ELEN 50 Class 24 –More Phasor Analysis Examples

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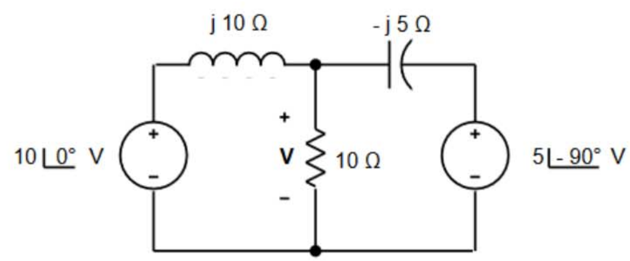
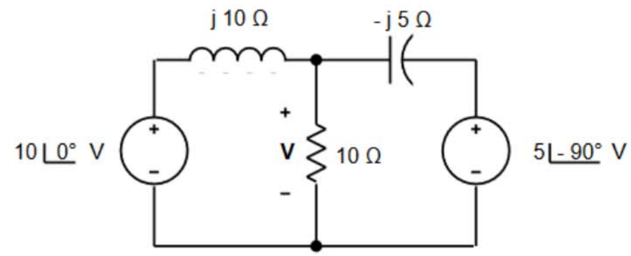
Use the node-voltage method to find the voltage V across the 10-Ω resistor, expressed as a phasor.



Notice: the circuit has already been transformed into the phasor domain. What is the next step in a node voltage solution. What would you have done if this were a simple DC resistive circuit?

Use the node-voltage method to find the voltage V across the 10-Ω resistor, expressed as a phasor.



Writing KCL at the upper node

$$\frac{10\,\angle 0^\circ - \mathbf{V}}{j\,10} + \frac{5\,\angle{-90^\circ} - \mathbf{V}}{-j\,5} = \frac{\mathbf{V}}{10}$$

Which simplifies to:

$$\frac{10-\mathbf{V}}{j10} + \frac{-j5-\mathbf{V}}{-j5} = \frac{\mathbf{V}}{10}$$

Because 10 <0^{O} is 10 and 5 < -90^{o} is –j5

Putting everything over j10

$$10 - \mathbf{V} + j10 + 2\mathbf{V} = j\mathbf{V}$$

Solving for **V**

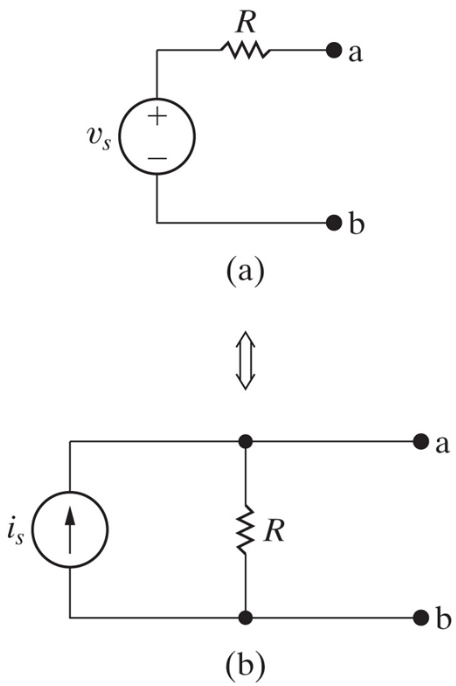
$$\mathbf{V} = -10\frac{(1+j)}{(1-j)}$$

Multiplying the fraction, top and bottom, by the complex conjugate of the denominator:

$$\mathbf{V} = \frac{-10}{2}2j = -j10 = 10\angle -90^\circ$$

Could we have solved this circuit using source transforms?

For DC resistive circuits we learned :



These two circuits are equivalent, if:

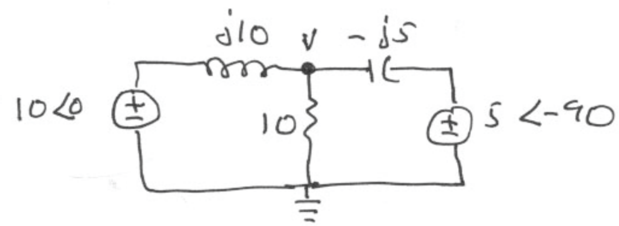
$$\boxed{i_s = v_s/R}$$

You can show this by attaching a load resistor to a and b and calculating the current flowing

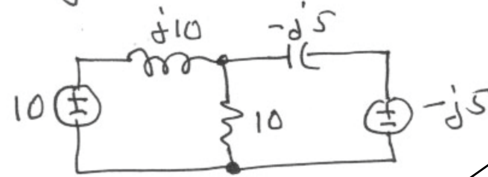
For AC steady-state circuits, the formula has to be changed to: $\mathbf{I} = \dfrac{\mathbf{v}_s}{\mathbf{Z}}$

where $\mathbf{I}$, $\mathbf{Z}$ and $\mathbf{v}_s$ are phasors or complex impedances
…..otherwise everything else is the same.

source transform solution



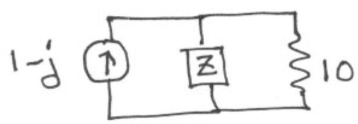
Solution by source transform



Source transform both voltage sources

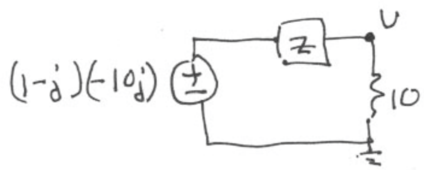
$$\frac{-j5}{-j5} = 1$$

combine current sources $\frac{10}{j10} + 1 = 1 - j$



$$Z = j10 \parallel -j5 = \frac{(j10)(-j5)}{j5}$$

$$= -10j$$

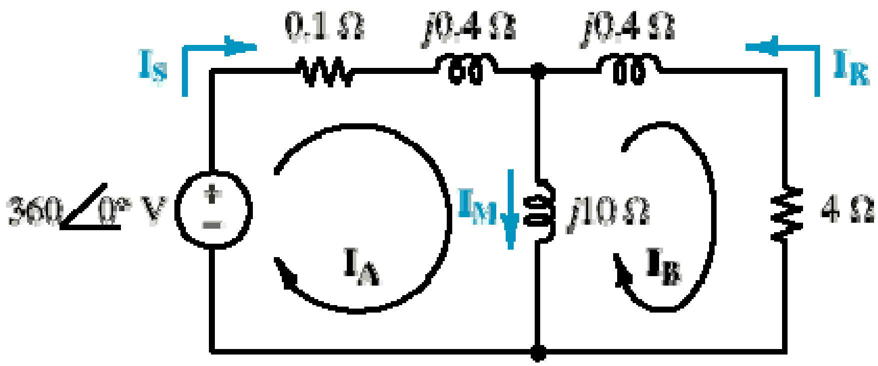


Source transform back to a voltage source ..and notice that v is part of a voltage divider

$$\text{so } V = \frac{(1-j)(-10j)\,10}{10-10j} = \frac{(-10j)(10)(1-j)}{(10)(1-j)}$$

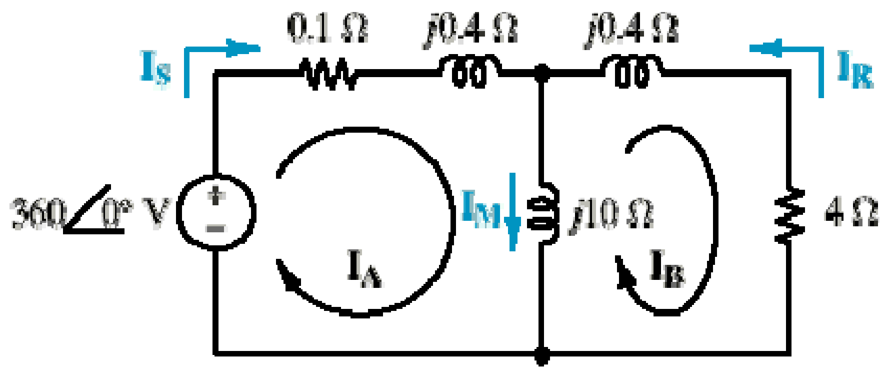
$$V = -10j$$

How about mesh current by inspection solution. Can we use this method in the phasor domain?



We want to solve for the branch currents, I_S, I_B, and I_M in this circuit – it has already been transformed into the phasor domain as you can see.

We'll write the mesh current equations for the phasor mesh currents, $\mathbf{I}_A$ and $\mathbf{I}_B$

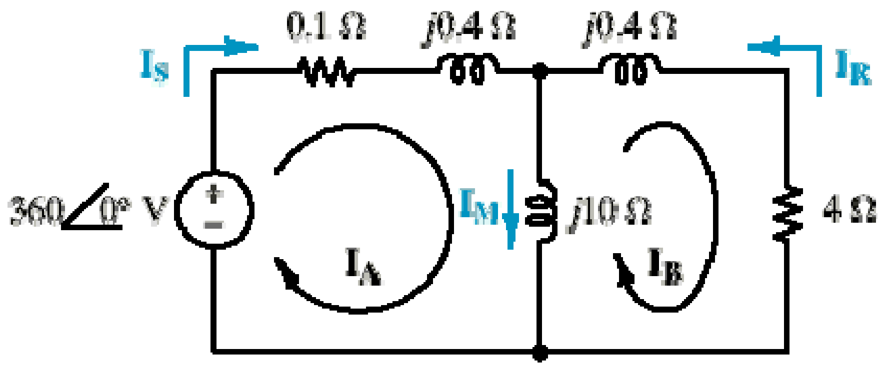


$$\text{Mesh A}: -360\angle 0° + [0.1 + j0.4]I_A + j10(I_A - I_B) = 0$$
$$\text{Mesh B}: j10(I_B - I_A) + [4 + j0.4]I_B = 0$$

In standard form:

$$(0.1 + j10.4)I_A - (j10)I_B = 360\angle 0°$$
$$-(j10)I_A + (4 + j10.4)I_B = 0$$

If we had written down the matrix using a “by inspection” technique – what would we have gotten?



MATLAB solution gives:

$$I_A = 79.0 - j48.2A$$
$$I_B = 81.7 - j14.9A$$

So ….. $\mathbf{I}_m = \mathbf{I}_A - \mathbf{I}_B =$ - 2.7 - j33.3 A

$\mathbf{I}_S = \mathbf{I}_A$

$\mathbf{I}_R = -\mathbf{I}_B$

KCL requires that $\mathbf{I}_S+\mathbf{I}_R=\mathbf{I}_M$ and this is satisfied by the solution

Just to review

In the last two classes we discussed the technique for analyzing sinusoidal steady state circuits called **phasor analysis.**

Phasors are a way of representing the amplitude and phase of a sinusoidal signal (current or voltage) that permits solution of circuits that contain inductors and capacitors – while completely avoiding having to solve differential equations.

We define the phasor as a complex number containing amplitude and phase information:

$$\mathbf{V} = V_m e^{j\phi}$$

In other words, if a voltage in the time domain is :

$$v = V_m \cos(\omega t + \phi)$$

transforming to the phasor domain gives us:

$$\mathbf{V} = V_m e^{j\phi}$$

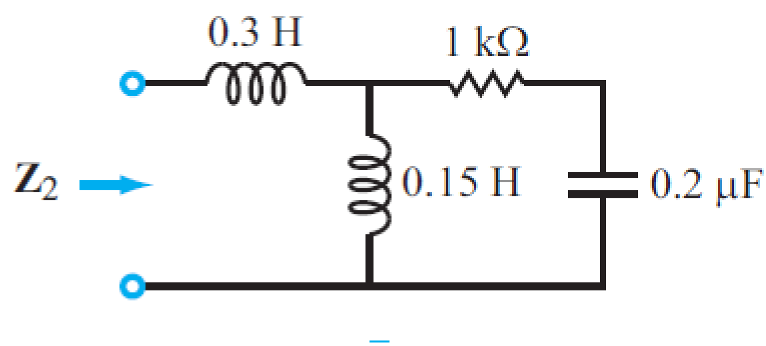
and, in the phasor domain:

Circuit Element	Impedance	Reactance
Resistor	R	none
Inductor	jωL	ωL
Capacitor	j(-1/ωC)	-1/ωC

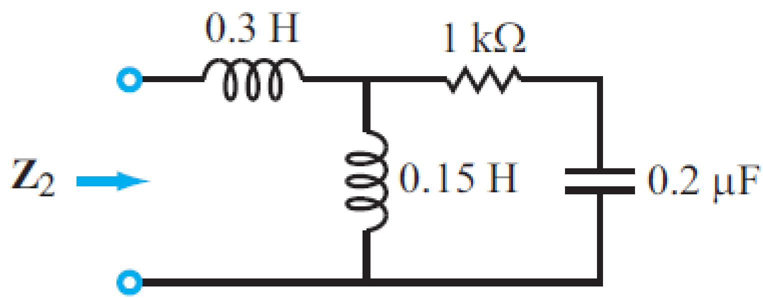
$$\mathbf{V} = Z\mathbf{I}$$

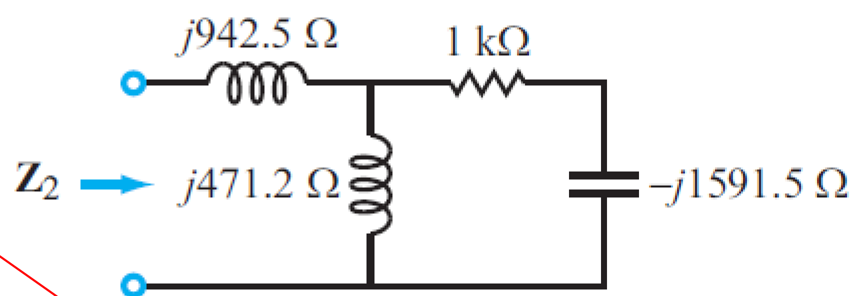
Here are some problems just using the impedance model
(so no KCL/KVL equations)

… determine Z_2 at 500 Hz for this circuit:



This is a much easier problem to solve than the previous one since it's only asking for the equivalent impedance. Here we need only to convert the circuit elements into their phasor domain equivalent impedances …..and then <u>combine them in series and parallel</u>.





Notice – **be careful** ---the frequency in this problem was specified in Hz. …….ω is in units or radians/s. ω (rad/s) = 2π f (Hz)

$$\mathbf{Z}_{L_1} = j\omega L_1 = j2\pi \times 500 \times 0.3 = j942.5\ \Omega$$

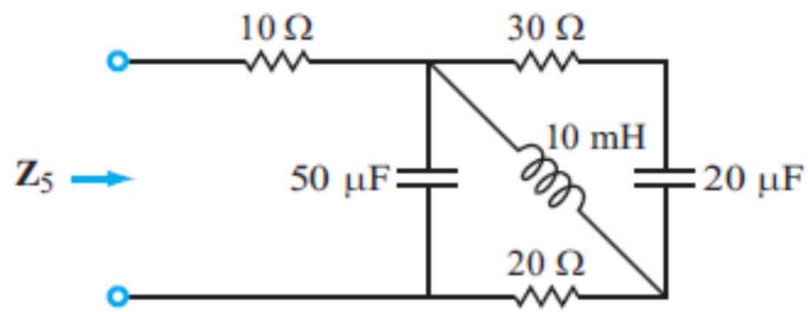
$$\mathbf{Z}_{L_2} = j\omega L_2 = j471.2\ \Omega$$

$$\mathbf{Z}_C = \frac{-j}{\omega C} = \frac{-j}{2\pi \times 500 \times 0.2 \times 10^{-6}} = -j1591.5\ \Omega$$

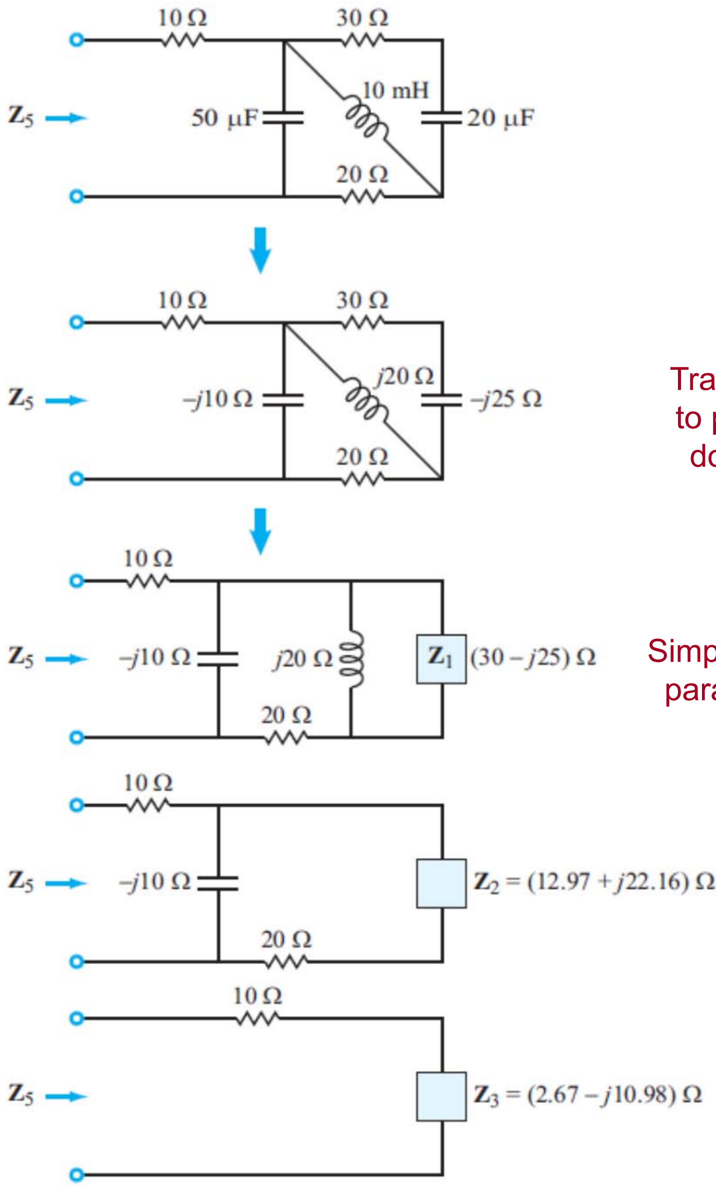
How did I get this?

so $$\mathbf{Z}_2 = j942.5 + j471.2 \parallel (1000 - j1591.5)$$

$$= j942.5 + \frac{j471.2(1000 - j1591.5)}{1000 - j1591.5 + j471.2} = (98.5 + j1524.0)\ \Omega$$



Calculate Z_5 at ω = 2000 rad/s



Transform to phasor domain

$$\mathbf{Z}_{C_1} = \frac{-j}{\omega C_1} = \frac{-j}{2000 \times 50 \times 10^{-6}} = -j10\ \Omega$$

$$\mathbf{Z}_{C_2} = \frac{-j}{\omega C_2} = \frac{-j}{2000 \times 20 \times 10^{-6}} = -j25\ \Omega$$

$$\mathbf{Z}_{\mathrm{L}} = j\omega L = j2000 \times 10 \times 10^{-3} = j20\ \Omega$$

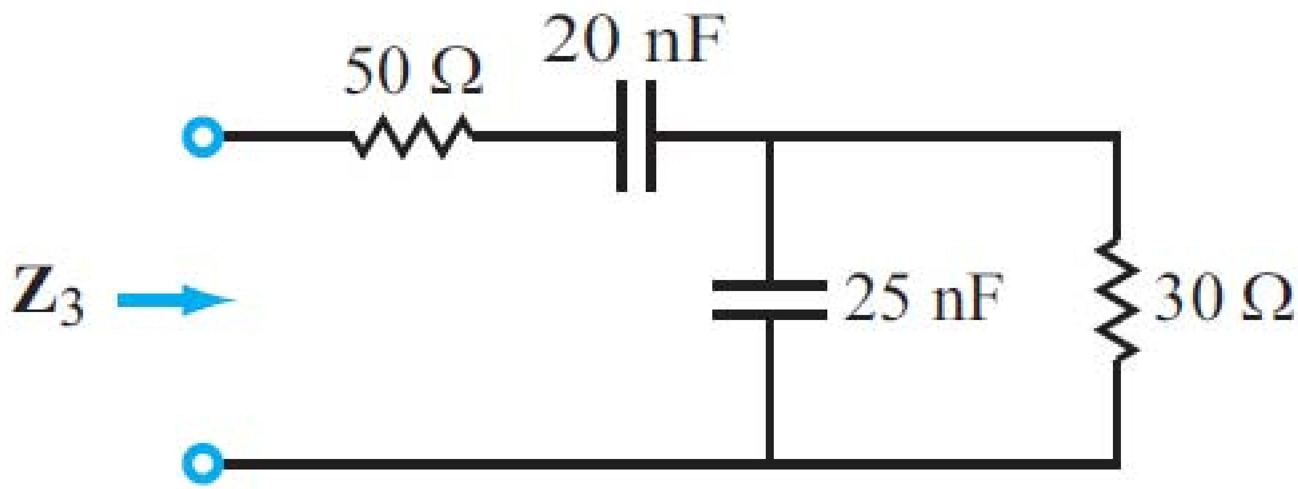
Simplify with series and parallel combinations

$$\begin{aligned}
\mathbf{Z}_2 &= \mathbf{Z}_1 \parallel (j20) \\
&= \frac{(30 - j25)(j20)}{30 - j25 + j20} \\
&= \frac{500 + j600}{30 - j5} \cdot \frac{(30 + j5)}{(30 + j5)} \\
&= \frac{12000 + j20500}{925} = (12.97 + j22.16)\ \Omega
\end{aligned}$$

$$\begin{aligned}
\mathbf{Z}_3 &= (20 + \mathbf{Z}_2) \parallel (-j10) \\
&= \frac{(32.97 + j22.16)(-j10)}{32.97 + j22.16 - j10} \\
&= \frac{221.6 - j329.7}{32.97 + j12.16} \cdot \frac{(32.97 - j12.16)}{(32.97 - j12.16)} \\
&= (2.67 - j10.98)\ \Omega
\end{aligned}$$

$$\mathbf{Z}_5 = 10 + \mathbf{Z}_3 = (12.67 - j10.98)\ \Omega.$$

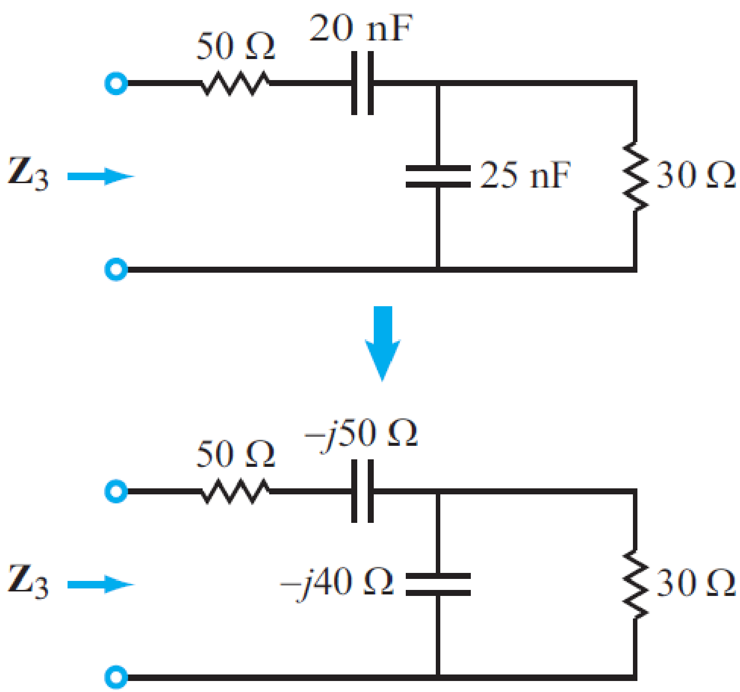
What is the value of Z_3 for this circuit



Is Z_3 going to be real or complex? Have you been given enough information to know this …or to calculate it's value?

Actually Z_3 is going to be a complex value (why is this true?) …but to calculate it's value you need to know the frequency because it will also be frequency dependent…so you haven't been given enough information.

OK…for practice, lets evaluate Z_3 at a frequency of 10^6 rad/s

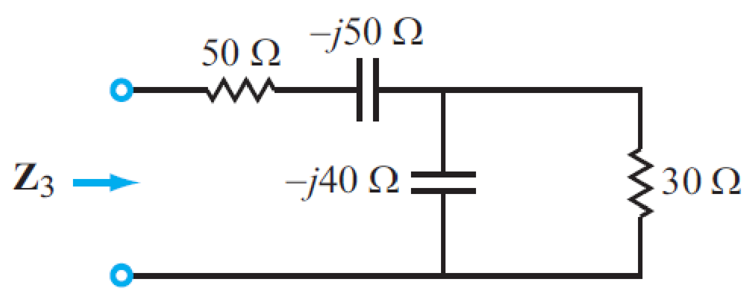


First we transfer to the phasor domain using:

$$R \rightarrow \mathbf{Z}_R = R$$
$$L \rightarrow \mathbf{Z}_L = j\omega L$$
$$C \rightarrow \mathbf{Z}_C = 1/j\omega C$$

All resistances, capacitances, and inductances are converted to complex impedances – at the specified frequency

Now we combine the impedances using well-known techniques



$$\mathbf{Z}_3 = 50 - j50 + \frac{30 \times (-j40)}{30 - j40} = (69.2 - j64.4)\ \Omega.$$

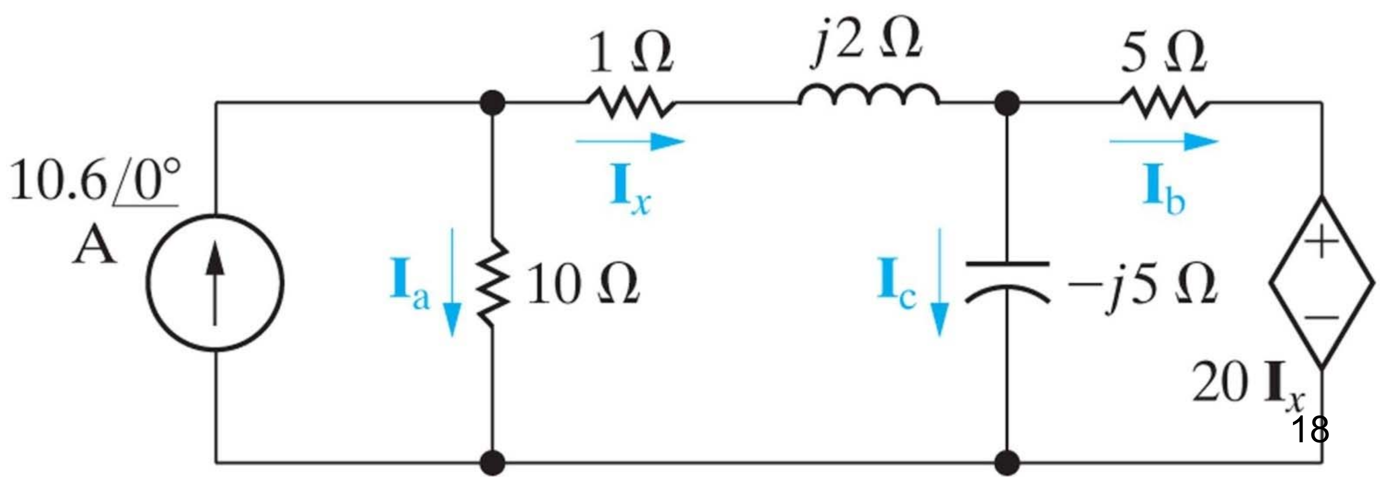
What would the impedance be at zero frequency (DC)
…or at very high frequency? You should be able to do this without doing any calculations.

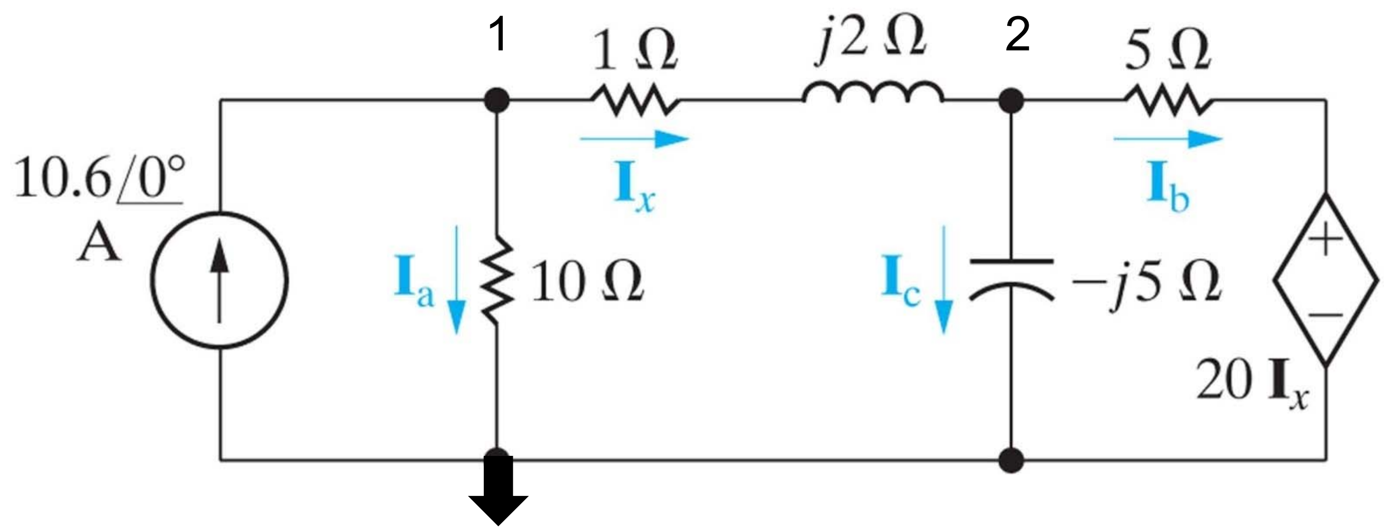
Obviously, as we just saw, if you can solve a sinusoidal steady state circuit problem by simply using the “impedance model” it is much easier than solving for phasor voltages or currents using a node voltage or mesh current approach.

As we discussed before, this is the equivalent of solving a DC circuit by series and parallel combinations of resistors and then using Ohms law to calculate the current or voltage as required.

It is possible, however, to use phasors to do a complete node voltage analysis in the frequency domain as we saw before.

Consider for example this circuit – where we’ve already transformed to the phasor domain:





We define voltages at the two extraordinary nodes and define a reference node as shown. Now, we write the node voltage equations at the two nodes using phasor voltages.

At node 1:

$$-10.6+\frac{\mathbf{V}_1}{10}+\frac{\mathbf{V}_1-\mathbf{V}_2}{1+j2}=0$$

This looks just like an ordinary node-voltage equation ….except the voltages are phasors ..and the equation has constants that are complex numbers.

$$-10.6+\frac{\mathbf{V}_1}{10}+\frac{\mathbf{V}_1-\mathbf{V}_2}{1+j2}=0$$

Multiplying by (1+j2) and collecting coefficients of the $\mathbf{V}_1$ and $\mathbf{V}_2$ phasors:

$$\mathbf{V}_1(1.1+j0.2)-\mathbf{V}_2=10.6+j21.2$$

Now we write the node voltage equation at the second node:

$$\frac{\mathbf{V}_2-\mathbf{V}_1}{1+j2}+\frac{\mathbf{V}_2}{-j5}+\frac{\mathbf{V}_2-20\mathbf{I}_x}{5}=0$$

$$and \quad \mathbf{I}_x=\frac{\mathbf{V}_1-\mathbf{V}_2}{1+j2}$$

Substituting the expression for $\mathbf{I}_x$ into the 2nd node equation, and multiplying through by (1+j2) …and collecting coefficients for the two phasor voltages:

$$-5\mathbf{V}_1 + (4.8 + j0.6)\mathbf{V}_2 = 0$$

So this equation along with the 1st equation:

$$\mathbf{V}_1(1.1 + j0.2) - \mathbf{V}_2 = 10.6 + j21.2$$

has the simultaneous solution for the phasor voltage: (by substitution)

$$\mathbf{V}_1 = 68.4 - j16.8V$$

$$\mathbf{V}_2 = 68 - j26V$$

from which the branch current phasors can be calculated:

In MATLAB ..the code looks like this:

```
>> A=[1.1+0.2*j, -1;-5,4.8+0.6*j]

A =

   1.1000 + 0.2000i  -1.0000
  -5.0000             4.8000 + 0.6000i

>> B=[10.6+21.2*j;0]

B =

  10.6000 +21.2000i
        0

>> x=A\B

x =

  68.4000 -16.8000i
  68.0000 -26.0000i
```

Notice: MATLAB <u>easily does matrix algebra with complex numbers</u>

Note: MATLAB uses “i” for the imaginary number and not “j”