ELEN 50 Class 6 – Source Transformations

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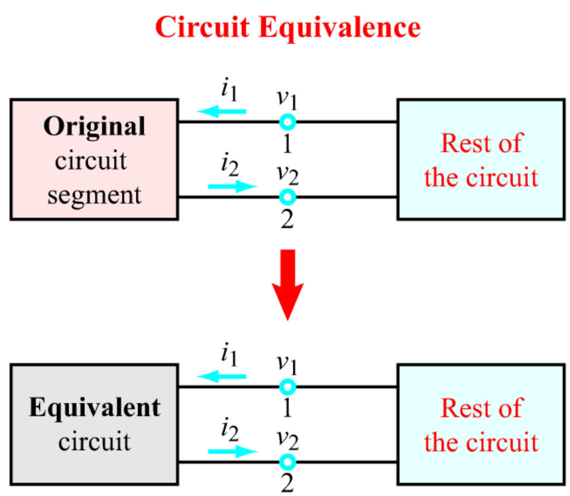
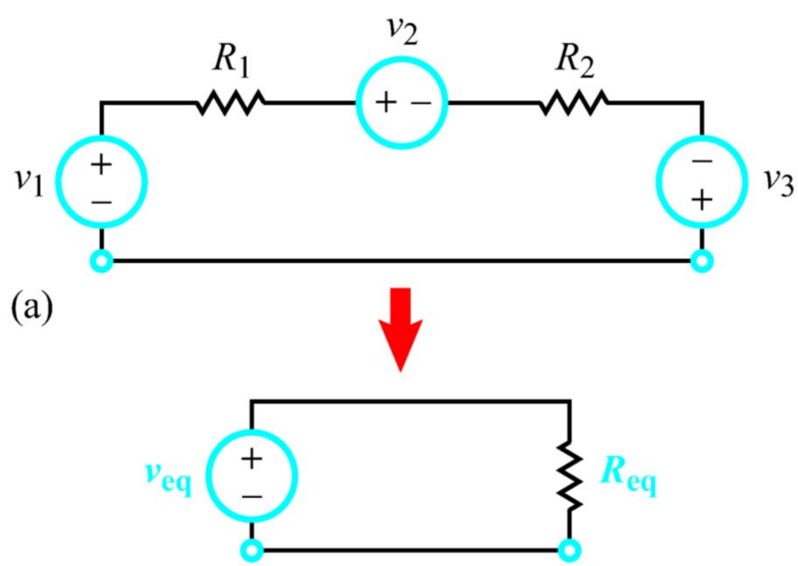
In the last lecture, we briefly introduced the concept of an **equivalent circuit**:

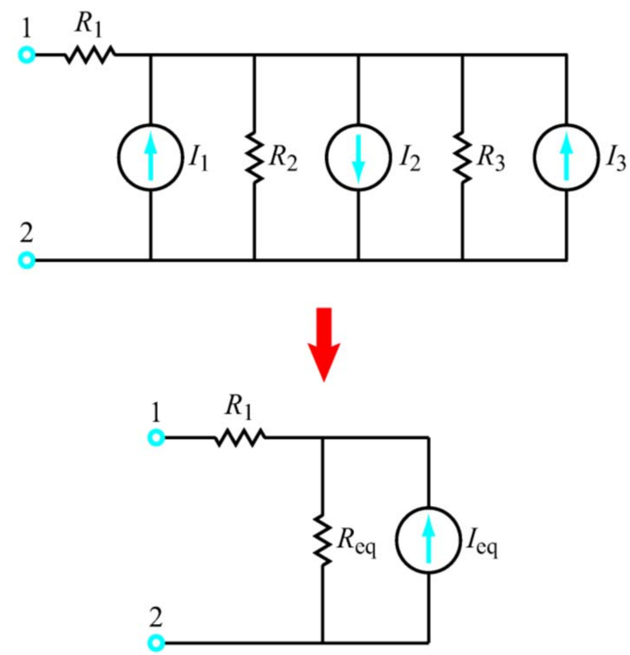


Where the equivalent circuit is defined to be equivalent to the original circuit if the <u>i-v characteristics at nodes 1 and 2 are identical to the i-v characteristics at these nodes in the original circuit.</u>

Replacing series and parallel combinations of resistors and sources with an equivalent resistor and source is one way of creating an equivalent circuit. Another important way to do this is with a <u>source transformation</u>.

And we discussed the simplest kind of equivalent circuits – where we could combine resistors and/or sources in series or parallel.

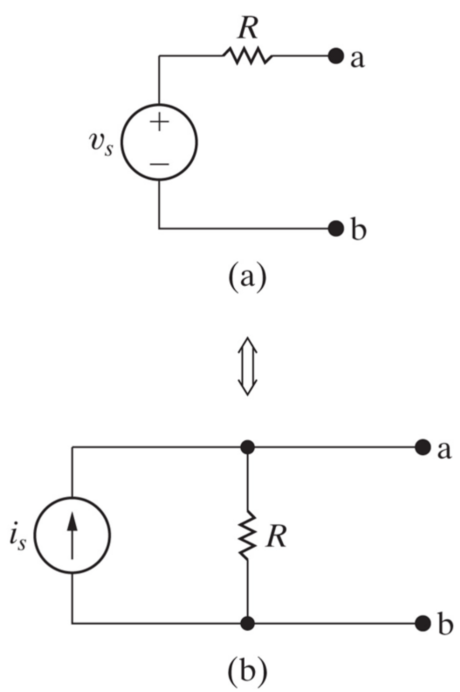




<u>This is a very useful technique for circuit analysis!</u>

Today we're going to talk about deriving equivalent circuits through <u>source transforms</u>.

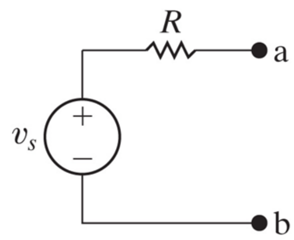
A source transform allows a <u>voltage source in series with a resistor to be transformed into a current source in parallel with a resistor.</u> This can be very useful in simplifying a circuit prior to doing a circuit analysis.



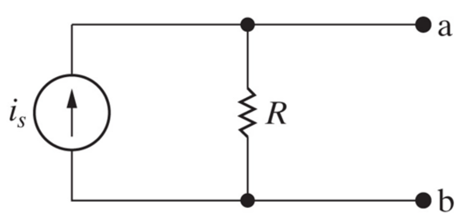
These two circuits are equivalent, if:

$$\boxed{i_s = v_s/R}$$

You can show this by attaching a load resistor to a and b and calculating the current flowing



(a)



(b)

Suppose we attach a load resistor, R_L across terminals a and b in the top figure. The current flowing through the resistor will be:

$i_L = v_s/(R + R_L)$ this is just Ohms law – since R_L and R are in series

Now if we attach the same resistor to terminals a and b in the bottom figure, the current in R_L will be:

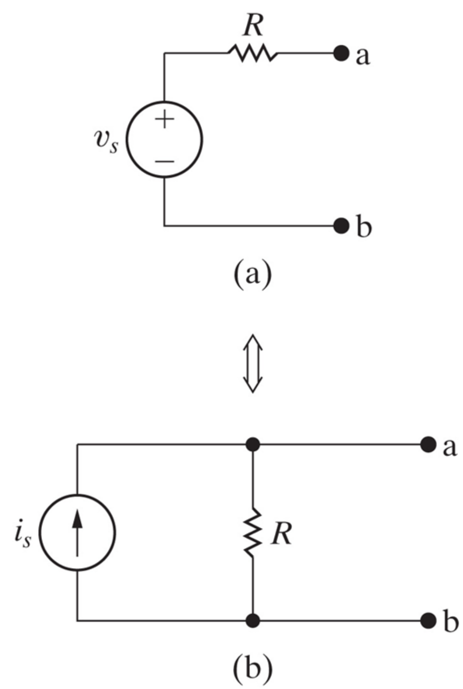
$i_L = R\, i_s /(R + R_L)$ this is a current divider

If we set the current through the load resistor in both cases to be the same, then this requires:

$$i_s = v_s/R$$

This will be true, regardless of the value of R_L …so if you had a circuit like (a) you can always replace it with (b) and if the current source obeys this equation …the circuits will be exactly equivalent.

Another way of saying it is a voltage source in series with a source resistance, R_s , is equivalent to the combination of a current source, $i_s = v_s/R_s$ in parallel with a shunt resistance, R_s.



where: $i_s = v_s/R_s$

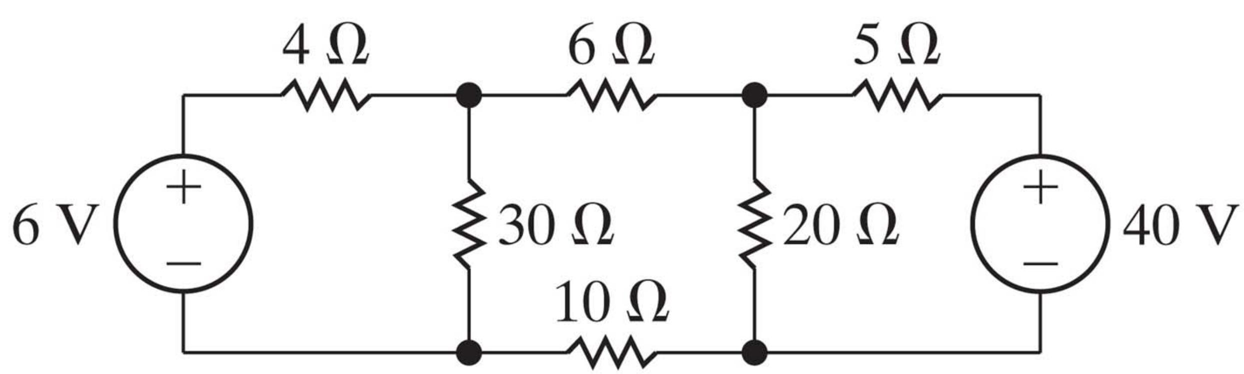
This relationship may not seem to be intuitive – but it is correct as we just saw --- and now I'll show you that <u>it's very useful</u>

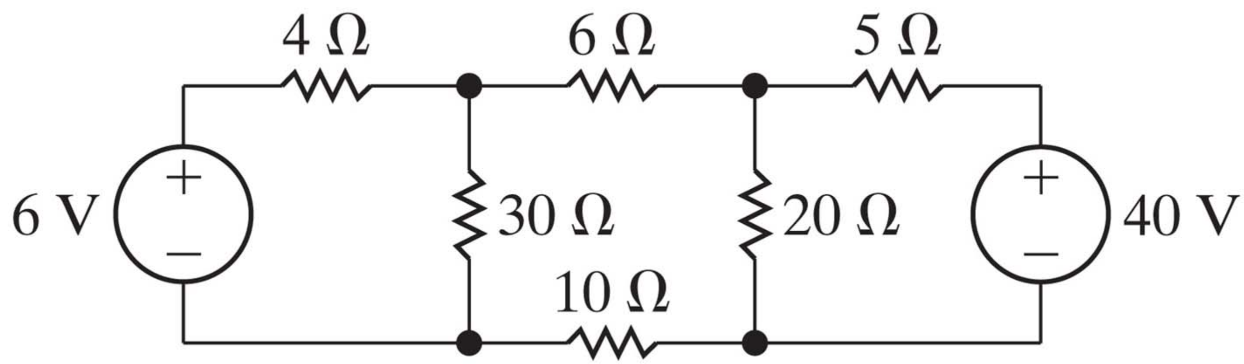
OK…. Why would you ever want to transform a series connected voltage source and a resistor into a parallel combination of a current source and a resistor?

Answer: you can often use this trick to greatly simplify a circuit you are trying to analyze.

Here's an example :

We want to find the <u>power (absorbed or supplied) by the 6V source</u>



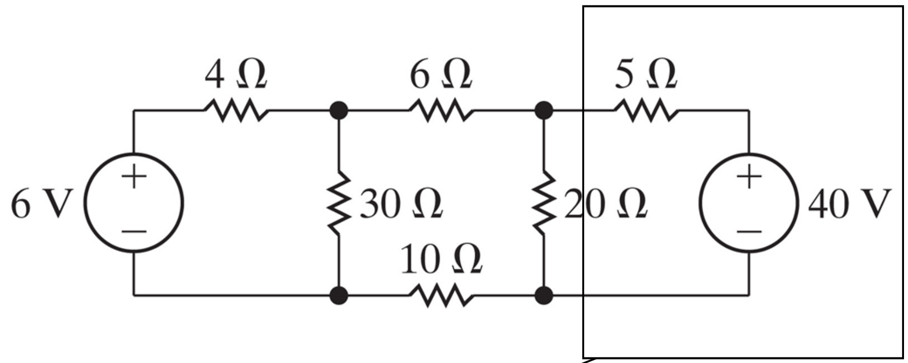


• There's no way to simplify the circuit by series and parallel combinations of the resistors (you should convince yourself of this)

• You could analyze the circuit using KCL – there are four essential nodes …so this would require solving three simultaneous equations (three nodes and a reference, "ground" node)….matrix algebra!

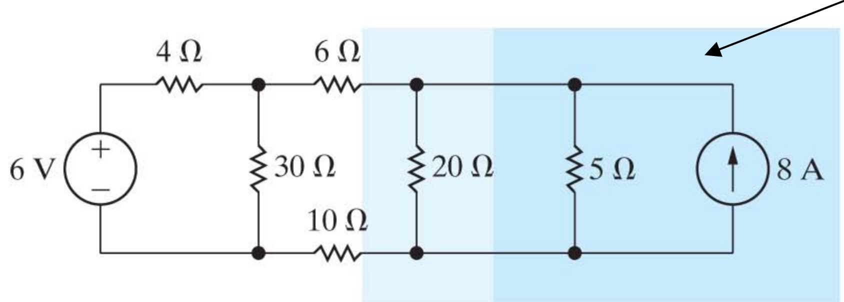
• Or you could use KVL – there are three loop currents, so, again, you'd need to solve three simultaneous mesh current equations…..again, matrix algebra.

Or …since to calculate the source power you care only about the <u>current flowing through the 6V source</u>, you could replace all of the rest of the circuit other than the branch containing the 6V source with an equivalent circuit ----- using source transformations.

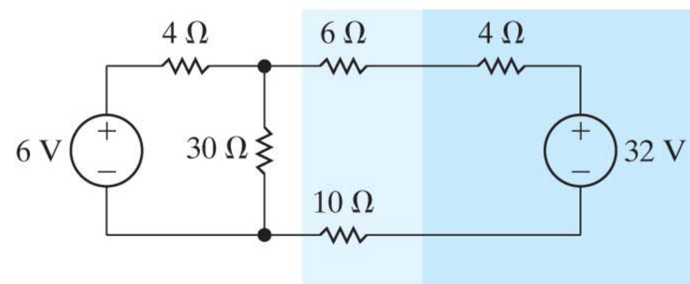


Remember

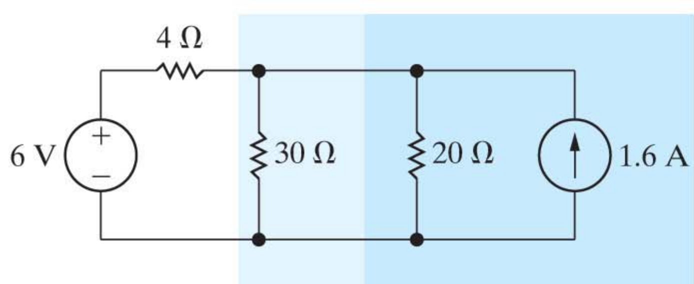
$$\boxed{i_s = v_s/R}$$



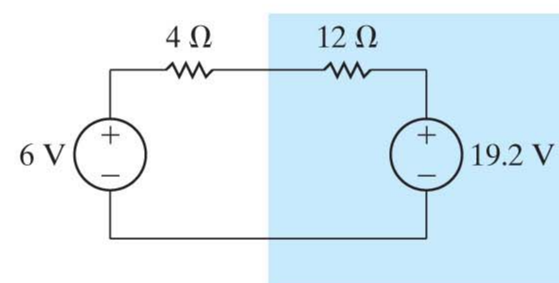
(a) First step



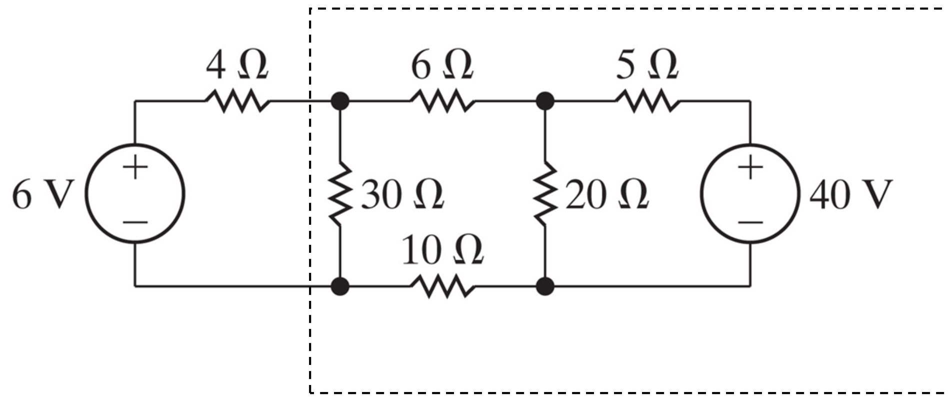
(b) Second step



(c) Third step



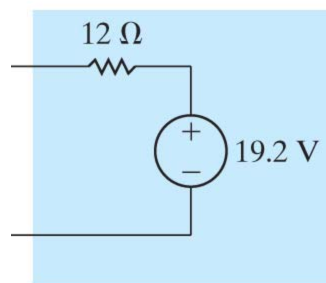
(d) Fourth step

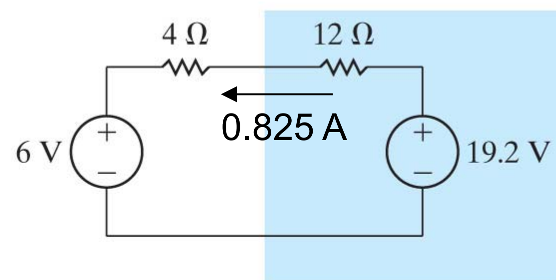


This whole thing has been replaced by:

Now it's trivially easy to determine <u>the current</u> through the 6V source – using Ohm's Law – No KCL…no KVL !.. it's (19.2 -6)/16 = 0.825A

flowing <u>backwards</u> through the voltage source so the source <u>absorbs</u> 4.95 W

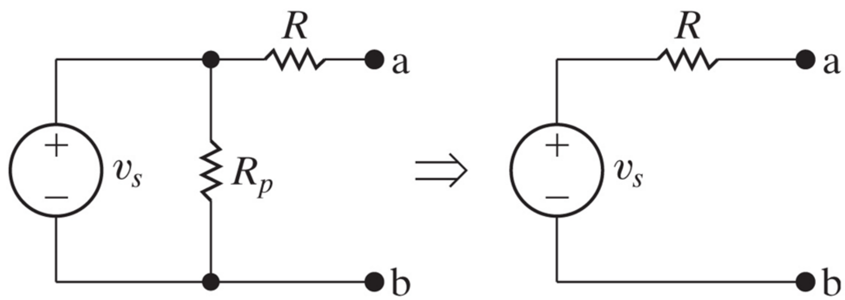




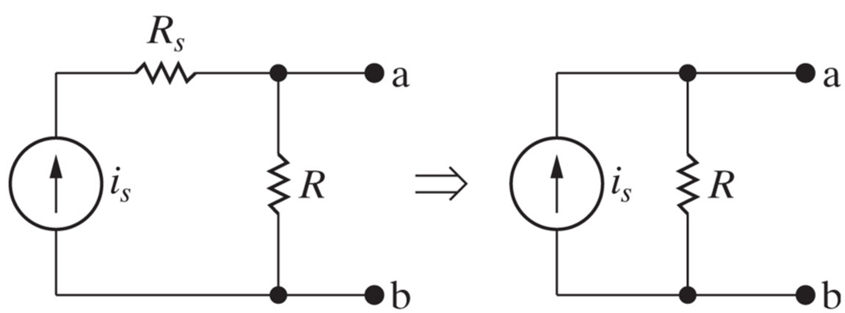
(d) Fourth step

This is an Important Trick!

What happens if there is already a resistor in parallel with a voltage source or a resistor in series with a current source when you do a source transform?



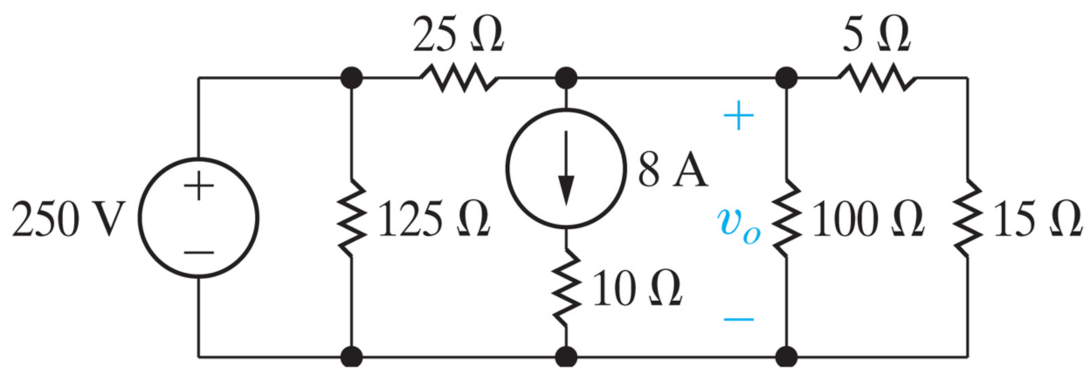
(a)



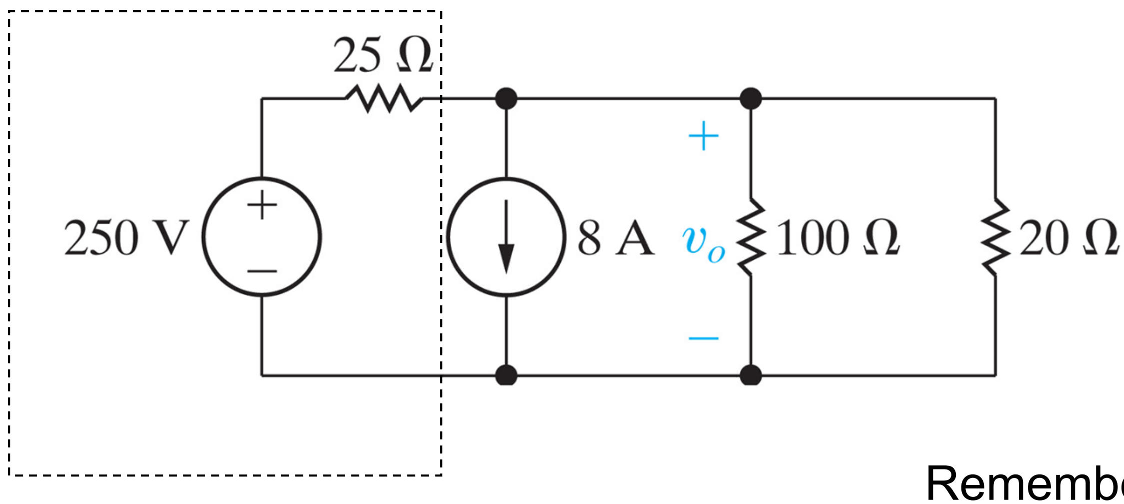
(b)

<u>Nothing!</u> The series or parallel resistor <u>can be removed</u>….it has no effect as regards the circuit to the left of terminals a and b. Can you figure out why this is true?

An example – find the voltage v0 in this circuit:



First we eliminate all resistors in <u>parallel with voltage sources</u> or resistors in <u>series with current sources</u> (the trick we just learned)

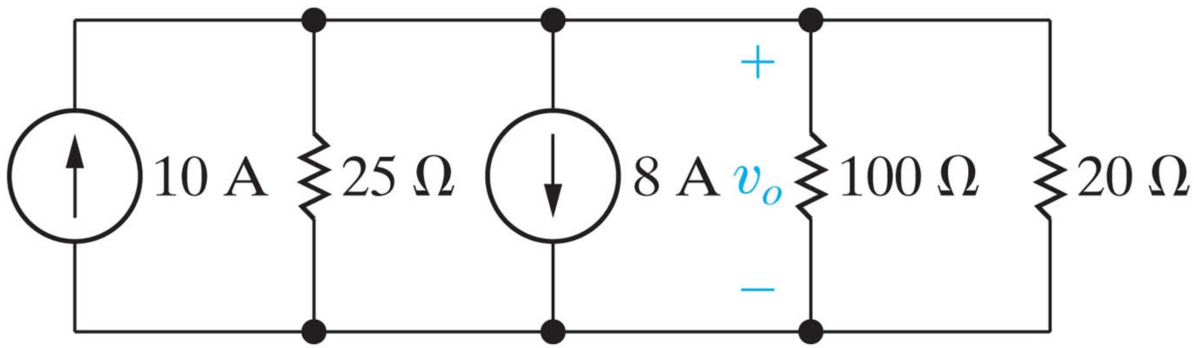


Then we replace this with a current source in parallel with a resistor

Remember

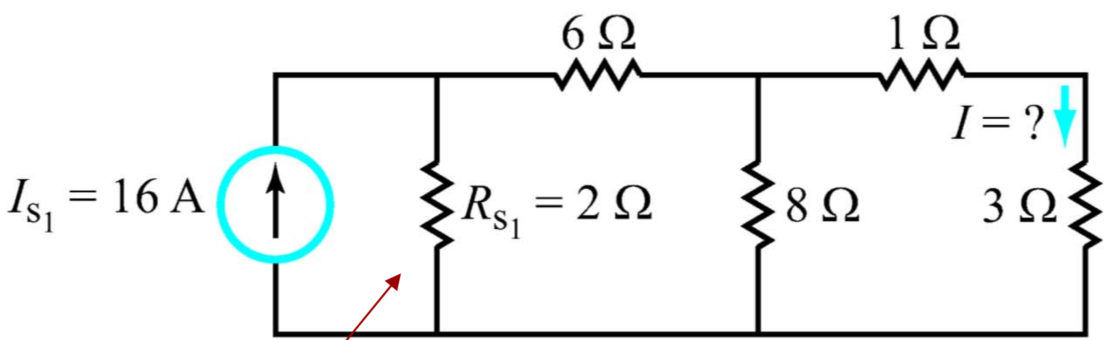
$$\boxed{i_s = v_s/R}$$

So the circuit becomes:

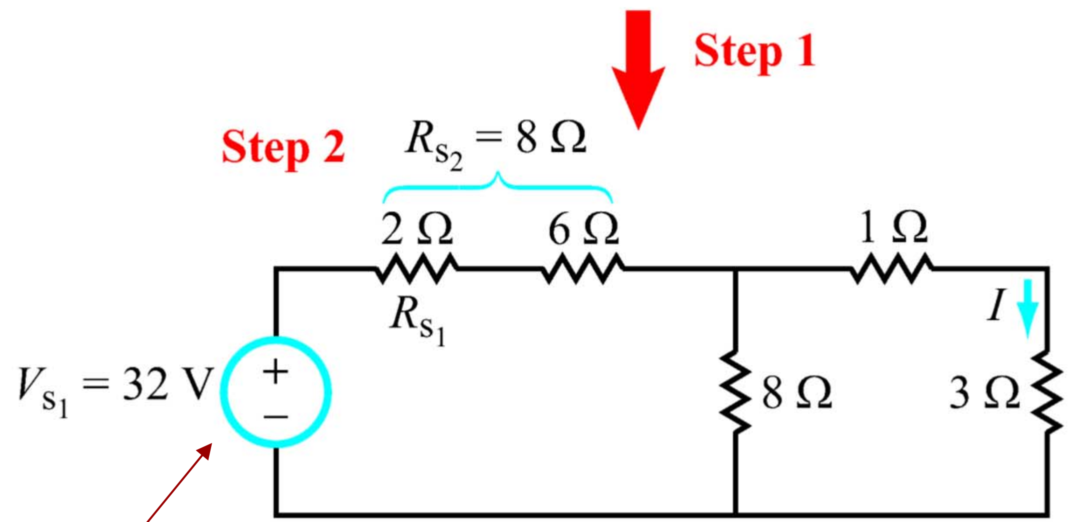


The three resistors in parallel with the current sources can be replaced by a single resistor of 10Ω and the current sources can be combined so the current flowing through the resistor is (10 -8) = 2A …so, Ohm's Law requires: v_0 = 20V.

We want to determine the current, I in this circuit. Can we simplify it by source transformations?

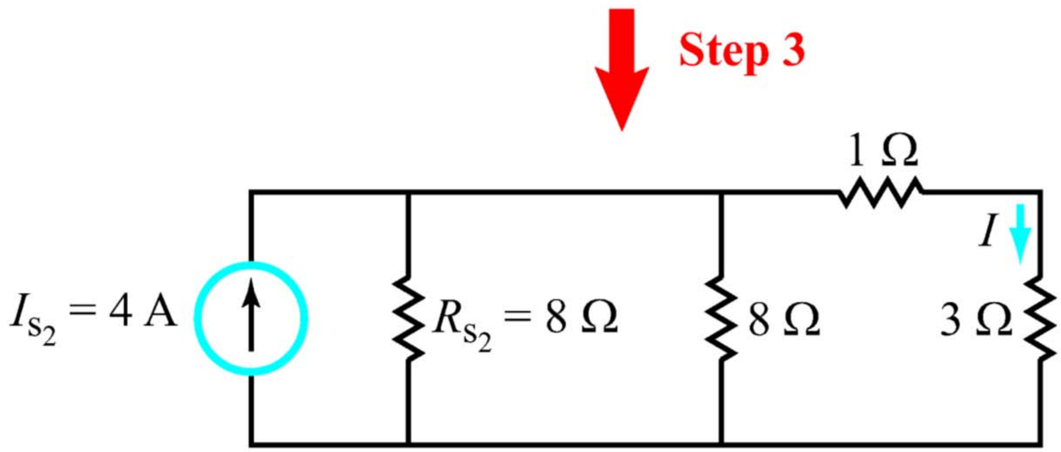


Let’s start by transforming this shunt resistance into a series resistance …which we can add to the 6Ω resistor.

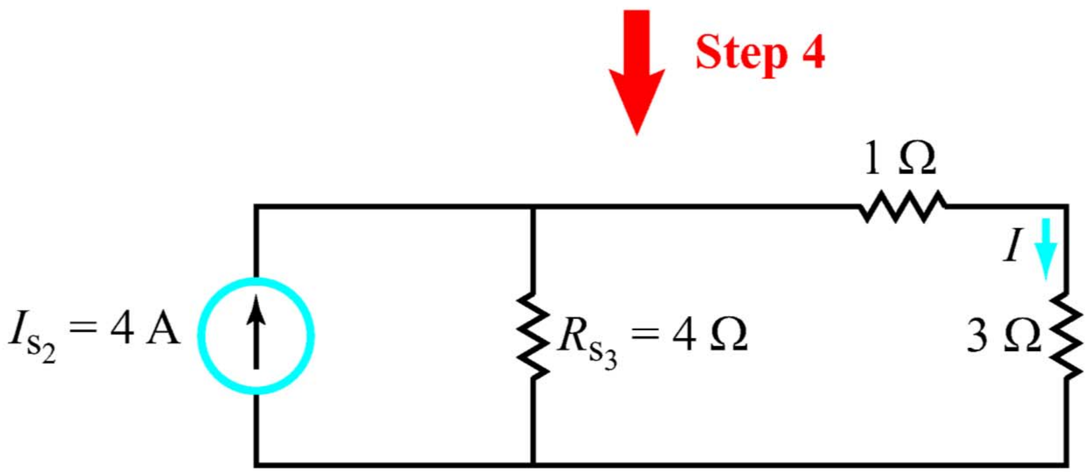


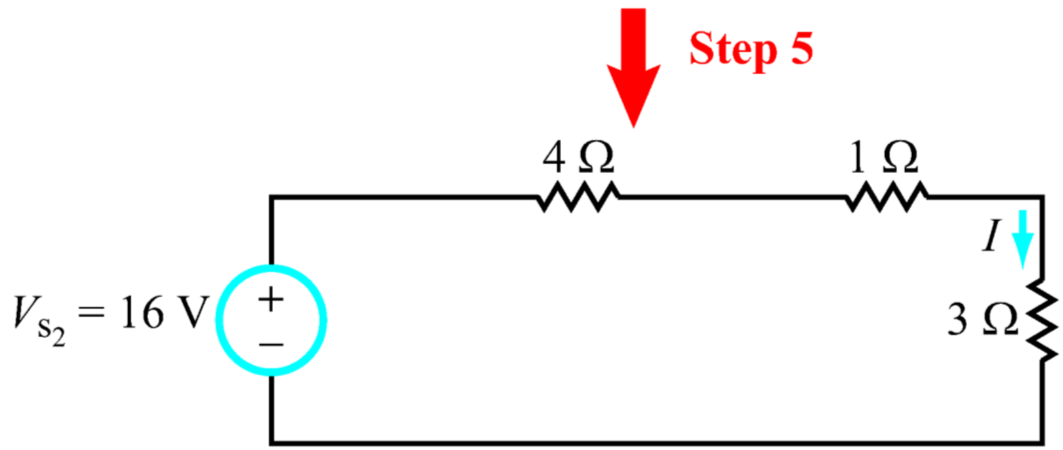
Remember ….$v_s = i_s R$

Now transforming the 8Ω series resistor and 32V voltage source into a current source with a shunt resistor



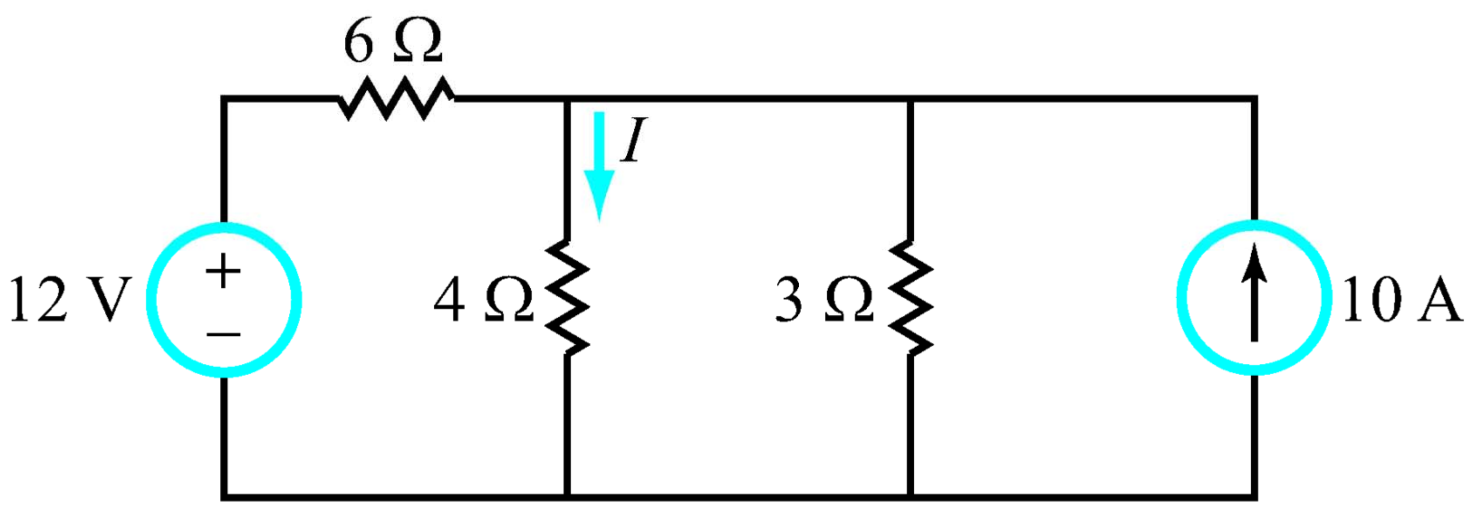
Remember ….$v_s = i_s R$

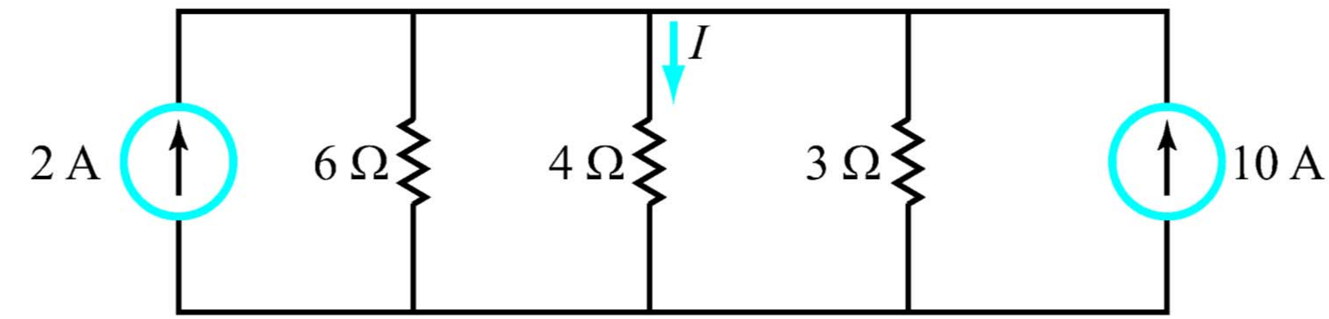

Step 4
1 Ω
I
$I_{S_2} = 4$ A
$R_{S_3} = 4\ \Omega$
3 Ω



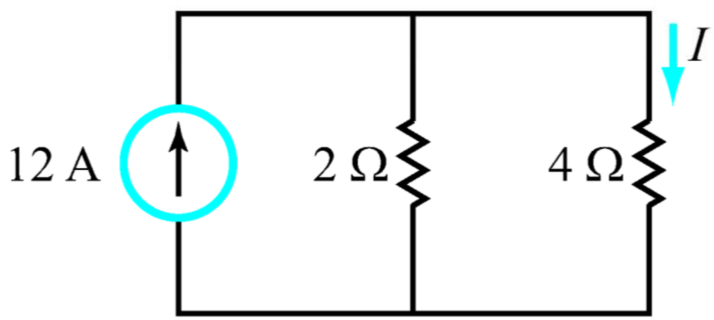
A total of 8Ω resistance across a 16V source results in 2A of current flowing. I = 2A

How would you solve for I in this circuit? Series and parallel combinations won't help. If you combine the parallel 4Ω and 3Ω resistors you won't be able to answer the question about the current flowing through the 4Ω resistor – because it will have disappeared!. You could always try using KCL and KVL – this will always work….but it's not very elegant. How about using source transforms?

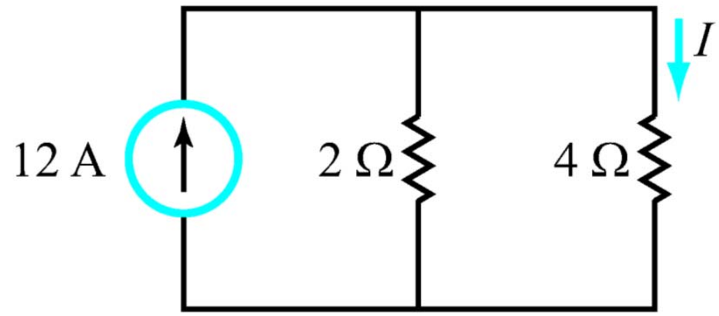




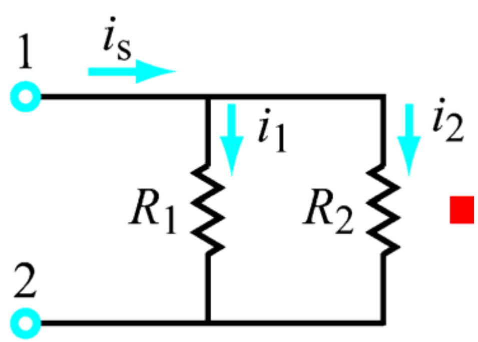
Combine current sources and combine 3-Ω and 6-Ω resistors, while leaving 4-Ω alone - why leave the 4Ω resistor alone?



This is a current divider for the 12A current source – do you remember the formula for a current divider?



For a current divider:

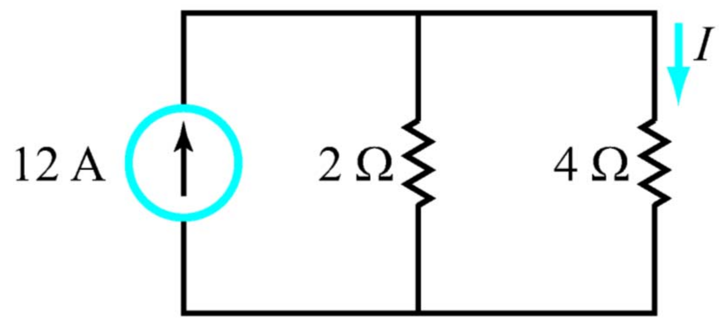


$$i_1 = \left(\frac{R_2}{R_1 + R_2} \right) i_s$$

$$i_2 = \left(\frac{R_1}{R_1 + R_2} \right) i_s$$

So: I= 12A * 2/(4+2) = 4A

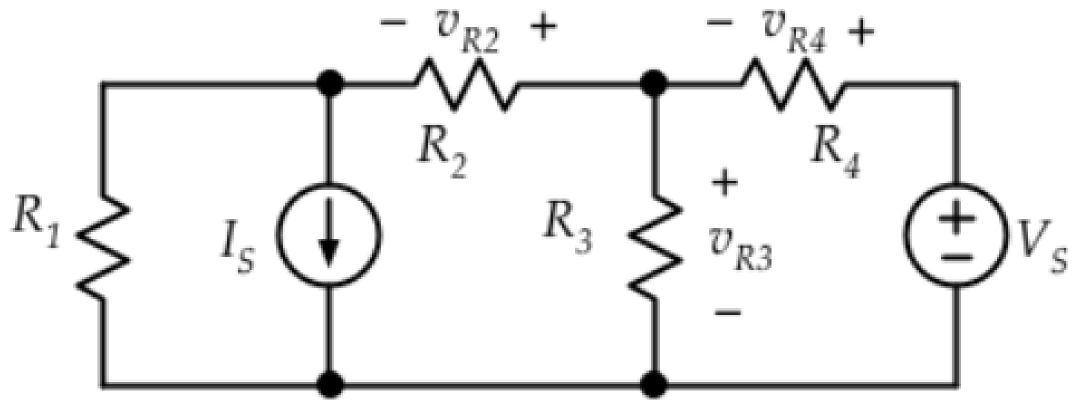
Instead of treating this as a current divider, could you have used another source transform to make the circuit even easier?



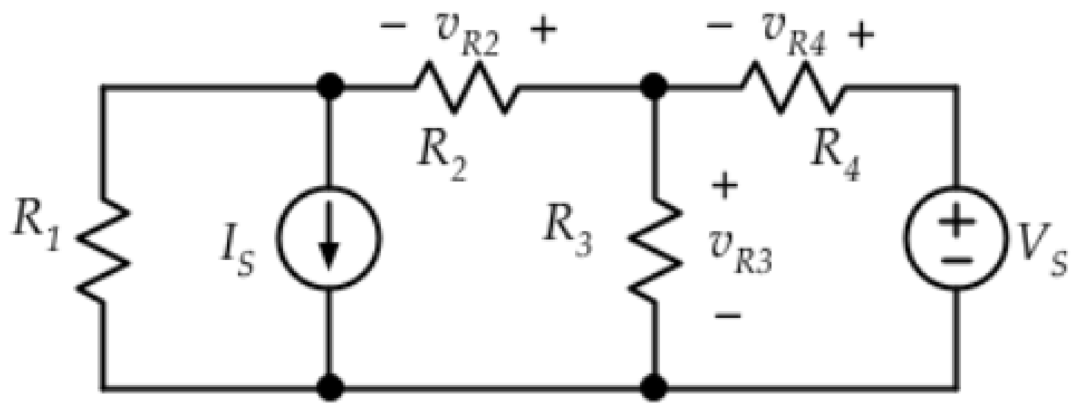
Yes….the 12 A source in parallel with the 2Ω resistor can become a 24V source in series with a 2Ω resistor. Then the entire circuit is a 24 V source in series with a total resistance of 6Ω.

Ohm's Law then gives I = 4A …which agrees with our earlier answer.

Here's another circuit that is easy to solve with source transforms. What is v_{R2} in terms of the other circuit elements?

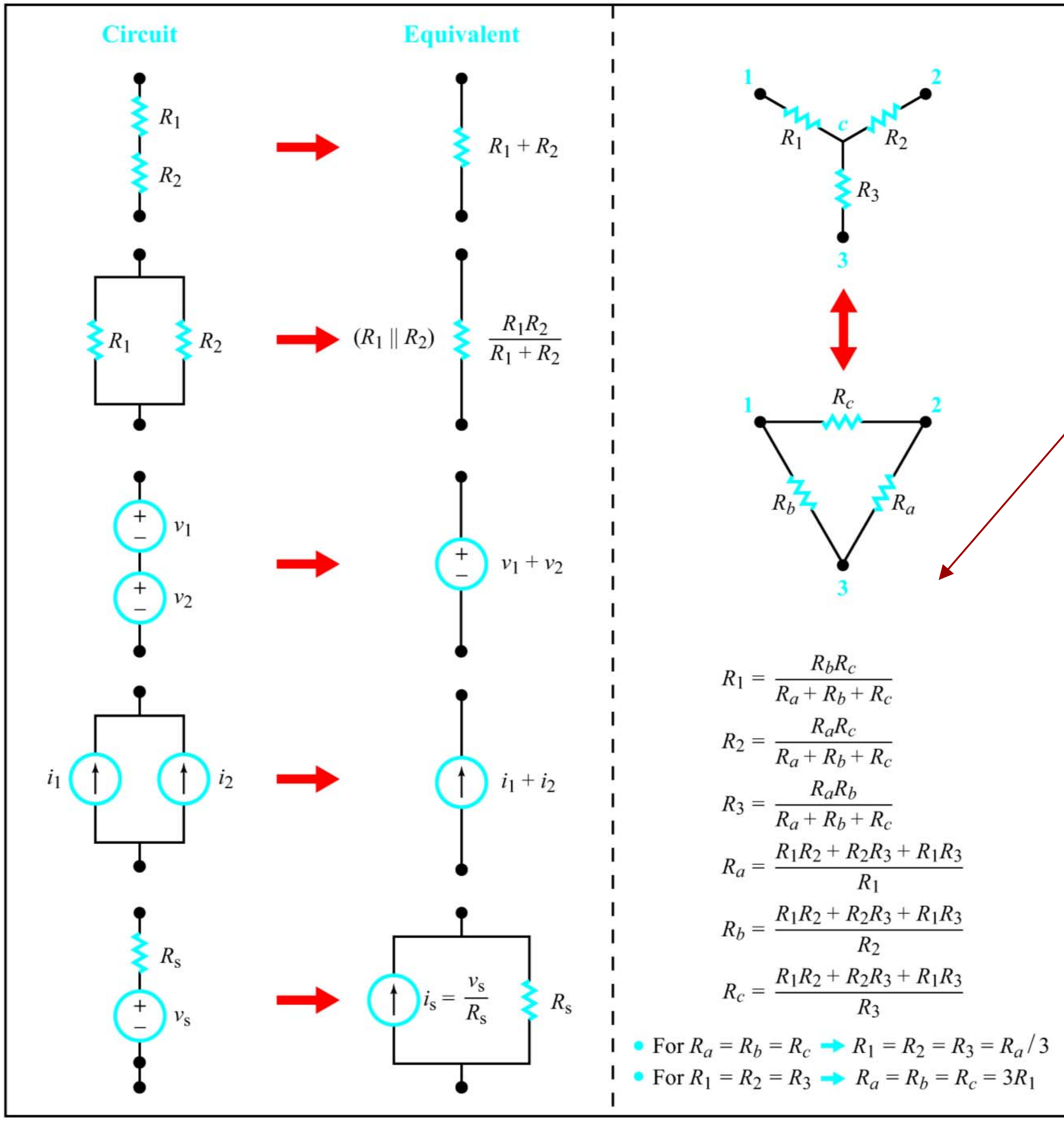


Where would you start?


− v_{R2} +
− v_{R4} +
R_2
R_4
R_1
I_S
R_3
+
v_{R3}
−
V_S

Equivalent Circuits

OK…we've talked about all of these so far



We'll talk about this mess -- Wye to Delta transformations and bridge circuits next time. You'll need to know this for Lab 3.