6

PLANE FIGURE GEOMETRY

BY THE PULL OF A CORD

Phoenix, Arizona **April 30, 1987**

In April of 1987, Skydive Arizona sponsored a convention for parachutists. About 420 daring men and women came to the convention, among them a thirty-one-year-old woman named Debbie Williams. One of the exercises planned for the day was a formation jump in which the participants linked hands in the air. Since taking up her hobby of skydiving, Debbie had performed fifty jumps. Because her past success made her confident in her abilities, she decided to take part in the formation jump. Waving to the watching crowd of friends and fellow parachutists, she and the other divers boarded the airplane that took them 9,000 feet into the air for the jump.

One of the parachutists aboard the plane was thirty-five-year-old Gregory Robertson. He was a seasoned skydiver and instructor with 1,700 jumps to his credit. When the time came to drop through the open hatch into the wide expanse of sky beneath the plane, he was ready. He waited his turn while several others jumped. Then he dived, controlling his speed with his outstretched arms and legs and preparing to link up with the formation. Suddenly he noticed that one of the other skydivers was in trouble. Debbie, who had jumped shortly before him, was tumbling too quickly toward the earth. She had been knocked unconscious in a collision with another diver. Instead of falling spread-eagle like the others, she was plummeting to the ground.

Gregory did not hesitate. He slammed his arms against his body, crossed his ankles, aimed his head at the ground, and dove after the unconscious woman. In this position he could travel about 200 miles per hour. Gregory plunged 5,500 feet before he caught up with Debbie. Immediately he pulled his body out of the dive into a froglike position to avoid hitting her. Debbie's reserve parachute, which should have opened by now, had malfunctioned. He turned her upright in the air so that her chute could open. By this time the two divers had dropped to 2,000 feet. Gregory yanked the ripcord on Debbie's emergency parachute and then on his own. Their chutes snapped open and brought them floating to the ground. Gregory's ability to calculate the speed and distance with which he and Debbie were falling allowed him to carefully plan how he moved through the air. The rescue, the most courageous one in the history of skydiving, had been made with only six seconds to spare before they both would have crashed to their deaths.

A skydiver in free fall

The largest free-fall formation jump took place over Udon Thani, Thailand, in the year 2006. Four hundred people from thirty-one countries jumped from 23,000 feet and held their flowerlike formation for five seconds. These skydivers plan to attempt a new record in 2012 with 500 jumpers.

In 1960 a man bailed out of a balloon at 102,800 feet and made a free fall of 84,700 feet, reaching a speed of almost 626 miles per hour.

Since a skydiver must open his parachute at least 2,500 feet above the ground, he wears on his wrist an altitude measuring device called an *altimeter*.

After 1,000 feet of free fall, a skydiver reaches his terminal velocity, which is a constant speed achieved when the air resistance is equal to gravity's force.

Plane Figure Geometry

Lesson	Topic	Lesson Objectives	Chapter Materials
49	**Basic Geometric Figures**	• Identify, name, and draw points, lines, and planes • Distinguish between collinear and noncollinear points • Identify the location of a point on a coordinate plane by naming the coordinates and the quadrant • Graph points on a coordinate plane • Recognize representations of points, lines, and planes in everyday life	**Teacher Manipulatives Packet:** • 2 Rays: ray A and ray BC **Student Manipulatives Packet:** • 2 Rays: ray A and ray BC **Instructional Aids (Teacher's Toolkit CD):** • Cumulative Review Answer Sheet (page IA9) for each student • Coordinate Plane (page IA24) • Coordinate Planes (page IA25) • Coordinate Planes (page IA25), 4 copies for each student • Angles (page IA26) • Angles (page IA26) for each student • Supplementary Angles (page IA27) • Complementary Angles (page IA28) • Complementary & Supplementary Angles (page IA29) • Triangles (IA30 CD) • Triangles (IA30 CD) for each student • Hierarchy of Quadrilaterals (page IA31) • Quadrilaterals (page IA32) • Quadrilaterals (page IA32) for each student • Polygon Angle Measure (page IA33) for each student (optional) • Congruent & Similar Polygons (page IA34) • Congruent & Similar Polygons (page IA34) for each student • Transformations (page IA35) • Transformations (page IA35) for each student • Block Letters (page IA36), one half page for each pair of students • Circle (page IA37) • Polyhedrons (page IA38) for each student **Christian Worldview Shaping (Teacher's Toolkit CD):** • Pages 18–19 **Other Teaching Aids:** • A blank page for display, for overlay with Coordinate Plane (page (IA24) • 2 lengths of string (each at least 1 yard in length) • 2 sheets of colored paper (2 different colors) • A small rectangular poster or teaching chart • An overhead protractor • Chalk or markers (3 different colors) • Construction paper (at least 3 different colors) • Tracing paper (optional) • An object to represent each of the following: cone, cylinder, rectangular prism, sphere, square prism, square pyramid, triangular prism, triangular pyramid (optional) • A purchased set of 3-dimensional figures • Three 3 × 5 cards • A ruler for each student and the teacher • A brass fastener for each student and the teacher • A protractor for each student and the teacher • Graph paper for each student • 8 toothpicks or craft sticks for each student • A sheet of notebook paper for each student • A blank sheet of paper for each student • Transparent tape for each student
50	**Types of Lines**	• Graph points on a coordinate plane to form a line, intersecting lines, perpendicular lines, and parallel lines • Identify lines: intersecting, perpendicular, parallel • Use variables to represent coordinates on a coordinate plane • Complete an input/output table • Relate lines to real-life situations	
51	**Classifying & Measuring Angles**	• Identify and name rays and angles • Classify angles: right, acute, obtuse, straight • Use a protractor to measure and draw angles • Relate geometry to everyday life	
52	**Angle Relationships**	• Develop an understanding of supplementary and complementary angles • Find the unknown measure of an angle in a pair of supplementary angles and in a pair of complementary angles • Recognize angles: right, acute, obtuse, straight • Use a protractor to measure angles	
53	**Polygons**	• Identify and name line segments • Demonstrate an understanding of regular and irregular polygons • Identify the number of sides and interior angles in a polygon • Graph points on a coordinate plane to form a polygon	
54	**Triangles**	• Develop the understanding that the sum of the measures of the angles in a triangle equals 180° • Find the unknown measure of an angle in a triangle • Classify triangles by their angles: right, acute, obtuse • Classify triangles by their sides: equilateral, isosceles, scalene • Measure the angles in a triangle using a protractor	
55	**Quadrilaterals**	• Distinguish between regular and irregular quadrilaterals • Develop an understanding of the classification of quadrilaterals • Develop the understanding that the sum of the angles in a quadrilateral equals 360° • Find the unknown measure of an angle in a quadrilateral • Measure the angles in a quadrilateral using a protractor	
56	**Congruent & Similar Figures**	• Identify congruent and similar polygons • Identify corresponding angles and line segments • Write a ratio for corresponding sides in a pair of polygons • Draw congruent and similar polygons	
57	**Transformations & Symmetry**	• Develop an understanding of transformations: translation, rotation, reflection • Identify a transformation • Draw a transformed figure • Identify symmetrical figures • Draw lines of symmetry on a figure	

Plane Figure Geometry

Lesson	Topic	Lesson Objectives	Chapter Materials
58	**Circles**	• Find the length of the diameter of a circle given the radius • Find the length of the radius of a circle given the diameter • Identify parts of a circle: center, radius, chord, diameter, central angle • Draw a circle using a protractor • Measure the central angles of a circle using a protractor • Relate fractions of a circle to degrees in a circle • Make a circle graph to represent given data	**Math 6 Tests and Answer Key** **Optional (Teacher's Toolkit CD):** • Fact Review pages • Application pages • Calculator Activities
59	**3-Dimensional Figures**	• Develop an understanding of polyhedrons • Identify 3-dimensonal figures that are not polyhedrons • Classify 3-dimensional figures: spherical, conical, cylindrical • Identify the number of faces, vertices, and edges in polyhedrons • Construct polyhedrons	
60	**Chapter 6 Review**	• Review	
61	**Chapter 6 Test** **Cumulative Review**	• Compare fractions • Multiply and divide decimals by a multiple of 10 • Add and subtract fractions • Identify kinds of angles and lines • Find the unknown measure of an angle in a triangle • Multiply by a 2-digit multiplier • Identify a prime number • Determine a common factor • Find the missing subtrahend in a subtraction equation • Add decimals and whole numbers • Compare integers • Use a chart to answer questions and to solve problems	

The ruler that is called for in this chapter and in the following chapters (Chapters 6–17) is a customary/metric ruler that can be used to measure inches, centimeters, and millimeters.

A Little Extra Help

Use the following to provide "a little extra help" for the student that is experiencing difficulty with the concepts taught in Chapter 6.

Use a protractor to measure angles—Keep a classroom supply of smaller protractors available for students who have difficulty using a large protractor to measure angles in the textbook and on worksheets. Another alternative is to allow them to place an index card along the outside of the ray to extend it so that they can clearly see the intersection of the ray with the scale on the protractor.

Identify the rotation of an irregular polygon—A student may have difficulty recognizing that when an irregular polygon rotates around a point, only the location of the figure changes; the figure does not reflect (flip). Prepare 2 trapezoids similar to the ones prepared for Lesson 57. Attach or draw an arrow along the longest side of one trapezoid. Follow a procedure similar to the one used to demonstrate a rotation in Lesson 57. Direct the student's attention to the position of the arrow as he rotates the trapezoid, clockwise and counterclockwise, $\frac{1}{4}$ turn, $\frac{1}{2}$ turn, $\frac{3}{4}$ turn, and 1 whole turn.

Math Facts

Throughout this chapter, review addition, subtraction, multiplication, and division facts using Fact Review pages or a Fact Fun activity on the Teacher's Toolkit CD, or you may use flashcards.

Lesson 49

Student Text pp. 118–21
Daily Review p. 421a

Objectives

- Identify, name, and draw points, lines, and planes
- Distinguish between collinear and noncollinear points
- Identify the location of a point on a coordinate plane by naming the coordinates and the quadrant
- Graph points on a coordinate plane
- Recognize representations of points, lines, and planes in everyday life

Teacher Materials

- Coordinate Plane, page IA24 (CD)
- A blank page for display
- 2 lengths of string (each at least 1 yard in length)
- 2 sheets of colored card stock (2 different colors)

Student Materials

- Coordinate Planes, page IA25 (CD)

Notes

Throughout this chapter, you may choose to have the students draw coordinate planes on graph paper rather than use the Coordinate Planes page.

Preview the Fact Review pages, the Application pages, and the Calculator Activities located on the Teacher's Toolkit CD.

Introduce the Lesson

Guide the students in reading aloud the story and facts on pages 118–19 of the Student Text (pages 116–17 of this Teacher's Edition).

Teach for Understanding

Identify and name points, lines, and planes

1. Explain *geometry* as noted on Student Text page 120.
2. Draw a dot for display. Explain that the dot represents a *point*, the smallest geometric figure. A point has no length, width, or thickness; it is a location in space that we cannot see.

   ➤ **What items in the classroom could represent a point?** ***possible answers: a tip of a pencil, a thumbtack hole, a period in a sentence***
3. Write the letter *A* next to the point. Explain that uppercase letters are used to name points. Write *point A*.
4. Choose a student to draw another point for display and label it *B*.

   ➤ **How many lines can you draw that contain point *A* and point *B*?** ***1 line***

   Draw a line with an arrow at each end that connects point *A* and point *B* and extends in opposite directions. Write $\overleftrightarrow{AB}$.

   ➤ **Why do you think there is an arrow at each end of the line?** ***Elicit that a line goes on without end in opposite directions.***

   Explain that a *line* is a string of invisible points; it is an abstract concept. A drawing with arrows at each end is used as a representation of the line. It shows a straight path that connects two points and extends endlessly in both directions.

   ➤ **What examples of a line might you see every day?** ***Answers will vary, but emphasize that a line extends endlessly in opposite directions.***
5. Write *collinear* for display. Explain that the prefix *co-* means "together." Point *A* and point *B* are collinear because they are together on the same line; one line can be drawn through both points, connecting them and extending in opposite directions.
6. Draw a third point that is not contained in $\overleftrightarrow{AB}$ and label it *M*.

   ➤ **Is point *M* contained in $\overleftrightarrow{AB}$?** ***no***

   ➤ **How many lines can you draw that contain point *A* and point *M*?** ***1 line*** **that contain point *B* and point *M*?** ***1 line*** **that contain point *A*, point *B*, and point *M*?** ***0 lines***

   Write *noncollinear* for display. Explain that the prefix *non-* means "not"; therefore, *noncollinear* means "not collinear." Since there is no one line that contains all 3 points (*A*, *B*, and *M*), the points are noncollinear.
7. Explain that although points *A*, *B*, and *M* are not all contained in the same line, they are all contained in the same *plane*, a flat surface extending endlessly in all directions. Point out that a line has length and a plane has length and width.

   ➤ **What flat surface contains point *A*, point *B*, and point *M*?** ***possible answers: a chalkboard or a dry erase board***

   Explain that a plane can be named by using a lowercase letter, but is frequently named by using 3 noncollinear points on the plane. Write *plane ABM*, *plane AMB*, and *plane MBA* and explain that the points can be named in any order.

   ➤ **What other items in the classroom represent a part of a plane?** ***possible answers: the floor, the ceiling, the wall, a desktop***
8. Ask students to predict how many different lines can contain point *A*. ***Accept any answer.*** Lightly draw lines going through point *A* in different directions. Allow students to discuss whether their predictions were correct.

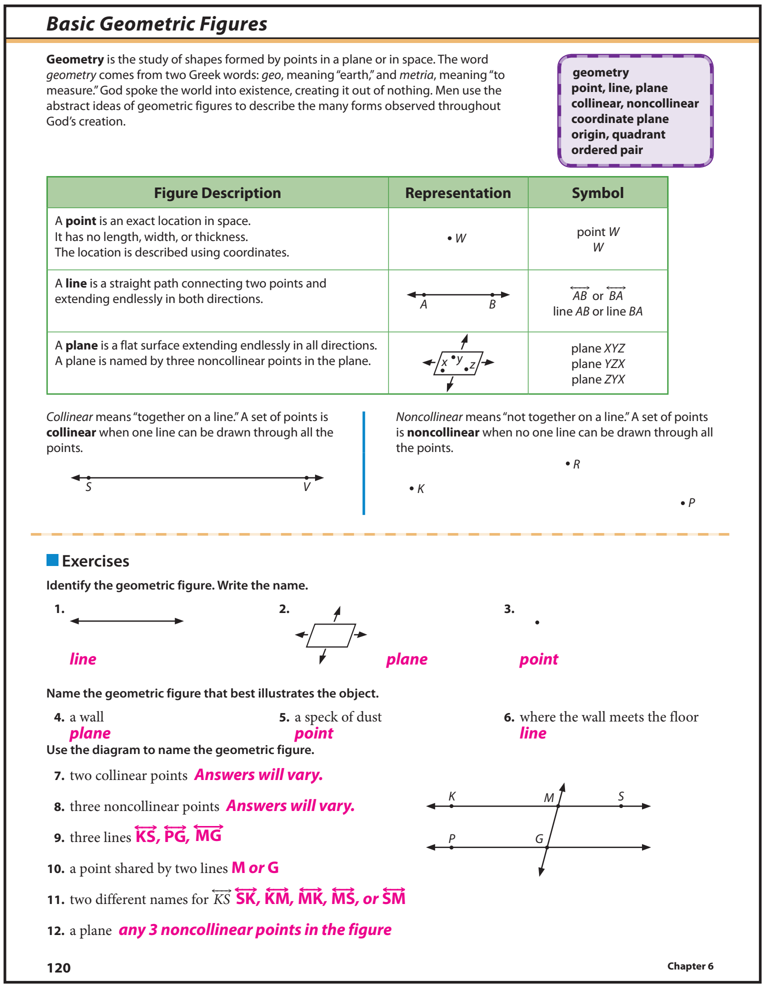
   ➤ **How many different lines in the same plane can contain point *A*?** ***an infinite number***
9. Attach (or hold) one end of a string on point *A*. Choose a student to hold the other end of the string out from the board. Name a point on the line (the string) as point *O*. Explain that although it looks like line *OA* stops at the board, it actually goes through plane *ABM* and continues without end; line *AB* and the new line intersect at point *A*.

   Attach (or hold) one end of the other string on point *A*. Select another student to sit on the floor and hold the other end of the string. Name a point on the line as point *T*. Explain that although it looks like line *TA* stops at the board, it too goes through the plane and continues without end.

   ➤ **Where does this new line intersect $\overleftrightarrow{AB}$?** ***point A***

   ➤ **Are the new lines contained in plane *ABM*? Why?** ***No; elicit that all the other points in each new line are not contained in plane ABM; only point A is contained in the plane.*** Continue to display both lines represented by the lengths of string.
10. Display 2 colored sheets of card stock and explain that they represent planes. Hold each sheet of card stock at an angle beside point *A* so that each string is lying on a different plane.

    ➤ **How many different planes can contain point *A*?** ***an infinite number***



Basic Geometric Figures

Geometry is the study of shapes formed by points in a plane or in space. The word *geometry* comes from two Greek words: *geo*, meaning "earth," and *metria*, meaning "to measure." God spoke the world into existence, creating it out of nothing. Men use the abstract ideas of geometric figures to describe the many forms observed throughout God's creation.

geometry
point, line, plane
collinear, noncollinear
coordinate plane
origin, quadrant
ordered pair

Figure Description	Representation	Symbol
A **point** is an exact location in space. It has no length, width, or thickness. The location is described using coordinates.	• *W*	point *W* *W*
A **line** is a straight path connecting two points and extending endlessly in both directions.	*A* *B*	$\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$ line *AB* or line *BA*
A **plane** is a flat surface extending endlessly in all directions. A plane is named by three noncollinear points in the plane.	*x* *y* *z*	plane *XYZ* plane *YZX* plane *ZYX*

Collinear means "together on a line." A set of points is **collinear** when one line can be drawn through all the points.

Noncollinear means "not together on a line." A set of points is **noncollinear** when no one line can be drawn through all the points.

Exercises

Identify the geometric figure. Write the name.

1. *line*

2. *plane*

3. *point*

Name the geometric figure that best illustrates the object.

4. a wall *plane*

5. a speck of dust *point*

6. where the wall meets the floor *line*

Use the diagram to name the geometric figure.

7. two collinear points ***Answers will vary.***

8. three noncollinear points ***Answers will vary.***

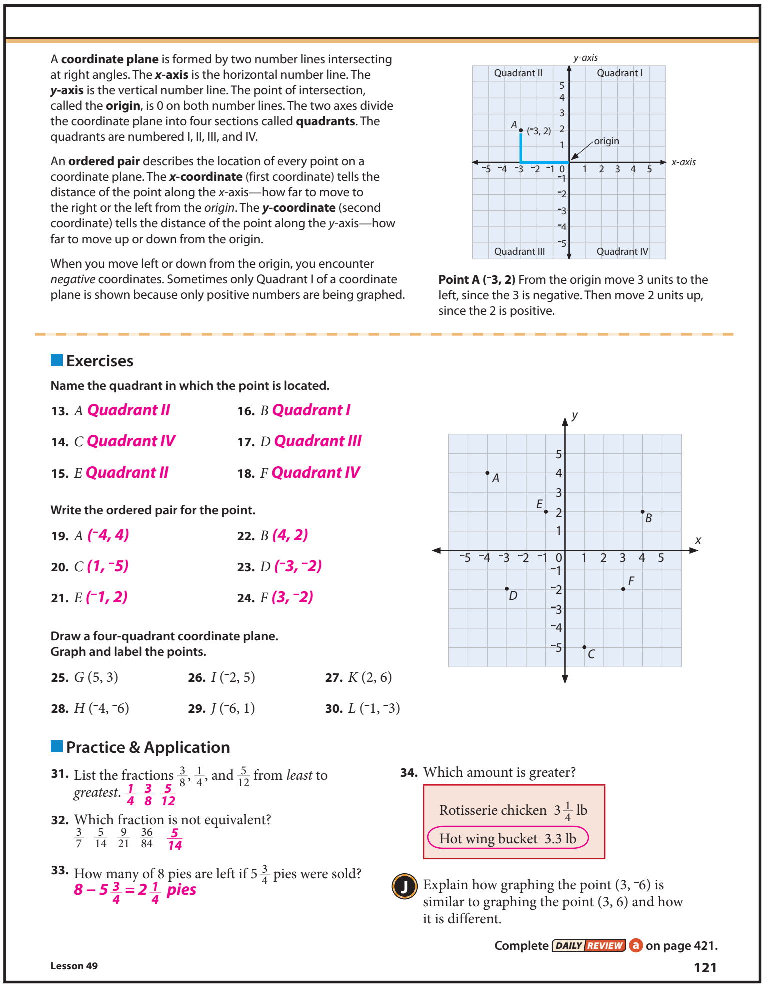
9. three lines $\overleftrightarrow{KS}$, $\overleftrightarrow{PG}$, $\overleftrightarrow{MG}$

10. a point shared by two lines **M *or* G**

11. two different names for $\overleftrightarrow{KS}$ $\overleftrightarrow{SK}$, $\overleftrightarrow{KM}$, $\overleftrightarrow{MK}$, $\overleftrightarrow{MS}$, *or* $\overleftrightarrow{SM}$

12. a plane ***any 3 noncollinear points in the figure***

120 Chapter 6



A **coordinate plane** is formed by two number lines intersecting at right angles. The ***x*-axis** is the horizontal number line. The ***y*-axis** is the vertical number line. The point of intersection, called the **origin**, is 0 on both number lines. The two axes divide the coordinate plane into four sections called **quadrants**. The quadrants are numbered I, II, III, and IV.

An **ordered pair** describes the location of every point on a coordinate plane. The ***x*-coordinate** (first coordinate) tells the distance of the point along the *x*-axis—how far to move to the right or the left from the *origin*. The ***y*-coordinate** (second coordinate) tells the distance of the point along the *y*-axis—how far to move up or down from the origin.

When you move left or down from the origin, you encounter *negative* coordinates. Sometimes only Quadrant I of a coordinate plane is shown because only positive numbers are being graphed.

Point A ($^-3$, 2) From the origin move 3 units to the left, since the 3 is negative. Then move 2 units up, since the 2 is positive.

Exercises

Name the quadrant in which the point is located.

13. *A* ***Quadrant II***

14. *C* ***Quadrant IV***

15. *E* ***Quadrant II***

16. *B* ***Quadrant I***

17. *D* ***Quadrant III***

18. *F* ***Quadrant IV***

Write the ordered pair for the point.

19. *A* **($^-4$, 4)**

20. *C* **(1, $^-5$)**

21. *E* **($^-1$, 2)**

22. *B* **(4, 2)**

23. *D* **($^-3$, $^-2$)**

24. *F* **(3, $^-2$)**

Draw a four-quadrant coordinate plane.
Graph and label the points.

25. *G* (5, 3)
26. *I* ($^-2$, 5)
27. *K* (2, 6)
28. *H* ($^-4$, $^-6$)
29. *J* ($^-6$, 1)
30. *L* ($^-1$, $^-3$)

Practice & Application

31. List the fractions $\frac{3}{8}$, $\frac{1}{4}$, and $\frac{5}{12}$ from *least* to *greatest*. $\frac{1}{4}$ $\frac{3}{8}$ $\frac{5}{12}$

32. Which fraction is not equivalent?
$\frac{3}{7}$ $\frac{5}{14}$ $\frac{9}{21}$ $\frac{36}{84}$ $\frac{5}{14}$

33. How many of 8 pies are left if $5\frac{3}{4}$ pies were sold?
$8 - 5\frac{3}{4} = 2\frac{1}{4}$ ***pies***

34. Which amount is greater?

Rotisserie chicken	$3\frac{1}{4}$ lb
Hot wing bucket	3.3 lb

Ⓙ Explain how graphing the point (3, $^-6$) is similar to graphing the point (3, 6) and how it is different.

Complete DAILY REVIEW ⓐ on page 421.

Lesson 49 **121**

Identify the location of a point on a coordinate plane

1. Draw for display a point on a blank page (in the top right section of the page) and label it *J*.

➤ **How can you describe the location of point *J* in this plane?** ***Accept any reasonable descriptions.***

2. Place the Coordinate Plane page on top of the blank page; align point *J* so that it can be identified using the coordinates (4, 7).

➤ **What flat surface could this coordinate graph represent?** ***a plane*** Explain that a coordinate graph is a *coordinate plane* that is used to identify the location of a point on a plane.

3. Use the information on Student Text page 121 to review *x-axis*, *y-axis*, and *ordered pair* and to explain the *origin* and *quadrants* on a coordinate plane.

➤ **How can you now describe the location of point *J*?** ***Name the coordinates or the ordered pair.***

➤ **What is the location of point *J*?** ***(4, 7) in Quadrant I***

4. Draw points in all four quadrants on the Coordinate Plane. Guide the students in identifying the quadrant in which each point is located and in writing each point's location using an ordered pair.

➤ **What direction do you move along the *x*-axis if the *x*-coordinate is negative?** ***Move left of the origin.***

➤ **What direction do you move along the *y*-axis if the *y*-coordinate is negative?** ***Move down from the origin.***

Graph points on a coordinate plane

1. Distribute the Coordinate Planes page. Point out that a coordinate plane is formed by drawing two lines that intersect at right angles and that the axes divide the coordinate plane into the four quadrants. Write the Roman numerals I–IV to label the quadrants and instruct the students to label their coordinate planes; begin with the top right quadrant and proceed in a counterclockwise direction.

2. Direct the students to graph ordered pairs as you name each one and to tell the quadrant in which the point is located.

3. Repeat the process for several ordered pairs, naming ordered pairs in each of the quadrants.

➤ **Which quadrant contains negative *x*-coordinates and positive *y*-coordinates?** ***Quadrant II***

➤ **Which quadrant contains positive *x*-coordinates and negative *y*-coordinates?** ***Quadrant IV***

➤ **Which quadrant contains negative *x*-coordinates and negative *y*-coordinates?** ***Quadrant III***

➤ **Which quadrant contains positive *x*-coordinates and positive *y*-coordinates?** ***Quadrant I***

Student Text pp. 120–21

Lesson
50

Student Text pp. 122–23

Daily Review p. 421b

Objectives

- Graph points on a coordinate plane to form a line, intersecting lines, perpendicular lines, and parallel lines
- Identify lines: intersecting, perpendicular, parallel
- Use variables to represent coordinates on a coordinate plane
- Complete an input/output table
- Relate lines to real-life situations

Teacher Materials

- Coordinate Planes, page IA25 (CD)
- A ruler
- A small rectangular poster or teaching chart

Student Materials

- Coordinate Planes, page IA25 (CD) or graph paper
- A ruler

Note

Since functions are not taught until seventh grade, tables used throughout Math 6 will be referred to as *input/output tables.*

Teach for Understanding

Graph points on a coordinate plane to form intersecting lines

1. Display and distribute the Coordinate Planes page. Write *A (−4, −3)* and *B (6, 5)* for display. Direct the students to graph and label the points on the first coordinate plane.
   (***Note:*** Throughout the lesson, model the drawing of points and lines.)

➤ **How many points do you need to draw a line?** *2*

2. Instruct the students to draw $\overleftrightarrow{AB}$ by connecting point *A* and point *B*, using a ruler as a guide. Remind them that a line is a string of invisible points that continues in both directions, so they need to extend the line in opposite directions and draw an arrow at each end.

➤ **How many points are on line *AB*?** *an infinite number*

3. Write *G (5, 8)* and *H (−2, −6)* for display. Direct the students to graph and label these points on the same coordinate plane and to draw $\overleftrightarrow{GH}$.
   (***Note:*** Drawing two lines accurately enough to get the correct point of intersection can be challenging. Encourage the students to draw their lines very carefully. Give help as needed.)

➤ **What are the coordinates for a point that is common to line *AB* and line *GH*?** *(2, 2)*

➤ **What type of lines are $\overleftrightarrow{AB}$ and $\overleftrightarrow{GH}$?** *intersecting lines*

4. Write $\overleftrightarrow{AB}$ *intersects* $\overleftrightarrow{GH}$. Explain that there is no symbol that means "intersects," so they must write the word.
   Write *intersecting lines* for display and guide the students in developing a definition. *lines that share a common point*
5. Repeat the procedure for *J (5, −1)* and *K (8, −4)*.

➤ **Does $\overleftrightarrow{JK}$ intersect with $\overleftrightarrow{AB}$ or $\overleftrightarrow{GH}$? Why?** *Elicit that $\overleftrightarrow{JK}$ intersects both $\overleftrightarrow{AB}$ and $\overleftrightarrow{GH}$ because lines continue without end.*

6. Direct the students to extend $\overleftrightarrow{JK}$ so that it intersects $\overleftrightarrow{AB}$ and $\overleftrightarrow{GH}$.

➤ **What coordinates identify the point of intersection?** *(2, 2)*

➤ **What examples of intersecting lines might you see every day?** *possible answer: two roads on a map*

Graph points on a coordinate plane to form perpendicular lines

1. Write *L (−4, 7)* and *M (−4, −3)*. Instruct the students to graph and label the points and draw $\overleftrightarrow{LM}$ on the second coordinate plane.
2. Repeat the procedure for *R (5, 5)*, *S (−7, 5)*, and $\overleftrightarrow{RS}$, using the same coordinate plane.

➤ **What are the coordinates for the point that is common to $\overleftrightarrow{LM}$ and $\overleftrightarrow{RS}$?** *(−4, 5)*

➤ **Are these lines intersecting lines? Why?** *Yes; the lines share a common point at (−4, 5).*

3. Explain that $\overleftrightarrow{LM}$ and $\overleftrightarrow{RS}$ are also *perpendicular lines* because right angles (angles with a measure of 90°) are formed at the point of intersection. Explain that because 90° angles form square corners, a small square is often drawn in the corner to indicate a right angle. Demonstrate drawing small squares to indicate the right angles formed where $\overleftrightarrow{LM}$ and $\overleftrightarrow{RS}$ intersect. Instruct the students to draw the squares on their coordinate planes.

➤ **How many right angles are formed at the point where $\overleftrightarrow{LM}$ and $\overleftrightarrow{RS}$ intersect?** *4*

4. Write $\overleftrightarrow{LM} \perp \overleftrightarrow{RS}$. Explain that the symbol ⊥ means "is perpendicular to."
   Write *perpendicular lines* for display and guide the students in developing a definition. *intersecting lines that form right angles at the point of intersection*

➤ **What examples of perpendicular lines might you see every day?** *possible answer: the top and side edges of a sheet of paper*

Graph points on a coordinate plane to form parallel lines

1. Write *C (−3, 6)* and *D (−3, −4)*. Direct the students to graph the points and draw $\overleftrightarrow{CD}$ on the third coordinate plane.
2. Repeat the procedure for *E (4, 5)*, *F (4, −2)*, and $\overleftrightarrow{EF}$, using the same coordinate plane.

➤ **Are these lines intersecting lines? Why?** *No; the lines do not share a common point.*

3. Direct the students to count the number of units between (−3, 5) and (4, 5); (−3, 2) and (4, 2); (−3, −2) and (4, −2); and (−3, −4) and (4, −4).

➤ **How many units apart are the lines?** *7 units*

➤ **Do you think these lines will ever intersect? Why?** *No; elicit that the distance between the lines will always be 7 units.*

4. Write $\overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$. Explain that the symbol ∥ means "is parallel to."
   Write *parallel lines* for display and guide the students in developing a definition. *a pair of lines in the same plane that will never intersect*

➤ **What examples of parallel lines might you see every day?** *possible answer: opposite edges of a sheet of paper*

➤ **Do you think parallel lines are always vertical?** *no*

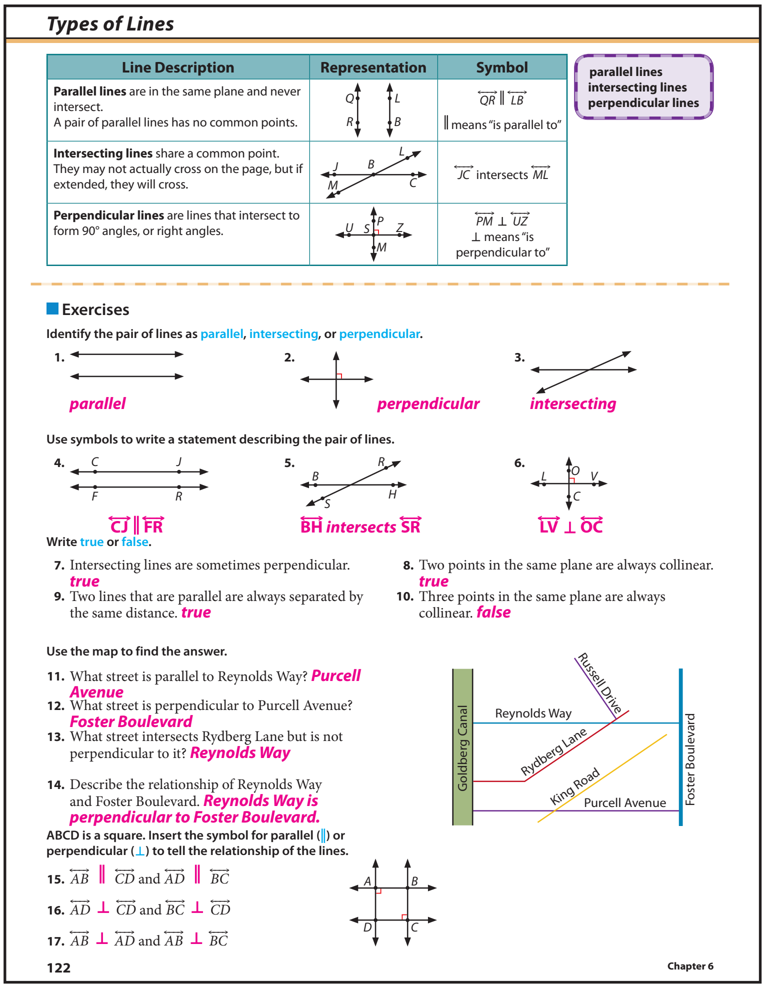
5. Direct the students to plot a pair of horizontal lines that are parallel on the same coordinate plane.

Use variables to represent coordinates

1. Draw an input/output table as shown. Explain that the variable *y* (output) is dependent on the variable *x* (input): if $x = 0$, then $y = 4$ and if $x = 1$, then $y = 5$.

x	*y*
0	4
1	5
2	
3	

➤ **What pattern do you see in this table?** *Answers will vary, but elicit that each value for y is 4 more than the corresponding value for x.*



Types of Lines

Line Description	Representation	Symbol
Parallel lines are in the same plane and never intersect. A pair of parallel lines has no common points.	Q R L B	$\overleftrightarrow{QR} \parallel \overleftrightarrow{LB}$ $\parallel$ means "is parallel to"
Intersecting lines share a common point. They may not actually cross on the page, but if extended, they will cross.	J B L M C	$\overleftrightarrow{JC}$ intersects $\overleftrightarrow{ML}$
Perpendicular lines are lines that intersect to form 90° angles, or right angles.	U S P Z M	$\overleftrightarrow{PM} \perp \overleftrightarrow{UZ}$ $\perp$ means "is perpendicular to"

parallel lines
intersecting lines
perpendicular lines

Exercises

Identify the pair of lines as parallel, intersecting, or perpendicular.

1. *parallel*
2. *perpendicular*
3. *intersecting*

Use symbols to write a statement describing the pair of lines.

4. C J F R — $\overleftrightarrow{CJ} \parallel \overleftrightarrow{FR}$
5. B R S H — $\overleftrightarrow{BH}$ *intersects* $\overleftrightarrow{SR}$
6. L O V C — $\overleftrightarrow{LV} \perp \overleftrightarrow{OC}$

Write true or false.

7. Intersecting lines are sometimes perpendicular. *true*
8. Two points in the same plane are always collinear. *true*
9. Two lines that are parallel are always separated by the same distance. *true*
10. Three points in the same plane are always collinear. *false*

Use the map to find the answer.

11. What street is parallel to Reynolds Way? *Purcell Avenue*
12. What street is perpendicular to Purcell Avenue? *Foster Boulevard*
13. What street intersects Rydberg Lane but is not perpendicular to it? *Reynolds Way*
14. Describe the relationship of Reynolds Way and Foster Boulevard. *Reynolds Way is perpendicular to Foster Boulevard.*

ABCD is a square. Insert the symbol for parallel ($\parallel$) or perpendicular ($\perp$) to tell the relationship of the lines.

15. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$
16. $\overleftrightarrow{AD} \perp \overleftrightarrow{CD}$ and $\overleftrightarrow{BC} \perp \overleftrightarrow{CD}$
17. $\overleftrightarrow{AB} \perp \overleftrightarrow{AD}$ and $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

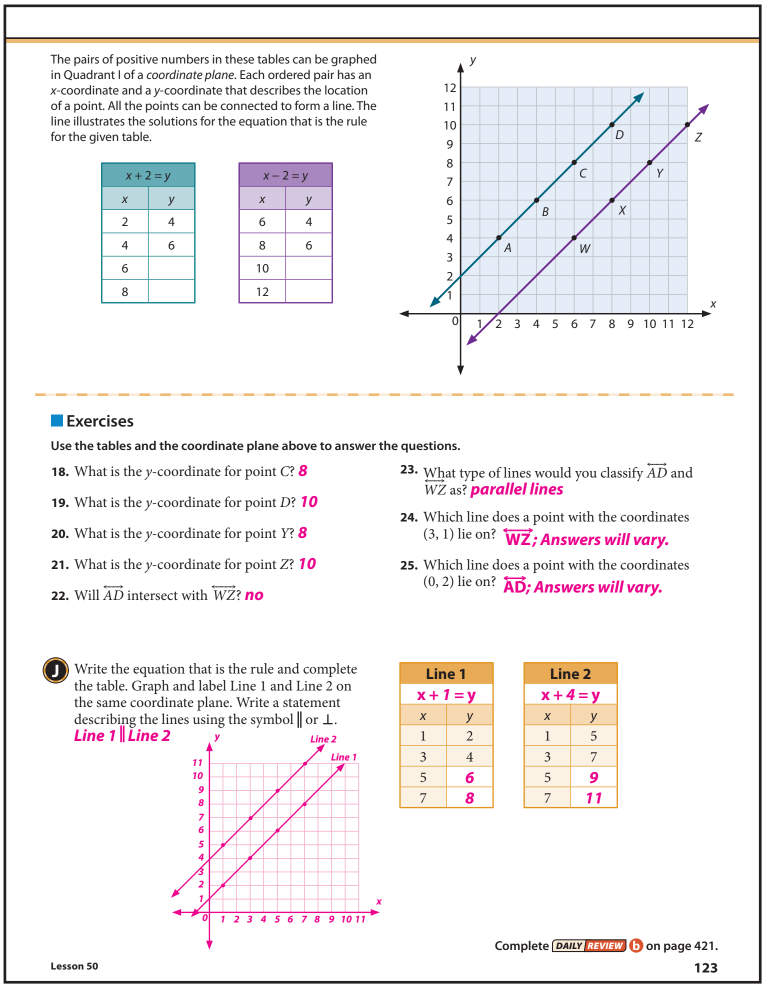
122 Chapter 6

- **When x is 2, what is the value of y? *6* when x is 3? *7***
  Write the answers in the table.
- **What equation can you write to show the relationship between x and y or the rule for the table? $x + 4 = y$**
  Write the rule at the top of the table.
  Select students to substitute each value for x to see if the corresponding value for y is correct for the equation. (e.g., $0 + 4 = 4$)

2. Explain that you can use the values for x and y as ordered pairs and plot the points on a coordinate plane. Choose students to tell the ordered pair as you plot each point on the fourth coordinate plane. Label the points A, B, C, and D, and connect them. (Refer to the example on Student Text page 123.)
   - **What was formed when the points represented by the ordered pairs in the table were connected? *a line***
3. Follow a similar procedure for a table with the same x values and the rule $x + 2 = y$, labeling the points M, N, O, and P. Elicit that the points form a line that is parallel to $\overleftrightarrow{AD}$.

Relate lines to real-life situations

1. Attach the poster to the board so that the bottom edge of the poster is parallel to the bottom edge of the board. Remind the students that each edge of the poster represents part of a line that continues without end in opposite directions.
   - **What is the relationship between the left and right edges of the poster? Why?** *They are parallel; elicit that the lines that the left and right edges are part of will never intersect.*
   - **What is the relationship between the top and bottom edges of the poster? Why?** *They are parallel; the lines that the top and bottom edges are part of will never intersect.*



The pairs of positive numbers in these tables can be graphed in Quadrant I of a *coordinate plane*. Each ordered pair has an *x*-coordinate and a *y*-coordinate that describes the location of a point. All the points can be connected to form a line. The line illustrates the solutions for the equation that is the rule for the given table.

$x + 2 = y$	
x	y
2	4
4	6
6	
8	

$x - 2 = y$	
x	y
6	4
8	6
10	
12	

Exercises

Use the tables and the coordinate plane above to answer the questions.

18. What is the *y*-coordinate for point *C*? *8*
19. What is the *y*-coordinate for point *D*? *10*
20. What is the *y*-coordinate for point *Y*? *8*
21. What is the *y*-coordinate for point *Z*? *10*
22. Will $\overleftrightarrow{AD}$ intersect with $\overleftrightarrow{WZ}$? *no*
23. What type of lines would you classify $\overleftrightarrow{AD}$ and $\overleftrightarrow{WZ}$ as? *parallel lines*
24. Which line does a point with the coordinates (3, 1) lie on? $\overleftrightarrow{WZ}$*; Answers will vary.*
25. Which line does a point with the coordinates (0, 2) lie on? $\overleftrightarrow{AD}$*; Answers will vary.*

Ⓙ Write the equation that is the rule and complete the table. Graph and label Line 1 and Line 2 on the same coordinate plane. Write a statement describing the lines using the symbol $\parallel$ or $\perp$.
Line 1 $\parallel$ Line 2

Line 1	
$x + 1 = y$	
x	y
1	2
3	4
5	*6*
7	*8*

Line 2	
$x + 4 = y$	
x	y
1	5
3	7
5	*9*
7	*11*

Complete DAILY REVIEW ⓑ on page 421.

Lesson 50 123

- **What is the relationship between the bottom edge of the poster and the bottom edge of the board? Why?** *They are parallel; the line that the bottom edge of the poster is part of and the line that the bottom edge of the board is part of will never intersect.*

2. Slightly tilt the poster so that one corner is pointing to the bottom of the board.
   - **Is the bottom edge of the poster parallel to the bottom of the board? Why?** *No; the line that the bottom edge of the poster is part of and the line that the bottom edge of the board is part of will intersect at some point.*
   - **With the poster in this position, what do you notice about the relationship between the left and right edges of the poster?** *They are still parallel.* **the relationship between the top and bottom edges of the poster?** *They are still parallel.*
3. Turn the poster so that the lines represented by opposite edges of the poster are perpendicular to the line represented by the bottom edge of the board. Elicit a brief explanation of the relationship between the lines.

Student Text pp. 122–23

Lesson 51

Student Text pp. 124–25
Daily Review p. 422c

Objectives

- Identify and name rays and angles
- Classify angles: right, acute, obtuse, straight
- Use a protractor to measure and draw angles
- Relate geometry to everyday life

Teacher Materials

- 2 Rays: ray *A* and ray *BC*
- Angles IA26 (CD)
- A brass fastener
- A blank page for display (from Lesson 49)
- An overhead protractor
- Chalk or markers (3 different colors)

Student Materials

- 2 Rays: ray *A* and ray *BC*
- Angles, page IA26 (CD)
- A brass fastener
- A ruler
- A protractor

Note

Beginning with this lesson, the 2 fastened Rays from the manipulatives packets will be referred to as an *Angler*.

Teach for Understanding

Identify and name rays

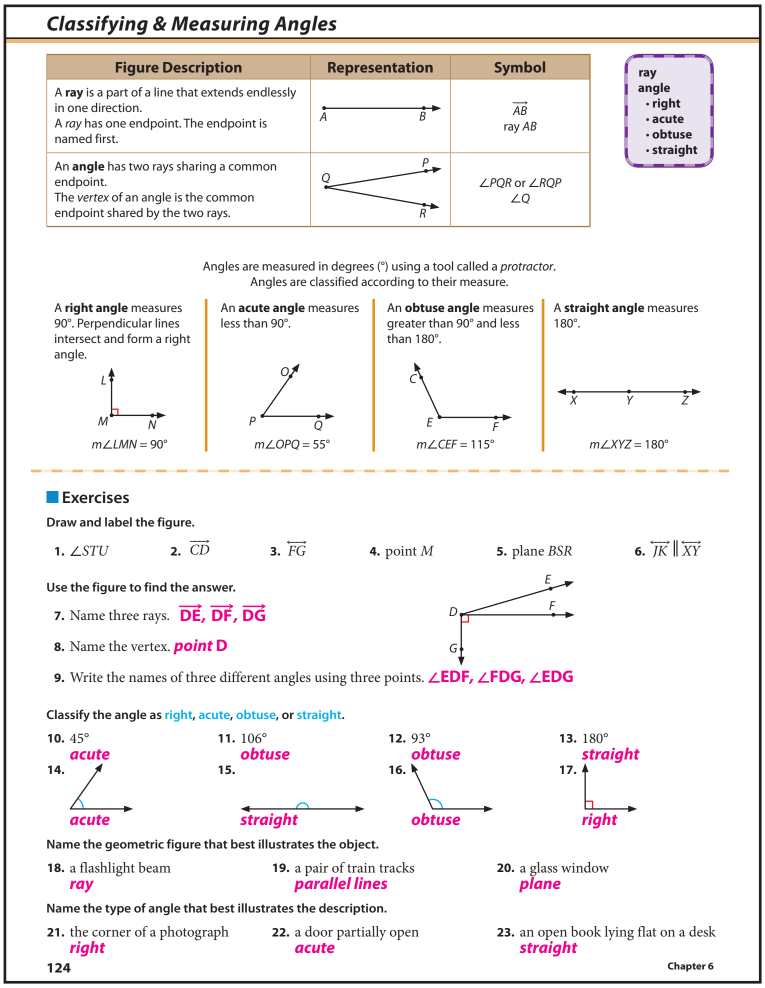
1. Draw two points and label them *J* and *K*. Remind the students that a point has no length, width, or thickness.
   ➤ How many lines can you draw that contain point *J* and point *K*? *1 line*
   Draw a line that contains and extends beyond each point. Draw an arrow at each end.
   ➤ What is the name of this line? *line JK or line KJ* Write either $\overleftrightarrow{JK}$ or $\overleftrightarrow{KJ}$.
   ➤ Does it matter which point you name first? Why? *No; elicit that since a line has no beginning and no end, the 2 points used to name a line can be given in any order.* Write the other name for the line.
2. Write the word *ray* for display. Explain that a ray is a geometric figure that is part of a line. It extends endlessly in one direction and has one endpoint.
3. Use a different color to draw ray *JK* above $\overleftrightarrow{JK}$. Explain that ray *JK* begins at point *J* and extends endlessly in one direction. Write $\overrightarrow{JK}$ for display. Explain that the symbol for a ray always points to the right and that the endpoint is always given first, using 2 points to name a ray.
4. Use a third color to draw $\overrightarrow{KJ}$ and write the name. Point out that the names $\overrightarrow{JK}$ and $\overrightarrow{KJ}$ are not interchangeable because the rays have a different endpoint, and they extend in opposite directions.
5. Direct the students to draw a ray on paper and to name it.

Identify and name angles

1. Distribute the Rays and the brass fasteners. Demonstrate as you guide the students in fastening the 2 Rays together. Explain that the joined Rays will be called an *Angler*.
   ➤ How many rays do you have? *2* What is formed when 2 rays share the same endpoint? *an angle*
2. Direct each student to close his Angler and place it in a horizontal position as you display your closed Angler. Demonstrate as you guide the students in slightly opening the Angler to form a narrow angle. Then guide them in gradually opening the Angler to form increasingly wider angles. Explain that the space between 2 rays is the interior of the angle.
   ➤ What is the point where 2 rays meet to form an angle called? *Elicit that it is the vertex.*
   ➤ What is the name for this angle? *Elicit that the angle can be named by its vertex (angle B) or by using 3 points, a point on one ray followed by the vertex and a point on the other ray (angle ABC or angle CBA).*
   Write $\angle B$, $\angle ABC$, and $\angle CBA$. Point out that each of these names the angle.
   ➤ Why is point *B* in the center? *It is the vertex of the angle.*
3. Direct each student to draw an angle, label the vertex and a point on each ray, and write the name of his angle.

Classify and measure angles

1. Distribute the protractors. Explain that angles are classified by their measure and are measured in units called *degrees*. Explain that the concept of a degree to measure angles was developed by the ancient Babylonians. They divided a circle into 360 equal units and called the units *degrees*. One degree is $\frac{1}{360}$ of a circle. Point out that a protractor has degree markings, a guide mark (center point) to align with the vertex, and 2 scales for measuring. One scale is used to measure angles with rays that open to the right, and the other scale is used to measure angles with rays that open to the left.
   ➤ What do you know about a right angle? *Elicit that a right angle measures 90°, forming a square corner.*
   ➤ Where in the classroom do you see right angles? *possible answers: where sides of the wall meet, where a wall meets the floor, where the top and side of a desk (table or window) meet*
2. Direct the students to lay their Angler on a sheet of paper and to open the Angler to form a right angle. Instruct them to trace the inside or interior of the angle and label 3 points to name the angle, using any letters. Lay the blank page on top of your Angler and demonstrate drawing the right angle.
   ➤ How can you make sure that you drew a right angle? *Elicit that you can use a protractor to see if the angle measures 90°.*
3. Demonstrate as you guide the students in measuring their angles; use these steps.
   Step 1: Align the center point of the base of the protractor with the vertex of the angle.
   Step 2: Align one ray of the angle with the 0° line on the protractor.
   Step 3: Read the angle measurement indicated by the other ray of the angle.
4. Write the measure of your angle and direct each student to write the measure of his angle; e.g., $m\angle PQR = 90°$. Point out that the *m* before the angle symbol means "the measure of." *Answers may vary, but should be within a few degrees of 90°.*
   ➤ What do you remember about an acute angle? *Elicit that an acute angle measures less than a right angle (90°).*
   ➤ What items in the classroom form acute angles? *possible answers: a partially opened door or book, chair legs*



Classifying & Measuring Angles

Figure Description	Representation	Symbol
A **ray** is a part of a line that extends endlessly in one direction. A *ray* has one endpoint. The endpoint is named first.	A B	$\overrightarrow{AB}$ ray *AB*
An **angle** has two rays sharing a common endpoint. The *vertex* of an angle is the common endpoint shared by the two rays.	Q P R	$\angle PQR$ or $\angle RQP$ $\angle Q$

ray
angle
- **right**
- **acute**
- **obtuse**
- **straight**

Angles are measured in degrees (°) using a tool called a *protractor*.
Angles are classified according to their measure.

A **right angle** measures 90°. Perpendicular lines intersect and form a right angle.

$m\angle LMN = 90°$

An **acute angle** measures less than 90°.

$m\angle OPQ = 55°$

An **obtuse angle** measures greater than 90° and less than 180°.

$m\angle CEF = 115°$

A **straight angle** measures 180°.

$m\angle XYZ = 180°$

Exercises

Draw and label the figure.

1. $\angle STU$ **2.** $\overrightarrow{CD}$ **3.** $\overleftrightarrow{FG}$ **4.** point *M* **5.** plane *BSR* **6.** $\overleftrightarrow{JK} \parallel \overleftrightarrow{XY}$

Use the figure to find the answer.

7. Name three rays. ***$\overrightarrow{DE}$, $\overrightarrow{DF}$, $\overrightarrow{DG}$***

8. Name the vertex. ***point D***

9. Write the names of three different angles using three points. ***$\angle EDF$, $\angle FDG$, $\angle EDG$***

Classify the angle as right, acute, obtuse, or straight.

10. 45° ***acute***
11. 106° ***obtuse***
12. 93° ***obtuse***
13. 180° ***straight***
14. ***acute***
15. ***straight***
16. ***obtuse***
17. ***right***

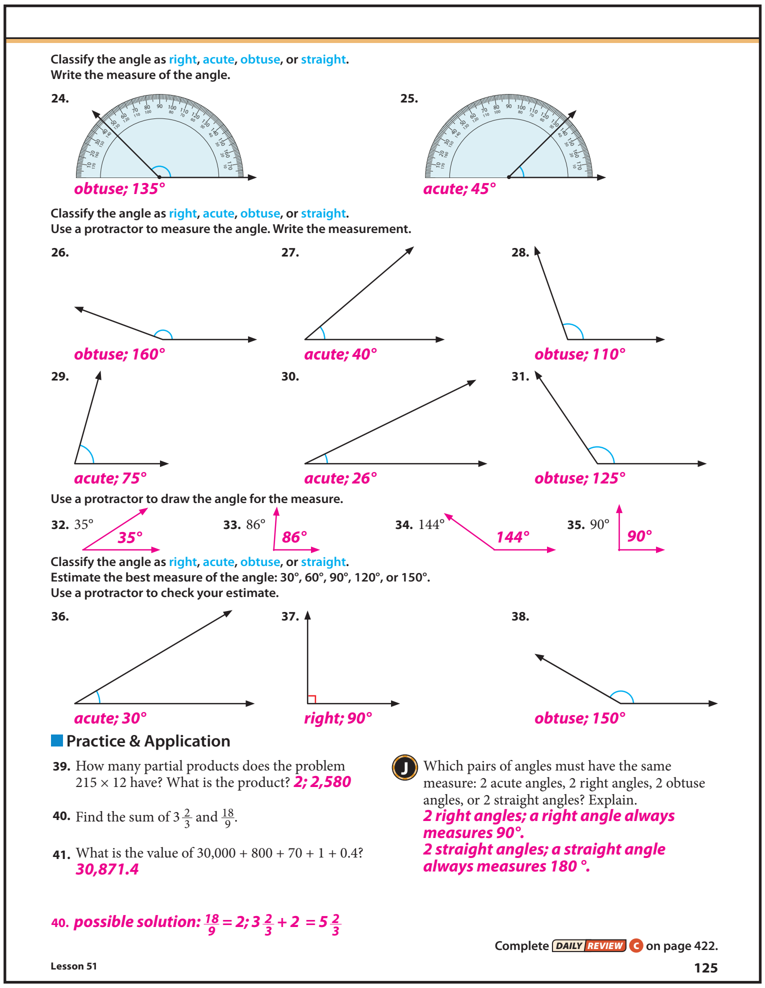
Name the geometric figure that best illustrates the object.

18. a flashlight beam ***ray***
19. a pair of train tracks ***parallel lines***
20. a glass window ***plane***

Name the type of angle that best illustrates the description.

21. the corner of a photograph ***right***
22. a door partially open ***acute***
23. an open book lying flat on a desk ***straight***

124 Chapter 6



Classify the angle as right, acute, obtuse, or straight.
Write the measure of the angle.

24. ***obtuse; 135°***
25. ***acute; 45°***

Classify the angle as right, acute, obtuse, or straight.
Use a protractor to measure the angle. Write the measurement.

26. ***obtuse; 160°***
27. ***acute; 40°***
28. ***obtuse; 110°***
29. ***acute; 75°***
30. ***acute; 26°***
31. ***obtuse; 125°***

Use a protractor to draw the angle for the measure.

32. 35° ***35°***
33. 86° ***86°***
34. 144° ***144°***
35. 90° ***90°***

Classify the angle as right, acute, obtuse, or straight.
Estimate the best measure of the angle: 30°, 60°, 90°, 120°, or 150°.
Use a protractor to check your estimate.

36. ***acute; 30°***
37. ***right; 90°***
38. ***obtuse; 150°***

Practice & Application

39. How many partial products does the problem 215×12 have? What is the product? ***2; 2,580***

40. Find the sum of $3\frac{2}{3}$ and $\frac{18}{9}$.

41. What is the value of $30{,}000 + 800 + 70 + 1 + 0.4$? ***30,871.4***

Ⓙ Which pairs of angles must have the same measure: 2 acute angles, 2 right angles, 2 obtuse angles, or 2 straight angles? Explain.
2 right angles; a right angle always measures 90°.
2 straight angles; a straight angle always measures 180 °.

40. possible solution: $\frac{18}{9} = 2; 3\frac{2}{3} + 2 = 5\frac{2}{3}$

Complete DAILY REVIEW Ⓒ on page 422.

Lesson 51 125

5. Direct the students to form an acute angle with their Anglers, trace the inside of the angle, and label 3 points to name the angle.

   ➤ **How can you make sure that you drew an acute angle? *Elicit that you can use a protractor to see if the angle measures less than 90°.***

6. Instruct each student to measure his angle and to write the measure. Give guidance as needed. ***Answers will vary but should be less than 90°.***

7. Follow a similar procedure for straight angles (180°) and obtuse angles (> 90° but < 180°).

   ➤ **Why is it helpful to classify an angle before finding its measure? *Elicit that if you know the angle is obtuse (greater than 90°) or acute (less than 90°), you can avoid making the mistake of using the wrong scale of numbers on the protractor, and you can check the reasonableness of your angle measurement.***

8. Display and distribute the Angles page. Write for display the classifications *right*, *acute*, *obtuse*, and *straight*, and the measurements 30°, 45°, 60°, 90°, 120°, 150°, and 180°. Choose students to classify each angle listed on the displayed page and to estimate its measure using the measurements written for display.

   Direct each student to measure the angle and to write the measurement on his page.

   $m\angle ABC$ = ***180°*** $m\angle FBC$ = ***90°*** $m\angle DBC$ = ***45°***
   $m\angle EBF$ = ***30°*** $m\angle ABD$ = ***135°*** $m\angle CBE$ = ***120°***
   $m\angle ABF$ = ***90°*** $m\angle EBD$ = ***165°*** $m\angle ABE$ = ***60°***

Draw angles

Guide the students in using their protractors to draw angles that measure 45°, 60°, and 120°; read the following instructions.

➤ **Draw a ray.**
➤ **Align the center point of the base of the protractor with the endpoint of the ray.**
➤ **Align the ray with the 0° line on the protractor.**
➤ **Draw a point at the desired degree mark.**
➤ **Draw the second ray, connecting the point marking the desired degree with the endpoint of the first ray.**
➤ **Check the angle measurement using the protractor.**

Student Text pp. 124–25

Lesson 52

Student Text pp. 126–27

Daily Review p. 422d

Objectives

- Develop an understanding of supplementary and complementary angles
- Find the unknown measure of an angle in a pair of supplementary angles and in a pair of complementary angles
- Recognize angles: right, acute, obtuse, straight
- Use a protractor to measure angles

Teacher Materials

- Supplementary Angles, page IA27 (CD)
- Complementary Angles, page IA28 (CD)
- Complementary & Supplementary Angles, page IA29 (CD)
- An overhead protractor

Student Materials

- A protractor

Introduce the Lesson

Pairs of angles are related to their measures. The relationship of angles is studied in science when learning about light. When light bounces off a smooth surface, the light reflects at the same angle.

Understanding the relationship of angles is also useful when playing some sports, such as miniature golf, pool, and hockey. Athletes who play these sports can use their knowledge of angle relationships to aim the ball or the puck so that it hits the side of the playing area and rebounds at the desired angle.

Teach for Understanding

Develop an understanding of supplementary angles

1. Write $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and select a student to explain the expression. ***Line AB is perpendicular to line CD. The lines intersect, forming right angles at the point of intersection.***
2. Direct the students to use their protractors to draw $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ and to label the point of intersection as point *G*. Give guidance if needed.
   - ➤ **What is the measure of $\angle AGC$?** ***90°*** **$\angle CGB$?** ***90°***

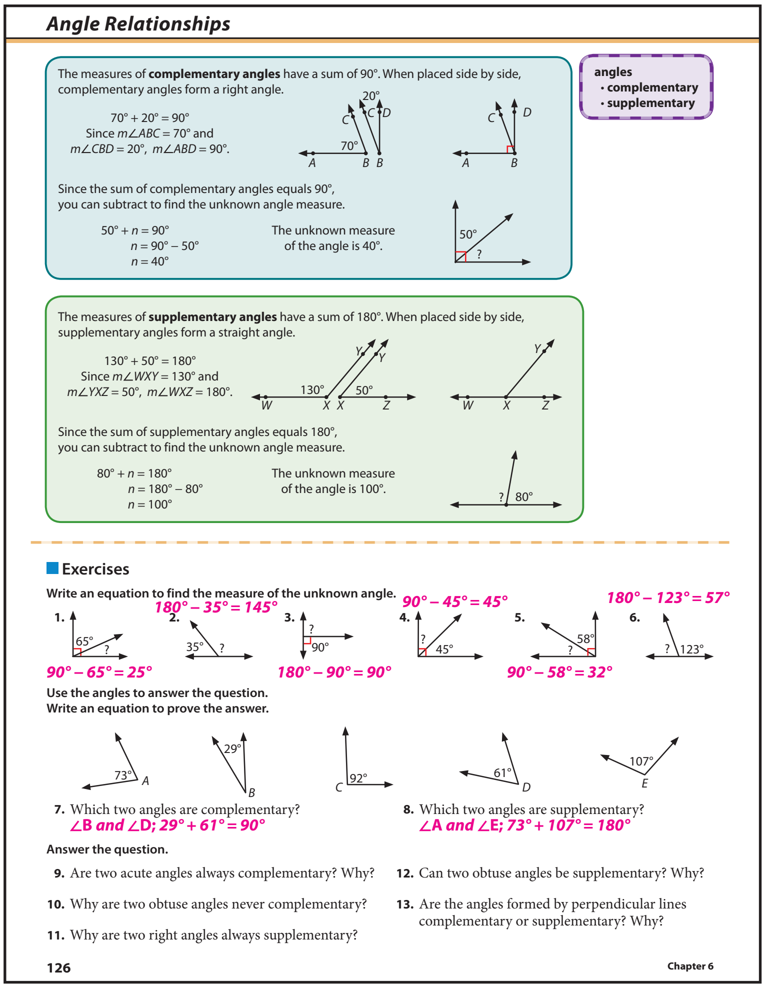
   Instruct the students to draw a small square in the corner of $\angle AGC$. Remind them that a small square in an angle indicates a right angle, a measure of 90°.
   - ➤ **What is the measure of $\angle AGB$? How do you know?** ***180°; answers will vary, but elicit $90° + 90° = 180°$.*** Write $90° + 90° = 180°$ for display.
3. Explain that a pair of angles whose combined measures equal 180° are *supplementary angles*. Since the sum of the measures of $\angle AGC$ and $\angle CGB$ equals 180°, they are supplementary angles. Point out that any two right angles are supplementary because the sum of their measures is 180°.
   - ➤ **How many right angles were formed where your perpendicular lines intersect?** ***4*** **How many pairs of right angles were formed? Name them.** ***4; $\angle AGC$ and $\angle CGB$, $\angle CGB$ and $\angle BGD$, $\angle BGD$ and $\angle DGA$, $\angle DGA$ and $\angle AGC$***
   - ➤ **What do you know about each pair of right angles?** ***They are supplementary angles; the sum of their measures is 180°.***
4. Display the Supplementary Angles page with problems 1–4 covered. Choose a student to read aloud the statement.
5. Select a student to measure $\angle JFL$. Instruct him to draw an arc to indicate the measured angle and to write the measure beside the arc. ***130°*** Choose another student to do the same for $\angle LFK$. ***50°***
   - ➤ **Are $\angle JFL$ and $\angle LFK$ supplementary angles? Why?** ***Yes; the sum of their measures is 180°.***
   - ➤ **What type of angle is $\angle JFK$? How do you know?** ***A straight angle; its measure is 180°.*** Explain that when two supplementary angles are placed side by side and share a common vertex and a common ray, they form a straight angle.

Find the unknown measure of an angle in a pair of supplementary angles

1. Elicit that $\overleftrightarrow{LM}$ on the Supplementary Angles page intersects $\overleftrightarrow{JK}$.
   - ➤ **What do you know about $\angle MFL$?** ***It is a straight angle.***
   - ➤ **How can you find the unknown measure of $\angle MFJ$ without using a protractor? Why?** ***Subtract the known measure (130°) from 180°; elicit that since the sum of the measures of two supplementary angles equals 180°, you can subtract the known measure of one angle from 180° to determine the unknown measure of the other angle.***

   Write for display *since $m\angle JFL = 130°$, $m\angle MFJ = 180° - 130° = \_\_°$* and lead the students in reading the statement aloud. Remind them that an *m* before an angle symbol name means "the measure of." Explain that $\angle MFJ$ is the *supplement* of $\angle JFL$. Its measure added to the measure of $\angle JFL$ equals 180°. Choose a student to write the equation to find the measure of $\angle MFJ$ and solve it. ***$180° - 130° = 50°$*** Draw an arc and write the measure in the angle.
2. Follow a similar procedure for $\angle KFM$. ***130°*** Point out that $\angle KFM$ is the supplement of both $\angle JFM$ and $\angle LFK$.
   - ➤ **Is $\angle JFL$ an acute angle or an obtuse angle? Why?** ***obtuse angle; $m\angle JFL > 90°$*** **Is $\angle LFK$ an acute angle or an obtuse angle? Why?** ***acute angle; $m\angle LFK < 90°$***

   Point out that supplementary angles can be 2 right angles $(90° + 90°)$ or 1 acute angle and 1 obtuse angle (e.g., $89° + 91°$).
3. Direct attention to problem 1. Point out that angles do not need to be placed beside each other in order to be supplementary.
   - ➤ **Are these two angles supplementary angles? Why?** ***Yes; the measures of the angles have a sum of 180°.***
   - ➤ **Are the two angles in problem 2 supplementary angles?** ***No; the sum of the measures is 140°, not the 180° necessary for angles to be supplementary.***
4. For problems 3 and 4, choose students to tell which pair of angles in each problem are supplementary and to explain their answers. ***85° and 95°; 146° and 34°***
   - ➤ **Can supplementary angles consist of a right angle and an acute angle? Why? Give an example to support your answer.** ***No; the sum of the measures would be less than 180°. Examples will vary.*** Elicit that the sum of the measures of a right angle (90°) and the greatest possible acute angle (89°) would be less than 180°.
   - ➤ **Can supplementary angles consist of a right angle and an obtuse angle? Why? Give an example to support your answer.** ***No; the sum of the measures would be greater than 180°. Examples will vary.*** Elicit that the sum of the measures of a right angle (90°) and the least possible obtuse angle (91°) would be greater than 180°.



Angle Relationships

The measures of **complementary angles** have a sum of 90°. When placed side by side, complementary angles form a right angle.

70° + 20° = 90°
Since m∠ABC = 70° and
m∠CBD = 20°, m∠ABD = 90°.

Since the sum of complementary angles equals 90°, you can subtract to find the unknown angle measure.

$50° + n = 90°$
$n = 90° - 50°$
$n = 40°$

The unknown measure of the angle is 40°.

angles
- complementary
- supplementary

The measures of **supplementary angles** have a sum of 180°. When placed side by side, supplementary angles form a straight angle.

130° + 50° = 180°
Since m∠WXY = 130° and
m∠YXZ = 50°, m∠WXZ = 180°.

Since the sum of supplementary angles equals 180°, you can subtract to find the unknown angle measure.

$80° + n = 180°$
$n = 180° - 80°$
$n = 100°$

The unknown measure of the angle is 100°.

Exercises

Write an equation to find the measure of the unknown angle.

1. *90° – 65° = 25°*
2. *180° – 35° = 145°*
3. *180° – 90° = 90°*
4. *90° – 45° = 45°*
5. *90° – 58° = 32°*
6. *180° – 123° = 57°*

Use the angles to answer the question.
Write an equation to prove the answer.

7. Which two angles are complementary? *∠B and ∠D; 29° + 61° = 90°*
8. Which two angles are supplementary? *∠A and ∠E; 73° + 107° = 180°*

Answer the question.

9. Are two acute angles always complementary? Why?
10. Why are two obtuse angles never complementary?
11. Why are two right angles always supplementary?
12. Can two obtuse angles be supplementary? Why?
13. Are the angles formed by perpendicular lines complementary or supplementary? Why?

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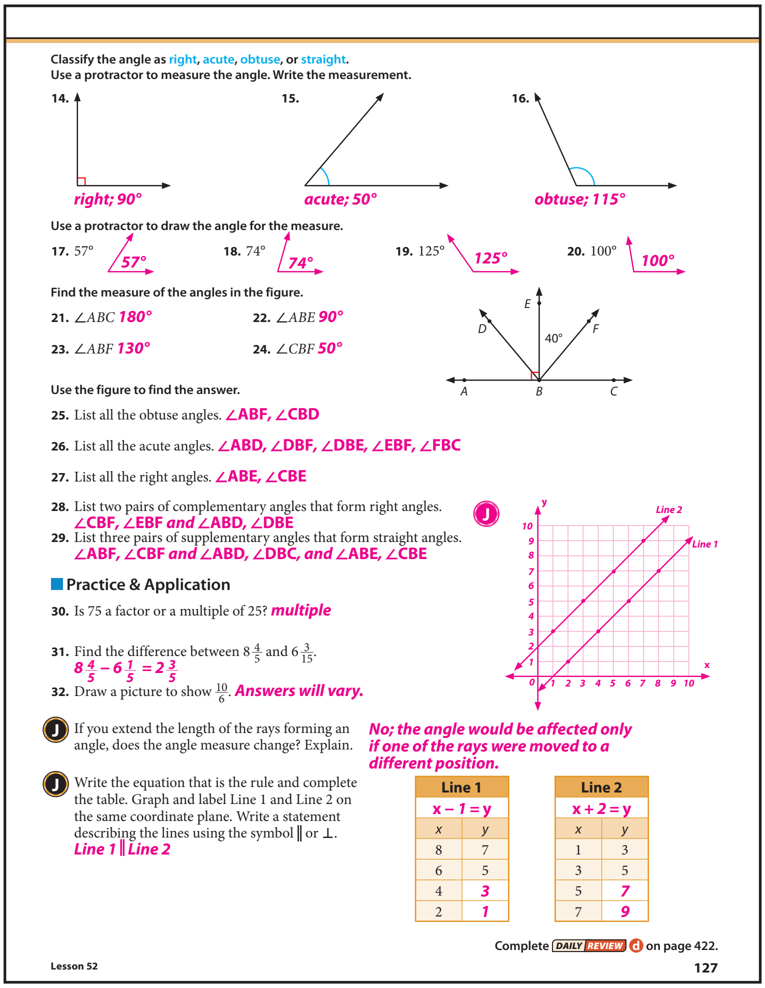


Classify the angle as right, acute, obtuse, or straight.
Use a protractor to measure the angle. Write the measurement.

14. *right; 90°*
15. *acute; 50°*
16. *obtuse; 115°*

Use a protractor to draw the angle for the measure.

17. 57° *57°*
18. 74° *74°*
19. 125° *125°*
20. 100° *100°*

Find the measure of the angles in the figure.

21. ∠ABC *180°*
22. ∠ABE *90°*
23. ∠ABF *130°*
24. ∠CBF *50°*

Use the figure to find the answer.

25. List all the obtuse angles. *∠ABF, ∠CBD*
26. List all the acute angles. *∠ABD, ∠DBF, ∠DBE, ∠EBF, ∠FBC*
27. List all the right angles. *∠ABE, ∠CBE*
28. List two pairs of complementary angles that form right angles. *∠CBF, ∠EBF and ∠ABD, ∠DBE*
29. List three pairs of supplementary angles that form straight angles. *∠ABF, ∠CBF and ∠ABD, ∠DBC, and ∠ABE, ∠CBE*

Practice & Application

30. Is 75 a factor or a multiple of 25? *multiple*
31. Find the difference between $8\frac{4}{5}$ and $6\frac{3}{15}$. *$8\frac{4}{5} - 6\frac{1}{5} = 2\frac{3}{5}$*
32. Draw a picture to show $\frac{10}{6}$. *Answers will vary.*

Ⓙ If you extend the length of the rays forming an angle, does the angle measure change? Explain. *No; the angle would be affected only if one of the rays were moved to a different position.*

Ⓙ Write the equation that is the rule and complete the table. Graph and label Line 1 and Line 2 on the same coordinate plane. Write a statement describing the lines using the symbol ∥ or ⊥. *Line 1 ∥ Line 2*

Line 1	
x – 1 = y	
x	y
8	7
6	5
4	*3*
2	*1*

Line 2	
x + 2 = y	
x	y
1	3
3	5
5	*7*
7	*9*

Complete DAILY REVIEW d on page 422.

Lesson 52 127

Develop an understanding of complementary angles

1. On the Supplementary Angles page, draw a line that intersects $\overleftrightarrow{JK}$ at point F and forms a right angle. Draw point *G* and point *H* on the line to show ∠*GFL* and ∠*HFK*. Mark the four right angles.
   - ➤ **Are ∠GFL and ∠LFK supplementary angles? Why?** ***No; elicit that the sum of the measures of supplementary angles is 180°, and the sum of the measure of ∠GFL and ∠LFK is 90°; two 90° angles (right angles) are supplementary angles.***
2. Explain that a pair of angles whose combined measures equal 90° are *complementary angles*. Since the sum of the measures of ∠*GFL* and ∠*LFK* equals 90°, they are complementary angles.
   - ➤ **When two complementary angles are placed side by side and share a common vertex and a common ray, what kind of angle do they form?** ***a right angle*** **What other pair of complementary angles do you see?** ***∠JFM and ∠MFH***

Find the unknown measure of an angle in a pair of complementary angles

1. Display the Complementary Angles page and choose a student to read aloud the statement.
   - ➤ **What complementary angles are in the top figure? Why?** ***∠RSU and ∠UST, and ∠PSQ and ∠QSR; elicit that the sum of the measures for each pair of angles is 90°.***
2. Choose a student to measure ∠*PSQ* and write the measure in the angle. ***70°*** Select another student to measure ∠*UST* and write its measure. ***25°*** Elicit that the complement of ∠*PSQ* is ∠*QSR* and the complement of ∠*UST* is ∠*RSU*.
3. Elicit that you can determine each unknown measure by subtracting the known measure of its complement from 90°. Write *since m∠PSQ = 70°, m∠QSR = 90° – 70° =* __°; lead in reading the statement aloud. Guide the students in finding both unknown measures. ***m∠QSR = 20°; m∠RSU = 90° – 25° = 65°.***
4. Follow a procedure similar to the one used for supplementary angles to guide the students in determining the answers for problems 1–4: 1. ***not complementary***; 2. ***complementary***; 3. ***50° and 40°***; 4. ***45° and 45°***.
   - ➤ **Can complementary angles consist of an acute angle and an obtuse angle? Why? Give an example to support your answer.** ***No; the sum of their measures would be greater than 90°. Examples will vary.*** Elicit that the sum of the measures of the least possible acute angle (1°) and the least possible obtuse angle (91°) would be greater than 90°.
   - ➤ **Can complementary angles consist of 2 obtuse angles? Why?** ***No; elicit that the measure of 1 least possible obtuse angle is 91°.***
   - ➤ **Can complementary angles consist of 2 acute angles? Why? Give an example to support your answer.** ***Yes; the sum of the measures could equal 90°. Examples will vary.*** Point out that complementary angles are always 2 acute angles, but 2 acute angles are not always complementary.
5. Display the Complementary & Supplementary Angles page. Explain that sometimes angles are referred to by number rather than labeled points. Select students to name complementary and supplementary angles in the top figure and in the angles at the bottom of the page.

Student Text pp. 126–27

(**Note**: *Assessment* available on Teacher's Toolkit CD.)

Lesson 53

Student Text pp. 128–29

Daily Review p. 423e

Objectives

- Identify and name line segments
- Demonstrate an understanding of regular and irregular polygons
- Identify the number of sides and interior angles in a polygon
- Graph points on a coordinate plane to form a polygon

Teacher Materials

- Coordinate Plane (from Lesson 49)

Student Materials

- Coordinate Planes, page IA25 (CD)
- 8 toothpicks or craft sticks
- A ruler

Note

The *Polygons* chart from the Teaching Visuals section of the Teacher's Toolkit CD is available for review in this lesson. Other supplemental charts are also included on the CD.

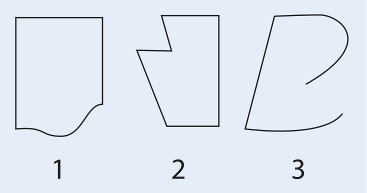
Teach for Understanding

Identify and name line segments

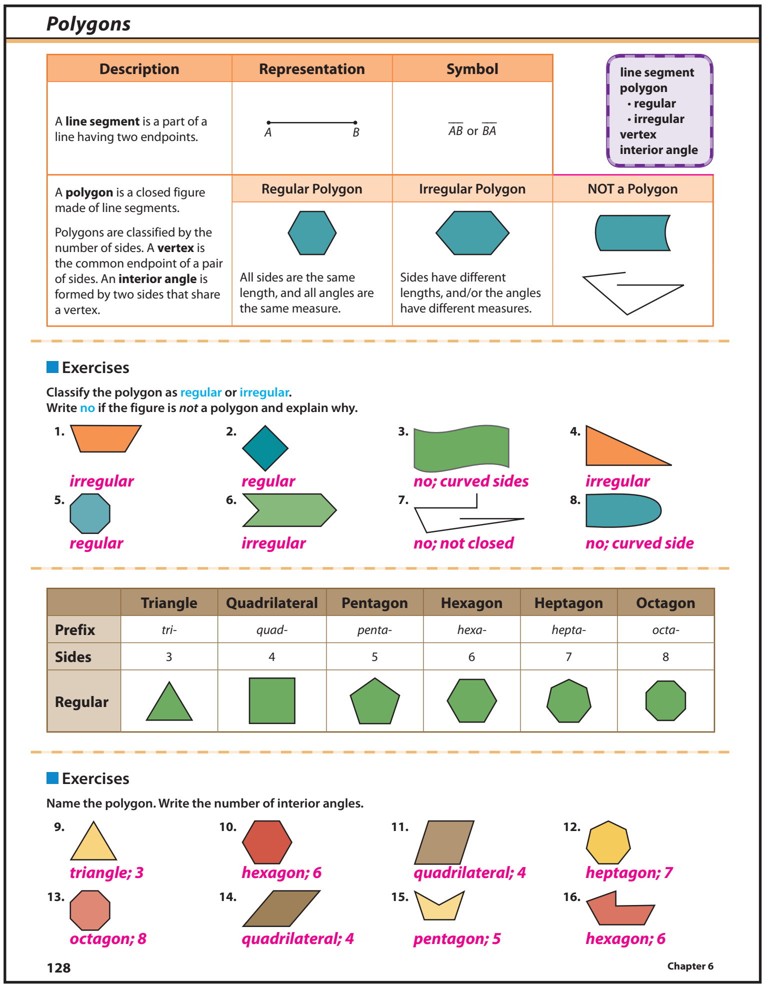
1. Draw point A and point B for display. Remind the students that a point has no length, no width, and no thickness.
   ➤ How many lines can you draw that contain point A and point B? *1 line*
   Draw a line that contains and extends beyond the points; draw an arrow at each end.
   ➤ What is the name of this line? *line* AB *or line* BA Write $\overleftrightarrow{AB}$ and $\overleftrightarrow{BA}$ for display.
   ➤ Does it matter which point you name first? Why? *No; elicit that since a line has no beginning and no end, any 2 points on the line can be used in any order to name the line.*
   ➤ What is a ray? *part of a line that has one endpoint and extends endlessly in one direction*
   ➤ What 2 rays are on this line? *ray* AB *and ray* BA Write $\overrightarrow{AB}$ and $\overrightarrow{BA}$.
   ➤ Do these names refer to the same ray? Why? *No; elicit that since the rays extend in opposite directions from different endpoints and since the endpoint is always given first, the names are not interchangeable.*
2. Write *line segment* for display. Explain that a line segment is a geometric figure that is part of a line and has 2 endpoints.
   ➤ What line segment is part of line AB? *line segment* AB *or line segment* BA Write $\overline{AB}$ and $\overline{BA}$.
   ➤ How does a line segment differ from a line? *Elicit that a line segment does not go on endlessly in opposite directions; it is a distinct part of a line.* Lead a discussion about the written symbol used to name a line segment.
3. Direct the students to draw a line segment and to name it.

Demonstrate an understanding of polygons

1. Write *polygon* for display. Tell the students that the word *polygon* comes from two Greek words: *poly* meaning "many" and *gon* meaning "angles." Explain that a polygon is a closed figure made up of line segments.
2. Draw and number figures similar to the ones below.



   ➤ Is figure 1 a polygon? Why? *No; the curved side is not a line segment.*
   ➤ Is figure 2 a polygon? Why? *Yes; it is a closed figure made up of line segments.*
   ➤ Is figure 3 a polygon? Why? *No; it is not a closed figure.*
3. Distribute the toothpicks or craft sticks. Direct the students to make a polygon with the fewest number of sides.
   ➤ What polygon has the fewest number of sides? *a triangle*
   Write *triangle* for display.
   ➤ How many sides are in a triangle? *3*
   Explain that each pair of sides in a polygon forms an angle, and the common endpoint is a *vertex*. The angle that is formed inside a polygon is an *interior angle*.
   ➤ How many interior angles are in a triangle? *3*
   ➤ What does the prefix *tri* mean? *three*
   ➤ What other words have the prefix *tri*? *possible answers: tricycle, tripod, triathlon, triple, triplets*
   ➤ Are the sides of your triangle the same length? Why? *Yes; all the toothpicks are the same length.*
   ➤ Do triangles always have sides that are the same length? *no*
4. Explain that polygons with sides that are the same length and angles that have the same measure are *regular polygons*, and polygons with sides of different lengths or angles of different measures are *irregular polygons*. Point out that every regular triangle is an *equilateral* triangle. Explain that the word *equilateral* comes from two Latin words: *equi* meaning "equal" and *lateralis* meaning "sides."
5. Instruct a student to draw an irregular triangle for display while the other students draw one on paper.
   ➤ Using 3 toothpicks, what figure could you make that is not a polygon? *a 3-sided open figure*
6. Direct the students to use 4 toothpicks to make a polygon with 4 sides.
   ➤ What polygon did you make? *possible answers: square, parallelogram, rhombus, quadrilateral*
   Write *quadrilateral* for display. Tell the students that the word *quadrilateral* contains the prefix *quad* meaning "four" and the base word *lateral* meaning "side." Explain that a quadrilateral is a four-sided polygon. Point out that every regular quadrilateral (4 sides with the same length and 4 angles with the same measure) is a *square*.
   ➤ What other words have the prefix *quad*? *possible answers: quadruplets, quadrillion, quadriceps, quadriplegic, quadrangle*
7. Direct the students to use 5 toothpicks to make a polygon with 5 sides.
   ➤ What is the name of this polygon? *Elicit that the figure is a pentagon.* Write *pentagon* for display.
   ➤ How many sides does a pentagon have? *5* interior angles? *5*
   ➤ Are the sides of your pentagon the same length? *yes*
   ➤ Are the angles of your pentagon the same measure? *Answers will vary.*
   Instruct a student to draw an irregular pentagon for display while the other students draw one on paper.
8. Continue the activity for hexagon (6 sides and 6 angles), heptagon (7 sides and 7 angles), and octagon (8 sides and 8 angles).



Polygons

Description	Representation	Symbol	
A **line segment** is a part of a line having two endpoints.	A B	$\overline{AB}$ or $\overline{BA}$	**line segment** **polygon** **• regular** **• irregular** **vertex** **interior angle**
A **polygon** is a closed figure made of line segments. Polygons are classified by the number of sides. A **vertex** is the common endpoint of a pair of sides. An **interior angle** is formed by two sides that share a vertex.	**Regular Polygon** All sides are the same length, and all angles are the same measure.	**Irregular Polygon** Sides have different lengths, and/or the angles have different measures.	**NOT a Polygon**

Exercises

Classify the polygon as regular or irregular.
Write no if the figure is *not* a polygon and explain why.

1. *irregular*
2. *regular*
3. *no; curved sides*
4. *irregular*
5. *regular*
6. *irregular*
7. *no; not closed*
8. *no; curved side*

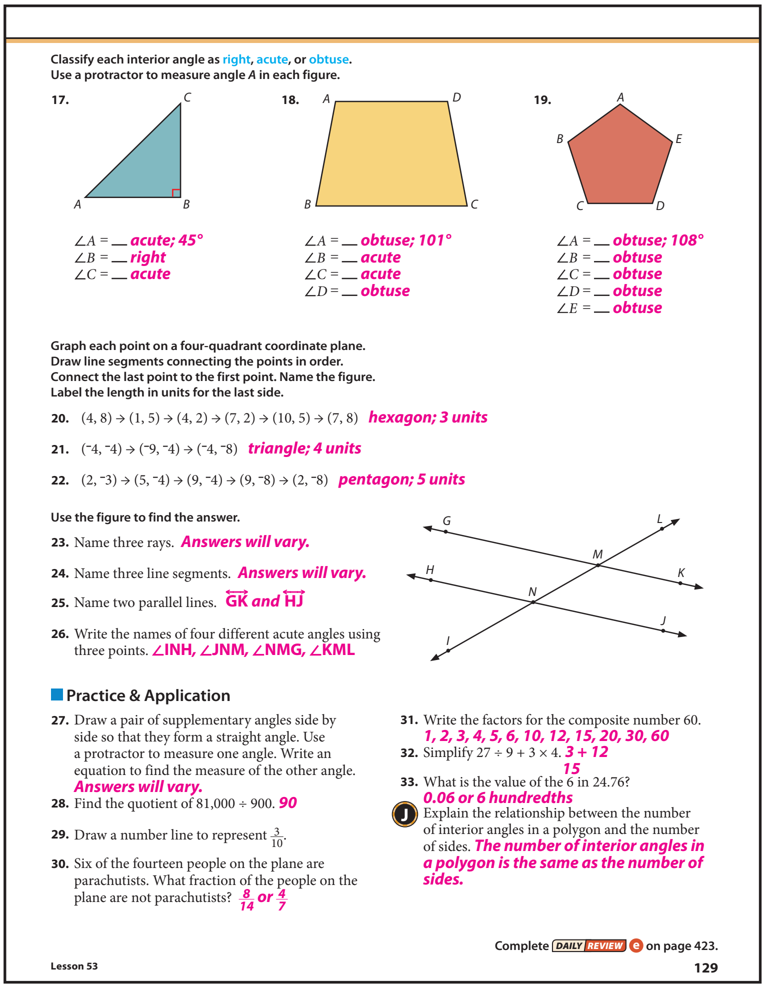
	Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon
Prefix	*tri-*	*quad-*	*penta-*	*hexa-*	*hepta-*	*octa-*
Sides	3	4	5	6	7	8
Regular						

Exercises

Name the polygon. Write the number of interior angles.

9. *triangle; 3*
10. *hexagon; 6*
11. *quadrilateral; 4*
12. *heptagon; 7*
13. *octagon; 8*
14. *quadrilateral; 4*
15. *pentagon; 5*
16. *hexagon; 6*

128 Chapter 6



Classify each interior angle as right, acute, or obtuse.
Use a protractor to measure angle *A* in each figure.

17.
$\angle A$ = __ *acute; 45°*
$\angle B$ = __ *right*
$\angle C$ = __ *acute*

18.
$\angle A$ = __ *obtuse; 101°*
$\angle B$ = __ *acute*
$\angle C$ = __ *acute*
$\angle D$ = __ *obtuse*

19.
$\angle A$ = __ *obtuse; 108°*
$\angle B$ = __ *obtuse*
$\angle C$ = __ *obtuse*
$\angle D$ = __ *obtuse*
$\angle E$ = __ *obtuse*

Graph each point on a four-quadrant coordinate plane.
Draw line segments connecting the points in order.
Connect the last point to the first point. Name the figure.
Label the length in units for the last side.

20. $(4, 8) \rightarrow (1, 5) \rightarrow (4, 2) \rightarrow (7, 2) \rightarrow (10, 5) \rightarrow (7, 8)$ *hexagon; 3 units*

21. $(^{-}4, ^{-}4) \rightarrow (^{-}9, ^{-}4) \rightarrow (^{-}4, ^{-}8)$ *triangle; 4 units*

22. $(2, ^{-}3) \rightarrow (5, ^{-}4) \rightarrow (9, ^{-}4) \rightarrow (9, ^{-}8) \rightarrow (2, ^{-}8)$ *pentagon; 5 units*

Use the figure to find the answer.

23. Name three rays. ***Answers will vary.***
24. Name three line segments. ***Answers will vary.***
25. Name two parallel lines. ***$\overleftrightarrow{GK}$ and $\overleftrightarrow{HJ}$***
26. Write the names of four different acute angles using three points. ***∠INH, ∠JNM, ∠NMG, ∠KML***

Practice & Application

27. Draw a pair of supplementary angles side by side so that they form a straight angle. Use a protractor to measure one angle. Write an equation to find the measure of the other angle. ***Answers will vary.***
28. Find the quotient of 81,000 ÷ 900. ***90***
29. Draw a number line to represent $\frac{3}{10}$.
30. Six of the fourteen people on the plane are parachutists. What fraction of the people on the plane are not parachutists? ***$\frac{8}{14}$ or $\frac{4}{7}$***
31. Write the factors for the composite number 60. ***1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60***
32. Simplify 27 ÷ 9 + 3 × 4. ***3 + 12***
15
33. What is the value of the 6 in 24.76? ***0.06 or 6 hundredths***

Ⓙ Explain the relationship between the number of interior angles in a polygon and the number of sides. ***The number of interior angles in a polygon is the same as the number of sides.***

Complete DAILY REVIEW e on page 423.

Lesson 53 129

Graph points on a coordinate plane to form a polygon

1. Display the Coordinate Plane page and distribute the Coordinate Planes page. Write for display: $(^{-}4, 3) \rightarrow (^{-}2, 8) \rightarrow (^{-}7, 8) \rightarrow (^{-}9, 3)$.
2. Direct the students to graph the first 2 points on the first coordinate plane and to draw a line segment connecting the points, using a ruler as a guide. Explain that the arrows indicate the order in which to graph the points and to draw a line to connect the points. Instruct the students to graph the third point and to draw a line segment connecting it to the second point; then graph the fourth point and draw a line segment connecting it to the third point. Finally, direct the students to draw a line segment connecting the fourth point to the first point.

   (*Note:* Demonstrate graphing the points and drawing the line segments as needed.)

   - ➤ How many sides does your polygon have? *4* vertices? *4*
   - ➤ How many interior angles are in your polygon? *4*
   - ➤ What type of polygon did you graph? *quadrilateral*
   - ➤ How can you find the length in units of the final side that you drew? *Elicit that you can count the number of units.* What is the length? *5 units*

   (*Note:* Allow the students to use the other coordinate planes on the page to graph polygons while completing Student Text page 129, problems 20–22.)

Student Text pp. 128–29

Lesson 54

Student Text pp. 130–31

Daily Review p. 423f

Objectives

- Develop the understanding that the sum of the measures of the angles in a triangle equals 180°
- Find the unknown measure of an angle in a triangle
- Classify triangles by their angles: right, acute, obtuse
- Classify triangles by their sides: equilateral, isosceles, scalene
- Measure the angles in a triangle using a protractor

Teacher Materials

- Triangles, page IA30 (CD)
- An overhead protractor

Student Materials

- Triangles, page IA30 (CD)
- A protractor
- A ruler

Teach for Understanding

Develop the understanding that the sum of the measures of the angles in a triangle equals 180°

1. Display and distribute the Triangles page. Write for display the coordinates for these points: L ($^-7$, 8); M ($^-7$, $^-8$); N (9, $^-8$).

➤ **What polygon do you predict you will form using these coordinates? Why?** ***A triangle; there will be 3 vertices.***

2. Direct the students to plot and label the points on the coordinate plane at the bottom of the page and to draw line segments connecting the points. Demonstrate on the displayed page. (***Note:*** Since the figure will be cut out, tell the students to write the labels inside the figure.)

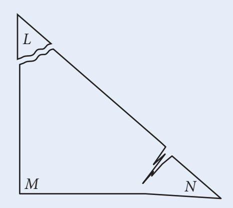
➤ **Was your prediction correct?**

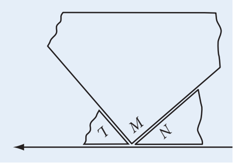
➤ **What angle measure in the triangle do you know without measuring it? Why?** ***∠M; elicit that ∠M is a square corner so it is a right angle, and right angles measure 90°.***

3. Instruct the students to first remove the bottom section of the page by cutting along the dashed line. Then direct them to cut out the graphed triangle and to tear ∠L and ∠N off of the triangle.

(***Note:*** Do not cut out the triangle on the displayed page.)

Direct the students to place the 3 angles side by side to form one larger angle.





➤ **What larger angle is formed when the 3 angles of Δ*LMN* are placed side by side?** ***a straight angle***

➤ **What is the measure of a straight angle?** ***180°***

➤ **What can you conclude about the sum of the 3 interior angle measures of a triangle?** ***The combined measure of the angles in a triangle equals 180°.***

Find the unknown measure of an angle in a triangle

1. Direct attention to ∠LMN on the displayed Triangles page. Remind the students that since ∠M is a right angle, they know that the measure of the angle is 90°. Label ∠M 90°.

➤ **What type of angle is *∠L*?** ***an acute angle***

➤ **What do you predict the measure of *∠L* is?** ***Answers will vary.***

2. Demonstrate measuring ∠L and write the measure (45°) in the angle.

➤ **How can you find the measure of *∠N* mathematically rather than by measuring the angle?** ***Elicit that you can subtract the sum of the measures of ∠L and ∠M from 180°.***

3. Write $m\angle L + m\angle M + m\angle N = 180°$ for display. Remind the students that the lowercase *m* means "the measure of." Lead in reading the equation aloud: *the measure of ∠L + the measure of ∠M + the measure of ∠N = 180°.*

Substitute the measures for the known angles and demonstrate solving the equation:

$90° + 45° + m\angle N = 180°$,

$135° + m\angle N = 180°$, $m\angle N = 180° - 135° = 45°$;

or $90° + 45° + m\angle N = 180°$,

$m\angle N = 180° - (90° + 45°) = 180° - 135° = 45°$.

Write 45° in ∠N on the page.

4. Choose students to write and solve an equation to find the unknown angle measures for triangles with these known angle measures.

140° and 15° ***180° − 155° = 25°***

70° and 50° ***180° − 120° = 60°***

➤ **Could you have a triangle with angle measures of 100°, 50°, and 40°? Why?** ***No; the combined angle measures of a triangle will always total 180°, and 100° + 50° + 40° = 190°.***

Classify triangles by their angles

1. Direct attention to ΔABC on the Triangles page.

(***Note:*** Throughout this activity, write each answer on the appropriate blank after the answer is given or determined and instruct each student to write the answer on his page.)

➤ **What type of angle is *∠B*?** ***a right angle***

➤ **What is the measure of a right angle?** ***90°***

➤ **What type of angle is *∠A*?** ***an acute angle***

➤ **How can you find the measure of *∠A*?** ***Elicit that you must use a protractor to measure the angle because you know only the measure of 1 angle in the triangle.***

Instruct the students to measure ∠A. ***25°***

➤ **How can you find the measure of *∠C* mathematically?** ***Subtract the sum of the 2 known angle measures from 180°.***

Direct the students to find the measure of ∠C mathematically. ***180° − (90° + 25°) = 65°***

2. Explain that triangles can be classified as acute, right, or obtuse, according to their angles.

➤ **Would you classify Δ*ABC* as an acute, right, or obtuse triangle? Why?** ***Elicit that the triangle is a right triangle because 1 of its angles is a right angle (90°).***

➤ **Can a triangle have more than 1 right angle? Why?** ***No; elicit that the sum of the 3 angle measures in any triangle is 180°; therefore, the combined measures of 2 right angles (180°) would not allow for a third angle.***

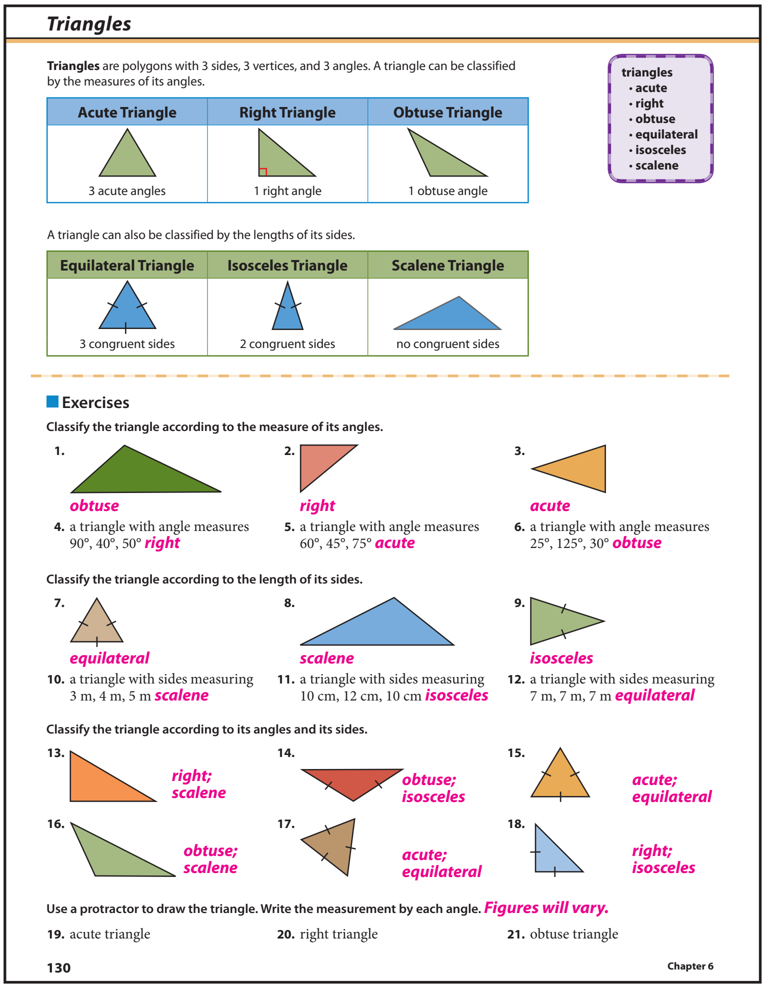
3. Direct the students to find the angle measures for ΔDEF.

➤ **What are the measures of the angles in Δ*DEF*?** ***Each of the angles is 60°.***

➤ **What is the least number of angles you had to measure with your protractor? Why?** ***2; accept any correct explanations.***

➤ **Would you classify Δ*DEF* as an acute, right, or obtuse triangle? Why?** ***Elicit that the triangle is an acute triangle because all of its angles are acute (less than 90°).***

➤ **How would you classify Δ*GHI*? Why?** ***Elicit that the triangle is an obtuse triangle because 1 of its angles is obtuse (greater than 90°).***



Triangles

Triangles are polygons with 3 sides, 3 vertices, and 3 angles. A triangle can be classified by the measures of its angles.

Acute Triangle	Right Triangle	Obtuse Triangle
3 acute angles	1 right angle	1 obtuse angle

triangles
- **acute**
- **right**
- **obtuse**
- **equilateral**
- **isosceles**
- **scalene**

A triangle can also be classified by the lengths of its sides.

Equilateral Triangle	Isosceles Triangle	Scalene Triangle
3 congruent sides	2 congruent sides	no congruent sides

Exercises

Classify the triangle according to the measure of its angles.

1. *obtuse*

2. *right*

3. *acute*

4. a triangle with angle measures 90°, 40°, 50° ***right***

5. a triangle with angle measures 60°, 45°, 75° ***acute***

6. a triangle with angle measures 25°, 125°, 30° ***obtuse***

Classify the triangle according to the length of its sides.

7. *equilateral*

8. *scalene*

9. *isosceles*

10. a triangle with sides measuring 3 m, 4 m, 5 m ***scalene***

11. a triangle with sides measuring 10 cm, 12 cm, 10 cm ***isosceles***

12. a triangle with sides measuring 7 m, 7 m, 7 m ***equilateral***

Classify the triangle according to its angles and its sides.

13. *right; scalene*

14. *obtuse; isosceles*

15. *acute; equilateral*

16. *obtuse; scalene*

17. *acute; equilateral*

18. *right; isosceles*

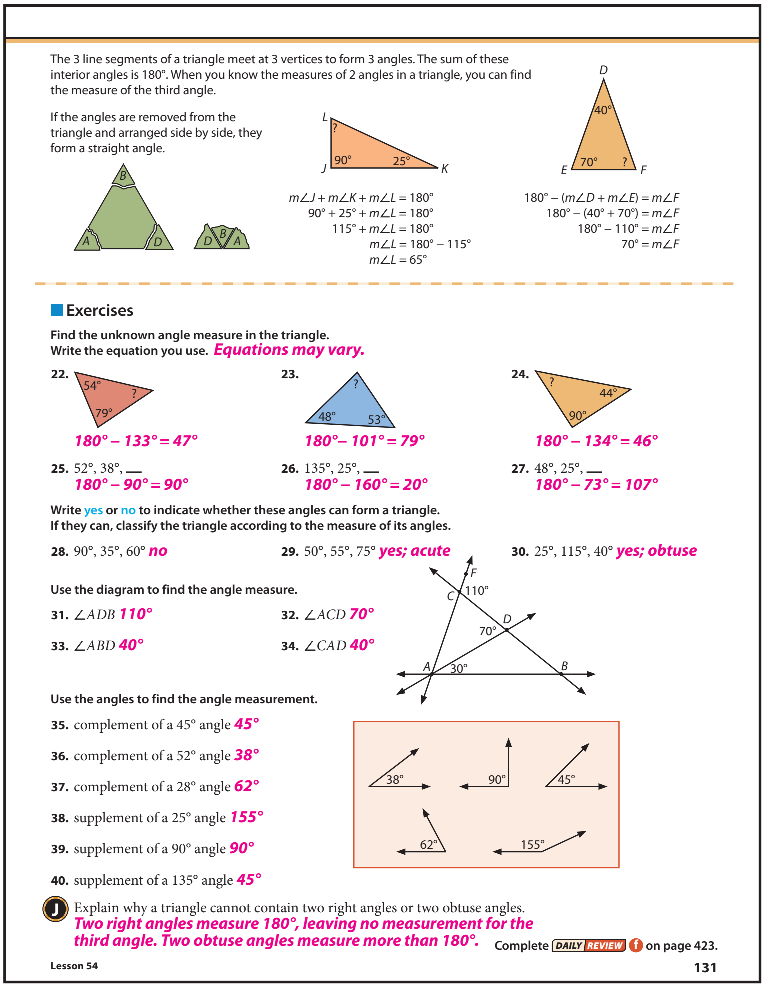
Use a protractor to draw the triangle. Write the measurement by each angle. ***Figures will vary.***

19. acute triangle

20. right triangle

21. obtuse triangle

130 Chapter 6



The 3 line segments of a triangle meet at 3 vertices to form 3 angles. The sum of these interior angles is 180°. When you know the measures of 2 angles in a triangle, you can find the measure of the third angle.

If the angles are removed from the triangle and arranged side by side, they form a straight angle.

$$\begin{aligned} m\angle J + m\angle K + m\angle L &= 180^\circ \\ 90^\circ + 25^\circ + m\angle L &= 180^\circ \\ 115^\circ + m\angle L &= 180^\circ \\ m\angle L &= 180^\circ - 115^\circ \\ m\angle L &= 65^\circ \end{aligned}$$

$$\begin{aligned} 180^\circ - (m\angle D + m\angle E) &= m\angle F \\ 180^\circ - (40^\circ + 70^\circ) &= m\angle F \\ 180^\circ - 110^\circ &= m\angle F \\ 70^\circ &= m\angle F \end{aligned}$$

Exercises

Find the unknown angle measure in the triangle.
Write the equation you use. ***Equations may vary.***

22. *180° – 133° = 47°*

23. *180°– 101° = 79°*

24. *180° – 134° = 46°*

25. 52°, 38°, __ *180° – 90° = 90°*

26. 135°, 25°, __ *180° – 160° = 20°*

27. 48°, 25°, __ *180° – 73° = 107°*

Write yes or no to indicate whether these angles can form a triangle.
If they can, classify the triangle according to the measure of its angles.

28. 90°, 35°, 60° ***no***

29. 50°, 55°, 75° ***yes; acute***

30. 25°, 115°, 40° ***yes; obtuse***

Use the diagram to find the angle measure.

31. ∠*ADB* ***110°***

32. ∠*ACD* ***70°***

33. ∠*ABD* ***40°***

34. ∠*CAD* ***40°***

Use the angles to find the angle measurement.

35. complement of a 45° angle ***45°***

36. complement of a 52° angle ***38°***

37. complement of a 28° angle ***62°***

38. supplement of a 25° angle ***155°***

39. supplement of a 90° angle ***90°***

40. supplement of a 135° angle ***45°***

J Explain why a triangle cannot contain two right angles or two obtuse angles. ***Two right angles measure 180°, leaving no measurement for the third angle. Two obtuse angles measure more than 180°.***

Complete DAILY REVIEW f on page 423.

Lesson 54 131

- ➤ **Can a triangle have more than 1 obtuse angle? Why?** ***No; elicit that the combined measures of 2 obtuse angles are greater than 180°, and 180° is the sum of the 3 angle measures in any triangle.***

4. Direct the students to find the angle measures for Δ*GHI*. ***20°, 20°, 140°***
5. Write the following angle measures for display. Select students to classify the triangles represented by the measurements.

   50°, 70°, 60° ***Acute; all 3 angles are acute.***
   19°, 117°, 44° ***Obtuse; one angle is obtuse.***
   90°, 27°, 63° ***Right; one angle is a right angle.***

Classify triangles by sides

1. Explain that a triangle can also be classified by the lengths of its sides. Write *equilateral*, *isosceles*, and *scalene* for display. Explain that an equilateral triangle has 3 congruent sides (i.e., 3 sides that are the same length), an isosceles triangle has 2 congruent sides, and a scalene triangle has no congruent sides.
2. Instruct each student to measure in millimeters the length of each side in Δ*ABC* and to write the measurements on his page.
   - ➤ **What is the length of $\overline{AB}$?** ***64 mm*** **$\overline{BC}$?** ***30 mm*** **$\overline{AC}$?** ***71 mm***
   - ➤ **Are any of the sides the same length?** ***no***
   - ➤ **Based on the lengths of its sides, what type of triangle is Δ*ABC*?** ***a scalene triangle*** Write *scalene* on the displayed page and instruct the students to write it.
3. Repeat the procedure for Δ*DEF*, measuring the length of each side in centimeters: $\overline{DF}$ ***3.7 cm***, $\overline{DE}$ ***3.7 cm***, $\overline{FE}$ ***3.7 cm; all 3 sides have the same length; an equilateral triangle***.

   Draw a slash mark through each side of Δ*DEF*. Explain that the slash marks show that the 3 sides are congruent. Direct the students to draw the slash marks on their triangles.
4. Follow a similar procedure for Δ*GHI*: $\overline{GH}$ ***6.9 cm***, $\overline{GI}$ ***3.7 cm***, $\overline{HI}$ ***3.7 cm; 2 sides have the same length; an isosceles triangle***.
5. Write the following side lengths for display. Select students to classify the triangles represented by the measurements.

   23 cm, 18 cm, 20 cm ***Scalene; there are no congruent sides.***
   8 m, 4 m, 8 m ***Isosceles; 2 sides are congruent.***
   12 cm, 12 cm, 12 cm ***Equilateral; all 3 sides are congruent.***

   - ➤ **Based on the measure of its angles and the length of its sides, what are the 2 ways to classify Δ*ABC*?** ***right triangle and scalene triangle*** **Δ*DEF*?** ***acute triangle and equilateral triangle*** **Δ*GHI*?** ***obtuse triangle and isosceles triangle***
6. You may choose to allow students to work in groups to answer the following questions, discuss the reasons for their answers, and draw examples of triangles to prove or disprove their answers.
   - ➤ **Is an acute triangle always an equilateral triangle?** ***no***
   - ➤ **Is an equilateral triangle always an acute triangle?** ***yes***
   - ➤ **Can a right triangle be an isosceles triangle?** ***yes***
   - ➤ **Can an obtuse triangle be a scalene triangle?** ***yes***

Student Text pp. 130–31

Lesson 55

Student Text pp. 132–33
Daily Review p. 424g

Objectives

- Distinguish between regular and irregular quadrilaterals
- Develop an understanding of the classification of quadrilaterals
- Develop the understanding that the sum of the angles of a quadrilateral equals 360°
- Find the unknown measure of an angle in a quadrilateral
- Measure the angles in a quadrilateral using a protractor

Teacher Materials

- Hierarchy of Quadrilaterals, page IA31 (CD)
- Quadrilaterals, page IA32 (CD)
- An overhead protractor

Student Materials

- Quadrilaterals, page IA32 (CD)
- Polygon Angle Measure, page IA33 (CD) (optional)
- A protractor
- A ruler

Teach for Understanding

Distinguish between regular and irregular quadrilaterals

1. Display the Hierarchy of Quadrilaterals page showing only the word *quadrilateral* at the top of the flow chart.

➤ **What are the distinguishing characteristics of a polygon that is a quadrilateral? Why?** *4 sides, 4 angles, and 4 vertices; elicit that the prefix* quad *means "four."*

2. Draw a regular and an irregular quadrilateral.

➤ **Which of these figures is the regular quadrilateral and which is the irregular quadrilateral? Why?** *The square is the regular quadrilateral, and the other figure is the irregular quadrilateral; the 4 sides of a regular quadrilateral are congruent (equal length), and its 4 angles are congruent (equal measure).*

Develop an understanding of the classification of quadrilaterals

➤ **How are triangles classified?** *by their angle measures (acute, right, obtuse) and by the lengths of their sides (scalene, equilateral, isosceles)*

1. Explain that quadrilaterals are classified by their angle measures, by the lengths of their sides, and whether opposite sides are parallel.
2. Uncover the trapezoid on the Hierarchy of Quadrilaterals page.

➤ **What do you notice about the sides and the angles of the trapezoid?** *1 pair of opposite sides is parallel.*

Explain that any quadrilateral with at least 1 pair of opposite sides that are parallel can be classified as a *trapezoid*.

Choose students to draw other trapezoids for display. Instruct each student to point out a pair of parallel lines in his figure. *Accept any quadrilateral with 1 pair of opposite sides that are parallel.*

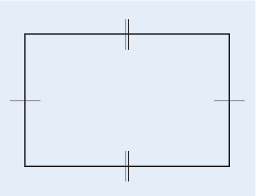
3. Uncover the parallelogram.

➤ **Can a parallelogram be classified as a trapezoid? Why?** *Yes; elicit that a parallelogram has 2 pairs of opposite sides that are parallel, so it meets the qualifications for a trapezoid.*

➤ **Besides having 2 pairs of opposite sides that are parallel, how does a parallelogram differ from a trapezoid?** *Elicit that the opposite sides are congruent.*

Point out that since both pairs of opposite sides in a parallelogram are parallel and congruent, the opposite angles are congruent.

4. Draw slash marks as shown. Remind the students that slash marks can be used to show congruency. Point out that the single slash marks indicate one pair of congruent sides, and the double slash marks indicate a second pair of congruent sides.



5. Explain that any quadrilateral with 2 pairs of opposite sides that are parallel and congruent can be classified as a *parallelogram*.
6. Follow a similar procedure to elicit the characteristics of a rectangle, a rhombus, and a square. Draw small squares in the right angles of the rectangle and the square as you discuss the figures. Refer to the characteristics of the figures listed on Student Text page 132 if needed.
7. Ask the following questions as you guide the students in analyzing the quadrilaterals. Point out that each classification is more specific than the classification(s) that comes before it and that geometric figures should be classified as specifically as possible.

➤ **Why are all of these polygons classified as quadrilaterals?** *They all have 4 sides, 4 angles, and 4 vertices.*

➤ **Which quadrilaterals have opposite sides that are congruent and opposite sides that are parallel?** *parallelogram, rhombus, rectangle, square*

➤ **Which quadrilaterals have all congruent sides?** *square and rhombus*

➤ **Which quadrilaterals have all right angles?** *rectangle and square*

➤ **Are all of these quadrilaterals trapezoids? How do you know?** *Yes; elicit that all of the quadrilaterals have at least 1 pair of parallel sides.*

➤ **Why are a rectangle and a rhombus parallelograms?** *Elicit that both the rectangle and the rhombus have 2 pairs of opposite sides that are parallel, opposite sides that are congruent, and opposite angles that are congruent.*

➤ **Why is a square a parallelogram?** *It has 2 pairs of opposite sides that are parallel, opposite sides that are congruent, and opposite angles that are congruent.*

➤ **Why is a square a rectangle?** *It has 2 pairs of opposite sides that are parallel, opposite sides that are congruent, and all right angles.*

➤ **Why is a square a rhombus?** *It has 2 pairs of opposite sides that are parallel, all congruent sides, and opposite angles that are congruent.*

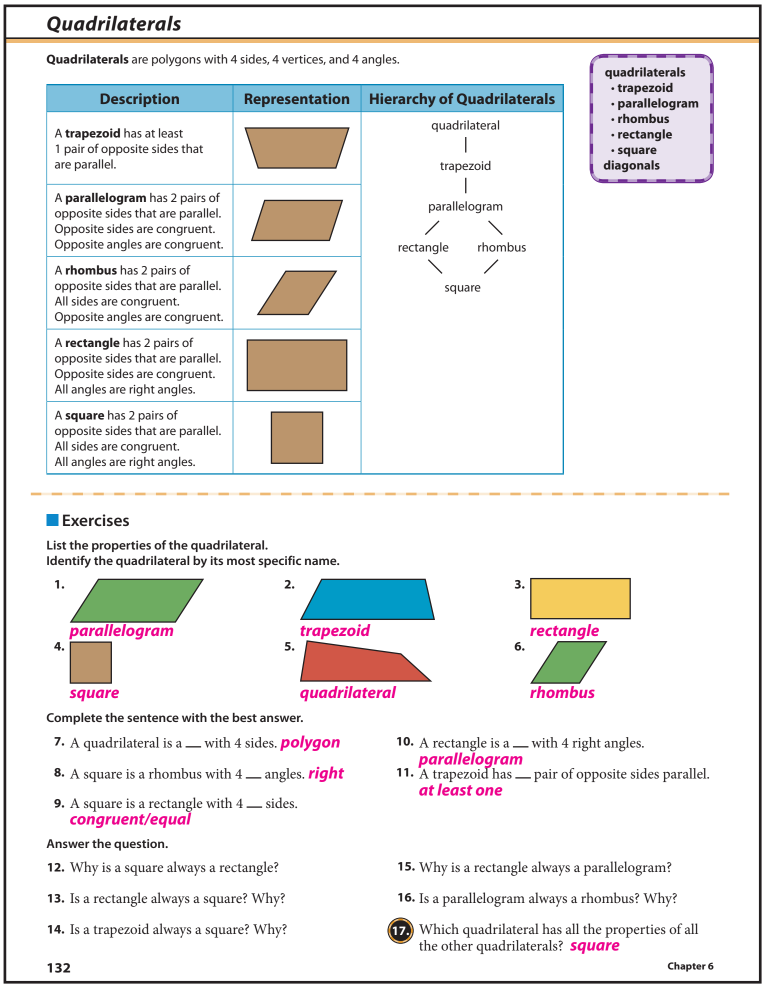
(Continue to display the Hierarchy of Quadrilaterals page.)

Develop the understanding that the sum of the angles in a quadrilateral equals 360°

➤ **What would be the result of drawing a diagonal line segment connecting opposite vertices in each quadrilateral on the displayed page?** *Answers may vary, but elicit that 2 triangles would be made in each quadrilateral.*

1. Draw a diagonal in each quadrilateral. Explain that a line segment that connects 2 nonadjacent vertices of a polygon is a *diagonal*; it divides a quadrilateral into 2 triangles.

➤ **What is the sum of the angles in a triangle?** *180°*



Quadrilaterals

Quadrilaterals are polygons with 4 sides, 4 vertices, and 4 angles.

Description	Representation	Hierarchy of Quadrilaterals
A **trapezoid** has at least 1 pair of opposite sides that are parallel.		quadrilateral trapezoid parallelogram rectangle rhombus square
A **parallelogram** has 2 pairs of opposite sides that are parallel. Opposite sides are congruent. Opposite angles are congruent.		
A **rhombus** has 2 pairs of opposite sides that are parallel. All sides are congruent. Opposite angles are congruent.		
A **rectangle** has 2 pairs of opposite sides that are parallel. Opposite sides are congruent. All angles are right angles.		
A **square** has 2 pairs of opposite sides that are parallel. All sides are congruent. All angles are right angles.		

quadrilaterals
- **trapezoid**
- **parallelogram**
- **rhombus**
- **rectangle**
- **square**

diagonals

Exercises

List the properties of the quadrilateral. Identify the quadrilateral by its most specific name.

1. *parallelogram*
2. *trapezoid*
3. *rectangle*
4. *square*
5. *quadrilateral*
6. *rhombus*

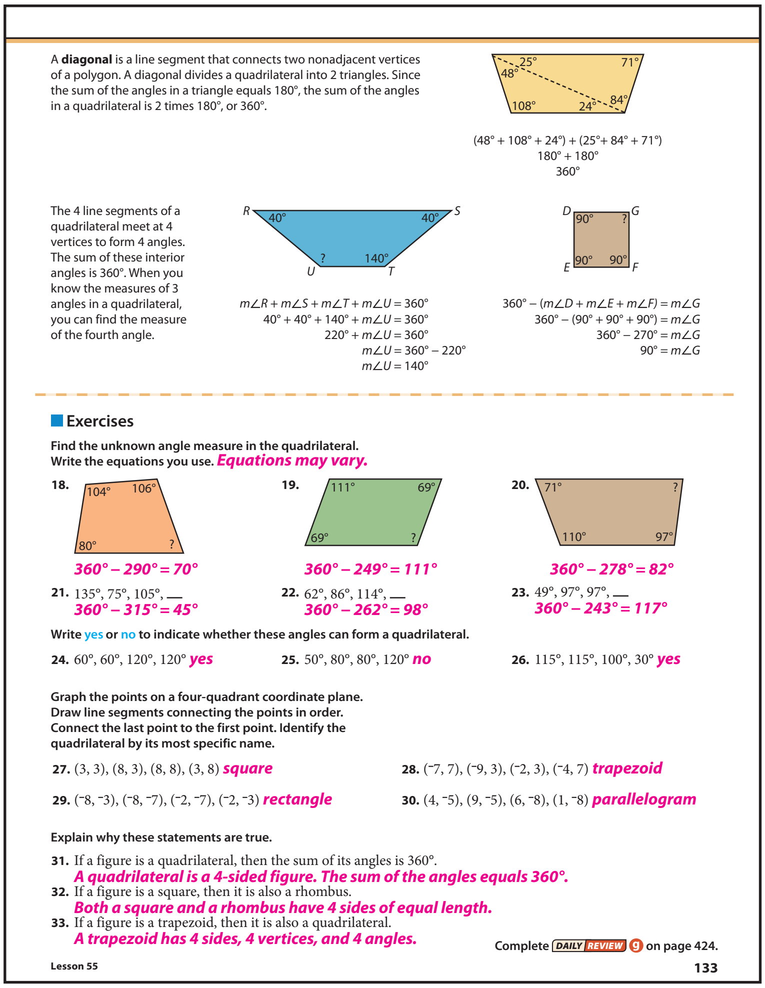
Complete the sentence with the best answer.

7. A quadrilateral is a ___ with 4 sides. *polygon*
8. A square is a rhombus with 4 ___ angles. *right*
9. A square is a rectangle with 4 ___ sides. *congruent/equal*
10. A rectangle is a ___ with 4 right angles. *parallelogram*
11. A trapezoid has ___ pair of opposite sides parallel. *at least one*

Answer the question.

12. Why is a square always a rectangle?
13. Is a rectangle always a square? Why?
14. Is a trapezoid always a square? Why?
15. Why is a rectangle always a parallelogram?
16. Is a parallelogram always a rhombus? Why?
17. Which quadrilateral has all the properties of all the other quadrilaterals? *square*

132 Chapter 6



A **diagonal** is a line segment that connects two nonadjacent vertices of a polygon. A diagonal divides a quadrilateral into 2 triangles. Since the sum of the angles in a triangle equals 180°, the sum of the angles in a quadrilateral is 2 times 180°, or 360°.

$$(48° + 108° + 24°) + (25° + 84° + 71°)$$
$$180° + 180°$$
$$360°$$

The 4 line segments of a quadrilateral meet at 4 vertices to form 4 angles. The sum of these interior angles is 360°. When you know the measures of 3 angles in a quadrilateral, you can find the measure of the fourth angle.

$$\begin{aligned} m\angle R + m\angle S + m\angle T + m\angle U &= 360° \\ 40° + 40° + 140° + m\angle U &= 360° \\ 220° + m\angle U &= 360° \\ m\angle U &= 360° - 220° \\ m\angle U &= 140° \end{aligned}$$

$$\begin{aligned} 360° - (m\angle D + m\angle E + m\angle F) &= m\angle G \\ 360° - (90° + 90° + 90°) &= m\angle G \\ 360° - 270° &= m\angle G \\ 90° &= m\angle G \end{aligned}$$

Exercises

Find the unknown angle measure in the quadrilateral. Write the equations you use. *Equations may vary.*

18. *360° − 290° = 70°*
19. *360° − 249° = 111°*
20. *360° − 278° = 82°*
21. 135°, 75°, 105°, ___ *360° − 315° = 45°*
22. 62°, 86°, 114°, ___ *360° − 262° = 98°*
23. 49°, 97°, 97°, ___ *360° − 243° = 117°*

Write yes or no to indicate whether these angles can form a quadrilateral.

24. 60°, 60°, 120°, 120° *yes*
25. 50°, 80°, 80°, 120° *no*
26. 115°, 115°, 100°, 30° *yes*

Graph the points on a four-quadrant coordinate plane. Draw line segments connecting the points in order. Connect the last point to the first point. Identify the quadrilateral by its most specific name.

27. (3, 3), (8, 3), (8, 8), (3, 8) *square*
28. (⁻7, 7), (⁻9, 3), (⁻2, 3), (⁻4, 7) *trapezoid*
29. (⁻8, ⁻3), (⁻8, ⁻7), (⁻2, ⁻7), (⁻2, ⁻3) *rectangle*
30. (4, ⁻5), (9, ⁻5), (6, ⁻8), (1, ⁻8) *parallelogram*

Explain why these statements are true.

31. If a figure is a quadrilateral, then the sum of its angles is 360°. *A quadrilateral is a 4-sided figure. The sum of the angles equals 360°.*
32. If a figure is a square, then it is also a rhombus. *Both a square and a rhombus have 4 sides of equal length.*
33. If a figure is a trapezoid, then it is also a quadrilateral. *A trapezoid has 4 sides, 4 vertices, and 4 angles.*

Complete DAILY REVIEW g on page 424.

Lesson 55 133

- ➤ **What do you predict is the sum of the angles in a quadrilateral? Why?** ***Answers may vary.***
- ➤ **How could you prove your prediction?** ***Possible answers: Measure the 4 angles in a quadrilateral and add the measures; since there are 2 triangles in each quadrilateral, you can multiply 2 × 180° (or add 180° + 180°); since a rectangle and a square each have 4 right angles, you can multiply 4 × 90°.***

2. Select students to demonstrate the 3 ways of proving the prediction.

- ➤ **What can you conclude about the sum of the 4 angles in a quadrilateral?** ***The combined measure of the angles equals 360°.***

Find the unknown measure of an angle in a quadrilateral

1. Display and distribute the Quadrilaterals page.

- ➤ **What is the classification of the first figure? Why?** ***A trapezoid; 1 pair of opposite sides is parallel.***
- ➤ **How can you find the measure mathematically of ∠D in the trapezoid?** ***Elicit that you can find the sum of ∠A, ∠B, and ∠C and subtract the total from 360°.***

2. Write $m\angle A + m\angle B + m\angle C + m\angle D = 360°$ for display. Rewrite the equation, substituting the known angle measures: $115° + 65° + 85° + m\angle D = 360°$.

   Guide the students in solving the equation. *$m\angle D = 360° - (115° + 65° + 85°) = 360° - 265° = 95°$* Direct them to write the angle measure of angle *D* as you write it on the displayed page.

3. Instruct each student to measure the lengths of the sides in millimeters and to write the measurements on his page: $\overline{AB}$ *28 mm*, $\overline{BC}$ *38 mm*, $\overline{CD}$ *25 mm*, $\overline{AD}$ *24 mm*.

- ➤ **Are any sides congruent?** ***no***
- ➤ **Which sides are parallel?** ***$\overline{AD}$ and $\overline{BC}$***

   Direct the students to complete the statement to tell which line segments are parallel: $\overline{AD} \parallel \overline{BC}$.

4. Instruct the students to complete the page.

   parallelogram: ∠*E* *105°*, ∠*F* *75°*, ∠*G* *105°*, ∠*H* *75°*; $\overline{EF}$ *36 mm*, $\overline{FG}$ *26 mm*, $\overline{GH}$ *36 mm*, $\overline{EH}$ *26 mm*

   rhombus: ∠*I* *70°*, ∠*J* *110°*, ∠*K* *70°*, ∠*L* *110°*; $\overline{IJ}$ *30 mm*, $\overline{JK}$ *30 mm*, $\overline{KL}$ *30 mm*, $\overline{IL}$ *30 mm*

- ➤ **Can a quadrilateral have angle measures of 100°, 50°, 90°, and 40°? How do you know?** ***No; the sum of 100°, 50°, 90°, and 40° is 280°, which is less than 360°, the sum of the angle measures in a quadrilateral.***
- ➤ **Can a quadrilateral have angle measures of 150°, 30°, 90°, and 90°? Why?** ***Yes; the measures total 360°.***

   (***Note:*** You may choose to use the Polygon Angle Measure page to guide the students in finding the sum of the angle measures and the measure of the unknown angle in more complex polygons. The page may also be used as a challenge activity. You may choose to allow the students to use a calculator to solve the problems.)

Student Text pp. 132–33

Lesson 56

Student Text pp. 134–35

Daily Review p. 424h

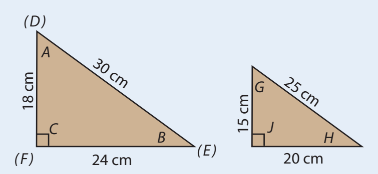
Objectives
- Identify congruent and similar polygons
- Identify corresponding angles and line segments
- Write a ratio for corresponding sides in a pair of polygons
- Draw congruent and similar polygons

Teacher Materials
- Congruent & Similar Polygons, page IA34 (CD)
- 3 sheets of construction paper (3 different colors)
- A protractor
- A ruler
- Tracing paper (optional)

Student Materials
- Congruent & Similar Polygons, page IA34 (CD)
- A protractor
- Graph paper

Preparation

From each of 2 sheets of the construction paper, cut a right triangle with sides measuring 18 cm, 24 cm, and 30 cm. Label one of the triangles with the vertices *A*, *B*, and *C* and label the corresponding vertices of the second triangle *D*, *E*, and *F*. Cut a third right triangle with sides measuring 15 cm, 20 cm, and 25 cm. Label its corresponding vertices *G*, *H*, and *J*.



Teach for Understanding

Identify congruent and similar polygons

1. Display the prepared ΔABC and ΔGHJ.
   ➤ **How are these triangles similar?** ***Elicit that ΔGHJ has the same shape as ΔABC.*** **How are they different?** ***Elicit that ΔGHJ is not the same size as ΔABC.***
2. Write *similar* for display. Explain that similar figures are geometric figures with the same shape but not necessarily the same size. Similar figures can be placed one on top of the other so that the larger figure frames the smaller figure equally on all sides. Demonstrate.

   Write $\Delta ABC \sim \Delta GHJ$ for display. Explain that the symbol $\sim$ is read "is similar to." Lead in reading the expression: *triangle ABC is similar to triangle GHI.*
3. Remove ΔGHI and display ΔDEF beside ΔABC.
   ➤ **How are these triangles similar?** ***Elicit that ΔDEF is the same shape and the same size as ΔABC.*** **Are they different in any way?** ***no***
4. Write *congruent* for display. Explain that congruent figures are geometric figures with exactly the same size and the same shape. Congruent figures can be placed one on top of the other so that all of the corresponding points on both figures match. Demonstrate.

   Write $\Delta ABC \cong \Delta DEF$ for display. Explain that the symbol $\cong$ is read "is congruent to." Lead in reading the expression: *triangle ABC is congruent to triangle DEF.*

   Point out that the symbol for congruency contains a similar sign and an equal sign; it shows that a congruent figure is a similar figure that is also equal in size.
5. Rotate ΔDEF.
   ➤ **If ΔDEF faces a different direction, is it congruent to ΔABC? Why?** ***Yes; elicit that the figures are still the same size and shape, no matter what direction they face.***

Identify corresponding angles and line segments

1. Display all 3 triangles.
   ➤ **Which angle on ΔDEF has the same measure as $\angle A$ on ΔABC?** ***∠D***

   Choose a student to explain how he knows that $\angle D$'s position in ΔDEF is the same as $\angle A$'s position in ΔABC, and then to measure $\angle A$ and $\angle D$ with a protractor. Direct him to write a mathematical statement that states that the angles are congruent. **$\angle A \cong \angle D$**

   Point out that $\angle A$ and $\angle D$ are *corresponding angles*; they occupy the same position in similar figures.
2. Select another student to demonstrate that the other corresponding angles are congruent by measuring the angles or by placing one on top of the other and to write the statements, stating that the angles are congruent. **$\angle B \cong \angle E$; $\angle C \cong \angle F$**
3. Follow a similar procedure for ΔABC and ΔGHJ. **$\angle A \cong \angle G$; $\angle B \cong \angle H$; $\angle C \cong \angle J$** Explain that all similar figures have corresponding angles of equal measure.
4. Choose a student to use a metric ruler to measure $\overline{AB}$ in centimeters. ***30 cm***
   ➤ **Which line segment on ΔDEF is congruent to $\overline{AB}$ on ΔABC?** **$\overline{DE}$**

   Choose a student to explain how he knows $\overline{DE}$'s position corresponds with $\overline{AB}$'s position and then to measure $\overline{DE}$. Direct him to write a mathematical statement to show the relationship between the *corresponding line segments.* **$\overline{AB} \cong \overline{DE}$**
5. Select another student to demonstrate that the other corresponding line segments in a pair of congruent polygons (the triangles) are congruent and to write the statements, stating that the line segments are congruent. **$\overline{BC} \cong \overline{EF}$; $\overline{CA} \cong \overline{FD}$**
6. Choose other students to point out the corresponding line segments in ΔDEF and ΔGHJ.

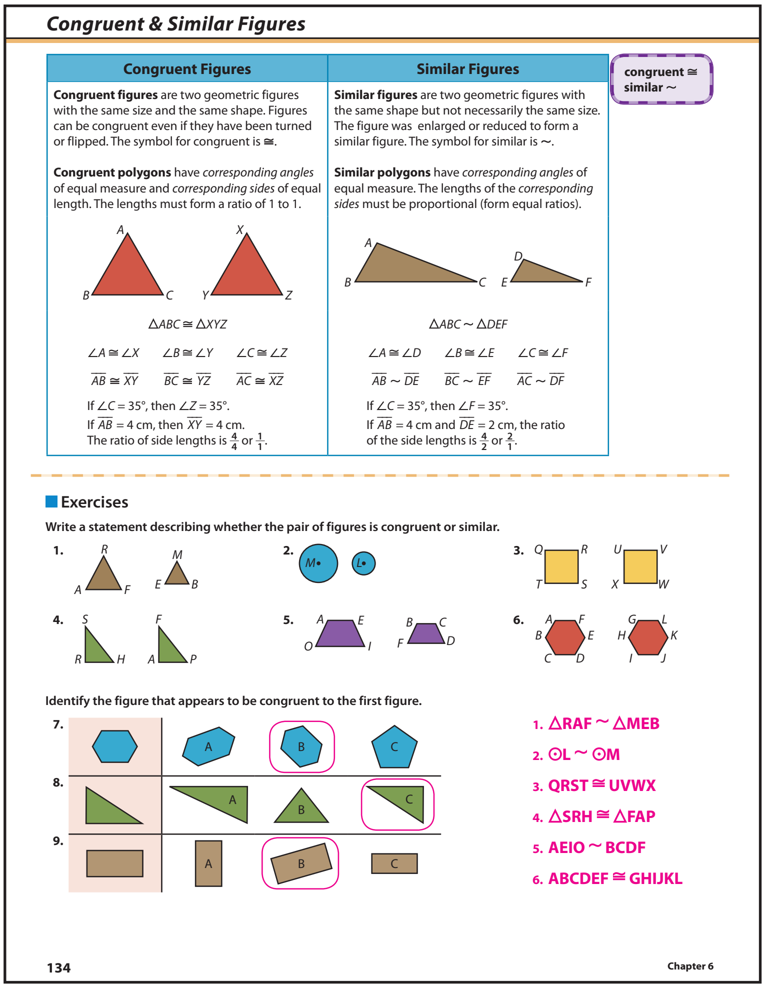
   (***Note:*** Encourage the students to begin at a distinguishable point on two seemingly similar figures and to work clockwise or counterclockwise around the figures to name corresponding parts.)

   (Continue to display the 3 triangles.)

Write a ratio for corresponding sides

(***Note:*** The use of ratios to identify figures as similar or congruent is introduced in this lesson but will not be tested. Ratios will be taught in Chapter 13.)

1. Explain that ratios can be written to show the relationship of one measurement to another. Write *$\overline{AB}$ to $\overline{DE}$, 30 cm to 30 cm, 30:30, and* $\frac{30}{30}$. Point out that the ratio $\frac{30}{30}$ (30 to 30) can be renamed to a $\frac{1}{1}$ (1 to 1) ratio. In congruent figures, corresponding angles and corresponding sides will always form a 1 to 1 ratio when renamed to lowest terms because each of the corresponding sides and angles are congruent.



Congruent & Similar Figures

Congruent Figures	Similar Figures
Congruent figures are two geometric figures with the same size and the same shape. Figures can be congruent even if they have been turned or flipped. The symbol for congruent is ≅.	**Similar figures** are two geometric figures with the same shape but not necessarily the same size. The figure was enlarged or reduced to form a similar figure. The symbol for similar is ∼.
Congruent polygons have *corresponding angles* of equal measure and *corresponding sides* of equal length. The lengths must form a ratio of 1 to 1.	**Similar polygons** have *corresponding angles* of equal measure. The lengths of the *corresponding sides* must be proportional (form equal ratios).
$\triangle ABC \cong \triangle XYZ$	$\triangle ABC \sim \triangle DEF$
$\angle A \cong \angle X$ $\angle B \cong \angle Y$ $\angle C \cong \angle Z$	$\angle A \cong \angle D$ $\angle B \cong \angle E$ $\angle C \cong \angle F$
$\overline{AB} \cong \overline{XY}$ $\overline{BC} \cong \overline{YZ}$ $\overline{AC} \cong \overline{XZ}$	$\overline{AB} \sim \overline{DE}$ $\overline{BC} \sim \overline{EF}$ $\overline{AC} \sim \overline{DF}$
If $\angle C = 35°$, then $\angle Z = 35°$. If $\overline{AB} = 4$ cm, then $\overline{XY} = 4$ cm. The ratio of side lengths is $\frac{4}{4}$ or $\frac{1}{1}$.	If $\angle C = 35°$, then $\angle F = 35°$. If $\overline{AB} = 4$ cm and $\overline{DE} = 2$ cm, the ratio of the side lengths is $\frac{4}{2}$ or $\frac{2}{1}$.

congruent ≅
similar ∼

Exercises

Write a statement describing whether the pair of figures is congruent or similar.

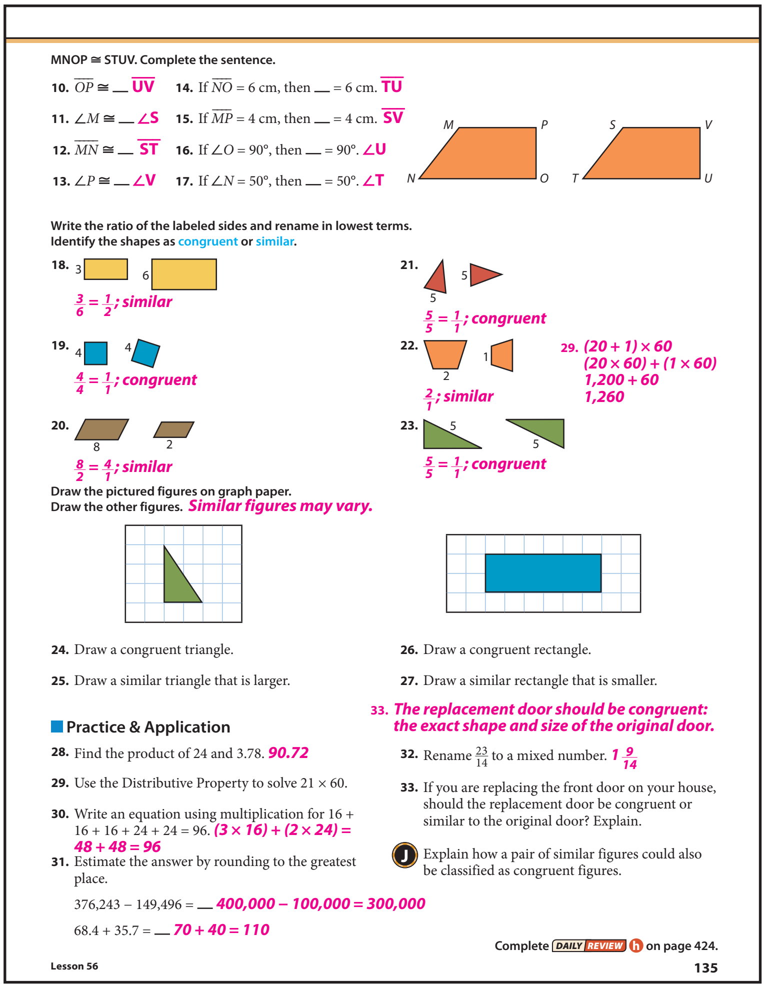
1. 2. 3. 4. 5. 6.

Identify the figure that appears to be congruent to the first figure.

7. 8. 9.

1. $\triangle$RAF ∼ $\triangle$MEB
2. ⊙L ∼ ⊙M
3. QRST ≅ UVWX
4. $\triangle$SRH ≅ $\triangle$FAP
5. AEIO ∼ BCDF
6. ABCDEF ≅ GHIJKL

134 Chapter 6



MNOP ≅ STUV. Complete the sentence.

10. $\overline{OP} \cong$ ___ **UV**
11. $\angle M \cong$ ___ **∠S**
12. $\overline{MN} \cong$ ___ **ST**
13. $\angle P \cong$ ___ **∠V**
14. If $\overline{NO} = 6$ cm, then ___ = 6 cm. **TU**
15. If $\overline{MP} = 4$ cm, then ___ = 4 cm. **SV**
16. If $\angle O = 90°$, then ___ = 90°. **∠U**
17. If $\angle N = 50°$, then ___ = 50°. **∠T**

Write the ratio of the labeled sides and rename in lowest terms. Identify the shapes as congruent or similar.

18. $\frac{3}{6} = \frac{1}{2}$; *similar*
19. $\frac{4}{4} = \frac{1}{1}$; *congruent*
20. $\frac{8}{2} = \frac{4}{1}$; *similar*
21. $\frac{5}{5} = \frac{1}{1}$; *congruent*
22. $\frac{2}{1}$; *similar*
23. $\frac{5}{5} = \frac{1}{1}$; *congruent*

29. $(20 + 1) \times 60$
$(20 \times 60) + (1 \times 60)$
$1{,}200 + 60$
$1{,}260$

Draw the pictured figures on graph paper.
Draw the other figures. *Similar figures may vary.*

24. Draw a congruent triangle.
25. Draw a similar triangle that is larger.
26. Draw a congruent rectangle.
27. Draw a similar rectangle that is smaller.

33. *The replacement door should be congruent: the exact shape and size of the original door.*

Practice & Application

28. Find the product of 24 and 3.78. **90.72**
29. Use the Distributive Property to solve 21 × 60.
30. Write an equation using multiplication for 16 + 16 + 16 + 24 + 24 = 96. $(3 \times 16) + (2 \times 24) = 48 + 48 = 96$
31. Estimate the answer by rounding to the greatest place.
376,243 − 149,496 = ___ $400{,}000 - 100{,}000 = 300{,}000$
68.4 + 35.7 = ___ $70 + 40 = 110$
32. Rename $\frac{23}{14}$ to a mixed number. $1\frac{9}{14}$
33. If you are replacing the front door on your house, should the replacement door be congruent or similar to the original door? Explain.

J Explain how a pair of similar figures could also be classified as congruent figures.

Complete DAILY REVIEW h on page 424.

Lesson 56 135

2. Follow a similar procedure to guide the students in writing ratios for $\overline{BC}$ to $\overline{EF}$ and $\overline{AC}$ to $\overline{DF}$. $\frac{24}{24}$ ***renamed to*** $\frac{1}{1}$; $\frac{18}{18}$ ***renamed to*** $\frac{1}{1}$
3. Repeat the procedure to guide the students in writing and identifying ratios for the corresponding sides of ΔABC and ΔGHJ. $\frac{15}{18}$, $\frac{20}{24}$, ***and*** $\frac{25}{30}$ ***renamed to*** $\frac{5}{6}$
4. Explain that when a figure is copied exactly, a figure congruent to the original figure is produced; however, a similar figure is produced by enlarging or reducing the original figure. Point out that the reduced (i.e., renamed to lowest terms) ratios of the sides show that ΔGHJ is $\frac{5}{6}$ the size of ΔABC.
5. Remind the students that the lengths of all the pairs of corresponding sides in a pair of similar or congruent figures must form equal ratios. Emphasize that for a pair of polygons to be similar, each pair of corresponding angles must have the same measure; and for a pair of polygons to be congruent, each pair of corresponding angles must have the same measure and each pair of corresponding line segments must have the same length.
6. Display and distribute the Congruent & Similar Polygons page.
   - ➤ **What kind of polygon is figure *EFGH*?** ***square or quadrilateral***
   - ➤ **How many units long is *EF*?** ***2 units***
   - ➤ **Which figures in the first row are similar to figure *EFGH*? Why?** ***1, 3, and 4; they are all squares, the same shape, but not necessarily the same size.***
   - ➤ **Which of the figures in the first row is congruent to figure *EFGH*? Why?** ***3; it is the same shape and the same size as figure* EFGH.**

   Elicit that the corresponding sides and the corresponding angles of figure *EFGH* and figure *3* are equal in measure.
7. Guide the students in determining the ratio of the corresponding sides of figure *EFGH* and each similar figure: 1 ***1:2,*** 3 ***1:1,*** 4 ***2:3***.
   - ➤ **How can you quickly tell that figure 2 is neither similar nor congruent to figure *EFGH*?** ***Figure 2 does not have the same square shape as figure* EFGH.**
8. Follow a similar procedure for the remaining figures. Instruct the students to count the number of units in each of the 2 shorter sides of the triangles to determine the ratio of corresponding sides. Instead of measuring the angles, you may choose to trace the original figure onto tracing paper to check corresponding angles for congruency.

   figure *JKLM*: similar figures ***2, 3***; ratios ***2:1, 1:1***;
   congruent figures ***3***; ratios ***1:1***
   ΔNOP: similar figures ***1, 2***; ratios ***1:2, 1:1***;
   congruent figures ***2***; ratios ***1:1***
   ΔQRS: similar figures ***1, 2, 3, 4***; ratios ***2:1, 4:3, 1:1, 2:3***;
   congruent figures ***3***; ratios ***1:1***

Draw congruent and similar polygons

Direct the students to draw a polygon on graph paper and label it "original." Then direct them to draw 2 more polygons, one that is congruent to the original polygon and one that is similar, and to label them accordingly.

Student Text pp. 134–35

Lesson
57

Student Text pp. 136–37

Daily Review p. 425i

Objectives

- Develop an understanding of transformations: translation, rotation, reflection
- Identify a transformation
- Draw a transformed figure
- Identify symmetrical figures
- Draw lines of symmetry on a figure

Teacher Materials

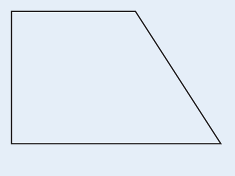
- Transformations, page IA35 (CD)
- 2 sheets of construction paper

Student Materials

- Transformations, page IA35 (CD)
- Block Letters, page IA36 (CD), one half page for each pair of students
- A sheet of notebook paper

Preparation

Prepare for display 2 identical trapezoids similar to the one pictured. Draw the same pattern (e.g., stripes, dots, and so on) on the front of each trapezoid.



Cut a large circle from one of the sheets of construction paper.

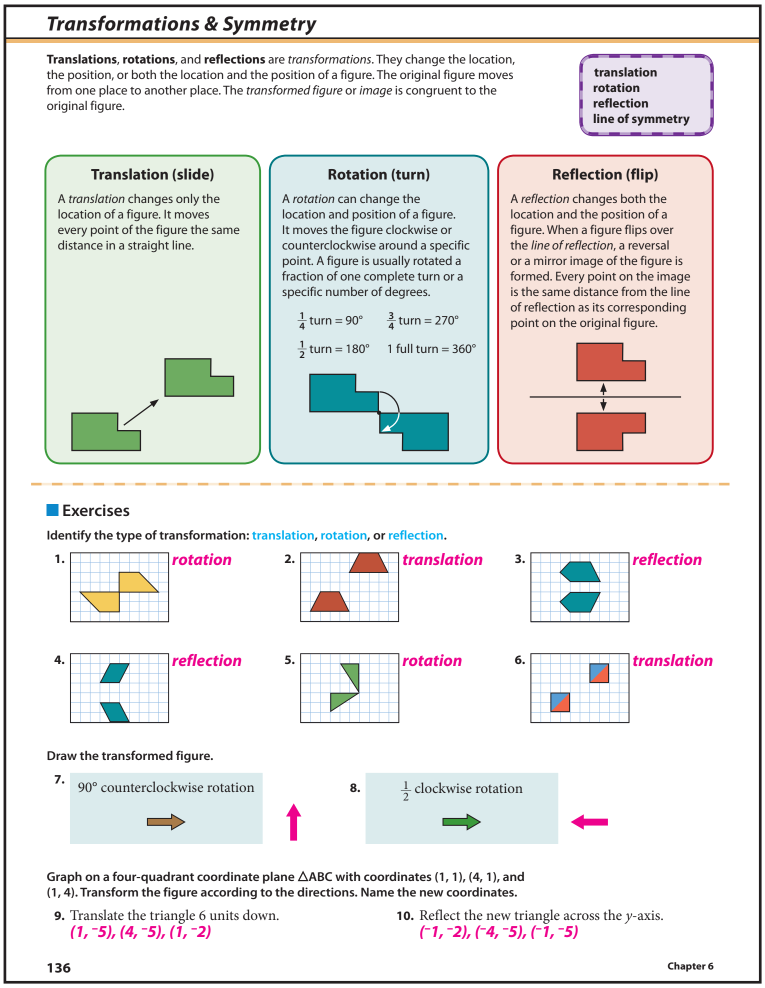
Teach for Understanding

Develop an understanding of transformations

1. Write *transformation* for display as a heading. Explain that a transformation changes the location and/or the position of a figure by moving the figure from one place to another.
   Attach 1 of the prepared trapezoids to the board, showing the drawn pattern. Throughout this activity, place the second trapezoid on top of the fixed trapezoid and use the loose trapezoid to model each transformation.
2. Write *translation* below *transformation*. Explain that a translation is one type of transformation; it changes only the location of a figure by sliding the figure. Demonstrate sliding the loose trapezoid to the right. Explain that every point of the trapezoid moved in a straight line exactly the same distance from its original position. Point out that the transformed trapezoid is congruent to the original one.
   Slide the trapezoid in several different directions. Point out that focusing on the direction of the movement of the vertices in a figure helps you determine the movement of all of the figure's points.
3. Write *reflection* for display. Explain that a reflection is another type of transformation; it changes the location and the position of a figure by flipping the figure over a *line of reflection*. Draw a vertical line to the right of the fixed trapezoid. Flip the loose trapezoid over the line, placing it the same distance from the line. Explain that every point in the figure is the same distance from the line of reflection as the original figure, forming a reversal or a mirror image with the back side of the figure showing.
4. Write *rotation* for display. Explain that a rotation is a third type of transformation; it can change the location and the position of a figure by moving the figure clockwise or counterclockwise around a specific point on, inside, or outside the figure. Hold the loose trapezoid in place by placing a finger on the endpoint of the longest side that is the vertex of the 90° angle. Turn the figure $\frac{1}{4}$ turn in a clockwise direction. Explain that the point that the figure rotated around is the *point of rotation*; it is the one point that remained in place while all the other points moved $\frac{1}{4}$ (90°) turn from their original locations.
5. Demonstrate a $\frac{1}{2}$ (180°) turn, a $\frac{3}{4}$ (270°) turn, and 1 whole (360°) turn of the figure. Point out that 1 whole turn, clockwise or counterclockwise, always brings the figure back to its original position.
6. Direct each student to place on his desk a sheet of notebook paper with the holes to the left and to label the angles (corners) *A*, *B*, *C*, and *D* in a clockwise direction, beginning at the top left corner.
7. Arrange the students in pairs and direct each student to show the following transformations with his paper. After each transformation, allow each student to check the position of his partner's paper.
   ➤ **Place the paper at the top of your desk so $\overline{AB}$ is parallel to the top edge of your desk.**
   ➤ **Rotate the paper $\frac{1}{4}$ turn in a clockwise direction, using point *A* as the point of rotation.**
   ➤ **Make a horizontal reflection. (Reflect the paper across an imaginary *y*-axis.)**
   ➤ **Translate the paper down.**
   Direct the students to find the location of point *B*, being careful not to move the paper as they lift the corners. ***bottom left corner***
   ➤ **What transformations can you use to return point *B* to its original position?** ***Answers will vary, but elicit that reversing the process will return the sheet of notebook paper to its original position.*** Discuss whether any of the other processes given uses fewer transformations than reversing the process. Choose students to give directions for transformations that will move the paper back to its original position.
8. Repeat the activity, allowing students to give directions for transforming the sheet of notebook paper.
9. Lead a discussion about using transformations in real life. For example, when helping a stranded motorist change a flat tire, both the tire iron and a lug nut are rotated around each bolt as the lug nuts are removed. Then translation is used to slide the flat tire off the axle and to slide the spare tire on. Rotation is again used to replace the lug nuts.

Identify a transformation; draw a transformed figure

1. Display and distribute the Transformations page. Choose students to identify the transformations for numbers 1–6 and to describe the movement.
   1. ***Translation; all the points moved 5 units to the right.***
   2. ***Rotation; $\frac{1}{4}$ (90°) turn clockwise or $\frac{3}{4}$ (270°) turn counterclockwise on the point of rotation inside the figure***
   3. ***Reflection; the figure was flipped, showing a reversed figure the same distance from the point of reflection.***



Transformations & Symmetry

Translations, **rotations**, and **reflections** are *transformations*. They change the location, the position, or both the location and the position of a figure. The original figure moves from one place to another place. The *transformed figure* or *image* is congruent to the original figure.

translation
rotation
reflection
line of symmetry

Translation (slide)

A *translation* changes only the location of a figure. It moves every point of the figure the same distance in a straight line.

Rotation (turn)

A *rotation* can change the location and position of a figure. It moves the figure clockwise or counterclockwise around a specific point. A figure is usually rotated a fraction of one complete turn or a specific number of degrees.

$\frac{1}{4}$ turn = 90° $\frac{3}{4}$ turn = 270°
$\frac{1}{2}$ turn = 180° 1 full turn = 360°

Reflection (flip)

A *reflection* changes both the location and the position of a figure. When a figure flips over the *line of reflection*, a reversal or a mirror image of the figure is formed. Every point on the image is the same distance from the line of reflection as its corresponding point on the original figure.

Exercises

Identify the type of transformation: translation, rotation, or reflection.

1. *rotation* **2.** *translation* **3.** *reflection*

4. *reflection* **5.** *rotation* **6.** *translation*

Draw the transformed figure.

7. 90° counterclockwise rotation

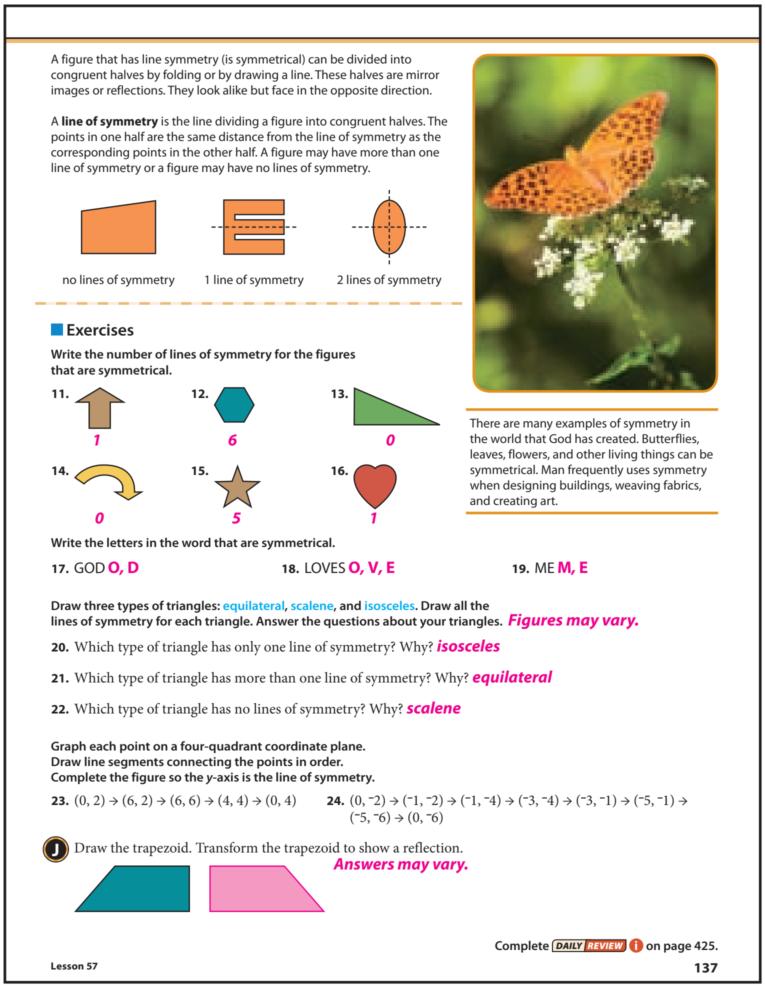
8. $\frac{1}{2}$ clockwise rotation

Graph on a four-quadrant coordinate plane △ABC with coordinates (1, 1), (4, 1), and (1, 4). Transform the figure according to the directions. Name the new coordinates.

9. Translate the triangle 6 units down.
$(1, {}^{-}5), (4, {}^{-}5), (1, {}^{-}2)$

10. Reflect the new triangle across the *y*-axis.
$({}^{-}1, {}^{-}2), ({}^{-}4, {}^{-}5), ({}^{-}1, {}^{-}5)$

136 Chapter 6



A figure that has line symmetry (is symmetrical) can be divided into congruent halves by folding or by drawing a line. These halves are mirror images or reflections. They look alike but face in the opposite direction.

A **line of symmetry** is the line dividing a figure into congruent halves. The points in one half are the same distance from the line of symmetry as the corresponding points in the other half. A figure may have more than one line of symmetry or a figure may have no lines of symmetry.

no lines of symmetry 1 line of symmetry 2 lines of symmetry

There are many examples of symmetry in the world that God has created. Butterflies, leaves, flowers, and other living things can be symmetrical. Man frequently uses symmetry when designing buildings, weaving fabrics, and creating art.

Exercises

Write the number of lines of symmetry for the figures that are symmetrical.

11. *1* **12.** *6* **13.** *0*

14. *0* **15.** *5* **16.** *1*

Write the letters in the word that are symmetrical.

17. GOD **O, D** **18.** LOVES **O, V, E** **19.** ME **M, E**

Draw three types of triangles: equilateral, scalene, and isosceles. Draw all the lines of symmetry for each triangle. Answer the questions about your triangles. *Figures may vary.*

20. Which type of triangle has only one line of symmetry? Why? ***isosceles***

21. Which type of triangle has more than one line of symmetry? Why? ***equilateral***

22. Which type of triangle has no lines of symmetry? Why? ***scalene***

Graph each point on a four-quadrant coordinate plane.
Draw line segments connecting the points in order.
Complete the figure so the *y*-axis is the line of symmetry.

23. $(0, 2) \rightarrow (6, 2) \rightarrow (6, 6) \rightarrow (4, 4) \rightarrow (0, 4)$

24. $(0, {}^{-}2) \rightarrow ({}^{-}1, {}^{-}2) \rightarrow ({}^{-}1, {}^{-}4) \rightarrow ({}^{-}3, {}^{-}4) \rightarrow ({}^{-}3, {}^{-}1) \rightarrow ({}^{-}5, {}^{-}1) \rightarrow ({}^{-}5, {}^{-}6) \rightarrow (0, {}^{-}6)$

Ⓙ Draw the trapezoid. Transform the trapezoid to show a reflection.
Answers may vary.

Complete DAILY REVIEW ⓘ on page 425.

Lesson 57 137

4. ***Rotation; $\frac{1}{4}$ (90°) turn clockwise or $\frac{3}{4}$ (270°) turn counterclockwise on point V (the point of rotation)***
5. ***Reflection; elicit that even though you do not see the line of reflection, there is no way to slide or rotate the figure from its original position to its transformed position; the figure must have been flipped.*** Choose a student to draw the line of reflection.
6. ***Rotation and translation; $\frac{1}{2}$ (180°) turn clockwise or counterclockwise on its center point and all the points moved 5 units to the right***

2. Direct the students to draw the transformed figures for numbers 7–12, using the given point of rotation for number 8 and number 10. Give assistance as needed.

Identify symmetrical figures; draw lines of symmetry

1. Explain that a *line of symmetry* divides a figure into congruent halves.
2. Fold a sheet of construction paper in half lengthwise and cut out a heart and a rectangle so that the fold in each figure represents a line of symmetry. Display the heart and the rectangle.

➤ **How many ways can you fold the heart so that the points on 1 half match exactly with the points on the other half?** *1 way*

➤ **How many ways can you fold the rectangle so that the points on 1 half match exactly with the points on the other half?** *2 ways*

Choose a student to fold the rectangle the other way to form another pair of symmetrical halves.

3. Display the prepared circle.

➤ **How many lines of symmetry do you think are in this circle? Why?** ***Elicit that a circle has an infinite number of lines of symmetry; a line of symmetry passes through the center of the circle just as the diameter does, and there are an infinite number of diameters in a circle.***

Fold the circle along a line of symmetry. Unfold the circle and display it.

➤ **What transformation do you think this line of symmetry can represent? Why?** ***Elicit that the line of symmetry can be a line of reflection because when half of a symmetrical figure is reflected (flipped), a mirror image or a reversal is formed.***

4. Distribute the Block Letters page. Instruct the students to work in pairs to identify the letters that are symmetrical and the line(s) of symmetry in each symmetrical letter. Allow students to share their findings.

Student Text pp. 136–37

Lesson 58

Student Text pp. 138–39
Daily Review p. 425j

Objectives

- Find the length of the diameter of a circle given the radius
- Find the length of the radius of a circle given the diameter
- Identify the parts of a circle: center, radius, chord, diameter, central angle
- Draw a circle using a protractor
- Measure the central angles of a circle using a protractor
- Relate fractions of a circle to degrees in a circle
- Make a circle graph to represent given data

Teacher Materials

- Circle, page IA37 (CD)
- A protractor
- A ruler
- A blank sheet of paper

Student Materials

- A protractor
- A ruler
- A blank sheet of paper

Notes

In this lesson, since the students have not yet learned to solve proportions, they will rename fractions to higher and lower terms when relating degree measures to fractions of a circle.

Throughout this lesson, when drawing line segments, model using an inch ruler or the straight edge of a protractor as a guide.

Teach for Understanding

Find the length of the radius and the diameter

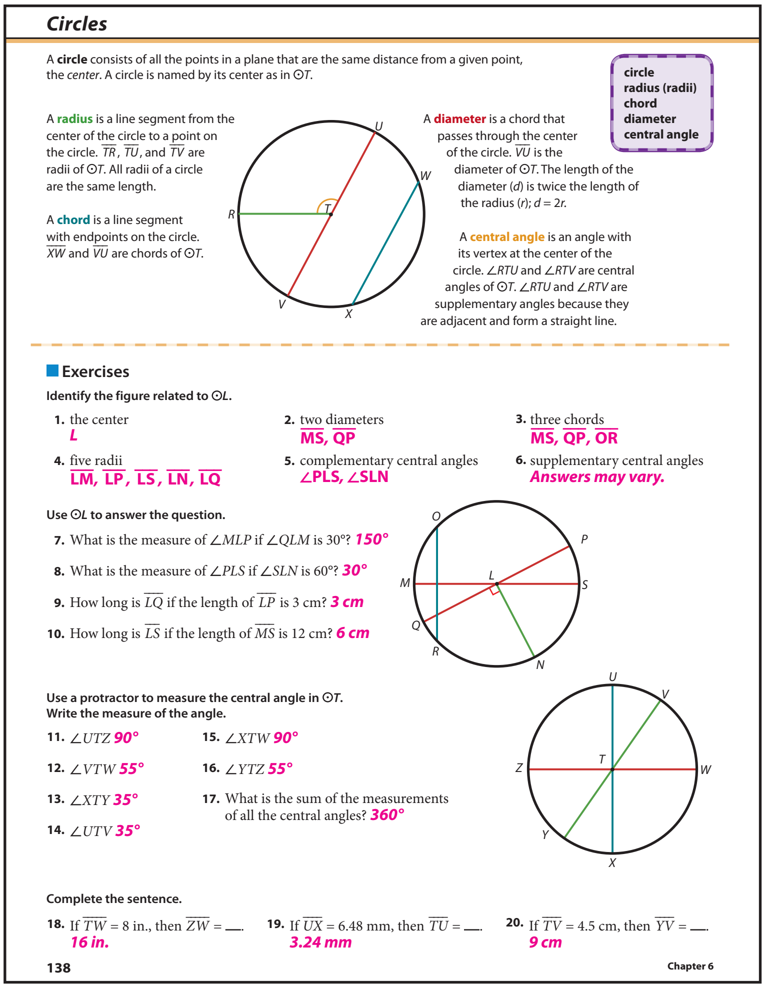
1. Display the Circle page. Explain that a *circle* consists of all the points in a plane that are the same distance from a given point, the *center.* A circle is named by its center although the center is not on the circle.
   ➤ **What is the name of this circle?** *circle A* Write $\odot A$ for display.
2. Draw points *B*, *C*, and *D* on the circle (not directly across from each other). Draw $\overline{AB}$ and $\overline{AC}$. Write *radius* and explain that a radius is a line segment from the center of the circle to a point on the circle; the center is always given first in the name of a radius.

   Point out that a circle can have an endless number of radii. All the radii of a circle are the same length, measuring an equal distance between the center and all the points that lie on the circle.

   Write $r =$ __ for display. Choose a student to use your ruler to measure in inches the length of $\overline{AB}$ and $\overline{AC}$ and complete the expression. *$3\frac{1}{2}$ in.*
3. Draw $\overline{BC}$, $\overline{BD}$, and $\overline{CD}$. Write *chord* and explain that a chord is a line segment with endpoints on the circle; $\overline{BC}$, $\overline{BD}$, and $\overline{CD}$ are chords of $\odot A$.
4. Draw point *E* on the circle; then draw a line segment that passes through the center from point *E*, dividing the circle in half. Label the other endpoint *F*. Write *diameter* and explain that the diameter is a chord that passes through the center of the circle.
   ➤ **How do you think you can find the length of the diameter ($\overline{EF}$)?** *Possible answers: measure the length of $\overline{EF}$; add the length of the known radius to itself; multiply the radius by 2.*
5. Write for display $d = r + r$ and $d = 2r$. Direct attention to $r = 3\frac{1}{2}$ *in.* previously written for display.
   ➤ **What is the diameter of $\odot A$?** *7 in.* Choose a student to check the calculated length by measuring $\overline{EF}$.
   ➤ **What would be the diameter of a circle if the radius is 5 in.?** *10 in.* **9 cm?** *18 cm* **12 cm?** *24 cm*
   ➤ **What would be the radius of a circle if the diameter is 12 cm?** *6 cm* **20 cm?** *10 cm* **15 in.?** *$7\frac{1}{2}$ in.*
6. Write *central angle* and explain that a central angle is an angle with its vertex at the center of a circle.
   ➤ **What central angles do you see in $\odot A$?** *Accept any correct answers.*
   ➤ **What is the measure of $\angle EAF$? How do you know?** *180°; $\angle$EAF is a straight angle.*

Draw a circle; measure central angles

1. Distribute the blank sheets of paper. Instruct the students to follow each direction after you read it. Demonstrate each step.
   (***Note:*** Folding the paper in half vertically and horizontally before you begin to draw the circle will provide points of reference.)
   ➤ **Draw point *C* in the center of the sheet of paper.**
   ➤ **Place the guide mark of the protractor on point *C*.**
   ➤ **Draw point *D* at 0° and draw point *E* at 180°.**
   ➤ **Trace the curve of the protractor from point *D* to point *E*.**
   ➤ **Use a ruler or the straight edge of the protractor as a guide to draw a line segment connecting point *D* and point *E*.**
   ➤ **What has been formed?** *a half circle*
   ➤ **How many degrees are in the half circle?** *180°*
   Rotate the paper, inverting the half circle, and direct the students to turn their papers.
   ➤ **Place the guide mark over point *C* so that point *D* is at 180° and point *E* is at 0°.**
   ➤ **Trace the curve of the protractor from point *E* to point *D*.**
   ➤ **What has been formed?** *the other half of the circle*
   ➤ **How many degrees are in this half circle?** *180°*
   ➤ **What equation can you write to show how many degrees are in the whole circle?** *180° + 180° = 360° or 2 × 180° = 360°*
2. Direct the students to draw point *F* anywhere on the circle; then direct them to draw a line segment connecting point *F* and point *D*.
   ➤ **What is $\overline{FD}$ in $\odot C$?** *a chord*
   ➤ **What do you call the chord with the endpoints *D* and *E*? Why?** *A diameter; elicit that $\overline{DE}$ passes through the center of $\odot C$.*
3. Direct the students to draw a line segment to connect point *C* and point *F*.
   ➤ **What is $\overline{CF}$ in $\odot C$?** *a radius*
   ➤ **What other radii are in $\odot C$?** *$\overline{CE}$ and $\overline{CD}$*
   ➤ **How many central angles have you formed in $\odot C$?** *3* **Name them.** *$\angle$DCE, $\angle$DCF, $\angle$ECF*
4. Instruct the students to measure each central angle and to write the measure inside the angle: $\angle ECD$ *180°*; $\angle DCF$ and $\angle ECF$ *Answers will vary*.
   ➤ **What should be the total measure of the 3 central angles?** *360°*
   ➤ **Why might the measure of your central angles total more or less than 360°?** *Possible answers: measured incorrectly, line segments and circle drawings might not be exact.*



Circles

A **circle** consists of all the points in a plane that are the same distance from a given point, the *center*. A circle is named by its center as in ⊙*T*.

circle
radius (radii)
chord
diameter
central angle

A **radius** is a line segment from the center of the circle to a point on the circle. $\overline{TR}$, $\overline{TU}$, and $\overline{TV}$ are radii of ⊙*T*. All radii of a circle are the same length.

A **chord** is a line segment with endpoints on the circle. $\overline{XW}$ and $\overline{VU}$ are chords of ⊙*T*.

A **diameter** is a chord that passes through the center of the circle. $\overline{VU}$ is the diameter of ⊙*T*. The length of the diameter (*d*) is twice the length of the radius (*r*); $d = 2r$.

A **central angle** is an angle with its vertex at the center of the circle. ∠*RTU* and ∠*RTV* are central angles of ⊙*T*. ∠*RTU* and ∠*RTV* are supplementary angles because they are adjacent and form a straight line.

Exercises

Identify the figure related to ⊙*L*.

1. the center
   L
2. two diameters
   $\overline{MS}$, $\overline{QP}$
3. three chords
   $\overline{MS}$, $\overline{QP}$, $\overline{OR}$
4. five radii
   $\overline{LM}$, $\overline{LP}$, $\overline{LS}$, $\overline{LN}$, $\overline{LQ}$
5. complementary central angles
   ∠PLS, ∠SLN
6. supplementary central angles
   Answers may vary.

Use ⊙*L* to answer the question.

7. What is the measure of ∠*MLP* if ∠*QLM* is 30°? ***150°***
8. What is the measure of ∠*PLS* if ∠*SLN* is 60°? ***30°***
9. How long is $\overline{LQ}$ if the length of $\overline{LP}$ is 3 cm? ***3 cm***
10. How long is $\overline{LS}$ if the length of $\overline{MS}$ is 12 cm? ***6 cm***

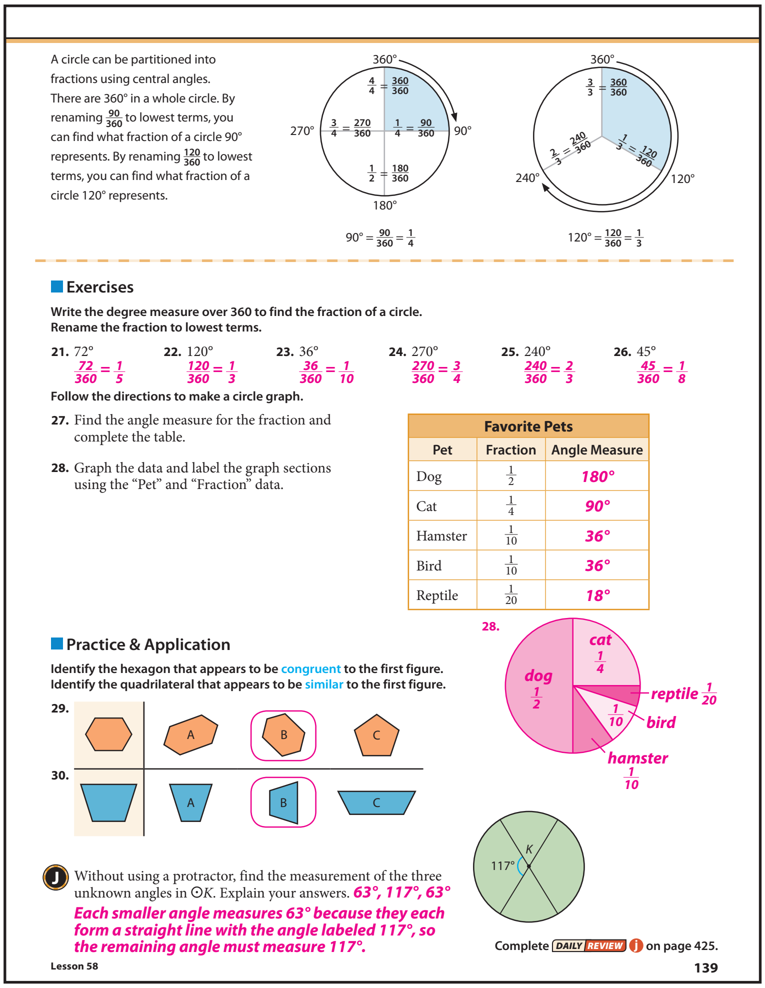
Use a protractor to measure the central angle in ⊙*T*. Write the measure of the angle.

11. ∠*UTZ* ***90°***
12. ∠*VTW* ***55°***
13. ∠*XTY* ***35°***
14. ∠*UTV* ***35°***
15. ∠*XTW* ***90°***
16. ∠*YTZ* ***55°***
17. What is the sum of the measurements of all the central angles? ***360°***

Complete the sentence.

18. If $\overline{TW}$ = 8 in., then $\overline{ZW}$ = ___. ***16 in.***
19. If $\overline{UX}$ = 6.48 mm, then $\overline{TU}$ = ___. ***3.24 mm***
20. If $\overline{TV}$ = 4.5 cm, then $\overline{YV}$ = ___. ***9 cm***

138 Chapter 6



A circle can be partitioned into fractions using central angles. There are 360° in a whole circle. By renaming $\frac{90}{360}$ to lowest terms, you can find what fraction of a circle 90° represents. By renaming $\frac{120}{360}$ to lowest terms, you can find what fraction of a circle 120° represents.

$90° = \frac{90}{360} = \frac{1}{4}$

$120° = \frac{120}{360} = \frac{1}{3}$

Exercises

Write the degree measure over 360 to find the fraction of a circle. Rename the fraction to lowest terms.

21. 72° $\frac{72}{360} = \frac{1}{5}$
22. 120° $\frac{120}{360} = \frac{1}{3}$
23. 36° $\frac{36}{360} = \frac{1}{10}$
24. 270° $\frac{270}{360} = \frac{3}{4}$
25. 240° $\frac{240}{360} = \frac{2}{3}$
26. 45° $\frac{45}{360} = \frac{1}{8}$

Follow the directions to make a circle graph.

27. Find the angle measure for the fraction and complete the table.
28. Graph the data and label the graph sections using the "Pet" and "Fraction" data.

Favorite Pets

Pet	Fraction	Angle Measure
Dog	$\frac{1}{2}$	***180°***
Cat	$\frac{1}{4}$	***90°***
Hamster	$\frac{1}{10}$	***36°***
Bird	$\frac{1}{10}$	***36°***
Reptile	$\frac{1}{20}$	***18°***

Practice & Application

Identify the hexagon that appears to be congruent to the first figure. Identify the quadrilateral that appears to be similar to the first figure.

Ⓙ Without using a protractor, find the measurement of the three unknown angles in ⊙*K*. Explain your answers. ***63°, 117°, 63°***
Each smaller angle measures 63° because they each form a straight line with the angle labeled 117°, so the remaining angle must measure 117°.

Complete DAILY REVIEW ⓙ on page 425.

Lesson 58 139

Relate fractions of a circle to degrees in a circle

1. Direct the students to cut out ⊙*C*. Throughout the activity, direct students to crease each fold.
   - ➤ **Fold your circle in half, matching the edges.**
   - ➤ **Use your protractor to measure the half circle.**
   - ➤ **How many degrees are in one half of a circle?** ***180°***

   Write for display $\frac{1}{2}$ *circle* = *180°*.
   - ➤ **Fold your half circle in half.**
   - ➤ **Use your protractor to measure the $\frac{1}{4}$ circle.**
   - ➤ **How many degrees are in a fourth of a circle?** ***90°***

   Write $\frac{1}{4}$ *circle* = *90°*.
   - ➤ **Fold your $\frac{1}{4}$ circle in half.**
   - ➤ **Use your protractor to measure the $\frac{1}{8}$ circle.**
   - ➤ **How many degrees are in an eighth of a circle?** ***45°***

   Write $\frac{1}{8}$ *circle* = *45°*.
   (***Note:*** Some students may find it difficult to fold their circles into eighths and get the correct measure. Assist them as needed.)
2. Point out that a circle can be partitioned into fractional parts using central angles.
   - ➤ **What part of a circle do you predict is 120°?**

   Elicit that knowing there are 360° in a whole circle and using your knowledge of fractions, you can find the fraction of a circle that is equivalent to 120° by renaming the fraction $\frac{120}{360}$ to lowest terms. Write $\frac{120}{360}$ for display.
   - ➤ **How can you rename $\frac{120}{360}$ to lowest terms?** ***Possible answers: Divide by the GCF (120), use cancellation, or use repeated division.***

   Guide the students in simplifying $\frac{120}{360}$ as $\frac{1}{3}$. (See Lesson 35 of Chapter 4.)
   - ➤ **How can you find what part of a circle is 45°?** ***Rename $\frac{45}{360}$ to lowest terms.***
3. Guide the students in simplifying $\frac{45}{360}$ as $\frac{1}{8}$.
   - ➤ **How can you find what part of a circle is 36°?** ***Rename $\frac{36}{360}$ to lowest terms.***
   - ➤ **What is $\frac{36}{360}$ in lowest terms?** $\frac{1}{10}$
   - ➤ **Since $\frac{1}{10}$ of a circle is 36°, how can you find the number of degrees in $\frac{3}{10}$ of a circle?** ***Elicit that you can multiply 36° by 3 to get 108°.***
   - ➤ **How can you find the number of degrees in $\frac{3}{4}$ of a circle?** ***Elicit that you can multiply 90° ($\frac{1}{4}$ of a circle) by 3 to get 270° or you can rename $\frac{3}{4}$ to an equivalent fraction with the denominator 360 (higher terms).***
4. Write $\frac{3}{4} = \frac{n}{360}$ and guide the students in renaming $\frac{3}{4}$ as $\frac{270}{360}$. (See Lessons 32 and 35 of Chapter 4.) $\frac{3}{4}$ *circle* = *270°*
5. Repeat the procedure to find the number of degrees in $\frac{1}{5}$ of a circle. $\frac{1}{5} = \frac{n}{360}$; $\frac{1}{5}$ *circle* = *72°*

Make a circle graph to represent given data

Guide the students in making the circle graph for problem 28 on Student Text page 139.

Student Text pp. 138–39

(***Note***: *Assessment* available on Teacher's Toolkit CD.)

Lesson 59

Student Text pp. 140–41

Daily Review p. 426k

Objectives

- Develop an understanding of polyhedrons
- Identify 3-dimensonal figures that are not polyhedrons
- Classify 3-dimensional figures: spherical, conical, cylindrical
- Identify the number of faces, vertices, and edges in polyhedrons
- Construct polyhedrons

Teacher Materials

- A purchased set of 3-dimensional figures
- The following objects (optional):
  cone: party hat or ice cream cone
  cylinder: oatmeal container, soup can, or candle
  rectangular prism: cereal box or tissue box
  sphere: ball or orange
  square prism (cube): photo cube or candle
  square pyramid: candle, paper weight, or origami pyramid
  triangular prism: light prism or Toblerone® chocolate box
  triangular pyramid: candle, paper weight, or origami pyramid
- Three 3 × 5 cards

Student Materials

- Polyhedrons, page IA35 (CD)
- Transparent tape

Notes

You may choose to enlarge the Polyhedrons page and, during the lesson, arrange the students in small groups. Instruct the students in each group to construct 1 or 2 of the figures rather than have each student construct all of the polyhedrons.

Label the three 3 × 5 cards: *Spherical Figures*, *Conical Figures*, and *Cylindrical Figures*.

The 3-dimensional figures used in this lesson will be used again in Lesson 103.

Teach for Understanding

Develop an understanding of polyhedrons

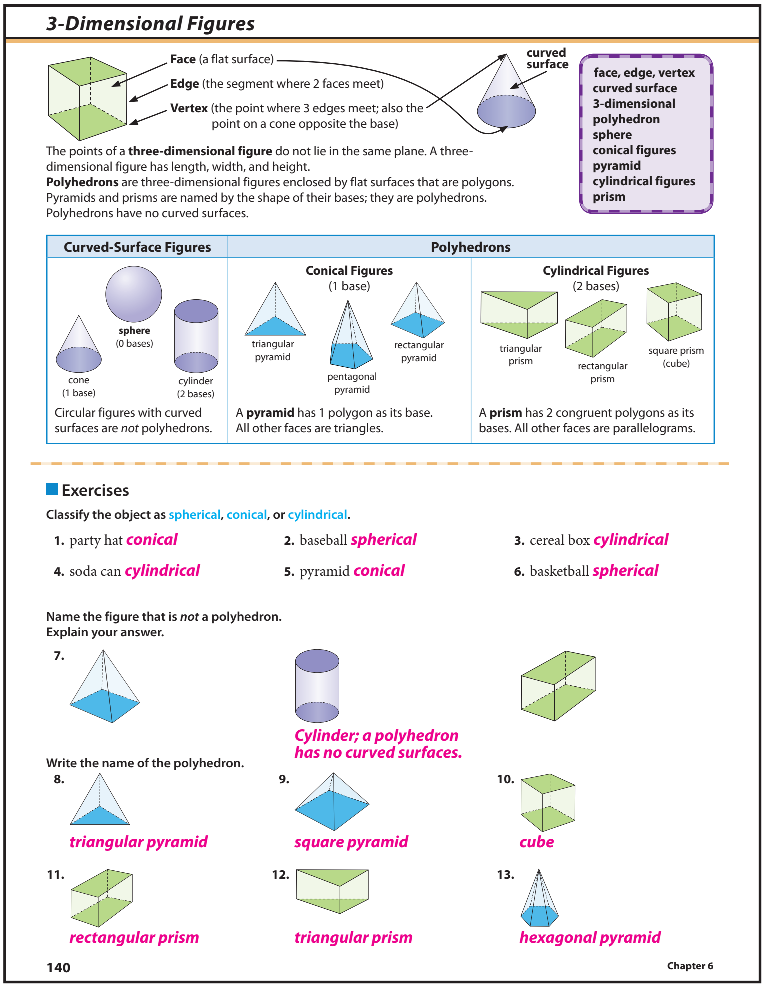
1. Distribute the Polyhedrons page.
   ➤ **What polygons do you see on the page?** ***triangle, quadrilateral, square, rectangle***
   ➤ **Are these polygons plane figures? How do you know?** ***Yes; all the points of each figure lie in the same plane.*** Point out that the worksheet represents the plane in which each of the figures lies.
2. Direct attention to the title of the page and explain that a *polyhedron* is a 3-dimensional figure enclosed by flat surfaces that are polygons; the flat surfaces are called *faces*.
   ➤ **If the triangles (polygons) in section I are used to form a closed 3-dimensional figure, how many faces will the formed polyhedron have?** ***Elicit that since there are 4 polygons to form the polyhedron, it will have 4 faces that are polygons.***
3. Direct each student to cut out the 4 triangles in section I and tape them together (grid side out) to form a closed 3-dimensional figure.
   ➤ **What object does this polyhedron resemble?** ***possible answers: Egyptian pyramid, mountain***
4. Follow a similar procedure to guide the students in forming the polyhedrons in sections II–V. Explain that a polyhedron (3-dimensional figure) consists of all the points on its surfaces; it does not include the points within the figure.
   ➤ **Are these polyhedrons plane figures? How do you know?** ***No; elicit that all of the points in each figure cannot lie in the same plane.***
5. Remind the students that plane figures are 2-dimensional figures; they have length and width. Three-dimensional figures, such as the polyhedrons they made, cannot lie in a single plane since they have a height dimension in addition to length and width dimensions.
   ➤ **What similarities do you see in these polyhedrons?** ***All faces are polygons.*** **What differences do you see?** ***Possible answers: The faces vary in shape; the faces in some of the figures form a point at one end of the figure; some of the figures have parallel faces.***

Identify 3-dimensional figures that are not polyhedrons

1. Display all of the 3-dimensional objects. Write *polyhedron* for display and elicit from the students that a polyhedron is a closed 3-dimensional figure made of flat surfaces (faces) that are polygons. Point out that 3-dimensional figures with curved surfaces are not polyhedrons.
2. Choose a student to identify the displayed items that are not polyhedrons. ***sphere, cone, and cylinder***
   ➤ **Why are these objects not polyhedrons?** ***The sphere, the cone, and the cylinder all have a curved surface.***
3. Write *base* for display and hold up the cylinder. Explain that, along with a curved surface, a cylinder also has 2 flat surfaces that are parallel and congruent. The 2 flat surfaces are bases.
   ➤ **What shape are the 2 bases of a cylinder?** ***circles***
   Point to the bases as you invert the cylinder. Explain that either base can be the bottom of the cylinder; the cylinder's appearance does not change when it is inverted so that the opposite base becomes the bottom.
4. Write *vertex* for display.
   ➤ **What is the vertex of an angle?** ***It is the point shared by 2 rays.***
   Hold up the cone. Explain that the point of a cone is also a vertex; it is opposite the base of the cone.
   ➤ **Is a cone a polyhedron? Why?** ***No; a cone has a curved surface.***
   ➤ **How many bases does a cone have?** ***1*** **What is the shape of the base?** ***circle***
   Point out that an ice-cream cone and a party hat are common examples of cones, even though they do not show the base. However, the 3-dimensional figure defined as a cone has 1 circular base opposite the vertex.
5. Hold up the sphere. Point out that a sphere is a continuous curved surface; since it has no flat surfaces, it has no bases.

Classify 3-dimensional figures

1. Display the labeled 3 × 5 cards. Explain that 3-dimensional figures can be classified as *spherical*, *conical*, or *cylindrical*. *Spherical figures* are similar to a sphere; they have only curved surfaces. *Conical figures* are similar to a cone; they have 1 polygon as a base and 1 of the vertices is opposite the base, but all the other faces are triangles. *Cylindrical figures* are similar to a cylinder; they are prisms that have 2 congruent bases, but all the other faces are parallelograms.
2. Choose students to classify each 3-dimensional object by placing the object with the appropriate label.



3-Dimensional Figures

Face (a flat surface)
Edge (the segment where 2 faces meet)
Vertex (the point where 3 edges meet; also the point on a cone opposite the base)
curved surface

face, edge, vertex
curved surface
3-dimensional
polyhedron
sphere
conical figures
pyramid
cylindrical figures
prism

The points of a **three-dimensional figure** do not lie in the same plane. A three-dimensional figure has length, width, and height.
Polyhedrons are three-dimensional figures enclosed by flat surfaces that are polygons. Pyramids and prisms are named by the shape of their bases; they are polyhedrons. Polyhedrons have no curved surfaces.

Curved-Surface Figures	Polyhedrons	
	Conical Figures (1 base)	**Cylindrical Figures** (2 bases)
sphere (0 bases) cone (1 base) cylinder (2 bases)	triangular pyramid pentagonal pyramid rectangular pyramid	triangular prism rectangular prism square prism (cube)
Circular figures with curved surfaces are *not* polyhedrons.	A **pyramid** has 1 polygon as its base. All other faces are triangles.	A **prism** has 2 congruent polygons as its bases. All other faces are parallelograms.

Exercises

Classify the object as spherical, conical, or cylindrical.

1. party hat *conical*
2. baseball *spherical*
3. cereal box *cylindrical*
4. soda can *cylindrical*
5. pyramid *conical*
6. basketball *spherical*

Name the figure that is *not* a polyhedron. Explain your answer.

7. *Cylinder; a polyhedron has no curved surfaces.*

Write the name of the polyhedron.

8. *triangular pyramid*
9. *square pyramid*
10. *cube*
11. *rectangular prism*
12. *triangular prism*
13. *hexagonal pyramid*

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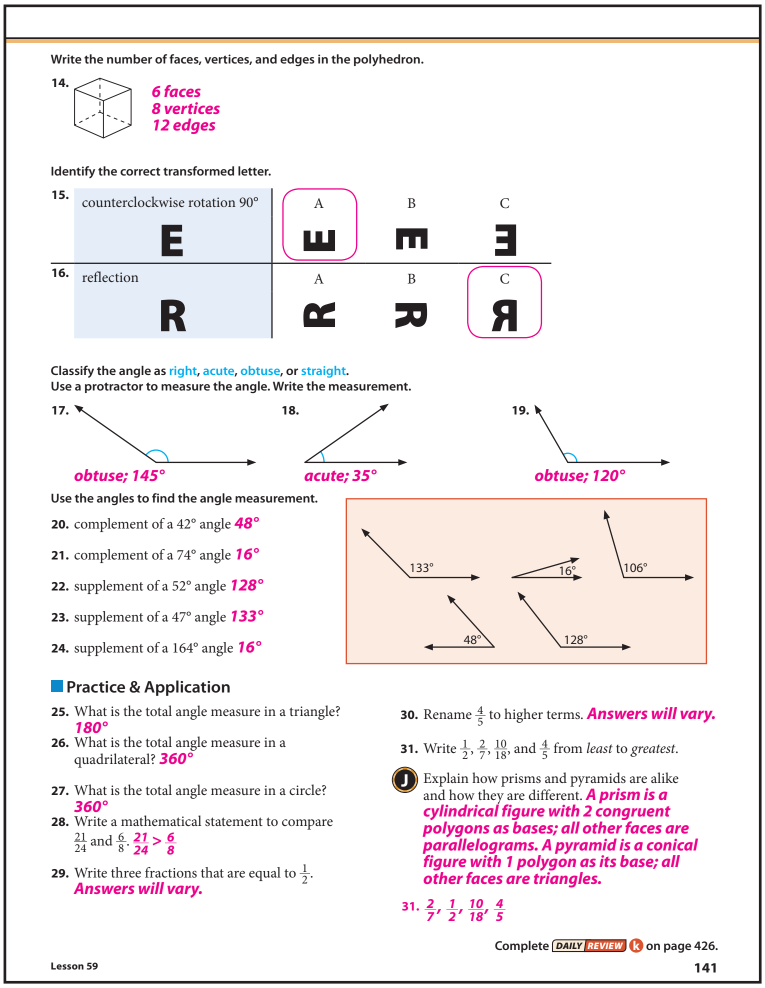


Write the number of faces, vertices, and edges in the polyhedron.

14. *6 faces*
8 vertices
12 edges

Identify the correct transformed letter.

		A	B	C
15.	counterclockwise rotation 90° E	[✓ Ш]	ᗰ	Ǝ
16.	reflection R	ᴚ	ꓤ	[✓ Я]

Classify the angle as right, acute, obtuse, or straight.
Use a protractor to measure the angle. Write the measurement.

17. *obtuse; 145°*
18. *acute; 35°*
19. *obtuse; 120°*

Use the angles to find the angle measurement.

20. complement of a 42° angle *48°*
21. complement of a 74° angle *16°*
22. supplement of a 52° angle *128°*
23. supplement of a 47° angle *133°*
24. supplement of a 164° angle *16°*

133° 16° 106°
48° 128°

Practice & Application

25. What is the total angle measure in a triangle? *180°*
26. What is the total angle measure in a quadrilateral? *360°*
27. What is the total angle measure in a circle? *360°*
28. Write a mathematical statement to compare $\frac{21}{24}$ and $\frac{6}{8}$. $\frac{21}{24} > \frac{6}{8}$
29. Write three fractions that are equal to $\frac{1}{2}$. *Answers will vary.*
30. Rename $\frac{4}{5}$ to higher terms. *Answers will vary.*
31. Write $\frac{1}{2}$, $\frac{2}{7}$, $\frac{10}{18}$, and $\frac{4}{5}$ from *least* to *greatest*.

Ⓙ Explain how prisms and pyramids are alike and how they are different. *A prism is a cylindrical figure with 2 congruent polygons as bases; all other faces are parallelograms. A pyramid is a conical figure with 1 polygon as its base; all other faces are triangles.*

31. $\frac{2}{7}, \frac{1}{2}, \frac{10}{18}, \frac{4}{5}$

Complete DAILY REVIEW ⓚ on page 426.

Lesson 59 141

➤ **Why are there no examples of polyhedrons that are spherical?** *Elicit that all polyhedrons have flat surfaces (faces) and no curved surfaces; to be spherical a figure must have only a curved surface.*

➤ **Why are a triangular prism, a rectangular prism, and a cube classified as cylindrical?** *Elicit that each of these figures has 2 congruent bases, similar to a cylinder.*

➤ **Why are pyramids classified as conical?** *Elicit that pyramids each have 1 base and a vertex opposite the base, similar to a cone.*

3. Direct the students to sort their polyhedrons as conical *I and II* or cylindrical *III – V*. Remind them that pyramids are conical polyhedrons, and that prisms are cylindrical polyhedrons.

➤ **Other than the base, what shapes are the other faces of your pyramids?** *triangles*

➤ **Other than the bases, what shapes are the other faces of your prisms?** *Elicit that the other faces are rectangles.* Point out that if a prism has non-rectangular sides, those sides are the bases; however, if all the sides are rectangular, any pair of opposite sides can be the bases.

4. Explain that pyramids and prisms are named according to the shape of their bases (e.g., triangular pyramid, square pyramid, rectangular prism). Elicit the name for each constructed polyhedron: I *triangular pyramid*, II *square pyramid*, III *square prism or cube*, IV *rectangular prism*, and V *triangular prism*.

➤ **What would you call a pyramid with a base that is a rectangle?** *rectangular pyramid* **a pentagon?** *pentagonal pyramid* **a hexagon?** *hexagonal pyramid*

➤ **What would you call a prism that has 2 bases that are hexagons?** *hexagonal prism* **pentagons?** *pentagonal prism* **octagons?** *octagonal prism*

Identify the number of faces, vertices, and edges in polyhedrons

1. Write *face* and *edge* for display. Remind the students that a *face* is a flat surface. Explain that an *edge* is the line segment where 2 faces meet and a *vertex* in a polyhedron is the point where 3 or more edges meet. Point out that the circumference of a circular base(s) of a cone or a cylinder is not considered an edge because a face and a curved surface meet rather than 2 faces meeting to form a line segment.
2. Direct the students to hold up the triangular pyramid that they constructed.

➤ **How many faces does a triangular pyramid have?** *4* **edges?** *6* **vertices?** *4*

➤ **What other pyramid did you make?** *a square pyramid*

3. Direct the students to hold up the square pyramid.

➤ **How many faces does a square pyramid have?** *5* **edges?** *8* **vertices?** *5*

4. Follow a similar procedure for each prism: square prism (cube) *6 faces, 12 edges, 8 vertices*; rectangular prism *6 faces, 12 edges, 8 vertices*; triangular prism *5 faces, 9 edges, 6 vertices*. (***Note:*** Store the figures made by the students for use in Chapter 11.)

Student Text pp. 140–41

Lesson 60

Student Text pp. 142–43

Chapter Review

Objectives

- Draw and name basic geometric figures: points, lines, line segments, rays, angles, planes
- Identify lines: intersecting, perpendicular, parallel
- Draw and measure angles: right, acute, obtuse, straight
- Identify supplementary and complementary angles
- Demonstrate an understanding of polygons: regular, irregular, number of sides, number of interior angles
- Identify congruent and similar figures
- Classify triangles by their angles and by their sides
- Find the unknown measure of an angle in a triangle and in a quadrilateral
- Identify the parts of a circle
- Find the length of the radius and the diameter of a circle
- Identify a transformation
- Identify symmetrical figures

Teacher Materials

- Coordinate Planes (from Lesson 50)
- Complementary & Supplementary Angles (from Lesson 52)
- Triangles (from Lesson 54)
- Quadrilaterals (from Lesson 55)
- Transformations (from Lesson 57)
- Circle (from Lesson 58)
- Christian Worldview Shaping, pages 18–19 (CD)
- A ruler

Student Materials

- Coordinate Planes, page IA25 (CD)
- A protractor

Note

This lesson reviews the concepts presented in Chapter 6 to prepare the students for the Chapter 6 Test. Student Text pages 142–43 provide the students with an excellent study guide.

Check for Understanding

Draw and name basic geometric figures

(*Note:* Throughout this activity, select students to plot the points and draw the lines for display while the other students plot the points and draw the lines on their worksheets.)

1. Display and distribute the Coordinate Planes page. Direct the students to plot and label points $A\ (6, 4)$, $B\ (^-5, 4)$, $C\ (2, ^-4)$, and $D\ (^-5, ^-4)$ on the first coordinate plane. Choose students to identify the quadrant in which each point is located. Instruct the students to label each quadrant. (See Lesson 49.)
2. Direct the students to draw $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{BD}$. Remind them that each line should connect the given points and extend beyond the points in both directions. Select students to write statements telling which lines are parallel and describing the lines that intersect. Instruct each student to explain his statement. **$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$; $\overleftrightarrow{AB} \perp \overleftrightarrow{BD}$; $\overleftrightarrow{BD} \perp \overleftrightarrow{CD}$**
3. Elicit from the students that 3 noncollinear points are frequently used to name a plane. Choose a student to name the plane containing the displayed points.

Draw and measure angles

1. Direct the students to draw a right angle on the back of the Coordinate Planes page. Elicit that a right angle measures 90°, forming a square corner. Instruct the students to measure their angles.
2. Follow a similar procedure for an acute angle, an obtuse angle, and a straight angle. Encourage the students to estimate the measure of the acute and the obtuse angles before measuring them. Allow students to show their angles and explain the classification of each. (See Lesson 51.)
3. Select students to tell 3 names for each right angle drawn on the front of the page and to name the rays that form each angle. **$\angle B$, $\angle ABD$, $\angle DBA$, $\overrightarrow{BA}$ *and* $\overrightarrow{BD}$; $\angle D$, $\angle BDC$, $\angle CDB$, $\overrightarrow{DB}$ *and* $\overrightarrow{DC}$**

   (*Note:* The Coordinate Planes pages will be used again in the lesson.)

Identify supplementary angles and complementary angles

1. Display the Complementary & Supplementary Angles page. Elicit from the students that the measures of complementary angles have a sum of 90° and the measures of supplementary angles have a sum of 180°.
2. Choose students to identify complementary angles and supplementary angles in each section on the page. Instruct each student to explain his answer.

Demonstrate an understanding of polygons

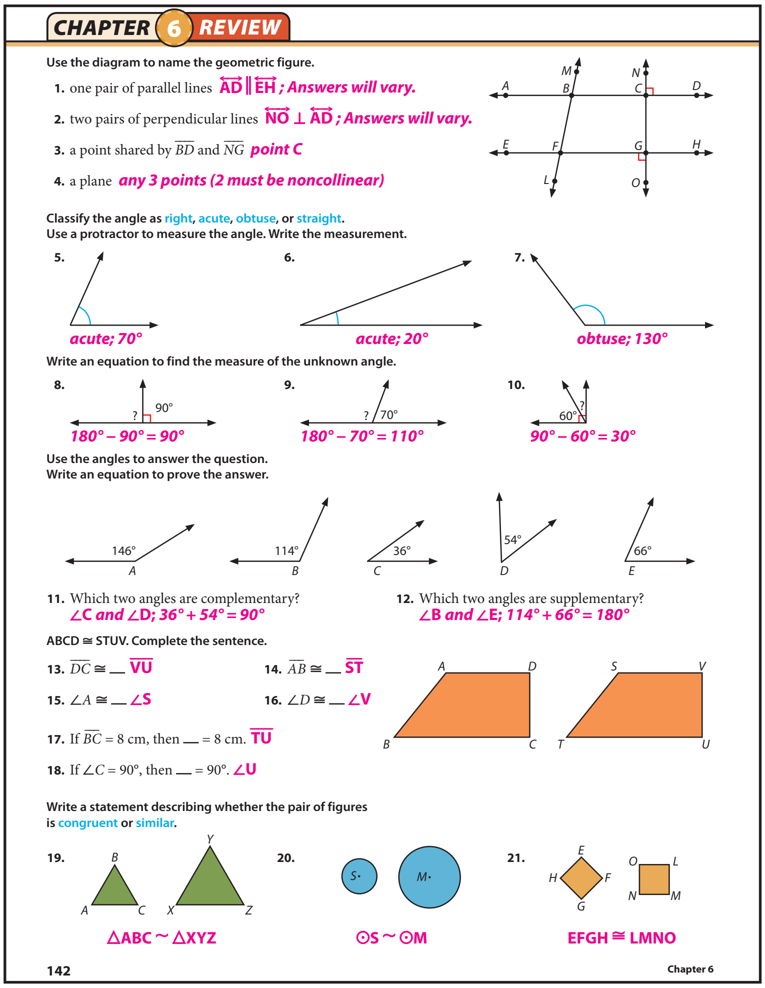
1. Direct the students' attention to the first plane on their Coordinate Planes page. Instruct them to draw a line segment to form a quadrilateral, using the points plotted earlier.
   - ➤ **What line segment did you draw? $\overline{AC}$**
   - ➤ **Is the quadrilateral regular or irregular? Why? *Irregular; elicit that neither the 4 sides nor the 4 angles are congruent.***
2. Direct the students to draw another line segment to form a triangle, using the plotted points.
   - ➤ **What line segments did you draw to make a triangle? *possible answers: line segment* AD *or line segment* BC**
3. Choose students to draw polygons such as an *irregular octagon* and a *regular quadrilateral* for display. Instruct each student to tell the characteristics of his figure and why it is regular or irregular. (See Lesson 53.)

Identify congruent and similar figures

1. Write ≅ and ∼ for display and elicit the meaning of each symbol. ***is congruent to; is similar to***
2. Draw for display pairs of congruent and similar figures and select students to write an expression that describes the relationship between the figures in each pair. (See Lesson 56.)

Classify triangles by angles and sides; find the unknown measure of an angle in a triangle

1. Display the Triangles page. Review classifying triangles according to their angles (acute, obtuse, or right), and according to their sides (equilateral, isosceles, scalene). Remind the students that slash marks through the sides of a triangle indicate congruent sides. Draw slash marks through the congruent sides in ΔDEF and in ΔGHI. (See Lesson 54.)



CHAPTER 6 REVIEW

Use the diagram to name the geometric figure.

1. one pair of parallel lines $\overleftrightarrow{AD} \parallel \overleftrightarrow{EH}$ ***; Answers will vary.***

2. two pairs of perpendicular lines $\overleftrightarrow{NO} \perp \overleftrightarrow{AD}$ ***; Answers will vary.***

3. a point shared by $\overline{BD}$ and $\overline{NG}$ ***point C***

4. a plane ***any 3 points (2 must be noncollinear)***

Classify the angle as right, acute, obtuse, or straight.
Use a protractor to measure the angle. Write the measurement.

5. ***acute; 70°***

6. ***acute; 20°***

7. ***obtuse; 130°***

Write an equation to find the measure of the unknown angle.

8. ? 90° ***180° – 90° = 90°***

9. ? 70° ***180° – 70° = 110°***

10. 60° ? ***90° – 60° = 30°***

Use the angles to answer the question.
Write an equation to prove the answer.

11. Which two angles are complementary?
∠C and ∠D; 36° + 54° = 90°

12. Which two angles are supplementary?
∠B and ∠E; 114° + 66° = 180°

ABCD ≅ STUV. Complete the sentence.

13. $\overline{DC} \cong$ __ $\overline{\mathbf{VU}}$

14. $\overline{AB} \cong$ __ $\overline{\mathbf{ST}}$

15. ∠A ≅ __ **∠S**

16. ∠D ≅ __ **∠V**

17. If $\overline{BC}$ = 8 cm, then __ = 8 cm. $\overline{\mathbf{TU}}$

18. If ∠C = 90°, then __ = 90°. **∠U**

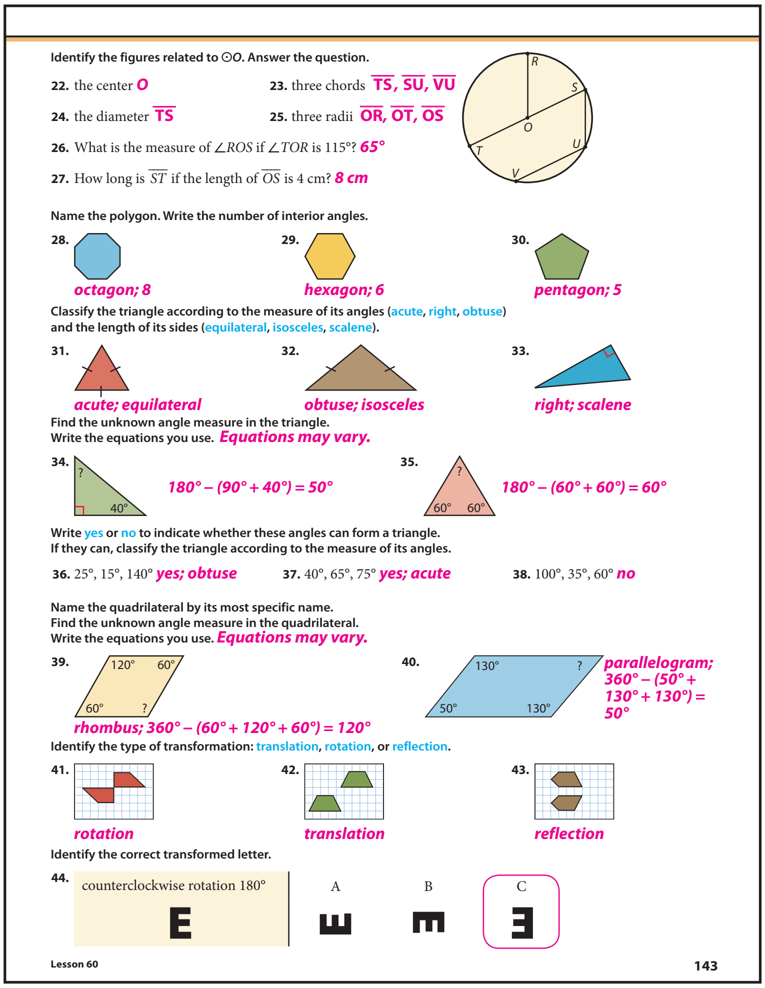
Write a statement describing whether the pair of figures is congruent or similar.

19. **△ABC ~ △XYZ**

20. **⊙S ~ ⊙M**

21. **EFGH ≅ LMNO**

142 Chapter 6



Identify the figures related to ⊙O. Answer the question.

22. the center ***O***

23. three chords $\overline{\mathbf{TS}}, \overline{\mathbf{SU}}, \overline{\mathbf{VU}}$

24. the diameter $\overline{\mathbf{TS}}$

25. three radii $\overline{\mathbf{OR}}, \overline{\mathbf{OT}}, \overline{\mathbf{OS}}$

26. What is the measure of ∠ROS if ∠TOR is 115°? ***65°***

27. How long is $\overline{ST}$ if the length of $\overline{OS}$ is 4 cm? ***8 cm***

Name the polygon. Write the number of interior angles.

28. ***octagon; 8***

29. ***hexagon; 6***

30. ***pentagon; 5***

Classify the triangle according to the measure of its angles (acute, right, obtuse) and the length of its sides (equilateral, isosceles, scalene).

31. ***acute; equilateral***

32. ***obtuse; isosceles***

33. ***right; scalene***

Find the unknown angle measure in the triangle.
Write the equations you use. ***Equations may vary.***

34. ? 40° ***180° – (90° + 40°) = 50°***

35. ? 60° 60° ***180° – (60° + 60°) = 60°***

Write yes or no to indicate whether these angles can form a triangle.
If they can, classify the triangle according to the measure of its angles.

36. 25°, 15°, 140° ***yes; obtuse***

37. 40°, 65°, 75° ***yes; acute***

38. 100°, 35°, 60° ***no***

Name the quadrilateral by its most specific name.
Find the unknown angle measure in the quadrilateral.
Write the equations you use. ***Equations may vary.***

39. 120° 60° 60° ?
rhombus; 360° – (60° + 120° + 60°) = 120°

40. 130° ? 50° 130°
parallelogram; 360° – (50° + 130° + 130°) = 50°

Identify the type of transformation: translation, rotation, or reflection.

41. ***rotation***

42. ***translation***

43. ***reflection***

Identify the correct transformed letter.

44. counterclockwise rotation 180°

Lesson 60 143

2. Write the following measures in the angles of the triangles. Guide the students in writing an equation to find the unknown measure in each triangle. Elicit that the total measure of the angles in a triangle is 180°.

   $m\angle A = 25°, m\angle B = 90°$ ***180° – (25° + 90°) = 65°***

   $m\angle D = 60°, m\angle E = 60°$ ***180° – (60° + 60°) = 60°***

   $m\angle G = 20°, m\angle H = 20°$ ***180° – (20° + 20°) = 140°***

Classify quadrilaterals; find the unknown measure of an angle in a quadrilateral

1. Write *parallelogram*, *rectangle*, *rhombus*, *square*, and *trapezoid* for display. Choose students to draw the figures. Instruct each student to tell the characteristics of his drawing such as number of parallel sides, congruent sides, and/or congruent angles. (See Lesson 55.)
2. Display the Quadrilaterals page. Write the following measures in the angles of the quadrilaterals. Guide the students in writing an equation to find the unknown measure in each quadrilateral. Elicit that the total measure of the angles in a quadrilateral is 360°.

   $m\angle A = 115°, m\angle B = 65°, m\angle C = 85°$
   360° – (115° + 65° + 85°) = 95°

   $m\angle F = 77°, m\angle G = 103°, m\angle H = 77°$
   360° – (77° + 103° + 77°) = 103°

   $m\angle I = 70°, m\angle J = 110°, m\angle L = 110°$
   360° – (70° + 110° + 110°) = 70°

Identify the parts of a circle; find the length of the radius and the diameter of a circle

1. Display the Circle page and review the following: center, chord, diameter, radius, and central angle. Elicit that a circle is named by its center. (See Lesson 58.)
2. Review finding the length of the diameter if the radius is 3 in. ***6 in.***, 12 cm ***24 cm***, or 15 cm ***30 cm***.
3. Review finding the length of the radius if the diameter is 8 ft ***4 ft***, 16 in. ***8 in.***, or 22 cm ***11 cm***.

Identify a transformation; identify symmetrical figures

1. Display the Transformations page and review translation, rotation, and reflection. (See Lesson 57.)
2. Choose students to tell which figures on the page are symmetrical. Instruct each student to explain his answer and to draw a line of symmetry in the figure. ***figures 2, 6, 9, 10, 12***

Student Text pp. 142–43

Christian Worldview Shaping (CD)

Lesson
61

Student Text pp. 144–47

Chapter 6 Test

Cumulative Review

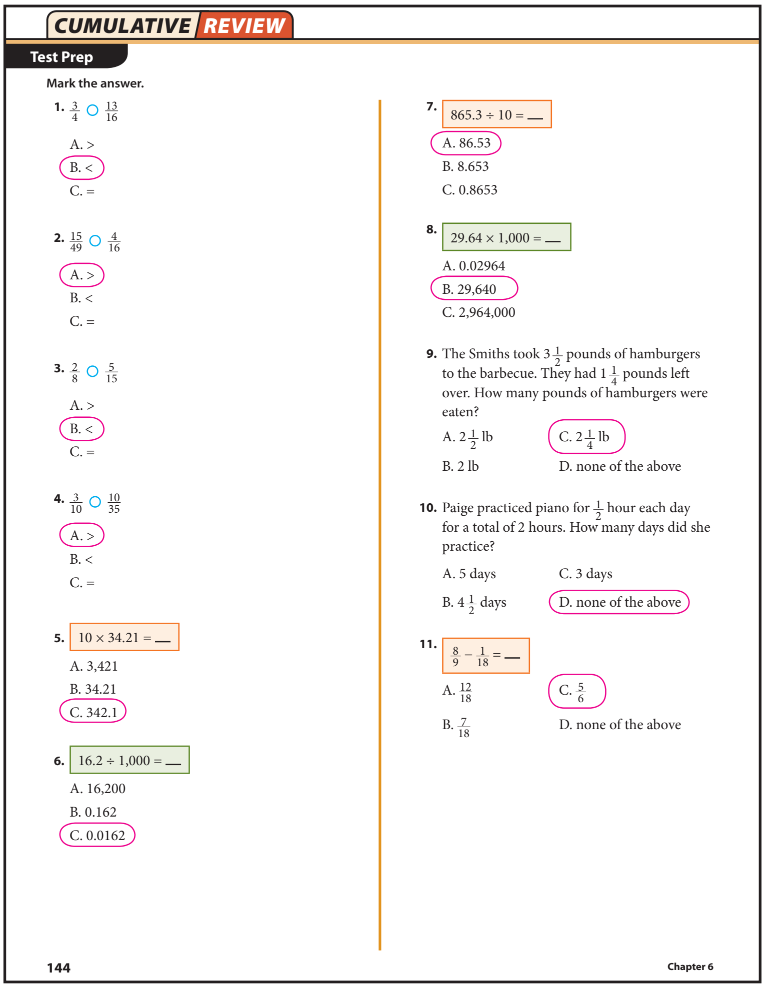
For a list of the skills reviewed in the Cumulative Review, see the Lesson Objectives for Lesson 61 in the Chapter Overview on page 119 of this Teacher's Edition.

Student Materials

- Cumulative Review Answer Sheet, page IA9 (CD)

Use the Cumulative Review on Student Text pages 144–46 to review previously taught concepts and to determine which students would benefit from your reteaching of the concepts. To prepare the students for the format of achievement tests, instruct them to work on a separate sheet of paper, if necessary, and to mark the answers on the Cumulative Review Answer Sheet.

Read aloud the Career Link on Student Text page 147 (page 145 of this Teacher's Edition) and discuss the value of math as it relates to a computer systems analyst.



CUMULATIVE REVIEW

Test Prep

Mark the answer.

1. $\frac{3}{4}$ ○ $\frac{13}{16}$
   A. >
   [✓ B. <]
   C. =

2. $\frac{15}{49}$ ○ $\frac{4}{16}$
   [✓ A. >]
   B. <
   C. =

3. $\frac{2}{8}$ ○ $\frac{5}{15}$
   A. >
   [✓ B. <]
   C. =

4. $\frac{3}{10}$ ○ $\frac{10}{35}$
   [✓ A. >]
   B. <
   C. =

5. $10 \times 34.21 = \_\_$
   A. 3,421
   B. 34.21
   [✓ C. 342.1]

6. $16.2 \div 1{,}000 = \_\_$
   A. 16,200
   B. 0.162
   [✓ C. 0.0162]

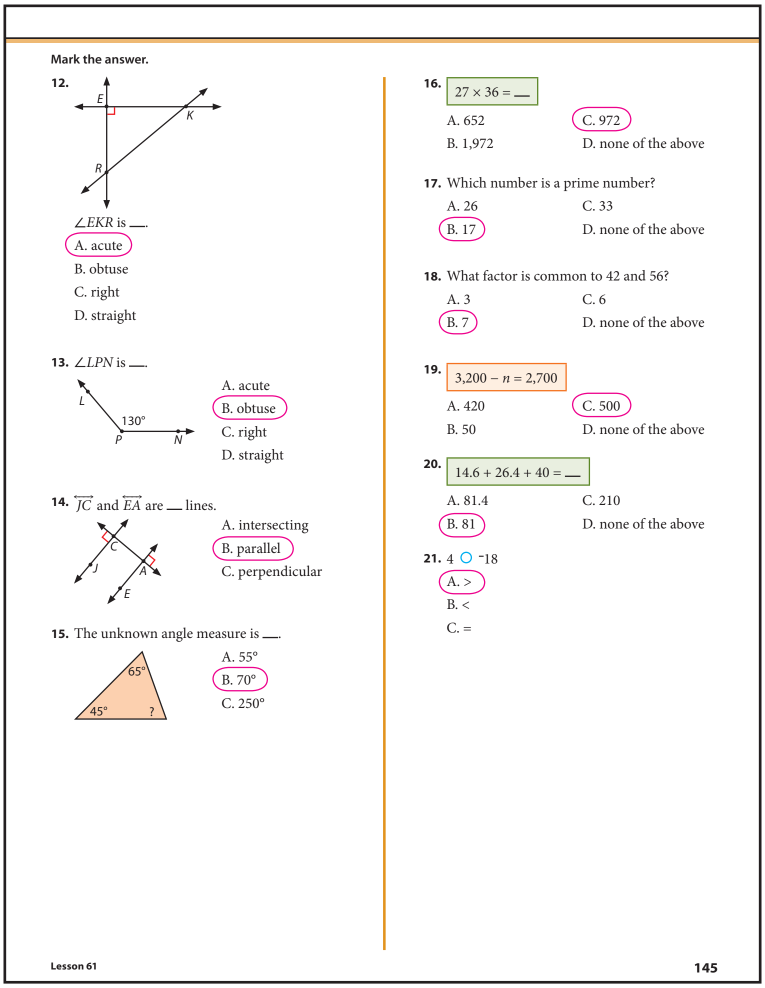
7. $865.3 \div 10 = \_\_$
   [✓ A. 86.53]
   B. 8.653
   C. 0.8653

8. $29.64 \times 1{,}000 = \_\_$
   A. 0.02964
   [✓ B. 29,640]
   C. 2,964,000

9. The Smiths took $3\frac{1}{2}$ pounds of hamburgers to the barbecue. They had $1\frac{1}{4}$ pounds left over. How many pounds of hamburgers were eaten?
   A. $2\frac{1}{2}$ lb
   [✓ C. $2\frac{1}{4}$ lb]
   B. 2 lb
   D. none of the above

10. Paige practiced piano for $\frac{1}{2}$ hour each day for a total of 2 hours. How many days did she practice?
    A. 5 days
    C. 3 days
    B. $4\frac{1}{2}$ days
    [✓ D. none of the above]

11. $\frac{8}{9} - \frac{1}{18} = \_\_$
    A. $\frac{12}{18}$
    [✓ C. $\frac{5}{6}$]
    B. $\frac{7}{18}$
    D. none of the above



Mark the answer.

12. ∠EKR is __.
    [✓ A. acute]
    B. obtuse
    C. right
    D. straight

13. ∠LPN is __.
    A. acute
    [✓ B. obtuse]
    C. right
    D. straight

14. $\overleftrightarrow{JC}$ and $\overleftrightarrow{EA}$ are __ lines.
    A. intersecting
    [✓ B. parallel]
    C. perpendicular

15. The unknown angle measure is __.
    A. 55°
    [✓ B. 70°]
    C. 250°

16. $27 \times 36 = \_\_$
    A. 652
    [✓ C. 972]
    B. 1,972
    D. none of the above

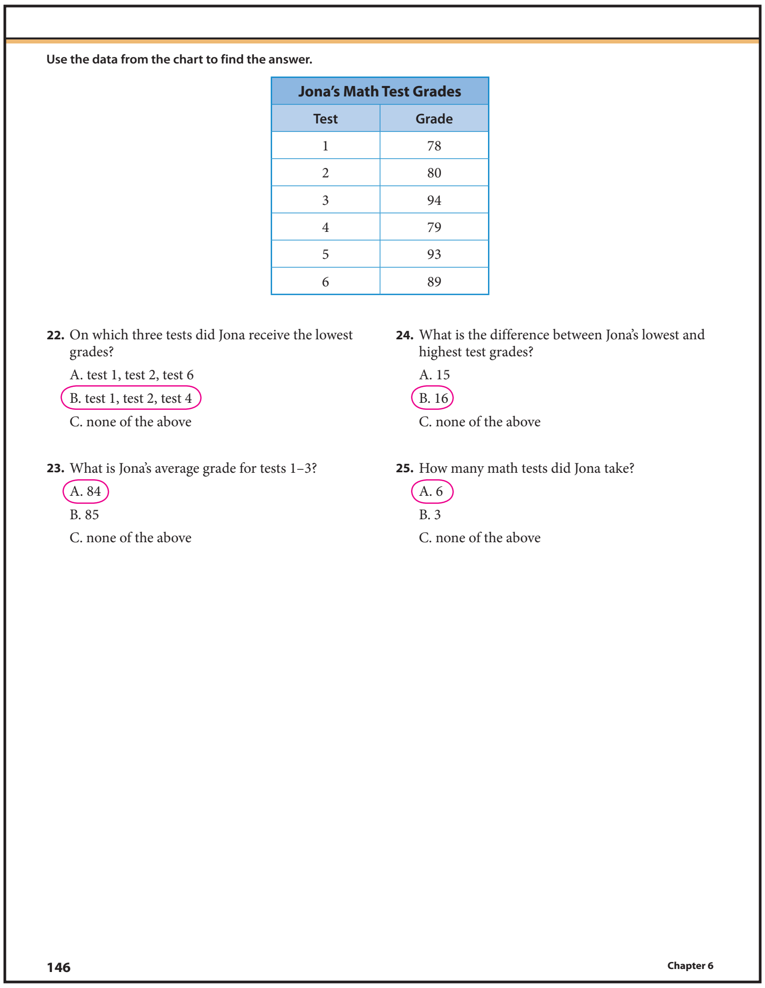
17. Which number is a prime number?
    A. 26
    C. 33
    [✓ B. 17]
    D. none of the above

18. What factor is common to 42 and 56?
    A. 3
    C. 6
    [✓ B. 7]
    D. none of the above

19. $3{,}200 - n = 2{,}700$
    A. 420
    [✓ C. 500]
    B. 50
    D. none of the above

20. $14.6 + 26.4 + 40 = \_\_$
    A. 81.4
    C. 210
    [✓ B. 81]
    D. none of the above

21. 4 ○ $^{-}18$
    [✓ A. >]
    B. <
    C. =



Use the data from the chart to find the answer.

Jona's Math Test Grades

Test	Grade
1	78
2	80
3	94
4	79
5	93
6	89

22. On which three tests did Jona receive the lowest grades?
    A. test 1, test 2, test 6
    [✓ B. test 1, test 2, test 4]
    C. none of the above

23. What is Jona's average grade for tests 1–3?
    [✓ A. 84]
    B. 85
    C. none of the above

24. What is the difference between Jona's lowest and highest test grades?
    A. 15
    [✓ B. 16]
    C. none of the above

25. How many math tests did Jona take?
    [✓ A. 6]
    B. 3
    C. none of the above

CAREER LINK

Computer Systems Analyst

A computer systems analyst's training gives him the skills to design and analyze computer systems; program and design software; and evaluate and install software, hardware, and other equipment needed to construct a network that meets the needs of a company. For example, an analyst may meet with a store manager about installing a computer system for his growing business. The manager, realizing that his current equipment is slowing down the productivity of his employees, relies on the computer systems analyst to supply him with the right system and products to maintain a productive business.

A computer systems analyst is aware of the company's current needs while looking for ways to make the system efficient. He must think about the future of the company and decide whether his employer's money is better spent upgrading his current programs and software or replacing the entire operating system. Because technology is constantly changing, the analyst is always looking for the best way to maximize his client's computer budget. Sometimes he is able to simply plug ready-made hardware or software into an existing system and then train personnel how to use and maintain it. Sometimes he must customize the system to meet the company's specific needs.

The analyst has a working knowledge of the mathematical programs of the system and products he recommends in order to determine which one best meets the needs of his client. He relies on logic in choosing, programming, and using computer solutions. His mathematical problem-solving skills equip him to do his job effectively and to help his client run his business efficiently.