ELEN 50 Class 04 – Kirchoff's Laws

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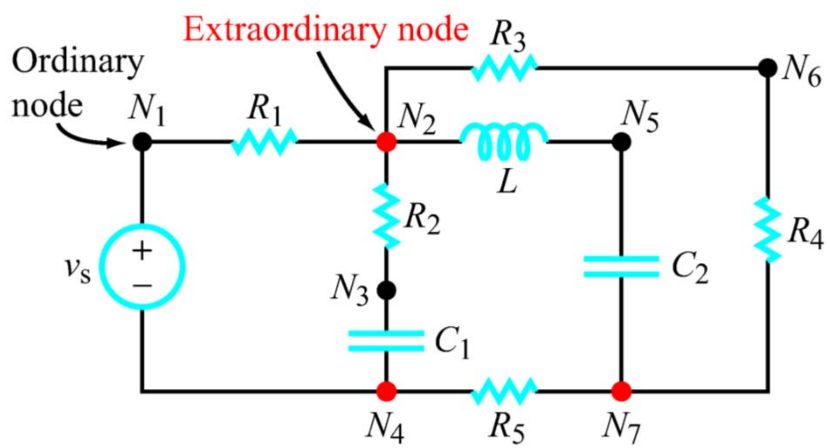
Reviewing the topology definitions from last lecture



$$b = n + l_{ind} - 1$$

branches

nodes

independent loops

You'll definitely need to be able to figure out where branches, nodes, and loops are in order to do circuit analysis …but it won't be necessary for you to use this numerical relationship very often (if at all!)

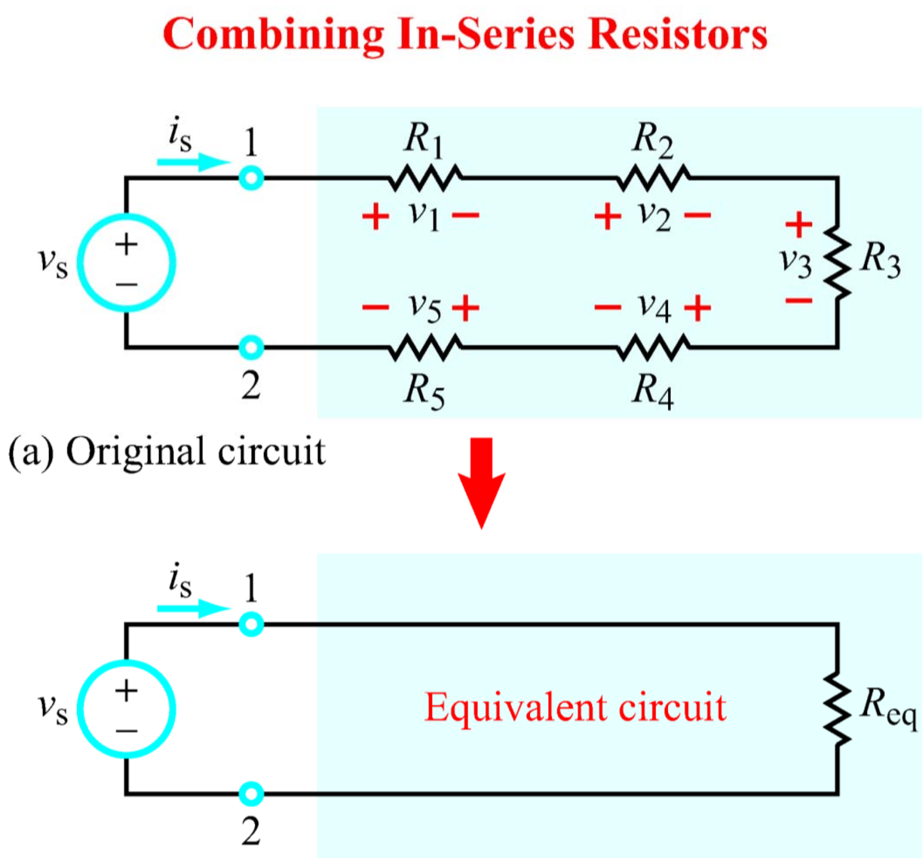
Reviewing Ohms Law and Series and Parallel Resistors

- The voltage difference, V, across a resistor of value, R, due to a current, I, flowing through it , is given by Ohms Law: $V = IR$.
- Two or more devices are in **series** if the same current flows through all of them.
- Two or more devices are in **parallel** if they share the same pair of nodes, thereby having the same voltage across them

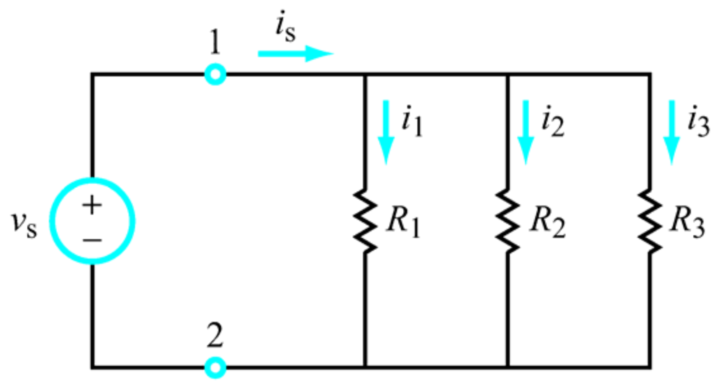
We also talked about how to combine resistors in series and parallel …to produce a single, equivalent resistor. Because of the definitions of series and parallel and Ohm's Law we were able to come up with some formulas.

The equivalent resistor for a number of resistors in series is just the sum of the resistances. The equivalent resistor for a number of resistors in parallel is obtained by noticing that the inverse of the equivalent resistor is the sum of the inverses of the individual resistors. This means the equivalent resistor is equal to the "product over the sum" for two resistors in parallel.

Resistors in series all have to have the same current passing through them – that's the definition of a series connection.



$$R_{eq} = R1 + R2 + R3 + R4 + R5$$



Resistors in parallel all have to have the same voltage drop across them – that's the definition of a parallel connection.

This means $$i_1 R_1 = i_2 R_2 = i_3 R_3 = v_s$$

and also $$i_1 + i_2 + i_3 = i_s$$

Ohm's Law says $$v_s = i_s R_{eq} = (i_1 + i_2 + i_3) R_{eq}$$

so $$\frac{1}{R_{eq}} = \left(\frac{i_1}{v_s} + \frac{i_2}{v_s} + \frac{i_3}{v_s} \right) = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

The Kirchoff Laws

Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL) are two of the **principle foundations of circuit theory**. They were first stated in a paper by Gustav Kirchoff in 1848 and they rest on very fundamental principles of physics – namely the conservation of electrical charge and the conservation of energy.

<u>The Kirchhoff Current and Voltage Laws in combination with Ohm's Law actually provide the theoretical basis of pretty much everything we will be doing in ELEN 50 this quarter!</u> So by the end of today's class you'll know, more or less, the fundamentals of everything you'll need to learn in ELEN 50! The rest of the stuff we'll do this quarter will be just applications and solution strategies.

Kirchhoff Current and Voltage Circuit Laws

KVL

- Conservation of energy requires that <u>the algebraic sum of all of the voltage drops around any closed path in a circuit equals zero</u>. This is Kirchhoff's Voltage Law (KVL).

$$\sum_{k=1}^{n} V_k = 0$$

where n is the total number of voltages measured.

Conservation of energy is as fundamental as charge conservation and it means that, in a closed system, energy (actually mass – energy) cannot be created or destroyed. It is related to a fundamental symmetry of time and is also known as the first law of thermodynamics. It's the reason why perpetual motion machines are impossible.

- Conservation of electric charge requires that <u>the algebraic sum of all of the currents at any node in a circuit equals zero</u>. This is Kirchhoff's Current Law (KCL).

KCL

$$\sum_{k=1}^{n} I_k = 0$$

where n is the total number of circuit branches with current flowing towards or away from a circuit node.

- There are some nice animations on the Web illustrating KVL and KCL
http://www.facstaff.bucknell.edu/mastascu/elessonshtml/TOC_BasicConcepts.html

Notice:

the Kirchhoff current law (**KCL**) is about currents and nodes.

The algebraic sum of the **currents** entering and leaving a **node** is zero

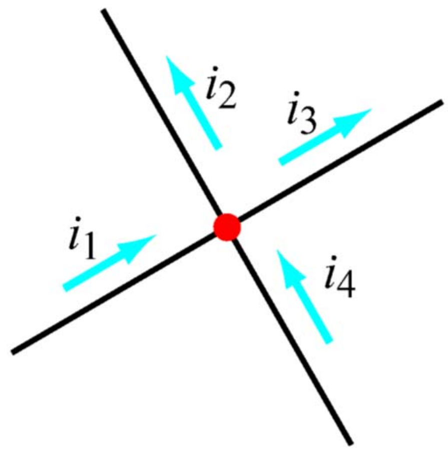
The Kirchhoff voltage law (**KVL**) is about voltages and loops).

The sum of all of the **voltage** differences encountered while moving around a closed **loop** is zero.

The Kirchhoff laws are fundamental and always true. If you have a circuit that doesn't obey the Kirchhoff Laws …it's an invalid circuit!

Graphically

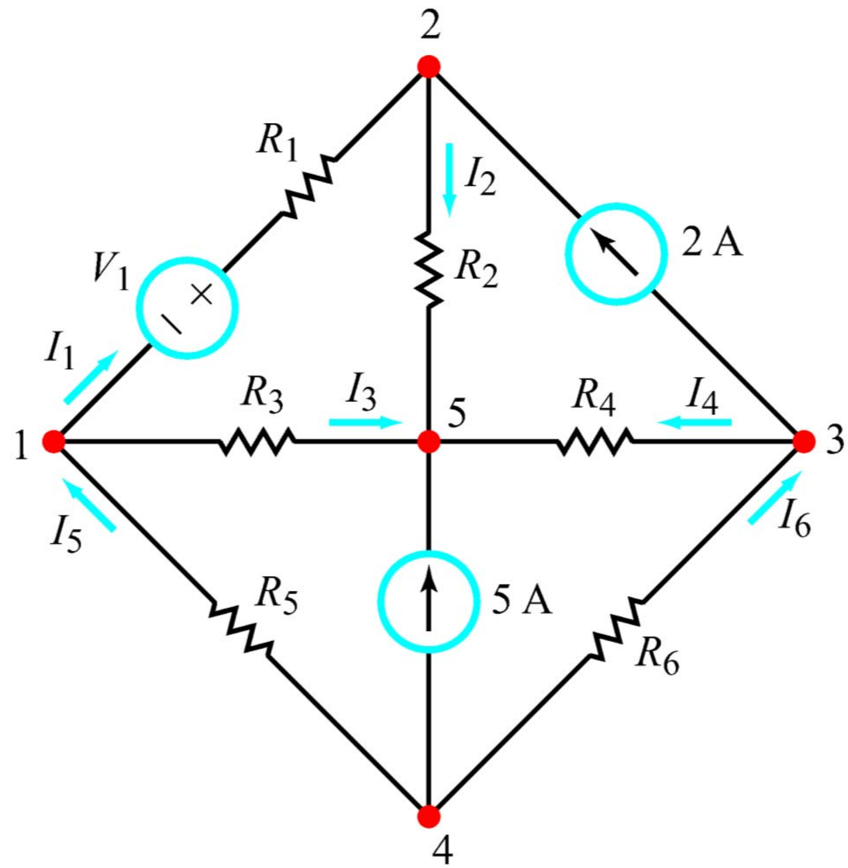
<u>Kirchhoff Current Law</u> (KCL) – the total current entering <u>a node</u> must be equal to the total current leaving <u>a node</u> …i.e. charge doesn't "build up" at a node.



$$I_1 + i_4 = i_2 + i_3$$

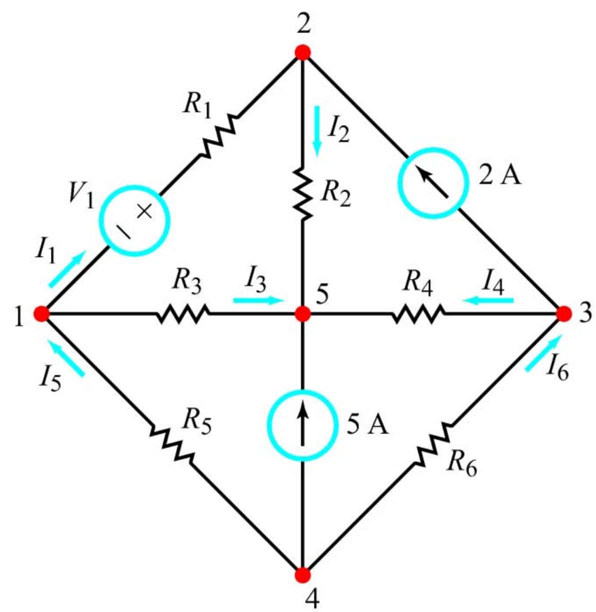
Note: when writing the KCL at a node, there will be one term for each branch connecting to the node. <u>As a convention, we consider current leaving a node as positive.</u>

<u>Kirchhoff Current Law</u> (KCL)



What are the KCL equations for nodes 1 through 5? How many branches are there in this circuit? Nodes? Loops? Does it satisfy $b = n + l_{ind} - 1$

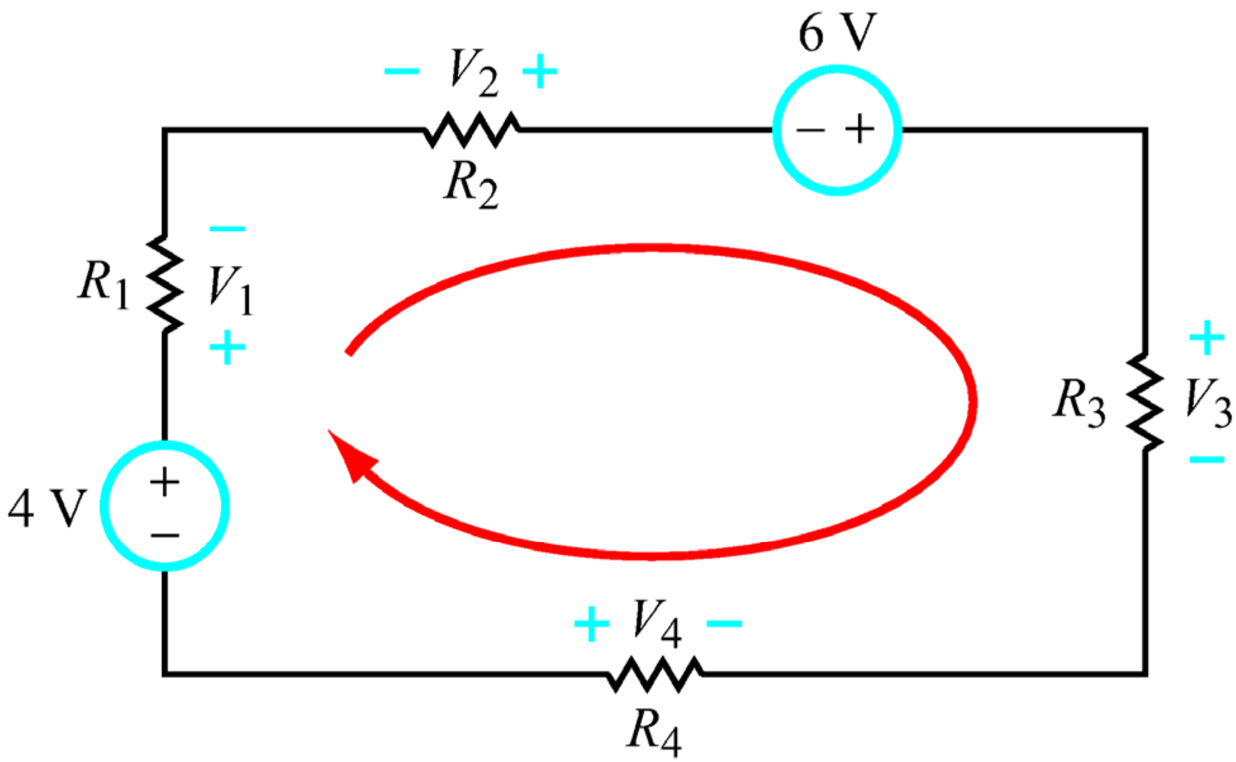
Kirchhoff Current Law (KCL)



At node 1:	I1 +I3 -I5 =0
At node 2:	-I1 +I2 -2 =0
At node 3:	2 +I4 -I6 =0
At node 4:	5 +I5 +I6 =0
At node 5:	-I3 -I4 -I2 -5 =0

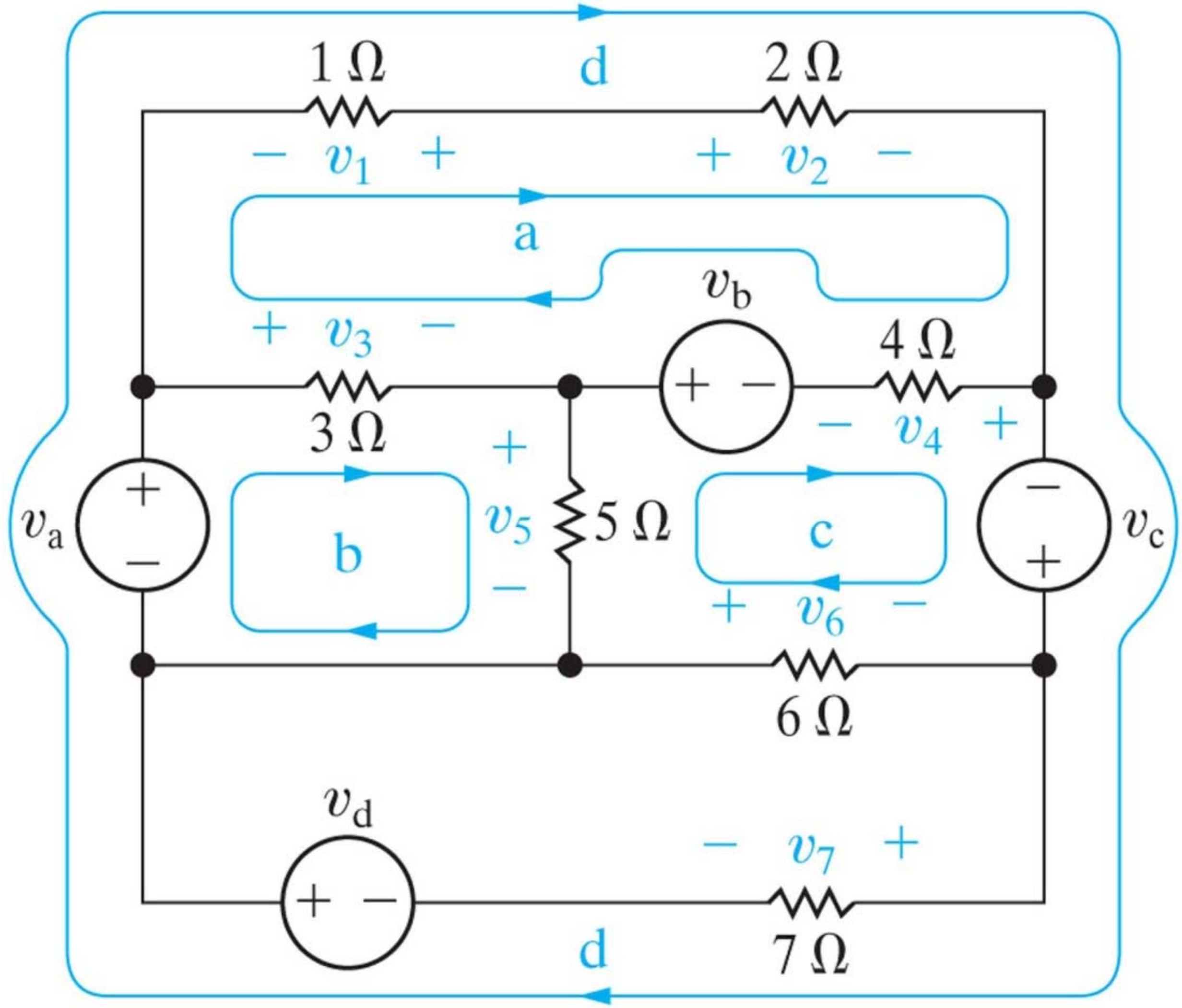
Graphically

Kirchhoff Voltage Law (KVL) – the algebraic sum of the voltages around a closed loop is zero.

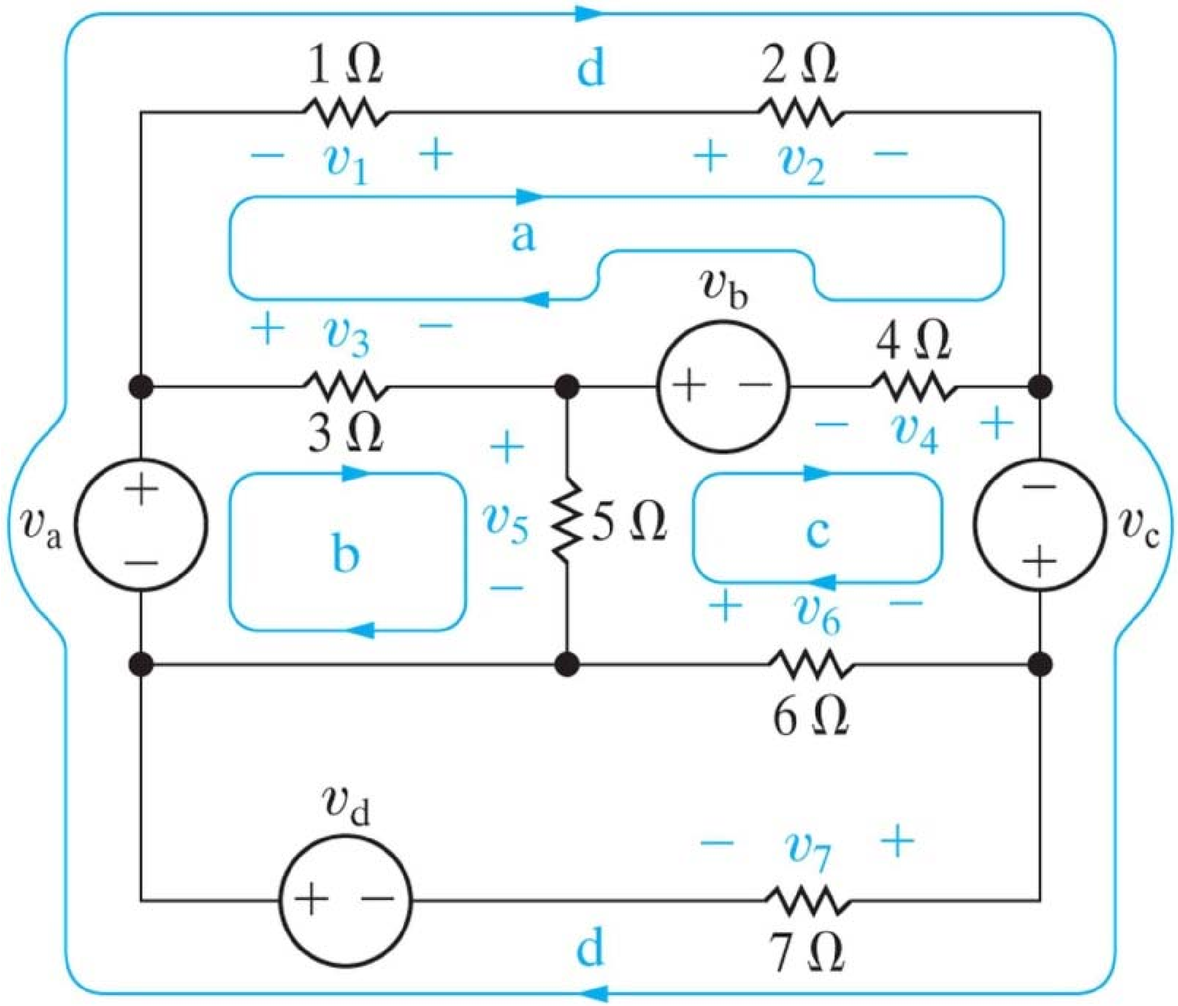


-4 + V1 – V2 -6 +V3 –V4 = 0

Note: a common convention is to assign a + sign to the voltage across an element if the (+) sign is the voltage encountered "entering the element" as you go around the loop.



What are the KVL equations for loops **a** through **d**?



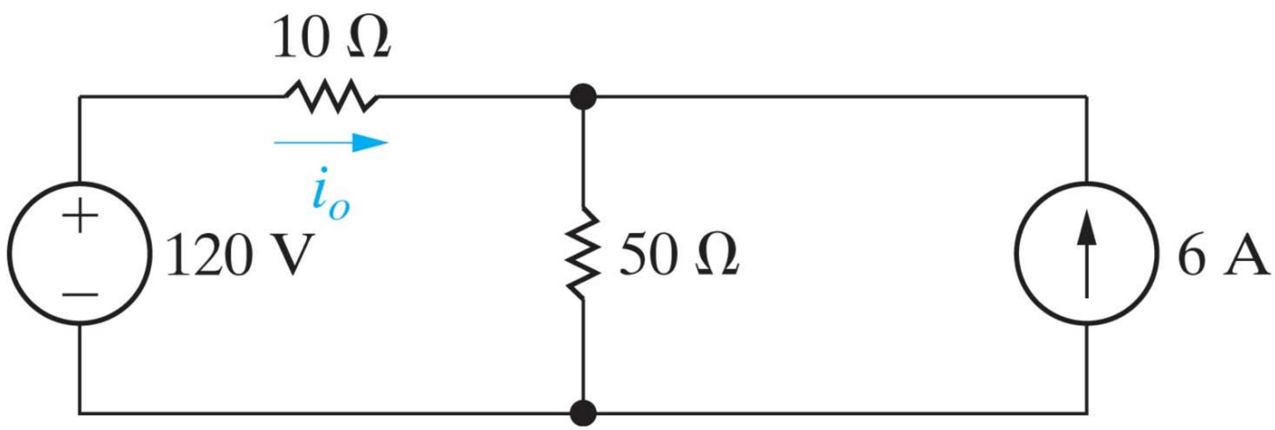
For loop a: -v1 +v2 +v4 –vb -v3 =0

For loop b: -va +v3 +v5 =0

For loop c: vb –v4 –vc –v6 –v5 =0

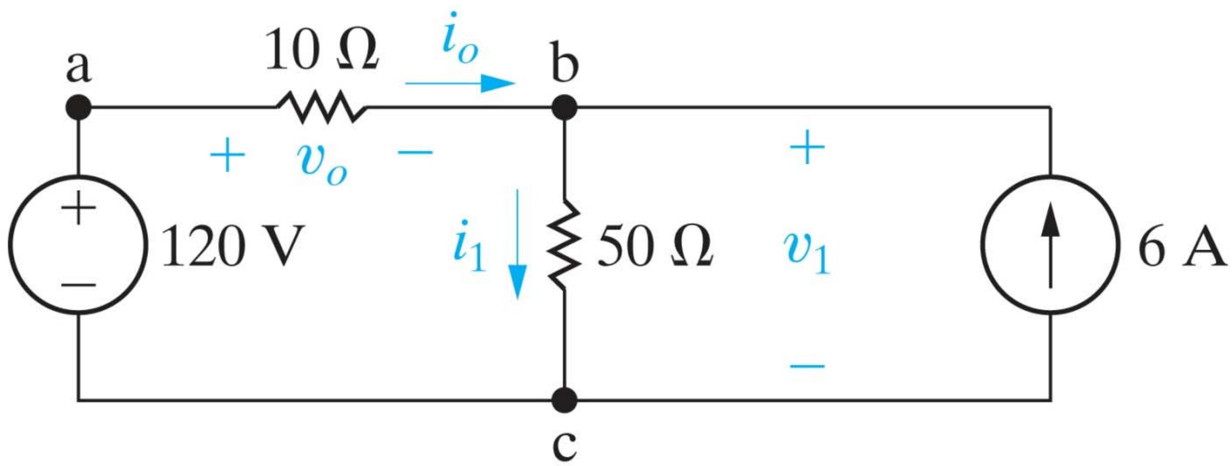
For loop d: -va –v1 +v2 –vc +v7 –vd = 0

Use Kirchhoff's Laws and Ohms Law to find i_0 in this circuit



There are two extraordinary nodes in the circuit and three loops, so, in principal we could write two equations using KCL (the two nodes) and three equations using KVL (the three loops). In other words, we'd have 5 equations. Since there are only two unknowns, we won't need to write all the equations. Which equations should we write?

Labeling the unknown currents and voltages:



Use KCL on node b: i_1 - i_0 -6 = 0 <u>by convention, we write the current leaving the node as positive.</u>

We can write KVL on the loop a-b-c: - 120 + v_0 + v_1 = 0
Or, by using Ohms Law: -120 + 10 i_0+ 50 i_1 = 0

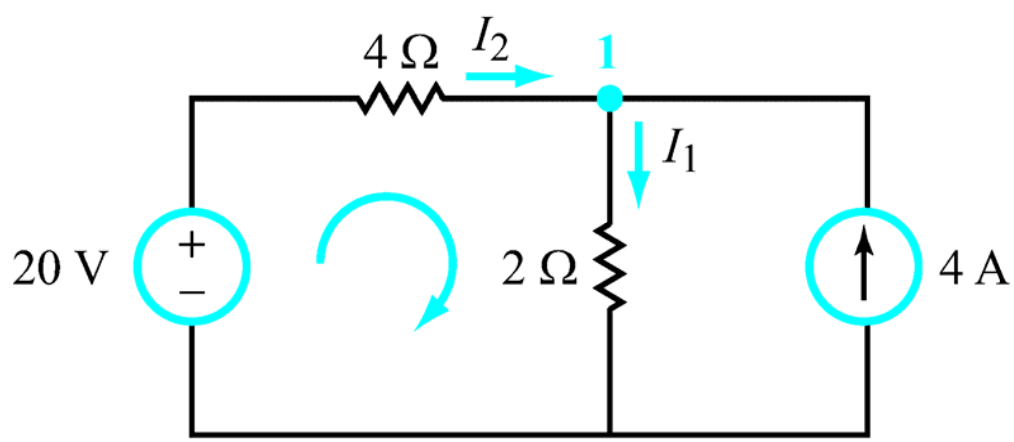
So we've got two equations in two unknowns ..and we solve to get:

i_0 = - 3A
i_1 = 3A

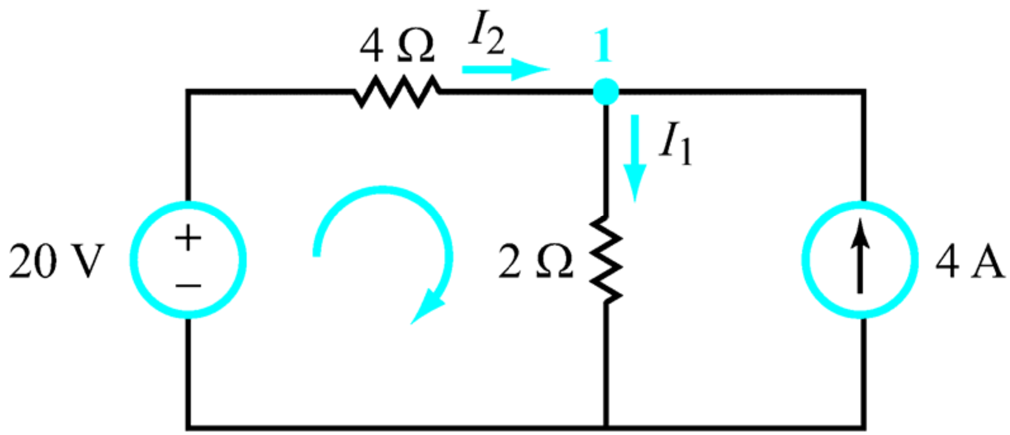
How can we get the voltages at the nodes? Are these already determined by i_0 and i_1?

Here’s another one:

Find I_1 and I_2



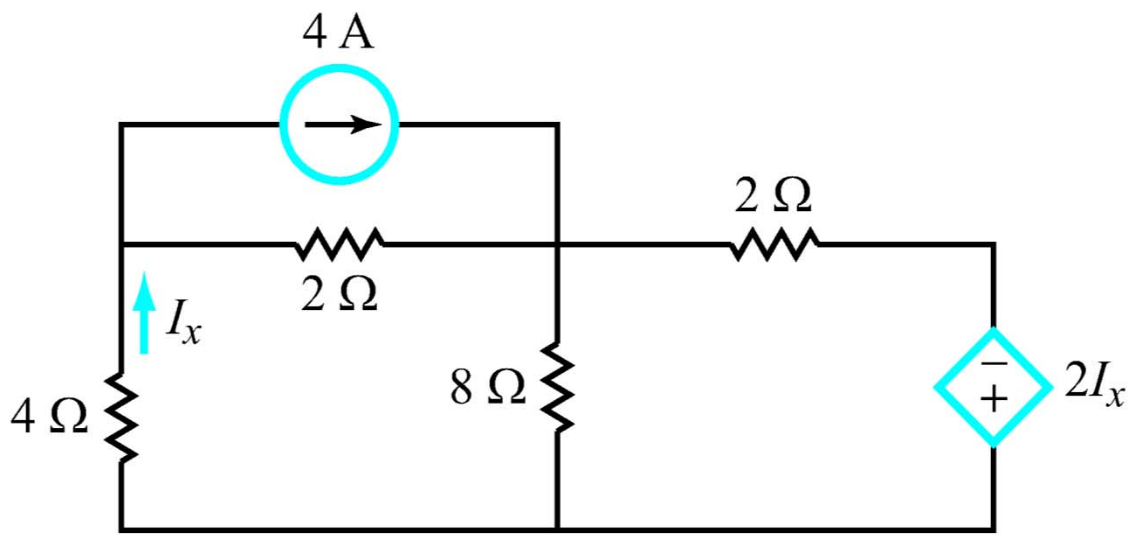
KCL at node #1 will involve I_1 and I_2 (and the 4A source) and KVL around the loop shown with the blue arrow will also involve I_1 and I_2 – giving us two equations in two unknowns.



KCL at node 1 requires that $I_1 = I_2 + 4$

KVL for the left loop is $-20 + 4I_2 + 2I_1 = 0$

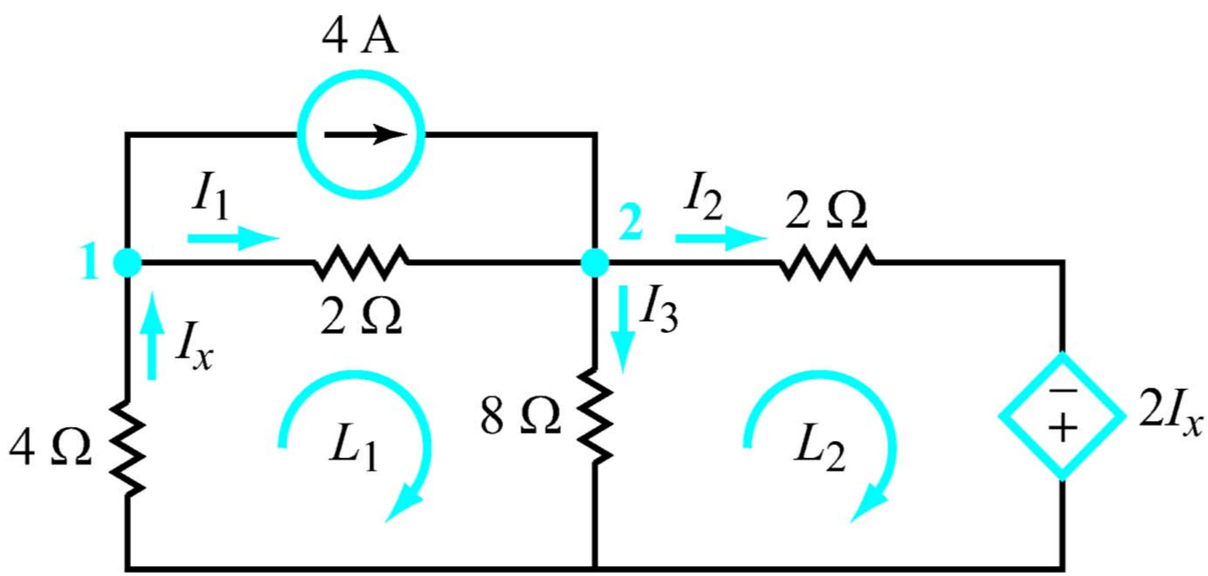
Simultaneous solution leads to $I_1 = 6$ A; $I_2 = 2$ A:



Determine I_x

….this is a more complicated circuit!

Labeling two nodes and two loops, we can write KCL and KVL



KCL @ node 1: $I_x = I_1+4$

KCL @ node 2: $I_1+4 = I_2+I_3$

KVL Loop 1: $4I_x+2I_1+8I_3 = 0$

KVL Loop 2: $-8I_3+2I_2- 2I_x = 0$

Four equations in four unknowns – probably should use matrix methods to solve this

How do we do that? Here's a description of one approach from the ELEN lab 1 document for this week's lab. For the circuit in the lab, KVL and KCL resulted in:

The result of a KVL/KCL analysis of a circuit is the set of simultaneous equations:

$$\begin{aligned} V1 &+ & & & V3 &= 10 \\ 3V1 &+ & 3V2 &+ & 4V3 &= 12 \\ 2V1 &+ & 2V2 &+ & 3V3 &= 5 \end{aligned}$$

which can be written using matrices as follows:

$$\begin{bmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} V1 \\ V2 \\ V3 \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 5 \end{bmatrix} = C \cdot V = S$$

$$C = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix} \qquad V = \begin{bmatrix} V1 \\ V2 \\ V3 \end{bmatrix} \qquad S = \begin{bmatrix} 10 \\ 12 \\ 5 \end{bmatrix}$$

To solve this 3x3 system of equations we invert the coefficients matrix C and multiply it by the source matrix S.

Invert the matrix C and solve the system for the voltage matrix V

So using MATLAB to solve the set of four equations we got for the circuit:

KCL @ node 1: $I_x = I_1+4$

KCL @ node 2: $I_1+4 = I_2+I_3$

KVL Loop 1: $4I_x+2I_1+8I_3 = 0$

KVL Loop 2: $-8I_3+2I_2- 2I_x = 0$

First we put the equations in standard form:

Ix	-I1			= 4
	I1	- I2	-I3	= -4
4Ix	2I1		8I3	= 0
-2Ix		2I2	-8I3	= 0

Now we can invert the coefficients matrix and multiply by the source matrix

```
A=[1,-1,0,0;0,1,-1,-1;4,2,0,8;-2,0,2,-8]

A =

     1    -1     0     0
     0     1    -1    -1
     4     2     0     8
    -2     0     2    -8


b=[4;-4;0;0]

b =

     4
    -4
     0
     0
```

A * X = b, and we can solve for the X vector by multiplying the b vector by the inverse of the coefficients matrix

```
C=inv(A)

C =

    0.6000    0.2667    0.1667    0.1333
   -0.4000    0.2667    0.1667    0.1333
   -0.2000   -0.5333    0.1667    0.2333
   -0.2000   -0.2000         0   -0.1000

C*b

ans =

    1.3333
   -2.6667
    1.3333
         0
```

This is the X vector with components Ix, I1, I2,and I3

We could also solve the set of simultaneous equations by forming what is called the “augmented matrix”, A:
A=[1,-1,0,0,4;0,1,-1,-1,-4;4,2,0,8,0;-2,0,2,-8,0]

A =

1	-1	0	0	4
0	1	-1	-1	-4
4	2	0	8	0
-2	0	2	-8	0

Then the MATLAB command rref(A) will produce:

```
>> rref(A)

ans =

    1.0000         0         0         0    1.3333
         0    1.0000         0         0   -2.6667
         0         0    1.0000         0    1.3333
         0         0         0    1.0000         0
```

So Ix = 1.3333, I1= -2.66667, I2= 1.3333, and I3= 0

Probably the easiest way to solve a set of linear equations in MATLAB is by using the “backslash operation”

For this matrix equation:

A * **X** = **b**

We can solve for the vector **X** using the statement in MATLAB:

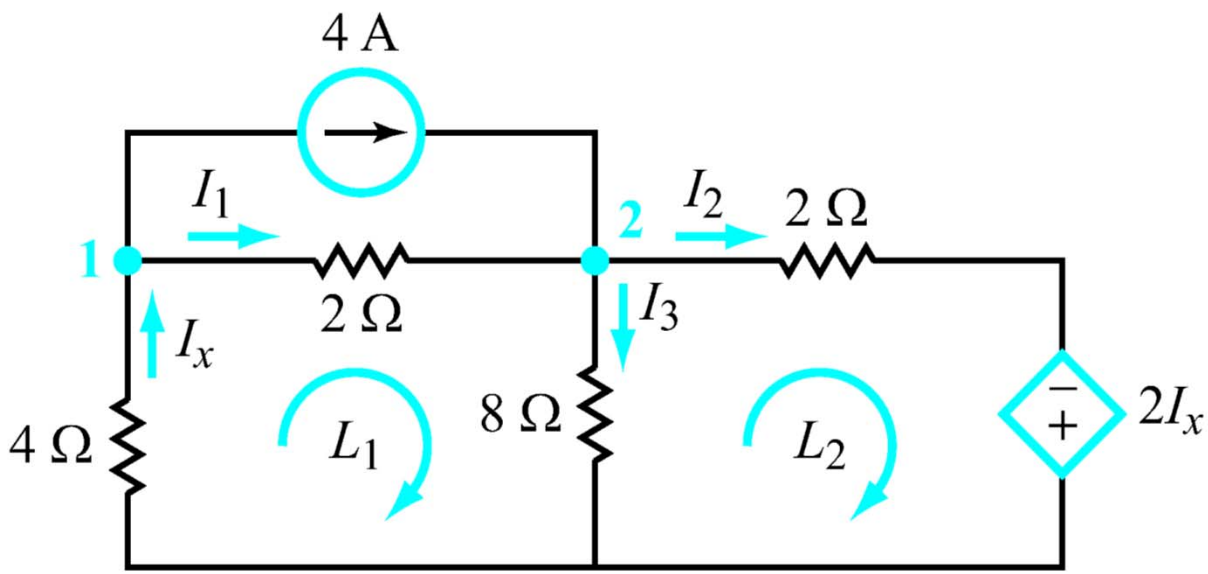
X = **A****b**

Notice, this obviously doesn’t mean that the vector, **X** is equal to the matrix, **A** divided by the vector, **b**. It’s just a way of writing a MATLAB operation

Here it is in MATLAB:

```
 A=[1,-1,0,0;0,1,-1,-1;4,2,0,8;-2,0,2,-8]

A =

     1    -1     0     0
     0     1    -1    -1
     4     2     0     8
    -2     0     2    -8

>>
>> b=[4;-4;0;0]

b =

     4
    -4
     0
     0

>> X=A\b

X =

    1.3333
   -2.6667
    1.3333
    0.0000
```

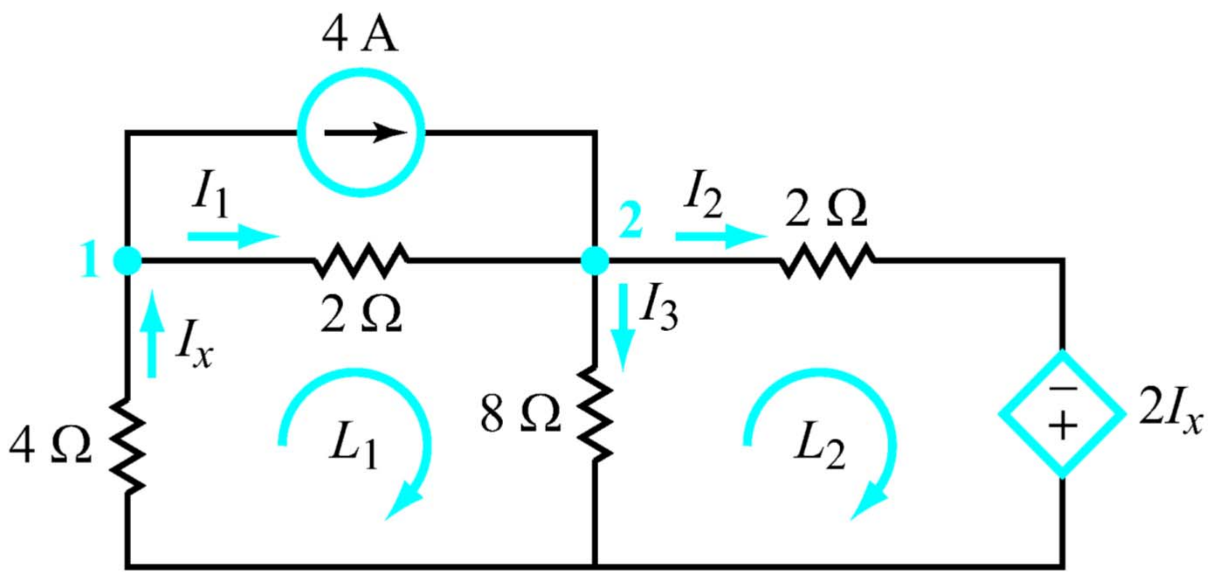
The backslash operation



Do I really need to solve a 4 x 4 matrix to solve this circuit?

Remember I told you earlier that either solving for the voltages at all of the extraordinary nodes or the loop currents in all of the meshes of a circuit will completely determine a circuit?

It seems that I could solve this circuit by solving a 2 x 2 matrix – either for the node voltages or for the mesh currents.



Can you convince yourself that knowing the voltages at node 1 and node 2 …or the mesh currents in L1 and L2 will give you all the information you need?