ELEN 50 Class 16 – Midterm II Review

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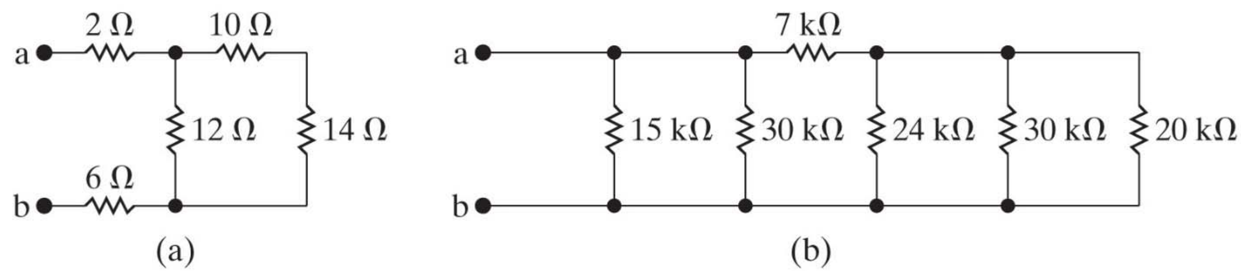
Since the Midterm I exam we have covered these new topics:

- Mesh current method
- Mesh current solutions with supermeshes
- Mesh current solution by inspection
- Operational Amplifiers

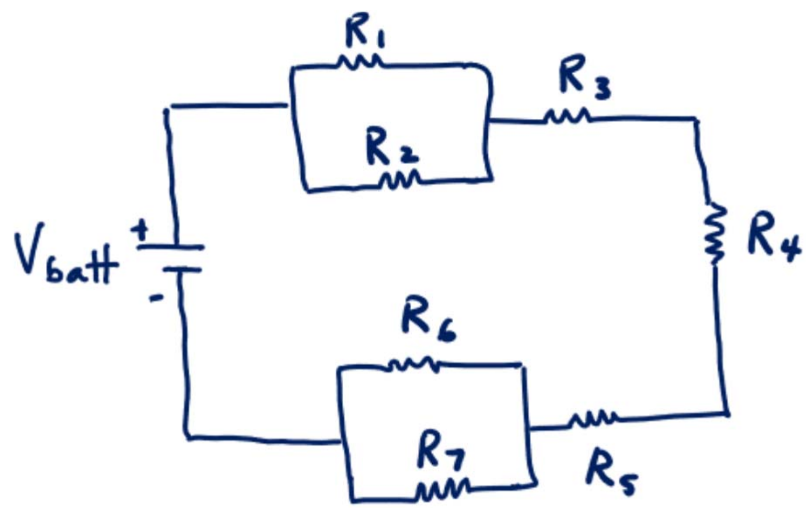
Midterm II will include questions from these areas as well as questions like those in the first mid-term exam.

Here is a review of the new topics with some typical problems. You should look at the Midterm I review for additional review of the earlier stuff. If you can do problems like these, you will do fine on the Midterm II exam.

Find the equivalent resistance R_{ab} for each of these circuits:



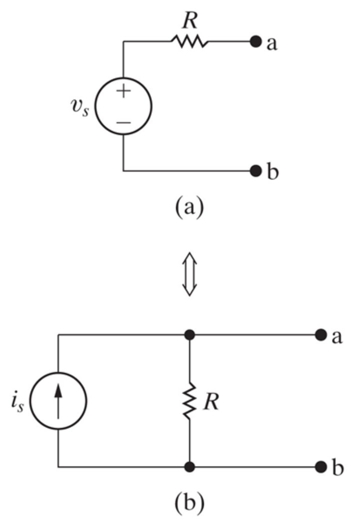
Is it clear which of these circuit elements are in series and which are in parallel?



How would you replace all these resistors with a single, equivalent resistor? Which resistors are in parallel and which are in series?

Another important kind of circuit simplification is a Source Transform

A source transform allows a voltage source in series with a resistor to be transformed into a current source in parallel with a resistor. This can be very useful in simplifying a circuit prior to doing a circuit analysis.



these two circuits are equivalent, if:

$$\boxed{i_s = v_s/R}$$

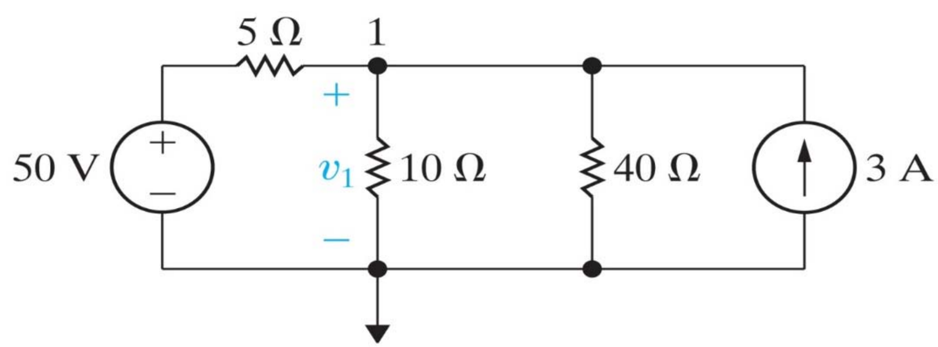
You can show this relationship by attaching a load resistor to a and b and calculating the current flowing

- As we’ve discussed many times, the mesh current method is completely complementary to the node voltage method …but it uses mesh currents instead of node voltages….and it explicitly uses KVL instead of KCL.

Node Voltage Method	**Mesh Current Method**
• Identify essential nodes • Select reference node • Label voltages at remaining essential nodes (v1, v2, ….vn) • Write equations for KCL <u>at these nodes</u> in terms of node voltages referenced to reference node. • Solve n equations in n unknowns	• Identify mesh currents • no reference node needed since we’re explicitly calculating currents and not voltages. • Label mesh currents (ia, ib, …in) • Write equations for KVL <u>around the mesh current paths.</u> • Solve n equations in n unknowns

Circuit Analysis with the Node Voltage Method

- Choose a reference node and label the other essential nodes
- Well…there is only one other essential node so we can solve this circuit with a single equation! How many terms will the equation have?

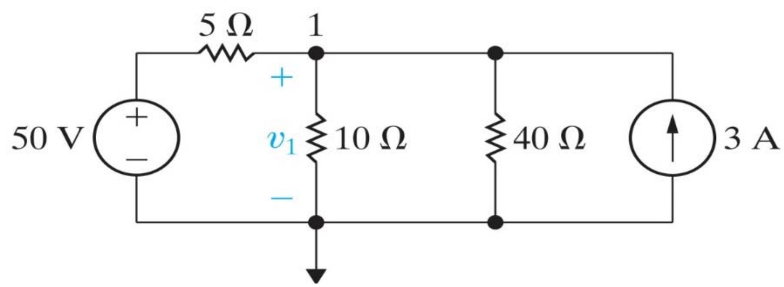


- Now we write the KCL for currents leaving the single essential node:

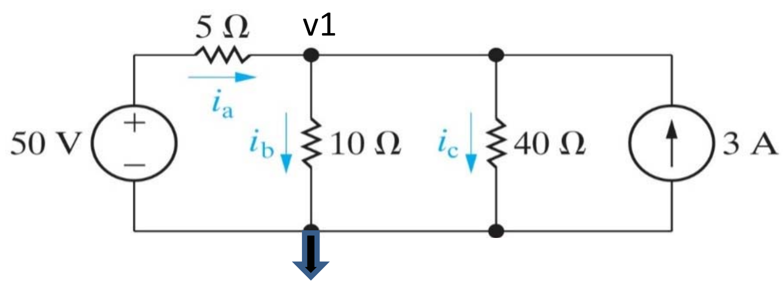
•If we assume that current is leaving the node through all the branches except the one with the current source:

$$(v1 – 50)/5 + v1/10 + v1/40 – 3 = 0$$

$$\rightarrow v1 = 40V$$



If, instead, we had assumed that the branch currents were flowing as shown below:



The KCL equation would be:

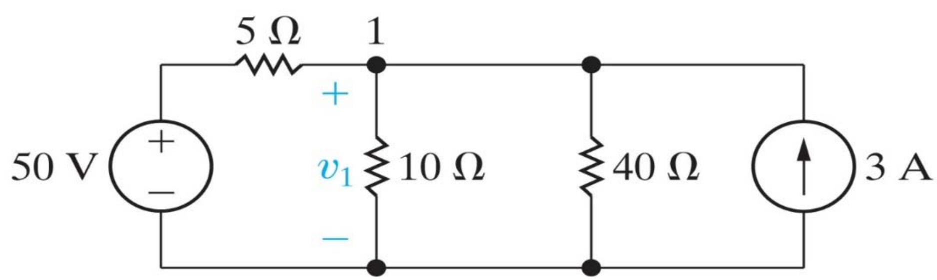
-(50- v1)/5 + v1/10 + v1/40 – 3 = 0

(v1 – 50)/5 + v1/10 + v1/40 – 3 = 0   the same as before!

…and we get the same answer:

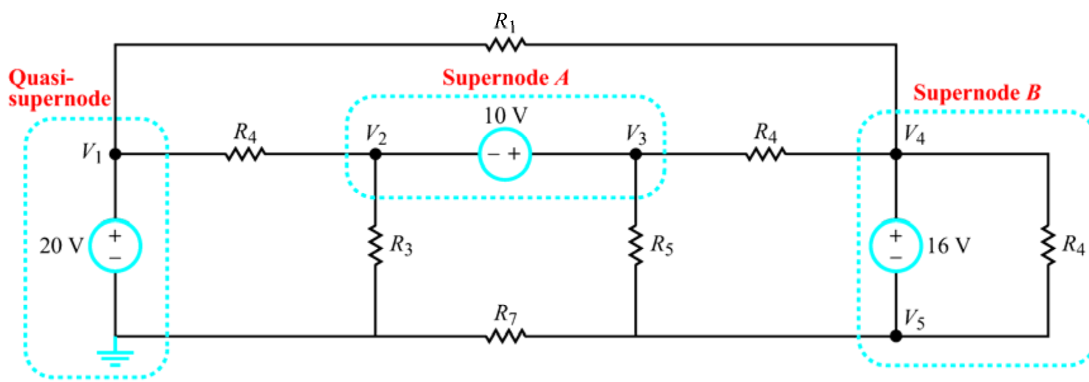
→ v1 = 40V

What if you wanted to solve for v1 using source transforms?



Would this be easier than a node voltage solution? Why?

Nodes, Supernodes, and Quasi-Supernodes



Here's an exercise from another textbook that we saw before …apply the supernode concept to determine I in the circuit.

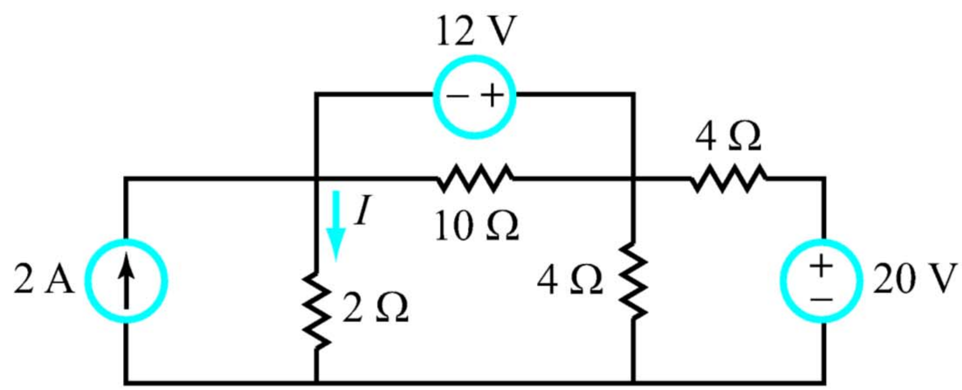
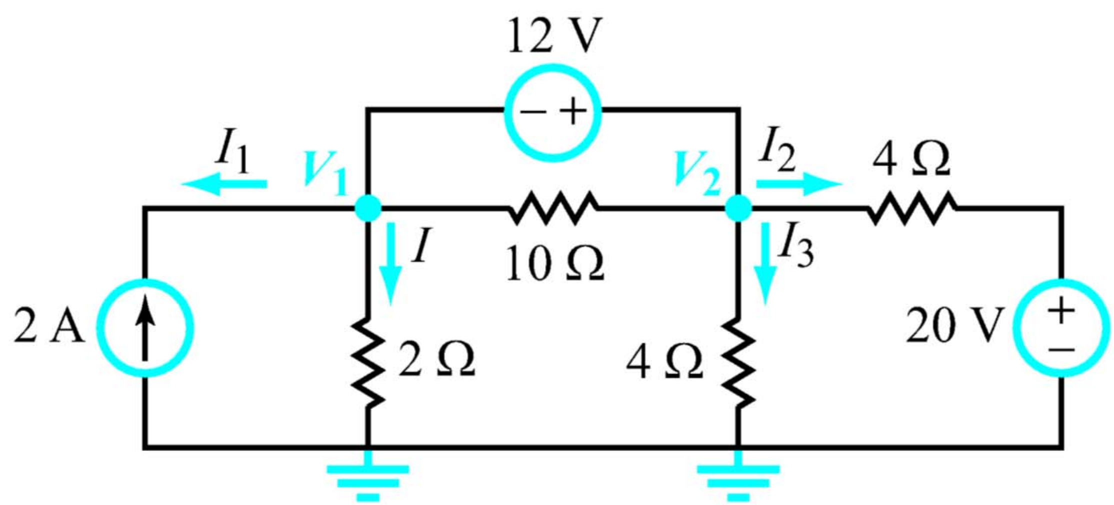
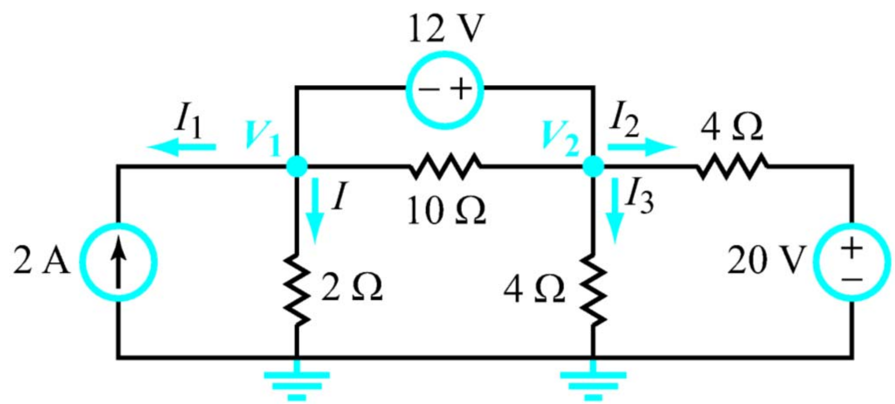


Figure E3.3

Number essential nodes and chose a reference node. Is there a supernode in this circuit?





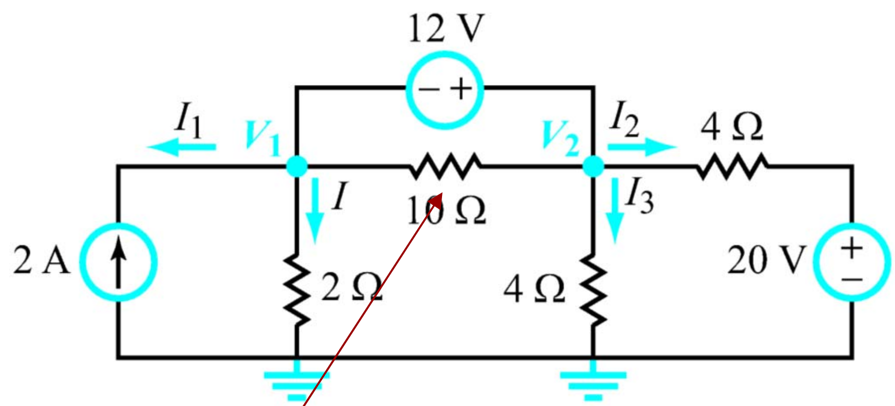
V1 and V2 form a supernode so we can write a single KCL equation for it:

$I_1 + I + I_2 + I_3 = 0$

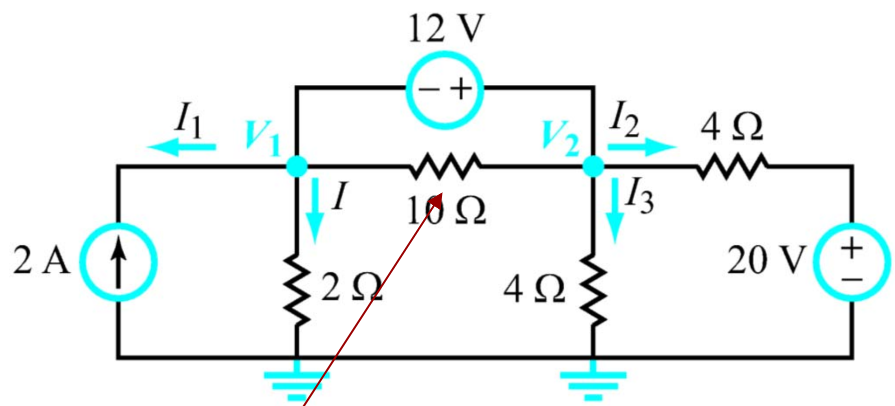
$I_1 = 2$, $I = V_1/2$, $I_3 = V_2/4$, $I_2 = (V_2-20)/4$ and KVL gives us $V_2-V_1=12$

So, solving for V_1 and substituting:

$I = 0.5A$, $V_1=1$, $V_2 = 13$

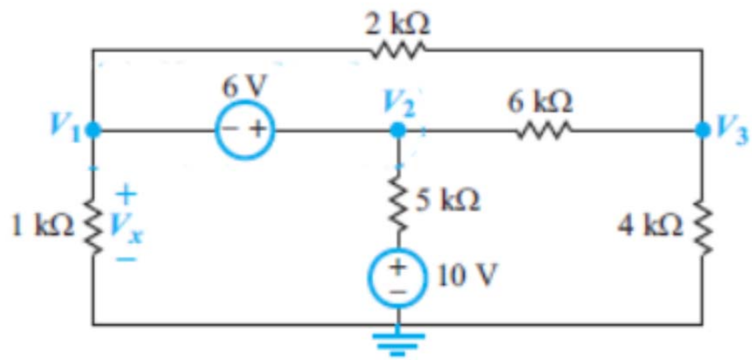


How about the current through this resistor – we didn't account for it in any of the calculations? Does the behavior of this circuit depend on the value of this resistor?

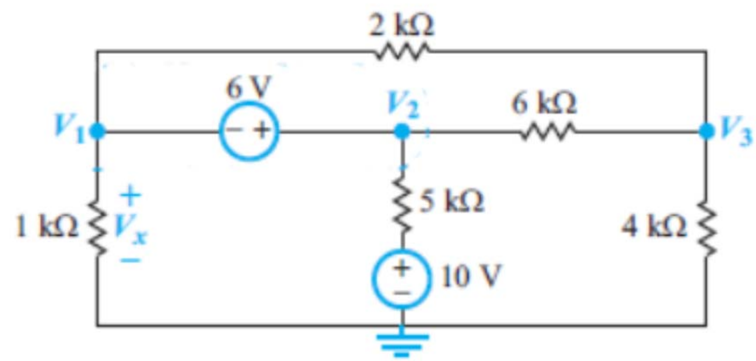


This is one of these “superfluous resistors” we’ve talked about before… it’s a resistor in parallel with a voltage source that can be removed from a circuit without affecting anything. Another way to look at it -- if we had included it in the supernode equation it would have added one current term to the v1 part of the equation and the negative of the same current term to the v2 part of the equation ….cancelling each other out.

Here’s another node voltage problem



What is Vx …the voltage drop across the 1K resistor?



For V1 and V2 of the supernode: $$\frac{V_1}{10^3}+\frac{V_1-V_3}{2\times10^3}+\frac{V_2-10}{5\times10^3}+\frac{V_2-V_3}{6\times10^3}=0$$

And for V3: $$\frac{V_3-V_1}{2\times10^3}+\frac{V_3-V_2}{6\times10^3}+\frac{V_3}{4\times10^3}=0.$$

Also …there is the auxiliary equation: V2-V1 =6

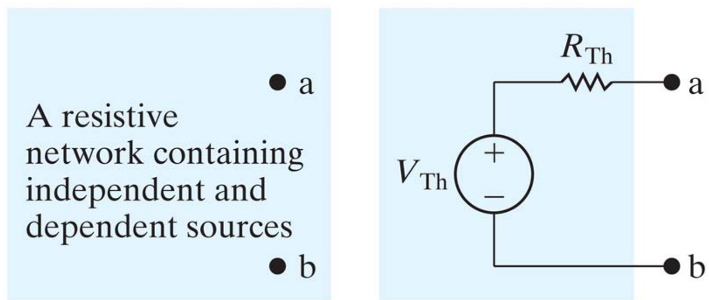
MATLAB gives:

V1= 0.38V
V2= 6.38V
V3= 1.37V

So Vx = V1 = 0.38V

Thevenin Equivalent Circuit

Thevenin (and Norton) equivalent circuits are ways of replacing complicated and/or irrelevant parts of a circuit with a simple circuit based on the behavior of the circuit at a particular pair of terminals using source transformations.



(a) (b)

The Thevenin equivalent circuit for a resistive network is a voltage source, V_{th} in series with a resistor, R_{th}. We chose these values by measuring the open circuit voltage for the network …this is V_{th} …then we measure the short circuit current out of the network. Since this will be V_{th}/R_{th}, the value of R_{th} is: $R_{th} = V_{th}/i_{sc}$. The Norton equivalent is just the source transform of the Thevenin circuit into a current source in parallel with a resistor.

Thevenin Equivalent Circuit – General Calculation Strategy

- **Obtain vth** by calculating the voltage across the two specified terminals when no load is present (open circuit voltage)

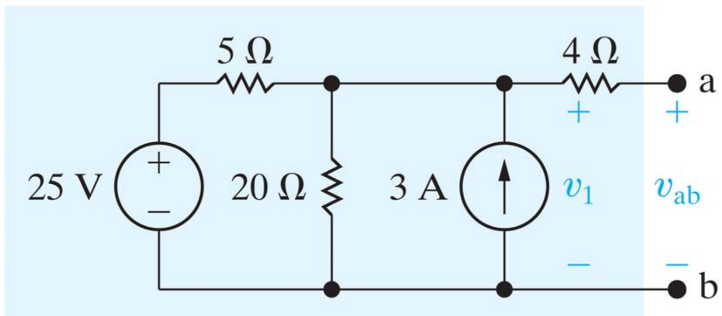
- **Obtain Rth** by either:

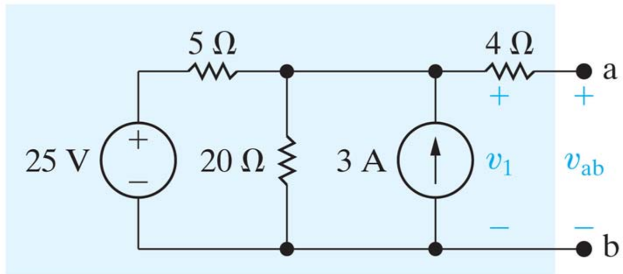
  - Calculating the current that will flow between the specified terminals in a short circuit. Rth is obtained from Rth = vth/Isc OR

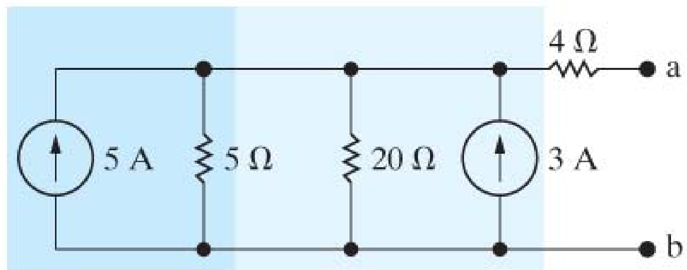
  - If the circuit doesn't contain dependent sources, you can calculate the equivalent resistance between the specified terminals after all independent voltage sources are deactivated (replaced with short circuits)and all independent current sources are deactivated (replaced with open circuits). The equivalent resistance is Rth, the Thevenin resistance. OR

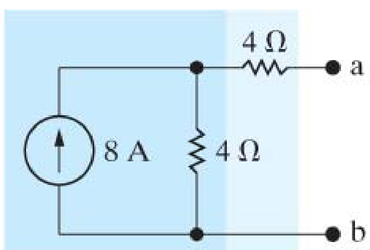
  - If the circuit contains independent and dependent sources, Rth can be determined by deactivating independent sources, and adding an external source (vex or iex)…then solve the circuit to determine the current iex or voltage vex supplied by the external source. Rth = vex/iex

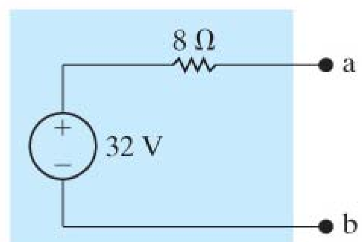
Sometimes it will be possible to obtain a Thevenin equivalent circuit simply by doing a series of source transforms. For example – what is the Thevenin equivalent circuit at terminals a and b? You've seen these kinds of solutions many times before.



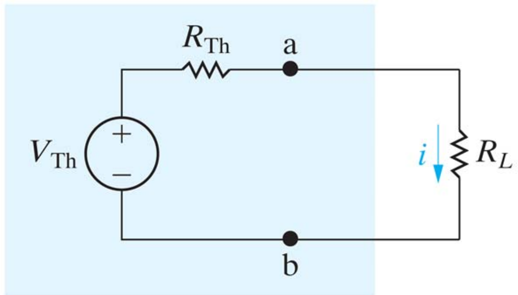








The Thevenin equivalent!



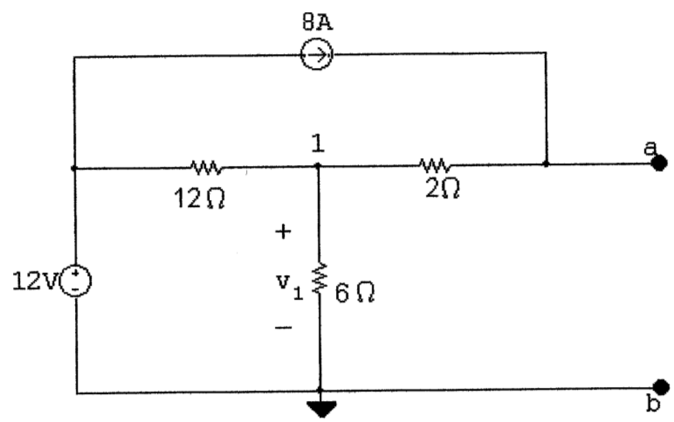
power transfer is maximum when the load resistance, $R_L = R_{Th}$

and we can determine the value of the maximum power by substituting into:

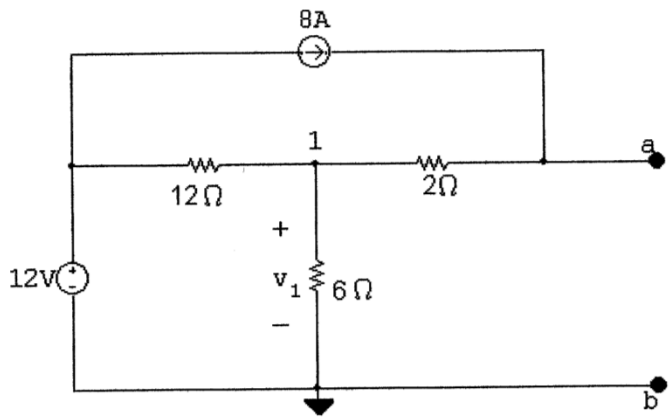
$p = i2\ R_L = (Vth / [R_{Th} + R_L])^2\ R_L$

$p_{max} = V_{Th}^2 / 4R_L$

Notice: if the problem asks only for the load resistance that will transfer maximum power – you need to calculate Rth only – not Vth also.



What load resistor at terminals a and b will result in maximum power transfer to the load? What will that power be?



We can get Rth by method # 2 – deactivating the sources and combining resistances.

$$R_{\text{Th}} = 2 + \frac{(12)(6)}{18} = 6\,\Omega$$

We can calculate v1 using node voltage analysis….and Vth is v1 plus the voltage drop across the 2Ω resistor.

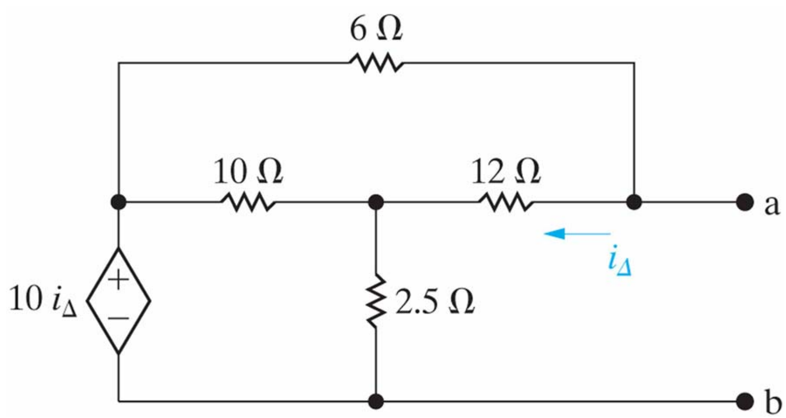
$$\frac{v_1 - 12}{12} + \frac{v_1}{6} - 8 = 0$$

$$v_1 = 36 \text{ V}$$

So $p_{max}=(52)^2/4*6 = 112.67$ W

$$v_{\text{Th}} = v_1 + (2)(8) = 52 \text{ V}$$

Same question ……what load resistor at terminals a and b will result in maximum power transfer to the load? What will that power be? Notice – this circuit has only a dependent source! What does this tell you about Vth? What does this tell you about methods for calculating Rth?

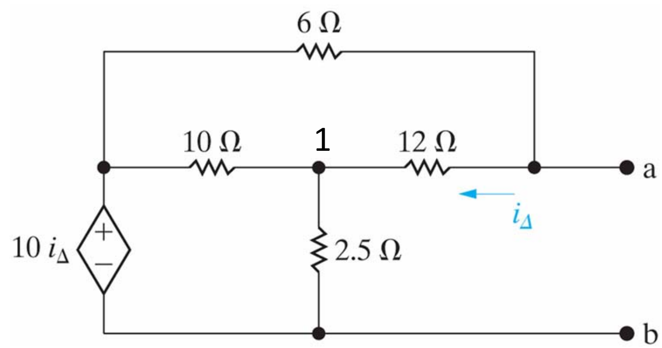


Well…..if you remembered that circuits with only dependent sources have Vth = 0 ..
..and you also remembered that:

Pmax = $(V_{th})^2/4R_{th}$

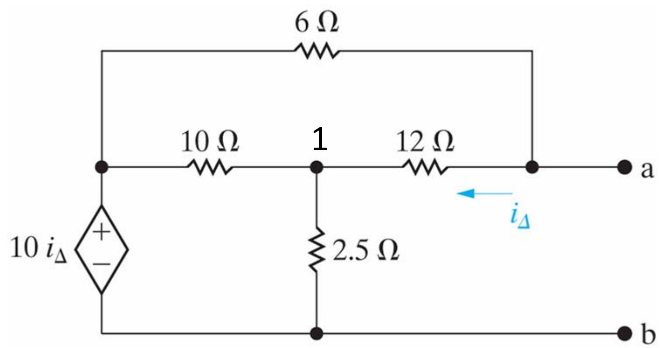
then you'd know that the maximum power transferred to the load resistor would have to be zero!

But, if you had forgotten this …you can always solve for Vth and Rth. To get V_{th} we calculate the open circuit voltage at terminals a and b. We get this by solving the single node voltage equation at node 1.



$$\frac{v_1 - 10 i_\Delta}{10} + \frac{v_1}{2.5} - i_\Delta = 0$$

We also see there is another relationship between v_1 and i_Δ because of the voltage divider created by the 6Ω, 12Ω and 25Ω resistor across the dependent voltage source.

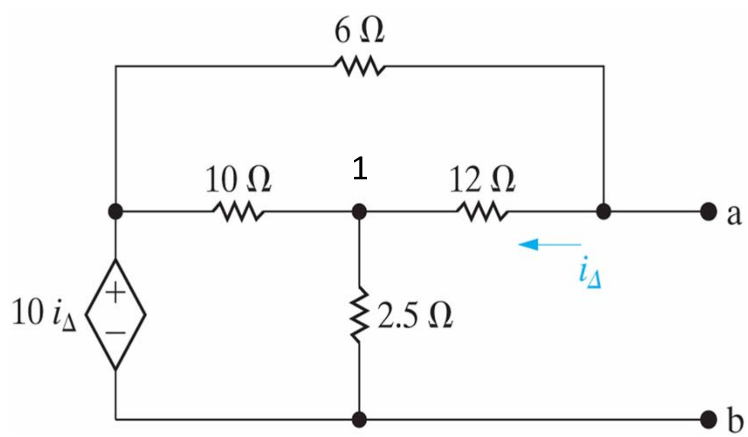


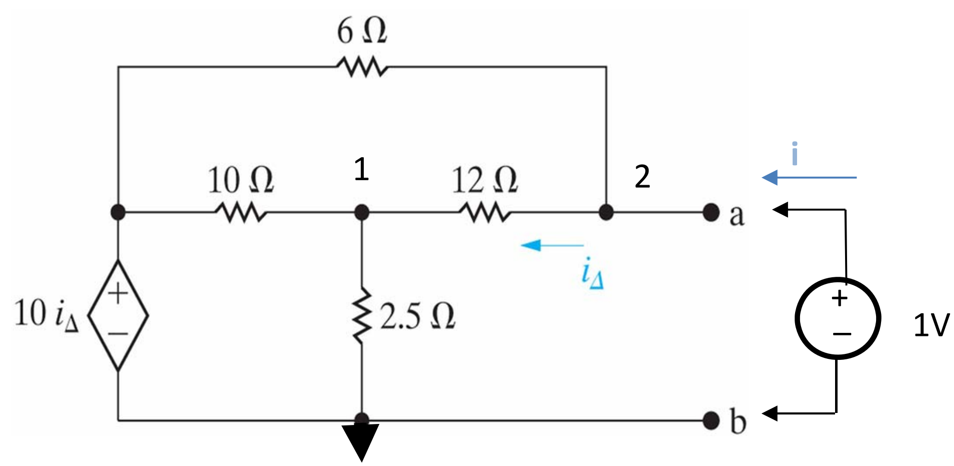
So: $$v_1 = 10 i_\Delta \frac{25}{6+12+25} = 12.195 i_\Delta$$

From the first KCL equation: $$\frac{v_1 - 10 i_\Delta}{10} + \frac{v_1}{2.5} - i_\Delta = 0$$ so $$v_1 = 3.64 i_\Delta$$

The value of i_Δ that satisfies both equations is zero…so 10 i_Δ = 0 and Vth =0 and since Pmax = $(V_{th})^2/4R_{th}$ the maximum power transferred is also zero.

What if, instead of maximum power, the problem asked you to determine the Thevenin equivalent circuit for this thing. You already know Vth = 0 …how would you calculate Rth for the Thevenin equivalent? Which method would you use?

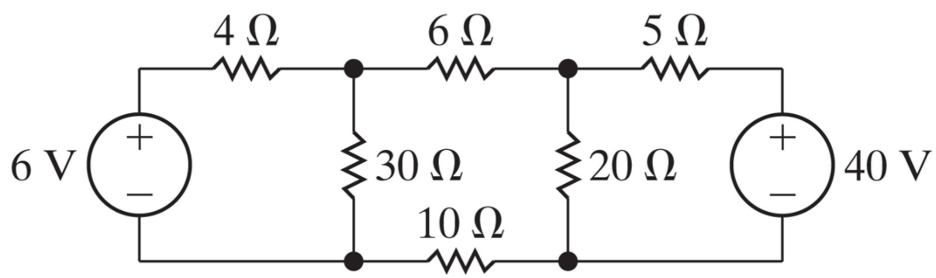




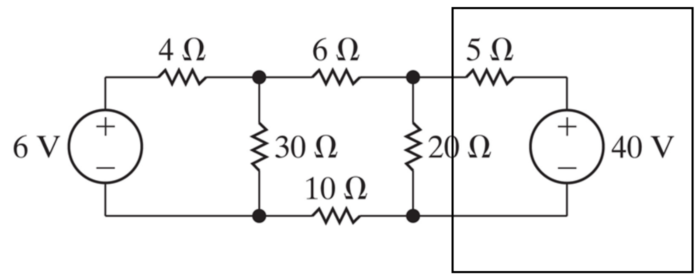
We'll use method #3 …and attach a test voltage source – here we've chosen it to be 1V. We want to calculate the current, i, flowing out of the test source …since we know from Ohm's law that the resistance from a to b will be the test voltage, 1V, divided by the current, i. If there were any independent sources in the circuit, we would deactivate them …but we leave dependent sources alone.

I'll leave the algebra to you. You'll have to solve one KCL equation at node 1 since node 2 will be a quasi-supernode (because of the 1V source) and the other node is also a quasi-supernode.

We want to find the power (absorbed or supplied) by the 6V source

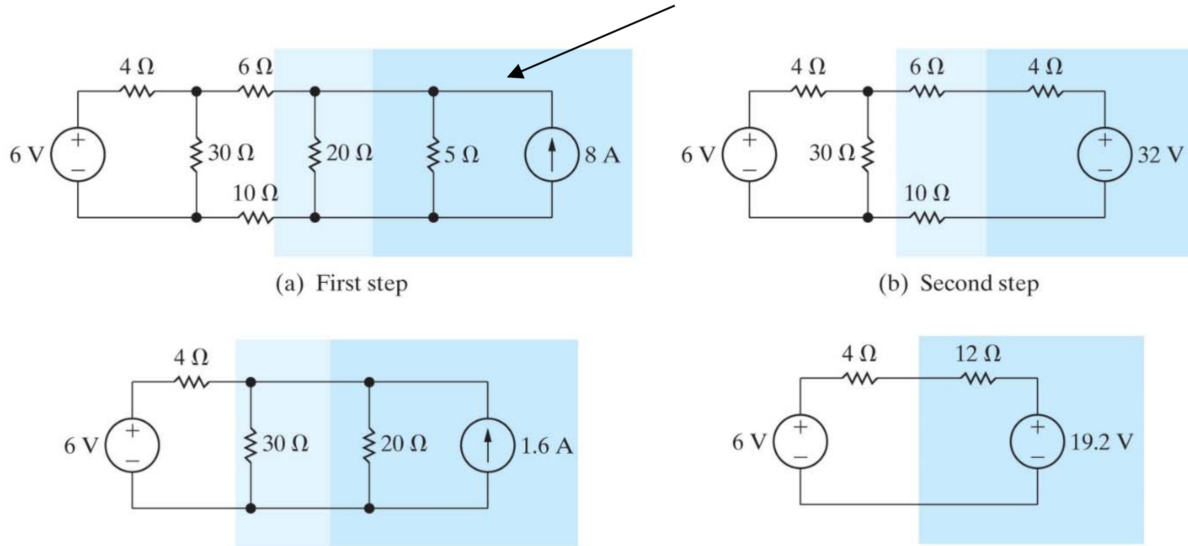


Since we only care about the current flowing into or out of the 6V source, probably the easiest way to solve this circuit is through source transforms on the 40V source – leaving the 6V source alone.



Remember

$$\boxed{i_s = v_s/R}$$



(a) First step

(b) Second step

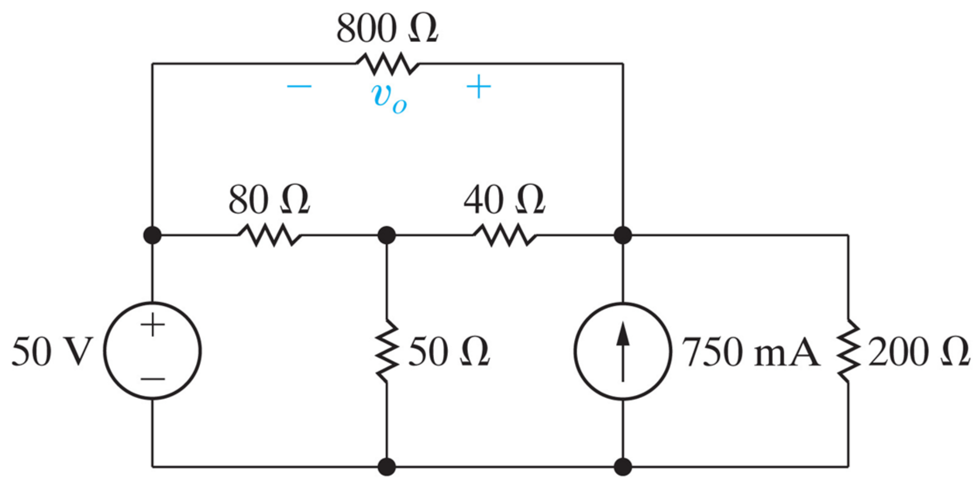
(c) Third step

(d) Fourth step

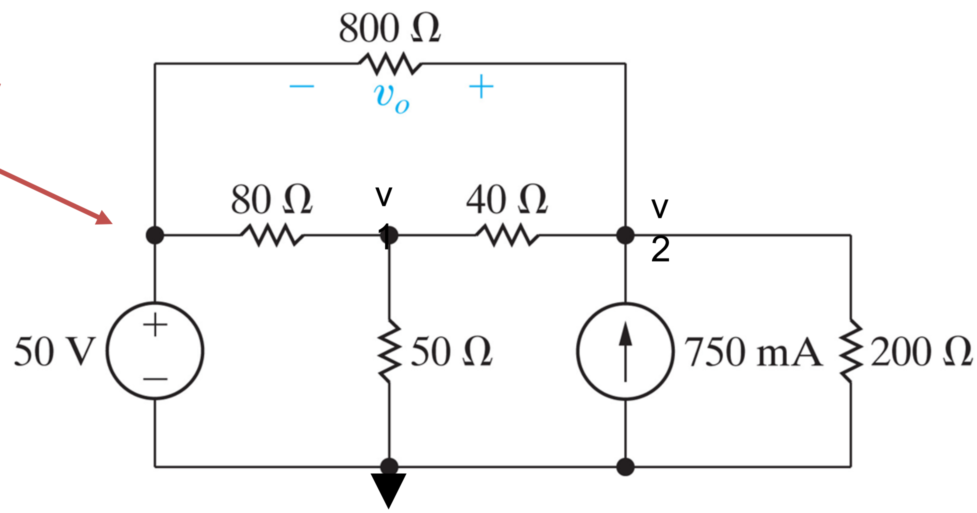
Now it's trivially easy to determine the current through the 6V source – it's (19.2 -6)/16 = 0.825A

flowing <u>backwards</u> through the voltage source so the source <u>absorbs</u> 4.95 W

Find v_0 using Node Voltage Method



Why only two node voltage equations? Why are we ignoring this node?



Number the nodes, pick a reference node, and write the KCL equations at the two remaining essential nodes.

(v1 -50)/80 + v1/50 + (v1-v2)/40 = 0 at node v1

v2/200 – 0.75 + (v2-v1)/40 + (v2-50)/800 = 0 at node v2

In standard form:

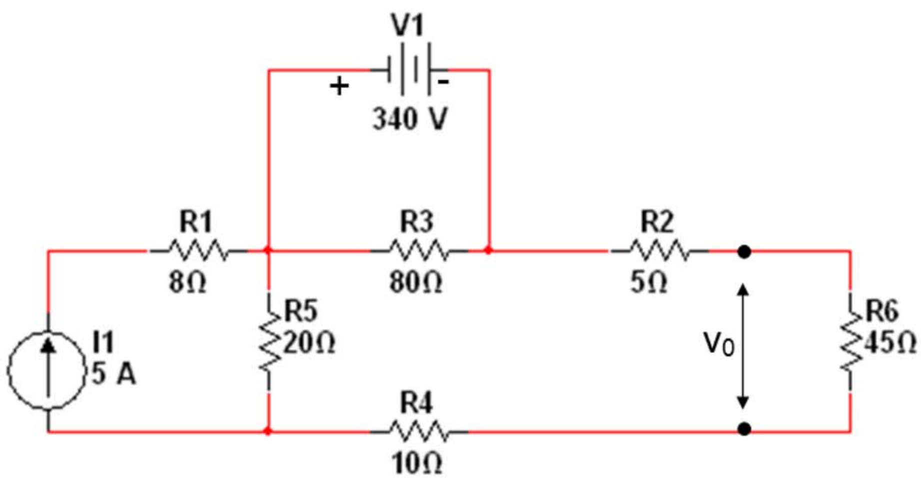
$$\begin{aligned} v_1\left(\frac{1}{80}+\frac{1}{50}+\frac{1}{40}\right) &+ v_2\left(-\frac{1}{40}\right) &&= \frac{50}{80} \\ v_1\left(-\frac{1}{40}\right) &+ v_2\left(\frac{1}{40}+\frac{1}{200}+\frac{1}{800}\right) &&= 0.75+\frac{50}{800} \end{aligned}$$

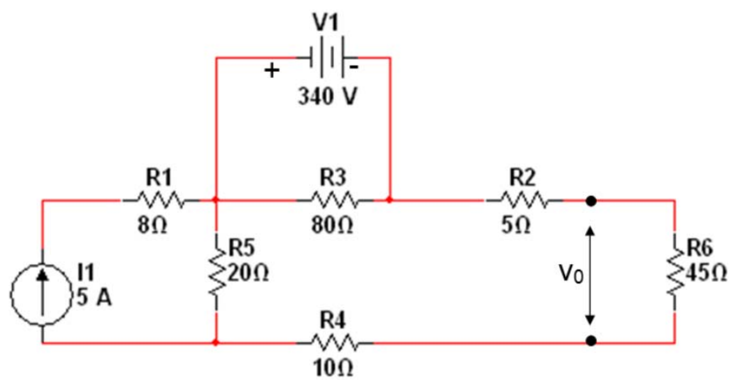
v1 = 34V;

v2 = 53.2V

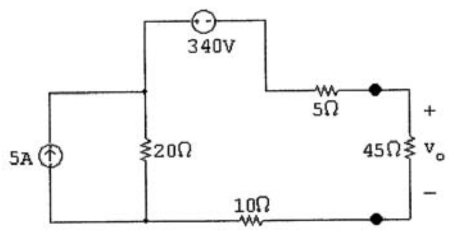
So …… v0 = v2 – 50 = 3.2V

Solve for V_0 Are there any simplifications that can be done before starting the solution?

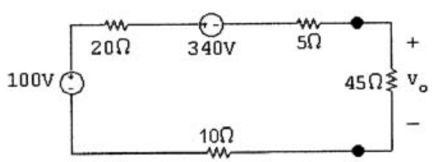




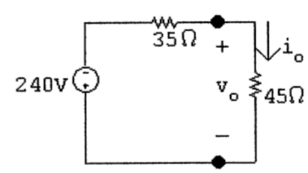
P 4.61 [a] First remove the 8 Ω and 80 Ω resistors:



Next use a source transformation to convert the 5 A current source and 20 Ω resistor:

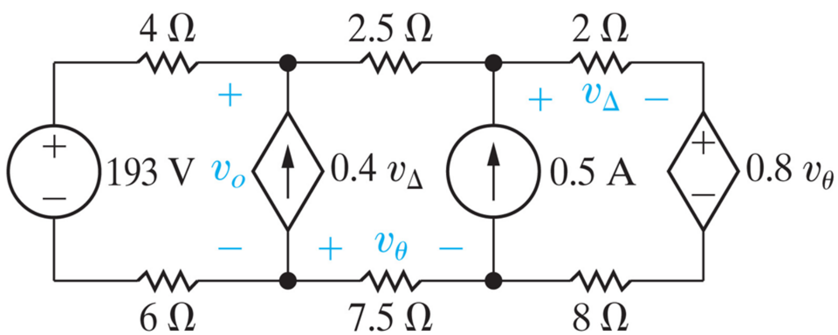


which simplifies to



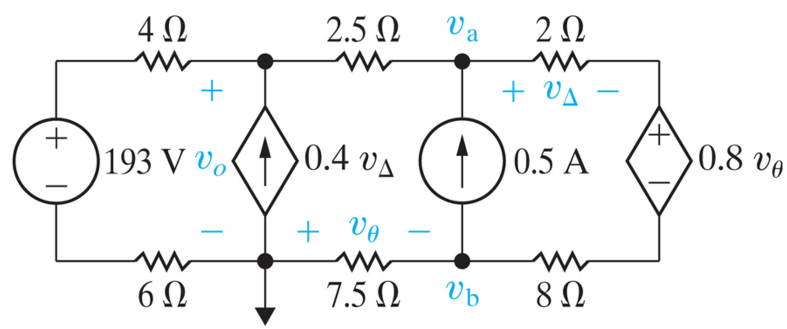
$$\therefore \quad v_o = \frac{45}{80}(-240) = -135 \text{ V};$$

Can you do a node voltage analysis of this circuit to determine v0?



How many nodes are present? What is a good choice for reference node?

node voltage analysis



(v0 – 193)/10 – 0.4 v_Δ + (v0-va)/2.5 = 0

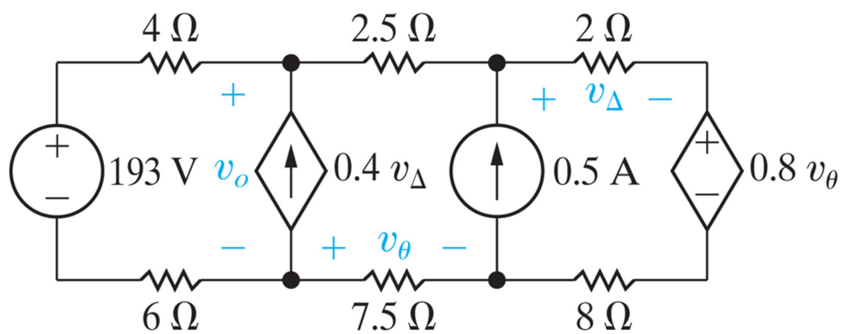
(va-v0)/2.5 + 0.5 + [va – (vb + 0.8vθ)]/10 = 0

vb/7.5 + 0.5 + (vb + 0.8 vθ – va)/10 = 0

And the constraint equations are:

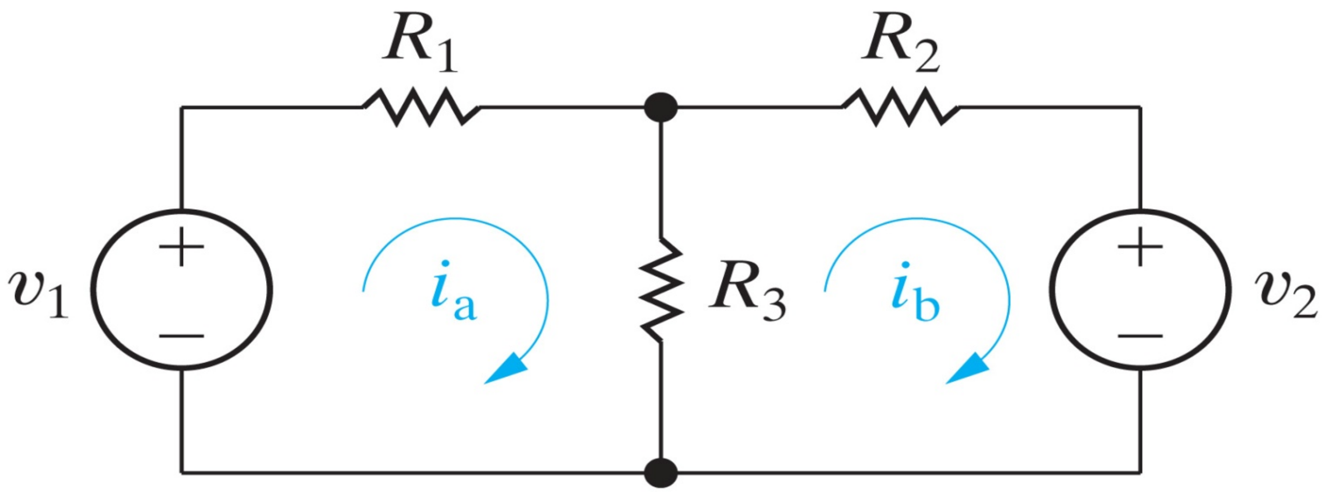
vθ = -vb

v_Δ = [va – (vb + 0.8 vθ)]/10 * 2



Could you have done some simplifications ? It's clear that if the two dependent sources had been independent sources, source transforms would have allowed solution of the circuit using only Ohms Law. However, we need to preserve the 7.5 Ohm and 2 Ohm resistor (not combine them) because they define the values of the dependent current and voltage sources. Probably the best strategy would be to do the ordinary node voltage solution and solve the 3 x 3 matrix.

Solve this circuit by the mesh current method:



For the ia mesh: ia R1 + (ia – ib) R3 – v1 = 0

For the ib mesh: (ib – ia) R3 + ib R2 + v2 = 0

Remember the sign convention for voltage sources!

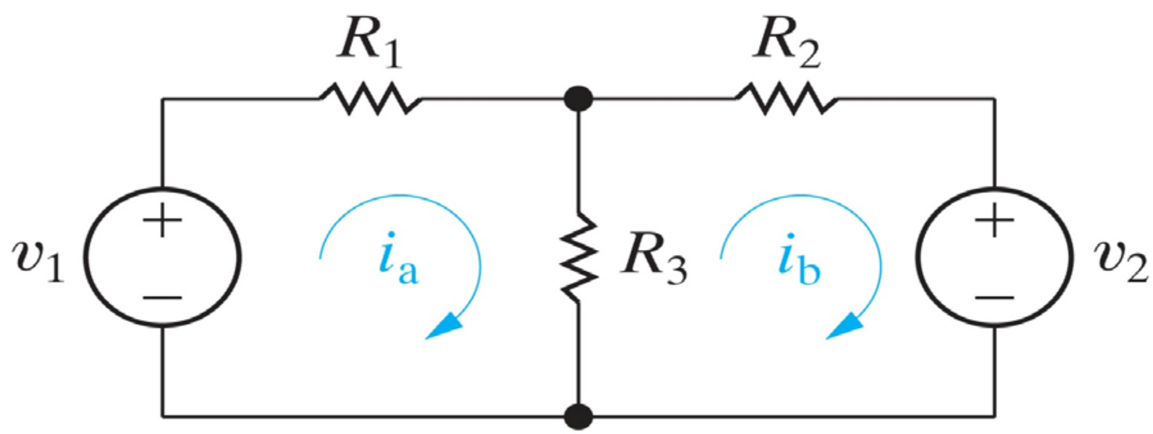
So, in standard form:

ia (R1+ R3) – ib (R3) = v1

ia (-R3) + ib (R2 + R3) = -v2

And we can solve the circuit.

Why didn't we choose a reference node in the circuit?



We could also write down the matrix equation for this circuit directly by inspection. Why does this circuit qualify for this approach?

In general:

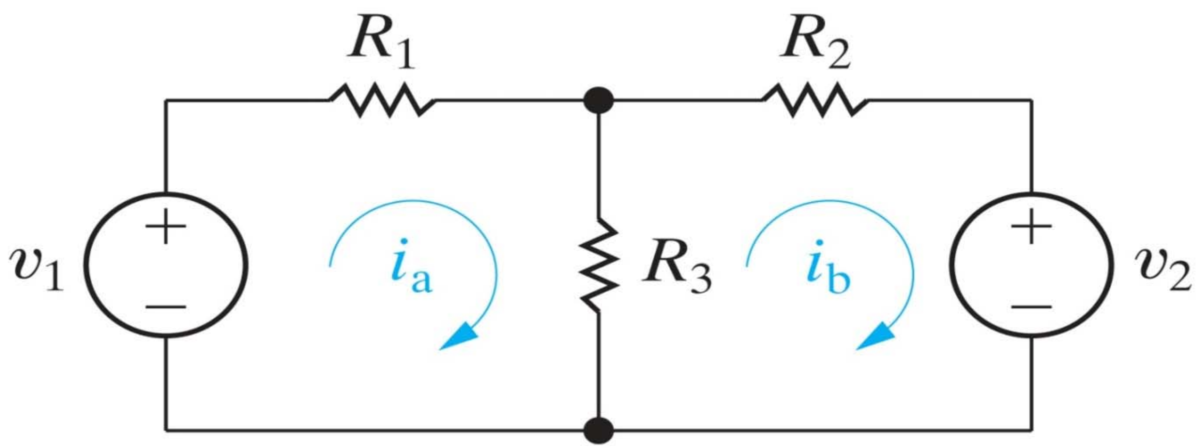
$$\mathbf{V} = \mathbf{RI}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & R_{22} & R_{23} & R_{24} \\ R_{31} & R_{32} & R_{33} & R_{34} \\ R_{41} & R_{42} & R_{43} & R_{44} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

The **R** matrix is symmetric ($r_{jk} = r_{kj}$) and that all off-diagonal terms are either negative or zero.

- the R_{kk} terms are the sum of all of the resistances in the k mesh
- the R_{jk} terms are the negative sum of all of the resistances common to the k and the j mesh
- the v_k term (k^{th} component of the vector **V**) is equal to the algebraic sum of all of the **independent** voltages in the k^{th} mesh.

If the circuit contains dependent voltage sources you can certainly do a mesh current solution …but you can't write it down by inspection!



So, by inspection:

$$\mathbf{V} = \mathbf{RI}$$

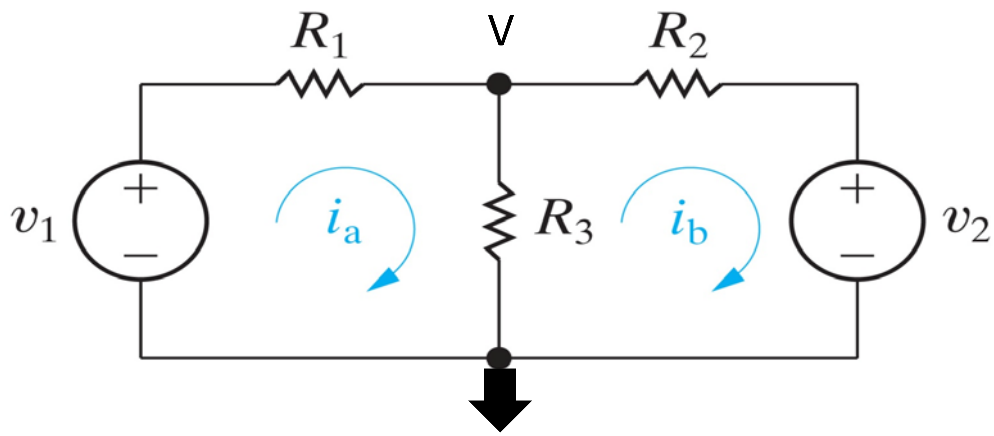
$$\begin{bmatrix} v_1 \\ -v_2 \end{bmatrix} = \begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix}$$

Compare to the standard form equation we got from writing KVL:

ia (R1+ R3) – ib (R3) = v1

ia (-R3) + ib (R2 + R3) = -v2

Just for fun – let's solve this circuit by the node voltage method:



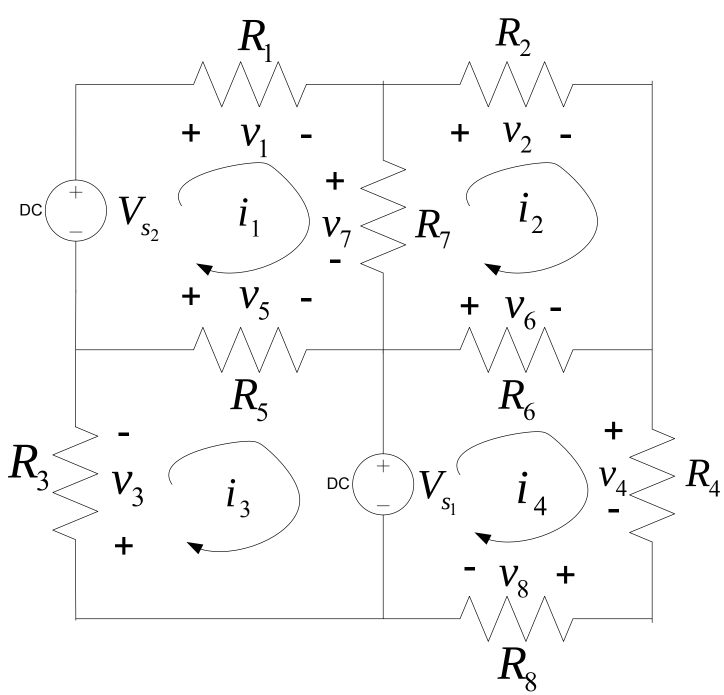
$$\frac{V - v_1}{R_1} + \frac{V - v_2}{R_2} + \frac{V}{R_3} = 0$$

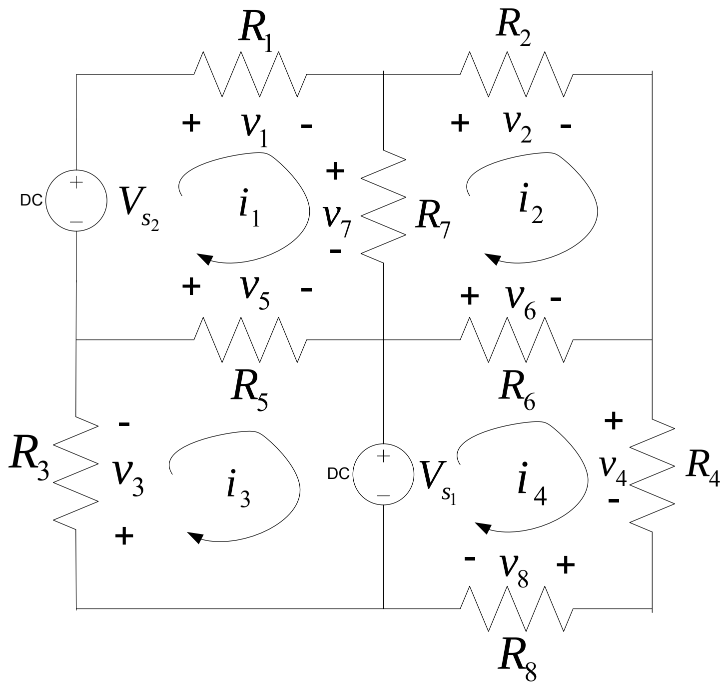
Once we solve this equation for V, how would we get ia and ib?

Could we get a solution for V by source transforms and Ohms Law?

Let’s try a solution by inspection for a bigger circuit:

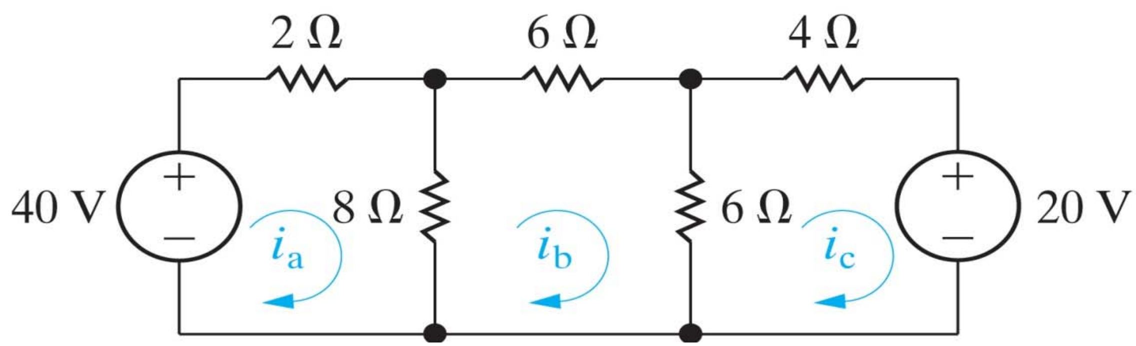
All of the voltage sources are independent so it should be OK – how big will the resistance matrix be …….i.e. what will the matrix rank be ?





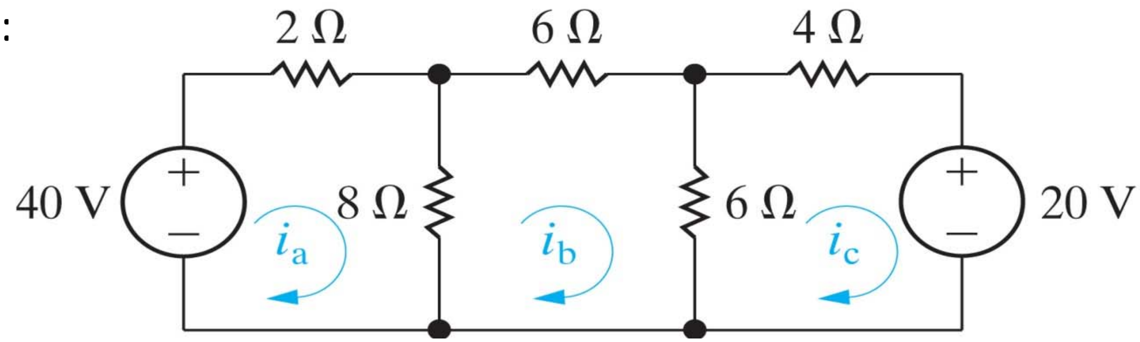
$$\begin{pmatrix} R_1+R_5+R_7 & -R_7 & -R_5 & 0 \\ -R_7 & R_2+R_6+R_7 & 0 & -R_6 \\ -R_5 & 0 & R_3+R_5 & 0 \\ 0 & -R_6 & 0 & R_4+R_6+R_8 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} V_{s_2} \\ 0 \\ -V_{s_1} \\ V_{s_1} \end{pmatrix}$$

Here’s another one we did earlier:



What is the rank of the resistance matrix (i.e. how big is it?)

By inspection:

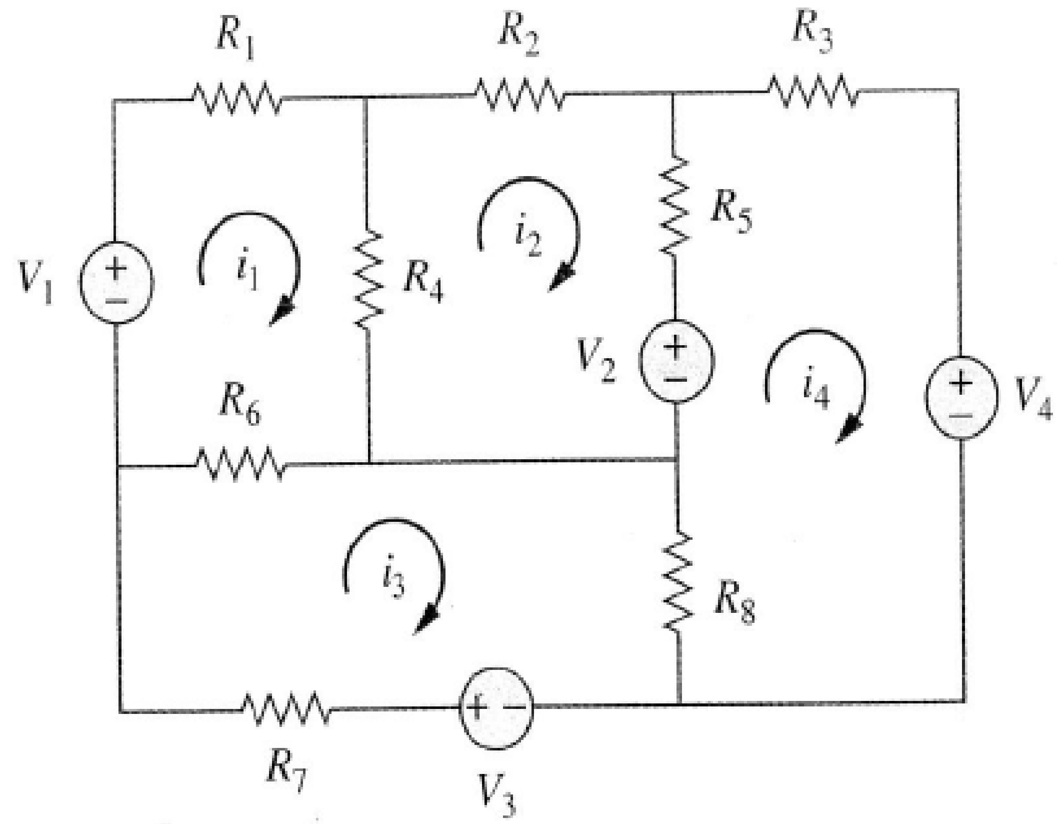


$$\begin{bmatrix} 10 & -8 & 0 \\ -8 & 20 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 40 \\ 0 \\ -20 \end{bmatrix}$$

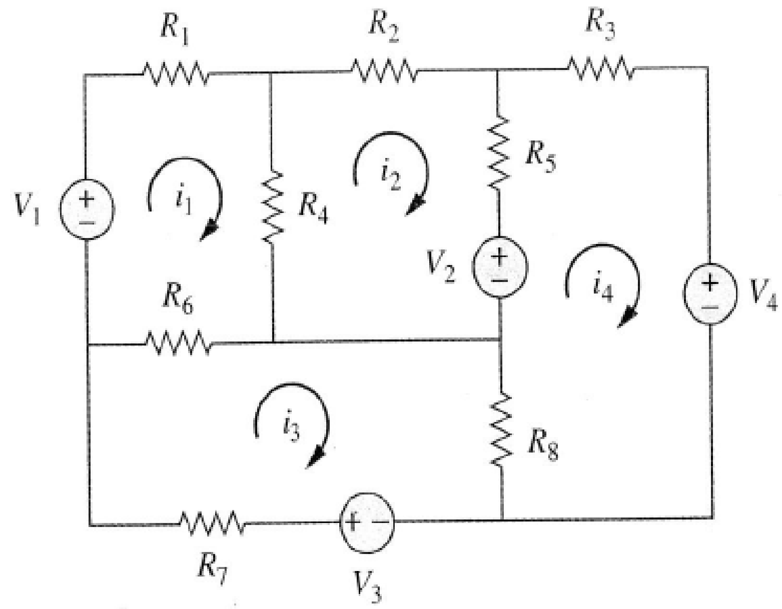
From MATLAB:

```
>> I=R\v

I =

    5.6000
    2.0000
   -0.8000
```

Here's another one. All the sources are independent voltage sources and no supermeshes……so it's a good candidate for solution by inspection.



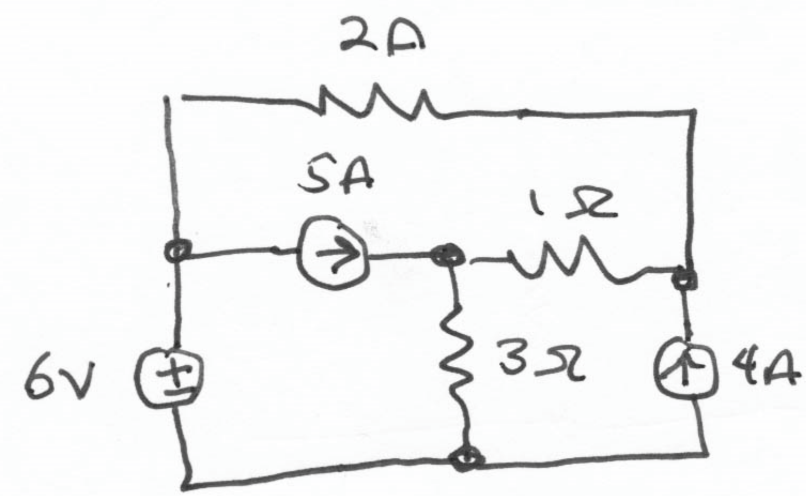
What does the matrix equation look like? How big is it?



$$\begin{bmatrix} V_1 \\ -V_2 \\ V_3 \\ -V_4+V_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \begin{bmatrix} R_1+R_4+R_6 & -R_4 & -R_6 & 0 \\ -R_4 & R_4+R_2+R_5 & 0 & -R_5 \\ -R_6 & 0 & R_6+R_7+R_8 & -R_8 \\ 0 & -R_5 & -R_8 & R_3+R_5+R_8 \end{bmatrix}$$

It's straightforward to do a mesh current solution (by inspection or otherwise) for a circuit that contains independent voltage sources. We know that circuits with dependent voltage sources can't be solved by inspection and must be addressed by the normal mesh current method.

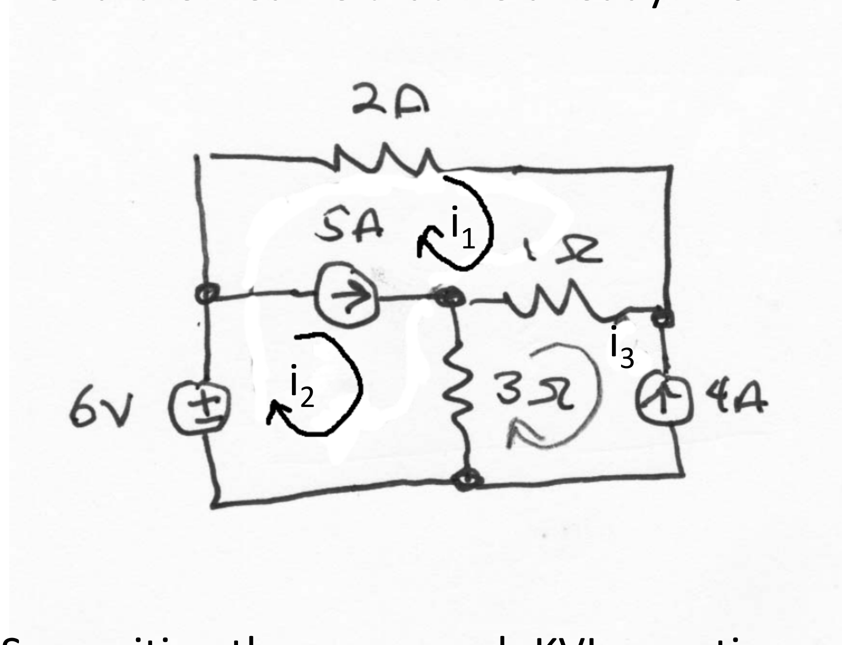
How about circuits that contain independent voltage and current sources:



How do we solve this kind of problem?

We recognize that the top mesh and the left hand mesh are separated by a current source …so these two meshes get combined into a supermesh, incorporating i_1 and i_2

We further realize that we already know mesh current, i_3 .. it is – 4A.



$$i_3 = -4$$

$$i_2 - i_1 = 5$$

So , writing the supermesh KVL equation: $2i_1 + 1(i_1 - i_3) + 3(i_2 - i_3) - 6 = 0$

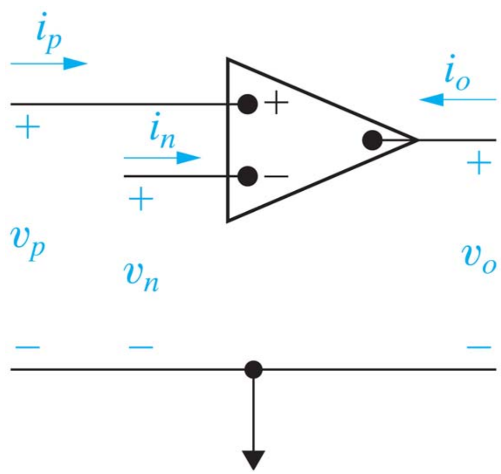
$$3i_1 + 3ii_2 - 4i_3 - 6 = 0$$

so $3i_1 + 3(5 + i_1) - 4(-4) - 6 = 0$ $\quad i_1 = \frac{25}{6} \quad i_2 = \frac{55}{6} \quad i_3 = -4$

Operational Amplifiers

$v_p = v_n$ and $i_p = i_n = 0$

The op amp "Golden Rules" for ideal operational amplifier circuits with negative feedback



These assumptions plus whatever else you've learned about circuit theory (KCL, KVL, etc.) will allow you to solve every op amp circuit!

The relationship $v_n = v_p$ for an ideal op amp with negative feedback comes about because the open loop gain (without feedback) for the op amp is:

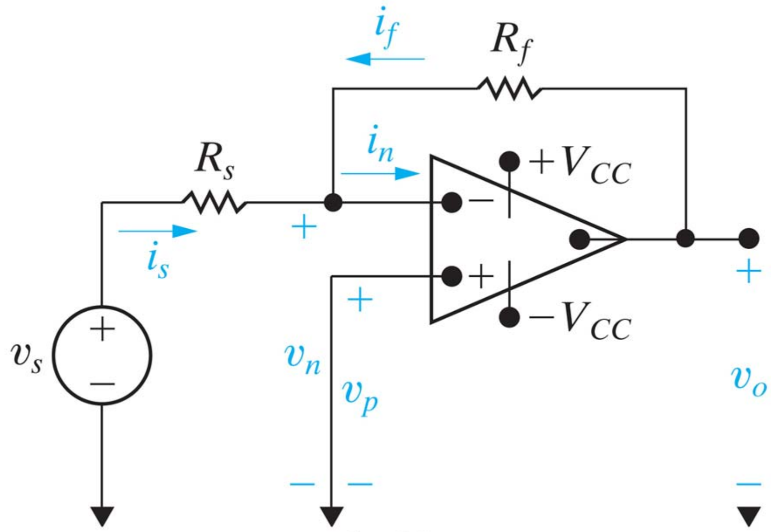
$$\boxed{v_0 = A\,(v_p - v_n)}$$

…. and A is a very large (almost infinite) number. Consequently, the feedback from the op amp output to the negative (inverting) input of the op amp will act to reduce any voltage difference that exists between the two inputs. In the ideal op amp, we assume that negative feedback causes the difference in voltage between the two inputs to be zero….although we know if it were actually zero …then the output voltage would be zero also.

The other “golden rule” relation, $i_p = i_n = 0$ is not because of feedback …rather it is because we assume in an ideal op amp that the input terminals are “inert” ..they have infinite input impedance and don’t draw any current.

You really don’t need to understand the origin of the op amp “Golden Rules” to solve op amp circuits – just use them along with KCL, KVL, and Ohms Law.

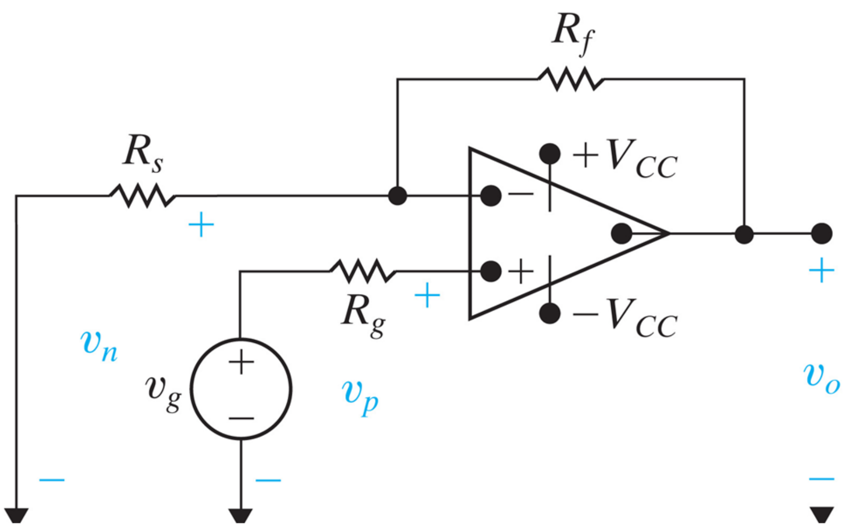
Basic Inverting Amplifier   can you derive the gain equation?



$$v_0 = -(R_f / R_s)\, v_s$$

Notice: output is inverted from input

Basic Non-inverting Amplifier can you derive the gain equation?

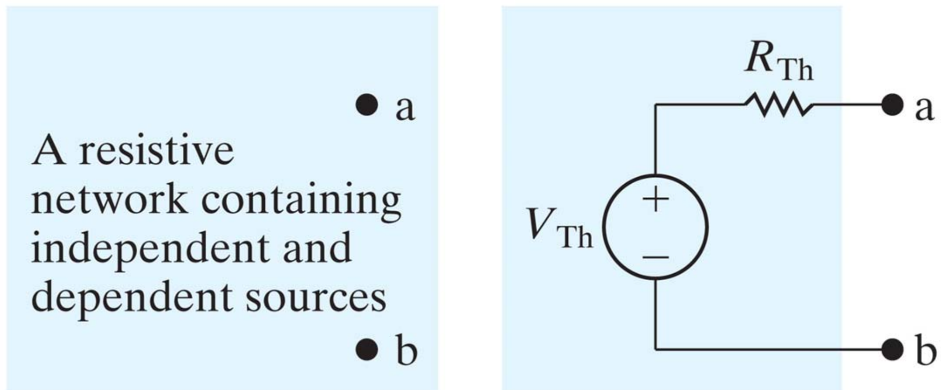


$$v_0 = v_g(R_s + R_f) / R_s$$
$$= v_g(1 + R_f/R_s)$$

Notice: (1+Rf/Rs) ..not (Rf/Rs) as with the inverting amplifier

Just for one more time ---a quick review of Thevenin and Norton Equivalent Circuits

Any resistive network containing independent and dependent sources can be replaced at a specified pair of terminals by a single voltage source (Vth) in series with a single resistor (Rth)



(a) (b)

How do we determine the values of Vth and Rth?

Thevenin Equivalent Circuit – General Calculation Strategy (this is important)

- **Obtain Vth** by calculating the voltage across the two specified terminals when no load is present (open circuit voltage) – sometimes you can do this by using source transforms.
- **Obtain Rth** by:

1. Calculating the current that will flow between the specified terminals in a short circuit. Rth is obtained from Rth = Vth/Isc

   …….or

2. If the circuit doesn't contain dependent sources, you can calculate the equivalent resistance between the specified terminals after all independent voltage sources are replaced with short circuits and all independent current sources are replaced with open circuits. This in effect is "deactivating" the independent sources and and equivalent resistance in this case is Rth, the Thevenin resistance. ….or

3. If the circuit contains independent and dependent sources, Rth can be determined by deactivating independent sources, and adding an external source (vex)…then solve the circuit to determine the current iex supplied by the external source. Rth = vex/iex

So …you always calculate the open circuit voltage at the two terminals to get Vth (if you can't use source transforms) …..and then you can use one of three methods to calculate Rth.

If there are no dependent sources present …method 2 is usually the easiest way to get Rth.

If dependent sources are present …method 1 or method 2 will work.