

Introduction to Supersymmetry

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Outline

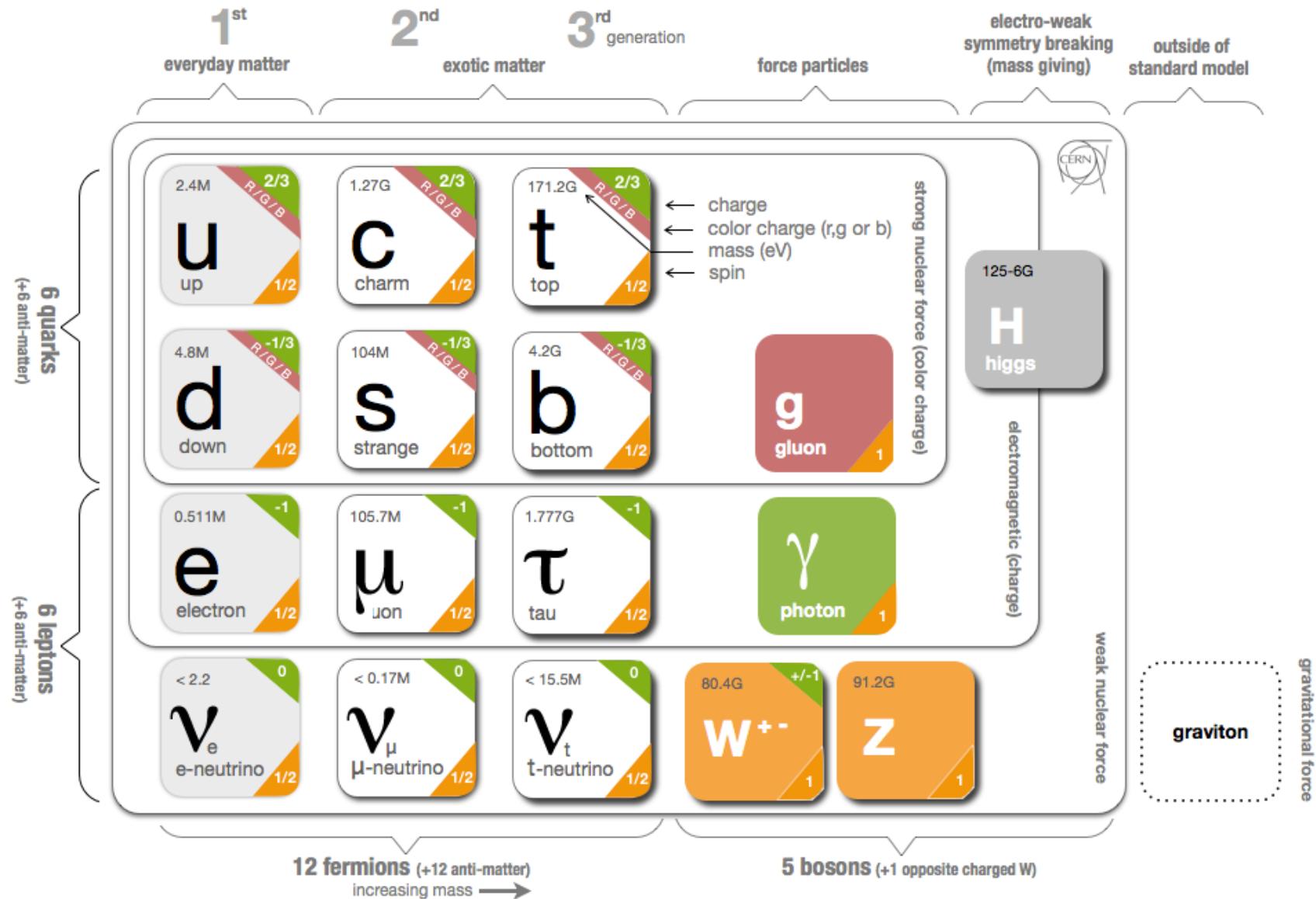
- Basic
 - ✓ Standard Model
 - ✓ Supersymmetry
- Supersymmetry Theory
 - ✓ The Wess-Zumino model
 - ✓ Superspace & superfields
 - ✓ Supersymmetry breaking
- Summary

Basic

Standard Model

- The Standard Model (SM)
 - ✓ was developed during 1970s.
 - ✓ is the most successful theory of particle physics.
 - ✓ explains elementary particles and the interactions.
 - ✓ cannot be the ultimate theory.

Standard Model



Standard Model

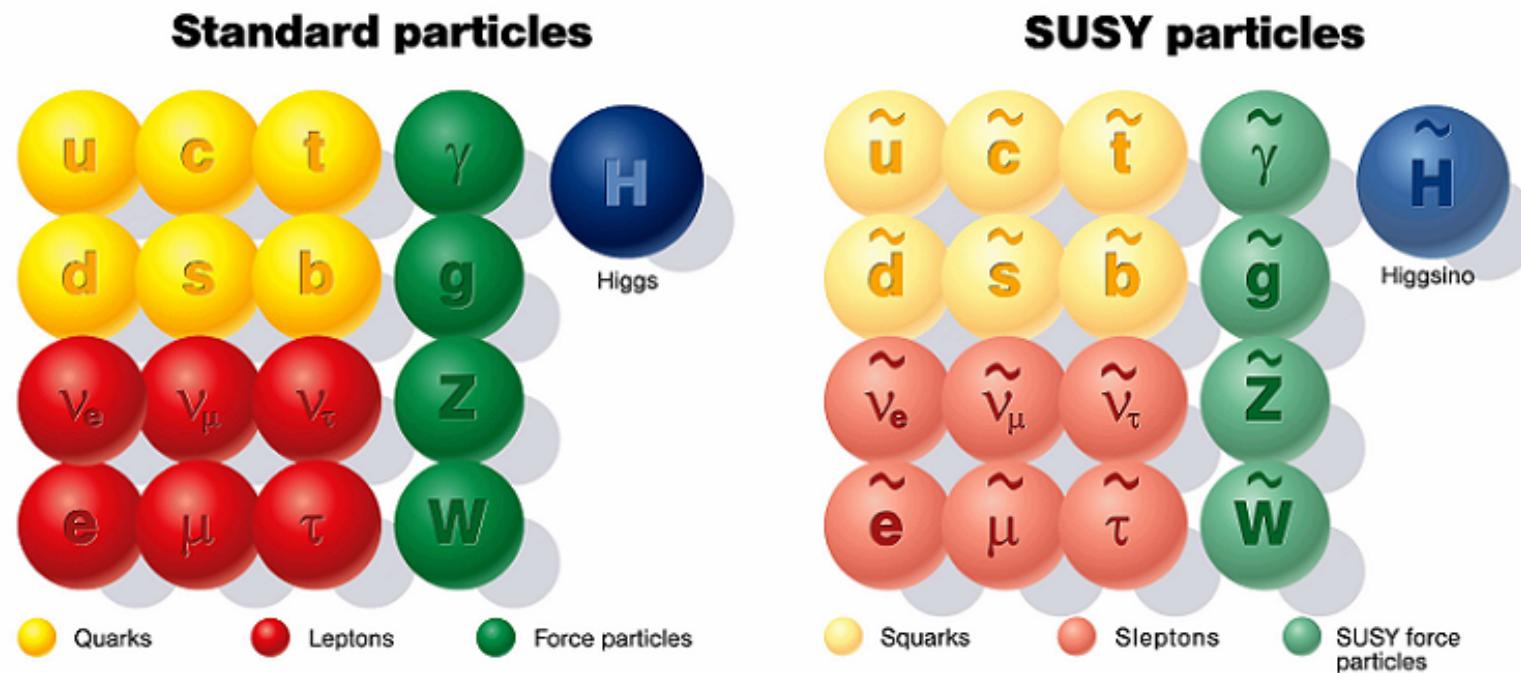
- Problems with the Standard Model:
 - ✓ Hierarchy problem
 - ✓ Why coupling constants do not converge?
 - ✓ Doesn't include dark matters and dark energy.
 - ✓ ...
- Standard Model needs to be extended.

Supersymmetry

- Motivations for supersymmetry
 - ✓ SUSY solves the hierarchy problem.
 - ✓ The coupling constants converge.
 - ✓ SUSY provides dark matter candidate.
 - ✓ ...

Supersymmetry

- New symmetry between fermions and bosons
 - ✓ Every SM particle has a superpartner differing by half of spin.
 - ✓ $\mathbf{Q} |\text{boson}\rangle = |\text{fermion}\rangle$, $\mathbf{Q} |\text{fermion}\rangle = |\text{boson}\rangle$

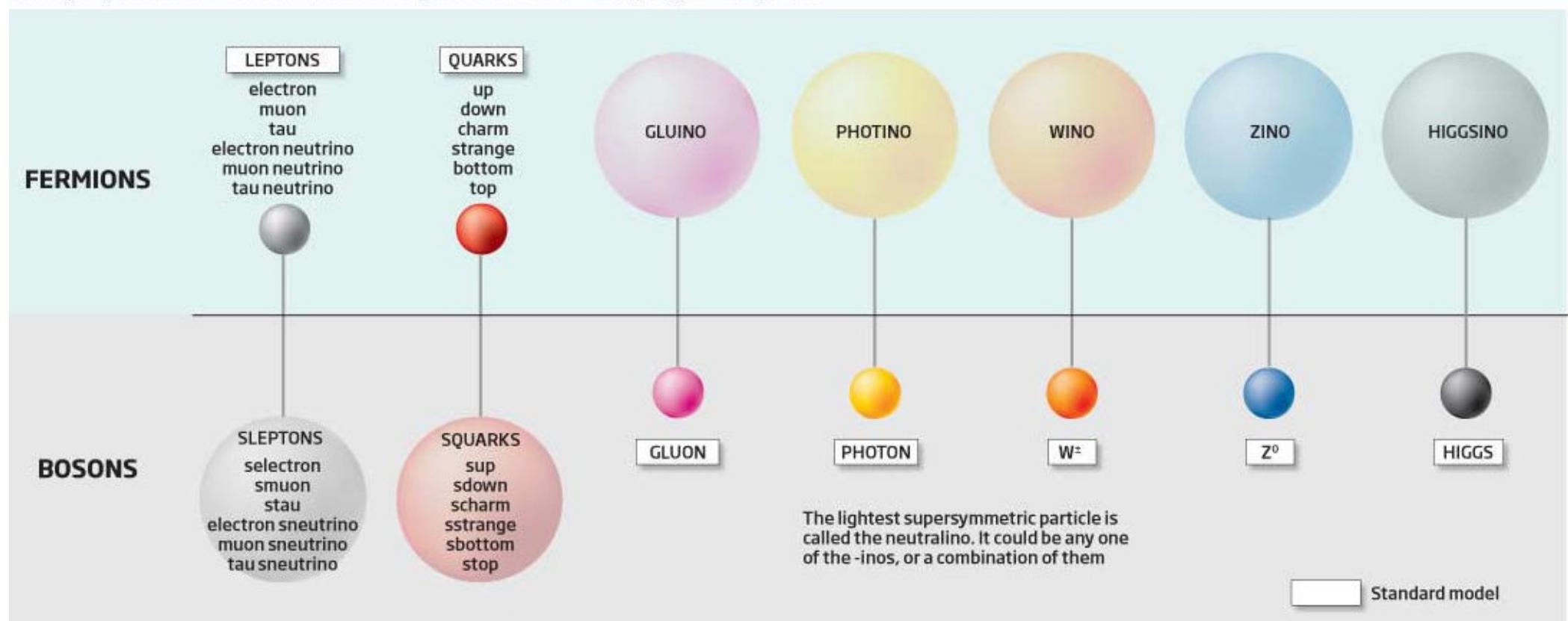


Supersymmetry

Particle zoo

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Particles are divided into two families called bosons and fermions. Among them are groups known as leptons, quarks and force-carrying particles like the photon. Supersymmetry doubles the number of particles, giving each fermion a massive boson as a super-partner and vice versa. The LHC is expected to find the first supersymmetric particle



Supersymmetry

- Supermultiplets
 - ✓ contain bosons and fermions.
 - ▶ have the same number of degrees of freedom.
 - ✓ all particles in the same supermultiplet have the same mass.
 - ✓ two kinds of supermultiplets
 - ▶ Chiral supermultiplet: $s = 0$ bosons (Higgs) & $s = 1/2$ fermions (quarks & leptons)
 - ▶ Vector supermultiplet: $s = 1$ gauge bosons

Supersymmetry

- R-Parity
 - ✓ is a new quantum number introduced by SUSY.
 - ✓ $R = (-1)^{3(B-L)+2s}$
 - ▶ B, L and s are Baryon number, Lepton number and spin
 - ✓ R = +1: SM particles, R = -1: SUSY particles
- If R-parity is conserved
 - ✓ there is no mixing between sparticles and Standard Model particles.
 - ✓ SUSY predicts the sparticles produced in pairs.

Supersymmetry

- Higgs mass $m_H^2(\text{phys}) = m_H^2(\text{bare}) + \delta m_H^2$
 - ✓ Radiative corrections: there are quadratically divergent contributions to the mass.



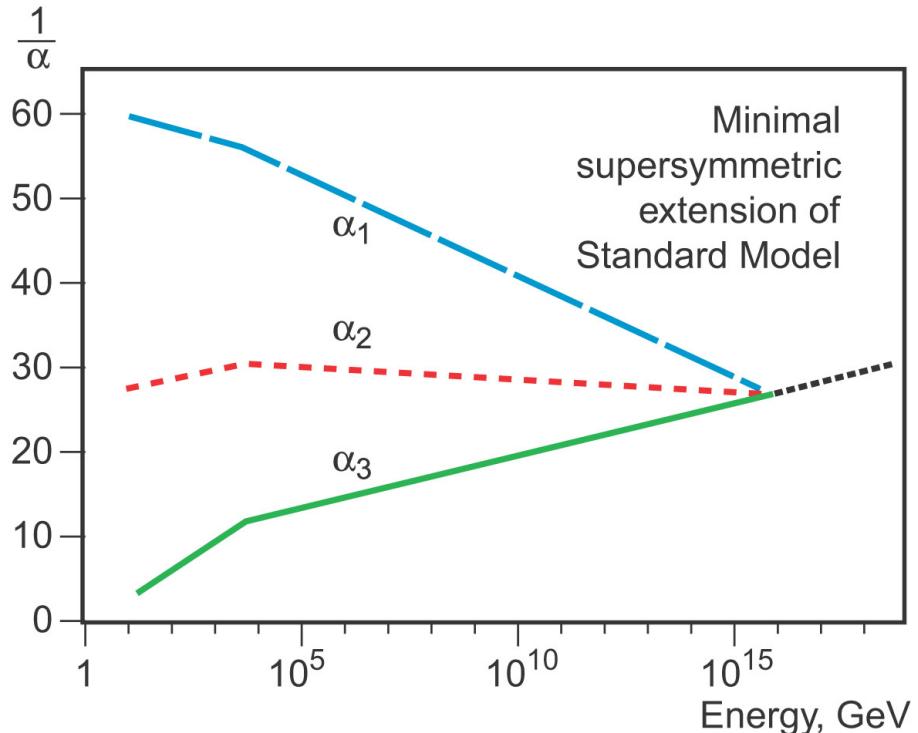
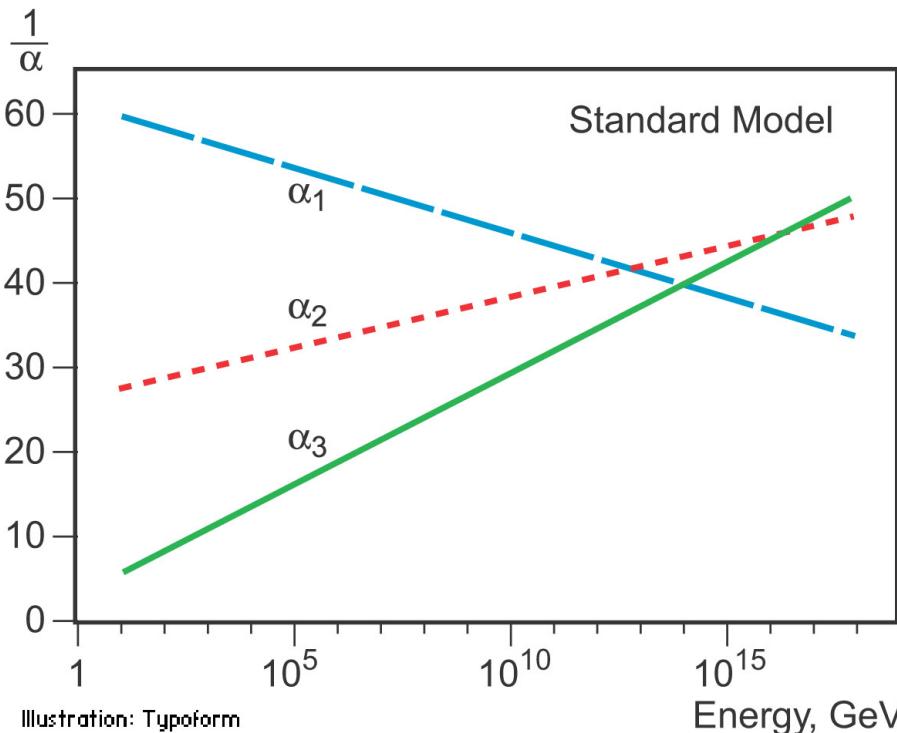
- ✓ From fermions: $\delta m_H^2 = \frac{|\lambda_f|^2}{8\pi^2} [-\Lambda^2 + 6m_f^2 \ln \frac{\Lambda}{m_f}]$
- ✓ From scalars: $\delta m_H^2 = \frac{\lambda_S}{16\pi^2} [\Lambda^2 - 2m_S^2 \ln \frac{\Lambda}{m_S}]$
- ✓ If Λ goes to infinity $\rightarrow \delta m_H^2$ quadratically divergent.

Supersymmetry

- Higgs mass $m_H^2(\text{phys}) = m_H^2(\text{bare}) + \delta m_H^2$
 - ✓ $m_H^2(\text{phys})$ is finite ($m_H \sim 125$ GeV).
→ We need to fine tune parameters to get a finite Higgs mass.
 - ✓ This is called the hierarchy problem.
- In SUSY
 - ✓ If every fermion is accompanied by two scalars with couplings $\lambda_S = \lambda_f^2$ the quadratic divergences cancel.
 - ✓ Fermions: $\delta m_H^2 = \frac{|\lambda_f|^2}{8\pi^2} [-\Lambda^2 + 6m_f^2 \ln \frac{\Lambda}{m_f}]$
 - ✓ Scalars: $\delta m_H^2 = \frac{\lambda_S}{16\pi^2} [\Lambda^2 - 2m_S^2 \ln \frac{\Lambda}{m_S}]$
 - ✓ The correction reduces to $\delta m_H^2 \approx \frac{\lambda_f^2}{4\pi^2} (m_S^2 - m_f^2) \ln \frac{\Lambda}{m_S}$

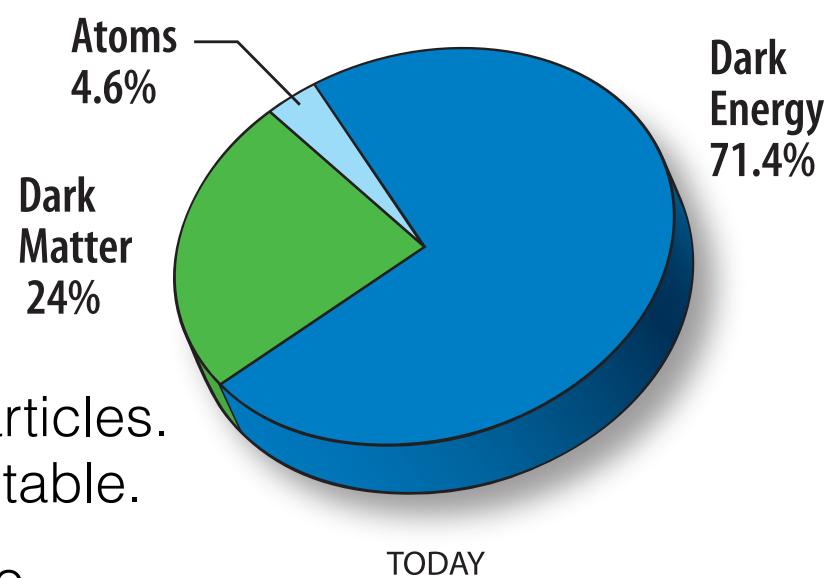
Supersymmetry

- The running coupling constants
 - ✓ α_1 : the electromagnetic coupling
 - ✓ α_2 : the weak coupling
 - ✓ α_3 : the strong coupling
 - ✓ are functions of energy.
 - ✓ are identical in strength at grand unification scale ($\sim 10^{16}$ GeV).



Supersymmetry

- Dark Matter (DM)
 - ✓ The strong evidence of DM was discovered in 1960s - 1970s.
 - ✓ Dark matter and dark energy consist ~95% of the universe
 - ✓ doesn't join strong or electromagnetic interactions.
 - ✓ is not included in Standard Model.
- SUSY
 - ✓ can provide a dark matter candidate.
 - ✓ R-parity must be conserved (RPC).
 - ✓ SUSY particles cannot decay to SM particles.
→ The lightest SUSY particle (LSP) is stable.
 - ✓ LSP can be lightest neutralino, gravitino...



Supersymmetry Theory

The Wess-Zumino Model

- Symmetry transformation:
 - ✓ Equation of motion is invariant under a transformation.
- Action $S = \int d^4x \mathcal{L}$ is invariant under symmetry transformation.
 - ✓ Lagrangian \mathcal{L} is invariant
 - ✓ Lagrangian changes by total derivative

$$\mathcal{L} \rightarrow \mathcal{L}' = \mathcal{L} + \partial_\mu \Lambda^\mu$$

Λ^μ vanishes at the boundary

The Wess-Zumino Model

- The simplest SUSY Lagrangian: the free Wess-Zumino model
- Particle content:
 - ✓ a massless complex scalar field Φ
 - ✓ a massless left-handed two-component Weyl fermion Ψ
 - ✓ without interaction between Φ and Ψ
- The simplest action

$$S = \int d^4x (\mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}})$$

$$\mathcal{L}_{\text{scalar}} = \partial^\mu \phi^* \partial_\mu \phi, \quad \mathcal{L}_{\text{fermion}} = i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi$$

$$\bar{\sigma}^\mu = (1, -\vec{\sigma})$$

Pauli matrix

The Wess-Zumino Model

- SUSY changes bosons into fermions and vice versa.

$$\delta\phi = \epsilon\psi$$

$$\delta\phi^* = \epsilon^\dagger\psi^\dagger$$

$$\delta\psi_\alpha = -i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi$$

$$\delta\psi_{\dot{\alpha}}^\dagger = i(\epsilon\sigma^\mu)_{\dot{\alpha}}\partial_\mu\phi^*$$

- The variations of the Lagrangian are

$$\delta\mathcal{L}_{\text{scalar}} = \partial^\mu(\delta\phi^*)\partial_\mu\phi + \partial^\mu\phi^*\partial_\mu(\delta\phi)$$

$$= \epsilon\partial^\mu\psi\partial_\mu\phi^* + \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi$$

$$\delta\mathcal{L}_{\text{fermion}} = i(\delta\psi^\dagger)\bar{\sigma}^\mu\partial_\mu\psi + i\psi^\dagger\bar{\sigma}^\mu\partial_\mu(\delta\psi)$$

$$= -\epsilon\sigma^\nu\partial_\nu\phi^*\bar{\sigma}^\mu\partial_\mu\psi + \psi^\dagger\bar{\sigma}^\mu\sigma^\nu\epsilon^\dagger\partial_\mu\partial_\nu\phi$$

$$= -\epsilon\partial^\mu\psi\partial_\mu\phi^* - \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi$$

$$+ \partial_\mu(\epsilon\sigma^\mu\bar{\sigma}^\nu\psi\partial_\nu\phi^* - \epsilon\psi\partial^\mu\phi^* + \epsilon^\dagger\psi^\dagger\partial^\mu\phi)$$

total derivative term

- The action S is invariant.

The Wess-Zumino Model

- Closure: the commutator of two SUSY transformations must itself be a symmetry transformation

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\phi = -i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)\partial_\mu\phi$$

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\psi_\alpha = -i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)\partial_\mu\psi_\alpha$$

$$+ i(\epsilon_{1\alpha}\epsilon_2^\dagger\bar{\sigma}^\mu\partial_\mu\psi - \epsilon_{2\alpha}\epsilon_1^\dagger\bar{\sigma}^\mu\partial_\mu\psi)$$

✓ on-shell

$$\bar{\sigma}^\mu\partial_\mu\psi = 0$$

unwanted term

✓ off-shell

- introduce a scalar auxiliary field \mathcal{F}

$$\delta\mathcal{F} = -i\epsilon^\dagger\bar{\sigma}^\mu\partial_\mu\psi, \quad \delta\mathcal{F}^* = i\partial_\mu\psi^\dagger\bar{\sigma}^\mu\epsilon$$

- The Lagrangian $\mathcal{L}_{\text{auxiliary}} = \mathcal{F}^*\mathcal{F}$

- fermionic field

$$\delta\psi_\alpha = -i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi + \epsilon_\alpha\mathcal{F}, \quad \delta\psi_{\dot{\alpha}}^\dagger = i(\epsilon\sigma^\mu)_{\dot{\alpha}}\partial_\mu\phi^* + \epsilon_{\dot{\alpha}}^\dagger\mathcal{F}^*$$

The Wess-Zumino Model

- The variations of Lagrangians are

$$\delta\mathcal{L}_{\text{auxiliary}} = i\partial_\mu\psi^\dagger\bar{\sigma}^\mu\epsilon\mathcal{F} - i\epsilon^\dagger\bar{\sigma}^\mu\partial_\mu\psi\mathcal{F}^*$$

$$\begin{aligned}\delta\mathcal{L}_{\text{fermion}} &= -\epsilon\sigma^\nu\partial_\nu\phi^*\bar{\sigma}^\mu\partial_\mu\psi + \psi^\dagger\bar{\sigma}^\mu\sigma^\nu\epsilon^\dagger\partial_\mu\partial_\nu\phi + i\epsilon^\dagger\bar{\sigma}^\mu\partial_\mu\psi\mathcal{F}^* + i\psi^\dagger\bar{\sigma}^\mu\partial_\mu(\epsilon\mathcal{F}) \\ &= \delta^{\text{old}}\mathcal{L}_{\text{fermion}} + i\epsilon^\dagger\bar{\sigma}^\mu\partial_\mu\psi\mathcal{F}^* - i(\partial_\mu\psi^\dagger)\bar{\sigma}^\mu\epsilon\mathcal{F} + \boxed{\partial_\mu(i\psi^\dagger\bar{\sigma}^\mu\epsilon\mathcal{F})}\end{aligned}$$

total derivative term

- $S = \int d^4x \mathcal{L} = \int d^4x (\mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{auxiliary}})$ is invariant under SUSY transformations.
→ $\delta S = 0$
- Field $X = \Phi, \Phi^*, \Psi, \Psi^\dagger, \mathcal{F}, \mathcal{F}^*$
 - ✓ SUSY algebra closes $(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})X = i(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger)\partial_\mu X$

The Wess-Zumino Model

- The multi-fields free Lagrangian

$$\mathcal{L}_{\text{free}} = \partial^\mu \phi^{*i} \partial_\mu \phi_i + i \psi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \psi_i + \mathcal{F}^{*i} \mathcal{F}_i$$

✓ $i = 1 \sim N$, N is number of particle

- On-shell/Off-shell SUSY transformation for muticompONENTS

$$\delta \phi_i = \epsilon \psi_i$$

$$\delta \psi_{i\alpha} = -i(\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi_i + \epsilon_\alpha \mathcal{F}_i$$

$$\delta \mathcal{F}_i = -i \epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \psi_i$$

$$\delta \phi^{*i} = \epsilon^\dagger \psi^{\dagger i}$$

$$\delta \psi_{\dot{\alpha}}^{\dagger i} = i(\epsilon \sigma^\mu)_{\dot{\alpha}} \partial_\mu \phi^{*i} + \epsilon_{\dot{\alpha}}^\dagger \mathcal{F}^{*i}$$

$$\delta \mathcal{F}^{*i} = i \partial_\mu \psi^{\dagger i} \bar{\sigma}^\mu \epsilon$$

The Wess-Zumino Model

- Consider interactions

- ✓ General form of the interaction Lagrangian

$$\mathcal{L}_{\text{int}} = \left(-\frac{1}{2}W^{ij}\psi_i\psi_j + W^i\mathcal{F}_i \right) + \text{c.c.}$$

- ✓ Superpotential W

- ▶ function of Φ_i only

$$W^i \equiv \frac{\partial W}{\partial \phi_i}, \quad W^{ij} \equiv \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$$

- ✓ The Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}} &= \partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi_i + \mathcal{F}^{*i} \mathcal{F}_i \\ &\quad + \left(-\frac{1}{2}W^{ij}\psi_i\psi_j + W^i\mathcal{F}_i - \frac{1}{2}W_{ij}^*\psi_i^\dagger\psi_j^\dagger + W_i^*\mathcal{F}^{*i} \right) \end{aligned}$$

The Wess-Zumino Model

- Consider interactions
 - ✓ The auxiliary fields
 - ▶ can be replaced by superpotential W using equations of motion.

$$\frac{\partial \mathcal{L}}{\partial \mathcal{F}^{i*}} = 0 \implies \mathcal{F}_i = -W_i^*$$

$$\frac{\partial \mathcal{L}}{\partial \mathcal{F}_i} = 0 \implies \mathcal{F}^{*i} = -W^i$$

- ✓ The Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$$

$$= \partial^\mu \phi^{*i} \partial_\mu \phi_i + i \psi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \psi_i - \frac{1}{2} \left(W^{ij} \psi_i \psi_j + W_{ij}^* \psi^{\dagger i} \psi^{\dagger j} \right) - W^i W_i^*$$

- ✓ Scalar potential

$$V(\phi, \phi^*) = W^i W_i^* = \mathcal{F}^{*i} \mathcal{F}_i \geq 0$$

Superspace & Superfields

- Supersymmetry is a symmetry in superspace.
- Superspace coordinate $(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$
 - ✓ $\theta_\alpha, \bar{\theta}_{\dot{\alpha}}$ are fermionic coordinates
 - ▶ two-component spinors.
 - ▶ Anticommute
 - $$\{\theta_\alpha, \theta_\beta\} = \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = \{\theta_\alpha, \bar{\theta}_{\dot{\beta}}\} = 0$$
 - ▶ the spinor indices $\alpha, \beta, \dot{\alpha}, \dot{\beta}$ go from 1 to 2.

Superspace & Superfields

- Superfield $S(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$
 - ✓ is a function of superspace coordinate.
 - ✓ is the representation of supermultiplet in superspace.
 - ✓ general form:

$$S(x, \theta, \bar{\theta}) = a + \theta \xi + \bar{\theta} \bar{\chi} + \theta^2 b + \bar{\theta}^2 c + \theta \sigma^\mu \bar{\theta} v_\mu + \theta^2 \bar{\theta} \zeta + \bar{\theta}^2 \theta \eta + \theta^2 \bar{\theta}^2 d$$

bosonic fields

fermionic fields

$$\theta^2 = \theta \theta = \theta^\alpha \theta_\alpha$$

$$\bar{\theta}^2 = \bar{\theta} \bar{\theta} = \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}}$$

Superspace & Superfields

- SUSY generators

$$Q_\alpha = -i \frac{\partial}{\partial \theta^\alpha} - \sigma_{\alpha\dot{\beta}}^\mu \bar{\theta}^{\dot{\beta}} \dot{\partial}_\mu$$

$$Q^\alpha = i \frac{\partial}{\partial \theta_\alpha} + (\bar{\theta} \bar{\sigma}^\mu)^\alpha \partial_\mu$$

$$\bar{Q}_{\dot{\alpha}} = i \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + \theta^\beta \sigma_{\beta\dot{\alpha}}^\mu \partial_\mu$$

$$\bar{Q}^{\dot{\alpha}} = -i \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - (\bar{\sigma}^\mu \theta)^{\dot{\alpha}} \partial_\mu$$

- ✓ anticommutation relations

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma_{\alpha\dot{\beta}}^\mu P_\mu = -2i\sigma_{\alpha\dot{\beta}}^\mu \partial_\mu$$

$$\{Q_\alpha, Q_\beta\} = 0$$

$$\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$

Superspace & Superfields

- SUSY transformation

$$\begin{aligned}\delta_\epsilon S(x, \theta, \bar{\theta}) &= S(x^\mu + i\epsilon\sigma^\mu\bar{\theta} + i\bar{\epsilon}\bar{\sigma}^\mu\theta, \theta + \epsilon, \bar{\theta} + \bar{\epsilon}) - S(x, \theta, \bar{\theta}) \\ &= (i\epsilon Q + i\bar{\epsilon}\bar{Q})S(x, \theta, \bar{\theta}) \\ &= \left[\epsilon^\alpha \frac{\partial}{\partial \theta^\alpha} + \bar{\epsilon}_{\dot{\alpha}} \frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}} - i(\epsilon\sigma^\mu\bar{\theta} + \bar{\epsilon}\bar{\sigma}^\mu\theta) \right] S(x, \theta, \bar{\theta})\end{aligned}$$

Superspace & Superfields

- SUSY covariant derivatives

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma_{\alpha\dot{\beta}}^\mu \bar{\theta}^{\dot{\beta}} \partial_\mu$$

$$\overline{D}_{\dot{\alpha}} = (D_\alpha)^\dagger = \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i \theta^\beta \sigma_{\beta\dot{\alpha}}^\mu \partial_\mu$$

✓ anticommutation relations

$$\{D_\alpha, \overline{D}_{\dot{\beta}}\} = 2i \sigma_{\alpha\dot{\beta}}^\mu \partial_\mu$$

✓ SUSY transformation

$$\{D_\alpha, D_\beta\} = \{\overline{D}_{\dot{\alpha}}, \overline{D}_{\dot{\beta}}\} = 0$$

$$\delta_\epsilon(D_\alpha S) = D_\alpha(\delta_\epsilon S)$$

$$\{D_\alpha, Q_\beta\} = \{D_\alpha, \overline{Q}_{\dot{\beta}}\} = 0$$

$$\delta_\epsilon(\overline{D}_{\dot{\alpha}} S) = \overline{D}_{\dot{\alpha}}(\delta_\epsilon S)$$

$$\{\overline{D}_{\dot{\alpha}}, Q_\beta\} = \{\overline{D}_{\dot{\alpha}}, \overline{Q}_{\dot{\beta}}\} = 0$$

Superspace & Superfields

- Chiral superfields
 - ✓ describe spin 0 bosons and spin 1/2 fermions
 - ▶ Higgs bosons, quarks, and leptons
 - ✓ Superfield $\Phi(x, \theta, \bar{\theta})$
 - ▶ chiral (left-chiral) superfield
$$\overline{D}_{\dot{\alpha}} \Phi = 0$$
 - ▶ antichiral (right-chiral) superfield
$$D_{\alpha} \Phi^* = 0$$

Superspace & Superfields

- Chiral superfields

✓ Change coordinate $(x^\mu, \theta, \bar{\theta}) \rightarrow (y^\mu, \theta)$

- chiral coordinate

$$(y^\mu, \theta) \quad y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$$

- antichiral coordinate

$$(\bar{y}^\mu, \bar{\theta}) \quad \bar{y}^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$$

- Covariant derivative

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + 2i\sigma^\mu_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\frac{\partial}{\partial y^\mu}$$

$$\bar{D}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}$$

Superspace & Superfields

- Chiral superfields

- ✓ The general form in chiral coordinate

$$\Phi(y, \theta) = \phi(y) + \sqrt{2}\theta\phi(y) + \theta\bar{\theta}\mathcal{F}(y)$$

- ▶ function of (y^μ, θ)
 - ▶ SUSY transformation $\delta_\epsilon \Phi = (i\epsilon Q + i\bar{\epsilon}\bar{Q})\Phi$
- component fields

$$\delta_\epsilon \phi = \sqrt{2}\epsilon\psi$$

$$\delta_\epsilon \psi_\alpha = i\sqrt{2}\partial_\mu \phi (\sigma^\mu \bar{\epsilon})_\alpha - \sqrt{2}\epsilon_\alpha \mathcal{F}$$

$$\delta_\epsilon \mathcal{F} = i\sqrt{2}\partial_\mu \psi \sigma^\mu \bar{\epsilon}$$

Superspace & Superfields

- Vector superfields
 - ✓ describe spin 1 gauge bosons.

✓ are real superfields $V(x, \theta, \bar{\theta}) = V^\dagger(x, \theta, \bar{\theta})$

✓ The general form

$$\begin{aligned} V(x, \theta, \bar{\theta}) = & C + i\theta\chi - i\bar{\theta}\bar{\chi} + \theta\sigma^\mu\bar{\theta}v_\mu \\ & + \frac{i}{2}\theta^2(M + iN) - \frac{i}{2}\bar{\theta}^2(\bar{M} - i\bar{N}) \\ & + i\theta^2\bar{\theta}(\bar{\lambda} + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi) - i\bar{\theta}^2\theta(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}) \\ & + \frac{1}{2}\theta^2\bar{\theta}^2(D - \frac{1}{2}\partial^2C) \end{aligned}$$

real scalar fields
Weyl spinor fields
vector field

Superspace & Superfields

- Vector superfields
 - ✓ a chiral superfield Φ can construct a vector field.

► example

$$\begin{aligned}\Phi + \Phi^\dagger = & (\phi + \phi^*) + \sqrt{2}\theta\psi + \sqrt{2}\bar{\theta} \bar{\psi} \\ & - i\theta\sigma^\mu\bar{\theta}\partial_\mu(\phi - \phi^*) + \theta^2\mathcal{F} + \bar{\theta}^2\mathcal{F}^* \\ & + \frac{i}{\sqrt{2}}\theta^2\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi - \frac{i}{\sqrt{2}}\bar{\theta}^2\theta\sigma^\mu\partial_\mu\bar{\psi} \\ & - \frac{1}{4}\theta^2\bar{\theta}^2\partial^2(\phi + \phi^*)\end{aligned}$$

Superspace & Superfields

- Vector superfields
 - ✓ SUSY gauge transformation $V \rightarrow V + i(\Phi - \Phi^\dagger)$
 - ✓ Wess-Zumino gauge
 - ▶ $C(x) = M(x) = N(x) = \chi(x) = \chi^\dagger(x) = 0$
 - ▶ reduces the number of the components
 - ▶ vector superfield under WZ gauge

$$V_{WZ} = \theta\sigma^\mu\bar{\theta}v_\mu(x) + i\theta^2\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}^2\theta\lambda(x) + \frac{1}{2}\theta^2\bar{\theta}^2D(x)$$

$$V_{WZ}^2 = [\theta\sigma^\mu\bar{\theta}v_\mu(x)][\theta\sigma^\nu\bar{\theta}v_\nu(x)] = \frac{1}{2}\theta^2\bar{\theta}^2v_\mu v^\mu$$

$$V_{WZ}^n = 0, n \geq 3$$

Supersymmetry Breaking

- What is symmetry breaking?



symmetry



symmetry is broken

Supersymmetry Breaking

- Symmetry breaking
 - ✓ spontaneous symmetry breaking: $E_{\text{ground state}} \neq 0$ 
 - ✓ soft symmetry breaking: non-symmetric term in E.O.M
- SUSY theory
 - ✓ In the same supermultiplet: $M_{\text{particle}} = M_{\text{sparticle}}$
 - No selectrons $m = 0.51 \text{ MeV}$ are found.
 - No massless fermionic partner of photon.
 - ✓ Nature is not exactly supersymmetry \rightarrow SUSY is broken.
 - $E_{\text{ground state}} > 0$

Supersymmetry Breaking

- Energy
 - ✓ is the expectation of the Hamiltonian H .
 - ▶ $E_\psi = \langle \psi | H | \psi \rangle$ for an arbitrary state $|\psi\rangle$.
 - ✓ $|\Omega\rangle$: vacuum state (ground state)
 - ▶ vacuum expectation value (VEV)

$$E_{\text{ground state}} = \langle \Omega | H | \Omega \rangle$$

Supersymmetry Breaking

- Superalgebra

$$H = \frac{1}{4}(\{Q_1, \bar{Q}_1\} + \{Q_2, \bar{Q}_2\})$$

- Energy

$$\begin{aligned} E_\Psi &= \langle \Psi | H | \Psi \rangle \\ &= \frac{1}{4} \sum_{\alpha, \dot{\alpha}} (\| Q_\alpha | \Psi \rangle \|^2 + \| \bar{Q}_{\dot{\alpha}} | \Psi \rangle \|^2) \end{aligned}$$

✓ non-negative

✓ VEV

$$\triangleright E = 0: \quad Q_\alpha | \Omega \rangle = \bar{Q}_{\dot{\alpha}} | \Omega \rangle = 0$$

$$\triangleright E \neq 0: \quad Q_\alpha | \Omega \rangle \neq 0 \quad \text{or} \quad \bar{Q}_{\dot{\alpha}} | \Omega \rangle \neq 0$$

Supersymmetry Breaking

- The scalar potential V

$$V(\phi, \phi^*) = \mathcal{F}^{*i} \mathcal{F}_i + \frac{1}{2} \sum_a \mathcal{D}^a \mathcal{D}_a$$

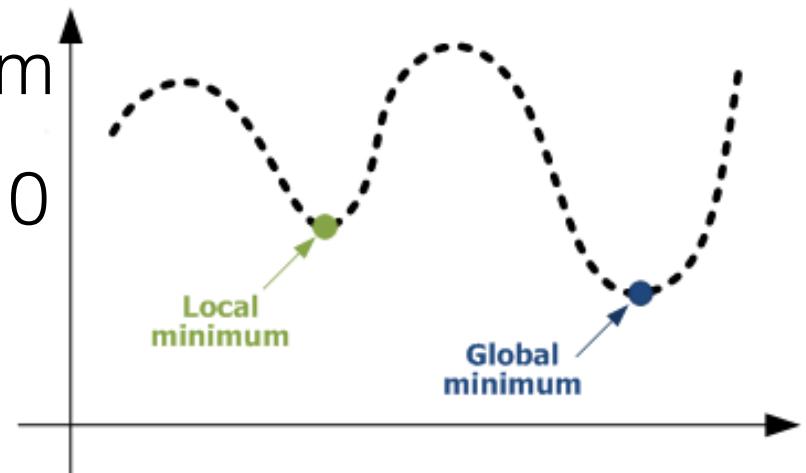
✓ Minimum of V and vacuum state $|\Omega\rangle$

▶ true vacuum: global minimum

example: $\mathcal{F}_i = \mathcal{D}_a = 0 \rightarrow V = 0$

▶ false vacuum: local minimum

example: $\langle \mathcal{F}_i \rangle \neq 0$ or $\langle \mathcal{D}_a \rangle \neq 0$



Supersymmetry Breaking

- O'Raifeartaigh model (\mathcal{F} -term breaking)

- ✓ 3 chiral supermultiplets: Φ_1, Φ_2, Φ_3
- ✓ superpotential W

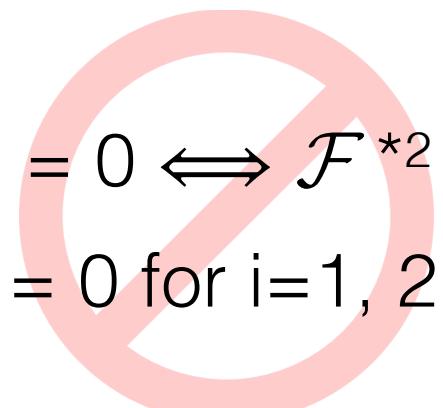
$$W = g \Phi_1 (\Phi_3^2 - m^2) + M \Phi_2 \Phi_3, M \gg m$$

- ✓ The auxiliary field $\mathcal{F}^{*i} = -W^i$

$$\mathcal{F}^{*1} = -W^1 = \frac{\partial W}{\partial \phi_1} = g(\phi_3^2 - m^2)$$

$$\mathcal{F}^{*2} = -W^2 = \frac{\partial W}{\partial \phi_2} = M\phi_3$$

$$\mathcal{F}^{*3} = -W^3 = \frac{\partial W}{\partial \phi_3} = 2g\phi_1\phi_3 + M\phi_2$$


$$\mathcal{F}^{*1} = 0 \iff \mathcal{F}^{*2} = 0$$
$$\mathcal{F}^{*i} = 0 \text{ for } i=1, 2, 3$$

Summary

- SUSY
 - ✓ expects the supersymmetric particles exist.
 - ▶ $R = -1$ for superparticles
 - ✓ two supermultiplets
 - ▶ chiral supermultiplet: $s=0$ bosons and $s=1/2$ fermions.
 - ▶ vector supermultiplet: gauge bosons
 - ✓ must be broken.

Summary

- SUSY
 - ✓ superspace & superfields
 - ✓ supermultiplets → superfields
 - ✓ transformation $(i\epsilon Q + i\bar{\epsilon}\bar{Q})S(x, \theta, \bar{\theta})$
 - $S(x, \theta, \bar{\theta})$ is a general superfield
 - ϵ is an anticommuting constant spinor
 - Q is a generator

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma_{\alpha\dot{\beta}}^\mu P_\mu, \quad \{Q_\alpha, Q_\beta\} = 0 \quad \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$

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Backup

Lorentz Group

- Lorentz transformation = rotation + boost
- The infinitesimal Lorentz transformation

$$U(\vec{\theta}, \vec{\phi}) \simeq 1 + i\vec{\theta} \cdot \vec{J} + i\vec{\phi} \cdot \vec{K}$$

\vec{J} rotation generator, \vec{K} boost generator

- Commutation relations

$$[J_i, J_j] = i \epsilon_{ijk} J_k$$

$$[K_i, K_j] = -i \epsilon_{ijk} J_k$$

$$[J_i, K_j] = i \epsilon_{ijk} K_k$$

$$i, j, k = 1, 2, 3$$

If we define new generators as

$$J_i^\pm = \frac{1}{2}(J_i \pm iK_i)$$

the commutation relations become

$$[J_i^+, J_j^+] = i\epsilon_{ijk} J_k^+$$

$$[J_i^-, J_j^-] = i\epsilon_{ijk} J_k^-$$

$$[J_i^+, J_j^-] = 0$$

► Two independent $SU(2)$ groups!

Poincaré Group

- Poincaré group = Lorentz group \otimes translation group (in space-time)
 - ✓ is the underlying symmetry group of particle physics.
- Commutation relations

$$[P_\mu, P_\nu] = 0$$

$$[M_{\mu\nu}, P_\lambda] = i(g_{\nu\lambda} P_\mu - g_{\mu\lambda} P_\nu)$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(g_{\mu\rho} M_{\nu\sigma} - g_{\mu\sigma} M_{\nu\rho} - g_{\nu\rho} M_{\mu\sigma} + g_{\nu\sigma} M_{\mu\rho})$$

where $M_{\mu\nu}$ is an antisymmetric tensor with rank 2.

- ✓ The six components are the six Lorentz group generators.
- ✓ $M_{ij} = \epsilon_{ijk} J_k$
- ✓ $M_{0i} = -M_{i0} = -K_i$
- ✓ The metric $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$.

Spinors

- Spinors
 - ✓ The general state of a spin-1/2 particle can be expressed as a two component column matrix.

$$\chi = c_+ \chi_+ + c_- \chi_- = \begin{pmatrix} c_+ \\ c_- \end{pmatrix} \quad \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Bi-spinors
 - ✓ consist of two spinors which belong to two different $SU(2)$ group.
 - ✓ Ψ_D is a solution of the Dirac equation $(i \gamma^\mu \partial_\mu - m)\Psi = 0$
 - is a 4-component field

$$\psi_D = \boxed{\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}} = \boxed{\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}}$$

4-component form
bi-spinor form

Spinors

- Weyl spinors (Weyl fermion fields)
 - ✓ are the building blocks for any fermion field.
 - ✓ Ψ_L and Ψ_R are Weyl spinors.
- Majorana spinors
 - ✓ Majorana condition: $\psi = C\bar{\psi}^T$
 - ✓ bi-spinor form
$$\psi = \begin{pmatrix} \xi^\alpha \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix}$$

Hermitian conjugate
- Identities
 - ✓ Spinors Ψ and χ are anticommuting objects.
$$\begin{array}{lll} \psi\chi = \chi\psi & \chi\sigma^\mu\bar{\psi} = -\bar{\psi}\bar{\sigma}^\mu\chi & \chi\sigma^\mu\bar{\sigma}^\nu\psi = \psi\sigma^\nu\bar{\sigma}^\mu\chi \\ \bar{\psi}\bar{\chi} = \bar{\chi}\bar{\psi} & (\chi\sigma^\mu\bar{\psi})^\dagger = \psi\sigma^\mu\bar{\chi} & (\chi\sigma^\mu\bar{\sigma}^\nu\psi)^\dagger = \bar{\psi}\bar{\sigma}^\nu\sigma^\mu\bar{\chi} \\ (\psi\chi)^\dagger = \bar{\psi}\bar{\chi} & & \end{array}$$

Helicity

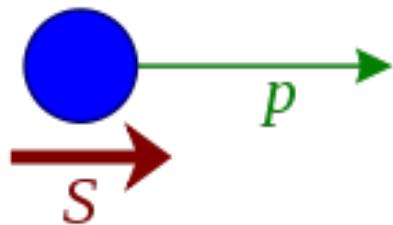
- the projection of angular momentum \mathbf{J} onto the direction of momentum \mathbf{P} .

$$h = \vec{J} \cdot \hat{p} = (\vec{L} + \vec{S}) \cdot \hat{p} = \vec{S} \cdot \hat{p}, \quad \hat{p} = \frac{\vec{p}}{|\vec{p}|}$$

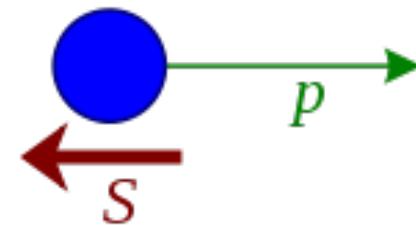
- ✓ eigenvalues of \mathbf{h} are ± 1

$h = +1$: right-handed, $h = -1$: left-handed

Right-handed:



Left-handed:



- is invariant under rotations but not invariant under boost.
 - ✓ For a massless particle, the helicity is Lorentz invariant.

Chirality

- Chirality \neq Helicity
 - ✓ For a mass less particle, chirality = helicity
- Chirality operators $P_L = \frac{1 - \gamma_5}{2}$ $P_R = \frac{1 + \gamma_5}{2}$
- Bi-spinor Ψ
 - ✓ $\Psi = \Psi_L + \Psi_R$
 - ✓ $\Psi_L = P_L \Psi, \Psi_R = P_R \Psi$
 - ✓ $P_L \Psi_R = 0, P_R \Psi_L = 0$
- Chirality is preserved under Lorentz transformation.

Grassmann Numbers

- Anticommute $\{\theta_\alpha, \theta_\beta\} = \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = \{\theta_\alpha, \bar{\theta}_{\dot{\beta}}\} = 0$

- Identities

$$\theta_\alpha \theta_\beta = \frac{1}{2} \epsilon_{\alpha\beta} \theta \theta$$

$$(\theta\psi)(\theta\chi) = -\frac{1}{2} \theta \theta \psi \chi$$

$$(\theta\sigma^\mu \bar{\theta})(\theta\sigma^\nu \bar{\theta}) = \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} g^{\mu\nu}$$

$$\bar{\theta}_{\dot{\alpha}} \bar{\theta}_{\dot{\beta}} = -\frac{1}{2} \epsilon_{\dot{\alpha}\dot{\beta}} \bar{\theta} \bar{\theta}$$

$$(\bar{\theta}\psi)(\bar{\theta}\chi) = -\frac{1}{2} \bar{\theta} \bar{\theta} \psi \chi$$

$$\theta_\alpha \bar{\theta}_{\dot{\beta}} = \frac{1}{2} \sigma_{\alpha\dot{\beta}}^\mu (\bar{\theta} \bar{\sigma}_\mu \theta)$$

$$(\theta\psi)(\bar{\theta}\chi) = \frac{1}{2} (\bar{\theta} \bar{\sigma}^\mu \theta)(\psi \sigma_\mu \chi)$$

- Derivatives $\frac{\partial}{\partial \theta^\alpha}(\theta^\beta) = \delta_\alpha^\beta, \quad \frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}}(\bar{\theta}_{\dot{\beta}}) = \delta_{\dot{\alpha}}^{\dot{\beta}}, \quad \frac{\partial}{\partial \theta^\alpha}(\bar{\theta}_{\dot{\beta}}) = 0, \quad \frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}}(\theta^\beta) = 0$

- Integrations $\int d\theta_\alpha = 0, \quad \int d\theta_\alpha \theta_\alpha = 1$

$$\int d^2\theta \theta \theta = \int d^2\bar{\theta} \bar{\theta} \bar{\theta} = 1 \quad \text{where} \quad d^2\theta = \frac{1}{2} d\theta^1 d\theta^2, \quad d^2\bar{\theta} = \frac{1}{2} d\bar{\theta}^1 d\bar{\theta}^2 = (d^2\theta)^\dagger$$

Super-Poincaré Group

- Commutation relations

$$\begin{array}{lll} [P_\mu, Q_\alpha^I] = 0 & [M_{\mu\nu}, Q_\alpha^I] = i(\sigma_{\mu\nu})_\alpha{}^\beta Q_\beta^I & \{Q_\alpha^I, \bar{Q}_{\dot{\beta}}^J\} = 2\sigma_{\alpha\dot{\beta}}^\mu P_\mu \delta^{IJ} \\ [P_\mu, \bar{Q}_{\dot{\alpha}}^I] = 0 & [M_{\mu\nu}, \bar{Q}^{I\dot{\alpha}}] = i(\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}{}_\beta \bar{Q}^{I\dot{\beta}} & \{Q_\alpha^I, Q_\beta^J\} = \epsilon_{\alpha\beta} Z^{IJ} \\ & & \{\bar{Q}_{\dot{\alpha}}^I, \bar{Q}_{\dot{\beta}}^J\} = \epsilon_{\dot{\alpha}\dot{\beta}} (Z^{IJ})^* \end{array}$$

✓ $Z^{IJ} = -Z^{JI}$ central charge ($\neq 0$ for $N > 1$).

‣ commutes with all generators

✓ $I, J = 1, 2, \dots, N$

‣ $N \leq 8$ physical meaning

‣ $N = 1$ SUSY

‣ $N > 1$ extended SUSY

Supermultiplets

- Supermultiplets
 - ✓ are SUSY single particle states \Leftrightarrow irreducible representations of super-Poincaré algebra
 - ✓ contain bosons and fermions.
 - ▶ equal number of degree of freedom.
 - ✓ all particles in the same supermultiplet have the same mass.
 - ▶ $P^2 = m^2$, commutes with all generators
 - chiral supermultiplet: a complex scalar and a Weyl fermion
 - vector supermultiplet: a gauge boson and a Weyl fermion

Standard Model

- Extensions:
 - ✓ Technicolor
 - ▶ The role of Higgs boson is replaced by a composite state of tightly bound fermions which is called technifermions.
 - ✓ Little Higgs models
 - ▶ use global symmetries to stabilize the mass of the Higgs bosons.
 - ✓ Supersymmetry (SUSY)
 - ✓ ...

The Wess-Zumino Model

- Noether's theorem
 - ✓ symmetry \Leftrightarrow conserved current
 - ✓ symmetry transformation $\Phi(x) \rightarrow \Phi'(x) + \alpha \Delta\Phi(x)$
 - ▶ Equation of motion $\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$ is invariant.
 - ▶ The Lagrangian $\mathcal{L}(x) \rightarrow \mathcal{L}(x) + \alpha \partial_\mu \mathcal{J}^\mu(x)$ is invariant.
 - ✓ conserved current
 - ▶ $\partial_\mu \mathcal{J}^\mu(x) = 0$, for $\mathcal{J}^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi - \mathcal{J}^\mu$
 - ✓ conserved charge

$$Q \equiv \int_{\text{all space}} j^0 d^3x$$

The Wess-Zumino Model

- SUSY is a symmetry in superspace.
 - ✓ Noether's theorem
 - ▶ conserved currents (supercurrents)

$$J_\alpha^\mu = (\sigma^\nu \bar{\sigma}^\mu \psi)_\alpha \partial_\nu \phi^*$$

$$J_{\dot{\alpha}}^{\dagger\mu} = (\psi^\dagger \bar{\sigma}^\mu \sigma^\nu)_{\dot{\alpha}} \partial_\nu \phi$$

- ▶ conserved charges (supercharges)

$$Q_\alpha = \sqrt{2} \int d^3x J_\alpha^0$$

$$Q_{\dot{\alpha}}^\dagger = \sqrt{2} \int d^3x J_{\dot{\alpha}}^{\dagger 0}$$

Superspace & Superfields

- Superfield $S(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$
 - ✓ is a function of superspace coordinate.
 - ✓ is the representation of supermultiplet in superspace.
 - ✓ general form:

$$S(x, \theta, \bar{\theta}) = a + \theta \xi + \bar{\theta} \bar{\chi} + \theta \theta b + \bar{\theta} \bar{\theta} c + \bar{\theta} \bar{\sigma}^\mu \theta v_\mu + \theta \theta \bar{\theta} \bar{\zeta} + \bar{\theta} \bar{\theta} \theta \eta + \theta \theta \bar{\theta} \bar{\theta} d$$

bosonic fields

fermionic fields

Superspace & Superfields

- Vector superfields
 - ✓ describe spin 1 gauge bosons.

✓ are real superfields $V(x, \theta, \bar{\theta}) = V^\dagger(x, \theta, \bar{\theta})$

✓ The general form

$$\begin{aligned} V(x, \theta, \bar{\theta}) = & C + i\theta\chi - i\bar{\theta}\bar{\chi} + \theta\sigma^\mu\bar{\theta}v_\mu \\ & + \frac{i}{2}\theta\theta(M + iN) - \frac{i}{2}\bar{\theta}\bar{\theta}(M - iN) \\ & + i\theta\theta\bar{\theta}(\bar{\lambda} + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi) - i\bar{\theta}\bar{\theta}\theta(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}) \\ & + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}(D - \frac{1}{2}\partial^2C) \end{aligned}$$

real scalar field
Weyl spinor field
vector field

Superspace & Superfields

- Vector superfields
 - ✓ a chiral superfield Φ can construct a vector field.
 - ▶ example

$$\begin{aligned}\Phi + \Phi^\dagger = & (\phi + \phi^*) + \sqrt{2}\theta\psi + \sqrt{2}\bar{\theta}\bar{\psi} \\ & - i\theta\sigma^\mu\bar{\theta}\partial_\mu(\phi - \phi^*) + \theta\theta\mathcal{F} + \bar{\theta}\bar{\theta}\mathcal{F}^* \\ & + \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi - \frac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}\theta\sigma^\mu\partial_\mu\bar{\psi} \\ & - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial^2(\phi + \phi^*)\end{aligned}$$

Superspace & Superfields

- Vector superfields
 - ✓ SUSY gauge transformation $V \rightarrow V + i(\Phi - \Phi^\dagger)$
 - ✓ Wess-Zumino gauge
 - ▶ $C(x) = M(x) = N(x) = \chi(x) = \chi^\dagger(x) = 0$
 - ▶ reduces the number of the components
 - ▶ vector superfield under WZ gauge

$$V_{WZ} = \theta\sigma^\mu\bar{\theta}v_\mu(x) + i\theta\bar{\theta}\bar{\theta}\lambda(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\bar{\theta}\bar{\theta}\bar{\theta}D(x)$$

$$V_{WZ}^2 = [\theta\sigma^\mu\bar{\theta}v_\mu(x)][\theta\sigma^\nu\bar{\theta}v_\nu(x)] = \frac{1}{2}\theta\bar{\theta}\bar{\theta}\bar{\theta}v_\mu v^\mu$$

$$V_{WZ}^n = 0, n \geq 3$$

Supersymmetry Breaking

- Superalgebra

- anticommutation relation $\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma_{\alpha\dot{\beta}}^\mu P_\mu$

- ▶ Pauli matrices

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\{Q_1, \bar{Q}_1\} = 2(\sigma_{11}^0 P_0 + \sigma_{11}^1 P_1 + \sigma_{11}^2 P_2 + \sigma_{11}^3 P_3) = 2(P_0 + P_3)$$

$$\{Q_2, \bar{Q}_2\} = 2(\sigma_{22}^0 P_0 + \sigma_{22}^1 P_1 + \sigma_{22}^2 P_2 + \sigma_{22}^3 P_3) = 2(P_0 - P_3)$$

- Hamiltonian

$$\begin{aligned} H \equiv P^0 &= g^{00} P_0 = P_0 \\ &= \frac{1}{4}(\{Q_1 + \bar{Q}_1\} + \{Q_2, \bar{Q}_2\}) \end{aligned}$$

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Supersymmetry Breaking

- \mathcal{F} -term breaking and \mathcal{D} -term breaking

MSSM

- The **m**inimal **s**upersymmetry **s**tandard **m**odel
- vector fields → vector supermultiplets

Names	spin 1/2	spin 1	$SU(3)_C, SU(2)_L, U(1)_Y$
gluino, gluon	\tilde{g}	g	(8, , 1, 0)
winos, W bosons	$\widetilde{W}^\pm, \widetilde{W}^0$	W^\pm, W^0	(1, 3, 0)
bino, B boson	\widetilde{B}^0	B^0	(1, 1, 0)

MSSM

- matter fields → chiral supermultiplets
- 5 left-chiral supermultiplets for fermions
- 2 Higgs supermultiplets $Q = T_3 + Y$

Names		spin 0	spin 1/2	$SU(3)_C, SU(2)_L, U(1)_Y$
squarks, quarks (×3 families)	Q	$(\tilde{u}_L, \quad \tilde{d}_L)$	$(u_L, \quad d_L)$	$(\mathbf{3}, \quad \mathbf{2}, \quad \frac{1}{6})$
	\bar{u}	$\tilde{\bar{u}}_L = \tilde{u}_R^*$	$\bar{u}_L = u_R^\dagger$	$(\overline{\mathbf{3}}, \quad \mathbf{1}, \quad -\frac{2}{3})$
	\bar{d}	$\tilde{\bar{d}}_L = \tilde{d}_R^*$	$\bar{d}_L = d_R^\dagger$	$(\overline{\mathbf{3}}, \quad \mathbf{1}, \quad \frac{1}{3})$
sleptons, leptons (×3 families)	L	$(\tilde{\nu}, \quad \tilde{e}_L)$	$(\nu, \quad e_L)$	$(\mathbf{1}, \quad \mathbf{2}, \quad -\frac{1}{2})$
	\bar{e}	$\tilde{\bar{e}}_L = \tilde{e}_R^*$	$\bar{e}_L = e_R^\dagger$	$(\mathbf{1}, \quad \mathbf{1}, \quad 1)$
Higgs, higgsinos	H_u	$(H_u^+, \quad H_u^0)$	$(\tilde{H}_u^+, \quad \tilde{H}_u^0)$	$(\mathbf{1}, \quad \mathbf{2}, \quad +\frac{1}{2})$
	H_d	$(H_d^0, \quad H_d^-)$	$(\tilde{H}_d^0, \quad \tilde{H}_d^-)$	$(\mathbf{1}, \quad \mathbf{2}, \quad -\frac{1}{2})$

Hypercharge (Y)

- Gell-Mann-Nishijima formula
 - ✓ $Q = I^3 + Y/2$
 - ✓ Q : electric charge (in units of e)
 - ✓ I^3 : the 3rd component of isospin
- $Y = S + A$
 - ✓ S : strangeness
 - ✓ A : baryon number

Isospin

- Heisenberg
 - ✓ Neutron and proton are the two states of a single particle, the nucleon.
- I is a vector in isospin space
 - ✓ I_1, I_2, I_3 are three components
 - ✓ The nucleon carries isospin $1/2$, and the 3rd component I_3 has eigenvalues $+1/2$ (the proton) and $-1/2$ (the neutron)
 $p = | 1/2, 1/2 \rangle$: isospin up
 $n = | 1/2, -1/2 \rangle$: isospin down
- The strong interactions are invariant under rotation in isospin space.

MSSM

- Superpotential

$$W_{\text{MSSM}} = \sum_{i,j=1}^3 (\mathbf{y_u})_{ij} H_u Q_i \bar{u}_j - (\mathbf{y_d})_{ij} H_d Q_i \bar{d}_j \\ - (\mathbf{y_e})_{ij} H_d L_i \bar{e}_j + \mu H_u H_d$$

$$Q_i \bar{u}_j = Q_i^\alpha \bar{u}_{\alpha j}$$

$\alpha = 1, 2, 3$
color indices

- ✓ y_u, y_d, y_e : Yukawa couplings
 - 3x3 matrices
 - masses (up quarks, down quarks, and leptons)
- ✓ μ -term $\mu H_u H_d = \mu (H_u)_\alpha (H_d)_\beta \epsilon^{\alpha\beta}$

MSSM

- Example

- ✓ top quark, bottom quark, τ lepton

$$\mathbf{y_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}, \quad \mathbf{y_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad \mathbf{y_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$W_{\text{MSSM}} = y_t (\tilde{\bar{t}}_L \tilde{t}_L H_u^0 - \tilde{\bar{t}}_L \tilde{b}_L H_u^+) - y_b (\tilde{\bar{b}}_L \tilde{t}_L H_d^- - \tilde{\bar{b}}_L \tilde{b}_L H_d^0) - y_\tau (\tilde{\bar{\tau}}_L \tilde{\nu}_{\tau L} H_d^- - \tilde{\bar{\tau}}_L \tilde{\tau}_L H_d^0) + \mu (H_u^+ H_d^- - H_u^0 H_d^0)$$

- ✓ When $\langle \Omega | H_u^0 | \Omega \rangle > 0$, $\langle \Omega | H_d^0 | \Omega \rangle > 0$

- Yukawa couplings become masses.

$$y_t \tilde{\bar{t}}_L \tilde{t}_L H_u^0, \quad y_b \tilde{\bar{b}}_L \tilde{b}_L H_d^0, \quad y_\tau \tilde{\bar{\tau}}_L \tilde{\tau}_L H_d^0$$

MSSM

- Superpotential

$$W_{\Delta L=1} = \frac{1}{2} \lambda_e^{ijk} L_i L_j \bar{e}_k + \lambda_L^{ijk} L_i Q_j \bar{d}_k + \mu_L L_i H_u$$

$$W_{\Delta B=1} = \frac{1}{2} \lambda_B^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k$$

✓ SUSY

- baryon number and lepton number are conserved.
- $W_{\Delta L=1}$ and $W_{\Delta B=1}$ are not allowed
- introduce R-parity $R = (-1)^{3(B-L)+2s}$

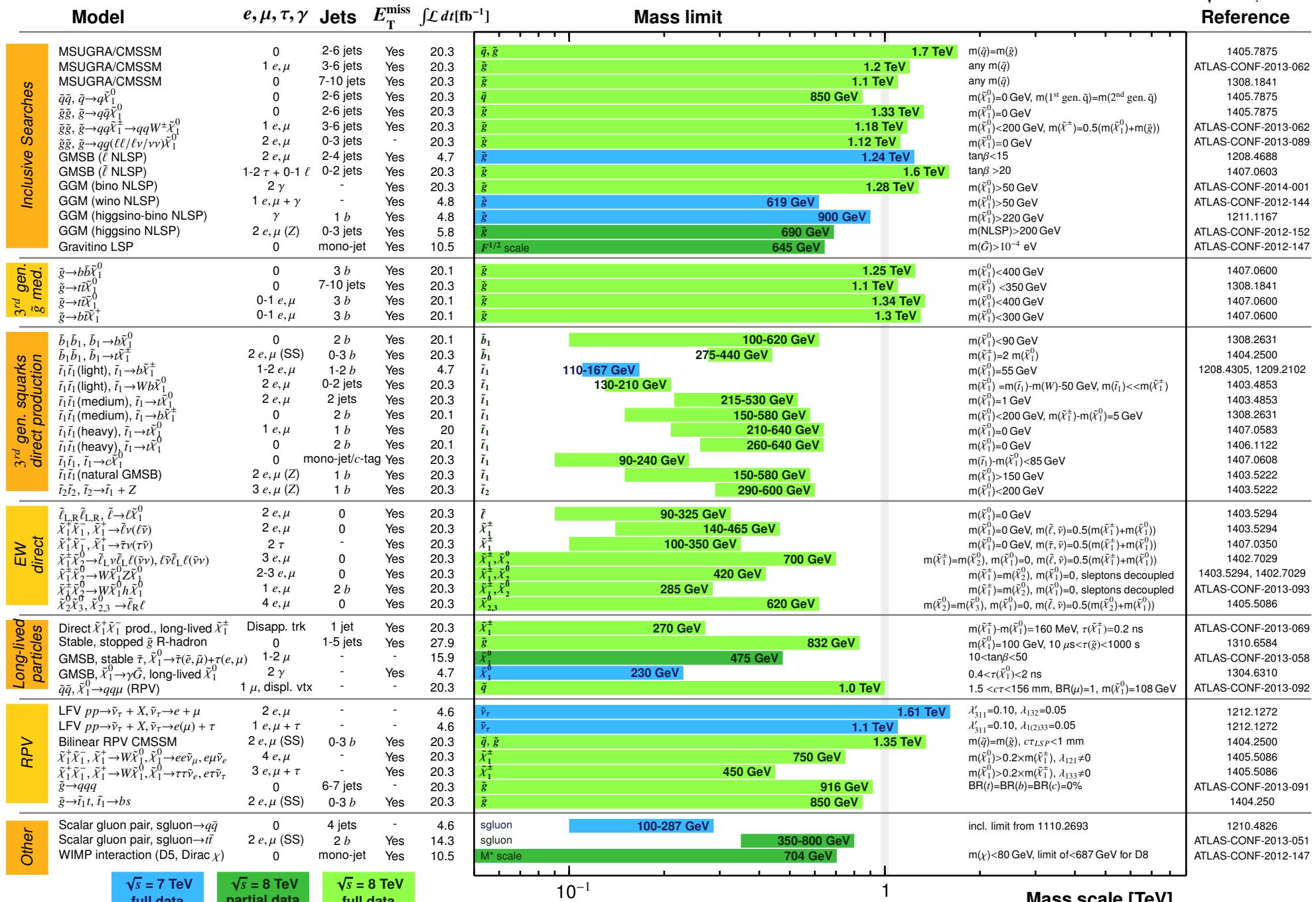
ATLAS SUSY Searches* - 95% CL Lower Limits

Status: ICHEP 2014

ATLAS Preliminary

$\sqrt{s} = 7, 8 \text{ TeV}$

Reference



$\sqrt{s} = 7 \text{ TeV}$
full data

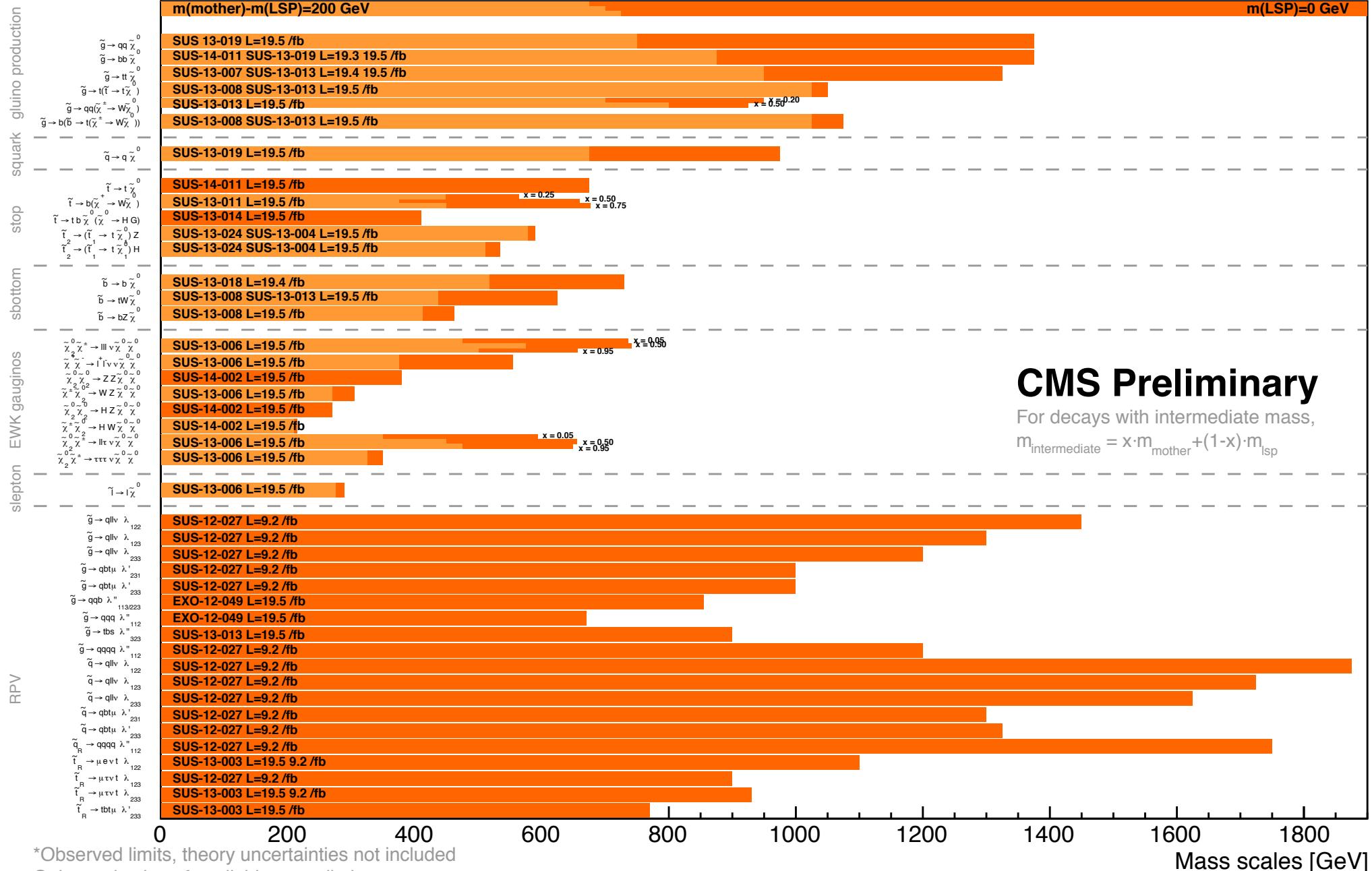
$\sqrt{s} = 8 \text{ TeV}$
partial data

$\sqrt{s} = 8 \text{ TeV}$
full data

*Only a selection of the available mass limits on new states or phenomena is shown. All limits quoted are observed minus 1σ theoretical signal cross section uncertainty.

Summary of CMS SUSY Results* in SMS framework

ICHEP 2014



CMS Preliminary

For decays with intermediate mass,
 $m_{\text{intermediate}} = x \cdot m_{\text{mother}} + (1-x) \cdot m_{\text{LSP}}$