

Phylogenomic Branch Length Estimation using Quartets

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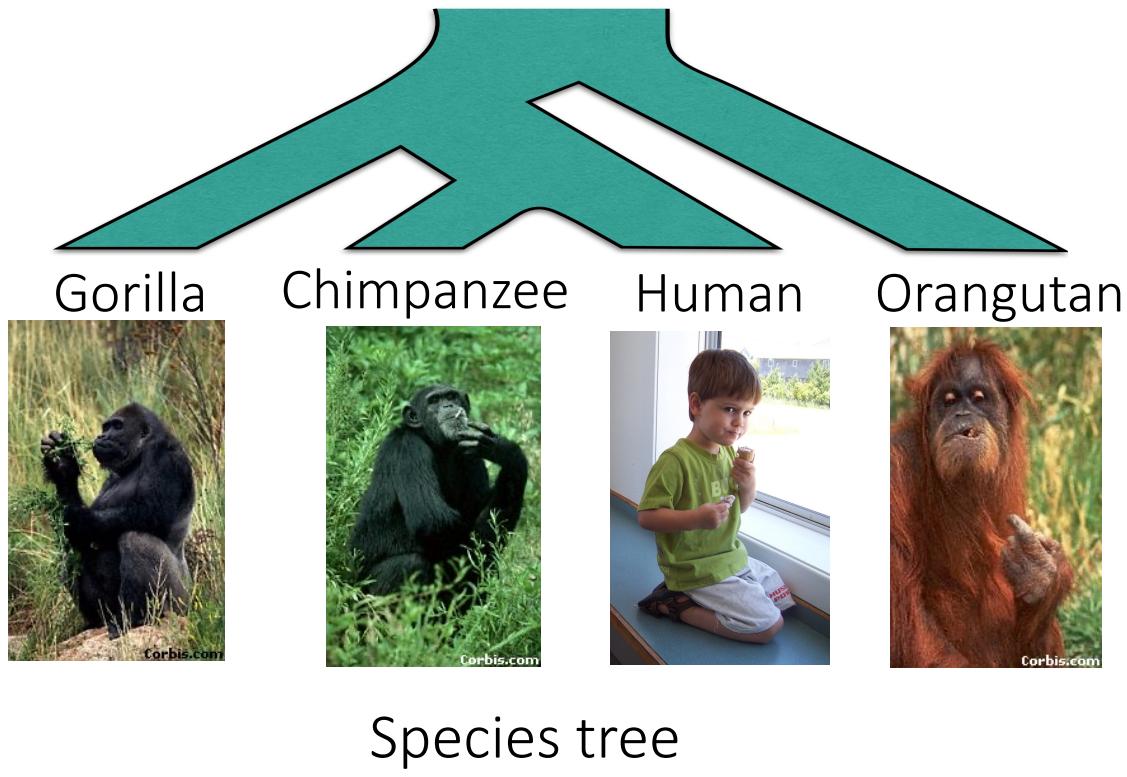
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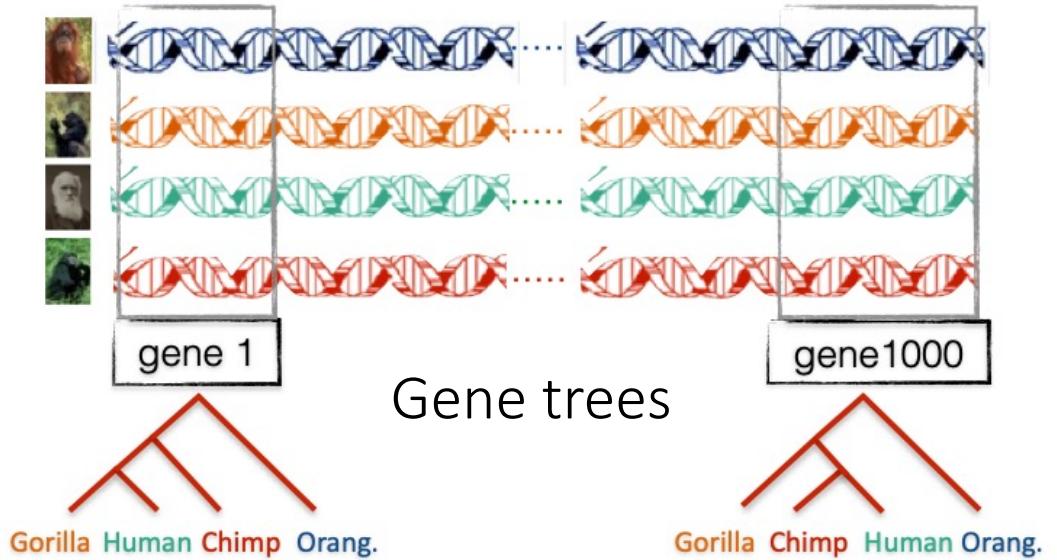
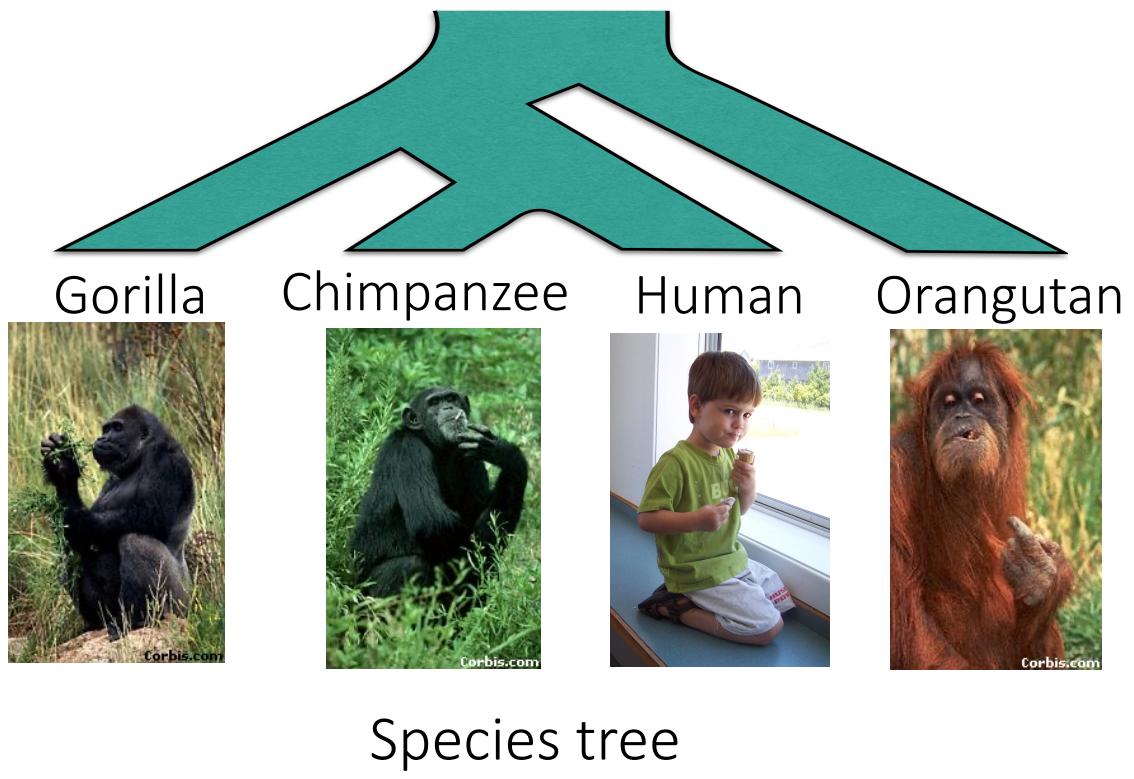
ISMB/ECCB

July 27, 2023

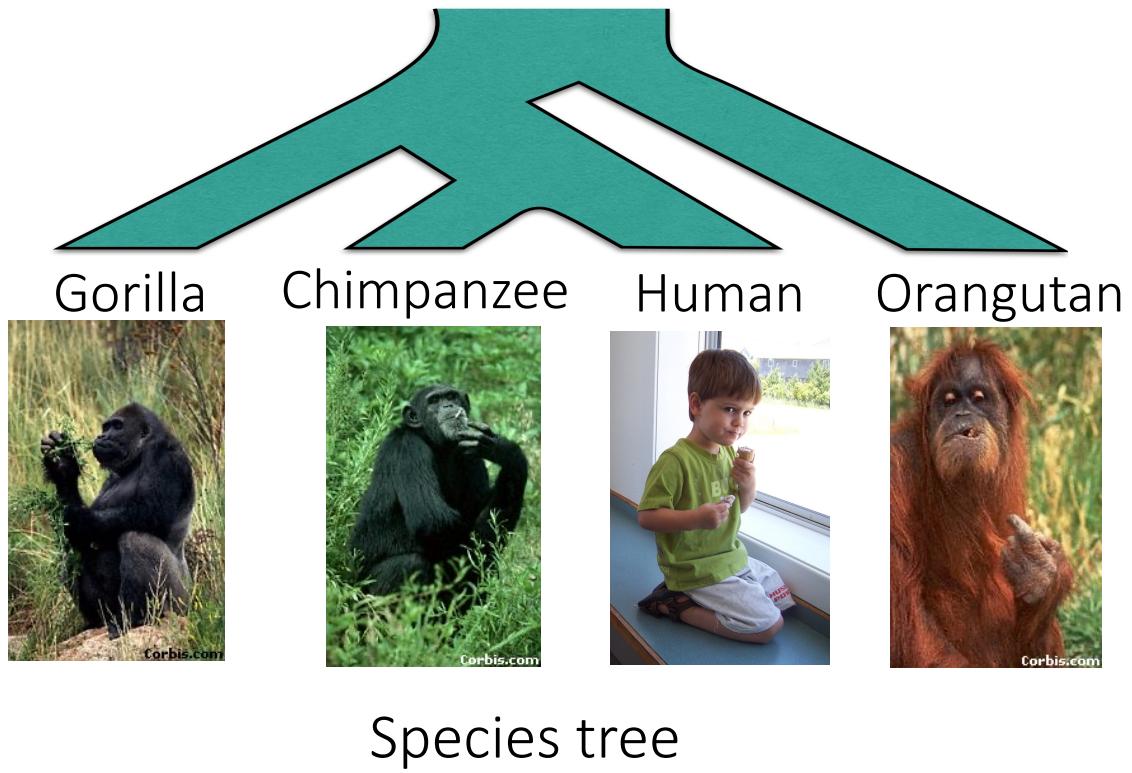
Phylogenomics and gene tree discordance



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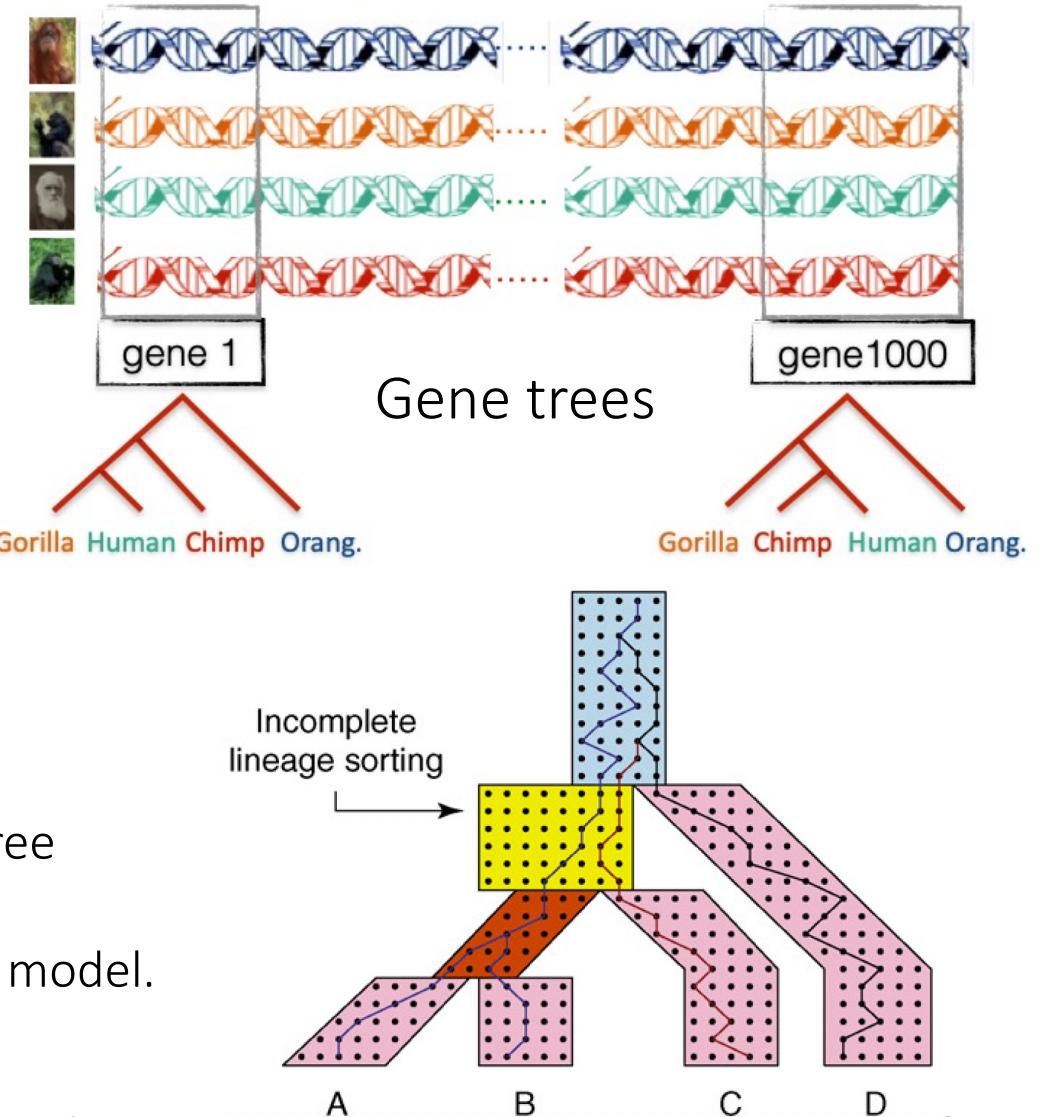


Phylogenomics and gene tree discordance

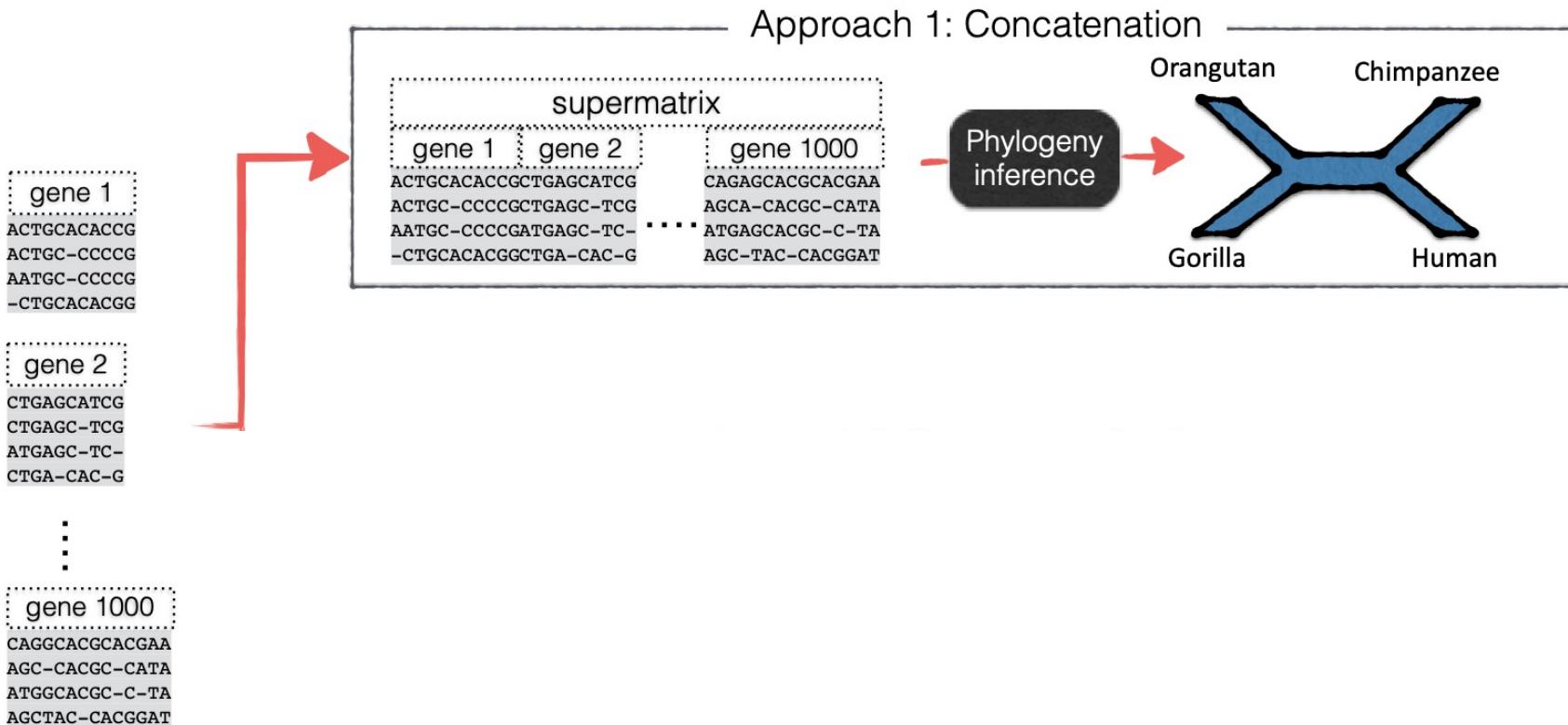


- Incomplete lineage sorting (ILS) is a major cause of gene tree discordance.
- ILS can be modeled by the multi-species coalescent (MSC) model.

Image Credit: Degnan and Rosenberg, 2009, Trends in Ecology and Evolution

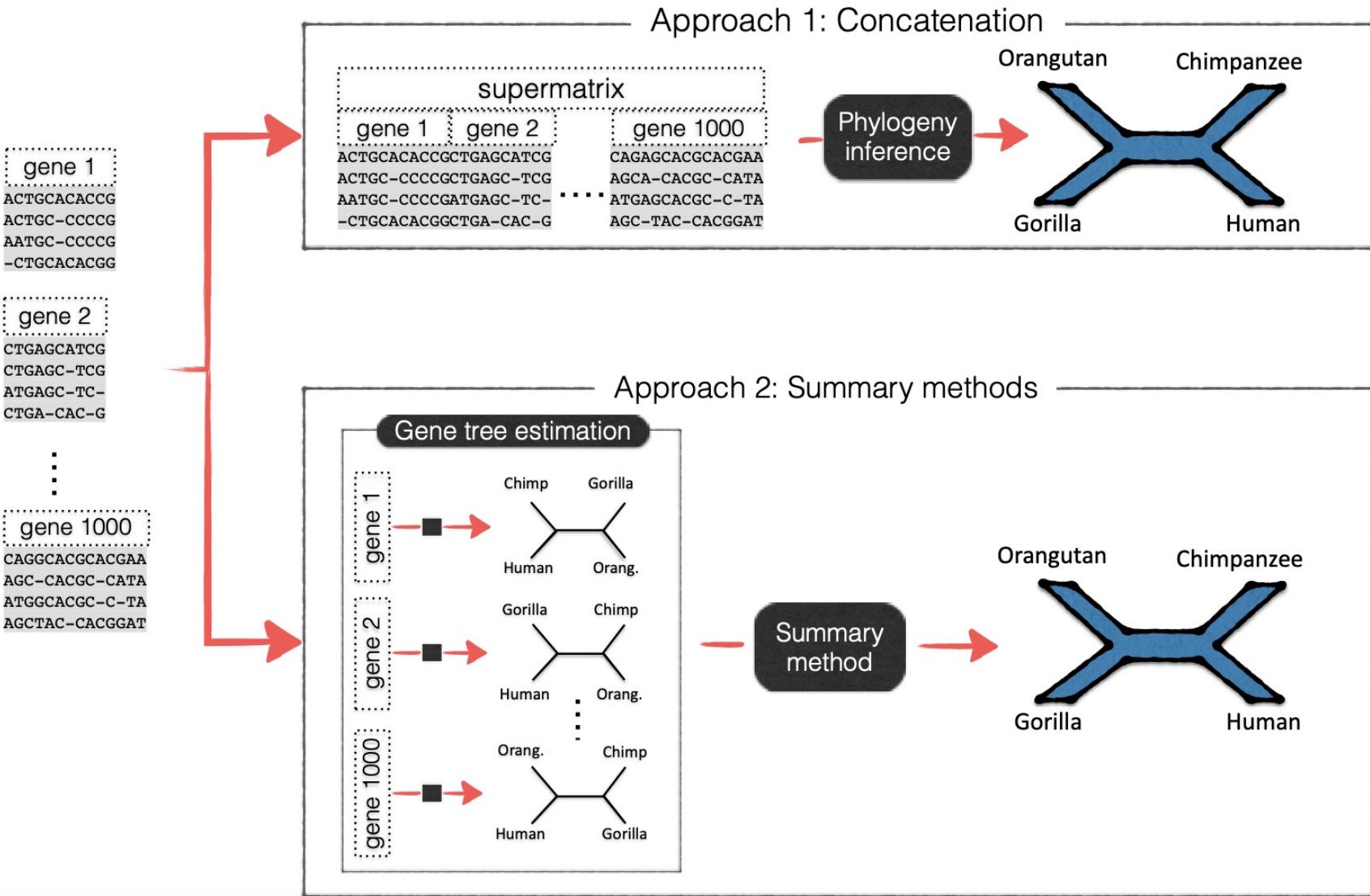


Species tree and branch length estimation



Maximum Likelihood, e.g.
RAxML [Stamatakis, 2014]
FastTree [Price et al, 2010]

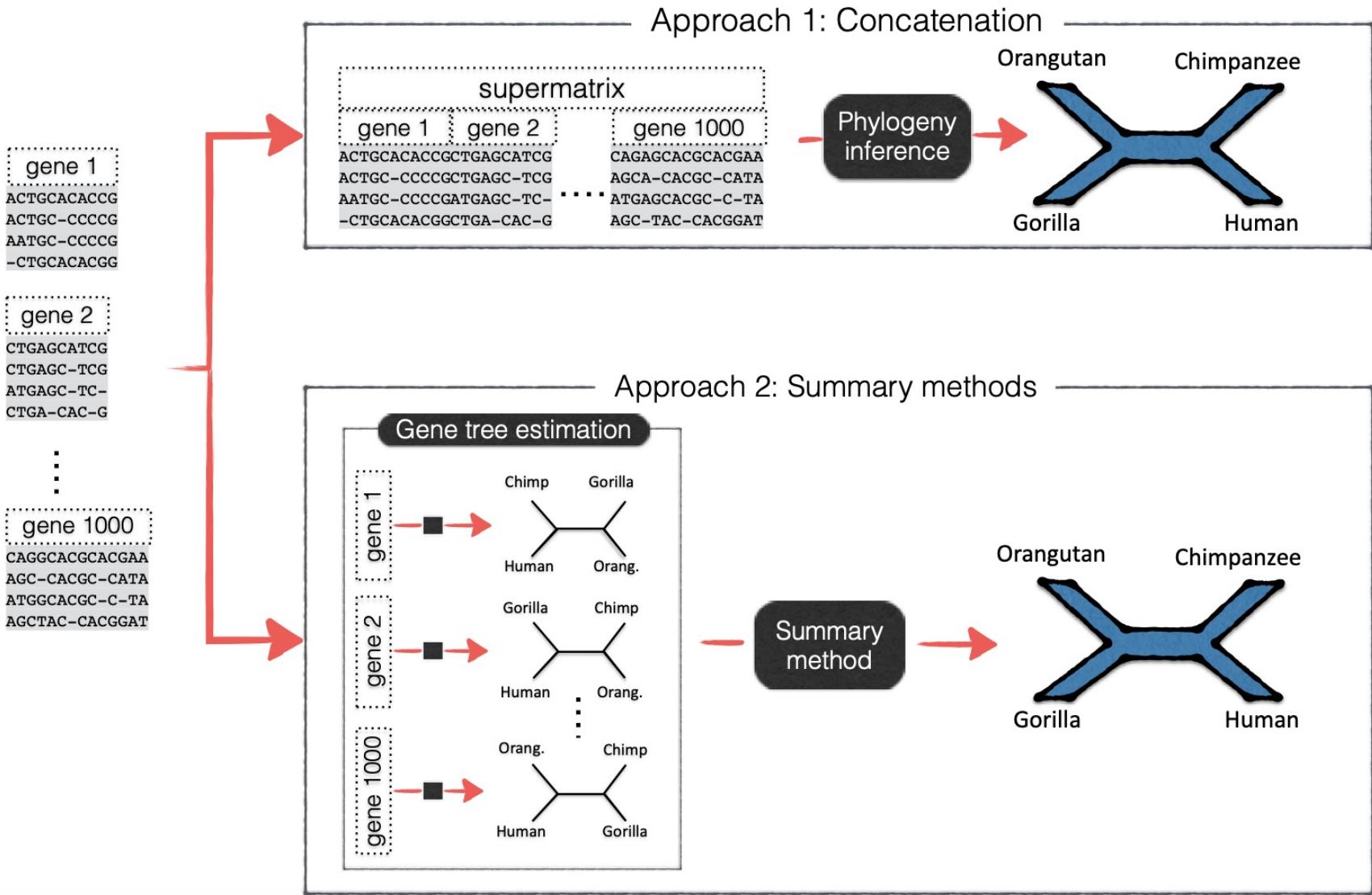
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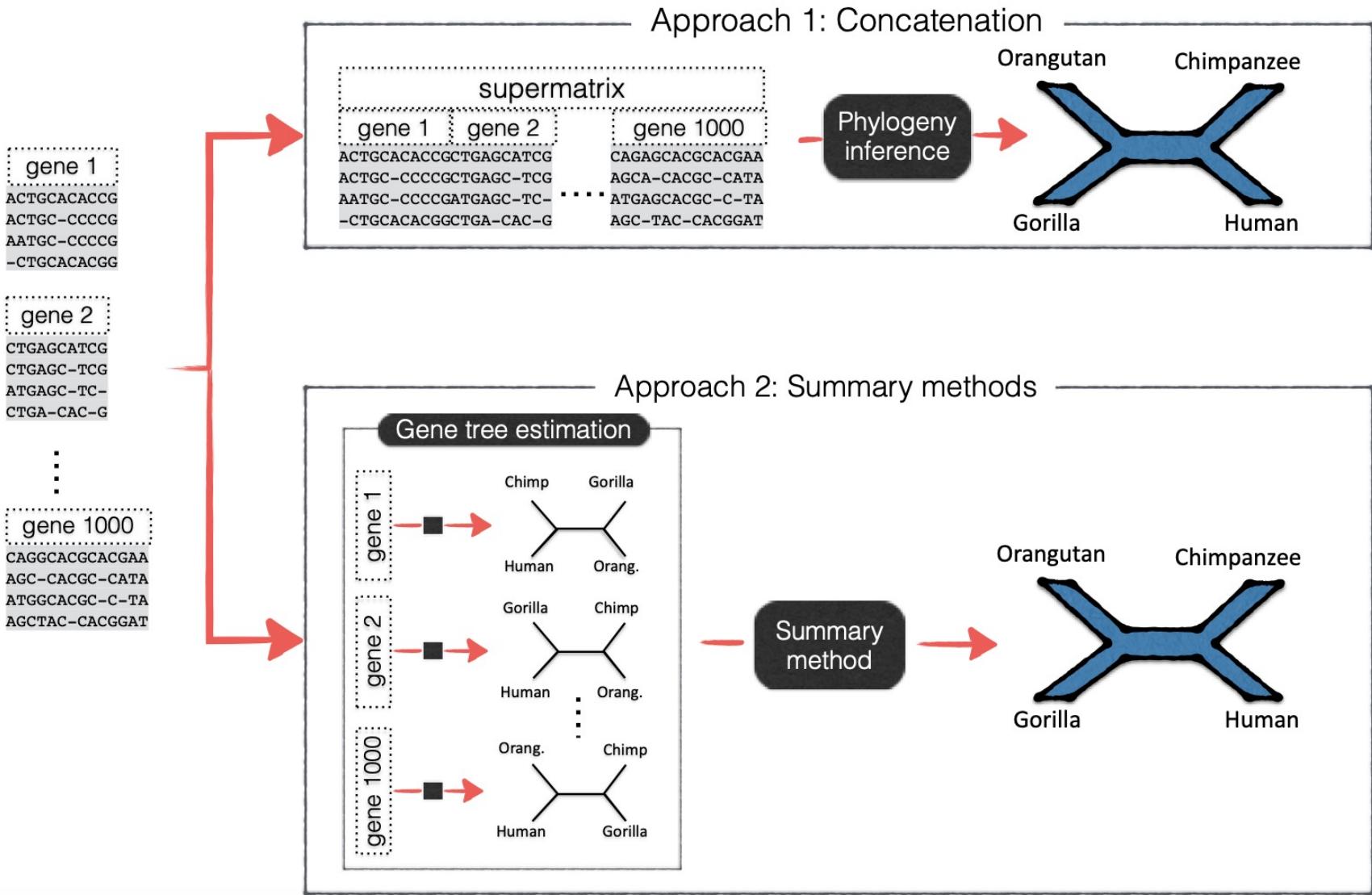
ASTRAL [Mirarab et al, 2014]
MP-EST [Liu et al, 2010]
...

Species tree and branch length estimation



- Summary methods are more scalable and more accurate when ILS is high, but produce branch lengths in **coalescent units (CU)**
- CU branch lengths are not useful for most downstream analysis
- **Two-step approach:**
 1. infer the topology with summary methods (e.g. ASTRAL, MP-EST)
 2. infer branch lengths on that **fixed topology** with concatenation

Species tree and branch length estimation

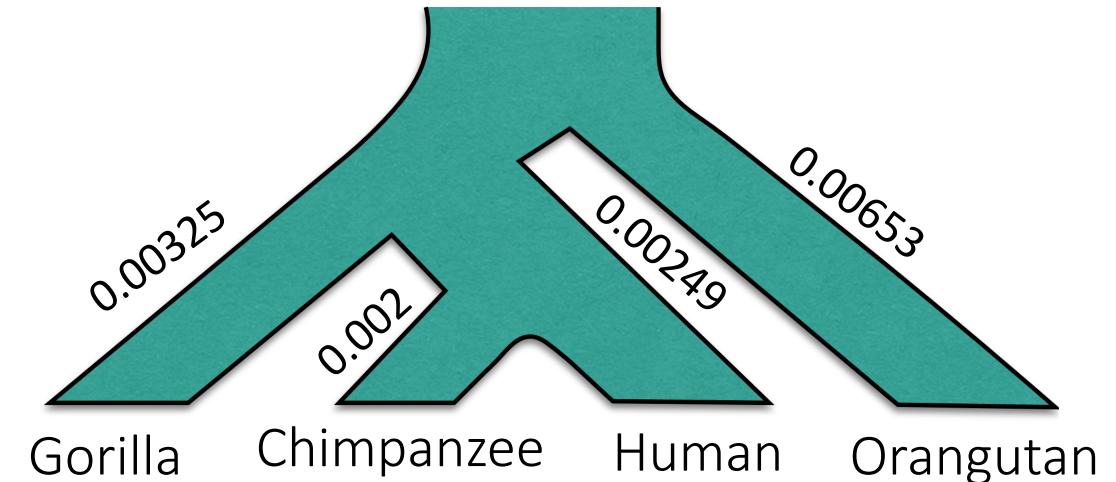


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ignores heterogeneity
across the genome

Branch lengths are necessary for downstream analysis

- Most downstream analysis need branch lengths in the unit of the expected number of substitutions per sites (SU)
- Applications of SU branch lengths
 - Dating
 - Comparative genomics
 - Species delimitation
 - Detecting and characterizing selection
 - ...
- CU branch lengths do not directly lead to SU branch lengths and are only inferable for **internal** branches

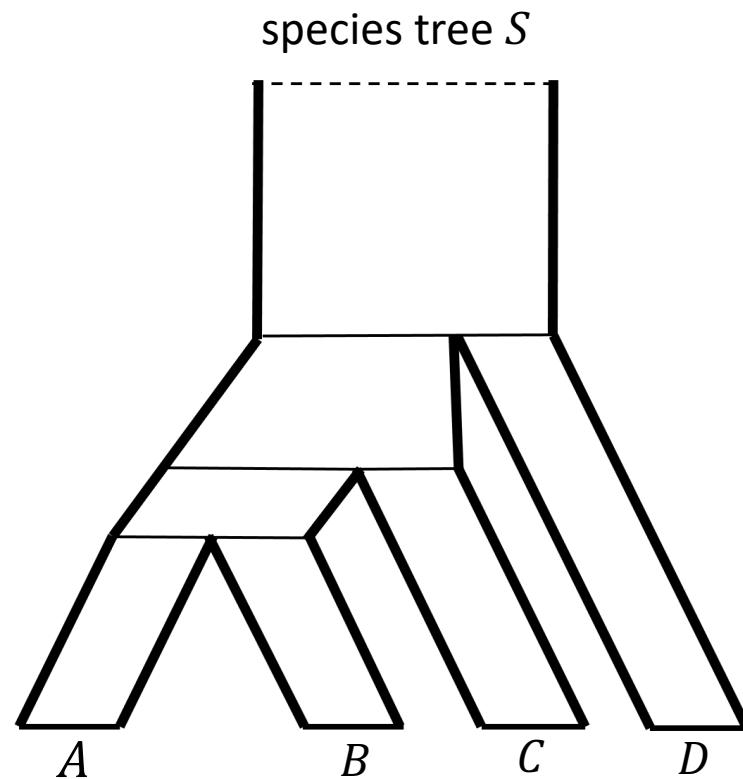


Our motivation

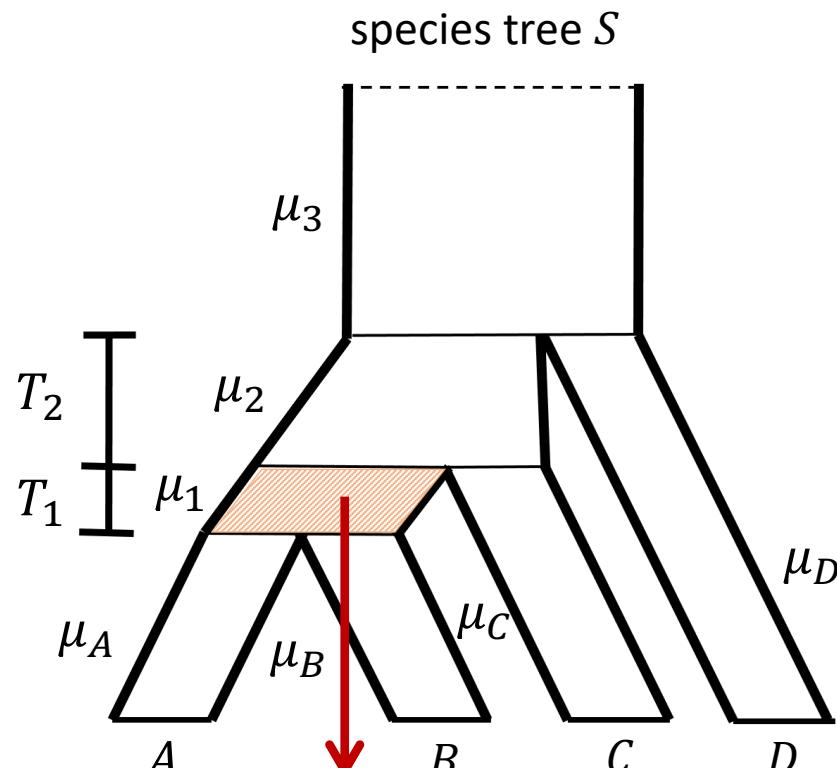
Can we design a branch length estimation method that...

- estimates branch lengths in substitution units (SU)
- addresses gene tree heterogeneity due to ILS and variation in mutation rates
- has strong theoretical foundation based on the MSC
- is scalable to large genome-wide datasets with hundreds to thousands of genes and species?

MSC+Substitution model



MSC+Substitution model



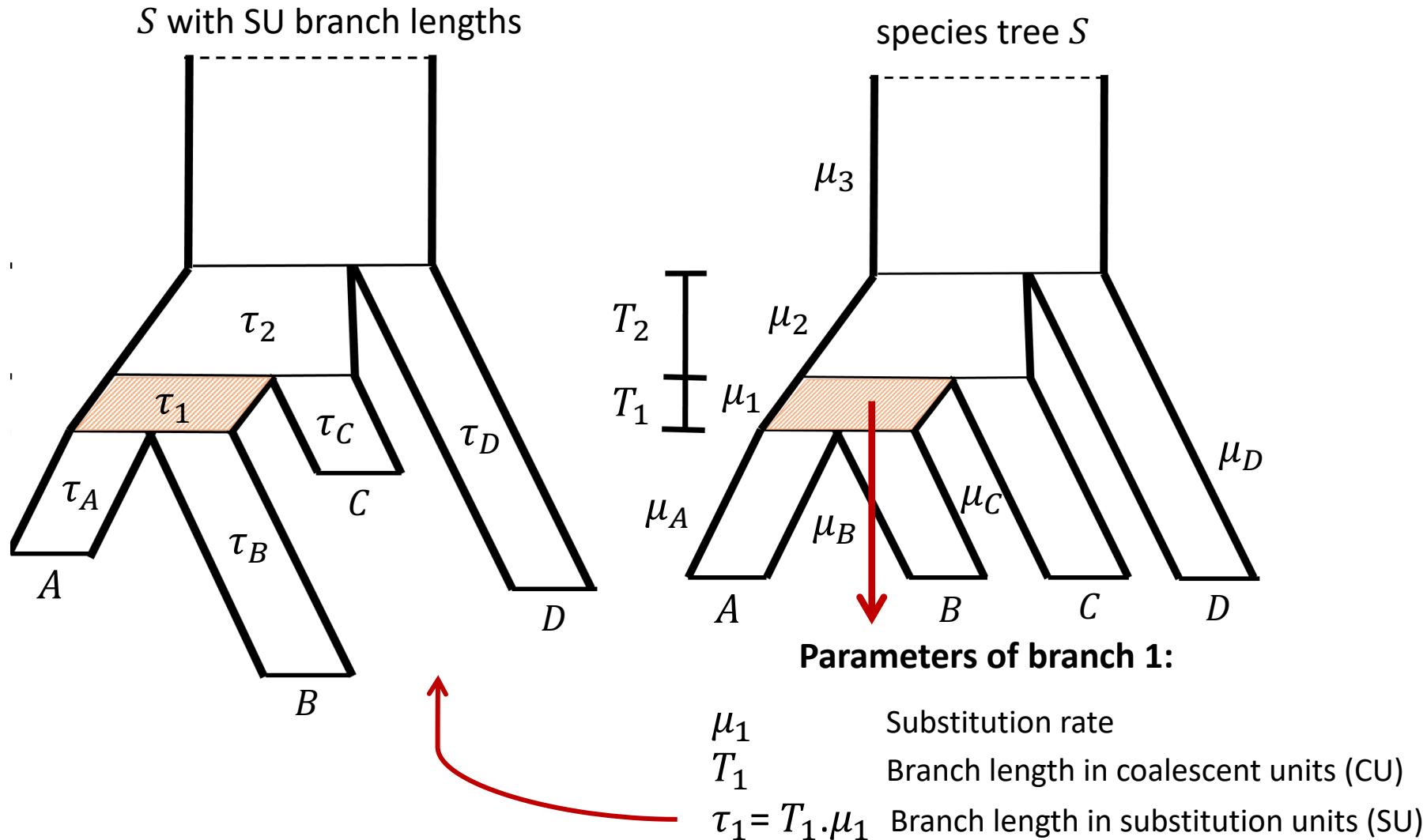
Parameters of branch 1:

μ_1 Substitution rate

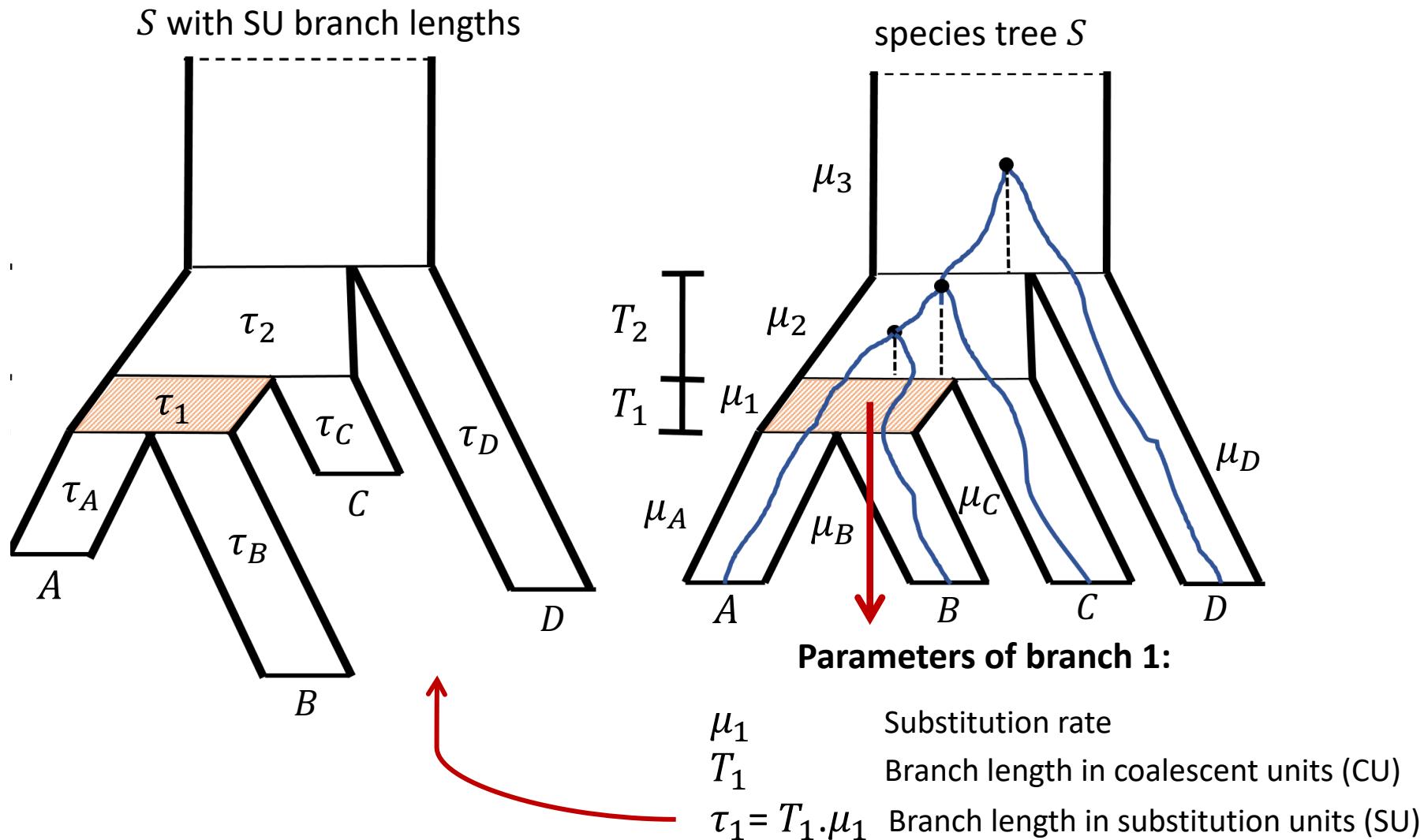
T_1 Branch length in coalescent units (CU)

$\tau_1 = T_1 \cdot \mu_1$ Branch length in substitution units (SU)

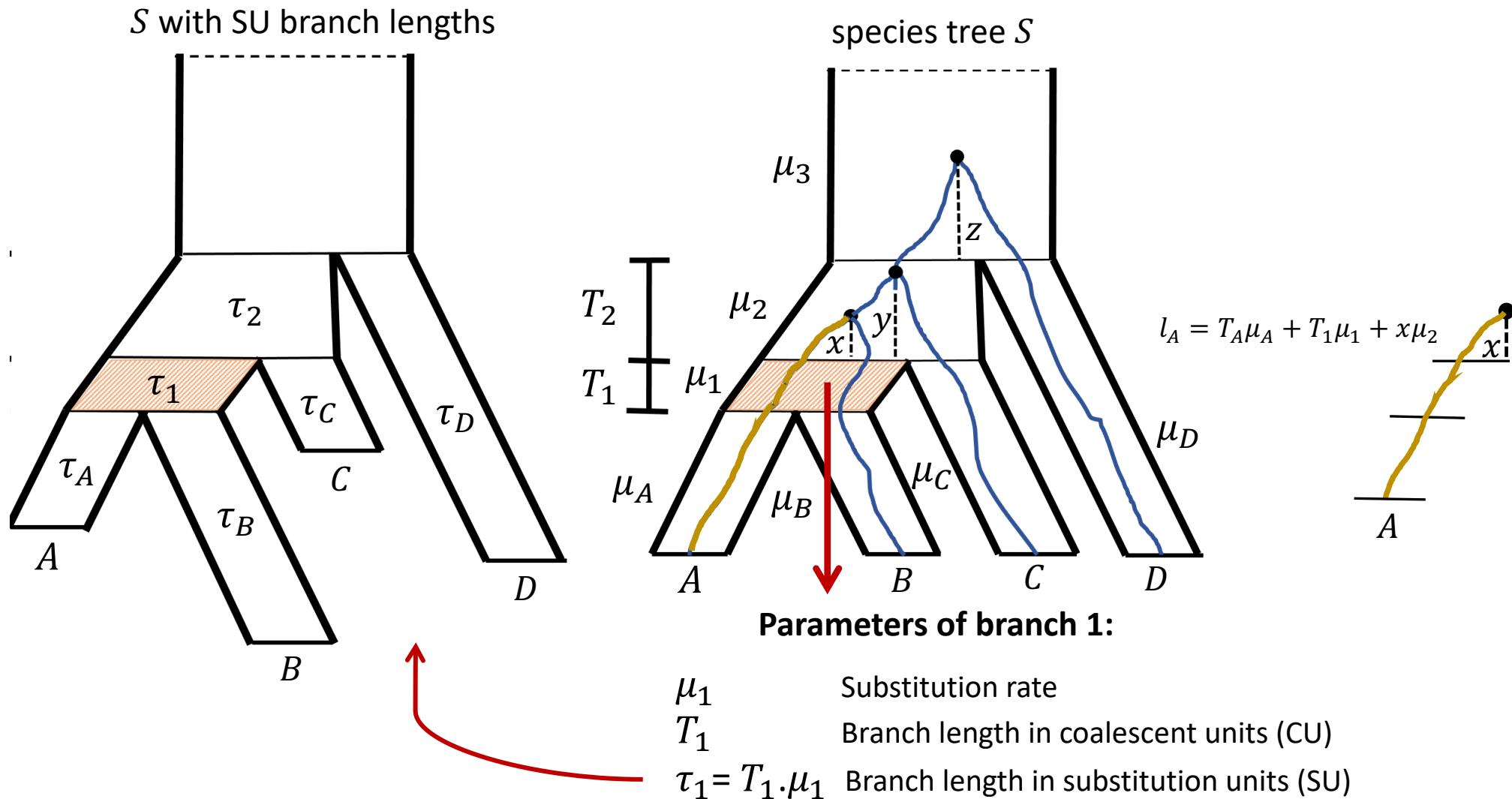
MSC+Substitution model



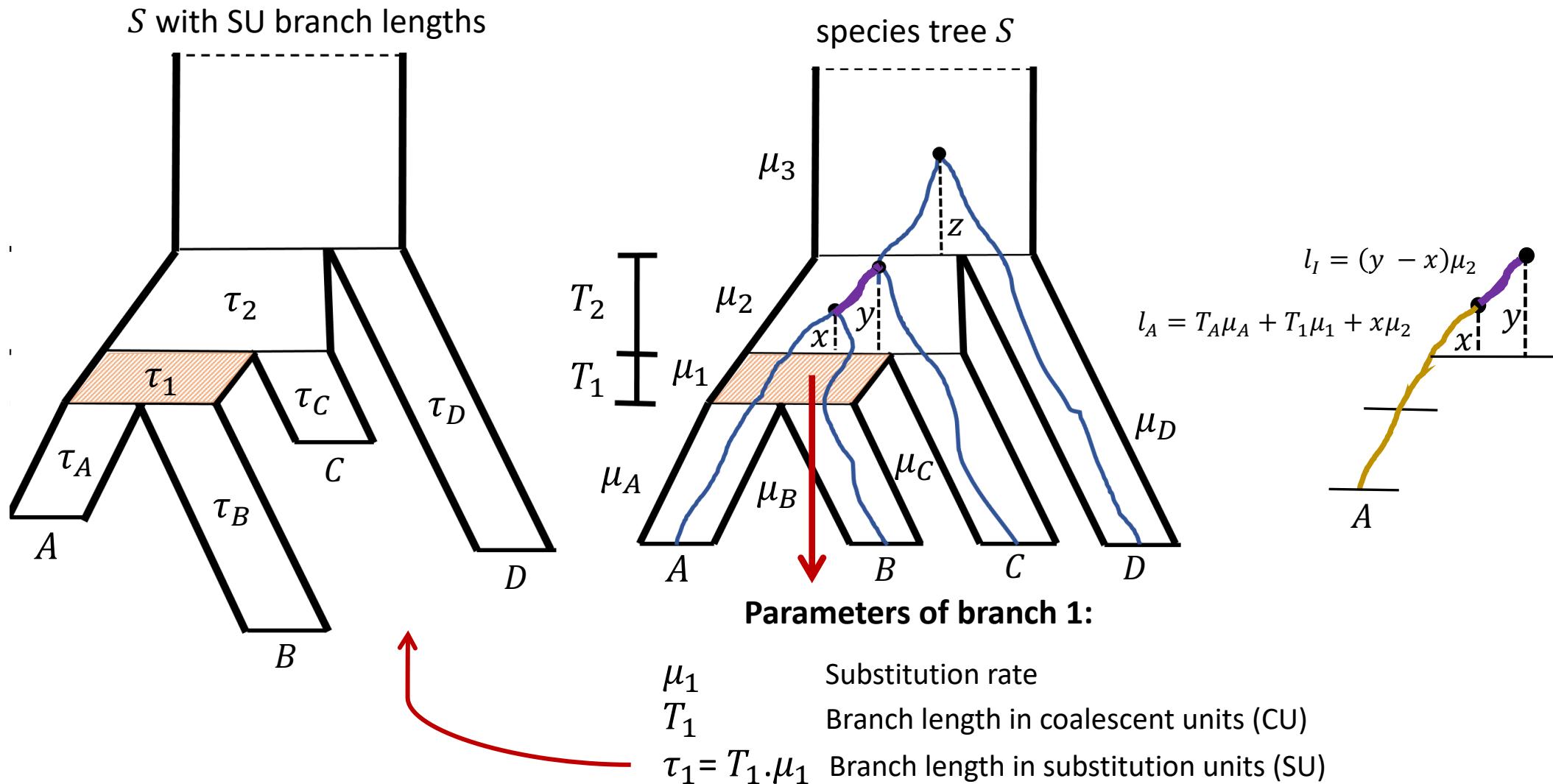
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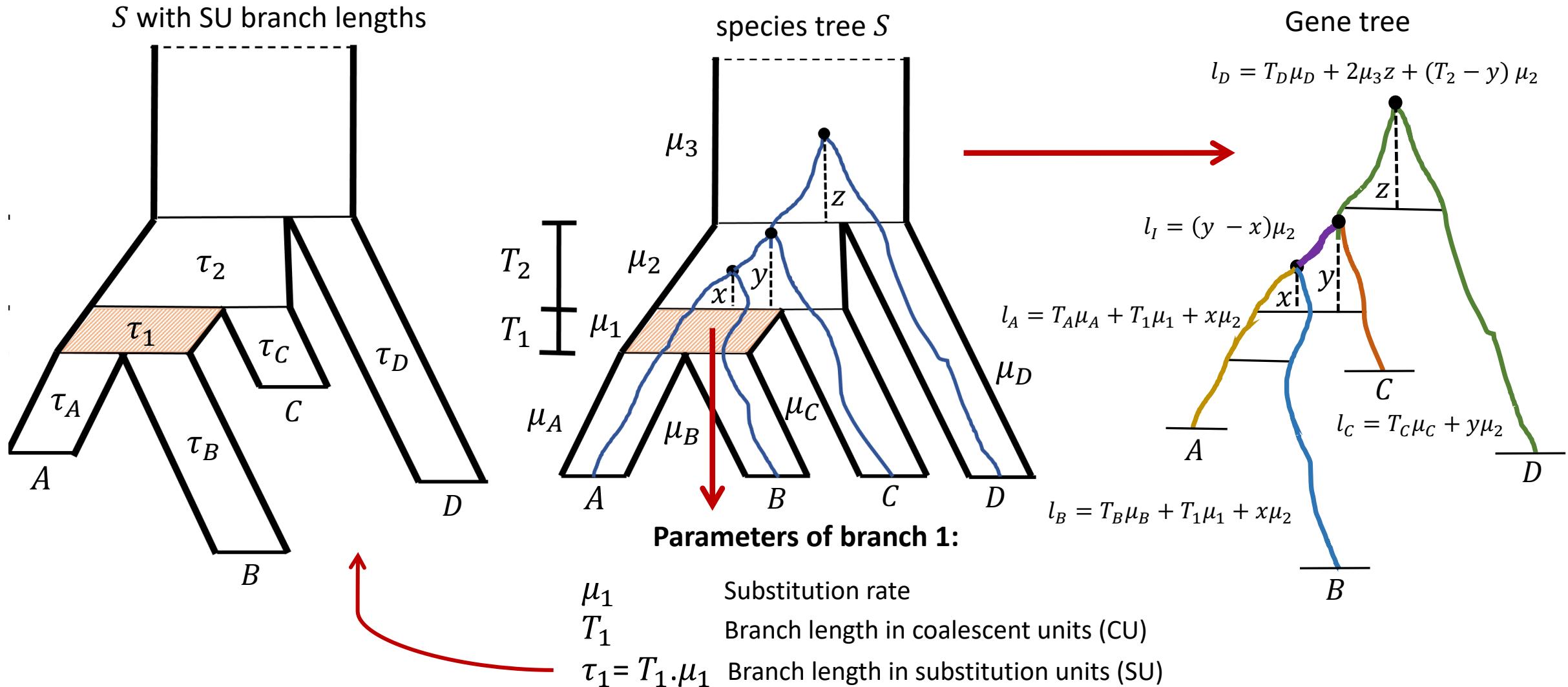
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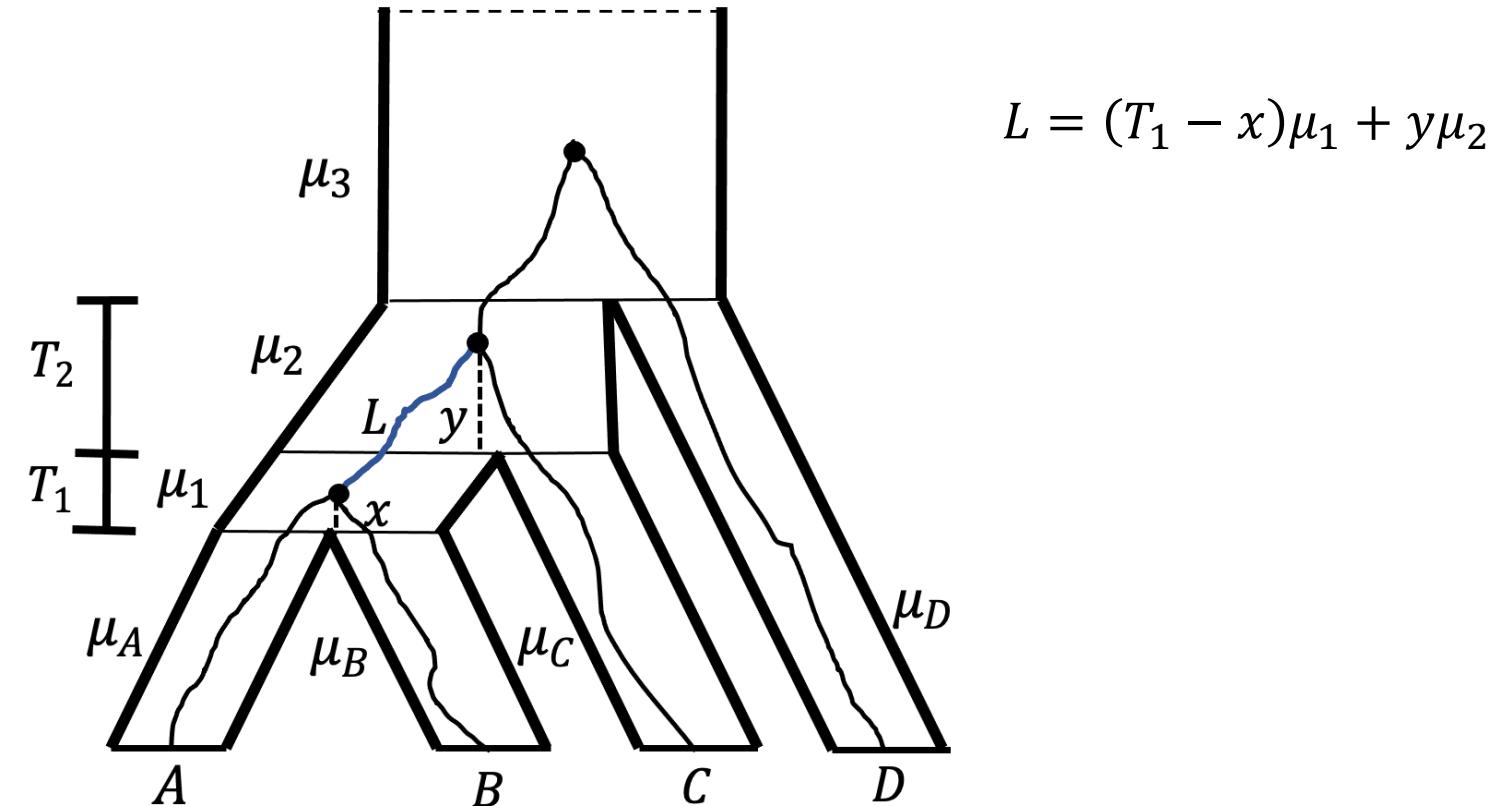
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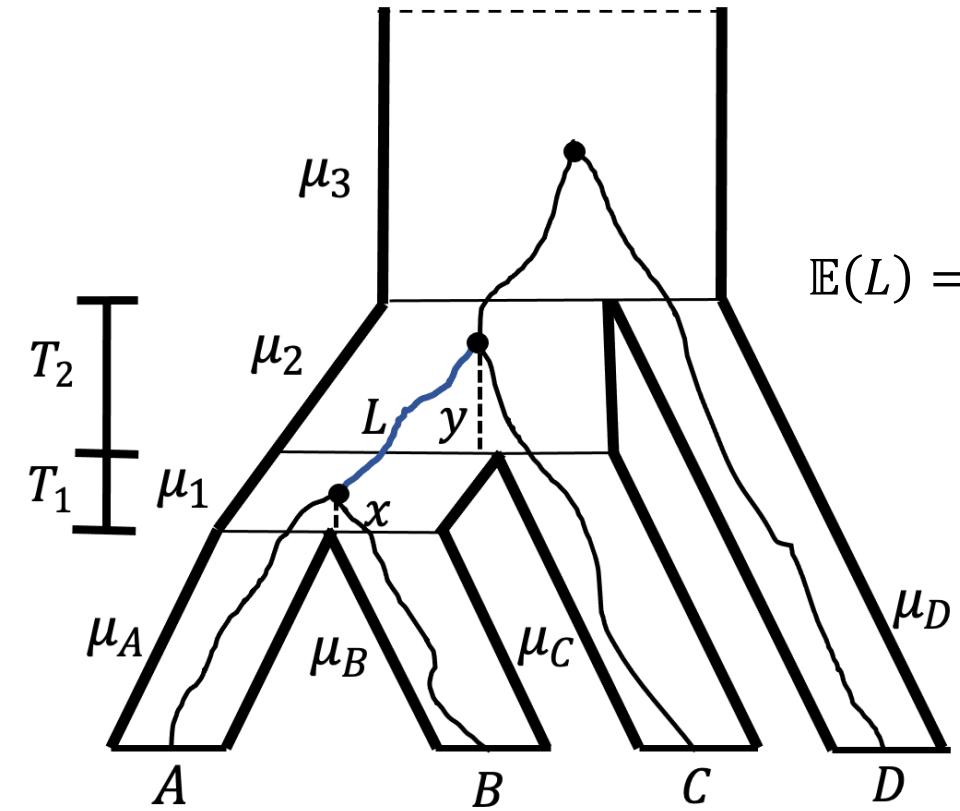
MSC+Substitution model



Expected quartet branch lengths under MSC



Expected quartet branch lengths under MSC



$$L = (T_1 - x)\mu_1 + y\mu_2$$

$$\mathbb{E}(L) = \int_0^{T_1} \int_0^{T_2} e^{-x} e^{-y} ((T_1 - x)\mu_1 + y\mu_2) dy dx$$

↑

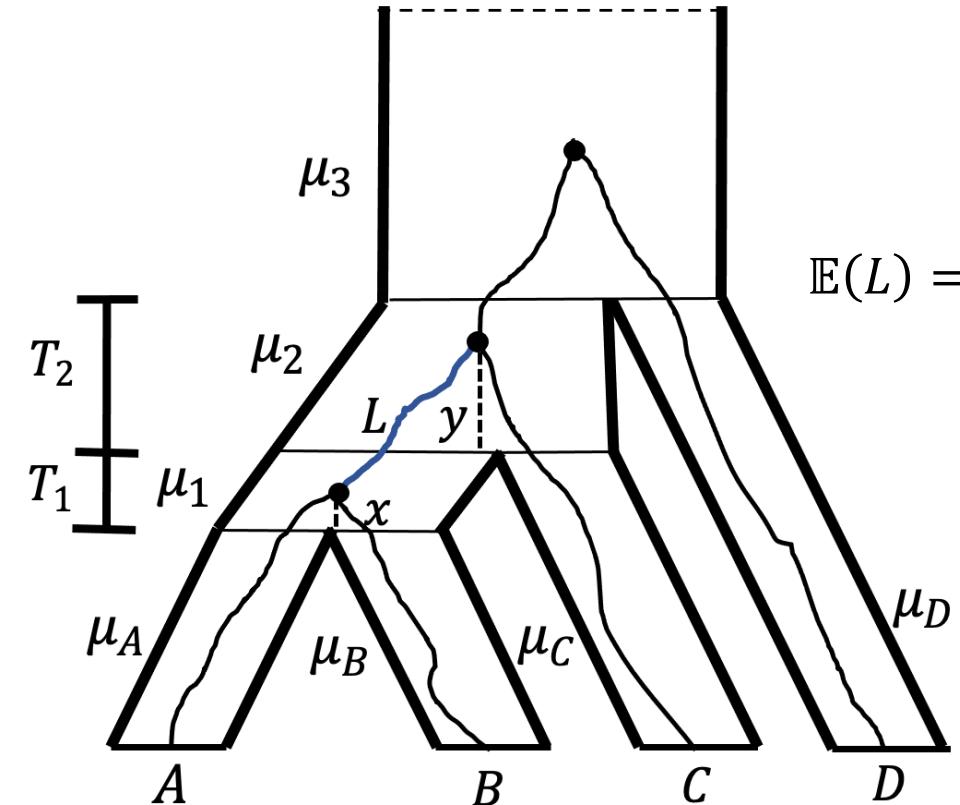
$k = 2$ lineages not coalescing
in an interval with length x

- Under MSC, waiting times before coalescent events are **exponential** random variables with rate $\lambda = \binom{k}{2}$ where k is the number of lineages entering an interval



$$f_X(x) = \binom{k}{2} e^{-(\binom{k}{2})x}$$

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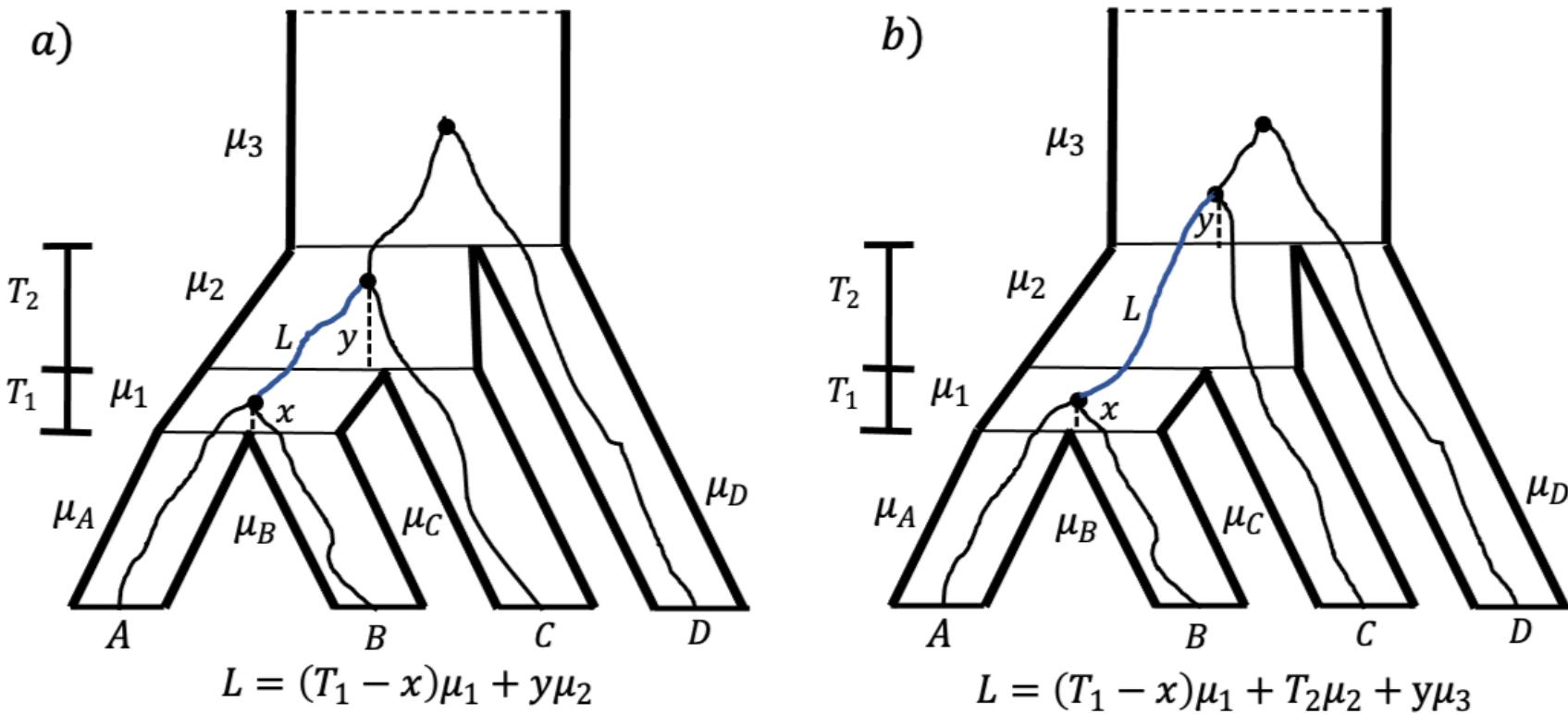
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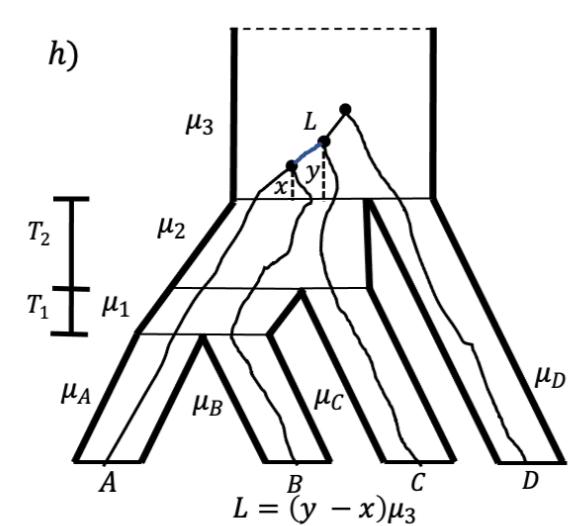
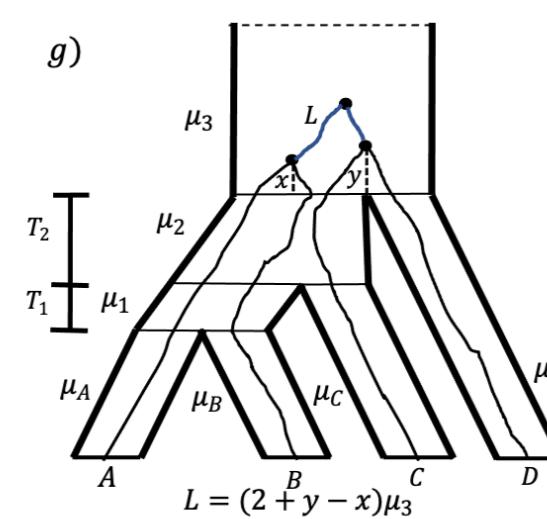
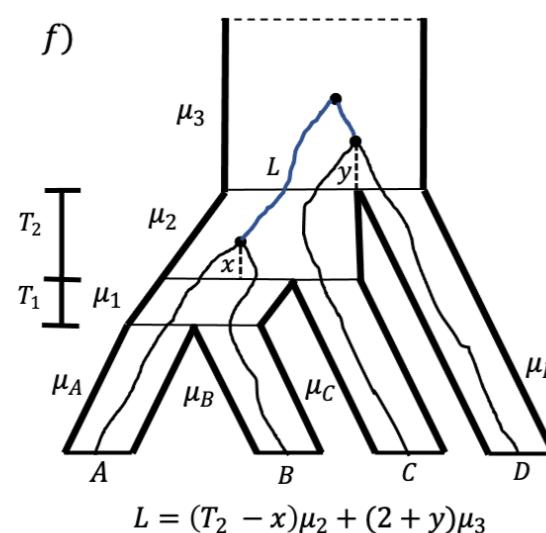
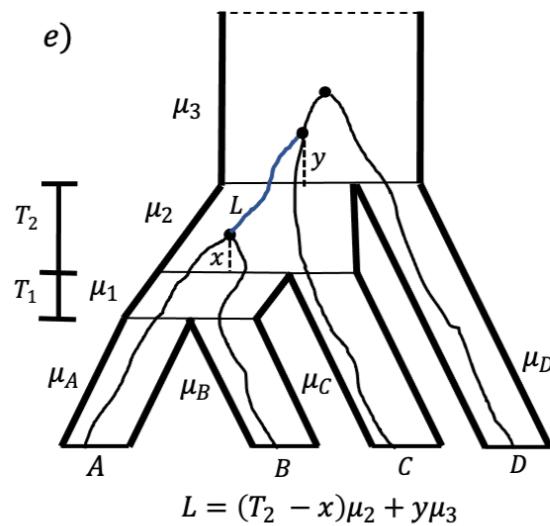
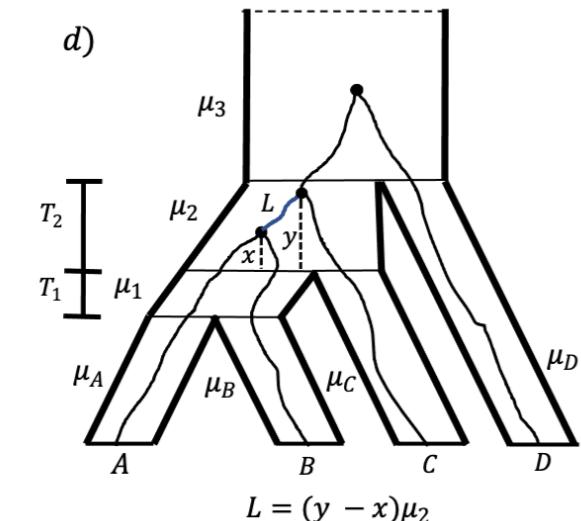
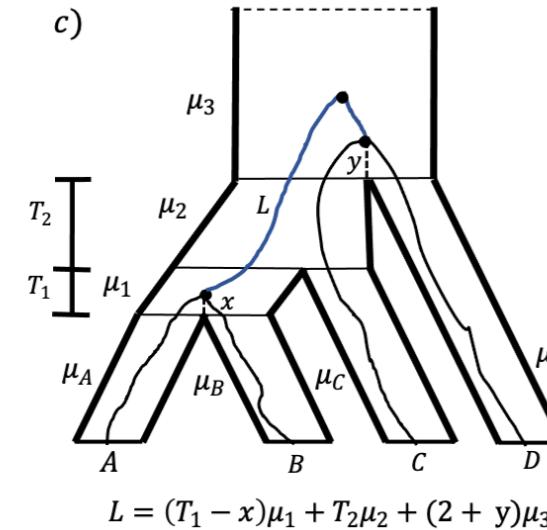
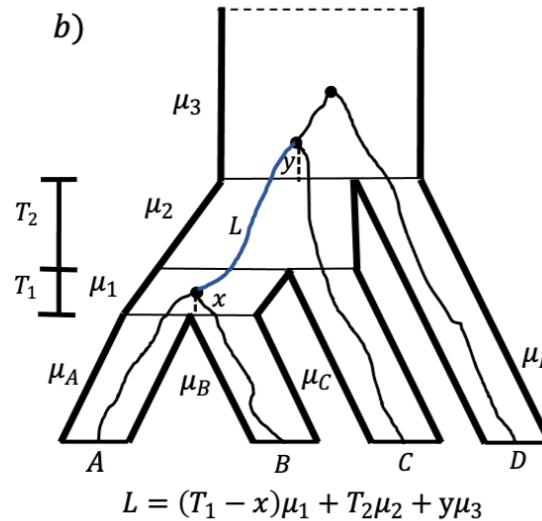
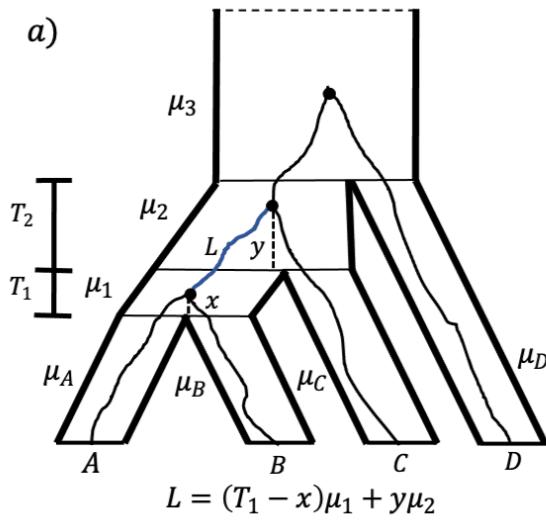
What about other patterns of coalescence?

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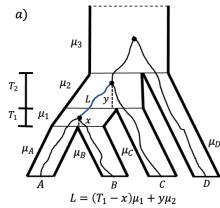


different patterns → different expected lengths

Scenarios for gene tree matching the unbalanced species tree



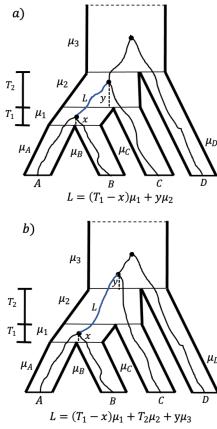
Expected quartet branch lengths under MSC



$$L_I = \left(\int_0^{T_1} \int_0^{T_2} e^{-x} e^{-y} ((T_1 - x)\mu_1 + y\mu_2) dy dx \right)$$

scenario (a)

Expected quartet branch lengths under MSC



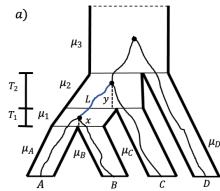
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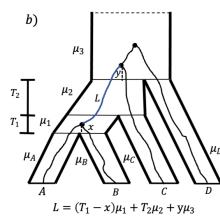
$$+ 2e^{-T_2} \int_0^{T_1} \int_0^{\infty} e^{-x} e^{-3y} ((T_1 - x)\mu_1 + T_2\mu_2 + y\mu_3) dy dx$$

scenario (b)

Expected quartet branch lengths under MSC

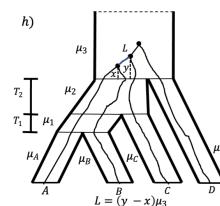


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⋮
⋮



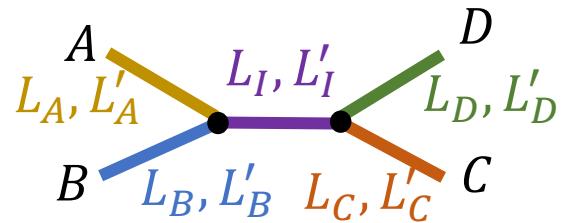
$$+ 4e^{-T_1} e^{-3T_2} \int_0^{\infty} \int_x^{\infty} e^{-6x} e^{-3(y-x)} (y - x)\mu_3 dy dx / (1 - \frac{2}{3}e^{-T_1}) \text{ scenario (h)}$$

Expected value of [internal](#)
branch length conditioned
on gene tree [matching](#)
the species tree

$$= \boxed{\frac{(e^{-3T_2} + 3e^{-T_2} - 6e^{T_1-T_2})(\mu_2 - \mu_3) + 6(1 - e^{T_1} + T_1 e^{T_1})\mu_1}{2(3e^{T_1} - 2)} + \mu_2}$$

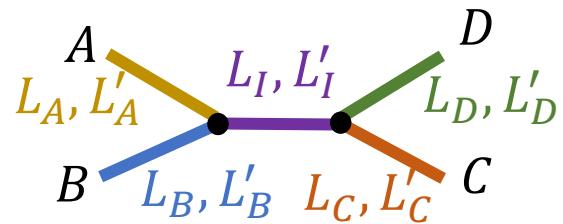
How do we infer the species tree parameters?

- We derive expected values for all branches (internal and terminal), for both matching and non-matching gene trees.
- The parameters of the species tree can be estimated from these 10 equations.



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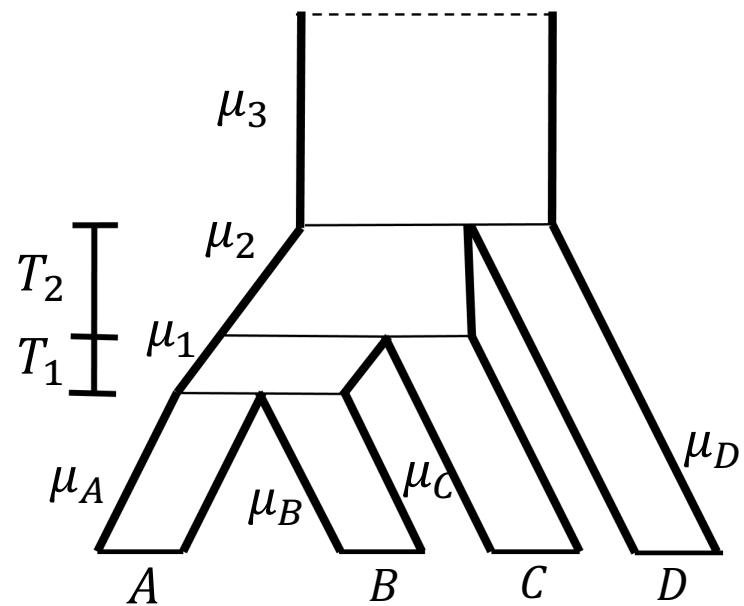


- **Challenge:** Solving these systems of equations directly can cause numerical instabilities and may not produce optimal solutions.
- We use **simplifications** that give **analytical formulas** for every branch of a quartet tree.

Simplifications (example)

Expected lengths
for internal branch

$$\left\{ \begin{array}{l} L_I = \frac{(e^{-3T_2} + 3e^{-T_2} - 6e^{T_1-T_2})(\mu_2 - \mu_3) + 6(1 - e^{T_1} + T_1 e^{T_1})\mu_1}{2(3e^{T_1} - 2)} + \mu_2 \\ L'_I = \mu_2 + \frac{1}{2}(\mu_2 - \mu_3)(e^{-3T_2} - e^{-T_2}) \end{array} \right.$$

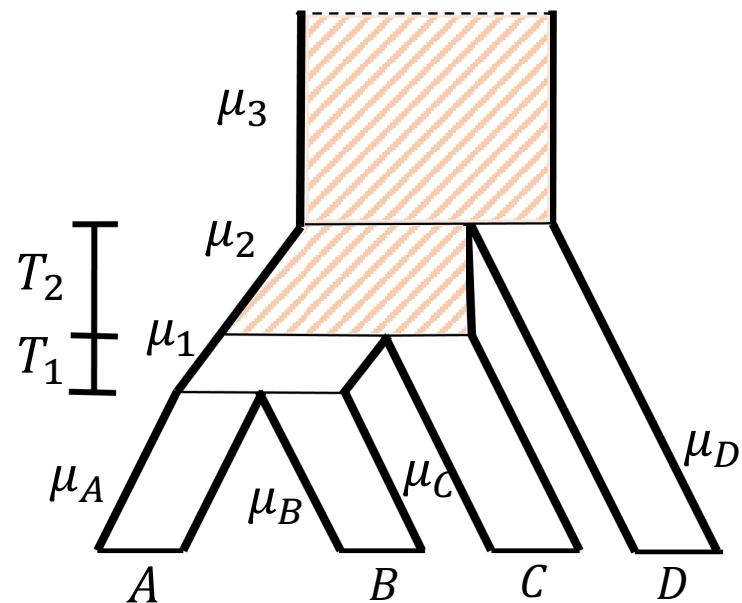


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Local clock assumption: $\mu_2 = \mu_3$



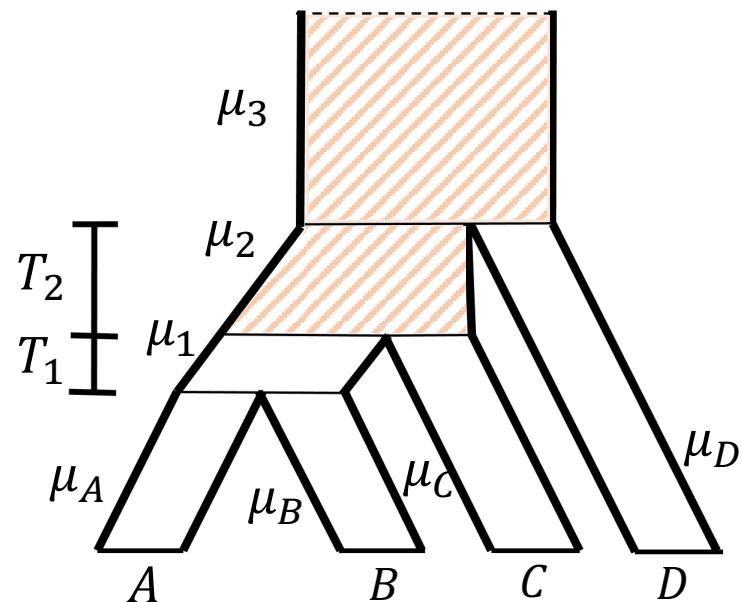
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Local clock assumption: $\mu_2 = \mu_3$

$$\left\{ \begin{array}{l} \lim_{\mu_2 \rightarrow \mu_3} L_I = \frac{3\mu_1(e^{-T_1} - 1 + T_1)}{3 - 2e^{-T_1}} + \mu_2 \\ \lim_{\mu_2 \rightarrow \mu_3} L'_I = \mu_2 \end{array} \right.$$



Summary of SU branch length formulas

Unbalanced

Parameter	Estimation formula	Simplifying assumption(s)
t_1	$\hat{t}_1 = \bar{L}'_I \left(\frac{1}{2} \bar{\delta} + \frac{1}{6} \sqrt{3\bar{\delta}(3\bar{\delta} + 4)} \right)$	$\mu_3 \rightarrow \mu_2; \mu_1 \rightarrow \mu_2$
t_A	$\hat{t}_A = \bar{L}'_A + \frac{\mu_1(e^{-T_1} - 1 + T_1) + \bar{\Delta}_A(1 - 2/3e^{-T_1})}{1 - 4/5e^{-T_1}} - T_1\mu_1$	$T_2 \rightarrow \infty$
t_B	$\hat{t}_B = \bar{L}'_B + \frac{\mu_1(e^{-T_1} - 1 + T_1) + \bar{\Delta}_B(1 - 2/3e^{-T_1})}{1 - 4/5e^{-T_1}} - T_1\mu_1$	$T_2 \rightarrow \infty$
t_C	$\hat{t}_C = \bar{L}'_C - \frac{1}{3}(2 - \frac{1}{2-e^{-T_1}})\bar{\Delta}_C$	$T_2 \rightarrow \infty$
$t_2 + t_D$	$\hat{t}_2 + \hat{t}_D = \bar{L}'_D - \frac{2}{3}(2 + \frac{1}{1-e^{-T_1}})\bar{\Delta}_D$	$\mu_3 \rightarrow \mu_2$

Balanced

$t_1 + t_2$	$\hat{t}_1 + \hat{t}_2 = \bar{L}'_I \left(\frac{1}{2} \bar{\delta} + \frac{1}{6} \sqrt{3\bar{\delta}(3\bar{\delta} + 4)} \right)$	$T_2 \rightarrow 0; \mu_1 \rightarrow \mu_3$
t_A	$\hat{t}_A = \bar{L}'_A - \frac{2}{3}\mu_1 - \frac{1}{3} \left\{ \mu_1 \left(1 - e^{-(T_1+T_2)} \right) - \bar{\Delta}_A \left(3 - 2e^{-(T_1+T_2)} \right) \right\}$	$\mu_3 \rightarrow \mu_1$
t_B	$\hat{t}_B = \bar{L}'_B - \frac{2}{3}\mu_1 - \frac{1}{3} \left\{ \mu_1 \left(1 - e^{-(T_1+T_2)} \right) - \bar{\Delta}_B \left(3 - 2e^{-(T_1+T_2)} \right) \right\}$	$\mu_3 \rightarrow \mu_1$
t_C	$\hat{t}_C = \bar{L}'_C - \frac{2}{3}\mu_2 - \frac{1}{3} \left\{ \mu_2 \left(1 - e^{-(T_1+T_2)} \right) - \bar{\Delta}_C \left(3 - 2e^{-(T_1+T_2)} \right) \right\}$	$\mu_3 \rightarrow \mu_2$
t_D	$\hat{t}_D = \bar{L}'_D - \frac{2}{3}\mu_2 - \frac{1}{3} \left\{ \mu_2 \left(1 - e^{-(T_1+T_2)} \right) - \bar{\Delta}_D \left(3 - 2e^{-(T_1+T_2)} \right) \right\}$	$\mu_3 \rightarrow \mu_2$



CASTLES

Coalescent-Aware Species Tree Length Estimation in Substitution-units

Input:

- Rooted species tree *topology S*
- A set of gene trees \mathcal{G} with SU branch lengths

Output:

- Species tree *S* with SU branch lengths

CASTLES

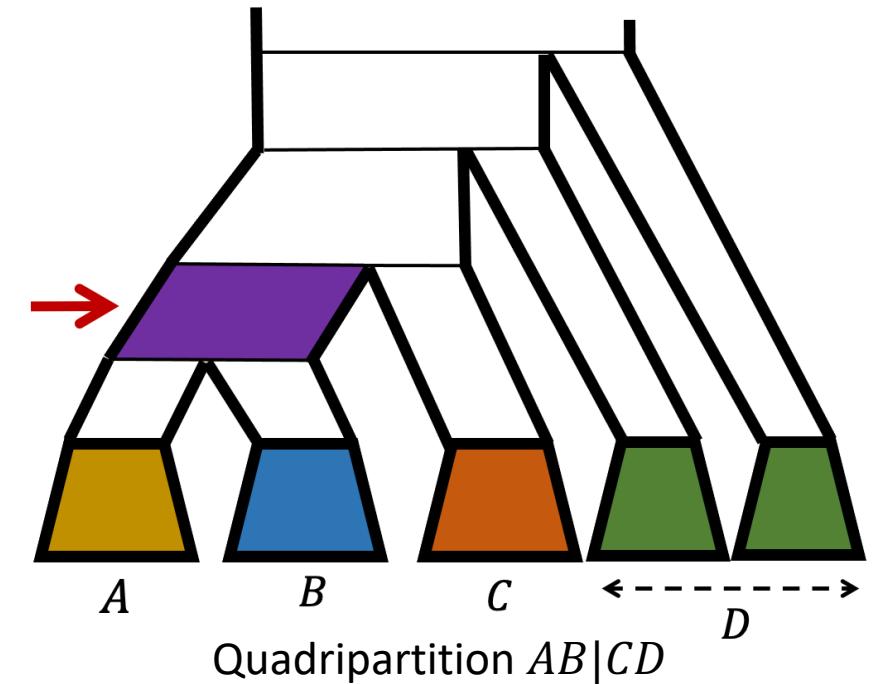
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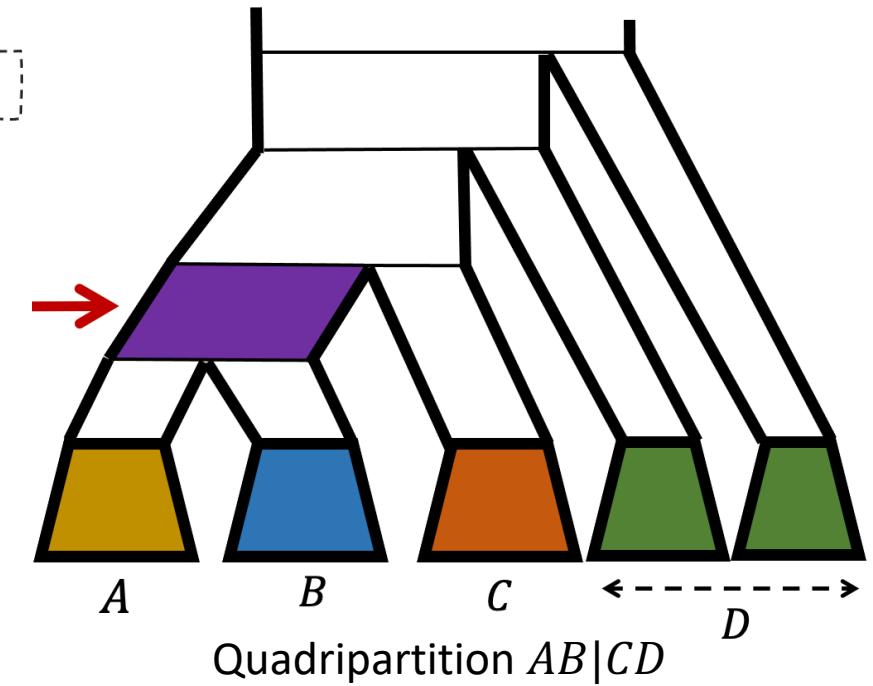
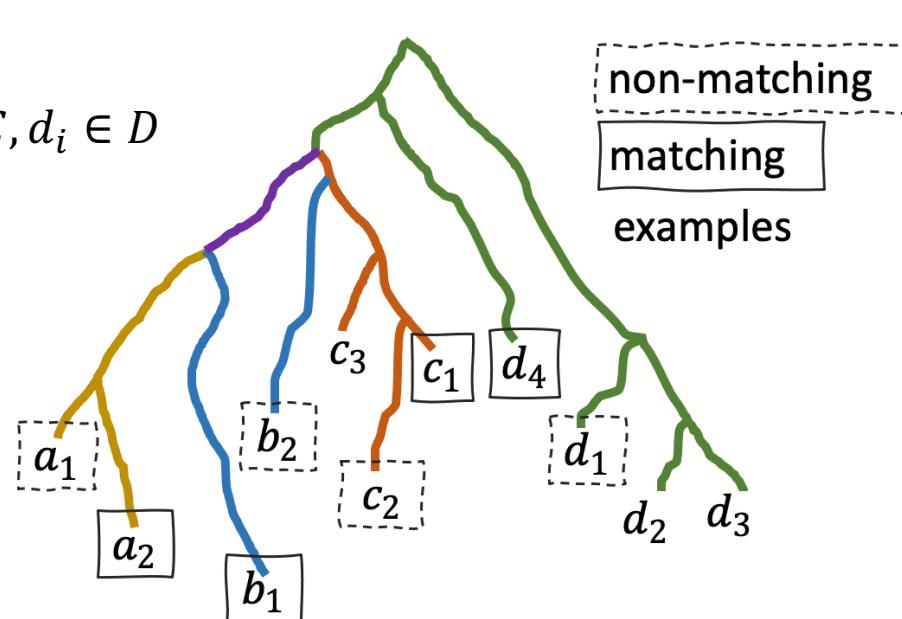
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Quartets: $a_i \in A, b_i \in B, c_i \in C, d_i \in D$



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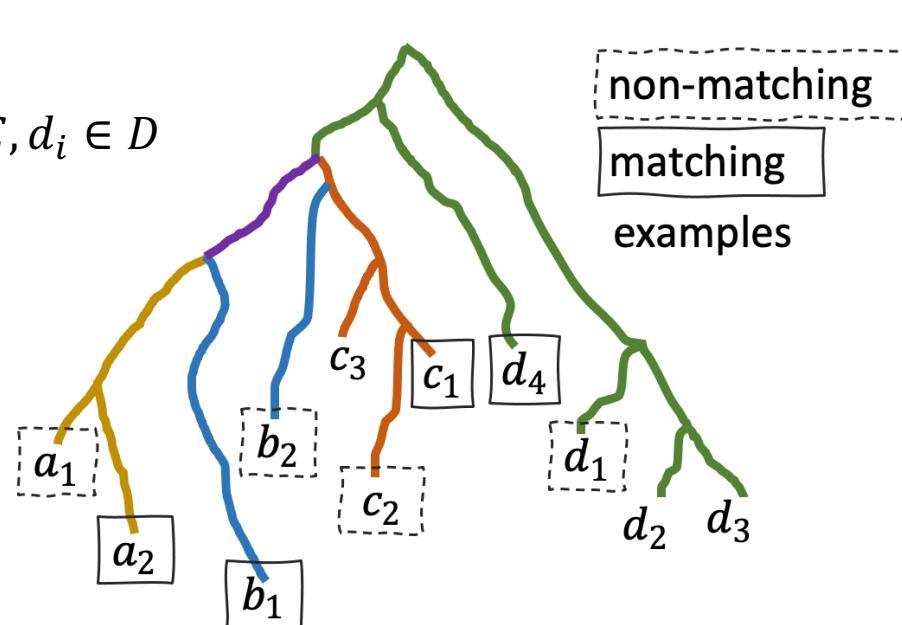
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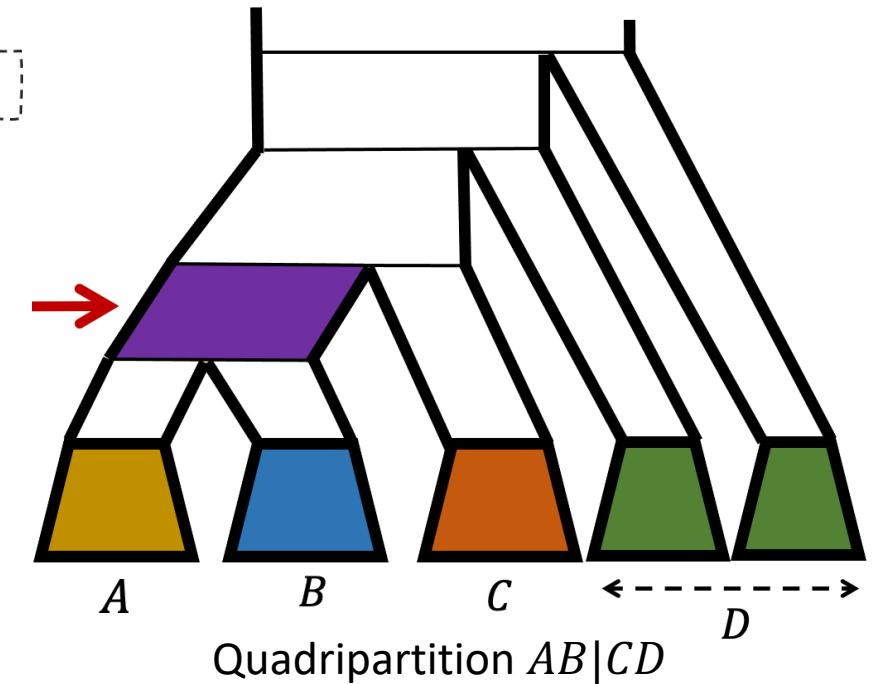
Quartets: $a_i \in A, b_i \in B, c_i \in C, d_i \in D$

- We average branch lengths over all quartets with an $O(n^2k)$ dynamic programming

n species, k genes

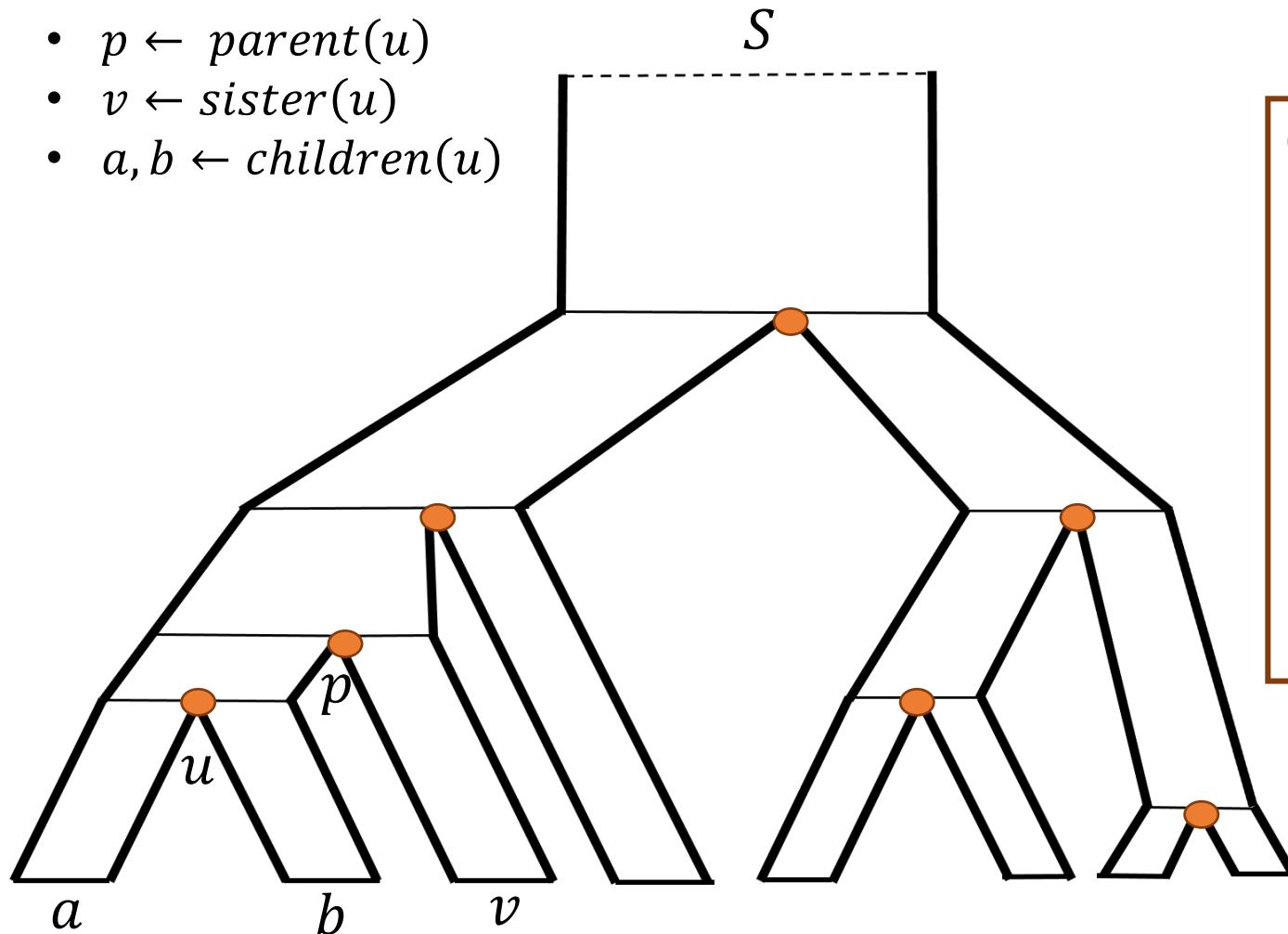


Phylogenomic branch length estimation using quartets



Large tree algorithm

- $p \leftarrow \text{parent}(u)$
- $v \leftarrow \text{sister}(u)$
- $a, b \leftarrow \text{children}(u)$

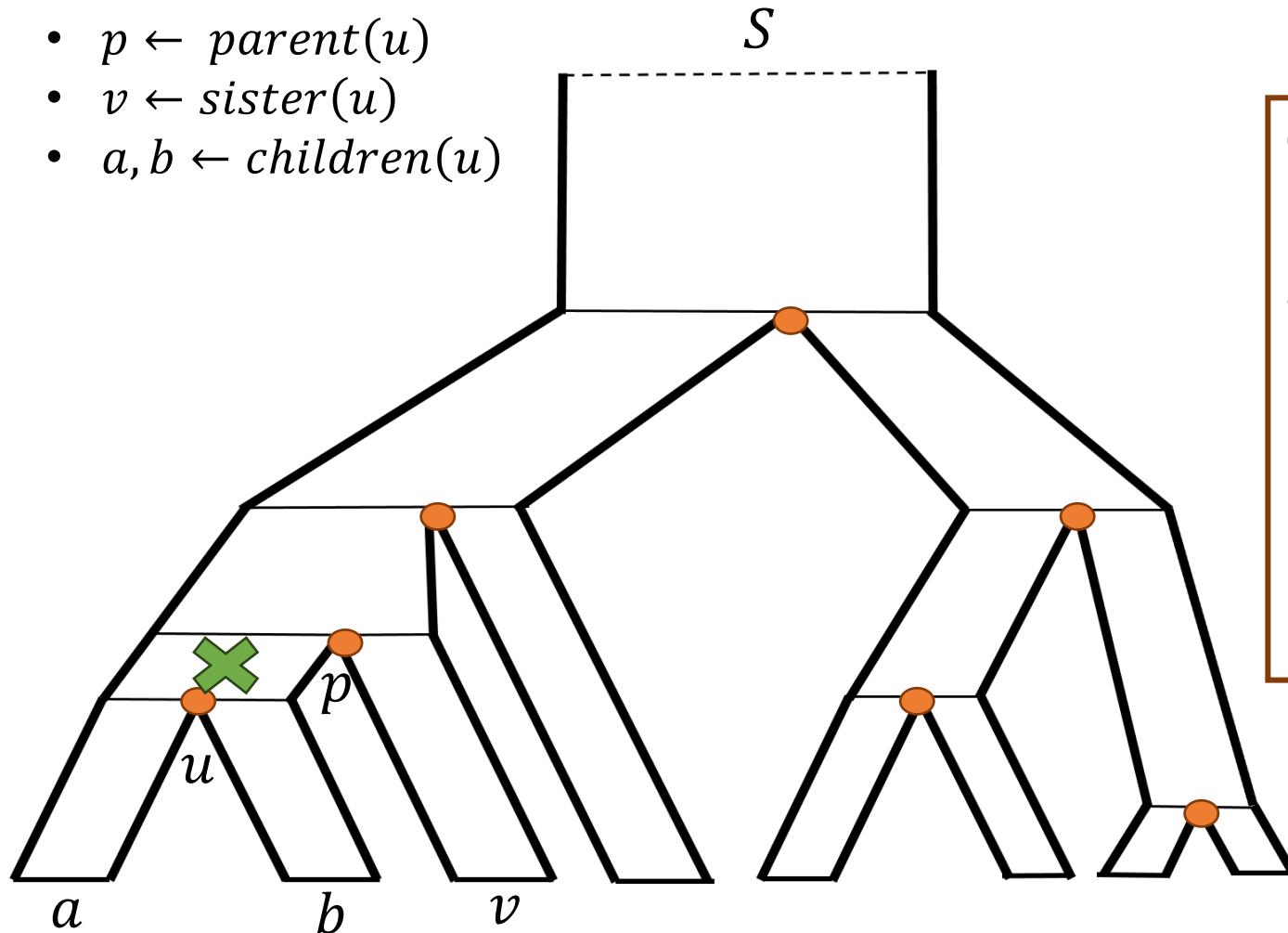


$O(n^2k)$ dynamic programming

Calculate $\overline{L_a}, \overline{L_b}, \overline{L_v}, \overline{L_p}, \overline{L'_a}, \overline{L'_b}, \overline{L'_v}, \overline{L'_p}$ for each branch

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$O(n^2k)$ dynamic programming

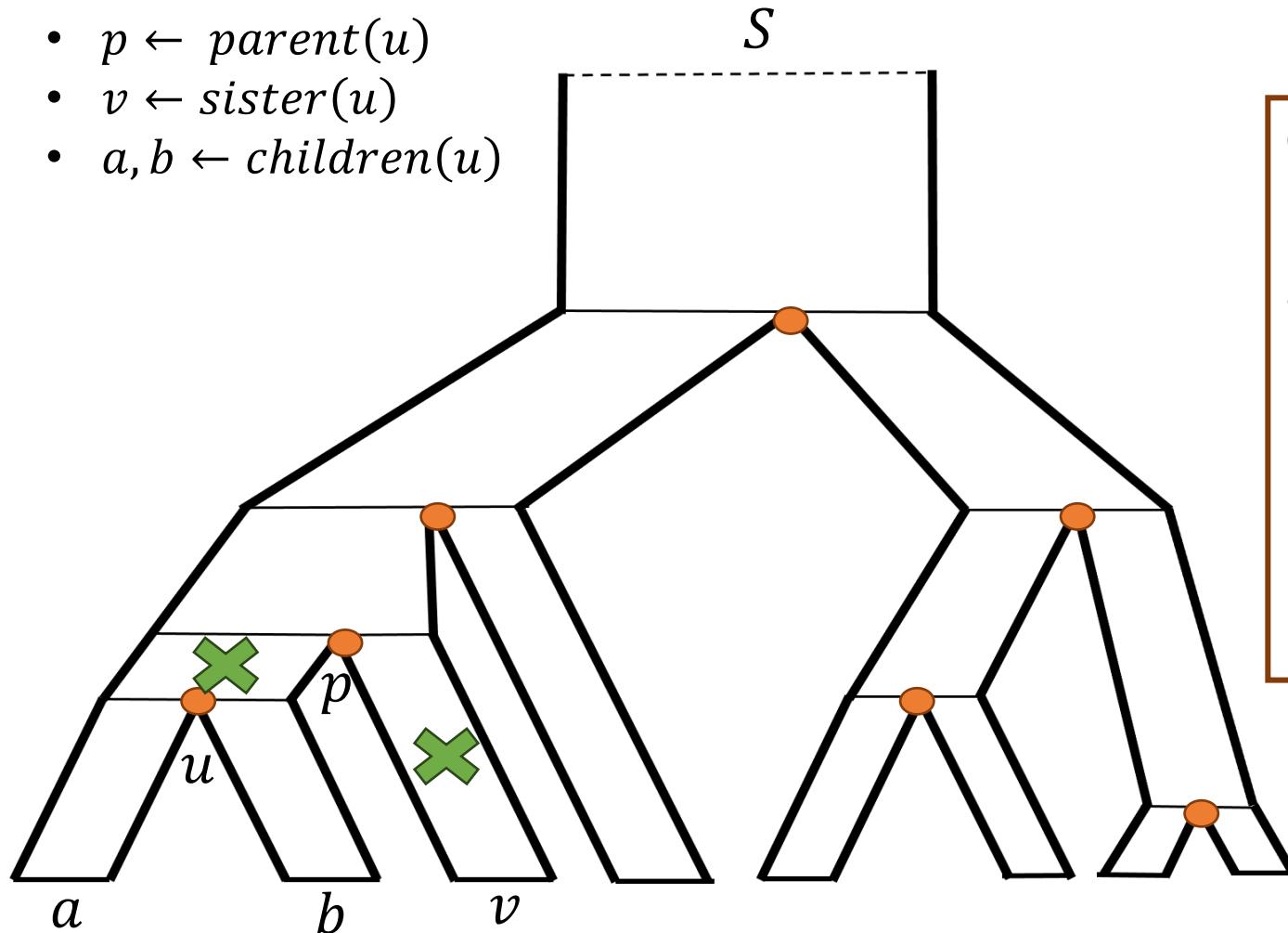
Calculate $\overline{L}_a, \overline{L}_b, \overline{L}_v, \overline{L}_p, \overline{L}'_a, \overline{L}'_b, \overline{L}'_v, \overline{L}'_p$ for each branch

For $u \in$ post order traversal of internal nodes of S :

- if p is not root:
 - $t_{p \rightarrow u} \leftarrow$ internal branch equation

Large tree algorithm

- $p \leftarrow \text{parent}(u)$
- $v \leftarrow \text{sister}(u)$
- $a, b \leftarrow \text{children}(u)$



$O(n^2k)$ dynamic programming

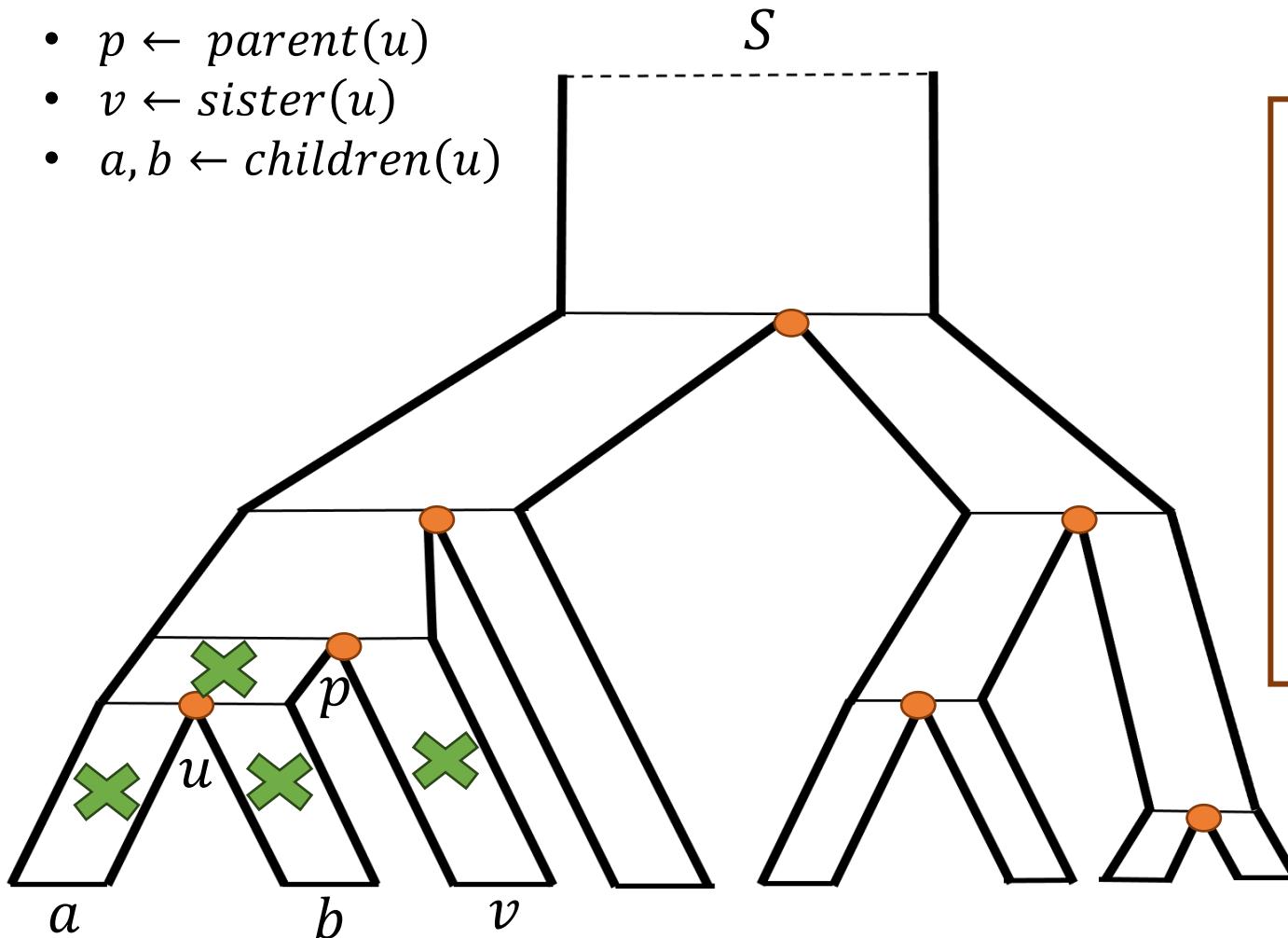
Calculate $\overline{L_a}, \overline{L_b}, \overline{L_v}, \overline{L_p}, \overline{L'_a}, \overline{L'_b}, \overline{L'_v}, \overline{L'_p}$ for each branch

For $u \in$ post order traversal of internal nodes of S :

- if p is not root:
 - $t_{p \rightarrow u} \leftarrow$ internal branch equation
 - If v is leaf:
- $t_{p \rightarrow v} \leftarrow$ terminal middle branch equation

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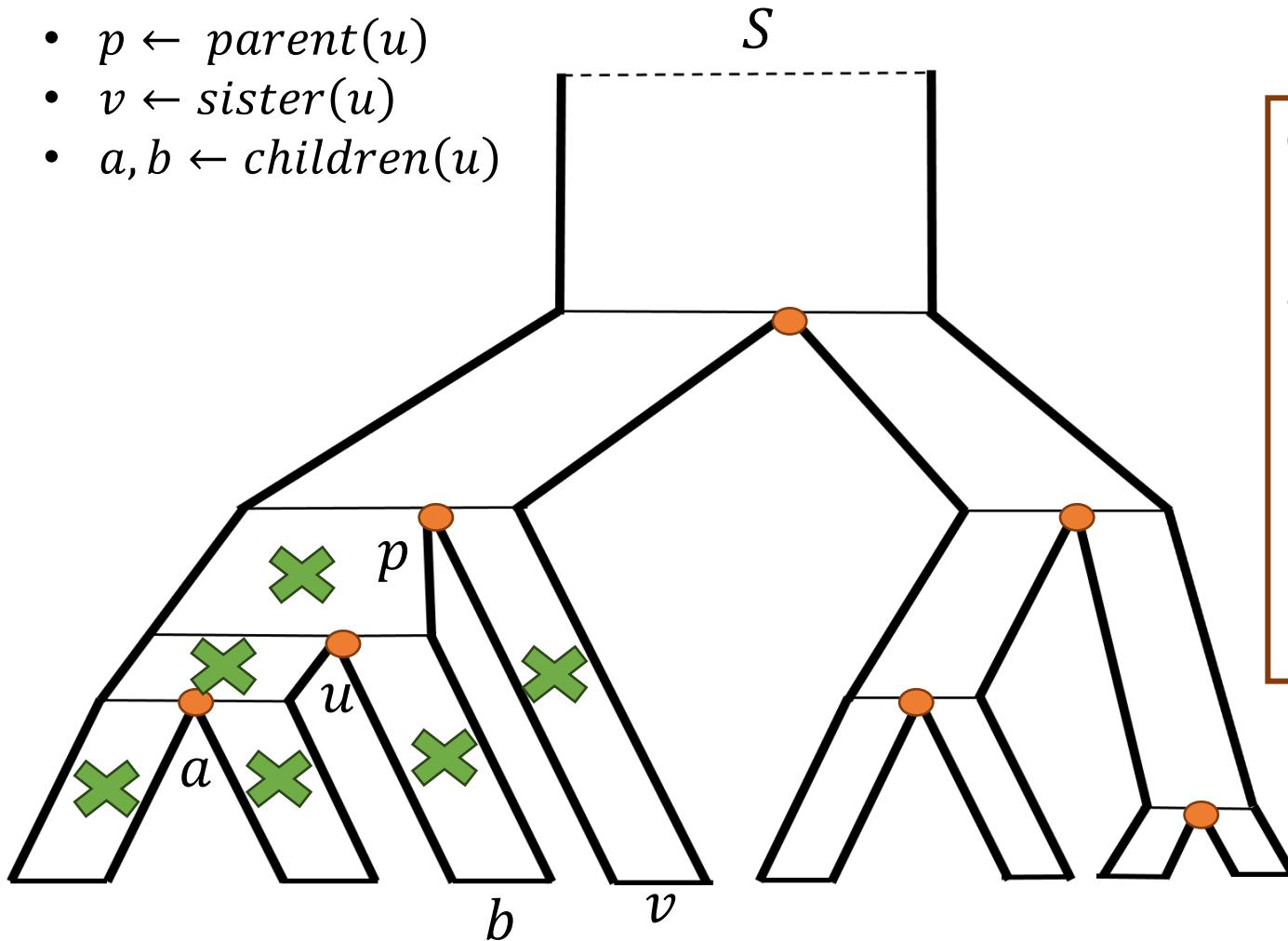
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 - $t_{u \rightarrow w} \leftarrow$ terminal cherry branch equation

Large tree algorithm

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$O(n^2k)$ dynamic programming

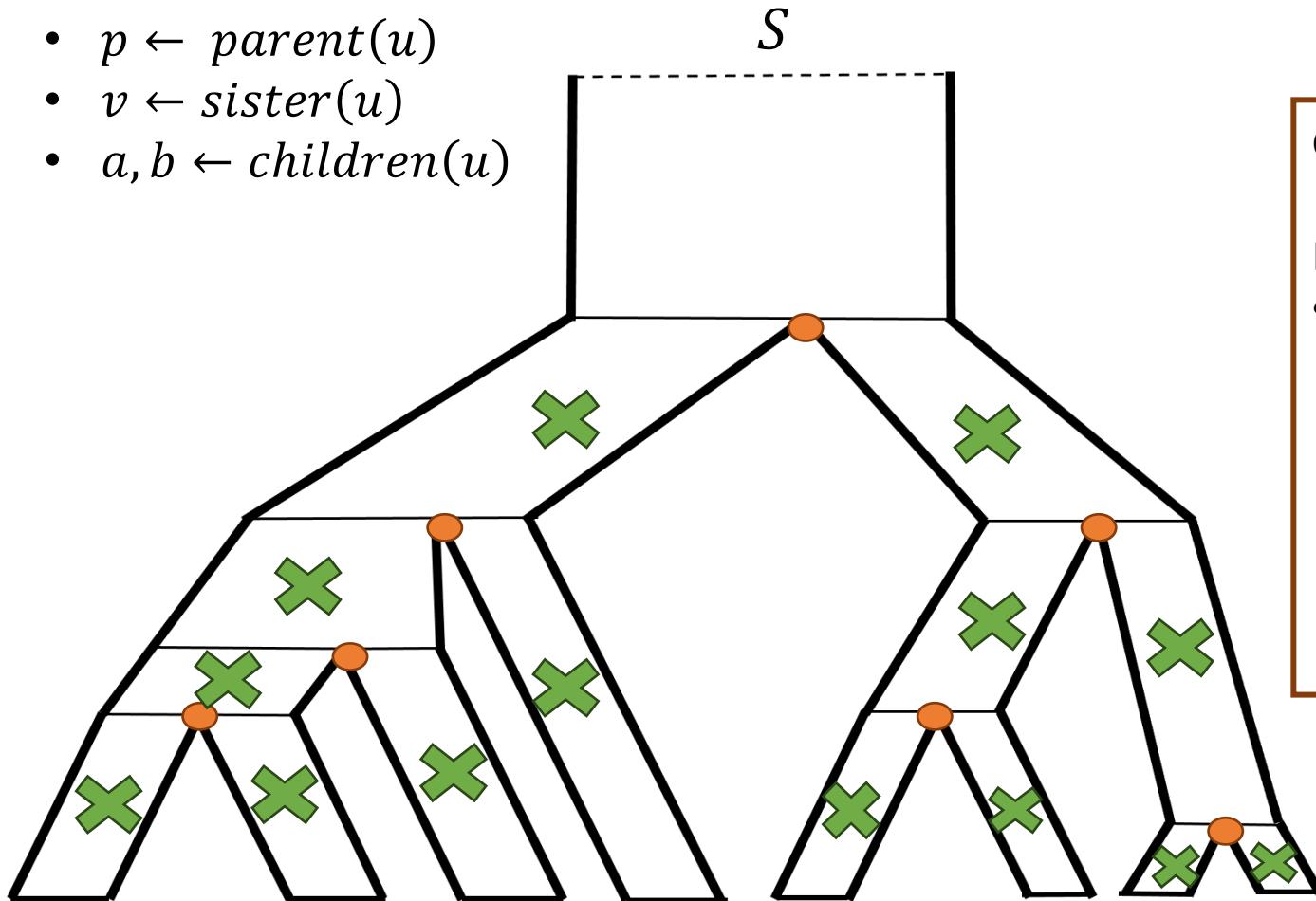
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Phylogenomic branch length estimation using quartets

$O(n^2k)$ dynamic programming

Calculate $\overline{L}_a, \overline{L}_b, \overline{L}_v, \overline{L}_p, \overline{L}'_a, \overline{L}'_b, \overline{L}'_v, \overline{L}'_p$ for each branch

For $u \in$ post order traversal of internal nodes of S :

- if p is not root:
 - $t_{p \rightarrow u} \leftarrow$ internal branch equation
 - If v is leaf:
 $t_{p \rightarrow v} \leftarrow$ terminal middle branch equation
- For $w \in \text{children}(u)$:
 - If w is leaf:
 $t_{u \rightarrow w} \leftarrow$ terminal cherry branch equation

Total runtime: $O(n^2k)$

in practice: ~ 50 s on 100-taxon tree with 1000 genes

Summary of results so far

- We derived expected branch lengths for matching/non-matching gene trees for an unbalanced/balanced quartet species tree under MSC+Substitution model.
- We presented simplifications that lead to analytical formulas for each branch in the species tree.
- We introduced CASTLES that uses these formulas to estimate branch lengths on a species tree in $O(n^2k)$.

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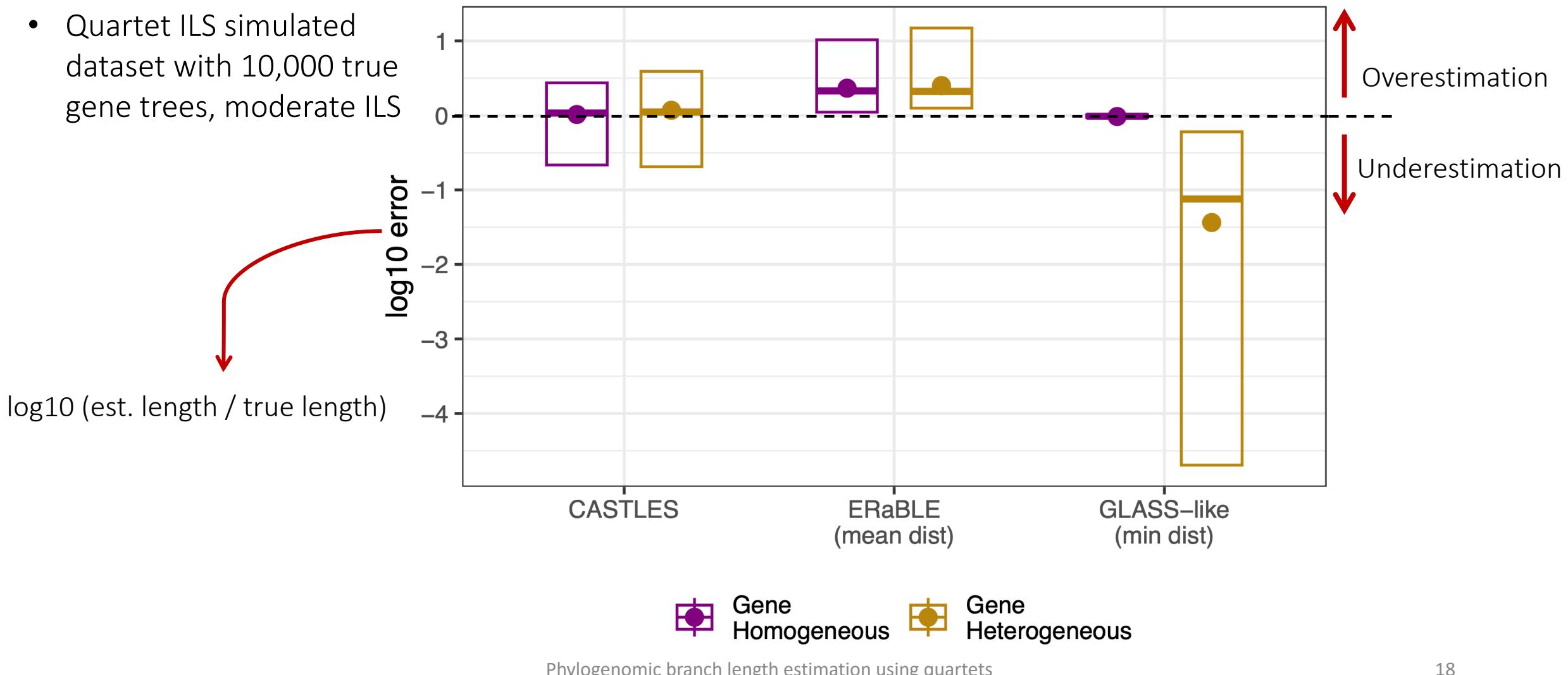
How well does it work in practice?

Experimental study

- Modified SimPhy [Mallo et al., 2016] to generate species trees with SU lengths
- Estimating branch lengths on the *true* species tree topology
- Three ILS simulated datasets and a mammalian biological dataset
- Evaluating using bias, absolute error, RMSE, and log error
- **Methods:** Concatenation with RAxML [Stamatakis, 2014], FastME [Lefort et al, 2015] with minimum and average distance matrices and ERaBLE [Binet et al., 2016]

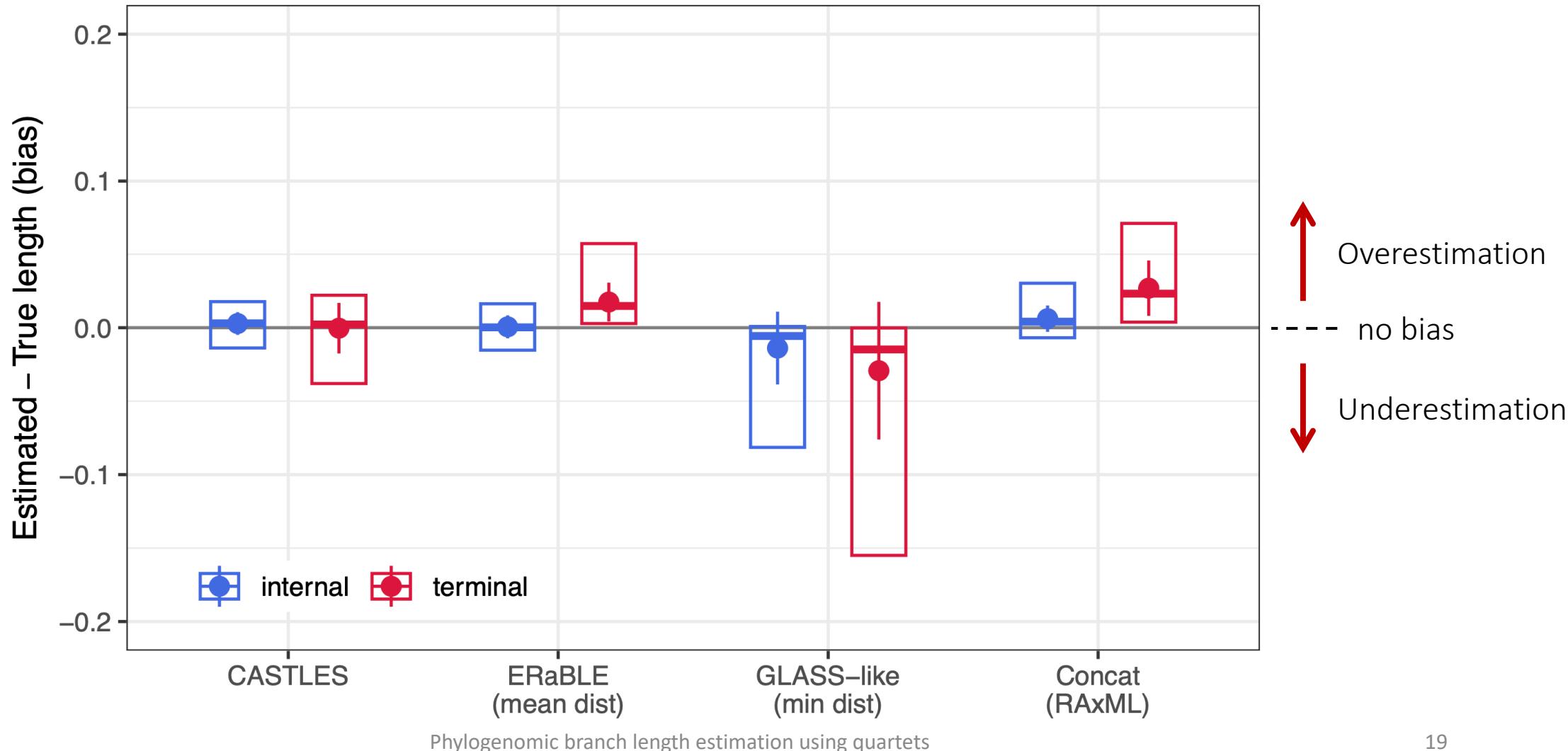
CASTLES is robust to rate heterogeneity across genes

- Quartet ILS simulated dataset with 10,000 true gene trees, moderate ILS



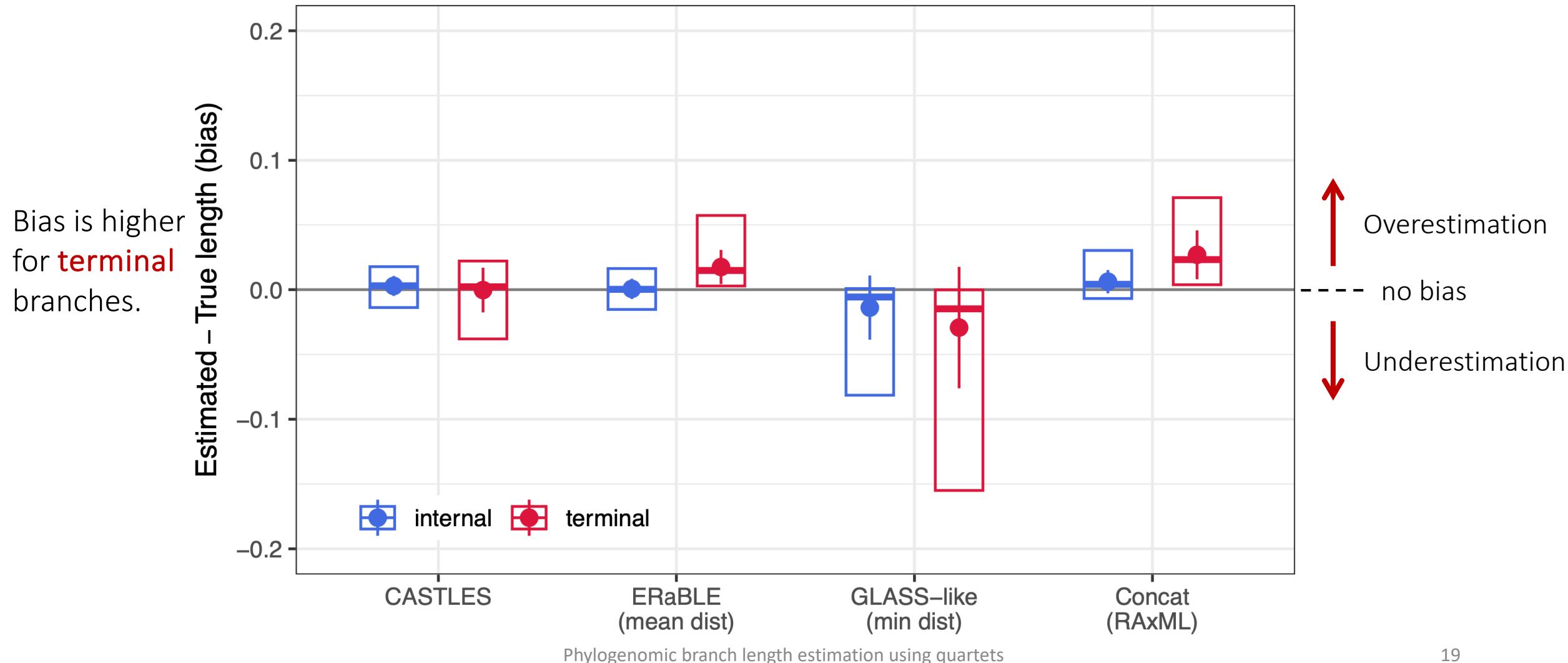
CASTLES is less biased than other methods

- 100-taxon ILS simulated dataset with 1000 genes, moderate ILS, 200bp sequence length [Zhang et al (2018)]



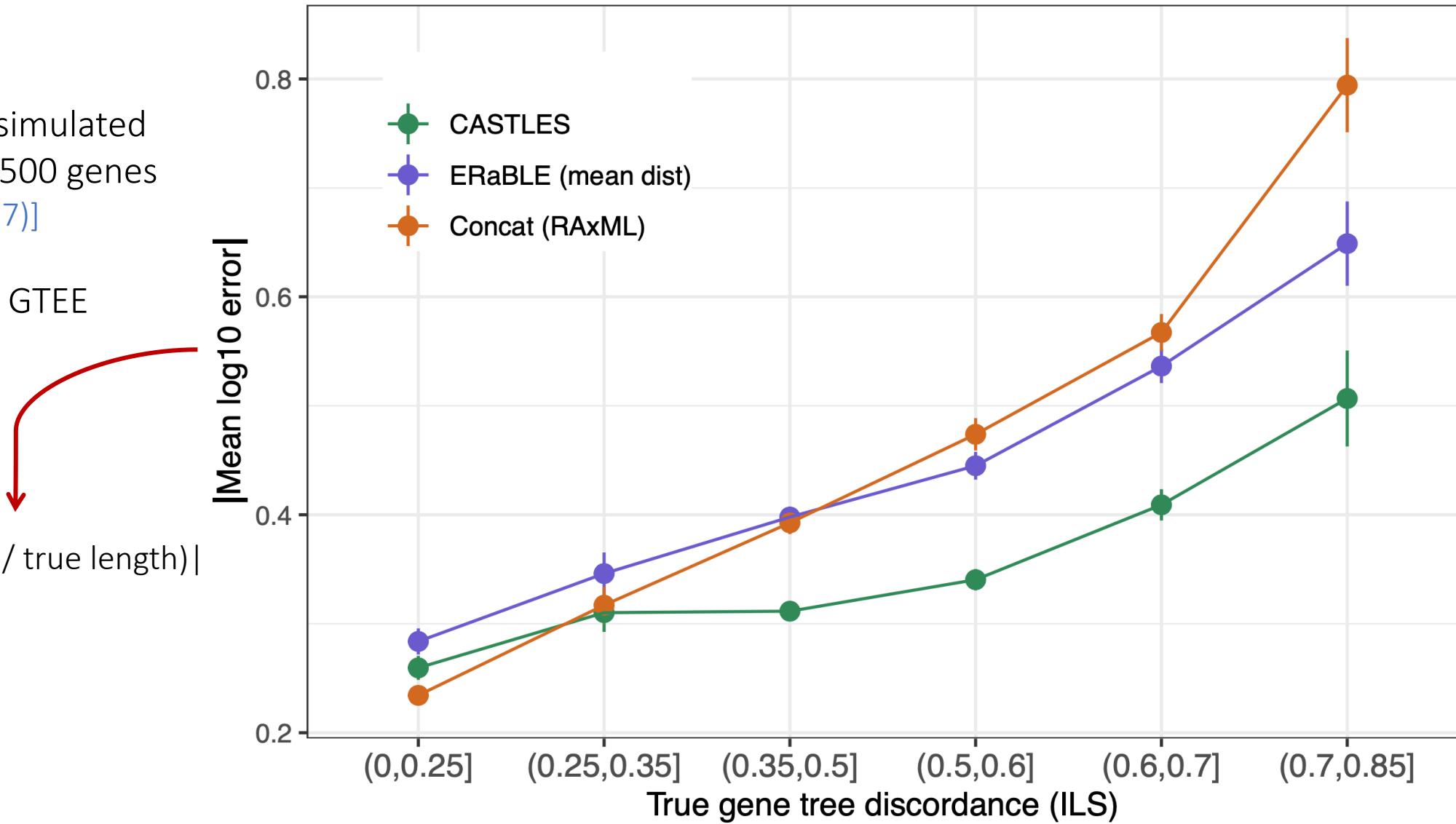
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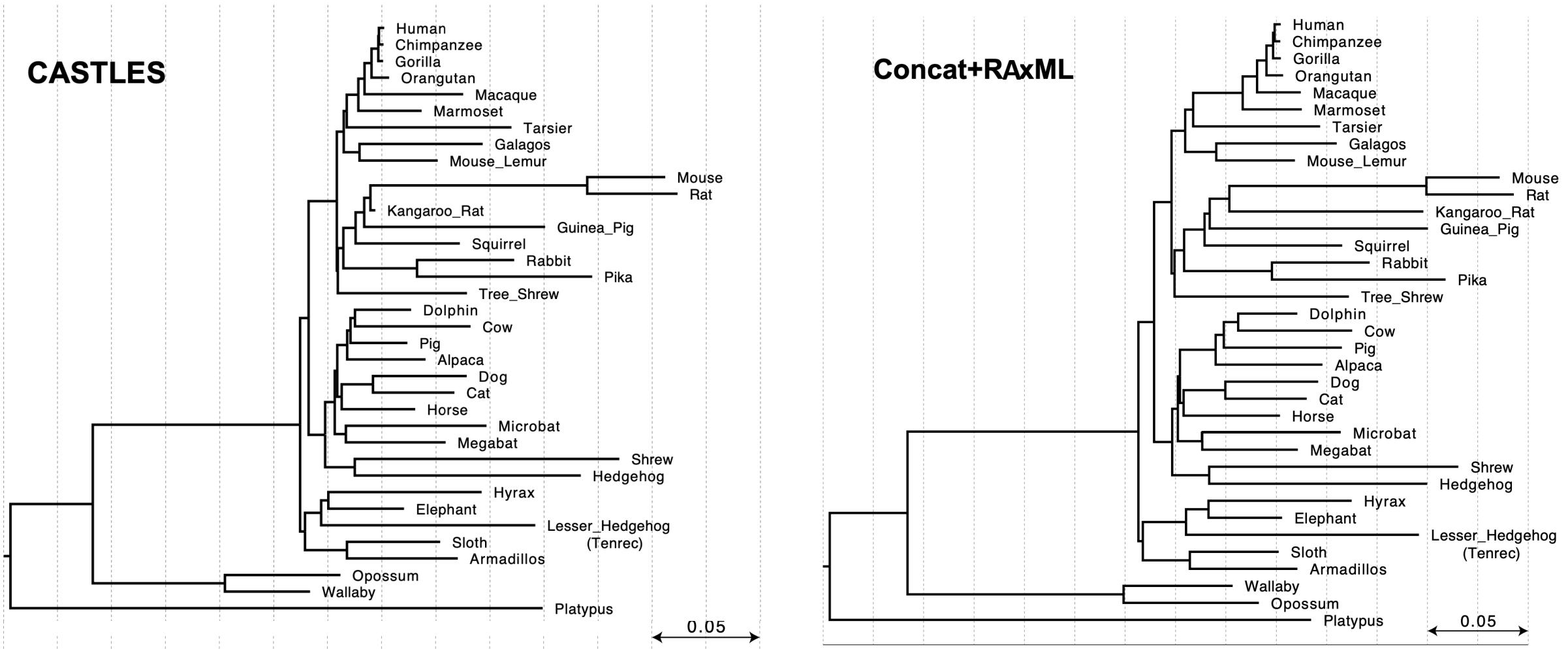
CASTLES's advantage increases with ILS

- 30-taxon ILS simulated dataset with 500 genes [Mai et al (2017)]
- Average 0.38 GTEE



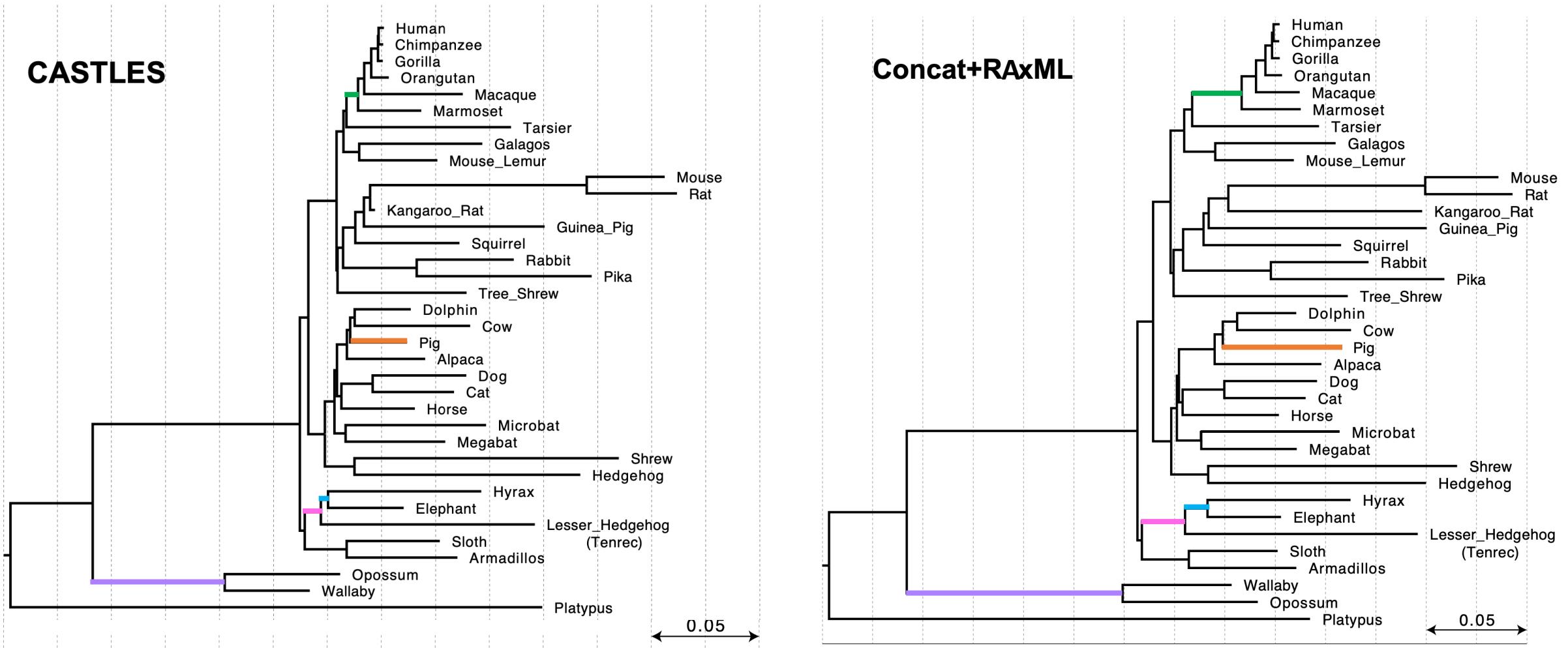
CASTLES produces shorter branches than concatenation on mammalian dataset

- 37-taxon mammalian biological dataset with 424 genes [Song et al (2012)], ASTRAL species tree



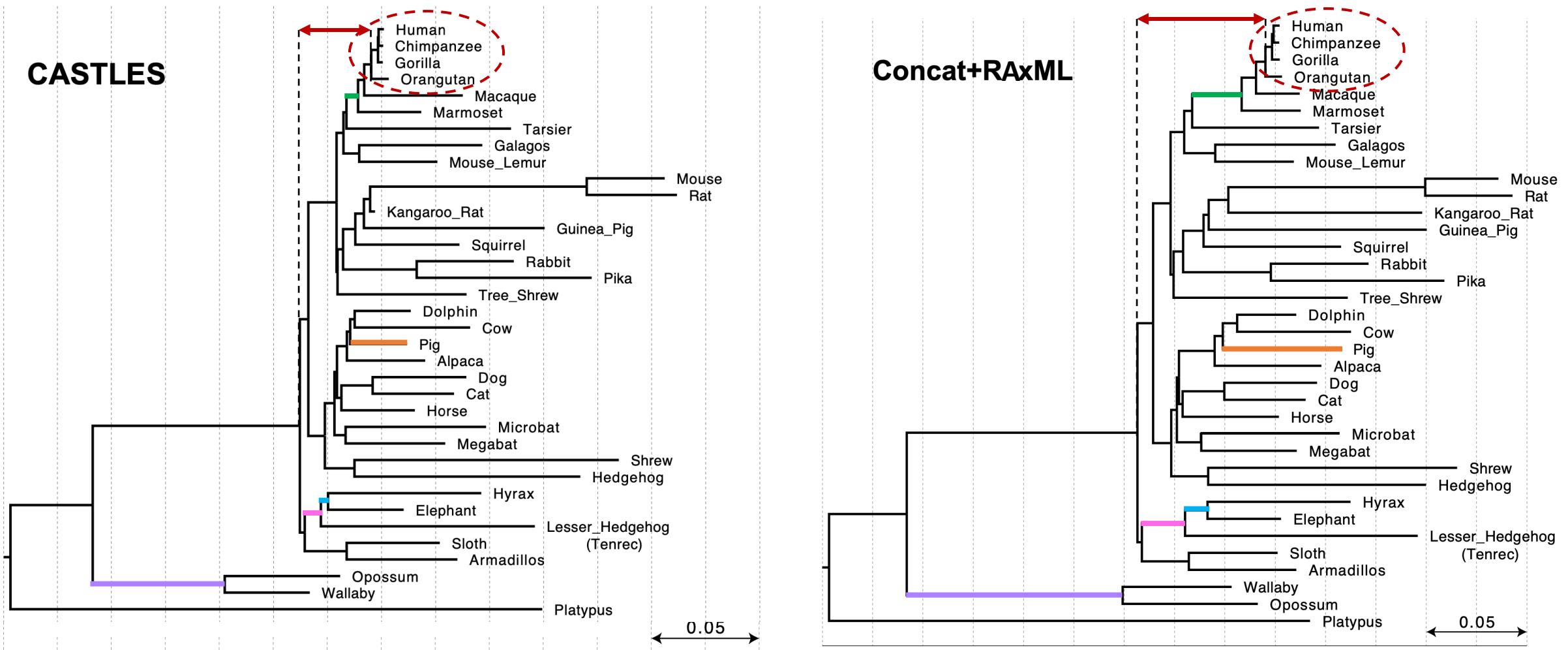
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Summary & Future Directions

Summary

- CASTLES is a scalable method for estimating branch lengths of a species tree given gene tree branch lengths
- CASTLES addresses **gene tree heterogeneity** due to ILS, and naturally occurring **variations in mutation rates**
- CASTLES produces more accurate and less biased branch lengths than prior methods in many model conditions

Future Directions

- Addressing other sources of gene tree discordance, such as gene duplication and loss and horizontal gene transfer
- Evaluating CASTLES on datasets with model misspecification, missing data, etc
- Are SU lengths identifiable under MSC+Substitution model, and is CASTLES statistically consistent?

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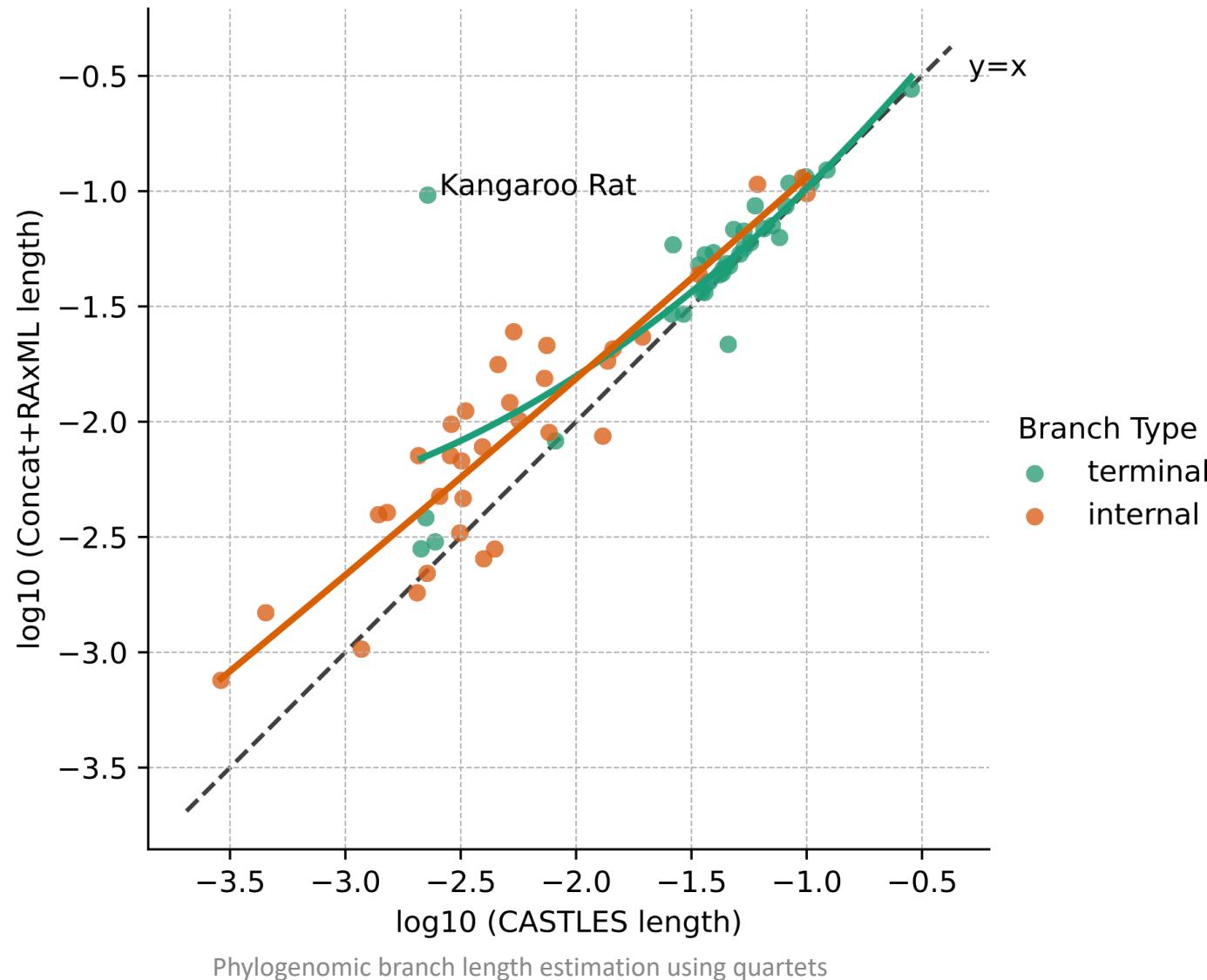
CASTLES is available on Github:

<https://github.com/ytabatabae/CSTLES>

CASTLES produces shorter branches than concatenation on mammalian dataset

- 37-taxon mammalian biological dataset with 424 genes [Song et al (2012)], ASTRAL species tree

Shorter branches
↓



Phylogenetic signal has relatively small impact on branch length accuracy

- 100-taxon ILS simulated dataset with 1000 genes, moderate ILS [Zhang et al (2018)]

