ELEN 50 Class 09 – The Node Voltage Method: Dependent Sources and Supernodes

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To Recap the Node Voltage Method from Last Lecture:

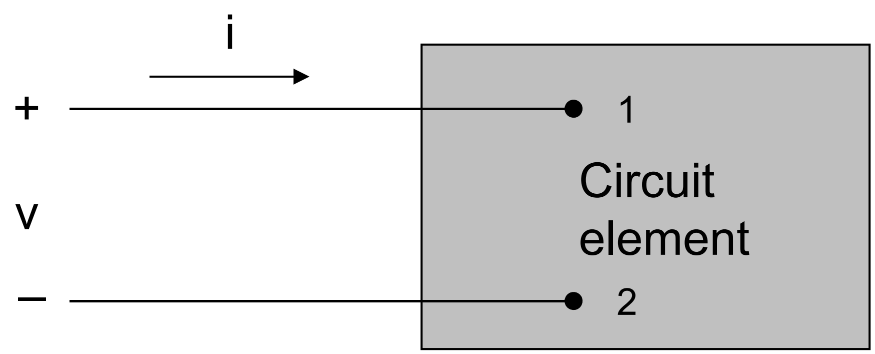
- Identify all extraordinary nodes, set one as the reference (ground) node, and assign node voltages (v1, v2, v3, etc.) to the $n_{ex} - 1$ remaining nodes.

- At each of the n_{ex}-1 nodes, write the KCL equation requiring the sum of all currents leaving the node to be zero.

- Solve the n_{ex}-1 independent simultaneous equations to determine the unknown node voltages.

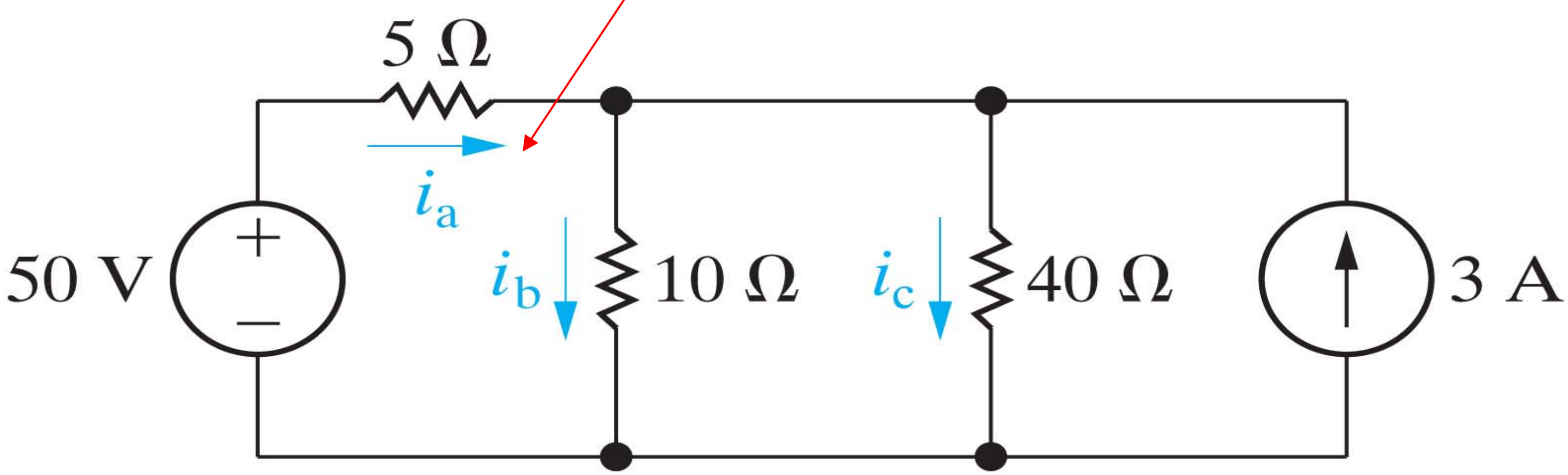
let's talk, one more time, about the issue of how to know which way the current is flowing in a circuit.

• As we know, the "passive sign convention" states: whenever the current in an element is flowing in the direction of the reference voltage drop across an element, we choose to use a positive sign in any expression that relates the voltage to the current.



Here's a circuit we analyzed earlier -- we want to determine all of the branch currents using the node voltage method.

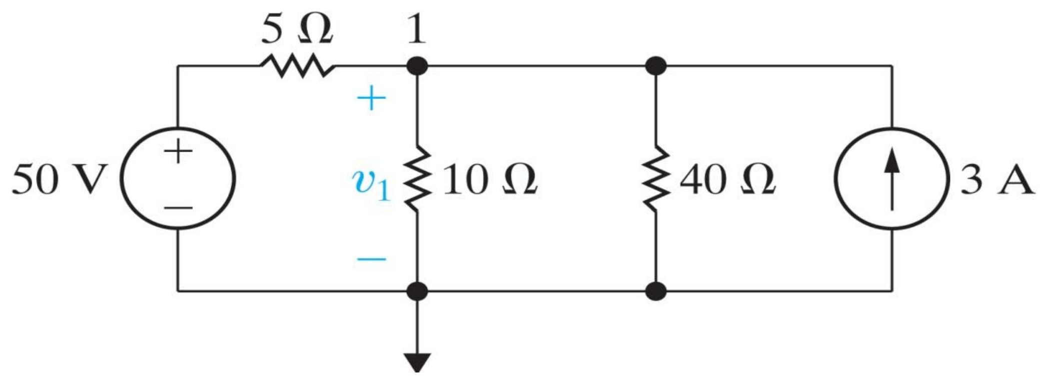
Notice – in the problem current is shown flowing into the node



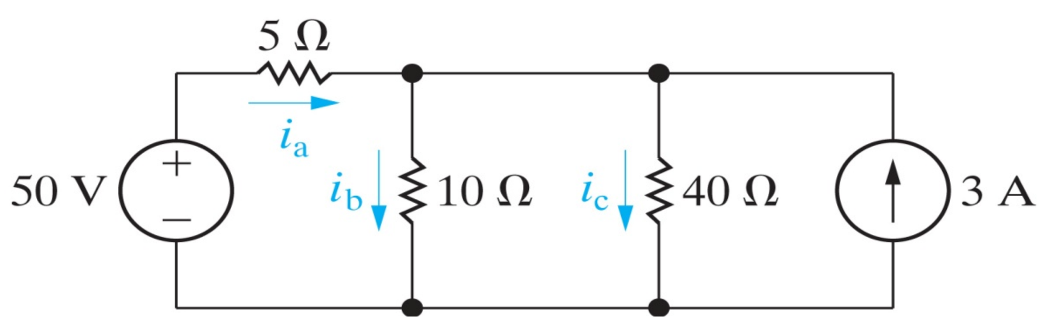
•We wrote KCL for currents leaving the single essential node as:

(v1 – 50)/5 + v1/10 + v1/40 – 3 = 0

→ v1 = 40V



So, for the current directions shown below :1a = (50-40)/5 = 2A, ib = 40/10 = 4A, and ic = 40/40 = 1A



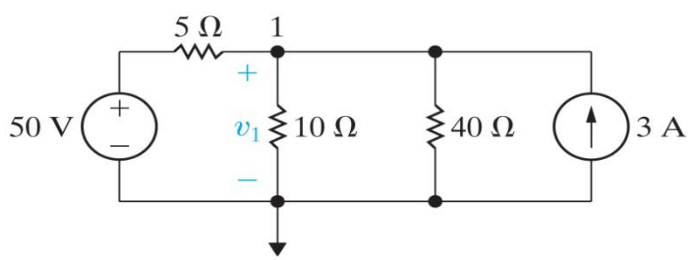
- Some might object that if we are supposed to write the KCL for currents leaving the single essential node the equation should have been:

$$-(50 - v1)/5 + v1/10 + v1/40 - 3 = 0$$

We should write the first term as -(50 – v1)/5 because the current is flowing into the node from this branch according to the passive sign convention.

However…this is just:

$(v1 - 50)/5 + v1/10 + v1/40 - 3 = 0$ which is the equation we saw earlier …and we get v1 = 40V and ia is flowing into the node



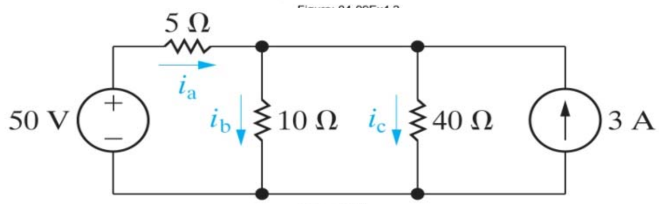
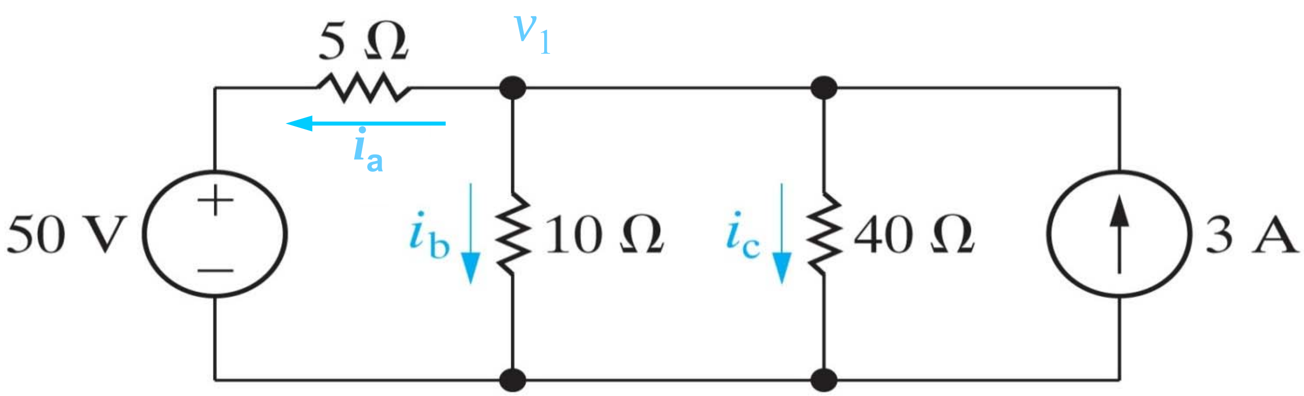


Figure: 04-08



If we had drawn the current going the other direction (as shown above), then the passive sign convention would require that v1 is higher than 50V (since current “flows downhill) and the KCL equation for this node would have been:

$$(v1- 50)/5 + v1/10 +v1/40 - 3 = 0$$

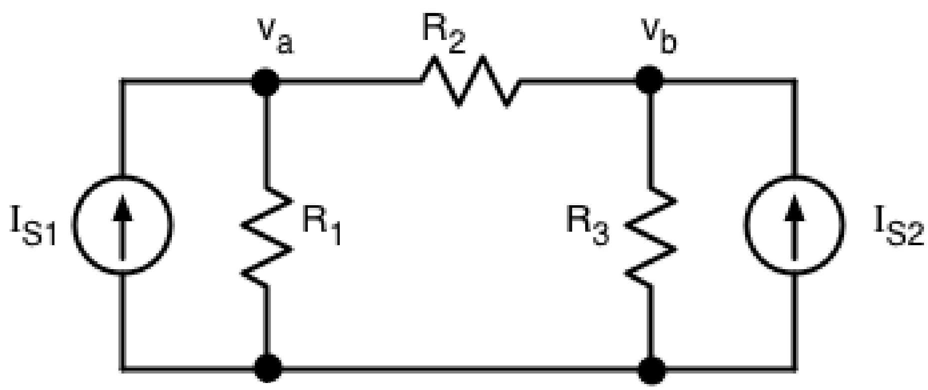
Which is exactly the same equation we used previously – and we get v1 = 40V. Now, using Ohm’s law we calculate $i_a = (v_1\text{-}50V)/5\Omega$. Since v1 =40V, this gives ia = -2A. The minus sign means the current is actually flowing the other direction….the direction we had shown it flowing initially….so everything is OK.

Here's another one:

For the circuit shown below

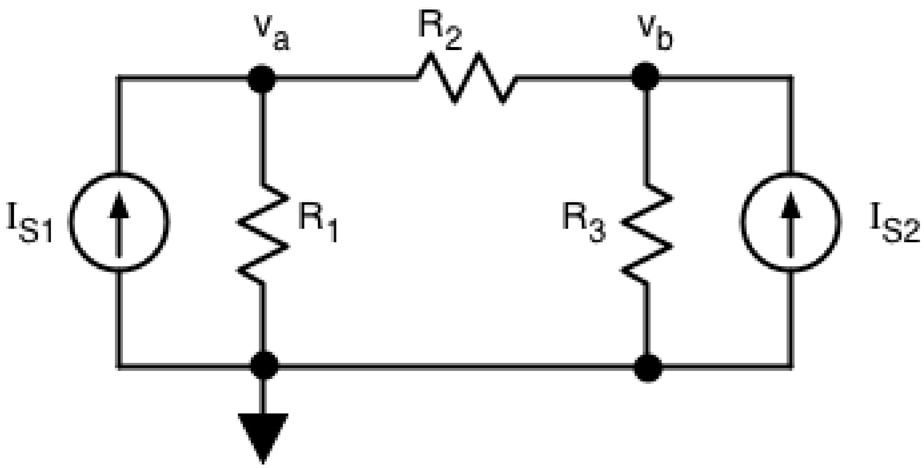
$R_1 = 6\ \Omega$, $R_2 = 6\ \Omega$, $R_3 = 9\ \Omega$, $I_{S1} = 2$ A, and $I_{S2} = 0$ A.

Use the node-voltage technique to find v_a and v_b.



What is the first step? What is the second step?

Pick a ground reference and label remaining essential nodes



R_1 = 6 Ω, R_2 = 6 Ω, R_3 = 9 Ω , I_{S1} = 2 A, and I_{S2} = 0 A.

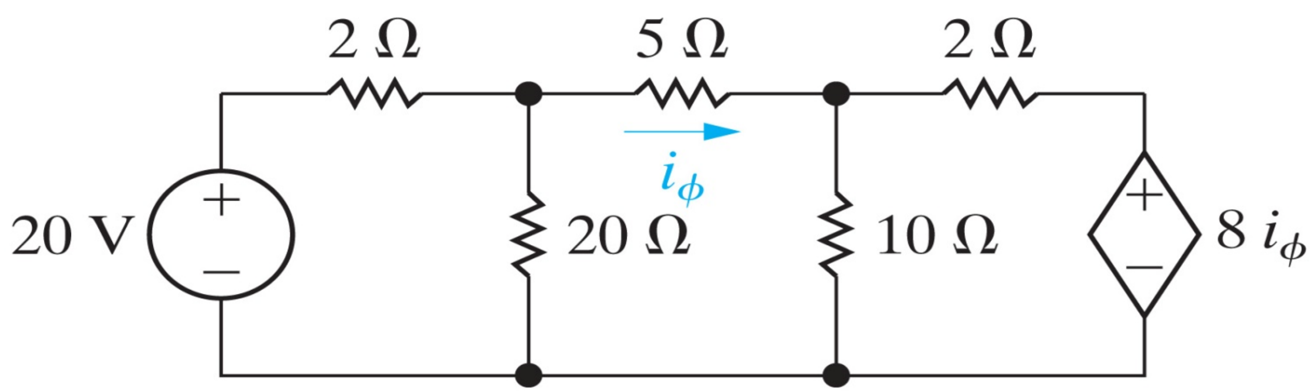
$$-2+\frac{v_a}{6}+\frac{v_a-v_b}{6}=0$$

$$\frac{v_b}{9}-\left(\frac{v_a-v_b}{6}\right)+0=0$$

v_a = 8.571 V, and v_b = 5.143 V

- Now, let's talk about something new -- using the node voltage method with dependent sources.

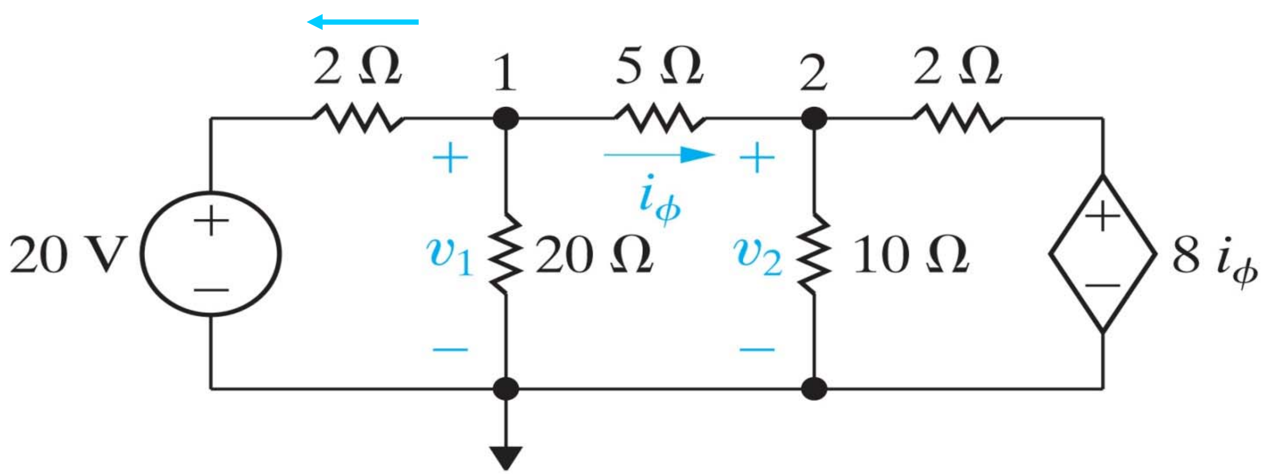
- Consider the following circuit:



- It's similar to some we have analyzed and we want to solve for i_ϕ using the node voltage method -- but notice, the circuit now contains a dependent voltage source…..and it depends on i_ϕ. Will this introduce any complications in using the node voltage method?

- As usual, we identify the essential nodes, select a reference node, and label the appropriate voltages and currents.
- Summing all of the current flowing <u>out</u> of node 1 gives:

$(v1 – 20)/2 + v1/20 + (v1-v2)/5 = 0$ (notice here again, if we drew the current through the 2Ω resistor going in the other direction, then the term for this current would have been $– (20 – v1)/2$ ….and the equation would be unchanged.



now we can write the KCL equation for node 2

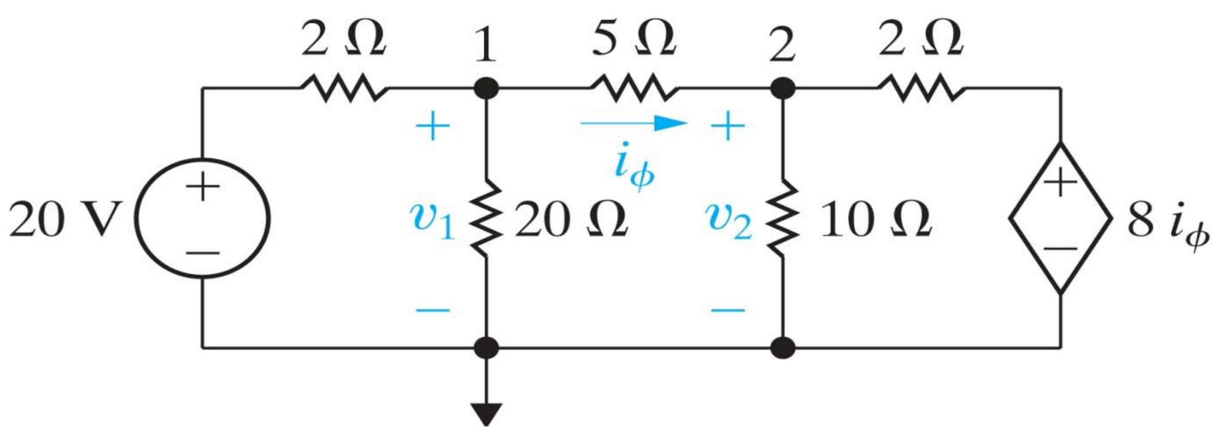
Node 1: $(v1 - 20)/2 + v1/20 + (v1-v2)/5 = 0$

Node 2: $-(v1 - v2)/5 + v2/10 + (v2 - 8i_\phi)/2 = 0$

So it appears we've got two equations in three unknowns (v1, v2, and i_ϕ). However, these aren't independent variables and we can express i_ϕ in terms of v1 and v2 using Ohms law: $i_\phi = (v1-v2)/5$

So the equations become: $0.75\ v1 - 0.2\ v2 = 10$ → $v2 = 10$ V

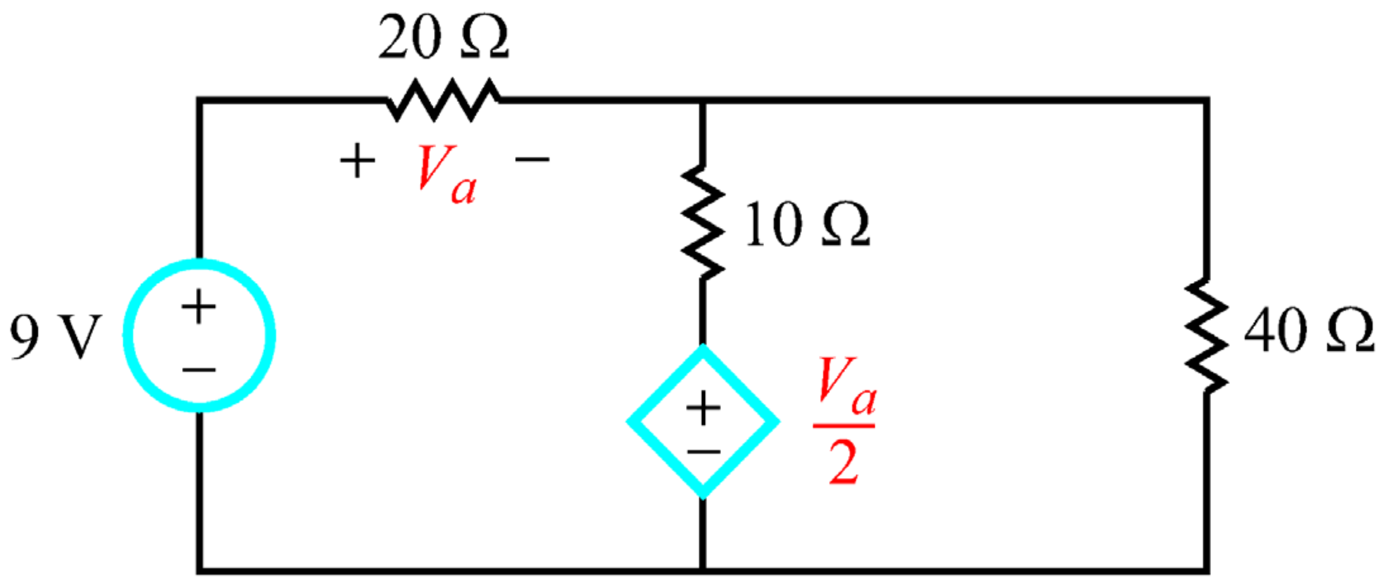
$- v1 + 1.6\ v2 = 0$ → $v1 = 16$ V



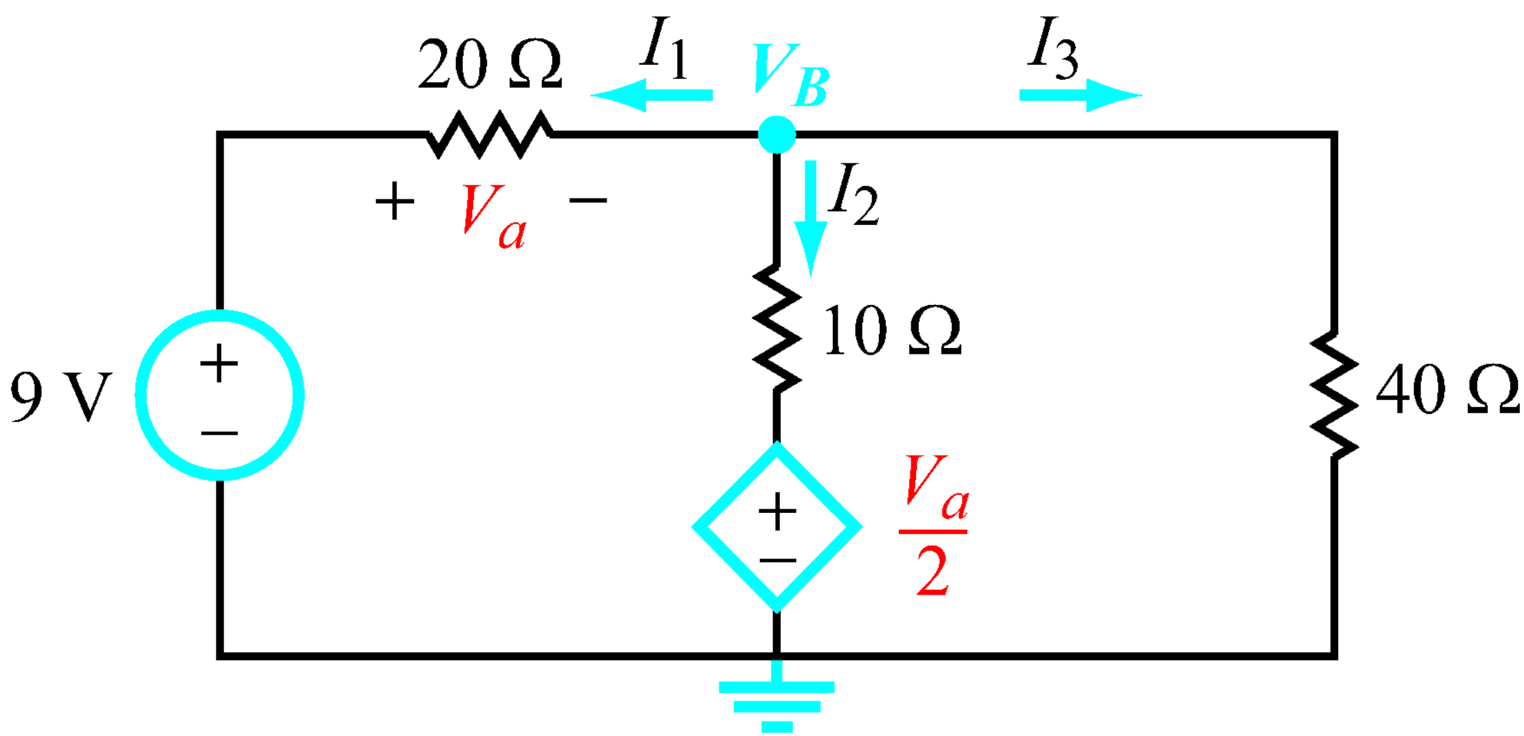
$i_\phi = 1.2$ A

If we have a dependent source, it's <u>constraint equation must be used along with the KCL equations.</u>

Use Node Voltage Analysis to determine V_a



What are the steps for node voltage analysis?



Just one equation

$$\frac{V_B - 9}{20} + \frac{V_B - \frac{V_a}{2}}{10} + \frac{V_B}{40} = 0$$

But since V_a=9-V_B

V_a =5V

why the minus sign

Special Cases: Quasi-Supernodes and Supernodes

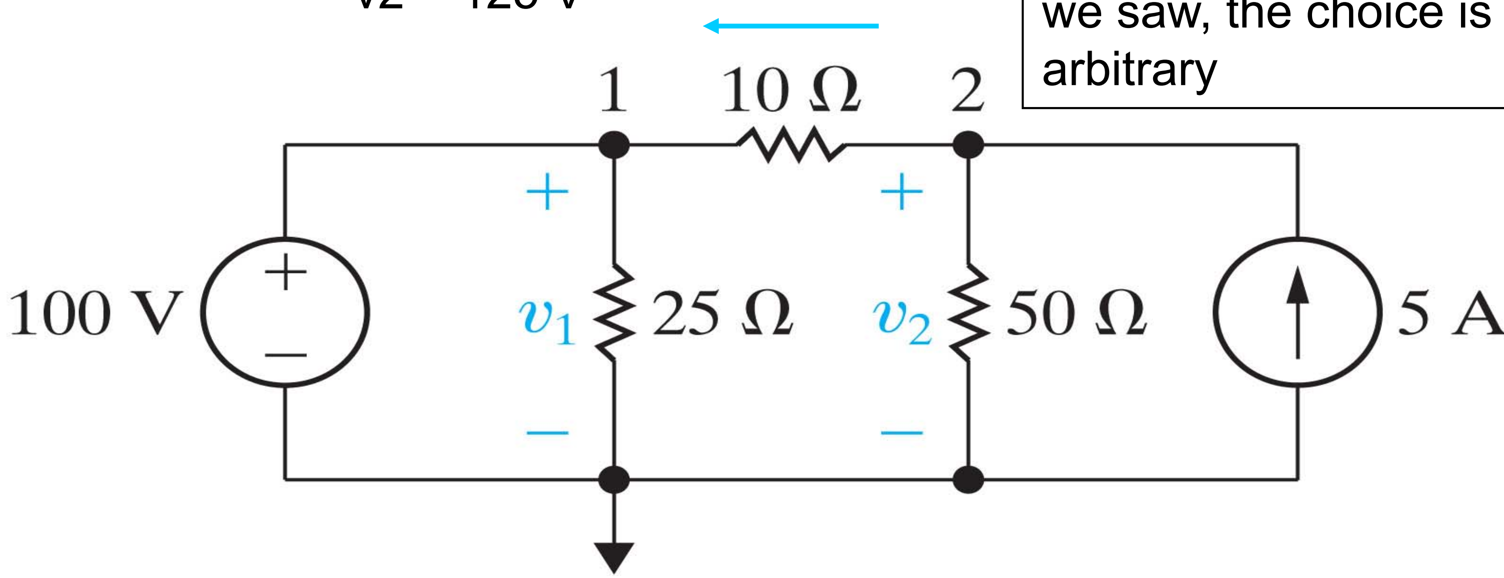
- In this circuit there are three essential nodes …but two of them (node 1 and the reference node) are connected by a voltage source …which constrains v1….so, basically, we already know v1! Node 1 is called a **quasi-supernode**!

- Consequently, although we'd expect two equations in two unknowns in the solution, there is really only one unknown voltage ….v2.

at node 2: (v2-v1)/10 + v2/50 – 5 = 0

v2 = 125 V

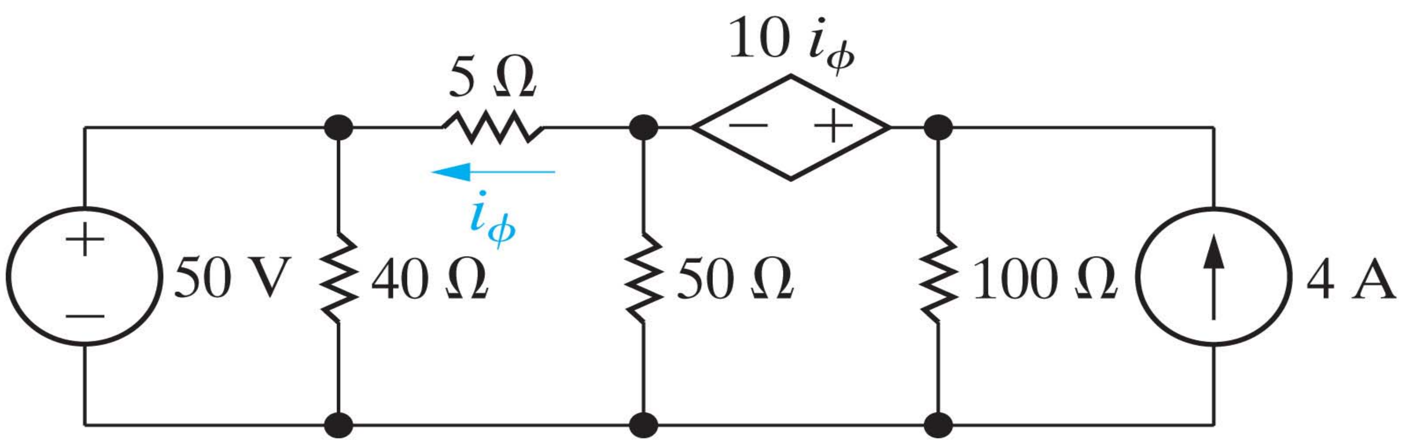
We're assuming current is flowing as shown .. but as we saw, the choice is arbitrary



Special Cases: Quasi-Supernodes and Supernodes

Here’s another example of a circuit with voltage sources between essential nodes – in this case, we have four essential nodes, but two of them are connected by voltage sources – one independent and one dependent.

Consequently, we can use two equations rather than the four we would otherwise expect with four essential nodes. One of the nodes is a quasi-supernode if we pick the bottom nodes as ground..as we discussed earlier. We already know the voltage at that node --- it’s 50V.



Node 2 and 3 together create a special situation. This configuration is called a "supernode." The voltages at the two nodes are not independent and we can't write the normal node voltage equations in terms of voltages and the resistance in the branches.

For node 2 we could label the unknown current through the dependent voltage source as i …and then write for node 2:

$(v2 – v1)/5 + v2/50 + i = 0$

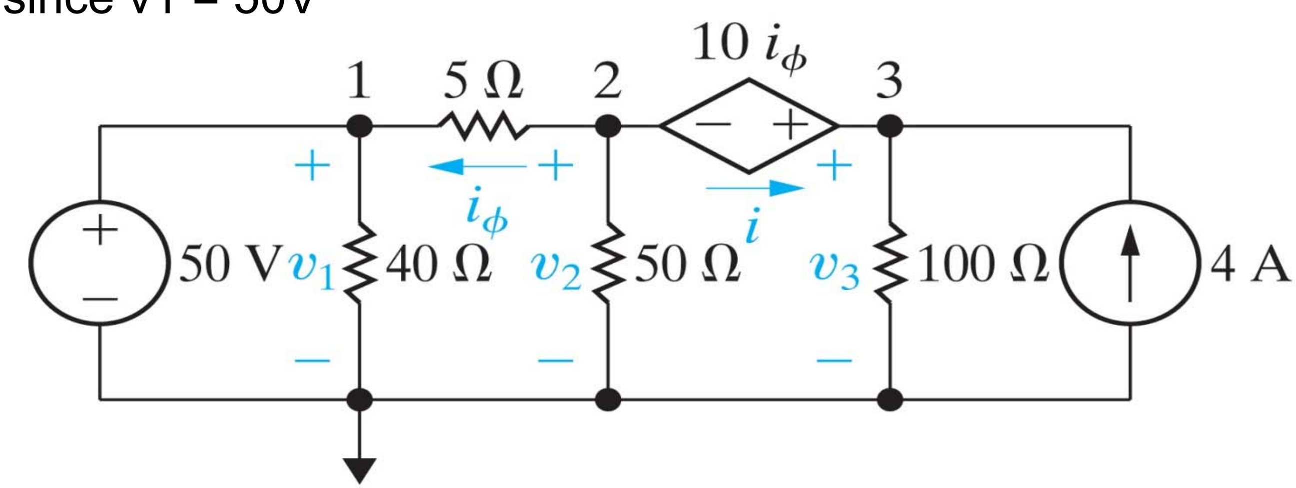
Then for node 3 we can write:

$v3/100 – i – 4 = 0$

Adding these two equations eliminates i

$(v2 – v1)/5 + v2/50 + v3/100 – 4 = 0$ or $(v2 – 50)/5 + v2/50 + v3/100 – 4 = 0$

since v1 = 50V



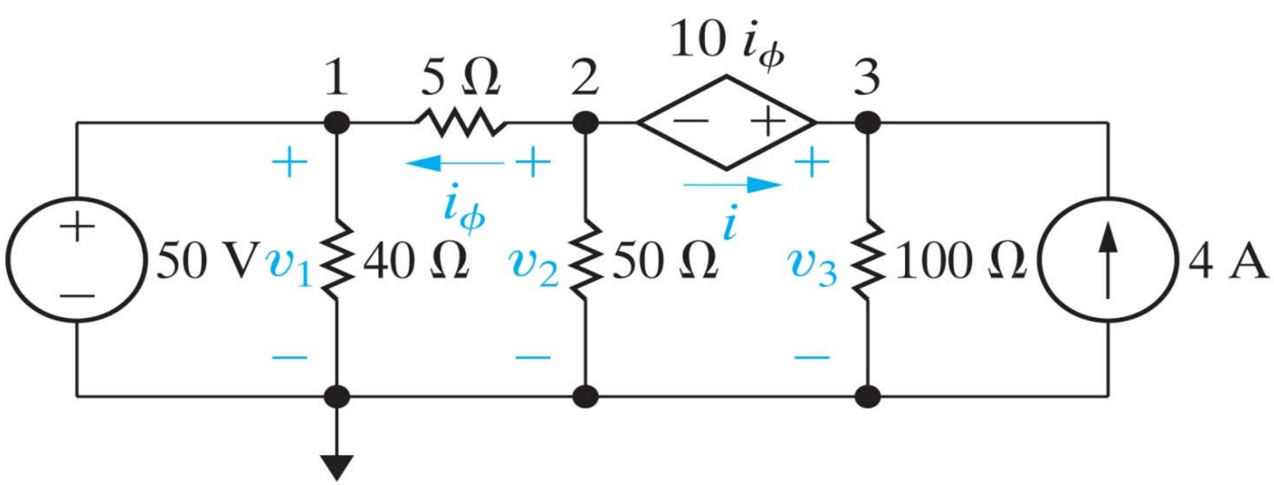
Now we can express v3 in terms of v2 because of the dependent voltage source between these two essential nodes:

v3 = v2 + 10 i_ϕ

and i_ϕ = (v2 – 50)/5 so v3 = v2 + 10(v2 -50)/5 = 2v2 – 100

Substituting for v3 in: (v2 – 50)/5 + v2/50 + v3/100 – 4 = 0

Gives v2 = 60 V so i_ϕ = (60 – 50)/5 = 2 A and v3 = 60V + 20V = 80V

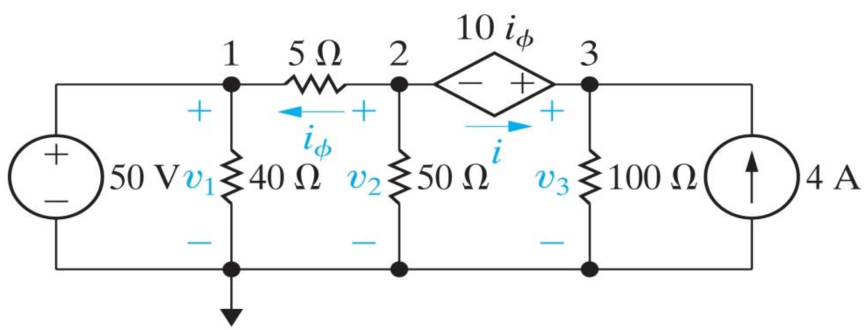


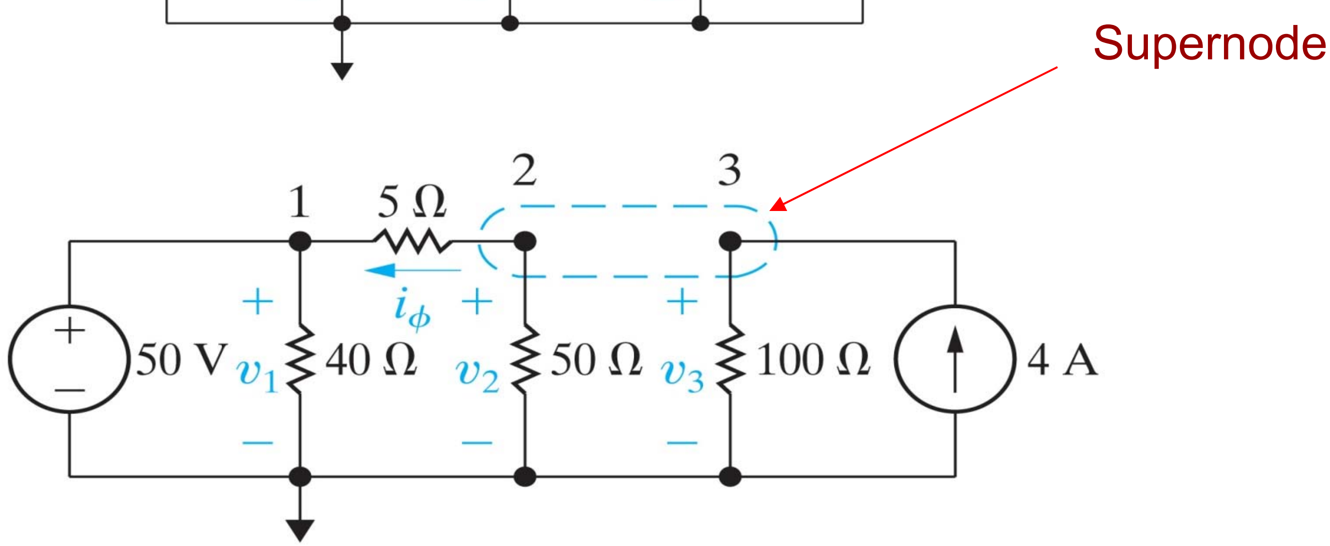
There’s a simpler way to get the equation:

(v2 – 50)/5 + v2/50 + v3/100 – 4 = 0

We can write it down directly by using the concept of a **supernode**.

A supernode is two essential nodes connected by a voltage source --- and KCL holds for the composite supernode just like for a regular node ---- so we can write a single KCL equation for it.

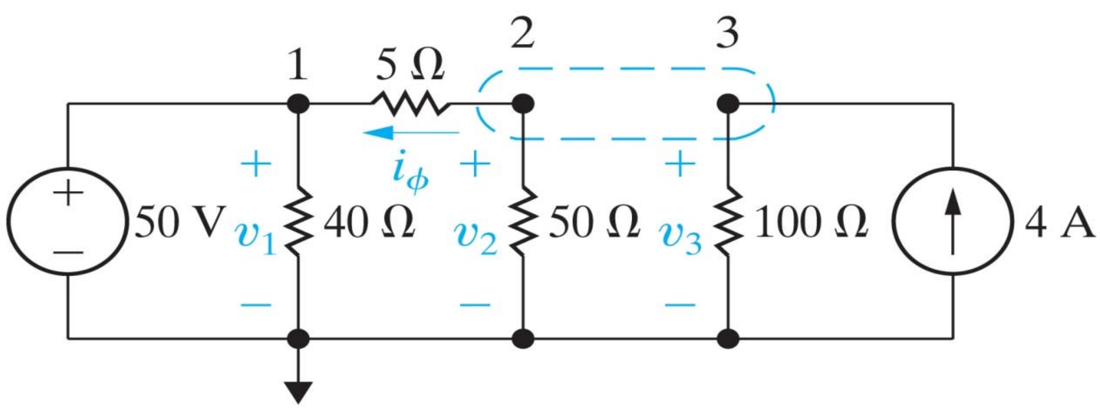




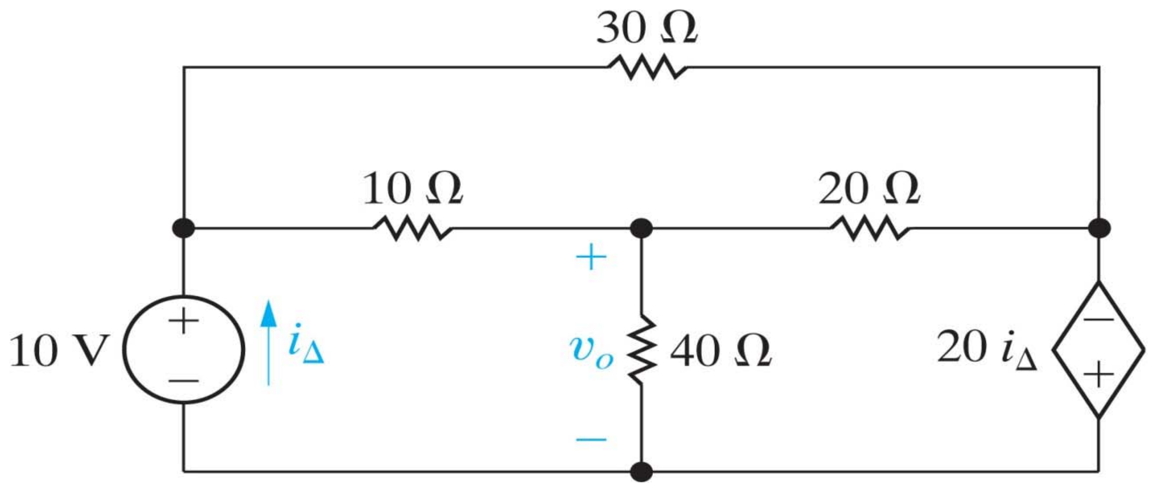
Instead of writing equations for the current out of nodes 2 and 3 separately, we can write a single equation for the current out of the supernode as:

$$(v2 - 50)/\ 5 + v2/50 + v3/\ 100 - 4 = 0$$

Then we proceed as before to solve the circuit

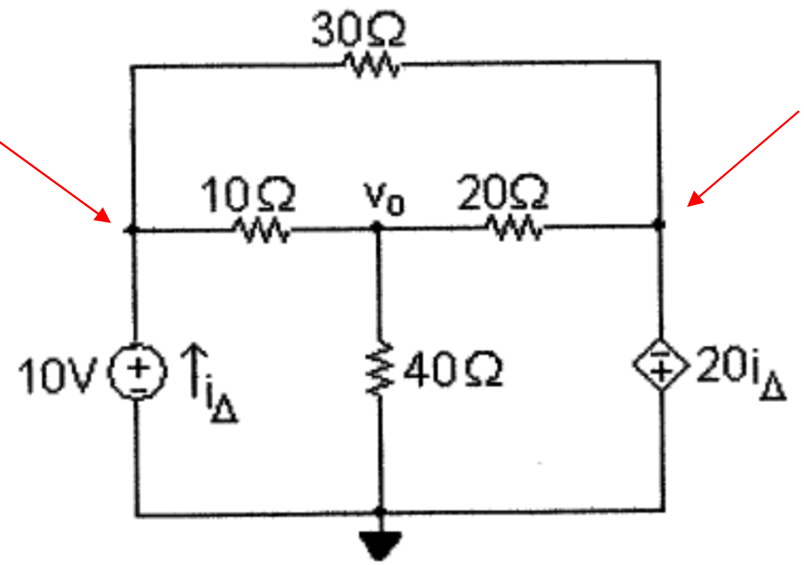


Here's another circuit to analyze – find the voltage, v0 and the current, iΔ



How many essential nodes are there? Which one is a good choice for a reference node? Does any pair of essential nodes qualify as a supernode?

How about this node?



How about this node?

There is only one (useful) essential node (other than the reference node)….and we can write the node voltage equation for it:

Why the plus sign?

$$\frac{v_o}{40} + \frac{v_o - 10}{10} + \frac{v_o + 20i_\Delta}{20} = 0$$

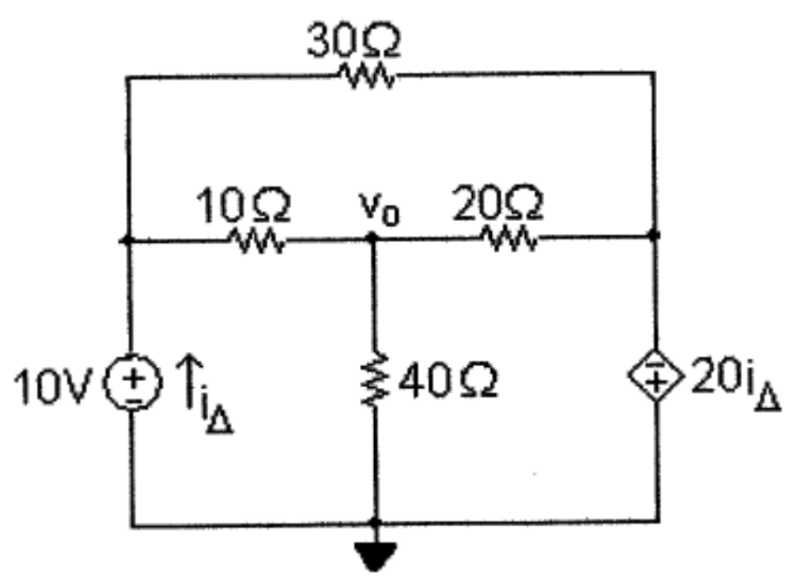
We've got two unknowns …we need another equation involving i_Δ

We can write for the constraint equation on i∆:

i∆ = (10 – v0)/10 + (10 + 20 i∆)/30

So we have two equations in v0 and I∆:

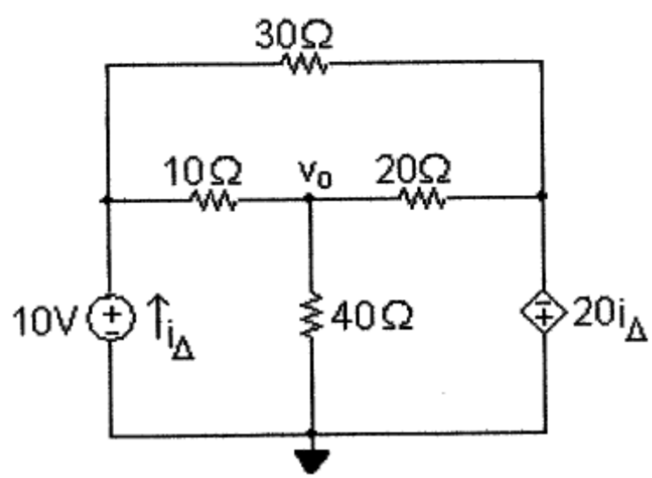
$$
\begin{aligned}
v_o\left(\frac{1}{40}+\frac{1}{10}+\frac{1}{20}\right) &+ i_\Delta(1) &&= 1 \\
v_o\left(\frac{1}{10}\right) &+ i_\Delta\left(1-\frac{20}{30}\right) &&= 1+\frac{10}{30}
\end{aligned}
$$



Which we can solve:

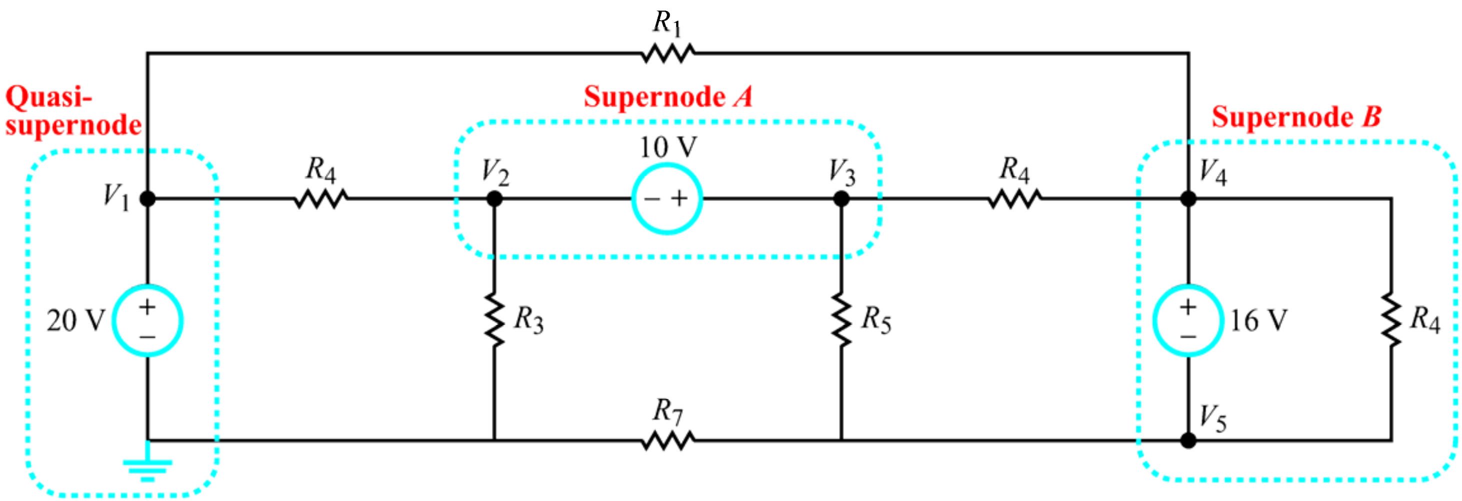
$i\Delta$ = - 3.2A and v0 = 24 V

Let's check the results - the current ($i\Delta$) through the 10Ω resistor from the 10V source is (10 -24)/10 = -1.4A and through the 30Ω resistor is (10 – 3.2*20)/30 = - 1.8A, so the sum is -3.2 A



Attributes of Supernodes

- At a supernode, Kirchoff's current law (KCL) can be applied to the combination of the two nodes as if it were a single node.
- KVL is used to express the voltage difference between the two nodes in terms of the voltage source between them
- If a supernode contains a resistor in parallel with the voltage source – as usual – the resistor can be removed since it will have no impact on other parts of the circuit as we discussed earlier.
- For a quasi-supernode (a supernode where one of the nodes is the reference node) the node voltage of the non reference node is equal to the voltage magnitude of the source.


R1
Quasi-
supernode
Supernode A
Supernode B
10 V
V1
R4
V2
V3
R4
V4
20 V
R3
R5
16 V
R4
R7
V5