

1. Give the entire count sequence of binary patterns for 4-bit unsigned integers.

0000  
0001  
0010  
0011  
0100  
0101  
0110  
0111  
1000  
1001  
1010  
1011  
1100  
1101  
1110  
1111

At which transition point does an overflow occur if the patterns represent:

- (a) An unsigned integer? From 1111 to 0000  
(b) A 2's complement integer? From 0111 to 1000
2. What is the decimal value represented by the 12-bit binary number  $11001001.0101_2$  when interpreted as:
- (a) An unsigned number? 201.3125  
(b) A 2's-complement number? -54.6875
3. Use polynomial evaluation to convert:
- (a)  $101101_2$  to base 10. 45  
(b)  $BEEF_{16}$  to base 10. 48879
4. Use repeated division to convert:
- (a)  $150_{10}$  to base 2. 10010110  
(b)  $1500_{10}$  to base 16. 5DC

5. Use repeated multiplication to convert: *Note: If the process repeats indefinitely, calculate as few digits as necessary to provide same or better resolution.*

(a)	$0.9_{10}$ to base 2.	0.1 1100 1100 1100 1100 .....	→ 0.1110
(b)	$0.9_{10}$ to base 16.	0.E666666 .....	→ 0.E

6. Use shortcuts based on power relationships to convert:

(a)	$ACE5_{16}$ to base 2.	1010 1100 1110 0101
(b)	$FA.CE_{16}$ to base 2.	1111 1010 . 1100 1110
(c)	$101011.01101_2$ to base 16.	2B.68

7. Convert the following 2's complement numbers to decimal:

(a)	01010101.	+85
(b)	10101010.	-86
(c)	1000.0001	-7.9375
(d)	1001.0110	-6.625
(e)	0111.1110	+7.875

8. Convert the following decimal numbers to 2's complement:

(a)	-6.7	1001.0101
(b)	-37.1	1011010.1111
(c)	-100	10011100
(d)	-7.7	1000.0101

9. Find the 2's complement of the following binary numbers:

(a)	01010101.	10101011.
(b)	10101010.	01010110.
(c)	1000.0001	0111.1111
(d)	1001.0110	0110.1010
(e)	0111.1110	1000.0010

10. Consider a 2's complement number represented by n bits, with two bits to the left of the binary point

(e.g.,  $b_1b_0.b_{-1}b_{-2}\cdots b_{n-2}$ ).

- |     |  |               |
|-----|--|---------------|
| (a) | Give an algebraic expression in terms of n for the <u>positive</u> value that has the smallest non-zero magnitude. | $+2^{-(n-2)}$ |
| (b) | Give the binary representation of (a), where n is 8.   | 00.000001     |
| (c) | Give an algebraic expression in terms of n for the <u>negative</u> value that has the smallest magnitude.          | $-2^{-(n-2)}$ |
| (d) | Give the binary representation of (c), where n is 8.   | 11.111111     |

11. What are the most positive (+31) and most negative (-32) decimal values of a 6-bit 2's-complement number?
12. What are the minimum (0) and maximum (63) decimal values of a 6-bit unsigned number?
13. Under what condition does adding 1 to a binary integer consisting of all 1's cause an overflow? **When the representation is unsigned.**
14. In 2's complement, are the absolute values of full-scale negative and full-scale positive are identical or not? **No** Explain why. **E.g.,  $0111_2 = +7_{10}$ ,  $1000_2 = -8_{10}$**
15. Convert:
- |     |                        |                               |
|-----|------------------------|-------------------------------|
| (a) | $0.324_7$ to base 10.  | <b>0.481049563</b>            |
| (b) | $400_{10}$ to base 7.  | <b>1111</b>                   |
| (c) | $0.9_{10}$ to base 3.  | <b>0.2200 2200 2200 .....</b> |
| (d) | $12.34_5$ to base 7.   | <b>10.5214 5214 5214 ...</b>  |
| (e) | $35.2_7$ to base 10.   | <b>26.285714286</b>           |
| (f) | $35.2_{10}$ to base 7. | <b>50.1254 1254 1254 ...</b>  |
16. Use shortcuts based on power relationships to convert:
- |     |                              |               |
|-----|------------------------------|---------------|
| (a) | $FACE_{16}$ to base 8.       | <b>175316</b> |
| (b) | $1011.0111_2$ to base 8.     | <b>13.34</b>  |
| (c) | $232.1_4$ to base 8.         | <b>56.2</b>   |
| (d) | $17.6_9$ to base 3.          | <b>121.2</b>  |
| (e) | $1100011.11001_2$ to base 8. | <b>143.62</b> |
| (f) | $71.3_8$ to base 4.          | <b>321.12</b> |
17. The exact binary representation of one-sixth (1/6) requires an infinite number of digits. Truncating it (discarding extra bits) to make it fit within a fixed-precision representation creates a representational error. What is the absolute error that results from storing one-sixth without rounding using 8 fractional bits?

$$\begin{array}{llll}
 2 \times \frac{1}{6} = 0 \frac{1}{3} & \rightarrow .0 & \text{Abs. Error} & = \left| \frac{42}{256} - \frac{1}{6} \right| = \left| \frac{42 \times 6 - 1 \times 256}{6 \times 256} \right| \\
 2 \times \frac{1}{3} = 0 \frac{2}{3} & \rightarrow .00 & & = \frac{1}{384} \\
 2 \times \frac{2}{3} = 0 \frac{4}{3} = 1 \frac{1}{3} & \rightarrow .001 & & = \mathbf{0.0026041666666667_{10}} \\
 2 \times \frac{1}{3} = 0 \frac{2}{3} \text{ (repeats)} & \rightarrow .00101010_2 = 42/256 & & 
 \end{array}$$

18. What is the minimum number of fractional binary digits required to have at least the same resolution as 3 fractional decimal digits? **Answer = 10 bits**

**Note:**  $10^3 = 1000_{10}$ . Need k such that  $2^{k-1} < 1000_{10} \leq 2^k$ :  $2^9 = 512$  and  $2^{10} = 1024$