Introduction to Supersymmetry

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Chapter 1

Motivations for supersymmetry

The Standard Model (SM) of particle physics was developed during the 1970s and it can explain much about elementary particles and the interactions that occur in our world. However, the SM is still incomplete and needs to be extended. In order to extend the SM, many different models of new physics have been proposed. Among all the possible theories, supersymmetry (SUSY) [1] plays an important role.

Many of the symmetries we know relate bosons to bosons and fermions to fermions but not bosons to fermions. SUSY is a kind of symmetry between bosons and fermions. The SUSY theory requires every boson to have a fermion partner, and vice versa. We can extend the SM into its SUSY version which says every SM boson (or fermion) has a fermonic (or bosonic) supersymmetry partner. In other words, the generators Q of SUSY must turn a bosonic state into a fermonic state, with

$$Q|\text{boson}\rangle = |\text{fermion}\rangle, \qquad Q|\text{fermion}\rangle = |\text{boson}\rangle \tag{1.1}$$

which also implies the generators of SUSY are anticommuting operators.

SUSY has the potential to provide explanations for some of the phenomena which the SM cannot explain, for example, the hierarchy problem, nonconvergent of the running coupling constants, and candidates of dark matter.

The running coupling constants are functions of energy as shown in figure 1.1. When energy increases, the strong coupling α_3 and the weak coupling α_2 go down, while the electromagnetic coupling α_1 goes up. In Grand Unified Theory (GUT), these three constants should be identical in strength when the energy is above the grand unification scale ($\approx 10^{16}$ GeV). Although there is no direct evidence showing the coupling constants converge, the theorists still believe it is true. However, these three constants do not converge in the Minimal Standard Model (MSM). If we extend the MSM into its supersymmetric version, Minimal Supersymmetric Standard Model (MSSM), they do converge well.

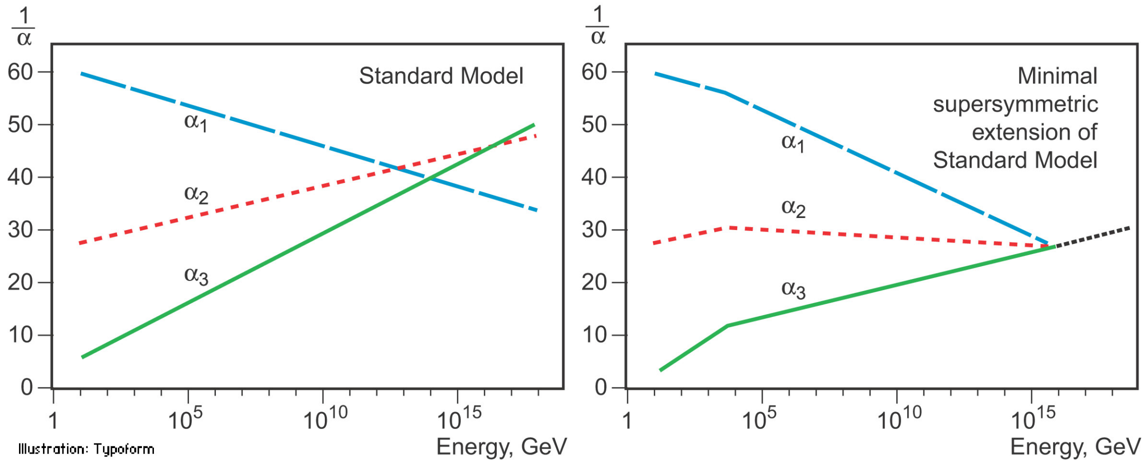

Figure 1.1: The coupling constants at the GUT scale in the Minimal Standard Model (left) and in Minimal supersymmetric Standard Model (right) [2].

Chapter 2

The Wess-Zumino model

2.1 Supersymmetry transformation

If the equation of motion is invariant under a transformation, we call this is a symmetry transformation. In other words, the action $S = \int d^4x\mathcal{L}$ is invariant under symmetry transformation. There are two ways to keep the action S invariant: the Lagrangian $\mathcal{L}$ is either invariant or changes by a total derivative $\mathcal{L} \to \mathcal{L}' = \mathcal{L} + \partial_\mu \Lambda^\mu$ where we assume the quantity Λ^μ vanishes at the boundary.

The simplest example of a supersymmetric theory in four dimensions consists of a massless complex scalar boson field ϕ and its superpartner which is a massless left-handed two-component Weyl fermion ψ without interactions between fields. We can write down the simplest action immediately [1]

$$S = \int d^4x\ (\mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}}) \tag{2.1}$$

$$\mathcal{L}_{\text{scalar}} = \partial^\mu\phi^*\partial_\mu\phi, \quad \mathcal{L}_{\text{fermion}} = i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\psi. \tag{2.2}$$

A SUSY transformation changes a boson field ϕ into a fermion field ψ. The simplest infinitesimal transformation of the scalar fields is

$$\delta\phi = \epsilon\psi, \quad \delta\phi^* = \epsilon^\dagger\psi^\dagger, \tag{2.3}$$

where ϵ^α is a constant, infinitesimal, anticommuting, two-component object with mass dimension $-\frac{1}{2}$. Similarly, a fermion field is turned into a scalar field under SUSY transformation

$$\delta\psi_\alpha = -i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi, \quad \delta\psi^\dagger_{\dot\alpha} = i(\epsilon\sigma^\mu)_{\dot\alpha}\partial_\mu\phi^*. \tag{2.4}$$

The variations of the scalar and fermion Lagrangian are

$$\begin{aligned}
\delta\mathcal{L}_{\text{scalar}} &= \partial^\mu(\delta\phi^*)\partial_\mu\phi + \partial^\mu\phi^*\partial_\mu(\delta\phi) \\
&= \epsilon\partial^\mu\psi\partial_\mu\phi^* + \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
\delta\mathcal{L}_{\text{fermion}} &= i(\delta\psi^\dagger)\overline{\sigma}^\mu\partial_\mu\psi + i\psi^\dagger\overline{\sigma}^\mu\partial_\mu(\delta\psi) \\
&= -\epsilon\sigma^\nu\partial_\nu\phi^*\overline{\sigma}^\mu\partial_\mu\psi + \psi^\dagger\overline{\sigma}^\mu\sigma^\nu\epsilon^\dagger\partial_\mu\partial_\nu\phi \\
&= -\epsilon\partial^\mu\psi\partial_\mu\phi^* - \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi + \partial_\mu(\epsilon\sigma^\mu\overline{\sigma}^\nu\psi\partial_\nu\phi^* - \epsilon\psi\partial^\mu\phi^* + \epsilon^\dagger\psi^\dagger\partial^\mu\phi).
\end{aligned} \tag{2.6}$$

[1] We define $\sigma^\mu \equiv (1, \vec{\sigma})$ and $\overline{\sigma}^\mu \equiv (1, -\vec{\sigma})$ where $\vec{\sigma}$ is the Pauli matrices.

The sum of $\delta\mathcal{L}_{\text{scalar}}$ and $\delta\mathcal{L}_{\text{fermion}}$ gives a total derivative so the action is invariant. Besides showing the action is invariant under SUSY transformation, we also need to show the SUSY algebra closes. However, equations (2.3) and (2.4) only valid for on-shell, which means $\overline{\sigma}^\mu \partial_\mu \psi = 0$. We have to introduce a new complex scalar field F, called the auxiliary field. The Lagrangian of the auxiliary field is defined as $\mathcal{L}_{\text{auxiliary}} = F^* F$. The auxiliary field transforms under SUSY as

$$\delta F = -i\epsilon^\dagger \overline{\sigma}^\mu \partial_\mu \psi, \quad \delta F^* = i\partial_\mu \psi^\dagger \overline{\sigma}^\mu \epsilon \tag{2.7}$$

and equation (2.4) has to be modified to

$$\delta\psi_\alpha = -i(\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi + \epsilon_\alpha F, \quad \delta\psi^\dagger_{\dot{\alpha}} = i(\epsilon\sigma^\mu)_{\dot{\alpha}} \partial_\mu \phi^* + \epsilon^\dagger_{\dot{\alpha}} F^*. \tag{2.8}$$

Then we can show that fields $X = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$ satisfies

$$(\delta_{\epsilon 2}\delta_{\epsilon 1} - \delta_{\epsilon 1}\delta_{\epsilon 2})X = i(-\epsilon_1 \sigma^\mu \epsilon_2^\dagger + \epsilon_2 \sigma^\mu \epsilon_1^\dagger)\partial_\mu X \tag{2.9}$$

and keeps the Lagrangian $\mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{auxiliary}}$ invariant under SUSY transformation for the off-shell case.

2.2 Supersymmetric Lagrangian with interactions

We have considered the free Lagrangian defined as

$$\mathcal{L}_{\text{free}} = \partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i \quad , \tag{2.10}$$

now let us consider the realistic theory which contains the interactions between fields. The general non-gauge interaction Lagrangian for chiral supermultiplets can be written as [2]

$$\mathcal{L}_{\text{int}} = \Big(-\frac{1}{2} W^{ij} \psi_i \psi_j + W^i F_i \Big) + \text{c.c} \tag{2.11}$$

where W^i and W^{ij} are defined as

$$W^i \equiv \frac{\partial W}{\partial \phi_i} \quad W^{ij} \equiv \frac{\partial W}{\partial \phi_i \partial \phi_j} \tag{2.12}$$

and W is the superpotential which is defined as

$$W = \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k. \tag{2.13}$$

The auxiliary fields F_i and F^{*i} can be replaced using equations of motion. From equations (2.10) and (2.11), we find the part of the $\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$ contains the auxiliary fields is $F^{*i} F_i + W^i F_i + W_i^* F^{*i}$, leading to the equations of motion

$$F_i = -W_i^*, \quad F^{*i} = -W^i. \tag{2.14}$$

Thus the Lagrangian becomes

$$\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}} = \partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i - \frac{1}{2}\Big(W^{ij} \psi_i \psi_j + W^*_{ij} \psi^{\dagger i} \psi^{\dagger j} \Big) - W^i W_i^*. \tag{2.15}$$

From equations (2.14) and (2.15), we know the scalar potential is given in terms of superpotential by

$$V(\phi, \phi^*) = W^i W_i^* = F^{*i} F_i \tag{2.16}$$

which is non-negative.

[2] c.c stands for complex conjugate.

Chapter 3

Superspace and superfields

The supersymmetric formalism can be recast in an elegant way by introducing superspace and superfields. A superspace is described by the four ordinary space-time coordinates x^μ and four anticommuting fermionic coordinates θ_α and $\overline{\theta}_{\dot{\alpha}}$ [1], where the spinor indices $\alpha = 1, 2$ and $\dot{\alpha} = 1, 2$. A superfield is a function of superspace coordinates: $S(x^\mu, \theta_\alpha, \overline{\theta}_{\dot{\alpha}})$, which contains bosonic, fermonic and auxiliary fields in the corresponding supermultiplet.

3.1 Superspace

Since θ_α and $\overline{\theta}_{\dot{\alpha}}$ anticommute among themselves, any product cannot contain more than two θ's or more than two $\overline{\theta}$'s. A general superfield can be expanded in terms of θ and $\overline{\theta}$

$$S(x, \theta, \overline{\theta}) = a + \theta\xi + \overline{\theta}\overline{\chi} + \theta\theta b + \overline{\theta}\overline{\theta} c + \overline{\theta}\overline{\sigma}^\mu \theta v_\mu + \theta\theta\overline{\theta}\overline{\zeta} + \overline{\theta}\overline{\theta}\theta\eta + \theta\theta\overline{\theta}\overline{\theta} d \tag{3.1}$$

where all spinor indices are suppressed. The 8 bosonic component fields a, b, c, d, and v_μ and 4 two-component fermion component fields $\xi, \overline{\chi}, \eta, \overline{\zeta}$ are complex functions of x^μ. To formulate SUSY transformation in superspace, we define the SUSY generators [2]

$$Q_\alpha = -i\frac{\partial}{\partial\theta^\alpha} - \sigma^\mu_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_\mu, \quad \overline{Q}_{\dot{\alpha}} = i\frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}} + \theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu \tag{3.2}$$

and they follow the commutation relations

$$\{Q_\alpha, \overline{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}}P_\mu = -2i\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu \tag{3.3}$$

$$\{Q_\alpha, Q_\beta\} = 0, \quad \{\overline{Q}_{\dot{\alpha}}, \overline{Q}_{\dot{\beta}}\} = 0. \tag{3.4}$$

Then the infinitesimal SUSY transformation can be written as

$$\begin{aligned}
\delta_\epsilon S(x, \theta, \overline{\theta}) &= S(x^\mu + i\epsilon\sigma^\mu\overline{\theta} + i\overline{\epsilon}\overline{\sigma}^\mu\theta, \theta + \epsilon, \overline{\theta} + \overline{\epsilon}) - S(x, \theta, \overline{\theta}) && (3.5)\\
&= (i\epsilon Q + i\overline{\epsilon}\overline{Q})S(x, \theta, \overline{\theta}) && (3.6)\\
&= \Big[\epsilon^\alpha\frac{\partial}{\partial\theta^\alpha} + \overline{\epsilon}_{\dot{\alpha}}\frac{\partial}{\partial\overline{\theta}_{\dot{\alpha}}} - i\big(\epsilon\sigma^\mu\overline{\theta} + \overline{\epsilon}\overline{\sigma}^\mu\theta\big)\Big]S(x, \theta, \overline{\theta}) && (3.7)
\end{aligned}$$

[1] Please refer to A.5.

[2] We also can define $Q^\alpha = i\frac{\partial}{\partial\theta_\alpha} + (\overline{\theta}\overline{\sigma}^\mu)^\alpha\partial_\mu$ and $\overline{Q}^{\dot{\alpha}} = -i\frac{\partial}{\partial\overline{\theta}_{\dot{\alpha}}} - (\overline{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu$

Now we introduce SUSY covariant derivatives D_α and $\overline{D}_{\dot{\alpha}}$ that anticommute with SUSY generators Q and $\overline{Q}$

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_\mu, \quad \overline{D}_{\dot{\alpha}} = (D_\alpha)^\dagger = \frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}} + i\theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu \tag{3.8}$$

with the commutation relations

$$\{D_\alpha, \overline{D}_{\dot{\beta}}\} = 2i\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu, \quad \{D_\alpha, D_\beta\} = \{\overline{D}_{\dot{\alpha}}, \overline{D}_{\dot{\beta}}\} = 0$$
$$\{D_\alpha, Q_\beta\} = \{D_\alpha, \overline{Q}_{\dot{\beta}}\} = \{\overline{D}_{\dot{\alpha}}, Q_\beta\} = \{\overline{D}_{\dot{\alpha}}, \overline{Q}_{\dot{\beta}}\} = 0$$

Then we can find $\delta_\epsilon(D_\alpha S) = D_\alpha(\delta_\epsilon S)$ and $\delta_\epsilon(\overline{D}_{\dot{\alpha}} S) = \overline{D}_{\dot{\alpha}}(\delta_\epsilon S)$.

3.2 Chiral superfields

The chiral superfields, which are irreducible representations of the SUSY algebra, can describe spin 0 bosons and spin 1/2 fermions – for example, the Higgs boson and the quarks and leptons. A superfield $\Phi(x, \theta, \overline{\theta})$ satisfies the condition

$$\overline{D}_{\dot{\alpha}}\Phi = 0 \tag{3.9}$$

is called a chiral (or left-chiral) superfield.[3] If we define the chiral coordinates (y^μ, θ) and antichiral coordinates $(\overline{y}^\mu, \overline{\theta})$, where

$$y^\mu = x^\mu + i\theta\sigma^\mu\overline{\theta}, \quad \overline{y}^\mu = x^\mu - i\theta\sigma^\mu\overline{\theta} \tag{3.11}$$

and the covariant derivative in (anti)chiral coordinates becomes

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + 2i\sigma^\mu_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\frac{\partial}{\partial y^\mu}, \quad \overline{D}_{\dot{\alpha}} = \frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}}. \tag{3.12}$$

then the general form of a chiral superfield is a function of y^μ and θ only. We can get rid of the $\overline{\theta}$ dependence in the chiral coordinates (y^μ, θ). Hence the general form of a chiral superfield can be expanded as

$$\Phi(y, \theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y). \tag{3.13}$$

By applying SUSY transformation on the chiral superfield Φ, through this $\delta_\epsilon\Phi = (i\epsilon Q + i\overline{\epsilon}\overline{Q})\Phi$, the component fields are found to be

$$\delta_\epsilon\phi = \sqrt{2}\epsilon\psi \tag{3.14}$$
$$\delta_\epsilon\psi_\alpha = i\sqrt{2}\partial_\mu\phi(\sigma^\mu\overline{\epsilon})_\alpha - \sqrt{2}\epsilon_\alpha F \tag{3.15}$$
$$\delta_\epsilon F = i\sqrt{2}\partial_\mu\psi\sigma^\mu\overline{\epsilon}. \tag{3.16}$$

[3] The complex conjugate of Φ, which is denoted as Φ^*, is called antichiral (or right-chiral) superfield and satisfies

$$D_\alpha\Phi^* = 0. \tag{3.10}$$

3.3 Vector superfields

In order to describe the spin 1 gauge bosons, we have to introduce the vector (real) superfields V which are defined as

$$V(x,\theta,\overline{\theta}) = V^{\dagger}(x,\theta,\overline{\theta}). \tag{3.17}$$

The general vector superfield $V(x,\theta,\overline{\theta})$ has the expansion

$$\begin{aligned} V(x,\theta,\overline{\theta}) = {} & C + i\theta\chi - i\overline{\theta}\overline{\chi} + \theta\sigma^{\mu}\overline{\theta}v_{\mu} + \frac{i}{2}\theta\theta(M+iN) - \frac{i}{2}\overline{\theta\theta}(M-iN) \\ & + i\theta\theta\overline{\theta}(\overline{\lambda} + \frac{i}{2}\overline{\sigma}^{\mu}\partial_{\mu}\chi) - i\overline{\theta\theta}\theta(\lambda - \frac{i}{2}\sigma^{\mu}\partial_{\mu}\overline{\chi}) + \frac{1}{2}\theta\theta\overline{\theta\theta}(D - \frac{1}{2}\partial^2 C) \end{aligned} \tag{3.18}$$

where $C, M, N,$ and D are real scalars, χ and λ are Weyl spinors, and v^{μ} is a vector field. Since there are too many component fields in a single supermultiplet, we want to reduce the number of them such that $V(x,\theta,\overline{\theta})$ is invariant under gauge transformation. Consider an abelian SUSY gauge transformation for a vector superfield

$$V \rightarrow V + i(\Phi - \Phi^{\dagger}) \tag{3.19}$$

where the gauge transformation parameter Φ is a chiral superfield. Then the component fields C, χ, M, N and one component of v_{μ} can be gauged away. This is called the Wess-Zumino gauge. By introducing Wess-Zumino gauge, we can reduce the number of components of the vector superfield as follows

$$V_{WZ} = \theta\sigma^{\mu}\overline{\theta}v_{\mu}(x) + i\theta\theta\overline{\theta\lambda}(x) - i\overline{\theta\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\overline{\theta\theta}D(x). \tag{3.20}$$

Since there is at least one θ in the V_{WZ}, the only non-vanishing power of V_{WZ} is

$$V_{WZ}^2 = \left[\theta\sigma^{\mu}\overline{\theta}v_{\mu}(x)\right]\left[\theta\sigma^{\nu}\overline{\theta}v_{\nu}(x)\right] = \frac{1}{2}\theta\theta\overline{\theta\theta}v_{\mu}v^{\mu} \tag{3.21}$$

and $V_{WZ}^n = 0, n \geq 3$.

Chapter 4

Supersymmetry breaking

SUSY theory says that all particles belonging to the same supermultiplet have the same mass. That is to say, the masses of SM particles and their superpartners are identical. However, it is not true in reality because we didn't find any sparticles, for example, there is no selectron with mass 0.51 MeV and a massless fermionic partner of the photon. Hence the supersymmetry must be spontaneously broken.

4.1 Vacuum expectation value in SUSY

The superalgebra tells us the anticommutation relation $\{Q_\alpha, \overline{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu$. So we have

$$\begin{aligned} \{Q_1, \overline{Q}_1\} &= 2(P_0 + P_3) \\ \{Q_2, \overline{Q}_2\} &= 2(P_0 - P_3) \end{aligned} \tag{4.1}$$

Then the Hamiltonian for SUSY is given by

$$H \equiv P^0 = \frac{1}{4}(\{Q_1, \overline{Q}_1\} + \{Q_2, \overline{Q}_2\}). \tag{4.2}$$

It follows that the expectation value of the Hamiltonian for an arbitrary state $|\Psi\rangle$ is a sum of squares and therefore non-negative

$$E_\Psi = \langle\Psi|H|\Psi\rangle = \frac{1}{4}\sum_{\alpha,\dot{\alpha}}(\|Q_\alpha|\Psi\rangle\|^2 + \|\overline{Q}_{\dot{\alpha}}|\Psi\rangle\|^2) \geq 0. \tag{4.3}$$

For a vacuum state (ground state) $|\Psi\rangle = |\Omega\rangle$, $E = 0$ if and only if $Q_\alpha|\Omega\rangle = \overline{Q}_{\dot{\alpha}}|\Omega\rangle = 0$. If a ground state has non-vanishing positive energy, the SUSY is broken spontaneously.

4.2 The F-term breaking and D-term breaking

A state $|\Omega\rangle$ is called a vacuum state when the scalar potential V has a minimum. There are two kinds of vacuums, the true vacuum and the false vacuum. If the minimum of V is the global minimum, then the vacuum is the true vacuum; otherwise it is a false vacuum. The scalar potential is

$$V(\phi, \phi^*) = F^{*i}F_i + \frac{1}{2}\sum_a D^a D_a \geq 0$$

where D_a is a real bosonic auxiliary field. [1] It is clear that when $F_i = D_a = 0$ then $V = 0$ is a global minimum.

Consider the SUSY transformation in equations (3.14), (3.15), (3.16), SUSY is broken if one of the $\delta\phi$, $\delta\psi$, $\delta F \neq 0$. A vacuum state is Lorentz invariant which implies all the space-time derivatives and all non-scalar fields must vanish. SUSY breaking conditions can be recast as $\langle F \rangle \neq 0$ which is called F-term breaking.

Now let's consider the simplest model, the O'Raifeartaigh model [3], of SUSY breaking. Given three chiral supermultiplets ϕ_1, ϕ_2, and ϕ_3, the Kähler potential and superpotential are

$$K = \phi_i^\dagger \phi_i, \quad W = g\phi_1(\phi_3^2 - m^2) + M\phi_2\phi_3, \quad M \gg m \tag{4.4}$$

From (2.14), we can find the auxiliary fields are

$$F^{*1} = -W^1 = \frac{\partial W}{\partial \phi_1} = g(\phi_3^2 - m^2) \tag{4.5}$$

$$F^{*2} = -W^2 = \frac{\partial W}{\partial \phi_2} = M\phi_3 \tag{4.6}$$

$$F^{*3} = -W^3 = \frac{\partial W}{\partial \phi_3} = 2g\phi_1\phi_3 + M\phi_2. \tag{4.7}$$

It is impossible to have $F^{*i} = 0$ simultaneously because $F^{*2} = 0$ contradicts $F^{*1} = 0$. Therefore SUSY must break.

[1] For more details about D^a, refer to appendix B.

Chapter 5

The minimal supersymmetry standard model

The Minimal Supersymmetry Standard Model (MSSM) is a supersymmetrization of the SM by introducing SUSY partners to every particle in the SM spectrum. The key word "minimal" means that we want to keep the number of superfields and interactions as small as possible.

5.1 The particle content of the MSSM

In the MSSM, the vector fields of the SM are assigned to vector supermultiplets and the matter fields of the SM are assigned to chiral supermultiplets. The scalar superpartners of fermions are called *squarks* and *sleptons* or collectively called *sfermions*, where the prefix '*s*' stands for scalar. For the fermionic superpartners of bosons, we append "-ino" to the SM particles. We usually add a tilde ($\widetilde{}$) on the symbol of a SM particle to denote its superpartner.

The SM fermions are described by five left-chiral superfields: Q_i contains the (s)quark $SU(2)$ doublets, $\overline{u}_i$ and $\overline{d}_i$ contain the (s)quark singlets [1], L_i contains the (s)lepton doublets, and $\overline{e}_i$ contains the (s)lepton singlets. The index $i = 1, 2, 3$ is the generation index because there are 3 generations of SM particles.

We introduce vector supermultiplets to describe the gauge sector of the SM. The fermonic superpartners of the gauge bosons are generally referred to as *gauginos* which include 8 glunios $\widetilde{g}$ as superpartners of 8 gluons of QCD, three winos $\widetilde{W}^+$, $\widetilde{W}^0$, $\widetilde{W}^-$ as superpartners of the $SU(2)_L$ gauge bosons, and a bino $\widetilde{B}^0$ as $U(1)_Y$ gaugino. Since the W^0 and B^0 gauge eigenstates mix to give mass eigenstates Z^0 and γ after the electroweak symmetry breaking, the corresponding gaugino mixtures are called zinos $\widetilde{Z}^0$ and photinos $\widetilde{\gamma}$.

Since the Higgs scalar boson has spin 0, it must be described by a chiral supermultiplet. For the fermionic partner of a Higgs chiral supermultiplet, we introduce two weak isospin doublets with weak hypercharge $Y = \frac{1}{2}$ and $Y = -\frac{1}{2}$. There are two main reasons to have two Higgs supermultiplets. If there were only one Higgs supermultiplet, the model will suffer from quadratic divergence because the condition for cancellation of gauge anomalies $Tr(T_3^2 Y) = Tr(Y^3) = 0$ isn't satisfied. However, the total contributions to the anomaly traces from two Higgs supermultiplets, one with each $Y = \pm\frac{1}{2}$, can cancel.

[1] The $SU(2)$ singlet super fields contain left-handed antifermions.

The other reason is that it is impossible to give masses to both up-type quarks [2] and down-type quarks [3] if there were only one Higgs supermultiplet. The two complex scalar fields are called H_u (with $Y = \frac{1}{2}$) and H_d (with $Y = -\frac{1}{2}$). Because the electric charge is the summation of the third component of weak isospin and the weak hypercharge, the components of H_u and H_d with $T_3 = (\frac{1}{2}, -\frac{1}{2})$ are denoted as (H_u^+, H_u^0) and (H_d^0, H_d^-).

Table 5.1 and table 5.2 summarise the chiral and gauge supermultiplets in the MSSM. In these two tables, the 'bar' on the fields is merely a label, signifying 'antiparticles'.The unbarred fields are the left-handed pieces of a Dirac spinor and barred fields the conjugate of the right-handed pieces of a Dirac spinor. For example, e stands for e_L and $\overline{e}$ stands for $e_R^\dagger$. They form a Dirac spinor

$$\begin{pmatrix} e_L \\ e_R \end{pmatrix} = \begin{pmatrix} e \\ \overline{e}^\dagger \end{pmatrix} \tag{5.1}$$

Names		spin 0	spin 1/2	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
squarks, quarks ($\times 3$ families)	Q	$(\widetilde{u}_L, \quad \widetilde{d}_L)$	$(u_L, \quad d_L)$	$(\mathbf{3}, \quad \mathbf{2}, \quad \frac{1}{6})$
	$\overline{u}$	$\widetilde{\overline{u}}_L = \widetilde{u}_R^*$	$\overline{u}_L = u_R^\dagger$	$(\overline{\mathbf{3}}, \quad \mathbf{1}, \quad -\frac{2}{3})$
	$\overline{d}$	$\widetilde{\overline{d}}_L = \widetilde{d}_R^*$	$\overline{d}_L = d_R^\dagger$	$(\overline{\mathbf{3}}, \quad \mathbf{1}, \quad \frac{1}{3})$
sleptons, leptons ($\times 3$ families)	L	$(\widetilde{\nu}, \quad \widetilde{e}_L)$	$(\nu, \quad e_L)$	$(\mathbf{1}, \quad \mathbf{2}, \quad -\frac{1}{2})$
	$\overline{e}$	$\widetilde{\overline{e}}_L = \widetilde{e}_R^*$	$\overline{e}_L = e_R^\dagger$	$(\mathbf{1}, \quad \mathbf{1}, \quad 1)$
Higgs, higgsinos	H_u	$(H_u^+, \quad H_u^0)$	$(\widetilde{H}_u^+, \quad \widetilde{H}_u^0)$	$(\mathbf{1}, \quad \mathbf{2}, \quad +\frac{1}{2})$
	H_d	$(H_d^0, \quad H_d^-)$	$(\widetilde{H}_d^0, \quad \widetilde{H}_d^-)$	$(\mathbf{1}, \quad \mathbf{2}, \quad -\frac{1}{2})$

Table 5.1: Chiral supermultiplet fields in the MSSM

Names	spin 1/2	spin 1	$SU(3)_C, SU(2)_L, U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\mathbf{8}, \quad ,\mathbf{1}, \quad 0)$
winos, W bosons	$\widetilde{W}^\pm$, $\widetilde{W}^0$	$W^\pm$, W^0	$(\mathbf{1}, \quad \mathbf{3}, \quad 0)$
bino, B boson	$\widetilde{B}^0$	B^0	$(\mathbf{1}, \quad \mathbf{1}, \quad 0)$

Table 5.2: Gauge supermultiplet fields in the MSSM

5.2 The superpotential of MSSM

The superpotential of the MSSM is

$$W_{\text{MSSM}} = \sum_{i,j=1}^{3} (\mathbf{y_u})_{ij} H_u Q_i \overline{u}_j - (\mathbf{y_d})_{ij} H_d Q_i \overline{d}_j - (\mathbf{y_e})_{ij} H_d L_i \overline{e}_j + \mu H_u H_d \tag{5.2}$$

[2]The up-type quarks are up, charm, and top quark.

[3]The down-type quarks are down, strange, and bottom quark.

where i and j are generation indecies and the dimensionless Yukawa couplings $\mathbf{y_u}$, $\mathbf{y_d}$, $\mathbf{y_e}$ are 3×3 matrices in family (generation) space. The colour indices have been suppressed, so that $Q_i \overline{u}_j$ is really $Q_i^\alpha \overline{u}_{\alpha j}$ where the superscript $\alpha = 1, 2, 3$ is the colour $\mathbf{3}$ (triplet) index and the subscript α is $\overline{\mathbf{3}}$ (anti-triplet) index. In the superpotential , W_{MSSM}, the first three terms give the mass to up quarks, down quarks and leptons via Yukawa couplings. And the μ-term, which is the supersymmetric version of the Higgs boson mass in the SM, is defined as $\mu H_u H_d = \mu (H_u)_\alpha (H_d)_\beta \epsilon^{\alpha\beta}$.

For example, since the top quark, bottom quark, and τ lepton are the heaviest fermions in the SM, we can consider an approximation that only the $(3,3)$ components of $\mathbf{y_u}$, $\mathbf{y_d}$, and $\mathbf{y_e}$ are non-zero.

$$\mathbf{y_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}, \quad \mathbf{y_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad \mathbf{y_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \tag{5.3}$$

The superpotential W is

$$\begin{aligned} W_{\mathrm{MSSM}} &= y_t(\widetilde{\overline{t}}_L \widetilde{t}_L H_u^0 - \widetilde{\overline{t}}_L \widetilde{b}_L H_u^+) - y_b(\widetilde{\overline{b}}_L \widetilde{t}_L H_d^- - \widetilde{\overline{b}}_L \widetilde{b}_L H_d^0) - y_\tau(\widetilde{\overline{\tau}}_L \widetilde{\nu}_{\tau L} H_d^- - \widetilde{\overline{\tau}}_L \widetilde{\tau}_L H_d^0) \\ &+ \mu(H_u^+ H_d^- - H_u^0 H_d^0) \end{aligned} \tag{5.4}$$

The minus signs in W were chosen as a convention so that the terms $y_t \widetilde{\overline{t}}_L \widetilde{t}_L H_u^0$, $y_b \widetilde{\overline{b}}_L \widetilde{b}_L H_d^0$, and $y_\tau \widetilde{\overline{\tau}}_L \widetilde{\tau}_L H_d^0$ are positive. These three terms will become the top, bottom, and τ lepton masses when H_u^0 and H_d^0 get VEVs [4].

In order to get the most general gauge-invariant and renormalizable superpotential, we should also consider $W_{\Delta L=1}$ and $W_{\Delta B=1}$ terms

$$W_{\Delta L=1} = \frac{1}{2} \lambda_e^{ijk} L_i L_j \overline{e}_k + \lambda_L^{ijk} L_i Q_j \overline{d}_k + \mu_L L_i H_u \tag{5.5}$$

$$W_{\Delta B=1} = \frac{1}{2} \lambda_B^{ijk} \overline{u}_i \overline{d}_j \overline{d}_k \tag{5.6}$$

The superfields Q_i carry baryon number $B = 1/3$, $\overline{u}, \overline{d}$ carry $B = -1/3$, and $B = 0$ for all others. The L_i carries lepton number $L = 1$, $\overline{e}_i$ carries $L = -1$, and $L = 0$ for all others. The superpotentials $W_{\Delta L=1}$ and $W_{\Delta B=1}$ break lepton number L and baryon number B, respectively. However, we have never observed B-violating and L-violating processes experimentally. For example, if both B and L were broken, the proton would decay very rapidly, but we have not seen this process experimentally [5]. Hence, we have to assume the baryon number and lepton number are conserved in the MSSM.

5.3 R-parity

Since the superpotentials $W_{\Delta L=1}$ and $W_{\Delta B=1}$ are not allowed in constructing SUSY models, we introduce a new symmetry called R-parity which has the effect to eliminate the B and L violating terms while allowing all the interactions of the MSSM. The R-party is defined as

$$R \equiv (-1)^{3(B-L)+2s} \tag{5.7}$$

[4]VEV stands for vacuum expectation value.

[5]The estimated half-life of proton is no less than 10^{32} years.

where s is the spin of the particle. All of the SM particles and the Higgs bosons have even R-parity ($R = +1$), while all of the sparticles have odd R-parity ($R = -1$). If the R-parity is conserved, SUSY predicts that sparticles are produced in pairs in collider experiments.

5.4 The lightest supersymmetric particle

The lightest sparticle in the MSSM is called lightest supersymmetric particle (LSP), which has $R = -1$ and is absolutely stable. Usually, LSP is electrically neutral; it doesn't join strong and electromagnetic interactions so it can be the candidate for dark matter. Since LSP is the lightest sparticle, all other sparticles decay into a state that contains at least one LSP [6] and any number of SM particles. In collider experiments, a LSP event will look like a neutrino, which cannot be detected, and carry away some energy and momentum. If the mass of the LSP is $m_{\tilde{\chi}_1^0}$, the missing energy for each SUSY event will be at least $2m_{\tilde{\chi}_1^0}$.

[6] Or an odd number of LSPs.

Chapter 6

Conclusion

Although the SM describes the world successfully, it still cannot answer some fundamental questions such as why the coupling constants do not converge at high energy. However, if the SM incorporates SUSY, the coupling constants do converge. The supersymmetric version of the SM requires that every SM particle has a superpartner with a different spin. A SM particle and its superpartner can be expressed in a supermultiplet. Every supermultiplet contains bosons and fermions with an equal number of degrees of freedom. There are two kinds of supermultiplets, a chiral and a vector supermultiplet. The chiral supermultiplets can describe spin 0 and spin 1/2 particles and the vector supermultiplet can express gauge bosons and gaugions. In the superspace, the supermultiplets are represented by superfields. For a general superfield $S(x, \theta, \overline{\theta})$, the SUSY transformation is $(i\epsilon Q + i\overline{\epsilon}\overline{Q})S(x, \theta, \overline{\theta})$ where ϵ is an anticommuting parameter and Q is the generator of SUSY transformation. If nature is exactly supersymmetric then the SM particles and its superpartners have the same mass. Because we didn't find any sparticles, then SUSY must be broken spontaneously.

Appendix A

Superalgebra

Since supersymmetric theory is based on superalgebra which is an extension of space-time Poincaré algebra, we first review some basic concepts of the Lorentz group, Poincaré group and spinor notation. At the end, we will present representations of the superalgebra, called supermultiplets.

A.1 Lorentz group

Since the Lorentz transformation combines rotations and boost, the infinitesimal Lorentz transformation matrix $U(\vec{\theta}, \vec{\phi})$ can be written as

$$U(\vec{\theta}, \vec{\phi}) \simeq 1 + i\vec{\theta} \cdot \vec{J} + i\vec{\phi} \cdot \vec{K} \tag{A.1}$$

where $\vec{\theta}$ is rotation angle, $\vec{\phi}$ is azimuth angle, and $\vec{J}$ and $\vec{K}$ are rotation and boost generators, respectively. The generators of the Lorentz group must satisfy the commutation relations

$$[J_i, J_j] = i\epsilon_{ijk}J_k, \quad [K_i, K_j] = -i\epsilon_{ijk}J_k, \quad [J_i, K_j] = i\epsilon_{ijk}K_k \tag{A.2}$$

where $i, j, k = 1, 2, 3$. It is very convenient to redefine the generators as

$$J_i^{\pm} = \frac{1}{2}(J_i \pm iK_i) \tag{A.3}$$

and the commutation relations become

$$[J_i^+, J_j^+] = i\epsilon_{ijk}J_k^+, \quad [J_i^-, J_j^-] = i\epsilon_{ijk}J_k^-, \quad [J_i^+, J_j^-] = 0 \tag{A.4}$$

which tell us the Lorentz group can be decomposed into the product of two independent $SU(2)$ groups.

A.2 Poincaré group

We need the unitary representations of a symmetry group which can preserve the translation probabilities between two eigenstates as measured in different reference frames. The irreducible representations of the Lorentz group is not unitary, the underlying symmetry group for particle physics is the Poincaré group. The Poincaré group is a product of the Lorentz group and the group of translations in space-time. We usually use the energy-momentum operator P_μ to denote the generators of the translation groups. If we define an antisymmetric second rank tensor generator $M_{\mu\nu}$ where the six components are the six Lorentz group generators, with $M_{ij} = \epsilon_{ijk}J_k$ and $M_{0i} = -M_{i0} = -K_i$. Then the commutation relations of the Poincaré group are

$$[P_\mu, P_\nu] = 0, \tag{A.5}$$

$$[M_{\mu\nu}, P_\lambda] = i(g_{\nu\lambda}P_\mu - g_{\mu\lambda}P_\nu), \tag{A.6}$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(g_{\mu\rho}M_{\nu\sigma} - g_{\mu\sigma}M_{\nu\rho} - g_{\nu\rho}M_{\mu\sigma} + g_{\nu\sigma}M_{\mu\rho}). \tag{A.7}$$

where the metric is

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

A.3 Spinors: Dirac, Magorana and Weyl fermion fields

The general state of a spin-1/2 particle can be expressed as a two-component column matrix, called a **spinor**:

$$\chi = c_+\chi_+ + c_-\chi_- = \begin{pmatrix} c_+ \\ c_- \end{pmatrix} \tag{A.8}$$

where c_+ and c_- are coefficients and

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{A.9}$$

representing spin up and spin down, respectively.

A **bi-spinor** is an object that consists of two spinors which belong to two different SU(2) groups. If a four-component field ψ_D is a solution of the Dirac equation $(i\gamma^\mu\partial_\mu - m)\psi = 0$, we call ψ_D a **Dirac spinor**. Instead of using a four-component column matrix, we can express ψ_D in terms of bi-spinors:

$$\psi_D = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{A.10}$$

where

$$\psi_L = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \psi_R = \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix} \tag{A.11}$$

are the left-handed and right-handed **Weyl spinors**. They transform under infinitesimal rotations $\vec{\theta}$ and boots $\vec{\beta}$ as

$$\psi_L \to \psi'_L = (1 - i\vec{\theta}\cdot\frac{\vec{\sigma}}{2} - \vec{\beta}\cdot\frac{\vec{\sigma}}{2})\psi_L; \tag{A.12}$$

$$\psi_R \to \psi'_R = (1 - i\vec{\theta}\cdot\frac{\vec{\sigma}}{2} + \vec{\beta}\cdot\frac{\vec{\sigma}}{2})\psi_R. \tag{A.13}$$

It is very convenient to choose the Weyl representation such that Weyl fermion fields (i.e. Weyl spinors) can be seen as the building blocks for any fermion filed.

Majorana found that if we choose all non-zero elements in all γ matrices as purely imaginary, we can obtain a real solution $\widetilde{\psi}_M$ of the Dirac equation [4]. A Majorana fermion field is defined through the Majorana condition

$$\widetilde{\psi}_M = \widetilde{\psi}_M^*. \tag{A.14}$$

in the Majorana representation.[1] We can use similarity transformation to rewrite the Majorana condition in a general representation as

$$\psi = C\overline{\psi}^T \tag{A.15}$$

which is saying a Majorana spinor is its own charge conjugate [2]. We can express a Majorana fermion field in terms of left-handed and right-handed Weyl spinors

$$\psi_M = \begin{pmatrix} \xi_\alpha \\ \overline{\xi}^{\dot{\alpha}} \end{pmatrix} = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \overline{\xi}^{\dot{\alpha}} \end{pmatrix} = \psi_{W,L} + \psi_{W,R} \tag{A.16}$$

where the right-handed Weyl spinor $\overline{\xi}^{\dot{\alpha}}$ is the Hermitian conjugate of the left-handed Weyl spinor ξ_α. [3]

Because the spinors anti-commute, we can derive the following identities for two spinors ψ and χ:

$$\begin{aligned}
&\psi\chi = \chi\psi, \quad \overline{\psi}\overline{\chi} = \overline{\chi}\overline{\psi}, \quad (\psi\chi)^\dagger = \overline{\psi}\overline{\chi} \\
&\chi\sigma^\mu\overline{\psi} = -\overline{\psi}\overline{\sigma}^\mu\chi, \quad (\chi\sigma^\mu\overline{\psi})^\dagger = \psi\sigma^\mu\overline{\chi} \\
&\chi\sigma^\mu\overline{\sigma}^\nu\psi = \psi\sigma^\nu\overline{\sigma}^\mu\chi, \quad (\chi\sigma^\mu\overline{\sigma}^\nu\psi)^\dagger = \overline{\psi}\overline{\sigma}^\nu\sigma^\mu\overline{\chi}
\end{aligned} \tag{A.17}$$

A.4 Helicity and chirality

Both helicity and chirality mean "handedness". Helicity is the projection of angular momentum $\vec{J}$ onto the direction of momentum $\hat{p}$. Since the orbital angular momentum $\vec{L}$ is perpendicular to the direction of momentum, it therefore does not contribute to helicity.

$$h = \vec{J}\cdot\hat{p} = (\vec{L}+\vec{S})\cdot\hat{p} = \vec{S}\cdot\hat{p}, \quad \hat{p} = \frac{\vec{p}}{|\vec{p}|} \tag{A.18}$$

It can be easily seen that the eigenvalues of h are ± 1. An eigenstate with eigenvalue -1 is called "left-handed" and an eigenstate with eigenvalue $+1$ is called "right-handed" as shown in figure A.1. Although helicity is invariant under rotations, it is not invariant under boosts. The only exception is when particles are massless, the helicity is Lorentz invariant because massless particles would move at the speed of light.

The chirality is related to the matrix γ_5 which is defined as

$$\gamma_5 = \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{A.19}$$

[1] We use tilde notation on the top to represent Majorana states.

[2] The charge conjugation, C, converts each particle into its antiparticle: $C|\psi\rangle = |\overline{\psi}\rangle$. For C can be applied to a neutral particle, and it changes the sign of all the internal quantum numbers – charge, baryon number, lepton number, strangeness, charm, beauty, truth – while leaving mass, energy, momentum and spin unchanged

[3] The two distinct types of spinor indices $\alpha = 1, 2$ and $\dot{\alpha} = 1, 2$.

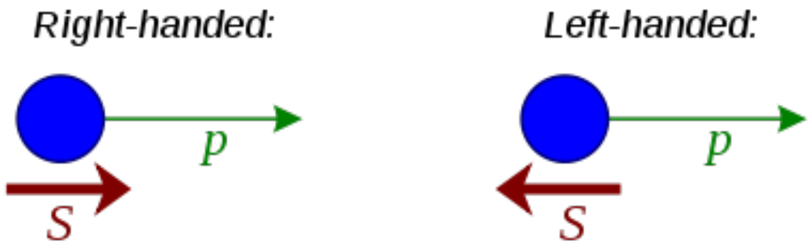

Figure A.1: Helicity. On the left hand side, spin and momentum are parallel (helicity +1); on the right hand side they are antiparallel (helicity −1) [5].

and has the following properties

$$\{\gamma_5, \gamma_\mu\} = 0, \quad \gamma_5^\dagger = \gamma_5, \quad (\gamma_5)^2 = 1 \tag{A.20}$$

where the γ matrices are defined in [6]. The chirality operators are defined as

$$P_L = \frac{1-\gamma_5}{2}, \quad P_R = \frac{1+\gamma_5}{2} \tag{A.21}$$

A particle bi-spinor ψ consists of left-chiral and right-chiral parts,

$$\psi = \psi_L + \psi_R \tag{A.22}$$

where

$$\psi_L = P_L\psi, \quad \psi_R = P_R\psi \tag{A.23}$$

It is easy to show that $P_L\psi_R = 0$ and $P_R\psi_L = 0$. The chirality of the field has the property that a left-chiral solution of the Dirac equation remains left-chiral under Lorentz transformations and likewise for a right-chiral field. Since γ_5 anticommutes with all the other γ matrices, the chirality is not conserved because of the mass term in the Dirac Hamiltonian $H = \gamma^0(\gamma^i p_i + m)$. If we are considering massless particles, the difference between helicity and chirality disappears [7].

A.5 Grassmann numbers

The Grassmann numbers are anticommuting objects

$$\{\theta_\alpha, \theta_\beta\} = \{\overline{\theta}_{\dot\alpha}, \overline{\theta}_{\dot\beta}\} = \{\theta_\alpha, \overline{\theta}_{\dot\beta}\} = 0 \tag{A.24}$$

where $\alpha, \beta, \dot\alpha$, and $\dot\beta = 1, 2$ [4] We can easily prove the following identities:

$$\begin{aligned}
&\theta_\alpha\theta_\beta = \frac{1}{2}\epsilon_{\alpha\beta}\theta\theta, \quad \overline{\theta}_{\dot\alpha}\overline{\theta}_{\dot\beta} = -\frac{1}{2}\epsilon_{\dot\alpha\dot\beta}\overline{\theta\theta}, \quad \theta_\alpha\overline{\theta}_{\dot\beta} = \frac{1}{2}\sigma^\mu_{\alpha\dot\beta}(\overline{\theta}\overline{\sigma}_\mu\theta) \\
&(\theta\psi)(\theta\chi) = -\frac{1}{2}\theta\theta\psi\chi, \quad (\overline{\theta\psi})(\overline{\theta\chi}) = -\frac{1}{2}\overline{\theta\theta\psi\chi}, \quad (\theta\psi)(\overline{\theta\chi}) = \frac{1}{2}(\overline{\theta}\overline{\sigma}^\mu\theta)(\psi\sigma_\mu\overline{\chi}) \\
&(\theta\sigma^\mu\overline{\theta})(\theta\sigma^\nu\overline{\theta}) = \frac{1}{2}\theta\theta\overline{\theta\theta}g^{\mu\nu}.
\end{aligned} \tag{A.25}$$

[4] $\overline{\theta}_{\dot\alpha} = \theta^\dagger_\alpha$

Derivatives with respect to the Grassmann numbers are defined by

$$\frac{\partial}{\partial\theta^\alpha}(\theta^\beta) = \delta_\alpha^\beta, \quad \frac{\partial}{\partial\overline{\theta}_{\dot\alpha}}(\overline{\theta}_{\dot\beta}) = \delta_{\dot\beta}^{\dot\alpha}, \quad \frac{\partial}{\partial\theta^\alpha}(\overline{\theta}_{\dot\beta}) = 0, \quad \frac{\partial}{\partial\overline{\theta}_{\dot\alpha}}(\theta^\beta) = 0 \tag{A.26}$$

and the integrations are defined as

$$\int d\theta_\alpha = 0, \quad \int d\theta_\alpha \ \theta_\alpha = 1. \tag{A.27}$$

If we have a general function f which is linear in θ_α:

$$f(\theta_\alpha) = f_0 + \theta_\alpha f_1, \tag{A.28}$$

where f_0 and f_1 can be functions of other commuting or anticommuting variables but not functions of θ_α. We can find differentiation and integration give the same results for Grassmann numbers

$$\frac{df}{d\theta_\alpha} = f_1, \quad \int d\theta_\alpha f(\theta_\alpha) = f_1. \tag{A.29}$$

We also define

$$d^2\theta = \frac{1}{2}d\theta^1 d\theta^2, \quad d^2\overline{\theta} = \frac{1}{2}d\overline{\theta}^2 d\overline{\theta}^1 = (d^2\theta)^\dagger \tag{A.30}$$

such that

$$\int d^2\theta \ \theta\theta = \int d^2\overline{\theta} \ \overline{\theta\theta} = 1. \tag{A.31}$$

A.6 Supermultiplets

The enlarged Poincaré group, called super-Poincaré group, has generators Q_α^I and its conjugate $\overline{Q}_{\dot\alpha}^I$ which obey the commutation relations

$$[P_\mu, Q_\alpha^I] = 0, \quad [P_\mu, \overline{Q}_{\dot\alpha}^I] = 0, \tag{A.32}$$

$$[M_{\mu\nu}, Q_\alpha^I] = i(\sigma_{\mu\nu})_\alpha{}^\beta Q_\beta^I, \quad [M_{\mu\nu}, \overline{Q}^{I\dot\alpha}] = i(\overline{\sigma}_{\mu\nu})^{\dot\alpha}{}_{\dot\beta}\overline{Q}^{I\dot\beta} \tag{A.33}$$

$$\{Q_\alpha^I, \overline{Q}_{\dot\beta}^J\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \delta^{IJ} \tag{A.34}$$

$$\{Q_\alpha^I, Q_\beta^J\} = \epsilon_{\alpha\beta} Z^{IJ}, \quad \{\overline{Q}_{\dot\alpha}^I, \overline{Q}_{\dot\beta}^J\} = \epsilon_{\dot\alpha\dot\beta}(Z^{IJ})^* \tag{A.35}$$

where $I, J = 1, \ldots, N$ and the central charge $Z^{IJ} = -Z^{JI}$ is nonvanishing only for $N > 1$ and commutes with all generators. Although there is no upper limit on N from the algebraic point of view, N must be less than or equal to 8 in order to have physical meaning. We only discuss the simplest case $N = 1$.

From the commutation relations listed above, it is easy to show that $[J_3, Q_1^I] = \frac{1}{2}Q_1^I$ and $[J_3, Q_2^I] = -\frac{1}{2}Q_2^I$. Similarly, $[J_3, (Q_1^I)^\dagger] = -\frac{1}{2}(Q_1^I)^\dagger$ and $[J_3, (Q_2^I)^\dagger] = \frac{1}{2}(Q_2^I)^\dagger$. Hence we can conclude that Q_1^I and $(Q_2^I)^\dagger$ raise the z-component of the spin (helicity) by $\frac{1}{2}$ and Q_2^I and $(Q_1^I)^\dagger$ lower it by $\frac{1}{2}$.

The single particle states of a SUSY theory correspond to irreducible representations of super-Poincaré algebra, called supermultiplets. Every supermultiplet contains both bosons and fermions with an equal number of degrees of freedom; and all particles belonging to the same supermultiplet have the same mass because P^2 commutes with all generators of the SUSY algebra. If we consider a massless supermultiplet in unextended

SUSY ($N = 1$), than there are three kinds of multiplets: $(0, \frac{1}{2}) \otimes (-\frac{1}{2}, 0)$, the chiral multiplet, which contains a complex scalar and a Weyl fermion; $(\frac{1}{2}, 1) \otimes (-1, -\frac{1}{2})$, the vector multiplet, which contains a gauge boson and a Weyl fermion; $(\frac{3}{2}, 2) \otimes (-2, -\frac{3}{2})$, the graviton multiplet, which contains a graviton and a gravitino.

For a massive case, there are 4 states, labeled by their helicities (or rather the z component of the angular momentum): $(-\frac{1}{2}, 0, 0, \frac{1}{2})$ and the same for its CPT conjugate $(-\frac{1}{2}, 0, 0, \frac{1}{2})$, and $(-1, -\frac{1}{2}, -\frac{1}{2}, 0)$ and its CPT conjugate $(1, \frac{1}{2}, \frac{1}{2}, 0)$. The detailed discussions about massless and massive supermultiplets can be found in many references such as [8] and [9].

Appendix B

Supersymmetric gauge theories

Since all the known interactions are gauge interactions, we have to extend gauge interactions to their supersymmetric version. This can be divided into two cases, one for Abelian and one for non-Abelian.

B.1 Lagrangians for gauge superfields

A vector superfield V which is used to describe the gauge bosons contains a massless gauge boson field A^a_μ [1] and a gaugino field λ^a. Under the gauge transformation, they transform as

$$A^a_\mu \to A^a_\mu + \partial_\mu \Lambda^a + g f^{abc} A^b_\mu \Lambda^c, \tag{B.1}$$

$$\lambda^a \to \lambda^a + g f^{abc} \lambda^b \Lambda^c \tag{B.2}$$

where Λ^a is an infinitesimal gauge transformation parameter, g is the gauge coupling constant, and f^{abc} is the antisymmetric structure constant of the gauge group. Because gauge transformation removes one degree of freedom from A^a_μ, we have to introduce a real bosonic auxiliary field D^a. This gauge auxiliary field has mass dimension 2 and no kinetic term, so it can be eliminated using the equation of motion. The Lagrangian density of a gauge superfield is

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + i\lambda^{\dagger a} \overline{\sigma}^\mu \nabla_\mu \lambda^a + \frac{1}{2} D^a D^a, \tag{B.3}$$

where the gauge field strength $F^a_{\mu\nu}$ is defined as

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{B.4}$$

and

$$\nabla_\mu \lambda^a = \partial_\mu \lambda^a + g f^{abc} A^b_\mu \lambda^c \tag{B.5}$$

is the covariant derivative of the gaugino field. If we apply SUSY transformation to the fields A^a_μ, λ^a, and D^a, we can find the $\mathcal{L}_{\text{gauge}}$ is supersymmetric.

[1] The field A^μ is defined as $(V, \vec{A})$ which is the conventions in electrodynamics.

B.2 Supersymmetric gauge interactions

When gauge interactions are involved, we have to promote the ordinary derivative to a gauge-covariant derivative

$$\partial_\mu \psi_i \to \nabla_\mu \psi_i = \partial_\mu \psi_i - i g_a A^a_\mu T^{aj}_i \psi_j \tag{B.6}$$

where g_a is the gauge coupling, T^a is the generator matrix, and A^a_μ is the vector field. And the SUSY transformations are modified to

$$\delta\phi_i = \epsilon\psi_i \tag{B.7}$$

$$\delta(\psi_i)_\alpha = -i(\sigma^\mu \epsilon^\dagger)_\alpha \nabla_\mu \phi_i + \epsilon_\alpha F_i \tag{B.8}$$

$$\delta F_i = -i\epsilon^\dagger \overline{\sigma}^\mu \nabla_\mu \psi_i + \sqrt{2} g (T^a \phi)_i \epsilon^\dagger \lambda^{\dagger a} \tag{B.9}$$

With the help of the extra term $\sqrt{2}g(T^a\phi)_i\epsilon^\dagger\lambda^{\dagger a}$ in δF_i, the total Lagrangian density

$$\begin{aligned}\mathcal{L} = {} & \nabla^\mu \phi^{*i} \nabla_\mu \phi_i + i\psi^{\dagger i}\overline{\sigma}^\mu \nabla_\mu \psi_i - \frac{1}{2}\Big(W^{ij}\psi_i\psi_j + W^*_{ij}\psi^{\dagger i}\psi^{\dagger j}\Big) - W^i W^*_i \\ & - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\lambda^{\dagger a}\overline{\sigma}^\mu \nabla_\mu \lambda^a + \frac{1}{2}D^a D^a \\ & - \sqrt{2}g(\phi^* T^a \psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger T^a \phi) + g(\phi^* T^a \phi) D^a\end{aligned} \tag{B.10}$$

is invariant under the SUSY transformations. From this Lagrangian, we can combine the $D^a D^a/2$ term with $g(\phi^* T^a \phi)D^a$ to find the equations of motion for the auxiliary field D^a

$$D^a = -g(\phi^* T^a \phi). \tag{B.11}$$

Replacing the auxiliary field in (B.7) using (B.11), we can find that the scalar potential is

$$V(\phi,\phi^*) = F^{*i}F_i + \frac{1}{2}\sum_a D^a D_a = W^i W^*_i + \frac{1}{2}\sum_a g_a^2 (\phi^* T^a \phi)^2. \tag{B.12}$$

Since $V(\phi,\phi^*)$ is a sum of square terms and it is non-negative.

B.3 Superspace Lagrangians for Abelian and non-Abelian gauge theory

Although we have obtained the Lagrangian for Abelian gauge theory in (B.10), there is another way to compute the Lagrangians using the gauge-invariant Abelian field strength superfields which is defined as

$$W_\alpha = -\frac{1}{4}\overline{D}\overline{D}D_\alpha V, \quad \overline{W}_{\dot\alpha} = -\frac{1}{4}DD\overline{D}_{\dot\alpha} \tag{B.13}$$

with mass dimension 3/2. Since $D^3 = \overline{D}^3 = 0$, from (3.9) and (3.10) we know W_α is chiral and $\overline{W}_{\dot\alpha}$ is antichiral. Because W_α is gauge invariant, the easiest way to derive the general expression is using the Wess-Zumino gauge. After converting the vector superfields V in (3.20) to $(y^\mu, \theta_\alpha, \overline{\theta}_{\dot\alpha})$ coordinate, we get

$$V(y,\theta,\overline{\theta})|_{\mathrm{WZ}} = \theta\sigma^\mu\overline{\theta}A_\mu(y) + i\theta\theta\overline{\theta\lambda}(y) - i\overline{\theta\theta}\theta\lambda(y) + \frac{1}{2}\theta\theta\overline{\theta\theta}[D(y) - i\partial_\mu A^\mu(y)] \tag{B.14}$$

and

$$W_\alpha = -i\lambda_\alpha(y) + \theta_\alpha D(y) + \frac{i}{2}(\sigma^\mu\overline{\sigma}^\nu\theta)_\alpha F_{\mu\nu}(y) - i\theta\theta(\sigma^\mu\partial_\mu\overline{\lambda}(y))_\alpha \tag{B.15}$$

where $F_{\mu\nu}$ is the field strength. Since W_α is a chiral superfield, we can calculate the SUSY invariant Lagrangian

$$\mathcal{L} = \int d^2\theta\ \frac{1}{4}[W^\alpha W_\alpha] + c.c = \frac{1}{2}D^2 + i\overline{\lambda}\overline{\sigma}^\mu\partial_\mu\lambda - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \tag{B.16}$$

and agree with (B.3).

For a non-Abelian gauge group, the vector superfields transform as

$$e^V \to e^{i\Lambda^\dagger}e^V e^{-i\Lambda} \tag{B.17}$$

$$e^{-V} \to e^{i\Lambda}e^{-V}e^{-i\Lambda^\dagger} \tag{B.18}$$

and the field strength chiral superfield becomes

$$W_\alpha = -\frac{1}{4}\overline{DD}(e^{-V}D_\alpha e^V) \tag{B.19}$$

and it transforms under supergauge transformations as

$$W_\alpha \to e^{i\Lambda}W_\alpha e^{-i\Lambda}. \tag{B.20}$$

After applying Wess-Zumino gauge, the component expansion of W_α is

$$W_\alpha = -i\lambda_\alpha(y) + \theta_\alpha D(y) + \frac{i}{2}(\sigma^\mu\overline{\sigma}^\nu\theta)_\alpha F_{\mu\nu}(y) - i\theta\theta(\sigma^\mu\nabla_\mu\overline{\lambda}(y))_\alpha \tag{B.21}$$

which is the same as (B.15) except the ordinary derivatives of the field turn into gauge corivant derivatives and the gauge field strength $F_{\mu\nu}$ is defined in (B.4). If we introduce complex coupling constant

$$\tau = \frac{\Theta}{2\pi} + \frac{4\pi i}{g^2} \tag{B.22}$$

where g is the gauge coupling constant and Θ is Θ-angle. Then the general Lagrangian for supersymmetric gauge theory is

$$\begin{aligned}\mathcal{L} &= \frac{1}{32\pi}\Im(\tau\int d^2\theta\ \mathrm{Tr}W^\alpha W_\alpha) \\ &= \mathrm{Tr}\big(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\lambda\sigma^\mu D_\mu\overline{\lambda} + \frac{1}{2}D^2\big) + \frac{\Theta}{32\pi^2}g^2\ \mathrm{Tr}F_{\mu\nu}\widetilde{F}^{\mu\nu}\end{aligned} \tag{B.23}$$

where $\widetilde{F}^{\mu\nu}$ is the dual field strength

$$\widetilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}. \tag{B.24}$$

Appendix C

The search for supersymmetry

The large hadron collider (LHC) is a proton-proton collider with centre of mass energy up to 14 TeV. The LHC was built to search for Higgs bosons but it also has potential to probe new physics beyond the SM. If the predictions of SUSY theories were correct, we might find sparticles produced by the LHC. ATLAS and CMS experiments are two of the four important experiments at the LHC and the current SUSY status will be described for both experiment below.

C.1 SUSY in ATLAS

The ATLAS detector consists of inner detector, calorimeter, muon detector, and magnetic system. The inner detector gives accurate momentum and vertex measurements, the calorimeters have excellent energy and position resolutions for charged particles, and the muon detector measures the energy of mouns. More details about the ATLAS detector can be found in Ref. [10]. If R-parity is conserved, SUSY predicts that sparticles are produced in pairs in collider experiments and all heavy sparticles must decay to a state with an odd number of LSP. The ATLAS collaboration can use the events with multiple leptons and missing transverse energy from LSP in the final state to search charginos [1], neutralinos [2], and sleptons in ATLAS.

Since a typical SUSY model involves many sparticles with different masses and there are different possible ways for each to decay, we have to simplify the models to study a specific decay chain with the assumed branching ratio. We also need to understand and accurately model the SM background because the SM processes can mimic the SUSY signal. The background can be reducible or irreducible, where the irreducible background has the same final state as signal. The main backgrounds are W/Z + jets, $t\bar{t}$, and dibosons events. Several methodologies were developed to model the possible backgrounds. So far, there is no evidence for SUSY has been observed with the dataset collected by ATLAS. Figure C.1 shows the newest SUSY research at ATLAS. If SUSY does exist at the TeV scale, the signals can be found when the LHC operates at higher energy.

[1] Chargions $\widetilde{\chi}^{\pm}_{1,2}$ are the mass eigenstates of the charged higgsino and charged winos.

[2] Neutralinos $\widetilde{\chi}^{0}_{1,2,3,4}$ are the mass eigenstates of the neutral gauginos and neutral higgsinos.

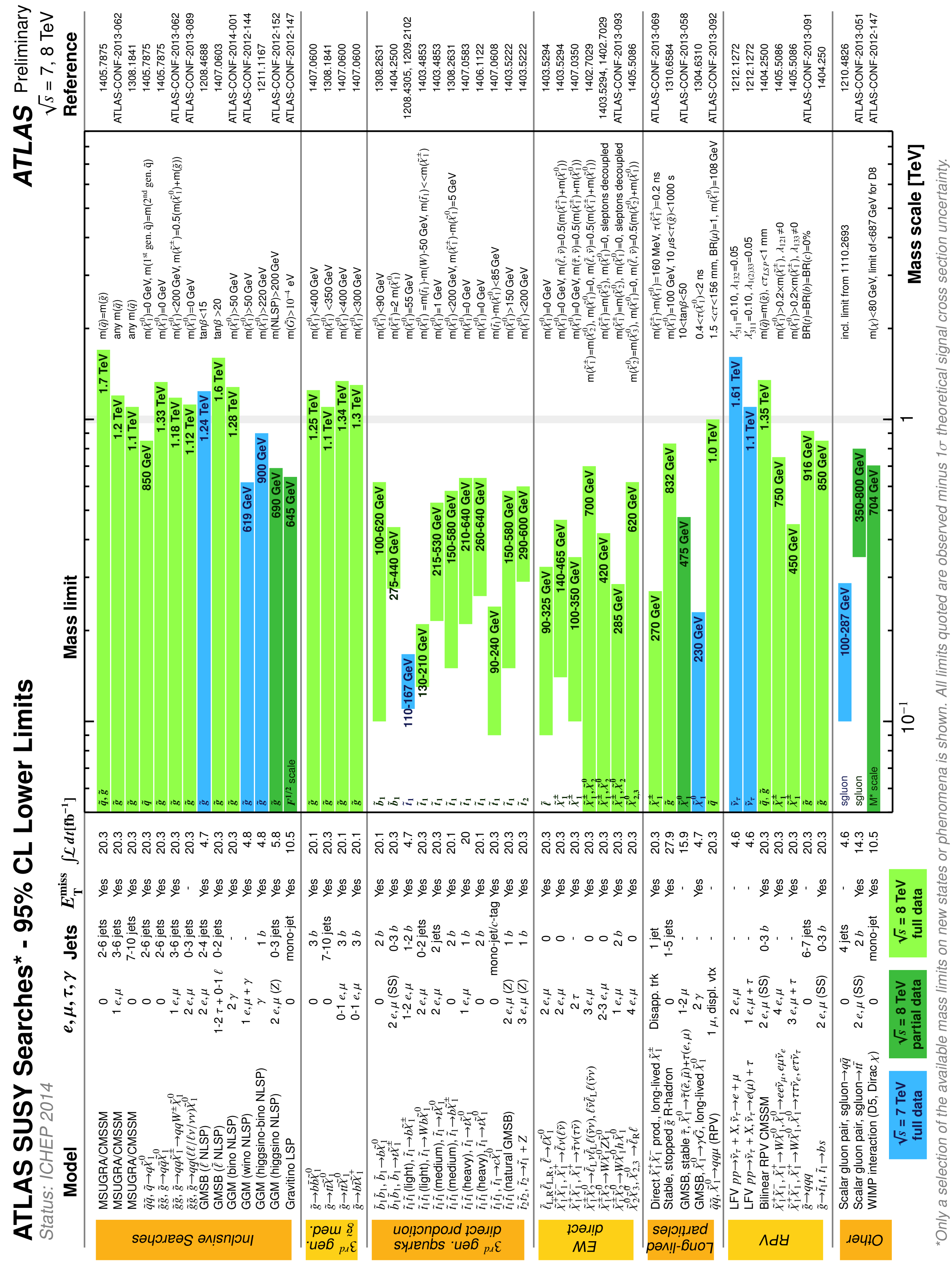

Figure C.1: The mass limits of sparticles in the ATLAS SUSY searches with 95% confidence level [11].

C.2 SUSY in CMS

Similar to the ATLAS detector, the Compact Muon Solenoid (CMS) particle detector is a general-purpose detector at LHC. The central feature of the CMS apparatus is a superconducting solenoid, of 6 m internal diameter, providing a magnetic field of 3.8 T. Within the magnetic field volume are a silicon pixel and strip tracker, a crystal electromagnetic calorimeter, and a brass-scintillator hadron calorimeter. Muons are measured in gas-ionization detectors embedded in the steel flux-return yoke located outside the solenoid. A detailed description of the CMS detector can be found in Ref. [12]. Although the CMS experiment has similar scientific goals as the ATLAS experiment, it has different design in the detector and might use different methods to analyse the collected data. The CMS experiment doesn't find any sparticles but it provides the limits for some possible channel as shown in figure C.2.

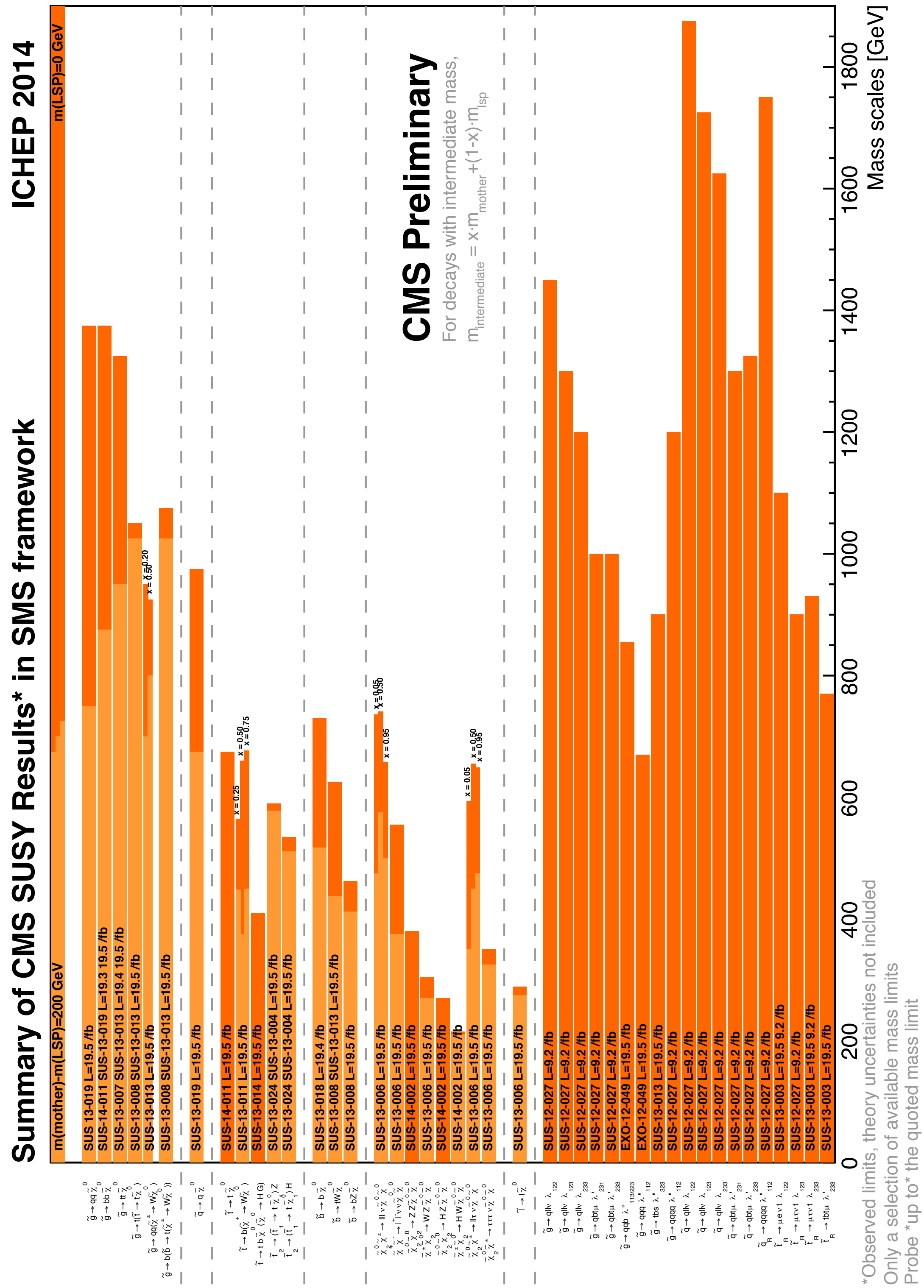

Figure C.2: The exclusion limits for the masses of the mother particles for m(LSP) = 0 GeV (dark shades) and m(mother) − m(LSP) = 200 GeV (light shades), for all results [13].

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