Problem-Solving Plan

Read

What is the question asking you to find?

Analyze

What information is given?
- Is there enough information?
- What information is needed?

Plan

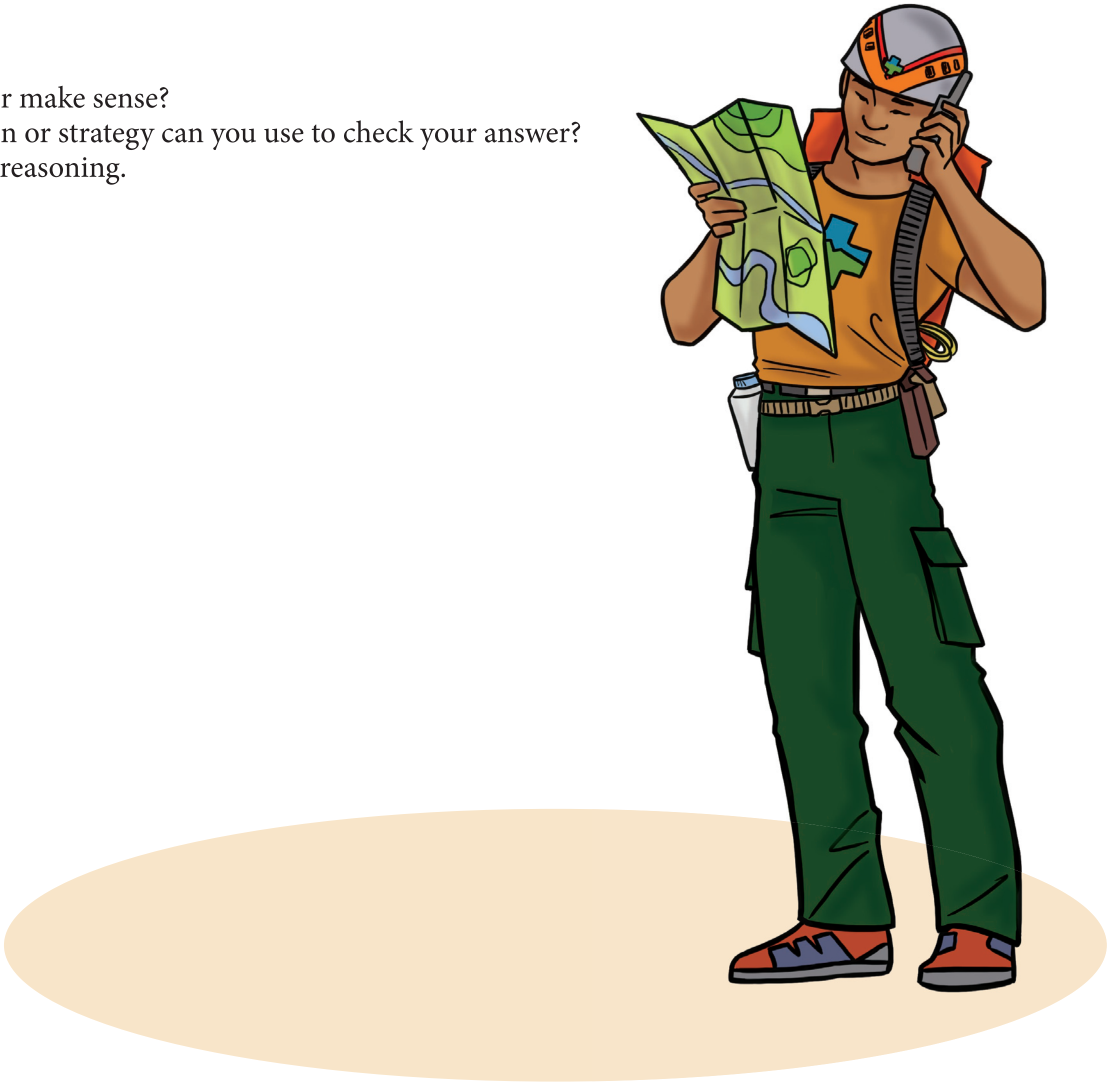
Can you write one or more equations?
If not, choose a different strategy.
- Draw a diagram or picture.
- Make a graph, list, or table.
- Use manipulatives.

Solve

Apply your plan to solve the problem.

Check

Does your answer make sense?
- What equation or strategy can you use to check your answer?
- Explain your reasoning.



Addition & Subtraction

Addends The numbers added to find the sum.

Associative Property of Addition The grouping of addends can be changed without changing the sum.

$(5 + 6) + 3 = 14$

$5 + (6 + 3) = 14$

Commutative Property of Addition The order of addends can be changed without changing the sum.

$8 + 3 = 11$

$3 + 8 = 11$

difference The answer to a subtraction problem.

fact family A group of related addition and subtraction or multiplication and division facts using the same numbers.

$\boxed{2 \quad 3 \quad 5}$

$2 + 3 = 5$ $\quad$ $5 - 2 = 3$

$3 + 2 = 5$ $\quad$ $5 - 3 = 2$

Identity Property of Addition When zero is added to an addend, the sum is the other addend.

$49 + 0 = 49$

minuend The number from which another number (subtrahend) is subtracted.

subtrahend The number that is subtracted from another number (minuend).

sum The answer to an addition problem.

variable A letter used to represent a number.

$15 + n = 18$

$n = 3$

Commutative Property

The order of addends can be changed without changing the sum.

$4 + 9 = 9 + 4$

$a + b = b + a$

Thinking of 9 + 4 allows me to mentally *count on* the smaller number to find the sum of 13.

Associative Property

The grouping of addends can be changed without changing the sum.

$(25 + 33) + 17 = 25 + (33 + 17)$

$(a + b) + c = a + (b + c)$

Grouping the 33 and 17 allows me to mentally make 10 in the Ones place.

$25 + 50 = 75$

Identity Property

When 0 is added to an addend, the sum is the other addend.

$37 + 0 = 37$

$a + 0 = a$

Nothing is added to 37.

Examples of Subtraction Situations

There are different kinds of real-life situations for using subtraction.

Take away—solving to find out how much is left over after some have been removed from a set

Andrew had $675.26. He gave $150.00 to a missionary. How much money does he have left?

$675.26 – $150.00 = $525.26

Comparing—solving to find out how many more (or fewer) one set has than another

Cora's camp team earned 10,364 points. Hector's team earned 7,854. How many fewer points did Hector's team have?

10,364 – 7,854 = 2,510 fewer points

Missing addend—solving to find out how many are needed

Abi has read 14 books so far. She will read a total of 50 by the end of the year to earn her new bike. How many books must she read to meet this goal?

14 + 36 = 50 books

Unknown part—solving to find the other part of a set that has been separated into parts

Coach has 75 balls in his bucket. Forty-eight of the balls are baseballs, and the rest are wiffle balls. How many wiffle balls are in the bucket?

75 – 48 = 27 wiffle balls

Missing subtrahend—solving to find out how many have been removed from a set

Eli had 875 pens to give away at the convention. There are 216 pens left. How many did he give away?

875 – 659 = 216 pens

Decimals

decimal A number expressing a part of a whole that is written with 1 or more places to the right of the decimal point. 0.15

decimal point (1) A dot that separates dollars from cents. $23.15 (2) A dot that separates the Ones place from the Tenths place. 5.7

hundredth (1) One of one hundred equal parts. (2) The place value 2 spaces to the right of the decimal point. 0.01

one thousandth (1) One of one thousand equal parts. (2) The place value 3 spaces to the right of the decimal point. 0.001

tenth (1) One of ten equal parts. (2) The place value to the right of the decimal point. 0.1

ten thousandth (1) One of ten thousand equal parts. (2) The place value 4 spaces to the right of the decimal point. 0.0001

Place Value Chart

Hundreds	Tens	Ones	.	Tenths	Hundredths	One Thousandths	Ten Thousandths
		4	.	3	5	8	6

Number Forms

Standard form	4.3586
Fraction form	$4\frac{3,586}{10,000}$
Word form	four and three thousand, five hundred eighty-six ten thousandths
Expanded form	4 + 0.3 + 0.05 + 0.008 + 0.0006
Expanded form with multiplication	(4 × 1) + (3 × 0.1) + (5 × 0.01) + (8 × 0.001) + (6 × 0.0001)

Compare Decimals

3.15 3.5 3.01 0.3

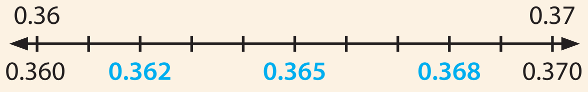
$$\begin{array}{rcl} & & 3.15 \\ 3.5 & = & 3.50 \\ 3.01 & = & 3.01 \\ 0.3 & = & 0.30 \end{array}$$

0.3	3.01	3.15	3.5
least			greatest

Round Decimals

Thinking of the location of a decimal on a number line can help you round the decimal.

0.362 rounds *down* to 0.36 0.368 rounds *up* to 0.37



Add & Subtract Decimals

- Align the numbers according to place value.
- Begin adding or subtracting in the place with least value and continue to the place with greatest value, renaming as needed.

4.3 + 2.9 = ?

$$\begin{array}{r} {\scriptstyle 1\ \ } \\ 4.3 \\ +\ 2.9 \\ \hline 7.2 \end{array}$$

4.3 – 2.9 = ?

$$\begin{array}{r} {\scriptstyle 3\ 13} \\ 4.3 \\ -\ 2.9 \\ \hline 1.4 \end{array}$$

15.3 + 12.75 = ?

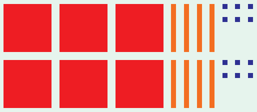
$$\begin{array}{r} {\scriptstyle 1\ \ \ \ \ \ } \\ 15.30 \\ +\ 12.75 \\ \hline 28.05 \end{array}$$

15.3 – 12.75 = ?

$$\begin{array}{r} {\scriptstyle 12\ \ \ } \\ {\scriptstyle 4\ \ 2\ 10} \\ 15.30 \\ -\ 12.75 \\ \hline 2.55 \end{array}$$

Multiply a Whole Number by a Decimal

2 sets of 3.46

Picture	Estimate	Add	Multiply	Distributive Property
	$\begin{array}{r} 3 \\ \times 2 \\ \hline 6 \end{array}$	$\begin{array}{r} 3.46 \\ +3.46 \\ \hline 6.92 \end{array}$	$\begin{array}{r} 3.46 \\ \times \quad 2 \\ \hline 6.92 \end{array}$	$2 \times 3.46 =$ $2 \times (3 + 0.46) =$ $(2 \times 3) + (2 \times 0.46) =$ $6 + 0.92 = 6.92$

Multiply a Decimal by a Decimal

0.4 × 0.3 = ___ means finding 4 tenths of 3 tenths.

$0.4 \times 0.3 = \underline{0.12}$

$\frac{4}{10} \times \frac{3}{10} = \frac{12}{100}$

0.5 × 3.46	Estimate	Multiply	Distributive Property
	$\begin{array}{r} 3 \\ \times 1 \\ \hline 3 \end{array}$	$\begin{array}{r} 3.46 \\ \times \ 0.5 \\ \hline 1.730 \end{array}$	$0.5 \times 3.46 =$ $0.5 \times (3 + 0.46) =$ $(0.5 \times 3) + (0.5 \times 0.46) =$ $1.5 + 0.23 = 1.73$

Read the multiplication sign as "sets of" or "of."

2.5 sets × 3.46	Estimate	Multiply	Fraction Form
	$\begin{array}{r} 3 \\ \times 3 \\ \hline 9 \end{array}$	$\begin{array}{r} 3.46 \\ \times \ 2.5 \\ \hline 1730 \\ +6920 \\ \hline 8.650 \end{array}$	$2\frac{5}{10} \times 3\frac{46}{100} =$ $\frac{\overset{1}{\cancel{25}}}{\underset{5}{\cancel{10}}} \times \frac{\overset{173}{\cancel{346}}}{\underset{4}{\cancel{100}}} =$ $\frac{173}{20} = 8\frac{13}{20}$

Remember a decimal can be renamed as a fraction or a mixed number.

$8.65 = 8\frac{13}{20}$

Multiply by a Power of 10

Write the digits of the decimal factor in the answer. The number of zeros in the **power of 10** or the *exponent* tells how many places to move the decimal point. Annex zeros as needed.

$\begin{array}{r} 5.68 \\ \times \quad 10 \\ \hline 5.68 \end{array}$	$\begin{array}{r} 5.68 \\ \times \ 100 \\ \hline 5.68 \end{array}$	$\begin{array}{r} 5.68 \\ \times 1{,}000 \\ \hline 5.680 \end{array}$
$10 \times 5.68 = 56.8$	$100 \times 5.68 = 568$	$1{,}000 \times 5.68 = 5{,}680$
$10^1 \times 23.75 =$ $10 \times 23.75 = 237.5$	$10^2 \times 0.956 =$ $100 \times 0.956 = 95.6$	$10^3 \times 368.1 =$ $1{,}000 \times 368.1 = 368{,}100$
Think 23.75	Think 0.956	Think 368.100

Divide a Decimal by a Whole Number

When **dividing decimals** by a whole number, annex zeros to continue renaming to find a more accurate answer. Decimal quotients with one or more digits that repeat endlessly are **repeating decimals**. The repeating digits are identified by the bar above them. Use the approximate symbol (≈) when writing rounded answers.

$41.85 \div 6 = \_\_$

Solve

$$\begin{array}{r}
6.975\\
6\overline{)41.850}\\
\underline{-36}\\
58\\
\underline{-54}\\
45\\
\underline{-42}\\
30\\
\underline{-30}\\
0
\end{array}$$

$8 \div 9 = \_\_$

$$\begin{array}{r}
0.88\\
9\overline{)8.00}\\
\underline{-72}\\
80\\
\underline{-72}\\
8
\end{array}$$

$8 \div 9 = 0.\overline{8}$

$8 \div 9 \approx 0.9$

$20 \div 14 = \_\_$

$$\begin{array}{r}
1.4285\ldots\\
14\overline{)20.0000}\\
\underline{-14}\\
60\\
\underline{-56}\\
40\\
\underline{-28}\\
120\\
\underline{-112}\\
80\\
\underline{-70}\\
10
\end{array}$$

$20 \div 14 \approx 1.429$

Divide by a Decimal

Multiply decimal divisors by the power of 10 that makes the divisor a whole number. Multiply the dividend by the same power of 10.

× 10 × 10

$12.5\overline{)175.0}$

$10 \times 12.5 = 125$ $10 \times 175 = 1{,}750$

$$\begin{array}{r}
14\\
125\overline{)1{,}750}\\
\underline{-125}\\
500\\
\underline{-500}\\
0
\end{array}$$

$175 \div 12.5 = 14$

Divide by a Power of 10

When dividing a whole number or a decimal by a power of 10, move the decimal point one place to the left for each zero in the divisor. Annex zeros as needed.

$$\begin{array}{r} 5\\ 10\overline{)50} \end{array}$$

Think 50.

$50 \div 10 = 5$

$$\begin{array}{r} 4.12\\ 10\overline{)41.20} \end{array}$$

Think 50.

$41.2 \div 10 = 4.12$

$$\begin{array}{r} 0.06\\ 10\overline{)0.60} \end{array}$$

Think 00.6

$0.6 \div 10 = 0.06$

$375 \div 10 = 37.5$

Think 375.

$37.51 \div 100 = 0.3751$

Think 037.51

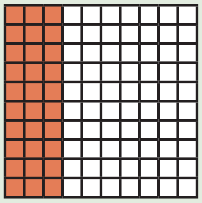
$37.51 \div 1{,}000 = 0.03751$

Think 0037.51

Fractions as Decimals

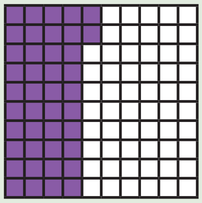
Some fractions have an exact equivalent decimal. Other fractions result in a repeating decimal that divides endlessly. All decimals can be rounded to a given place.

If the denominator is a power of 10, the fraction names the decimal.



$\frac{3}{10} = 0.3$

three tenths



$\frac{42}{100} = 0.42$

forty-two hundredths

If the denominator is a factor of a power of 10, rename the fraction. The fraction will name the decimal.

5 is a factor of 10.

$$\frac{3}{5} \overset{\times 2}{\underset{\times 2}{=}} \frac{6}{10} = 0.6$$

six tenths

4 is a factor of 100.

$$\frac{3}{4} \overset{\times 25}{\underset{\times 25}{=}} \frac{75}{100} = 0.75$$

seventy-five hundredths

If the denominator is not a power of 10 or a factor of a power of 10, divide the numerator by the denominator. The quotient is the equivalent decimal.

$\frac{1}{6} = 1 \div 6 = 0.1\overline{6}$

$\frac{1}{6} \approx 0.17$

$$\begin{array}{r}
0.166 \\
6\overline{)1.000} \\
-\underline{6} \\
40 \\
-\underline{36} \\
40 \\
-\underline{36} \\
4
\end{array}$$

$\frac{9}{23} = 9 \div 23 \approx 0.391$

$$\begin{array}{r}
0.3913\ldots \\
23\overline{)9.00000} \\
-\underline{69} \\
210 \\
-\underline{207} \\
30 \\
-\underline{23} \\
70 \\
-\underline{69} \\
10
\end{array}$$

Equations

algebraic expression A mathematical sentence which contains a variable.

equation A mathematical sentence stating that two expressions are equal.

$4 \times 12 = 48$ $\qquad$ $7 + x = 7 + 3$

expression A mathematical phrase made up of numbers, operation signs, and sometimes variables.

$45 + n$ $\qquad$ 27×6

inequality A mathematical sentence that states that two expressions are not equal.

substitution Replacing a variable with a quantity.

$n = 9$

$15 + n$ $\qquad$ $n \times 6$

$15 + 9$ $\qquad$ 9×6

variable A letter used to represent a number.

$36 \div x = 9$

$x = 4$

Expressions

An **expression** is a mathematical phrase made up of numbers, operation signs, and sometimes variables. A **variable** is a letter used to represent an unknown value. An expression can be written by interpreting a word phrase.

Two added to thirty: **$30 + 2$** $\qquad$ A number divided by 3: **$n \div 3$**

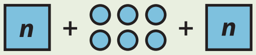
An **algebraic expression** always uses a variable. When the operation is multiplication, an algebraic expression is written using a **coefficient** (number) before the variable. A multiplication sign is *not* needed.

The product of a number and 12: **$12n$**

$12n = 12 \times n$
12 is the coefficient of the variable n.

Simplify Expressions

Simplify expressions by combining **like terms**. The Commutative and Associative Properties of Addition or Multiplication allow you to rewrite the expression to organize the *like terms*.



$n + 6 + n =$
$n + n + 6 =$
$2n + 6$

Commutative Property:
Change the order of addends.

Associative Property:
Group like terms.

$4 + x + 3 =$
$x + 4 + 3 =$
$x + 7$

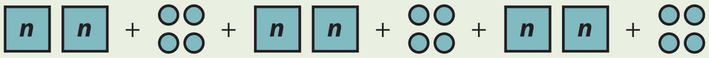
Repeated addends are combined using multiplication.

$x + x + x + x =$
$4x$

$2x + 2x + 2x + 2x =$
$4(2x) =$
$8x$

The Distributive Property of Multiplication over Addition can also be used to simplify expressions. Or it can be used to find an equivalent expression by finding a common factor in the terms of the equation.

Simplify



$3(2n + 4) =$ There are 3 sets of each addend.
$3(2n) + 3(4) =$ Multiply each addend by the multiplier, 3.
$6n + 12$ Write the simplified expression.

Equivalent Expression

$9 + 15b$

3 is a common factor of 9 and 15.

$9 + 15b =$ **$3(3 + 5b)$**

Addition & Subtraction Equations

An equation with a variable is solved by finding the value of the variable.
The value must make the sentence true to be called a **solution**.

Solve

Isolate the variable on one side of the equal sign by using the **inverse operation**. Keep the equation **balanced** by performing the exact same operation on both sides of the equation.

Check

Substitute (replace) the variable with the solution and evaluate.

$n + 6 = 10$

$n + 6 - 6 = 10 - 6$

$n = 4$

$\boxed{4 + 6 = 10}$

Isolate the variable using the inverse operation.

Check using substitution.

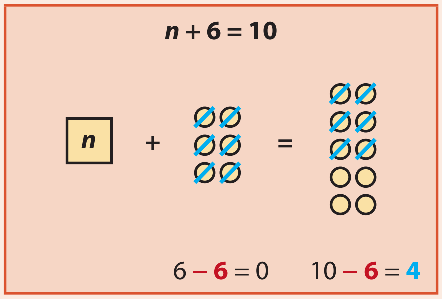
$n - 3 = 8$

$n - 3 + 3 = 8 + 3$

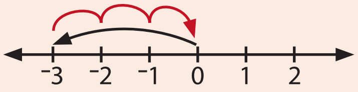
$n = 11$

$\boxed{11 - 3 = 8}$

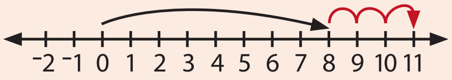
The equation mat shows the result of subtracting 6 from both sides of the equation.



The number lines show the result of adding 3 to both sides of the equation.



$^{-}3 + 3 = 0$



$8 + 3 = 11$

Multiplication & Division Equations

Multiplication and division are inverse operations.

$36 \div 4 \times 4 = 36$

$\frac{36}{\underset{1}{\cancel{4}}} \times \overset{1}{\cancel{4}} = 36$

$8 \cdot 2 \div 2 = 8$

$\frac{8 \cdot \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}}} = 8$

Isolate the variable on one side of the equal sign using the inverse operation. An equation is much like a balanced scale: when you perform an operation on the left side of the equation, you must perform the exact same operation on the right side of the equation.

The inverse of multiplying by 5 is dividing by 5.

$5x = 35$

$\frac{\overset{1}{\cancel{5}}x}{\underset{1}{\cancel{5}}} = \frac{35}{5}$

$x = 7$

$\boxed{5 \cdot 7 = 35}$

The inverse of dividing by 8 is multiplying by 8.

$n \div 8 = 4$

$\frac{n}{\underset{1}{\cancel{8}}} \cdot \overset{1}{\cancel{8}} = 4 \cdot 8$

$n = 32$

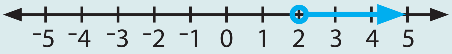
$\boxed{\frac{32}{8} = 4}$

Inequality

An **inequality** is a mathematical sentence in which two expressions are not equal. The *greater than* (>) and *less than* (<) symbols can be used to express the inequality. A number line shows all solutions for the inequality.

The open circle on the number line indicates that the circled number is *not* included in the solution.

$x > 2$



The number line shows that x is any value *greater than* 2.

$x < 3 - 1$

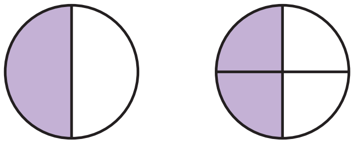
The number line shows that x is any value *less than* 2.

Fractions

composite number A composite number is a multiple of more than two factors, other than 1 and itself.

denominator The number below the fraction line; it names the equal parts of the whole.

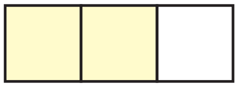
equivalent fractions Fractions that name the same part of a whole or set.



$\frac{1}{2} = \frac{2}{4}$

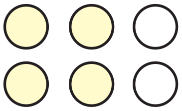
fraction A number that names a part of a whole, a part of a set, or a part of a length expressed as a numerator and denominator.

part of a whole



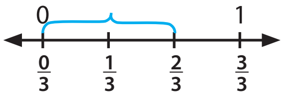
$\frac{2}{3}$

part of a set



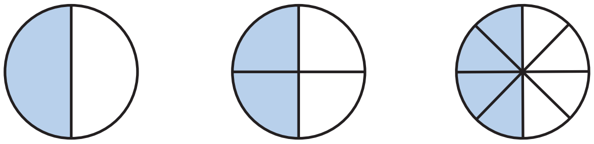
$\frac{2}{3}$

part of a length



$\frac{2}{3}$

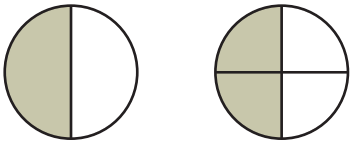
fractional terms Digits used to write a fraction.



$\frac{1}{2}$ $\frac{2}{4}$ $\frac{4}{8}$

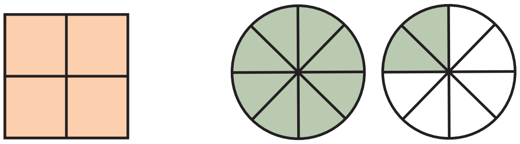
greatest common factor (GCF) The largest factor that is the same for two or more numbers.

higher terms An equivalent fraction expresses the same part with larger digits.



$\frac{1}{2} = \frac{2}{4}$ ← higher terms

improper fraction A fraction that has a value equal to or greater than 1 whole. The numerator is the same as or greater than the denominator.



$\frac{4}{4} = 1$ $\frac{10}{8} > 1$

least common denominator (LCD) The lowest shared denominator of renamed fractions.

least common multiple (LCM) The lowest multiple, other than 0, that is the same for 2 or more numbers.

multiples of 3: 3, 6, 9, ⑫, 15, 18, 21, 24, 27, 30

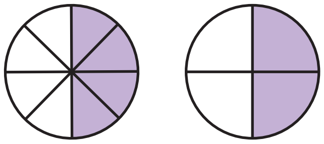
multiples of 4: 4, 8, ⑫, 16, 20, 24, 28, 32, 36, 40

12 is the least common multiple of 3 and 4

like fractions Fractions that have the same denominator.

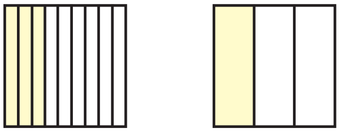
$\frac{6}{8}$ $\frac{1}{8}$

lower terms An equivalent fraction expressed with smaller digits.



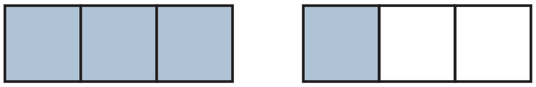
$\frac{4}{8} = \frac{2}{4}$ ← lower terms

lowest terms (simplest form) An equivalent fraction expressed with the least total number of parts, where the numerator and denominator have no common factor greater than 1.



$\frac{3}{9} = \frac{1}{3}$ ← lowest terms

mixed number A number that is the sum of a whole number and a fraction.



one and one-third

$1\frac{1}{3}$

numerator The number above the fraction line in a fraction; it is the number of parts selected.

prime number A prime number has only two factors: 1 and itself.

reciprocals Two numbers whose product equals 1.

$$\frac{2}{3} \times \frac{3}{2} = 1$$

reciprocals

related fractions Unlike fractions where one denominator is a multiple of the other denominator.

$\frac{2}{3}$ $\frac{3}{12}$ 12 is a multiple of 3.

unlike fractions Fractions that have different denominators.

$\frac{2}{5}$ $\frac{2}{3}$

Mathematical Properties for Fractions

Commutative Property of Addition

$$\frac{1}{6} + \frac{2}{6} = \frac{2}{6} + \frac{1}{6}$$

$$\frac{3}{6} = \frac{3}{6}$$

Commutative Property of Multiplication

$$\frac{2}{3} \times \frac{2}{5} = \frac{2}{5} \times \frac{2}{3}$$

$$\frac{4}{15} = \frac{4}{15}$$

The order of addends or factors can be changed without changing the sum or product.

Associative Property of Addition

$$\left(\frac{2}{8} + \frac{1}{8}\right) + \frac{4}{8} = \frac{2}{8} + \left(\frac{1}{8} + \frac{4}{8}\right)$$

$$\frac{7}{8} = \frac{7}{8}$$

Associative Property of Multiplication

$$\frac{1}{5} \times \left(\frac{2}{5} \times \frac{3}{5}\right) = \left(\frac{1}{5} \times \frac{2}{5}\right) \times \frac{3}{5}$$

$$\frac{6}{125} = \frac{6}{125}$$

The grouping of addends or factors may be changed without changing the sum or product.

Distributive Property of Multiplication over Addition

$$2 \times 1\frac{2}{3} = (2 \times 1) + \left(2 \times \frac{2}{3}\right)$$

$$3\frac{1}{3} = 3\frac{1}{3}$$

The product of any two factors can be found by separating one factor into parts, multiplying each part by the other factor, and adding the partial products.

Identity Property of Addition

$$\frac{2}{7} + 0 = \frac{2}{7}$$

When 0 is an addend, the sum is the other addend.

Identity Property of Multiplication

$$1 \times \frac{4}{9} = \frac{4}{9}$$

When 1 is a factor, the product is the other factor.

Zero Principle of Subtraction

$$\frac{7}{12} - 0 = \frac{7}{12}$$

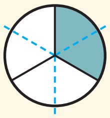
When 0 is subtracted from a number (the minuend), the answer is that number.

Zero Property of Multiplication

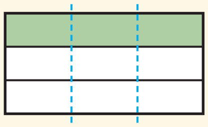
$$\frac{5}{6} \times 0 = 0$$

When 0 is a factor, the product is 0.

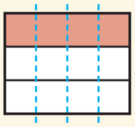
Multiply by a name for 1 to rename a fraction to higher terms.



$\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$

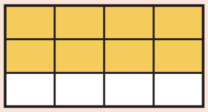


$\frac{1 \times 3}{3 \times 3} = \frac{3}{9}$

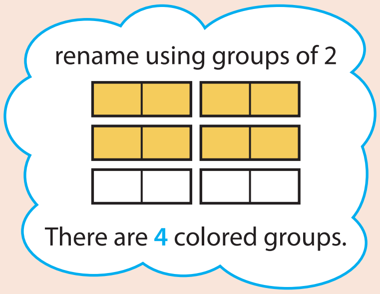


$\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$

Divide by a name for 1 to rename a fraction to lower terms.

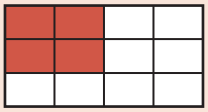


$\frac{8 \div 2}{12 \div 2} = \frac{4}{6}$

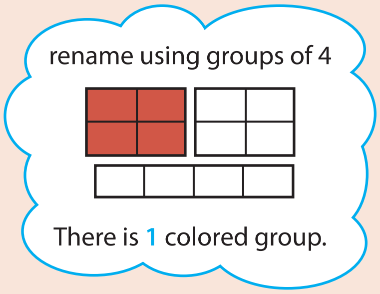


rename using groups of 2

There are 4 colored groups.



$\frac{4 \div 4}{12 \div 4} = \frac{1}{3}$



rename using groups of 4

There is 1 colored group.

Use strategies to rename a fraction to lowest terms (simplest form).

Divide by the GCF

30: 1, 2, 3, 5, 6, 10, 15, 30

36: 1, 2, 3, 4, 6, 9, 12, 18, 36

GCF of 30 and 36: 6

$\frac{30 \div 6}{36 \div 6} = \frac{5}{6}$

Repeated division

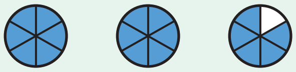
$\frac{30 \div 2}{36 \div 2} = \frac{15 \div 3}{18 \div 3} = \frac{5}{6}$

Improper Fractions & Mixed Numbers

An **improper fraction** has a value *equal to* or *greater than* 1. $\frac{6}{6}$ $\frac{17}{6}$

A **mixed number** is the sum of a whole number and a fraction. $2\frac{5}{6} = 2 + \frac{5}{6}$

$\frac{17}{6} = 2\frac{5}{6}$



$\frac{17}{6} = \frac{6}{6} + \frac{6}{6} + \frac{5}{6} = 1 + 1 + \frac{5}{6} = 2\frac{5}{6}$

Rename mixed numbers as improper fractions.

$2\frac{3}{8} = \_\_$

1. Multiply to find the number of parts in the wholes. $2 \cdot \frac{8}{8} = \frac{16}{8}$
2. Add the additional parts. $\frac{16}{8} + \frac{3}{8} = \frac{19}{8}$

Rename improper fractions as mixed numbers.

$\frac{19}{8} = \_\_$

1. Divide the numerator by the denominator to find the number of wholes.
2. Write the remainder as a fraction to tell how many parts of the next whole there are.

$$\begin{array}{r} 2\frac{3}{8} \\ 8\overline{)19} \\ -16 \\ \hline 3 \end{array}$$

Add & Subtract Like Fractions

Fractions with like denominators can be added or subtracted. Remember to rename the answer to lowest terms. Rename an improper fraction as a mixed number.

1. Add or subtract the fractions.
2. Add or subtract the whole numbers.
3. Simplify the answer, if needed.

Add or subtract fractions.

$$\frac{2}{8} + \frac{5}{8} = \frac{7}{8}$$

$$\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$$

$$\frac{5}{8} + \frac{2}{8} + \frac{3}{8} = \frac{10}{8} = 1\frac{2 \div 2}{8 \div 2} = 1\frac{1}{4}$$

Add or subtract mixed numbers.

$$\begin{array}{r} 3\frac{5}{8} \\ +\ 2\frac{1}{8} \\ \hline 5\frac{6 \div 2}{8 \div 2} = 5\frac{3}{4} \end{array} \qquad \begin{array}{r} 7\frac{5}{8} \\ -\ 1\frac{1}{8} \\ \hline 6\frac{4 \div 4}{8 \div 4} = 6\frac{1}{2} \end{array}$$

Rename a whole to subtract.

$$\begin{array}{r} 4\phantom{\frac{8}{8}} \\ \cancel{5}\frac{8}{8} \\ -\ 1\frac{3}{8} \\ \hline 3\frac{5}{8} \end{array} \qquad \begin{array}{r} 3\phantom{\frac{3}{8}} \\ \cancel{4}\frac{3}{8} \\ -\ 1\frac{5}{8} \\ \hline \end{array} = 3 + \frac{8}{8} + \frac{3}{8} = 3\frac{11}{8} \qquad \xrightarrow{} \qquad \begin{array}{r} 3\frac{11}{8} \\ -\ 1\frac{5}{8} \\ \hline 2\frac{6 \div 2}{8 \div 2} = 2\frac{3}{4} \end{array}$$

Add & Subtract Related Fractions

Unlike fractions are fractions with different denominators (different parts in the whole). Some unlike fractions are related fractions. **Related fractions** are fractions in which one denominator is a multiple of the other denominator. To add or subtract related fractions, **rename** one fraction so that the fractions have the same denominator. Complete the operation. Write the answer in lowest terms.

$$\begin{array}{rl} \frac{8}{10} & = \frac{8}{10} \\ +\ \frac{3 \times 2}{5 \times 2} & = \frac{6}{10} \\ \hline & \frac{14}{10} = 1\frac{4}{10} = 1\frac{2}{5} \end{array}$$

10 is a multiple of 5.

1. Rename. $\frac{3}{5} = \frac{6}{10}$
2. Add.
3. Write the answer in lowest terms.

$$\begin{array}{rl} 3\frac{1 \times 3}{4 \times 3} & = \cancel{3}\frac{\cancel{3}}{12} = 2\frac{15}{12} \\ -\ 1\frac{11}{12} & = \phantom{\cancel{3}\frac{3}{12} =}\ 1\frac{11}{12} \\ \hline & \phantom{\cancel{3}\frac{3}{12} =}\ 1\frac{4}{12} = 1\frac{1}{3} \end{array}$$

12 is a multiple of 4.

1. Rename. $\frac{1}{4} = \frac{3}{12}$
2. Subtract. Rename a whole when necessary. $3\frac{3}{12} = 2\frac{15}{12}$
3. Write the answer in lowest terms.

Add & Subtract Unlike Fractions

When fractions with **unlike** denominators are *not* related, rename both fractions before adding or subtracting. Rename the fractions by finding a **common denominator**. Remember to rename the sum or difference to lowest terms.

Multiply the denominators.

$3 \times 5 = 15$

$$\begin{array}{rl} 6\frac{2 \times 3}{5 \times 3} = 6\frac{6}{15} & = 5\frac{21}{15} \\ -\ 4\frac{2 \times 5}{3 \times 5} = 4\frac{10}{15} & = 4\frac{10}{15} \\ \hline & 1\frac{11}{15} \end{array}$$

List multiples.

$$\begin{array}{rl} \frac{5 \times 2}{6 \times 2} & = \frac{10}{12} \\ -\ \frac{1 \times 3}{4 \times 3} & = \frac{3}{12} \\ \hline & \frac{7}{12} \end{array}$$

4: 4, 8, 12
6: 6, 12

LCM of 4 and 6: 12

Use prime factorization.

$$\begin{array}{rl} \frac{5 \times 9}{8 \times 9} & = \frac{45}{72} \\ +\ \frac{7 \times 4}{18 \times 4} & = \frac{28}{72} \\ \hline & \frac{73}{72} = 1\frac{1}{72} \end{array}$$

8: $2 \times 2 \times 2 = 2^3$
18: $2 \times 3 \times 3 = 2 \times 3^2$

LCM of 8 and 18:
$2^3 \times 3^2 = 8 \times 9 = 72$

Find the Greatest Common Factor by Listing Factors

List the Factors

28: 1 × 28, 2 × 14, 4 × 7
40: 1 × 40, 2 × 20, 4 × 10, 5 × 8

28: 1, 2, 4, 7, 14, 28
40: 1, 2, 4, 5, 8, 10, 20, 40

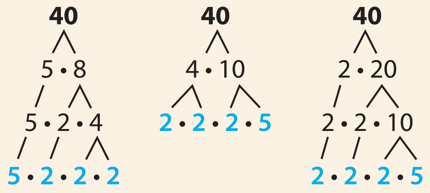
common factors of **28** and **40:** 1, 2, 4
greatest common factor of **28** and **40:** 4

Find the Prime Factors Using a Factor Tree

A composite number can be expressed as the product of a set of **prime factors**.

1. Write the number to be factored at the top.
2. Choose any pair of factors.
3. Continue to factor any composite number until all factors are prime.

Ways to Factor 40

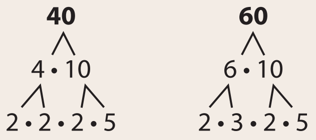


40 = 2 • 2 • 2 • 5

Find the Greatest Common Factor with a Factor Tree

Use **prime factorization** to find the greatest common factor (GCF) of two numbers.

1. List the prime factors of each number in ascending order.
2. Select the factors that are common to both lists.
3. Multiply the common factors.



40: 2 • 2 • 2 • 5
60: 2 • 2 • 3 • 5

GCF of 40 and 60: 2 • 2 • 5 = 20

Cancellation

A fraction can be renamed to lowest terms, or simplified, by a process called **cancellation**. For this process, use these steps to cancel out fractional names for 1.

1. Use mental math to list the prime factors of the numerator and the denominator from *least* to *greatest*.

4 × 9

$$\frac{36}{40} = \frac{2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 5}$$

8 × 5

2. Identify and cancel all the fractional names for 1.

$$\frac{36}{40} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3 \cdot 3}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2 \cdot 5}$$

3. The canceled numbers removed the GCF (4). Multiply the remaining factors in the numerator and in the denominator. The result is the simplified fraction.

$$\frac{36}{40} = \frac{9}{10}$$

Least Common Multiple

A common multiple of 6 and 9 is 54 because 6 × 9 = 54.
The following list shows the nonzero multiples of 6 and 9 *less than* 54.
The **least common multiple (LCM)** of 6 and 9 is **18**.

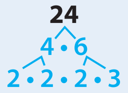
6: 6, 12, 18, 24, 30, 36, 42, 48
9: 9, 18, 27, 36, 45

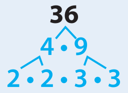
Methods for Finding the Least Common Multiple

For numbers that are easy to count by, list the multiples.

20: 20, 40, 60
15: 15, 30, 45, 60
LCM of 15 and 20:
60

For numbers that are not easy to list the multiples of, write the **prime factorization with exponents**. Multiply the highest power of each prime factor listed.





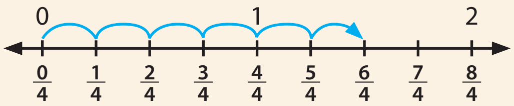
24: $2^3 \cdot 3$
36: $2^2 \cdot 3^2$
LCM of 24 and 36: $2^3 \cdot 3^2 = 72$

Multiply a Whole Number by a Fraction

Multiplying a fraction by a whole number follows the pattern of whole number multiplication. It can be solved using repeated addition or by drawing an array. The denominator tells how many equal sets to make. The numerator tells how many sets to select to find the answer.

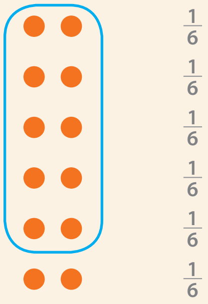
$6 \times \frac{1}{4}$ is 6 sets of $\frac{1}{4}$.

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$



$\frac{5}{6} \times 12$ is $\frac{5}{6}$ of 12.

$\frac{5}{6} \times 12 = 10$



When a multiplication equation includes a whole number and a fraction, rename the whole number as an improper fraction. Multiply the numerators and multiply the denominators.

$6 \times \frac{1}{4} = \frac{6}{1} \times \frac{1}{4} = \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$

$\frac{5}{6} \times 12 = \frac{5}{6} \times \frac{12}{1} = \frac{60}{6} = 10$

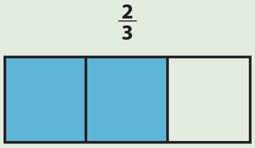
The denominator of a whole number is 1.

$6 = \frac{6}{1}$ $12 = \frac{12}{1}$

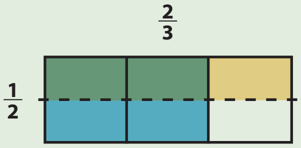
Multiply a Fraction by a Fraction

Multiplying a fraction by a fraction is finding *a part of a part*. The product will be smaller than either factor because the answer is only a part of the original unit.

A picture can be drawn to show the product of two fractions. Find $\frac{1}{2}$ of $\frac{2}{3}$.

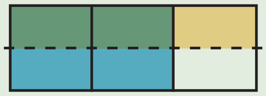


Draw a figure. Color two-thirds.



Draw a line the other way to show $\frac{1}{2}$ of the figure. Color one-half.

$\frac{1}{2}$ of $\frac{2}{3} = \frac{2}{6} = \frac{1}{3}$



The double shaded area represents the product.

The product of two fractions can also be found by multiplying the numerators and multiplying the denominators.

$\frac{1}{2}$ of $\frac{2}{3} = \frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$

Simplify Fractions

Cancellation Using Prime Factorization

- List the prime factors of each term.
- Cancel fractional names for 1.
- Multiply the simplified numerators and the simplified denominators.

$$\frac{3}{4} \times \frac{8}{9} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 3} = \frac{2}{3}$$

Cancellation Finding a Common Factor

- Divide a numerator and a denominator by a common factor.
- Multiply the simplified numerators and the simplified denominators.

$$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}} = \frac{2}{3}$$

GCF of 3 and 9: 3
GCF of 4 and 8: 4

Multiply Mixed Numbers

Rename a mixed number as an improper fraction to multiply.

- Multiply the numerators and then multiply the denominators.
- Simplify the fractions using cancellation when possible.

$$5 \times 9\frac{3}{10} = \frac{\overset{1}{\cancel{5}}}{1} \times \frac{93}{\underset{2}{\cancel{10}}} = \frac{93}{2} = 46\frac{1}{2}$$

$$\begin{array}{r} 46\frac{1}{2} \\ 2\overline{)93} \\ -8 \\ 13 \\ -12 \\ 1 \end{array}$$

Use the Distributive Property to multiply.

- Multiply each part of the mixed number by the whole number.
- Simplify. Write the answer in lowest terms.

$$5 \times 9\frac{3}{10} =$$

$$5 \times (9 + \tfrac{3}{10}) =$$

$$(5 \times 9) + (\tfrac{5}{1} \times \tfrac{3}{10}) =$$

$$45 + \tfrac{3}{2} =$$

$$45 + 1\tfrac{1}{2} = 46\tfrac{1}{2}$$

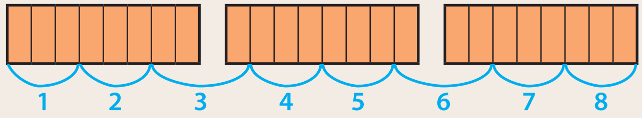
$$\begin{array}{r} 1\frac{1}{2} \\ 2\overline{)3} \\ -2 \\ 1 \end{array}$$

Dividing by a fraction is finding the number of fractional units.

Divide a Whole Number by a Fraction

$3 \div \frac{3}{8}$ is finding how many $\frac{3}{8}$ units are in 3.

Partition each whole (3) into equal parts ($\frac{1}{8}$). Count the parts.



There are 8 sets of $\frac{3}{8}$ or $\frac{24}{8}$.

Divide the 24 parts into $\frac{3}{8}$.

Solve the division equation.

$$3 \div \frac{3}{8} = \frac{24}{8} \div \frac{3}{8} = 8$$

Check using multiplication.

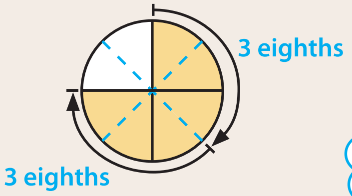
$$8 \times \frac{3}{8} = \frac{8 \times 3}{1 \times 8} = \frac{24}{8} = 3$$

Divide a Fraction by a Fraction

$\frac{3}{4} \div \frac{3}{8}$ is finding how many $\frac{3}{8}$ units are in $\frac{3}{4}$.

Rename the fractions being divided using a common denominator. Divide the numerators. Rename mixed numbers as improper fractions. Then find the common denominator and divide the numerators.

Repartition the parts to show the common denominator.



$$\frac{3}{4} \div \frac{3}{8} =$$

$$\frac{6}{8} \div \frac{3}{8} = \frac{6}{3} = 2$$

$$\frac{6 \div 3}{8 \div 8} = \frac{6 \div 3}{1} = \frac{2}{1}$$

Divide Fractions

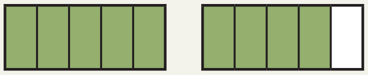
When renaming fractions using a common denominator, it is not always possible to divide the numerators evenly. If the numerators are not compatible, the quotient will be a fraction or a mixed number. Drawing a picture can help you solve the equation. Remember to write the final answer in lowest terms.

$1\frac{4}{5} \div \frac{1}{2} =$

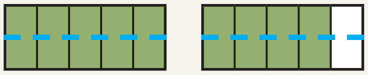
$\frac{9}{5} \div \frac{1}{2} =$

$\frac{18}{10} \div \frac{5}{10} = \frac{18}{5} = 3\frac{3}{5}$

$$\begin{array}{r} 3\tfrac{3}{5} \\ 5\overline{)18} \\ -15 \\ \hline 3 \end{array}$$

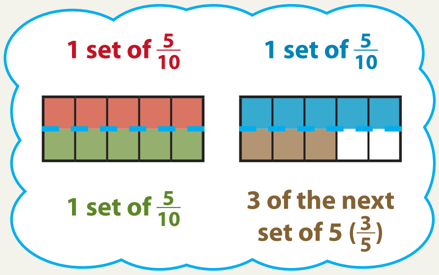


$1\frac{4}{5} = \frac{9}{5}$



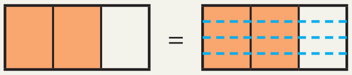
$\frac{18}{10} \div \frac{5}{10}$

There are $3\frac{3}{5}$ sets of $\frac{1}{2}$ in $1\frac{4}{5}$.



$\frac{2}{3} \div \frac{3}{4} =$

$\frac{8}{12} \div \frac{9}{12} = \frac{8}{9}$



$\frac{2}{3} = \frac{8}{12}$

There is $\frac{8}{9}$ of a set.

There are only 8 of the 9 parts needed to make 1 whole set.
There is only part of a set of $\frac{3}{4}$ in $\frac{2}{3}$.

Divide: Multiply by the Reciprocal

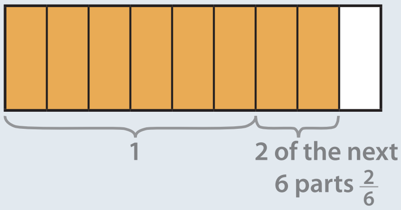
Dividing by a fraction is the same as multiplying by the reciprocal of the divisor. The **reciprocal** of the divisor is found by **inverting** the numerator and the denominator.

$3 \div \frac{1}{2}$ is the same as $3 \times \frac{2}{1} = \frac{3 \times 2}{1} = 6$

$3 \div \frac{1}{2} = 3 \times \frac{2}{1} = \frac{3 \times 2}{1} = 6$

$\frac{8}{9} \div \frac{2}{3}$

How many sets of $\frac{2}{3}$ are in $\frac{8}{9}$?



$8\frac{1}{3} \div 1\frac{2}{3} =$

$\frac{25}{3} \div \frac{5}{3} =$

$\frac{\overset{5}{\cancel{25}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{5}}} = \frac{5}{1} = 5$

How many sets of $1\frac{2}{3}$ are in $8\frac{1}{3}$?

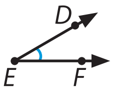
Divide

$\frac{8}{9} \div \frac{6}{9} = \frac{8}{6} = 1\frac{2}{6} = 1\frac{1}{3}$

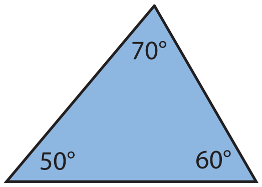
Multiply

$\frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}} = \frac{4}{3} = 1\frac{1}{3}$

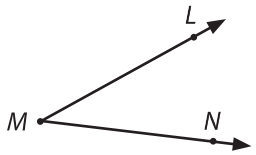
acute angle An angle that measures less than 90°.



acute triangle A triangle with three acute angles.

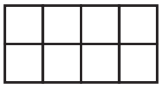


angle A figure formed when 2 rays share the same endpoint.



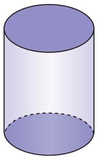
symbol	read
$\angle M$	angle M
$\angle LMN$	angle LMN
$\angle NML$	angle NML

area The space within a figure that is measured in square units.

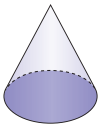


8 square units

base (1) Either of two congruent and parallel faces of a cylindrical figure. (2) The bottom face of a conical figure.

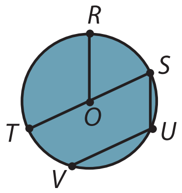


cylinder
(2 bases)



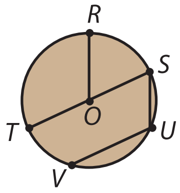
cone
(1 base)

central angle An angle with its vertex in the center of the circle.



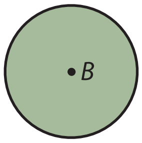
$\angle ROS$ and $\angle ROT$ are central angles.

chord A line segment that connects any two points on a circle.



$\overline{SU}$, $\overline{VU}$, and $\overline{TS}$ are chords.

circle A closed curve where each point on the curve is the same distance from a center point.



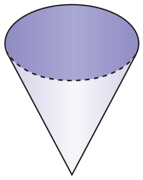
read: circle B

circumference The distance around a circle.

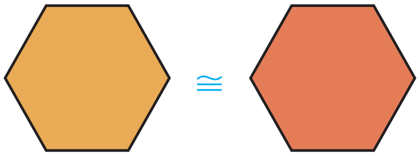
collinear A set of points when one line can be drawn through all the points.

complementary angles Two angles whose measure have a sum of 90°. When placed side by side, complementary angles form a right angle.

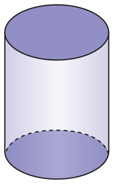
cone A three-dimensional figure with 1 circular face, 1 curved surface, and 1 vertex.



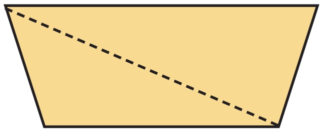
congruent figures Figures that are the same shape and the same size.



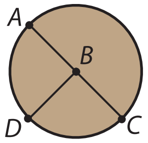
cylinder A three-dimensional figure with 2 circular bases separated by 1 curved surface.



diagonal A line segment that connects two nonadjacent vertices of a polygon. A diagonal divides a quadrilateral into 2 triangles.

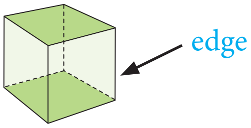


diameter A line segment that connects 2 points on a circle and passes through the center point.



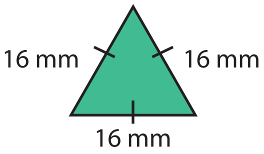
$\overline{AC}$ or $\overline{CA}$ is the diameter of circle B.

edge Where two faces meet on a three-dimensional figure.

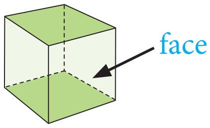


endpoint (1) A point that indicates the end of a ray. (2) One of 2 points that marks the end of a line segment.

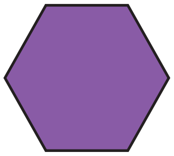
equilateral triangle A triangle with all sides congruent in length.



face A flat surface on a three-dimensional figure.

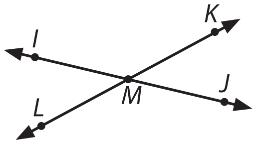


hexagon A polygon with 6 sides.



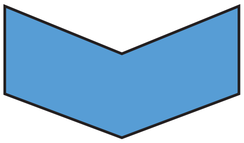
horizontal line A line that is straight across.

intersecting lines Lines that share a common point.

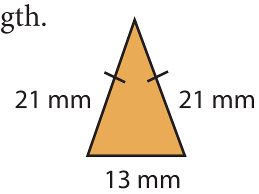


$\overleftrightarrow{KL}$ intersects $\overleftrightarrow{IJ}$.

irregular polygon A polygon with sides of different lengths and angles with different measurements.



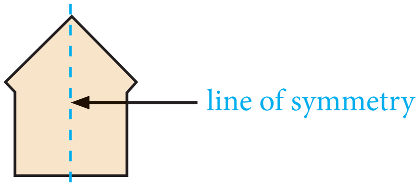
isosceles triangle A triangle with at least 2 sides congruent in length.



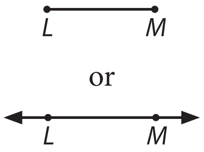
line A straight path that goes on without end in 2 directions.

symbol	read
$\overleftrightarrow{LM}$	line *LM*
$\overleftrightarrow{ML}$	line *ML*

line of symmetry A line dividing a figure into congruent halves.

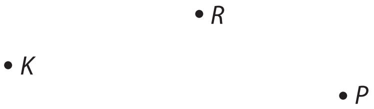


line segment A part of a line having 2 endpoints.



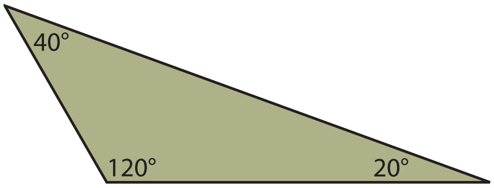
symbol	read
$\overline{LM}$	line segment *LM*
$\overline{ML}$	line segment *ML*

noncollinear A set of points when no line can be drawn through all the points.

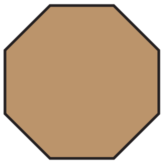


obtuse angle An angle that measures greater than 90° and less than 180°.

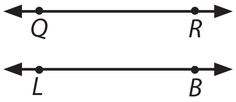
obtuse triangle A triangle with one obtuse angle.



octagon A polygon with 8 sides.



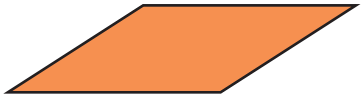
parallel lines Lines in the same plane that never intersect.



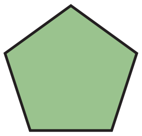
$\overleftrightarrow{QR} \parallel \overleftrightarrow{LB}$

∥ means "is parallel to"

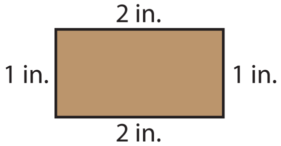
parallelogram A quadrilateral whose opposite sides are parallel.



pentagon A polygon with 5 sides.

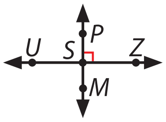


perimeter The distance around a figure.



1 in. + 2 in. + 1 in. + 2 in. = 6 in.

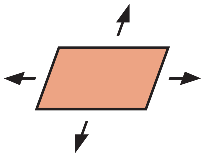
perpendicular lines Intersecting lines that form 4 right angles.



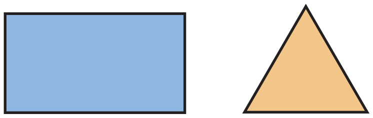
$\overleftrightarrow{PM} \perp \overleftrightarrow{UZ}$

⊥ means "is perpendicular to"

plane A flat surface that goes on endlessly in all directions.



plane figure A flat shape; a two-dimensional figure.

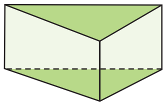


point An exact location in space represented by a dot.

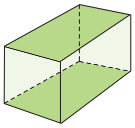
read: point *C*

polygon A closed flat shape made of 3 or more line segments.

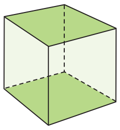
prism A three-dimensional figure that has at least 2 faces identical and parallel to each other.



triangular prism

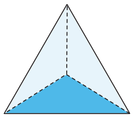


rectangular prism

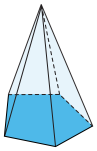


square prism (cube)

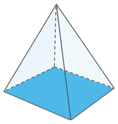
pyramid A three-dimensional figure that has a polygon as its base and at least 3 triangular faces that meet at a common vertex.



triangular pyramid

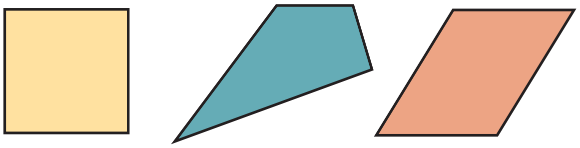


pentagonal pyramid

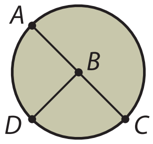


rectangular pyramid

quadrilateral Any polygon with 4 sides.



radius A line segment from the center point to a point on the circle.



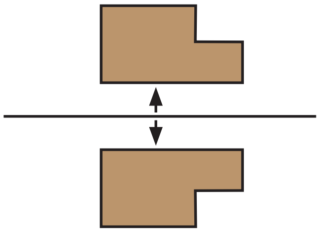
$\overline{BA}$, $\overline{BC}$, and $\overline{BD}$ are radii of circle *B*.

ray A part of a line that has 1 endpoint and goes on without end in one direction.

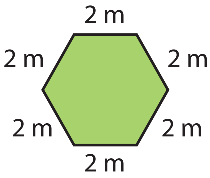
symbol	read
$\overrightarrow{LM}$	ray *LM*

rectangle A quadrilateral with opposite sides that are equal in length and 4 right angles.

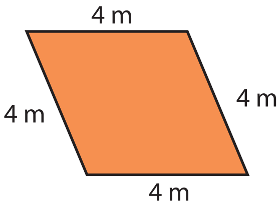
reflection (flip) A movement of a figure made by flipping the figure across a line of reflection.



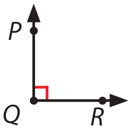
regular polygon A polygon with sides that are the same length and angles with the same measure.



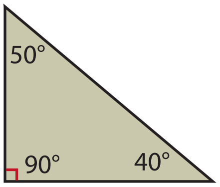
rhombus A quadrilateral with opposite sides that are parallel and 4 congruent sides.



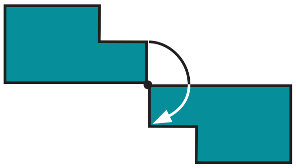
right angle An angle that measures 90° and forms a square corner.



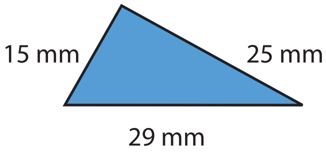
right triangle A triangle with one right angle.



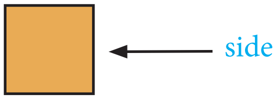
rotation (turn) A movement of a figure to a new position by rotating the figure clockwise or counterclockwise around a specific point.



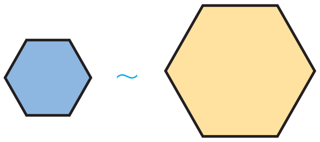
scalene triangle A triangle with no sides that are congruent.



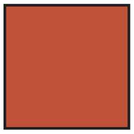
side A line segment that is part of a polygon.



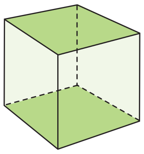
similar figures Figures that are the same shape but not necessarily the same size.



square A quadrilateral with 4 sides that are equal in length and 4 right angles.



square prism (cube) A three-dimensional figure with 6 square faces.

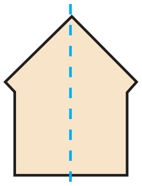


straight angle An angle that measures 180°.

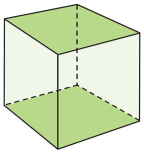
supplementary angles Two angles whose measure have a sum of 180°. When placed side by side, supplementary angles form a straight angle.

surface area The sum of all the areas of all the faces of a three-dimensional figure.

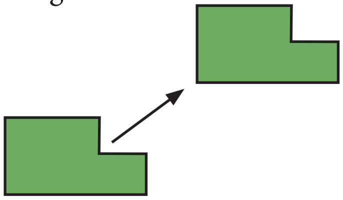
symmetrical figure A figure that, when folded along the line of symmetry, is divided into congruent halves.



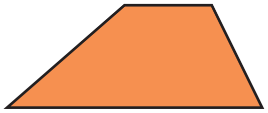
three-dimensional figures Figures that have length, width, and height.



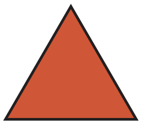
translation (slide) A movement of a figure to a new position in a straight line.



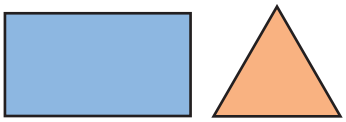
trapezoid A quadrilateral that has at least one pair of opposite sides that are parallel.



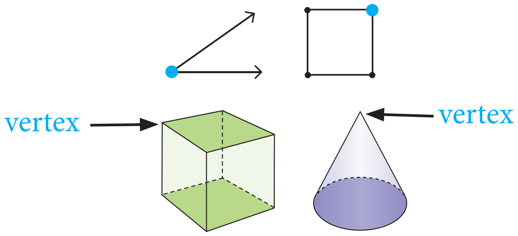
triangle A polygon with 3 sides.



two-dimensional figures Figures that have length and width.



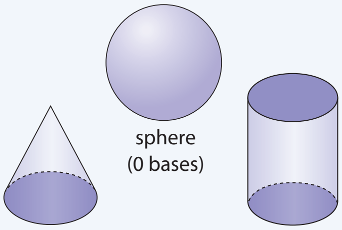
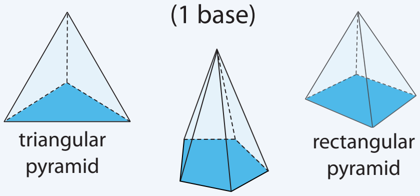
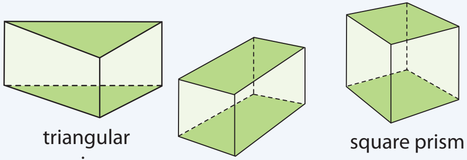
vertex (1) Where 2 rays meet and form an angle. (2) The point where 2 sides of a polygon meet. (3) The point where 3 or more edges of a prism or a pyramid meet. (4) The point formed by the curved surface of a cone.



vertical line A line that goes straight up and down.

volume The number of cubic units within a closed three-dimensional figure.

3-Dimensional Figures

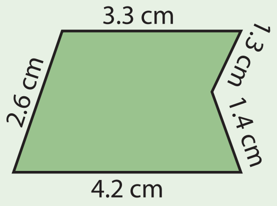
Curved-Surface Figures	Polyhedrons	
	Conical Figures (1 base)	**Cylindrical Figures** (2 bases)
 sphere (0 bases) cone (1 base) cylinder (2 bases)	 triangular pyramid pentagonal pyramid rectangular pyramid	 triangular prism rectangular prism square prism (cube)
Circular figures with curved surfaces are *not* polyhedrons.	A **pyramid** has 1 polygon as its base. All other faces are triangles.	A **prism** has 2 congruent polygons as its bases. All other faces are parallelograms.

Perimeter

Perimeter is the distance around a geometric figure and is represented by *P*.

Any Polygon

Add the lengths of the sides.



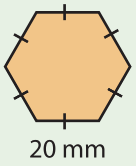
$\boldsymbol{P = a + b + c + d + e}$

$P = 2.6\text{ cm} + 4.2\text{ cm} + 1.4\text{ cm} + 1.3\text{ cm} + 3.3\text{ cm}$

$P = 12.8\text{ cm}$

Regular Polygon

Multiply:
number of sides • length of side



$\boldsymbol{P = n \cdot s}$

$P = 6 \cdot 20\text{ mm}$

$P = 120\text{ mm}$

Rectangle

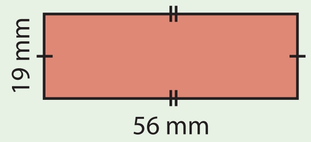
Multiply the length by 2 and the width by 2; add the products.

$\boldsymbol{P = (2 \cdot l) + (2 \cdot w)}$

$P = (2 \cdot 56\text{ mm}) + (2 \cdot 19\text{ mm})$

$P = 112\text{ mm} + 38\text{ mm}$

$P = 150\text{ mm}$



Multiply the sum of the length and the width by 2.

$\boldsymbol{P = 2(l + w)}$

$P = 2(56\text{ mm} + 19\text{ mm})$

$P = 2 \cdot 75\text{ mm}$

$P = 150\text{ mm}$

Circumference

The **circumference** of a circle is a little more than 3 times its diameter. The ratio $\frac{C}{d}$ has a value of **π (pi)**. Pi is a non-repeating and non-terminating decimal with an approximate value of **3.14** or $\mathbf{\frac{22}{7}}$. Use the approximate value of pi to find an unknown circumference or diameter.

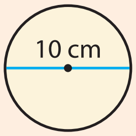
Find the Circumference Given the Diameter

$\boldsymbol{C = \pi d}$

$\boldsymbol{C = \pi d}$

$C = 3.14 \times 10$

$C = 31.4\text{ cm}$

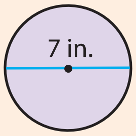


$\boldsymbol{C = \pi d}$

$C = \frac{22}{7} \times 7$

$C = \frac{22}{\underset{1}{\cancel{7}}} \times \frac{\overset{1}{\cancel{7}}}{1}$

$C = 22\text{ in.}$



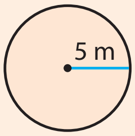
Find the Circumference Given the Radius

$\boldsymbol{C = 2\pi r}$

$\boldsymbol{C = 2\pi r}$

$C = 2 \times 3.14 \times 5$

$C = 31.4\text{ m}$

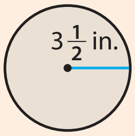


$\boldsymbol{C = 2\pi r}$

$C = 2 \times \frac{22}{7} \times 3\frac{1}{2}$

$C = 2 \times \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}}}$

$C = 22\text{ in.}$



Find the Diameter Given the Circumference

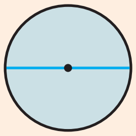
Since $\boldsymbol{C = \pi d}$**, then** $\boldsymbol{\frac{C}{\pi} = d}$**.**

$\boldsymbol{\frac{C}{\pi} = d}$

$C = 28.26$

$\frac{28.26}{3.14} = d$

$d = 9\text{ cm}$



Area

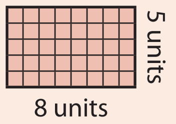
Area is the space within a region. The area of a region is the number of square units needed to cover its surface.

Area of a Rectangle

$\boldsymbol{A = l \cdot w}$

$A = 8 \cdot 5$

$A = 40\ \text{units}^2$

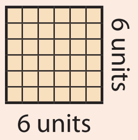


Area of a Square

$\boldsymbol{A = l \cdot w}$ **or** $\boldsymbol{A = s^2}$

$A = 6 \times 6$ or 6^2

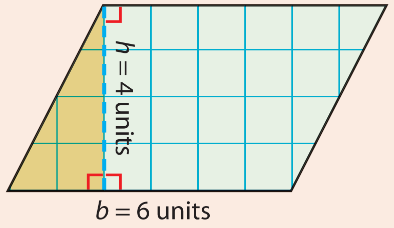
$A = 36\ \text{units}^2$



Area of a Parallelogram

$$\boldsymbol{A = b \cdot h}$$

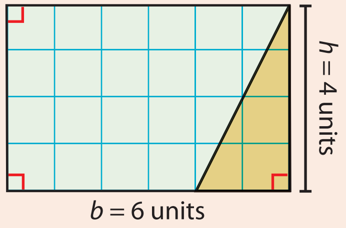
To change a parallelogram to a rectangle, you can remove a triangle from one side of the parallelogram and connect it to the other side.



$\boldsymbol{A = b \cdot h}$

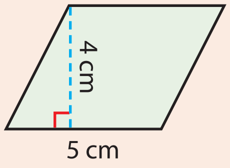
$A = 6 \cdot 4$

$A = 24\ \text{units}^2$



Area of a Triangle

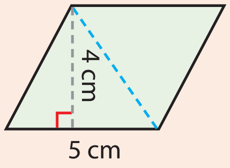
A diagonal divides a parallelogram into two congruent triangles. The area of each triangle is $\frac{1}{2}$ of the area of the parallelogram.



$\boldsymbol{A = b \cdot h}$

$A = 5 \cdot 4$

$A = 20\ \text{cm}^2$



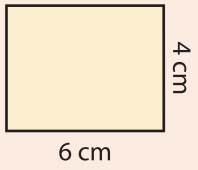
$\boldsymbol{A = \frac{1}{2}(b \cdot h)}$

$A = \frac{1}{2}(5 \cdot 4)$

$A = \frac{1}{2}(20)$

$A = 10\ \text{cm}^2$

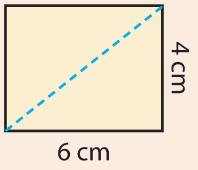
Each triangle has an area of 10 cm².



$\boldsymbol{A = l \cdot w}$

$A = 6 \cdot 4$

$A = 24\ \text{cm}^2$



$\boldsymbol{A = \frac{1}{2}(b \cdot h)}$

$A = \frac{1}{2}(6 \cdot 4)$

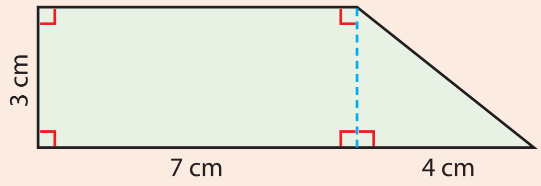
$A = \frac{1}{2}(24)$

$A = 12\ \text{cm}^2$

Each triangle has an area of 12 cm².

Area of a Complex Figure

The area of an irregular polygon is determined by finding the area of each smaller region in the figure and then adding the areas.



$A = (7 \cdot 3) + \frac{1}{2}(4 \cdot 3)$

$A = 21 + \frac{1}{2}(12)$

$A = 21 + 6$

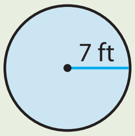
$A = 27\ \text{cm}^2$

Area of Circles

The area of a circle is π times the radius squared: $\boldsymbol{A = \pi r^2}$.

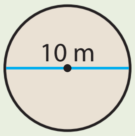
Substitute the length of the radius for r and 3.14 for π.

$r = 7$ ft
$A = \pi r^2$
$A = 3.14(7^2)$
$A = 3.14(49)$
$A = 153.86$ ft^2



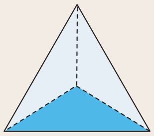
Remember that the radius is half the length of the diameter.

If $d = 10$ m, then $r = 5$ m.
$A = \pi r^2$
$A = 3.14(5^2)$
$A = 3.14(25)$
$A = 78.5$ m^2

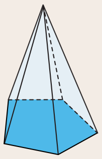


Surface Area of Prisms

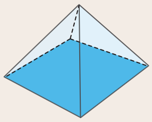
A **pyramid** is a cone with a polygon instead of a circle as a base. A pyramid is named for the shape of its base. All other faces of a pyramid are triangles.



Triangular pyramid

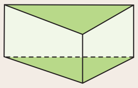


Pentagonal pyramid

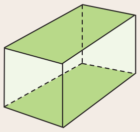


Square pyramid

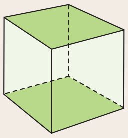
A **prism** is a type of cylinder with 2 congruent polygon bases that are parallel. A prism is named for the shape of its bases. All other faces of a prism are parallelograms.



Triangular prism



Rectangular prism

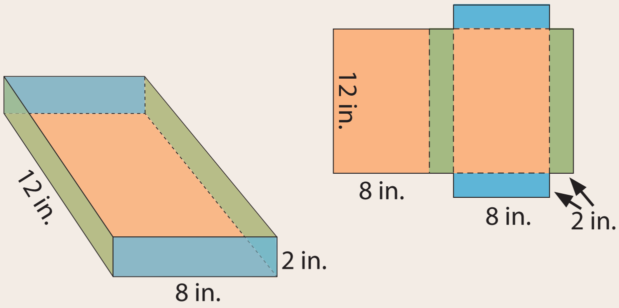


Cube (square prism)

The **surface area** of a 3-dimensional figure is the sum of the areas of all its surfaces.

A **rectangular prism** has 3 sets of congruent faces.

$S = 2(l \cdot w) + 2(w \cdot h) + 2(l \cdot h)$

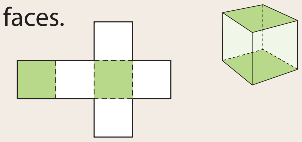


top and bottom	$2(12 \cdot 8) =$	**192 in.^{2}**
front and back	$2(12 \cdot 2) =$	**48 in.^{2}**
sides	$2(8 \cdot 2) =$	**32 in.^{2}**

Total Surface Area = $\boxed{272 \text{ in.}^2}$

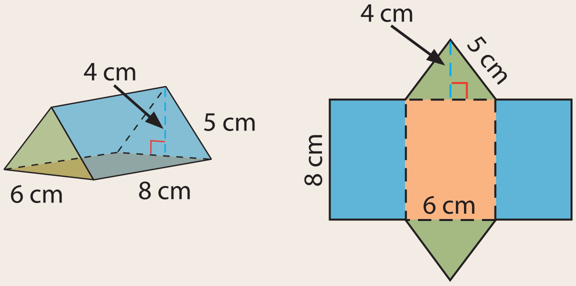
A **cube** has 6 congruent faces.

$S = 6(l \cdot w)$ or $S = 6s^2$



A **triangular prism** has 5 faces.

Calculate the area of the 3 rectangular faces.
$A = l \cdot w$
Calculate the area of the 2 triangular bases.
$A = \frac{1}{2}(b \cdot h)$
Add the areas of the 5 faces.



bottom face	$8 \cdot 6 =$	**48 cm^2**
slanted sides	$2(8 \cdot 5) =$	**80 cm^2**
triangular bases	$2[\frac{1}{2}(4 \cdot 6)] =$	**24 cm^2**

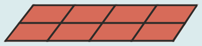
Total Surface Area = $\boxed{152 \text{ cm}^2}$

Volume of Rectangular Prisms

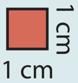
The area of a figure is the number of square units a flat space covers. **Volume** builds on the area of a figure. Multiply the area of the **base** ($B = l \times w$) by the number of cubic unit layers (**height**) of the three-dimensional figure. Volume is the number of cubic units a figure contains. The formula is $\boldsymbol{V = Bh}$.

Area is measured using square units. units^2

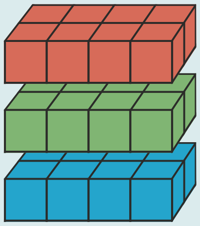
Volume is measured using cubic units. units^3



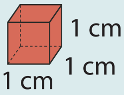
1 square cm or cm^2



$1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^2$

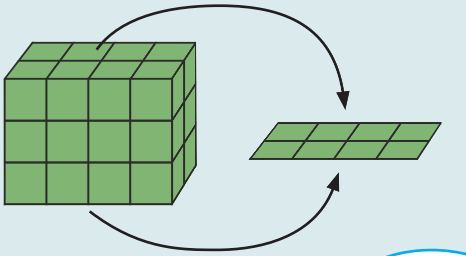


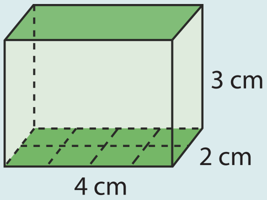
1 cubic cm or cm^3



$1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^3$

The volume of any prism can be found using the volume formula. Because prism bases are parallel and congruent, opposite bases will have the same area.





$\boldsymbol{V = Bh}$
$\boldsymbol{V = (l \times w) \times h}$
$\boldsymbol{V = (\textbf{4 rows of 2 cubes}) \times \textbf{3 layers}}$
$V = 8 \text{ cubes} \times 3 \text{ layers}$
$V = 24 \text{ cubes or } 24 \text{ cubic units}$

B (base) is found using the area formula ***l* × *w***.

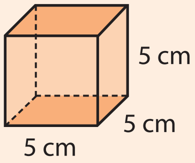
$\boldsymbol{V = Bh}$
$\boldsymbol{V = (l \times w) \times h}$
$V = (4 \times 2) \times 3$
$V = 8 \times 3$
$V = 24 \text{ cm}^3$

Volume of Cubes

A cube or square prism is a special rectangular prism where all six faces are congruent squares and all sides measure the same.

The formula for volume, $V = Bh$, can be modified for a cube.

$$\text{Volume of a cube} = \overset{\text{Base}}{(\text{side} \times \text{side})} \times \overset{\text{height}}{\text{side}}$$
$$V = s^3$$



$\boldsymbol{V = (s \times s) \times s}$
$V = (5 \times 5) \times 5$
$V = 25 \times 5$
$\boldsymbol{V = 125 \text{ cm}^3}$

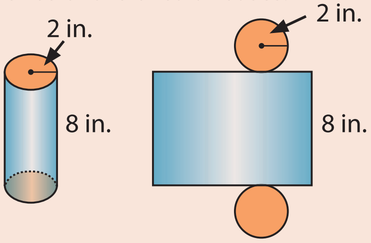
or

$\boldsymbol{V = s^3}$
$V = 5^3$
$V = 5 \times 5 \times 5$
$\boldsymbol{V = 125 \text{ cm}^3}$

Surface Area of Cylinders

A **cylinder** has 2 congruent circular bases and 1 curved surface.

- Calculate the area of one circle using $\boldsymbol{A = \pi r^2}$. Multiply by 2 to find the area of both circular bases.
- Calculate the area of the curved surface. The curved surface is a rectangle when lying flat. Use $\boldsymbol{A = l \bullet w}$. The width of the rectangle is the height of the cylinder. The length of the rectangle is the circumference of the circular bases.



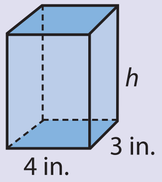
circular bases	$2(3.14 \bullet 2^2) =$	**25.12 in.²**
curved surface	$(3.14 \bullet 4)8 =$	**100.48 in.²**

Total Surface Area = $\boxed{125.60 \text{ in.}^2}$

Find an Unknown Measurement

The formula for volume of a prism is $V = Bh$. Since the bases of the figure are rectangles, use $V = (l \cdot w) \cdot h$.

When the volume is given and any two of the volume dimensions are known, you can find the unknown third dimension of a figure.



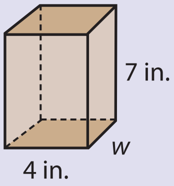
$V = Bh$
$V = (l \cdot w) \cdot h$

$V = 84$ in.^{3}
$l = 4$ in.
$w = 3$ in.
$h =$ ___ in.

$84 \text{ in.}^3 = (4 \cdot 3) \cdot h$
$84 \text{ in.}^3 = 12 \cdot h$

$\frac{84 \text{ in.}^3}{12} = \frac{\cancel{12}^{1}h}{\cancel{12}_{1}}$

$7 \text{ in.} = h$



$V = Bh$
$V = (l \cdot w) \cdot h$

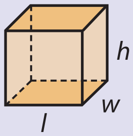
$V = 84$ in.^{3}
$l = 4$ in.
$w =$ ___ in.
$h = 7$ in.

$84 \text{ in.}^3 = (4 \cdot w) \cdot 7$
$84 \text{ in.}^3 = 4 \cdot w \cdot 7$
$84 \text{ in.}^3 = 4 \cdot 7 \cdot w$
$84 \text{ in.}^3 = 28 \cdot w$

$\frac{84 \text{ in.}^3}{28} = \frac{\cancel{28}^{1}w}{\cancel{28}_{1}}$

$3 \text{ in.} = w$

What is the measure of the length, the width, and the height of a cube whose volume is 27 units?



V (of a cube) $= s^3$
$27 = s \cdot s \cdot s$

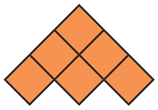
If $s = 2$: $2 \cdot 2 \cdot 2 = 8$
If $s = 3$: $3 \cdot 3 \cdot 3 = 27$
Each side is 3 units.

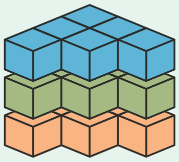
Volume of an Irregular Prism

Count the square units to find the area of an irregular base. Multiply the base by the height to find the volume of the irregular prism.

1. Count the square units in the base. $B = 6$ **square units**
2. Substitute the area of the base for B in the volume formula.
   $V = Bh$
   $V = (6)(3)$
   $V = 18$ **units**^{3}

The base of the irregular prism.

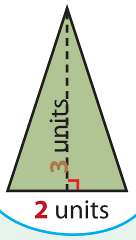


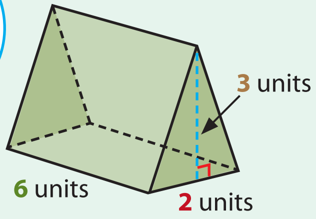


Volume of a Triangular Prism

1. Find the area of the triangular base.
   $B = \frac{1}{2}bh$
   $B = \frac{1}{2}(2 \cdot 3)$
   $B = \frac{1}{2}(6)$
   $B = 3$ **units**^{2}
2. Substitute the area of the base for B in the volume formula.
   $V = Bh$ or $(\frac{1}{2}bh_1)h_2$
   $V = (3)(6)$
   $V = 18$ **units**^{3}

Use $A = \frac{1}{2}bh$ to find the area of a triangle.

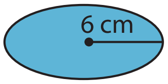


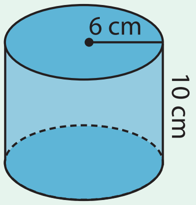


Volume of a Cylinder

1. Find the area of the circular base given the radius. (Remember that $r = \frac{1}{2}d$ if a diameter is given.)
   $B = \pi r^2$
   $B = (3.14)(6^2)$
   $B = (3.14)(36)$
   $B = 113.04$ **cm**^{2}
2. Substitute the area of the circular base for B in the volume formula.
   $V = Bh$ or $(\pi r^2)h$
   $V = 113.04 \cdot 10$
   $V = 1130.4$ **cm**^{3}

Use $A = \pi r^2$ to find the area of a circular base.





Integers

absolute value The distance of a number from 0 on a number line. (See below.)

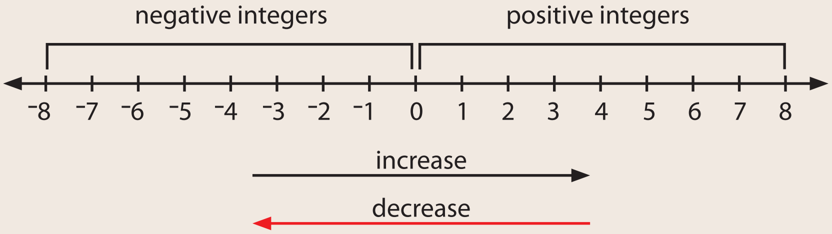
integer A whole number or its opposites.
(. . . $^-2$, $^-1$, 0, 1, 2, . . .)

negative number A number whose value is less than 0.

positive number A number whose value is greater than 0.

Positive & Negative Numbers

Integers consist of the whole numbers and their opposites.
{. . . $^-3$, $^-2$, $^-1$, 0, 1, 2, 3 . . .}



The values of negative numbers continue to *decrease* as you move left on a number line.

$^-6 < ^-3$

$^-8 < ^-7$

$^-2 > ^-4$

The value of a negative number will always be *less than* the value of a positive number.

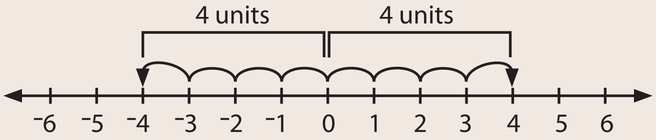
$1 > ^-5$

$^-3 < 2$

$7 > ^-7$

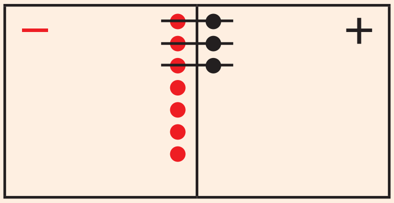
The numbers 4 and $^-4$ are **opposites** because they are the same distance from zero in opposite directions. **Absolute value** is the distance from zero. Distance is expressed as a positive value. It is indicated by the symbol $|n|$ and is read "the absolute value of *n*."

$|4|$ The *absolute value of 4* is 4.
$|^-4|$ The *absolute value of negative 4* is 4.

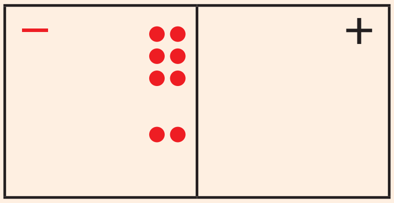


Add Integers Using an Algebra Mat

1. Draw the first addend on the mat.
2. Draw the second addend on the mat.
3. A positive counter and a negative counter cancel each other out to make zero. $1 + ^-1 = 0$
4. The answer is the number of counters that have *not* been cancelled out.



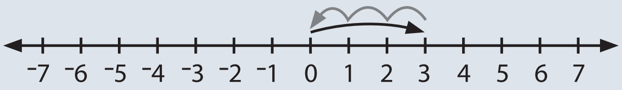
$^-7 + 3 = ^-4$



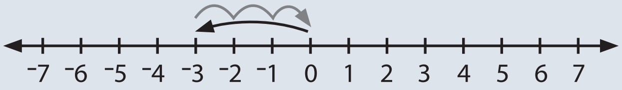
$^-6 + ^-2 = ^-8$

When combining ***opposites***, the sum is always zero.

$3 + {}^{-}3 = 0$

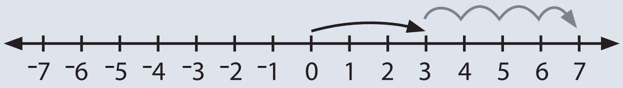


$^{-}3 + 3 = 0$

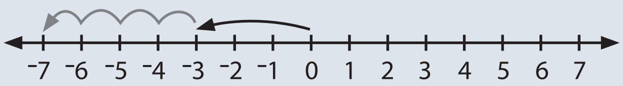


These addition equations combine numbers with ***like*** signs.

$3 + 4 = 7$

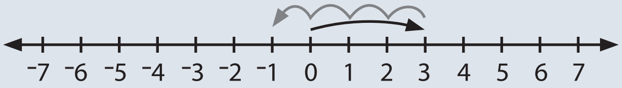


$^{-}3 + {}^{-}4 = {}^{-}7$

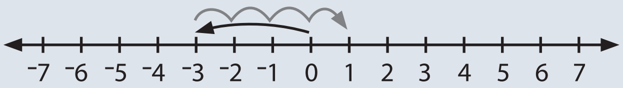


These addition equations combine numbers with ***unlike*** signs.

$3 + {}^{-}4 = {}^{-}1$

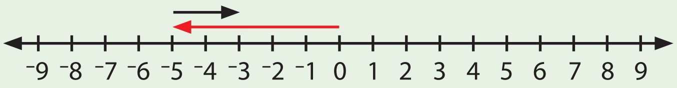


$^{-}3 + 4 = 1$

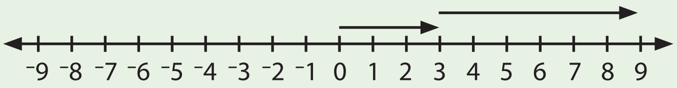


Subtract Integers Using a Number Line

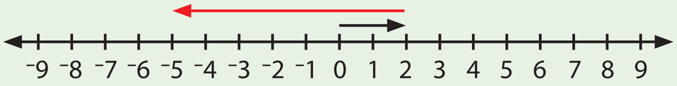
1. Begin at 0 and draw an arrow to the minuend (the total).
2. Draw a second arrow from the minuend. Draw the arrow *left* of the minuend to subtract a positive number or *right* of the minuend to subtract a negative number.
3. The final stopping place is the difference.



$^{-}5 - {}^{-}2 = {}^{-}3$



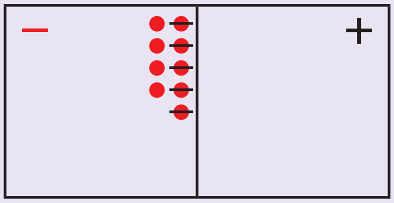
$3 - {}^{-}6 = 9$



$2 - 7 = {}^{-}5$

Subtract Integers Using an Algebra Mat

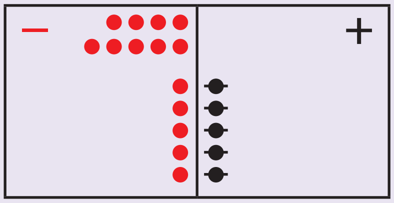
1. Put the minuend (the total) on the mat.
2. Cross out the subtrahend counters (the number being subtracted).
3. If there are *not* enough counters to subtract, draw pairs of positive and negative counters on the mat until the subtrahend can be subtracted (the Zero Principle).
4. The answer is the number of counters that have *not* been cancelled out.



$^{-}9 - ^{-}5 = ^{-}4$

Zero Principle

A number may be renamed by adding or subtracting 0 without changing the value of that number.

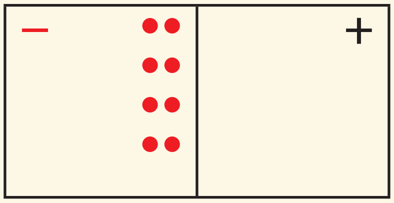


$^{-}9 - 5 = ^{-}14$

Multiply Integers

$^{-}2 \times 4 = ^{-}8$

$^{-}2$ sets of 4 counters cannot be shown on the algebra mat, but the Commutative Property states that the product for $^{-}2 \times 4$ is the same as $4 \times ^{-}2$.



$4 \times ^{-}2 = ^{-}8$

4 sets of 2 negative counters can be shown on an algebra mat.

Commutative Property of Multiplication

The order of factors may be changed without changing the product. $4 \times ^{-}2 = ^{-}2 \times 4$

Rules for Multiplying Integers

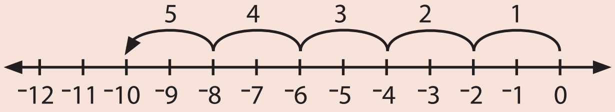
When two factors have the same sign, the product is positive.

positive factor × positive factor = positive product
negative factor × negative factor = positive product

When two factors have different signs, the product is negative.

positive factor × negative factor = negative product
negative factor × positive factor = negative product

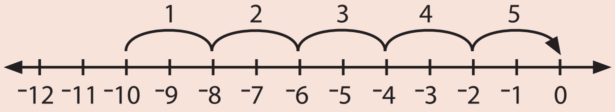
Multiply and Divide Integers



$5 \times ^{-}2 = ^{-}10$

1. Begin at 0.
2. Add five sets of $^{-}2$.

$5 \times ^{-}2 = ^{-}10$;
therefore, $^{-}10 \div ^{-}2 = 5$ and $^{-}10 \div 5 = ^{-}2$.



$^{-}10 \div ^{-}2 = 5$

3. Begin at $^{-}10$.
4. Subtract sets of $^{-}2$ until you reach 0.

$^{-}10 \div ^{-}2 = 5$
Check: $5 \times ^{-}2 = ^{-}10$

Customary Measurement

Linear Equivalents

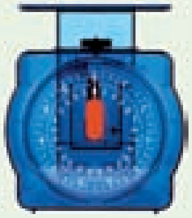
12 inches (in.) = 1 foot (ft)

36 inches = 1 yard (yd)
3 feet = 1 yard

5,280 feet = 1 mile (mi)
1,760 yards = 1 mile

Weight Equivalents

1 pound (lb) = 16 ounces (oz)
1 ton (tn) = 2,000 pounds



Capacity Equivalents

1 cup (c) = 8 fluid ounces (fl oz)
1 pint (pt) = 2 cups
1 quart (qt) = 2 pints
1 gallon (gal) = 4 quarts

Rename Measurements

Rename larger units as smaller units. Determine the equivalency and then *multiply*.

30 lb = ___ oz

1 lb = 16 oz

$$\begin{array}{r} 30 \\ \times 16 \\ \hline 480 \end{array}$$

30 × 16 = 480

30 lb = 480 oz

Rename smaller units as larger units. Determine the equivalency and then *divide*.

19 qt = ___ gal

4 qt = 1 gal

$$\begin{array}{r} 4\frac{3}{4} \\ 4\overline{)19} \\ -16 \\ \hline 3 \end{array}$$

19 ÷ 4 = 4 r3

19 qt = $4\frac{3}{4}$ gal
or
19 qt = 4 gal 3 qt

Determine the equivalency and solve.

$\frac{3}{4}$ of a mile = ___ ft

1 mi = 5,280 ft

$$\begin{array}{r} 1,320 \\ \times \quad 3 \\ \hline 3,960 \end{array}$$

$\frac{3}{\underset{1}{\cancel{4}}} \times \overset{1,320}{\cancel{5,280}}$ ft =

$\frac{3}{4}$ mi = 3,960 ft

Metric Measurement

The **meter** (m) is the basic unit of length in the metric system.

Metric Equivalents

1 kilometer (km) = 1000 m
1 m = 100 centimeters (cm)
1 m = 1000 millimeters (mm)

Decimal Equivalents

1 kilometer (km) = 1000 m
1 centimeter (cm) = 0.01 m
1 millimeter (mm) = 0.001 m

Mass is the amount of matter an object has. The **gram** (g) is a basic unit of mass.

1000 g = 1 kilogram (kg)
1 g = 0.001 kg
1 g = 1000 milligrams (mg)

Capacity is the amount of liquid a container will hold. The **liter** (L) is the basic unit of capacity in the metric system.

1 L = 1000 milliliters (mL)
1 mL = 0.001 L

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Rename Metric Measurements

Rename larger units as smaller units. Determine the equivalency and then *multiply*.

4 km = ___ m

$\boxed{1 \text{ km} = 1000 \text{ m}}$

4 × 1000 = 4000

4 km = 4000 m

$$\begin{array}{r} 1000 \\ \times \quad 4 \\ \hline 4000 \end{array}$$

Rename smaller units as larger units. Determine the equivalency and then *divide*.

150 cm = ___ m

$\boxed{100 \text{ cm} = 1 \text{ m}}$

150 ÷ 100 = 1.5

150 cm = 1.5 m

$$\begin{array}{r} 1.5 \\ 100\overline{)150.0} \\ -100 \\ \hline 500 \\ -500 \\ \hline 0 \end{array}$$

Determine the equivalency and solve.

$\frac{3}{4}$ km = ___ m

$\boxed{1 \text{ km} = 1000 \text{ m}}$

$\frac{3}{\underset{1}{\cancel{4}}} \times \overset{250}{\cancel{1000}} = 750$

$\frac{3}{4}$ km = 750 m

$$\begin{array}{r} 250 \\ \times \quad 3 \\ \hline 750 \end{array}$$

Use the facts for temperature to convert between **Fahrenheit** and **Celsius**.

Convert Temperature

Celsius to Fahrenheit

$F = (\frac{9}{5} \times C) + 32$

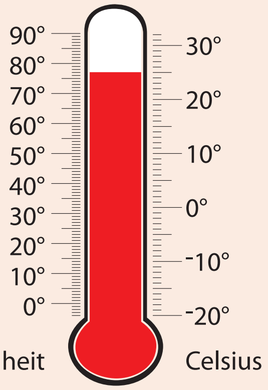
___°F = 25°C

$F = (\frac{9}{5} \times 25) + 32$

$F = (\frac{9}{5} \times \frac{25}{1}) + 32$

$F = 45 + 32$

$F = 77°$



77°F = 25°C

Fahrenheit to Celsius

$C = \frac{5}{9} \times (F - 32)$

___°C = 77°F

$C = \frac{5}{9} \times (77 - 32)$

$C = \frac{5}{9} \times 45$

$C = \frac{5}{\underset{1}{\cancel{9}}} \times \frac{\overset{5}{\cancel{45}}}{1}$

$C = 25°$

Standard Temperatures

Freezing Point of Water

32°F
0°C

Boiling Point of Water

212°F
100°C

Normal Body Temperature

98.6°F
37°C

Time Equivalents

1 **minute** (min) = 60 **seconds** (sec)

1 **hour** (hr) = 60 min

1 **day** (d) = 24 hr

Rename Time

Rename larger units as smaller units. Determine the equivalency and then *multiply*.

7 hr = ___ min

$\boxed{1 \text{ hr} = 60 \text{ min}}$

7 × 60 = 420

7 hr = 420 min

Rename smaller units as larger units. Determine the equivalency and then *divide*.

72 hr = ___ d

$\boxed{24 \text{ hr} = 1 \text{ d}}$

72 ÷ 24 = 3

72 hr = 3 d

Rename a fraction of time.

$\frac{1}{4}$ hr = ___ min

$\boxed{1 \text{ hr} = 60 \text{ min}}$

$\frac{1}{4} \times 60 = 15$ min

Add or Subtract Time

$$\begin{array}{r} 3 \text{ hr } 25 \text{ min} \\ +\ 4 \text{ hr } 50 \text{ min} \\ \hline 7 \text{ hr } 75 \text{ min} = \\ 8 \text{ hr } 15 \text{ min} \end{array}$$

Add the minutes.
Add the hours.
Rename the answer if possible.

$$\begin{array}{r} \overset{22}{\cancel{23}} \text{ hr } \overset{90}{\cancel{30}} \text{ min} \\ -\ 16 \text{ hr } 40 \text{ min} \\ \hline 6 \text{ hr } 50 \text{ min} \end{array}$$

Subtract the minutes.
Rename if needed.
Subtract the hours.

Multiplication & Division

Associative Property of Multiplication The grouping of factors may be changed without changing the product.

$2 \times (3 \times 4) = 24$

$(2 \times 3) \times 4 = 24$

average (mean) The number found by adding two or more quantities in a set of numbers and then dividing the sum by the number of addends.

> scores: 9, 12, 15, 8
> Add the scores to find the sum.
> $9 + 12 + 15 + 8 = 44$
> Divide the sum by the number of addends.
> $44 \div 4 = 11$
> The average score is 11.

base The number used as a factor in exponent form.

$$\underset{\substack{\uparrow \\ \text{base}}}{3}^4 = (3 \times 3 \times 3 \times 3)$$

Commutative Property of Multiplication The order of factors in a multiplication equation can be changed without changing the product.

$8 \times 2 = 16$

$2 \times 8 = 16$

composite number A number greater than 1 that has more than 2 factors.

$1 \times 6 = 6$ $2 \times 3 = 6$

6 has 4 factors: 1, 2, 3, and 6.

6 is composite.

Distributive Property of Multiplication over Addition The product of 2 factors can be found by separating 1 factor into parts, multiplying each part by the other factor, and adding the partial products.

dividend The number to be divided.

$56 \div 7 = 8$ $\begin{array}{r} 8 \\ 7\overline{)56} \end{array}$ $\frac{56}{7} = 8$

divisible A number that can be equally divided with a remainder of 0 (or with none remaining).

divisor The number by which another number (dividend) is divided.

$56 \div 7 = 8$ $\begin{array}{r} 8 \\ 7\overline{)56} \end{array}$ $\frac{56}{7} = 8$

exponent A small raised number that indicates how many times a number is to be used as a factor.

$$3^{\overset{\text{exponent}}{\uparrow}\,4} = (3 \times 3 \times 3 \times 3)$$

expression A mathematical phrase made up of numbers, operation signs, and sometimes variables.

$45 + n$ 27×6

fact family A group of related addition and subtraction or multiplication and division facts using the same numbers.

$\boxed{3 \quad 4 \quad 12}$

$3 \times 4 = 12$ $12 \div 3 = 4$

$4 \times 3 = 12$ $12 \div 4 = 3$

factors The numbers multiplied to find a product.

$3 \times 6 = 18$

greatest common factor (GCF) The greatest factor that is the same for two or more numbers.

Identity Property of Multiplication When 1 is a factor, the product is the same as the other factor.

$5 \times 1 = 5$

least common multiple (LCM) The lowest multiple, other than 0, that is the same for 2 or more numbers.

multiples of 3: 3, 6, 9, ⑫, 15, 18, 21, **24**, 27, 30

multiples of 4: 4, 8, ⑫, 16, 20, **24**, 28, 32, 36, 40

12 is the least common multiple of 3 and 4.

long division A series of math operations used to solve a division problem.

multiple The product of two whole numbers. The first 4 multiples of 4 are 0, 4, 8, and 12.

multiplicand The factor in a multiplication equation that tells the number in each set.

$3 \times 16 = 48$

multiplier The first factor in a multiplication equation; it tells how many times the other factor will be multiplied.

$3 \times 16 = 48$

prime number A number with exactly two different factors (the number itself and 1).

$1 \times 5 = 5$

5 has exactly 2 factors: 1 and 5.

5 is prime.

product The answer to a multiplication problem.

$3 \times 7 = 21$

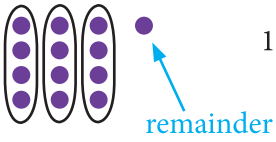
quotient The answer to a division problem.

$56 \div 7 = 8$

$7\overline{)56}^{\,8}$

$\frac{56}{7} = 8$

remainder The part left over after dividing a number (dividend) by another number (divisor).



$13 \div 4 = 3\text{ r}1$

variable A letter used to represent a number.

$36 \div x = 9$

$x = 4$

Zero Property of Multiplication When 0 is a factor, the product is always 0.

$7 \times 0 = 0$

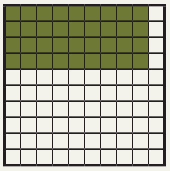
Divisibility Rules

A number is divisible by		Example
2	if the ones digit is even (0, 2, 4, 6, or 8).	$13\underline{4}$ $50\underline{6}$ $2{,}77\underline{8}$
3	if the sum of the digits is divisible by 3.	27 (2 + 7 = 9) 561 (5 + 6 + 1 = 12)
4	if the last two digits form a number divisible by 4.	624 (24 ÷ 4 = 6) 7,932 (32 ÷ 4 = 8)
5	if the ones digit is 0 or 5.	690 2,115
6	if the number is divisible by 2 and 3.	978 2: Ones place is even; 3: 9 + 7 + 8 = 24
9	if the sum of the digits is divisible by 9.	4,473 (4 + 4 + 7 + 3 = 18)
10	if the ones digit is 0.	990 6,000 12,630

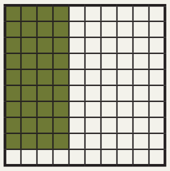
Multiplication

Multiplication is a form of addition. It is used to find the **product** (total) when equal sets are joined. The first **factor** of a multiplication equation tells the number of sets; the second **factor** tells the size of each set. When illustrating or writing an equation for a word problem or phrase, determine the number of sets and how many are in each set.

$4 \times 9 = 36$
(4 sets of 9)



$9 \times 4 = 36$
(9 sets of 4)



A **multiple** is the product of a whole number and any given number.

- The first four nonzero multiples of 3 are 3, 6, 9, and 12.
- The first four nonzero multiples of 4 are 4, 8, 12, and 16.
- 12 is a multiple of both 3 and 4.

A multiplication equation can be written several ways. In the following problems, 3 and 4 are known factors, *a* is an unknown factor, and 12 is the product.

$4 \times 3 = 12$ $3 \cdot 4 = 12$ $3a = 12$ $3(4) = 12$

Properties of Multiplication

Commutative Property

The order of factors can be changed without changing the product.

$$4 \times 6 = 6 \times 4$$
$$24 = 24$$
$$a \bullet b = b \bullet a$$

Identity Property

When 1 is a factor, the product is the same as the other factor.

$$6 \times 1 = 6$$
$$a \bullet 1 = a$$

Zero Property

When 0 is a factor, the product is always 0.

$$4 \times 0 = 0$$
$$a \bullet 0 = 0$$

Associative Property

The grouping of factors can be changed without changing the product.

$$(6 \times 4) \times 5 = 6 \times (4 \times 5)$$
$$24 \times 5 = 6 \times 20$$
$$120 = 120$$
$$(a \bullet b) \bullet c = a \bullet (b \bullet c)$$

Distributive Property

The product of any 2 factors can be found by separating 1 factor into parts or addends. Multiply each part or addend by the other factor and add the partial products.

$$\begin{aligned} 6 \times 27 &= \\ 6 \times (20 + 7) &= \\ (6 \times 20) + (6 \times 7) &= \\ 120 + 42 &= 162 \end{aligned}$$

Multiplication

The short form of multiplication combines the steps of the **Distributive Property**.

Distributive Property	Short Form
4 × 5,280 = 4 × (5,000 + 200 + 80) = (4 × 5,000) + (4 × 200) + (4 × 80) = 20,000 + 800 + 320 = 21,120	$\begin{array}{r} {}^{1}\;{}^{3} \\ 5{,}280 \\ \times\quad 4 \\ \hline 21{,}120 \end{array}$

Multiply the **ones** by 4.
Multiply the **tens** by 4.
    Rename 30 of the 32 tens as 3 hundreds.
Multiply the **hundreds** by 4.
    Add the renamed 3 hundreds.
    Rename 10 of the 11 hundreds as 1 thousand.
Multiply the **thousands** by 4.
    Add the renamed 1 thousand.

Multiplication Terms

7 × 3 = 21

$$\begin{array}{r} 3 \\ \times 7 \\ \hline 21 \end{array}$$

The factors are 7 and 3.
The product is 21.

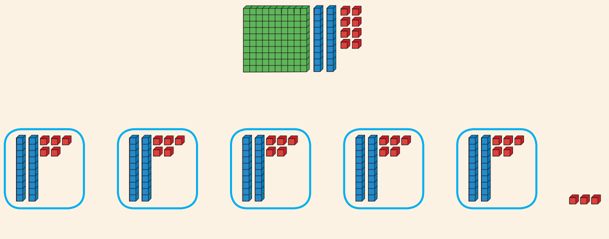
Division

Divide 128 into 5 sets.

- Rename 1 hundred as 10 tens.
- **Divide 12 tens by 5.**
- Rename 2 tens as 20 ones.
- **Divide 28 ones by 5.**

$$\begin{array}{r} 25\text{ r3} \\ 5\overline{)128}\phantom{\text{ r3}} \\ -10\phantom{\text{ r3}} \\ \hline 28\phantom{\text{ r3}} \\ -25\phantom{\text{ r3}} \\ \hline 3\phantom{\text{ r3}} \end{array}$$

There are 25 in each of 5 sets with 3 remaining.



128 ÷ 5 = 25 r3

Division Terms

36 ÷ 4 = 9

$$\begin{array}{r} 9 \\ 4\overline{)36} \end{array}$$

$$\frac{36}{4} = 9$$

The **dividend** is 36, the **divisor** is 4, and the **quotient** is 9.

Divide by Multiples of 10

A whole number is **divisible** by another whole number if there is no remainder. You can use **divisibility rules** to determine whether a whole number is divisible by 2, 3, 4, 5, 6, 9, or 10.

Use mental math to find the quotient when the dividend and the divisor are multiples of 10. When only the divisor is a multiple of 10, think of **compatible numbers** as you use the long division process.

1. Decide where to start.

$$\begin{array}{r} \text{xx} \\ 50\overline{)4{,}500} \end{array}$$

$$\begin{array}{r} \text{xx} \\ 40\overline{)1{,}651} \end{array}$$

2. Think of the basic fact or think of the compatible numbers.

$$\underline{9}0 \times \underline{5}0 = \underline{4{,}5}00$$

$$\underline{4}0 \times \underline{4}0 = \underline{1{,}6}00$$

3. Solve using mental math or solve using the long division process.

$$\begin{array}{r} 90 \\ 50\overline{)4{,}500} \end{array}$$

$$\begin{array}{r} 41\ \text{r}11 \\ 40\overline{)1{,}651} \\ -\ \underline{160} \\ 51 \\ -\underline{40} \\ 11 \end{array}$$

Order of Operations

1. Do operations in **parentheses**.
2. Find the value of **exponents**.
3. **Multiply** and **divide** from left to right.
4. **Add** and **subtract** from left to right.

This sentence can help you remember the order of operations.

Please Excuse My Dear Aunt Sally.

Parentheses Exponents Multiplication Division Addition Subtraction

To simplify an expression, analyze it and use the necessary steps.

$8 - \mathbf{3 \times 4} \div 4$ → Multiply.
$8 - \mathbf{12 \div 4}$ → Divide.
$\mathbf{8 - 3}$ → Subtract.
5

$3^2 - \mathbf{(5 - 3)} + 6$ → Subtract within parentheses.
$\mathbf{3^2} - 2 + 6$ → Find the value of the exponent.
$\mathbf{9 - 2} + 6$ → Subtract.
$\mathbf{7 + 6}$ → Add.
13

Exponents

Multiplication is a short way to write a repeated addition equation. When a factor is repeated in a multiplication equation, it can be written in exponent form.

$7 + 7 + 7 + 7 + 7 + 7 + 7 = 7 \times 7$ $\quad$ $7 \times 7 = 7^2$

The **base** tells what number is repeated as a factor.
The **exponent** tells the number of times the base is repeated as a factor.

Standard Form	Factored Form	Exponent Form
1,000,000	$10 \times 10 \times 10 \times 10 \times 10 \times 10$	10^6
100,000	$10 \times 10 \times 10 \times 10 \times 10$	10^5
10,000	$10 \times 10 \times 10 \times 10$	10^4
1,000	$10 \times 10 \times 10$	10^3
100	10×10	10^2
10	10	10^1
1	1	10^0
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10^1}$
$\frac{1}{100}$	$\frac{1}{10 \times 10}$	$\frac{1}{10^2}$

When 10 is the base, the exponent is the same as the number of zeros in the standard form.

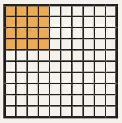
Exponents can be used to express the **powers of 10** in the expanded form of a number.

$$546.32 = (5 \times 100) + (4 \times 10) + (6 \times 1) + (3 \times \tfrac{1}{10}) + (2 \times \tfrac{1}{100}) =$$
$$(5 \times 10^2) + (4 \times 10^1) + (6 \times 10^0) + (3 \times \tfrac{1}{10^1}) + (2 \times \tfrac{1}{10^2})$$

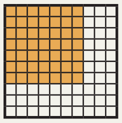
Numbers written in exponent form can be factored to find the standard form.

Exponent Form	Word Form	Factored Form	Standard Form
2^5	two to the fifth power	$2 \times 2 \times 2 \times 2 \times 2$	32
3^7	three to the seventh power	$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$	2,187
9^4	nine to the fourth power	$9 \times 9 \times 9 \times 9$	6,561
4^2	four to the second power, or four squared	4×4	16

When a number has an exponent of 2, it is called a **squared number**. An array created for a squared number always forms a square.



$4^2 = 4 \times 4$



$7^2 = 7 \times 7$

Probability

complementary events Two events that cannot occur at the same time.

compound event A single event that cannot occur unless two or more other events have occurred.

dependent events Two events in which the outcome of the first event affects the outcome of the second event.

experimental probability The number of outcomes of an event divided by the total number of trials.

independent events Two events where the first event does not affect the outcome of the second event.

probability The chance that an event will occur.

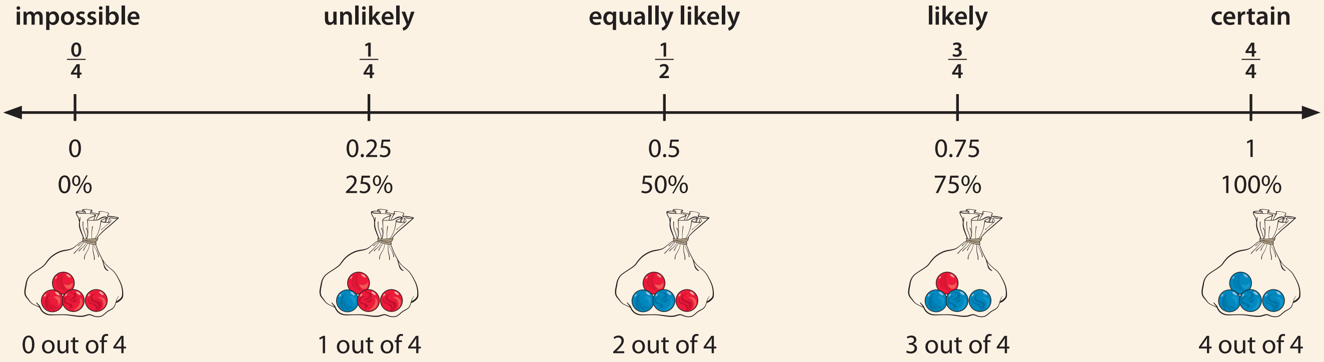
sample space The set of all possible outcomes of an event.

theoretical probability The number of favorable outcomes that an event will occur.

Probability is the likelihood that an event will occur. **Theoretical probability** is found when the total possible outcomes of an event are known and all outcomes are equally likely to occur. Probability is written as a ratio or a percent.

What is the probability of drawing a blue marble from each bag?

$$P(\text{event}) = \frac{\textbf{number of favorable outcomes}}{\textbf{number of possible outcomes}}$$



Complementary events are two events that could happen, but both events cannot happen at the same time. The sum of the two events must equal 1 or 100%.

The complement of it snowing today is *not* snowing today.

25% + P(*not* snow) = 100%
25% + 75% = 100%

Think 25% + ___ = 100%

When there is a 25% chance that it will snow, there is a 75% chance that it will *not* snow.

Experimental probability is found using data collected from an experiment or a survey. The experimental probability of an event is the number of observed occurrences of an event in relation to the total number of trials (or people surveyed).

$$P(\text{event}) = \frac{\textbf{number of outcomes in the event}}{\textbf{total number of trials}}$$

Knowing the number of possible outcomes is necessary when calculating probability. The **sample space** for an event is the set of all possible outcomes. A **tree diagram** is an organized way to show all the possible outcomes.

Use the **Multiplication Counting Principle** to find the number of outcomes when choices are given.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

The sample spaces show the outcomes of flipping 1, 2, and 3 coins. Each coin has a head (**H**) and a tail (**T**).

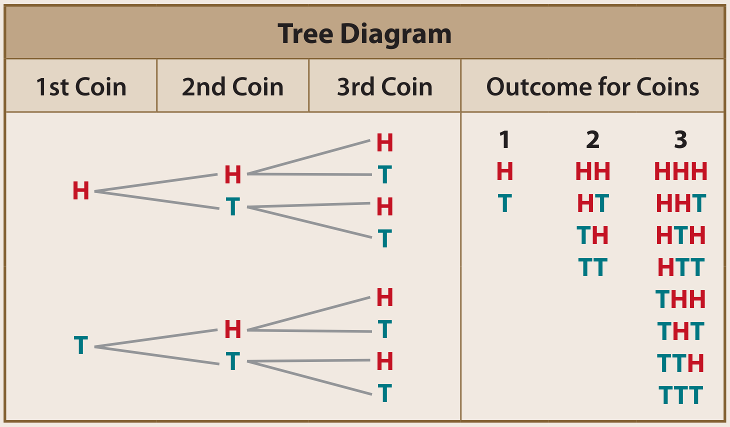
flipping 1 coin: 2 possible outcomes
{**H**, **T**}

flipping 2 coins: 4 possible outcomes
{**HH**, **HT**, **TH**, **TT**}

$2 \times 2 = 4$ possible outcomes

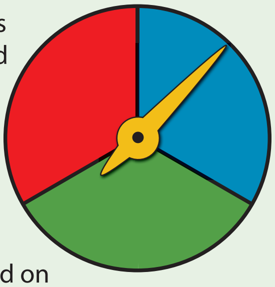
flipping 3 coins: 8 possible outcomes
{**HHH**, **HHT**, **HTH**, **HTT**, **THH**, **THT**, **TTH**, **TTT**}

$2 \times 2 \times 2 = 8$ possible outcomes



A **compound event** involves two or more simple events; they can be independent or dependent. An **independent event** occurs when the sample space of one event remains the same regardless of the outcome of a previous event. A **dependent event** occurs when the outcome of one event affects the sample space of a later event. You can find the probability of a compound event by multiplying the individual theoretical probabilities.

Independent Event



Alberto spins the spinner. He lands on a red section. He spins a second time and lands on blue.

Sample space for each spin:
{r, b, g}

$P(\text{red}) = \frac{1}{3}$ $P(\text{blue}) = \frac{1}{3}$

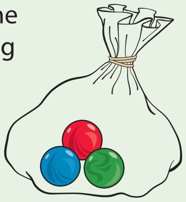
Find the probability of spinning red on the first spin and blue on the second spin.

$\boldsymbol{P(A, B) = P(A) \times P(B)}$

Sample space for 2 spins: {rr, rb, rg, bb, br, bg, gg, gr, gb}

$P(\text{red, blue}) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Dependent Event



Rosalie draws a marble from the bag. She keeps the red marble and passes the bag to Jolene to draw a marble. Jolene chooses a blue marble.

Sample space for the first draw: {r, b, g}

$P(\text{red}) = \frac{1}{3}$

Sample space for the second draw: {b, g}

$P(\text{blue after 1 red marble drawn}) = \frac{1}{2}$

Find the probability of drawing a red marble followed by a blue marble.

$\boldsymbol{P(A, B) = P(A) \times P(B \text{ after } A)}$

$P(\text{red, blue}) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

Ratios, Percents & Proportions

percent Per hundred; a ratio comparing a number to 100.

$5:100$	=	$\frac{5}{100}$	=	0.05	=	5%
5 to 100		*5 hundredths*		*5 hundredths*		*5 percent*

proportion An equation stating that two ratios are equal.

$$\frac{8}{12} = \frac{16}{24}$$

ratio A comparison of two quantities. (See below for more information.)

rate A ratio that compares two quantities that have different measuring units.

scale A ratio comparing the size of a map, model, or drawing, and the size of the actual object.

unit rate A rate in which the second term is 1. (See below for more information.)

A **ratio** is a mathematical comparison of two quantities. The **terms** represent the items being compared. A ratio is read using the word *to*.

There are 4 girls to every 6 boys that play in the county soccer league. The ratio of girls to boys can be written three ways: **4 to 6**, **4 : 6**, or $\frac{4}{6}$.

Quantities can be compared differently; therefore, the order of the terms must match the order of the quantities being compared in the written statement.

Quantities	Ratio	Word Form	Ratio Form	Fraction Form
part to part	girls to boys	4 to 6	4 : 6	$\frac{4}{6}$
part to whole	girls to players	4 to 10	4 : 10	$\frac{4}{10}$
whole to part	players to girls	10 to 4	10 : 4	$\frac{10}{4}$

Equivalent ratios can be found by multiplying or dividing both terms of the ratio by the same nonzero number (a form of 1). It is similar to renaming a fraction into higher or lower terms. To simplify a ratio, rename it to lowest terms.

$$\overset{\times 2}{\underset{\times 2}{4 \text{ to } 6 = 8 \text{ to } 12}} \qquad \overset{\div 2}{\underset{\div 2}{4 \text{ to } 6 = 2 \text{ to } 3}} \qquad \overset{\times 3}{\underset{\times 3}{4:10 = 12:30}} \qquad \overset{\div 2}{\underset{\div 2}{4:10 = 2:5}} \qquad \overset{\times 4}{\underset{\times 4}{\frac{10}{4} = \frac{40}{16}}} \qquad \overset{\div 2}{\underset{\div 2}{\frac{10}{4} = \frac{5}{2}}}$$

A **rate** is a special ratio comparing two quantities having different measuring units. The **unit rate** tells how many of a quantity there are *per* one unit of another quantity.

To find the unit rate, rename the ratio using a denominator of 1.

$$\overset{\div 4}{\underset{\div 4}{\frac{11}{4} = \frac{2.75}{1}}}$$

Multiply the terms of the unit rate to find an equivalent ratio.

$$\overset{\times 4}{\underset{\times 4}{\frac{12}{1} = \frac{48}{4}}}$$

Finding the Unknown Measure

Solve a proportion to find an unknown measure in similar figures.

Ratios Between Figures

Write a ratio for the corresponding sides of the two figures.

$6 \cdot 10$ $\quad \frac{10}{4} = \frac{n}{6} \quad$ $4 \cdot n$

$$60 = 4n$$
$$\frac{60}{4} = \frac{\overset{1}{\cancel{4}}n}{\underset{1}{\cancel{4}}}$$
$$15\text{ cm} = n$$

Ratios Within a Figure

Write a ratio for the sides *within* the same figure.

$6 \cdot 10$ $\quad \frac{10}{n} = \frac{4}{6} \quad$ $n \cdot 4$

$$60 = 4n$$
$$\frac{60}{4} = \frac{\overset{1}{\cancel{4}}n}{\underset{1}{\cancel{4}}}$$
$$15\text{ cm} = n$$

A **proportion** is an equation stating that two ratios are equivalent. Ratios are **proportional** when they are equivalent. The terms can be compared vertically, horizontally, or diagonally to test for equivalency.

$$\overset{\times 3}{\underset{\times 3}{\frac{1}{2} = \frac{3}{6}}}$$

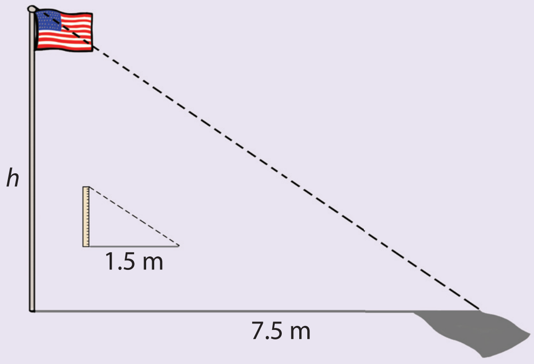
These ratios are equivalent and form a proportion.

$$\overset{\times 4}{\underset{\times 4}{\frac{2}{3} \neq \frac{8}{10}}}$$

$\frac{2}{3}$ is **not proportional** to $\frac{8}{10}$.

Indirect measurement uses similar objects to find the measurement of an object difficult to measure. Solve a proportion to find the unknown measurement.

The sixth-grade class wanted to know the height of the flagpole in the schoolyard. The teacher taught them how to determine the height of the flagpole without measuring it using the length of the flagpole's shadow and the length of the shadow of a meter stick.



Ratios Between Figures

$$\frac{\text{flagpole } h}{\text{meter } h} = \frac{\text{flagpole } s}{\text{meter } s}$$

$1.5 \cdot h$ $\quad \frac{h}{1} = \frac{7.5}{1.5} \quad$ $1 \cdot 7.5$

$$1.5h = 7.5$$
$$\frac{\overset{1}{\cancel{1.5}}h}{\underset{1}{\cancel{1.5}}} = \frac{7.5}{1.5}$$
$$h = 5\text{ m}$$

Ratios Within a Figure

$$\frac{\text{flagpole } h}{\text{flagpole } s} = \frac{\text{meter } h}{\text{meter } s}$$

$1.5 \cdot h$ $\quad \frac{h}{7.5} = \frac{1}{1.5} \quad$ $7.5 \cdot 1$

$$1.5h = 7.5$$
$$\frac{\overset{1}{\cancel{1.5}}h}{\underset{1}{\cancel{1.5}}} = \frac{7.5}{1.5}$$
$$h = 5\text{ m}$$

A **scale** is a ratio of measurements that compares the size of a drawing, a map, or a model with the size of the actual object.

Actual Measurement

The distance between two cities on a map is 2.5 centimeters. Given a map scale of 1 cm : 100 km, what is the actual distance?

$$\frac{\text{map distance (cm)}}{\text{actual distance (km)}} = \frac{1}{100} \times \frac{2.5}{n}$$

$$n = 250\text{ km}$$

Drawing or Model Measurement

The length of a car is 156 inches. Find the length of a model car using the scale 1 in. : 52 in.

$$\frac{\text{model length}}{\text{actual length}} = \frac{1}{52} \times \frac{n}{156}$$

$$156 = 52n$$

$$3\text{ in.} = n$$

Percent is a ratio in which a quantity is compared to 100. The symbol for percent is **%**. Percent means "out of 100," "per 100," or "÷ 100."

Change a Decimal to a Percent

Use mental math: move the decimal point two places to the right when multiplying by 100.

$$0.71 \times 100 = 71\%$$

$$0.2 \times 100 = 20\%$$

Change a Percent to a Decimal

Use mental math: move the decimal point two places to the left when dividing by 100.

$$60\% = \frac{60}{100} = 60 \div 100 = 0.6$$

$$4\% = \frac{4}{100} = 4 \div 100 = 0.04$$

Change a Fraction to a Percent

1. Divide. Round the answer to the nearest hundredth.
2. Change the decimal to a percent.

$$\frac{3}{5} = 3 \div 5 = 0.6 = 60\%$$

Change a Percent to a Fraction

1. Rename the percent as a fraction with 100 as the denominator.
2. Simplify the fraction by renaming to lowest terms.

$$40\% = \frac{40}{100} = \frac{2}{5}$$

Problems involving percents can be solved by setting up a proportion. An unknown in a proportion can be found by cross-multiplying or by finding the equivalent ratios.

$$\frac{\text{part}}{100} = \frac{\text{part}}{\text{whole}}$$

Cross-Multiply

$$\frac{n}{100} = \frac{6}{23}$$

$$23 \cdot n = 100 \cdot 6$$

$$23n = 600$$

$$\frac{{}^{1}\cancel{23}n}{\cancel{23}_{1}} = \frac{600}{23}$$

$$n = 26.09$$

About 26% of the sixth-grade class preferred Gooey Cluster bars.

Equivalent Ratios

$$\underset{\div 4}{\overset{\div 4}{\frac{n}{100} = \frac{8}{25}}}$$

$$n \div 4 = 8$$

$$\frac{n}{{}_{1}\cancel{4}} \cdot \overset{1}{\cancel{4}} = 8 \cdot 4$$

$$n = 32$$

32% of the fifth-grade class preferred Gooey Cluster bars.

Write an Equation

Find the unknown whole when given the percent and the part by substituting known information into the formula.

percent × whole = part

$$30\% \times n = \$24$$

$$0.3n = \$24$$

$$\frac{{}^{1}\cancel{0.3}n}{\cancel{0.3}_{1}} = \$24 \div 0.3$$

$$n = \$80$$

whole = part ÷ percent

$$n = \$24 \div 30\%$$

$$n = \$24 \div 0.3$$

$$n = \$80$$

The original amount was $80.

Write a Proportion

Find the unknown whole by solving a proportion. Solve by cross-multiplying or by finding the equivalent ratio.

$$\frac{\textbf{part}}{\textbf{100}} = \frac{\textbf{part}}{\textbf{whole}}$$

$$\frac{30}{100} = \frac{\$24}{n}$$

$$30n = \$2{,}400$$

$$\frac{{}^{1}\cancel{30}n}{\cancel{30}_{1}} = \frac{\$2{,}400}{30}$$

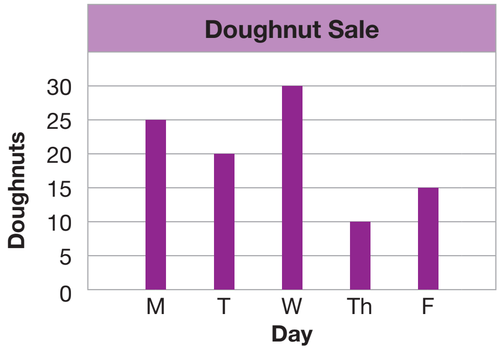
$$n = \$80$$

The original amount was $80.

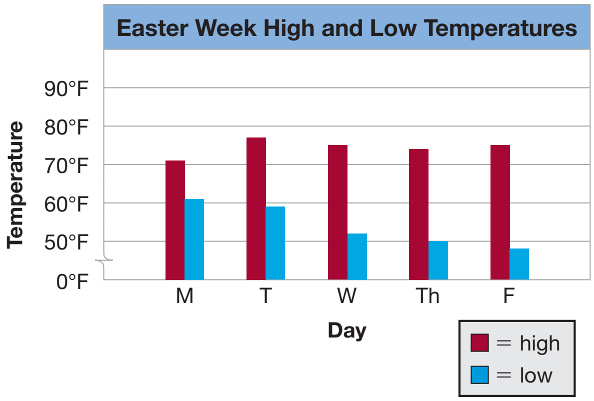
Statistics

average (mean) The number found by adding two or more quantities in a set of numbers and then dividing the sum by the number of addends.

bar graph A picture that compares data by using bars.

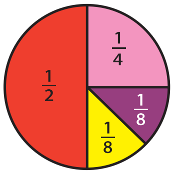


bar graph



double bar graph

circle graph A picture that shows data using a whole circle divided into parts.



cluster A tight grouping of data on the line plot.

coordinate plane A graph formed by two perpendicular number lines, one horizontal and one vertical. The point of intersection is zero for both number lines.

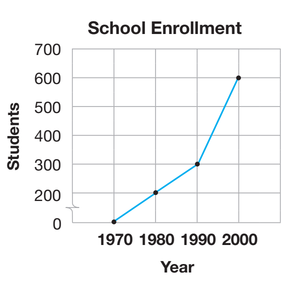
data Information or facts collected.

frequency The number of times a particular item of data occurs.

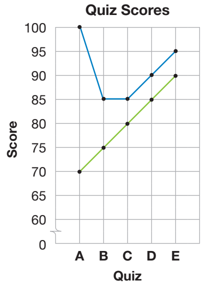
gap An empty space with no data on the line plot.

graphs Pictorial forms used to summarize data.

line graph A picture that shows changes over time by connecting points on a grid.

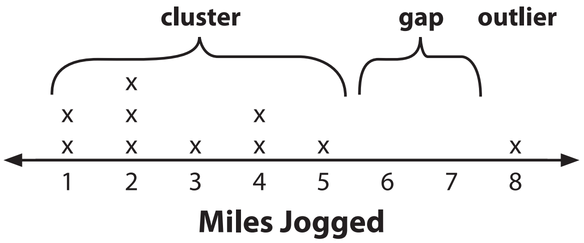


line graph



double line graph

line plot A picture that uses the range of data as its scale. Each piece of data is indicated by an *X* above the number it represents.



mean See *average*.

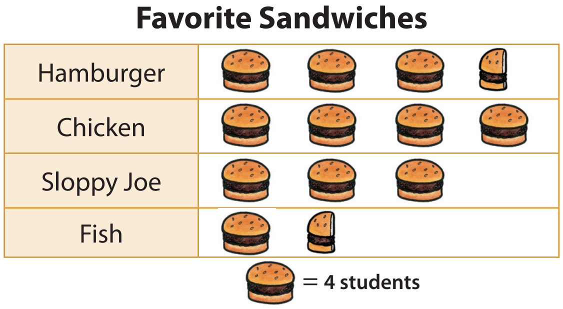
median The middle value (or an average of the two middle values) of a set of data when ordered from least to greatest.

mode The value that occurs most often or has the greatest frequency. Some sets may have more than one mode, and some sets may not have a mode.

ordered pair A pair of numbers that names the location of a point on a coordinate graph. (2, 7)

outlier A piece of data that is much greater or much less than the other data.

pictograph A picture that shows data using pictures and a key.



range The difference between the largest and smallest numbers in a set of data.

statistics The branch of mathematics that deals with the collection, organization, analysis, and interpretation of data. The data is more easily interpreted when displayed in a table, a chart, or a graph.

stem and leaf plot A picture that displays frequency of data using the tens digit of the data as its stems and the ones digit of the data as its leaves.

scores:

79, 87, 89, 89, 92, 92, 92, 94, 95, 98

Math Test Scores

Stem	Leaf
7	9
8	7 9 9
9	2 2 2 4 5 8

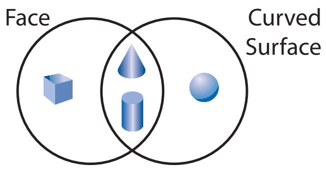
Key	8|7 = 87

tally table A table that uses tally marks to record data.

Children Drinking Milk on Monday

White	𝍸 ||||
Chocolate	𝍸 𝍸

Venn Diagram A diagram that uses circles or squares to show the relationship of sets.



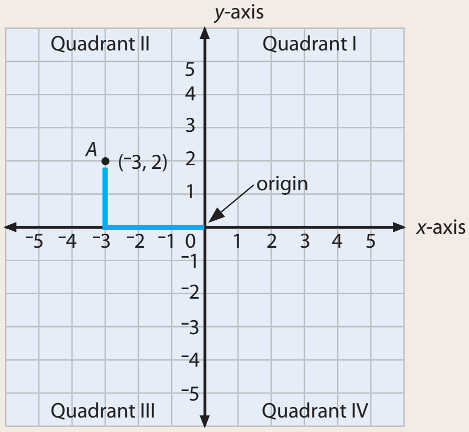
***x*-axis** The horizontal number line in a coordinate graph.

***y*-axis** The vertical number line in a coordinate graph.

A **coordinate plane** is formed by two number lines intersecting at right angles. The ***x*-axis** is the horizontal number line. The ***y*-axis** is the vertical number line. The point of intersection, called the **origin**, is 0 on both number lines. The two axes divide the coordinate plane into four sections called **quadrants**. The quadrants are numbered I, II, III, and IV.

An **ordered pair** describes the location of every point on a coordinate plane. The ***x*-coordinate** (first coordinate) tells the distance of the point along the *x*-axis—how far to move to the right or the left from the *origin*. The ***y*-coordinate** (second coordinate) tells the distance of the point along the *y*-axis—how far to move up or down from the origin.

When you move left or down from the origin, you encounter *negative* coordinates. Sometimes only Quadrant I of a coordinate plane is shown because only positive numbers are being graphed.



Point A (⁻3, 2) From the origin move 3 units to the left, since the 3 is negative. Then move 2 units up, since the 2 is positive.

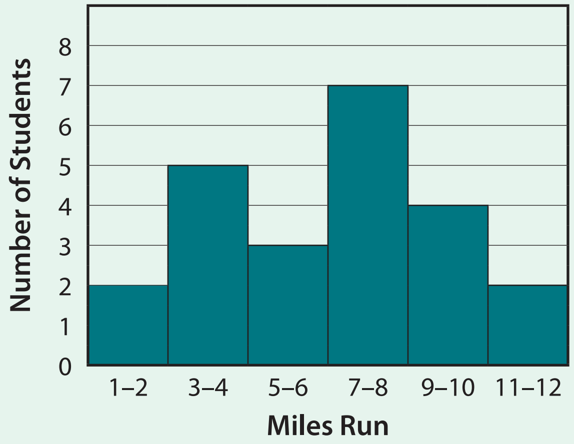
Histograms

When there is a greater amount of data to graph, **histograms** are used instead of the typical bar graph. The data is separated into equal **intervals** on the horizontal axis. The vertical axis is used to show the frequency. A histogram always uses vertical bars with no spaces between them. A frequency table helps to organize the data before the histogram is constructed.

Data: 8, 7, 12, 7, 1, 5, 10, 2, 3, 10, 12, 9, 4, 6, 9, 4, 3, 7, 8, 5, 7, 4, 8

Interval	Tally	Frequency
1–2	||	2
3–4	𝍸	5
5–6	|||	3
7–8	𝍸 ||	7
9–10	||||	4
11–12	||	2

P. E. Class Weekly Running



Box-and-Whisker Plot

A **box-and-whisker plot** summarizes data on a number line using a list that is organized from *least* to *greatest*.

Use the least value of the data as the beginning point of the number line and the greatest value of the data as the ending point.

Find the median of the whole set of data and plot that point on the number line. This is the **middle quartile**. The median separates the data into two sets.

Plot the **lower quartile** by finding the median for the lower half of the data. Plot the **upper quartile** by finding the median for the upper half of the data.

Draw a box from the lower quartile to the upper quartile. The whiskers are the lines that extend from the box to the least value and the greatest value of the data.

Remember to average the two middle numbers to find the median when there is an even number of data.

Number of points scored by the boy's basketball team: 72, 85, 91, 87, 79, 78, 93

Plot the middle, upper, and lower quartiles.

$\cancel{72}, \cancel{78}, \cancel{79}, \boxed{85}, \cancel{87}, \cancel{91}, \cancel{93}$

Median of data: 85

Lower quartile: $\cancel{72}\ \boxed{78}\ \cancel{79}$ 78

Upper quartile: $\cancel{87}\ \boxed{91}\ \cancel{93}$ 91