ELEN 50 Class 8 – Node Voltage method

S. Hudgens

Introduction to the Node Voltage Method

The node voltage method of circuit analysis is a **systematic method** to solve for the voltages at nodes in a circuit through the repeated application of the Kirchhoff Current Law (KCL) and solution of the resulting simultaneous linear equations. Notice – we won't use KVL at all …we'll just use KCL at every essential node in the circuit.

It is the "workhorse" circuit analysis method and probably the one technique you will use more than any other in your professional life (assuming you ever have to solve circuits in your professional life!)

Of course, you should also use all of the equivalent circuit tricks we've discussed to simplify a circuit before you start on a node voltage analysis. Sometimes, as you know, it is possible to simplify a circuit sufficiently to be able to solve it using only Ohm's Law.

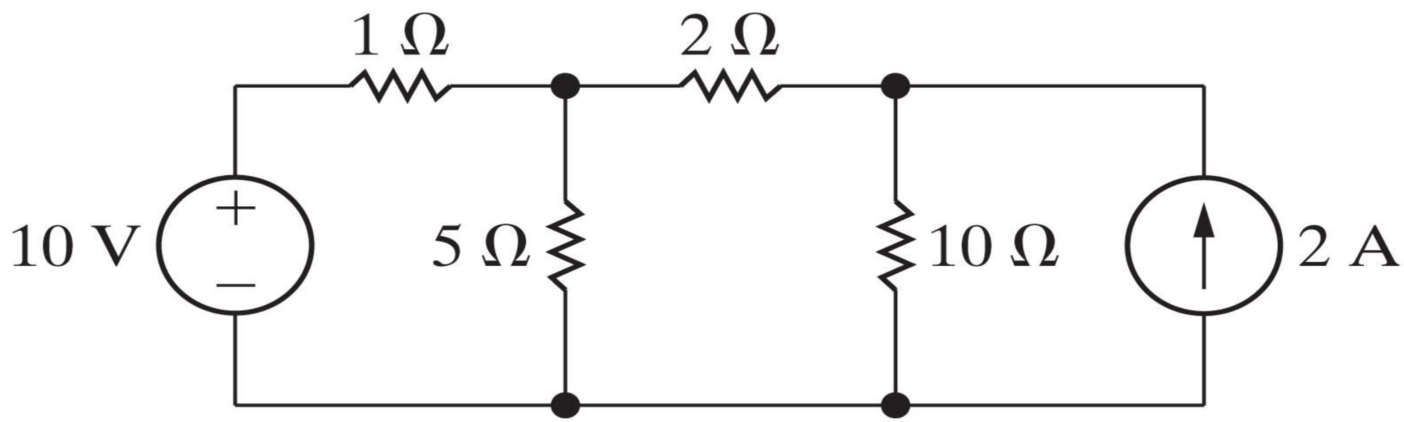
If circuit simplification isn't enough …….

The node voltage method of circuit analysis determines the voltages present on all of the essential nodes in a circuit through systematic application of the Kirchhoff Current Law. You should be able to convince yourself that knowing these voltages will completely determine the state of the circuit ….since with these voltages and the values of all the resistors in the circuit, you should be able to calculate all of the current values through all of the branches of the circuit.

Obviously, you could have, instead, chosen to determine all of the loop currents flowing in the independent loops in a circuit by applying the Kirchhoff Voltage Law….and this would have equally well completely determined the state of the circuit. From the loop currents you could calculate all of the voltages on all of the essential nodes. This complementary analytical technique is called the mesh current method and we'll discuss it in a later class.

Introduction to the Node Voltage Method

• The node-voltage method allows us to solve the circuit with n_e-1 equations where n_e is the number of essential nodes – remember, an <u>essential node is a node where three or more circuit elements join.</u>

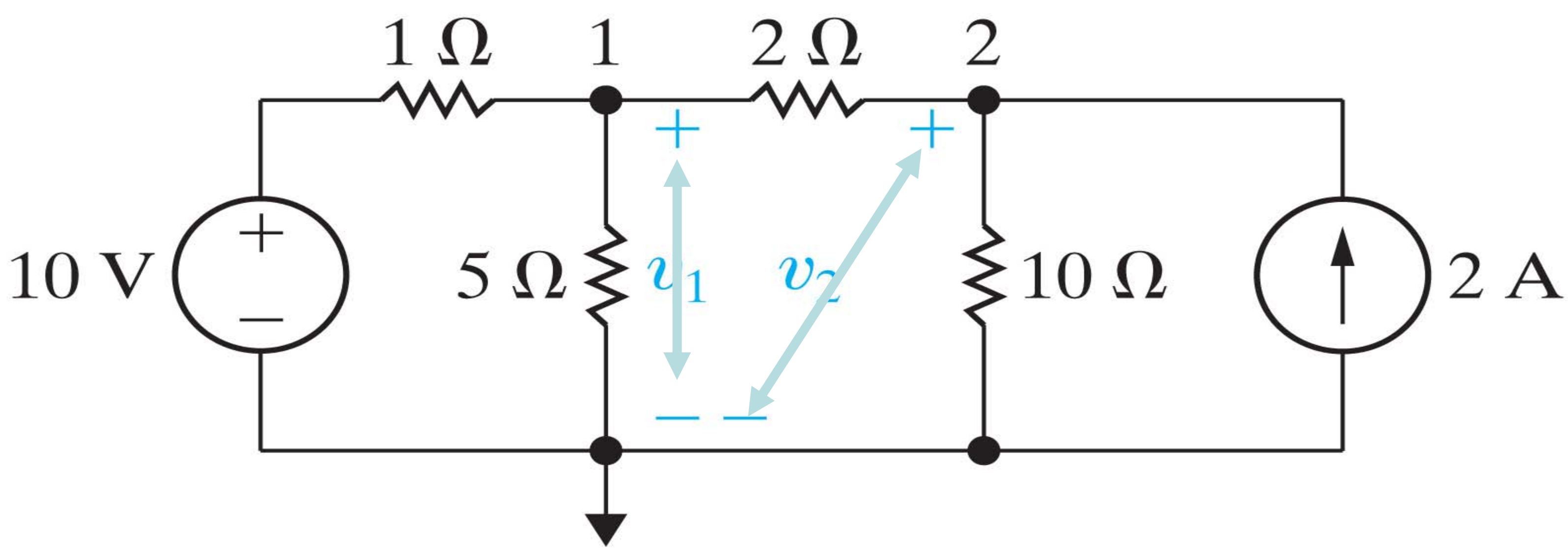


- How many essential nodes are present in this circuit?
- Note: you should also always be sure before you start that the circuit is valid …and you should look for possibilities to simplify the circuit through the use of equivalent circuits, series and parallel combinations, etc.

Why do you suppose the node voltage method results in n_e-1 equations – even though there are n_e essential nodes present?

- The next step is to number the essential nodes and pick one of them as the reference node (shown here with the ground symbol). Usually, you'll pick the node that connects the most branches as the reference node – but the choice is not critical and it's entirely up to you.

- Next, we define node voltages – the voltage rise from the reference node to the essential nodes in the circuit ….shown here as v1 and v2. These voltages are the unknowns in the problem and, of course, you'll need to have the same number of equations as you have unknowns to be able to solve the problem.



- Now we apply KCL for the current leaving each branch connected to an essential node as a function of the node voltages in the circuit (using Ohm's Law) …and we're done!

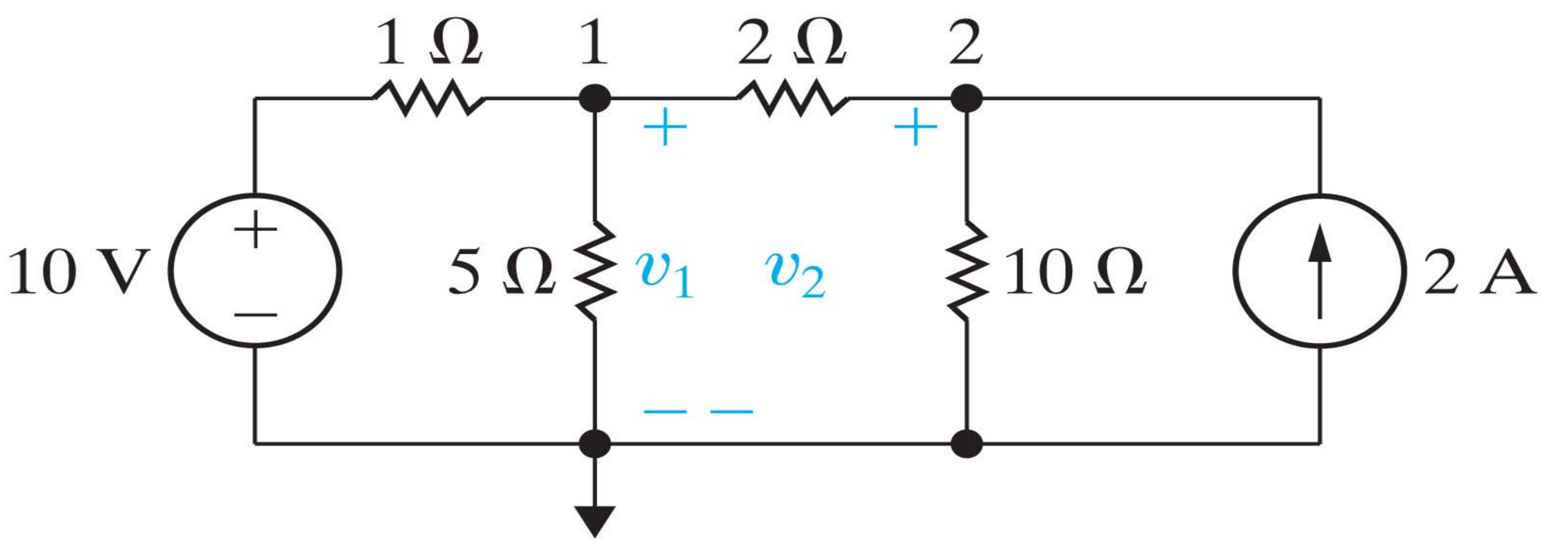
- For node 1 KCL would give:

$$(v_1-10)/1 + v_1/5 + (v_1-v_2)/2 = 0$$

- For node 2 KCL would give:

$$-(v_1-v_2)/2 + v_2/10 - 2 = 0$$

- We've now got two equations in two unknowns…..the unknowns are the node voltages….and knowing these node voltages completely solves the circuit.

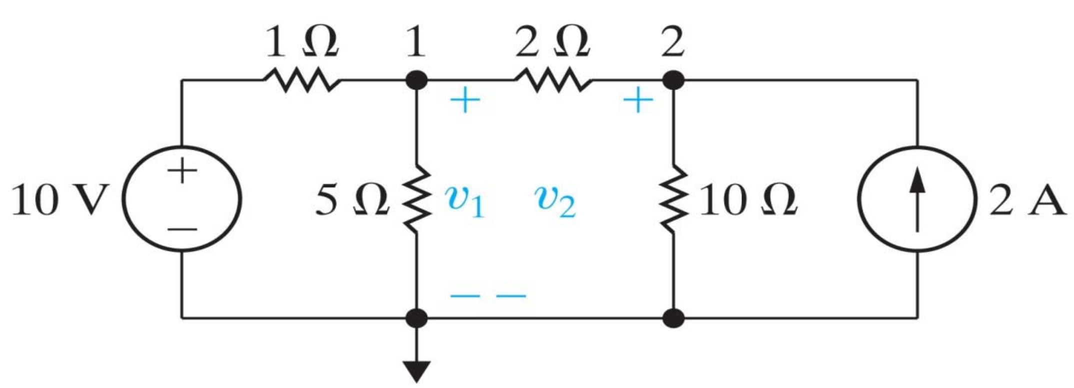


We can solve these two equations by substitution

v_1 = 100/11 = 9.09 V

v_2 = 120/11 = 10.91 V

- Now we could calculate all the branch currents from the node voltages we just obtained … if we wanted to.
- Notice if we had just plowed ahead with this circuit, randomly applying KCL and KVL, we would have seen that there are 2 nodes and 2 loops (we already know the loop current in the loop on the right). We would have gotten 2 equations from KCL for the 3 nodes, and 2 from applying KVL to 2 of the loops or meshes.
- Two equations in two unknowns is much easier to solve than 4 equations in 4 unknowns!



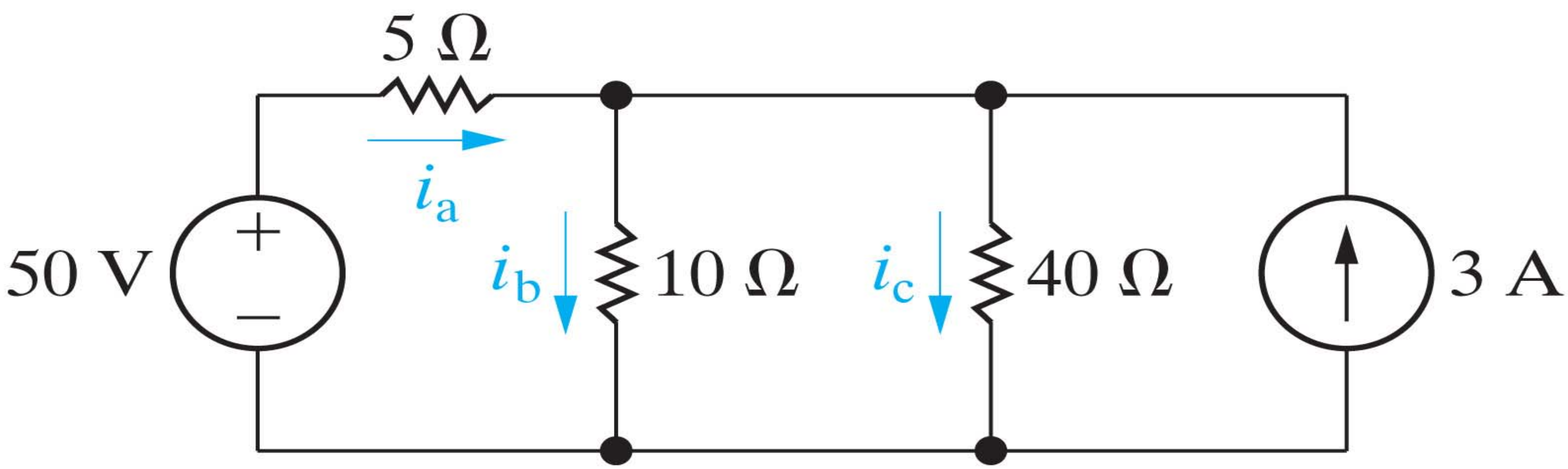
3 Steps for the Node Voltage Method

- Identify all extraordinary nodes, set one as the reference (ground) node, and assign node voltages (v1, v2, v3, etc.) to the $n_{ex} - 1$ remaining nodes.

- At each of the n_{ex}-1 nodes, write the KCL equation requiring the sum of all currents leaving the node to be zero.

- Solve the n_{ex}-1 independent simultaneous equations to determine the unknown node voltages.

• Here's another example …use the node voltage method to calculate the branch currents i_a, i_b, and i_c.

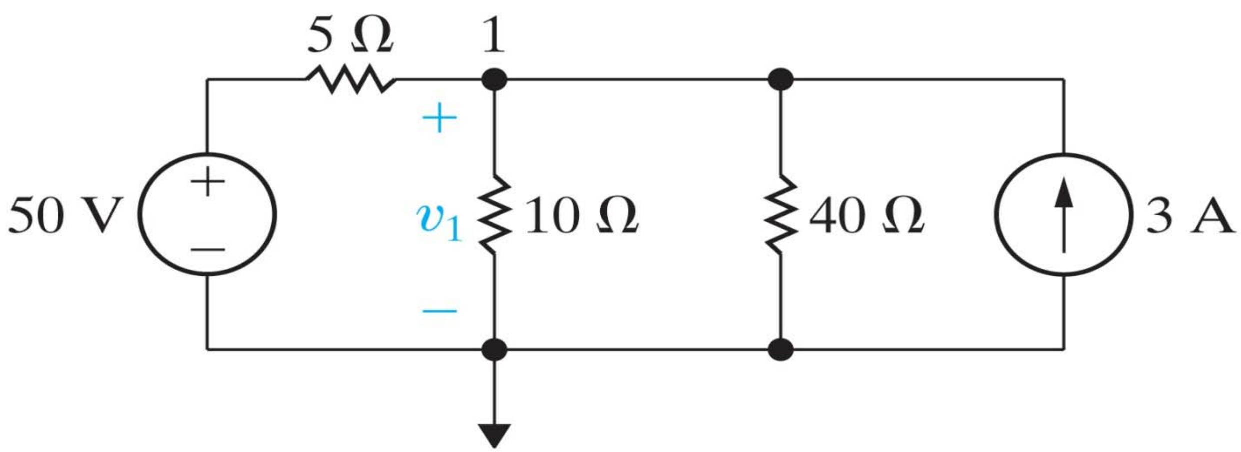


• What's the first step?

• Notice …you could do a source transform on the 50V source – then the circuit would be two current sources in parallel with three resistors – which could give you i_b and i_c directly (it's a current divider) …but how about i_a??? The values for i_b and i_c would be correct but the source transform would have eliminated i_a.

Doing it with the Node Voltage Method

- Choose a reference node and label the other essential nodes
- Well…there is only one other essential node so we can solve this circuit with a single equation!

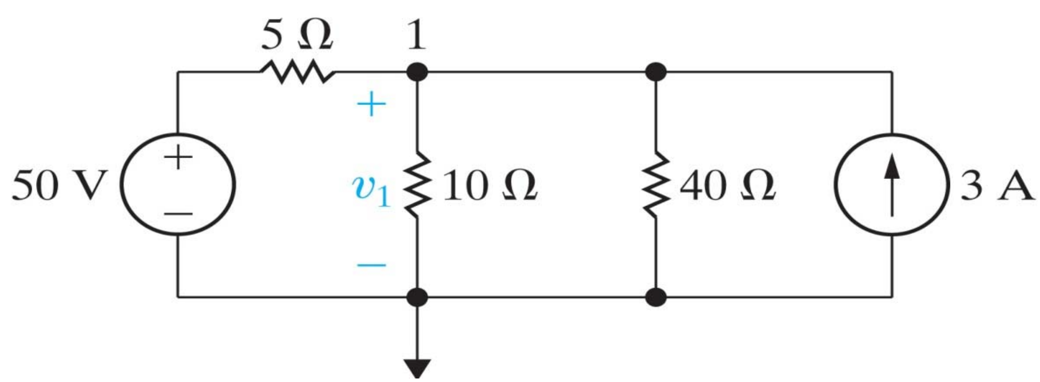


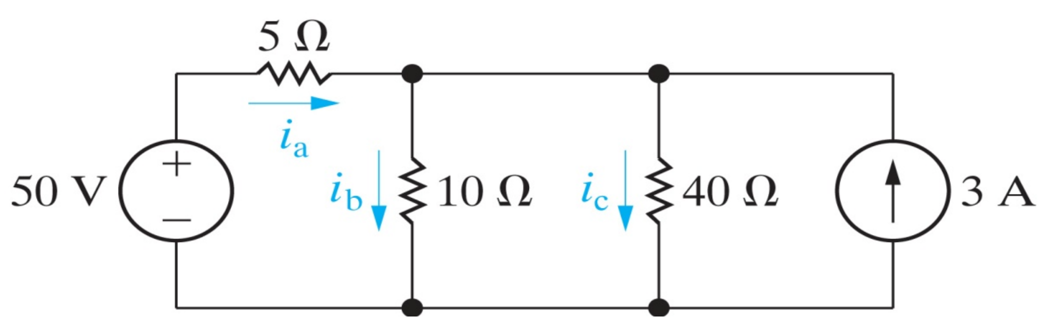
- Now we write the KCL for currents leaving the single essential node:

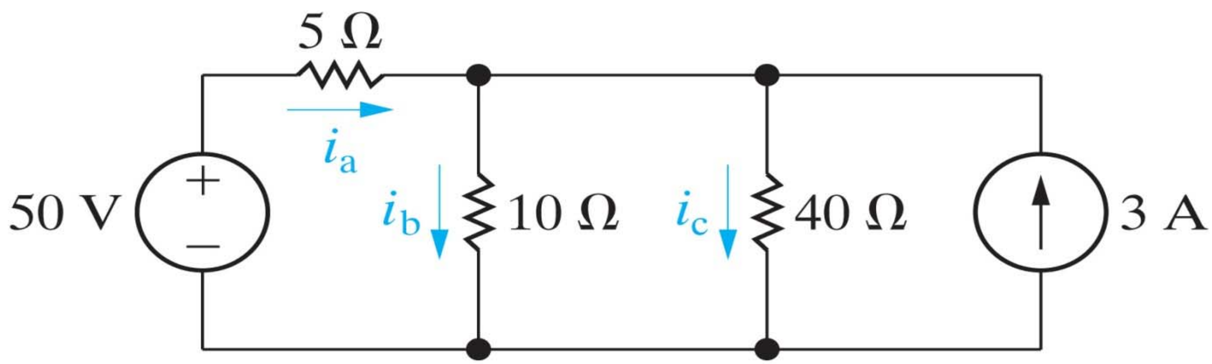
$(v1 - 50)/5 + v1/10 + v1/40 - 3 = 0$

$\rightarrow$ v1 = 40V

so 1a = (50-40)/5 = 2A, ib = 40/10 = 4A, and ic = 40/40 = 1A





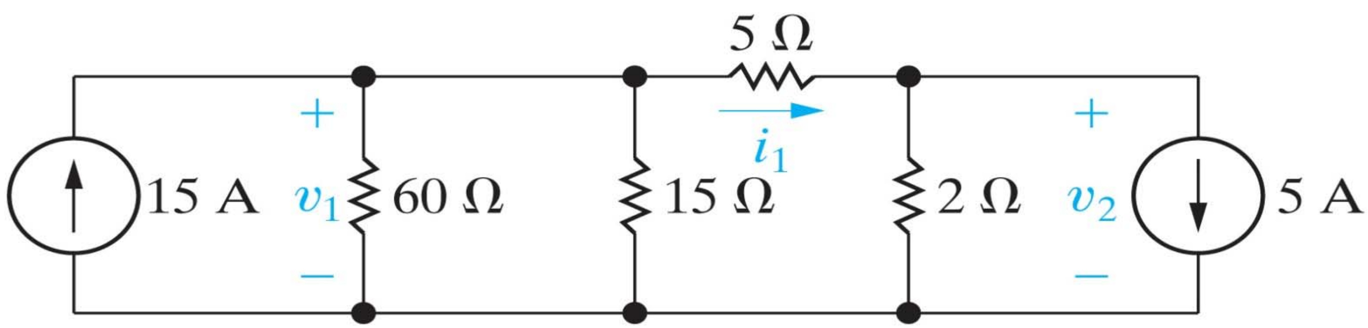


If we had done a source transform first on the voltage source …it would have transformed into a current source of 10A in parallel with a 5Ω resistor. We would, therefore, have had a total current of 13A divided in a current divider among a 40Ω, a 10Ω and a 5Ω resistor. The currents through the 40Ω and 10Ω resistors are ic and ib. The combined conductance of these three resistors is given by:

$$\frac{1}{5}+\frac{1}{10}+\frac{1}{40}$$

So the current through the 40Ω is the ratio of its conductance to the combined conductance times the total current, 13A = 1A as we saw before. Similarly for the 10Ω resistor we get 4A …These are the right values, but we have no way, in principle, of determining the current through the 5Ω series resistor <u>because we would have eliminated it with the source transform!</u>

- One more ….find v1, v2, and i1



- What’s the first step?

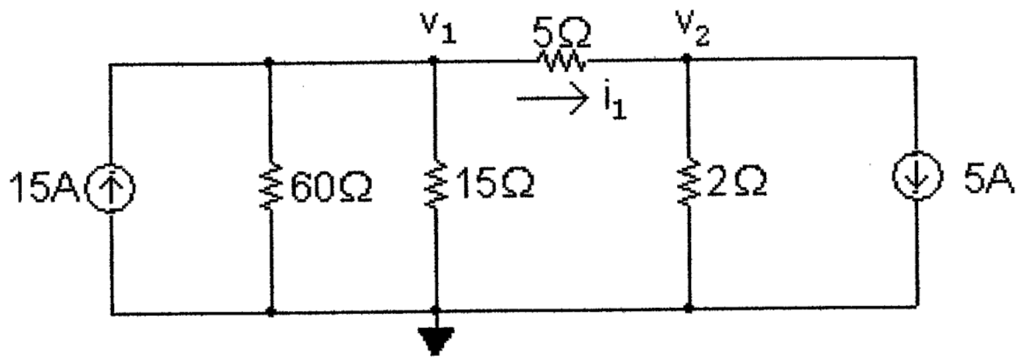
Remember!

- Identify all extraordinary nodes, set one as the reference (ground) node, and assign node voltages (v1, v2, v3, etc.) to the $n_{ex} - 1$ remaining nodes.

- At each of the n_{ex}-1 nodes, write the KCL equation requiring the sum of all currents leaving the node to be zero.

- Solve the n_{ex}-1 independent simultaneous equations to determine the unknown node voltages.

Redraw the circuit, labeling the reference node and the two node voltages:



- apply KCL on the currents from the two nodes:

The two node voltage equations are

$$
\begin{aligned}
-15 + \frac{v_1}{60} + \frac{v_1}{15} + \frac{v_1 - v_2}{5} &= 0 \\
5 + \frac{v_2}{2} + \frac{v_2 - v_1}{5} &= 0
\end{aligned}
$$

- turn the crank

Place these equations in standard form:

$$\begin{aligned} v_1\left(\frac{1}{60}+\frac{1}{15}+\frac{1}{5}\right) \quad &+ \quad v_2\left(-\frac{1}{5}\right) &= \quad 15 \\ v_1\left(-\frac{1}{5}\right) \quad &+ \quad v_2\left(\frac{1}{2}+\frac{1}{5}\right) &= \quad -5 \end{aligned}$$

Solving, $v_1 = 60$ V and $v_2 = 10$ V;
Therefore, $i_1 = (v_1 - v_2)/5 = 10$ A

You can do this in MATLAB using the following:

After getting rid of the fractions …we can write the two equations as a single matrix equation:

C * V = S

$$\begin{bmatrix} 17 & -12 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 900 \\ -50 \end{bmatrix}$$

```
C=[17,-12;-2,7]

C =

   17   -12
   -2     7


S=[900;-50]

S =

  900
  -50
```

So I can get the V vector by multiplying the inverse of the C matrix by the S vector

```
V=inv(C)*S

V =

   60.0000
   10.0000
```

You could, obviously, also solve something as simple as two equations in two unknowns by substitution…but this is cumbersome for a 3X3 or larger matrix and it’s very convenient to use MATLAB.

Notice it’s also possible to solve a system of linear equations in MATLAB using the backslash operation. We talked about this before.

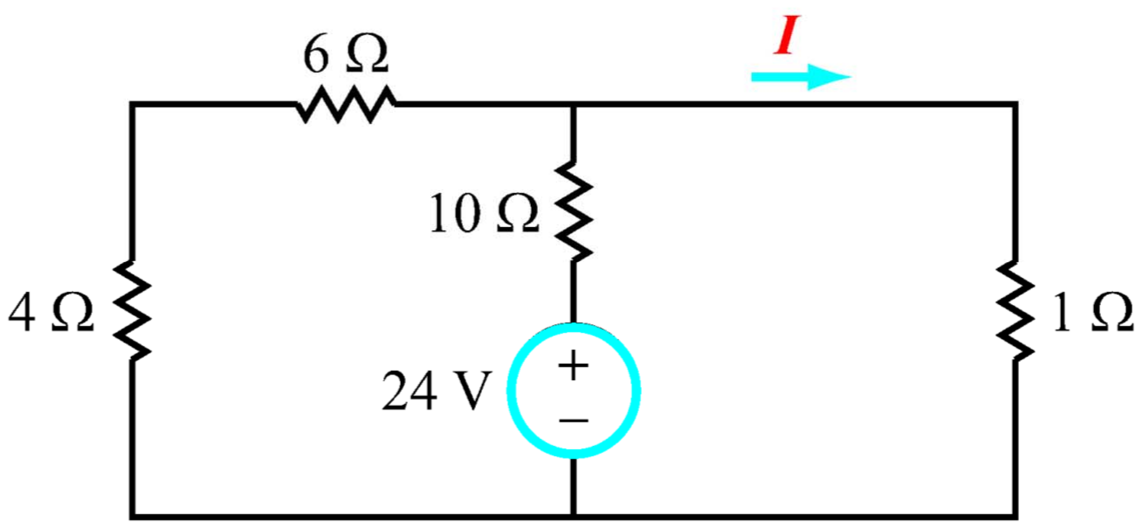
C * V = S

So we can solve for the vector V using the statement in MATLAB:

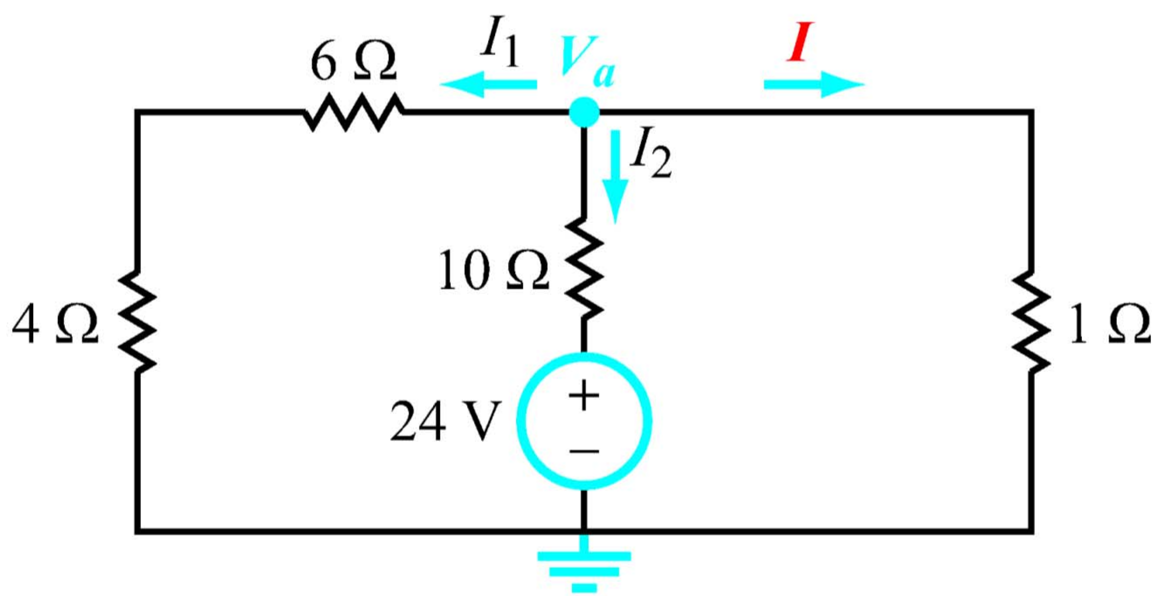
V = C\S

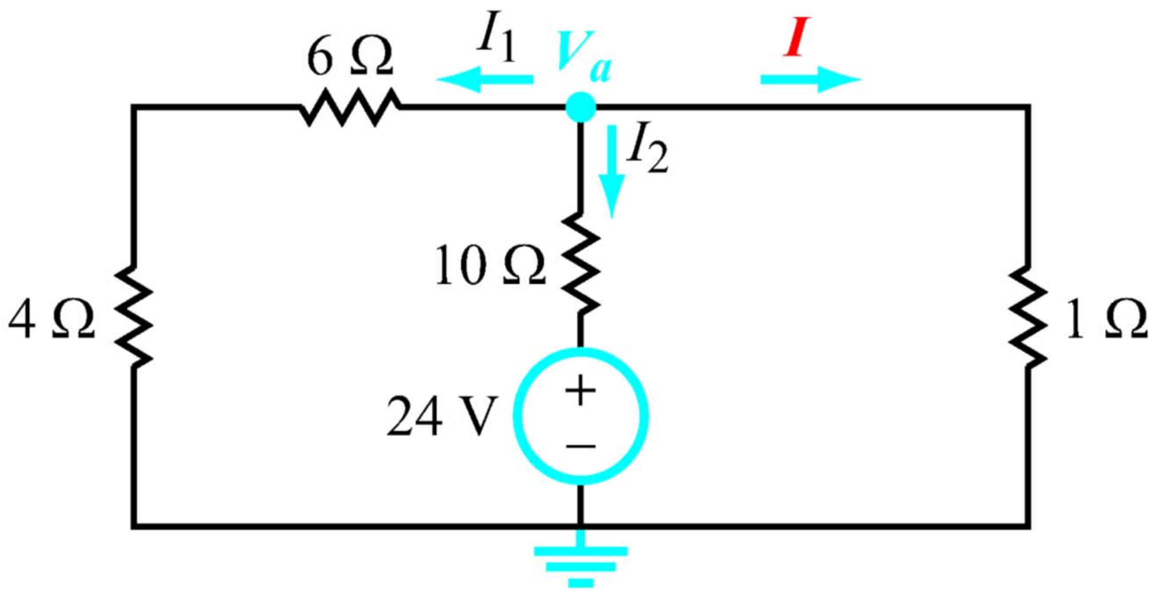
```
V=C\S

V =

    60
    10
```

Here's another circuit. We want to determine the current, I using the Node Voltage Method. How many essential nodes are there and what is the first step in the Node Voltage Method?



There are two essential nodes …so we chose a reference node and label the unknown voltage (and branch currents) at the remaining essential node.

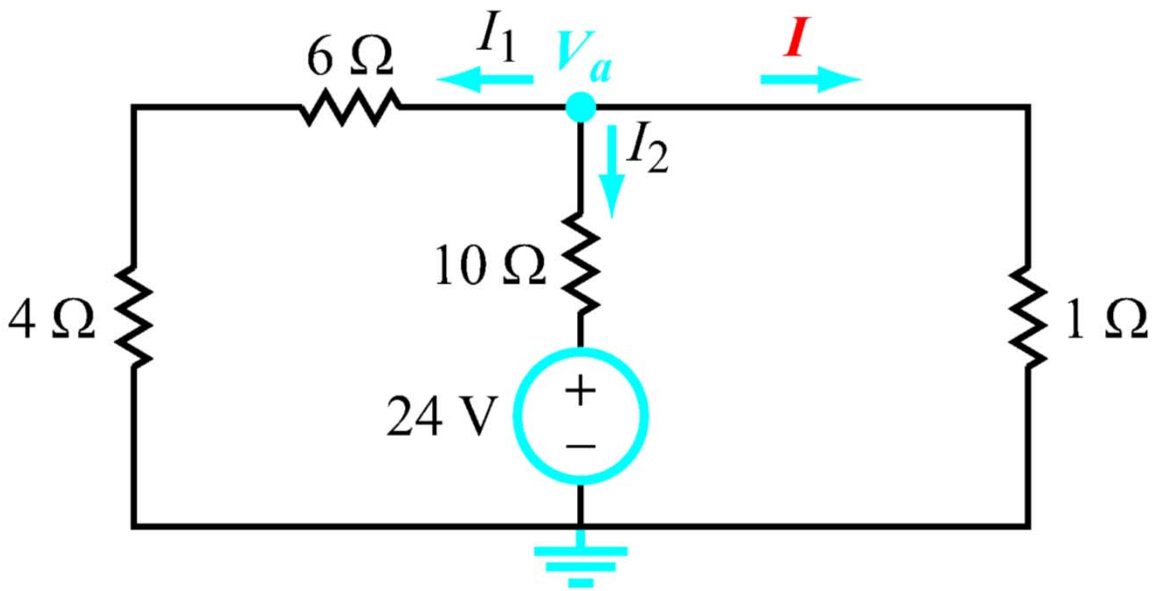




$$I_1 + I_2 + I = 0$$

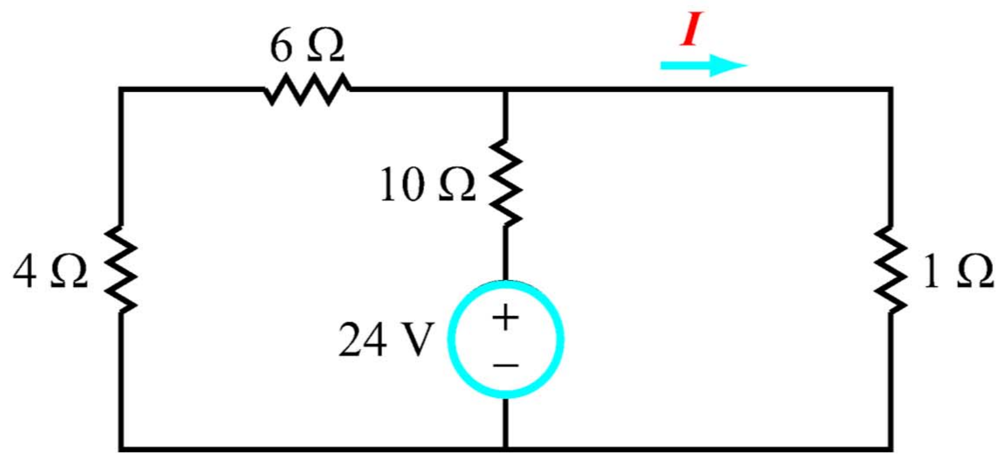
$$I_1 = \frac{V_a}{10}, \quad I_2 = \frac{(V_a - 24)}{10}, \quad I = \frac{V_a}{1}$$

$$so.... \quad \frac{V_a}{10} + \frac{(V_a - 24)}{10} + \frac{V_a}{1} = 0$$

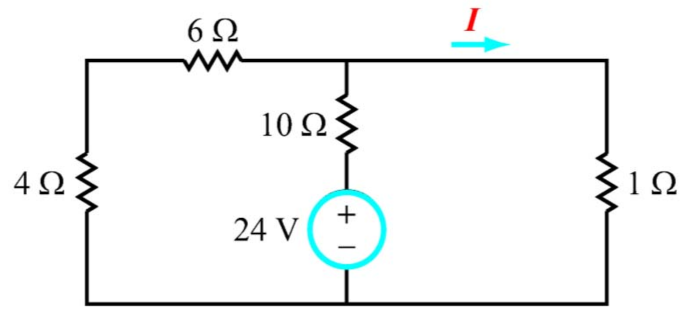


$$V_a\left(\frac{1}{10}+\frac{1}{10}+1\right)=\frac{24}{10}$$

$$V_a = 2V \quad I=\frac{V_a}{1}=2A$$



We wanted to use this circuit to demonstrate a node voltage solution …but, we could have also solved this circuit more simply by a source transformation. Can you see how to do it?



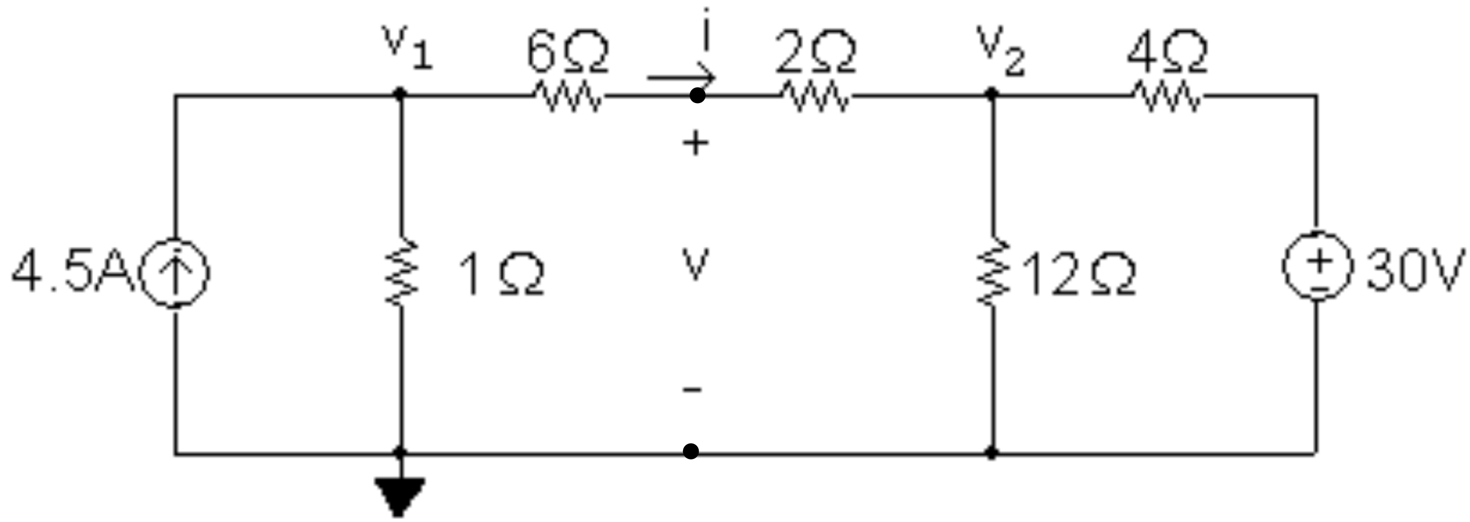
First we can combine the 4Ω and 6Ω resistor in series – then we can transform the 24V source and the 10Ω series resistor into a 2.4A current source in parallel with a 10Ω resistor. None of these equivalent circuits have affected the part of the circuit where the current, I, is flowing, so we should be free to do it.

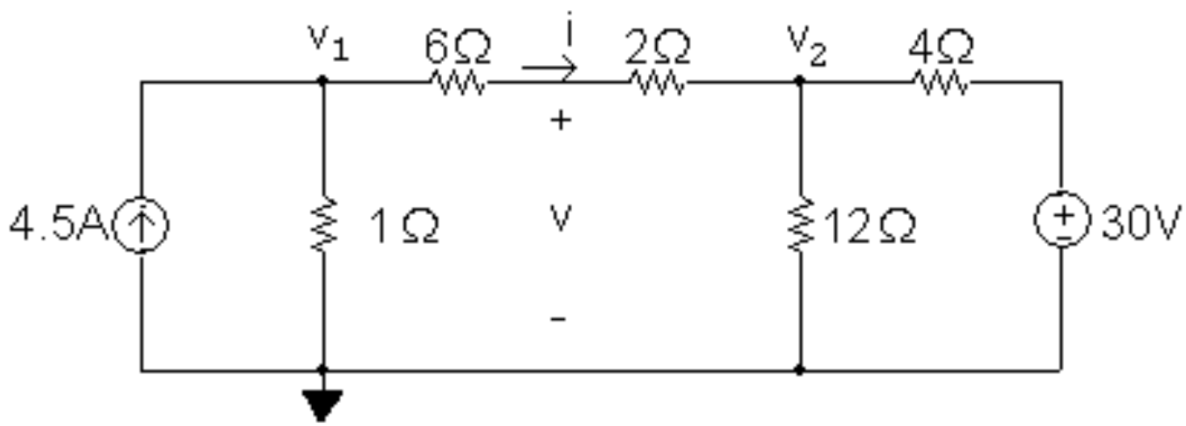
Now we can combine the two 10Ω resistors in parallel so we end up with a current divider …one 5Ω resistor and one 1Ω resistor dividing up a 2.4A current source. The current division is proportional to the conductance (inverse resistance) of the resistors …so 5/6 of the current flows through the 1Ω resistor and 1/6 flows through the 5Ω resistor. 5/6 of 2.4A = 2A …the answer we got from the node voltage analysis. (not surprisingly!)

Or….as we saw earlier, we could do one more source transform …causing the 2.4A current source and 5 Ohm resistor in parallel to become a 12V source in series with a 5 Ohm resistor …and the 1 Ohm resistor where we are trying to calculate the current. The current flowing through this resistor is the current flowing through the 6 Ohm series combination

and, from Ohm's Law, this is just 2 A ….the answer we got earlier.

Find v in this circuit





The two node voltage equations are:

$$-4.5 + \frac{v_1}{1} + \frac{v_1 - v_2}{6+2} = 0$$

$$\frac{v_2}{12} + \frac{v_2 - v_1}{6+2} + \frac{v_2 - 30}{4} = 0$$

Place these equations in standard form:

$$v_1\left(1 + \frac{1}{8}\right) + v_2\left(-\frac{1}{8}\right) = 4.5$$

$$v_1\left(-\frac{1}{8}\right) + v_2\left(\frac{1}{12} + \frac{1}{8} + \frac{1}{4}\right) = 7.5$$

Solving, $v_1 = 6$ V $\quad v_2 = 18$ V

notice

So we can calculate i from Ohms Law i = (v1 – v2)/8Ω = -1.5A
...and, finally, v is i x 2Ω + v2 = 15V.

There are some circuits to solve with the node voltage method on the next problem set.

In the next class, we'll consider what happens when we do a node voltage analysis of a circuit with dependent sources (hint …not very much). And we'll also talk about supernodes and quasi-supernodes!