Can We Stop Climate Change?

MATH3041 Group Assignment Report

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1 Executive Summary

This report, commissioned by the Global Institute for Climate Change and the Environment (GICCE), assessed the feasibility and implications of stopping climate change using mathematical modelling. We aimed to determine how the carbon emissions, and consequently, the economy and technology, need to change under the three scenarios:

1. Global mean temperature was projected to rise to 4°C above pre-industrial levels and stabilize at that level from 2100;

2. Global mean temperature was expected to rise to 2°C above pre-industrial levels and stabilize at that level from 2050; and

3. Global mean temperature was to rise no more than 1.5°C above pre-industrial levels this century, with other climate impacts, such as sea level rise, halting by 2100.

Assuming that global warming and associated changes were directly linked to anthropogenic carbon dioxide (CO_2) emissions from fossil fuel combustion, we developed a series of mathematical models to project CO_2 emissions, temperature changes, sea level rise, and ocean pH under each scenario. The results were as follows:

- **Scenario 1**: Projected sea levels rising by 1051.16 mm and ocean pH dropping to 7.98, with relatively steady growth in GDP per capita.

- **Scenario 2**: Projected a sea level rise of 406.27 mm and a pH of 8.03, with much slower GDP per capita growth compared to Scenario 1.

- **Scenario 3**: Projected a maximum temperature rise of 1.73°C, sea level rise of 587.32 mm, and a pH increase to 8.27, with GDP reaching zero in 2045, indicating the impracticality of this emission scenario given the current rate of carbon and energy intensity reductions.

To achieve these emission scenarios, significant investments in renewable energy and carbon pricing policies are required. While implementing Negative Emission Technologies (NETs) is essential to achieve negative carbon emissions in Scenario 3.

2 Introduction

2.1 Background Information

Climate change is one of the most pressing issues of the century, causing significant alterations in weather patterns, rising sea levels, ocean acidification and more frequent extreme weather events. Such a phenomenon is primarily driven by carbon dioxide emission from anthropogenic activities since the industrial revolution.

The Paris Agreement, adopted at the UN Climate Change Conference in 2015, established the overarching goal of pursuing efforts 'to limit the temperature increase to 1.5°C above pre-industrial levels' [1]. To align with this target, it is insufficient for humans merely to reduce the emission of greenhouse gases; it is also imperative to remove a substantial quantity of historical emissions from the atmosphere [2]. Such an action will require changes in the economy and technology, which in turn indicates varying outcomes in global mean temperature, sea level rise, and the degree of ocean acidification, depending on the extent and efficacy of emission management strategies.

2.2 Literature Review

2.2.1 Economy and Carbon Emissions

Historical records have shown heavy dependence of economic activities on energy, with the predominant source of current energy generation being the combustion of fossil fuels. The Kaya's identity is a conceptual framework employed to characterise the various driving force of anthropogenic carbon emission at a global scale.

The non-differential form of the identity is [3]

$$C = P \cdot \frac{Y}{P} \cdot \frac{E}{Y} \cdot \frac{C}{E}, \tag{1}$$

where C is the global CO_2 emission, P is the global population, Y is the global GDP, E is the global energy consumption. While decoupling economic growth and carbon emissions implies that significant emission reductions are possible with little or no effect on growth, Deutch's work [4] suggests that the extent of decoupling is entirely dependent on reductions in energy intensity ($\frac{E}{Y}$) and carbon intensity ($\frac{C}{E}$); however, the current decline is insufficient to avoid a significant average global temperature increase in the second half of this century.

2.2.2 Temperature Change

The global temperature response to rising carbon emissions is a complex system that takes into account the interactions of carbon sensitivity, climate sensitivity, and climate-carbon feedback. Carbon sensitivity is the extent to which atmospheric carbon increases as a result of CO_2 emissions, whereas climate sensitivity is the extent to which temperature changes respond to an increase in atmospheric carbon.

Such a sophisticated interaction system was simplified in Matthews' work [5] using a robust metric of the carbon-climate response (CCR), measured in °C per trillion tonnes of carbon, which gives the ratio of temperature change to cumulative carbon emission as

$$CCR = \frac{\Delta T}{E_T}, \tag{2}$$

where ΔT is the global mean temperature change over some period of time and E_T is the total cumulative carbon dioxide emitted over that period in teratonne (1 Tt of carbon is equivalent to 3.7 trillion tonnes of carbon dioxide). As it aggregates both climate feedback and carbon cycle feedback, the CCR can be treated as a constant with respect to time and emission scenarios in a given model. This produced an estimate of 1.25 Tt C of allowable carbon emission (cumulative emission until now is approximately 0.5 Tt C) for 2 °C warming relative to preindustrial temperature.

2.2.3 Sea Level Rise

Global sea level changes are caused by complex mechanisms including thermal expansion of water due to heat uptake and penetration into the oceans, melting of glaciers and ice sheets, and changed water storage on land. Rahmstorf proposed a semi-empirical model [6] in which sea level rises as ice melts and the ocean takes up head, until a new asymptotic equilibrium level is reached. While the equilibration time scale is on the order of millennia, when exposed to a change in global mean temperature, such as anthropogenic warming caused by carbon emissions, over a short period of time, the rate of sea level rise is proportional to the temperature increase, expressed as

$$\frac{dH}{dt} = a\left(T - T_0\right), \tag{3}$$

where H is the global mean sea level, T is the global mean temperature, and T_0 is the previous equilibrium temperature value. This approximation is expected to hold for a few centuries after the period of anthropogenic warming.

2.2.4 Ocean Acidification

Ocean acidification is the process by which seawater pH decreases as it absorbs CO_2 from the atmosphere. By considering the carbonate chemistry of seawater, Caldeira and Berner suggested that the mean partial pressure of CO_2 on the surface of the ocean is nearly equal to that of the atmosphere [7]. As a result, the atmospheric pressure of carbon dioxide (pCO_2) can be quantified by hydrogen ion concentration using the equation

$$\mathrm{pCO_2} = \frac{K_H\left[\mathrm{H^+}\right]^2\left[\mathrm{CO_3}^{2-}\right]}{K_1 K_2}, \tag{4}$$

where K_H is the Henry's law constant for CO_2, and K_1 and K_2 are the first and second dissociation constants for carbonic acid respectively.

3 Data and Methodology

3.1 Data Pre-processing

Considering the availability of different sources of data, the historical data used for fitting were extracted from the time period 1880-2021, with the value in 1880 treated as the pre-industrial value to provide a baseline for comparison.

The data for carbon emissions was published by Global Carbon Budget [8], which contains the global emissions of carbon dioxide measured in tonnes. The cumulative carbon emissions since 1880 were computed by summing all previous annual emissions up to the current year.

The temperature data was published by the Met Office Hadley Centre [9], which records global temperature anomalies in °C relative to the average temperature from 1961 to 1990. To compute the global mean temperature rise from the pre-industrial level, the temperature anomaly in 1880 was subtracted from the anomaly of the current year.

The data for sea level rise and ocean pH were published by National Oceanic and Atmospheric Administration (NOAA) [10]. The sea level rise data set gives the global mean sea level rise measured relative to the average sea level from 1993 to 2008. To compute the annual rate of change in sea level, we subtracted the previous year difference from the current year. The ocean pH data set contains the monthly average pH from 1988 to 2021, with some months missing. To compute the global annual average pH for each year, the average of available monthly measurements from that year was used.

The data for economy projection was obtained from Our World in Data [8], containing the percentage increase of population, energy and carbon intensity for each year.

3.2 Model Proposal

A structured approach was taken to determine the required change in carbon emissions to achieve each target, and to project the environmental and economic outcomes for each emission scenario. Such an approach first converts the target temperature, or the target of plateauing sea level in Scenario 3, to a target cumulative carbon emission level to be met in the designated year. This target was then used to plan an annual carbon emission scheme for each scenario over the specified time period. This annual emission plan was converted to a cumulative emission trajectory by summing up the annual emissions up until the current year, which then allowed the computation of global mean temperature, sea level rise, and ocean pH over the same time frame.

3.2.1 Annual Emission Planning

Scenario 1 aimed for the temperature to rise to and plateau at 4 degrees above the pre-industrial in 2100. According to equation (2), the cumulative carbon emissions must remain constant after 2100, implying that annual emissions should drop to zero by then. We assumed that the annual emissions increase at the average rate observed over the past 20 years until 2050, then remain constant for a period before decreasing to zero by 2100. By equating the area under the annual emission curve to the target cumulative emission, we determined when emissions should transition from constant to decreasing.

Scenario 2 aimed for a temperature rise of 2°C, plateauing after 2050, which required more stringent emission controls. We assumed that annual emissions stay constant from 2021 for some time before decreasing to zero by 2050. An exact time for the shift was computed using the same approach as Scenario 1.

Scenario 3 focused on stopping sea level rise by 2100, which, according to equation (3), means that the global mean temperature must return to the pre-industrial equilibrium level, and consequently the cumulative emission should reduce to zero. We assumed that annual emissions start decreasing at a constant rate from 2021, eventually becoming negative, meaning that past carbon emissions would be removed from the atmosphere. By equating the area under the annual emission curve before reaching zero-emission (total carbon emission) with that after reaching zero emission (total amount of carbon being removed), we found the exact time for zero-emission to be achieved.

3.2.2 Temperature Change Model

Given that the CCR is approximately constant over time, we converted the cumulative carbon emission data to Tt of carbon and divided the temperature change by it to compute the CCR for each year. We took the average of CCR values over the last 20 years, yielding a point estimate of 2.8933°C per Tt C and a 95% confidence interval of 2.7891-2.9975°C per Tt C (Figure 1).

3.2.3 Sea Level Rise Model

Assuming that the previous temperature equilibrium T_0 is the pre-industrial temperature, the proportionality model between the rate of change of sea level and the temperature rise was fitted using data and gave an estimate of proportionality constant a being 3.7267 (Figure 2) with a R^2 value of 0.0008. The sea level rise in relation to warming was then

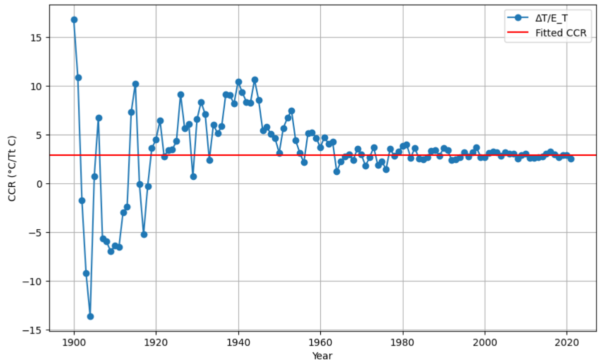

Figure 1: Fitting climate-carbon response (CCR)

computed using the integral form derived from equation (3)

$$H(t) = 3.7267 \int_{t_0}^{t} (T(t') - T_0)\, dt. \tag{5}$$

3.2.4 Ocean pH Model

Given that pCO_2 is directly proportional to the hydrogen-ion concentration squared (derived from equation (4)), we can assume that the cumulative carbon emission is an equivalent measure as pCO2, such that, when the definition $\mathrm{pH} = -\log_{10}\left[\mathrm{H}^+\right]$ is incorporated, the relationship between cumulative emission and pH can be modelled by the simple logarithmic equation

$$\mathrm{pH} = \beta_1 \log_{10}(E_T) + \beta_0, \tag{6}$$

where β_0 and β_1 are model parameters to be determined. Transforming the cumulative emission data using base 10 logarithmic operation and fitting with the ocean pH data (Figure 2) gave a model of

$$\mathrm{pH} = -0.1648 \log_{10}(\mathrm{E_T}) + 10.0737, \tag{7}$$

with a R^2 value of 0.9536.

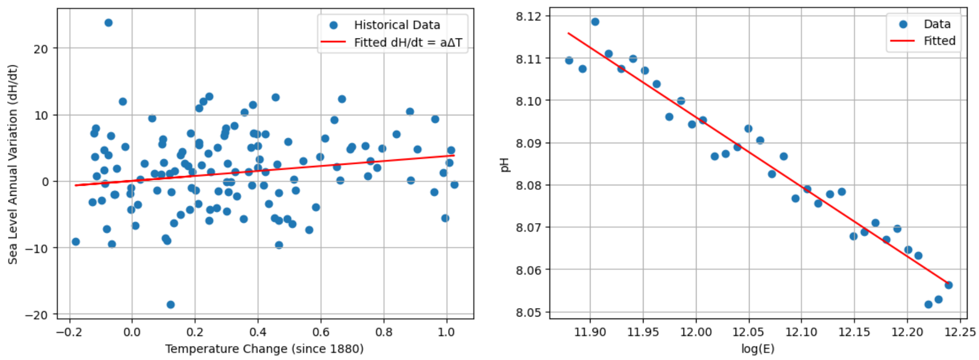

Figure 2: (left) Fitting proportionality constant a; (right) Fitting logarithmic pH model

4 Results

4.1 Emission Scenario Outcomes

The 4°C temperature target was translated into a cumulative emission target of 5.12 trillion tonnes. Assuming we continue to emit at the same annual rate of increasing 555.73 million tonnes per year until 2050, we can only maintain a constant annual emission of 53.24 billion tonnes until 2077 and must reduce annual emissions from such a level to zero by 2100 at a constant rate of decreasing 2.40 billion tonnes per year. As a result, sea levels will rise to 1051.16 mm above pre-industrial levels in 2100 and 1778.33 mm by 2150 (Figure 3). While the ocean pH plateaus at 7.98 by 2100. The GDP per capita was projected to reach 40692.22 US dollars in 2077, right before the significant reduction of annual emissions to zero.

The 2050 temperature target of 2°C has been translated into a target cumulative emission of 2.56 trillion tonnes. We can only allow annual emissions to remain constant beginning in 2021, at 37.12 billion tonnes per year, until 2035, and then reduce emissions to zero by 2050 at a constant rate of 2.74 billion tonnes per year. This projected a sea level rise of 406.27 mm in 2050 and 770.26 mm in 2100 (Figure 3). At the same time, the ocean's pH remains constant at 8.03 until 2050. The GDP per capita was predicted to be 17038.21 US dollars in 2035, before annual emissions start to drop to zero.

To achieve zero cumulative emissions by 2100, we need to reduce annual emissions at a steady rate of 1.50 billion tonnes per year. Under such an emission plan, zero annual emission is achieved in 2045, and emissions will become negative after as carbon-removal strategies are implemented. This projected a maximum global mean temperature rise of 1.73°C (Figure 3) with a confidence interval of 1.66-1.79°C (calculated using the CCR estimate's confidence interval). While sea level was expected to rise to and plateau at 587.32 mm above pre-industrial levels by 2100, pH drops to a minimum of 8.04 in 2045 and begins to rise to a near-pre-industrial level of 8.27 when cumulative emissions reach zero in 2100. However, with an immediate action of reducing annual emissions from now, the GDP per capita was predicted to reach zero when emission drops to zero in 2045.

5 Discussion

5.1 Emission Scenario Implications

Scenarios 1 to 3 represent varying degrees of emission controls, ranging from minimal to stringent. The corresponding environmental implications, including warming, sea level rise, and ocean acidification, are most severe under Scenario 1 and least significant under Scenario 3. Rising sea levels cause coastal flooding and erosion, threatening both human communities and wildlife, particularly in low-lying areas. While, a lowering ocean pH is expected to have the greatest impact near the surface, particularly on organisms such as coral reefs and calcareous plankton that have calcium carbonate skeletons or shells [11], consequently reducing marine biodiversity.

In Scenarios 1 and 2, GDP per capita is projected to grow during the period where annual emission growths are regulated, with Scenario 1 showing greater growth due to its looser emission controls. Although neither scenario achieves the economic growth projected with a 2.50% increase in GDP per capita per year (approximately the average

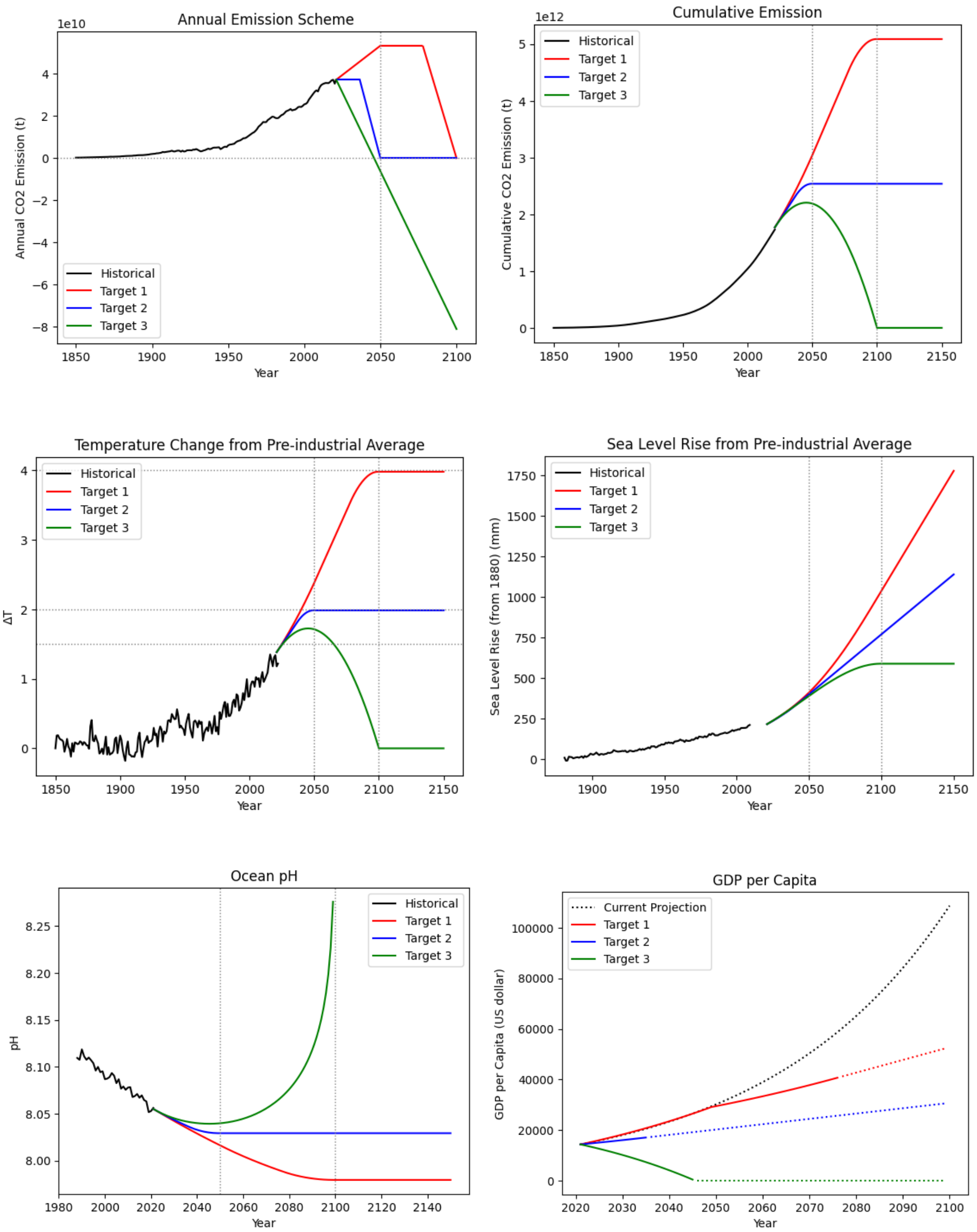

Figure 3: Projection outcomes based on Scenario 1, 2, and 3

growth over the last 20 years), both scenarios indicate a continued increase in the economy. This assumes that policies have successfully enforced a global decoupling between carbon emissions and energy generation, meaning carbon intensity is zero and GDP growth no longer depends on carbon emissions.

To foster the decoupling process, transitioning from traditional to renewable energy methods—such as wind, solar, and geothermal power—is essential on a global scale. This transition requires substantial financial investments and strict policy reforms for effective implementation. Increasing government funding for research and development of new clean energy technologies, coupled with building partnerships with private companies and academic institutions, will accelerate the commercialization of innovative clean energy solutions [2]. Simultaneously, imposing a carbon tax on emission-heavy industries while providing subsidies, tax credits, and other financial incentives to encourage renewable energy generation will establish a global regulatory system. This system will enforce the mandatory replacement of fossil fuels with clean energy, laying the practical foundation for completely decoupling carbon emissions from energy production.

Different to the first two scenarios, the requirement for the sea level to plateaus, aligned with the theory in literature, forces the cumulative emission to reach zero with negative annual emissions. To achieve negative emissions, the innovation and implementation of Negative Emission Technologies (NETs) are essential for removing past emissions from the atmosphere. Key NETs include Direct Air Capture (DAC) [12], where carbon dioxide is captured directly from the ambient air using chemical processes, then compressed for underground storage (geological sequestration) or industrial applications. Additionally, afforestation and reforestation—planting trees in areas without previous tree coverage or in deforested areas—significantly increase carbon sequestration as atmospheric carbon is absorbed then stored in biomass and soil through photosynthesis.

Despite having a more desirable outcomes on the environment, such an emission scenario projected to a zero GDP per capita and implies that following the current rate of decoupling economic growth from carbon emission is insufficient to avoid large scale economic downturns as a result of an aggressive emission controls. Also note that, the range of maximum projected temperature (1.66-1.79°C) was still above the outlined target, implying that, despite such efforts, global warming cannot be limited to 1.5°C, and the zero cumulative emission target must be shifted to an earlier year to keep warming below such a target.

5.2 Model Limitations

While the structured approach to project the emission and consequent environmental and economic implications of each target was effective, individual models contained limitation which contribute to an oversimplified prediction lacking in accuracy.

5.2.1 Kaya's Identity

Kaya's identity is frequently adopted due to its simplicity and reliance on readily available data. However, its limitation lies in the fact that the four factors it considers should not be viewed as fundamental driving forces in themselves, nor as entirely independent from one another [13]. This limitation becomes evident in its inability to project the economy when annual emissions reach zero, as the model breaks down with both carbon emissions and carbon intensity being zero. Furthermore, its failure to identify the fundamental drivers of economic growth restricts its accuracy, making it perform well only for short-term predictions that heavily rely on historical data. As a result, the projected GDP per capita under highly hypothetical assumptions of technological advancement and policy reforms becomes inaccurate as it fails to account for the impact of complex structural changes in sectors of the economy beyond the energy industry. Despite this, Kaya's identity still holds value for qualitative comparisons between scenarios.

5.2.2 CCR

The CCR is a simple representation of the temperature response to anthropogenic emissions, however, calculated uncertainties is quite large with a possibility of a higher value of CCR resulting in lower values of allowable emission prediction [5]. This limitation stems from poorly quantified uncertainties in land-use change emissions and the exclusion of other greenhouse gases or aerosols. Consequently, the CCR struggles to accurately predict temperature changes during rapid changes in emissions, as seen in the historical fluctuations of the CCR from 1900 to 1960, which only stabilize over long periods. This suggests potential inaccuracies in temperature projections, especially under scenarios involving aggressive emission changes and controls.

5.2.3 Sea Level Rise Model

Ice sheets may respond more strongly to warming this century than predicted by the fitted proportionality constant due to time-lagged positive feedback effects, such as ice bed lubrication, loss of supporting ice shelves, and ocean warming at the grounding line of the ice stream [14]. This discrepancy is evident from the extremely low R^2 value of the fit, indicating that the linear relationship fails to capture variations in sea level rise relative to temperature increases. A pathway analysis, as demonstrated in Song's work [15], can incorporate intermediate variables like humidity, sea ice coverage, and glacier mass to more accurately capture sea level responses to warming over both short and long timeframes.

5.2.4 Ocean pH Model

The simple logistic model characterising the ocean pH response to emission was based on the assumption that the dissociation constants, K1 and K2 in equation (4), of carbonate equilibria are constant under all circumstances. However, Zeebe and Wolf-Gladrow's work served as critique to such a naive assumption, which outlined that varying temperature and pressure of the carbonate system results in a change of energy in this thermodynamic system and the change in the value of the equilibrium constants [16]. Such a criticism can explain the unlikely sharp reverse of ocean pH in a short period of time after 2045 in the projection of Scenario 3, as the projected pH should exhibit a much lower level and gradual increase given a different dissociation constant value corresponding to the warming temperature after 2045.

6 Conclusion

Although the projections of emissions and related variables involved many assumptions and simplifications, they offer valuable insights into the environmental impacts of different emission scenarios and the necessary direction of policies to achieve each target. This structured approach can be adapted for future research, where advanced models like Earth System Models (ESMs) and sophisticated computational tools can provide more precise projections of temperature rise, sea level changes, and ocean pH levels, aiding in more detailed policy-making and implementation.

References

[1] Jos Delbeke, Artur Runge-Metzger, Yvon Slingenberg, and Jake Werksman. The paris agreement. In *Towards a climate-neutral Europe*, pages 24–45. Routledge, 2019.

[2] Paul Ekins. *Stopping climate change: policies for real zero.* Taylor & Francis, 2023.

[3] Azadeh Tavakoli. How precisely" kaya identity" can estimate ghg emissions: A global review. *Jordan Journal of Earth and Environmental Sciences*, 8(2):91–96, 2017.

[4] John Deutch. Decoupling economic growth and carbon emissions. *Joule*, 1(1):3–5, 2017.

[5] H Damon Matthews, Nathan P Gillett, Peter A Stott, and Kirsten Zickfeld. The proportionality of global warming to cumulative carbon emissions. *Nature*, 459(7248):829–832, 2009.

[6] Sea-Level Rise. A semi-empirical approach to projecting future. *Mineral. Petrol*, 147:155, 2004.

[7] Ken Caldeira and Robert Berner. Seawater ph and atmospheric carbon dioxide. *Science*, 286(5447):2043–2043, 1999.

[8] Global Carbon Project. Co emissions – gcb. Dataset, 2023. Major processing by Our World in Data. Original data from "Global Carbon Budget".

[9] Met Office Hadley Centre and J. et al. Kennedy. sea_temperature_anomaly. Dataset, 2019. Processed by Our World in Data. Original data from "The Met Office Hadley Centre HadSST.4.0.0.0 data set".

[10] National Oceanic and Atmospheric Administration (NOAA). Average. Dataset, n.d. Processed by Our World in Data. Original data from "National Oceanic and Atmospheric Administration (NOAA)".

[11] Joan A Kleypas, Robert W Buddemeier, David Archer, Jean-Pierre Gattuso, Chris Langdon, and Bradley N Opdyke. Geochemical consequences of increased atmospheric carbon dioxide on coral reefs. *science*, 284(5411):118–120, 1999.

[12] Christian Breyer, Mahdi Fasihi, Cyril Bajamundi, and Felix Creutzig. Direct air capture of co2: a key technology for ambitious climate change mitigation. *Joule*, 3(9):2053–2057, 2019.

[13] YUAN Lu and Pan Jiahua. Disaggregation of carbon emission drivers in kaya identity and its limitations with regard to policy implications. *Advances in Climate Change Research*, 9(3):210, 2013.

[14] Stefan Rahmstorf. A semi-empirical approach to projecting future sea-level rise. *Science*, 315(5810):368–370, 2007.

[15] Tong G. Chao J. Song, J. Data driven pathway analysis and forecast of global warming and sea level rise. *Sci Rep*, 13(5536), 2023.

[16] Richard E Zeebe and Dieter Wolf-Gladrow. *CO2 in seawater: equilibrium, kinetics, isotopes*, volume 65. Gulf Professional Publishing, 2001.

Appendices

A Python Code

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import fsolve
import sympy as sp
import math
import scipy.stats

data_1 = 'https://raw.githubusercontent.com/janzika/MATH3041/main/data/
temperature-anomaly.csv'
tem_change_1 = pd.read_csv(data_1)
tem_change_1 = tem_change_1[tem_change_1['Entity'] == 'Global']
tem_change_1= tem_change_1.rename(columns={'Global average temperature
anomaly relative to 1961-1990': 'temperature anomaly'})
tem_change_1 = tem_change_1[['Year','temperature anomaly']]


base_year_value = tem_change_1[tem_change_1['Year'] == 1850]
['temperature anomaly'].values[0]
# temperature change from preindustrial
tem_change_1['temperature change'] = tem_change_1['temperature anomaly']
- base_year_value
tem_change = tem_change_1[['Year', 'temperature change']]

data_2 = 'https://raw.githubusercontent.com/janzika/MATH3041/main/data/
annual-co-emissions-by-region.csv'
emission_1 = pd.read_csv(data_2)
emission_1 = emission_1[(emission_1['Entity'] == 'World') &
(emission_1['Year'] >= 1850)]
# cumulative carbon emission
emission_1['Cumulative CO emissions'] =
emission_1['Annual CO emissions (zero filled)'].cumsum()
emission_2 = emission_1[['Year','Cumulative CO emissions']]
emission_2['Cumulative CO emissions'] =
emission_2['Cumulative CO emissions']/(3.7*10**12)
emission = emission_2[['Year','Cumulative CO emissions']]


# CCR
def calculate_ratio(tem_change, emission):

    merged_dT_E = pd.merge(tem_change, emission , on='Year')
    merged_dT_E['T/E_T'] =  (merged_dT_E['temperature change'] /
    merged_dT_E['Cumulative CO emissions'])
    return  merged_dT_E[['Year', 'T/E_T']]
result = calculate_ratio(tem_change, emission)

# last 20 year mean CCR
result_2000 = result[(result['Year'] >= 2000)]
mean_value_since_2000 = result_2000['T/E_T'].mean()
year = np.linspace(1900, 2021, num=1000)
plt.figure(figsize=(10, 6))
plt.plot(result['Year'], result['T/E_T'], marker='o', label='T/E_T')
plt.axhline(y = mean_value_since_2000, color = "r", label = "Fitted CCR")
```

```
plt.xlabel('Year')
plt.ylabel('CCR (°C/Tt C)')
plt.legend()
plt.grid(True)
plt.show()


data = pd.read_csv('https://raw.githubusercontent.com/janzika/MATH3041/
main/data/climate-change_2.csv')
# format date
data[['Year', 'Month', 'Day']] = data['Date'].str.split('-', expand=True)
data = data.drop(columns=['Day', 'Month'])
data['Year'] = data['Year'].astype(int)
# annual average sea level rise
sea_level_yearly_mean = data.groupby('Year').mean(numeric_only=True).reset_index()
sea_level_yearly_mean['sea_level_difference'] =
sea_level_yearly_mean['Church & White'].diff()
sea_level_yearly_diff = sea_level_yearly_mean.dropna(subset=
['sea_level_difference'])

# fitting proportionality constant
merged_data = pd.merge(sea_level_yearly_diff, tem_change, on='Year')
x = merged_data['temperature change'].values
y = merged_data['sea_level_difference'].values
coefficients = np.polyfit(x, y, 1)
slope, intercept = coefficients
a_temp = np.sum(x*y)/ np.sum(x**2)
y_fit = a_temp * x

plt.scatter(x, y, label='Historical Data')
plt.plot(x, y_fit, color='red', label='Fitted dH/dt = aT')
plt.xlabel('Temperature Change (since 1880)')
plt.ylabel('Sea Level Annual Variation (dH/dt)')
plt.legend()
plt.grid(True)
plt.show()

# cumulative carbon emission increase to meet 4 degree in 2100
C_tar1 = (4/mean_value_since_2000)*(3.7*10**12)
current_C = emission_1.iloc[-1]["Cumulative CO emissions"]
delta_C_tar1 = C_tar1 - current_C
print(C_tar1)
print(delta_C_tar1)

# last 20 year average rate of emission (2015-2021)
m_last5 = np.diff(emission_1[emission_1["Year"] >= 2000]
["Annual CO emissions (zero filled)"]).mean()
print(m_last5)

# current annual emission
a = emission_1.iloc[-1]["Annual CO emissions (zero filled)"]
print(a)

# annual emission if follwo m_last5 to 2050
a1 = m_last5*(2050-2021) + a

# cumulative carbon emission increase from 2050 to 2100 if to meet 4 degree
A1 = 0.5*(2050-2021)*(a + a1)
delta_C2050_tar1 = delta_C_tar1 - A1
```

```
# find how many years after 2050 to start decrease annual emission
x_tar1 = 2*(delta_C2050_tar1/a1 - 50/2)
print(x_tar1)
# plot three steps for annual emission
x1_tar1 = np.linspace(2021, 2050, num = (2050-2021))
y1_tar1 = m_last5*x1_tar1 + (a - m_last5*2021)

x2_tar1 = np.linspace(2051, math.floor(2050+x_tar1), num = math.floor(x_tar1))
y2_tar1 = a1 + 0*x2_tar1

x3_tar1 = np.linspace(math.ceil(2050+x_tar1), 2100, num =
(50 - math.floor(x_tar1)))
y3_tar1 = -(a1/(50-x_tar1))*x3_tar1 + a1*2100/(50-x_tar1)

x4_tar1 = np.linspace(2101, 2150, num = (2150-2100))
y4_tar1 = 0*x4_tar1

x_pred_tar1 = np.concatenate([x1_tar1, x2_tar1, x3_tar1, x4_tar1])
y_pred_tar1 = current_C +
np.concatenate([y1_tar1, y2_tar1, y3_tar1, y4_tar1]).cumsum()
historical_cum = emission_1["Cumulative CO emissions"]

# cumulative emission to meet 2 degree in 2050
C_tar2 = (2/mean_value_since_2000)*(3.7*10**12)
current_C = emission_1.iloc[-1]["Cumulative CO emissions"]
delta_C_tar2 = C_tar2 - current_C

# current annual emission
a = emission_1.iloc[-1]["Annual CO emissions (zero filled)"]
print(a)
# find how many years after 2021 to start decrease annual emission
x_tar2 = 2*(delta_C_tar2/a - 29/2)
print(x_tar2)

# plot step for annual emission
x1_tar2 = np.linspace(2021, math.floor(2021 + x_tar2), num = math.floor(x_tar2))
y1_tar2 = a + 0*x1_tar2

x2_tar2 = np.linspace(math.ceil(2021 + x_tar2), 2050, num =
29 - math.floor(x_tar2))
y2_tar2 = -a*x2_tar2/(29 - x_tar2) + a*2050/(29 - x_tar2)

x3_tar2 = np.linspace(2051, 2150, num = (2150-2050))
y3_tar2 = 0*x3_tar2

x_pred_tar2 = np.concatenate([x1_tar2, x2_tar2, x3_tar2])
y_pred_tar2 = current_C + np.concatenate([y1_tar2, y2_tar2, y3_tar2]).cumsum()

# cumulative emission to meet 1.5 degree
C_tar3 = (1.5/mean_value_since_2000)*(3.7*10**12)
current_C = emission_1.iloc[-1]["Cumulative CO emissions"]

# current annual emission
a = emission_1.iloc[-1]["Annual CO emissions (zero filled)"]

# find time when annual emission goes negative
x_tar3 = (0.5*a*79**2)/(current_C + 79*a)
print(x_tar3)

# plot annual emission
```

```
x1_tar3 = np.linspace(2021, 2100, num = (2100 - 2021))
y1_tar3 = -a*x1_tar3/x_tar3 + a*(2021+x_tar3)/x_tar3

x2_tar3 = np.linspace(2101, 2150, num = (2150-2100))
y2_tar3 = 0*x4_tar1

x_pred_tar3 = np.concatenate([x1_tar3, x2_tar3])
y_pred_tar3 = current_C + np.concatenate([y1_tar3, y2_tar3]).cumsum()

# Target 1
# temperature change from pre-industrial value (1880)

historical_temp = tem_change["temperature change"]
temp_pred_tar1 = (y_pred_tar1/(3.7*10**12))*mean_value_since_2000

# sea level rise from pre-industrial value (1880)
historical_H = sea_level_yearly_diff["sea_level_difference"].cumsum()
print(historical_H)
H_pred_tar1 = historical_H.iloc[-1] + a_temp*temp_pred_tar1.cumsum()

# Target 2
# temperature change from pre-industrial value (1880)
temp_pred_tar2 = (y_pred_tar2/(3.7*10**12))*mean_value_since_2000

# sea level rise from pre-industrial value (1880)
H_pred_tar2 = historical_H.iloc[-1] + a_temp*temp_pred_tar2.cumsum()

# Target 3
# temperature change from pre-industrial value (1880)
temp_pred_tar3 = (y_pred_tar3/(3.7*10**12))*mean_value_since_2000
x_add_tar3 = np.linspace(2100,2150, num = 50)
temp_add_tar3 = 0*x_add_tar3

# sea level rise from pre-industrial value (1880)
H_pred_tar3 = historical_H.iloc[-1] + a_temp*temp_pred_tar3.cumsum()
H_add_tar3 = H_pred_tar3[-1] + a_temp*temp_add_tar3

data = pd.read_csv("https://raw.githubusercontent.com/janzika/MATH3041/
618718923297ec99e11cd3cd25af5ba1173d4a33/data/climate-change_2.csv")
data_global = data[data["Entity"] == "World"]
pH = data_global[["Date", "Monthly pH measurement"]]

pH["Date"] = pd.to_datetime(pH["Date"])
pH = pH.dropna(subset=['Monthly pH measurement'])
pH['Year'] = pH['Date'].dt.year
avgpH = pH.groupby('Year')['Monthly pH measurement'].mean().reset_index()
avgpH.columns = ['Year', 'avg_pH']

pH_CO2 = pd.merge(avgpH, emission_1, on='Year')
[["Year", "avg_pH", "Cumulative CO emissions"]]
print(pH_CO2)
x_pH = np.log10(pH_CO2["Cumulative CO emissions"])
m,b = np.polyfit(x_pH, pH_CO2["avg_pH"], 1)
print(m,b)

plt.scatter(x_pH, pH_CO2["avg_pH"], label = "Data")
plt.plot(x_pH, m*x_pH + b, color = "r", label = "Fitted")
plt.xlabel("log(E)")
plt.ylabel("pH")
plt.grid(True)
```

```
plt.legend()
plt.show()

# Target 1 pH
historical_pH = pH_CO2["avg_pH"]
pH_pred_tar1 = m*np.log10(y_pred_tar1) + b

# Target 2 pH
pH_pred_tar2 = m*np.log10(y_pred_tar2) + b

# Target 3 pH
pH_pred_tar3 = m*np.log10(y_pred_tar3) + b

# Annual emission comparison
plt.plot(historical_years, historical_annual, color = "black",
label = "Historical")
plt.plot(x1_tar1, y1_tar1, color = "r", label = "Target 1")
plt.plot(x2_tar1, y2_tar1, color = "r")
plt.plot(x3_tar1, y3_tar1, color = "r")

plt.plot(x1_tar2, y1_tar2, color = "blue", label = "Target 2")
plt.plot(x2_tar2, y2_tar2, color = "blue")
plt.plot(x3_tar2, y3_tar2, color = "blue")

plt.plot(x1_tar3, y1_tar3, label = "Target 3", color = "green")

plt.ylabel("Annual CO2 Emission (t)")
plt.xlabel("Year")
plt.legend(loc = 3)
plt.axvline(2050, color='grey', linestyle=':', linewidth=1)
plt.axhline(0, color='grey', linestyle=':', linewidth=1)
plt.title("Annual Emission Scheme")

# Cumulative emission comparison
plt.plot(historical_years, historical_cum, color = "black",
label = "Historical")
plt.plot(x_pred_tar1, y_pred_tar1, color = "r", label = "Target 1")

plt.plot(x_pred_tar2, y_pred_tar2, label = "Target 2", color = "blue")

plt.plot(x_pred_tar3, y_pred_tar3, label = "Target 3", color = "green")

plt.ylabel("Cumulative CO2 Emission (t)")
plt.xlabel("Year")
plt.legend()
plt.axvline(2050, color='grey', linestyle=':', linewidth=1)
plt.axvline(2100, color='grey', linestyle=':', linewidth=1)
plt.title("Cumulative Emission")

# Temperature comparison
plt.plot(tem_change["Year"], historical_temp, color = "black",
label = "Historical")
plt.plot(x_pred_tar1, temp_pred_tar1, color = "r", label = "Target 1")

plt.plot(x_pred_tar2, temp_pred_tar2, label = "Target 2", color = "blue")

plt.plot(x_pred_tar3, temp_pred_tar3, label = "Target 3", color = "green")
plt.xlabel("Year")
plt.ylabel("T")
plt.legend()
```

```
plt.axvline(2050, color='grey', linestyle=':', linewidth=1)
plt.axvline(2100, color='grey', linestyle=':', linewidth=1)
plt.axhline(4, color='grey', linestyle=':', linewidth=1)
plt.axhline(2, color='grey', linestyle=':', linewidth=1)
plt.axhline(1.5, color='grey', linestyle=':', linewidth=1)
plt.title("Temperature Change from Pre-industrial Average")
plt.show()

# Sea level comparison
plt.plot(sea_level_yearly_diff["Year"], historical_H, color = "black",
label = "Historical")
plt.plot(x_pred_tar1, H_pred_tar1, color = "r", label = "Target 1")

plt.plot(x_pred_tar2, H_pred_tar2, color = "blue", label = "Target 2")

plt.plot(x_pred_tar3, H_pred_tar3, color = "green", label = "Target 3")
plt.xlabel("Year")
plt.ylabel("Sea Level Rise (from 1880) (mm)")
plt.axvline(2050, color='grey', linestyle=':', linewidth=1)
plt.axvline(2100, color='grey', linestyle=':', linewidth=1)
plt.title("Sea Level Rise from Pre-industrial Average")
plt.legend()
plt.show()

# pH comparison
plt.plot(pH_CO2["Year"], historical_pH, color = "black",
label = "Historical")
plt.plot(x_pred_tar1, pH_pred_tar1, color = "r", label = "Target 1")

plt.plot(x_pred_tar2, pH_pred_tar2, color = "blue", label = "Target 2")

plt.plot(x_pred_tar3, pH_pred_tar3, color = "green", label = "Target 3")
plt.xlabel("Year")
plt.ylabel("pH")
plt.axvline(2050, color='grey', linestyle=':', linewidth=1)
plt.axvline(2100, color='grey', linestyle=':', linewidth=1)
plt.title("Ocean pH")
plt.legend()
plt.show()

def compute_r_squared(x, y, coefficients):
    polynomial = np.poly1d(coefficients)
    y_pred = polynomial(x)
    ss_res = np.sum((y - y_pred) ** 2)
    ss_tot = np.sum((y - np.mean(y)) ** 2)
    r_squared = 1 - (ss_res / ss_tot)
    return r_squared

# Sea Level model R squared
sea_coef = a_temp, 0
r_sea = compute_r_squared(merged_data['temperature change'].values, merged_data['sea_
print(r_sea)

# pH model R squared
ph_coef = np.polyfit(x_pH, pH_CO2["avg_pH"], 1)
r_ph = compute_r_squared(x_pH, pH_CO2["avg_pH"], ph_coef)
print(r_ph)

def mean_confidence_interval(data, confidence=0.95):
    a = 1.0 * np.array(data)
```

```
    n = len(a)
    m, se = np.mean(a), scipy.stats.sem(a)
    h = se * scipy.stats.t.ppf((1 + confidence) / 2., n-1)
    return m, m-h, m+h

# CCR confidence interval
ccr_interval = mean_confidence_interval(result_2000['T/E_T'])
print(ccr_interval)

pop_rate = 1.2779**(1/20)
energy_intensity_rate = (1-0.2785)**(1/20)
carbon_intensity_rate = (1-0.1543)**(1/20)

curr_pop = 7909295151
curr_energy_intensity = 4540000
curr_carbon_intensity = 0.26/3600000000

def calculate_gdp_per_capita(year_X, annual_emission):
    years = np.arange(2021, 2100)
    num_years = len(years)

    population = curr_pop * (pop_rate ** (years - 2021))

    energy_intensity = curr_energy_intensity *
    (energy_intensity_rate ** (years - 2021))

    carbon_intensity = curr_carbon_intensity *
    (carbon_intensity_rate ** (years - 2021))


    GDP = np.zeros(num_years)
    for i, year in enumerate(years):
        if year <= year_X:
            GDP[i] = annual_emission[i] /
            (population[i] * energy_intensity[i] * carbon_intensity[i])
        else:
            GDP[i] = GDP[i] = GDP[i-1] + (GDP[year_X-2021]
            - GDP[year_X-2022])
    GDP[GDP < 0] = 0
    return years, GDP

# Target 1
X_tar1 = 2076
year_tar1, GDP_tar1 = calculate_gdp_per_capita(X_tar1,
np.concatenate([y1_tar1, y2_tar1, y3_tar1, y4_tar1]))

# Target 2
X_tar2 = 2035
year_tar2, GDP_tar2 = calculate_gdp_per_capita(X_tar2,
np.concatenate([y1_tar2, y2_tar2, y3_tar2]))

# Target 3
X_tar3 = 2045
year_tar3, GDP_tar3 = calculate_gdp_per_capita(X_tar3,
np.concatenate([y1_tar3, y2_tar3]))

# Current Projection
year_norm = np.linspace(2021, 2100, (2100-2021))
GDP_norm = GDP_tar1[0]*(1.026)**(year_norm-2021)
```

```
# Plot GDP per capita versus year
plt.plot(year_norm, GDP_norm, color = "black",
label = "Current Projection", linestyle=':')
plt.plot(year_tar1[year_tar1 <= X_tar1],
GDP_tar1[year_tar1 <= X_tar1], color = "red", label = "Target 1")
plt.plot(year_tar1[year_tar1 > X_tar1],
GDP_tar1[year_tar1 > X_tar1], linestyle=':', color = "r")

plt.plot(year_tar2[year_tar2 <= X_tar2],
GDP_tar2[year_tar2 <= X_tar2], color = "blue", label = "Target 2")
plt.plot(year_tar2[year_tar2 > X_tar2],
GDP_tar2[year_tar2 > X_tar2], linestyle=':', color = "blue")

plt.plot(year_tar3[year_tar3 <= X_tar3],
GDP_tar3[year_tar3 <= X_tar3], color = "green", label = "Target 3")
plt.plot(year_tar3[year_tar3 > X_tar3],
GDP_tar3[year_tar3 > X_tar3], linestyle=':', color = "green")
plt.xlabel('Year')
plt.ylabel('GDP per Capita (US dollar)')
plt.title('GDP per Capita')
plt.legend()
plt.show()

print(GDP_tar1[year_tar1 == X_tar1])
print(GDP_tar2[year_tar2 == X_tar2])
print(GDP_tar3[year_tar3 == X_tar3])
```