ELEN 21/COEN 21

Homework 5 and 6 solution

*3.1 Determine the decimal values of the following unsigned numbers:

(a) $(0111011110)_2$

(b) $(1011100111)_2$

(c) $(3751)_8$

(d) $(A25F)_{16}$

(e) $(F0F0)_{16}$

3.2 Determine the decimal values of the following 1's complement numbers:

(a) 0111011110

(b) 1011100111

(c) 1111111110

3.3 Determine the decimal values of the following 2's complement numbers:

(a) 0111011110

(b) 1011100111

(c) 1111111110

3.1. (*a*) 478
(*b*) 743
(*c*) 2025
(*d*) 41567
(*e*) 61680

3.2. (*a*) 478
(*b*) -280
(*c*) -1

3.3. (*a*) 478
(*b*) -281
(*c*) -2

3.4 Convert the decimal numbers 73, 1906,−95, and−1630 into signed 12-bit numbers in the following representations:

(a) Sign and magnitude

(b) 1's complement

(c) 2's complement

3.4. The numbers are represented as follows:

Decimal	Sign and Magnitude	1's Complement	2's Complement
73	000001001001	000001001001	000001001001
1906	011101110010	011101110010	011101110010
-95	100001011111	111110100000	111110100001
-1630	111001011110	100110100001	100110100010

3.5 Perform the following operations involving eight-bit 2's complement numbers and indicate whether arithmetic overflow occurs. Check your answers by converting to decimal sign-and-magnitude representation.

$$\begin{array}{r} 00110110 \\ +\ 01000101 \\ \hline \end{array} \qquad \begin{array}{r} 01110101 \\ +\ 11011110 \\ \hline \end{array} \qquad \begin{array}{r} 11011111 \\ +\ 10111000 \\ \hline \end{array}$$

$$\begin{array}{r} 00110110 \\ -\ 00101011 \\ \hline \end{array} \qquad \begin{array}{r} 01110101 \\ -\ 11010110 \\ \hline \end{array} \qquad \begin{array}{r} 11010011 \\ -\ 11101100 \\ \hline \end{array}$$

3.5. The results of the operations are:

$$\begin{array}{lrrlrrlrr}
(a): & 00110110 & 54 & (b): & 01110101 & 117 & (c): & 11011111 & (-33) \\
 & \underline{+01000101} & \underline{+69} & & \underline{+11011110} & \underline{-34} & & \underline{+10111000} & \underline{+(-72)} \\
 & 01111011 & 123 & & 01010011 & 83 & & 10010111 & (-105) \\
 & & & & & & & & \\
(d): & 00110110 & 54 & (e): & 01110101 & (117) & (f): & 11010011 & (-45) \\
 & \underline{-00101011} & \underline{-43} & & \underline{-11010110} & \underline{-(-42)} & & \underline{-11101100} & \underline{-(-20)} \\
 & 00001011 & 11 & & 10011111 & (159) & & 11100111 & (-25)
\end{array}$$

Arithmetic overflow occurs in example e; note that the pattern 10011111 represents −97 rather than +159.

3.6

The associativity of the XOR operation can be shown as follows:

$$\begin{aligned}
x \oplus (y \oplus z) &= x \oplus (\overline{y}z + y\overline{z}) \\
&= \overline{x}(\overline{y}z + y\overline{z}) + x(\overline{y} \cdot \overline{z} + yz) \\
&= \overline{x} \cdot \overline{y}z + \overline{x}y\overline{z} + x\overline{y} \cdot \overline{z} + xyz
\end{aligned}$$

$$\begin{aligned}
(x \oplus y) \oplus z &= (\overline{x}y + x\overline{y}) \oplus z \\
&= (\overline{x} \cdot \overline{y} + xy)z + (\overline{x}y + x\overline{y})\overline{z} \\
&= \overline{x} \cdot \overline{y}z + xyz + \overline{x}y\overline{z} + x\overline{y} \cdot \overline{z}
\end{aligned}$$

The two SOP expressions are the same.

3.7

$$
\begin{aligned}
s_i &= (x_i \oplus y_i) \oplus c_i \\
&= x_i \oplus y_i \oplus c_i
\end{aligned}
$$

$$
\begin{aligned}
c_{i+1} &= (x_i \oplus y_i)c_i + x_i y_i \\
&= (\overline{x}_i y_i + x_i \overline{y}_i)c_i + x_i y_i \\
&= \overline{x}_i y_i c_i + x_i \overline{y}_i c_i + x_i y_i \\
&= y_i c_i + x_i c_i + x_i y_i
\end{aligned}
$$

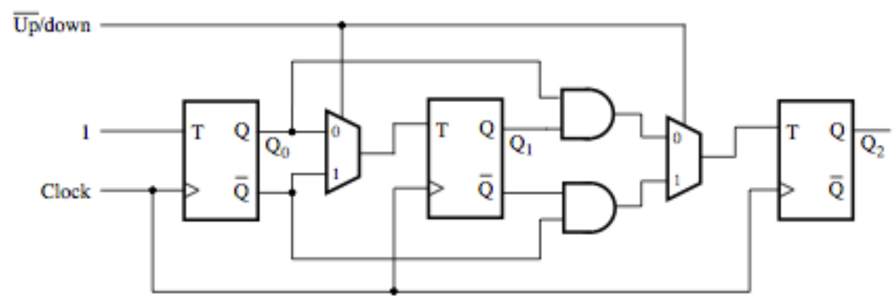
The expressions for s_i and c_{i+1} are the same as those derived in Figure .

3.10

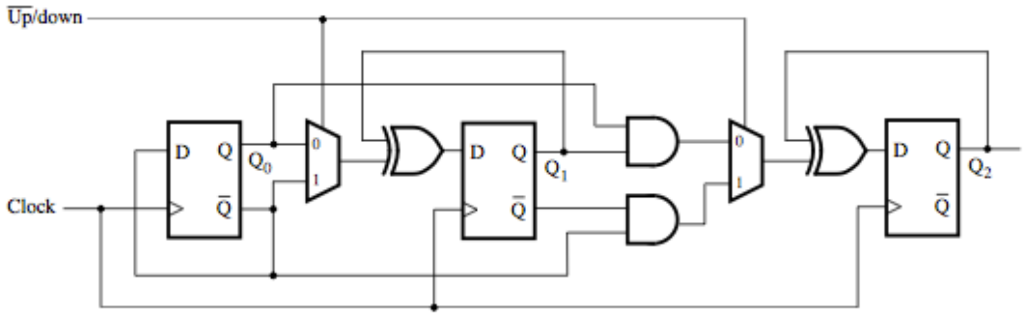
Since $s_k = x_k \oplus y_k \oplus c_k$, it follows that

$$
\begin{aligned}
x_k \oplus y_k \oplus s_k &= (x_k \oplus y_k) \oplus (x_k \oplus y_k \oplus c_k) \\
&= (x_k \oplus y_k) \oplus (x_k \oplus y_k) \oplus c_k \\
&= 0 \oplus c_k \\
&= c_k
\end{aligned}
$$

5.15.



5.16.



$\overline{\text{Up}}$/down
Clock
D
Q
$\overline{\text{Q}}$
Q_0
0
1
D
Q
$\overline{\text{Q}}$
Q_1
0
1
D
Q
$\overline{\text{Q}}$
Q_2