

Project 2: Basic Filter Design

1. Introduction

Through this lab, we were able to learn how to make high, low and all pass filters using Op Amps, resistors, capacitors, and more. We additionally observed how they operated when we varied the frequencies, and observed the general trends by using graphs. Furthermore, the lab tests our skills in MATLAB and ELVIS, which we will need in future labs and work in electrical engineering.

2. Objective

- Learn about how adding a capacitor can change Op Amps frequency response.
- Learn about how to build low pass filter, all pass filter, and high pass filter and how they work.

3. Laboratory Procedure

Problem 1. Derive an expression for the amplitude of the output voltage as a function of R_1 , R_2 , C and ω (in the following we will refer to this amplitude as $A(\omega)$).

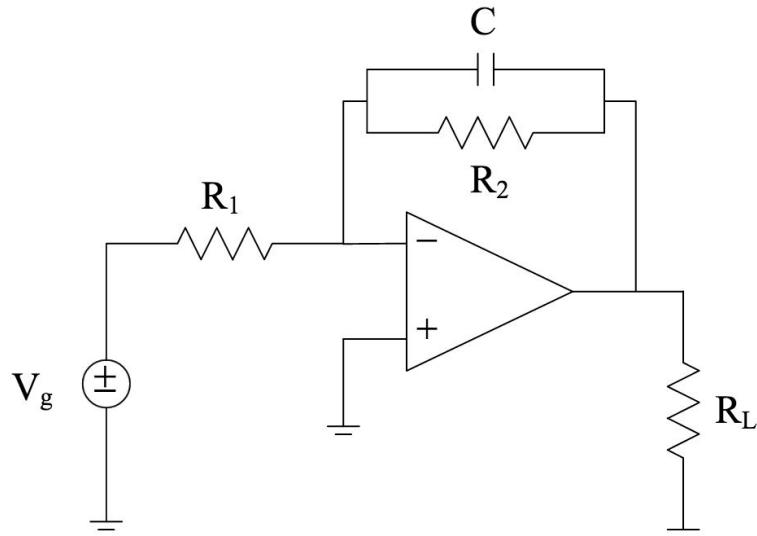
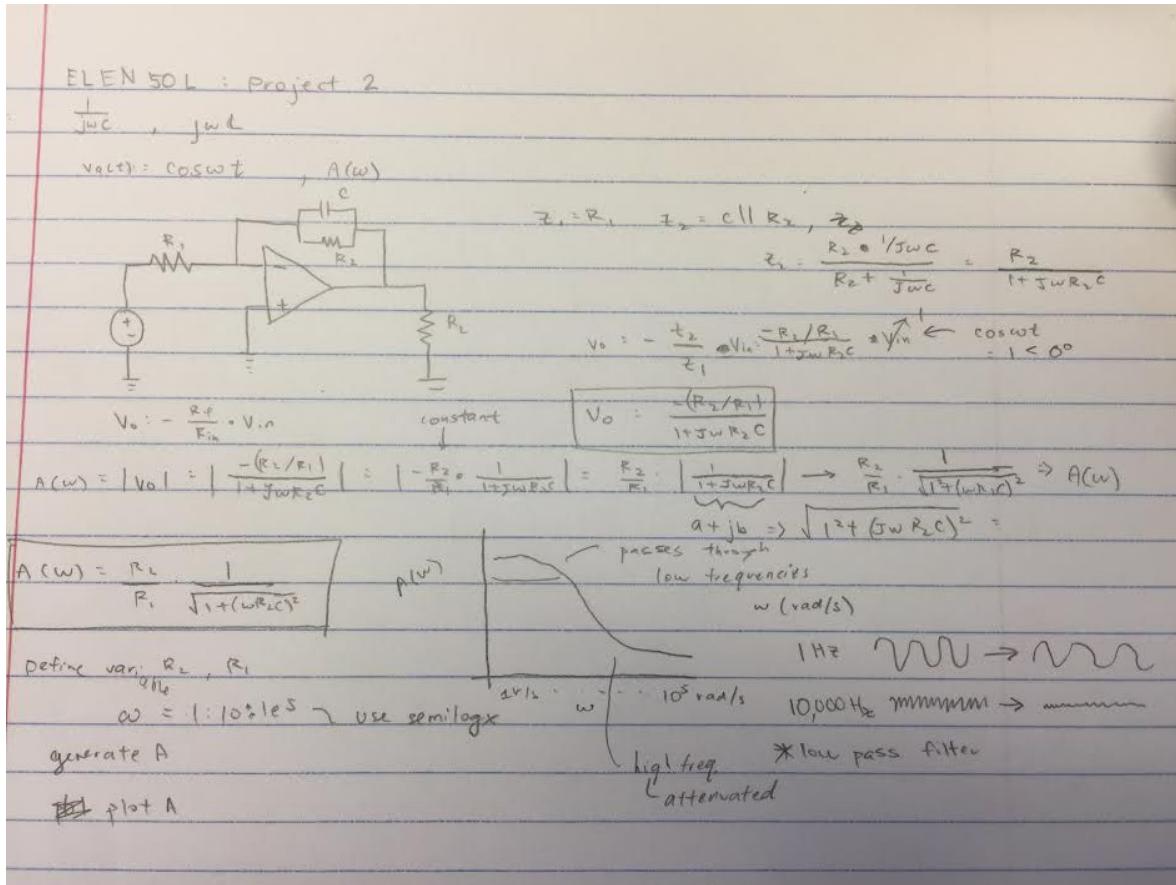


Fig. 1. A simple low-pass filter.



Problem 2. Based on the results of problem 1, design a low-pass filter that satisfies the following requirements.

We decide to use $1\mu\text{F}(1\text{e}^{-6}\text{F})$ as the value of our capacitor.

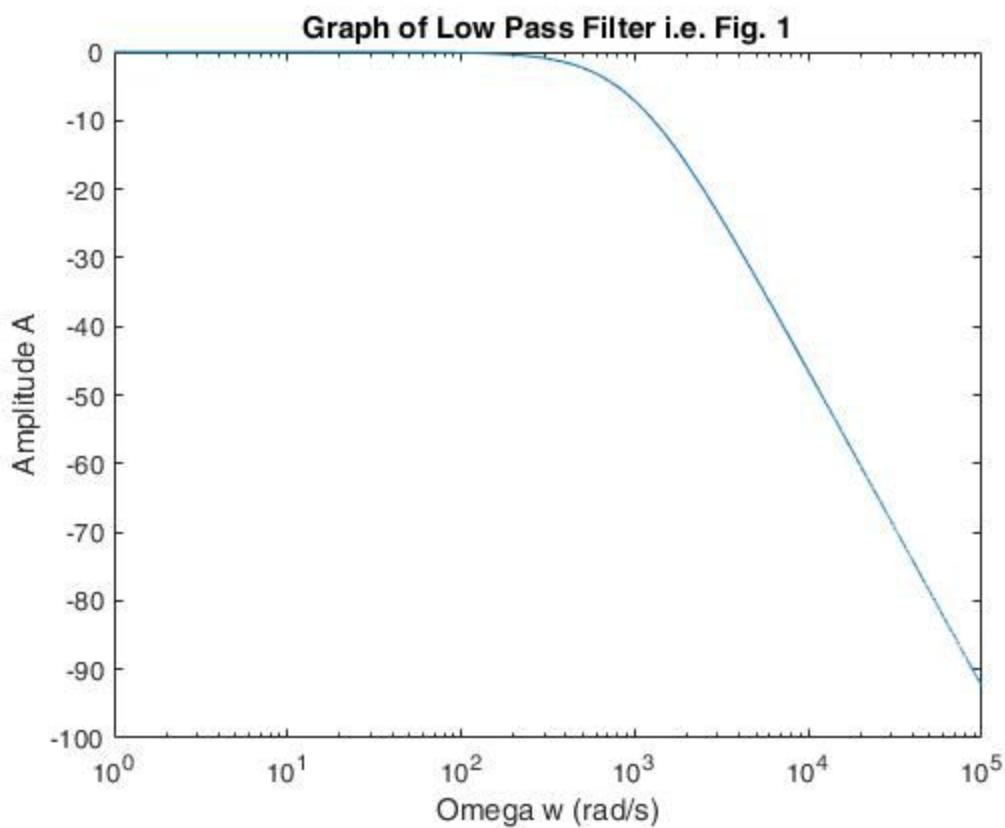
- 1) For $\omega = 0$ (which corresponds to DC), the amplitude must be $A(0) = 1$.
- 2) For $\omega = 1,000 \text{ rad/s}$, the amplitude must be $A(1000) = 0.7$.
- 3) Your element values must be physically realistic.

R_1 needs to be equal to R_2

$$R_1 = R_2 = 1020.2\Omega$$

Problem 3. Write an m-file that solves the circuit in Fig.1 for different frequencies. Use it to plot $20 \log A(\omega)$ for the values chosen in Problem 2, and verify that the design requirements are satisfied.

Code in Appendix under Problem 3



Problem 4. Repeat Problem 1 for the circuit shown in Fig.2.

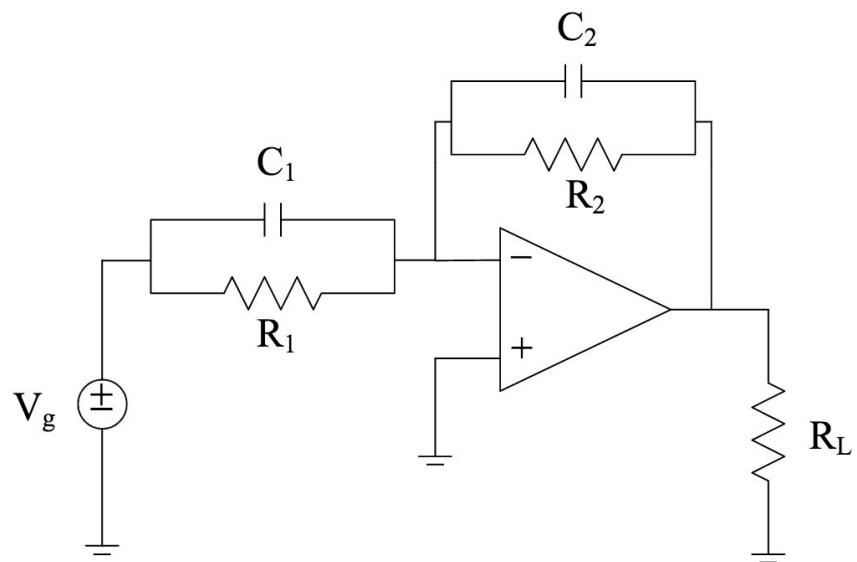


Fig. 2. An alternative circuit for filter design.

$$\begin{aligned}
 V_o &= -\frac{R_2}{R_1} \cdot V_s \\
 &= -\frac{Z_f}{Z_s} \cdot V_s \\
 &= -\frac{\frac{1}{j\omega C_2 || R_2}}{\frac{1}{j\omega C_1 || R_1}} \cdot V_s \\
 &= -\frac{\frac{R_2}{1 + j\omega R_2 C_2}}{\frac{R_1}{1 + j\omega R_1 C_1}} V_s \cos(\omega t) \\
 &= -\frac{R_2}{R_1} \cdot \frac{1 + j\omega R_1 C_1}{1 + j\omega R_2 C_2} \cdot V_s = 1
 \end{aligned}$$

$$\begin{aligned}
 A(\omega) &= |V_o| = \left| -\frac{R_2}{R_1} \cdot \frac{1 + j\omega R_1 C_1}{1 + j\omega R_2 C_2} \right| \\
 &= \frac{R_2}{R_1} \cdot \left| \frac{1 + j\omega R_1 C_1}{1 + j\omega R_2 C_2} \right| \\
 &= \frac{R_2}{R_1} \cdot \sqrt{\frac{1 + (R_1 C_1)^2}{1 + (R_2 C_2)^2}}
 \end{aligned}$$

Problem 5. Choose physically realistic values for R_1 , R_2 , C_1 , and C_2 so that $A(\omega) = 10$ for all values for ω (this would correspond to an all-pass filter).

$$R_2/R_1 = C_1/C_2 = 10$$

Therefore, $C_1 = 1\text{uF}$, $C_2 = 0.1\text{uF}$ and $R_1 = 1000\Omega$, $R_2 = 10000\Omega$

Problem 6. Choose physically realistic values for R_1 , R_2 , C_1 , and C_2 so that the following two requirements are met:

For $\omega = 0$ the amplitude must be $A(0) = 0.01$

$$R_1 = 100 * R_2$$

For $\omega = 10,000 \text{ rad/x}$, the amplitude must be $A(10,000) = 0.9$

$$\text{Let } C_1 = C_2 = 1\text{uF}$$

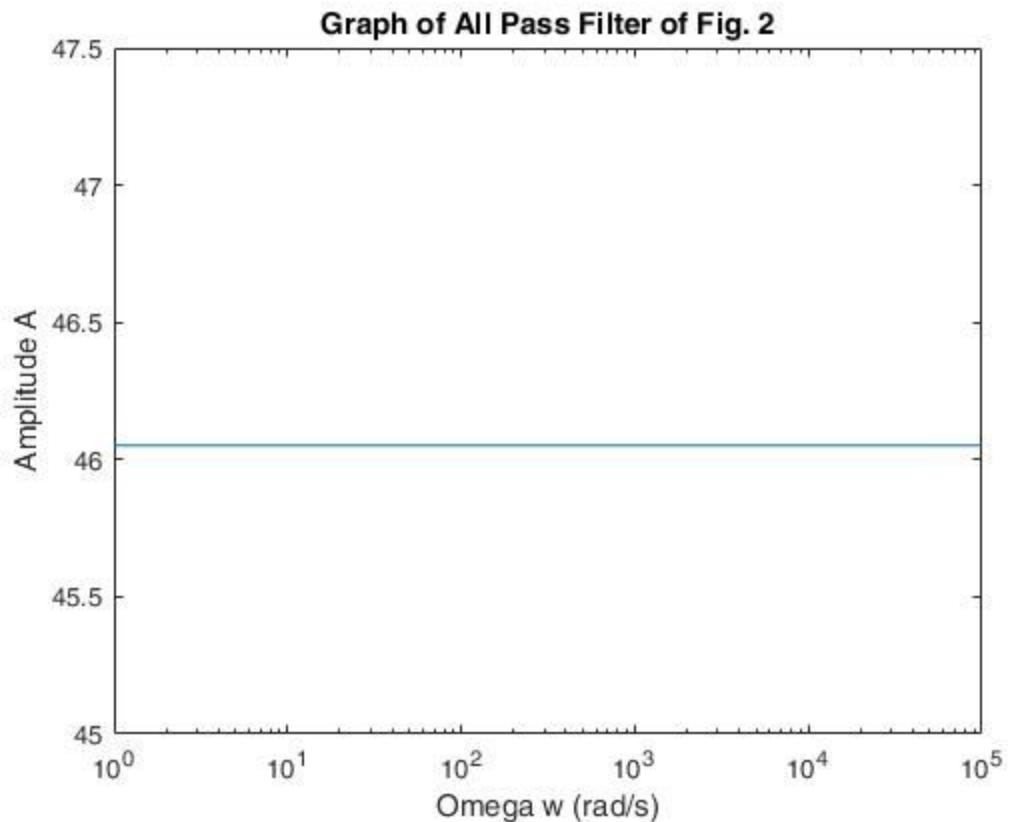
$$R_2 = 206.5\Omega, R_1 = 20650\Omega$$

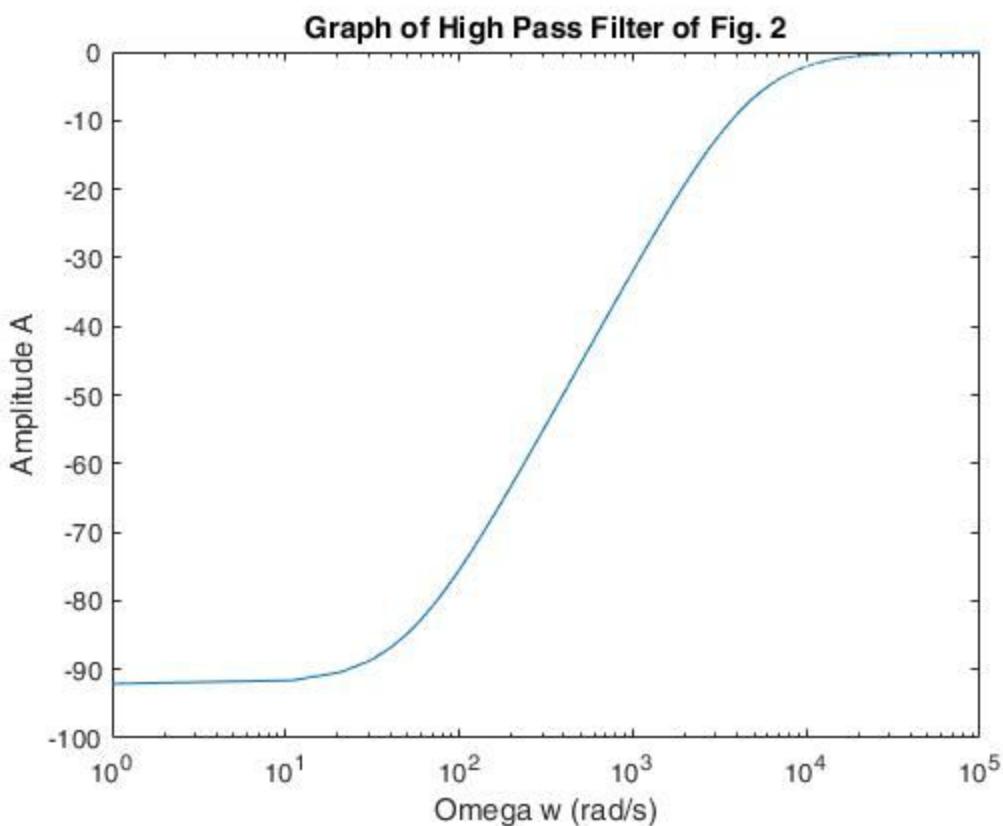
What kind of filter is this? Explain.

This filter is a high pass filter because the amplitude is close to 1 when the frequency is high, while when the frequency is low, the amplitude is attenuated.

Problem 7. Write an m-file that solves the circuit in Fig.2 for different frequencies, and plot $20\log A(\omega)$ for the values chosen in Problems 5 and 6. Use these plots to verify the two designs.

Code in Appendix under Problem 5 and 7





Problem 8. Assemble the circuits in Figs. 1 and 2 with the element values obtained in Problems 2,5 and 6. In all three cases measure $A(\omega)$ for a range of relevant frequencies, and use the data to plot $20\log A(\omega)$ (do this in Matlab). Compare the measurement with your simulation results.

Low Pass Filter:

$$R_1 = R_2 = 1000 \Omega$$

All Pass Filter:

$$R_2/R_1 = C_1/C_2 = 10$$

Therefore, $C_1 = 1\mu F$, $C_2 = 0.1\mu F$ and $R_1 = 1000 \Omega$, $R_2 = 10000 \Omega$

High Pass Filter:

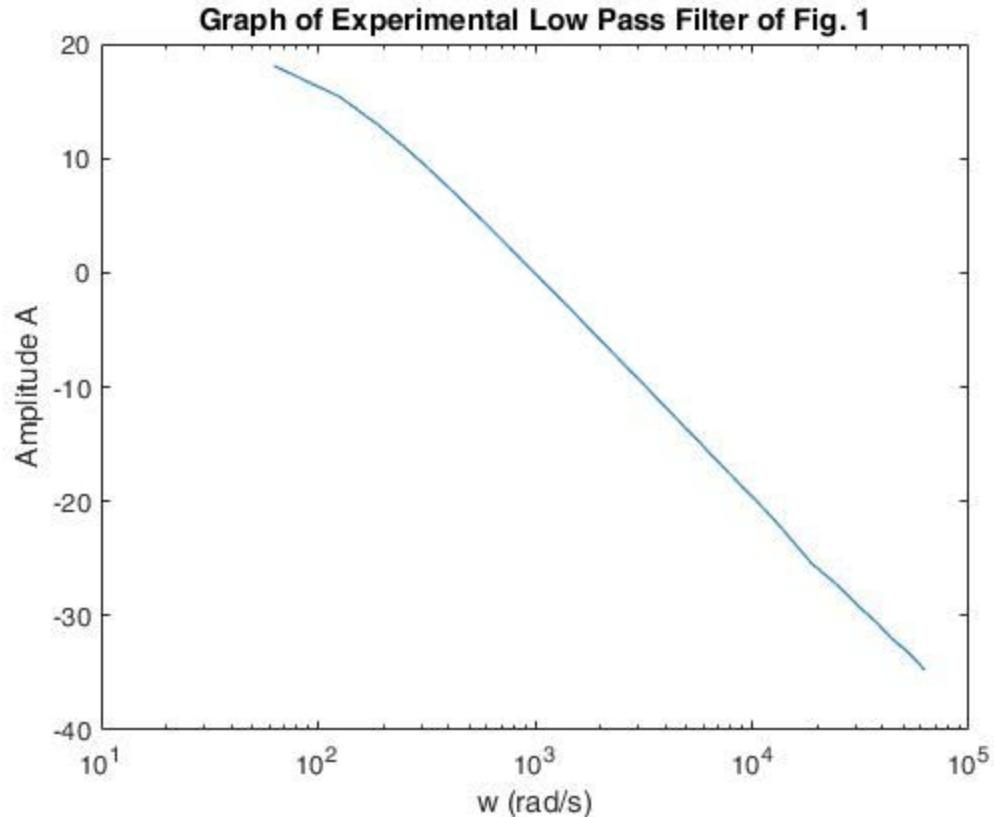
$$R_1 = 20k\Omega, R_2 = 200\Omega, C_1 = C_2 = 1\mu F$$

Frequencies (Hz)	Low Pass Filter	All Pass Filter	High Pass Filter
10	8.020 V	9.78 V	11 mV

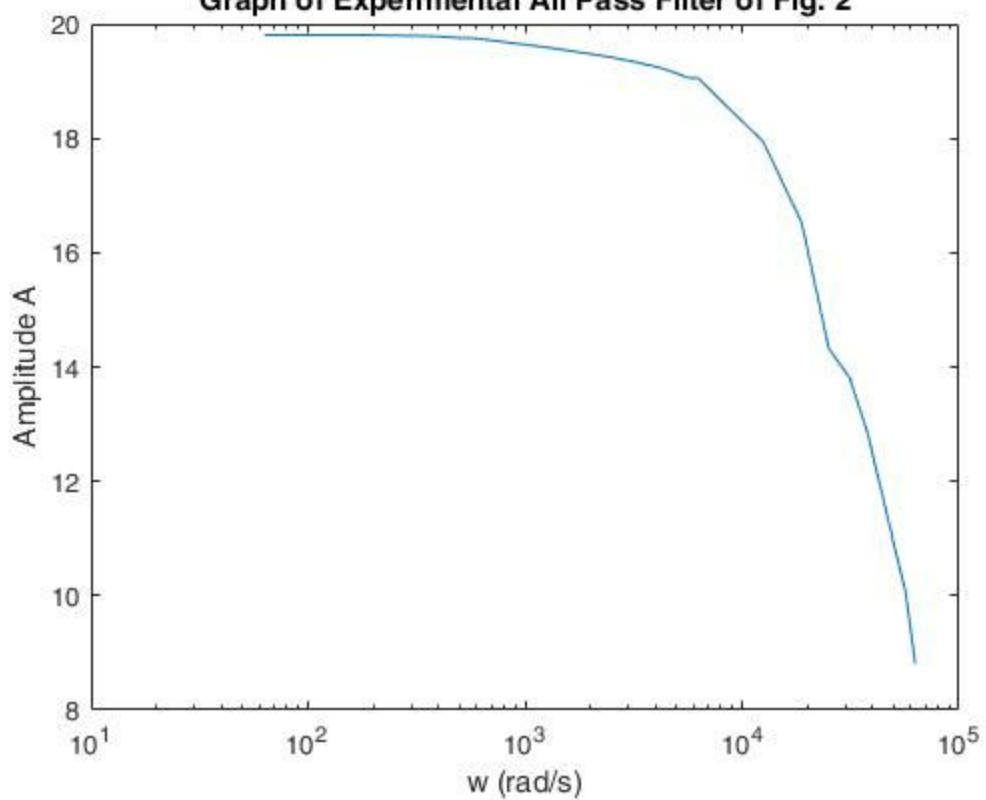
20	5.882 V	9.78 V	20 mV
30	4.444 V	9.78 V	30 mV
40	3.530 V	9.77 V	56 mV
50	2.924 V	9.765 V	67 mV
60	2.484 V	9.756 V	77.8 mV
70	2.158 V	9.74 V	90 mV
80	1.911 V	9.72 V	101 mV
90	1.710 V	9.72 V	111 mV
100	1.550 V	9.7 V	123 mV
200	798.7 mV	9.54 V	238 mV
300	537.83 mV	9.43 V	340 mV
400	406.6 mV	9.35 V	423 mV
500	326.8 mV	9.27 V	505 mV
600	273.1 mV	9.2 V	565 mV
700	235 mV	9.13 V	620 mV
800	206.1 mV	9.05 V	661 mV
900	183.7 mV	8.97 V	693 mV
1000	165.2 mV	8.97 V	722 mV
2000	83.8 mV	7.88 V	783 mV
3000	53.4 mV	6.7 V	717 mV
4000	42.5 mV	5.2 V	633 mV
5000	34.3 mV	4.9 V	555 mV
6000	29.4 mV	4.4 V	491 mV
7000	25.1 mV	3.9 V	439 mV
8000	22.66 mV	3.5 V	395 mV

9000	20.3 mV	3.2 V	362 mV
10000	18.2 mV	2.76 V	331 mV

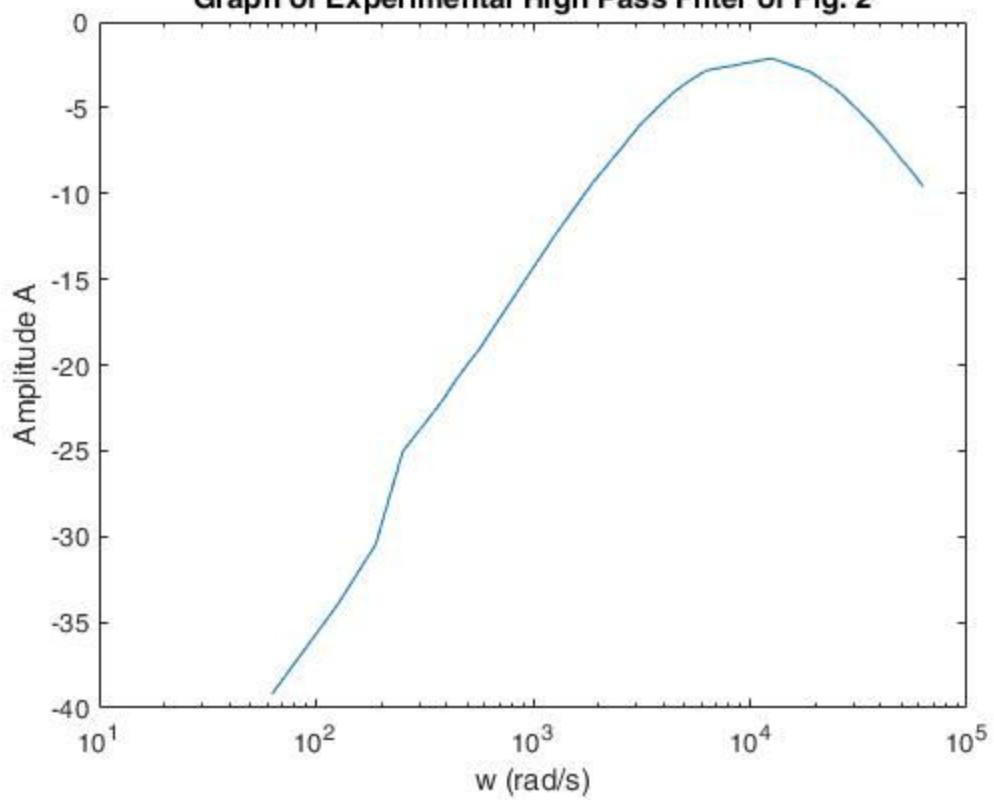
Our results do not necessarily match our theoretical values, but it is probably because of the operational amplifier cannot handle extremely low and high frequencies. Also for the low pass filter and the high pass filter, we use resistors whose values are different from our calculation.



Graph of Experimental All Pass Filter of Fig. 2



Graph of Experimental High Pass Filter of Fig. 2



4. Observation and Analysis

We notice that low pass filter only passes signals with relatively low frequency and attenuates signals with relatively high frequency, while high pass filter works in the opposite way. In the all pass filter, we can see that it passes both low and high frequency signals. Because Op-Amps doesn't function well at low frequency and very high frequency, our graphs of the experimental amplitude do not match with our calculation/expectation perfectly.

5. Questions

Questions answered in their respective sections.

6. Conclusions

Through this lab, we were able to integrate our knowledge of Operational Amplifiers and AC circuits in order to observe different kind of pass filters. Using software and hardware tools such as MATLAB and ELVIS II, we measured, calculated and graphed our results from creating low, pass, and high filters. Additionally, the project included technical skills for electrical engineers by allowing us to calculate values and equations by hand, and code our data. Overall, this lab tested our knowledge of electrical engineering in general, and was extremely helpful in developing sed skills. We learned a lot of hardware and software skills in this lab, which will help us later in the future. NATE WAS GREAT AND WE COULDN'T DO THIS LAB WITHOUT HIM :DDDDD

Appendix:

```
%% ELEN 50 Lab Project 2
% Wed 2:15 - 5:00 PM
% Yutong Li and Zenichi Yaskawa
% March 10th, 2017
```

%% Problem 3

```
% m file that solves the circuit in Fig. 1 and use it to plot 20log(A(w))
% for the values chosen in Problem 2
```

```
R1 = 1020.2;
R2 = 1020.2;
C = 1e-6;
```

```
% frequency values for w
w = [1:10:1e5];
figure(1);
```

```
% Equation of A(w)
A = (R2/R1) * (1./sqrt(1+(w*R2*C).^2));
semilogx(w, 20*log(A));
```

```
title('Graph of Low Pass Filter i.e. Fig. 1')
xlabel('Omega w (rad/s)')
ylabel('Amplitude A')
```

```
% Checking values of w and A(w)
w = 0
A = (R2/R1) * (1./sqrt(1+(w*R2*C).^2))
w = 1000
A = (R2/R1) * (1./sqrt(1+(w*R2*C).^2))
```

%% Problem 5

```
% Choose physically realistic values for R1, R2, C1, and C2 so that A(w) =
% 10 for all values of w for Problem 5
```

```
% Assigning variables
R1 = 1000;
R2 = 10*R1;
C1 = 1e-6;
C2 = 0.1e-6;
```

```
% frequency values for w
w = [1:10:1e5];
```

```

figure(2);

% Equation of A(w)
A = (R2/R1) * (sqrt((1+(w*R1*C1).^2)./(1+(w*R2*C2).^2)));
semilogx(w, 20*log(A));

% Graph
title('Graph of All Pass Filter of Fig. 2')
xlabel('Omega w (rad/s)')
ylabel('Amplitude A')

% Testing values
w = 0
A = (R2/R1) * (sqrt((1+(w*R1*C1).^2)./(1+(w*R2*C2).^2)))

w = 10000
A = (R2/R1) * (sqrt((1+(w*R1*C1).^2)./(1+(w*R2*C2).^2)))

%% Problem 7
% m file that solves the circuit in Fig. 2 for different frequencies, and
% plot 20*log(A) for the values chosen in Problem 6.
% Assigning variables
R2 = 206.5;
R1 = 100*R2;
C1 = 1e-6;
C2 = 1e-6;

% frequency values for w
w = [1:10:1e5];
figure(3);

% Equation of A(w)
A = (R2/R1) * (sqrt((1+(w*R1*C1).^2)./(1+(w*R2*C2).^2)));
semilogx(w, 20*log(A));

% Graph
title('Graph of High Pass Filter of Fig. 2')
xlabel('Omega w (rad/s)')
ylabel('Amplitude A')

% Checking values of w and A(w)
w = 0
A = (R2/R1) * (sqrt((1+(w*R1*C1).^2)./(1+(w*R2*C2).^2)))

```

```

w = 10000
A = (R2/R1) * (sqrt((1+(w*R1*C1).^2)./(1+(w*R2*C2).^2)))

%% Problem 8
% Assemble the circuits in Figs. 1 and 2 with the element values obtained
% in Problem 2, 5, and 6.
% In all three cases measure A(?) for a range of relevant frequencies,
% use the data to plot 20logA(?)  

% LOW PASS FILTER
f = [10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 2000, 3000,
4000, 5000, 6000, 7000, 8000, 9000, 10000];

w = 2*pi*f;  

V = [8.020, 5.882, 4.444, 3.530, 2.924, 2.484, 2.158, 1.911, 1.710, 1.550, 798.7e-3, 537.83e-3, 406.6e-3,
326.8e-3, 273.1e-3, 235e-3, 206.1e-3, 183.7e-3, 165.2e-3, 83.8e-3, 53.4e-3, 42.5e-3, 34.3e-3, 29.4e-3,
25.1e-3, 22.66e-3, 20.3e-3, 18.2e-3];  

A = 20*log10(V);  

% Graph
figure(4);
semilogx(w, A);
title('Graph of Experimental Low Pass Filter of Fig. 1')
xlabel('w (rad/s)')
ylabel('Amplitude A')  

% ALL PASS FILTER
f = [10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 2000, 3000,
4000, 5000, 6000, 7000, 8000, 9000, 10000];
w = 2*pi*f;  

V = [9.78, 9.78, 9.78, 9.77, 9.765, 9.756, 9.74, 9.72, 9.72, 9.7, 9.54, 9.43, 9.35, 9.27, 9.2, 9.13, 9.05, 8.97,
8.97, 7.88, 6.7, 5.2, 4.9, 4.4, 3.9, 3.5, 3.2, 2.76];  

A = 20*log10(V);  

% Graph
figure(5);
semilogx(w, A);

```

```

title('Graph of Experimental All Pass Filter of Fig. 2')
xlabel('w (rad/s)')
ylabel('Amplitude A')

% High PASS FILTER
f = [10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 2000, 3000,
4000, 5000, 6000, 7000, 8000, 9000, 10000];
w = 2*pi*f;

V = [11e-3, 20e-3, 30e-3, 56e-3, 67e-3, 77.8e-3, 90e-3, 101e-3, 111e-3, 123e-3, 238e-3, 340e-3, 423e-3,
505e-3, 565e-3, 620e-3, 661e-3, 693e-3, 722e-3, 783e-3, 717e-3, 633e-3, 555e-3, 491e-3, 439e-3, 395e-3,
362e-3, 331e-3];

A = 20*log10(V);

% Graph
figure(6);
semilogx(w, A);
title('Graph of Experimental High Pass Filter of Fig. 2')
xlabel('w (rad/s)')
ylabel('Amplitude A')

```