

LIGHTS AT SUNSET

Carriacou

July 16, 1992

The sun was setting on the Caribbean island of Carriacou on July 16, 1992, as a young missionary named Randy Cornelius climbed up the ham radio tower on the side of his home. Randy was in the process of setting up a Christian radio station on the island, and work on the new station was going well. Today he had devoted some time to repairing the antenna on his own small ham radio tower at home. Now he was ready to put it back in place. He climbed carefully, holding the repaired antenna in his hand as he went. Fifty feet in the air, he stopped and clipped the nylon rope onto the safety belt at his waist, draped it around the tower, and attached it to the clip on the other side of his waist. The safety belt and rope would allow him to lean back away from the tower and keep his hands free to work.

Then something unusual happened. The safety rope broke! Randy was never quite sure what caused the rope to break, but the next thing he knew, he was falling. His feet hit the ground first, and he lay on his

back in his yard, looking up at the setting sun. He felt numb from the waist down, and he did not know how badly he was hurt. But one thought surfaced in his mind: something he had said to the Lord when he was a fifteen-year-old boy. He spoke it aloud. “Lord, whatever you want to do with me is fine.”

Randy’s neighbors saw his fall and rushed to help him. Because the island had no lighted airstrip, people from all over Carriacou parked their cars in two lines and shone their headlights on the runway so that an emergency plane from the larger island of Grenada could land. Randy and his wife were flown to a hospital on Grenada. They waited there all night while the doctors and nurses decided what to do. When morning came, the doctors set his broken foot and wrist. Then they told him that he would have to go to the United States for more treatment. Randy’s parents called all the commercial airlines, but none were willing to take responsibility for an injured man. Finally, Randy’s brother contacted a friend in Greenville, South Carolina, and asked to use his private jet. The man gave his permission, and Randy was flown out of Grenada the next day.

Doctors in the States found serious internal injuries that would have cost Randy his life if the Lord had not provided a way for him to leave Grenada to receive proper treatment. Miraculously, Randy is back in Carriacou today, working at the radio station and walking well. He praises the Lord daily for protecting him and giving him courage to trust Him throughout the ordeal.



Randy being moved from an ambulance in Grenada to a private jet.



Ham radio originated around the beginning of the 1900s when inventor Guglielmo Marconi sent radio signals from England to Canada.

Over 600,000 ham radio amateurs operate in the United States and over 2,000,000 worldwide.

Carriacou has an area of thirteen square miles and a population of about 7,000 people. Fewer than 1,000 inhabitants live in the capital city of Hillsborough.

The radio station where Randy Cornelius ministers, Harbour Light of the Windwards, broadcasts to a 250-mile radius. The station has received signal reports from all of the islands in the coverage area and from as far away as South America and Europe.

By God's grace, Harbour Light was the only radio station in Grenada to remain on the air during Hurricane Ivan on September 7, 2004.

Equations

| Lesson | Topic | Lesson Objectives | Chapter Materials |
|-----------|---|---|---|
| 88 | Expressions | <ul style="list-style-type: none"> • Write a numerical or an algebraic expression for a word phrase • Demonstrate an understanding of algebraic expressions with more than one operation • Evaluate an expression using substitution • Use the Order of Operations to evaluate expressions | <p>Teacher Manipulatives Packet:</p> <ul style="list-style-type: none"> • Shapes Kit (optional) • Place Value Kit • Variable Cards <p>Student Manipulatives Packet:</p> <ul style="list-style-type: none"> • Equation Mat • Variable Cards <p>Instructional Aids (Teacher's Toolkit CD):</p> <ul style="list-style-type: none"> • Cumulative Review Answer Sheet (page IA9) for each student • Variable Cards (page IA45) (optional) • Equation Mat (page IA46) • Graph an Equation (page IA47) • Graph an Equation (page IA47) for each student <p>Christian Worldview Shaping (Teacher's Toolkit CD):</p> <ul style="list-style-type: none"> • Pages 25–27 <p>Other Teaching Aids:</p> <ul style="list-style-type: none"> • Round counters for each student and the teacher <p>Math 6 Tests and Answer Key</p> <p>Optional (Teacher's Toolkit CD):</p> <ul style="list-style-type: none"> • Fact Review pages • Application pages • Calculator Activities |
| 89 | Equations | <ul style="list-style-type: none"> • Demonstrate an understanding of equations • Write an equation with two equal expressions • Determine the unknown in a word problem and write it as a variable in an equation • Evaluate and relate expressions using $>$, $<$, or $=$ | |
| 90 | Simplify Expressions | <ul style="list-style-type: none"> • Demonstrate an understanding of the Commutative and Associative Properties of Addition and of Multiplication • Simplify algebraic expressions using manipulatives • Apply the Commutative and Associative Properties to simplify algebraic expressions | |
| 91 | Addition & Subtraction Equations | <ul style="list-style-type: none"> • Solve addition and subtraction equations using inverse operations • Check addition and subtraction equations using substitution | |
| 92 | Multiplication Equations | <ul style="list-style-type: none"> • Solve multiplication equations using division (the inverse operation) • Check multiplication equations using substitution • Write an equation with a variable to solve a word problem | |
| 93 | Multiplication & Division Equations | <ul style="list-style-type: none"> • Solve multiplication and division equations using inverse operations • Check multiplication and division equations using substitution • Write an equation with a variable to solve a word problem • Demonstrate an understanding of inequalities • Picture an inequality on a number line • Determine whether a given number is a solution to an inequality | |
| 94 | Equivalent Expressions | <ul style="list-style-type: none"> • Demonstrate an understanding of the Distributive Property of Multiplication over Addition • Apply the Distributive Property of Multiplication over Addition to find equivalent expressions • Solve equations using inverse operations | |
| 95 | Distance = Rate \times Time | <ul style="list-style-type: none"> • Calculate the distance traveled given the rate and the time, the rate of travel given the distance and the time, and the time traveled given the distance and the rate • Complete a table using the formula $d = r \times t$ • Create a line graph relating to the formula $d = r \times t$ | |
| 96 | Chapter 10 Review | <ul style="list-style-type: none"> • Review | |
| 97 | Chapter 10 Test Cumulative Review | <ul style="list-style-type: none"> • Read and interpret a bar graph • Identify plane figures when given the characteristics • Determine the perimeter of a figure • Solve a fraction word problem • Add, multiply, and divide fractions • Find a missing factor • Identify the reciprocal of a fraction • Multiply and divide decimals • Determine the value of an expression • Determine the lowest term of a fraction or a mixed number | |

A Little Extra Help

Use the following to provide “a little extra help” for the student that is experiencing difficulty with the concepts taught in Chapter 10.

Isolate the variable to solve an equation—Use an Equation Mat and several sets of number cards, operation symbol cards (+, −, ×, ÷), and variable cards from the Student Manipulatives Packets; or prepare sets of the cards using 3×5 cards. Use the cards to display an equation such as $n \times 3 = 27$. Ask the student to identify the operation that is being performed on the variable in the equation. *Multiplication; n is being multiplied by 3.* Ask the student to tell the inverse operation he should use to isolate the variable. *division*

Direct the student to place cards in the equation so that both sides of the equation are divided by 3: $n \times 3 \div 3 = 27 \div 3$. Remind him that any operation that is performed on one side of the equation must also be performed on the other side to keep the equation balanced (the same value on both sides of the equation). Instruct the student to remove the cards for $\times 3 \div 3$ from the left side of the equation. Remind him that since $3 \div 3 = 1$, dividing the left side of the equation by 3 isolates the variable because $n \times 1 = n$ (Identity Property of Multiplication). Direct the student to perform the same operation ($\div 3$) on the right side of the equation, simplifying the numerical expression by replacing $27 \div 3$ with the card that shows the answer. 9 Repeat the procedure, varying the numbers and operation in each equation.

Math Facts

Throughout this chapter, review fractions using Fact Review pages on the Teacher’s Toolkit CD. Also, review addition, subtraction, multiplication, and division facts using Fact Review pages or a Fact Fun activity on the Teacher’s Toolkit CD, or you may use flashcards.

Objectives

- Write a numerical or an algebraic expression for a word phrase
- Demonstrate an understanding of algebraic expressions with more than one operation
- Evaluate an expression using substitution
- Use the Order of Operations to evaluate expressions

Note

Preview the Fact Review pages, the Application pages, and the Calculator Activities located on the Teacher's Toolkit CD.

Introduce the Lesson

Guide the students in reading aloud the story and facts on pages 212–13 of the Student Text (pages 210–11 of this Teacher's Edition).

Teach for Understanding

Write a numerical or an algebraic expression for a word phrase

Five hikers were hiking on the nature trail. As they proceeded up the path, 2 other hikers caught up with them and joined their group.

- **How many hikers were on the trail at first?** 5
- **Which word phrase tells you what happened to the original 5 hikers?** 2 other hikers joined the group of 5 hikers.
- **What operation sign and number can you write to show that 2 hikers joined the group of 5?** $+ 2$

1. Write $5 + 2$ for display. Explain that *expressions* are mathematical phrases made up of numbers, operation signs, and sometimes variables. *Numerical expressions* are mathematical phrases such as $5 + 2$; the phrases contain numbers and operations. Recognizing a mathematical operation in written form can help you to translate a word phrase into a numerical expression.

- **What other word phrases can you think of that indicate the addition of 2 to 5?** *Elicit word phrases such as 5 increased by 2; 2 added to 5; the sum of 5 and 2; and 2 more than 5.*

After the 7 hikers had walked one mile, the path became very steep, so 3 hikers left the group and went back to the trail head.

- **How many hikers were walking together?** 7
- **Which word phrase tells you what happened to the 7 hikers?** 3 hikers left.
- **What operation sign and number can you write to show that 3 hikers left the group?** $- 3$

2. Write $7 - 3$ for display.

- **What other word phrases can you think of that indicate the subtraction of 3 from 7?** *Elicit word phrases such as 7 minus 3; 7 decreased by 3; the difference of 7 and 3; 3 subtracted from 7; and 3 less than 7.*

Glenn saw a large group of hikers walking together on the nature trail. When the trail became narrow, the guide divided the hikers into groups of 2.

- **How many hikers did Glenn see?** *Elicit that it was a large group; the number of hikers is unknown.*

- **Which word phrase tells you what happened to the large group of hikers?** *"divided the hikers into groups of 2"*

3. Remind students that any letter can be used as a variable. Point out that using an initial letter such as h to represent "hikers" can help them remember what the variable represents. Explain that when a mathematical phrase includes a variable, the phrase is an *algebraic expression*.

- **What algebraic expression can you write to show that an unknown number of hikers was divided into groups of 2?** $h \div 2$ or $\frac{h}{2}$

4. Write $h \div 2$ for display.

- **What other word phrases can you think of that indicate division by 2?** *Elicit phrases such as the quotient of a number and 2; grouped in pairs; cut in half; shared by 2 people.*

The trail guide led 3 times more hikers on the afternoon hike than he led on the morning hike.

- **What expression can you write for this situation? Why?** *Elicit $3 \times h$, $3 \cdot h$, or $3h$; since the number of hikers in the morning is unknown, the variable h can be used to represent the unknown number. The number of hikers in the afternoon is 3 times h .*

5. Write $3h$ for display. Remind the students that $3h$ is another way to write 3 times h . Explain that a number multiplied with a variable is a *coefficient*; 3 is the coefficient of the variable h . Point out that the known factor (the coefficient) precedes the variable, even for a word phrase such as "the product of h and 3," and that a multiplication sign is not needed.

6. Point out that $h \div 2$, $\frac{h}{2}$, $3 \times h$, $3 \cdot h$, and $3h$ are all algebraic expressions because they include a variable.

7. Guide the students in writing a numerical or an algebraic expression for each of the following word phrases. Point out that since addition and multiplication are commutative, the order of the addends or factors may vary. However, the numbers must be written in the correct order when the operation is subtraction or division because subtraction and division are not commutative. If the numbers are not written in the correct order, the expression will not be representative of the word phrase; its value will differ from the word phrase.

the quotient of 12 and 4 $12 \div 4$ or $\frac{12}{4}$

2 to the third power minus 3 $2^3 - 3$

16 less than 30 $30 - 16$

the sum of a number and 5 $n + 5$ or $5 + n$

6 times a number $6x$

the product of 8 and 7 8×7 or 7×8

a number divided by 6 $n \div 6$ or $\frac{n}{6}$

a number decreased by 4 $a - 4$

Demonstrate an understanding of algebraic expressions with more than one operation

1. Explain that word phrases may indicate more than one operation. You must read the words carefully to be sure that you perform the order of the operations correctly.

Mrs. Donner delivered a container of cookies to the church for a youth activity. Alisa brought 3 dozen more cookies. How many cookies did Mrs. Donner and Alisa bring to the church?

- **What is the question asking you to find?** *how many cookies Mrs. Donner and Alisa brought to the church*

Expressions

An **expression** is a mathematical phrase made up of numbers, operation signs, and sometimes variables. A **variable** is a letter used to represent an unknown value. An expression can be written by interpreting a word phrase.

Two added to thirty: $30 + 2$

A number divided by 3: $n \div 3$

An **algebraic expression** always uses a variable. When the operation is multiplication, an algebraic expression is written using a **coefficient** (number) before the variable. A multiplication sign is not needed.

The product of a number and 12: $12n$

$12n = 12 \times n$
12 is the coefficient of the variable n .

Expressions can be evaluated by **substituting** a value for the variable. Calculate to find a numerical value for the expression.

Home Zone offers a discount to customers who purchase 4 or more gallons of paint. Mr. Lehman purchased 4 gallons of paint for his music studio. Write an algebraic expression to show the cost of paint for Mr. Lehman. Find the cost if the discounted price is \$13.00 for each gallon.

$4g$
 $4(\$13.00) = \52.00

The cost of 4 gallons of paint is \$52.00.

expression
variable
algebraic expression
coefficient
substitution



Exercises

Complete the word phrase for the expression. **Answers may vary.**

- $15 + 32$: $\underline{\hspace{1cm}}$ added to $\underline{\hspace{1cm}}$ **32; 15**
- $14\frac{1}{2} \div 4 + 3$: $\underline{\hspace{1cm}}$ added to $\underline{\hspace{1cm}}$ divided by $\underline{\hspace{1cm}}$ **3; $14\frac{1}{2}$; 4**
- $6c$: $\underline{\hspace{1cm}}$ times $\underline{\hspace{1cm}}$ **6; a number**
- $8x - 2$: $\underline{\hspace{1cm}}$ less than $\underline{\hspace{1cm}}$ times $\underline{\hspace{1cm}}$ **2; 8; a number**

Write the numerical expression for the word phrase.

- 52 less than 96 **$96 - 52$**
- 8 multiplied by 2 **2×8**
- the product of 4 and 6 **4×6**
- 0.9 more than 1.06 **$1.06 + 0.9$**
- 12 decreased by 1.7 **$12 - 1.7$**
- the quotient of 32 and 4 **$32 \div 4$**
- 3 to the second power added to 9 **$9 + 3^2$**
- 17.3 increased by three-tenths **$17.3 + 0.3$**



214

Chapter 10

► **What information is given?** *A container of cookies and 3 dozen more cookies were brought to the church.*

► **What algebraic expression can you write to show an unknown number of cookies in the container and 3 dozen more cookies? Why?** *Elicit $c + 3 \times 12$; “ c ” represents the unknown number of cookies in the container, “more” indicates addition, and “3 dozen” can be interpreted as 3×12 .*

(Note: Some students may mentally calculate the 3 dozen cookies to be 36 cookies; therefore, accept $c + 36$ as a correct expression, but continue the lesson using $c + 3 \times 12$.)

2. Write $c + 3 \times 12$. Elicit that the Order of Operations must be considered when writing a multi-step expression.

► **What do you do first to solve $c + 3 \times 12$? Why?** *Multiply 3×12 ; elicit that there are no parentheses or exponents, and multiplication and division are performed before addition and subtraction.*

Remind the students that parentheses can be placed around the first step in a multi-step problem to indicate which operation should be performed first. However, because of the Order of Operations, parentheses are not always necessary. Write $c + (3 \times 12)$. Point out that with or without parentheses, you multiply before you add.

► **Can you find how many cookies there are? Why?** *No; the number of cookies in the container is not known.*

3. Explain that although you cannot know for sure how many cookies are in the container, you can use *substitution* to assign a value to the variable c and evaluate the expression; i.e., determine a possible total number of cookies.

► **What will be the total number of cookies if there are 27 cookies in the container or $c = 27$? How do you know?** *63; $27 + (3 \times 12) = 27 + 36 = 63$*

Write the algebraic expression for the word phrase.

- a number divided by 7 **$n \div 7$**
- 9 times a number **$9n$**
- 0.2 less than a number **$n - 0.2$**
- 16.8 more than n **$n + 16.8$**
- $\frac{3}{4}$ of a number **$\frac{3}{4}n$**
- 1.5 less than n **$n - 1.5$**
- n divided by 8, decreased by 4 **$n \div 8 - 4$**
- 2 less than 3 times n **$3n - 2$**
- the sum of 6 and a number, divided by 3 **$(6 + n) \div 3$**
- $\frac{2}{3}$ of a number divided by 12 **$\frac{2}{3} \times (n \div 12)$ or $(\frac{2}{3} \times n) \div 12$**

Write the algebraic expression.

Identify what the variable represents in the expression. **Variables and explanations may vary.**

- The pet shop sold 3 of the parakeets. **$p - 3$; p represents the number of parakeets.**
- Jillian grew 3 inches. **$5 + 3$; s represents the height of Jillian.**
- Cameron read $2\frac{1}{2}$ times as many books as Joseph.
- Matthew scored 10 fewer points than Alexa on the math test. **$a - 10$; a represents Alexa's math test score.**
- The apples were divided into 8 bags. **$a \div 8$; a represents the number of apples.**
- Mrs. Sullivan bought 3 dozen eggs at the Farmer's Market. By the time she arrived home, some of the eggs had broken. **$(3 \times 12) - n$; n represents the number of eggs broken on the way home.**
- $2\frac{1}{2} \cdot b$; **b represents the number of books read by Joseph.**

Evaluate the expression. Let $x = 3$.

- $2x - 5$ **$(2 \times 3) - 5 = 1$**
- $15 \div x + 4$ **$15 \div 3 + 4 = 9$**
- $x + 7 \times 3$ **$3 + 7 \times 3 = 24$**
- $12 - x + 2$ **$12 - 3 + 2 = 11$**

Practice & Application

- List the factors for 36 and 72. What is the greatest common factor? **GCF = 36**
- Draw and name two quadrilaterals with no right angles. **Answers will vary.**
- Write an equation showing that 42 is a multiple of 6. **$7 \times 6 = 42$**
- Rename $\frac{5}{6}$ and $\frac{7}{8}$ using a common denominator. Write a comparison statement using the greater than sign ($>$). **$\frac{5}{6} = \frac{20}{24}$; $\frac{7}{8} = \frac{21}{24}$; $\frac{20}{24} < \frac{21}{24}$ or $\frac{5}{6} < \frac{7}{8}$**
- Explain how the fractions $\frac{5}{16}$, $\frac{9}{24}$, and $\frac{12}{32}$ are related to $\frac{3}{8}$.
- Write the next three numbers in the pattern: 24, 48, 96. **192, 384, 768**
- Explain why it is important to use the order of operations to solve the problem $3x + 24 \div 3 - 2$ if $x = 10$. **$(3 \times 10) + (24 \div 3) - 2 = 30 + 8 - 2 = 36$; Following the order of operations is the only way to get the correct answer.**



- $3.8 + 2.05 = 5.85$ **$5.85 - 2.05 = 3.8$**
- $2.05 + 3.8 = 5.85$ **$5.85 - 3.8 = 2.05$**

Complete **DAILY REVIEW** on page 436.

Lesson 88

215

- Guide the students in finding the total number of cookies if $c = 12$, $c = 30$, and $c = 36$, using an input/output table.
- Write the sum of 8 and a number divided by 2.

► **What do the words “the sum” tell you?**

Elicit that there will be 2 addends.

► **What expressions can you write for this word phrase?** *Elicit $(8 + n) \div 2$ and $8 + (n \div 2)$.* Write both expressions. Point out that either expression is correct. The word phrase can be interpreted to mean addends 8 and n or addends 8 and $(n \div 2)$.

Explain that a comma can clarify the meaning of a word phrase. Insert a comma after the word *number* in the word phrase: *the sum of 8 and a number, divided by 2*. Point out that the comma separates the phrase to indicate that addition is the first operation and division is the second operation. Elicit that because of the Order of Operations, parentheses must be placed around $8 + n$ to show that the addition is performed first: $(8 + n) \div 2$.

- Guide the students in writing algebraic expressions for these word phrases.

2 times a number, divided by 4 **$2n \div 4$ or $(2 \times n) \div 4$**

a number increased by 7, multiplied by 3 **$(n + 7) \times 3$ or $3 \times (n + 7)$**

n divided by 5, decreased by 2 **$(n \div 5) - 2$ or $n \div 5 - 2$**

| c | $c + (3 \times 12)$ |
|-----|---------------------|
| 12 | 48 |
| 27 | 63 |
| 30 | 66 |
| 36 | 72 |

Student Text pp. 214–15

Objectives

- Demonstrate an understanding of equations
- Write an equation with two equal expressions
- Determine the unknown in a word problem and write it as a variable in an equation
- Evaluate and relate expressions using $>$, $<$, or $=$

Teach for Understanding

Demonstrate an understanding of equations

- Write 10 for display. Below the 10 write $9\frac{3}{4} + \frac{1}{4}$; $12.5 - 2.5$; 2×5 ; and $\frac{30}{3}$.
 > **What do you notice about these numerical expressions?**
Elicit that each expression represents the value 10.
- Direct the students to write the number 24 and then to write in one minute as many numerical expressions for 24 as possible. Encourage them to use all four operations and various types of numbers (fractions and decimals). Allow the students to share their expressions. *possible expressions:* $18 + 6$; $26 - 2$; 12×2 ; $48 \div 2$; $20\frac{1}{2} + 3\frac{1}{2}$; $22.5 + 1.5$
 > **How many different expressions for 24 do you think there are?** *Elicit that there is an infinite number of expressions.*
- Explain that any two expressions for 24 can be written with an equal sign ($=$) between them because each expression has a value of 24; they are equivalent.
- Choose pairs of students to write equations for display. Instruct each student in a pair to write one of his expressions for 24 so that the expressions are side by side, and then to write an equal sign ($=$) between the expressions to complete the equation.
 > **What is an equation?** *Answers may vary, but elicit that an equation is a mathematical sentence that states that two expressions are equal.*
- Write for display: *A number divided by 6 equals 2.* Direct attention to the phrase *equals 2*.
 > **What does the phrase “equals 2” tell you about the phrase before the word “equals”?** *Elicit that the phrase before the word “equals” has a value of 2.*
 Draw a simple picture of a balanced scale. Explain that the equal sign in an equation is like the fulcrum of a balanced scale, and the expressions on each side of the equal sign are like the equal weights on each side of a balanced scale.
 > **What word phrase represents the value on the left side of the equation?** *a number divided by 6*
 > **How can you mathematically write “a number divided by 6 equals 2”?** $n \div 6 = 2$ or $\frac{n}{6} = 2$
 Write $\frac{n}{6} = 2$ above the pictured scale so that the equal sign is directly above the fulcrum.
- Write for display: *12 decreased by a is 7.*
 > **What verb in this word phrase is similar to “equals”?** *is*
 > **What algebraic expression can you write for 12 decreased by a?** $12 - a$
 Select a student to write for display the algebraic equation for *12 decreased by a is 7.* $12 - a = 7$

- Follow a similar procedure for the following sentences. Remind the students that the words *equals* and *is* indicate where the equal sign should be written in the equation.
 - > **The product of 9 and a equals 54.** $9a = 54$
 - > **Dividing x by 7 is 4.** $\frac{x}{7} = 4$
 - > **The sum of 73 and b equals 97.** $73 + b = 97$
 - > **A number decreased by 2 to the power of 3 is 11.** $n - 2^3 = 11$

Determine the unknown in a word problem and write it as a variable in an equation

Last week, Mitchell earned \$115 for mowing lawns. That is \$35 more than he earned for mowing lawns this week. Write an equation to find how much money Mitchell earned for mowing this week. $\$115 = w + \35 [BAT: 2e Work]

- > **What is the word problem asking you to find?** *how much money Mitchell earned this week*
 - > **What statement tells you the amount of money Mitchell earned this week?** *That is \$35 more than he earned mowing lawns this week.*
- Write for display: *That is \$35 more than he earned this week.* Write the mathematical interpretation below the written statement as students answer the following questions.

That is \$35 more than he earned this week.
 $\$115 = w + \$35.$

 - > **What word in the statement indicates an equal sign in the equation for the word problem?** *is*
 - > **What does the word “that” refer to?** *the \$115 Mitchell earned last week* Point out that just as the word “that” comes before the word “is” in the written statement, the \$115 can be written before (to the left of) the equal sign ($=$) in the mathematical statement: $\$115 =$.
 - > **What is \$115 equal to?** *\$35 more than Mitchell earned this week*
 - > **What algebraic expression can you write to the right of the equal sign to show that the \$115 Mitchell earned last week is \$35 more than he earned this week?** $w + \$35$ Elicit that w , or any other variable given by the students, represents the unknown amount that Mitchell earned this week.

Three children from the sixth-grade class remained at school after 12 children from the class went home on the school bus. Write an equation to find how many students are in the class. *possible equations:* $s = 3 + 12$; $s = 12 + 3$; $s - 12 = 3$; $3 = s - 12$

- > **What is the word problem instructing you to do?** *Write an equation to find how many students are in the class.*
 - > **What phrases tell you the number of students in the class?** *“Three children from the sixth-grade class remained at school” and “after 12 children from the class went home.”*
 - > **What word indicates an equal sign?** *Elicit that there is no word similar to “equals” or “is.”*
- Draw 2 part-whole models side by side for display. Label the sections of the first model *whole*, *part*, and *part* as shown. Explain to the students that their understanding of part-whole models can help them write an equation when the words in a problem are unclear. Read the word problem aloud again.
 - > **What represents the whole in the word problem?** *the number of students in the class*

Equations

An **equation** is a mathematical sentence stating that two expressions are equal.
An equation can be written with or without variables.

equation

| | |
|-----------------------------|-----------------|
| 25 added to 75 is 100. | $75 + 25 = 100$ |
| Two less than y is 12. | $y - 2 = 12$ |
| Dividing c by 4 equals 9. | $c \div 4 = 9$ |

Exercises

Complete the sentence for the equation.

- $\frac{30}{y} = 15$: $\frac{30}{y}$ divided by y equals 15
- $3x + 8 = 20$: the sum of 3 times x and 8 is 20
- $n + 7 = 25$: a number increased by 7 equals 25
- $6p - 4 = 32$: 4 less than 6 times p equals 32
- $9s = 63$: the product of 9 and s is 63
- $\frac{m}{2} - 2 = 3$: the quotient of m and 2 decreased by 2 equals 3



Write an equation for the sentence.

- 5 less than a number is 17. $n - 5 = 17$
- 3 times s equals 27. $3s = 27$
- 8 less than 6 times r equals 4. $6r - 8 = 4$
- A number decreased by 2 is 27. $n - 2 = 27$
- A number divided by 7 is 6. $\frac{n}{7} = 6$
- 9 added to a number equals 21. $n + 9 = 21$

Evaluate the expression. Let $s = 8$.
Write a comparison sentence using $>$, $<$, or $=$.

- $2 + s < 15 - 3$ $10 < 12$
- $4s > 30$ $32 > 30$
- $12 - s = 2 \times 2$ $4 = 4$
- $\frac{24}{s} < 2^2$ $3 < 4$
- $\frac{1}{s} > 2 - 1$ $2 > 1$
- $s^2 = 60 \div 4$ $64 = 64$

Complete the table using the given values to evaluate the expression.

19.

| x | $\frac{x}{4} - 3$ |
|-----|-------------------|
| 12 | 0 |
| 16 | 1 |
| 20 | 2 |
| 24 | 3 |

20.

| x | $5x + 2$ |
|-----|----------|
| 2 | 12 |
| 3 | 17 |
| 5 | 27 |
| 7 | 37 |

21.

| x | $6 + x^2$ |
|-----|-----------|
| 2 | 10 |
| 3 | 15 |
| 4 | 22 |
| 5 | 31 |

216

Chapter 10

Write $s = \text{students in the class}$ for display. Then write s in the whole section of the second model.

- What represents the parts in the problem? *the 3 students who were still at school, and the 12 students who went home*
Write 3 and 12 in the part sections of the second model.

| | | | |
|-------|------|-----|----|
| whole | | s | |
| part | part | 3 | 12 |

- Using the part-whole model, what equation can you write to solve for s , the number of students in the class? *possible equations: $s = 3 + 12$; $s = 12 + 3$; $s - 12 = 3$*

- Instruct the students to write an equation for each of these word problems as you read the word problem aloud several times. Encourage them to sketch a part-whole model or a picture if needed.

The youth chamber orchestra has 23 members. Ten of the members are boys. Write an equation to find how many girls are in the orchestra. *possible equations: $23 - 10 = g$; $g + 10 = 23$*

Cal divided some apples into 5 bags. When he was finished dividing the apples, there were 8 apples in each bag. Write an equation to find how many apples he put in all of the bags. *possible equations: $a \div 5 = 8$; $5 \times 8 = a$*

At halftime of the basketball game, one team's score was 48. The team had tripled the score it had at the end of the first quarter. Write an equation to find the score at the end of the first quarter. *possible equations: $3s = 48$; $\frac{48}{3} = s$*

Write an addition and a subtraction equation for each word problem. Use a variable for the unknown part.

- Nine pieces of pizza were left after the family ate 7 pieces. How many pieces of pizza did the family begin with? $9 + 7 = p$; $p - 7 = 9$
- The basketball team won 8 of its 12 basketball games. How many games were lost? $8 + g = 12$; $12 - g = 8$ or $12 - 8 = g$

Evaluate the expression. Let $m = 6$.

- $30 \div m - 3$
 $30 \div 6 - 3 = 5 - 3 = 2$
- $m + 8 \times 3$
 $6 + 8 \times 3 = 6 + 24 = 30$
- $2(m + 1) \div 2$
 $2(6 + 1) \div 2 = 2(7) \div 2 = 14 \div 2 = 7$

Write the algebraic expression for the word phrase.

- 4 times a number divided by 3 $4n \div 3$ or $4(n \div 3)$
- x divided by 8, decreased by 2 $\frac{x}{8} - 2$

Practice & Application

The Hagan twins are planning a deep-sea fishing trip. The cost of the tickets is \$40 for each person. The Hagan twins figured the cost of the fishing trip and the cost of getting to the dock to be \$90.

- Write an equation using c for the unknown cost of getting to the boat dock. Solve for the value of c .
 $\$90 - (2 \times \$40) = c$; $\$90 - \$80 = \$10$; $c = \$10$
- Write an equation to show the cost of the deep-sea fishing trip for the boys if they split the cost evenly. $\$90 \div 2 = \45
- If Mr. Hagan goes with the boys on the trip, what will be the total cost of the trip?
 $(3 \times \$40) + \$10 = \$120 + \$10 = \$130$
Explain the difference between an expression and an equation.

An expression is a mathematical phrase made up of numbers and operation signs but not an equal sign; an equation is a mathematical sentence stating that 2 expressions are equal. Both expressions and equations can use variables.



Lesson 89

Complete **DAILY REVIEW** on page 437.

217

Evaluate and relate expressions using $>$, $<$, or $=$

- Explain that expressions are not always equal in value; therefore, other mathematical symbols such as a greater than sign ($>$) or a less than sign ($<$) are used to make a statement true.
- Write $a + 15 \underline{\hspace{1cm}} 19 - 3$ for display.
► If $a = 2$, what is the value of $a + 15$? 17
► What is the value of $19 - 3$? 16
► What mathematical symbol shows the relationship between the two expressions? How do you know? *the greater than sign; $17 > 16$* Complete the comparison sentence.
- Repeat the procedure for these comparison sentences, using $a = 6$.

$$\frac{48}{a} = 2^3, 8 = 8 \quad a - 2 = \frac{12}{3}, 4 = 4$$

$$11 + 3 < 3a; 14 < 18 \quad 7a + 3 > 6 \times 7; 45 > 42$$

Student Text pp. 216–17

Objectives

- Demonstrate an understanding of the Commutative and Associative Properties of Addition and of Multiplication
- Simplify algebraic expressions using manipulatives
- Apply the Commutative and Associative Properties to simplify algebraic expressions

Teacher Materials

- Variable Cards: 6 n cards; 15 x cards (from the Student Manipulatives Packets)
- Variable Cards, page IA45 (CD) (optional)
- 19 round counters

Student Materials

- Variable Cards: 6 n cards
- 3 round counters

Note

Rather than using x cards from Student Manipulatives Packets, you may choose to prepare x cards using the pattern provided on Variable Cards, page IA45 on the Teacher's Toolkit (CD).

Teach for Understanding

Demonstrate an understanding of the Commutative and the Associative Properties

- Write $85 + 25$ $25 + 85$ for display.
 - **What do you notice about the addends in the expressions?** *The addends are the same, but their order is different.*
 - Choose a student to write the sum below each of the expressions **110** and then write the correct symbol between the expressions $=$. Elicit that $85 + 25$ and $25 + 85$ are *equivalent expressions*; they have the same value.
 - **Did the order of the addends change the sum?** *no*
 - **What property of addition tells you that the order of the addends can be changed without changing the sum?** *the Commutative Property of Addition*
- Write for display $a + b = b + a$.
 - **What is different about the addends in this equation?** *They are letters or variables rather than numbers.*
 - **What property is shown using the variables? How do you know?** *The Commutative Property of Addition; the order of the addends was changed.*
- Write $(25 + 10) + 45$ $25 + (10 + 45)$ for display.
 - **What do the parentheses in the equation tell you?** *Add the numbers within the parentheses first.*
 - Choose a student to write the sum below each of the expressions **80** and then write the correct symbol between the expressions $=$.
 - **Did the grouping of addends change the sum?** *no*
 - **What property tells you that the grouping of addends can be changed without changing the sum?** *the Associative Property of Addition*
- Select a student to write for display an example of the Associative Property of Addition using variables. *possible equation: $(a + b) + c = a + (b + c)$*
- Write $12 \times 10 = 120$ and $10 \times 12 = 120$ for display.
 - **What do you notice about these multiplication equations?** *The factors and the products are the same, but the order of the factors is different.*

- **What property states that the order of the factors can be changed without changing the product?** *the Commutative Property of Multiplication*
- Select a student to write for display an example of the Commutative Property of Multiplication using variables. *possible equation: $a \times b = b \times a$*
 - **What does the Associative Property of Multiplication state?** *The grouping of the factors may be changed without changing the product.*
 - Choose a student to write for display an example of the Associative Property of Multiplication using numbers. Select another student to write an example using variables. *possible equations: $(10 \times 11) \times 5 = 10 \times (11 \times 5)$; $a \times (b \times c) = (a \times b) \times c$*
 - Point out that the Commutative Properties involve changing the *order* of the addends or the factors and the Associative Properties involve *regrouping* the addends or the factors. The properties can be helpful when doing mental calculations.
 - Write the following expressions for display. Guide the students in using the Commutative and the Associative Properties to make sums and products that are multiples of 10. Remind the students that multiples of 10 can be easily added and multiplied. Elicit the value of each expression. (*Note: The order of the addends and the factors may vary.*)

$$(8 + 23) + 2 \quad (8 + 2) + 23 = 10 + 23 = 33$$

$$(8 + 25) + (75 + 20) \quad (75 + 25) + (20 + 8) = 100 + 28 = 128$$

$$(7 \times 5) \times 6 \quad 7 \times (5 \times 6) = 7 \times 30 = 210$$

$$(5 \times 25) \times (2 \times 3) \quad (5 \times 2) \times (25 \times 3) = 10 \times 75 = 750$$

- **Which expression could you determine the value of by applying only one property? Explain.** *$(7 \times 5) \times 6$; elicit that only the Associative Property was used to regroup the factors by moving the parentheses. The Commutative Property (reordering the factors) was not needed to make a multiple of 10.*

Simplify algebraic expressions

- Distribute the n cards and the counters. Write for display $5n + 3 + n$.
- Display an illustration of the expression as shown below. Direct the students to display the illustration on their desks. Explain that the variable n , without a coefficient preceding it, is the same as $1n$.

$$\begin{array}{ccccccc} \boxed{n} & \boxed{n} & \boxed{n} & \boxed{n} & \boxed{n} & + & \bigcirc & \bigcirc & \bigcirc & + & \boxed{n} \\ 5n & & & & & & 3 & & & & n \end{array}$$

- **Which two addends do you think are similar? Why?** *Answers may vary, but elicit that $5n$ and n are similar because they both have the variable n .*
 - Explain that $5n$ and n are *like terms*; they can be combined to make 1 term or addend.
 - **If you added from left to right, would you be adding like terms? Why?** *No; elicit that $5n$ and 3 are beside each other in the expression, so unlike terms would be grouped together.*
 - **How can you reorder the addends so that the like terms are side by side? Why?** *$5n + n + 3$; elicit that the Commutative Property of Addition allows you to reorder the addends without changing the value of the expression.*
- Direct the students to reorder the 3 counters and the one n card. Demonstrate. Write $5n + n + 3$ below the picture.

$$\begin{array}{ccccccc} \boxed{n} & \boxed{n} & \boxed{n} & \boxed{n} & \boxed{n} & + & \boxed{n} & + & \bigcirc & \bigcirc & \bigcirc \\ 5n & & & & & & n & & 3 \end{array}$$

Simplify Expressions

Simplify expressions by combining like terms. The Commutative and Associative Properties of Addition or Multiplication allow you to rewrite the expression to organize the like terms.

simplify expressions like terms

$$\begin{aligned} n + 6 + n &= \\ n + n + 6 &= \\ 2n + 6 & \end{aligned}$$

Commutative Property:
Change the order of addends.

Associative Property:
Group like terms.

$$\begin{aligned} 4 + x + 3 &= \\ x + 4 + 3 &= \\ x + 7 & \end{aligned}$$

Repeated addends are combined using multiplication.

$$\begin{aligned} x + x + x + x &= \\ 4x & \end{aligned}$$

$$\begin{aligned} 2x + 2x + 2x + 2x &= \\ 4(2x) &= \\ 8x & \end{aligned}$$

Exercises

Write the missing number or variable. Name the property used.

- $(p + \underline{\quad}) + r = p + (q + r)$
q; Associative Property
- $4 + 9 = \underline{\quad} + 4$
9; Commutative Property
- $m + n = n + \underline{\quad}$
m; Commutative Property
- $5 \cdot (7 \cdot 4) = (\underline{\quad} \cdot 7) \cdot 4$
5; Associative Property
- $45 + (29 + 80) = (45 + 29) + \underline{\quad}$
80; Associative Property
- $2a + (a + 8) = (2a + \underline{\quad}) + 8$
a; Associative Property
- $(5 + 7x) + 8x = 5 + (7x + \underline{\quad})$
8x; Associative Property
- $3s \times 4 \times 7s = \underline{\quad} \times 3s \times 7s$
4; Commutative Property

Write the simplified expression for the picture.

| | |
|-----|---|
| 9. | $\begin{aligned} 4y + 9 + 5 \\ 4y + 14 \\ y + y + y + y + 14 \end{aligned}$ |
| 10. | $\begin{aligned} 6 + a + 2a + 2 \\ 8 + a + 2a \\ 8 + 3a \end{aligned}$ |
| 11. | $\begin{aligned} 2(2c) + 8 \\ 2c + 2c + 8 \\ c + c + c + c + 8 \end{aligned}$ |
| 12. | $\begin{aligned} x + x + 6 \\ 2x + 6 \\ x + x + 3 + 3 \end{aligned}$ |

Simplify the expression.

- $6 + 4x + 2$ **$4x + 8$**
- $x + 4x$ **$5x$**
- $2x + 3x$ **$5x$**
- $x + 5 + x$ **$2x + 5$**
- $3 + x + 6$ **$x + 9$**
- $x + 2x + 3$ **$3x + 3$**
- $2x + 1 + x$ **$3x + 1$**
- $5x + 2x + 3$ **$7x + 3$**
- $4 \cdot 2 \cdot x$ **$8x$**

Order of terms in the expression may vary.

218

Chapter 10

► How can you regroup the addends to combine addends of like terms? Why? Group $5n$ and n together to make $6n$; the Associative Property of Addition allows you to regroup the addends without changing the value of the expression.

Draw parentheses around $5n + n$.

- Instruct the students to combine the like terms (combine the n cards). Demonstrate. Explain that combining like terms is referred to as *simplifying* the expression. Point out that both the Commutative and the Associative Properties of Addition were used to organize the expression so that you could combine the like terms.

$$\begin{array}{ccccccc} \boxed{n} & \boxed{n} & \boxed{n} & \boxed{n} & \boxed{n} & \boxed{n} & \bigcirc \bigcirc \bigcirc \\ 6n & & & & & & + 3 \end{array}$$

- What is the simplified expression? $6n + 3$ Write $6n + 3$.
- Why do you think the expression is not simplified as $9n$? Elicit that $9n$ represents $9 \times n$ or 9 addends of n and there are only 6 n s in the original expression; therefore, the simplified expression of 6 n s and three counters is correct.

- Choose students to demonstrate simplifying the following expressions, using the x cards. Instruct each student to explain the property or properties he uses as he simplifies the expression. Remind the students that applying the Commutative and the Associative Properties to an expression does not change its value.

$$\begin{aligned} x + 8 + 5x & \mathbf{6x + 8} & 8 + 7x + 2 & \mathbf{7x + 10} \\ 3x + 12 + 6x & \mathbf{9x + 12} & 15 + 3 + 9x & \mathbf{9x + 18} \\ 6x + 10 + x & \mathbf{7x + 10} & & \end{aligned}$$

- Write $(36 + x) + 19$ for display.

► How do you think you can simplify this expression? Elicit that you can use the Commutative Property to change the order

Simplify the expression.

Show each step used to combine the like terms in the expression.

Name the property used in each step. Order of terms in the expression may vary.

- $8x + (2 + 4x)$ **$12x + 2$**
- $3(7x)$ **$21x$**
- $(6 + 5x) + 7$ **$5x + 13$**
- $5 \cdot x$ **$5x$**
- $8 + (2 + x)$ **$x + 10$**
- $(x \cdot 5) \cdot 6$ **$30x$**
- $2 + (3 + x)$ **$x + 5$**
- $x \cdot 8 \cdot 9$ **$72x$**
- $12(2x)$ **$24x$**

Write an equation for the sentence.

- 5 added to y equals 18. **$y + 5 = 18$**
- 12 decreased by 7 is 5. **$12 - 7 = 5$**
- The product of 9 and n is 36. **$9n = 36$**
- A number divided by 3 is 7. **$n \div 3 = 7$**
- 58 more than n is 134. **$n + 58 = 134$**
- The sum of b and 8 equals 25. **$b + 8 = 25$**

Evaluate the expression. Let $y = 4$.

- $40 \div y \times 3$
 $40 \div 4 \times 3 = 10 \times 3 = 30$
- $y + 7 - 6$
 $4 + 7 - 6 = 11 - 6 = 5$
- $(36 - y) \div 4$
 $(36 - 4) \div 4 = 32 \div 4 = 8$
- $2 + y - 3$
 $2 + 4 - 3 = 6 - 3 = 3$

Practice & Application

- Write an addition equation and a multiplication equation for the part-whole model.

| | | | |
|------|-----|-----|-----|
| 15.2 | | | |
| 3.8 | 3.8 | 3.8 | 3.8 |

$$3.8 + 3.8 + 3.8 + 3.8 = 15.2; 4 \times 3.8 = 15.2$$

- The diameter of one of the largest Ferris wheels in the world is 492 feet. What is the length of any radius? **$492 \div 2 = 246$ ft**

Use the equation $x + 45 = \$27$ for problems 43–45.

- Solve to find the value of x . Write the related multiplication equation to solve.
 $x = 45 \times \$27; x = \$1,215$
- If x represents the amount paid to the amusement park for entrance fees for 45 students, what does the quotient represent? **the cost for each student to get into the park**
- Each student paid the entrance fee and took an additional \$5 for gas and \$30 for food and games. How much money did each person take?
 $\$27 + \$5 + \$30 = \62
- Write a word problem showing how Jake spent his food and game money at the amusement park. **Answers will vary.**



The Singapore Flyer is 165 meters tall. The passengers ride in air-conditioned capsules that take about 37 minutes to make one full rotation on the wheel.

Complete **DAILY REVIEW** on page 437.

Lesson 90

219

of the addends in the parentheses and then use the Associative Property to regroup the addends so that 36 and 19 are grouped together.

Choose students to apply the properties and simplify the expression. **possible answers: $(x + 36) + 19$; $x + (36 + 19)$; $x + 55$** (Note: Although the variable is normally written first in a simplified addition or multiplication expression, at this introductory level of instruction, accept as correct answers in which the variable is written in another position [e.g., accept $2x + 6$ written as $6 + 2x$].)

- Write $5(3x)$ for display. Remind the students that multiplication can be indicated as $5 \times 3x$, $5 \cdot 3x$, or by parentheses around the second factor. Select a student to read the expression. **5 times 3x**

Choose a student to illustrate the expression. **5 sets of 3x**

- When you combine 5 sets of $3x$, what is the product? **$15x$**
- Direct attention to $5(3x)$ written for display. Elicit that the Associative Property of Multiplication can be applied to the expression, simplifying it without using manipulatives: $5(3x) = 5 \cdot (3 \cdot x) = (5 \cdot 3) \cdot x = 15 \cdot x = 15x$.
 - Guide the students in simplifying the following expressions, eliciting the property or properties to be applied. Discuss applying the properties as needed.

$$\begin{aligned} x + 8 + 2x + 7 & \mathbf{3x + 15} & (x \cdot 5) \cdot 12 & \mathbf{60x} \\ 8(2x) & \mathbf{16x} & 9x + (11 + 4x) & \mathbf{13x + 11} \end{aligned}$$

Student Text pp. 218–19

Objectives

- Solve addition and subtraction equations using inverse operations
- Check addition and subtraction equations using substitution

Teacher Materials

- Equation Mat, page IA46 (CD)
- Variable Card: 1 n card (or a rectangle from the Shapes Kit to represent the n card)
- 30 round counters

Student Materials

- Equation Mat
- Variable Cards: 1 n card and the x card
- 19 round counters

Note

Rather than using the Equation Mat provided for you on the Teacher's Toolkit CD, you may choose to prepare a large equation mat similar to one provided for the students in the Student Manipulatives Packet, Math 4–6. Use white poster board for the mat and cut circles from colored paper to use as counters. Use the x card from the Teacher Manipulatives Packet or cut a rectangle from colored paper to use as x .

Teach for Understanding

Solve equations using inverse operations

- Write $7 + 5 = 12$ for display.
 ➤ **What math symbol can you write to make the number sentence true? Why? An equal sign; both expressions have a value of 12.** Write the equal sign.
- Display the Equation Mat. Place a set of 7 counters and a set of 5 counters to the left of the equal sign and a set of 12 counters to the right of the equal sign. Remind the students that an equation is like a balanced scale; any operation that is performed on one side of the equation must also be performed on the other side to keep the expressions equal (the equation balanced).
- Remove the 5 counters that are to the left of the equal sign.
 ➤ **Are the expressions still equal? Why? No; 5 counters were removed from only one side of the equation; 7 does not equal 12.** Write $7 + 5 - 5 \neq 12$ for display and then write $7 \neq 12$ below it, aligning the not equal signs.
 ➤ **What must you do to keep the values on both sides of the Equation Mat equal? Elicit that you must also remove 5 of the counters that are to the right of the equal sign.** Remove 5 of the counters from the right of the equal sign.
 ➤ **Are the expressions equal? Why? Yes; the same amount (5 counters) was removed from both sides of the equation; 7 equals 7.** Write $7 + 5 - 5 = 12 - 5$ and then write $7 = 7$ below it, aligning the equal signs. Continue to display the equation.
- Repeat the procedure, using $7 + 5 = 12$. Allow students to remove or add an equal number of counters on both sides of the equal sign. Guide the students in writing equations for the operations that are performed (e.g., remove 7 counters from each side $7 + 5 - 7 = 12 - 7$; $5 = 5$; add 3 counters to both sides $7 + 5 + 3 = 12 + 3$; $15 = 15$).
 Direct attention to $7 + 5 - 5 = 12 - 5$ and $7 + 5 - 7 = 12 - 7$. Elicit that subtracting from both sides of the equation an amount that is equal to one of the addends will cancel out

the value of that addend on one side of the equation, making the value of that expression the other addend (e.g., $7 + 5 - 5 = 7 + 0 = 7$ [Identity Property of Addition]).

- Write for display: *The sum of a number and 5 equals 8.*
 ➤ **What algebraic equation can you write for this sentence? Why? $n + 5 = 8$; elicit that "equals" tells you that an equation is needed, and "sum" tells you to add n and 5.**
- Distribute the Equation Mats, the Variable Cards, and the counters. Write $n + 5 = 8$. Explain that to solve an equation with a variable, you need to find the value of the variable. The value of the variable must make the number sentence true; it is the *solution* to the equation.
 Point out that the goal when solving an equation with an unknown value is to isolate the variable so that it stands alone on one side of the equation, giving you the value of n (e.g., $n = \underline{\quad}$ or $\underline{\quad} = n$).
 ➤ **How can you illustrate $n + 5$ on your Equation Mat? Place 1 n card and 5 counters to the left of the equal sign. the sum 8? Place 8 counters to the right of the equal sign.**
- Direct the students to picture the equation on their mats as you picture it on your mat.
 ➤ **How do you think you can isolate the variable n on your mat? Elicit that you need to remove the 5 counters that are to the left of the equal sign.**
 ➤ **If you remove the 5 counters that are to the left of the equal sign, what must you do to the right of the equal sign? Remove 5 counters.**
 Instruct the students to remove 5 counters from both sides of the equal sign. Demonstrate.
 ➤ **What is the value of n ? $n = 3$**
- Direct attention to $n + 5 = 8$ that was written for display. Explain that to solve the equation you first need to identify the operation that is being performed on the variable.
 ➤ **What operation is being performed on the variable in the equation? Elicit that 5 is being added to n .**
 ➤ **What operation must you perform to isolate n ? Why? Elicit that you must subtract 5 from the left side of the equation because subtraction is the inverse operation of addition.**
 ➤ **If you subtract 5 from the left side of the equation, what must you do to the right side? Subtract 5.**
- Write $n + 5 - 5 = 8 - 5$ below $n + 5 = 8$. Point out that subtracting 5 from the left side of the equation cancels out the addend 5 ($5 - 5 = 0$). The value of the equation does not change because 5 is being subtracted from both sides of the equation. Write $n + 0 = 3$ below $n + 5 - 5 = 8 - 5$.
 ➤ **What does the Identity Property of Addition tell you about adding zero to any number? The sum will be the other addend.**
 ➤ **What is the value of n ? $n = 3$** Write $n = 3$ below $n + 0 = 3$. Explain that by subtracting 5 from both sides of the equation you isolated the variable (n) and determined that its value is 3.
 ➤ **How could you check the solution to this equation? Elicit that you can substitute 3 for the variable (n) in the original equation.**
 ➤ **What does $3 + 5$ equal? 8 Is this solution correct? yes**
- Write $4 + x = 15$ and instruct the students to solve it on their Equation Mat, using the x card and the counters.
 ➤ **What operation is being performed on the variable? Elicit that 4 is being added to x .**

Addition & Subtraction Equations

An equation with a variable is solved by finding the value of the variable. The value must make the sentence true to be called a **solution**.

Solve

Isolate the variable on one side of the equal sign by using the **inverse operation**. Keep the equation **balanced** by performing the exact same operation on both sides of the equation.

Check

Substitute (replace) the variable with the solution and evaluate.

$$n + 6 = 10$$

$$n + 6 - 6 = 10 - 6$$

$$n = 4$$

$$4 + 6 = 10$$

Isolate the variable using the inverse operation.

Check using substitution.

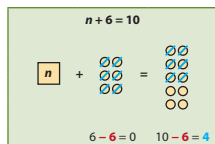
$$n - 3 = 8$$

$$n - 3 + 3 = 8 + 3$$

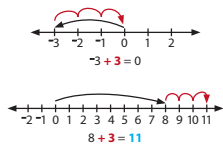
$$n = 11$$

$$11 - 3 = 8$$

The equation mat shows the result of subtracting 6 from both sides of the equation.



The number lines show the result of adding 3 to both sides of the equation.



Exercises

Solve the equation using the inverse operation. Check the solution.

1. $n + 5 = 21$ **$n = 16$**

5. $d + 45 = 90$ **$d = 45$**

9. $d + 43 + 17 = 85$ **$d = 25$**

2. $x + 12 = 40$ **$x = 28$**

6. $16 + f = 35$ **$f = 19$**

10. $s + 14 - 3 + 0.5 = 20$ **$s = 8.5$**

3. $c - 6 = 17$ **$c = 23$**

7. $s - 39 = 61$ **$s = 100$**

11. $3.8 + 16 + b = 29$ **$b = 9.2$**

4. $a - 4 = 36$ **$a = 40$**

8. $24 + n = 100$ **$n = 76$**

12. $\frac{3}{4} + \frac{1}{2} + f = 1\frac{1}{2}$ **$f = \frac{1}{4}$**

Determine whether the given value is the solution to the equation. Write **yes** or **no**. Solve the problems with incorrect values.

13. $a - 8 = 6$ $a = 14$ **yes**

16. $n + 0.8 = 1.7$ $n = 0.8$ **no; $n = 0.9$**

14. $x + 13 = 40$ $x = 17$ **no; $x = 27$**

17. $a - 1\frac{1}{2} = 6\frac{1}{2}$ $a = 8$ **yes**

15. $f - 17 = 9$ $f = 2.6$ **no; $f = 26$**

18. $b + 17 + 3.5 = 40.7$ $b = 20.2$ **yes**

Evaluate the expression. Let $x = 3$.

19. $15 \div x - 4$

$$15 \div 3 - 4 =$$

$$5 - 4 = 1$$

22. $(x + 15) \div 2$

$$(3 + 15) \div 2 =$$

$$18 \div 2 = 9$$

25. $3^2 + 7 - x$

$$9 + 7 - 3 =$$

$$16 - 3 = 13$$

20. $4 + x - 2$

$$4 + 3 - 2 =$$

$$7 - 2 = 5$$

23. $x(2.3)$

$$3(2.3) = 6.9$$

26. $x(2) - 6$

$$3(8) - 6 =$$

$$24 - 6 = 18$$

21. $(18 - x) \div 5$

$$(18 - 3) \div 5 =$$

$$15 \div 5 = 3$$

24. $2x + 3x$

$$2(3) + 3(3) =$$

$$6 + 9 = 15$$

27. $\sqrt{49} + 6x$

$$7 + 6(3) =$$

$$7 + 18 = 25$$

Write an equation for the sentence.

Solve the equation using the inverse operation.

Check the solution.

28. 5 more than n equals 12.

$$n + 5 = 12; n = 7$$

29. 8 less than n is 3.

$$n - 8 = 3; n = 11$$

30. The sum of 10 and a number is 17.

$$10 + n = 17; n = 7$$

31. The difference of a number and 2 equals 5.

$$n - 2 = 5; n = 7$$

32. 6 subtracted from a number is 12.

$$n - 6 = 12; n = 18$$

33. A number increased by 3 equals 16.

$$n + 3 = 16; n = 13$$

34. A number decreased by 7 is 9.

$$n - 7 = 9; n = 16$$

35. 7 and a number equals 10.

$$7 + n = 10; n = 3$$

Practice & Application

36. Find the product of 113 and 609 using only two partial products. **68,817**

37. Write equations to show that 468 is divisible by 2, 3, 4, and 6. **$468 \div 2 = 234$; $468 \div 3 = 156$; $468 \div 4 = 117$; $468 \div 6 = 78$**

38. Explain how you know that 317 is *not* divisible by 5 without dividing.

39. Use front-end estimation to estimate the sum of 398,640 and 954,207.

$390,000 + 950,000 = 1,340,000$

40. What is the value of x in $x + 5 = 14$?

$x + 5 - 5 = 14 - 5$; $x = 9$

41. What is the value of n in $17.3 + 16.8 + n = 35$?

$34.1 - 34.1 + n = 35 - 34.1$; $n = 0.9$

42. What whole number is equivalent to $\frac{5}{3}$? **17**

43. Three triangles represent $\frac{1}{2}$ of a set. How many triangles are in the whole set? **6 triangles**

44. Draw a number line to show the sum for $-9 + 13$.

45. Round each factor to the greatest place to estimate the product of 17.8 and 21.03.

$20 \times 20 = 400$

Explain how the inverse operation helps you find the value of n in $n + 3\frac{1}{2} = 6\frac{3}{4}$. Solve.

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

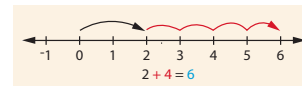
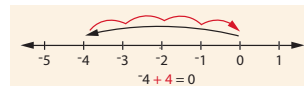
$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

$20 \times 20 = 400$

not change because 4 is being added to both sides of the equation.



Write $x - 0 = 6$ below $x - 4 + 4 = 2 + 4$.

► **What does the Zero Principle of Subtraction state? When you subtract zero from any number, the answer is that number.**

► **What is the value of x ? $x = 6$** Write $x = 6$.

► **Is the solution correct? How do you know? Yes; elicit that when you substitute 6 for the variable (x) in the original equation, the equation is true; $6 - 4 = 2$.**

13. Guide the students in solving these equations on paper without using manipulatives.

$$b + 12 = 18$$

$$j - 7 = 21$$

$$a + 3 + 7 = 12$$

$$c - 5 = 14$$

$$b = 6$$

$$j = 28$$

$$a = 2$$

$$c = 19$$

14. Guide the students in solving the following equations by first simplifying the expression to the left of the equal sign and then applying the inverse operation to find the value of the variable. Point out that solving an equation with decimals or fractions is similar to solving equations with whole numbers.

$$9.4 - 2 + d = 13$$

$$d = 5.6$$

$$t + 4 + 2.7 - 1.9 = 14.6$$

$$t = 9.8$$

$$f + \frac{2}{3} + \frac{5}{6} = 5\frac{1}{2}$$

$$f = 4$$

$$m - 3\frac{3}{4} = 2\frac{1}{4}$$

$$m = 6$$

Student Text pp. 220–21

Objectives

- Solve multiplication equations using division (inverse operation)
- Check multiplication equations using substitution
- Write an equation with a variable to solve a word problem

Teacher Materials

- Equation Mat (from Lesson 91)
- Variable Cards: 5 n cards (or 5 rectangles from the Shapes Kit to represent the n cards)
- Christian Worldview Shaping, pages 25–27 (CD)
- 15 round counters

Student Materials

- Equation Mat
- Variable Cards: 5 n cards
- 15 round counters

Teach for Understanding

Solve multiplication equations using division

- Distribute the Equation Mats, the n cards, and the counters. Display your Equation Mat and write: *The product of n and 3 equals 15.*
 - **What words indicate a mathematical operation or symbol?** Elicit that “product” indicates multiplication (\times) and “equals” indicates an equal sign ($=$). Choose students to write multiplication equations for the sentence. **possible answers:** $3 \times n = 15$; $3 \cdot n = 15$; $n \times 3 = 15$; $n \cdot 3 = 15$; $3n = 15$
 - Direct attention to $3n = 15$. (Write the equation if it was not written by a student.) Remind the students that when the multiplication is written as $3n$, the coefficient (multiplier) is written in front of the variable.
 - **How do you know that $3 \cdot n$, $n \times 3$, and $3n$ all have the same value?** Elicit that they all represent multiplication, and the **Commutative Property of Multiplication** states that the order of the factors can be changed without changing the product.
- Remind the students that an equation is a mathematical sentence that states that two expressions are equal; the expression to the left of the equal sign and the expression to the right of the equal sign have the same value. To solve $3n = 15$, you must isolate the variable (n) on one side of the equal sign to find the number that makes the sentence true when it is substituted for the variable. The value of n is the solution to the equation.
 - **How can you illustrate $3n = 15$ on the Equation Mat?** Elicit that you can place 3 n cards to the left of the equal sign and 15 counters to the right.
- Direct the students to picture the equation on their mats as you picture it on your mat. Remind them that to solve the equation they first need to identify the operation that is being performed on the variable.
 - **What operation is being performed on the variable in the equation?** Elicit that n is being multiplied by 3.
 - **How do you think you can isolate the variable n on your mat?** Why? Elicit that you can divide the 3 n cards by 3; division is the inverse operation of multiplication.

Instruct the students to divide the 3 n cards into 3 groups as you demonstrate.

- **How many n cards are in each of the 3 sets?** 1
- Elicit from the students that an equation is similar to a balanced scale. Any operation that is performed on one side of the equation must also be performed on the other side to keep the equation balanced (i.e., to keep the values on both sides of the equation equal).
 - **Since you divided the n cards to the left of the equal sign by 3, what must you do to the counters to the right of the equal sign?** Divide them by 3 or divide them into 3 equal groups. Direct the students to divide the 15 counters into 3 equal groups as you demonstrate.
 - **How many counters are in each of the 3 groups?** 5
 - Explain that each n card on the mat is equivalent to 1 set of 5 counters, showing that $n = 5$.
 - Direct attention to $3n = 15$ that was written for display and demonstrate solving the equation without using manipulatives: $\frac{3n}{3} = \frac{15}{3}$, $n = 5$. Elicit that the variable (n) is being multiplied by 3; therefore, you must use division (the inverse operation) and divide the value on each side of the equation by 3.
 - **How can you check the solution?** Substitute 5 for the variable (n) in the original equation. Choose a student to demonstrate solving the equation.
 $3(5) = 15$
 - **Is $n = 5$ the correct solution? Why?** Yes; elicit that $3(5) = 15$ is a true mathematical statement.
 - Follow a similar procedure to guide the students in illustrating the following equations on their mats. Direct them to solve the equations and check the solutions on paper.

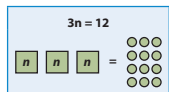
| | |
|-------------------------------|------------------------------|
| $5n = 10$ | $2n = 8$ |
| $\frac{5n}{5} = \frac{10}{5}$ | $\frac{2n}{2} = \frac{8}{2}$ |
| $n = 2$ | $n = 4$ |
 - Explain that an equation with a variable can be written to solve a word problem.

Mrs. Johnson is 3 times as old as her daughter, Kathryn. If Mrs. Johnson is 36 years old, how old is Kathryn? **12 years old**

- **What is the question asking you to find?** Kathryn’s age
 - **What information are you given?** Mrs. Johnson is 3 times as old as Kathryn, and Mrs. Johnson is 36 years old. Write for display these two sentences: Mrs. Johnson is 3 times Kathryn’s age. and Mrs. Johnson is 36 years old.
 - **What word could be written as an equal sign in each of these sentences?** is Erase “is” in each of the sentences and write an equal sign ($=$) in its place: Mrs. Johnson = 3 times Kathryn’s age. and Mrs. Johnson = 36 years old.
 - **Which of these word phrases expresses the same value?** 3 times Kathryn’s age and 36 years old What value do both phrases express? Mrs. Johnson’s age
- Explain that since 3 times Kathryn’s age and 36 years old both express Mrs. Johnson’s age, you can write them as equivalent expressions. Write 3 times Kathryn’s age = 36 years old for display.

Multiplication Equations

Solving Multiplication Equations



1. Identify the operation that has been performed on the variable:
 n is multiplied by **3**. $3n = 3 \cdot n$
2. Apply the inverse operation to isolate the variable:
divide both sides of the equation by **3**. $\frac{3n}{3} = \frac{12}{3}$
3. Check the solution by substituting the value of the variable into the original equation. $n = 4$
 $3 \cdot 4 = 12$

Exercises

Solve. Check the solution. **Answers are shown using cancellation.**

1. $a \times 5 = 30$ **$a = 6$**
2. $2m = 18$ **$m = 9$**
3. $f \cdot 3 = 24$ **$f = 8$**
4. $12 \cdot b = 36$ **$b = 3$**
5. $9p = 54$ **$p = 6$**
6. $8 \times p = 64$ **$p = 8$**
7. $4n = 32$ **$n = 8$**
8. $7h = 56$ **$n = 8$**
9. $c \cdot 7 = 49$ **$c = 7$**
10. $d \times 2^2 = 48$ **$d = 12$**
11. $a \cdot 0.9 = 3.6$ **$a = 4$**
12. $1.8 \times a = 36$ **$a = 20$**

Write an equation for the sentence. Solve.

13. The product of a and 7 equals 21.
 $a = 3$
14. y times 9 is 54.
 $y = 6$
15. b multiplied by 8 equals 48.
 $b = 6$
16. 3 times a number is 36.
 $n = 12$
17. The product of x and 7 equals 35.
 $x = 5$



This is the radio tower of Harbour Light of the Windwards, a Christian radio station that presents the gospel to Carriacou and surrounding islands.

222

Chapter 10

► How could you write “3 times Kathryn’s age” as an algebraic expression? Why? **possible expression: $3k$; Kathryn’s age is unknown and can be written using a variable.**

(Note: Accept any variable, but remind the students that using a variable that is related to the information can be helpful when solving a word problem.)

9. Write $3k = 36$ for display. Direct the students to write the equation on paper, solve it, and then check their solution.

► How old is Kathryn? **12 years old**

► How do you know that 12 years old is the correct solution?

Elicit that when 12 is substituted for k in $3k = 36$, the mathematical statement is true: $3(12) = 36$.

10. Direct the students to solve the following equations. Remind them to identify the operation that is being performed on the variable and then apply the inverse operation. Give guidance as needed.

$$\begin{array}{lll} 6a = 24 & 3 \times c = 21 & 4m = 36 \\ \mathbf{a = 4} & \mathbf{c = 7} & \mathbf{m = 9} \\ 54 = 9b & 6.3 = 0.7 \cdot h & \\ \mathbf{b = 6} & \mathbf{h = 9} & \end{array}$$

11. w Worldview Shaping (CD)

Student Text pp. 222–23

(Note: Assessment available on Teacher’s Toolkit CD.)

Evaluate the expression. Let $a = 4$.

Write a comparison sentence using $>$, $<$, or $=$.

18. $a + 8 > 20 \div 2$ **$12 > 10$**
19. $\frac{32}{a} = a \cdot 2$ **$8 = 8$**
20. $16 - a < 15 - 2$ **$12 < 13$**
21. $9a > 5 \cdot 7$ **$36 > 35$**
22. $\frac{a}{2} < 2 \cdot 2$ **$2 < 4$**
23. $28 \div a < 2^3$ **$7 < 8$**
24. $8 = x + 2$ **$x = 6$**
25. $x - 5 = 8$ **$x = 13$**
26. $b \times 8 = 48$ **$b = 6$**
27. $5m = 40$ **$m = 8$**
28. $16 = 9 + x$ **$x = 7$**
29. $a - 9 = 25$ **$a = 34$**
30. $a + 9 = 20$ **$a = 11$**
31. $6n = 36$ **$n = 6$**
32. $3 \cdot m = 30$ **$m = 10$**
33. $b - 3 = 12$ **$b = 15$**
34. $x + 2 = 25$ **$x = 23$**
35. $24 = 4x$ **$x = 6$**

Practice & Application

36. Write an equation showing that Lyla is 6 years younger than her brother.
 $l = b - 6$
37. Write an equation showing that Liam’s science test score was 6 points more than Zoe’s.
 $l = z + 6$
38. Write an equation to show 58 boys divided evenly among 4 baseball teams. Explain your answer.
39. What terms can be combined in the expression $2s + \frac{1}{4} + \frac{2}{3}$? **$\frac{1}{4}$ and $\frac{2}{3}$**
40. If $a = 4$ and $b = 6$, is $\frac{24}{a} = b$ a true statement? **yes; $\frac{24}{4} = 6$**
41. Write and solve the multiplication equation that can be used to find the value of c in $c \div 2.3 = 17$.
 $c = 17 \times 2.3$; $c = 39.1$
42. Find the product of $\frac{12}{25}$ and $\frac{5}{8}$ in lowest terms.
 $\frac{12}{25} \times \frac{5}{8} = \frac{3}{10}$
Answer is shown using cancellation.
43. Find the least common denominator for $\frac{1}{8}$ and $\frac{1}{12}$. What is the sum of these fractions in lowest terms? **$LCD = 24$; $\frac{3}{24} + \frac{2}{24} = \frac{5}{24}$**
44. Write an equation to show that factors r and s equal 36. **$r \times s = 36$**
45. Write an equation with numbers for problem 44. Explain why 6 cannot be a factor in this problem.
 $9 \times 4 = 36$; 6 cannot represent both r and s in the same equation.

1 A student solved the following equations incorrectly. Find the mistakes and correct them. Explain how to solve the problem correctly.

$$4x = 36$$

$$\frac{4x}{2} = \frac{36}{2}$$

$$2x = 18$$

$$\frac{4x}{4} = \frac{36}{4}$$

$$x = 9$$

Since x is multiplied by 4, both sides of the equation must be divided by 4.

$$8m = 24$$

$$\frac{8m}{8} = \frac{24}{8}$$

$$m = 6$$

$$\frac{8m}{8} = \frac{24}{8}$$

$$m = 3$$

Since m is multiplied by 8, both sides of the equation must be divided by 8.

38. **58 is not evenly divisible by 4; there is a remainder of 2. All 4 teams will have 14 boys. Two of the teams will have an extra player.**

Complete **DAILY REVIEW** on page 438.

Lesson 92

223

Objectives

- Solve multiplication and division equations using inverse operations
- Check multiplication and division equations using substitution
- Write an equation with a variable to solve a word problem
- Demonstrate an understanding of inequalities
- Picture an inequality on a number line
- Determine whether a given number is a solution to an inequality

Teach for Understanding

Solve equations using inverse operations

- Write for display: *The quotient of a number and 4 equals 12.*
 - What is an equation? a mathematical statement in which two expressions are equal**
 - Can the sentence be written as an equation? How do you know? Yes; elicit that “equals” indicates an equal sign and “quotient” indicates that division is being performed.**
 - What are the two equal expressions in the sentence? the quotient of a number and 4; 12**
 - How do you represent an unknown number in an equation? Use a variable to represent the unknown value.**
- Choose a student to write the equation for display. Select another student to write the equation in fraction form.
 $x \div 4 = 12$; $\frac{x}{4} = 12$
 - What operation is being performed on the variable? x is being divided by 4.**
 - How can you find the value of x? Elicit that since multiplication is the inverse operation of division, you can multiply both sides of the equation by 4 to isolate the variable.**
 - Why must you multiply both sides of the equation by 4? to keep the values on both sides of the equation equal (keep the equation balanced)**
- Explain each step as you demonstrate solving both equations as shown. For the equation written in fraction form, explain that $\frac{4x}{4}$ is the same as $(\frac{4}{4})x$, $1x$, or x . (Note: You may choose to remind the students that the whole number 4 can be written as the improper fraction $\frac{4}{1}$ [$\frac{4}{1} \cdot \frac{4}{4} = 12 \cdot 4$] and then demonstrate solving the equation.)

$$x \div 4 = 12$$

$$x \div \cancel{4} \times \cancel{4} = 12 \times 4$$

$$x = 48$$

$$\frac{x}{4} = 12$$

$$\frac{x}{\cancel{4}} \cdot \frac{4}{4} = 12 \cdot 4 \text{ (or } \frac{4x}{1} = 12 \cdot 4)$$

$$x = 48$$

 - Is $x = 48$ the correct solution? How do you know? Yes; elicit that when 48 is substituted for x in the original equation, the mathematical statement is true; $48 \div 4 = 12$ and $\frac{48}{4} = 12$.**

Point out that writing each step in the solution helps you to accurately solve an equation and makes it easier to identify possible errors.
- Follow a similar procedure for the following equations. Point out that solving equations with fractions or decimals is similar to solving equations with whole numbers. While solving $f \cdot \frac{3}{4} = 7$, discuss with the students that they can use what they know about division (any number divided by itself is 1; $\frac{3}{4} \div \frac{3}{4} = 1$) or multiply by the reciprocal ($\frac{3}{4} \cdot \frac{4}{3} = 1$).

$$\begin{aligned} f \cdot \frac{3}{4} &= 7 \\ f \cdot \cancel{\frac{3}{4}} \div \cancel{\frac{3}{4}} &= 7 \div \frac{3}{4} \\ f &= 7 \cdot \frac{4}{3} \\ f &= \frac{28}{3} = 9\frac{1}{3} \end{aligned}$$

$$\begin{aligned} f \cdot \frac{3}{4} &= 7 \\ f \cdot \cancel{\frac{3}{4}} \div \cancel{\frac{3}{4}} &= 7 \div \frac{3}{4} \\ f \cdot \frac{4}{\cancel{3}} \cdot \frac{1}{\cancel{4}} &= 7 \cdot \frac{4}{3} \\ f &= \frac{28}{3} = 9\frac{1}{3} \end{aligned}$$

$$\begin{aligned} d \div 0.4 &= 23.2 \\ d \div \cancel{0.4} \cdot \cancel{0.4} &= 23.2 \cdot 0.4 \\ d &= 23.2 \cdot 0.4 \\ d &= 9.28 \end{aligned}$$

$$\begin{aligned} d \div 0.4 &= 23.2 \\ \frac{d}{\cancel{0.4}} \cdot \cancel{0.4} &= 23.2 \cdot 0.4 \\ d &= 9.28 \end{aligned}$$

- Write the following equations for display and direct the students to solve the problems. Remind them to first identify the operation that is being performed on the variable and then apply the inverse operation. Give guidance as needed.

$$\begin{aligned} \frac{x}{6} &= 7 & n \div 5 &= 4 & 3y &= 2.7 \\ x &= 42 & n &= 20 & y &= 0.9 \end{aligned}$$

$$\begin{aligned} 7 &= t \cdot \frac{2}{5} & p \times 8 &= 32 & a \div \frac{4}{5} &= 10 \\ t &= 17\frac{1}{2} & p &= 4 & a &= 8 \end{aligned}$$

Nicole and Jane are making a photo album of the 165 vacation pictures they took. Nicole took twice as many pictures as Jane. How many vacation pictures did Nicole and Jane each take?

- Direct the students to write what they need to find and the given information. Reread the problem as needed. **The number of pictures Nicole and Jane each took; 165 pictures taken, and Nicole took twice as many as Jane.**
 - What word sentence can you write to find the total number of pictures taken? Elicit this sentence: The number of pictures Nicole took and the number of pictures Jane took equal 165 pictures.** Write the sentence for display.
 - What information do you need in order to find how many pictures Nicole took? Elicit that you need the number of pictures that Jane took.**
 - How can you represent the unknown number of pictures Jane took? Why? Elicit that you can use the variable j; a variable is used to represent an unknown value, and j is the first letter in Jane's name.**

Write $j = \text{number of pictures Jane took}$ for display. Point out that although an uppercase j begins Jane's name, variables are written using a lowercase letter.
 - If j represents the number of pictures Jane took, what algebraic expression could represent the number of pictures Nicole took? Why? 2j; Nicole took twice as many pictures as Jane.**
- Write $2j = \text{number of pictures Nicole took}$ for display.
- Guide the students in writing on paper a mathematical equation for the word sentence: $2j + j = 165$.
 - How can you simplify the equation? 3j = 165**
 - Choose a student to solve the equation and to find the number of pictures that Nicole took while the other students solve the problems on paper. **j = 55, Jane took 55 pictures; and 2j = 110, Nicole took 110 pictures.**
 - How can you check the answers to the word problem? Substitute 55 for the variables (j) in the original equation.** Elicit that $(2 \times 55) + 55 = 165$; therefore, $j = 55$ pictures and $2j = 110$ pictures are correct answers.

Multiplication & Division Equations

Multiplication and division are inverse operations.

$$36 \div 4 \times 4 = 36$$

$$36 \times 4 \div 4 = 36$$

$$8 \cdot 2 \div 2 = 8$$

$$8 \div 2 \cdot 2 = 8$$

inequality

Isolate the variable on one side of the equal sign using the inverse operation. An equation is much like a balanced scale: when you perform an operation on the left side of the equation, you must perform the exact same operation on the right side of the equation.



The inverse of multiplying by 5 is dividing by 5.

$$5x = 35$$

$$\frac{5x}{5} = \frac{35}{5}$$

$$x = 7$$

$$5 \cdot 7 = 35$$

The inverse of dividing by 8 is multiplying by 8.

$$n \div 8 = 4$$

$$\frac{n}{8} \cdot 8 = 4 \cdot 8$$

$$n = 32$$

$$\frac{32}{8} = 4$$

Exercises

Solve. Check the solution.

1. $n \div 7 = 63$ **$n = 441$**

5. $\frac{m}{4} = 3$ **$m = 12$**

9. $x + 8 = 14.5$ **$x = 116$**

2. $3 \cdot x = 15$ **$x = 5$**

6. $n \cdot \frac{1}{5} = 9$ **$n = 45$**

10. $r \div 0.3 = 57.5$ **$r = 17.25$**

3. $\frac{2}{3} \cdot x = 15$ **$x = 22\frac{1}{2}$**

7. $\frac{5}{9} = 18$ **$x = 162$**

11. $4 \cdot c = 32$ **$c = 8$**

4. $\frac{1}{2}p = 45$ **$p = 90$**

8. $3n = 78.12$ **$n = 26.04$**

12. $p \cdot \frac{3}{5} = 5$ **$p = 8\frac{1}{3}$**

Write an equation for the sentence.

Solve the equation using the inverse operation. Check the solution.

13. The quotient of a number divided by 12 is 3.
 $n \div 12 = 3$ or $\frac{n}{12} = 3$; $n = 36$

17. The quotient of y and 7 equals 2.
 $y \div 7 = 2$ or $\frac{y}{7} = 2$; $y = 14$

14. The product of x and 7 is 35.
 $x \cdot 7 = 35$ or $7x = 35$; $x = 5$

18. A number divided by 20 equals 3.
 $n \div 20 = 3$ or $\frac{n}{20} = 3$; $n = 60$

15. A number divided by 5 equals 11.
 $n \div 5 = 11$ or $\frac{n}{5} = 11$; $n = 55$

19. Twice a number is 18.
 $2n = 18$ or $2 \cdot n = 18$; $n = 9$

16. 6 times a number is 54.
 $6n = 54$ or $6 \cdot n = 54$; $n = 9$

20. The product of a number and 5 is 10.
 $n \cdot 5 = 10$ or $5n = 10$; $n = 2$

Complete the table.

| x | $2x + 3$ |
|-----|-----------|
| 11 | 25 |
| 13 | 29 |
| 15 | 33 |
| 17 | 37 |

| x | $x^2 + 5$ |
|-----|------------|
| 8 | 69 |
| 10 | 105 |
| 12 | 149 |
| 14 | 201 |

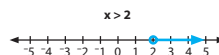
| x | $4(x)$ |
|-----|------------|
| 25 | 100 |
| 35 | 140 |
| 45 | 180 |
| 55 | 220 |

224

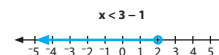
Chapter 10

An **inequality** is a mathematical sentence in which two expressions are not equal. The **greater than** ($>$) and **less than** ($<$) symbols can be used to express the inequality. A number line shows all solutions for the inequality.

The open circle on the number line indicates that the circled number is not included in the solution.



The number line shows that x is any value greater than 2.



The number line shows that x is any value less than 3.

Exercises

Draw a number line to illustrate the inequality. **Figures may vary.**

24. $x < 3$

25. $b > -2$

26. $y > 15 + 3$

27. $c < 5 - 4$

Determine whether the given value is a solution to the inequality. Write **yes** or **no**.

28. $x < 12$ if $x = 10$ **yes**

29. $y > 3$ if $y = 2$ **no**

30. $n < 2$ if $n = -1$ **no**

31. $b > 10$ if $b = 3^2$ **no**

32. $m < 5$ if $m = \frac{19}{4}$ **yes**

33. $s > 7$ if $s = \sqrt{9}$ **no**

Practice & Application Equations may vary.

34. There will be 23 people at Josiah and Jacob's birthday party. How many packages of hot dogs will need to be purchased if 2 hot dogs are cooked for each person at the party? (There are 10 hot dogs in each package.) **$(23 \times 2) \div 10 = 46 \div 10 = 4.6$; 5 packages**

Write an equation for the water balloon purchase. Use the variable t for the unknown amount.

Solve to find the amount of tax paid.
2 packages of balloons: \$2.94 each
2 balloon launchers: \$11.99 each
Shipping: \$5.00
Tax: —
Total: \$36.65

35. There are 8 hot dog buns in each package. How many packages of buns will need to be purchased for the number of hot dogs cooked? **$46 \div 8 = 5.75$; 6 packages**

$$(2 \times \$2.94) + (2 \times \$11.99) + \$5.00 + t = \$36.65$$

$$\$5.88 + \$23.98 + \$5.00 + t = \$36.65$$

$$\$34.86 + t = \$36.65$$

$$\$36.65 - \$34.86 = \$1.79$$

$$t = \$1.79$$

36. Two ice-cream cakes were made for the party. Each cake was cut into 12 equal pieces. There were 8 pieces of cake left at the end of the day. What fraction of the cake was left?
 $2 \times 12 = 24$; $\frac{8}{24} = \frac{1}{3}$ of the cake

37. Four hundred water balloons were filled. Jacob figured that each guest that was not an adult would have 26 balloons to throw. How many extra water balloons were there? (There were 8 adults at the party.) **$23 - 8 = 15$ guests; $400 - (15 \times 26) = 400 - 390 = 10$ extra water balloons**



Complete **DAILY REVIEW** on page 439.

Lesson 93

225

Demonstrate an understanding of inequalities

1. Write for display the word **inequality** and the symbols $>$ and $<$. Explain that an inequality is a mathematical sentence in which two expressions are not equal; it can be expressed using a greater than ($>$) or a less than ($<$) symbol. Point out that inequalities have more than one solution.

2. Write $x > 3$. Elicit fraction, decimal, and whole-number solutions such as $3\frac{1}{2}$, 4.2, and 75.

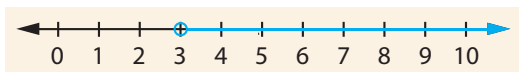
Explain that there are an infinite number of values greater than 3; therefore, it is impossible to name all of the values for x that will make the number sentence true. The best way to show the solutions for an inequality is to graph them on a number line.

3. Draw for display a number line for 0–10.

► **How do you show $x = 3$ on a number line? Plot a point at 3.**

► **Where on the number line are numbers greater than 3 located? to the right of 3**

Draw an open (unshaded) circle to mark 3 on the number line and shade the part of the number line to the right of 3. Explain that since the number sentence tells you that x is greater than 3, x is not equal to 3; therefore, 3 is not a solution to the inequality. The open circle shows that 3 is not included in the solution. The shaded part of the number line shows that any value greater than 3 is included in the solution, whether the value is slightly or significantly greater than 3.



4. Guide the students in identifying which of the following values are included in the set of solutions for $x > 3$. (**Note:** The magenta numbers indicate the values that are included in the set of solutions.)

6
3.00001
2.999999

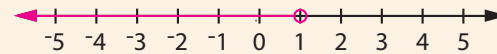
$\frac{7}{1}$
 $\sqrt{25}$

$\frac{12}{4}$
 $\frac{72}{8}$

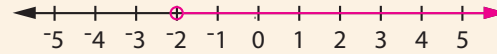
3.0
 $\sqrt{9}$
3.3

5. Draw 3 number lines for -5 – 5 . Choose students to graph the solutions for the following inequalities.

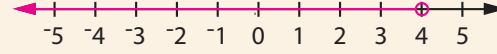
$y < 1$



$x > -2$



$n < 4$



6. Write the following inequalities for display. Choose students to tell whether the given value is a solution to the inequality and have them explain their answer.

$x > 7$ if $x = 6$ **no; 6 is less than 7**

$y < 4$ if $y = 2$ **yes; 2 is less than 4**

$n > -1$ if $n = -3$ **no; -3 is less than -1**

$x < 7$ if $x = 2^3$ **no; 2^3 equals 8, and 8 is greater than 7**

Student Text pp. 224–25

Objectives

- Demonstrate an understanding of the Distributive Property of Multiplication over Addition
- Apply the Distributive Property of Multiplication over Addition to find equivalent expressions
- Solve equations using inverse operations

Teacher Materials

- Place Value Kit: ones

Teach for Understanding

Demonstrate an understanding of the Distributive Property

- Write for display $a + b = b + a$ and $x \cdot y = y \cdot x$. Explain that properties are important when studying algebraic expressions and equations.
 - **What properties do these equations represent? the Commutative Property of Addition and the Commutative Property of Multiplication**
 - **What does the Commutative Property of Addition allow you to do? change the order of the addends without changing the sum the Commutative Property of Multiplication? change the order of the factors without changing the product**
- Write for display $(a + b) + c = a + (b + c)$ and $(a \times b) \times c = a \times (b \times c)$.
 - **What properties do these equations represent? the Associative Property of Addition and the Associative Property of Multiplication**
 - **What does the Associative Property of Addition allow you to do? change the grouping of the addends without changing the sum the Associative Property of Multiplication? change the grouping of the factors without changing the product**
- Write $(5 + 2x) + 3$ for display.
 - **How can you apply the Commutative and the Associative Properties of Addition to simplify the expression? Elicit that you can apply the Commutative Property to change the order of the addends within the parentheses: $(2x + 5) + 3$. Then you can apply the Associative Property to change the grouping of the addends: $2x + (5 + 3)$. Write the equations as they are given.**
 - **What is the expression in simplified form? $2x + 8$**
- Write for display *Distributive Property of Multiplication over Addition* and $a(b + c) = ab + ac$. Remind the students that the Distributive Property involves both multiplication and addition. Explain that this property is also important when studying algebraic expressions and equations.
 - **What is different about the expression to the right of the equal sign? Elicit that each addend is being multiplied by the multiplier, a .**
Write $2(3 + 5) = \underline{\hspace{2cm}}$ for display. Direct the students to apply the Distributive Property and to write an equivalent expression. $(2 \times 3) + (2 \times 5)$ Write the expression in the blank.
- Select a student to use ones from the Place Value Kit to picture 2 sets of 8 below the left side of the equation. Choose another student to picture 2 sets of 3 and 2 sets of 5 below the right side of the equation.
Lead a discussion about how the pictures are similar and how they are different.

- **How do you know that the expressions are equivalent? Elicit that both expressions have a value of 16.** Write an equal sign between the two pictures.
- Write $4(3 + x) =$ for display.
 - **How can you apply the Distributive Property to write an equivalent expression? Elicit that you can multiply each addend by the multiplier, 4.**
Choose a student to apply the Distributive Property and to write the equivalent expression below $4(3 + x) = (4 \cdot 3) + (4 \cdot x)$ Write an equal sign after the expression.
 - **How can you simplify $(4 \cdot 3) + (4 \cdot x)$ to write an equivalent expression? Elicit that you can multiply $4 \cdot 3$ and multiply $4 \cdot x$; then add the products.**
Select a student to simplify the expression and to write the equivalent expression below $(4 \times 3) + (4 \times x) = 12 + 4x$
Elicit that because of the Commutative Property, $12 + 4x$ can also be written as $4x + 12$.
 - Write $4b + 24$ for display. Explain that you can also find an equivalent expression by factoring a common factor out of each term. Point out that factoring out the greatest common factor (GCF) will usually make it easier to find an equivalent expression.
 - **What are the common factors of 4 and 24? 2 and 4 What is the greatest common factor? 4**
 - **What times 4 equals 4b? 1b**
 - **What number times 4 equals 24? 6**
Write $4(b + 6)$ to the right of $4b + 24$, leaving a space between the 2 expressions. Explain that when a common factor is factored out of the addends, it is written outside parentheses to show that it is now a factor in the expression. Remind the students that when the coefficient of a variable is 1, the 1 does not need to be written (Identity Property of Multiplication).
 - **Is $4(b + 6)$ equal to $4b + 24$? How do you know? Yes; elicit that b multiplied by 4 equals 4b, and 6 multiplied by 4 equals 24.** Complete the equation: $4b + 24 = 4(b + 6)$.
Select a student to simplify $4(b + 6)$, proving that the expressions are equivalent.
 - Write the following expressions for display. Instruct the students to apply the Distributive Property and to write the equivalent expressions. Encourage them to use the GCF when factoring out common factors. Give guidance as needed.

| | | | | | |
|------------|------------|------------|-------------|------------|-------------|
| $3(x + 2)$ | $3x + 6$ | $5(4 + y)$ | $20 + 5y$ | $4(n + 1)$ | $4n + 4$ |
| $18 + 6a$ | $6(3 + a)$ | $14x + 21$ | $7(2x + 3)$ | $9x + 24$ | $3(3x + 8)$ |
 - Write $a + a + a$ for display.
 - **How could you write the repeated addends using multiplication? Why? Elicit $3a$; the coefficient 3 tells how many times the variable a is multiplied (added repeatedly).**
 - Write $x \cdot x \cdot x$ for display.
 - **How could you write the repeated factors using exponents? Why? x^3 ; the exponent tells how many times the factors are repeated.**
 - Write the following expressions and direct the students to write the equivalent expressions.

| | | |
|-----------------|------|---------------------|
| $y + y + y + y$ | $4y$ | $x \cdot x \cdot x$ |
|-----------------|------|---------------------|

Equivalent Expressions

The Associative and Commutative Properties of Addition or Multiplication are used to simplify expressions by reordering and regrouping like terms.

$$\begin{aligned} 4 + (n + 5) &= \\ 4 + (5 + n) &= \text{Commutative Property: order was changed} \\ (4 + 5) + n &= \text{Associative Property: grouped differently} \\ 9 + n &= \end{aligned}$$

Addition & Multiplication Properties

$$\begin{aligned} 6 \cdot (n \cdot 9) &= \\ 6 \cdot (9 \cdot n) &= \\ (6 \cdot 9) \cdot n &= \\ 54n &= \end{aligned}$$

The Distributive Property of Multiplication over Addition can also be used to simplify expressions. Or it can be used to find an equivalent expression by finding a common factor in the terms of the equation.

Simplify

$$\begin{aligned} 3(2n + 4) &= \\ 3(2n) + 3(4) &= \\ 6n + 12 &= \end{aligned}$$

There are 3 sets of each addend.
Multiply each addend by the multiplier, 3.
Write the simplified expression.

Equivalent Expression

$$9 + 15b$$

3 is a common factor of 9 and 15.

$$9 + 15b = 3(3 + 5b)$$

Exercises

Apply the Distributive Property to write an equivalent expression.

$$\begin{aligned} 1. 9(2 + x) &= 18 + 9x \\ 2. 5(n + 21) &= 5n + 105 \\ 3. 7(3 + 4y) &= 21 + 28y \\ 4. 2(5x + 3.5) &= 10x + 7 \\ 5. 16(5 + 3a) &= 80 + 48a \\ 6. 3(3x + 0.8) &= 9x + 2.4 \\ 7. 4(y + \frac{1}{8}) &= 4y + \frac{1}{2} \\ 8. 6(n + \frac{1}{4}) &= 6n + 1\frac{1}{2} \end{aligned}$$

Choose the expression with an equivalent value.

$$\begin{aligned} 9. 6x + 18 &= 3(2x + 6) \\ 10. 4(7 + 3a) &= 28 + 12a \\ 11. 12(y + 3) &= 12y + 36 \end{aligned}$$

$$\begin{aligned} 18. 156 \div 12 &= 13 \\ 156 \div 12 &= 13 \end{aligned}$$

Choose the inverse operation that would be used to solve the equation.

$$\begin{aligned} 12. 5 + y = 25 & \text{ subtraction} \\ 13. \frac{x}{7} = 7 & \text{ multiplication} \\ 14. 3y = 48 & \text{ division} \end{aligned}$$

$$19. \frac{1}{8}(50) = 6.25$$

Determine whether the given value is the solution to the equation. Steps to solve may vary. Write yes or no.

$$\begin{aligned} 15. r - 42 = 59 \text{ if } r = 17 & \text{ no;} \\ 17 - 42 &= 59 \\ 18. 156 \div h = 12 \text{ if } h = 12 & \text{ no} \\ 16. 3.5 + w = 17.9 \text{ if } w = 14.4 & \text{ yes;} \\ 3.5 + 14.4 &= 17.9 \\ 19. \frac{1}{8}x = 6 \text{ if } x = 50 & \text{ no} \\ 20. \frac{4}{3} = 13 \text{ if } x = 39 & \text{ yes;} \\ \frac{4}{3} &= 13 \end{aligned}$$

226

Chapter 10

Solve. Check the solution.

$$\begin{aligned} 21. a + 5 = 33 & \text{ a = 28} \\ 22. x - 1.2 = 10 & \text{ x = 11.2} \\ 23. \frac{a}{12} = 3 & \text{ a = 36} \\ 24. 8x = 1 & \text{ x = } \frac{1}{8} \\ 25. n - 16 = 140 & \text{ n = 156} \\ 26. 8x = 480 & \text{ x = 60} \\ 27. y - 43 = 129 & \text{ y = 172} \\ 28. \frac{3}{4}x = 6 & \text{ x = 8} \\ 29. 2x = 14.8 & \text{ x = 7.4} \\ 30. x - 6 = 1.4 & \text{ x = 7.4} \\ 31. 1.5w = 30 & \text{ w = 20} \\ 32. 3.8p = 64.6 & \text{ p = 17} \\ 33. \frac{x}{9} = 4 & \text{ x = 36} \\ 34. a + 1.7 = 1.9 & \text{ a = 0.2} \\ 35. x \div 12 = 62 & \text{ x = 744} \end{aligned}$$

Determine whether the given value is a solution to the inequality. Write yes or no.

$$\begin{aligned} 36. y > 3 \text{ if } y = 2 & \text{ no} \\ 37. x < 2 \text{ if } x = -1 & \text{ yes} \\ 38. a > 7 \text{ if } a = 3^2 & \text{ yes} \end{aligned}$$

Solve. Equations may vary.

39. How many pizzas will be ordered for the Sunday school picnic if each pizza is cut into 8 slices and 200 slices are needed? **25 pizzas**

40. On Monday Forrest found 9 more insects for his science project. Now he can make a display of all 20 insects. How many insects did he have before Monday? **11 insects**

Practice & Application

41. Simplify the expression $(2n + 4) + (3n + 6)$.

$$2n + 3n + 4 + 6 = 5n + 10$$

42. What operation will always undo addition?

subtraction

43. What is the product of $3n$ if $n = 36$?

$$3 \cdot 36 = 108$$

44. If $\frac{1}{2}n = 118$, what is the value of n ?

$$n = 236$$

45. Write an expression for a number plus 3, multiplied by 6. **$(n + 3) \times 6$**

$$46. y < 2$$

47. Write true or false: $3(8 + 12) = 24 + 36$. **true;**
 $3(8) + 3(12) = 24 + 36$

48. Which expression has a greater value:

$$30 \times \frac{1}{5} \text{ or } \frac{1}{3} \times 39? \quad 30 \times \frac{1}{5} = 6; \quad \frac{1}{3} \times 39 = 13; \quad \frac{1}{3} \times 39 \text{ has a greater value.}$$

Identify the steps shown as the Associative, the Commutative, and/or the Distributive Property. Simplify the expression.

$$\begin{aligned} 3x + 4 + 2x \\ 3x + 2x + 4 \\ 5x + 4 \end{aligned}$$

Commutative

$$\begin{aligned} n + (3 + 5n) \\ n + (5n + 3) \\ (n + 5n) + 3 \\ 6n + 3 \end{aligned}$$

Commutative

Associative

$$\begin{aligned} 7(y + 3y) \\ (7 \cdot y) + (7 \cdot 3y) \\ 7y + 21y \\ 28y \end{aligned}$$

Distributive

Complete **DAILY REVIEW** on page 439.

Lesson 94

227

Solve equations using inverse operations

1. Write $m - 3 = 17$ for display.

► **How can you solve the equation? Elicit that since 3 is being subtracted from m, you can apply the inverse operation and add 3 to both sides of the equation.**

► **Why is it necessary to apply the inverse operation to both sides of the equation? Elicit that you apply the inverse operation to both sides of the equation to keep the expressions equal or balanced while isolating the variable.**

2. Choose a student to demonstrate solving the equation.

Remind the students that the value of the variable (m) is the solution to the equation. **$m - 3 + 3 = 17 + 3$; $m = 20$**

► **Is 20 the correct solution? How do you know? Yes; elicit that when 20 is substituted for m in the original equation ($m - 3 = 17$), the mathematical statement is true.**

3. Write the following equations for display and direct the students to solve them, showing their work for each solution. Select students to write their solutions for display and to explain them.

$$\begin{aligned} 9 + a &= 12 \\ a + 9 - 9 &= 12 - 9 \\ a &= 3 \end{aligned}$$

$$\begin{aligned} y - 5 &= 13 \\ y - 5 + 5 &= 13 + 5 \\ y &= 18 \end{aligned}$$

$$\begin{aligned} 5x &= 35 \\ \frac{5x}{5} &= \frac{35}{5} \\ x &= 7 \end{aligned}$$

$$\begin{aligned} n \div 4 &= 8 \\ n \div 4 \times 4 &= 8 \times 4 \\ n &= 32 \end{aligned}$$

$$\begin{aligned} 6 \cdot x &= 48 \\ 6 \cdot \frac{x}{6} &= \frac{48}{6} \\ x &= 8 \end{aligned}$$

$$\begin{aligned} \frac{y}{7} &= 3 \\ \frac{y}{7} \cdot 7 &= 3 \cdot 7 \\ y &= 21 \end{aligned}$$

Student Text pp. 226–27

Lesson 95

Student Text pp. 228–29

Daily Review p. 440h

Objectives

- Calculate the distance traveled given the rate and the time, the rate of travel given the distance and the time, and the time traveled given the distance and the rate
- Complete a table using the formula $d = r \times t$
- Create a line graph relating to the formula $d = r \times t$

Teacher Materials

- Graph an Equation, page IA47 (CD)

Student Materials

- Graph an Equation, page IA47 (CD)

Teach for Understanding

Calculate distance, rate, and time

1. Write the words *distance*, *rate*, and *time* for display. Point out that rate often refers to speed.

- **How would you define these words?** *Elicit that distance is how far someone travels, rate is how fast someone travels, and time is how long it takes to travel a distance.*

Write for display $\text{distance} = \text{rate} \times \text{time}$. Explain that a formula is a method of calculating information in a way that works every time. Formulas are often written using mathematical symbols and variables, allowing you to substitute known information into an equation to find the unknown information.

Write $d = r \times t$ below $\text{distance} = \text{rate} \times \text{time}$. Point out that d represents distance, r represents rate, and t represents time.

- **How can you find the distance if you are given the rate and time?** *Multiply the rate times the time.*

If a car travels at a rate of 55 mph for 2 hours, how far will it travel? **110 miles**

- **How can you find how far the car will travel?** *Elicit that the distance can be found by multiplying the rate (speed) times the time, using the formula $d = r \times t$.*

- **What equation can you write?** $d = 55 \times 2$

Write the equation for display and choose a student to solve it without labeling the answer. $d = 110$

- **What label should you write for the answer? Why?** *Elicit that the label should be miles because "how far" indicates a distance, and the rate is given in miles per hour. Label the answer: $d = 110$ miles.*

- **What distance will the car travel?** **110 miles**

2. Follow a similar procedure for the following word problems. Point out that the word *speed* and the phrase *88 kilometers per hour* both represent rate.

How far will a car travel in 3 hours if its average speed is 65 mph? $d = 65 \times 3$; $d = 195$ miles

A car traveled 88 kilometers per hour for 2 hours. How far did it travel? $d = 88 \times 2$; $d = 176$ kilometers

3. Explain that you can also use the formula $d = r \times t$ to find the time it takes to travel a given distance when given the rate (speed).

The distance that Joseph, Mary, and the young child, Jesus, traveled from Bethlehem to Egypt may have been about 400 miles. If they had traveled 20 miles per day, how long would it have taken them to travel from Bethlehem to Egypt? **20 days**

4. Call attention to the formula $d = r \times t$.

- **What is the possible distance that Joseph, Mary, and Jesus traveled?** **400 miles** Write $d = 400$ miles for display.

- **At what rate would Joseph, Mary, and Jesus have traveled?** **20 miles per day** Write $r = 20$ miles per day.

- **What is the word problem asking you to find?** **how long it may have taken Joseph, Mary, and Jesus to travel from Bethlehem to Egypt**

- **Which variable in the formula will also be the variable representing the unknown value in the equation for solving the word problem? Why?** **t ; elicit that, in the formula, t represents time, and you need to find the amount of time it may have taken for Joseph, Mary, and Jesus to travel from Bethlehem to Egypt.**

5. Select a student to rewrite the formula substituting the known information for d (distance) and r (rate). $400 = 20 \times t$, $400 = 20 \cdot t$, or $400 = 20t$

- **How can you solve this equation?** *Elicit that since division is the inverse operation of multiplication, you can divide both sides of the equation by 20 to isolate the variable (t).*

Choose a student to solve the equation without labeling the answer. $\frac{400}{20} = \frac{20t}{20}$; $t = 20$

- **What label should you write for the answer? Why?** *Elicit that the label should be days because "how long did it take" indicates an amount of time and the rate is given in "miles per day." Label the answer: $t = 20$ days.*

- **How long would it have taken for Joseph, Mary, and Jesus to travel from Bethlehem to Egypt?** **20 days**

6. Point out that you can also use the formula $d = r \times t$ to find the rate at which someone or something traveled when given the distance and the time.

Henri Giffard built an airship in 1852. If the airship flew 17 miles in 3.4 hours, how many miles an hour did it travel? **5 mph**

- **What information is given in this problem?** *Elicit that the distance traveled was 17 miles, and the time it took to travel the distance was 3.4 hours.*

7. Direct the students to rewrite $d = r \times t$, substituting the given information for the appropriate variables.

- **What equation did you write?** $17 = r \times 3.4$ or $17 = 3.4r$

Write $17 = 3.4r$ for display. Point out that the Commutative Property of Multiplication allows you to write $r \times 3.4$ as $3.4r$.

- **How can you solve this equation?** *You can divide both sides of the equation by 3.4 to isolate the variable (r).*

Choose a student to solve the equation without labeling the answer. $\frac{17}{3.4} = \frac{3.4r}{3.4}$; $r = 5$

8. Explain that when finding the rate, two different units of measure are being compared in the division, and the division bar can be read *per* (e.g., 17 miles/3.4 hours can be read *17 miles per 3.4 hours*). Therefore, both units of measure are used in the label (e.g., miles per hour [mph] or kilometers per hour [km/hr]).

Chapter Review

Objectives

- Write algebraic expressions and equations
- Demonstrate an understanding of equations
- Simplify algebraic expressions using addition and multiplication properties: Commutative Property, Associative Property, and Distributive Property
- Evaluate expressions using substitution
- Solve equations using inverse operations
- Write an equation with a variable to solve a word problem
- Demonstrate an understanding of inequalities

Note

This lesson reviews the concepts presented in Chapter 10 to prepare the students for the Chapter 10 Test. Student Text pages 230–31 provide the students with an excellent study guide.

Check for Understanding

Write algebraic expressions and equations

- Write for display: *Joseph lost 2 of his pens.*
 - **How many pens did Joseph have to begin with?** *The amount is unknown.*
 - **How can you represent an unknown value?** *A variable can be used to write an unknown value.*
 - **When a mathematical expression includes a variable, is it an algebraic or a numerical expression? How do you know?** *An algebraic expression; algebraic expressions include a variable rather than just numbers as in a numerical expression.*
 - **If the letter p represents the number of pens Joseph had before losing 2, what algebraic expression can you write to express the number of pens Joseph has now?** $p - 2$
 - **Why is $p - 2$ an algebraic expression rather than an algebraic equation?** *Elicit that algebraic expressions are made up of numbers, operation signs, and variables, and an algebraic equation requires an equal sign to show that two algebraic expressions have the same value.*
 - **What words would indicate that an equal sign is needed when writing a word phrase or sentence mathematically?** *"is" or "equals"*
 - **How do you know what to write on each side of the equal sign in an algebraic equation?** *Elicit that since "is" or "equals" indicates that an equal sign is needed to write an algebraic equation, "is" or "equals" also separates the two equal word phrases in a sentence, indicating what should be written on each side of the equal sign.*
- Write the following word phrases and sentences for display. Choose students to tell whether the phrases and sentences represent an expression or an equation. Instruct each student to write the algebraic expression or equation for the word phrase or sentence. Elicit that when writing a multiplication expression, such as three times a number, the coefficient (multiplier) precedes the variable ($3n$).
 - a number divided by 6 **expression; $n \div 6$ or $\frac{n}{6}$**
 - the sum of a number and 2 **expression; $n + 2$**
 - the product of a number and 3 **expression; $3n$**
 - a number decreased by 4 **expression; $n - 4$**
 - 9 more than a number is 14 **equation; $n + 9 = 14$**
 - 5 less than a number equals 12 **equation; $n - 5 = 12$**

2 times a number equals 8 **equation; $2n = 8$**

a number divided by 3 is 4 **equation; $n \div 3 = 4$ or $\frac{n}{3} = 4$**

- Write 2 times a number, divided by 3.

➤ **How many operations are indicated in the word phrase? How do you know?** *2 operations; "times" indicates multiplication, and "divided by" indicates division.*

Direct the students to write an algebraic expression for the word phrase. **$2n \div 3$ or $\frac{2n}{3}$**

- Instruct the students to write algebraic expressions for the following word phrases. Remind them that using parentheses is helpful when writing expressions with more than one operation.

a number decreased by 7, multiplied by 3

$(n - 7) \times 3$, $3 \times (n - 7)$, or $3(n - 7)$

a number divided by 5, increased by 2

$(n \div 5) + 2$ or $n \div 5 + 2$

Simplify algebraic expressions

- Write $3(2n)$ for display.
 - **What property can you use to simplify the expression? Why?** *Elicit that you can use the Associative Property of Multiplication because the grouping of the factors can be changed without changing the value of the expression.*

Select a student to apply the Associative Property to $3(2n)$ and then simplify the expression. **$(3 \times 2) \times n$; $6n$**
- Follow a similar procedure for the following expressions. Elicit that like terms are combined when simplifying an expression such as $8x + 2 + 5x$.
 - $8x + 2 + 5x$ **Commutative Property of Addition; the order of the addends can be changed without changing the value of the expression; $8x + 5x + 2$; $13x + 2$.**
 - $4(3x + 2)$ **Distributive Property; each addend can be multiplied by 4 without changing the value of the expression; $(4 \times 3x) + (4 \times 2)$; $12x + 8$.**
- Write $s + s + s + s$ for display.
 - **What is the simplified expression for $s + s + s + s$? Why?** *$4s$; multiplication is repeated addition.*
- Direct the students to simplify the following expressions on paper. Choose students to write the answers for display and discuss the properties that were used to simplify each expression. Point out that both the Commutative and the Associative Properties of Addition were used to simplify $8a + (5 + 6a)$.

$$y + y$$

$$6 + 2y + 3$$

$$2(5x + 7)$$

$$4(3y)$$

$$8a + (5 + 6a)$$

$$4(3 + y)$$

Evaluate expressions using substitution

Write the following values and expressions for display. Select students to rewrite the expressions, substituting the given value for the variable, and to evaluate the expressions.

$$a = 4$$

$$b = 3$$

$$c = 2$$

$$(20 + a) \div 3$$

$$(20 + 4) \div 3$$

$$24 \div 3 = 8$$

$$5b - 9.7$$

$$(5 \times 3) - 9.7$$

$$15 - 9.7 = 5.3$$

$$34 - (14 \div c)$$

$$34 - (14 \div 2)$$

$$34 - 7 = 27$$

Solve equations using inverse operations

- Write $n + 9 = 14$; $m - 5 = 12$; $2a = 8$; and $\frac{x}{3} = 4$ for display.
 - **How can you solve these algebraic equations? Elicit that you can identify what operation is being performed on the**

Write the algebraic expression.
Identify what the variable represents in the expression.

- Eli read 3 times as many books as Nolan. **3b; b represents the number of books Nolan read.**
- Brynn broke 4 water glasses. **g - 4; g represents the total number of glasses.**
- The florist divided the roses among 5 vases. **r ÷ 5; r represents the total number of roses.**
- The product of a number and 5. **5n**
- Two less than 20 times n. **20n - 2**
- Sixty more than n, divided by 3. **(n + 60) ÷ 3**

Evaluate the expression. Let $n = 4$.

- $n + 7 - 3 = 4 + 7 - 3 = 11 - 3 = 8$
- $5n + 2 = (5 \cdot 4) + 2 = 20 + 2 = 10$
- $3 + (n + 2) = 3 + (4 + 2) = 3 + 2 = 5$
- $3n + 5 = (3 \cdot 4) + 5 = 12 + 5 = 17$
- $(2.7 \cdot n) - 3 = (2.7 \cdot 4) - 3 = 10.8 - 3 = 7.8$
- $100 - 12n = 100 - (12 \cdot 4) = 100 - 48 = 52$

Simplify the expression. **Order of terms in the expression may vary.**

- $3(6x) = 18x$
- $8x + (3 + 7x) = 8x + 7x + 3 = 15x + 3$
- $3 + x + 6 = x + 3 + 6 = x + 9$
- $(6 + 2x) + 3 = 2x + 6 + 3 = 2x + 9$
- $y + y + y = 3y$
- $2(8x) = 16x$

Simplify the expression using the Distributive Property. **Order of terms in the expression may vary.**

- $3(8 + 2a) = 3(8) + 3(2a) = 24 + 6a$
- $6(n + 2) = 6(n) + 6(2) = 6n + 12$
- $5(7x + 5.1) = 5(7x) + 5(5.1) = 35x + 25.5$

Complete the table using the given values to evaluate the expression.

22.

| b | $2b + 5$ |
|-----|----------|
| 6 | 17 |
| 12 | 29 |
| 25 | 55 |
| 49 | 103 |

23.

| x | $\frac{x}{3} - 2$ |
|-----|-------------------|
| 9 | 1 |
| 15 | 3 |
| 21 | 5 |
| 33 | 9 |

24.

| n | $n^2 + 9$ |
|-----|-----------|
| 5 | 34 |
| 9 | 90 |
| 12 | 153 |
| 20 | 409 |

variable and then apply the inverse operation to both sides of the equation.

- **Why is it necessary to apply the inverse operation to both sides of the equation?** *to keep the expressions equal or balanced while isolating the variable*
- Choose students to demonstrate solving the equations by applying the inverse operation. Ask each student to name the operation in the equation and the inverse operation he used to find the solution. **n = 5, subtraction; m = 17, addition; a = 4, division; x = 12, multiplication**
 - **How can you check each of the solutions?** *Elicit that you can substitute the solution for the variable in the original equation. If the equation is a true mathematical statement, the solution is correct.*
 - Write the following equations for display. Select students to solve the equations and check the solutions while the other students find the solutions and check them on paper.

$$\begin{array}{llll} a + 7 = 15 & n - 3 = 7 & 8a = 16 & \frac{x}{7} = 3 \\ a = 8 & n = 10 & a = 2 & x = 21 \end{array}$$

Samantha has 48 coins in her collection. This amount is 16 coins less than Mia has in her collection. How many coins does Mia have? **64 coins**

- Direct the students to write on paper what they need to find and the given information. Reread the problem as needed. **The number of coins that Mia has; Samantha has 48 coins, and Samantha has 16 fewer coins than Mia.**
- Write for display these two sentences: *Samantha has 48 coins, and Samantha has 16 fewer coins than Mia.*

Write an equation for the sentence.

- 8 less than a number is 14. **n - 8 = 14**
- The quotient of a number divided by 2 equals 9. **n ÷ 2 = 9**
- The sum of 2 times a number and 8 is 14. **2n + 8 = 14**

Determine whether the given value is the solution to the equation. Write **yes** or **no**.

- $n - 12 = 5$ if $n = 15$ **no**
- $\frac{x}{6} = 7$ if $x = 42$ **yes**
- $8a = 56$ if $a = 7$ **yes**
- $9 + n = 14$ if $n = 4$ **no**
- $m \div 8 = 2$ if $m = 16$ **yes**
- $5 \cdot a = 50$ if $a = 10$ **yes**

Solve. Write the inverse operation used to solve. Check the solution.

- $\frac{x}{4} = 9$ **x = 36; multiplication**
- $n - 8 = 61$ **n = 69; addition**
- $4a = 56$ **a = 14; division**
- $n + 3 = 32$ **n = 29; subtraction**
- $x - 1.6 = 1.4$ **x = 3; addition**
- $y + 5 = 25$ **y = 20; subtraction**
- $b + 5 = 48$ **b = 43; subtraction**
- $7n = 85.4$ **n = 12.2; division**

Determine whether the given value is a solution to the inequality. Write **yes** or **no**.

- $y < -1$ if $y = -2$ **yes**
- $x > 2$ if $x = 1$ **no**
- $y < 5$ if $y = 2$ **yes**

Draw a number line to illustrate the inequality.

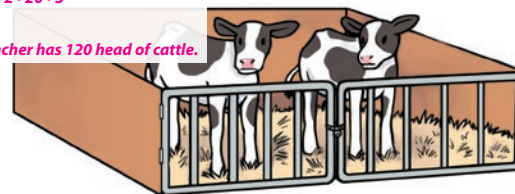
- $b < 4$
- $w > 1$

Write an equation with a variable. Solve. **Equations may vary.**

- One-third of the rancher's cattle are calves. If he puts 2 calves in each of his 20 stalls, how many head of cattle does the rancher have?

$$\begin{aligned} \frac{1}{3}x &= 2 \cdot 20 \\ 3 \cdot \frac{1}{3}x &= 2 \cdot 20 \cdot 3 \\ x &= 120 \end{aligned}$$

The rancher has 120 head of cattle.



Guide the students in using the information in the sentences to write an algebraic equation. (See Lesson 93.) **48 = m - 16**

- Direct the students to solve the equation. **48 + 16 = m - 16 + 16; m = 64**
- **How many coins does Mia have?** **64 coins**
- **How do you know that 64 coins is the correct answer?** *Elicit that 64 substituted for m in the original equation is a true mathematical statement; 48 = 64 - 16.*
- Guide the students in determining what information in the multi-step word problem on Student Text page 231 should be represented using a variable. Then guide them in solving the word problem. **x = 120 head of cattle**

Demonstrate an understanding of inequalities

- **What is an inequality?** *a mathematical sentence in which two expressions are not equal* Elicit that inequalities have an infinite number of solutions.
- Choose students to draw number lines to illustrate the solutions for $x < 6$, $y > -1$, and $x < 3$. **The number lines will vary, but the given value should be graphed using an open (unshaded) circle and the part of the number line showing the solutions should be shaded.** (See Lesson 93.)
 - Write the following inequalities. Choose students to tell whether the given value is a solution to the inequality and have them explain their answer.
 - $x > 2$ if $x = 7$ **yes; 7 is greater than 2**
 - $y < -2$ if $y = 1$ **no; 1 is greater than -2**
 - $n < 5$ if $n = 6$ **no; 6 is greater than 5**
 - $x > -1$ if $x = 1$ **yes; 1 is greater than -1**

Chapter 10 Test

Cumulative Review

For a list of the skills reviewed in the Cumulative Review, see the Lesson Objectives for Lesson 97 in the Chapter 10 Overview on page 212 of this Teacher's Edition.

Student Materials

- Cumulative Review Answer Sheet, page IA9 (CD)

Use the Cumulative Review on Student Text pages 232–34 to review previously taught concepts and to determine which students would benefit from your reteaching of the concepts. To prepare the students for the format of achievement tests, instruct them to work on a separate sheet of paper, if necessary, and to mark the answers on the Cumulative Review Answer Sheet.

Use the Exploring Ideas on Student Text page 235 (page 233 of this Teacher's Edition) any time after this chapter.

Mark the answer.

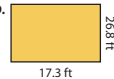
7. The Cape Henry lighthouse tower has an octagonal shape. How many sides does the lighthouse have?

A. 4
B. 5
C. 6
D. 8

8. What specific name is given to a quadrilateral with 4 congruent sides and 4 right angles?

A. rhombus
B. rectangle
C. square
D. parallelogram

9.



What is the perimeter of the figure?

A. 44.1 ft
B. 88.2 ft
C. 100.3 ft
D. 122.4 ft

10. What shapes are created when a diagonal line is drawn in a rectangle?

A. congruent equilateral triangles
B. congruent right triangles
C. congruent squares
D. similar rectangles

11. Three pizzas were ordered for family night. Each pizza had 8 slices. Three-fourths of the pizza was eaten. How many slices of pizza were left?

A. 6
B. 8
C. 10
D. 12

12. Which statement is true?

A. $1\frac{1}{2} = \frac{3}{4} + \frac{3}{4}$
B. $1\frac{1}{2} = \frac{3}{4} - \frac{3}{4}$
C. $1\frac{1}{2} = \frac{3}{4} \times \frac{3}{4}$
D. $1\frac{1}{2} = \frac{3}{4} \div \frac{3}{4}$

13. $7 \times n = 469$

A. $n = 64$
B. $n = 65$
C. $n = 66$
D. $n = 67$

14. $7 \overline{)64}$

A. $8\frac{5}{7}$
B. $9\frac{1}{7}$
C. $9\frac{5}{7}$
D. $10\frac{1}{7}$

Lesson 97

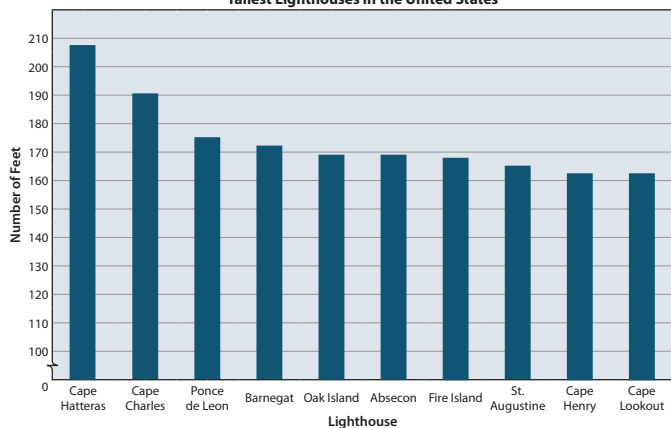
233

Come unto me, all ye that labour and are heavy laden, and I will give you rest.
Matthew 11:28

CUMULATIVE REVIEW

Test Prep

Tallest Lighthouses in the United States



Use the data from the bar graph to find the answer.

- This bar graph displays
 - the amount of material used to construct each lighthouse.
 - the height of 10 lighthouses.
 - the number of tourists that visit lighthouses.
 - the number of ships saved by each lighthouse.
- Estimate the difference in height between the tallest and shortest lighthouses on the graph.
 - 10 ft
 - 20 ft
 - 30 ft
 - 40 ft
- Which expression shows the height of the Ponce de Leon lighthouse?
 - 1.75×10^1 ft
 - 1.75×10^2 ft
 - 17×10^2 ft
 - 1.7×10^1 ft
- According to the graph, Cape Henry is
 - approximately 180 feet tall.
 - taller than the Oak Island lighthouse.
 - the same height as the Cape Lookout lighthouse.
 - the tallest lighthouse in the United States.
- About how much taller is Barnegat than Absecon?
 - less than 10 ft
 - about 10 ft
 - more than 20 ft
 - about 20 ft
- Which lighthouse measures 165 feet?
 - Barnegat
 - Ponce de Leon
 - Cape Charles
 - St. Augustine

232

Chapter 10

Mark the answer.

15. $3\frac{7}{8} + 4\frac{2}{3}$

A. $7\frac{9}{11}$
B. $8\frac{1}{8}$
C. $8\frac{13}{24}$
D. 9

16. $\frac{3}{9} \times \underline{\quad} = 1$

A. $\frac{3}{9}$
B. $\frac{3}{3}$
C. $\frac{9}{3}$
D. $\frac{9}{9}$

17. $\frac{1}{3} \div \frac{1}{2}$

A. $\frac{2}{3}$
B. $\frac{1}{6}$
C. $1\frac{1}{2}$
D. 2

18. 1.63×100

A. 0.0163
B. 0.163
C. 16.3
D. 163

19. $1.5 \overline{)4.05}$

A. 0.27
B. 2.7
C. 27
D. 270

20. $3^2 + 2^3$

A. 12
B. 17
C. 36
D. 72

21. $3 + 7 \times 4 - 8$

A. 23
B. 30
C. 45
D. 80

22. $4 \times (7.8 - 3.3) \div 2$

A. 4
B. 9
C. 9.8
D. 12

23. Rename $17\frac{15}{10}$ to lowest terms.

A. $17\frac{1}{2}$
B. $18\frac{1}{2}$
C. $18\frac{3}{4}$
D. 19

24. Which fraction is in lowest terms?

A. $\frac{4}{10}$
B. $\frac{6}{8}$
C. $\frac{7}{14}$
D. $\frac{9}{16}$

25. What is the sum of $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{1}{2}$?

A. 1
B. $1\frac{1}{2}$
C. $1\frac{7}{8}$
D. $2\frac{1}{4}$

234

Chapter 10



BIBLE MEASUREMENT

The chart shows the approximate values of some of the measurements found in the Bible. Use the data from the chart to find the answer.

- Noah's ark was 30 cubits high (Genesis 6:15). What was the height of the ark in inches? **$30 \times 17.5 = 525$ inches**
- King Og of Bashan slept in a bed that was 9 cubits long and 4 cubits wide (Deuteronomy 3:11). How long and wide was his bed in inches? **$9 \times 17.5 = 157.5$ inches long; $4 \times 17.5 = 70$ inches wide**
- Read 1 Samuel 17:7 to find the weight of the head of Goliath's spear. How much did the head of his spear weigh in pounds? **$600 \times 0.4 = 240$ ounces; $240 \div 16 = 15$ pounds**
- Read Numbers 11:32 to find how much quail was gathered by the person who gathered the least amount. How many bushels did this person gather? **$10 \times 5.2 = 52$ bushels**
- Read 1 Kings 10:14 to find how much gold King Solomon collected in a year. How many pounds of gold is this? (Hint: A score equals 20.) **$666 \times 75.6 = 50,349.6$ pounds**
- Read Ruth 2:17 to find how much barley Ruth gleaned from Boaz's field the first day. If she gathered about the same amount each day, how many bushels of barley did she gather in 6 days? **$6 \times \frac{3}{8} = 2 \frac{1}{4}$ bushels**

Modern Equivalents

| | |
|--------------|----------------------|
| cubit | 17.5 inches |
| shekel | 0.4 ounces |
| homer..... | 5.2 bushels |
| talent..... | 75.6 pounds |
| ephah | $\frac{3}{8}$ bushel |

