7 Multiply Fractions & Decimals

A RELAY OF HOPE

Nome, Alaska **February 26, 1925**

In the winter of 1925, the townspeople of Nome, Alaska, on the coast of the Bering Sea, were troubled. Already several children had become ill and died from diphtheria. The disease was very contagious, and those who contracted it usually died unless treated with a special serum. Dr. Curtis Welch asked that a supply of the serum be sent from a hospital in Anchorage. He had only a limited supply and feared the deadly diphtheria would spread quickly through his little town. The hospital had enough serum to protect Nome and sent the medicine by train from Anchorage to Nenana, where the railroad ended. The only way left to cover the almost 700 miles of ice and snow between Nenana and Nome was to follow the mail carriers' trail with dogsled teams.

Statue of Balto in Central Park, New York

Twenty dogsled teams answered the plea to join in a relay to bring the medicine to Nome. The dogs and their drivers traveled to various stations along the route and waited their turn in the race against death. They traveled many miles in harsh weather and at top speeds along the lonely, silent trail to deliver the needed serum to the next team, miles away. Team after team fought the terrain and the weather until they were able to hand off the medicine to the next brave pack of dogs and their courageous musher.

One very strong dog, Togo, fought his way through the harsh winter winds. His musher was Leonhard Seppala, a champion racer. He led his team along the icy shoreline, which sometimes broke apart, but they braved the harsh run until they were able to pass along the serum to Charlie Olsen. On his leg of the trip, Olsen was blown off the trail and endured painful frostbite before he arrived at Gunnar Kassen's station.

Kassen and his black husky, Balto, ran through blustery winds so strong that their sled fell over, causing them to almost lose the container that held the precious serum. Kassen used his bare hands to find the serum in the snow and suffered frostbite for his bravery. Within hours he arrived in Nome, where the serum was thawed and ready to be used.

The dogsled teams covered the 674-mile trail in about 127 hours (six days) through wind-chill temperatures below −50°F, limited daylight, and winds up to 65 miles per hour.

In 1927 President Calvin Coolidge presented each musher with a gold medal.

A bronze statue of the Siberian husky Balto stands in New York's Central Park.

A display in the Iditarod Museum in Wasilla, Alaska, honors the diligent race of Seppala's dog Togo.

Today the Iditarod Trail Sled Dog Race, which covers over 1,150 Alaskan miles, commemorates the famous Serum Run of 1925.

Diphtheria is now rare in the United States due to vaccines given to most newborn babies.

Multiply Fractions & Decimals

Lesson	Topic	Lesson Objectives	Chapter Materials
62	**Multiply Fractions**	• Multiply a whole number and a fraction • Multiply to find a fraction of a whole number • Multiply to find a fraction of a fraction • Rename a whole number as an improper fraction • Write and solve an equation for a multiplication word problem	**Teacher Manipulatives Packet:** • Decimal Place Value Pocket Chart (B) • Fraction Kit • Fraction Number Line (yellow) **Student Manipulatives Packet:** • Fraction Kit • Fraction Number Line (yellow) **Instructional Aids (Teacher's Toolkit CD):** • Cumulative Review Answer Sheet (page IA9) for each student • Fraction Paper Folding (page IA39) (optional) • Decimal Grids (page IA40) • In-Between Numbers (page IA41) **Christian Worldview Shaping (Teacher's Toolkit CD):** • Pages 20–21 **Other Teaching Aids:** • 12 counters • A blank sheet of paper (unlined) for the teacher • 2 blank sheets of paper (unlined) for each student • Colored markers: red and blue **Math 6 Tests and Answer Key** **Optional (Teacher's Toolkit CD):** • Fact Review pages • Application pages • Calculator Activities
63	**Simplify**	• Multiply to find a fraction of a fraction • Use cancellation when multiplying fractions • Identify the reciprocal of a fraction • Write and solve an equation for a word problem	
64	**Multiply Mixed Numbers**	• Multiply mixed numbers • Rename an improper fraction as a mixed number • Rename a mixed number as an improper fraction • Apply the Distributive Property of Multiplication over Addition when multiplying mixed numbers • Use cancellation when multiplying mixed numbers • Write and solve an equation for a word problem • Estimate products of mixed numbers and fractions	
65	**Multiply Decimals**	• Multiply a decimal by a decimal • Demonstrate an understanding of the relationship between decimals and fractions • Estimate a decimal product by rounding to the nearest whole number or to the place of greatest value • Apply the Distributive Property of Multiplication over Addition when multiplying decimals • Use mental math to multiply a decimal and a power of 10	
66	**More Multiplying Decimals**	• Multiply a decimal by a decimal • Demonstrate an understanding of the relationship between decimals and fractions • Annex zeros in the product • Multiply money by a decimal • Estimate a decimal product by rounding	
67	**Between Numbers**	• Develop an understanding of our infinite number system • Identify fractions and decimals that come between two numbers • Solve a multi-step word problem	
68	**Chapter 7 Review**	• Review	
69	**Chapter 7 Test Cumulative Review**	• Find the unknown measure of an angle in a quadrilateral • Identify the parts of a circle • Classify triangles: equilateral, isosceles, scalene • Solve money word problems • Add and subtract mixed numbers • Identify the value of a digit in a decimal • Multiply fractions and decimals • Read and interpret a double line graph	

A Little Extra Help

Use the following to provide "a little extra help" for the student that is experiencing difficulty with the concepts taught in Chapter 7.

Multiply to find a fraction of a fraction—The students have learned that to add and subtract fractions the denominators must be the same, and the sum also has the same denominator. It may be difficult for them to understand that when they multiply fractions, the denominators can be different. The use of manipulatives to teach multiplication will help their understanding of finding a part of a part. However, the actual computation may still be confusing for some students. Writing out each step in solving an equation may help these students. The students may need to see the steps often until they can eventually do some of the steps mentally. An example is shown below. The parts of the equation in the shaded box are "mental" steps that students may need to write out to help them understand the process.

$$\frac{2}{3} \times \frac{1}{6} = \frac{2 \times 1}{3 \times 6} = \frac{2}{18}$$

Rename a mixed number as an improper fraction—Students needing extra help with renaming mixed numbers may try the following methods. Write $2\frac{1}{4}$ for display. As you give the following explanations, write the corresponding equations.

Point out that the mixed number $2\frac{1}{4}$ can be renamed: $2 \times \frac{4}{4} = \frac{8}{4}$, $\frac{8}{4} + \frac{1}{4} = \frac{9}{4}$.

Explain that $2\frac{1}{4}$ can also be renamed by multiplying the whole number times the denominator of the fraction and then adding the numerator; this process provides you with the numerator for the improper fraction, and the denominator of the improper fraction is the same as the denominator of the mixed number: $(2 \times 4) + 1 = 9$, the numerator is 9 and the denominator is 4, $\frac{9}{4}$.

Repeat the procedures with other mixed numbers if necessary.

Multiply a decimal by a decimal—Instruct the student who is experiencing difficulty multiplying decimals to rewrite each multiplication problem without the decimal points. Direct him to multiply as if the factors are whole numbers and to solve the problem. Next, direct the student to combine the number of decimal places in the factors of the original problem and to write the decimal point in the product so that the number of decimal places in the product is the same as the total number of decimal places in the factors.

Math Facts

Throughout this chapter, review fractions using Fact Review pages on the Teacher's Toolkit CD. Also, review addition, subtraction, multiplication, and division facts using Fact Review pages or a Fact Fun activity on the Teacher's Toolkit CD, or you may use flashcards.

Lesson 62

Student Text pp. 148–51
Daily Review p. 426a

Objectives
- Multiply a whole number and a fraction
- Multiply to find a fraction of a whole number
- Multiply to find a fraction of a fraction
- Rename a whole number as an improper fraction
- Write and solve an equation for a multiplication word problem

Teacher Materials
- Fraction Kit: fraction bars (fifths)
- Fraction Paper Folding, page IA39 (CD) (optional)
- 12 counters
- A blank sheet of paper (unlined)

Student Materials
- Fraction Kit
- 2 blank sheets of paper (unlined) (1 sheet optional)

Note
Preview the Fact Review pages, the Application pages, and the Calculator Activities located on the Teacher's Toolkit CD.

Introduce the Lesson

Guide the students in reading aloud the story and facts on pages 148–49 of the Student Text (pages 146–47 of this Teacher's Edition).

Teach for Understanding

Multiply a whole number and a fraction

1. Write $1 \times \frac{1}{5} =$ ___ and display 1 fifth fraction bar.

➤ **What is $1 \times \frac{1}{5}$?** $\frac{1}{5}$

Repeat the procedure, displaying 1 additional fifth for each of these equations: $2 \times \frac{1}{5} = \frac{2}{5}$, $3 \times \frac{1}{5} = \frac{3}{5}$, $4 \times \frac{1}{5} = \frac{4}{5}$, $5 \times \frac{1}{5} = \frac{5}{5} = 1$. Remind the students that multiplication is repeated addition.

2. Distribute the Fraction Kits and arrange the students in groups. Write *5 sets of* $\frac{3}{4}$ for display. Direct each group to use the fraction bars or fraction circles to show 5 sets of $\frac{3}{4}$ and to write 2 equations for their fraction picture.

➤ **What equations are true for this picture?** *Elicit* $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{15}{4}$ *and* $5 \times \frac{3}{4} = \frac{15}{4}$. Write both equations for display.

3. Write *3 fourths + 3 fourths + 3 fourths + 3 fourths + 3 fourths = 15 fourths* and *5 × 3 fourths = 15 fourths*.

➤ **What do you notice about these equations?** *Elicit that all the labels are the same (fourths).*

4. Remind the students that addition is the combining of like objects to get more of the same objects (e.g., When adding 3 apples to 3 oranges, you need to rename them both as pieces of fruit: 3 pieces of fruit + 3 pieces of fruit = 6 pieces of fruit). Point out that you added fourths to fourths and got more fourths. Explain that multiplication is the combining of same-size sets of like objects to get more of the same object. When you combined the sets of fourths, the total was more fourths.

➤ **What do you know about $\frac{15}{4}$?** *Elicit that $\frac{15}{4}$ is an improper fraction; it should be renamed to lowest (simplest) terms.*

➤ **What is $\frac{15}{4}$ in lowest terms? How do you know?** $3\frac{3}{4}$*; elicit that there are 3 sets of 4 fourths (3 wholes) and 3 fourths remaining in* $\frac{15}{4}$, $\frac{15}{4} = \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{3}{4}$.

5. Follow a similar procedure to solve these problems.

$5 \times \frac{1}{2} = \frac{5}{2} = 2\frac{1}{2}$ $\quad$ $3 \times \frac{6}{7} = \frac{18}{7} = 2\frac{4}{7}$

$8 \times \frac{3}{4} = \frac{24}{4} = 6$ $\quad$ $6 \times \frac{7}{8} = \frac{42}{8} = 5\frac{2}{8} = 5\frac{1}{4}$

➤ **What do you notice about multiplying a whole number and a fraction?** *Elicit that you multiply the whole number times the numerator (the number of parts selected); the denominator (the name of the equal parts) does not change.* Point out that the denominator does not change because the denominator of a whole number is 1.

6. Write $\frac{2}{3} \times 12 =$ ___ and display 12 counters.

➤ **Does this multiplication equation represent a whole set of 12? Why?** *No; elicit that since the multiplier $\frac{2}{3}$ (the first factor) is a fraction, you are multiplying to find a part of the whole set of 12, so the product will be less than 12.*

Explain that the multiplication sign in the equation means *of*, so the equation is read $\frac{2}{3}$ *of 12*. You are multiplying to find a part of the multiplicand (12) rather than repeating the multiplicand as you do when the first factor or both factors are whole numbers.

➤ **How could you find $\frac{1}{3}$ of the set of 12?** *Elicit that 1 of 3 equal parts in 12 is found by dividing 12 by 3.*

Choose a student to divide the 12 counters into 3 equal sets. Point out that since you know that $\frac{1}{3}$ of a set of 12 is 4, you can find any number of thirds.

➤ **How many counters are in each equal set?** *4* **in 2 sets?** *8* **in 3 sets?** *12*

➤ **What is $\frac{1}{3}$ of a set of 12?** *4* $\frac{2}{3}$ **of a set of 12?** *8* $\frac{3}{3}$ **of a set of 12?** *12*

7. Direct attention to the multiplication equation $\frac{2}{3} \times 12$. Point out that the whole number 12 is multiplied by the numerator 2 to find the number of thirds in a part ($\frac{2}{3}$) of the set. Choose a student to multiply $\frac{2}{3} \times 12$. $\frac{24}{3} = 8$

8. Write the following equations for display and lead in reading aloud each equation (e.g., $\frac{1}{3}$ *of 9*). Direct the students to solve the equations on paper, giving guidance as needed. Remind them that when you multiply a whole number by a fraction, the product is a part of the whole number.

$\frac{1}{3} \times 9 = \frac{9}{3} = 3$ $\quad$ $\frac{2}{5} \times 6 = \frac{12}{5} = 2\frac{2}{5}$

$\frac{6}{8} \times 5 = \frac{30}{8} = 3\frac{6}{8} = 3\frac{3}{4}$ $\quad$ $\frac{9}{10} \times 3 = \frac{27}{10} = 2\frac{7}{10}$

Multiply to find a fraction of a fraction

1. Distribute the blank sheets of paper.

> Jeff and his brother painted $\frac{1}{4}$ of the fence on Tuesday. Jeff did $\frac{1}{2}$ of the work. How much of the entire fence did Jeff paint? $\frac{1}{8}$ *of the fence*

➤ **What is the problem asking you to find?** *how much of the fence Jeff painted*

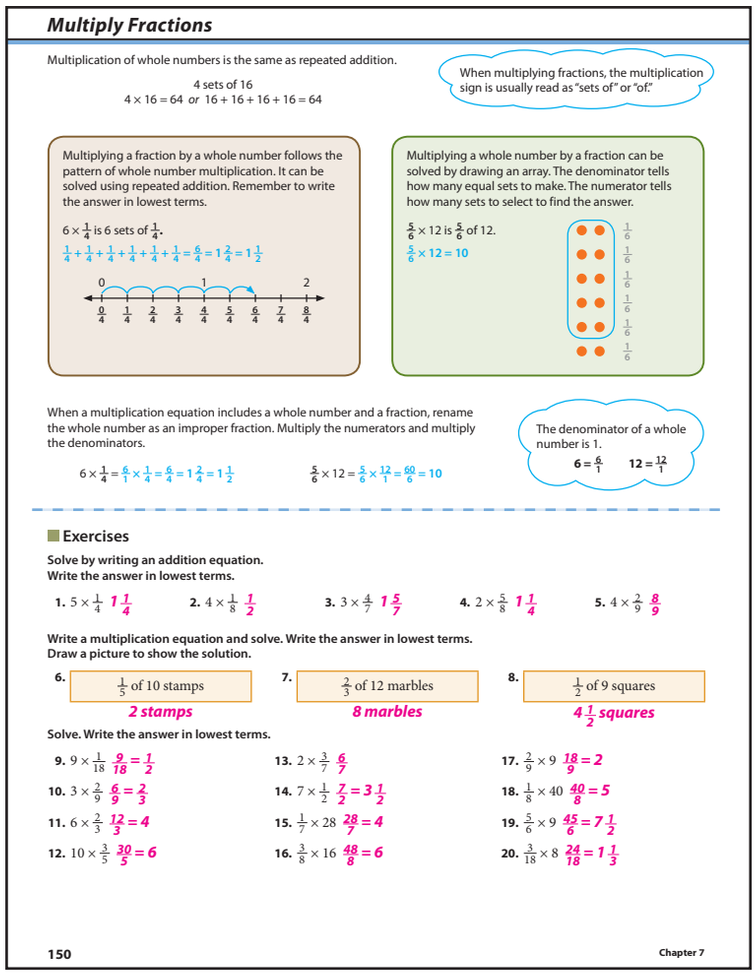
➤ **What information is given?** *The brothers painted $\frac{1}{4}$ of the fence; Jeff painted $\frac{1}{2}$ of the $\frac{1}{4}$ of the fence.*

2. Tell the students to pretend that one sheet of paper represents the whole length of the fence.

➤ **How could you fold the paper to show the part of the fence that the boys painted?** *Elicit that you can fold the paper in fourths lengthwise and shade 1 of the 4 equal parts.*

Direct the students to fold their whole "fence" lengthwise into 4 equal parts and to lightly shade one part ($\frac{1}{4}$). Demonstrate each step throughout this activity.

➤ **How could you show the $\frac{1}{2}$ of the $\frac{1}{4}$ of the fence that Jeff painted?** *Elicit that you could shade $\frac{1}{2}$ of the $\frac{1}{4}$ of the fence.*



Multiply Fractions

Multiplication of whole numbers is the same as repeated addition.

4 sets of 16
$4 \times 16 = 64$ *or* $16 + 16 + 16 + 16 = 64$

When multiplying fractions, the multiplication sign is usually read as "sets of" or "of."

Multiplying a fraction by a whole number follows the pattern of whole number multiplication. It can be solved using repeated addition. Remember to write the answer in lowest terms.

$6 \times \frac{1}{4}$ is 6 sets of $\frac{1}{4}$.

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$

Multiplying a whole number by a fraction can be solved by drawing an array. The denominator tells how many equal sets to make. The numerator tells how many sets to select to find the answer.

$\frac{5}{6} \times 12$ is $\frac{5}{6}$ of 12.

$\frac{5}{6} \times 12 = 10$

When a multiplication equation includes a whole number and a fraction, rename the whole number as an improper fraction. Multiply the numerators and multiply the denominators.

$6 \times \frac{1}{4} = \frac{6}{1} \times \frac{1}{4} = \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$

$\frac{5}{6} \times 12 = \frac{5}{6} \times \frac{12}{1} = \frac{60}{6} = 10$

The denominator of a whole number is 1.

$6 = \frac{6}{1}$ $12 = \frac{12}{1}$

Exercises

Solve by writing an addition equation.
Write the answer in lowest terms.

1. $5 \times \frac{1}{4}$ *1 $\frac{1}{4}$*
2. $4 \times \frac{1}{8}$ *$\frac{1}{2}$*
3. $3 \times \frac{4}{7}$ *1 $\frac{5}{7}$*
4. $2 \times \frac{5}{8}$ *1 $\frac{1}{4}$*
5. $4 \times \frac{2}{9}$ *$\frac{8}{9}$*

Write a multiplication equation and solve. Write the answer in lowest terms.
Draw a picture to show the solution.

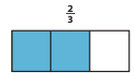
6. $\frac{1}{5}$ of 10 stamps — *2 stamps*
7. $\frac{2}{3}$ of 12 marbles — *8 marbles*
8. $\frac{1}{2}$ of 9 squares — *4 $\frac{1}{2}$ squares*

Solve. Write the answer in lowest terms.

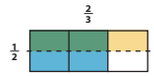
9. $9 \times \frac{1}{18}$ *$\frac{9}{18} = \frac{1}{2}$*
10. $3 \times \frac{2}{9}$ *$\frac{6}{9} = \frac{2}{3}$*
11. $6 \times \frac{2}{3}$ *$\frac{12}{3} = 4$*
12. $10 \times \frac{3}{5}$ *$\frac{30}{5} = 6$*
13. $2 \times \frac{3}{7}$ *$\frac{6}{7}$*
14. $7 \times \frac{1}{2}$ *$\frac{7}{2} = 3\frac{1}{2}$*
15. $\frac{1}{7} \times 28$ *$\frac{28}{7} = 4$*
16. $\frac{3}{8} \times 16$ *$\frac{48}{8} = 6$*
17. $\frac{2}{9} \times 9$ *$\frac{18}{9} = 2$*
18. $\frac{1}{8} \times 40$ *$\frac{40}{8} = 5$*
19. $\frac{5}{6} \times 9$ *$\frac{45}{6} = 7\frac{1}{2}$*
20. $\frac{3}{18} \times 8$ *$\frac{24}{18} = 1\frac{1}{3}$*

Multiplying a fraction by a fraction is finding *a part of a part*. The product will be smaller than either factor because the answer is only a part of the original unit.

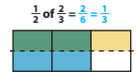
A picture can be drawn to show the product of two fractions. Find $\frac{1}{2}$ of $\frac{2}{3}$.



Draw a figure. Color two-thirds.



Draw a line the other way to show $\frac{1}{2}$ of the figure. Color one-half.



The double shaded area represents the product.

The product of two fractions can also be found by multiplying the numerators and multiplying the denominators.

$\frac{1}{2}$ of $\frac{2}{3} = \frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$

Exercises

Solve. Draw a picture to show the solution.

21. A banana bread recipe calls for $\frac{3}{4}$ of a cup of sugar. If the baker uses only $\frac{1}{2}$ of the amount of sugar called for, how much sugar is needed? *$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$*

Draw a picture to show the product.
Write an equation to solve. Write the answer in lowest terms.

22. $\frac{1}{2}$ of $\frac{1}{4}$ *$\frac{1}{8}$*
23. $\frac{1}{2}$ of $\frac{4}{7}$ *$\frac{2}{7}$*
24. $\frac{2}{3}$ of $\frac{3}{5}$ *$\frac{2}{5}$*
25. $\frac{3}{4}$ of $\frac{1}{2}$ *$\frac{3}{8}$*

Solve. Write the answer in lowest terms.

26. $\frac{5}{6} \times \frac{2}{3}$ *$\frac{10}{18} = \frac{5}{9}$*
27. $\frac{1}{4} \times \frac{3}{5}$ *$\frac{3}{20}$*
28. $\frac{1}{6} \times \frac{6}{7}$ *$\frac{6}{42} = \frac{1}{7}$*
29. $\frac{5}{6} \times \frac{3}{10}$ *$\frac{15}{60} = \frac{1}{4}$*
30. $\frac{2}{9} \times \frac{3}{5}$ *$\frac{6}{45} = \frac{2}{15}$*
31. $\frac{2}{3} \times \frac{2}{3}$ *$\frac{4}{9}$*
32. $\frac{1}{4} \times \frac{1}{2}$ *$\frac{1}{8}$*
33. $\frac{8}{9} \times \frac{6}{7}$ *$\frac{48}{63} = \frac{16}{21}$*

Solve. Write the answer in lowest terms.

34. Andrew ran $\frac{7}{10}$ of a mile each day for 5 days. How far did he run? *$\frac{5}{1} \times \frac{7}{10} = \frac{35}{10} = 3\frac{5}{10} = 3\frac{1}{2}$ miles*

35. Miranda has $\frac{5}{8}$ of a yard of ribbon. If she used $\frac{1}{3}$ of it to make a hair bow, how much ribbon is left? *$\frac{1}{3} \times \frac{5}{8} = \frac{5}{24}$; $\frac{5}{8} - \frac{5}{24} = \frac{15}{24} - \frac{5}{24} = \frac{10}{24} = \frac{5}{12}$ of a yard*

Practice & Application

36. Use front-end estimation to find the sum of 78,356; 39,785; and 35,609.
78,000 + 39,000 + 35,000 = 152,000
37. Use front-end estimation to find the difference of 53,486 and 38,750.
53,000 – 38,000 = 15,000
38. Estimate the quotient. 49,317 ÷ 73
49,000 ÷ 70 = 700
39. Find the LCM of the denominators in $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{6}$.
12
40. Which fraction is *not* equivalent to the others?
$\frac{1}{3}$ $\frac{3}{6}$ $\frac{4}{12}$ $\frac{5}{15}$
$\frac{3}{6}$

41. In which pair of fractions are the denominators related?
$\frac{3}{4}, \frac{5}{10}$ $\frac{4}{5}, \frac{3}{15}$ $\frac{2}{3}, \frac{2}{7}$
$\frac{4}{5}, \frac{3}{15}$

J Explain how you could use the picture to find $\frac{3}{4}$ of $\frac{5}{8}$.

Complete DAILY REVIEW 3 on page 426.

Instruct the students to shade the top $\frac{1}{2}$ of the "painted $\frac{1}{4}$ of the fence." Point out that the part of the fence Jeff painted (the double-shaded part) is not shown as an equal part of the whole fence.

➤ **How could you fold your whole "fence" to show the $\frac{1}{2}$ of the $\frac{1}{4}$ painted by Jeff as an equal part?** ***Elicit that you would have to fold each fourth in half.***

Direct the students to make 1 fold to partition each fourth into 2 parts.

➤ **What equal part of the whole fence did Jeff paint?** $\frac{1}{8}$

3. Write $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ for display. Remind the students that the multiplication sign means *of*. Point out that you are multiplying to find a part of a fraction ($\frac{1}{2}$ of a set of $\frac{1}{4}$), so the product will be less than either factor. Lead in reading the equation aloud: $\frac{1}{2}$ *of* $\frac{1}{4}$ *equals* $\frac{1}{8}$.

➤ **Is $\frac{1}{8}$ a reasonable answer? Why?** ***Yes; elicit that $\frac{1}{8}$ is $\frac{1}{2}$ (1 of 2 equal parts) of $\frac{1}{4}$.***

➤ **How can you solve $\frac{1}{2} \times \frac{1}{4}$ without folding paper?** ***Elicit that you can multiply the numerators and multiply the denominators.***

4. Remind the students that a fraction represents the division of 2 whole numbers, and that every whole number can be written as a fraction with 1 as the denominator, without changing the value of the whole number (e.g., $5 = \frac{5}{1}$; $5 \div 1 = 5$).

5. Write $5 \times \frac{1}{2}$ again for display.

➤ **Does multiplying the numerators and multiplying the denominators apply to $5 \times \frac{1}{2}$? Why?** ***Yes; elicit that 5 can be written as $\frac{5}{1}$ and that $\frac{5}{1} \times \frac{1}{2} = \frac{5}{2} = 2\frac{1}{2}$, just as $5 \times \frac{1}{2} = \frac{5}{2} = 2\frac{1}{2}$.***

6. Write $\frac{1}{3} \times \frac{3}{4}$. Choose a student to first multiply the numerators and then multiply the denominators. $\frac{3}{12} = \frac{1}{4}$

Direct the students to prove that $\frac{1}{4}$ is a reasonable answer by picturing $\frac{1}{3} \times \frac{3}{4}$ using the Fraction Kit or by folding paper. (***Note:*** You many choose to display the Fraction Paper Folding page as a guide.)

Solve a multiplication word problem

Brent bought 6 kinds of candy for a party. He bought $\frac{1}{4}$ of a pound of each kind. How many pounds of candy did he buy? *$1\frac{1}{2}$ pounds*

➤ **What is the question asking you to find?** ***how many pounds of candy Brent bought***

➤ **What information is given?** ***Brent bought $\frac{1}{4}$ of a pound of each of the six kinds of candy.***

➤ **What equation can you write to solve the word problem? Why?** ***$6 \times \frac{1}{4} =$ ___; there are 6 sets of $\frac{1}{4}$ of a pound of candy.***

1. Write $6 \times \frac{1}{4} =$ ___ for display.

➤ **How can you solve the equation?** ***Elicit that since 1 would be the denominator of the whole number 6, you can multiply the whole number 6 times the numerator 1 and write 4 in the denominator of the product; $\frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$ pounds.***

➤ **How many pounds of candy did Brent buy?** ***$1\frac{1}{2}$ pounds***

2. Follow a similar procedure for this word problem.

Cara practices playing the piano for $\frac{3}{4}$ of an hour each day. How many hours does she practice in 7 days?
$7 \times \frac{3}{4} = \frac{21}{4} = 5\frac{1}{4}$ hours

Student Text pp. 150–51

Lesson 63

Student Text pp. 152–53
Daily Review p. 427b

Objectives
- Multiply to find a fraction of a fraction
- Use cancellation when multiplying fractions
- Identify the reciprocal of a fraction
- Write and solve an equation for a word problem

Student Materials
- Fraction Kit: fraction circles (halves and thirds)

Teach for Understanding

Use cancellation when multiplying fractions

1. Write $\frac{5}{6} \times \frac{3}{4} = \_\_$ for display. Choose a student to solve the equation, renaming the product to lowest terms. $\frac{5}{6} \times \frac{3}{4} = \frac{15}{24} = \frac{5}{8}$ Instruct the student to explain what he did to rename the product to lowest terms. ***Elicit that he divided the numerator and the denominator by 3 because 15 and 24 are both divisible by 3.***
2. Remind the students that *cancellation* allows you to simplify fractions by canceling out fractional names for 1 from the prime factorization of the numerator and the denominator. (See Lesson 35.) Explain that cancellation can also be used to simplify fraction factors by canceling out prime factors with a value of 1 before you multiply rather than after you multiply, allowing you to work with smaller numbers.
3. Write for display the following example of using cancellation to solve $\frac{5}{6} \times \frac{3}{4}$. Point out that box A shows the process of multiplying the numerators and the denominators.

$$\frac{5}{6} \times \frac{3}{4} = \underset{\text{A}}{\boxed{\frac{5 \times 3}{6 \times 4}}} = \underset{\text{B}}{\boxed{\frac{5 \times 3}{(2 \times 3) \times (2 \times 2)}}} = \underset{\text{C}}{\boxed{\frac{5 \times \overset{1}{\cancel{3}}}{2 \times \underset{1}{\cancel{3}} \times 2 \times 2}}} = \underset{\text{D}}{\boxed{\frac{5}{8}}}$$

➤ **What do you notice about the expression in box B?** ***Elicit that the denominators have been factored to prime numbers.***
➤ **Are the fractions in box B equivalent to the original fractions?** ***yes***
Direct attention to the numbers crossed out in box C.
➤ **Why have the 3 in the numerator and the 3 in the denominator been canceled?** $\frac{3}{3} = 1$
Explain that because of the Identity Property of Multiplication, any fractional names equal to 1 can be canceled (crossed out) without changing the product. Point out that only the 3s have been canceled since there are no other fractional names equal to 1.
Choose a student to multiply the expression in box C and write the product in box D. $\frac{5}{8}$ Point out that in both processes (simplifying the product after multiplying and using cancellation to simplify before multiplying) the numerator and the denominator were divided by 3. Continue to display the example.
4. Write $\frac{5}{6} \times \frac{3}{4}$ again. Explain that you do not need to write out all of the steps every time you use cancellation. Since you know the numerator 3 is a factor of the denominator 6, you can mentally divide both numbers by 3, the greatest common factor (GCF). Demonstrate the cancellation as shown.

$$\frac{5}{\underset{2}{\cancel{6}}} \times \frac{\overset{1}{\cancel{3}}}{4} = \frac{5}{8}$$

➤ **What does the 1 written above the 3 in the numerator represent?** ***the number of times the common factor 3 is a factor in the number 3*** **the 2 written below the 6 in the denominator?** ***the number of times the common factor 3 is a factor in the number 6***
Point out that the equation now represents the equation shown in box C and that the product is the same.
5. Repeat the procedure for $\frac{3}{4} \times \frac{8}{9}$. Point out that it is faster to do the cancellation mentally. Emphasize that you must divide both a numerator and a denominator by the same common factor when canceling. Also point out that you cannot cancel a numerator and a numerator nor a denominator and a denominator.

$$\frac{3}{4} \times \frac{8}{9} = \underset{\text{A}}{\boxed{\frac{3 \times 8}{4 \times 9}}} = \underset{\text{B}}{\boxed{\frac{3 \times (2 \times 2 \times 2)}{(2 \times 2) \times (3 \times 3)}}} = \underset{\text{C}}{\boxed{\frac{(\cancel{2} \times \cancel{2} \times 2) \times \cancel{3}}{(\cancel{2} \times \cancel{2}) \times (3 \times \cancel{3})}}} = \underset{\text{D}}{\boxed{\frac{2}{3}}}$$

or

$$\frac{\overset{1}{\cancel{3}}}{4} \times \frac{2}{\underset{3}{\cancel{9}}} = \frac{2}{3}$$

6. Write the following solution for $\frac{12}{15} \times \frac{7}{8}$.

$$\frac{\overset{6}{\cancel{12}}}{15} \times \frac{7}{\underset{4}{\cancel{8}}} = \frac{\overset{3}{\cancel{6}}}{15} \times \frac{7}{\underset{2}{\cancel{4}}} = \frac{3 \times 7}{15 \times 2} = \frac{21}{30} = \frac{7}{10}$$

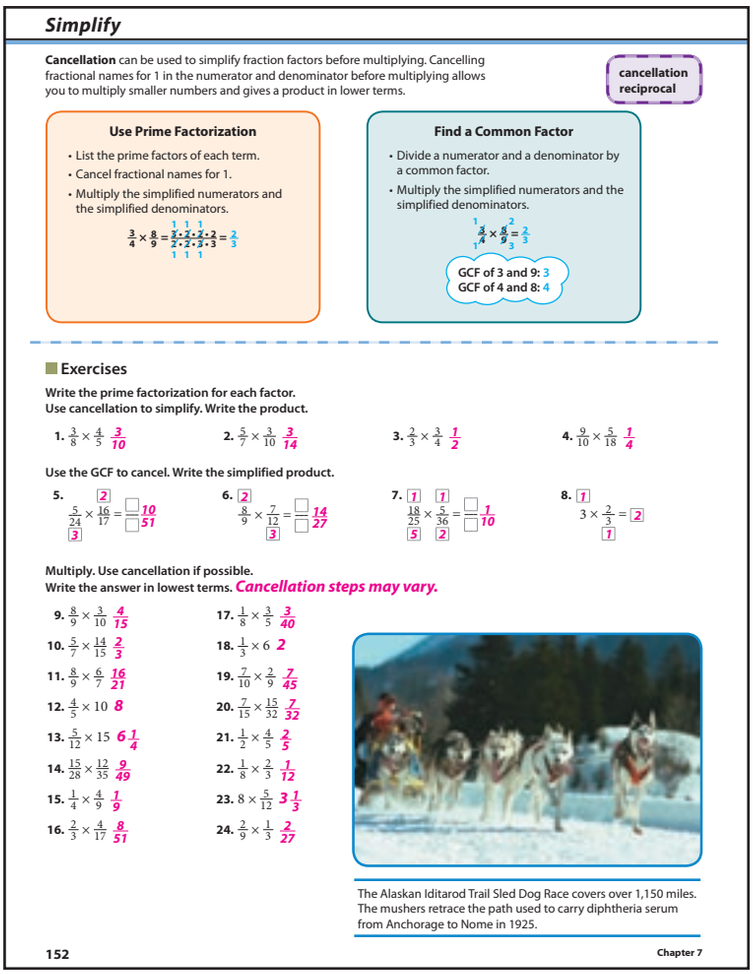
➤ **What do you notice about the cancellation process in this equation?** ***Elicit that the equation was simplified by dividing by a common factor rather than by the GCF. The numerator 12 and the denominator 8 were divided by 2, making the equation*** $\frac{6}{15} \times \frac{7}{4}$. ***Then the numerator 6 and the denominator 4 needed to be divided again by the common factor of 2.***
Explain that simplifying fraction factors using the GCF will minimize the number of times you need to divide. Point out that you still need to make sure the product is in lowest terms.
7. Guide the students in solving these equations.

$\frac{5}{9} \times \frac{3}{5} = \frac{1}{3}$ $\quad\quad$ $\frac{1}{2} \times \frac{8}{9} = \frac{4}{9}$
$\frac{4}{15} \times \frac{10}{11} = \frac{8}{33}$ $\quad\quad$ $\frac{3}{8} \times \frac{7}{9} = \frac{7}{24}$
$\frac{4}{9} \times \frac{9}{20} = \frac{1}{5}$ $\quad\quad$ $\frac{5}{18} \times \frac{3}{4} = \frac{5}{24}$

8. Write $\frac{2}{9} \times \frac{5}{7}$ for display.
➤ **What do you notice about the factors in this problem?** ***Elicit that it is not possible to use cancellation to simplify the factors.***
Choose a student to solve the problem. $\frac{10}{63}$ Point out that it is not always possible to use cancellation to simplify the factors before multiplying.

Identify the reciprocal of a fraction

1. Distribute the Fraction Kits and write $\frac{3}{4} \times \frac{4}{3} = \_\_$ and $\frac{2}{3} \times \frac{3}{2} = \_\_$ for display.
➤ **What do you notice about these equations?** ***The second fraction in each equation is the reverse of the first fraction.***
2. Remind the students that $\frac{3}{4} \times \frac{4}{3}$ means $\frac{3}{4}$ of $\frac{4}{3}$. Instruct them to first place 4 thirds on their desks and then to remove $\frac{3}{4}$ of the set.
➤ **How many thirds did you remove?** ***3 thirds*** **What do you know about** $\frac{3}{3}$**?** $\frac{3}{3} = 1$ ***whole***
3. Repeat the procedure for $\frac{2}{3}$ of $\frac{3}{2}$. ***2 halves;*** $\frac{2}{2} = 1$ ***whole***



Simplify

Cancellation can be used to simplify fraction factors before multiplying. Cancelling fractional names for 1 in the numerator and denominator before multiplying allows you to multiply smaller numbers and gives a product in lower terms.

cancellation
reciprocal

Use Prime Factorization

- List the prime factors of each term.
- Cancel fractional names for 1.
- Multiply the simplified numerators and the simplified denominators.

$\frac{3}{4} \times \frac{8}{9} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 3} = \frac{2}{3}$

Find a Common Factor

- Divide a numerator and a denominator by a common factor.
- Multiply the simplified numerators and the simplified denominators.

$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}} = \frac{2}{3}$

GCF of 3 and 9: 3
GCF of 4 and 8: 4

Exercises

Write the prime factorization for each factor. Use cancellation to simplify. Write the product.

1. $\frac{3}{8} \times \frac{4}{5}$ $\frac{3}{10}$
2. $\frac{5}{7} \times \frac{3}{10}$ $\frac{3}{14}$
3. $\frac{2}{3} \times \frac{3}{4}$ $\frac{1}{2}$
4. $\frac{9}{10} \times \frac{5}{18}$ $\frac{1}{4}$

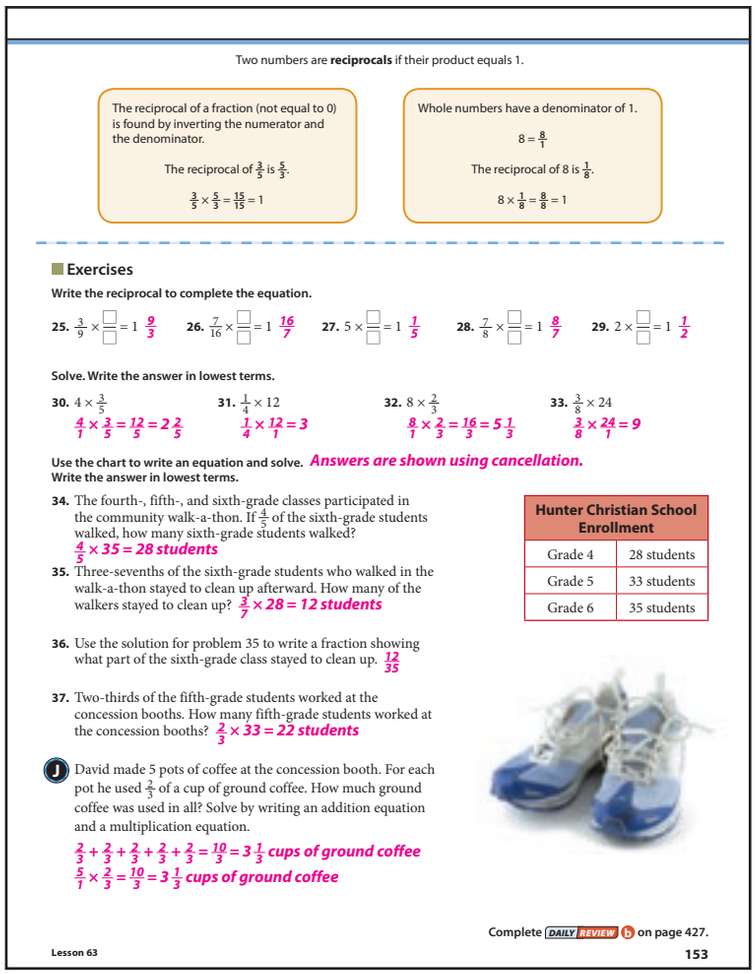
Use the GCF to cancel. Write the simplified product.

5. $\frac{\boxed{2}}{\frac{5}{\boxed{3}}} \times \frac{16}{17} = \frac{\square}{\square}$ $\frac{10}{51}$
6. $\frac{\boxed{2}}{\frac{8}{9}} \times \frac{7}{\frac{12}{\boxed{3}}} = \frac{\square}{\square}$ $\frac{14}{27}$
7. $\frac{\boxed{1}}{\frac{18}{25}} \times \frac{\boxed{1}}{\frac{5}{36}} = \frac{\square}{\square}$ $\frac{1}{10}$
8. $\frac{\boxed{1}}{3} \times \frac{2}{\boxed{3}} = \boxed{2}$

Multiply. Use cancellation if possible. Write the answer in lowest terms. *Cancellation steps may vary.*

9. $\frac{8}{9} \times \frac{3}{10}$ $\frac{4}{15}$
10. $\frac{5}{7} \times \frac{14}{15}$ $\frac{2}{3}$
11. $\frac{8}{9} \times \frac{6}{7}$ $\frac{16}{21}$
12. $\frac{4}{5} \times 10$ 8
13. $\frac{5}{12} \times 15$ $6\frac{1}{4}$
14. $\frac{15}{28} \times \frac{12}{35}$ $\frac{9}{49}$
15. $\frac{1}{4} \times \frac{4}{9}$ $\frac{1}{9}$
16. $\frac{2}{3} \times \frac{4}{17}$ $\frac{8}{51}$
17. $\frac{1}{8} \times \frac{3}{5}$ $\frac{3}{40}$
18. $\frac{1}{3} \times 6$ 2
19. $\frac{7}{10} \times \frac{2}{9}$ $\frac{7}{45}$
20. $\frac{7}{15} \times \frac{15}{32}$ $\frac{7}{32}$
21. $\frac{1}{2} \times \frac{4}{5}$ $\frac{2}{5}$
22. $\frac{1}{8} \times \frac{2}{3}$ $\frac{1}{12}$
23. $8 \times \frac{5}{12}$ $3\frac{1}{3}$
24. $\frac{2}{9} \times \frac{1}{3}$ $\frac{2}{27}$

The Alaskan Iditarod Trail Sled Dog Race covers over 1,150 miles. The mushers retrace the path used to carry diphtheria serum from Anchorage to Nome in 1925.



Two numbers are **reciprocals** if their product equals 1.

The reciprocal of a fraction (not equal to 0) is found by inverting the numerator and the denominator.

The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.

$\frac{3}{5} \times \frac{5}{3} = \frac{15}{15} = 1$

Whole numbers have a denominator of 1.

$8 = \frac{8}{1}$

The reciprocal of 8 is $\frac{1}{8}$.

$8 \times \frac{1}{8} = \frac{8}{8} = 1$

Exercises

Write the reciprocal to complete the equation.

25. $\frac{3}{9} \times \frac{\square}{\square} = 1$ $\frac{9}{3}$
26. $\frac{7}{16} \times \frac{\square}{\square} = 1$ $\frac{16}{7}$
27. $5 \times \frac{\square}{\square} = 1$ $\frac{1}{5}$
28. $\frac{7}{8} \times \frac{\square}{\square} = 1$ $\frac{8}{7}$
29. $2 \times \frac{\square}{\square} = 1$ $\frac{1}{2}$

Solve. Write the answer in lowest terms.

30. $4 \times \frac{3}{5}$ $\frac{4}{1} \times \frac{3}{5} = \frac{12}{5} = 2\frac{2}{5}$
31. $\frac{1}{4} \times 12$ $\frac{1}{4} \times \frac{12}{1} = 3$
32. $8 \times \frac{2}{3}$ $\frac{8}{1} \times \frac{2}{3} = \frac{16}{3} = 5\frac{1}{3}$
33. $\frac{3}{8} \times 24$ $\frac{3}{8} \times \frac{24}{1} = 9$

Use the chart to write an equation and solve. Write the answer in lowest terms. *Answers are shown using cancellation.*

34. The fourth-, fifth-, and sixth-grade classes participated in the community walk-a-thon. If $\frac{4}{5}$ of the sixth-grade students walked, how many sixth-grade students walked? $\frac{4}{5} \times 35 = 28$ students
35. Three-sevenths of the sixth-grade students who walked in the walk-a-thon stayed to clean up afterward. How many of the walkers stayed to clean up? $\frac{3}{7} \times 28 = 12$ students

Hunter Christian School Enrollment	
Grade 4	28 students
Grade 5	33 students
Grade 6	35 students

36. Use the solution for problem 35 to write a fraction showing what part of the sixth-grade class stayed to clean up. $\frac{12}{35}$
37. Two-thirds of the fifth-grade students worked at the concession booths. How many fifth-grade students worked at the concession booths? $\frac{2}{3} \times 33 = 22$ students

J. David made 5 pots of coffee at the concession booth. For each pot he used $\frac{2}{3}$ of a cup of ground coffee. How much ground coffee was used in all? Solve by writing an addition equation and a multiplication equation.

$\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$ cups of ground coffee
$\frac{5}{1} \times \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$ cups of ground coffee

Complete DAILY REVIEW b on page 427.

4. Write the word *reciprocals* and elicit that reciprocals are two numbers whose product is 1; $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals and $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals.
5. Write $\frac{5}{6} \times \_\_ = \frac{30}{30}$ for display.
   - ➤ **What fraction completes the equation?** $\frac{6}{5}$ **What is the reciprocal of** $\frac{5}{6}$**?** $\frac{6}{5}$
6. Write $6 \times \_\_ = 1$ for display.
   - ➤ **What fraction completes the equation? Why?** $\frac{1}{6}$**;** *elicit that since the whole number 6 can be expressed as the fraction* $\frac{6}{1}$*, the reciprocal of 6 is* $\frac{1}{6}$**.**
7. Guide the students in completing these equations.

   $\frac{1}{5} \times \_\_ = 1$ *5 or* $\frac{5}{1}$ $\quad\quad$ $\frac{7}{10} \times \_\_ = 1$ $\frac{10}{7}$

   $8 \times \_\_ = 1$ $\frac{1}{8}$ $\quad\quad$ $\frac{3}{9} \times \_\_ = 1$ $\frac{9}{3}$

Solve a word problem

Amanda has 6 pieces of ribbon that are each $\frac{2}{3}$ of a yard long. How many yards of ribbon does she have? *4 yards*

- ➤ **What is the question asking you to find?** *how many yards of ribbon Amanda has*
- ➤ **What information is given?** *There are 6 pieces of ribbon, and each piece of ribbon is* $\frac{2}{3}$ *of a yard long.*
- ➤ **What equation can you write to solve this problem?** $6 \times \frac{2}{3} = \_\_$
- ➤ **How many yards of ribbon does Amanda have?** *4 yards*

Direct the students to write the equations for these word problems and solve them.

Ron cleaned out some of the boxes in the garage. His mother wanted him to sort out the boxes of books and the boxes of old toys. If $\frac{1}{3}$ of the boxes were books and $\frac{1}{4}$ were toys, what part of all the boxes did Ron sort out? $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$ *of the boxes* [BAT: 2e Work]

Ron's mother asked him to put some of the toys and books on a table for a yard sale. He covered $\frac{5}{6}$ of the table. If $\frac{3}{4}$ of the items were books, what part of the table did he cover with books? $\frac{3}{4} \times \frac{5}{6} = \frac{5}{8}$ *of the table*

Before the yard sale, there were 12 boxes of items to sell. After the yard sale, Ron filled up $\frac{1}{3}$ of the boxes with items that did not sell. How many boxes of items did not sell? $\frac{1}{3} \times 12 = 4$ *boxes*

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Lesson 64

Student Text pp. 154–55
Daily Review p. 427c

Objectives

- Multiply mixed numbers
- Rename an improper fraction as a mixed number
- Rename a mixed number as an improper fraction
- Apply the Distributive Property of Multiplication over Addition when multiplying mixed numbers
- Use cancellation when multiplying mixed numbers
- Write and solve an equation for a word problem
- Estimate products of mixed numbers and fractions

Teach for Understanding

Multiply mixed numbers

1. Draw the following model for display to represent $2\frac{1}{4}$.

➤ **What improper fraction is shown in this picture?** $\frac{9}{4}$

➤ **What mixed number does the picture represent? How do you know?** *$2\frac{1}{4}$; 2 whole rectangles and $\frac{1}{4}$ of another rectangle are shaded.*

Elicit from the students that a *proper fraction* always has a value that is less than 1. When the terms of a proper fraction such as $\frac{4}{8}$ have a common factor, the fraction can be renamed to lower terms. An *improper fraction* has a value that is equal to or greater than 1. When the numerator of an improper fraction such as $\frac{8}{4}$ is a multiple of its denominator, the improper fraction can be renamed as a whole number. However, when the numerator of an improper fraction such as $\frac{9}{4}$ is not a multiple of its denominator, the improper fraction can be renamed as a mixed number.

➤ **How can you use addition to rename $2\frac{1}{4}$ as an improper fraction?** *Elicit that you can rename each whole as $\frac{4}{4}$ and add $\frac{4}{4} + \frac{4}{4} + \frac{1}{4} = \frac{9}{4}$.*

2. Write $2\frac{1}{4} = 2 \times \frac{4}{4} = \frac{8}{4} + \frac{1}{4} = \frac{9}{4}$ for display. Explain that multiplication can be used to rename mixed numbers as improper fractions. First, you multiply to find the number of parts in the wholes and then add the additional parts.

3. Follow a similar procedure as you guide the students in writing these mixed numbers as improper fractions.

$5\frac{5}{8} = \frac{45}{8}$ $2\frac{7}{9} = \frac{25}{9}$ $8\frac{2}{3} = \frac{26}{3}$

➤ **How can you rename $\frac{9}{4}$ as a mixed number?** *Elicit that you can divide the numerator by the denominator to find the number of wholes, and the remainder is written as a fraction because it tells how many parts of the next whole there are.*

4. Instruct the students to write each of these improper fractions as a mixed number.

$\frac{29}{8} = 3\frac{5}{8}$ $\frac{22}{3} = 7\frac{1}{3}$ $\frac{11}{4} = 2\frac{3}{4}$

Mr. Whitemount worked on his dogsled for $2\frac{1}{2}$ hours each day for 5 days. How much time did he spend working on the dogsled? *$12\frac{1}{2}$ hours* [BAT: 2e Work]

➤ **What equation can you write to solve this word problem? Why?** *$5 \times 2\frac{1}{2}$; Mr. Whitemount worked on the dogsled the same amount of time ($2\frac{1}{2}$ hours) on each of the 5 days.*

5. Write $5 \times 2\frac{1}{2}$ for display. Explain that one way to solve this problem is to use the Distributive Property of Multiplication over Addition. You must think of the mixed number as the sum of the whole number 2 and the fraction $\frac{1}{2}$ and then multiply each part of the mixed number by the factor 5. Guide the students in writing the problem using the Distributive Property and then in solving it. Remind them that every whole number has a denominator of 1.
$5 \times (2 + \frac{1}{2}) = (5 \times 2) + (5 \times \frac{1}{2}) = 10 + (\frac{5 \times 1}{1 \times 2}) = 10 + \frac{5}{2} = 10 + 2\frac{1}{2}$ $= 12\frac{1}{2}$
Elicit that the answer is in lowest terms.

6. Explain that another way to solve $5 \times 2\frac{1}{2}$ is to rename the mixed number as an improper fraction and then multiply.

➤ **What is $2\frac{1}{2}$ renamed as an improper fraction?** $\frac{5}{2}$

➤ **What equation can you write using the improper fraction?** *Accept $5 \times \frac{5}{2} = \frac{25}{2} = 12\frac{1}{2}$ or $\frac{5}{1} \times \frac{5}{2} = \frac{25}{2} = 12\frac{1}{2}$.*

7. Point out that the answer is $12\frac{1}{2}$ hours, whether the equation is solved using the Distributive Property or by renaming the mixed number as an improper fraction and then multiplying.

One morning during a race in 1925, a dogsled team traveled 28 miles in blizzard conditions. That afternoon, the team was able to travel $2\frac{1}{7}$ times farther than in the morning. How many miles did the team travel in the afternoon? *60 miles*

➤ **What is the question asking you to find?** *how many miles the team traveled in the afternoon*

➤ **What information is given?** *The team traveled 28 miles in the morning and traveled $2\frac{1}{7}$ times farther in the afternoon.*

➤ **What equation can you write to solve this word problem?** *$2\frac{1}{7} \times 28 =$ ___*

8. Write $2\frac{1}{7} \times 28 =$ ___ and select a student to rewrite the equation, renaming the mixed number as an improper fraction. *$\frac{15}{7} \times 28$ or $\frac{15}{7} \times \frac{28}{1}$*

➤ **Can you use cancellation to solve this equation? Why?** *Yes; elicit that both the denominator 7 and the numerator 28 are divisible by 7.*

Choose a student to demonstrate the cancellation and solve the equation. *$\frac{15}{7} \times 28 = 15 \times 4 = 60$ miles or $\frac{15}{7} \times \frac{28}{1} = \frac{15}{1} \times \frac{4}{1} =$ 60 miles*

Two dogsled teams ran in a race. Jack's team finished the race in $20\frac{3}{4}$ minutes. Joe's team took $1\frac{1}{3}$ times longer than Jack's team. How long did Joe's team take to finish the race? *$27\frac{2}{3}$ minutes*

➤ **What is the question asking you to find?** *how many minutes it took Joe's team to finish the race*

➤ **What information is given?** *Jack's team finished the race in $20\frac{3}{4}$ minutes, and Joe's team took $1\frac{1}{3}$ times longer than Jack's team.*

➤ **What equation can you write to solve this word problem?** *$1\frac{1}{3} \times 20\frac{3}{4} =$ ___* Write the equation.

➤ **What do you think is the best way to solve this equation?** *Answers may vary, but elicit that using the Distributive Property to solve this equation would be a long process; therefore, it would be better to rename the mixed numbers as improper fractions and use cancellation if possible.*

9. Direct the students to write the equation on paper, renaming the mixed numbers as improper fractions, and to solve it, using cancellation if possible.

Multiply Mixed Numbers

You can **multiply mixed numbers** by renaming them as improper fractions or by using the **Distributive Property**.

multiply mixed numbers
Distributive Property

Rename a mixed number as an improper fraction to multiply.

- Multiply the numerators and then multiply the denominators.
- Simplify the fractions using cancellation when possible.

$5 \times 9\frac{3}{10} = \frac{\cancel{5}^{1}}{1} \times \frac{93}{\cancel{10}_{2}} = \frac{93}{2} = 46\frac{1}{2}$

$2\overline{)93}$ $46\frac{1}{2}$, -8, 13, -12, 1

Use the Distributive Property to multiply.

- Multiply each part of the mixed number by the whole number.
- Simplify. Write the answer in lowest terms.

$5 \times 9\frac{3}{10} =$
$5 \times (9 + \frac{3}{10}) =$
$(5 \times 9) + (\frac{5}{1} \times \frac{3}{10}) =$
$45 + \frac{3}{2} =$
$45 + 1\frac{1}{2} = 46\frac{1}{2}$

$2\overline{)3}$ $1\frac{1}{2}$, -2, 1

Exercises

Write the mixed number as an improper fraction and the improper fraction as a mixed number in lowest terms.

1. $5\frac{3}{4}$ *23/4*
2. $3\frac{1}{4}$ *13/4*
3. $7\frac{5}{16}$ *117/16*
4. $\frac{17}{8}$ *2 1/8*
5. $\frac{15}{5}$ *3*
6. $\frac{6}{4}$ *1 1/2*

Multiply. Use cancellation if possible. Write the answer in lowest terms. *Cancellation steps may vary.*

7. $4\frac{1}{10} \times \frac{5}{6}$ *3 5/12*
8. $1\frac{1}{4} \times 3\frac{1}{3}$ *4 1/6*
9. $\frac{3}{8} \times 7\frac{1}{4}$ *2 23/32*
10. $\frac{1}{2} \times 8\frac{2}{9}$ *4 1/9*
11. $2\frac{1}{4} \times \frac{2}{3}$ *1 1/2*
12. $\frac{6}{7} \times 8\frac{2}{5}$ *7 1/5*
13. $2\frac{1}{5} \times 6\frac{2}{3}$ *14 2/3*
14. $5\frac{3}{5} \times 4$ *22 2/5*
15. $\frac{7}{8} \times 7\frac{3}{7}$ *6 1/2*

Multiply *length* × *width* to find the area of the rectangle. Label the answer as square feet or square yards.

16. $1\frac{1}{2}$ ft by $7\frac{3}{10}$ ft — *10 19/20 square feet*
17. $1\frac{9}{16}$ yd by $2\frac{1}{5}$ yd — *3 7/16 square yards*

Use the Distributive Property to solve. *Steps used to solve may vary.*

18. $3 \times 5\frac{1}{2}$ *16 1/2*
19. $\frac{3}{4} \times 4\frac{5}{8}$ *3 15/32*
20. $16\frac{4}{5} \times 10$ *168*
21. $9 \times 12\frac{2}{3}$ *114*
22. $2\frac{1}{3} \times \frac{5}{8}$ *1 11/24*
23. $7\frac{1}{2} \times \frac{6}{7}$ *6 3/7*

Sled dogs are selected according to their endurance, strength, and speed.

Estimate the product by rounding each factor and multiplying.

Round factors *less than* 1 to $\frac{1}{2}$ or 1.

Round mixed numbers to the nearest whole number by determining if the fractional part is *greater than*, *less than*, or *equal to* $\frac{1}{2}$.

- If the fraction is *less than* $\frac{1}{2}$, the whole number stays the same.
- If the fraction is *greater than* or *equal to* $\frac{1}{2}$, the whole number increases by 1.

$\frac{1}{2}$ 1 — $\frac{3}{8} \times 3\frac{10}{16} = \_\_$ — $\frac{1}{2} \times 4 = 2$

1 $\frac{1}{4} < \frac{1}{2}$ — $6\frac{1}{2} \times 5\frac{1}{4} = \_\_$ — $7 \times 5 = 35$

The Siberian husky Balto led his dogsled team on the final leg of the 1925 Serum Run to Nome, Alaska.

Exercises

Estimate the product.

24. $\frac{4}{10} \times 9\frac{7}{8}$ *$\frac{1}{2} \times 10 = 5$*
25. $\frac{2}{5} \times 6\frac{1}{2}$ *$\frac{1}{2} \times 7 = 3\frac{1}{2}$*
26. $2\frac{3}{4} \times 5\frac{2}{8}$ *$3 \times 5 = 15$*
27. $6\frac{5}{7} \times 2\frac{1}{4}$ *$7 \times 2 = 14$*

Solve. *Answers are shown using cancellation.*

28. Taylor took $3\frac{1}{2}$ quarts of chocolate milk to the class party. The students drank $\frac{4}{5}$ of the milk. How many quarts of chocolate milk did the class drink? *$\frac{4}{5} \times 3\frac{1}{2} = \frac{4}{5} \times \frac{7}{2} = \frac{14}{5} = 2\frac{4}{5}$ qt*
29. The distance from home to school is $1\frac{2}{3}$ miles. Heather walked this distance 4 times today. How far did she walk? *$4 \times 1\frac{2}{3} = \frac{4}{1} \times \frac{5}{3} = \frac{20}{3} = 6\frac{2}{3}$ mi*
30. Nate plowed a garden plot that measured $8\frac{2}{3}$ yards long and $4\frac{1}{2}$ yards wide. What is the area of his garden plot? Label the answer as *square yards*. *$8\frac{2}{3} \times 4\frac{1}{2} = \frac{26}{3} \times \frac{9}{2} = 39$ square yards*
31. Nate is installing trellis fencing along one long side of his garden. He was able to complete $\frac{3}{4}$ of the fence on Saturday. How many yards of fence did he install? *$\frac{3}{4} \times 8\frac{2}{3} = \frac{3}{4} \times \frac{26}{3} = 6\frac{1}{2}$ yd*

Practice & Application

32. $3\frac{7}{9} + 4\frac{5}{9}$ *$7\frac{12}{9} = 8\frac{1}{3}$*
33. $4\frac{5}{8} + 6\frac{1}{2}$ *$4\frac{5}{8} + 6\frac{4}{8} = 10\frac{9}{8} = 11\frac{1}{8}$*
34. $\frac{5}{9} - \frac{1}{3}$ *$\frac{5}{9} - \frac{3}{9} = \frac{2}{9}$*
35. $2\frac{1}{2} - \frac{3}{4}$ *$2\frac{2}{4} - \frac{3}{4} = 1\frac{6}{4} - \frac{3}{4} = 1\frac{3}{4}$*
36. $5\frac{1}{8} - 1\frac{2}{3}$ *$5\frac{3}{24} - 1\frac{16}{24} = 4\frac{27}{24} - 1\frac{16}{24} = 3\frac{11}{24}$*
37. 17×2.81 *47.77*
38. 16.5×407 *6,715.5*
39. $174.62 \div 5$ *34.924*
40. One side of a square measures $23\frac{3}{4}$ inches. Find the perimeter using multiplication. *$4 \times 23\frac{3}{4} = 92\frac{12}{4} = 95$ in.*

J Draw a picture to represent each phrase. Which phrase can be solved using repeated addition?

8 sets of $1\frac{1}{2}$ — $1\frac{1}{2}$ sets of 8

Complete DAILY REVIEW C on page 427.

➤ **Did you use cancellation to solve the equation? Why?** *Yes; the numerator 4 in $\frac{4}{3}$ and the denominator 4 in $\frac{83}{4}$ cancel each other, $4 \div 4 = 1$.*

Select a student to write his solution for display. Lead a discussion about the solution. *$\frac{4}{3} \times \frac{83}{4} = \frac{1}{3} \times 83 = \frac{83}{3} = 27\frac{2}{3}$ minutes*

10. Guide the students in solving these equations on paper. Allow students to write 1 as the denominator of a whole number if needed.

$3\frac{1}{5} \times 2\frac{1}{10} = \frac{16}{5} \times \frac{21}{10} = \frac{8}{5} \times \frac{21}{5} = \frac{168}{25} = 6\frac{18}{25}$

$2\frac{1}{2} \times 3\frac{1}{3} = \frac{5}{2} \times \frac{10}{3} = 5 \times \frac{5}{3} = \frac{25}{3} = 8\frac{1}{3}$

11. Guide the students in using the Distributive Property to solve these equations.

$2\frac{3}{4} \times \frac{2}{5} =$

$(2 + \frac{3}{4}) \times \frac{2}{5} = (2 \times \frac{2}{5}) + (\frac{3}{4} \times \frac{2}{5}) = \frac{4}{5} + \frac{3}{10} = \frac{8}{10} + \frac{3}{10} = \frac{11}{10} = 1\frac{1}{10}$

$7 \times 3\frac{5}{8} =$

$7 \times (3 + \frac{5}{8}) = (7 \times 3) + (7 \times \frac{5}{8}) = 21 + \frac{35}{8} = 21 + 4\frac{3}{8} = 25\frac{3}{8}$

Estimate products

1. Remind the students that estimation is useful for determining an approximate answer before solving an equation or when an exact answer is not needed. It is also useful for checking the accuracy of your work. Point out that estimation can make mental calculations easy. For an estimate to be useful, it should be easily determined; otherwise, finding the actual answer is more desirable.

Colin runs an average of $5\frac{1}{2}$ miles an hour. About how far can he run in $2\frac{5}{6}$ hours?

➤ **How could you estimate the answer to this word problem? Explain your answer.** *Accept reasonable estimates and explanations, allowing for one or both of the factors to be rounded; possible answers: $3 \times 6 = 18$ miles, $3 \times 5\frac{1}{2} = 16\frac{1}{2}$ miles, $3 \times 5 = 15$ miles (using compensation).*

2. Repeat the procedure for the following word problem. Elicit that only 1 factor needs to be rounded to estimate the answer. You would not round $\frac{2}{10}$ to 0 because multiplying a factor of 0 gives you a product of 0, which is not a reasonable estimate.

Caden runs an average of $\frac{2}{10}$ of a mile per minute. About how far can he run in $6\frac{1}{2}$ minutes? *$7 \times \frac{2}{10} = \frac{14}{10} = 1\frac{4}{10} = 1\frac{2}{5}$ miles*

3. Follow a similar procedure to estimate the product of $\frac{3}{8} \times 19\frac{7}{10}$. *possible estimates: $\frac{3}{8} \times 20 = \frac{60}{8} = 7\frac{4}{8} = 7\frac{1}{2}$; $\frac{1}{2} \times 20 = \frac{20}{2} = 10$; $\frac{1}{2} \times 19 = 9\frac{1}{2}$*
4. Write $\frac{1}{3} \times 10$ for display.

➤ **Do you think it would be useful to estimate the product of this equation? Why?** *Answers may vary, but elicit that since $\frac{1}{3} \times 10 = \frac{10}{3} = 3\frac{1}{3}$, it is just as easy to mentally solve for the exact product as to find an estimate.*

5. Direct the students to estimate the products for these problems. Select students to write their estimate for display and to explain their reasoning.

$5\frac{6}{8} \times 2\frac{1}{4}$ *12* — $8\frac{3}{4} \times 7\frac{1}{3}$ *63*

$\frac{10}{12} \times 8\frac{2}{7}$ *8* — $\frac{1}{6} \times 11\frac{7}{9}$ *2*

Student Text pp. 154–55

Lesson 65

Student Text pp. 156–57
Daily Review p. 428d

Objectives
- Multiply a decimal by a decimal
- Demonstrate an understanding of the relationship between decimals and fractions
- Estimate a decimal product by rounding to the nearest whole number or to the place of greatest value
- Apply the Distributive Property of Multiplication over Addition when multiplying decimals
- Use mental math to multiply a decimal and a power of 10

Teacher Materials
- Decimal Place Value Pocket Chart (B)

Note
The counting of decimal places in the factors for determining the number of decimal places in the product is a common practice. However, to emphasize number sense, the focus of Lessons 65 and 66 is the use of fractional equivalents and estimation for determining the placement of the decimal point in the product.

Introduce the Lesson

1. Display the Decimal Place Value Pocket Chart (B). Remind the students that in our base ten number system, the places on each side of the Ones place reflect a similar pattern. Point out that each place has a value that is 10 times greater than the place to its right and $\frac{1}{10}$ of the value of the place to its left. (See Lesson 4.)
2. Write $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1{,}000}$ and choose students to write the equivalent decimals. Remind them that decimals use powers of 10 to represent fractions. ***0.1, 0.01, and 0.001*** As you lead the following discussion, write the corresponding equations for display.

➤ **What is the product of 10 × 10? *100* the fractions $\frac{1}{10} \times \frac{1}{10}$? $\frac{1}{100}$**

➤ **What do you think is the product of the decimals 0.1 × 0.1? Why? *0.01; elicit that multiplying the decimal forms of $\frac{1}{10}$ equals the decimal form of $\frac{1}{100}$.***

Ask similar questions for 10 × 100 ***1,000; $\frac{1}{1{,}000}$; 0.001***, 10 × 1,000 ***10,000; $\frac{1}{10{,}000}$; 0.0001***, and 100 × 100 ***10,000; $\frac{1}{10{,}000}$; 0.0001***.

Teach for Understanding

Multiply a decimal by a decimal

> Sophia has 8 oz of chocolate chips to make two pans of brownies. She plans to sprinkle one-half of the chocolate chips on top of the batter in each pan of brownies. How many ounces of chocolate chips will she use for each pan? ***4 oz***

➤ **What equation can you write for this word problem? Why? *Answers may vary, but elicit $\frac{1}{2} \times 8 = 4$ oz; using the information given, you need to find $\frac{1}{2}$ of 8 to determine the ounces of chocolate chips that Sophia will sprinkle on the brownie batter in each pan.***

1. Write $\frac{1}{2} \times 8 = 4$ oz and lead in reading it: *$\frac{1}{2}$ of 8 equals 4 oz.*

➤ **Why is the product less than the multiplicand 8? *You are finding a part ($\frac{1}{2}$) of a set of 8.***

➤ **What decimal can you substitute for $\frac{1}{2}$ in the equation? Why? *0.5; elicit that $\frac{1}{2}$ can be renamed as $\frac{5}{10}$, which is 0.5 in decimal form.***

2. Write 0.5 × 8 for display.

➤ **Will the product be greater than or less than the multiplicand 8? Why? *Less than; you are finding a part (5 tenths or one-half) of 8.***

➤ **What does 0.5 × 8 equal? How do you know? *4; elicit that since multiplying decimals is similar to multiplying whole numbers, 5 × 8 = 40, and a decimal point is needed to indicate tenths in the product (4.0) because there is a decimal point showing tenths in the multiplier. There are no tenths, so the product is 4 whole units (ounces).*** Complete the problem.

3. Write $\frac{5}{10} \times \frac{8}{10}$ for display and lead in reading it: *5 tenths of 8 tenths.* Select a student to write the expression in decimal form. ***0.5 × 0.8***

➤ **What does 5 tenths × 8 tenths equal? How do you know? *40 hundredths; elicit that when you multiply tenths × tenths, the product is hundredths; whether you are multiplying fractions or decimals, the product will be 40 hundredths.*** Complete both problems: $\frac{5}{10} \times \frac{8}{10} = \frac{40}{100}$ and 0.5 × 0.8 = 0.40.

➤ **What can you simplify $\frac{40}{100}$ and 0.40 to so that they represent the same value? *$\frac{4}{10}$ and 0.4*** Write = $\frac{4}{10}$ and = 0.4 after the equations. Remind the students that although the fraction $\frac{4}{10}$ can be renamed to $\frac{2}{5}$, decimals represent only powers of 10.

➤ **Do you think the fraction $\frac{2}{5}$ is equivalent to the decimal 0.4? Why? *Elicit that since the fraction $\frac{4}{10}$ is equal to the decimal 0.4, both $\frac{4}{10}$ and 0.4 are equivalent to $\frac{2}{5}$.*** Choose a student to divide 2 by 5 to prove that $\frac{2}{5}$ equals 0.4. (See Lesson 24 of Chapter 3.)

4. Follow a similar procedure for $\frac{3}{10} \times \frac{9}{10}$ ***$\frac{27}{100}$; 0.3 × 0.9 = 0.27*** and $3\frac{2}{10} \times 2\frac{6}{10}$ ***$8\frac{32}{100}$; 3.2 × 2.6 = 8.32***.

➤ **What do you notice about the number of decimal places in the product? *Elicit that the number of decimal places in the product is the same as the combined number of decimal places in the factors.*** Point out that thinking of the fraction form of the decimal can help you determine the placement of the decimal point.

5. Repeat the procedure for the following equations. Explain that sometimes it is necessary to annex 1 or more zeros to place the decimal point correctly in the product.

$\frac{4}{10} \times \frac{7}{100} = \frac{28}{1{,}000}$; ***0.4 × 0.07 = 0.028***

$2\frac{6}{10} \times \frac{25}{100} = \frac{650}{1{,}000} = \frac{65}{100}$; ***2.6 × 0.25 = 0.650 = 0.65***

$\frac{3}{10} \times 2\frac{12}{100} = \frac{636}{1{,}000}$; ***0.3 × 2.12 = 0.636***

$\frac{2}{100} \times \frac{4}{100} = \frac{8}{10{,}000}$; ***0.02 × 0.04 = 0.0008***

$\frac{12}{100} \times \frac{9}{100} = \frac{108}{10{,}000}$; ***0.12 × 0.09 = 0.0108***

Estimate a decimal product

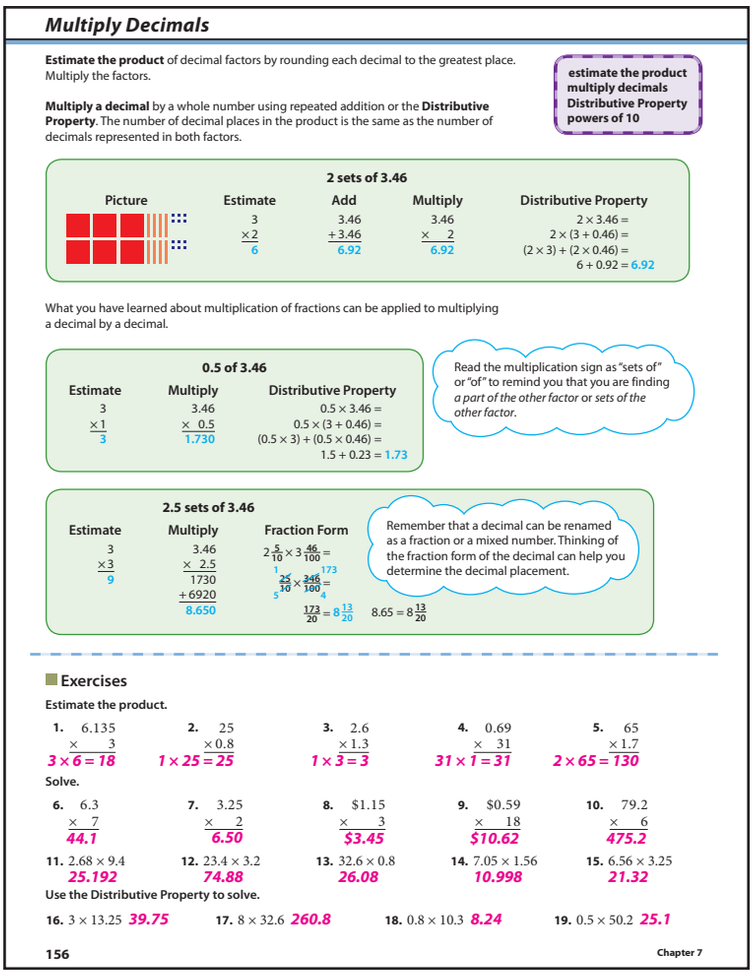
Write the following problems for display. Explain that estimating is especially helpful in determining the placement of the decimal point when multiplying decimals. You can estimate the product by rounding to the nearest whole number or to the place of greatest value. Guide the students in mentally estimating the products and solving the problems on paper. Emphasize using the estimates and fractional equivalents for determining the placement of the decimal point in the exact product (e.g., When multiplying tenths and hundredths, there will be one thousandths in the exact product). ***Accept reasonable estimates that differ from the given estimates.***

4 × 23.9 = ***95.6 (4 × 20 = 80)***

7.7 × 5.6 = ***43.12 (8 × 6 = 48)***

0.8 × 62 = ***49.6 (1 × 62 = 62)***

9.8 × 2.46 = ***24.108 (10 × 2 = 20)***



Multiply Decimals

Estimate the product of decimal factors by rounding each decimal to the greatest place. Multiply the factors.

estimate the product
multiply decimals
Distributive Property
powers of 10

Multiply a decimal by a whole number using repeated addition or the **Distributive Property**. The number of decimal places in the product is the same as the number of decimals represented in both factors.

2 sets of 3.46

Picture	Estimate	Add	Multiply	Distributive Property
	3 × 2 = 6	3.46 + 3.46 = 6.92	3.46 × 2 = 6.92	2 × 3.46 = 2 × (3 + 0.46) = (2 × 3) + (2 × 0.46) = 6 + 0.92 = 6.92

What you have learned about multiplication of fractions can be applied to multiplying a decimal by a decimal.

0.5 of 3.46

Estimate	Multiply	Distributive Property
3 × 1 = 3	3.46 × 0.5 = 1.730	0.5 × 3.46 = 0.5 × (3 + 0.46) = (0.5 × 3) + (0.5 × 0.46) = 1.5 + 0.23 = 1.73

Read the multiplication sign as "sets of" or "of" to remind you that you are finding *a part of the other factor* or *sets of the other factor*.

2.5 sets of 3.46

Estimate	Multiply	Fraction Form
3 × 3 = 9	3.46 × 2.5 = 1730 + 6920 = 8.650	$2\frac{5}{10} \times 3\frac{46}{100} = \frac{25}{10} \times \frac{346}{100} = \frac{173}{20} = 8\frac{13}{20}$ 8.65 = $8\frac{13}{20}$

Remember that a decimal can be renamed as a fraction or a mixed number. Thinking of the fraction form of the decimal can help you determine the decimal placement.

Exercises

Estimate the product.

1. 6.135 × 3 — 3 × 6 = 18
2. 25 × 0.8 — 1 × 25 = 25
3. 2.6 × 1.3 — 1 × 3 = 3
4. 0.69 × 31 — 31 × 1 = 31
5. 65 × 1.7 — 2 × 65 = 130

Solve.

6. 6.3 × 7 = 44.1
7. 3.25 × 2 = 6.50
8. $1.15 × 3 = $3.45
9. $0.59 × 18 = $10.62
10. 79.2 × 6 = 475.2
11. 2.68 × 9.4 = 25.192
12. 23.4 × 3.2 = 74.88
13. 32.6 × 0.8 = 26.08
14. 7.05 × 1.56 = 10.998
15. 6.56 × 3.25 = 21.32

Use the Distributive Property to solve.

16. 3 × 13.25 = 39.75
17. 8 × 32.6 = 260.8
18. 0.8 × 10.3 = 8.24
19. 0.5 × 50.2 = 25.1

Use Mental Math to Multiply a Decimal by Powers of 10

Write the digits of the decimal factor in the answer. Move the decimal point from its placement in the decimal factor one place to the right for each 0 in the **power of 10**. Annex zeros as needed.

5.68 × 10 = 5.68	5.68 × 100 = 5.68	5.68 × 1,000 = 5.680
10 × 5.68 = 56.8	100 × 5.68 = 568	1,000 × 5.68 = 5,680
The decimal moves 1 place to the right.	The decimal moves 2 places to the right.	The decimal moves 3 places to the right. Annex a 0.

When powers of 10 are written with exponents, the exponent tells how many places to move the decimal point.

$2.376 \times 10^1 = 23.76$ The decimal moves 1 place to the right.
$2.376 \times 10^2 = 237.6$ The decimal moves 2 places to the right.
$2.376 \times 10^3 = 2{,}376$ The decimal moves 3 places to the right.

Exercises

Use mental math to solve.

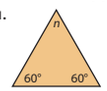
20. 10×14.9 = 149
21. 100×65.7 = 6,570
22. $1{,}000 \times 2.127$ = 2,127
23. $10^2 \times 6.259$ = 625.9
24. $10^1 \times 54.31$ = 543.1
25. 1.868×10^3 = 1,868
26. 10×0.012 = 0.12
27. $10^2 \times 12.314$ = 1,231.4
28. $1{,}000 \times 3.078$ = 3,078

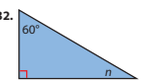
Solve.

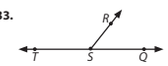
29. The grocery store put its holiday candy on sale. The 1.69-ounce bags are priced at 3 bags for $3.00. The 2.48-ounce bags are priced at 2 bags for $3.00. Which is the better deal? 1.69-oz bags

30. Mrs. Ray bought a shirt for her husband and a shirt for each of her 2 sons. The shirts cost $24.95 each. She had a coupon for $10.00 off. What was the cost of the shirts? $64.85

Practice & Application

31.  n = ___ 60°

32.  n = ___ 30°

33.  ∠RST = 100°, ∠RSQ = ___ 80°

34. If the angles of a triangle measure 115°, 25°, and 40°, would the triangle be classified as obtuse or acute? obtuse

35. On graph paper draw 2 congruent triangles and 2 similar rectangles. Pictures will vary.

36. On graph paper draw circle *B*. Label the diameter $\overline{CE}$. If the diameter measures 8 centimeters, what is the measure of $\overline{BE}$? 4 cm
Pictures will vary.
Explain how you know that the product of 0.6 × 45 is less than 45. Will the product be *greater than* or *less than* 20?

Complete DAILY REVIEW on page 428.

Elicit that checking a problem is the only way to know whether your exact answer is correct.

Apply the Distributive Property

1. Write 4 × 3.21 for display. Explain that the Distributive Property of Multiplication over Addition can be used to multiply decimals. Just as you can think of a mixed number as the sum of its whole number and its fraction, you can also think of the decimal 3.21 as the sum of its whole number and its decimal fraction (3 + 0.21).
   Elicit the expressions used in applying the Distributive Property to the problem. Write each expression as it is given.
   4 × (3 + 0.21) = (4 × 3) + (4 × 0.21) = 12 + 0.84 = 12.84
2. Follow a similar procedure for these problems.
   0.6 × 28.3 = *0.6 × (28 + 0.3) = (0.6 × 28) + (0.6 × 0.3) = 16.8 + 0.18 = 16.98*
   0.03 × 25 = *0.03 × (20 + 5) = (0.03 × 20) + (0.03 × 5) = 0.60 + 0.15 = 0.75*

Multiply a decimal and a power of 10

1. Write 10 × 3.76 for display.
   ➤ **How can you solve this problem?** *Elicit that you multiply as if you were multiplying a whole number by 10 and write the decimal point in the product.* **What is the product?** *Accept 37.60 or 37.6.*
   ➤ **Is it necessary to write a zero in the Hundredths place of the product? Why?** *No; 37.6 is equivalent to 37.60.*
2. Repeat the procedure for 100 × 3.76 = *376* and 1,000 × 3.76 = *3,760*. Remind the students that every whole number has a decimal point to the right of the Ones place, but the decimal point is not written.
3. Follow a similar procedure for 10 × 0.255 *2.55*; 100 × 0.255 *25.5*; and 1,000 × 0.255 *255*.
   ➤ **What conclusion can you make about the decimal point when multiplying any decimal by 10, 100, or 1,000?** *Elicit that for each 0 in the factors 10, 100, or 1,000, the decimal point in the other factor moves 1 place to the right.*
   Point out that each time the decimal point moves 1 place to the right, each digit in the decimal becomes 10 times greater and is renamed to the next greater place.
4. Elicit from the students that 10, 100, and 1,000 are powers of 10 and can be written 10^1, 10^2, and 10^3.
5. Guide the students in multiplying 4.3×10^1 *43*; 4.3×10^2 *430*; and 4.3×10^3 *4,300*. Point out that sometimes you need to annex a zero(s) to find the correct product. Elicit that the exponent indicates the number of zeros in a power of 10. Also elicit that the exponent indicates the number of decimal places a decimal point moves to the right when multiplying by a power of 10.
6. Choose students to solve these equations mentally.
   7.92 × 10 = *79.2*
   100 × 6.04 = *604*
   0.5376 × 1,000 = *537.6*
   $10^1 \times 3.84$ = *38.4*
   5.631×10^2 = *563.1*
   $10^3 \times 6.04$ = *6,040*

Student Text pp. 156–57

(*Note*: *Assessment* available on Teacher's Toolkit CD.)

Lesson 66

Student Text pp. 158–59
Daily Review p. 428e

Objectives
- Multiply a decimal by a decimal
- Demonstrate an understanding of the relationship between decimals and fractions
- Annex zeros in the product
- Multiply money by a decimal
- Estimate a decimal product by rounding

Teacher Materials
- Decimal Grids, page IA40 (CD)
- Christian Worldview Shaping, pages 20–21 (CD)
- Colored markers: red and blue

Teach for Understanding

Multiply a decimal by a decimal

1. Remind the students that decimals use powers of 10 to represent fractions. The decimal place tells the name of the denominator in an equivalent fraction.
2. Write the following decimals and fractions for display and choose students to write the fraction or decimal equivalents.

   $0.05 = \frac{5}{100}$ $\quad\quad$ $\frac{2}{10} = 0.2$

   $0.036 = \frac{36}{1{,}000}$ $\quad\quad$ $2\frac{403}{1{,}000} = 2.403$

   $1.0099 = 1\frac{99}{10{,}000}$ $\quad\quad$ $\frac{7}{1{,}000} = 0.007$
3. Write 0.6×1.4 horizontally for display.

➤ **How do you read this problem?** *6 tenths of 1 and 4 tenths*
Lead in reading the problem, emphasizing the word *of*.

➤ **Will the product be greater than or less than 1.4? Why?** *Less than; you are finding a part of 1.4.*

➤ **When multiplying decimals, what is important to remember about the product?** *A decimal point is needed.*

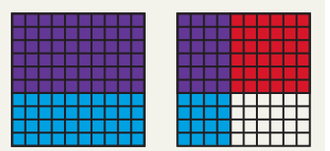
➤ **What are some ways you can determine where to place the decimal point in the product?** *Possible answers: Estimate the product; think of the fraction form for each decimal factor.* Elicit that the fraction form indicates the value of each decimal factor.

➤ **If you round these factors to the place of greatest value, what do you estimate the product to be? Why?** *1; both factors round to 1.*

➤ **When you multiply tenths by tenths, what decimal fraction will be in the product?** *hundredths*

4. Choose a student to write the problem in fraction form and solve it; select another student to solve the decimal problem. $\frac{6}{10} \times 1\frac{4}{10} = \frac{84}{100}$; *0.84* Point out that since you do not need to multiply 1.4 by the zero in the Ones place of the multiplier, it is important to remember to write or annex a zero in the Ones place of the product to show that the Ones place has no value. Elicit that the products are equivalent and lead a discussion about the reasonableness of the products.
5. Display the Decimal Grids page. Demonstrate the shading as you elicit the answer to the following question.

➤ **How could you picture 0.6×1.4?** *Elicit that you could use one color to shade all of one grid (10 tenths) and 4 columns (4 tenths) of another grid to picture the multiplicand, and then use another color to shade 6 rows (6 tenths) to picture the multiplier. The parts of the grids that are shaded twice represent the product (84 hundredths).*



Remind the students that tenths are equal parts. Point out that when 14 of those equal parts are multiplied by 6 of the same size parts, the product is 84 smaller parts (the double-shaded hundredths). A part of 1.4 was found.

6. Write 0.5×0.7 horizontally and lead in reading it: *5 tenths of 0.7.*

➤ **Will the product be greater than or less than 0.7? Why?** *Less than; you are finding a part (one-half of) of 0.7.*

➤ **What is the product of 5 tenths and 7 tenths? How do you know?** *35 hundredths; elicit that the basic fact 5×7 equals 35 and when both factors are tenths, the product will be hundredths.*

Choose a student to complete the problem. *0.35* Select another student to write the problem in fraction form. $\frac{5}{10} \times \frac{7}{10} = \frac{35}{100}$

7. Follow a similar procedure to illustrate 0.5×0.7 on the Decimal Grids page.

Annex zeros in the product

1. Write 0.2×0.3 vertically for display.

➤ **When solving this problem, do you think it will be necessary to multiply by the 0 in the Ones place to find the second partial product? Why?** *No; elicit that the second partial product would be 0 and adding it will not change the final product.*

➤ **Will the product be closer to 0 or to 1? Why?** *0; elicit that you are finding a part of 0.3, and 0.3 is less than one-half.*

2. Write 6 in the product of the problem.

➤ **Since you know that $2 \times 3 = 6$, what does 0.2×0.3 equal? Why?** *0.06; tenths $\times$ tenths = hundredths*

➤ **What do you need to do to show 6 hundredths in the product?** *Elicit that you need to write or annex 2 zeros in front of the 6 (a zero in the Tenths place and a zero in the Ones place) so that you can write the decimal point separating the Ones place from the decimal fraction.* Point out that the annexed zeros show that the Tenths place and the Ones place have no value.

3. Write 0.4×0.08 vertically for display.

➤ **Do you think the product will be closer to 0 or to 1? Why?** *0; elicit that you are finding a part of 0.08, and 0.08 is much closer to 0 than to 1.*

➤ **What do you think 0.4×0.08 equals? How do you know?** *0.032; accept any reasonable answers, but elicit that $4 \times 8 = 32$, and when you multiply hundredths by tenths there will be one thousandths in the product.* Select a student to complete the problem.

4. Write 0.03×0.03 vertically and choose a student to write the problem as an equation in fraction form. $\frac{3}{100} \times \frac{3}{100} = \_\_$

➤ **What is the product of the problem written in fraction form?** $\frac{9}{10{,}000}$ Complete the equation.

➤ **What is the product of the decimal problem?** *9 ten thousandths*

➤ **How many zeros do you need to annex to write 9 ten thousandths in the product? How do you know?** *4 zeros; elicit that you need to annex zeros in the One Thousandths, Hundredths, Tenths, and Ones places to show that those places have no value and to show that the 9 is in the Ten Thousandths place.*

More Multiplying Decimals

When multiplying decimals, you may need to annex zeros in order to place the decimal point.

0.04 tenths × hundredths = thousandths
× 0.6 3 decimal places are needed for thousandths.
0.024 Annex zeros to the left of the digits to show 24 thousandths.

Think of the equivalent fractions to make sense of the answer.
$\frac{6}{10} \times \frac{4}{100} = \frac{24}{1,000}$
tenths × hundredths = thousandths

Exercises

Write the fraction equivalent for each decimal. Multiply.

1. 0.09 × 0.2 **0.018** $\frac{9}{100} \times \frac{2}{10} = \frac{18}{1,000}$
2. 0.03 × 0.07 **0.0021** $\frac{3}{100} \times \frac{7}{100} = \frac{21}{10,000}$
3. 0.2 × 0.76 **0.152** $\frac{2}{10} \times \frac{76}{100} = \frac{152}{1,000}$
4. 0.5 × 0.017 **0.0085** $\frac{5}{10} \times \frac{17}{1,000} = \frac{85}{10,000}$

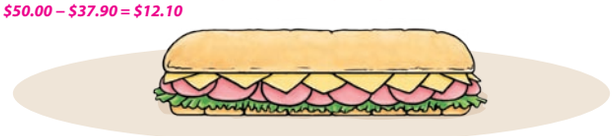
Solve.

5. 0.02 × 0.03 = **0.0006**
6. 23.4 × 3.2 = **74.88**
7. 0.07 × 0.8 = **0.056**
8. 0.02 × 0.4 = **0.008**
9. 4.9 × 0.5 = **2.45**
10. 2.68 × 9.4 = **25.192**
11. 6.1 × 0.03 = **0.183**
12. 32.6 × 0.8 = **26.08**
13. $7.05 × 6 = **$42.30**
14. 3.29 × 0.03 = **0.0987**
15. 10.01 × 2.1 = **21.021**
16. $1.15 × 3.4 = **$3.91**

Write the decimal equivalent for each fraction to solve. Round to the nearest hundredth.

Dean's Deli Specials	
Roast Beef	$6.49/lb
Ham	$3.79/lb
Salami	$4.19/lb

17. How much will $\frac{3}{4}$ of a pound of roast beef cost at today's special price? ***0.75 × $6.49 = $4.87***
18. Chloe bought 2 pounds of roast beef and $1\frac{1}{2}$ pounds of ham. How much did she spend on sandwich meat? ***2 × $6.49 = $12.98; 1.5 × $3.79 ≈ $5.69; $12.98 + $5.69 = $18.67***
19. How much cheaper is it to buy $1\frac{1}{2}$ pounds of ham than 1 pound of roast beef? ***$6.49 – $5.69 = $0.80***
20. Mrs. Hodge bought enough sandwich meat for her family's lunch. She bought $3\frac{1}{4}$ pounds of roast beef, $2\frac{1}{2}$ pounds of ham, and $1\frac{3}{4}$ pounds of salami. How much change will she receive from $50.00? ***3.25 × $6.49 ≈ $21.09; 2.5 × $3.79 ≈ $9.48; 1.75 × $4.19 ≈ $7.33; $21.09 + $9.48 + $7.33 = $37.90; $50.00 – $37.90 = $12.10***



Use the Distributive Property to solve.

21. 0.8 × 3.8 **3.04**
22. 5 × 27.2 **136**
23. 12 × 0.08 **0.96**
24. 6.8 × 10^2 **680**

Solve.

25. 2.39 × 2 = **4.78**
26. 0.03 × 0.8 = **0.024**
27. 23.28 × 0.7 = **16.296**
28. 0.09 × 0.08 = **0.0072**

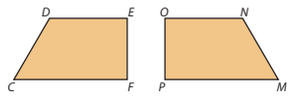
Solve. Write the answer in lowest terms.

29. $\frac{6}{7} \times 8$ **$\frac{48}{7} = 6\frac{6}{7}$**
30. $23 \times \frac{3}{4}$ **$\frac{69}{4} = 17\frac{1}{4}$**
31. $\frac{14}{15} \times \frac{2}{3}$ **$\frac{28}{45}$**
32. $2\frac{7}{8} \times 5\frac{4}{5}$ **$\frac{23}{8} \times \frac{29}{5} = \frac{667}{40} = 16\frac{27}{40}$**

Use mental math to solve.

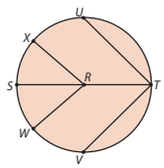
33. 6.12 × 1,000 **6,120**
34. 12.13 × 10^2 **1,213**
35. 0.78 × 10 **7.8**
36. 0.521 × 10^2 **52.1**

Practice & Application



Trapezoid *CDEF* ≅ Trapezoid *MNOP*

37. Name the corresponding congruent line segment to $\overline{CD}$. **$\overline{MN}$**
38. Name the corresponding congruent angle to ∠*D*. **∠N**
39. If $\overline{EF}$ = 12 mm, then ___ = 12 mm. **$\overline{OP}$**
40. If ∠*P* = 90°, then ___ = 90°. **∠F**
41. Does Trapezoid *MNOP* show a translation, reflection, or rotation of Trapezoid *CDEF*? ***reflection***



42. Name the diameter of circle *R*. **$\overline{ST}$**
43. Name the 4 radii of circle *R*. **$\overline{RT}$, $\overline{RX}$, $\overline{RS}$, $\overline{RW}$**
44. Name 3 chords shown on circle *R*. **$\overline{ST}$, $\overline{UT}$, $\overline{TV}$**
45. If ∠*SRX* measures 45°, what does ∠*XRT* measure? ***135°***
46. If $\overline{XW}$ were drawn, would it represent a chord or a radius? Explain your answer. ***A chord; a chord connects any two points on a circle.***

J Solve the problem $\frac{3}{4} \times 1.5 =$ ___. Explain your reasoning.
$1\frac{1}{8}$ or 1.125; Explanations may vary but should include something like the following: Rename 1.5 as $1\frac{1}{2}$, or rename $\frac{3}{4}$ as 0.75.

0.75
× 1.5
375
+750
1.125

21. ***(0.8 × 3) + (0.8 × 0.8) = 2.4 + 0.64 = 3.04***
22. ***(5 × 27) + (5 × 0.2) = 135 + 1 = 136***
23. ***(10 × 0.08) + (2 × 0.08) = 0.8 + 0.16 = 0.96***
24. ***6.8 × 100 = (6 × 100) + (0.8 × 100) = 600 + 80 = 680***

Complete DAILY REVIEW 6 on page 428.

5. Write 4 × 39 = 156 for display. Write the following problems below the equation. Choose students to mentally determine the exact products by using the product in the equation written for display and their knowledge of annexing zeros, estimation, and equivalent relationships of decimals and fractions. Instruct each student to explain his answer. Choose another student to demonstrate solving the problem.
   - 4 × 3.9 = ***15.6***
   - 0.04 × 3.9 = ***0.156***
   - 0.4 × 0.039 = ***0.0156***
   - 0.04 × 0.39 = ***0.0156***
6. Repeat the procedure with 3 × 27 = 81.
   - 0.3 × 0.27 = ***0.081***
   - 0.03 × 0.27 = ***0.0081***
   - 0.03 × 27 = ***0.81***
   - 0.03 × 2.7 = ***0.081***

Multiply money by a decimal

1. Guide the students in estimating and solving the following word problems. Remind the students that the cents in an amount of money represent hundredths of a dollar. Elicit that when you multiply an amount of money by a decimal, the product will have more than 2 decimal places. Explain that the decimal product in a money problem must be rounded to the nearest cent or hundredth of a dollar.

Vanessa bought 3.25 pounds of apples that were priced at $1.69 a pound. How much did she pay for the apples? ***3 × $2 = $6; 3.25 × $1.69 = 5.4925; $5.49***

If gasoline costs $2.97 per gallon, how much will it cost to purchase 9.2 gallons? ***9 × $3 = $27; 9.2 × $2.97 = 27.324; $27.32***

2. Direct attention to the word problems on page 158 of the Student Text and review the decimal equivalents for $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$. Arrange the students in groups and instruct each group to solve the problems. Allow students to share their group's answers and explain how they solved the problems.
3. **Christian Worldview Shaping (CD)**

Student Text pp. 158–59

Lesson 67

Student Text pp. 160–61
Daily Review p. 429f

Objectives
- Develop an understanding of our infinite number system
- Identify fractions and decimals that come between two numbers
- Solve a multi-step word problem

Teacher Materials
- Fraction Number Line (yellow)
- In-Between Numbers, page IA41 (CD)

Student Materials
- Fraction Number Line (yellow)

Teach for Understanding

Identify fractions that come between two numbers

1. Explain that our number system is infinite; it continues without end. Select a student to name a large number. Continue choosing students to name a number that is greater than the previous number that was named.

➤ **Whatever number is named, is there always a number that is greater? Why?** ***Yes; elicit that our infinite number system continues without end.***

➤ **What do you think is the largest number?** ***Answers will vary, but elicit that there is no largest number. Whatever number is named, 1 more can be added to it.***

Point out that it is difficult to understand the value of very large numbers and the vastness of our infinite number system; even more so, man cannot measure or understand the depth of God's infinite love and mercy. [Bible Promise: H. God as Father]

2. Distribute the Fraction Number Lines and display your Fraction Number Line. Select students to name the fractions between $\frac{1}{8}$ and $\frac{3}{8}$. $\frac{2}{10}$, $\frac{2}{8}$ *or* $\frac{1}{4}$, $\frac{3}{10}$

Choose other students to name the numbers between $\frac{7}{8}$ and $\frac{9}{8}$. $\frac{9}{10}$, *1 or the fractional names for 1,* $\frac{11}{10}$ *or* $1\frac{1}{10}$

3. Repeat the procedure using other pairs of fractions or mixed numbers.

➤ **What do you notice about every pair of numbers?** ***Elicit that there are numbers between every pair of numbers.***

Identify decimals that come between two numbers

1. Direct attention to the first number line on the In-Between Numbers page.

➤ **What value is the arrow above the number line pointing to?** ***1.3*** **the arrow below the number line?** ***1.4*** Write the values on the blanks.

➤ **What equivalent decimal can you write for 1.3?** ***Elicit 1.30.*** **1.4?** ***Elicit 1.40.*** Annex a 0 in each decimal.

➤ **Why can you annex a 0 in the Hundredths place of each of these decimals?** ***Answers may vary, but elicit that 3 tenths is equivalent to 30 hundredths and that 4 tenths is equivalent to 40 hundredths.***

2. Point out that the next number line shows the units that are between 1.3 and 1.4.

➤ **What units are pictured between 1.3 and 1.4?** ***hundredths***

➤ **What value is the arrow pointing to?** ***1.33*** Write the value.

3. Beginning with the least value, guide the students in naming the numbers that are represented between 1.3 and 1.4. Write each number on the appropriate blank: ***1.31, 1.32,*** 1.33, ***1.34, 1.35, 1.36,*** 1.37, ***1.38, 1.39***.

4. Repeat the procedure for the numbers between 1.33 and 1.34. ***1.331, 1.332, 1.333, 1.334, 1.335, 1.336, 1.337, 1.338, 1.339*** Elicit that a zero can be annexed in both 1.33 and 1.34 to show thousandths in the decimal fraction.

5. Select students to write the numbers that come between the numbers in the remaining problems.

2.4–2.5 ***2.41, 2.42, 2.43, 2.44, 2.45, 2.46, 2.47, 2.48, 2.49***

0.53–0.54 ***0.531, 0.532, 0.533, 0.534, 0.535, 0.536, 0.537, 0.538, 0.539***

1.627–1.628 ***1.6271, 1.6272, 1.6273, 1.6274, 1.6275, 1.6276, 1.6277, 1.6278, 1.6279***

Solve a multi-step word problem

1. Choose a student to read aloud this word problem (problem 22 on Student Text page 161).

> Max had 2 boxes of nails. One box weighed $\frac{3}{4}$ of a pound, and the other box weighed $\frac{2}{3}$ of a pound. How many pounds of nails did Max have after using $\frac{1}{2}$ of a pound of nails? $\frac{11}{12}$ *of a pound*

➤ **What is the question asking you to find?** ***how many pounds of nails Max had after he used*** $\frac{1}{2}$ ***of a pound***

➤ **What information is needed to solve the word problem?** ***One box of nails weighed*** $\frac{3}{4}$ ***of a pound, and the second box weighed*** $\frac{2}{3}$ ***of a pound;*** $\frac{1}{2}$ ***of a pound of nails was used.***

➤ **How could you solve this word problem?** ***Elicit that first you must add the weights of both boxes of nails to find the total weight. Then you can subtract the*** $\frac{1}{2}$ ***pound of nails used from the total weight to find the weight of the nails that Max had left.***

2. Write $\frac{3}{4} + \frac{2}{3}$ for display. Select a student to solve this first step of the problem while the other students solve it on paper. $1\frac{5}{12}$ *pounds*

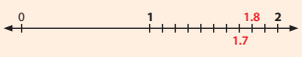
➤ **Now how can you find the weight of the nails that Max had left?** ***subtact*** $\frac{1}{2}$ ***of a pound from*** $1\frac{5}{12}$ ***pounds***

3. Write $- \frac{1}{2} =$ after $\frac{3}{4} + \frac{2}{3}$; insert parentheses around $\frac{3}{4} + \frac{2}{3}$. Choose a student to solve this second step of the problem while the other students solve the equation on paper. $1\frac{5}{12} - \frac{1}{2} = \frac{11}{12}$ *of a pound*

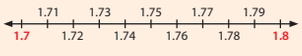
4. Guide the students in completing the word problems on page 161.

Between Numbers

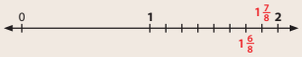
Any two numbers have an infinite number of numbers between them. These "in-between" numbers are found by partitioning the space between two numbers into fractions or decimals.



The space between **1** and **2** can be partitioned into tenths. 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, and 1.9 are between the whole numbers 1 and 2.



Tenths can be partitioned into hundredths. 1.71, 1.72, 1.73, 1.74, 1.75, 1.76, 1.77, 1.78, and 1.79 are between 1.7 and 1.8.



The space between **1** and **2** can be partitioned into eighths. $1\frac{1}{8}$, $1\frac{2}{8}$, $1\frac{3}{8}$, $1\frac{4}{8}$, $1\frac{5}{8}$, $1\frac{6}{8}$, and $1\frac{7}{8}$ are between the whole numbers 1 and 2.

Eighths can be partitioned into sixteenths. $1\frac{13}{16}$ is between $1\frac{6}{8}$ and $1\frac{7}{8}$.

Exercises

Choose the number that is between the given numbers.

1. 0.10, ___, 0.20

(0.11) 0.21 0.003

2. $1.00, ___, $1.05

($1.03) $1.10 $1.06

3. 35, ___, 36

34.9 (35.08) 36.3

4. $\frac{1}{2}$, ___, $\frac{3}{4}$

($\frac{5}{8}$) $\frac{6}{8}$ $\frac{7}{8}$

5. $^-4$, ___, $^-3$

$3\frac{1}{2}$ $^-4\frac{1}{2}$ ($^-3\frac{1}{2}$)

6. 7.8, ___, 7.9

(7.81) 7.08 7.91

7. $\frac{5}{6}$, ___, $\frac{6}{6}$

$\frac{4}{6}$ ($\frac{11}{12}$) $\frac{2}{3}$

8. 5, ___, $5\frac{1}{2}$

$\frac{9}{2}$ $\frac{10}{3}$ ($\frac{21}{4}$)

9. $\frac{11}{4}$, ___, $\frac{12}{4}$

$2\frac{3}{4}$ ($2\frac{7}{8}$) $3\frac{1}{4}$

Write the missing numbers.

10. 0.84, ___, ___, 0.843 ***0.841, 0.842***

11. 0.012, ___, ___, 0.0123 ***0.0121, 0.0122***

12. 1.726, ___, ___, 1.729 ***1.727, 1.728***

13. $90.99, ___, ___, $91.02 ***$91.00, $91.01***

14. 0.2, ___, ___, 0.23 ***0.21, 0.22***

15. ___, 3.001, ___, 3.003 ***3.000, 3.002***

16. 52.3, ___, ___, 52.33 ***52.31, 52.32***

17. 817.06, ___, ___, 817.063 ***817.061, 817.062***

18. 21.07, ___, 21.09, ___ ***21.08, 21.10***

19. $\frac{1}{2}$, ___, $\frac{5}{8}$ ***$\frac{9}{16}$***

20. $\frac{1}{8}$, ___, ___, $\frac{1}{2}$ ***$\frac{1}{4}$, $\frac{3}{8}$***

21. $-\frac{2}{3}$, ___, ___, $-\frac{1}{3}$ ***$-\frac{5}{9}$, $-\frac{4}{9}$***

Solve. ***Equations may vary.***

22. Max had 2 boxes of nails. One box weighed $\frac{3}{4}$ of a pound, and the other box weighed $\frac{2}{3}$ of a pound. How many pounds of nails did Max have after using $\frac{1}{2}$ of a pound of nails? ***$(\frac{3}{4}+\frac{2}{3})-\frac{1}{2}=(\frac{9}{12}+\frac{8}{12})-\frac{6}{12}=\frac{17}{12}-\frac{6}{12}=\frac{11}{12}$ of a pound of nails***

23. Mrs. Elmer is collecting money for her students' school pictures. Each package costs $8.50. Eighteen students have already paid. Nine students still need to pay. How much money will be collected when all the students have paid? ***(18 + 9) × $8.50 = 27 × $8.50 = $229.50***

24. Mother had 5 cups of sugar. She used $1\frac{1}{2}$ cups of sugar to make a cake and $\frac{3}{4}$ of a cup to make a pie. How much sugar did Mother have left? ***$5-(1\frac{1}{2}+\frac{3}{4})=5-2\frac{1}{4}=2\frac{3}{4}$ cups of sugar***

25. Jeffrey made a square table with a perimeter of 396 cm. Lou made a square table with a perimeter of 332 cm. What is the measurement of each side of Jeffrey's table? Of Lou's table? ***Jeffrey's table: 396 ÷ 4 = 99 cm***
Lou's table: 332 ÷ 4 = 83 cm

26. An average twelve-year-old's heart beats about 9,600 times in 2 hours. At this rate, how many times does the average twelve-year-old's heart beat in 1 minute? ***9,600 ÷ (2 × 60) = 9,600 ÷ 120 = 80 times***

27. Janet's 50-yard dash times were 8.4 seconds, 9.3 seconds, and 9 seconds. What is her average time? ***(8.4 + 9.3 + 9) ÷ 3 = 26.7 ÷ 3 = 8.9 seconds***

28. According to the doctor, Mr. Gates should eat 3 servings of vegetables each day. Mr. Gates made a vegetable tray with $\frac{3}{4}$ of a serving of cherry tomatoes, $1\frac{1}{2}$ servings of carrots, and 1 serving of celery. How many servings of vegetables did Mr. Gates prepare? ***$\frac{3}{4}+1\frac{1}{2}+1=3\frac{1}{4}$ servings***

29. The doctor also recommended that Mr. Gates walk 2 miles a day, five days each week. If 4 laps around his school track is 1 mile, how many laps must Mr. Gates walk in 5 days? ***5 × (2 × 4) = 5 × 8 = 40 laps***

Complete DAILY REVIEW 1 on page 429.

Student Text pp. 160–61

Lesson 68 **Student Text** pp. 162–63

Chapter Review

Objectives

- Multiply a whole number and a fraction
- Multiply to find a fraction of a whole number and a fraction of a fraction
- Use cancellation when multiplying fractions
- Identify the reciprocal of a fraction
- Multiply mixed numbers
- Multiply decimals
- Estimate the product by rounding
- Apply the Distributive Property of Multiplication over Addition when multiplying mixed numbers or decimals
- Use mental math to multiply a decimal and a power of 10
- Write and solve an equation for a word problem

Note

This lesson reviews the concepts presented in Chapter 7 to prepare the students for the Chapter 7 Test. Student Text pages 162–63 provide the students with an excellent study guide.

Check for Understanding

Multiply fractions and mixed numbers

1. Write for display $6 \times \frac{3}{4}$.

➤ **How do you read this problem?** ***6 sets of $\frac{3}{4}$***

➤ **What do you know about multiplying a whole number and a fraction?** ***Answers will vary, but elicit that since the denominator of every whole number is 1 and every whole number can be written as an improper fraction with 1 as the denominator, you multiply the whole number and the numerator; the denominator does not change.***

Choose a student to solve the problem while the other students solve it on paper. Remind them to simplify the product. $\frac{18}{4} = 4\frac{2}{4} = 4\frac{1}{2}$

Select another student to illustrate the problem. ***Accept any picture showing 6 sets with 3 fourths in each set.***

➤ **Since the fraction $\frac{3}{4}$ is being multiplied by a whole number, how else could you solve this problem?** ***Elicit that you could use repeated addition.***

Choose a student to demonstrate the repeated addition.
$\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{18}{4} = 4\frac{2}{4} = 4\frac{1}{2}$

2. Write $\frac{3}{4} \times 8$ for display.

➤ **How do you read this problem?** ***$\frac{3}{4}$ of 8*** Elicit that the product will represent a part or a fraction of the whole number 8.

Choose a student to solve the problem while the other students solve it on paper. $\frac{24}{4} = 6$

Select another student to illustrate the problem. ***Accept any picture showing one set of 8 figures with $\frac{3}{4}$ of the figures (6) circled.***

3. Write $\frac{3}{4} \times \frac{1}{2}$ and choose a student to read it aloud. ***$\frac{3}{4}$ of $\frac{1}{2}$***

➤ **Will the product be greater than or less than $\frac{1}{2}$? How do you know?** ***Less than $\frac{1}{2}$; you are finding a part of $\frac{1}{2}$.***

Choose a student to solve the problem while the other students solve it on paper. $\frac{3}{8}$

➤ **How could you picture this problem?** ***Answers will vary; possible answer: draw a figure such as a rectangle, partition the figure into halves, and shade $\frac{1}{2}$. Then partition each half into fourths; use a different color to shade 3 fourths of 1 of the halves. The product represents the part of the figure that is double shaded.***

Select a student to illustrate $\frac{3}{4} \times \frac{1}{2}$. Give guidance as needed.

4. Write $\frac{8}{12} \times \frac{3}{10}$ for display.

➤ **What could you do to make this problem easier to solve?** ***Elicit that you can use cancellation to simplify the factors so that the numbers you multiply will be smaller.*** (See Lesson 63.)

➤ **What can you simplify this problem as? How do you know?** ***$\frac{4}{4} \times \frac{1}{5}$; 2 is the greatest common factor (GCF) of the numerator 8 and the denominator 10, and 3 is the GCF of the numerator 3 and the denominator 12.***

Choose a student to demonstrate the cancellation and to solve the problem. $\frac{4}{20} = \frac{1}{5}$ Point out that since you know that $\frac{4}{4} = 1$, using the Identity Property of Multiplication to solve this problem will give you the product in lowest terms: $1 \times \frac{1}{5} = \frac{1}{5}$.

5. Write $\frac{2}{3} \times \frac{3}{2}$.

➤ **When the second fraction in a problem is the reverse of the first fraction, what is the product?** ***1*** Elicit that two numbers whose product is 1 are reciprocals.

➤ **What is the reciprocal of the whole number 6? Why?** ***$\frac{1}{6}$; a whole number can be expressed as an improper fraction with a denominator of 1 (e.g., $\frac{6}{1}$).***

6. Write $2\frac{1}{4} \times 1\frac{2}{3}$ for display.

➤ **What do you estimate the product of this problem to be? Why?** ***4; possible explanation: $2\frac{1}{4}$ rounds to the whole number 2, and $1\frac{2}{3}$ rounds to the whole number 2; $2 \times 2 = 4$.***

➤ **How could you multiply these mixed numbers?** ***Answers may vary, but elicit that since both factors are mixed numbers, renaming both factors as improper fractions is the easiest way to solve the equation.*** (See Lesson 64.)

➤ **What improper fraction does $2\frac{1}{4}$ rename to? $\frac{9}{4}$ $1\frac{2}{3}$? $\frac{5}{3}$** Write the renamed factors: $\frac{9}{4} \times \frac{5}{3}$.

➤ **What do you notice about the renamed factors?** ***Elicit that cancellation can be used to simplify the improper fractions before you multiply.***

Choose a student to solve $\frac{9}{4} \times \frac{5}{3}$ using cancellation, while the other students solve it on paper. $\frac{9}{4} \times \frac{5}{3} = \frac{3}{4} \times 5 = \frac{15}{4} = 3\frac{3}{4}$

➤ **Is the product $3\frac{3}{4}$ reasonable? Why?** ***Yes; $3\frac{3}{4}$ is close to the estimate 4.***

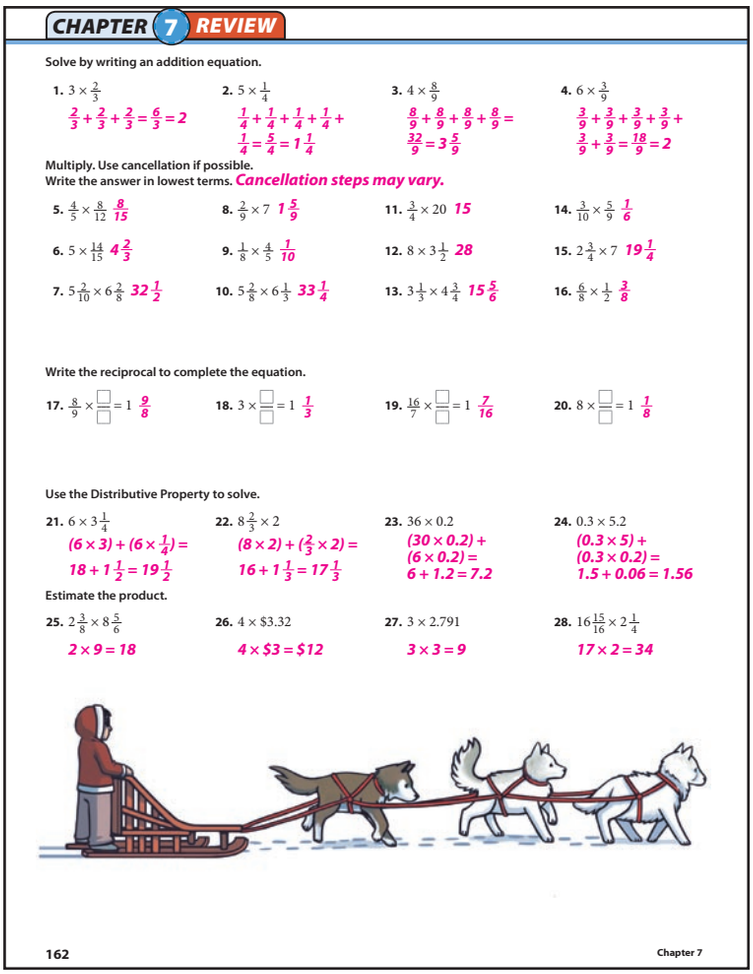
7. Direct the students to use the Distributive Property to solve $6 \times 4\frac{1}{4}$. Select a student to write his solution for display and explain it. Guide the explanation to include that first you think of the mixed number as the sum of the whole number and the fraction and then you multiply each part of the mixed number by the other factor. (See Lesson 64.)
$6 \times (4 + \frac{1}{4}) = (6 \times 4) + (6 \times \frac{1}{4}) = 24 + \frac{6}{4} = 24 + 1\frac{2}{4} = 25\frac{2}{4} = 25\frac{1}{2}$

➤ **How else could you solve this problem?** ***Elicit that you can change the mixed number to an improper fraction and multiply.*** Direct the students to rename the mixed number as an improper fraction and to solve the problem.
$6 \times \frac{17}{4} = 3 \times \frac{17}{2} = \frac{51}{2} = 25\frac{1}{2}$

8. Instruct the students to solve these problems. Choose students to write their solutions for display. Discuss the solutions as needed.

$\frac{5}{9} \times 8 = 4\frac{4}{9}$ $\quad 3 \times 3\frac{4}{5} = 11\frac{2}{5}$ $\quad \frac{2}{3} \times 2\frac{3}{4} = 1\frac{5}{6}$

$\frac{6}{7} \times \frac{14}{18} = \frac{2}{3}$ $\quad 5\frac{3}{4} \times 4\frac{2}{5} = 25\frac{3}{10}$ $\quad 2\frac{5}{8} \times \frac{10}{12} = 2\frac{3}{16}$



CHAPTER 7 REVIEW

Solve by writing an addition equation.

1. $3 \times \frac{2}{3}$ — $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{6}{3}=2$
2. $5 \times \frac{1}{4}$ — $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{5}{4}=1\frac{1}{4}$
3. $4 \times \frac{8}{9}$ — $\frac{8}{9}+\frac{8}{9}+\frac{8}{9}+\frac{8}{9}=\frac{32}{9}=3\frac{5}{9}$
4. $6 \times \frac{3}{9}$ — $\frac{3}{9}+\frac{3}{9}+\frac{3}{9}+\frac{3}{9}+\frac{3}{9}+\frac{3}{9}=\frac{18}{9}=2$

Multiply. Use cancellation if possible. Write the answer in lowest terms. *Cancellation steps may vary.*

5. $\frac{4}{5} \times \frac{8}{12}$ $\frac{8}{15}$
6. $5 \times \frac{14}{15}$ $4\frac{2}{3}$
7. $5\frac{2}{10} \times 6\frac{2}{8}$ $32\frac{1}{2}$
8. $\frac{2}{9} \times 7$ $1\frac{5}{9}$
9. $\frac{1}{8} \times \frac{4}{5}$ $\frac{1}{10}$
10. $5\frac{2}{8} \times 6\frac{1}{3}$ $33\frac{1}{4}$
11. $\frac{3}{4} \times 20$ 15
12. $8 \times 3\frac{1}{2}$ 28
13. $3\frac{1}{3} \times 4\frac{3}{4}$ $15\frac{5}{6}$
14. $\frac{3}{10} \times \frac{5}{9}$ $\frac{1}{6}$
15. $2\frac{3}{4} \times 7$ $19\frac{1}{4}$
16. $\frac{6}{8} \times \frac{1}{2}$ $\frac{3}{8}$

Write the reciprocal to complete the equation.

17. $\frac{8}{9} \times \square = 1$ $\frac{9}{8}$
18. $3 \times \square = 1$ $\frac{1}{3}$
19. $\frac{16}{7} \times \square = 1$ $\frac{7}{16}$
20. $8 \times \square = 1$ $\frac{1}{8}$

Use the Distributive Property to solve.

21. $6 \times 3\frac{1}{4}$ — $(6 \times 3) + (6 \times \frac{1}{4}) = 18 + 1\frac{1}{2} = 19\frac{1}{2}$
22. $8\frac{2}{3} \times 2$ — $(8 \times 2) + (\frac{2}{3} \times 2) = 16 + 1\frac{1}{3} = 17\frac{1}{3}$
23. 36×0.2 — $(30 \times 0.2) + (6 \times 0.2) = 6 + 1.2 = 7.2$
24. 0.3×5.2 — $(0.3 \times 5) + (0.3 \times 0.2) = 1.5 + 0.06 = 1.56$

Estimate the product.

25. $2\frac{3}{8} \times 8\frac{5}{6}$ — $2 \times 9 = 18$
26. $4 \times \$3.32$ — $4 \times \$3 = \$12$
27. 3×2.791 — $3 \times 3 = 9$
28. $16\frac{15}{16} \times 2\frac{1}{4}$ — $17 \times 2 = 34$

Solve.

29. 2.315×4 — 9.26 or 9.260
30. 7.3×0.05 — 0.365
31. 3.178×2.3 — 7.3094
32. 2.37×0.4 — 0.948
33. 0.08×0.4 — 0.032

Use mental math to solve.

34. 2.482×10^3 2,482
35. 27.314×100 2,731.4
36. 65.72×10^1 657.2

Solve. Write the answer in lowest terms. *Equations may vary.*

37. Roland runs $4\frac{7}{10}$ miles each day. How many miles does he run in 4 days?
$4 \times 4\frac{7}{10} = 4 \times \frac{47}{10} = \frac{94}{5} = 18\frac{4}{5}$ mi

38. Karis ran $3\frac{2}{3}$ miles on Monday. Benita ran $\frac{1}{2}$ of the distance that Karis ran. How far did Benita run?
$\frac{1}{2} \times \frac{11}{3} = \frac{11}{6} = 1\frac{5}{6}$ mi

39. What was the total number of miles run by Karis and Roland on Monday?
$4\frac{7}{10} + 3\frac{2}{3} = 4\frac{21}{30} + 3\frac{20}{30} = 7\frac{41}{30} = 8\frac{11}{30}$ mi

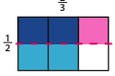
40. William bought 12.7 gallons of gasoline for $3.89 per gallon. How much did he pay for the gasoline?
$12.7 \times \$3.89 = \$49.40$

41. A recipe calls for $\frac{3}{4}$ cup butter and $\frac{1}{3}$ cup milk. If only $\frac{1}{2}$ of the recipe is made, how much butter and milk is needed?
$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ c butter; $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ c milk

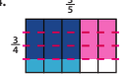
42. Mrs. O'Malley found background fabric for her bulletin board on sale for $3.75 per yard. How much would $4\frac{1}{4}$ yards cost?
$4.25 \times \$3.75 = \$15.94$

Solve. Complete the picture to show the product.

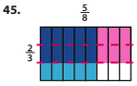
43. $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$



44. $\frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$

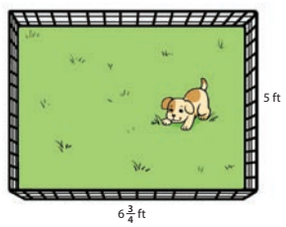


45. $\frac{2}{3} \times \frac{5}{8} = \frac{10}{24} = \frac{5}{12}$



Solve.

46. The dog pen measures $6\frac{3}{4}$ feet long by 5 feet wide. Lucas used the Commutative Property to estimate the area of the dog pen. Write the equation he wrote. Estimate the area. Write the label as *square feet*. $5 \times 7 = 35$ square feet



Multiply decimals

1. Write 3×2.34 for display.
➤ **What is an estimated product of 3×2.34? Explain your estimate.** *6; 2.34 rounded to the nearest whole number is 2, and $3 \times 2 = 6$.* (See Lesson 65.)
Instruct the students to solve the problem. *7.02*
➤ **How did you know where to place the decimal point in the product?** *Possible answers: Since the estimated product is 6, the only reasonable answer is 7.02; since the multiplier is a whole number, a decimal point is needed to indicate hundredths in the product because the decimal point in the multiplicand shows hundredths.*
2. Follow a similar procedure for 3×4.625. *13.875*
3. Write for display 5.23×10^1; 5.23×10^2; and 5.23×10^3.
➤ **What pattern do you know for multiplying a decimal by a power of 10?** *Elicit that each exponent indicates the number of zeros in a power of 10, and for each 0 in a power of 10 (10, 100, 1,000 . . .), the decimal point in the other factor moves 1 place to the right.* (See Lesson 65.)
Choose students to solve the problems using mental math. *52.3; 523; 5,230* Remind the students that every whole number has an unseen decimal point. Point out that a zero was annexed in the product 5,230 so that the decimal point could be moved 1 more place to the right.
4. Select students to tell the products for 0.071×10^2 *7.1*, $1{,}000 \times 8.33$ *8,330*, and 0.0046×10 *0.046*.
5. Write 0.04×0.2. Choose a student to solve the problem. Select another student to write the problem in fraction form and to solve it. *0.008;* $\frac{4}{100} \times \frac{2}{10} = \frac{8}{1{,}000}$ Remind the students that thinking of the fraction form of the decimal factors can help you determine the placement of the decimal point. Point out that 3 zeros were annexed in the decimal product so that the decimal point could be placed correctly.
6. Guide the students in solving on paper $0.97 \times 0.03 =$ *0.0291* and $28.5 \times 4.7 =$ *133.95*.
7. Guide the students in applying the Distributive Property to solve these equations.
$0.6 \times 2.13 =$ *$(0.6 \times 2) + (0.6 \times 0.13) = 1.2 + 0.078 = 1.278$*
$82 \times 0.04 =$ *$(80 \times 0.04) + (2 \times 0.04) = 3.2 + 0.08 = 3.28$*

Solve a word problem

Guide the students in estimating the product and in solving each word problem.

Mr. Stanley's square flower garden is $8\frac{3}{4}$ feet long on each side. How many feet of fencing does he need to enclose the garden? *$4 \times 8\frac{3}{4} = 35$ feet ($4 \times 9 = 36$)*

Jordan paid $2.69 per gallon for 8.4 gallons of gasoline. What was the total cost of the gasoline?
$8.4 \times \$2.69 = 22.596$; $22.60 ($8 \times \$3 = \$24$)

A roll of wallpaper border contains 5.5 meters of border. What is the total length of border on 3.5 rolls?
$3.5 \times 5.5 = 19.25$ meters (possible estimates: $4 \times 6 = 24$, $4 \times 5 = 20$, or $5 \times 4 = 20$)

Student Text pp. 162–63

Lesson 69

Student Text pp. 164–67

Chapter 7 Test

Cumulative Review

For a list of the skills reviewed in the Cumulative Review, see the Lesson Objectives for Lesson 69 in the Chapter 7 Overview on page 148 of this Teacher's Edition.

Student Materials

- Cumulative Review Answer Sheet, page IA9 (CD)

Use the Cumulative Review on Student Text pages 164–66 to review previously taught concepts and to determine which students would benefit from your reteaching of the concepts. To prepare the students for the format of achievement tests, instruct them to work on a separate sheet of paper, if necessary, and to mark the answers on the Cumulative Review Answer Sheet.

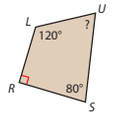
Read aloud the Career Link on Student Text page 167 (page 165 of this Teacher's Edition) and discuss the value of math as it relates to a travel agent.

CUMULATIVE REVIEW

Test Prep

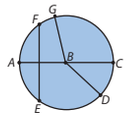
Mark the answer.

1.



$\angle LUS = \_\_$

A. 20°
C. 70°
B. 60°
D. none of the above

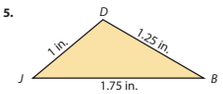


2. $\overline{BG}$ is a ___.
A. radius
B. diameter
C. chord

3. $\overline{FE}$ is a ___.
A. radius
B. diameter
C. chord

4. The diameter of circle B is ___.
A. $\overline{FE}$
B. $\overline{AC}$
C. $\overline{BD}$

5.

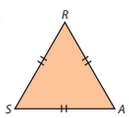


$\triangle JDB$ is a(n) ___ triangle.
A. scalene
B. equilateral
C. isosceles

6.

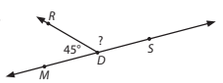
$\triangle EAC$ is a(n) ___ triangle.
A. scalene
B. equilateral
C. isosceles

7.



$\triangle SRA$ is a(n) ___ triangle.
A. scalene
B. equilateral
C. isosceles

8.



$\angle RDS = \_\_$

A. 315°
C. 45°
B. 135°
D. none of the above

Mark the answer.

9. Sunflower seeds cost $3.69 per pound. How much do 2 pounds of seeds cost?
A. $7.38
B. $1.85
C. $11.07

10. Micah paid $7.12 for 8 pounds of jellybeans. What is the price of the jellybeans per pound?
A. $0.65
B. $56.96
C. $0.89

11. $2\frac{2}{3} + 7\frac{2}{5} = \_\_$
A. $9\frac{4}{5}$
B. $10\frac{1}{15}$
C. $10\frac{1}{6}$

12. $8\frac{2}{9} - 1\frac{5}{6} = \_\_$
A. $6\frac{7}{18}$
B. $6\frac{11}{18}$
C. $6\frac{1}{3}$

13. $1\frac{2}{12} + 1\frac{4}{7} = \_\_$
A. $3\frac{1}{2}$
B. $2\frac{31}{42}$
C. $2\frac{18}{84}$

1,865.709

14. number of hundredths
A. 0
B. 9
C. 8

15. number of one thousands
A. 9
B. 1
C. 5

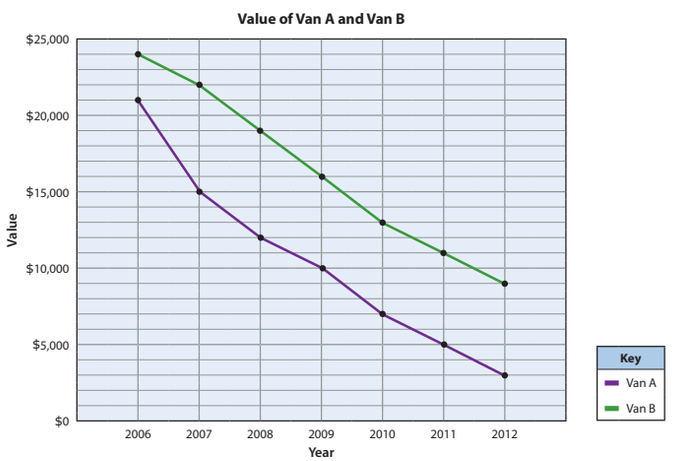
16. value of the Tenths place
A. 60
B. 0.7
C. 0.07

17. value of the Hundreds place
A. 0.009
B. 800
C. 1,800

18. $\frac{2}{3} \times \frac{3}{4} = \_\_$
A. $\frac{1}{2}$
C. $\frac{7}{12}$
B. $1\frac{5}{12}$
D. none of the above

19. $8.2 \times 0.03 = \_\_$
A. 0.246
C. 0.0246
B. 2.46
D. none of the above

Use the data from the line graph to find the answer.



20. Which van would be cheaper to purchase?
A. Van A
B. Van B

21. By how much did the value of Van B decrease between 2006 and 2007?
A. $6,000
B. $3,000
C. $2,000

22. What is the difference between the values of the two vans in 2006?
A. $3,000
B. $5,000
C. $4,000

23. What is the difference between the values of the two vans in 2012?
A. $6,000
B. $3,000
C. $12,000

24. What is the value of Van A in 2010?
A. $7,000
B. $5,000
C. $13,000

CAREER LINK

Travel Agent

A travel agent has the unique opportunity to help people with a variety of travel needs. Sometimes the agent helps with a special vacation or a business trip, aids a student traveling home for Christmas, or books an emergency trip for a family member to see a loved one who is sick. In each case, the agent's job is to find the best prices and provide superior service to those who ask for her help.

Someone traveling under emergency circumstances may be concerned about how quickly he can arrive at his destination. A college student may have limited funds and only a certain amount of time to spend with his friends and family during break. When helping a family plan a vacation, an agent must make arrangements for travel with children and suggest family-friendly hotels and attractions while staying within the family's budget. Therefore, a travel agent must be knowledgeable about hotels, destinations, car rentals, airlines, airports, regulations, policies, currency, and custom and cultural issues. She must also be ready with an alternative plan if the weather, political or economic conditions, or transportation malfunctions affect her client's travel.

An effective travel agent enjoys traveling and researching. She is expected to be organized, pay attention to details, display good writing and communication skills, and have a thorough working knowledge of geography. Speaking more than one language is also a bonus for her clients and the travel agency.

Math is an important part of an agent's daily work. Knowing how to convert money from one currency to another is essential. Since the market changes daily, the value of the dollar changes as well. Many people travel across time zones. Consequently, the travel agent must consider the time of departure and arrival and the length of layover between flights when booking itineraries. Skills in accounting help the agent to be accurate for both her clients and her employer.

An agent who is knowledgeable and considerate of a client's time and money is a good steward of the task she has been given as well as a valuable asset to her company.