ELEN 50 Class 07 – Delta to Wye and Wye to Delta Transformations – Bridge Circuits

S. Hudgens

Last time we introduced a very useful type of source transformation:

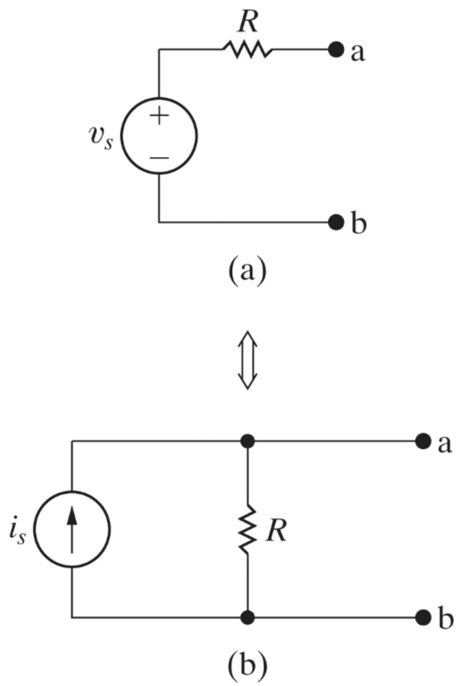


Figure: 04-36a,b

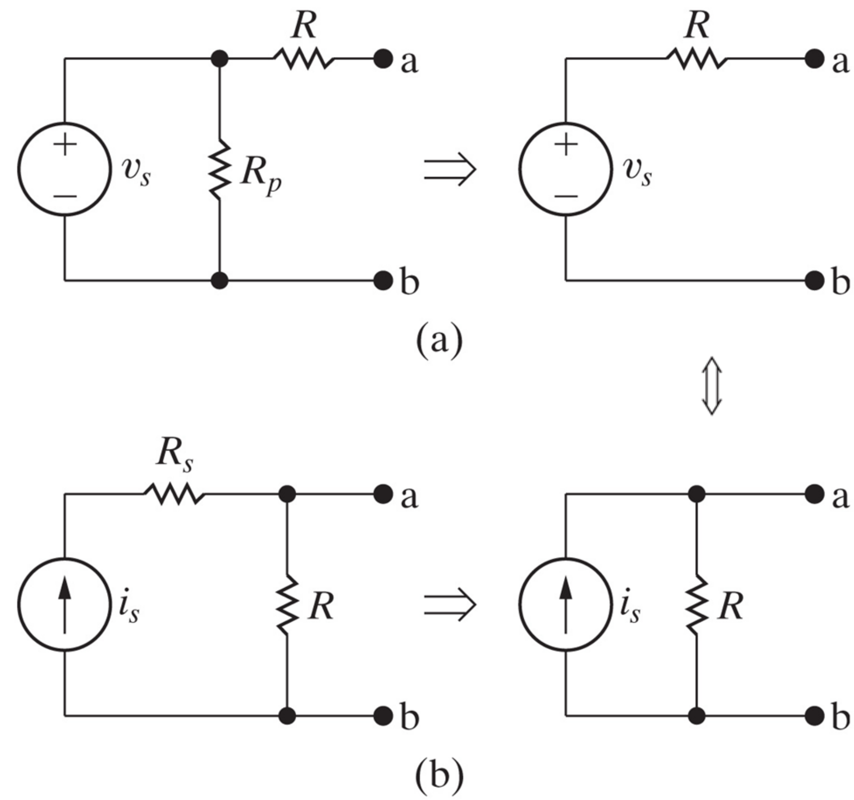
Here's the relationship between the equivalent voltage source and the equivalent current source

$$\boxed{i_s = v_s/R_s}$$

This can often permit significant simplification of a circuit

But, As Was Mentioned Last Time!

If there is already a resistor in parallel with a voltage source or a resistor in series with a current source it is redundant – remove it! You should not account for it in the source transformed circuit.



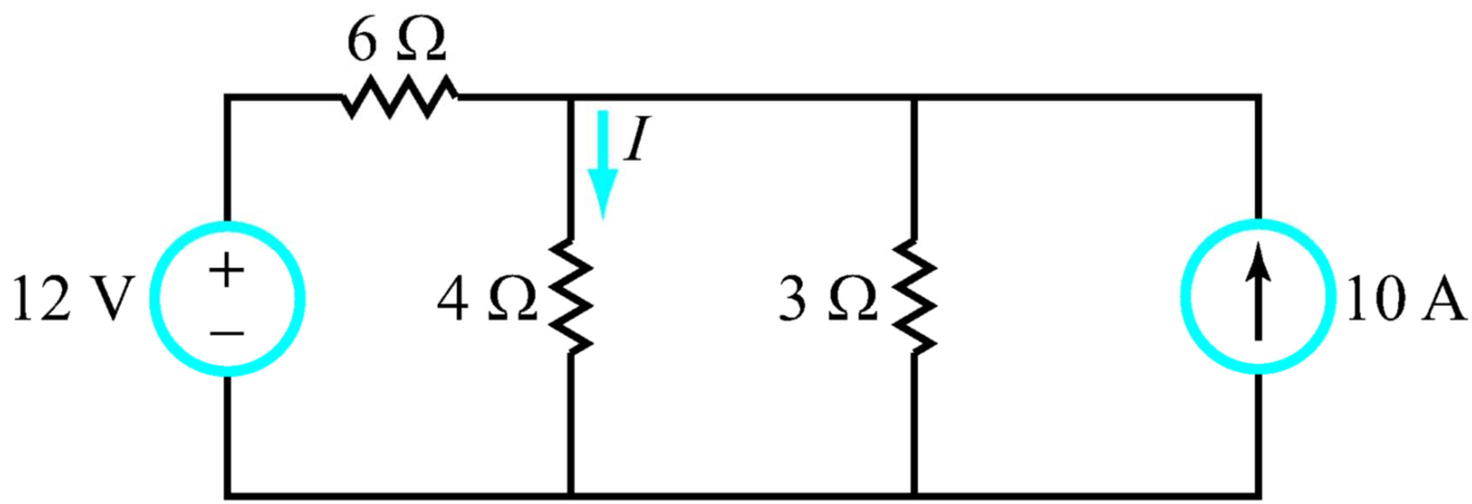
R_p is gone ….now go ahead and do the transform

R_s is gone ….now go ahead and do the transform

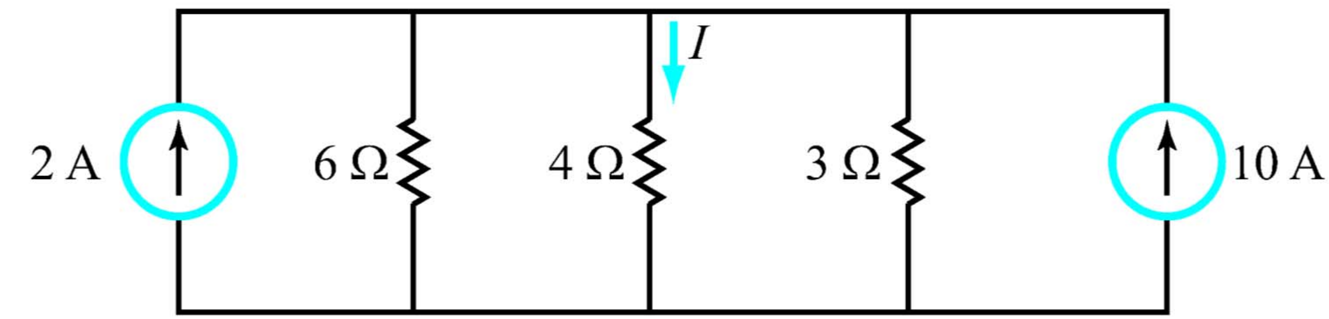
The series or parallel resistor can be removed ….it has no effect on the rest of the circuit attached to terminals a and b. If you are interested in the total current supplied by v_s or the voltage drop across i_s however…you can't remove the resistors.

Here's a circuit example from last time

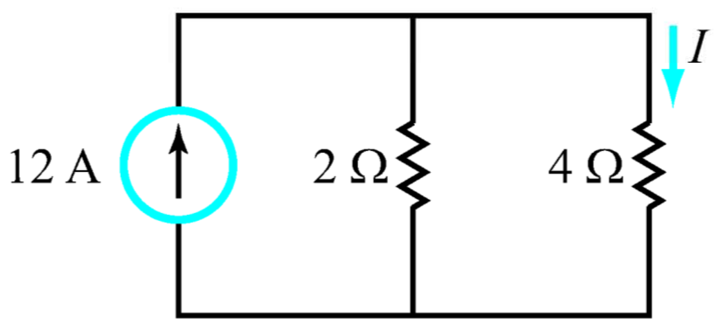
How would you solve for I in this circuit? Series and parallel combinations won't help. How about using source transforms?



The question asks about current through the 4 Ω resistor …so you want to **make sure that you don't combine this resistor with the 3Ω resistor**!

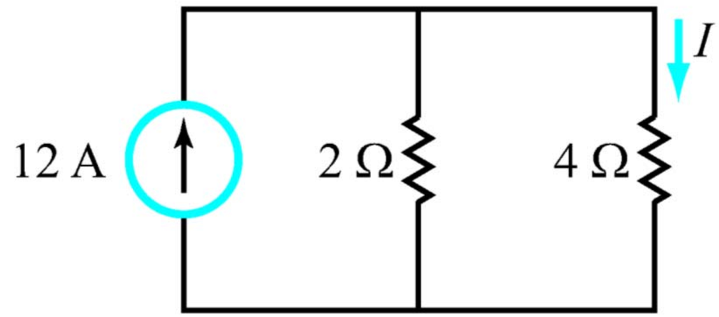


Combine current sources and combine 3-Ω and 6-Ω resistors, while leaving 4-Ω alone



This is a current divider for the 12A current source

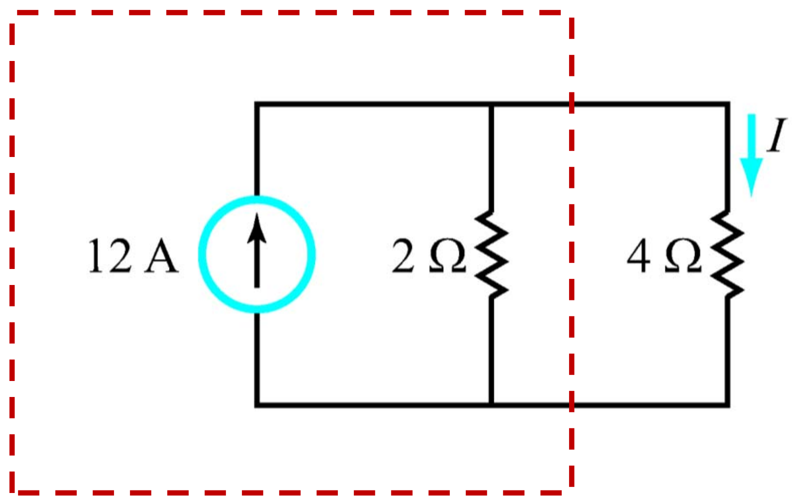
This is a current divider – as you can see.



$$I = 12A\frac{2}{2+4} = 4A$$

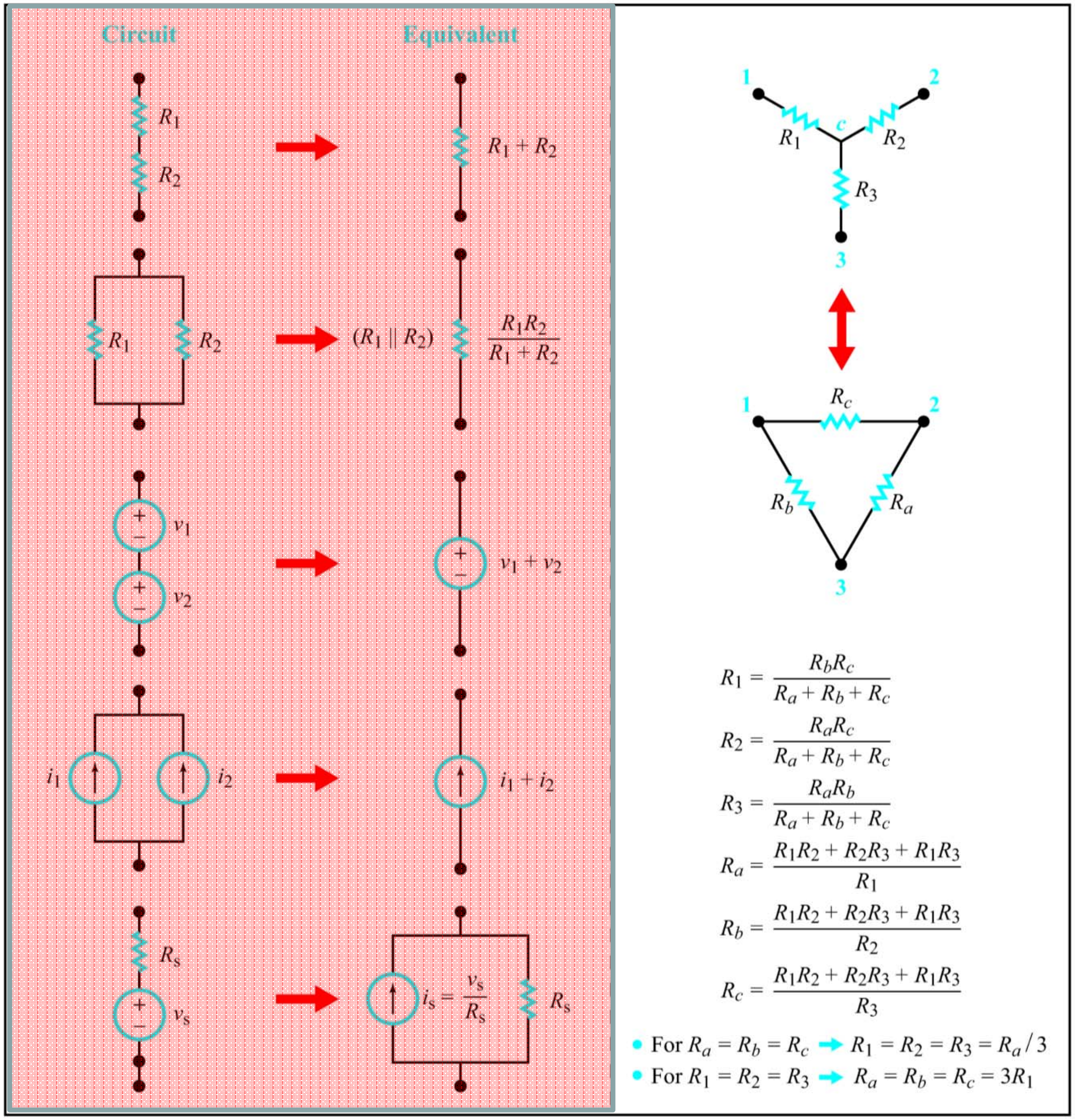
Remember the formula for a current divider?

If not …..as was pointed out last time…. it might even be easier to do another circuit transform on the current source and the parallel 2Ω resistor (the other resistor from the one whose current you are trying to solve). Then you'll have a 24 V voltage source and two resistors in series (6 Ω) …and you can get the current you want from Ohm's Law.



Equivalent Circuits

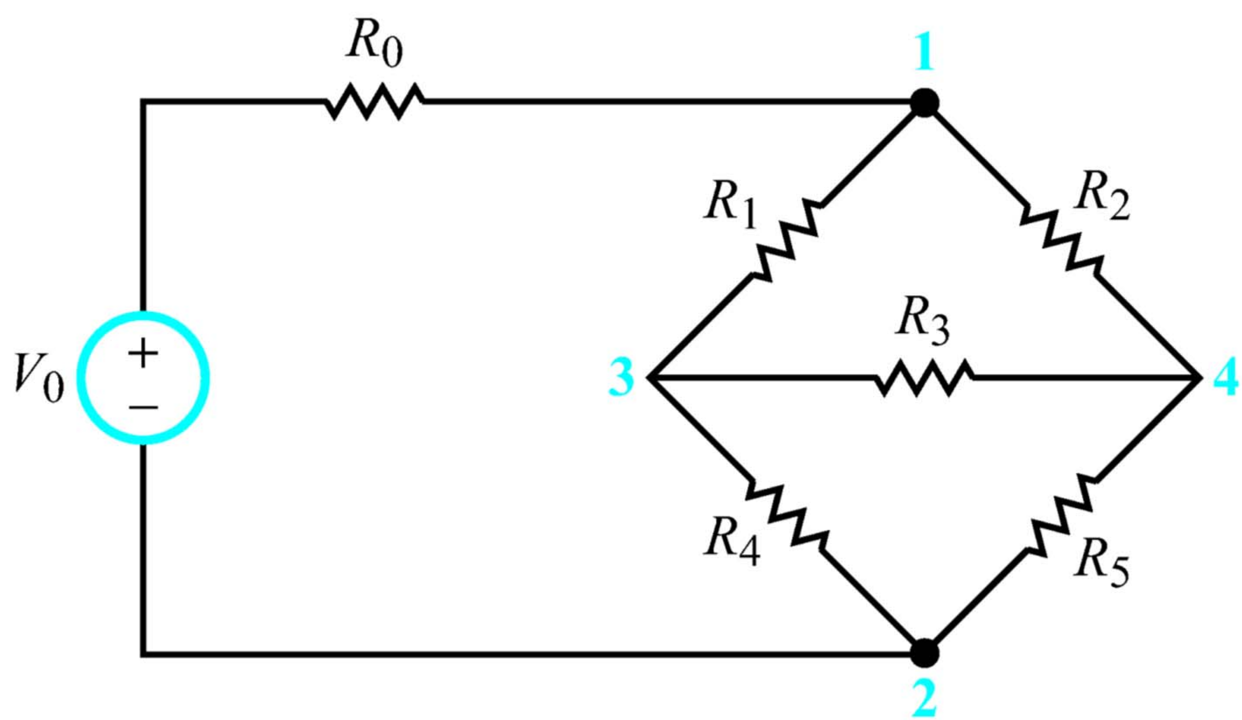
OK, we've discussed all of these types of equivalent circuits …some are just combinations and the last one is a transformation



Now, we'll discuss this thing!

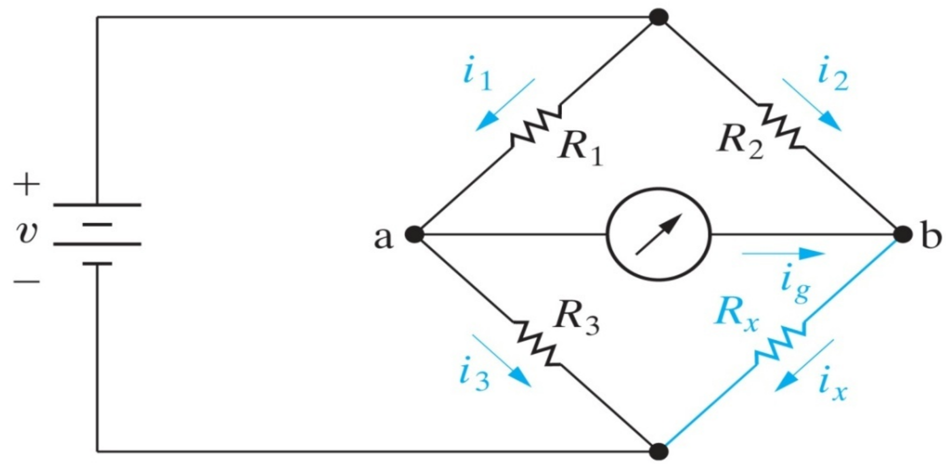
As you can see, it is also a transformation

We will see later (the Thevenin Theorem) that it is always possible to simplify the behavior of any resistive circuit – measured across two nodes – by replacing it with an equivalent circuit composed of a voltage source and a series resistor. However, we can't always do this by simple series and parallel combinations or by source transformations.



For example, no two resistors in this circuit share the same voltage or current – i.e. no series or parallel combinations are possible. We could do a source transform …but it wouldn't help.

This kind of topological arrangement is used in the famous Wheatstone Bridge circuit to measure unknown resistance – in strain gauges, thermistor temperature sensors, etc.



When i_g is zero, nodes a and b have to be at the same potential – and KVL around the upper and lower triangles requires that:

(a.) $i_3R_3 = i_xR_x$

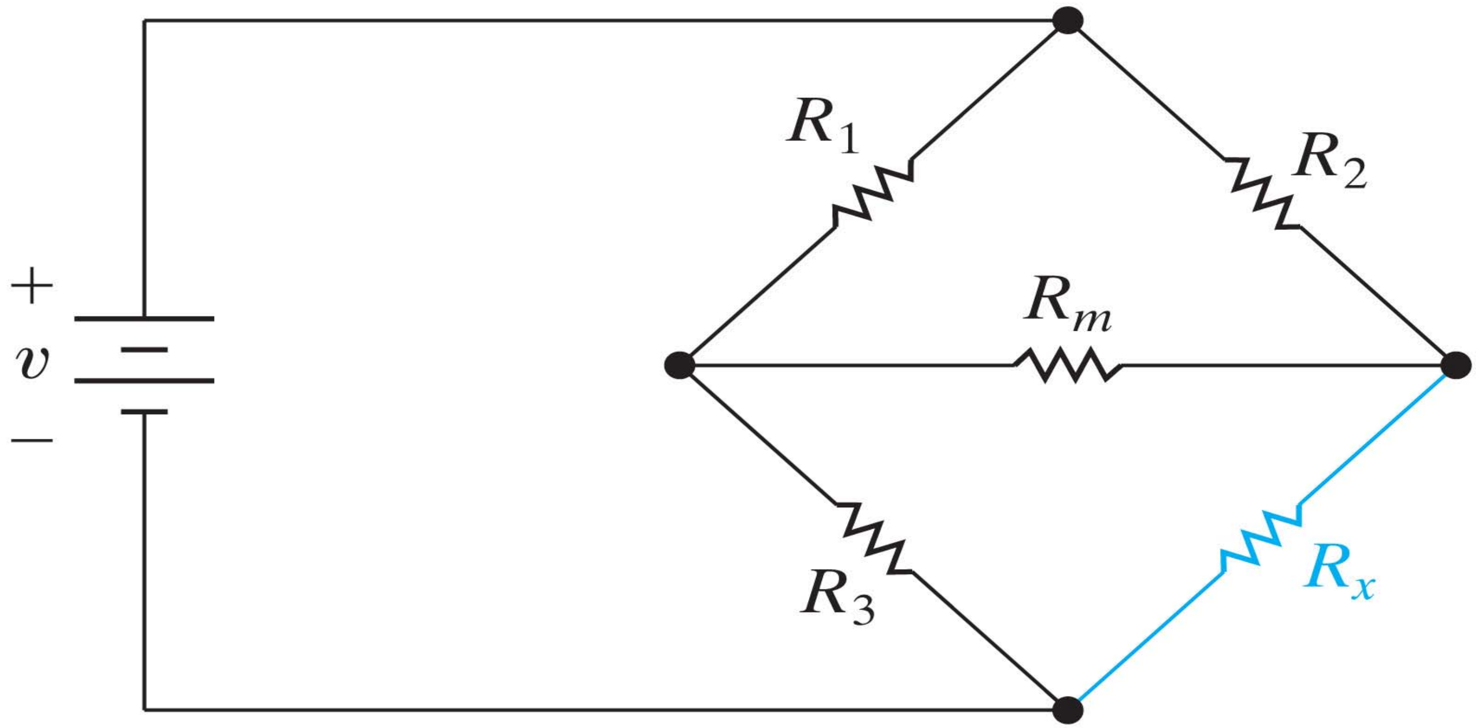
(b.) $i_1R_1 = i_2R_2$

…and KCL requires that $i_1 = 1_3$ and $i_2 = i_x$ therefore, $i_1R_3 = i_2R_x$ when there is no current through the meter.

Dividing by (b.) gives $R_3/R_1 = R_x/R_2$ so Rx = (R2/R1) R3 --- this is the condition for which the "bridge is balanced." Notice that the balance condition doesn't depend on v. You could make this voltage variable …. as a kind of sensitivity control. The out of balance current, ig, depends on v in general, but when the bridge is balanced, ig=0 …regardless of the voltage, v.

The Wheatstone bridge circuit was universally used to make accurate and precise measurements of electrical resistance from ~ 1840 until in the 1960s and 1970s it became possible to make precise digital resistance measurements. Wheatstone bridge circuits are still widely used …. e.g. for strain gauge measurements.

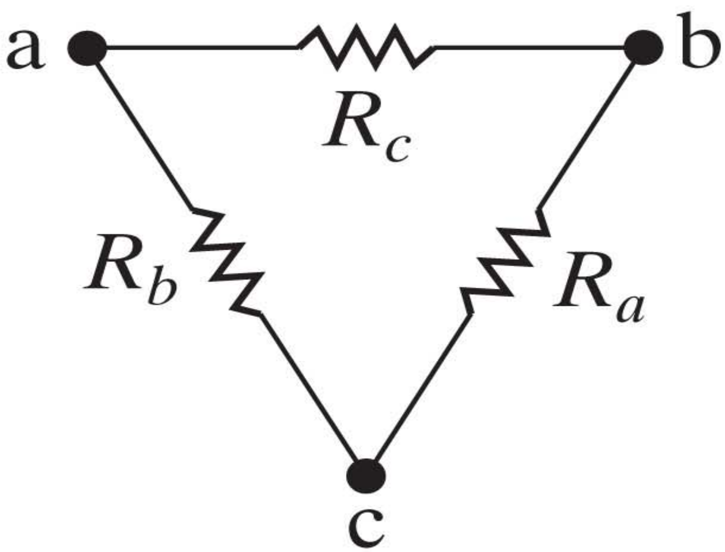
Here is a similar circuit with the galvanometer replaced by a resistor.



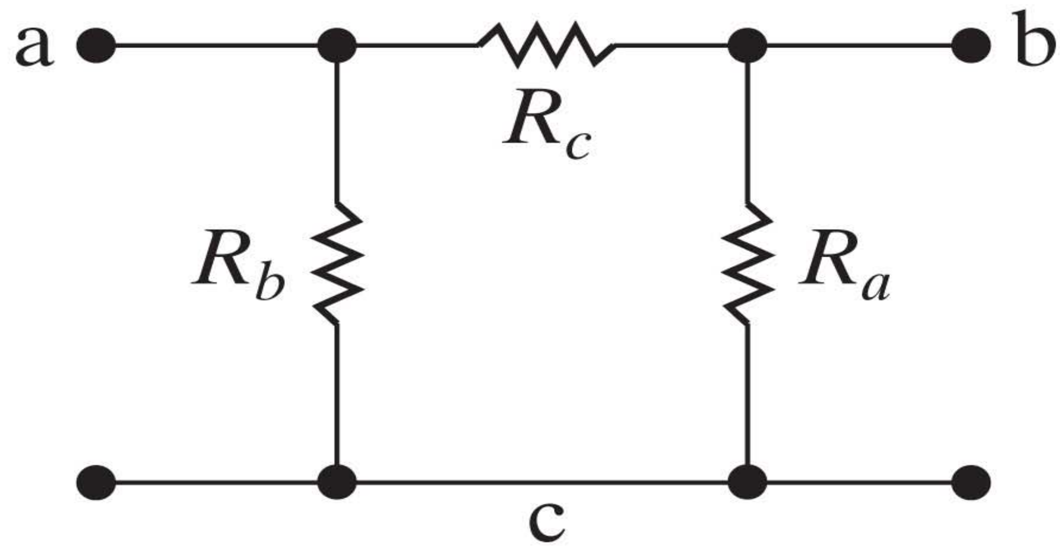
There is no way to reduce this circuit to a single equivalent resistor using series and parallel combinations of the resistors shown. Is there some other way to simplify it?

Yes…..we can do this by means of a Δ to Y or Π to T equivalent circuit

Delta

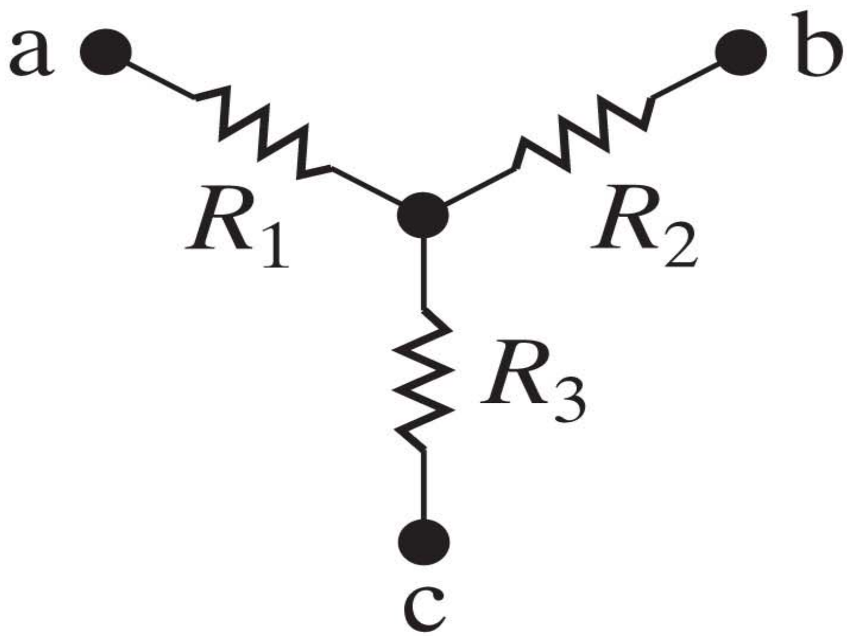


Pi

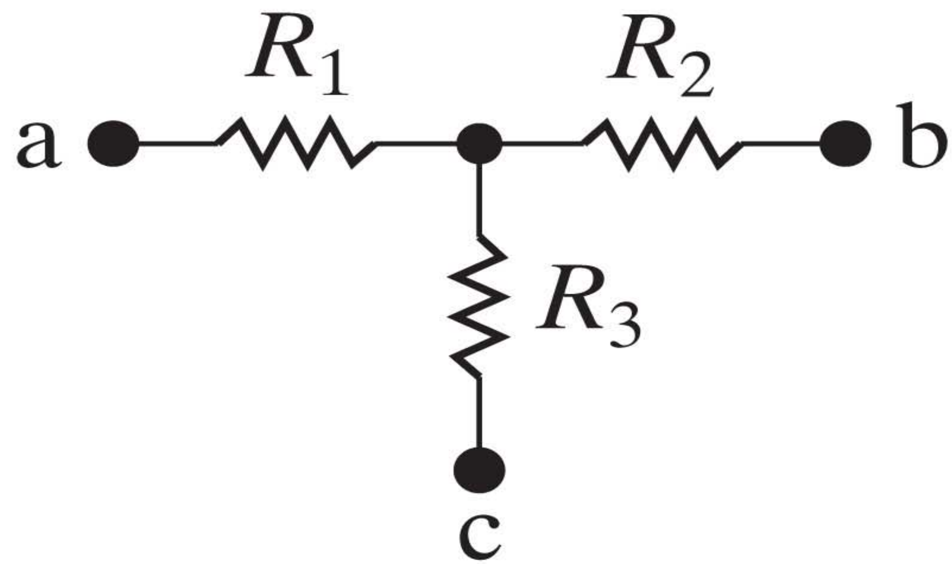


The top of the bridge circuit forms a delta interconnection – which, by the way, can also be redrawn as a “Π” interconnection. A “Π” configuration is the same as a “Δ” !

Wye

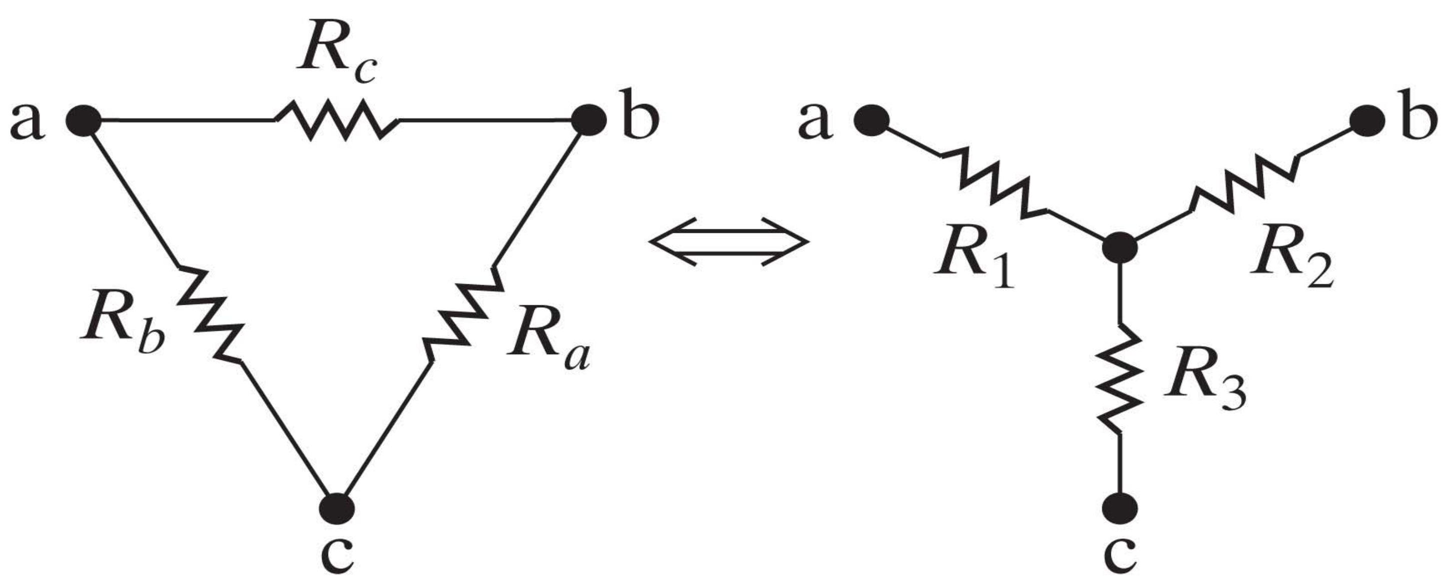


Tee

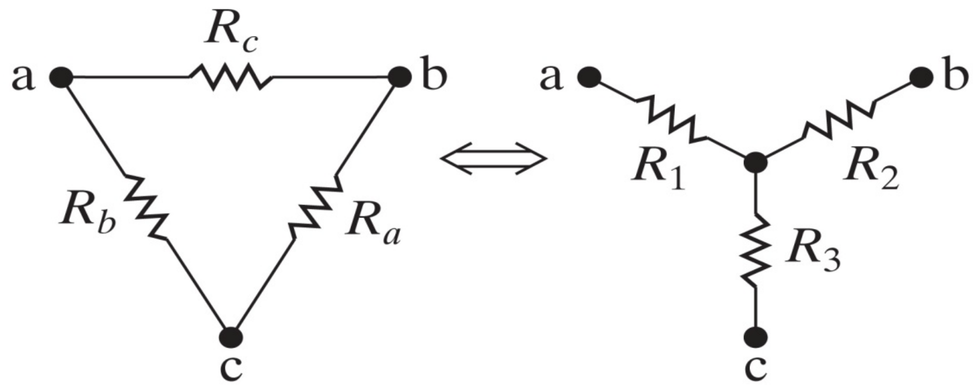


The left and right sides of the bridge circuit form a “Y” interconnection ….which can be trivially redrawn as a “T.” We’re going to derive the formula that allows a “Wye” or “Tee” to be transformed to a “Pi” or “Delta” configuration.

Since both the Delta and the Wye interconnection have three nodes (the central node in the Wye configuration is assumed not to be accessible …since it's not labeled), it should be possible to construct an equivalent Wye network from a Delta network ….and vice versa. We can't do this by just moving nodes around and redrawing. In other words, <u>the transformation is not topological</u> …the equivalent circuit transformation <u>replaces</u> the Δ interconnection with a new Y interconnection which is <u>an equivalent circuit</u>.



We want to establish a relationship between the resistors in the Delta circuit and the resistors in the equivalent Wye circuit.



We can use series and parallel simplifications to write the following relations between the nodes in the two circuit configurations:

$$R_{ab} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} = R_1 + R_2$$

$$R_{bc} = \frac{R_a(R_c + R_b)}{R_a + R_b + R_c} = R_3 + R_2$$

$$R_{ca} = \frac{R_b(R_c + R_a)}{R_a + R_b + R_c} = R_1 + R_3$$

so we have three equations in three unknowns

Solving for R_1, R_2, and R_3 in terms of R_a, R_b, and R_c gives:

$$\boxed{\begin{aligned} R_1 &= \frac{R_b R_c}{R_a + R_b + R_c} \\ R_2 &= \frac{R_c R_a}{R_a + R_b + R_c} \\ R_3 &= \frac{R_a R_b}{R_a + R_b + R_c} \end{aligned}}$$

These give the <u>Δ to Y</u> transformation relations

We can also solve for R_a, R_b, and R_c in terms of R_1, R_2, and R_3

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

These give the Y to Δ transformation relations

Its really not necessary to memorize these relationships. In any case where you need to use them you can always look them up – or in the worst case, derive them!

If the resistors in a Delta circuit are all equal the circuit is said to be balanced. The equivalent Y circuit will also be balanced and have equal resistors given by:

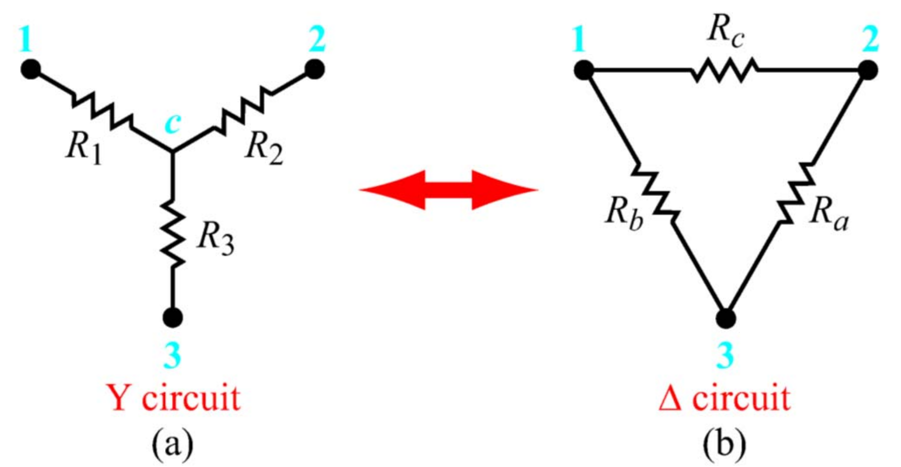
$$R_1 = R_2 = R_3 = \frac{R_a}{3}$$

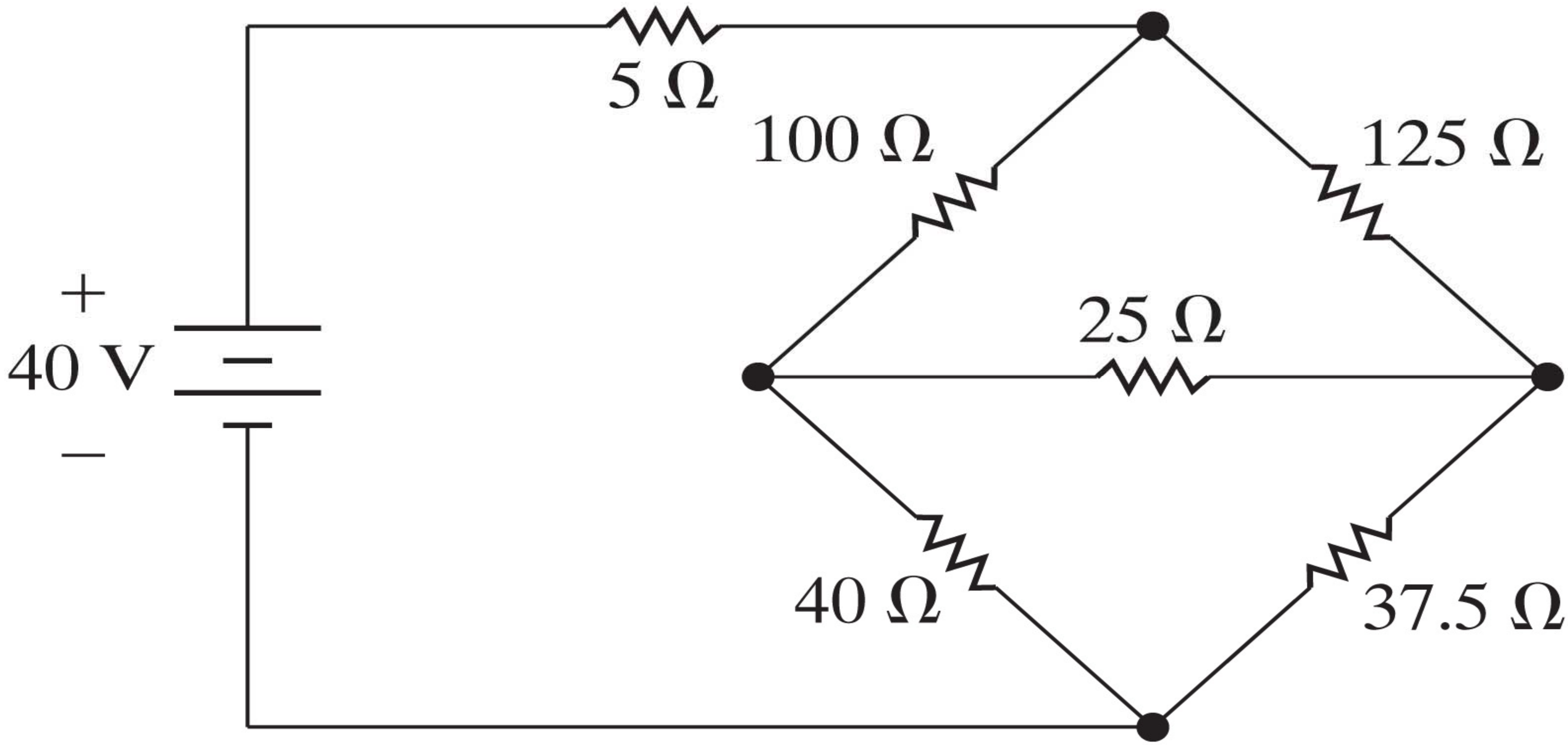
$$if \quad R_a = R_b = R_c$$

also

$$R_a = R_b = R_c = 3R_1$$

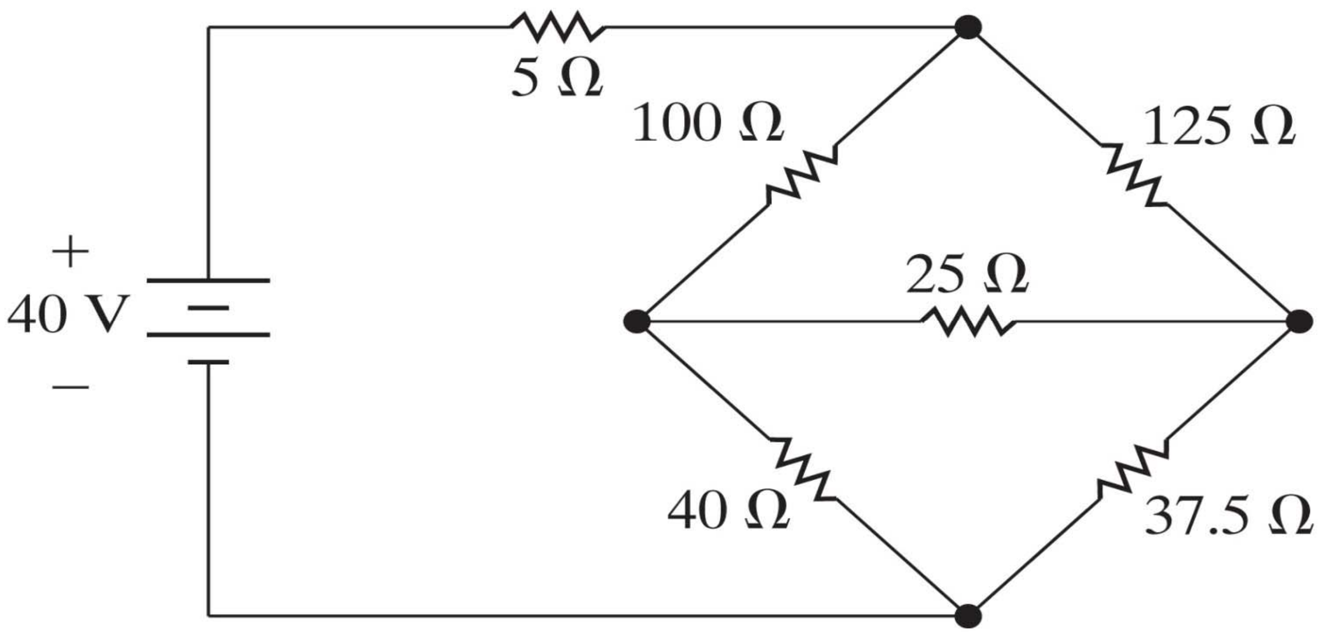
$$if \quad R_1 = R_2 = R_3$$





Find the current and power supplied by the 40V source.

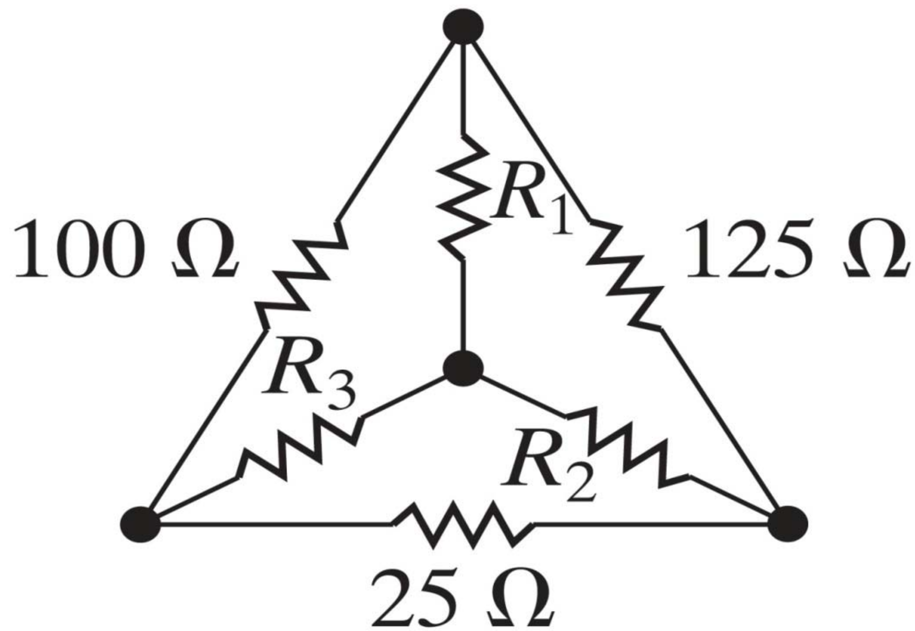
First, you should convince yourself that you can't simplify this circuit by series and parallel resistor combinations. You should also convince yourself that you can't do anything with source transforms that will help.



You could, of course, write a bunch of KCL and KVL equations …………

Or you could find the values of R1, R2, and R3 which can convert the upper Delta circuit into a Wye circuit.

Here's the upper delta redrawn. with the currently unknown Y-circuit (R1,R2, and R3) drawn over it.



Can you see why transforming the upper delta into a wye would help solve the circuit?

using the formulas …

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

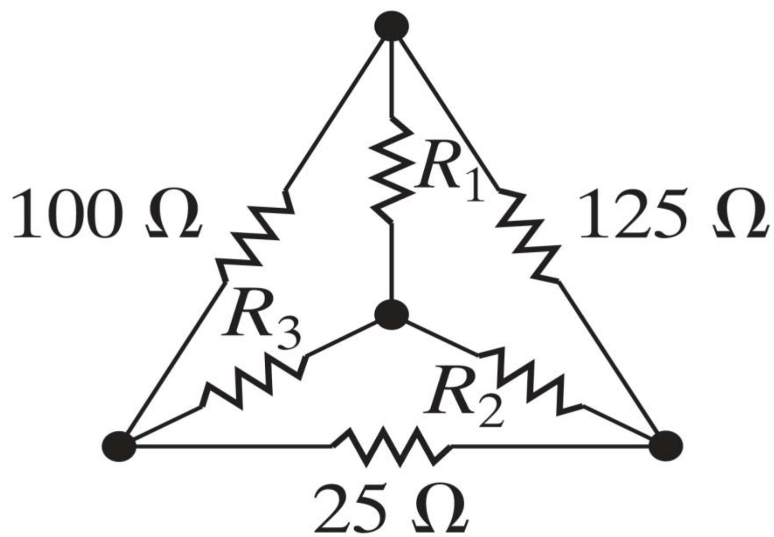


Figure: 03-33

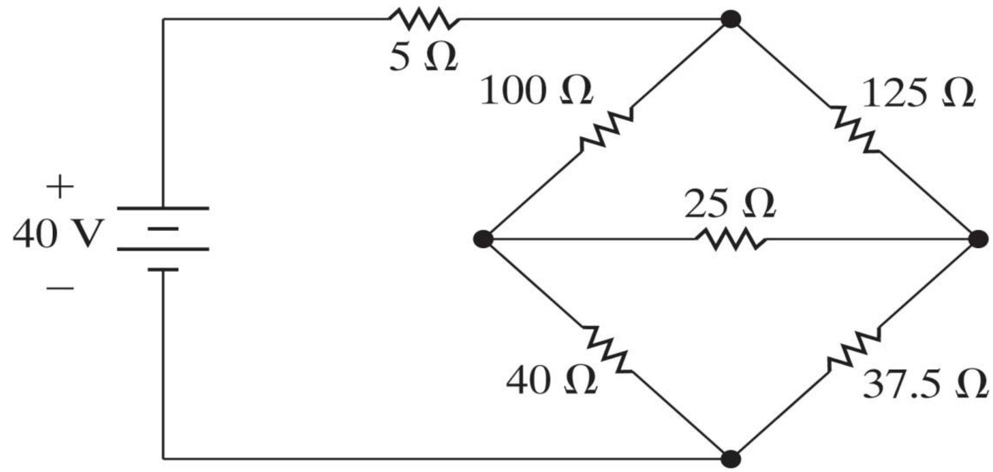
So R_1 = (100 x 125)/250 = 50 Ω

R_2 = (125 x 25) / 250 = 12.5 Ω

R_3 = (100 x 25) / 250 = 10 Ω

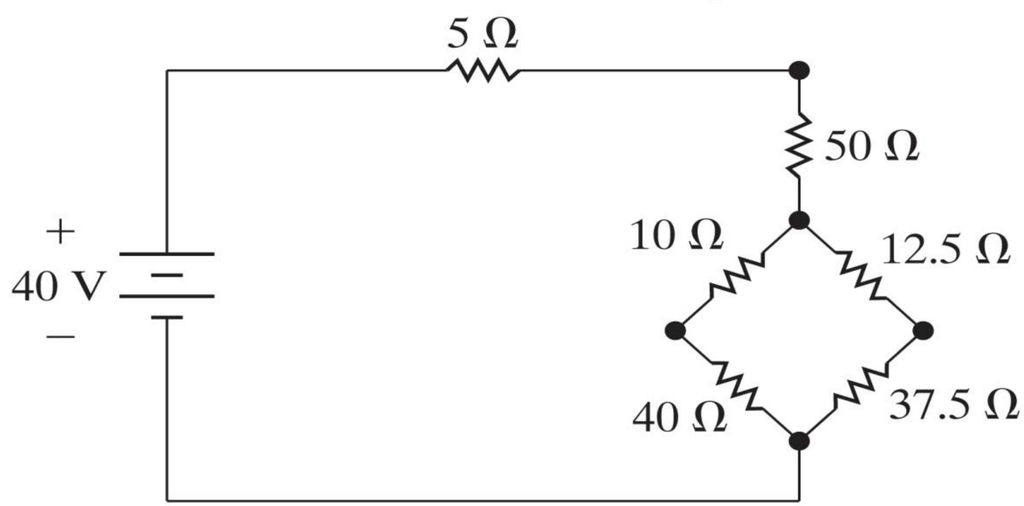
replacing the upper delta with a Y gives a circuit that we can simplify by parallel and series combinations of resistors

This



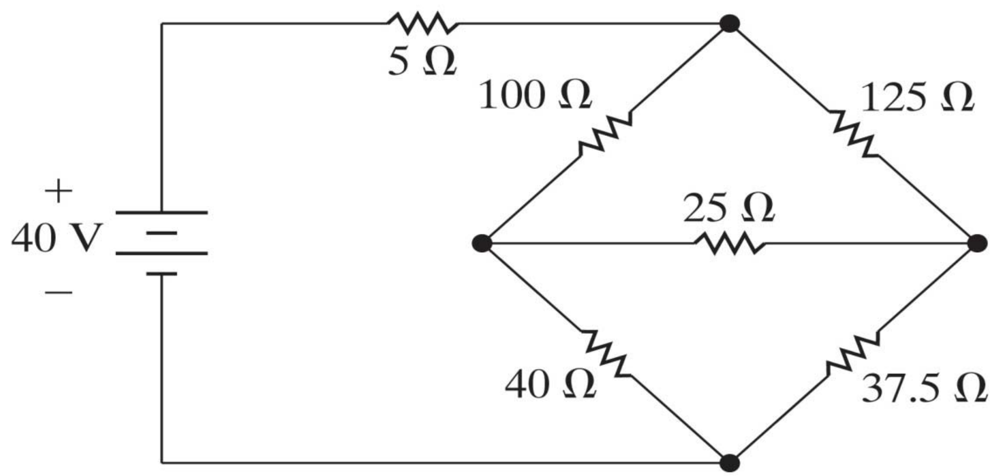
becomes

This



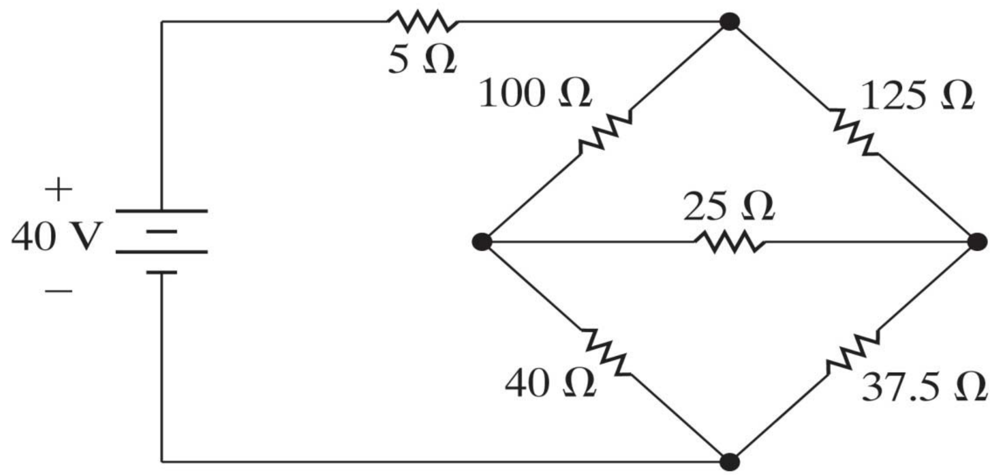
And it’s trivial to reduce this to a single resistor (of 80Ω) – so you can calculate the power delivered to it by the 40 V source….= 20W

Notice -- even though you can't simplify the original circuit by source transforms or parallel and series combinations of resistors …there is something simpler than a Y to Delta transform you can do to get an <u>estimate</u> of the power delivered to the circuit. Sometimes you may only want an estimate ..so this could be a useful trick.



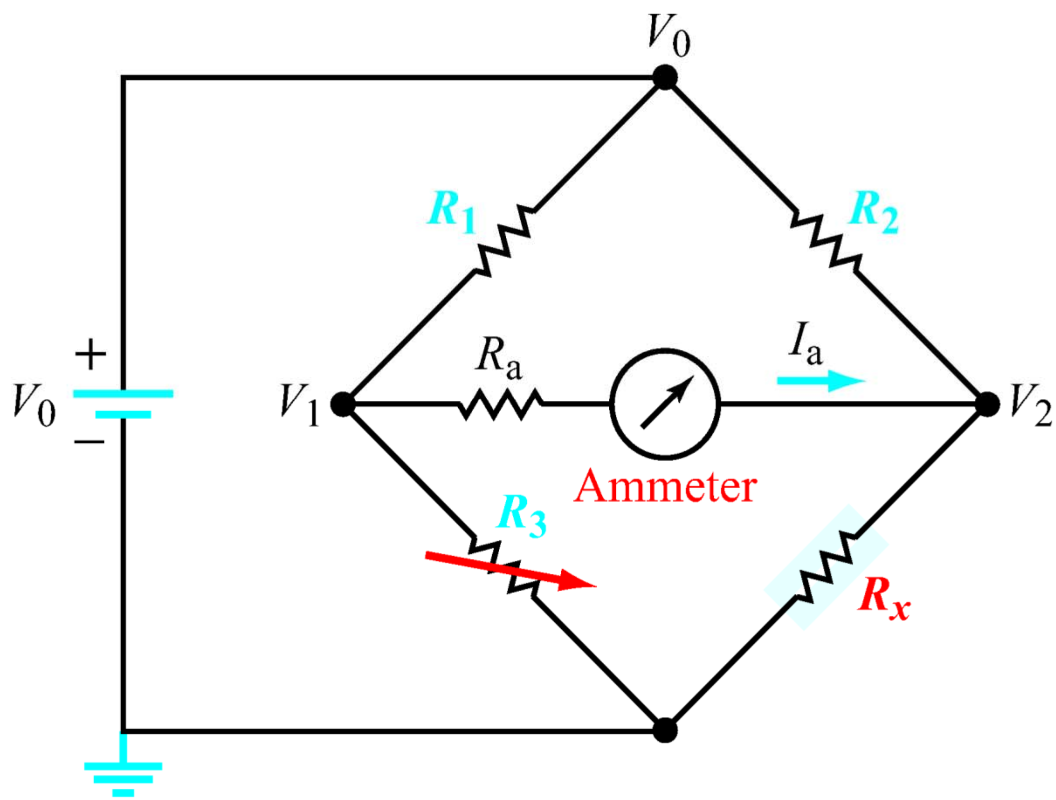
Let's look at the 25Ω resistor that is causing all the complexity to this circuit. If instead of 25Ω, it was a short circuit, then the whole circuit could be simplified by series and parallel resistors. The 100Ω and 125Ω resistors would be in parallel and in series with the 5Ω resistor and the parallel combination of the 40Ω and the 37.5Ω resistor. This would give a 79.91Ω resistor across the 40V source

Now, instead, what if the 25Ω resistor were an open circuit? This would also remove the complexity from the circuit.

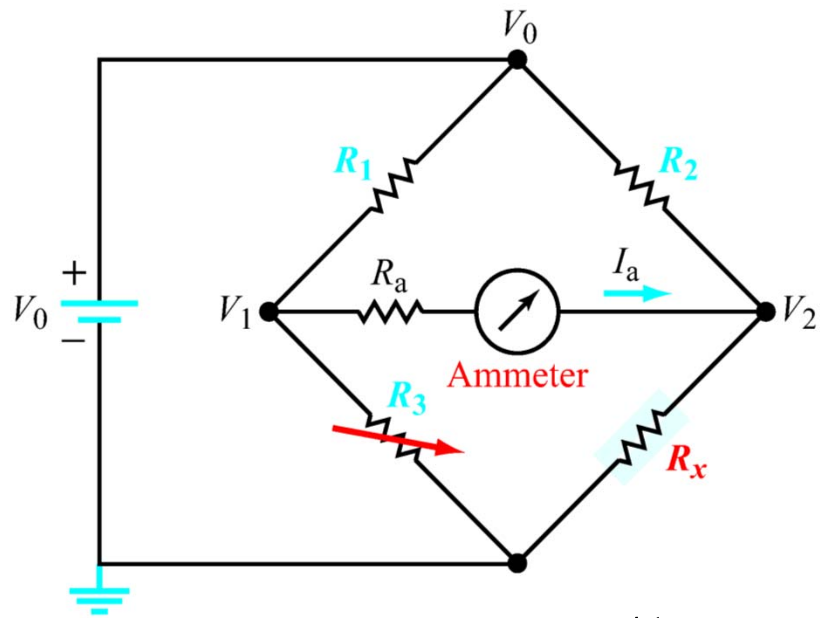


There would be a 140Ω resistor in parallel with a 162.5Ω resistor (giving 75.21Ω) …and this parallel combination would be in series with a 5Ω resistor --- giving a total resistance of 80.20 Ω. Since the 25Ω resistor has a value somewhere between 0Ω and infinity …we know the equivalent resistance of the network must be between 79.91Ω and 80.2Ω! Midway between these two values is 80.06 ..not a bad estimate for a system where the actual value is 80Ω.

The Wheatstone Bridge



In this famous circuit, R1 and R2 are known, fixed resistors, R3 is a (known) adjustable resistor, and Rx is the unknown resistor. V_0 is any convenient voltage source. R3 is adjusted until the ammeter shows I_a is zero –and we say that the bridge is “in balance.”



In balance, V1 = V2, so, since:

$$V_1 = R_3 V_0 /(R_1 + R_3)$$

$$V_2 = R_x V_0 /(R_2 + R_x)$$

$$\frac{R_3 V_0}{R_1 + R_3} = \frac{R_x V_0}{R_2 + R_x}$$

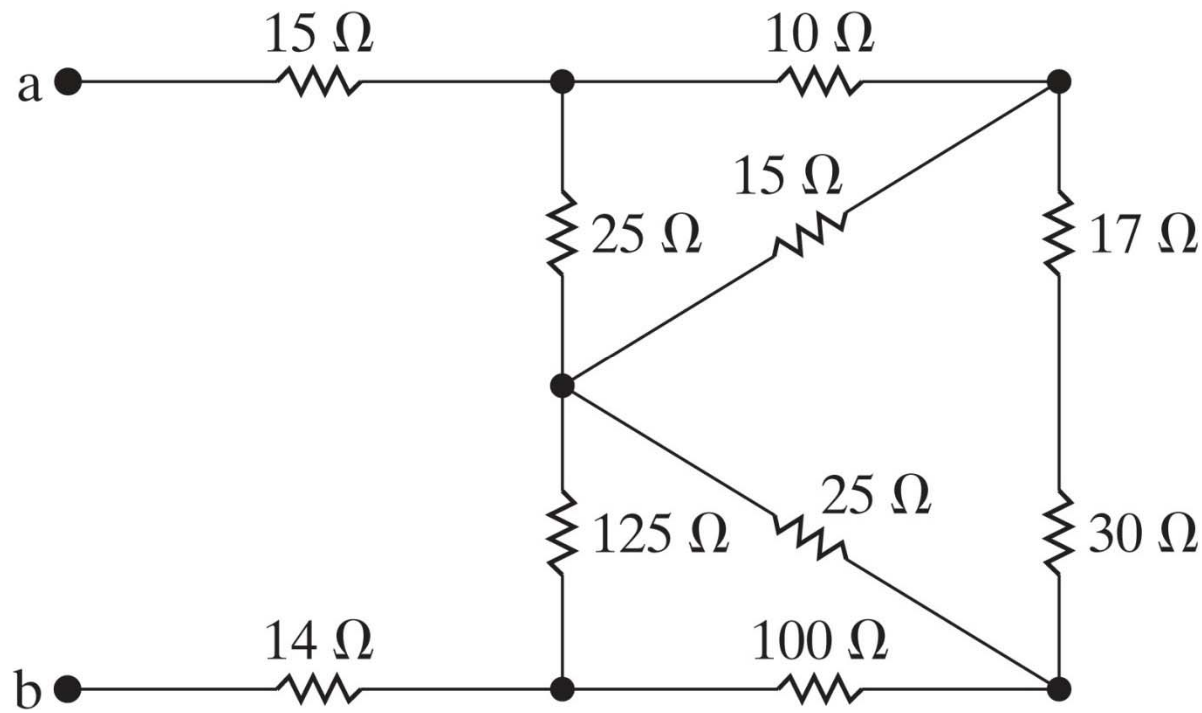
Also, since the voltages across R1 and R2 are equal:

$$\frac{R_1 V_0}{R_1 + R_3} = \frac{R_2 V_0}{R_2 + R_x}$$

So, dividing these equations:

$$\boxed{\frac{R_3}{R_1} = \frac{R_x}{R_2}}$$

We want to find the equivalent resistance R_{ab}?
What would be a good strategy for simplifying this mess?

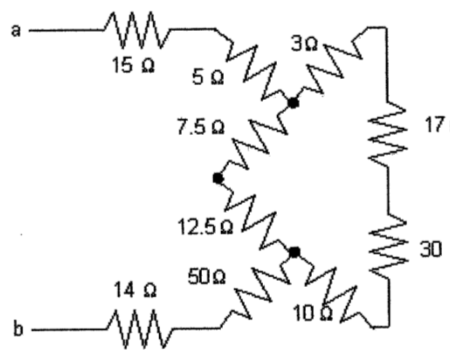


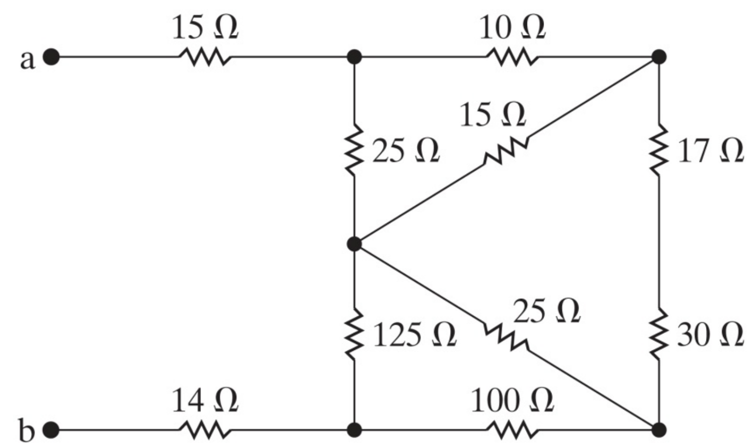
Replace the upper and lower deltas with the equivalent wyes:

$$R_{1U} = \frac{(25)(10)}{50} = 5\,\Omega;\ R_{2U} = \frac{(10)(15)}{50} = 3\,\Omega;\ R_{3U} = \frac{(25)(15)}{50} = 7.5\,\Omega$$

$$R_{1L} = \frac{(125)(25)}{250} = 12.5\,\Omega;\ R_{2L} = \frac{(25)(100)}{250} = 10\,\Omega;\ R_{3L} = \frac{(125)(100)}{250} = 50\,\Omega$$

The resulting circuit is shown below:



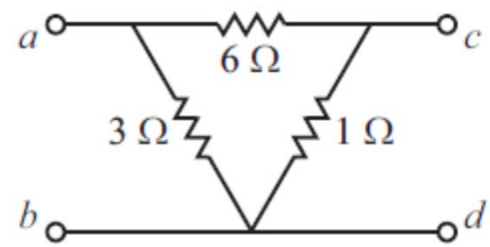


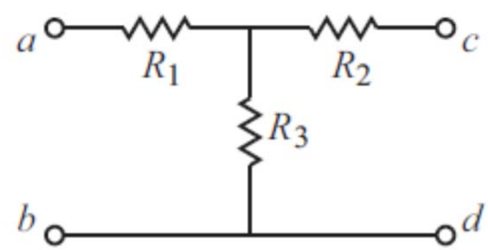
Now make series and parallel combinations of the resistors:

$(7.5 + 12.5)\|(3 + 17 + 30 + 10) = 20\|60 = 15\,\Omega$

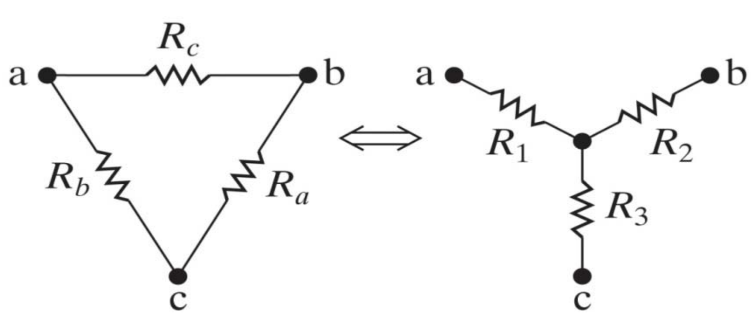
$R_{ab} = 15 + 5 + 15 + 50 + 14 = 99\,\Omega$

Convert this circuit into a Wye (or Tee) circuit:





We know for:



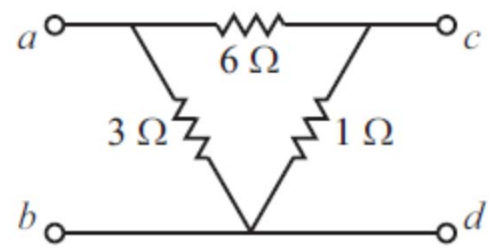
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

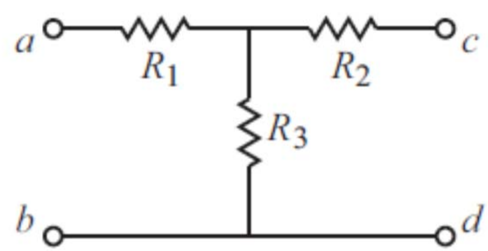
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

So:

$$R_1 = \frac{6 \times 3}{6+3+1} = 1.8\ \Omega$$
$$R_2 = \frac{6 \times 1}{10} = 0.6\ \Omega$$
$$R_3 = \frac{3 \times 1}{10} = 0.3\ \Omega.$$





In the next lecture we'll introduce the Node Voltage Circuit Analysis Method

It is probably the most widely used elementary circuit analysis method – and, as I told you, one of the three or four things (like Ohms Law, KCL and KVL) that I hope you will remember about this class – if you don't remember anything else!

If you are interviewing for a job (in EE or maybe in engineering in general) and you tell the interviewer you took an undergraduate circuit theory course …I guarantee that they will ask you what you know about Ohms Law, KCL, KVL, and the Node Voltage Method!