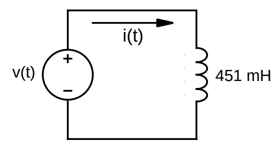
ECE 125 Solved and Practice Problems Related to the Final Exam

Note: This ECE-125 handout mainly covers material not included in the midterm examinations – steady-state ac circuit analysis (phasors and impedances) and basic operational-amplifier (op-amp) circuits. The study guide for final examination provides the expected level of knowledge in these subjects. It will be observed when reviewing the following that some material goes beyond what is expected.

Solved Problems

1. Find the phasor expression of the current I in the circuit below if v(t) = 170 sin (377 t + 60°) V. The circuit is under steady-state conditions.



Solution: The first necessary step is to convert the sinusoidal time-domain voltage of the source to a phasor voltage. This conversion must be on a cosine base; therefore,

$$v(t) = 170 \sin (377 t + 60°) = 170 \cos (377 t + 60° – 90°)$$

or

$$v(t) = 170 \cos (377 t – 30°) \text{ V}.$$

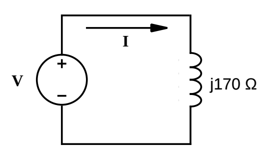
This voltage expressed as a phasor is

$$\mathbf{V} = \frac{170}{\sqrt{2}}\underline{/–30°} = 120.2\underline{/–30°} \text{ V}.$$

Now converting the inductance to its impedance,

$$Z_L = j\omega L = j\,(377)(0.451) = j\,170\ \Omega,$$

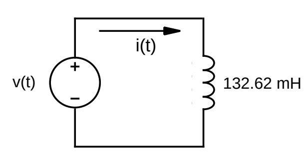
the above circuit in the phasor domain becomes,



Therefore, using Ohm's law to solve for circuit current,

$$\mathbf{I} = \frac{120.2\underline{/–30°}}{170\underline{/90°}} = 0.707\underline{/–120°} \text{ A}.$$

2. Given the circuit below and the steady-state time-domain expressions for v(t) and i(t), determine the radian frequency, ω, expressed radians/second.



$$v(t) = 100 \cos \omega t \quad \text{V}$$
$$i(t) = 2 \sin \omega t \quad \text{A}$$

Solution: Noting that 2 sin ω t is equal to 2 cos (ω t – 90°), the voltage and current phasors for this problem are:

$$\mathbf{V} = \frac{100}{\sqrt{2}}\underline{/0°} = 70.7\underline{/0°}\ \text{V} \quad\text{and}\quad \mathbf{I} = \frac{2}{\sqrt{2}}\underline{/-90°} = 1.414\underline{/-90°}\ \text{A}.$$

By Ohm's law, the impedance of the inductor (computed by the ratio of phasor voltage to phasor current) is

$$Z_L = \frac{70.7\underline{/-30°}}{1.414\underline{/-90°}} = 50\underline{/90°}\ \Omega .$$

However, the impedance of the inductor is also

$$Z_L = j\omega L$$

or

$$j\omega L = j\,50$$

or

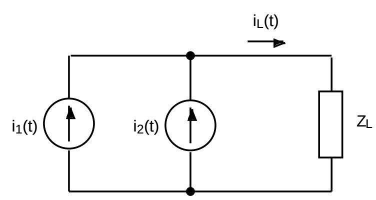
$$\omega L = 50$$

or

$$\omega = \frac{50}{L} = \frac{50}{0.133} = 377\ \text{rad/s} .$$

3. Find the expression for the steady-state sinusoidal current, i_L, in the circuit below if:

$i_1 = 30 \cos 377\, t$ and $i_2 = 40 \sin 377\, t$.



Solution: Use of the phasor domain is likely the easiest way to work this problem. Here, both sinusoidal expressions must be expressed as cosine functions. Thus,

$$i_2(t) = 40 \sin 377t = 40 \cos (377t - 90°) .$$

Now converting the sinusoids to phasors gives

$$\mathbf{I}_1 = \frac{30}{\sqrt{2}}\underline{/0°} = 21.2\underline{/0°}\ \text{A}$$

and

$$\mathbf{I}_2 = \frac{40}{\sqrt{2}}\underline{/-90°} = 28.28\underline{/-90°}\ \text{A} .$$

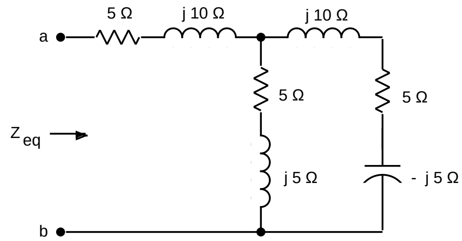
The load current expressed as a phasor is therefore

$$\mathbf{I}_1 + \mathbf{I}_2 = 21.2\underline{/0°} + 28.28\underline{/-90°} = 21.2 - j28.28 = 35.34\underline{/-53.13°}\ \text{A} .$$

Converting this phasor back to a sinusoidal function gives

$$i_L = 50 \cos (377\, t - 53.13°)\ \text{A} .$$

4. **Determine the equivalent impedance, Z_{eq}, for the circuit below.**



Solution: The right essential branch reduces to

$$Z_{right} = 5 + j\,10 - j\,5 = 5 + j\,5\,\Omega$$

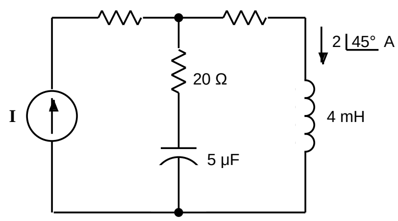
This branch is in parallel with the center branch containing the 5 + j 5 Ω impedance, and the impedance of the parallel combination is equal to

$$Z_{||} = (5 + j\,5)\,||\,(5 + j\,5) = \frac{(5 + j\,5)\,(5 + j\,5)}{5 + j\,5 + 5 + j\,5} = \frac{(5 + j\,5)}{2} = 2.5 + j\,2.5\,\Omega$$

This parallel combination is in series with the (5 + j 10)-Ω branch connected to terminal a; thus

$$Z_{eq} = 5 + j\,10 + 2.5 + j\,2.5 = 7.5 + j\,12.5\,\Omega\,.$$

5. **Consider that the circuit below is under steady-state conditions with the radian frequency, $\omega = 10^4$ rad/s. Determine the phasor current, I, from the current source, if the current through the 4-mH inductor is 2 <u>/ 45°</u> A.**



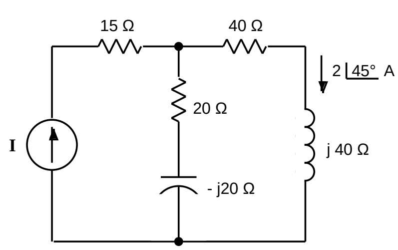
Solution: The capacitance and inductance must be changed to impedances, or

$$Z_C = \frac{1}{j\,\omega C} = \frac{-j}{10^4\,(5)} = -j\,20\,\Omega$$

and

$$Z_L = j\,\omega L = j\,10^4\,(4) = j\,40\,\Omega\,.$$

The circuit can now be expressed using these impedances.



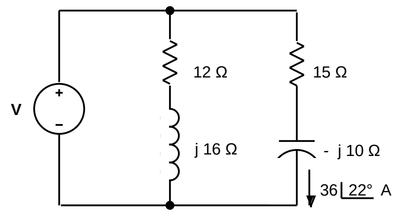
Now, using current division,

$$\mathbf{I}_2 = \mathbf{I}\left(\frac{Z_1}{Z_1 + Z_2}\right)$$

or

$$\mathbf{I} = \mathbf{I}_2 \left(\frac{Z_1 + Z_2}{Z_1}\right) = 2\,\underline{/45°}\left(\frac{40 + j\,40 + 20 - j\,20}{20 - j\,20}\right) = 4.47\,\underline{/108.44°}\text{ A}.$$

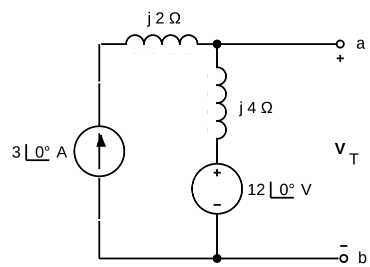
6. Find the phasor voltage, V, in the steady-state ac circuit below if the phasor current through the capacitor is 36 $\underline{/22°}$ A.



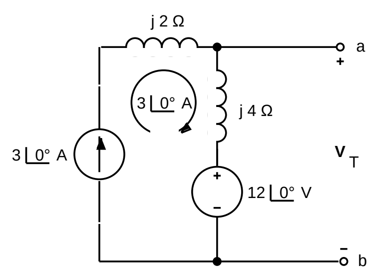
Solution: The given current flows through the 15 - j 10 Ω branch; the voltage source is connected across the branch. Therefore,

$$\mathbf{V} = 36\,\underline{/22°}\,(15 - j\,10) = 36\,\underline{/22°}\,(18.03\,\underline{/-33.69°}) = 649\,\underline{/-11.7°}\text{ V}.$$

7. Find the voltage, V_T, expressed as a phasor, across the terminals a and b in the following steady-state ac circuit.



Solution: Notice that the voltage between terminals a and b is the sum the voltage drop across the j 4-Ω inductor plus the output of the voltage source. Because the terminals are open, the current through the inductor is from the current source, as shown below.



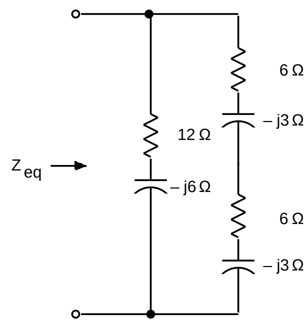
Therefore,

$$\mathbf{V}_T = \mathbf{V}_{j\,4} + 12\,\underline{/0°}$$

or

$$\mathbf{V}_T = 3\,\underline{/0°}\,(4\,\underline{/90°}) + 12\,\underline{/0°} = 12 + j\,12\text{ V} \quad\text{or}\quad 17\,\underline{/45°}\text{ V}$$

8. Determine the equivalent impedance, Z_{eq}, of the circuit shown below.



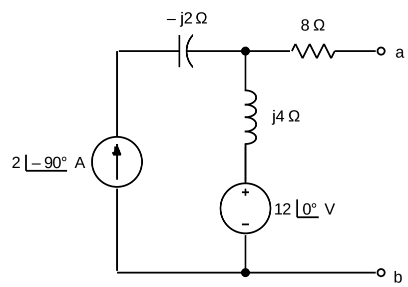
Solution: The equivalent impedance of this series-parallel circuit is therefore

$$Z_{eq} = (6 - j\,3 + 6 - j\,3)\,||\,(12 - j\,6) = (12 - j\,6)\,||\,(12 - j\,6)$$

thus,

$$Z_{eq} = 6 - j\,3\,\Omega \quad \text{or} \quad 6.71\angle{-26.57^\circ}\,\Omega\,.$$

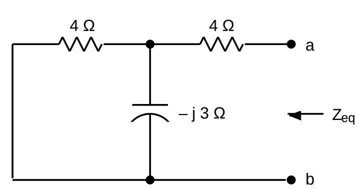
9. Determine the phasor voltage across the terminals a and b for the steady-state ac circuit below.



Solution: Note that the terminals a and b are open. Therefore, no current current flows through the 8-Ω resistor, and the voltage across that resistor is zero. Thus, the voltage across terminals a and b is the voltage across the j4-Ω impedance and the 12 $\angle 0^\circ$-V voltage source. In addition, the current through the j4-Ω impedance is set by the 2 $\angle{-90^\circ}$-A current source. Consequently, the phasor voltage across the terminals a and b in the circuit, where

$$\mathbf{V} = (2\angle{-90^\circ})(j\,4) + 12\angle 0^\circ = 8 + 12 = 20\angle 0^\circ \text{ V}\,.$$

10. For the illustrated diagrram, find the equivalent impedance between terminals a and b.



Solution: Note that the left 4-Ω resistor is in parallel with the – j3-Ω impedance of the capacitor, and the equivalent impedance of that parallel combination is:

$$4\,||{-j3} = \frac{(4)(-\,j3)}{4 - j3} = \frac{-\,j12}{4 - j3} = \frac{12\angle{-90^\circ}}{5\angle{-36.87^\circ}} = 2.4\angle{-53.13^\circ}$$

or

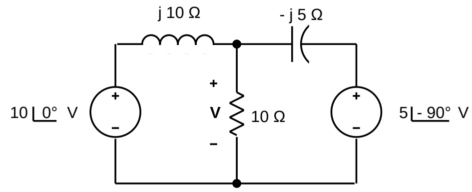
$$4\,||\,{-j3} = 1.44 - j1.92$$

This parallel combination is in series with the right-hand 4-Ω resistor, and the series combination is the answer to the problem:

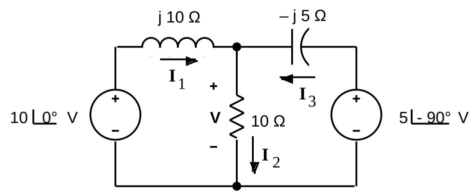
$$Z_{eq} = 4 + 4 \parallel -j3 = 4 + 1.44 - j1.92 = 5.44 - j1.92\ \Omega.$$

Note: The following two solved problems concern node-voltage analysis, and are beyond what is expected for the ECE-125 final, but give additional thoughts of using steady-state ac circuit analysis.

11. Use the node-voltage method to find the voltage V across the 10-Ω resistor, expressed as a phasor.



Solution: Essential branch currents in the above circuit can be assigned as below.



Employing Kirchhoff's current law for the top essential node,

$$\mathbf{I}_1 + \mathbf{I}_3 = \mathbf{I}_2$$

Using Ohm's law to express each current in terms of the unknown voltage, **V**, yields

$$\frac{10\,\underline{/0^\circ} - \mathbf{V}}{j\,10} + \frac{5\,\underline{/-90^\circ} - \mathbf{V}}{-j\,5} = \frac{\mathbf{V}}{10}$$

or

$$-\,j + j\,0.1\,\mathbf{V} = 1 - j\,0.2\,\mathbf{V} = 0.1\,\mathbf{V}$$

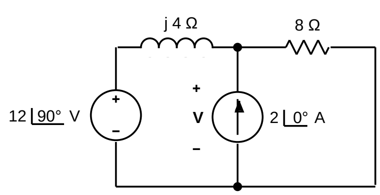
which reduces to

$$(0.1 + j\,0.1)\,\mathbf{V} = 1 - j\,1 = 1.414\,\underline{/-45^\circ}$$

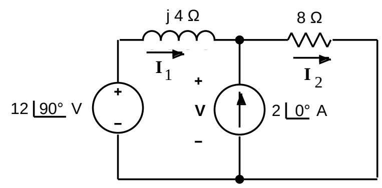
or

$$\mathbf{V} = 10\,\underline{/-90^\circ}\,\mathrm{V} \quad \text{or} \quad -\,j\,10\,\mathrm{V}.$$

12. Use the node-voltage method to find the voltage V across the current source. Express this voltage as a phasor.



Solution: Essential branch currents can be assigned as shown in the circuit below.



Writing Kirchhoff's current law for the top center node,

$$2\,\underline{/0^\circ} + \mathbf{I}_1 = \mathbf{I}_2$$

or

$$\mathbf{I}_1 - \mathbf{I}_2 = -2\,\underline{/0^\circ}$$

Using Ohm's law to express branch current in terms of the unknown voltage yields

$$\frac{12\,\underline{/90^\circ} - \mathbf{V}}{j\,4} - \frac{\mathbf{V}}{8} = -2\,\underline{/0^\circ}$$

Simplifying the above equation produces

$$(0.125 - j\,0.25)\,\mathbf{V} = -5$$

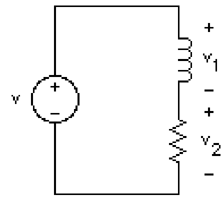
or

$$\mathbf{V} = 17.89\,\underline{/63.44^\circ}\ \text{V}.$$

Practice Problems

P1. Using phasor-domain calculations, find the expression for the steady-state sinosoidal voltage v (i.e., in the time domain) if

$$v_1 = 30 \cos 377\,t \quad \text{and} \quad v_2 = 40 \sin 377\,t\,.$$

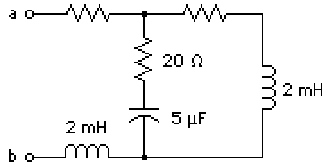


P2. A 100-Ω resistor, a 200-mH inductance, and a 5-μF capacitance are connected in series and are energized by a steady-state sinusoidal source. What is the equivalent impedance if the frequency is 159 Hz?

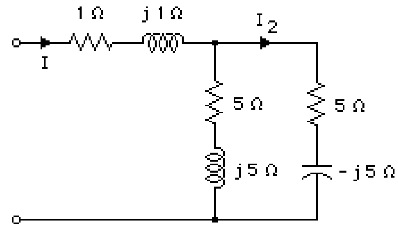
[ans. 100 Ω]

P3. Consider the circuit below is under steady-state conditions with the radian frequency, $\omega = 10^4$ rad/s. Determine the total equivalent impedance between the terminals a and b.

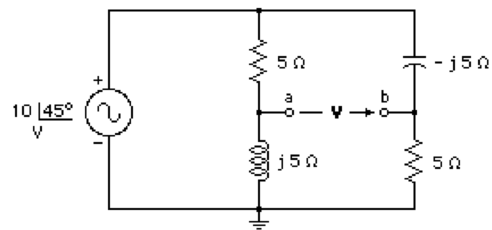
[ans. 40 + j20 Ω]



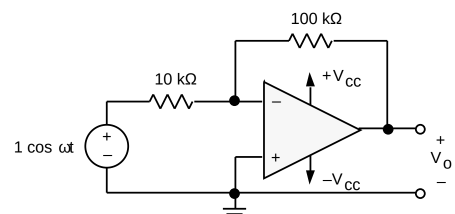
P4. **The phasor of current I in the circuit below is 50 $\underline{/0^\circ}$ A. By using the principle of current division, find I_2, expressed as a phasor.** **[ans. 35.4 $\underline{/45^\circ}$ A]**



P5. **For the illustrated circuit in, calculate the the voltage, V, expressed as a phasor.** **[ans. 0 V]**

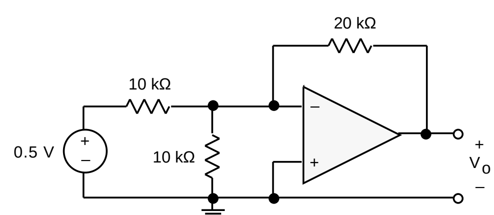


P6. **Consider the circuit below. Assuming an ideal operational amplifier, determine the the output voltage expressed as a sinusoid.** **[ans: – 10 cos ωt]**



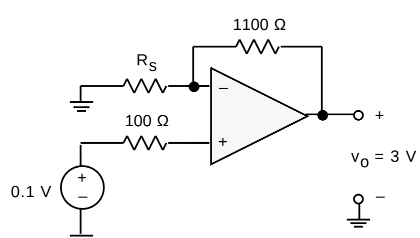
P7. **Determine V_o for the circuit shown below. Assume an ideal operational amplifier.**

[ans: – 1 V]



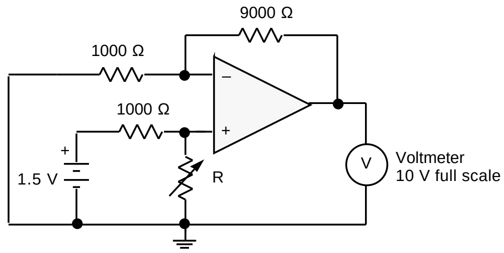
P8. **Find R_S such that the 0.1-V input voltage produces the 3-V output voltage. Assume an ideal operational amplifier.**

[ans: 37.9 Ω]



P9. **In the circuit below, the variable resistor, R, is 1,000 Ω when the voltmeter reads 7.5 V. What is the value of its resistance when the voltmeter reads 10 V? Assume an ideal operational amplifier.**

[ans: 2000 Ω]


9000 Ω
1000 Ω
1000 Ω
−
+
+
1.5 V
R
V
Voltmeter
10 V full scale