12 VOLUME

THE VOICE OF CALM

Colorado Springs, Colorado **January 7, 1990**

On January 7, 1990, the sun shone down on a snowy field in Colorado Springs where Tex Houston was preparing to launch his huge yellow hot air balloon. Tex's friends, the Nicholos family and a retired pilot named Dave Hollenbaugh, had come to help him with the launching. Eleven-year-old Alex Nicholos was to be Tex's passenger that morning in "The Yellow Rose of Tex's." As the balloon rose gently from the ground, Alex waved to his parents and then turned to watch Tex handling the instruments. Fascinated, he asked Tex question after question about the CB radio and the burner valve that shot bright flashes of flame up into the balloon to make it rise.

A hot-air balloon with its burner lit rises in the sky.

After about half an hour of flight, the winds picked up, and Tex told Alex that they were going to have to land. Alex squatted down inside the basket and held on to the sides. He closed his eyes and felt the entire balloon shudder as it thumped against the snowy ground and tipped sideways. Then he felt it rise quickly into the air again. He opened his eyes, and Tex was gone! Leaning over the edge of the basket, he saw Tex on the ground below, waving his arms. He had been knocked out of the basket by the sharp blow of the landing.

Alex's first thought was to grab the CB radio. "Help me! I'm scared!" he cried into it.

After a moment, Dave Hollenbaugh's calm voice answered him. "It's all right, Alex." He told Alex he was going to teach him how to land the balloon. Feeling calmer himself, Alex listened carefully to Dave's instructions. He caught on quickly to the use of the burner valve, and every time Dave told him to give the balloon another short blast of hot air, he obeyed. He gradually lost his panicky feeling and almost began to enjoy himself. The balloon sank lower and lower, barely clearing a power line. At fifty feet, Dave instructed him to pull the vent line. Alex sat on the floor of the basket and pulled with all his strength. He kept pulling even when he felt the balloon settle on the ground, making sure it had landed for good. Alex's family gathered around him with smiles and tears of relief on their faces. The National Aeronautic Association awarded Dave a Certificate of Honor for helping Alex land safely.

In order to carry two passengers, a balloon must have an inflatable volume of about 60,000 cubic feet.

The highest altitude attained by a balloon with a passenger is 123,800 feet.

In 1984 the first solo balloon flight across the Atlantic Ocean covered a distance of 3,543 miles.

The largest balloons ever made are 1,000 feet tall and have volumes of 70 million cubic feet when inflated.

A person may get his student pilot's license and fly alone in balloon races or rallies when he is as young as fourteen years old.

Both the Union and the Confederate armies used hot air balloons during the American Civil War to spy on the enemy.

The Anderson-Abruzzo Albuquerque International Balloon Museum is located in Albuquerque, New Mexico. It houses a 59,000-square-foot museum facility that showcases ballooning.

Volume

Lesson	Topic	Lesson Objectives	Chapter Materials
108	**Volume of Rectangular Prisms**	• Develop an understanding of volume • Find the volume of a rectangular prism using a model • Calculate the volume of a rectangular prism using a formula • Relate volume to real-life situations	**Instructional Aids (Teacher's Toolkit CD):** • Cumulative Review Answer Sheet (page IA9) for each student • Triangular Prisms & Cylinders (page IA59) • Fixed Volume (page IA60) • Fixed Volume (page IA60) for each group of students • Volume Review (page IA61) • Volume Review (page IA61) for each student • Volume Word Problems, page IA62 **Christian Worldview Shaping (Teacher's Toolkit CD):** • Page 30 **Other Teaching Aids:** • A clear storage container with lid (shoebox size) • Cube-shaped blocks • Several $8\frac{1}{2} \times 11$ blank sheets of paper • A rectangular prism-shaped block • A clear cube-shaped container with lid • A calculator for each student • A ruler for each student and the teacher • Four 9×12 sheets of construction paper for each group of students and the teacher • Transparent tape for each group of students and the teacher • 6 cups of rice, unpopped popcorn, or dried beans for each group of students and the teacher • A box lid or rectangular pan for each group of students and the teacher • A 1-cup liquid measuring cup for each group of students and the teacher **Math 6 Tests and Answer Key** **Optional (Teacher's Toolkit CD):** • Fact Review page • Application pages • Calculator Activities
109	**Volume of Cubes**	• Develop an understanding of the volume of a cube (square prism) • Find the volume of a cube using a model • Calculate the volume of a cube using a formula • Calculate the unknown measurement of a rectangular prism • Relate volume to real-life situations	
110	**Volume of Other 3-D Figures**	• Find the volume of an irregular prism using a model • Calculate the volume of a triangular prism and of a cylinder using formulas • Relate volume to real-life situations	
111	**Fixed Volumes & Fixed Lateral Surfaces**	• Recognize that surface area can vary for a fixed volume • Calculate the volume and the lateral surface area of a rectangular prism and of a cylinder using formulas • Recognize that volume can vary for a fixed lateral surface area	
112	**Chapter 12 Review**	• Review	
113	**Chapter 12 Test Cumulative Review**	• Add, subtract, multiply, and divide whole numbers, decimals, and fractions • Rename a fraction as a decimal • Identify the ordered pair for a point on a coordinate plane • Determine the radius and the diameter of a circle • Identify the formula used to find the area of a circle • Identify geometric figures: a parallelogram, parallel lines, perpendicular lines • Classify angles as *acute*, *obtuse*, or *right* • Read and interpret a line plot	

A Little Extra Help

Use the following to provide "a little extra help" for the student that is experiencing difficulty with the concepts taught in Chapter 12.

Visualize a 3-dimensional figure—Provide 3-dimensional figures for the student who needs assistance in "seeing" a 3-dimensional object when looking at a 2-dimensional representation. Allow the student to handle the objects as he completes his assignments.

Math Facts

Throughout this chapter, review fractions using Fact Review pages on the Teacher's Toolkit CD. Also, review addition, subtraction, multiplication, and division facts using Fact Review pages or a Fact Fun activity on the Teacher's Toolkit CD, or you may use flashcards.

Lesson **108** **Student Text** pp. 260–63
Daily Review p. 444a

Objectives

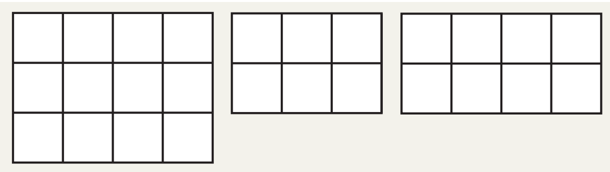
- Develop an understanding of volume
- Find the volume of a rectangular prism using a model
- Calculate the volume of a rectangular prism using a formula
- Relate volume to real-life situations

Teacher Materials

- A clear storage container with lid (shoebox size)
- Cube-shaped blocks (to fill the clear container and build a 60 unit^3 tower)
- Three $8\frac{1}{2} \times 11$ blank sheets of paper
- A rectangular prism-shaped block
- A ruler

Preparation

Prepare each of the following area drawings on separate sheets of blank paper. Make the squares in each drawing the same size as the cubes used in the lesson.



Notes

If enough cubes are available, you may choose to prepare copies of the area drawings so that groups of students can build the towers as you demonstrate.

Preview the Fact Review pages, the Application pages, and the Calculator Activities located on the Teacher's Toolkit CD.

Introduce the Lesson

Guide the students in reading aloud the story and facts on pages 260–61 of the Student Text (pages 258–59 of this Teacher's Edition).

Teach for Understanding

Develop an understanding of volume

1. Display the clear storage container.
   - ➤ **What 3-dimensional figure is this container?** ***a rectangular prism***
   - ➤ **What is a prism?** ***Elicit that it is a cylindrical figure having 2 congruent and parallel bases that are polygons; all of its other faces are parallelograms.***
   - ➤ **How can you calculate the surface area of this container?** ***Accept that you can add the areas of all the faces, but elicit that you can add the congruent top and bottom areas, the congruent front and back areas, and the side areas using the formula $S = 2(l \times w) + 2(w \times h) + 2(l \times h)$.***
2. Remind the students that perimeter and area are attributes of 2-dimensional figures. Surface area is an attribute of 3-dimensional figures. Explain that another attribute of 3-dimensional figures is *volume*, the capacity of a 3-dimensional object or the amount that the object can hold. Write *volume* for display.
3. Cover the interior bottom of the container with cubes. Point out that, by looking at the bottom of the cubes, the students can see the number of square units that form the bottom face of the prism.
   - ➤ **How many square units form the bottom face of the container?** ***Answer will vary based on the container's size.***
   - ➤ **How many cubes were needed to make one layer in the bottom of the container?** ***Answer will vary.***

   Explain that when you use cubes to find the volume of a 3-dimensional object, each cube is *1 cubic unit*.
   - ➤ **What do you notice about the number of cubic units needed to cover the bottom of the container and the number of square units for the bottom face?** ***The number of cubic units is the same as the number of square units.***
   - ➤ **How many layers of cubes do you estimate are needed to fill the container?** ***Answers will vary.***
4. Place a second layer of cubes in the container.
   - ➤ **How many cubic units are in the second layer?** ***Elicit the same number of cubic units that is in the first layer.***
   - ➤ **How many cubic units are now in the container? How do you know?** ***Answers will vary; elicit that the number of cubic units is 2 times the number in the first layer.***
5. Repeat the procedure until the container is filled.
   - ➤ **How many cubes were needed to fill the container?** ***Answers will vary.***
   - ➤ **What is the volume of the box in cubic units?** Write the volume for display (e.g., 28 units^3.) Point out that the exponent 3 indicates *cubic units*.
6. Elicit that volume is the measurement of a 3-dimensional object's capacity or the amount the object can hold.

Find the volume of rectangular prisms using models

1. Display the area drawing that shows 3 rows of 4 square units.
   - ➤ **What is the area of this figure? How do you know?** ***12 square units; elicit that $A = l \times w$; $A = 3 \times 4 = 12$ units^2.***
   - ➤ **If you are building a tower with blocks on this base plan, how many blocks will be needed for the first floor? Why?** ***12 blocks; each square unit in the base plan represents one face of each block (cubic unit) needed for the first floor.***
2. Choose a student to use cubes to build the first floor of the tower (3 rows of 4 cubes).
   - ➤ **What is the volume of the tower since it is one cube high? How do you know?** ***12 cubic units; elicit that a total of 12 cubes make up one floor with a base that is a 4×3 area.***
3. Draw for display a table similar to the one below and complete the first row using the data for the first floor of the tower.
4. Repeat the procedure until the tower has 5 floors (layers). Elicit that the volume of the tower after each floor is built is the area of the base multiplied by the number of layers or the height.

Area of Base (unit^2)	Number of Layers—Height (unit)	Volume (unit^3)
12 units^2	*1 unit*	*12 units^3*
12 units^2	*2 units*	*24 units^3*
12 units^2	*3 units*	*36 units^3*
12 units^2	*4 units*	*48 units^3*
12 units^2	*5 units*	*60 units^3*

5. Display the area drawing that shows 2 rows of 3 square units. Follow a similar procedure for building a tower. Elicit that the volume of the tower after each floor is built is the area of the base multiplied by the number of layers or the height.

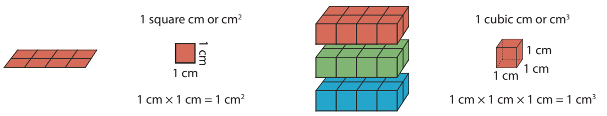
Volume of Rectangular Prisms

The area of a figure is the number of square units a flat space covers. **Volume** builds on the area of a figure. Multiply the area of the **base** ($B = l \times w$) by the number of cubic unit layers (**height**) of the three-dimensional figure. Volume is the number of cubic units a figure contains. The formula is $V = Bh$.

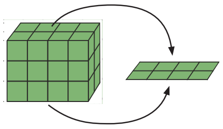
volume
base
height
V = Bh

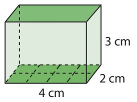
Area is measured using square units. units²

Volume is measured using cubic units. units³



The volume of any prism can be found using the volume formula. Because prism bases are parallel and congruent, opposite bases will have the same area.





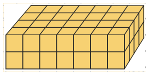
V = Bh
V = (l × w) × h
V = (4 rows of 2 cubes) × 3 layers
V = 8 cubes × 3 layers
V = 24 cubes or 24 cubic units

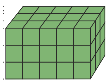
B (base) is found using the area formula ***l × w***.

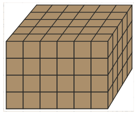
V = Bh
V = (l × w) × h
$V = (4 \times 2) \times 3$
$V = 8 \times 3$
$V = 24 \text{ cm}^3$

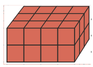
Exercises

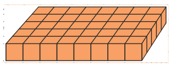
Write an equation to find the volume of the model. Use cubic units as the label.

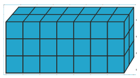
1. 
56 cubic units

2. 
45 cubic units

3. 
120 cubic units

4. 
24 cubic units

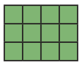
5. 
40 cubic units

6. 
42 cubic units

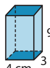
Find the area of the base: $B = l \times w$. Find the volume of the rectangular prism that could be built using centimeter cubes for the given height.

7. 
18 cm³
height = 3 cm

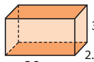
8. 
8 cm³
height = 2 cm

9. 
48 cm³
height = 4 cm

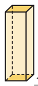
Write an equation to find the volume.
Round a decimal answer to the nearest tenth.

10. 

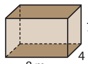
108 cm³

11. 

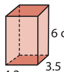
43.7 m³

12. 

$1\frac{1}{2}$ in.³

13. 

224 m³

14. 

88.2 cm³

15. 

32 in.³

Find the volume of a prism with the given dimensions.

16. $l = 5$ in., $w = 7$ in., $h = 2$ in.
70 in.³
17. $l = 12$ cm, $w = 8$ cm, $h = 10$ cm
960 cm³
18. $l = \frac{3}{8}$ in., $w = \frac{2}{3}$ in., $h = \frac{1}{2}$ in.
$\frac{1}{8}$ in.³
19. $B = 15 \text{ ft}^2$, $h = 2$ ft
30 ft³
20. $B = 25 \text{ m}^2$, $h = 8$ m
200 m³
21. $B = 46 \text{ cm}^2$, $h = 5.2$ cm
239.2 cm³

Practice & Application *Equations may vary.*

On Memorial Day weekend, Carmen and her family attended a hot air balloon festival.

22. Carmen selected a box in which she will store her souvenirs. The box is 24 inches long, 18 inches wide, and 12 inches high. What is the volume of the box? ***5,184 in.³***

23. Carmen wants to cover the outside of her box with decorative contact paper. What is the least amount of contact paper Carmen will need? ***1,872 in.²***

24. Mr. Fields has 4 feet of molding to make a rectangular frame for a photo he took at the festival. Does he have enough to make a 9-inch by 12-inch frame? Explain. ***yes; (2 • 9 in.) + (2 • 12 in.) = 42 in. needed; 4 × 12 = 48 in. available***

25. Mr. Fields purchased a 9-inch by 12-inch piece of glass for the frame. What is the area of the glass? ***9 in. × 12 in. = 108 in.²***

26. Mrs. Fields wants to move the shadow box displaying her new balloon figurines. How much space does a shadow box that measures 2 feet long, 1 foot wide, and $\frac{1}{2}$ of a foot high take up? ***2 ft × 1 ft × $\frac{1}{2}$ ft = 1 ft³***

J Write which math concept, *perimeter*, *area*, or *volume*, would be used to solve these problems. Explain your answer.

a. the amount of air to fill a balloon
b. the amount of material needed to make a balloon
c. the amount of space in a basket
d. the amount of cushioned edging to go around a basket

a. volume; find the amount of cubic units within a figure
b. area; find the square units in a figure
c. volume; find the amount of cubic space within a figure
d. perimeter; find the distance around a figure

Complete DAILY REVIEW a on page 444.

6. Display the area drawing that shows 2 rows of 4 square units.
➤ What is the area of this base? ***8 square units***
➤ If a tower with a height of 5 units was built on this base, what would its volume be? Why? ***40 cubic units; elicit that the volume equals the area of the base multiplied by the number of units high; 8 units² × 5 units = 40 units³.***

Calculate the volume of a rectangular prism using a formula

1. Display the rectangular prism-shaped block.
➤ What shape is the base of this prism? ***a rectangle***
➤ How could you find the volume of the prism? ***Elicit that you multiply the area of the base times the height of the prism.***
Explain that when finding the volume of a solid figure, you are finding the amount of space the object fills rather than how much the object can hold.
2. Write for display V = *area of the base (B)* × *height of the prism (h)* and write $V = Bh$ below it.
➤ What does the *h* in the formula for volume represent? ***the height of the prism*** the uppercase *B*? ***the area of the base***
3. Choose a student to measure the length and the width of the prism's base to the nearest centimeter and then calculate the area of the base using $A = l \times w$.
Select another student to measure the height to the nearest centimeter.
Guide the students in finding the volume of the solid figure by substituting into the formula $A = Bh$ the calculated area of the base and the measured height.
➤ How do you think you can find the volume of the prism using one equation? ***Elicit that since the B in the formula represents the area of the base, you can multiply the length of the base times its width and then multiply by the height.***
4. Write $V = (l \times w) \times h$ below $V = Bh$. Guide the students in substituting the prism's dimensions into $V = (l \times w) \times h$ and solving it.
5. Guide the students in using the formula $A = Bh$ to calculate the volume of prisms having the following dimensions.
$l = 2.5$ m, $w = 1.5$ m, $h = 3$ m **$V = Bh$; $V = (l \times w) \times h$; $V = (2.5 \text{ m} \times 1.5 \text{ m}) \times 3 \text{ m}$; $V = 3.75 \text{ m}^2 \times 3 \text{ m}$; $V = 11.25 \text{ m}^3$**
$B = 20 \text{ in.}^2$, $h = 6$ in. **$V = Bh$; $V = 20 \text{ in.}^2 \times 6 \text{ in.}$; $V = 120 \text{ in.}^3$**
6. Guide the students in solving these word problems by substituting the dimensions into the appropriate formula.

Kyle has an aquarium that is 40 centimeters long, 30 centimeters wide, and 20 centimeters high. What is the volume of Kyle's aquarium? **$V = Bh$; $V = (l \times w) \times h$; $V = (40 \text{ cm} \times 30 \text{ cm}) \times 20 \text{ cm}$; $V = 24{,}000 \text{ cm}^3$**

One of the fish tanks at the zoo is 3.2 m long, 2.6 m wide, and 1.5 m high. What is the volume of the aquarium tank? **$V = Bh$; $V = (l \times w) \times h$; $V = (3.2 \text{ m} \times 2.6 \text{ m}) \times 1.5 \text{ m}$; $V = 12.48 \text{ m}^3$**

At the zoo, the parrots are kept in a wire cage that is 3 yards long, 3 yards wide, and 4 yards high. What is the volume of the parrot cage? **$V = Bh$; $V = (l \times w) \times h$; $V = (3 \text{ yd} \times 3 \text{ yd}) \times 4 \text{ yd}$; $V = 36 \text{ yd}^3$**

The gift shop at the zoo is 25 feet long and 30 feet wide. What area does the gift shop cover? **$A = l \times w$; $A = 25 \text{ ft} \times 30 \text{ ft}$; $A = 750 \text{ ft}^2$**

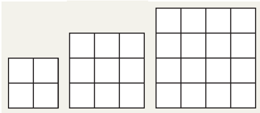
Student Text pp. 262–63

Lesson 109 — Student Text pp. 264–65

Daily Review p. 445b

Objectives
- Develop an understanding of the volume of a cube (square prism)
- Find the volume of a cube using a model
- Calculate the volume of a cube using a formula
- Calculate the unknown measurement of a rectangular prism
- Relate volume to real-life situations

Teacher Materials
- A clear cube-shaped container with lid
- Cube-shaped blocks (to fill the clear container and build a 64 unit^3 tower)
- Three $8\frac{1}{2} \times 11$ sheets of paper
- A ruler



Preparation

Prepare each area drawing on a separate sheet of paper. Make the squares in each drawing the same size as the cubes used in the lesson.

Note

If enough cubes are available, you may choose to prepare copies of the area drawings so that groups of students can build the towers as you demonstrate.

Teach for Understanding

Develop an understanding of the volume of a cube

➤ **What is a prism?** *Elicit that it is a cylindrical figure having 2 congruent and parallel bases that are polygons; all of its other faces are parallelograms.*

1. Display the cube-shaped container.
   ➤ **What is this 3-dimensional figure? How do you know?** *A square prism or cube; all 6 sides are congruent squares.*
   Remind the students that a square is a rectangle whose sides are all the same length, so a square prism or cube is a special type of rectangular prism whose sides (length, width, and height) are all the same length.
2. Choose a student to measure the length of any one side of the container to the nearest centimeter and then calculate the area of one face: $(l \times w)$ or (s^2). Guide the students in calculating the surface area of the cube: $S = 6(lw)$ or $6s^2$.
3. Hold up the container and remind the students that volume can refer to *the number of cubic units an object will hold* or *the amount of space an object fills.*
4. Cover the inside bottom of the container with the cubes.
   ➤ **How many cubes were needed to cover the bottom of the container?** *Answer will vary based on the container's size.*
   ➤ **How many layers of cubes do you predict will be needed to fill the container?** *Answer will vary.*
5. Place a second layer of cubes in the container.
   ➤ **How many cubic units are in the second layer?** *Answer will vary, but is the same as the number of cubic units in the first layer.*
   ➤ **How many cubic units are now in the container? How do you know?** *Answer will vary; elicit that the number of cubic units is 2 times the number in the first layer.*
6. Repeat the procedure until the container is filled with cubes.
   ➤ **How many cubes does the container hold?** *Answer will vary.*
   ➤ **What is the volume of the container in cubic units?** Write the volume for display. Remind the students that the exponent 3 indicates *cubic units.*
7. Write the terms *perimeter*, *area*, *surface area*, and *volume*.
   ➤ **Which terms are attributes of 2-dimensional figures?** *perimeter; area* **attributes of 3-dimensional figures?** *surface area; volume* Choose students to give an example to support their answer.

Find the volume of a cube

1. Display the area drawing of 4 square units.
   ➤ **What is the area of this figure? How do you know?** *4 square units; $A = l \times w$ or $A = s^2$; $A = 2 \times 2 = 4$ units^2 or $A = 2^2 = 4$ units^2.*
   ➤ **If you are building a tower with blocks on this base plan, how many blocks will be needed for the first floor? Why?** *4 blocks; each square unit in the base plan represents one face of each block (cubic unit) needed for the first floor.*
2. Choose a student to use cubes to build the first floor of the tower (2 rows of 2 cubes).
   ➤ **What is the volume of the tower since it is one cube high? How do you know?** *4 cubic units; elicit that a total of 4 cubes make up one floor with a base that is a 2×2 area.*
3. Draw for display a table similar to the one below. Complete the first row using the data for the first floor of the tower.
4. Repeat the procedure until the tower has 4 layers and forms a cube. Elicit that the volume of the tower after each floor is built is the area of the base multiplied by the number of layers or the height. Point out that the length, width, and height of the cube-shaped tower are equal.

Area of Base (unit^2)	Number of Layers—Height (unit)	Volume (unit^3)
4 units^2	*1 unit*	*4 units^3*
4 units^2	*2 units*	*8 units^3*
4 units^2	*3 units*	*12 units^3*
4 units^2	*4 units*	*16 units^3*

5. Display the area drawing of 9 square units.
   ➤ **To make a cube-shaped tower from this base, how many units high must the figure be? Why?** *3 units high; elicit that the length, width, and height measurements of a cube must be equal.*
6. Choose a student to place the blocks, layer by layer, on the area drawing to build the $3 \times 3 \times 3$ tower.
   ➤ **What is the volume of this tower? How do you know?** *27 cubic units; possible explanations: each 3×3 layer contains 9 cubes and there are 3 layers; volume is equal to the area of the base (9) multiplied by the height (3); $9 \times 3 = 27$.*
   ➤ **What formula can be used to find the volume of all prisms?** *Elicit* $V = Bh$. Write $V = Bh$ for display.
7. Guide the students in using the formula to write an equation to find the volume of the tower. Remind them that the factors in parentheses are the length and width, which are multiplied to find the area of the base. $V = (3 \times 3) \times 3$
   ➤ **How can you rewrite this equation using exponents?** $V = 3^3$ Write the new equation for display. Select a student to give the answer. $V = 27$ *units^3*
   ➤ **How could you modify the formula $V = Bh$ to find the volume of a cube? Why?** *Elicit $V = (s \times s) \times s$ and $V = s^3$; since the length, width, and height of a cube are all equal, you multiply the same measurement three times.* Write $V = (s \times s) \times s$ and $V = s^3$ for display. Elicit that $(s \times s)$ represents the area of the cube's base.

Volume of Cubes

A cube or square prism is a special rectangular prism where all six faces are congruent squares and all sides measure the same.

volume of a cube
$V = s^3$

The formula for volume, $V = Bh$, can be modified for a cube.

5 cm, 5 cm, 5 cm

Volume of a cube = (side × side) × side (Base, height)
$V = s^3$

$V = (s \times s) \times s$		$V = s^3$
$V = (5 \times 5) \times 5$	or	$V = 5^3$
$V = 25 \times 5$		$V = 5 \times 5 \times 5$
$V = 125\ cm^3$		$V = 125\ cm^3$

Exercises

Write an equation to find the volume of the cube. Round a decimal answer to the nearest tenth. *Formula used may vary.*

1. 4 cm, 4 cm, 4 cm — *64 cm³*
2. 10 in., 10 in., 10 in. — *1,000 in.³*
3. 3.2 m, 3.2 m, 3.2 m — *32.8 m³*
4. 6 ft, 6 ft, 6 ft — *216 ft³*
5. $1\frac{1}{2}$ yd, $1\frac{1}{2}$ yd, $1\frac{1}{2}$ yd — $3\frac{3}{8}$ *yd³*
6. 4.5 m, 4.5 m, 4.5 m — *91.1 m³*

Find the volume of a prism with the given dimensions. *Equations may vary.*

7. rectangular prism: $l = 5$ in., $w = 2$ in., $h = 4$ in. *(5 in. × 2 in.) × 4 in. = 40 in.³*
8. rectangular prism: $B = 12\ ft^2$, $h = 3$ ft *12 ft² × 3 ft = 36 ft³*
9. cube (square prism): $s = 7$ m *(7 m)³ or (7 m × 7 m) × 7 m = 343 m³*
10. cube (square prism): $B = 100\ cm^2$, $h = 10$ cm *100 cm² × 10 cm = 1000 cm³*

Find the area of the base: $B = l \times w$. Find the volume of the prism that could be built using centimeter cubes for the given height.

11. *8 cm³* — height = 2 cm
12. *27 cm³* — height = 3 cm
13. *60 cm³* — height = 5 cm

Find an Unknown Measurement

The formula for volume of a prism is $V = Bh$. Since the bases of the figure are rectangles, use $V = (l \cdot w) \cdot h$.

When the volume is given and any two of the volume dimensions are known, you can find the unknown third dimension of a figure.

4 in., 3 in., h

$V = Bh$
$V = (l \cdot w) \cdot h$
$V = 84$ in.³
$l = 4$ in.
$w = 3$ in.
$h =$ ___ in.

84 in.³ = (4 • 3) • h
84 in.³ = 12 • h
$\frac{84\text{ in.}^3}{12} = \frac{\cancel{12}h}{\cancel{12}}$
7 in. = h

4 in., w, 7 in.

$V = Bh$
$V = (l \cdot w) \cdot h$
$V = 84$ in.³
$l = 4$ in.
$w =$ ___ in.
$h = 7$ in.

84 in.³ = (4 • w) • 7
84 in.³ = 4 • w • 7
84 in.³ = 4 • 7 • w
84 in.³ = 28 • w
$\frac{84\text{ in.}^3}{28} = \frac{\cancel{28}w}{\cancel{28}}$
3 in. = w

What is the measure of the length, the width, and the height of a cube whose volume is 27 units?

l, w, h

V (of a cube) = s^3
$27 = s \cdot s \cdot s$

If $s = 2$: $2 \cdot 2 \cdot 2 = 8$
If $s = 3$: $3 \cdot 3 \cdot 3 = 27$
Each side is 3 units.

Exercises

Find the unknown measurement of the rectangular prism.

14. $V = 24\ ft^3$ **h = *4 ft*** (2 ft, 3 ft, h)
15. $V = 248\ m^3$ **l = *10 m*** (l, 4 m, 6.2 m)
16. $V = 144\ cm^3$ **w = *3 cm*** (6 cm, w, 8 cm)
17. $l = 5$ in., $w = 3$ in., $h =$ ___ in., $V = 75$ in.³ **h = 5**
18. $l = 2$ cm, $w =$ ___ cm, $h = 4$ cm, $V = 48\ cm^3$ **w = 6**
19. $B = 6$ in.², $h =$ ___ in., $V = 90$ in.³ **h = 15**
20. $B = 5.7\ cm^2$, $h =$ ___ cm, $V = 51.3\ cm^3$ **h = 9**

Practice & Application *Equations may vary.*

21. Mr. Cole is making a small rectangular pond in his backyard. The pond will be 7 meters long, 5 meters wide, and 1 meter deep. What is the volume of the pond? *7 m × 5 m × 1 m = 35 m³*
22. One cubic meter can hold about 264 gallons of water. Approximately how many gallons of water will the pond hold? *35 × 264 gal = 9,240 gal*
23. Mr. Cole is placing stones along the 4 sides of the pond. What is the distance around the pond's edge? *(2 × 7 m) + (2 × 5 m) = 24 m*
24. Mrs. Cole wants to purchase a tarp to cover the pond during the winter. What is the area of the pond? *7 m × 5 m = 35 m²*

J Find the surface area and the volume of the prisms.

Complete DAILY REVIEW b on page 445.

8. Display the area drawing of 16 square units and choose a student to build a cube-shaped tower on the base. Guide in using both formulas to find the volume of the tower.
   $V = (s \times s) \times s$ *V = (4 × 4) × 4; V = 16 × 4; V = 64 units³*
   $V = s^3$ *V = 4³; V = 4 × 4 × 4; V = 64 units³*
9. Guide the students in using the formulas to calculate the volume of cubes having the following dimensions. While solving the second problem, explain that when a factor is labeled, such as 6 *cm*, the exponent must be written outside the parentheses to indicate "6 centimeters cubed" rather than "6 cubic centimeters".
   $l = \frac{1}{2}$ in., $w = \frac{1}{2}$ in., $h = \frac{1}{2}$ in. *V = (s × s) × s; V = ($\frac{1}{2}$ in. × $\frac{1}{2}$ in.) × $\frac{1}{2}$ in.; V = $\frac{1}{4}$ in. × $\frac{1}{2}$ in; V = $\frac{1}{8}$ in.³*
   $s = 6$ cm *V = s³; V = (6 cm)³; V = 6 cm × 6 cm × 6 cm; V = 216 cm³*
   $B = 16\ m^2$, $h = 4$ m *V = Bh; V = 16 m² × 4 m; V = 64 m³*

Calculate the unknown measurement of a rectangular prism

A gift box for earrings has a volume of 24 cubic inches. If the area of the rectangular base is 12 square inches, what is the height of the gift box? *2 inches*

➤ **What is the question asking you to find?** *the unknown height of the gift box*
➤ **What type of figure is the gift box? Why?** *A rectangular prism; it has a rectangular base.*
➤ **How can you find the unknown height of the gift box?** *Elicit that you can think of the formula V = Bh; since the volume of the gift box is 24 in.³ and the area of its rectangular base is 12 in.², you can divide the volume by the area of the base to find the height.*

1. Guide the students in first substituting the given values into the volume formula, and then in solving the equation to find the unknown dimension of the prism (the value of the variable). *V = Bh; 24 in.³ = (12 in.²)h; $\frac{24\text{ in.}^3}{12\text{ in.}^2} = \frac{(12\text{ in.}^2)h}{12\text{ in.}^2}$; 2 in. = h*
2. Follow a similar procedure for these word problems. Elicit that when you multiply the width of a rectangular prism times the height, or multiply the length times the height, you are finding the area of one of the faces and area is measured in square units.

A gift box for a sweater has a volume of 648 cubic inches. If the width of the box is 12 inches and the height is 3 inches, what is the length of the box? *18 inches; V = Bh; V = (l • w) • h; 648 in.³ = (l • 12 in.) • 3 in.; 648 in.³ = l • 36 in.²; $\frac{648\text{ in.}^3}{36\text{ in.}^2} = \frac{l \cdot 36\text{ in.}^2}{36\text{ in.}^2}$; 18 in. = l*

A packing box has a volume of 4 cubic feet. If the length of the box is 2 feet and the height is 1 foot, what is the width of the box? *2 feet; V = Bh; V = (l • w) • h; 4 ft³ = (2 ft • w) • 1 ft; 4 ft³ = (2 ft • 1 ft) • w; $\frac{4\text{ ft}^3}{2\text{ ft}^2} = \frac{2\text{ ft}^2 \cdot w}{2\text{ ft}^2}$; 2 ft = w*

3. Follow a similar procedure for finding the unknown measurements of prisms with these dimensions.
   $V = 540\ cm^3$, $l = 20$ cm, $h = 3$ cm *width = 9 cm*
   $V = 15.2\ cm^3$, $B = 3.8\ cm^2$ *height = 4 cm*

Student Text pp. 264–65

Lesson 110

Student Text pp. 266–67

Daily Review p. 445c

Objectives

- Find the volume of an irregular prism using a model
- Calculate the volume of a triangular prism and of a cylinder using formulas
- Relate volume to real-life situations

Teacher Materials

- Triangular Prisms & Cylinders, page IA58 (CD)
- Cube-shaped blocks (to build a 25 unit^3 tower)
- Two $8\frac{1}{2} \times 11$ blank sheets of paper

Student Materials

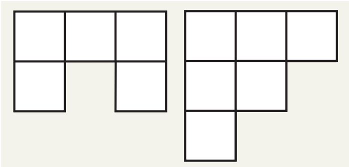
- A calculator

Preparation

Prepare each area drawing on a separate sheet of paper. Make the squares in each drawing the same size as the cubes used in the lesson.

Notes

If enough cubes are available, you may choose to prepare copies of the area drawings so that groups of students can build the towers as you demonstrate.



Allow the students to use calculators throughout the remainder of this chapter so that they can focus on finding the volume of the various figures rather than on doing lengthy calculations.

Teach for Understanding

Find the volume of an irregular prism

1. Display the area drawing of 5 square units.
   ➤ **If you are building a tower with blocks using this irregular base plan, how many blocks will be needed for the first floor? Why?** *5 blocks; elicit that you can count the number of square units in the figure to know how many cubes are needed to cover the first floor.*
   ➤ **What is the area of this irregular figure?** *5 square units*
2. Draw for display a table similar to the one below and complete the first row using the data for the first floor of the tower.
3. Repeat the procedure until the tower has 5 layers. Elicit that the volume of the tower after each floor is built is the area of the base multiplied by the number of layers or the height.

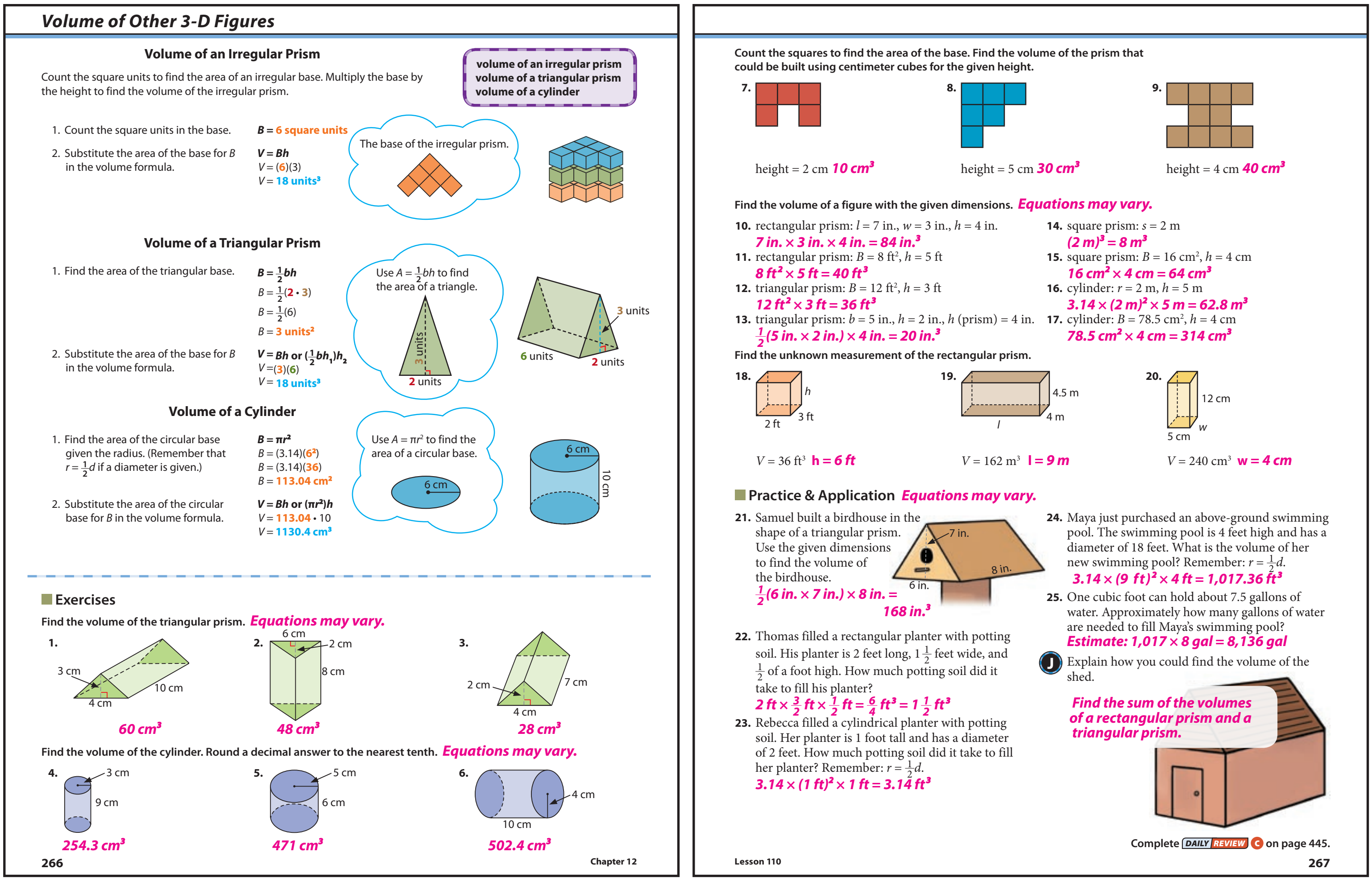
Area of Base (unit^2)	Number of Layers—Height (unit)	Volume (unit^3)
5 units^2	*1 unit*	*5 units^3*
5 units^2	*2 units*	*10 units^3*
5 units^2	*3 units*	*15 units^3*
5 units^2	*4 units*	*20 units^3*
5 units^2	*5 units*	*25 units^3*

4. Display the area drawing of 6 square units.
   ➤ **What formula can be used to find the volume of all prisms?** **V = Bh** Write the formula for display.
   ➤ **How can you use *V = Bh* to find the volume of a tower that is 3 units high built in this base?** *Elicit that since the area of the base is 6 square units, you can multiply the area of the base (6 square units) by the height (3 units).* **What is the volume of the tower?** *18 cubic units*
5. Choose a student to place the blocks, layer by layer, on the area drawing to build the tower.
   ➤ **Is this 3-dimensional figure a prism? How do you know?** *Yes; elicit that it has 2 congruent and parallel bases, and its other faces are parallelograms.*
6. Select a student to write and solve the equation to find the volume of the prism. *V = 6 units^2 × 3 units; V = 18 units^3*
7. Guide the students in solving the following word problems, using the appropriate formulas. For the first problem, elicit that a square prism can also be referred to as a cube; all of its edges have the same measure and all of its faces are congruent.
   (***Note:*** Allow the students to draw pictures for the word problems if needed.)

> Ben attended classes to learn how to make ice sculptures. During the first week, Ben learned to make geometric figures. On the first day, Ben made a square prism. Each edge of the prism measured 1.5 feet. To the nearest tenth of a foot, what was the volume of the ice sculpture? **V = s^3** *(or V = s × s × s); V = (1.5 ft)^3; V = 1.5 ft × 1.5 ft × 1.5 ft; V = 3.375 ft^3 < 3.4 ft^3*
>
> On the second day of class, Ben made an ice sculpture in the shape of a rectangular prism. The sculpture measured 2 feet long, 1.5 feet wide, and 3 feet high. What was the volume of this ice sculpture? **V = Bh; V = (l × w) × h;** *V = (2 ft × 1.5 ft) × 3 ft; V = 3 ft^2 × 3 ft; V = 9 ft^3*

Calculate the volume of a triangular prism

1. Display the Triangular Prisms and Cylinders page. Explain that although the formula $V = Bh$ is used to find the volume of any prism, the base of the prism determines how to find B (area of the base).
   ➤ **What is the shape of the congruent parallel bases of the first prism?** *triangle*
   ➤ **What formula do you use to calculate the area of a triangular base?** **A = $\frac{1}{2}$(bh)**
2. Write for display $B = \frac{1}{2}(bh)$ and guide the students in using the formula to find the area of the triangular base. Remind them that the height of a triangle forms a right angle where it intersects the triangle's base. *B = $\frac{1}{2}$(2 in. × 3 in.)^2; B = $\frac{1}{2}$(6 in.^{2}); B = 3 in.^{2}*
3. Guide the students in using $V = Bh$ to find the volume of the triangular prism. *V = (3 in.^{2})(5 in.); V = 15 in.^{3}*
4. Follow a similar procedure for finding the volume of the second triangular prism.
   B = $\frac{1}{2}$bh; B = $\frac{1}{2}$(4 in. × 5 in.); B = $\frac{1}{2}$(20 in.^{2}); B = 10 in.^{2}
   V = Bh; V = (10 in.^{2})(6 in.); V = 60 in.^{3}
5. Explain that you can also calculate the area of the triangular base of the prism while calculating the volume of the triangular prism. Write $V = (\frac{1}{2}bh_1)h_2$ for display. Point out that when calculating the area of the triangle, you use the base and height of the triangle. When multiplying the area of the triangular base (B) by the height (h), you use the height of the prism. The subscripts 1 and 2 help you to distinguish between the two heights: h_1 is the height of the *triangle* and h_2 is the height of the *prism*.



Volume of Other 3-D Figures

Volume of an Irregular Prism

Count the square units to find the area of an irregular base. Multiply the base by the height to find the volume of the irregular prism.

volume of an irregular prism
volume of a triangular prism
volume of a cylinder

1. Count the square units in the base. $B = 6$ square units
2. Substitute the area of the base for B in the volume formula. $V = Bh$; $V = (6)(3)$; $V = 18$ units³

The base of the irregular prism.

Volume of a Triangular Prism

1. Find the area of the triangular base. $B = \frac{1}{2}bh$; $B = \frac{1}{2}(2 \cdot 3)$; $B = \frac{1}{2}(6)$; $B = 3$ units²
2. Substitute the area of the base for B in the volume formula. $V = Bh$ or $(\frac{1}{2}bh_1)h_2$; $V = (3)(6)$; $V = 18$ units³

Use $A = \frac{1}{2}bh$ to find the area of a triangle.

3 units, 2 units, 6 units

Volume of a Cylinder

1. Find the area of the circular base given the radius. (Remember that $r = \frac{1}{2}d$ if a diameter is given.) $B = \pi r^2$; $B = (3.14)(6^2)$; $B = (3.14)(36)$; $B = 113.04$ cm²
2. Substitute the area of the circular base for B in the volume formula. $V = Bh$ or $(\pi r^2)h$; $V = 113.04 \cdot 10$; $V = 1130.4$ cm³

Use $A = \pi r^2$ to find the area of a circular base.

6 cm, 10 cm

Exercises

Find the volume of the triangular prism. *Equations may vary.*

1. 3 cm, 4 cm, 10 cm — *60 cm³*
2. 6 cm, 2 cm, 8 cm — *48 cm³*
3. 2 cm, 4 cm, 7 cm — *28 cm³*

Find the volume of the cylinder. Round a decimal answer to the nearest tenth. *Equations may vary.*

4. 3 cm, 9 cm — *254.3 cm³*
5. 5 cm, 6 cm — *471 cm³*
6. 4 cm, 10 cm — *502.4 cm³*

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Count the squares to find the area of the base. Find the volume of the prism that could be built using centimeter cubes for the given height.

7. height = 2 cm *10 cm³*
8. height = 5 cm *30 cm³*
9. height = 4 cm *40 cm³*

Find the volume of a figure with the given dimensions. *Equations may vary.*

10. rectangular prism: $l = 7$ in., $w = 3$ in., $h = 4$ in. *7 in. × 3 in. × 4 in. = 84 in.³*
11. rectangular prism: $B = 8$ ft², $h = 5$ ft *8 ft² × 5 ft = 40 ft³*
12. triangular prism: $B = 12$ ft², $h = 3$ ft *12 ft² × 3 ft = 36 ft³*
13. triangular prism: $b = 5$ in., $h = 2$ in., h (prism) = 4 in. *$\frac{1}{2}$(5 in. × 2 in.) × 4 in. = 20 in.³*
14. square prism: $s = 2$ m *(2 m)³ = 8 m³*
15. square prism: $B = 16$ cm², $h = 4$ cm *16 cm² × 4 cm = 64 cm³*
16. cylinder: $r = 2$ m, $h = 5$ m *3.14 × (2 m)² × 5 m = 62.8 m³*
17. cylinder: $B = 78.5$ cm², $h = 4$ cm *78.5 cm² × 4 cm = 314 cm³*

Find the unknown measurement of the rectangular prism.

18. h, 3 ft, 2 ft; $V = 36$ ft³ **h = 6 ft**
19. 4.5 m, 4 m, l; $V = 162$ m³ **l = 9 m**
20. 12 cm, w, 5 cm; $V = 240$ cm³ **w = 4 cm**

Practice & Application *Equations may vary.*

21. Samuel built a birdhouse in the shape of a triangular prism. Use the given dimensions to find the volume of the birdhouse. (7 in., 8 in., 6 in.) *$\frac{1}{2}$(6 in. × 7 in.) × 8 in. = 168 in.³*
22. Thomas filled a rectangular planter with potting soil. His planter is 2 feet long, $1\frac{1}{2}$ feet wide, and $\frac{1}{2}$ of a foot high. How much potting soil did it take to fill his planter? *2 ft × $\frac{3}{2}$ ft × $\frac{1}{2}$ ft = $\frac{6}{4}$ ft³ = $1\frac{1}{2}$ ft³*
23. Rebecca filled a cylindrical planter with potting soil. Her planter is 1 foot tall and has a diameter of 2 feet. How much potting soil did it take to fill her planter? Remember: $r = \frac{1}{2}d$. *3.14 × (1 ft)² × 1 ft = 3.14 ft³*
24. Maya just purchased an above-ground swimming pool. The swimming pool is 4 feet high and has a diameter of 18 feet. What is the volume of her new swimming pool? Remember: $r = \frac{1}{2}d$. *3.14 × (9 ft)² × 4 ft = 1,017.36 ft³*
25. One cubic foot can hold about 7.5 gallons of water. Approximately how many gallons of water are needed to fill Maya's swimming pool? *Estimate: 1,017 × 8 gal = 8,136 gal*

J Explain how you could find the volume of the shed. *Find the sum of the volumes of a rectangular prism and a triangular prism.*

Complete DAILY REVIEW C on page 445.

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6. Guide the students in finding the area of the second triangular prism using $V = (\frac{1}{2}bh_1)h_2$. **$V = (\frac{1}{2} \times 4$ in. $\times 5$ in.$)6$ in.; $V = (10$ in.$^2)6$ in.; $V = 60$ in.^{3}**
7. Guide the students in using $V = (\frac{1}{2}bh_1)h_2$ to solve this word problem.

> On the third day, Ben made an ice sculpture in the shape of a triangular prism. The triangular base had a length of 12 inches and a height of 10 inches. The prism had a height of 15 inches. What was the volume of this ice sculpture?
> **$V = (\frac{1}{2}bh_1)h_2$; $V = (\frac{1}{2} \times 12$ in. $\times 10$ in.$)$ 15 in.;**
> **$V = (60$ in.$^2)15$ in. $= 900$ in.^{3}**

Calculate the volume of a cylinder using a formula

1. Direct attention to the first cylinder on the Triangular Prisms & Cylinders page.
   - ➤ **What is the shape of the cylinder's base?** *a circle*
   - ➤ **What formula do you use to calculate the area of a circle?** **$A = \pi r^2$**
   - ➤ **What other letter could you substitute for A to indicate the area of a circular base?** *Elicit an uppercase* **B.**
2. Write $B = \pi r^2$ for display. Elicit 3.14 as the value of pi (accept $\frac{22}{7}$) and guide the students in finding the area of the base, using the formula. Remind them that when a factor is labeled, such as *4 in.*, the exponent must be written outside the parentheses so that the factor is 4 inches squared rather than 4 square inches. **$B = (3.14)(4$ in.$)^2$; $B = 3.14 \times 16$ in.^{2}; $B = 50.24$ in.^{2}**
   - ➤ **Since you know the area of the base, what formula do you think you can use to find the volume of the cylinder?** *Elicit* **$V = Bh$.**
3. Write $V = Bh$ and guide the students in using the formula to find the volume of the cylinder. Round the answer to the nearest tenth. **$V = (50.24$ in.$^2)3$ in.; $V = 150.72$ in.$^3 \approx 150.7$ in.^{3}**
4. Follow a similar procedure for the second cylinder. Elicit that half of the diameter (6 cm) is 3 cm.
   $B = \pi r^2$; $B = 3.14 \cdot (3$ cm$)^2$; $B = 3.14 \times 9$ cm^2; $B = 28.26$ cm^2
   $V = Bh$; $V = 28.26$ cm$^2 \times 5$ cm.; $V = 141.3$ cm^3
5. Explain that you can also calculate the area of the circular base of the cylinder while calculating the volume of the cylinder. Write $V = (\pi r^2)h$ for display and guide the students in finding the area of the second cylinder.
   $V = (\pi r^2)h$; $V = 3.14 \cdot (3$ cm$)^2 \cdot 5$ cm; $V = (3.14 \times 9$ cm$^2) \cdot 5$ cm;
   $V = 28.26$ cm$^2 \cdot 5$ cm; $V = 141.3$ cm^3
6. Guide the students in using $V = (\pi r^2)h$ to solve this word problem.

> The last ice sculpture Ben made was in the shape of a cylinder. The diameter of the base was 12 inches and the height of the cylinder was 24 inches. To the nearest inch, what was the volume of this sculpture?
> **$V = (\pi r^2)h$; $V = 3.14 \cdot (6$ in.$)^2 \cdot 24$ in.; $V = (3.14 \times 36$ in.$^2) \cdot 24$ in.;**
> **$V = 113.04$ in.$^2 \cdot 24$ in.; $V = 2{,}712.96$ in.$^3 \approx 2{,}713$ in.^{3}**

Student Text pp. 266–67

(*Note*: *Assessment* available on Teacher's Toolkit CD.)

Lesson 111 **Student Text** pp. 268–69
Daily Review p. 446d

Objectives
- Recognize that surface area can vary for a fixed volume
- Calculate the volume and the lateral surface area of a rectangular prism and a cylinder
- Recognize that volume can vary for a fixed lateral surface area

Teacher Materials
- Fixed Volume, page IA60 (CD)
- 18 cube-shaped blocks
- Rice, unpopped popcorn, or dried beans (to fill the long paper tube used in the activity—approximately 6 cups)
- A box lid or a rectangular pan
- A 1-cup liquid measuring cup
- Four 9 × 12 sheets of construction paper
- A ruler
- Transparent tape

Student Materials
- Fixed Volume, page IA60, for each group of 3 to 4 students
- 18 cube-shaped blocks for each group of students
- Rice, unpopped popcorn, or dried beans for each group of students (to fill the long paper tube used in the activity—approximately 6 cups)
- A box lid or a rectangular pan for each group of students
- A 1-cup liquid measuring cup for each group of students
- Four 9 × 12 sheets of construction paper for each group of students
- A ruler
- A calculator
- Transparent tape

Preparation
Cut a 1-inch strip off of 2 of the 4 sheets of construction paper, to be used by the teacher and students so that the 2 sheets measure 8" × 12".

Teach for Understanding

Recognize that surface area can vary for a fixed volume

➤ **What is surface area?** ***the sum of the areas of all the surfaces of a 3-dimensional figure***

➤ **What is volume?** ***the number of cubic units that a 3-dimensional figure holds or the amount of space that the figure fills***

1. Display the Fixed Volume page. Arrange the students into groups of 3 or 4 and distribute to each group a copy of the Fixed Volume page and 18 cubes.

➤ **How many different rectangular prisms do you think you can make with 8 cubes?** ***Answers will vary.***

2. Direct the students to make as many 8 cubic unit rectangular prisms as they can. Direct them to record the volume, dimensions, and surface area of each prism on the page. ***possible configurations: V = 8, l = 8, w = 1, h = 1, S = 34; V = 8, l = 4, w = 2, h = 1, S = 28; V = 8, l = 2, w = 2, h = 2, S = 24***

3. Choose students to share their group's findings and record the dimensions on the displayed page.

➤ **What do you notice about the volume and the surface area of your rectangular prisms?** ***Elicit that the prisms are equal in volume but have surface areas that are not equal.***

➤ **Why can the surface areas be different when rectangular prisms have the same fixed volume?** ***Elicit that the same number of cubic units can be arranged differently, changing the dimensions which determine the surface area of the figure.***

4. Select three students to each use 8 cubes to build a tower that has a base of 1 square unit. Direct one of the students to lay down his tower sideways and another student to lay down his tower backwards. Write the dimensions as the students answer the following questions.

➤ **What is the volume of each of these rectangular prisms?** ***8 cubic units***

➤ **What do you know about the surface areas of the prisms?** ***The surface areas are the same because the prisms are congruent.***

➤ **What are the dimensions of the tower that is still standing upright?** ***l = 1 unit, w = 1 unit, h = 8 units*** **the tower that is lying sideways?** ***l = 8 units, w = 1 unit, h = 1 unit*** **the tower that is lying backwards?** ***l = 1 unit, w = 8 units, h = 1 unit***

Elicit that congruent figures have the same dimensions (e.g., 1 unit × 1 unit × 8 units). How the figure is positioned or looked at determines whether a dimension is referred to as the length, the width, or the height.

5. Follow a procedure similar to Step 2 for rectangular prisms with the following volumes. Accept any configuration of the following dimensions.

V = 2 cubic units ***2 × 1 × 1, S = 10***
V = 4 cubic units ***4 × 1 × 1, S = 18; 2 × 2 × 1, S = 16***
V = 6 cubic units ***6 × 1 × 1, S = 26; 2 × 3 × 1, S = 22***
V = 18 cubic units ***18 × 1 × 1, S = 74; 9 × 2 × 1, S = 58; 6 × 3 × 1, S = 54***

Recognize volume can vary for a fixed lateral surface area

1. Distribute to each group of students these items: construction paper, rulers, tape, a box lid, and rice. Direct the students to measure the construction paper to the nearest inch and calculate the area. ***A = 9 in. × 12 in.; A = 108 in.²***

2. Demonstrate how to make a tube that is 12 inches tall by rolling one 9 × 12 sheet of construction paper lengthwise, overlapping the ends slightly, and taping the seam at the top, middle, and bottom. Make a 9-inch tube using the other 9 × 12 sheet of paper. Direct each group of students to do the same with two sheets of construction paper.

3. Write *lateral surface area* for display. Explain that the *lateral surface area* of a 3-dimensional figure is the sum of the surface areas excluding the area of the base(s). A cylinder's lateral surface area is the area of only the curved surface, without the top and bottom circular bases—similar to the 2 paper tubes.

➤ **What is the approximate lateral surface area of the taller tube? Why?** ***108 in.²; when the curved surface is flat, it is the same shape and size as the rectangular sheet of paper.***

➤ **What is the approximate lateral surface area of the shorter tube? Why?** ***108 in²; it has approximately the same area as the taller tube (108 in.²) because both tubes were made from congruent rectangular sheets of paper.***

➤ **Since the lateral surface area of a cylinder is the area of its curved surface, how could you mathematically find the lateral surface area of each tube?** ***Elicit that you can use the same formula that is used to find the surface area of the curved surface of a cylinder; (πd)h or (2πr)h.***

4. Write $LS = (\pi d)h$ for display. Point out that LS represents *lateral surface area*.

Direct the students to measure the diameter of each tube to the nearest inch. ***3 inches for the taller tube; 4 inches for the shorter tube*** Guide them in using the formula to calculate the lateral surface area of each tube to the nearest inch. Point out that the answers will be approximate because the diameters were measured to the nearest inch.

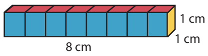
Fixed Volumes & Fixed Lateral Surfaces

Geometric figures can have the same volume but different shapes. *Volume* is the number of cubic units an object holds. *Surface area* is the sum of the areas of all the surfaces.

lateral surface area

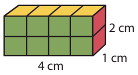
Shapes with a Volume of 8 cm³ | **Surface Area**

***V* = 8 cm³**
$V = 8 \cdot 1 \cdot 1$
$l \cdot w \cdot h$



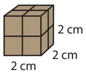
top and bottom $2 \cdot (8 \cdot 1) =$ 16 cm²
front and back $2 \cdot (8 \cdot 1) =$ 16 cm²
left and right sides $2 \cdot (1 \cdot 1) =$ 2 cm²
Total surface area = 34 cm²

***V* = 8 cm³**
$V = 4 \cdot 1 \cdot 2$
$l \cdot w \cdot h$



top and bottom $2 \cdot (4 \cdot 1) =$ 8 cm²
front and back $2 \cdot (4 \cdot 2) =$ 16 cm²
left and right sides $2 \cdot (1 \cdot 2) =$ 4 cm²
Total surface area = 28 cm²

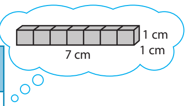
***V* = 8 cm³**
$V = 2 \cdot 2 \cdot 2$
$l \cdot w \cdot h$



All sides $6 \cdot (2 \cdot 2) =$ 24 cm²
Total surface area = 24 cm²

Exercises *Order of dimensions may vary within a row.*

The volume and the length of a rectangular prism are given. Use the given dimensions to arrange cubes to find the unknown values for the *width* and the *height*: $V = l \cdot w \cdot h$. Find the surface area using the 3 dimensions.



	Volume (cm³)	Length (cm)	Width (cm)	Height (cm)	Surface Area (cm²)
1.	7 cm³	7 cm	*1 cm*	*1 cm*	*30 cm²*
2.	10 cm³	10 cm	*1 cm*	*1 cm*	*42 cm²*
		5 cm	*2 cm*	*1 cm*	*34 cm²*
3.	12 cm³	12 cm	*1 cm*	*1 cm*	*50 cm²*
		6 cm	*2 cm*	*1 cm*	*40 cm²*
		4 cm	*3 cm*	*1 cm*	*38 cm²*
		3 cm	*2 cm*	*2 cm*	*32 cm²*
4.	16 cm³	16 cm	*1 cm*	*1 cm*	*66 cm²*
		8 cm	*2 cm*	*1 cm*	*52 cm²*
		4 cm	*2 cm*	*2 cm*	*40 cm²*
5.	24 cm³	24 cm	*1 cm*	*1 cm*	*98 cm²*
		12 cm	*2 cm*	*1 cm*	*76 cm²*
		8 cm	*3 cm*	*1 cm*	*70 cm²*
		6 cm	*4 cm*	*1 cm*	*68 cm²*
		6 cm	*2 cm*	*2 cm*	*56 cm²*
		4 cm	*3 cm*	*2 cm*	*52 cm²*

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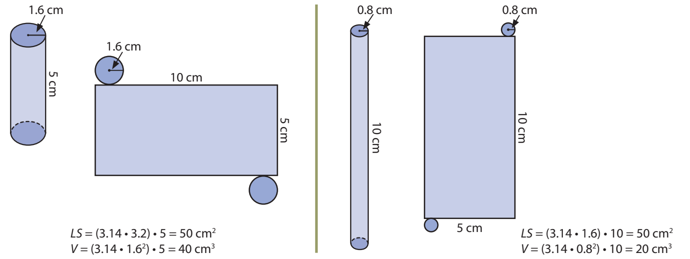
To find **lateral surface area**, find the surface area of all faces except the bases. Two geometric figures with different base areas and different heights can have the same lateral surface area but different volumes.

Lateral Surface Area of a Cylinder

LS = circumference ($2\pi r$) × base (*h*); *V* = Base (*B*) × height (*h*)



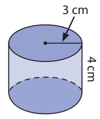
$LS = (3.14 \cdot 3.2) \cdot 5 = 50\text{ cm}^2$
$V = (3.14 \cdot 1.6^2) \cdot 5 = 40\text{ cm}^3$

$LS = (3.14 \cdot 1.6) \cdot 10 = 50\text{ cm}^2$
$V = (3.14 \cdot 0.8^2) \cdot 10 = 20\text{ cm}^3$

Exercises

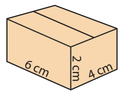
Find the lateral surface area and the volume of a cylinder using the given radius and height. Draw the shape if needed. Round the answer to the nearest unit.

	Lateral Surface Area (nearest cm²)	Radius (cm)	Height (cm)	Volume (nearest cm³)
6.	*75 cm²*	3 cm	4 cm	*113 cm³*
		1 cm	12 cm	*38 cm³*
7.	*132 cm²*	1.5 cm	14 cm	*99 cm³*
		1 cm	21 cm	*66 cm³*



Find the lateral surface area (*LS* = Area of side faces) and the volume ($V = [l \cdot w] \cdot h$) of a rectangular prism using the given dimensions.

	Lateral Surface Area (cm²)	Length (cm)	Width (cm)	Height (cm)	Volume (cm³)
8.	*40 cm²*	6 cm	4 cm	2 cm	*48 cm³*
		5 cm	5 cm	2 cm	*50 cm³*



J A toy manufacturer makes wooden blocks that are 1-inch cubes and packages them in sets. A box with the smallest surface area is the least expensive to produce. Find the length, width, and height for a box that gives the least surface area for each set of blocks.

Order of dimensions may vary.
a small set of 16 blocks
***l* = 4 in., *w* = 2 in., *h* = 2 in.**
a medium set of 24 blocks
***l* = 4 in., *w* = 3 in., *h* = 2 in.**
a large set of 36 blocks
***l* = 6 in., *w* = 3 in., *h* = 2 in.**

Complete DAILY REVIEW d on page 446.

Lesson 111 269

taller tube **LS = (πd)h; LS ≈ (3.14 • 3 in.)12 in.; LS ≈ 9.42 in. • 12 in.; LS ≈ 113.04 in.² ≈ 113 in.²**

shorter tube **LS = (πd)h; LS ≈ (3.14 • 4 in.)9 in.; LS ≈ 12.56 in. • 9 in.; LS ≈ 113.04 in.² ≈ 113 in.²**

5. Stand both tubes upright inside the box lid. Direct the students to do the same.

➤ **Do you think these tubes have the same volume? Why?** ***Accept all answers.***

6. As you demonstrate, direct the students to place the taller tube inside the shorter tube.

7. Demonstrate using the measuring cup to fill the entire taller tube with rice. Gently hold the taller tube in place so that the rice doesn't come out of the bottom as it is poured in from the top. Elicit that the amount of rice in the taller tube is equal to the volume of the tube.

   As you demonstrate, direct the students to remove the taller tube to let the rice fill the shorter tube. Instruct them to flatten the rice if necessary to keep the top level.

➤ **Do both tubes have the same volume? How do you know?** ***No; the rice that filled the taller tube does not fill the shorter tube.*** Point out that the rice that completely filled the taller tube fills only about $\frac{3}{4}$ of the shorter tube.

➤ **How can you calculate the volume of each tube?** ***Elicit that you measure the diameter of the circular base, calculate the area of the base (πr^2), and then multiply the area by the height.***

8. Guide the students in calculating the volume of each tube. Round the answers to the nearest tenth.

   taller tube **V = (πr²)h; V = 3.14 • (1.5 in.)² • 12 in.; V = (3.14 • 2.25 in.²) • 12 in.; V = 7.065 in.² • 12 in.; V = 84.78 in.³ ≈ 84.8 in.³**

   shorter tube **V = (πr²)h; V = 3.14 • (2 in.)² • 9 in.; V = (3.14 • 4 in.²) • 9 in.; V = 12.56 in.² • 9 in.; V = 113.04 in.³ ≈ 113 in.³**

➤ **Why is it possible for both of these tubes to have different volumes when they have the same lateral surface area?** ***Elicit that the different layout of each cylinder's curved surface creates different dimensions for the circular base and height, which determine the volume of the figure.***

9. Follow a similar procedure with rectangular prisms. Construct a tall and a short prism by folding one 8 × 12 sheet of construction paper lengthwise into fourths and folding the other sheet of paper widthwise into fourths. Tape the seams of each prism at the top, middle, and bottom.

   Explain that the lateral surface area of a prism is found by adding all the areas of its side faces (without the 2 bases), and elicit that the volume of a prism is found by multiplying its length, width, and height.

   lateral surface area of both prisms based on the paper size ***8 in. × 12 in. = 96 in.²***

   taller prism **LS = (2 in. × 12 in.) + (2 in. × 12 in.) + (2 in. × 12 in.) + (2 in. × 12 in.); LS = 24 in.² + 24 in.² + 24 in.² + 24 in.²; LS = 96 in.² or LS = 4 × 24 in.² = 96 in.²**

   V = (2 in. × 2 in.)12 in.; V = 4 in.² × 12 in.; V = 48 in.³

   shorter prism **LS = (3 in. × 8 in.) + (3 in. × 8 in.) + (3 in. × 8 in.) + (3 in. × 8 in.); LS = 24 in.² + 24 in.² + 24 in.² + 24 in.²; LS = 96 in.² or LS = 4 × 24 in.² = 96 in.²**

   V = (3 in. × 3 in.)8 in.; V = 9 in.² × 8 in.; V = 72 in.³

Student Text pp. 268–69

Lesson 112 Student Text pp. 270–71

Chapter Review

Objectives
- Find the volume of a rectangular prism, a cube (square prism), and an irregular prism using models
- Calculate the unknown measurement of a rectangular prism
- Calculate the volume of a rectangular prism, a cube (square prism), a triangular prism, and a cylinder using formulas
- Relate volume to real-life situations

Teacher Materials
- Volume Review, page IA61 (CD)
- Volume Word Problems, page IA62 (CD)
- Christian Worldview Shaping, page 30 (CD)
- Copies of the area drawings (from Lessons 108–10 Preparation)
- Cube-shaped blocks (to build towers of varied heights)

Student Materials
- A calculator

Note
This lesson reviews the concepts presented in Chapter 12 to prepare the students for the Chapter 12 Test. Student Text pages 270–71 provide the students with an excellent study guide.

Check for Understanding

Find the volume of a prism using a model
Display several of the area drawings, one at a time. Choose a student to tell the area of the base and build a tower on it with the height of your choice. Choose another student to tell the volume of the tower and explain his answer.

Calculate the unknown measurement of a rectangular prism
1. Draw a rectangular prism for display.
   ➤ **What formula do you use to calculate the volume of a rectangular prism?** $V = Bh$ *or* $V = (l \times w) \times h$
   Write $V = Bh$ and $V = (lw)h$ for display.
2. Direct the students to find the unknown measurement by substituting the given values into the volume formula. Review the solution as needed.
   $V = 360 \text{ cm}^3$, $l = 15$ cm, $h = 6$ cm
   $360 \text{ cm}^3 = (15 \text{ cm} \times w)6 \text{ cm}; 360 \text{ cm}^3 = (15 \text{ cm} \times 6 \text{ cm})w;$
   $360 \text{ cm}^3 = 90 \text{ cm}^2 \times w; \frac{360 \text{ cm}^3}{90 \text{ cm}^2} = \frac{90 \text{ cm}^2 \times w}{90 \text{ cm}^2}; 4 \text{ cm} = w$
   $V = 31.5 \text{ cm}^3$, $B = 4.5 \text{ cm}^2$
   $31.5 \text{ cm}^3 = 4.5 \text{ cm}^2 \times h; \frac{31.5 \text{ cm}^3}{4.5 \text{ cm}^2} = \frac{4.5 \text{ cm}^2 \times h}{4.5 \text{ cm}^2}; 7 \text{ cm} = h$
3. Repeat the procedure, allowing students to give values for the volume and 2 dimensions.

Calculate the volume of a 3-dimensional figure
1. Display the Volume Review page. Direct the students to calculate the volume of each figure. Discuss the solutions and review the formulas for finding volume. Or use the Volume Review page as a "Tic-tac-toe" board and arrange the students into 2 teams, Team X and Team O, to play a game of "Tic-tac-toe" using the following procedure.
2. Explain that you will choose one member from a team to select a section on the "Tic-tac-toe" board. Then each member of that team will use the appropriate formula to solve the problem at his desk. Direct the students to round the decimal answers to the nearest tenth. Points will be given for each team member who gets the answer correct.
3. Begin with a student from Team O. If the student correctly finds the volume of the figure in the selected section, write an O in that section. Also award 1 point for every member of Team O who has the correct answer. If the student answers incorrectly, the section is available for a student from Team X to give the answer, but award 1 point to Team O for each team member who has the correct answer. Continue the game by calling on Team X to choose a section of the board. Alternate teams until the game has been completed. Award 5 bonus points to the first team to achieve 3 correct answers in a row on the board (vertically, horizontally, or diagonally), and then add up the points earned. The team with the most points wins the game. You may choose to play the game again by varying the dimensions of each figure.
   1. rectangular prism $V = (lw)h; (12 \text{ in.} \cdot 4 \text{ in.})5 \text{ in.} = 240 \text{ in.}^3$
   2. cylinder $V = (\pi r^2)h; (3.14)(3 \text{ cm})^2 \cdot 9 \text{ cm} \approx 254.3 \text{ cm}^3$
   3. square prism (cube) $V = s^3; (8 \text{ in.})^3 = 512 \text{ in.}^3$
   4. triangular prism $V = (\frac{1}{2}bh_1)h_2; \frac{1}{2}(4 \text{ in.} \times 4 \text{ in.}) \times 8 \text{ in.} = 64 \text{ in.}^3$
   5. cylinder $V = (\pi r^2)h; (3.14)(2.5 \text{ m})^2 \cdot 5 \text{ m} \approx 98.1 \text{ m}^3$
   6. triangular prism $V = (\frac{1}{2}bh_1)h_2; \frac{1}{2}(6 \text{ in.} \times 5 \text{ in.}) \times 12 \text{ in.} = 180 \text{ in.}^3$
   7. cylinder $V = (\pi r^2)h; (3.14)(2\text{cm})^2 \cdot 6 \text{ cm} \approx 75.4 \text{ cm}^3$
   8. rectangular prism
      $V = (lw)h; (4.5 \text{ cm} \times 3.5 \text{ cm})9 \text{ cm} \approx 141.8 \text{ cm}^3$
   9. triangular prism $V = (\frac{1}{2}bh_1)h_2; \frac{1}{2}(4 \text{ in.} \cdot 5 \text{ in.}) \cdot 10 \text{in.} = 100 \text{ in.}^3$

Relate volume to real-life situations
1. Display the Volume Word Problems page and read aloud the first word problem. Discuss the type of container described in the first word problem and the formula needed to find the answer. Choose a student to write the formula for display. Direct each student to write and solve the equation for the word problem. Discuss the answer. *cube or square prism;* $V = s^3; V = (2 \text{ ft})^3; V = 8 \text{ ft}^3$
2. Follow a similar procedure for problems 2–4. Instruct the students to round the decimal answers to the nearest tenth.
   2. *cylinder;* $V = (\pi r^2)h; V = (3.14)(1 \text{ ft})^2 \times 2 \text{ ft}; V \approx 6.3 \text{ ft}^3$
   3. *rectangular prism;* $V = (lw)h; V = (4 \text{ ft} \times 2 \text{ ft}) \times 1.5 \text{ ft}; V = 12 \text{ ft}^3$
   4. *triangular prism,* $V = (\frac{1}{2}bh_1)h_2; V = \frac{1}{2}(4 \text{ ft} \times 4 \text{ ft}) \times 2 \text{ ft} = 16 \text{ ft}^3$
3. **Christian Worldview Shaping (CD)**

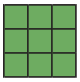
CHAPTER 12 REVIEW

Write the formula for calculating the volume of the figure.

$V = (lw)h$
$V = (\pi r^2)h$
$V = s^3$
$V = (\frac{1}{2}bh_1)h_2$

1. volume of a cube **$V = s^3$**
2. volume of a triangular prism **$V = (\frac{1}{2}bh_1)h_2$**
3. volume of a cylinder **$V = (\pi r^2)h$**
4. volume of a rectangular prism **$V = (lw)h$**

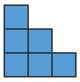
Find the area of the base. Find the volume of the prism that could be built using centimeter cubes for the given height.

5. 

height = 3 cm ***27 cm³***

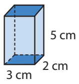
6. 

height = 5 cm ***20 cm³***

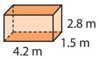
7. 

height = 2 cm ***12 cm³***

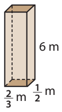
Find the volume of the figure. Round a decimal answer to the nearest tenth. ***Equations may vary.***

8. 

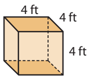
30 cm³

9. 

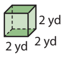
17.6 m³

10. 

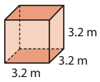
2 m³

11. 

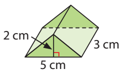
64 ft³

12. 

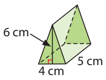
8 yd³

13. 

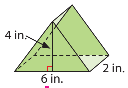
32.8 m³

14. 

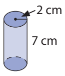
15 cm³

15. 

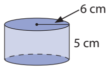
60 cm³

16. 

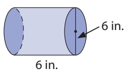
24 in.³

17. 

87.9 cm³

18. 

565.2 cm³

19. 

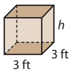
169.6 in.³

Find the volume of a figure with the given dimensions.
Round a decimal answer to the nearest tenth. ***Equations may vary.***

20. rectangular prism: $l = 5.3$ in., $w = 4$ in., $h = 6$ in. **$V = 5.3$ in. $\times$ 4 in. $\times$ 6 in.; $V = 127.2$ in.³**
21. cube: $s = 10$ m **$V = (10\text{ m})^3$; $V = 1000\text{ m}^3$**
22. triangular prism: $b = 4$ in., $h = 3$ in., h (prism) = 5 in. **$V = \frac{1}{2}(4$ in. $\times$ 3 in.$) \times 5$ in.; $V = 30$ in.³**
23. cylinder: $r = 3$ m, $h = 2$ m **$V = 3.14 \times (3\text{ m})^2 \times 2$ m; $V \approx 56.5\text{ m}^3$**

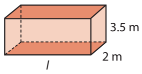
Find the unknown measurement of the rectangular prism.

24. $V = 36\text{ ft}^3$



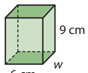
h = 4 ft

25. $V = 70\text{ m}^3$



l = 10 m

26. $V = 216\text{ cm}^3$



w = 4 cm

27. $l = 5$ in., $w = 2$ in., $h =$ ___ in., $V = 80\text{ in.}^3$ **h = 8 in.**

28. $l = 6$ cm, $w =$ ___ cm, $h = 4$ cm, $V = 144\text{ cm}^3$ **w = 6 cm**

Solve.
Round a decimal answer to the nearest tenth. ***Equations may vary.***

29. The Fernandez family is building raised garden beds to grow vegetables. The largest bed is 9 feet long, 4 feet wide, and 1.5 feet high. How many cubic feet of soil will it take to fill the largest bed? ***54 ft³***
30. The smallest bed the Fernandez family will build is 4 feet long, 4 feet wide, and 1.5 feet high. How many cubic feet of soil will it take to fill the smallest bed? ***24 ft³***
31. Mr. Fernandez wants to use rainwater to water his garden. He purchased a rain barrel that is 37 inches tall and has a 20-inch diameter. What is the volume of the rain barrel? ***11,618 in.³***
32. There are 1,728 cubic inches in 1 cubic foot. What is the volume of the rain barrel in cubic feet? ***11,618 in.³ ÷ 1,728 in.³ = 6.7 ft³***
33. Since 1 cubic foot can hold about 7.5 gallons of water, approximately how many gallons of water will the rain barrel hold? ***6.7 • 7.5 gal = 50.3 gal***

Student Text pp. 270–71

Lesson **113**

Student Text pp. 272–75

Chapter 12 Test

Cumulative Review

For a list of the skills reviewed in the Cumulative Review, see the Lesson Objectives for Lesson 113 in the Chapter 12 Overview on page 260 of this Teacher's Edition.

Student Materials

- Cumulative Review Answer Sheet, page IA9 (CD)

Use the Cumulative Review on Student Text pages 272–74 to review previously taught concepts and to determine which students would benefit from your reteaching of the concepts. To prepare the students for the format of achievement tests, instruct them to work on a separate sheet of paper, if necessary, and to mark the answers on the Cumulative Review Answer Sheet.

Read aloud the Career Link on Student Text page 275 (page 273 of this Teacher's Edition) and discuss the value of math as it relates to a mission board director.

CUMULATIVE REVIEW

Test Prep

Mark the answer.

1. 36,903 + 14,772 + 21,187

A. 71,682 C. 72,862
B. 72,753 D. 72,992

2. $3 \times 26 \times 16 =$ ___

A. 1,221 C. 1,336
B. 1,248 D. 1,428

3. 17.34 − 8.9286

A. 8.4114 C. 9.4114
B. 8.6286 D. 9.4214

4. $3\frac{3}{16} + 7\frac{1}{4} =$ ___

A. $10\frac{7}{16}$ C. $10\frac{3}{16}$
B. $10\frac{1}{4}$ D. 11

5. $10 - 3\frac{6}{7} =$ ___

A. $7\frac{3}{7}$ C. $6\frac{4}{7}$
B. 7 D. $6\frac{1}{7}$

6. Round to the nearest hundredth.

$112\overline{)4,318}$

A. 38.55 C. 40.51
B. 39.52 D. 45.01

7. $\frac{3}{4} \div \frac{1}{2} =$ ___

A. 1 C. 2
B. $1\frac{1}{2}$ D. $2\frac{1}{2}$

8. $\frac{4}{9} \times \frac{11}{12} =$ ___

A. $\frac{1}{3}$ C. $\frac{11}{108}$
B. $\frac{5}{7}$ D. $\frac{11}{27}$

9. Rename $\frac{5}{6}$ as a decimal. Round to the nearest thousandth.

A. 0.822 C. 0.932
B. 0.833 D. 0.983

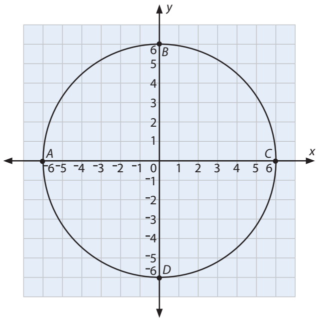
10. Estimate the difference.

$\frac{8}{9} - \frac{3}{4} =$ ___

A. 0 C. 1
B. $\frac{1}{2}$ D. $1\frac{1}{2}$

272 Chapter 12

Use the coordinate plane to find the answer.



11. What are the coordinates for point *C*?

A. (0, 6) C. (0, ⁻6)
B. (⁻6, 0) D. (6, 0)

12. The diameter of the circle is ___ units.

A. 6 C. 18
B. 12 D. 24

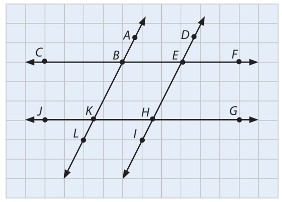
13. The radius is ___ units.

A. 6 C. 18
B. 12 D. 24

14. Which formula could be used to find the area of the circle?

A. $A = l \times w$ C. $A = \pi r^2$
B. $A = \frac{1}{2}bh$ D. $A = bh$

Use the figure to find the answer.



15. What figure is enclosed by the lines?

A. square C. parallelogram
B. trapezoid D. hexagon

16. Describe the relationship between $\overleftrightarrow{AL}$ and $\overleftrightarrow{DI}$.

A. intersecting lines C. perpendicular lines
B. parallel lines D. none of the above

17. Describe $\angle BKH$.

A. acute C. right
B. obtuse D. all of the above

18. Mark the true statement.

A. $\angle GHI = 90°$ C. $\angle GHI < 90°$
B. $\angle GHI > 90°$ D. $\angle GHI > 180°$

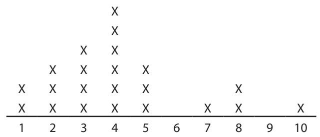
19. Which line is perpendicular to $\overleftrightarrow{CF}$?

A. $\overleftrightarrow{AL}$ C. $\overleftrightarrow{DI}$
B. $\overleftrightarrow{JG}$ D. none of the above

Lesson 113 273

Use the data from the line plot to find the answer.

Scott asked this survey question:
"How many people live in your home?"
He recorded the survey responses on a line plot.



Number of Occupants in Each Home

20. According to the survey, how many people lived alone?

A. 1 C. 6
B. 2 D. 10

21. How many people did Scott survey?

A. 10 C. 18
B. 15 D. 22

22. The largest group of responses was for how many occupants?

A. 2 C. 4
B. 3 D. 5

23. How many homes had 10 occupants?

A. 1 C. 3
B. 2 D. 4

24. Can you tell from this graph how many homes have school-age children?

A. yes
B. no

274 Chapter 12

CAREER LINK

Mission Board Director

A mission board director uses math every day to help missionaries and churches take the gospel all over the world. He keeps current on how much the dollar is worth in the countries in which his missionaries serve. He helps determine the amount of financial support a missionary needs by considering the cost of food and housing, the number of family members, health insurance, taxes and other expenses.

What will it cost for a missionary to do his work? The mission board director must assess a variety of information. Will the missionary translate the Bible? Will he need money to repair airplanes or fly to remote areas? Will the missionary use medical equipment to help the people? These work-related expenses must be figured into the missionary's financial support.

Insurance and taxes are important financial concerns. The director keeps records of medical expenses, such as payments made to doctors and hospitals. He also records all financial transactions needed for filing the missionary's income taxes.

Financial support collected by a church is designated for a particular missionary. The money is sent to the mission board and then given to the missionary. The mission board provides current information about the missionaries to their supporting churches. The director gives a regular report so that churches will know how to best help and pray for the missionaries they support. Each month the missionary and the director communicate about expenses and support. With several hundred missionaries in many countries, the director finds math a useful and important tool.