$$
\begin{aligned}
\delta\mathcal{L}_{\text{fermion}} &= i(\delta\psi^\dagger)\overline{\sigma}^\mu\partial_\mu\psi + i\psi^\dagger\overline{\sigma}^\mu\partial_\mu(\delta\psi) \\
&= -\epsilon\sigma^\nu\partial_\nu\phi^*\overline{\sigma}^\mu\partial_\mu\psi + \psi^\dagger\overline{\sigma}^\mu\sigma^\nu\epsilon^\dagger\partial_\mu\partial_\nu\phi \\
&= -\epsilon\partial^\mu\psi\partial_\mu\phi^* - \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi \\
&+ \partial_\mu(\epsilon\sigma^\mu\overline{\sigma}^\nu\psi\partial_\nu\phi^* - \epsilon\psi\partial^\mu\phi^* + \epsilon^\dagger\psi^\dagger\partial^\mu\phi)
\end{aligned}
$$