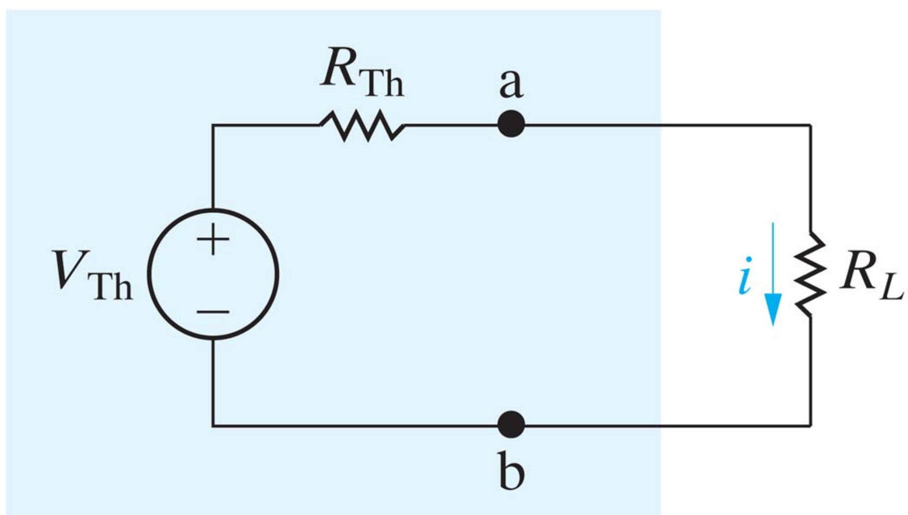
We can get Vth in a number of ways ..probably the easiest is to do a source transform on the 9V voltage source and the 1.5kΩ series resistor – replacing them with a 6 mA source in parallel with a 1.5kΩ resistor. This will result in a total current source of 12 mA in parallel with a 1 kΩ resistor ..producing a voltage of 12V. Vth=12V

To get the Thevenin resistance, the easiest method is probably method 2 – deactivate the sources and calculate the resistance across terminals a and b. We can do this because all the sources are independent. Replacing the voltage source with a short circuit and the current source with an open circuit, we have a 3 kΩ resistor in parallel with a 1.5 kΩ resistor across terminals a and b. Rth = I kΩ

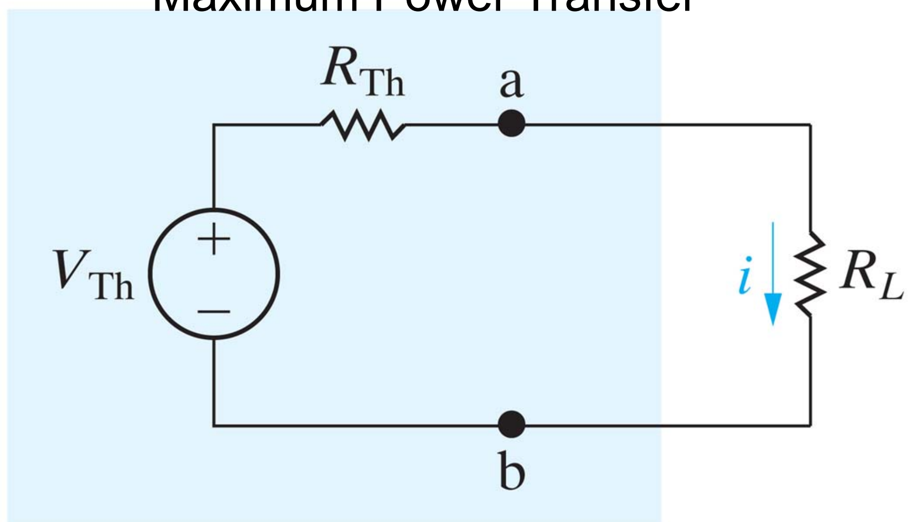
Maximum Power Transfer



A resistive (linear) network can always be replaced by its Thevenin equivalent



Maximum Power Transfer



The power dissipated in R_L : $p = i^2 R_L = (Vth / [Rth + R_L])^2 R_L$

To find out how the power varies as we vary R_L, we differentiate this expression with respect to R_L. The derivative will be zero at an extreme value of the function. We know that power is zero at both $R_L = 0$ and $R_L = \infty$, so, somewhere between 0 and ∞ is a value of R_L at which power dissipation is a maximum….and at this point the derivative will be zero.

Maximum Power Transfer

The derivative equals zero when: $(R_{Th} + R_L)^2 = 2R_L\ (R_{Th} + R_L)$

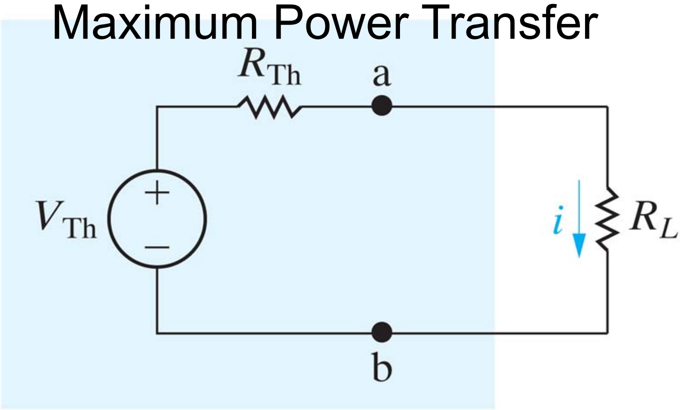
So, solving for R_L

Power transfer is maximum when the load resistance, $R_L = R_{Th}$

and we can determine the value of the maximum power by substituting into: $p = i2\ R_L = (Vth / [R_{Th} + R_L])^2\ R_L$

$$\boxed{p_{max} = V_{Th}^2 / 4R_L}$$

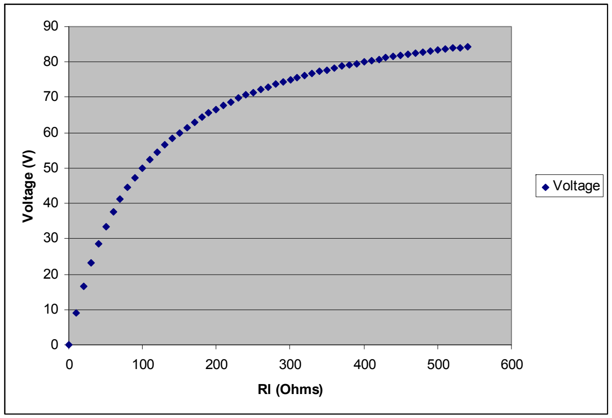
Maximum Power Transfer

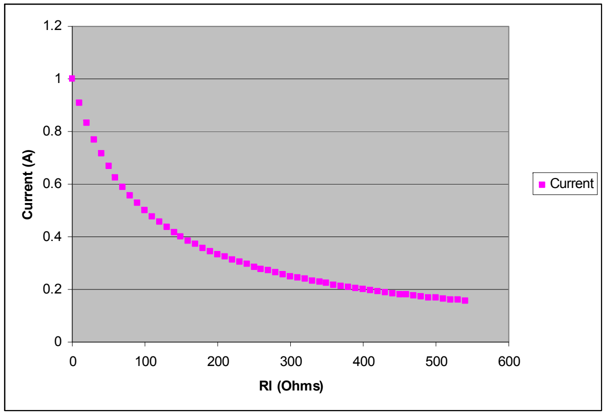


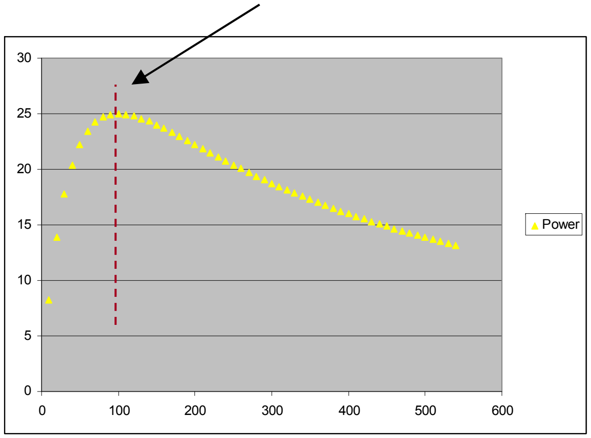
We can see this graphically by plotting voltage, current, and power for the load resistor. Let's consider, for example, Vth = 100V, and Rth = 100Ω

$$p_{max} = V_{Th}^2 / 4R_L = 25W$$

Peak power transfer







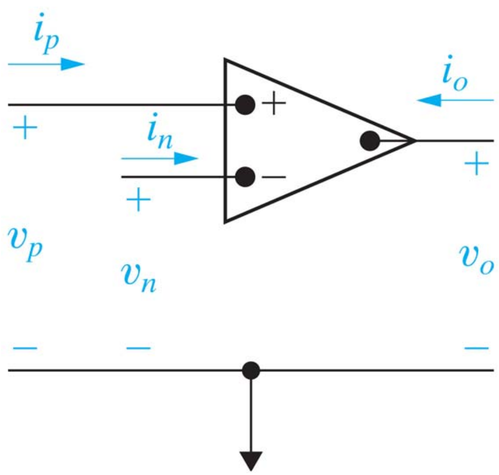
Op Amps

In circuits where the op amp is constrained by negative feedback to operate in the linear region, $(v_p - v_n) \sim 0$.

For circuit analysis, we'll assume the open loop gain is infinite so, for the analysis, we can assume that there is no voltage difference between v_p and v_n when the amplifier is operating in the linear region. We will also assume that the input impedance of the two inputs is infinite. These two conditions for an ideal op amp can be written:

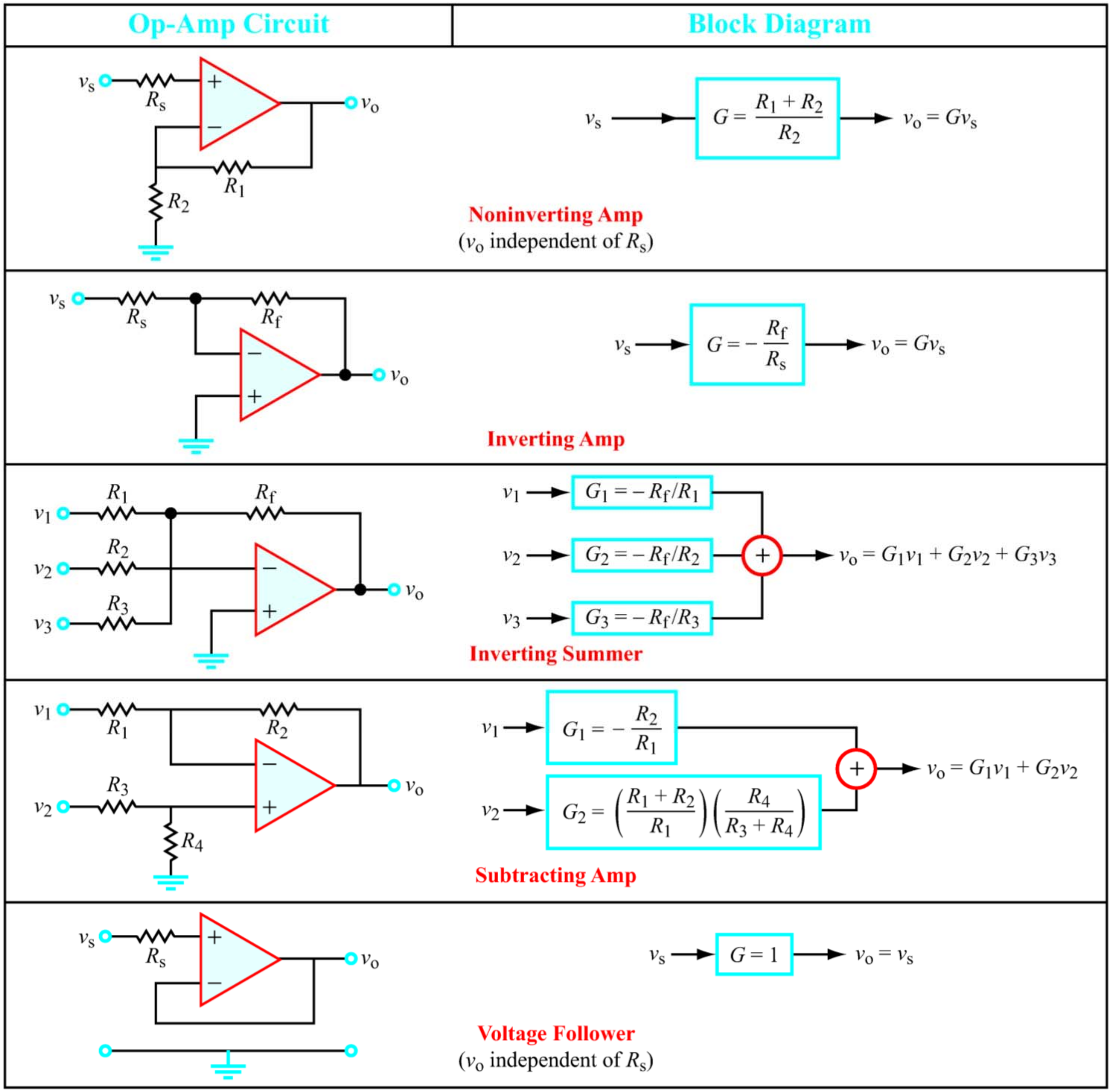
$v_p = v_n$ $\quad\quad$ $i_p = i_n = 0$

These are the op-amp "golden rules"



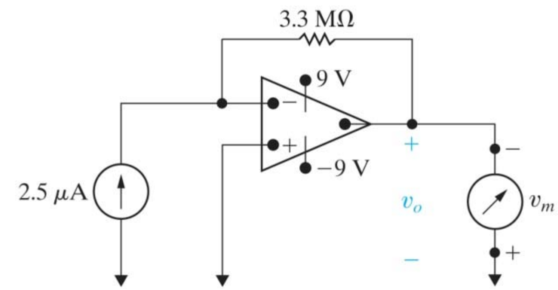
You can derive the equation for the gain of any op-amp circuit by doing one (or more) node voltage calculations at the extraordinary nodes in the circuit --- and applying the golden rules

To Review: Some Simple Op Amp Circuits



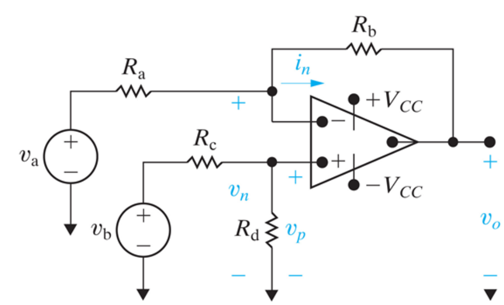
also remember the <u>current to voltage convertor</u> we saw earlier

$V_m i R_f$



and the difference amplifier

$$v_0 = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)} v_b - \frac{R_b}{R_a} v_a$$



Basic Properties of R, L and C

Property	R	L	C
i–v relation	$i = \frac{v}{R}$	$i = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)$	$i = C\,\frac{dv}{dt}$
v-i relation	$v = iR$	$v = L\,\frac{di}{dt}$	$v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$
p (power transfer in)	$p = i^2 R$	$p = Li\,\frac{di}{dt}$	$p = Cv\,\frac{dv}{dt}$
w (stored energy)	0	$w = \frac{1}{2}Li^2$	$w = \frac{1}{2}Cv^2$
Series combination	$R_{eq} = R_1 + R_2$	$L_{eq} = L_1 + L_2$	$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$
Parallel combination	$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$	$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$	$C_{eq} = C_1 + C_2$
dc behavior	no change	short circuit	open circuit
Can v change instantaneously?	yes	yes	no
Can i change instantaneously?	yes	no	yes

We can develop a general expression to describe both the **natural and step responses of RL and RC circuits:**

$$\boxed{x(t) = x_f + \left[x(t_0) - x_f\right] e^{-(t-t_0)/\tau}}$$

where x is either the current or voltage, x_f is the final value of the current or voltage, $x(t_0)$ is the initial value of the current or voltage, and τ is the appropriate time constant for the circuit.

RL and RC circuits are described by first order differential equations that are solved by this function. For the RC circuit, τ = RC and for the RL circuit, τ = L/R.

Phasors

The phasor is a complex number that carries the amplitude and phase angle information of a sinusoidal function. It is a convenient way of representing these properties and, as we'll see, it's possible to develop simple, powerful circuit analysis techniques for the response of circuits to <u>steady-state sinusoidal inputs</u> using phasors.

The phasor concept is based on the Euler identity which lets us express sine and cosine functions as an exponential function.

$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

The cosine function is the real part of the complex exponential function and the sine function is the imaginary part. Notice we are using the symbol, "j" to indicate imaginary numbers – i.e.

$$\boxed{j^2 = -1}$$

We do that to avoid confusion with the symbol, i, for current.

So if: $$v = V_m \cos(\omega t + \phi)$$

We can write $$= V_m \Re\left\{ e^{j\omega t} e^{j\phi} \right\}$$

$$= \Re\left\{ V_m e^{j\phi} e^{j\omega t} \right\}$$

symbol for phasor transform

We can define the phasor transform as:

$$\mathbf{V} = V_m e^{j\phi} = \boldsymbol{P}\left\{ V_m \cos(\omega t + \phi) \right\}$$

This is the representation for a phasor – it's written in <u>boldface</u> and it's a complex number containing <u>amplitude and phase</u> information.

Notice: <u>the phasor doesn't have any time dependence</u> – it is a transform from the time domain to the frequency or complex number domain.

So for: $$v = V_m \cos(\omega t + \phi)$$

The phasor transform is: $$\mathbf{V} = V_m e^{j\phi}$$

It lets us keep track of the magnitude and the phase of a voltage (or current) and greatly simplifies solving sinusoidal steady-state circuits

To reiterate: <u>the phasor doesn't have any time dependence</u> –the phasor written in <u>boldface</u> and it's a complex number containing <u>amplitude and phase</u> information.

It is a transform from the time domain to the frequency or complex number domain.

Passive Circuit Elements in the Frequency Domain

Resistors

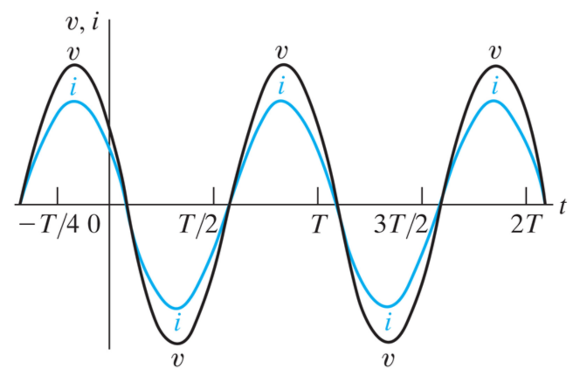
If the current in a resistor varies sinusoidally with time:

$$i = I_m \cos(\omega t + \theta_i) \;\; then \;\; from \; Ohms \; law:$$
$$v = R\left[I_m \cos(\omega t + \theta_i)\right]$$
$$= RI_m\left[\cos(\omega t + \theta_i)\right]$$

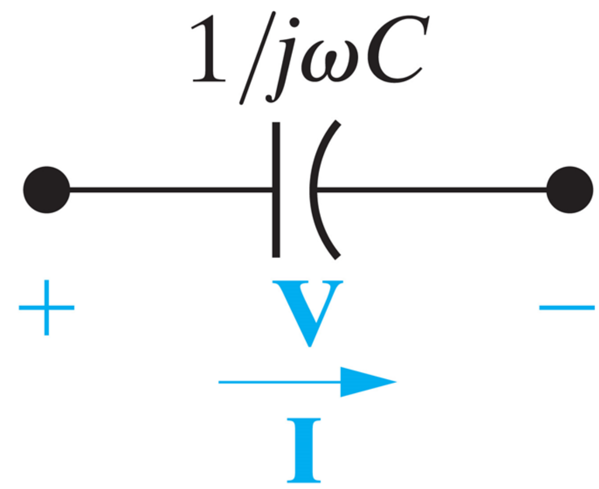
The phasor transform of this voltage: $\mathbf{V} = V_m e^{j\phi} = \mathcal{P}\left\{V_m \cos(\omega t + \phi)\right\}$

$$\mathbf{V} = RI_m e^{j\theta_i} = RI_m \angle \theta_i$$

So: $\boxed{\mathbf{V} = R\mathbf{I}}$ Ohm's law for phasors



Capacitors



If a sinusoidal voltage is applied to a capacitor:

$$v = V_m \cos(\omega t + \theta_i)$$

Then, since for a capacitor: $i = C\dfrac{dv}{dt}$

In phasor notation (using the trick we learned from the inductor example) .. we replace the time derivative of the voltage with $j\omega\mathbf{V}$

$$and\ \ we\ can\ write:$$

$$\mathbf{I} = j\omega C\mathbf{V}$$

$$so\ \boxed{\mathbf{V} = \frac{\mathbf{I}}{j\omega C}}$$

Also, we could write this as: $\mathbf{V} = \dfrac{-j\,\mathbf{I}}{\omega C}$

Is it clear why we can do this?

So by using phasor notation, we have been able to write the fundamental voltage-current relation for an inductor:

$$v = L\frac{di}{dt}$$

As:

$$\mathbf{V} = j\omega L\mathbf{I}$$

We've replaced the time derivative of the current with: $j\omega\mathbf{I}$

This is really important! Phasor transforms allow the use of all of the DC analysis tools we've developed for AC circuits …which contain inductors and capacitors …and with **no differential equations**!

We can summarize all of the stuff about phasor representations of passive circuit elements by writing a general phasor equation

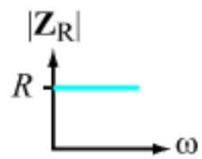
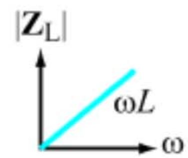
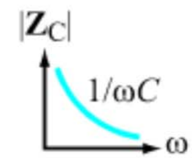
$$\boxed{\mathbf{V} = Z\mathbf{I}}$$

Where Z is the impedance of the circuit element

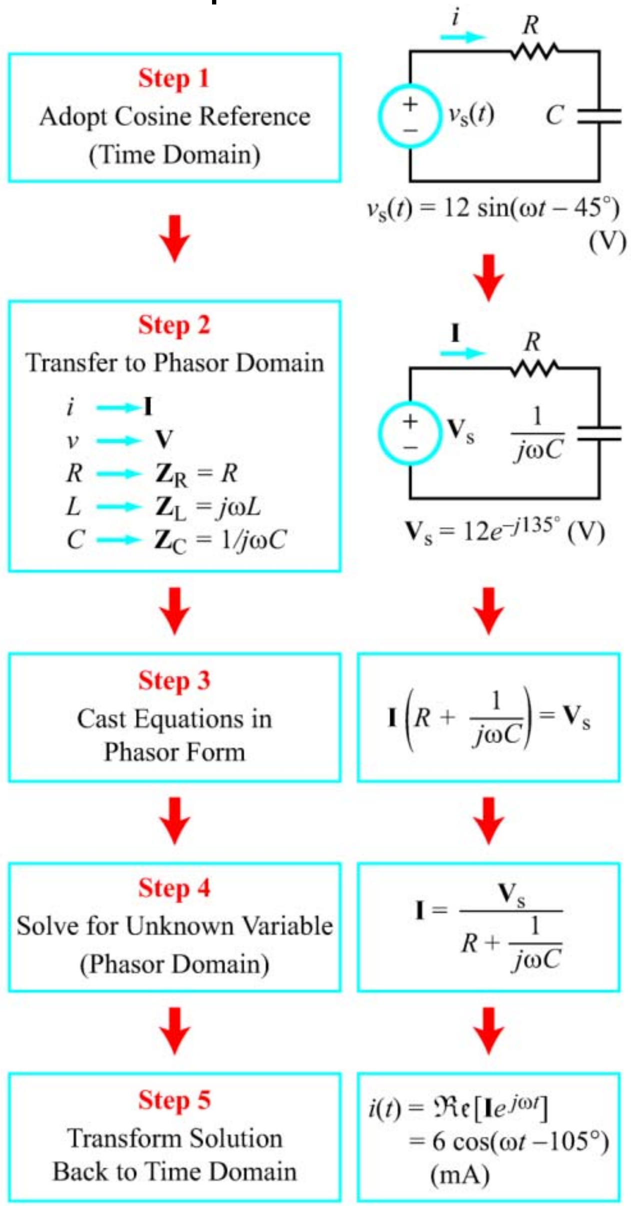
Z can be real, complex, or purely imaginary – it has units of Ohms ….and it is not a phasor…it multiplies the current phasor to give the voltage phasor. The imaginary part of the impedance (if it exists) is called the reactance.

Circuit Element	**Impedance**	**Reactance**
Resistor	R	none
Inductor	$j\omega L$	ωL
Capacitor	$j(-1/\omega C)$	$-1/\omega C$

Table 7-4: Summary of v–i properties for R, L, and C.

Property	R	L	C
v–i	$v = Ri$	$v = L\,\dfrac{di}{dt}$	$i = C\,\dfrac{dv}{dt}$
V–I	$\mathbf{V} = R\mathbf{I}$	$\mathbf{V} = j\omega L\mathbf{I}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$
Z	R	$j\omega L$	$\dfrac{1}{j\omega C}$
dc equivalent	R	short circuit	open circuit
High-frequency equivalent	R	open circuit	short circuit
Frequency response			

Here's the **5 –step procedure** for a phasor analysis of a simple RC circuit:



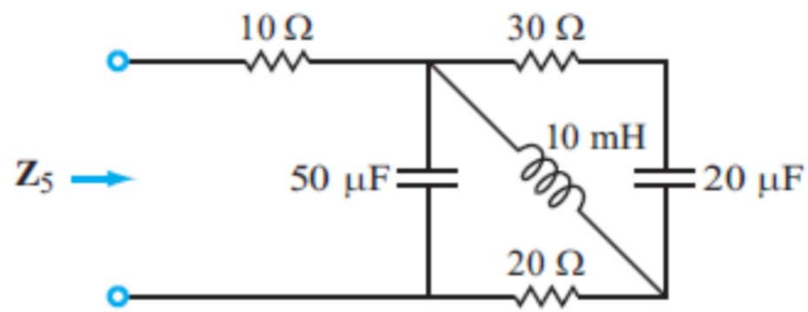
The Impedance Model

Many times it is possible to solve a circuit in the AC steady-state regime by simply replacing all of the circuit elements (capacitors, inductors, resistors) with their appropriate impedances – evaluated at the frequency specified for the problem.

Then – combine the impedances using appropriate series and parallel combinations (bearing in mind that these are now complex numbers in general).

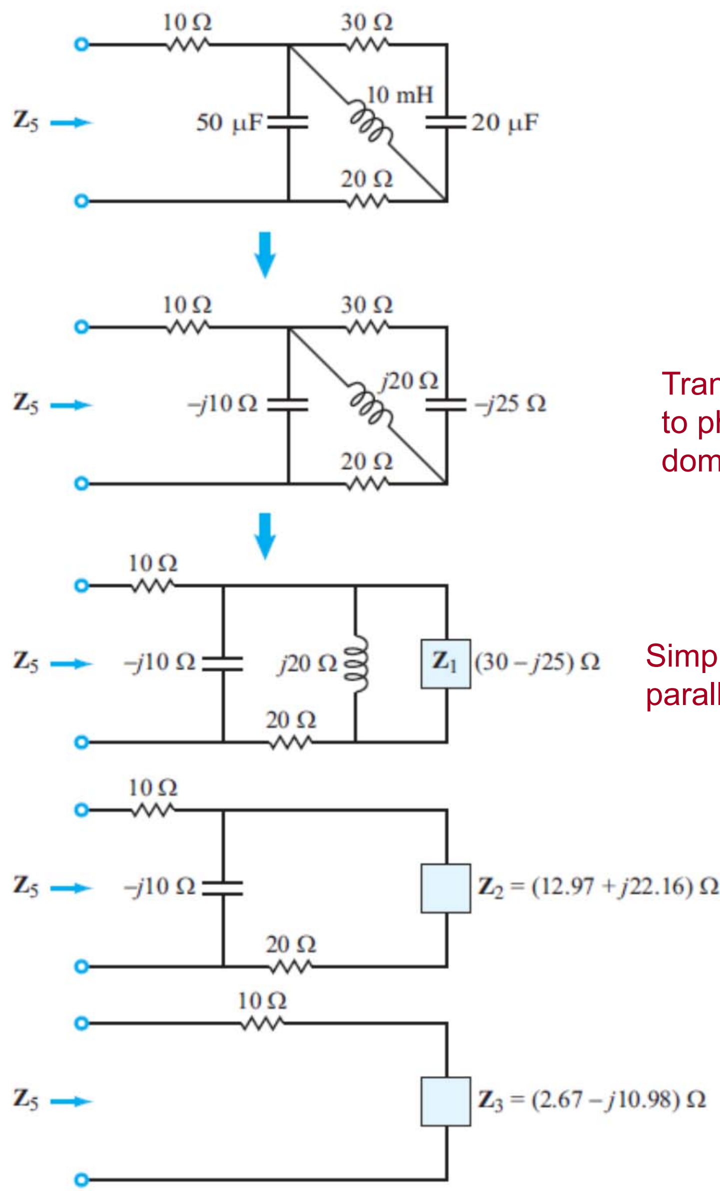
Finally, evaluate the current or voltage that is of interest using the equivalent impedance and the phasor representation of the source current or voltage.

Circuit Element	Impedance	Reactance
Resistor	R	none
Inductor	$j\omega L$	ωL
Capacitor	$j(-1/\omega C)$	$-1/\omega C$



Calculate Z_5 at ω = 2000 rad/s

We can use the impedance model to do this – as we saw in class last week



Transform to phasor domain

$$\mathbf{Z}_{C_1} = \frac{-j}{\omega C_1} = \frac{-j}{2000 \times 50 \times 10^{-6}} = -j10\ \Omega$$

$$\mathbf{Z}_{C_2} = \frac{-j}{\omega C_2} = \frac{-j}{2000 \times 20 \times 10^{-6}} = -j25\ \Omega$$

$$\mathbf{Z}_{\mathrm{L}} = j\omega L = j2000 \times 10 \times 10^{-3} = j20\ \Omega$$

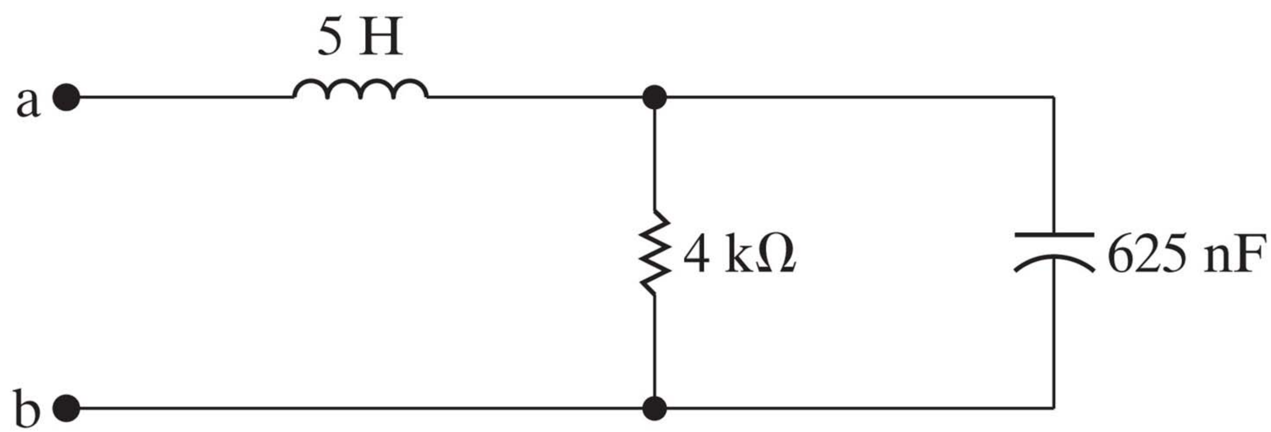
Simplify with series and parallel combinations

$$\begin{aligned}\mathbf{Z}_2 &= \mathbf{Z}_1 \parallel (j20) \\ &= \frac{(30 - j25)(j20)}{30 - j25 + j20} \\ &= \frac{500 + j600}{30 - j5} \cdot \frac{(30 + j5)}{(30 + j5)} \\ &= \frac{12000 + j20500}{925} = (12.97 + j22.16)\ \Omega\end{aligned}$$

$$\begin{aligned}\mathbf{Z}_3 &= (20 + \mathbf{Z}_2) \parallel (-j10) \\ &= \frac{(32.97 + j22.16)(-j10)}{32.97 + j22.16 - j10} \\ &= \frac{221.6 - j329.7}{32.97 + j12.16} \cdot \frac{(32.97 - j12.16)}{(32.97 - j12.16)} \\ &= (2.67 - j10.98)\ \Omega\end{aligned}$$

$$\mathbf{Z}_5 = 10 + \mathbf{Z}_3 = (12.67 - j10.98)\ \Omega.$$

Here's another phasor problem ---- Find the frequency (in radians per sec) at which the impedance Z_{ab} is purely resistive. Find the value of this impedance. (This problem was also on the ELEN 50 Final exam last quarter)



What is the general strategy to use to solve this problem?

[a] $$Z_{\mathrm{ab}} = j5\omega + \frac{(4000)(10^9/j\omega 625)}{4000 + (10^9/j625\omega)}$$

$$= j5\omega + \frac{4 \times 10^{12}}{25 \times 10^5 j\omega + 10^9}$$

$$= j5\omega + \frac{4 \times 10^7}{10^4 + j25\omega}$$

$$= j5\omega + \frac{4 \times 10^{11}}{10^8 + 625\omega^2} - j\frac{100 \times 10^7\omega}{10^8 + 625\omega^2}$$

$$\therefore \quad 5 = \frac{10^9}{10^8 + 625\omega^2}$$

In order to make the imaginary part of this expression zero

$$5 \times 10^8 + 3125\omega^2 = 10^9$$

$$\omega = 4 \times 10^2 = 400\,\mathrm{rad/s}$$

[b] $$Z_{\mathrm{ab}}(400) = j2000 + \frac{(4000)(-j4000)}{4000 - j4000} = 2\,\mathrm{k\Omega}$$

Notice …the impedance at 400 rad/s is entirely real (resistive) …as required

AC Power

Average and Reactive Power

$$p = P + P\cos 2\omega t - Q\sin 2\omega t$$

$$where:$$

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$

Average Power or
Real Power
(measured in Watts)

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$

Reactive Power
(measured in VAR –
volt amps reactive)

Power factor is the cosine of the phase angle between current and voltage

The final exam will last two hours and will have ~10 problems. The exam will be open book, open notes, with access to the design center for your laptop/tablet/phone if you need it.

Considering the expected learning outcomes for the course you can almost guess the types of questions on the exam.

Expected learning outcomes:

1. Formulate Kirchoff current and voltage law equations in a systematic manner.
2. Formulate and solve node voltage and loop current equations
3. Compute Thevenin equivalent circuits and apply them in circuit analysis
4. Analyze circuits using operational amplifiers
5. Use phasor techniques to compute sinusoidal steady state solutions in linear circuits.
6. Design and test circuits that meet a given set of specifications.

Good Luck on the Final!