17 Integers

UNSINKABLE SPIRIT

Denver, Colorado **April 14, 1912**

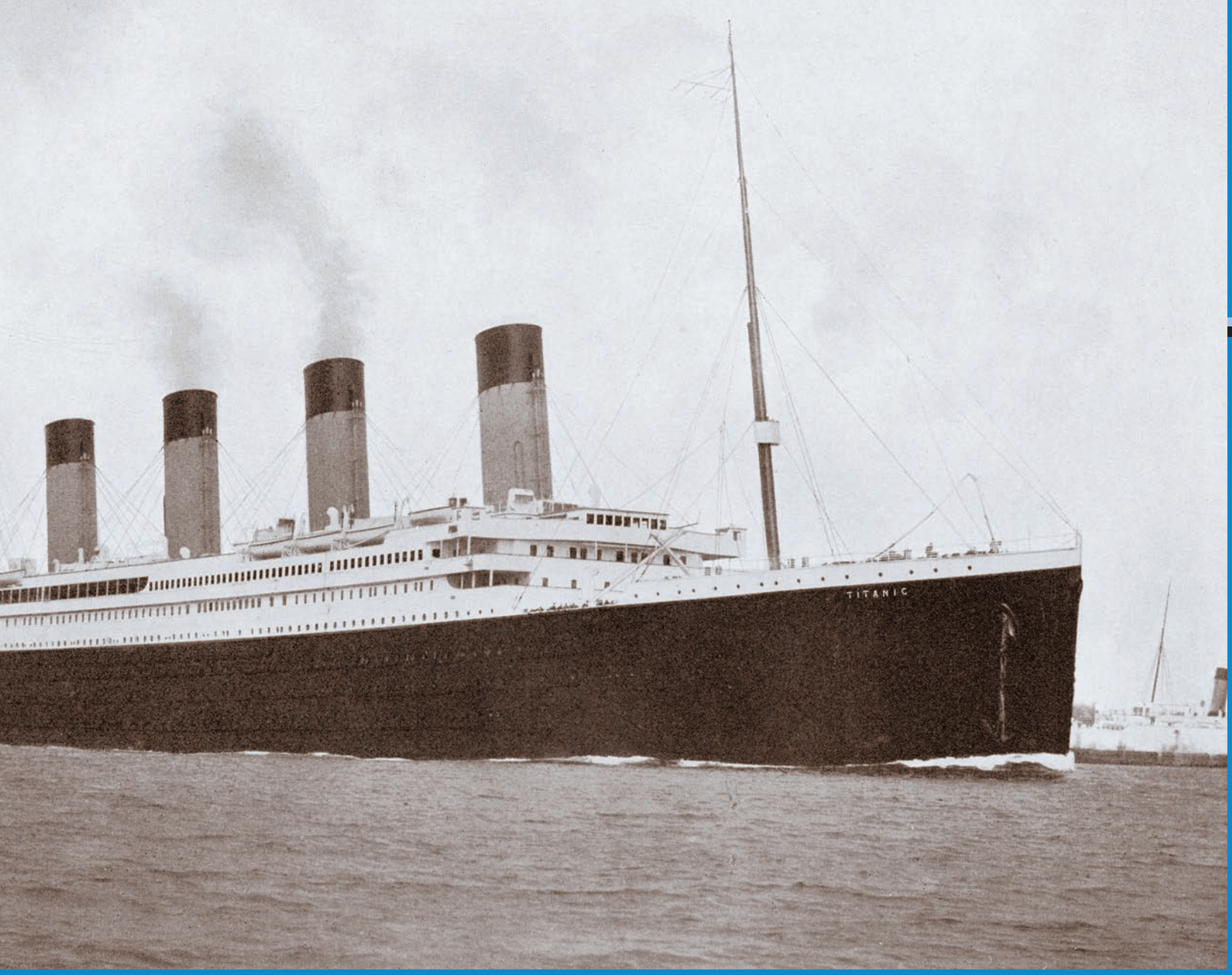
On April 10, 1912, the British steamer *Titanic* cast off on her maiden voyage from Southampton, England, to New York City. Aboard the ship were about 2,200 people: crew members, wealthy socialites traveling first class, and second- and third-class passengers enjoying their berths on such a highly publicized voyage. With 50,000-horsepower engines, she was as tall as an eleven-story building, and her length equaled four city blocks. The ship was said to be the greatest ever built. Newspapers declared her "unsinkable."

But on April 14, disturbing messages from other ships reached the *Titanic*. Icebergs had been sighted in the Atlantic much farther south than usual. The *Titanic*'s crew members heard the warnings without a great deal of concern and kept the ship on her course. It was traditional to not slow a ship until ice was actually spotted. Later that evening, a small ship named the *Mesaba* sent an urgent message to the *Titanic*'s radio room. A great number of icebergs lay directly in the path of the huge steamer. But the radio operator was so busy sending greetings from passengers to friends in the United States that the message went unnoticed. The watchmen did not see the icebergs until it was too late. About 11:40 PM, the ship's side was torn open by one of those icebergs. The crew soon realized that the *Titanic* was sinking.

Among the passengers aboard was Mrs. Margaret Tobin Brown, a prominent woman from Denver, Colorado, known for wearing fashionable clothes and for speaking her mind. Rather than scrambling to the safety of a lifeboat, "Molly," as her friends called her, calmly helped others into the lifeboats when the time came to fill them. As she turned from helping a frightened passenger, someone grabbed her from behind and dropped her into the lifeboat just as it was lowered over the ship's deck. Wrapped in her expensive furs, Molly rowed in unison with a team of survivors until they reached the rescue ship *Carpathia*. She even removed her fur stole and wrapped it around a man who was half-frozen from the icy water.

While on the *Carpathia*, Mrs. Brown established and was voted chairwoman for "The Survivor's Committee," which had raised a large sum of money for the survivors even before the rescue boat reached New York City. "Unsinkable Molly Brown," as she was later dubbed, was one of only about 700 survivors of the disaster.

Mrs. Brown presenting a trophy cup award to Captain Arthur Henry Rostron for his service in the rescue of passengers from the *Titanic*



In 1909 the cost to build the *Titanic* was $7,500,000. Today the cost to build it would be an estimated $400,000,000.

The dimensions of the *Titanic* were $882\frac{3}{4}$ feet long, $92\frac{1}{2}$ feet wide, and 175 feet high.

Approximately 3,000 men worked two years to complete the "largest manmade moving object in the world."

It took 20 draft horses to pull one of the *Titanic*'s 15-ton anchors in a wagon through Belfast, Ireland, to the shipyard.

The *Titanic* carried only enough lifeboats to hold approximately 53% of the people onboard.

A total of 1,517 people lost their lives on the *Titanic*: 130 first-class passengers, 166 second-class passengers, 536 third-class passengers, and 685 crew members.

Three hours after hitting the iceberg, the *Titanic* was completely underwater.

Integers

Lesson	Topic	Lesson Objectives
155	Integers	• Demonstrate an understanding of integers • Find the absolute value of a number • Compare and order integers
156	Add Integers	• Add integers using manipulatives • Add integers using a number line • Write an addition equation for a word problem
157	Subtract Integers	• Subtract integers using manipulatives • Subtract integers using a number line • Write a subtraction equation for a word problem
158	Add & Subtract Integers	• Subtract integers • Add and subtract integers using a number line and manipulatives • Demonstrate an understanding of the relationship between subtracting an integer and adding its opposite • Write an equation for a word problem
159	More Integers	• Solve real-life word problems • Add and subtract integers • Write an equation for a word problem
160	Multiply Integers	• Multiply integers using manipulatives • Write an equation for a word problem
161	Multiply & Divide Integers	• Multiply and divide integers • Write an equation for a word problem
162	Mixed Review	• Add, subtract, multiply, and divide integers • Subtract integers by adding the opposite • Apply the order of operations to integers
163	Coordinate Plane	• Graph points on a four-quadrant coordinate plane • Write ordered pairs to identify points on a four-quadrant coordinate plane
164	Chapter 17 Review	• Review
165	Chapter 17 Test Cumulative Review	• Read and interpret a double line graph • Identify the geometric figure • Find the area of a parallelogram • Find the surface area and volume of a rectangular prism • Calculate the unknown measure of an angle • Estimate an answer • Solve fraction problems

Chapter Materials

Teacher Manipulatives Packet:
- Thermometer
- Red Strip

Student Manipulatives Packet:
- Algebra Mat Kit
- Thermometer
- Red Strip

Instructional Aids (Teacher's Toolkit CD):
- Positive & Negative Number Line (page IA4)
- Cumulative Review Answer Sheet (page IA9) for each student
- Coordinate Plane (page IA24)
- Coordinate Planes (page IA25) for each student
- Algebra Mat (page IA102)
- Positive & Negative Number Lines (page IA103) 2 copies for each student
- Subtraction Patterns (page IA104)
- Subtraction Patterns (page IA104) for each student
- More Subtraction Patterns (page IA105)
- Multiplication Patterns (page IA106)
- Order of Operations (page IA107)
- Order of Operations (page IA107) for each student

Christian Worldview Shaping (Teacher's Toolkit CD):
- Pages 37–39

Other Teaching Aids:
- Plastic counters: opaque (to appear black when placed on the Algebra Mat) and transparent red

Math 6 Tests and Answer Key

Optional (Teacher's Toolkit CD):
- Fact Review pages
- Application pages
- Calculator Activities

A Little Extra Help

Use the following to provide "a little extra help" for the student that is experiencing difficulty with the concepts taught in Chapter 17.

Recognize an integer as being positive—Allow the student who has difficulty recognizing that an integer without a symbol is a positive integer to write positive symbols ($^{+}$) in front of positive integers. This will help him in comparing and ordering integers, in seeing that opposites cancel each other out within an equation (e.g., $5 + {}^{-}5 = 0$), and in solving equations (e.g., $3 + {}^{-}4 = {}^{+}3 + {}^{-}4 = {}^{-}1$ and ${}^{-}3 + 4 = {}^{-}3 + {}^{+}4 = {}^{+}1$).

Recognize plus (+) and minus (–) signs as an operation (action) or as an integer symbol—Solving addition and subtraction equations without the Algebra Mat may be helpful for the student who has difficulty in determining whether a plus (+) or minus (–) sign indicates an action in an equation or is an integer symbol. Write a positive symbol (+) on black counters, using a white marker, and write a negative symbol (–) on red counters, using a black marker. Guide the student in using the counters to solve addition and subtraction equations.

Mental Math

Throughout this chapter, select problems from the list of mental math problems provided on pages IA108–IA110 of the Teacher's Toolkit CD.

Lesson 155

Student Text pp. 374–77
Daily Review p. 463a

Objectives
- Demonstrate an understanding of integers
- Find the absolute value of a number
- Compare and order integers

Teacher Materials
- Thermometer
- Red Strip
- Positive & Negative Number Line, page IA4 (CD)

Student Materials
- Thermometer
- Red Strip

Preparation
Draw for display the number line pictured on this page but do not draw the jumps.

Note
Preview the Fact Review pages, the Application pages, and the Calculator Activities located on the Teacher's Toolkit CD.

Introduce the Lesson

Guide the students in reading aloud the story and facts on pages 374–75 of the Student Text (pages 372–73 of this Teacher's Edition).

Teach for Understanding

Demonstrate an understanding of integers and absolute value

1. Distribute a Thermometer and a Red Strip to each student.

➤ **What type of measuring unit does a thermometer use to measure temperature?** ***degrees***

> The weatherman reported that the temperature was 10°F during the day, but the temperature will drop 20 degrees during the night.

Direct the students to set the thermometer to 10°F.

➤ **How far from zero is 10°F?** ***10 degrees (units)*** **in what direction?** ***up***

➤ **How can you show a drop in temperature of 20°?** ***Move the red strip down 20 degrees (units).***

Direct the students to show a drop in temperature of 20°F on their thermometers.

➤ **What is the temperature now?** ***10°F below zero***

➤ **How far from zero is 10° below zero?** ***10 degrees (units)*** **in what direction?** ***down***

➤ **Besides the direction, how does the thermometer distinguish between 10° above zero and 10° below zero?** ***Elicit that a negative sign is used to indicate a temperature below zero.***

Choose a student to write the daytime temperature and the nighttime temperature for display. ***10°F; −10°F***

2. Explain that a thermometer is a type of vertical number line. Negative and positive numbers are represented on a number line. Point out that zero is neither positive nor negative but that it separates the positive numbers from the negative numbers.

➤ **Which numbers are positive on a vertical number line?** ***all the numbers above 0*** **which numbers are negative?** ***all the numbers below 0***

3. Two numbers are opposites if they are the same distance from zero on a number line but in opposite directions. Point out that the opposite of a negative number is positive, and the opposite of a positive number is negative. Zero is its own opposite.

➤ **What measurement is the opposite of 4°C? Why?** ***−4°C; 4°C is four units above 0, and its opposite is four units below 0.***

> Eric reported to the class that Death Valley is approximately 86 meters below sea level and that the Dead Sea depression is approximately 413 meters below sea level.

➤ **What numbers can you write to represent 86 meters below sea level and 413 meters below sea level? Why?** ***−86 meters and −439 meters; elicit that sea level is considered zero, and any elevation below sea level is represented by a negative number.***

➤ **What measurement is the opposite of −86 meters? Why?** ***86 meters; since −86 meters is eighty-six meters (units) below sea level (0), its opposite is eighty-six meters (units) above sea level (0).***

4. Explain that positive and negative numbers are used in everyday life. Negative numbers allow you to express a number with a value less than zero. Display the Positive and Negative Number Line page.

➤ **What do the arrows at either end of a number line represent?** ***Elicit that the number line extends infinitely in both directions with positive numbers to the right of zero and negative numbers to the left of zero.***

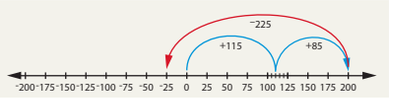
Remind the students that the counting numbers (natural numbers) and zero make the set of whole numbers. Explain that the set of whole numbers and their opposites make the set of integers. Write *negative integers* above the numbers to the left of zero and write *positive integers* above the numbers to the right of zero.

➤ **What is true about the numbers to the left of zero?** ***Elicit that they are the opposites of the numbers to the right of zero; they are all negative numbers; their values decrease as you move farther from zero (to the left).***

5. Direct attention to the −200 to 200 number line. Tell students to listen to the next situation and determine how they could use this number line to picture the changing amount in the bank account.

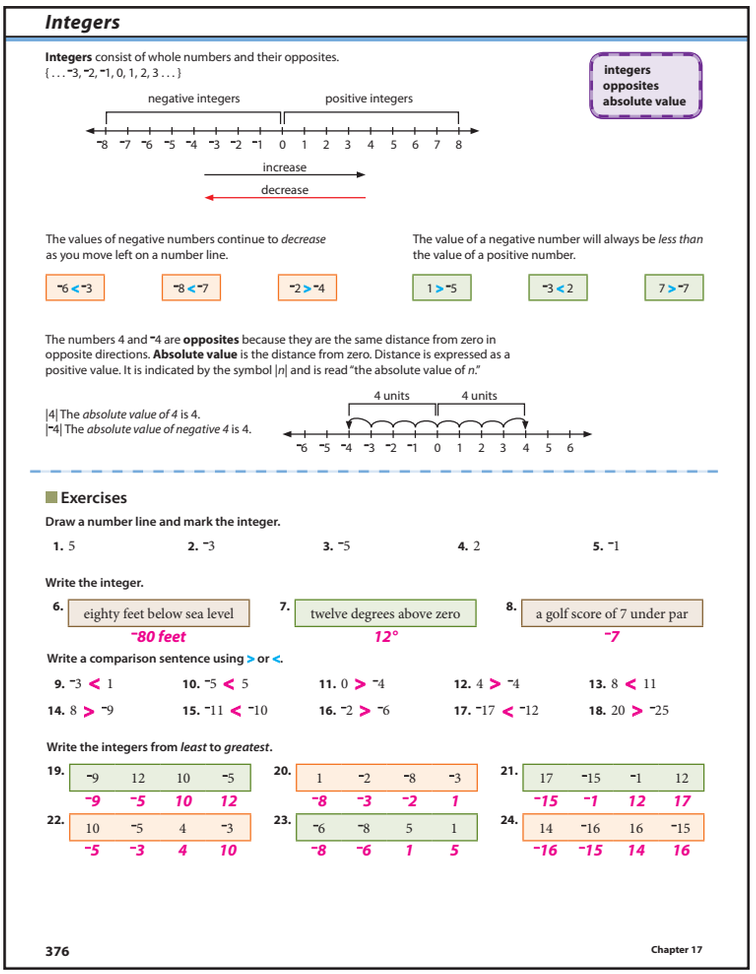
> Joe had $115 in his checking account. He deposited $85 more. Joe wants to purchase a $225 lawnmower for his business.

6. Choose students to show the positive and negative actions as jumps on the number line.



➤ **What will Joe's account balance be if he were to purchase the lawnmower? How do you know?** ***−$25; elicit that $115 + $85 − $225 = −$25***

➤ **Does Joe have enough money in his checking account to purchase the lawn mower? How do you know?** ***No; elicit that a negative balance is the amount shown to be in your account when you withdraw more than you deposit.***



Integers

Integers consist of whole numbers and their opposites.
{ . . . −3, −2, −1, 0, 1, 2, 3 . . . }

integers
opposites
absolute value

The values of negative numbers continue to *decrease* as you move left on a number line.

The value of a negative number will always be *less than* the value of a positive number.

−6 < −3 | −8 < −7 | −2 > −4 | 1 > −5 | −3 < 2 | 7 > −7

The numbers 4 and −4 are **opposites** because they are the same distance from zero in opposite directions. **Absolute value** is the distance from zero. Distance is expressed as a positive value. It is indicated by the symbol |*n*| and is read "the absolute value of *n*."

|4| The *absolute value of 4* is 4.
|−4| The *absolute value of negative 4* is 4.

Exercises

Draw a number line and mark the integer.

1. 5
2. −3
3. −5
4. 2
5. −1

Write the integer.

6. eighty feet below sea level — *−80 feet*
7. twelve degrees above zero — *12°*
8. a golf score of 7 under par — *−7*

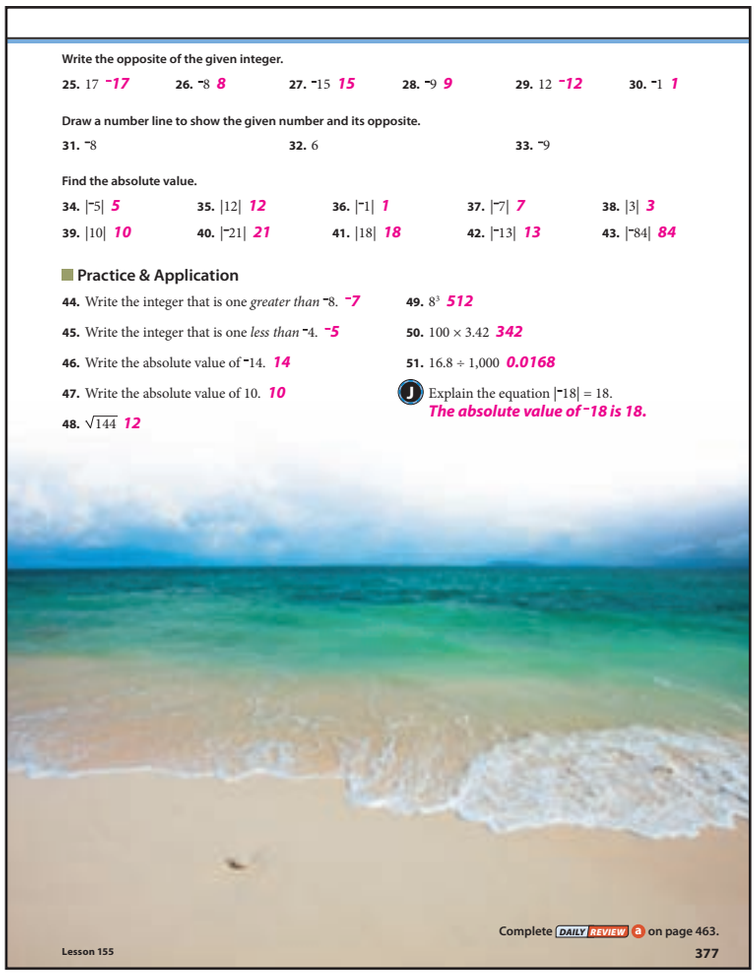
Write a comparison sentence using > or <.

9. −3 < 1
10. −5 < 5
11. 0 > −4
12. 4 > −4
13. 8 < 11
14. 8 > −9
15. −11 < −10
16. −2 > −6
17. −17 < −12
18. 20 > −25

Write the integers from *least* to *greatest*.

19. −9, 12, 10, −5 — *−9, −5, 10, 12*
20. 1, −2, −8, −3 — *−8, −3, −2, 1*
21. 17, −15, −1, 12 — *−15, −1, 12, 17*
22. 10, −5, 4, −3 — *−5, −3, 4, 10*
23. −6, −8, 5, 1 — *−8, −6, 1, 5*
24. 14, −16, 16, −15 — *−16, −15, 14, 16*

376 Chapter 17



Write the opposite of the given integer.

25. 17 *−17*
26. −8 *8*
27. −15 *15*
28. −9 *9*
29. 12 *−12*
30. −1 *1*

Draw a number line to show the given number and its opposite.

31. −8
32. 6
33. −9

Find the absolute value.

34. |−5| *5*
35. |12| *12*
36. |−1| *1*
37. |−7| *7*
38. |3| *3*
39. |10| *10*
40. |−21| *21*
41. |18| *18*
42. |−13| *13*
43. |−84| *84*

Practice & Application

44. Write the integer that is one *greater than* −8. *−7*
45. Write the integer that is one *less than* −4. *−5*
46. Write the absolute value of −14. *14*
47. Write the absolute value of 10. *10*
48. $\sqrt{144}$ *12*
49. 8^3 *512*
50. 100 × 3.42 *342*
51. 16.8 ÷ 1,000 *0.0168*

J Explain the equation |−18| = 18.
The absolute value of −18 is 18.

Complete DAILY REVIEW a on page 463.

Lesson 155 377

7. Write absolute value and explain that the absolute value of a number, *x*, is its distance from zero on the number line. It is indicated by the symbol |*x*| and is read the absolute value of x. Point out 4 and −4 on the number line and explain that they have the same absolute value. They are both four units from zero. Absolute value will always be a positive number since distance is always expressed as a positive number of units.

➤ **What type of integers always have the same absolute value?** ***opposites***

Write |6| and |−6| for display.

➤ **What is the absolute value of 6?** ***6*** **of −6?** ***6***

8. Write the following for display and direct the students to find the absolute values.

|12| *12* |−8| *8* |−1| *1* |9| *9* |−25| *25*

Compare and order integers

1. Explain that you can compare integers using the signs > and <. Display the Positive & Negative Number Line page.

➤ **What do you know about the value of the numbers as you move to the right on the number line?** ***The numbers increase in value.*** **as you move to the left on the number line?** ***The numbers decrease in value.***

2. Point out that as you move from 4 to 7 on the number line, you move in a positive direction and the value of the numbers increases; as you move from 7 to 4, you move in a negative direction and the value of the numbers decreases. Choose a student to write a comparison of 4 and 7 using the signs > and <. *7 > 4 or 4 < 7*

3. Direct the students to find 6 and −7 on their number line. Demonstrate on your number line.

➤ **Does 6 or −7 have the greater value? How do you know?** ***6; elicit that the numbers increase in value as you move to the right on the number line; therefore, a positive number always has a greater value than a negative number.***

➤ **Which number has the lesser value, −6 or −2? Why?** ***−6; elicit that the negative numbers decrease in value as you move to the left on the number line.***

Choose a student to write the comparisons using the signs > and <. *−6 < −2 and −2 > −6*

4. Follow a similar procedure to complete the following number sentences.

−1 __ 1 < −3 __ −4 > −6 __ 0 <
7 __ −7 > 0 __ 5 < 2 __ 1 >

5. Write the following sets of numbers for display. Direct the students to use the number line to order each set of numbers from least to greatest.

−2, −4, 7, −1 *−4, −2, −1, 7*
−3, −5, 5, 2 *−5, −3, 2, 5*
6, 4, −1, 2 *−1, 2, 4, 6*

6. Repeat the procedure without the number line.

−12, 8, −9, 3 *−12, −9, 3, 8*
18, −19, −20, 20 *−20, −19, 18, 20*
25, −26, 26, −25 *−26, −25, 25, 26*

Student Text pp. 376–77

Lesson 156

Student Text pp. 378–79
Daily Review p. 463b

Objectives
- Add integers using manipulatives
- Add integers using a number line
- Write an addition equation for a word problem

Teacher Materials
- Positive & Negative Number Line, page IA4 (CD)
- Algebra Mat, page IA102 (CD); color the positive sign black and the negative sign red.
- Plastic counters: opaque (to appear black when placed on the Algebra Mat) and transparent red

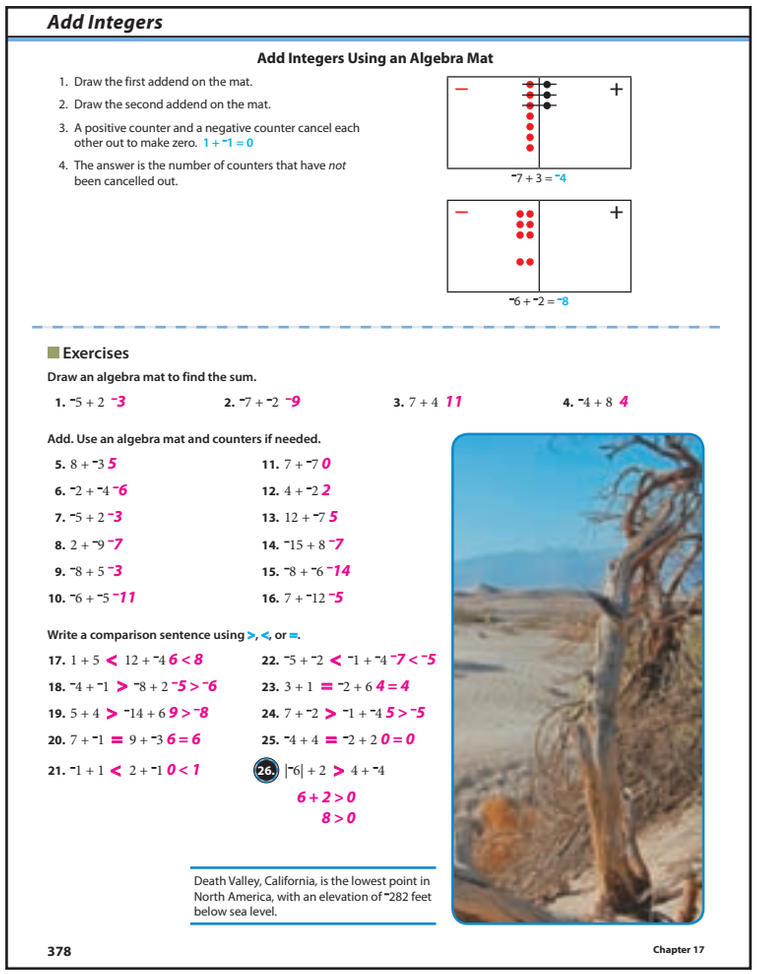
Student Materials
- Algebra Mat Kit
- Positive & Negative Number Lines, page IA103 (CD)

Note
As an alternative to using the provided Algebra Mat, you may choose to prepare a large algebra mat similar to one provided in the Student Manipulatives Packet, Math 4–6. Use white poster board for the mat and cut circles from red and black paper to use as counters.

Teach for Understanding

Add integers

1. Display the Algebra Mat and distribute the Algebra Mat Kits. Guide the students in positioning their mats so the negative side is on the left and the positive side is on the right. Explain that the Algebra Mat can be used to illustrate the addition of integers.
➤ **What are integers?** ***whole numbers and their opposites such as −3, −2, −1, 0, 1, 2, 3.***
2. Point out that the Algebra Mat has a + side for positive integers and a − side for negative integers. The black counters represent positive integers, and the red counters represent negative integers. (***Note:*** You may choose to explain that accountants often refer to a positive financial balance as being "in the black" and a negative financial balance as being "in the red.")
3. Write the equation 7 + 3 = __ for display and direct each student to display the equation on his Algebra Mat as you display it on your mat.
➤ **How could you show 7 + 3 on the mat? Why?** ***Elicit that you could place 7 black counters on the positive side of the mat and then add 3 more black counters on the positive side for a total of 10 positive counters.***
4. Guide the students in using their mats to solve these problems: 4 + 3 = ***7***; 6 + 2 = ***8***; 1 + 8 = ***9***
➤ **What is true about the sum when the addends are all positive integers?** ***Elicit that the sum is a positive integer of greater value.***
5. Display the Positive & Negative Number Line page. Explain that a number line can be used to illustrate the addition of integers by drawing arrows to represent each addend. Demonstrate finding 7 + 3 on the number line. Begin at zero and draw an arrow to the first addend (7 units to the right); then from the first addend, draw a second arrow (3 units to the right), ending at 10. (***Note:*** You may choose to draw red (negative) and black (positive) arrows as shown on Student Text page 379.)
Choose students to draw arrows above your number line to demonstrate the other problems.
6. Write the equation −3 + −8 = __ for display and direct each student to display the equation on his Algebra Mat.
➤ **How could you show −3 + −8 on the mat? Why?** ***Elicit that you could place 3 red counters on the negative side of the mat and then add 8 more red counters on the negative side for a total of 11 negative counters.***
7. Guide the students in using their mats to solve these problems: −7 + −2 = ***−9***; −5 + −3 = ***−8***; −4 + −6 = ***−10***
➤ **What is true about the sum when the addends are all negative integers?** ***Elicit that the sum is a negative integer of lesser value but of greater absolute value.***
➤ **How can you find the sum of −3 + −8 = __ on a number line?** ***Begin at 0 and move 3 units to the left, stopping at −3. Then move 8 more units to the left, ending at −11.***
Choose students to demonstrate −3 + −8 = −11 and the other addition problems, using a number line.
8. Write 1 + −1 = __ for display.
➤ **How do you solve this equation using the number line?** ***Elicit that you begin at zero and draw an arrow 1 space (unit) to the right (in a positive direction) from 0 to 1; then from the first addend, draw a second arrow 1 space (unit) to the left (in a negative direction), ending at 0.***
➤ **What does 1 + −1 equal?** ***0***
Demonstrate 1 + −1 = 0 on the number line.
9. Direct each student to display 1 + −1 = __ on his mat as you display it on your mat.
➤ **What do you think is the value of the 1 positive counter and the 1 negative counter? Why?** ***0; answers will vary, but guide the students to the conclusion that they cancel each other out.***
➤ **How do you think you can show that the 1 positive counter and the 1 negative counter cancel each other out to equal 0?** ***Accept any reasonable answers, but elicit that you can remove the pair of counters (1 red and 1 black) from the mat so that no counters remain on the mat.***
10. Demonstrate removing 1 red and 1 black counter. Remind the students that negative 1 is the opposite of positive 1 and that every number has an opposite positive or negative number, with the exception of zero, which is neither negative nor positive. Repeat the procedure to demonstrate 3 + −3 = 0, 5 + −5 = 0, and 7 + −7 = 0 on the mat.
➤ **What is true about the sum when the addends are a number and its opposite?** ***Elicit that the sum is zero.***
11. Remind the students that the Zero Principle allows you to add or subtract 0 without changing the value of an equation. Point out that when you removed a pair of counters (1 red and 1 black) from the mat, you were applying the Zero Principle by subtracting 0.
12. Write the equation −3 + 7 = __ for display and direct each student to display the equation on his mat as you display it on your mat.
➤ **How can you show −3 + 7 on the Algebra Mat?** ***Place 3 red counters on the negative side and 7 black counters on the positive side. Elicit that you remove pairs of red and black counters (1 + −1 = 0) until no more pairs remain.***
13. Demonstrate removing the pairs of counters until only 4 black counters remain. Point out that the remaining counters indicate the sum, positive 4.



Add Integers

Add Integers Using an Algebra Mat

1. Draw the first addend on the mat.
2. Draw the second addend on the mat.
3. A positive counter and a negative counter cancel each other out to make zero. 1 + −1 = 0
4. The answer is the number of counters that have *not* been cancelled out.

−7 + 3 = −4

−6 + −2 = −8

Exercises

Draw an algebra mat to find the sum.

1. −5 + 2 −3
2. −7 + −2 −9
3. 7 + 4 11
4. −4 + 8 4

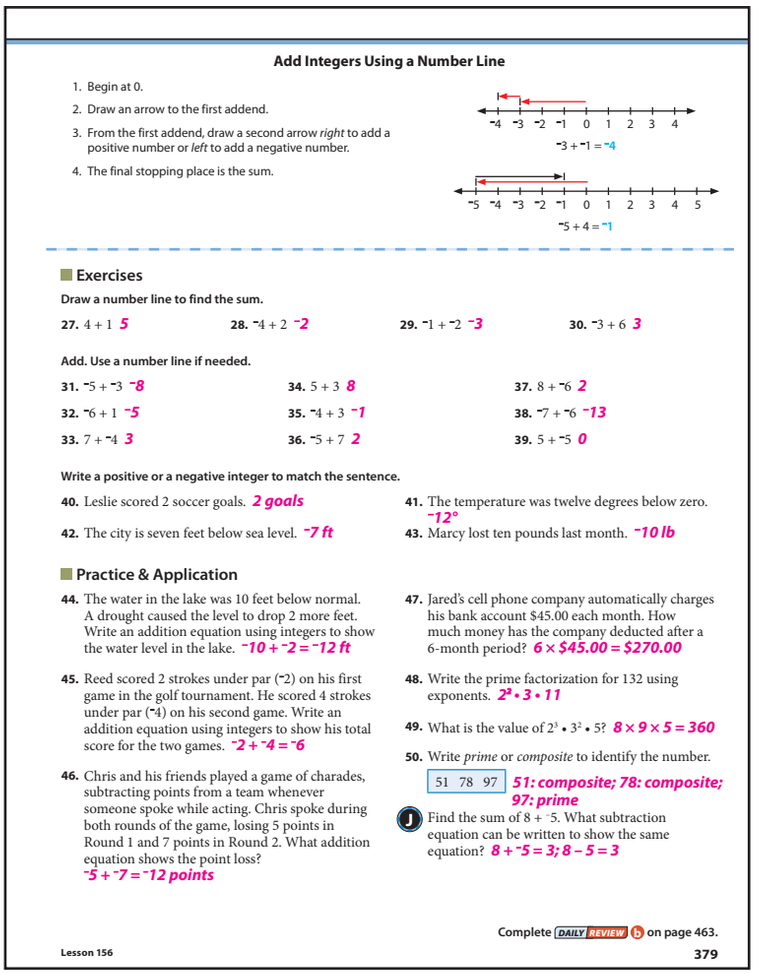
Add. Use an algebra mat and counters if needed.

5. 8 + −3 5
6. −2 + −4 −6
7. −5 + 2 −3
8. 2 + −9 −7
9. −8 + 5 −3
10. −6 + −5 −11
11. 7 + −7 0
12. 4 + −2 2
13. 12 + −7 5
14. −15 + 8 −7
15. −8 + −6 −14
16. 7 + −12 −5

Write a comparison sentence using >, <, or =.

17. 1 + 5 < 12 + −4 6 < 8
18. −4 + −1 > −8 + 2 −5 > −6
19. 5 + 4 > −14 + 6 9 > −8
20. 7 + −1 = 9 + −3 6 = 6
21. −1 + 1 < 2 + −1 0 < 1
22. −5 + −2 < −1 + −4 −7 < −5
23. 3 + 1 = −2 + 6 4 = 4
24. 7 + −2 > −1 + −4 5 > −5
25. −4 + 4 = −2 + 2 0 = 0
26. |−6| + 2 > 4 + −4 6 + 2 > 0 8 > 0

Death Valley, California, is the lowest point in North America, with an elevation of −282 feet below sea level.



Add Integers Using a Number Line

1. Begin at 0.
2. Draw an arrow to the first addend.
3. From the first addend, draw a second arrow *right* to add a positive number or *left* to add a negative number.
4. The final stopping place is the sum.

−3 + −1 = −4

−5 + 4 = −1

Exercises

Draw a number line to find the sum.

27. 4 + 1 5
28. −4 + 2 −2
29. −1 + −2 −3
30. −3 + 6 3

Add. Use a number line if needed.

31. −5 + −3 −8
32. −6 + 1 −5
33. 7 + −4 3
34. 5 + 3 8
35. −4 + 3 −1
36. −5 + 7 2
37. 8 + −6 2
38. −7 + −6 −13
39. 5 + −5 0

Write a positive or a negative integer to match the sentence.

40. Leslie scored 2 soccer goals. 2 goals
41. The temperature was twelve degrees below zero. −12°
42. The city is seven feet below sea level. −7 ft
43. Marcy lost ten pounds last month. −10 lb

Practice & Application

44. The water in the lake was 10 feet below normal. A drought caused the level to drop 2 more feet. Write an addition equation using integers to show the water level in the lake. −10 + −2 = −12 ft
45. Reed scored 2 strokes under par (−2) on his first game in the golf tournament. He scored 4 strokes under par (−4) on his second game. Write an addition equation using integers to show his total score for the two games. −2 + −4 = −6
46. Chris and his friends played a game of charades, subtracting points from a team whenever someone spoke while acting. Chris spoke during both rounds of the game, losing 5 points in Round 1 and 7 points in Round 2. What addition equation shows the point loss? −5 + −7 = −12 points
47. Jared's cell phone company automatically charges his bank account $45.00 each month. How much money has the company deducted after a 6-month period? 6 × $45.00 = $270.00
48. Write the prime factorization for 132 using exponents. $2^2 \cdot 3 \cdot 11$
49. What is the value of $2^3 \cdot 3^2 \cdot 5$? 8 × 9 × 5 = 360
50. Write *prime* or *composite* to identify the number.

51	78	97

51: composite; 78: composite; 97: prime

J Find the sum of 8 + −5. What subtraction equation can be written to show the same equation? 8 + −5 = 3; 8 − 5 = 3

Complete DAILY REVIEW b on page 463.

➤ What does −3 + 7 equal? *4*

➤ How can you show −3 + 7 on the Number Line? *Begin at 0 and draw an arrow 3 units to the left, stopping at −3. Then draw an arrow from 7 units to the right from −3 to 4.* Choose a student to demonstrate the equation on the number line.

14. Follow a similar procedure for −5 + 2 = ___, using the mat and the number line to illustrate the problem.

➤ What is the sum of −5 + 2? *−3*

15. Arrange the students in pairs. Distribute the Positive & Negative Number Lines page. Write the following equations for display. Direct one student in each pair to use the mat to solve the problem, while the other student uses the number line to solve the problem. Select students to demonstrate solving the problems. Direct the students to trade manipulatives after each problem.

    9 + −8 = *1* −2 + 1 = *−1* −4 + −3 = *−7*

    −4 + 8 = *4* −7 + 5 = *−2*

16. Follow a similar procedure to complete the following number sentences using >, <, or =.

    4 + −2 > −7 + 3 −13 + −5 < 4 + −9

    3 + −8 = −7 + 2 −6 + −4 < −12 + 3

Write an addition equation for a word problem

John competes on a school quiz team. The quiz teams receive 1 point for each question answered correctly, and the teams lose 1 point for each question answered incorrectly. John's team answered 12 questions correctly and 4 questions incorrectly. What was the team's final score? *8 points*

➤ What addition equation can you write to solve this word problem? *12 + −4 = ___* Write the equation for display.

➤ How can you solve the equation using the Number Line? *Elicit that you begin at 0 and move 12 units to the right (a positive direction) and then move 4 units to the left from 12 (a negative direction).*

1. Choose a student to demonstrate how to find the sum using a number line.

➤ What was the team's final score? *8 points* Write 8 to complete the question.

The opposing quiz team answered 9 questions correctly and 2 questions incorrectly. What was the opposing team's final score?

➤ What addition equation can you write to solve this word problem? *9 + −2 = ___*

2. Choose a student to write the equation for display and use the Algebra Mat to solve the equation.

➤ How can you use the mat to solve this equation? *Place 9 black counters on the positive side of the mat and 2 red counters on the negative side of the mat. Elicit that you can remove 2 pairs of counters, leaving 7 positive counters.*

➤ What was the opposing team's final score? *7*

➤ Who won the competition? *John's team won by 1 point.*

Student Text pp. 378–79

Lesson 157

Student Text pp. 380–81
Daily Review p. 464c

Objectives
- Subtract integers using manipulatives
- Subtract integers using a number line
- Write a subtraction equation for a word problem

Teacher Materials
- Algebra Mat (from Lesson 156)
- Positive & Negative Number Line (from Lesson 155)
- Plastic counters: opaque (black) and transparent red

Student Materials
- Algebra Mat Kit
- Positive & Negative Number Lines, page IA103 (CD)

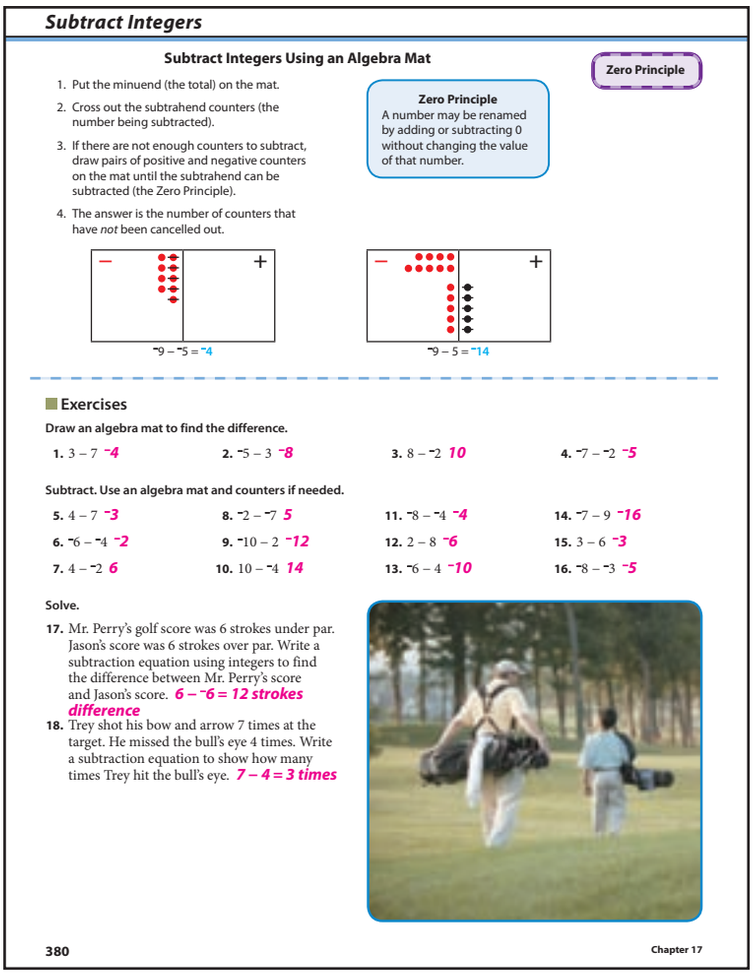
Teach for Understanding

Subtract integers using manipulatives

1. Distribute the Algebra Mat Kits and display the Algebra Mat page. Write 8 − 5 = 3 for display.
➤ **How can you show this subtraction equation using your Algebra Mat?** ***Place 8 black counters on the positive side of the mat and then remove 5 black counters so that 3 black counters remain on the positive side of the mat.***
2. Guide students in demonstrating this equation on your mat by placing counters to represent the minuend (first number), and then removing counters to represent the subtrahend (second number).
➤ **How many counters are left on your mat?** ***3***
Remind the students that the answer to a subtraction equation is called the *difference*.
3. Write $^{-}8 - {}^{-}5 =$ __ for display.
➤ **How could you solve this equation using your Algebra Mat?** ***Elicit that you place 8 red counters on the negative side of the mat and then remove 5 red counters so that 3 red counters remain on the negative side of the mat.***
Guide the students in solving the equation on their mats as you demonstrate on your mat.
➤ **What does $^{-}8 - {}^{-}5$ equal?** ***$^{-}3$*** Complete the equation.
4. Point out that the procedure for subtracting a negative number from a negative number is the same as the procedure for subtracting a positive number from a positive number. First, you place the number of counters representing the minuend on the mat, and then you remove the number of counters representing the subtrahend.
5. Write 8 − $^{-}5$ = __ for display.
➤ **How do you think you could solve this equation using your mat?** ***Elicit that you need to place 8 black counters on the positive side of the mat and remove 5 red counters from the negative side.***
Direct the students to place 8 black counters on the positive side of their mats as you do the same.
➤ **Can you remove 5 negative counters from the mat? Why?** ***No; there are no red counters on the negative side of the mat.***
6. Remind the students that the Identity Property of Addition or the Zero Principle allows you to add 0 without changing the value of the equation.
➤ **If you add 0 to the 8 counters on the mat, will the value of the counters change?** ***no***
7. Remind the students that they can "add zero" by adding the same number of counters to each side of the mat until there are enough counters to subtract the second number (subtrahend). Place 1 black counter on the positive side of the mat and 1 red counter on the negative side of the mat and direct the students to do the same.
➤ **Has the difference of the counters on the mat changed? How do you know?** ***No; elicit that the difference has not changed since there are still 5 more red counters than black counters.***
Repeat the procedure, adding 0 (1 negative counter and 1 positive counter) until 5 negative counters are on the mat.
8. Instruct the students to subtract (remove) the 5 negative counters from the mat.
➤ **What is 8 − $^{-}5$?** ***13*** Complete the equation.
9. Write $^{-}8 - 5 =$ __ for display.
➤ **How can you show this subtraction equation using the Algebra Mat?** ***Place 8 red counters on the negative side of the mat. Since there are not enough black counters to subtract 5 from the positive side of the mat, place pairs of 1 negative counter and 1 positive counter on the mat until there are 5 counters on the positive side of the mat. Then you can remove 5 black counters from the positive side of the mat.***
Demonstrate the subtraction equation on your mat as each student solves the equation using his Algebra Mat.
➤ **What is $^{-}8 - 5$?** ***$^{-}13$*** Complete the equation.
10. Write 5 − 8 = __ for display.
➤ **How do you think you could solve this equation using your mat?** ***Elicit that you can place 5 black counters on the positive side of the mat. Since there are not enough counters to subtract 8 from the positive side of the mat, you can add zero: place pairs of 1 positive and 1 negative counter on the mat until there are enough positive counters to remove 8.***
Demonstrate the subtraction equation on your mat as each student solves the equation using his Algebra Mat.
➤ **What is 5 − 8?** ***$^{-}3$*** Complete the equation.

Subtract integers using a number line

1. Display the Positive & Negative Number Line page and distribute the Positive & Negative Number Lines page to each student. Write the equation $^{-}7 - {}^{-}3 =$ __ for display.
➤ **When adding a negative number, what direction do you move on the number line?** ***negative direction (to the left)***
➤ **What direction should you move when subtracting a negative number?** ***Elicit that since subtraction is the inverse (opposite) operation of addition, you move in a positive direction (to the right) to subtract a negative number.***
➤ **Beginning at 0, how could you solve $^{-}7 - {}^{-}3$?** ***Elicit that first you move from 0 to $^{-}7$ and then move 3 units to the right (positive direction) to subtract $^{-}3$, stopping on $^{-}4$.*** Demonstrate on a number line and complete the equation.
2. Write 6 − $^{-}4$ = __ for display.
➤ **How can you solve 6 − $^{-}4$ using the Number Line?** ***Elicit that you begin at 0 and move 6 units to the right and then move 4 more units to the right (positive direction) to subtract $^{-}4$, ending on 10.***
3. Direct the students to solve the problem using a number line on the page. Choose a student to demonstrate solving the equation on a number line. Complete the equation. Remind the students that subtraction is finding the difference between two numbers.
➤ **What is the difference in the number of units between 6 and $^{-}4$ on the Number Line?** ***10*** Lead in counting the units.



Subtract Integers

Subtract Integers Using an Algebra Mat

Zero Principle

1. Put the minuend (the total) on the mat.
2. Cross out the subtrahend counters (the number being subtracted).
3. If there are not enough counters to subtract, draw pairs of positive and negative counters on the mat until the subtrahend can be subtracted (the Zero Principle).
4. The answer is the number of counters that have *not* been cancelled out.

Zero Principle
A number may be renamed by adding or subtracting 0 without changing the value of that number.

−9 − −5 = −4

−9 − 5 = −14

Exercises

Draw an algebra mat to find the difference.

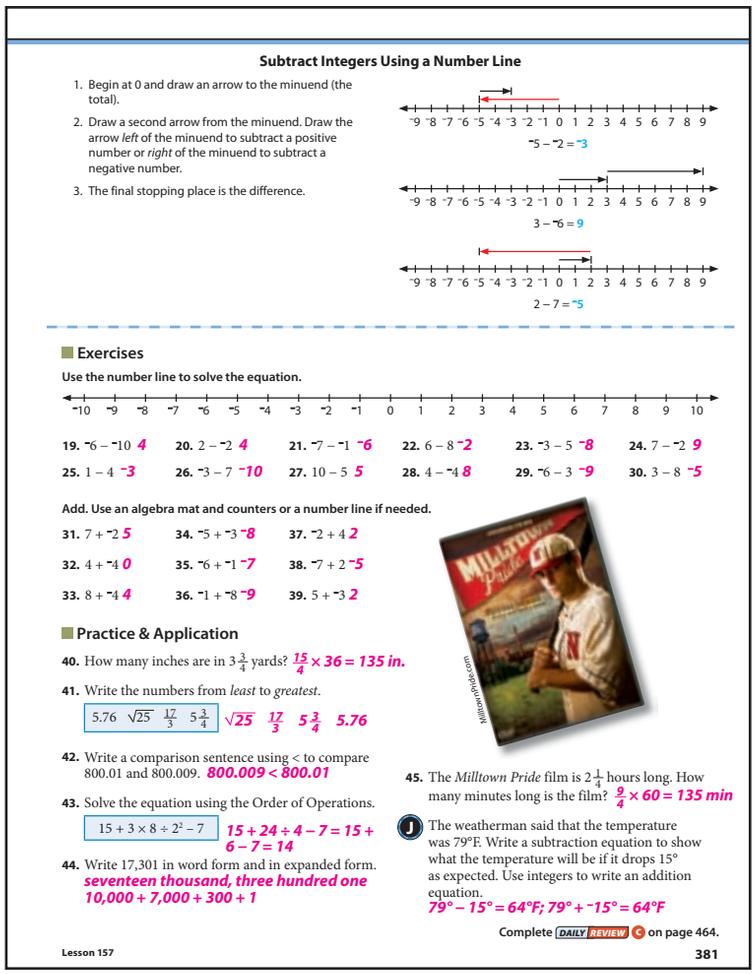
1. 3 − 7 −4
2. −5 − 3 −8
3. 8 − −2 10
4. −7 − −2 −5

Subtract. Use an algebra mat and counters if needed.

5. 4 − 7 −3
6. −6 − −4 −2
7. 4 − −2 6
8. −2 − −7 5
9. −10 − 2 −12
10. 10 − −4 14
11. −8 − −4 −4
12. 2 − 8 −6
13. −6 − 4 −10
14. −7 − 9 −16
15. 3 − 6 −3
16. −8 − −3 −5

Solve.

17. Mr. Perry's golf score was 6 strokes under par. Jason's score was 6 strokes over par. Write a subtraction equation using integers to find the difference between Mr. Perry's score and Jason's score. *6 − −6 = 12 strokes difference*
18. Trey shot his bow and arrow 7 times at the target. He missed the bull's eye 4 times. Write a subtraction equation to show how many times Trey hit the bull's eye. *7 − 4 = 3 times*



Subtract Integers Using a Number Line

1. Begin at 0 and draw an arrow to the minuend (the total).
2. Draw a second arrow from the minuend. Draw the arrow *left* of the minuend to subtract a positive number or *right* of the minuend to subtract a negative number.
3. The final stopping place is the difference.

−5 − −2 = −3

3 − −6 = 9

2 − 7 = −5

Exercises

Use the number line to solve the equation.

19. −6 − −10 4
20. 2 − −2 4
21. −7 − −1 −6
22. 6 − 8 −2
23. −3 − 5 −8
24. 7 − −2 9
25. 1 − 4 −3
26. −3 − 7 −10
27. 10 − 5 5
28. 4 − −4 8
29. −6 − 3 −9
30. 3 − 8 −5

Add. Use an algebra mat and counters or a number line if needed.

31. 7 + −2 5
32. 4 + −4 0
33. 8 + −4 4
34. −5 + −3 −8
35. −6 + −1 −7
36. −1 + −8 −9
37. −2 + 4 2
38. −7 + 2 −5
39. 5 + −3 2

Practice & Application

40. How many inches are in $3\frac{3}{4}$ yards? $\frac{15}{4} \times 36 = 135$ in.
41. Write the numbers from *least* to *greatest*.

5.76 $\sqrt{25}$ $\frac{17}{3}$ $5\frac{3}{4}$ $\sqrt{25}$ $\frac{17}{3}$ $5\frac{3}{4}$ 5.76

42. Write a comparison sentence using < to compare 800.01 and 800.009. *800.009 < 800.01*
43. Solve the equation using the Order of Operations.

$15 + 3 \times 8 \div 2^2 - 7$ *15 + 24 ÷ 4 − 7 = 15 + 6 − 7 = 14*

44. Write 17,301 in word form and in expanded form. *seventeen thousand, three hundred one 10,000 + 7,000 + 300 + 1*

45. The *Milltown Pride* film is $2\frac{1}{4}$ hours long. How many minutes long is the film? $\frac{9}{4} \times 60 = 135$ min

J The weatherman said that the temperature was 79°F. Write a subtraction equation to show what the temperature will be if it drops 15° as expected. Use integers to write an addition equation.
79° − 15° = 64°F; 79° + −15° = 64°F

Complete DAILY REVIEW C on page 464.

4. Explain that in the equation, $-7 - -3 = -4$, you moved 7 units to the left of 0 to locate −7. Then you moved 3 units to the right (positive direction), ending on −4. Remind the students that since subtraction is the inverse (opposite) operation of addition, you moved in this positive direction (to the right) to subtract a negative number. Likewise, point out that in the equation $6 - -4 = 10$, you moved to the right of 0 to locate 6. Then you moved to the right (positive direction) 4 more units, ending on 10. Point out that in both these equations you moved in a positive direction (to the right) to subtract a negative number.
5. Write 8 − 2 = ___ for display. Direct the students to solve using a number line on the page. Choose a student to demonstrate solving this equation on a number line. Instruct him to explain the process. *Begin at 0, move 8 units to the right, and then move 2 units to the left, stopping on 6.* Complete the equation.
6. Write 4 − 7 = ___ for display. Direct the students to solve the problem using a number line on the page.
   - ➤ **How can you solve this equation?** *Elicit that you begin at 0, move 4 units to the right, and then move 7 units to the left, stopping on −3.*
   - ➤ **What is 4 − 7?** *−3* Complete the equation.

   Point out that in the equation $4 - 7 = -3$, you moved right from 0 to 4, and then you moved to the left (negative direction) to subtract a positive number, since subtracting a positive number is the inverse (opposite) of adding a positive number.
7. Follow a similar procedure for −3 − 6 = ___. *Begin at 0, move 3 units to the left (a negative direction) and then move 6 units to the left (a negative direction) to subtract a positive 6, stopping on −9.*

Write an equation for a word problem

Sarah's score was −5. How many points must she score consecutively to reach a final score of 0?

- ➤ **What subtraction equation can you write for this problem? Why?** *0 − −5 = ___; subtraction is used to find the difference between two numbers* Write 0 − −5 = ___ for display.
- ➤ **How can you show 0 − −5 using the Number Line?** *Begin at zero and move 5 units right (positive direction) to subtract −5.*
- ➤ **How many points must Sarah score to reach a final score of 0?** *5 points* Write 5 to complete the equation.
- ➤ **What missing addend equation can you write for this problem?** *−5 + ___ = 0* Write −5 + ___ = 0 for display.
- ➤ **How can you show this problem using the Algebra Mat?** *Place 5 counters on the negative side of the mat; determine that adding 5 counters to the positive side of the mat will make zero (−5 + 5 = 0).*
- ➤ **What is the missing addend? How do you know?** *5; an integer and its opposite will always equal 0.*
- ➤ **How can you show the problem using the Number Line?** *Possible answer: begin at −5, and determine that you need to move 5 units right (positive 5) to arrive at 0.*
- ➤ **What conclusion can you reach about subtracting a number and adding its opposite?** *Elicit that subtracting a number is the same as adding its opposite.*

Student Text pp. 380–81

Lesson 158

Student Text pp. 382–83
Daily Review p. 464d

Objectives
- Subtract integers
- Add and subtract integers using a number line and manipulatives
- Demonstrate an understanding of the relationship between subtracting an integer and adding its opposite
- Write an equation for a word problem

Teacher Materials
- Subtraction Patterns, page IA104 (CD)
- More Subtraction Patterns, page IA105 (CD)
- Positive & Negative Number Line (from Lesson 155)
- Algebra Mat (from Lesson 156)
- Plastic counters: opaque (black) and transparent red

Student Materials
- Algebra Mat Kit
- Subtraction Patterns, page IA104 (CD) for each student

Teach for Understanding

Subtract integers

1. Display and distribute the Subtraction Patterns page. Review the steps at the top of the page on how to use a number line to subtract or add integers. Point out the first number line and the arrows above the number line.

➤ **How do the arrows above the number line illustrate 6 − 4?** ***Elicit that you began at 0 and moved 6 units to the right (positive direction) stopping at 6; from there you moved 4 units to the left (negative direction) since subtracting a positive number is the inverse (opposite) of adding a positive number. The final stopping place of 2 illustrates that 6 − 4 = 2.***

Write 2 to complete the equation and direct the students to do the same.

2. Direct the students to study the number line and arrows carefully since they also demonstrate another equation besides 6 − 4 = 2.

➤ **What addition equation do you think this number line illustrates? Why?** ***6 + −4 = 2; you began at 0 and moved right 6 units, indicating a positive 6; then you moved left 4 units, indicating that you added −4 to the 6. The final stopping place of 2 illustrates that 6 + −4 = 2.***

Write the new equation below the number line and direct the students to do the same.

3. Direct attention to the second number line and guide the students as they draw arrows to illustrate the equation −4 − −8 = ___.

➤ **How can you illustrate the minuend on the number line?** ***Elicit that you begin at 0 and move 4 units to the left (negative direction) to −4.***

➤ **How can you illustrate subtracting the subtrahend (−8) from the minuend (−4) on the number line?** ***Elicit that from −4 you move 8 units to the right (the opposite of a negative direction) ending at 4.***

➤ **What is −4 − −8?** ***4*** Complete the equation and direct the students to do the same.

➤ **What addition equation does this number line illustrate? Why?** ***−4 + 8 = 4; you began at 0 and moved 4 units (spaces) to the left (negative direction), to −4; from there you moved 8 units to the right (positive direction), ending at 4 to illustrate that −4 + 8 = 4.***

Write the new equation below the number line and direct the students to do the same.

4. Instruct the students to use the third number line to illustrate the equation 3 − −5 = ___. Guide the students as they draw arrows on their worksheet and demonstrate on the page.

➤ **How can you illustrate this equation on the number line?** ***Elicit that you draw an arrow 3 units long to the right from 0 to 3, and from there you draw another arrow 5 units long to the right, stopping on 8.***

➤ **Why did you move 5 units to the right?** ***Elicit that you would move left to add −5, so you moved the opposite direction to subtract −5.***

➤ **What is 3 − −5?** ***8*** Complete the equation and direct the students to do the same.

➤ **Do the arrows on this number line illustrate another equation? Which one?** ***Yes; 3 + 5 = 8.***

Direct the students to write this new equation below the number line as you do the same.

➤ **What is the same in these two equations: 3 − −5 = 8 and 3 + 5 = 8?** ***Elicit that the first number and the last numbers are the same (where you began and where you finished. Elicit further that to get from one point on the number line to a second point, the same action had to be performed.***

➤ **What action was the same that appears different in these two equations: 3 − −5 = 8 and 3 + 5 = 8?** ***Elicit that moving right to add an integer and moving right to subtract that integer's opposite is the same action.***

5. Instruct the students to use the fourth number line to illustrate the equation 2 − 6 = ___.

➤ **How can you illustrate this equation on the number line?** ***Elicit that you move right from 0 to 2, and then from 2 you move left 6 units to subtract a positive six, stopping on −4.***

➤ **What is 2 − 6?** ***−4*** Complete the equation and direct the students to do the same.

➤ **What other equation does this number line illustrate? Why?** ***2 + −6 = −4; you begin at 0 and move right, indicating a positive 2; then you move from there 6 units to the left to add a negative six. The final stopping place of −4 illustrates that 2 + −6 = −4.***

➤ **What pattern do you see in each pair of written equations?** ***Elicit that the operation sign changed to its inverse, and the integer following the operation sign changed to its opposite.*** **How are the subtraction equations related to the addition equations?** ***Elicit that subtracting an integer gives the same result as adding its opposite.***

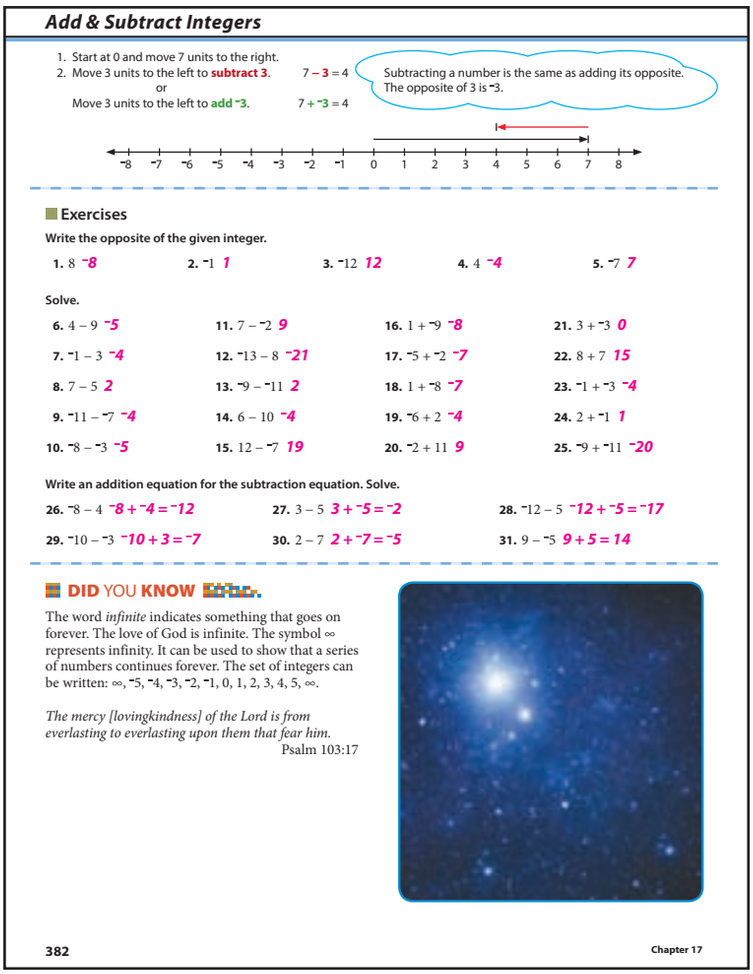
6. Display the More Subtraction Patterns page. Direct the students to study the number patterns to find the missing numbers needed to complete the equations. ***6 − −2 = 8 and 6 + 2 = 8; 5 − 6 = −1 and 5 + −6 = −1***

➤ **What new rule have you discovered?** ***Elicit that subtracting an integer is the same as adding its opposite.***

Write an equation for a word problem

> Sam's team has a score of 5 points. Jason's team has a score of −3 points. What is the difference between the scores? How many points separate their scores?

1. Display the Positive and Negative Number Line page. Choose a student to place dots on the number line to mark Sam's team score of 5 points and Jason's team score of −3 points.



Add & Subtract Integers

1. Start at 0 and move 7 units to the right.
2. Move 3 units to the left to **subtract 3**.
   or
   Move 3 units to the left to **add −3**.

$7 - 3 = 4$

$7 + {-3} = 4$

Subtracting a number is the same as adding its opposite. The opposite of 3 is −3.

Exercises

Write the opposite of the given integer.

1. 8 **−8**
2. −1 **1**
3. −12 **12**
4. 4 **−4**
5. −7 **7**

Solve.

6. $4 - 9$ **−5**
7. $-1 - 3$ **−4**
8. $7 - 5$ **2**
9. $-11 - -7$ **−4**
10. $-8 - -3$ **−5**
11. $7 - -2$ **9**
12. $-13 - 8$ **−21**
13. $-9 - -11$ **2**
14. $6 - 10$ **−4**
15. $12 - -7$ **19**
16. $1 + -9$ **−8**
17. $-5 + -2$ **−7**
18. $1 + -8$ **−7**
19. $-6 + 2$ **−4**
20. $-2 + 11$ **9**
21. $3 + -3$ **0**
22. $8 + 7$ **15**
23. $-1 + -3$ **−4**
24. $2 + -1$ **1**
25. $-9 + -11$ **−20**

Write an addition equation for the subtraction equation. Solve.

26. $-8 - 4$ **$-8 + -4 = -12$**
27. $3 - 5$ **$3 + -5 = -2$**
28. $-12 - 5$ **$-12 + -5 = -17$**
29. $-10 - -3$ **$-10 + 3 = -7$**
30. $2 - 7$ **$2 + -7 = -5$**
31. $9 - -5$ **$9 + 5 = 14$**

DID YOU KNOW

The word *infinite* indicates something that goes on forever. The love of God is infinite. The symbol ∞ represents infinity. It can be used to show that a series of numbers continues forever. The set of integers can be written: ∞, −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, ∞.

The mercy [lovingkindness] of the Lord is from everlasting to everlasting upon them that fear him.
Psalm 103:17

Write a comparison sentence using >, <, or =.

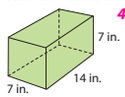
32. $3 + -1$ **>** $2 - 3$ **$2 > -1$**
33. $-8 + -2$ **<** $5 - 5$ **$-10 < 0$**
34. $3 - 4$ **>** $2 - 4$ **$-1 > -2$**
35. $-5 - 2$ **=** $-3 + -4$ **$-7 = -7$**
36. $1 - 6$ **>** $5 + -12$ **$-5 > -7$**
37. $-1 - 1$ **<** $4 + -4$ **$-2 < 0$**

Practice & Application

38. In three days a certain stock on the stock market lost 4 points, gained 2 points, and then lost 7 points. What was the total change in three days?
$-4 + 2 - 7 = -9$ points or $-4 + 2 + -7 = -9$ points

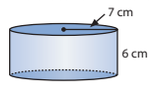
39. Stacie attended a coin show. She sold 10 of her duplicate coins in order to buy 2 coins she did not have in her collection. If Stacie began with 25 coins, how many does she have now?
$25 - 10 + 2 = 17$ coins or $25 + -10 + 2 = 17$ coins

40. Find the surface area of the rectangular prism.



$4(14 \text{ in.} \times 7 \text{ in.}) = 392 \text{ in.}^2$
$2(7 \text{ in.} \times 7 \text{ in.}) = 98 \text{ in.}^2$
Surface Area = 490 in.²

41. Find the surface area of the cylinder.



$2 \cdot 3.14 \cdot (7 \text{ cm})^2 = 307.72 \text{ cm}^2$
$(2 \cdot 3.14 \cdot 7 \text{ cm}) \cdot 6 \text{ cm} = 263.76 \text{ cm}^2$
Surface Area = 571.48 cm²

42. Find the volume of the rectangular prism.

10 cm × 4 cm × 6 cm = 240 cm³

43. Find the volume of the triangular prism.

V = Bh
$V = [\frac{1}{2}(12 \text{ in.} \times 8 \text{ in.})] \times 4 \text{ in.}$
$V = 48 \text{ in.} \times 4 \text{ in.}$
$V = 192 \text{ in.}^3$

44. Solve the equation $c \div 21 = 250$. **$c = 5{,}250$**

45. Find the product of 205×36. **7,380**

46. Write 9,843.067 in expanded form using multiplication and powers of 10.

47. Find the quotient of $1\frac{7}{8} \div \frac{3}{4}$. **$\frac{15}{8} \times \frac{4}{3} = \frac{5}{2} = 2\frac{1}{2}$**

J Explain the statement *The subtraction of an integer is the same as adding its opposite.* Use the equation $-10 - 6 = -16$. **$-10 - 6 = -16$ is the same as $-10 + -6 = -16$**

46. $(9 \times 10^4) + (8 \times 10^3) + (4 \times 10^1) + (3 \times 10^0) + (6 \times \frac{1}{10^2}) + (7 \times \frac{1}{10^3})$

Complete DAILY REVIEW d on page 464.

➤ **What is the difference between the two teams' scores? How do you know?** ***8 points; you can count the number of units that separate the two points.***

2. Remind students that the absolute value of a number is its distance from zero on the number line. Since distances are always positive (or zero), the absolute value of any number is never negative.

➤ **How many units is 5 from 0 on the number line?** ***5 units*** **−3 from 0?** ***3 units***

➤ **What equation can you write to find the difference between these two numbers? Why?** ***5 − −3 = ___; you always subtract the lesser number from the greater number to find the difference.***

Write the equation for display.

➤ **What do you get when you subtract a greater number from a lesser number?** ***elicit a negative number***

Elicit that $-3 - 5 = -8$. Point out that the difference of −8 still indicates that 8 units separate the scores, since distances are expressed as an absolute value: $|8|$ or $|-8|$ equals 8.

3. Direct the students to draw a similar number line on their own paper to the one displayed to illustrate $5 - -3 = 8$.

➤ **How can you solve this equation on a number line?** ***Elicit that you draw an arrow 5 units long to the right (positive direction) from 0 to 5, and from there you draw another arrow 3 units long from 5 to 8 (positive direction to indicate subtracting a negative 3). The final stopping place of 8 illustrates that $5 - -3 = 8$.***

Choose a student to demonstrate on the Positive & Negative Number Line page.

4. Display your Algebra Mat. Direct the students to draw a similar algebra mat on their own paper and then to draw counters to illustrate $5 - -3 = 8$.

➤ **How can you solve this problem using your Algebra Mat?** ***Elicit that you place 5 black counters on the positive side of the mat. Since there are no negative counters to subtract, you must add pairs of 1 negative counter and 1 positive counter until there are 3 negative counters to subtract or take away.*** (***Note:*** The students will need to draw lines through the pairs of counters, rather than remove them $(1 + -1 = 0)$.)

Choose a student to demonstrate on your mat.

➤ **What is left on the mat?** ***8 positive counters***

➤ **What addition equation can you write for $5 - -3 = \_\_$ using the new rule that you discovered in this lesson?** ***$5 + 3 = 8$; add the opposite***

➤ **What is the difference between the scores in this word problem?** ***8 points*** Complete the equation.

5. Follow a similar procedure for the following word problem.

> The temperature was 6° F at 8 PM. By midnight the temperature had dropped 10 degrees. What was the temperature at midnight? ***$6°F - 10°F = -4°F$; $6°F + -10°F = -4°F$***

Student Text pp. 382–83

(***Note***: *Assessment* available on Teacher's Toolkit CD.)

Lesson 159

Student Text pp. 384–85
Daily Review p. 465e

Objectives
- Solve real-life word problems
- Add and subtract integers
- Write an equation for a word problem

Teacher Materials
- Thermometer
- Red Strip
- Positive & Negative Number Line (from Lesson 155)
- Christian Worldview Shaping, pages 37–39 (CD)

Student Materials
- Thermometer
- Red Strip
- Christian Worldview Shaping, page 38 (CD)

Teach for Understanding

Solve real-life word problems

1. Display the Positive & Negative Number Line.

➤ **What is an integer?** ***Elicit that integers include the whole numbers (0, 1, 2, 3, . . .) and their opposites.***
➤ **What is the opposite of 3?** ***−3*** **12?** ***−12*** **−23?** ***23***
➤ **What does it mean when 3 is the opposite of −3?** ***The numbers are the same distance from zero on the number line but in opposite directions. The opposite of a negative number is a positive number, and the opposite of a positive number is a negative number.***
➤ **Which integers are to the right of 0?** ***positive integers*** **to the left of 0?** ***negative integers***
➤ **Is 0 negative or positive?** ***Elicit that it is neither positive nor negative; it has no value.***
➤ **How do negative numbers compare to 0?** ***They are less than 0.*** **How do the positive numbers compare to 0?** ***They are greater than 0.***
➤ **What do the arrows at the ends of the number line tell you about the set of integers?** ***Elicit that even as the set of whole numbers has infinite growth possibilities, the set of integers has infinite growing and decreasing possibilities.***

2. Display and distribute the Thermometers. Following each problem, demonstrate the equation on your Thermometer.

➤ **What do the integers on a thermometer represent?** ***Elicit that 0° is an absence of temperature; positive integers represent temperatures greater than or warmer than 0°, while negative integers represent temperatures less than or colder than 0°.***

> At 5 AM the temperature was −1°F. By 10 AM the temperature had risen 6 degrees. What was the temperature at 10 AM?

3. Direct the students to show the change in temperature on their Thermometers. ***Begin at −1°F then proceed up 6 degrees in a positive or warmer direction, stopping at 5°F.***

➤ **What was the temperature at 10 AM?** ***5°F***
➤ **What equation can you write for this problem?** ***−1°F + 6°F = 5°F*** Choose a student to write the equation.

> At 8:30 PM the temperature was 8°C. By 2 AM the next morning the temperature had dropped 11 degrees. What was the temperature at 2 AM?

4. Direct the students to show the change in temperature on their Thermometers. ***Begin at 8°C and then proceed down 11 degrees in a negative or colder direction, ending at −3°C.***

➤ **What was the temperature at 2 AM?** ***5°F***
➤ **What subtraction equation can you write for this problem?** ***8°C − 11°C = −3°C*** **What addition equation can you write for this problem?** ***8°C + −11°C = −3°C*** Choose a student to write the equations.

5. Explain that the set of integers are used in everyday tasks for recording information that is above or below 0; these tasks include recording temperature, financial balances, golf scores, and depths of the ocean.

> Nolan's checking account balance is −$5.00. If he deposits $10.00, what will his account balance be?

6. Direct the students to draw a simple number line using arrows to show the change in the checking account balance. ***Begin at 0 and move left 5 units (dollars) in a negative direction to −5. From −5 move right 10 units (dollars) in a positive direction, stopping at 5.*** **(Note:** ***Some students may draw vertical number lines moving up and down rather than right and left.)***

➤ **What will Nolan's account balance be if he makes the $10.00 deposit?** ***$5.00***
➤ **What equation can you write for this problem?** ***−$5.00 + $10.00 = $5.00*** Choose a student to write the equation.

Demonstrate on the Positive & Negative Number Line page. (***Note:*** Rotating the horizontal number line $\frac{1}{4}$ turn left will show the movement on a vertical number line.)

➤ **If Nolan's account balance is $5.00, can he afford to make a purchase greater than $5.00?** ***no*** **What would his bank balance read if he made a $7.00 purchase with only $5.00 in his account?** ***−$2.00*** **What does a negative bank balance indicate?** ***Elicit that a negative bank balance indicates that you have spent more money than you have, so you are now in debt or owe that amount to the bank.***

7. **Christian Worldview Shaping (CD)**

> Jake went scuba diving. He was 20 feet below the surface of the water when he decided to dive down 6 more feet. What is Jake's elevation?

8. Direct the students to draw a simple number line using arrows to show the change in Jake's depth. ***Begin at 0 and move left 20 units (feet) in a negative direction to −20. From −20 move left 6 units (feet) in a negative (deeper) direction stopping at −26.*** **(Note:** ***Some students may draw vertical number lines moving up and down rather than right and left.)***

➤ **What is Jake's elevation?** ***26 feet below sea level*** **Why does the number line read −26 feet?** ***Elicit that the negative sign indicates that the 26 feet are below sea level (0).***

9. Direct the students to write a subtraction equation and an addition equation for this problem. Choose a student to write the equations for display. ***−20 feet − 6 feet = −26 feet; −20 feet + −6 feet = −26 feet***

10. Remind the students that it is possible to have a negative score in a game in which points are taken away.

➤ **What would cause you to have a negative score in such a game?** ***Elicit that you would have had more points taken away from you than the number of points awarded to you.***

Exercises

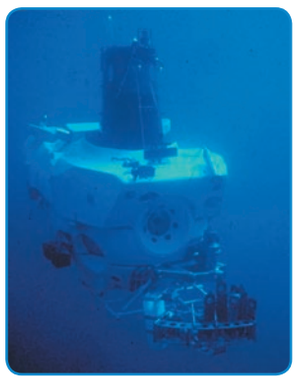
Use the data from the chart to find the answer.

Temperature Readings January 5	
Time	Temperature
6:30 AM	−10°F
7:30 AM	−7°F
8:30 AM	
9:30 AM	
10:30 AM	3°F
11:30 AM	8°F
12:30 PM	10°F
1:30 PM	12°F
2:30 PM	12°F
3:30 PM	10°F
4:30 PM	5°F
5:30 PM	0°F
6:30 PM	−2°F

1. At 6:30 AM the temperature was −10°F. Two hours later the temperature had risen 8 degrees. What was the temperature at 8:30 AM?
   −10° + 8° = −2°F
2. From 6:30 AM to 9:30 AM there was a rise in temperature of 10°. What was the temperature at 9:30 AM? *−10° + 10° = 0°F*
3. What was the difference in temperature from 12:30 PM to 6:30 PM? *10° − −2° = 12°*
4. At 2:30 PM the temperature was 12°F. How much did the temperature decrease by 6:30 PM?
   12° − −2° = 14°
5. Find the difference between the highest and the lowest temperatures recorded for January 5.
   12° − −10° = 22° difference

Solve.

6. Chad's golf score was 2 under par through the 16th hole. On the 17th hole, he shot 2 over par. What was Chad's net score?
   par or zero; −2 + 2 = 0
7. A football team lost 10 yards on their first down of the third quarter. They gained 5 yards on their next down. What was their total loss or gain?
   −10 + 5 = −5 yards; 5 yard loss
8. Ashley had 17 beads. She lost 3 beads and used 8 beads to make a bracelet. How many beads did Ashley have left? *17 − 3 − 8 = 6 beads*
9. A scuba diver went 25 feet below the surface of the water. If he swims up 8 feet, what will his depth be?
   −25 + 8 = −17; 17 feet below sea level



Alvin is the deep-sea submersible that carried three men 2.5 miles below sea level to the sunken *Titanic*.

Write a comparison sentence using > or <.

10. −3°F > −5°F
11. −23°F < 23°F
12. 0°F > −1°F
13. −1°C > −3°C
14. −18°C < 18°C
15. −2°C < −1°C
16. 0°F < 1°F
17. 5°C > 4°C
18. −1°F < 1°F

Practice & Application

19. Margo had $100 in her checking account. She made an error and wrote a check for $125. What was her balance after the check was written? What is her debt? *$100 − $125 = −$25; Margo's debt is $25*
20. If Margo's account requires a minimum balance of $25.00 at all times, how much money must she deposit to reach her minimum balance?
    $25.00 − −$25.00 = $50.00
21. In one possession a football team gained 8 yards, lost 4 yards, lost 3 yards, and gained 2 yards. What was the total loss or gain? *8 − 4 − 3 + 2 = 3-yd gain or 8 + −4 + −3 + 2 = 3-yd gain*
22. Brett's team scored 15 points and then lost 35. Hudson's team lost 45 points and then gained 30. Whose team won? By how many points did they win?
23. Rachel's baby sister weighed 8 pounds when she was born. When she came home from the hospital, she lost 2 pounds. At her first week check-up, she had gained back 1 pound. How much does Rachel's sister weigh now?
    8 − 2 + 1 = 7 lb or 8 + −2 + 1 = 7 lb
24. Evan played golf while on vacation. On his first game he scored 5 strokes over par, which is written as +5 on his score card. On his second game he scored 1 stroke under par, which is written as −1 on his score card. What is the difference between his two scores?
    +5 − −1 = 6 strokes
25. If there is a 40% chance of rain tomorrow, what is the chance of it *not* raining? *60%*
26. A spinner is divided into 5 equal sections. There are 2 green sections, 2 red sections, and 1 yellow section. Write a ratio to show the probability of landing on a green section. *P(green) = $\frac{2}{5}$*
27. Using the spinner from problem 26, does *certain*, *equally likely*, or *impossible* best describe the chance of landing on a blue section?
    impossible
28. Find the value of *n* to complete the proportion.
    $\frac{15}{51} = \frac{n}{17}$ *n = 5*
29. The ratio $\frac{12}{96}$ represents $\frac{\text{tables}}{\text{chairs}}$. Write the ratio in lowest terms and explain its terms. *$\frac{1}{8}$; there are 8 chairs for each table.*
30. Workmen were repairing the elevator at the building where Jesse makes deliveries. Jesse used the stairs to deliver a package to Mr. Samuels on Floor 31. Mr. Samuels gave him a package to deliver to Miss Murphy. Jesse descended 18 floors to Miss Murphy's office. She gave him a package to deliver to Mr. Hays, whose office is 22 floors above hers. After delivering Mr. Hays's package, Jesse took a break in the employee lounge on Floor 5. How many floors did he descend to get to the lounge? *31 − 18 = 13; 13 + 22 = 35; 35 − 5 = 30 floors*

20. *−$25.00 + n = $25.00*
    −$25.00 + $25.00 + n = $25.00 + $25.00
    n = $50.00
22. *Brett: 15 − 35 = −20 points;*
    Hudson: −45 + 30 = −15 points;
    Hudson; −15 − −20 = 5 points

Complete DAILY REVIEW e on page 465.

Carl scored 21 points, and then he lost 23 points during the course of the game. What is his score?

11. Direct the students to draw a simple number line using arrows to show the change in Carl's score. *Begin at 0 and move right 21 units (points) in a positive direction to 21. From 21 move left 23 units (points) in a negative direction, stopping at −2.* (***Note:*** Some students may draw vertical number lines to move up and down rather than right and left.)
    ➤ **What is Carl's score?** *−2 points*
12. Direct the students to write a subtraction equation and an addition equation for this problem. Choose a student to write the equations for display. *21 points − 23 points = −2 points; 21 points + −23 points = −2 points*

Student Text pp. 384–85

Lesson 160

Student Text pp. 386–87
Daily Review p. 465f

Objectives
- Multiply integers using manipulatives
- Write an equation for a word problem

Teacher Materials
- Algebra Mat (from Lesson 156)
- Multiplication Patterns, page IA106 (CD)
- Plastic counters: opaque (black) and transparent red

Student Materials
- Algebra Mat Kit

Teach for Understanding

Multiply positive and negative numbers using manipulatives

1. Write 4 + 4 + 4 = __ for display. Display the Algebra Mat and distribute the Algebra Mat Kits. Direct the students to show this equation by placing rows of counters on their mats. *3 rows of 4 black counters on the positive side of the mat* Demonstrate on yours.
   - ➤ **What is 4 + 4 + 4?** *12* Write 12 to complete the equation.
   - ➤ **How many rows of 4 counters are shown?** *3*
   - ➤ **What multiplication equation can you write for 3 sets (rows) of 4?** *3 × 4 = 12*

   Write the equations 3 × 4 = 12 and 3(4) = 12 for display. Point out that the mat shows both the repeated addition equation and the multiplication equation.
   - ➤ **What related fact does the Commutative Property of Multiplication allow you to write? Why?** *4 × 3 = 12; the order of the factors does not change the product.*
   - ➤ **How does changing the order of the factors change the picture?** *4 rows of 3 counters*

   (*Note:* You may choose to rotate the pictured 3 × 4 array to show the 4 × 3 array.)

   Write 4 × 3 = 12 and 4(3) = 12 for display.
2. Write −4 + −4 + −4 = __ for display. Direct the students to show this equation by placing rows of counters on their mats. *3 rows of 4 counters on the positive side of the mat* Choose a student to demonstrate on your mat.
   - ➤ **What is −4 + −4 + −4?** *−12* Write −12 to complete the equation.
   - ➤ **How many rows of −4 counters are shown?** *3*
   - ➤ **What multiplication equation can you write for 3 sets (rows) of 4?** *3 × −4 = −12*

   Write the equations 3 × −4 = −12 and 3(−4) = −12 for display. Point out that the mat shows both the repeated addition equation and the multiplication equation.
   - ➤ **What related fact does the Commutative Property of Multiplication allow you to write? Why?** *−4 × 3 = 12; the order of the factors does not affect the product.*

   Write −4 × 3 = −12 and −4(3) = −12 for display.
3. Write −3 × −4 =__ and −3(−4) =__; −4 × −3 = __ and −4(−3) = __

   Guide the students to the conclusion that a picture of negative sets of counters cannot be shown.
   - ➤ **What do you know about 3 and −3?** *Elicit that these numbers are opposites.*
   - ➤ **If 3 sets of −4 = −12, what do you think the opposite of 3 sets (−3 sets) of −4 would be?** *12; the opposite of −12*
   - ➤ **If 4 sets of −3 = −12, what do you think the opposite of 4 sets (−4 sets) of −3 would be?** *12*

   Write 12 to complete the equations.
4. Display the Multiplication Patterns page. Direct the students to study the first list of equations to find the pattern of multiplying a negative number and a positive number. Instruct them to use the pattern to complete the last 2 equations.
   - ➤ **What is −3 × 3?** *−9* **What is −4 × 3?** *−12*
   - ➤ **What pattern do you see in this list of equations?** *Decreasing the first factor by 1 decreases the product by 3. To continue the pattern, the last 2 products must be negative.*
   - ➤ **What property could also help you find −3 sets of 3? Why?** *Commutative; 3 × −3 = 9.*
5. Direct the students to study the second list of equations to find the pattern of multiplying a negative number by a negative number. Instruct them to use the pattern to complete the last 2 equations.
   - ➤ **What is −4 × −3?** *12* **What is −4 × −4?** *16*
   - ➤ **What pattern do you see in this list of equations?** *As the second factor decreases by 1, the product increases by 4. To continue the pattern, the last 2 products must be positive.*
   - ➤ **What do you notice about the product when the factors have the same sign?** *Elicit that the product is positive.* **When the factors have different signs?** *Elicit that the product is negative.*

Write an equation for a word problem

The value of stock in Davis Lumber Company dropped 12 points each day this week. What was the change in stock points at the end of the 5-day week? *−60 points*

- ➤ **What integer represents the change in stock points each day?** *−12*
- ➤ **How many times (days) was this same change in stock points repeated? How do you know?** *5; it was a 5-day week, and the stock fell 12 points each day this week.*
- ➤ **What multiplication equation can you write for this problem? Why?** *5 × −12 = __; there were 5 days of a loss of 12 points (5 sets of −12 points).*

1. Direct the students to show 5 × −12 on the mat and/or number line. *Place 5 sets (rows) of 12 counters on the negative side of the mat for a total of −60; begin at 0 and move 12 units (points) to the left; repeat the movement of 12 units left four more times, stopping at −60.*
2. Follow a similar procedure for the following problems.

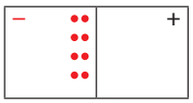
Samantha lost 2 points on each of her first 4 turns of the game. What was her score after 4 turns? *4 × −2 = −8 points*

A marathon runner in training lost 2 pounds a month for 6 months. What was the total change in his weight after the 6 months? *6 × −2 = −12 pounds*

Multiply Integers

$4 \times {}^{-}2 = {}^{-}8$ $\quad$ $^{-}2 \times 4 = {}^{-}8$

4 sets of 2 negative counters can be shown on an algebra mat.



$4 \times {}^{-}2 = {}^{-}8$

Commutative Property of Multiplication

The order of factors may be changed without changing the product. $4 \times {}^{-}2 = {}^{-}2 \times 4$

$^{-}2$ sets of 4 counters cannot be shown on the algebra mat, but the Commutative Property states that the product for $^{-}2 \times 4$ is the same as the product for $4 \times {}^{-}2$.

Rules for Multiplying Integers

When two factors have the same sign, the product is positive.
- positive factor × positive factor = positive product — $4 \times 2 = 8$
- negative factor × negative factor = positive product — ${}^{-}4 \times {}^{-}2 = 8$

When two factors have different signs, the product is negative.
- positive factor × negative factor = negative product — $4 \times {}^{-}2 = {}^{-}8$
- negative factor × positive factor = negative product — ${}^{-}4 \times 2 = {}^{-}8$

Exercises

Use the Commutative Property to write the related multiplication fact.

1. $5 \times {}^{-}2 = {}^{-}10$
   $^{-}2 \times 5 = {}^{-}10$
2. $^{-}10 \times 15 = {}^{-}150$
   $15 \times {}^{-}10 = {}^{-}150$
3. $^{-}12 \times {}^{-}11 = 132$
   $^{-}11 \times {}^{-}12 = 132$

Find the product using the rules for multiplying integers. Notice the pattern when one factor remains the same.

4.

4×2	*8*
3×2	*6*
2×2	*4*
1×2	*2*
0×2	*0*
$^{-}1 \times 2$	*$^{-}2$*
$^{-}2 \times 2$	*$^{-}4$*
$^{-}3 \times 2$	*$^{-}6$*
$^{-}4 \times 2$	*$^{-}8$*

5.

$4 \times {}^{-}3$	*$^{-}12$*
$3 \times {}^{-}3$	*$^{-}9$*
$2 \times {}^{-}3$	*$^{-}6$*
$1 \times {}^{-}3$	*$^{-}3$*
$0 \times {}^{-}3$	*0*
$^{-}1 \times {}^{-}3$	*3*
$^{-}2 \times {}^{-}3$	*6*
$^{-}3 \times {}^{-}3$	*9*
$^{-}4 \times {}^{-}3$	*12*

6.

$^{-}5 \times 4$	*$^{-}20$*
$^{-}5 \times 3$	*$^{-}15$*
$^{-}5 \times 2$	*$^{-}10$*
$^{-}5 \times 1$	*$^{-}5$*
$^{-}5 \times 0$	*0*
$^{-}5 \times {}^{-}1$	*5*
$^{-}5 \times {}^{-}2$	*10*
$^{-}5 \times {}^{-}3$	*15*
$^{-}5 \times {}^{-}4$	*20*

Multiply.

7. $5 \times {}^{-}5$ *$^{-}25$*
8. $6 \times {}^{-}3$ *$^{-}18$*
9. $^{-}6 \times 8$ *$^{-}48$*
10. 4×12 *48*
11. $^{-}6 \times {}^{-}3$ *18*
12. $2 \times {}^{-}5$ *$^{-}10$*
13. $^{-}4 \times {}^{-}3$ *12*
14. $5 \times {}^{-}10$ *$^{-}50$*
15. $^{-}5 \times {}^{-}2$ *10*
16. $^{-}1 \times 8$ *$^{-}8$*
17. $7 \times {}^{-}5$ *$^{-}35$*
18. $^{-}12 \times 2$ *$^{-}24$*

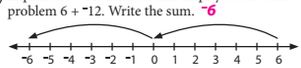
Write a comparison sentence using >, <, or =.

19. $9 \times {}^{-}2$ *< $^{-}5 \times 3$ $^{-}18 < {}^{-}15$*
20. $^{-}2 \times 7$ *= $7 \times {}^{-}2$ $^{-}14 = {}^{-}14$*
21. 0×2 *> $^{-}1 \times 2$ $0 > {}^{-}2$*
22. $^{-}5 \times {}^{-}8$ *> 6×6 $40 > 36$*
23. $^{-}4 \times {}^{-}6$ *= 2×12 $24 = 24$*
24. 6×3 *> $^{-}9 \times 2$ $18 > {}^{-}18$*

Practice & Application

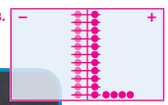
25. Mackenzie borrowed $10 from her brother to purchase her rocket model. Then she borrowed $5 more for lunch. Write an equation to show Mackenzie's debt. *$-\$10 + -\$5 = -\$15$ or $-\$10 + -\$5 = -\$15$*
26. Stock in the Davenport Sofa Company lost 12 points each day for 4 days. Write an equation to show the number of total points lost. *$4 \times {}^{-}12 = {}^{-}48$ points*
27. Show 4 sets of $^{-}3$ on a number line. Write a multiplication equation and solve. *$4 \times {}^{-}3 = {}^{-}12$*
28. Draw a picture to show how one positive counter cancels out one negative counter. Solve the equation $^{-}11 + 15$. *$^{-}11 + 15 = 4$*
29. Write an addition equation for $^{-}4 - 7$. Solve. (Remember that subtraction of an integer is the same as adding its opposite.) *$^{-}4 + {}^{-}7 = {}^{-}11$*
30. The quarterback was sacked on 3 consecutive plays. He lost 10 yards on each sack. Write an addition equation and a multiplication equation to show how many yards he lost in 3 plays. *$^{-}10 + {}^{-}10 + {}^{-}10 = {}^{-}30$ yards; $3 \times {}^{-}10 = {}^{-}30$ yards*

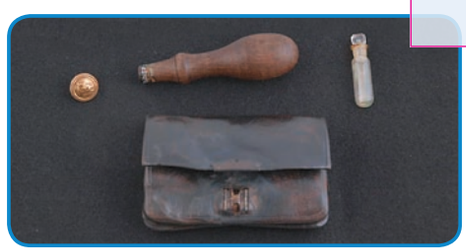
J Explain how the number line helps you solve the problem $6 + {}^{-}12$. Write the sum. *$^{-}6$*



12 jumps away from 6 are needed. Using the fact $6 + 6 = 12$ allows for 2 jumps instead of 12 individual jumps.
6 to 0 = 6 jumps
0 to $^{-}6$ = 6 jumps

28.





Over 5,000 artifacts have been recovered from the sunken *Titanic* by RMS Titanic, Inc.

Complete DAILY REVIEW on page 465.

Student Text pp. 386–87

Lesson 161

Student Text pp. 388–89
Daily Review p. 466g

Objectives
- Multiply and divide integers
- Write an equation for a word problem

Teacher Materials
- Algebra Mat (from Lesson 156)
- Positive & Negative Number Line (IA4)
- Plastic counters: opaque (black) and transparent (red)

Student Materials
- Algebra Mat Kit

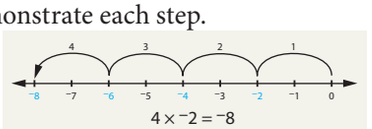
Teach for Understanding

Multiply and divide integers

1. Display the Algebra Mat.
2. Choose a student to place 2 red counters on the negative side of the mat. Repeat this procedure 3 more times leaving space between each group of 2 red counters.

➤ **What equation can you write to represent these counters? Why?** ***Elicit the equation −2 + −2 + −2 + −2 = −8 since you repeatedly added 2 negative counters to the mat 4 times for a total of −8 counters; also elicit 4 × −2 = −8 since there are 4 sets of −2 producing one large set of −8.*** Write $-2 + -2 + -2 + -2 = -8$ and $4 \times -2 = -8$ for display.

3. Display the Positive & Negative Number Line page. Direct the students to draw a number line similar to the one displayed and to use arrows to show that $4 \times -2 = -8$. ***Elicit that you can begin at 0; since you are adding 4 sets of −2, you can draw an arrow to show 4 jumps of 2 in a negative direction (left) from 0: −2, −4, −6 and −8.*** Demonstrate each step.
4. Remind the students that division is the inverse operation of multiplication. Write $4 \times 2 = 8$ for display.



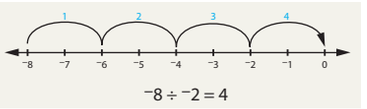
➤ **What are the related division equations for $4 \times 2 = 8$? Why?** ***8 ÷ 2 = 4 and 8 ÷ 4 = 2; When a product is divided by either of its factors, the quotient is the other factor.***

➤ **How can you show 8 ÷ 2 (in each set) = 4 (sets)?** ***8 positive counters can be dispersed into sets of 2 positive counters in each set to make 4 sets;*** **How can you show 8 ÷ 4 (sets) = 4 (in each set)?** ***8 positive counters can be fairly shared among 4 sets to find that there are 2 positive counters in each set.*** Write the equations and demonstrate on the Algebra Mat.

➤ **What are the related division equations for $4 \times -2 = -8$? Why?** ***−8 ÷ −2 = 4 and −8 ÷ 4 = −2; when a product is divided by either of its factors, the quotient is the other factor.***

➤ **How can you show −8 ÷ −2 (in each set) = 4 (sets)?** (***Note:*** Elicit that it is not possible to show negative sets.) ***8 negative counters can be dispersed into sets of 2 negative counters in each set to make 4 sets;*** **How can you show −8 ÷ 4 (sets) = −2 (in each set)?** ***8 positive counters can be fairly shared among 4 sets to find that there are 2 positive counters in each set.*** Write the equations and demonstrate on the Algebra Mat.

5. Direct the students to draw another −10 to 10 number line and to use arrows to show that $-8 \div -2 = 4$. ***Elicit that it is not possible to make negative sets. Begin at −8 and make jumps (sets) of −2 units until you reach zero.*** Demonstrate each step.
6. Point out that repeated addition of integers is to multiplication as repeated subtraction of integers is to division: $-8 - -2 - -2 - -2 - -2 = 0$.



➤ **How many sets (jumps) of −2 did you make?** ***4*** **How many times did you subtract −2?** ***4***

Explain that the quotient is the number of times you subtracted −2, so the quotient in the equation $-8 \div -2$ is 4. Display 8 red counters on the negative side of the mat. Remove sets of −2 (2 red counters) four times.

7. Write these equations for display: $4 \times -2 = -8$, $-8 \div -2 = 4$, and $-8 \div 4 = -2$; $-5 \times -3 = 15$, $15 \div -3 = -5$, and $-3 \times -5 = 15$. Elicit the missing equation in each fact family. ***$-2 \times 4 = -8$; $15 \div -5 = -3$***

➤ **What pattern do you see between the factors and products of these problems?** ***Elicit that multiplication and division with like signs result in a positive answer and that multiplication and division with unlike signs result in a negative answer.***

8. Guide the students in identifying a mental picture to represent a problem. Write $-24 \div -4 =$ __ for display.

➤ **What picture comes to your mind when you see this problem?** ***Possible answers: 24 red counters on the negative side of the mat can be dispersed into sets of 4 red counters in each set to make 6 sets; begin at −24 on the number line and make jumps of −4, with a total of 6 jumps (sets).***

9. Direct each student to write a multiplication or division equation including negative integers. Arrange the students in pairs; direct them to trade equations and then picture the equation on their mat. (***Note:*** You may also choose to have students picture the equation on a number line.)

Write an equation for a word problem

1. Direct the students to listen carefully to each of the following word problems to form a picture in their minds.

> During a board game each player had 5 turns. On each turn Katelyn lost 2 points. What was her score at the end of the game?

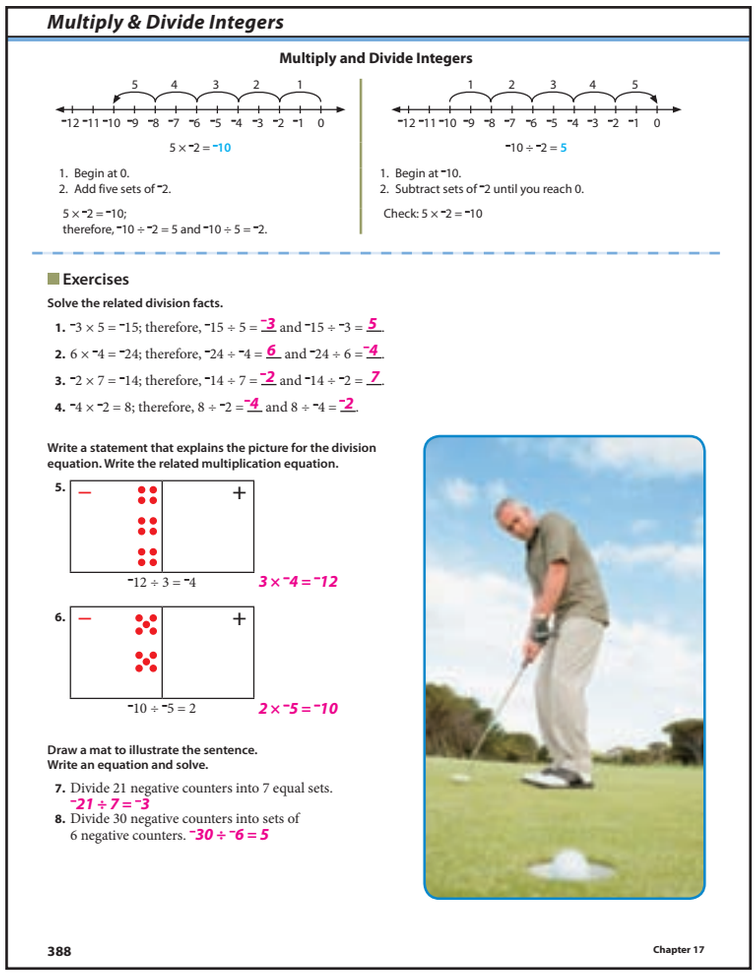
➤ **What picture came to mind?** ***Encourage any reasonable answer including 5 sets of −2.***

Explain that it is hard to picture a turn in a game; therefore, you may see only a loss of 2 points repeated 5 times in a row. Write −2 five times in one horizontal row, leaving space for an operation sign to be inserted between each integer.

➤ **Consider the four mathematical operations: addition, subtraction, multiplication, and division. Which of these four can be used to show this repetition? Why?** ***Elicit addition and/or multiplication; the −2 was added 5 times in a row, or there were 5 times −2.***

(***Note:*** The student who answers repeated subtraction is not incorrect, but he must establish that Katelyn began the game with a score of 0 and repeatedly subtracted 2. Division is not an option since the final score was not given, and −10 cannot be obtained without first performing addition or multiplication.)

➤ **Which operation sign (+, −, ×, or ÷) can be inserted between the five −2s to form the needed equation?** ***Elicit the + sign.***



Multiply & Divide Integers

Multiply and Divide Integers

$5 \times {}^{-}2 = {}^{-}10$

1. Begin at 0.
2. Add five sets of $^{-}2$.

$5 \times {}^{-}2 = {}^{-}10$;
therefore, $^{-}10 \div {}^{-}2 = 5$ and $^{-}10 \div 5 = {}^{-}2$.

$^{-}10 \div {}^{-}2 = 5$

1. Begin at $^{-}10$.
2. Subtract sets of $^{-}2$ until you reach 0.

Check: $5 \times {}^{-}2 = {}^{-}10$

Exercises

Solve the related division facts.

1. $^{-}3 \times 5 = {}^{-}15$; therefore, $^{-}15 \div 5 =$ *$^{-}3$* and $^{-}15 \div {}^{-}3 =$ *5*
2. $6 \times {}^{-}4 = {}^{-}24$; therefore, $^{-}24 \div {}^{-}4 =$ *6* and $^{-}24 \div 6 =$ *$^{-}4$*
3. $^{-}2 \times 7 = {}^{-}14$; therefore, $^{-}14 \div 7 =$ *$^{-}2$* and $^{-}14 \div {}^{-}2 =$ *7*
4. $^{-}4 \times {}^{-}2 = 8$; therefore, $8 \div {}^{-}2 =$ *$^{-}4$* and $8 \div {}^{-}4 =$ *$^{-}2$*

Write a statement that explains the picture for the division equation. Write the related multiplication equation.

5.

$^{-}12 \div 3 = {}^{-}4$ *$3 \times {}^{-}4 = {}^{-}12$*

6.

$^{-}10 \div {}^{-}5 = 2$ *$2 \times {}^{-}5 = {}^{-}10$*

Draw a mat to illustrate the sentence. Write an equation and solve.

7. Divide 21 negative counters into 7 equal sets. *$^{-}21 \div 7 = {}^{-}3$*
8. Divide 30 negative counters into sets of 6 negative counters. *$^{-}30 \div {}^{-}6 = 5$*

Multiply.

9. $5 \times {}^{-}7$ *$^{-}35$*	12. $^{-}7 \times {}^{-}9$ *63*	15. $2 \times {}^{-}12$ *$^{-}24$*	18. $^{-}8 \times {}^{-}1$ *8*
10. $^{-}2 \times 6$ *$^{-}12$*	13. $^{-}8 \times 6$ *$^{-}48$*	16. 4×10 *40*	19. 4×7 *28*
11. $3 \times {}^{-}12$ *$^{-}36$*	14. $^{-}5 \times {}^{-}15$ *75*	17. $^{-}6 \times 0$ *0*	20. $16 \times {}^{-}6$ *$^{-}96$*

Divide.

21. $^{-}15 \div 3$ *$^{-}5$*	24. $80 \div 4$ *20*	27. $42 \div 2$ *21*	30. $^{-}8 \div 4$ *$^{-}2$*
22. $^{-}100 \div 2$ *$^{-}50$*	25. $18 \div {}^{-}3$ *$^{-}6$*	28. $^{-}2 \div {}^{-}2$ *1*	31. $^{-}12 \div 4$ *$^{-}3$*
23. $99 \div {}^{-}9$ *$^{-}11$*	26. $^{-}16 \div {}^{-}2$ *8*	29. $^{-}12 \div 6$ *$^{-}2$*	32. $^{-}20 \div {}^{-}5$ *4*

Write a comparison sentence using >, <, or =.

33. $^{-}16 \div 4 = 2 \times {}^{-}2$ *$^{-}4 = {}^{-}4$*
34. $4 \div {}^{-}4 < 3 \div 3$ *$^{-}1 < 1$*
35. $0 \times 3 > {}^{-}1 \times 2$ *$0 > {}^{-}2$*
36. $^{-}2 \times {}^{-}3 > {}^{-}1 \times 5$ *$6 > {}^{-}5$*

Practice & Application

37. Marc's golf team scored 6 strokes under par ($^{-}6$) each day for 5 days. Write an equation to show the team's combined score. *$5 \times {}^{-}6 = {}^{-}30$*

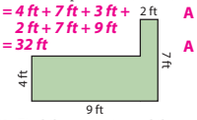
38. At the end of a 3-day golf tournament, Mr. Davis had a final score of $^{-}12$, or 12 strokes under par. Write an equation to show his average score each day. *$^{-}12 \div 3 = {}^{-}4$ strokes average per day*

39. Cody collected 36 blankets for the homeless. He gave away 6 blankets each day for 5 days. Write a multiplication and an addition equation using integers to show how many blankets Cody has left. *$36 - (5 \times 6) = 6$ blankets; $6 + 6 + 6 + 6 + 6 = 30$; $36 - 30 = 6$ blankets*

40. Clark made 3 clay pots each day during 4 days at Art Camp. While in the kiln, 5 of the pots cracked. How many pots does Clark have left? *$4 \times 3 - 5 = 12 - 5 = 7$ pots*

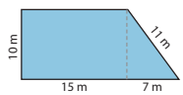
41. Lexie's lap times at the swim meet were 12 seconds slower than the best time. If her time decreases 2 seconds each day, how many days will it take her to tie the best time? *$^{-}12 \div {}^{-}2 = 6$ days or $12 \div 2 = 6$ days*

42. Find the perimeter and the area of the figure.

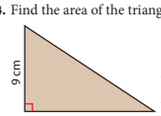


$P = 4 \text{ ft} + 7 \text{ ft} + 3 \text{ ft} + 2 \text{ ft} + 7 \text{ ft} + 9 \text{ ft}$
$P = 32 \text{ ft}$
$A = (4 \text{ ft} \times 9 \text{ ft}) + (2 \text{ ft} \times 3 \text{ ft})$
$A = 42 \text{ ft}^2$

43. Find the perimeter and the area of the trapezoid.



44. Find the area of the triangle.



$A = \frac{1}{2}(18 \text{ cm} \times 9 \text{ cm})$
$A = 81 \text{ cm}^2$

J Use $10 \times 18 = 180$ to explain the statement *The inverse operation of multiplication is division.* *Division is used to take apart what multiplication puts together. 180 items can be put into 10 sets of 18 or 18 sets of 10.*

43. *$P = 10 \text{ m} + 15 \text{ m} + 11 \text{ m} + 22 \text{ m}$*
$P = 58 \text{ m}$

$A = (10 \text{ m} \times 15 \text{ m}) + \frac{1}{2}(7 \text{ m} \times 10 \text{ m})$
$A = 185 \text{ m}^2$

Complete DAILY REVIEW 6 on page 466.

Write an addition sign (+) between the $^{-}2$s: $^{-}2 + {}^{-}2 + {}^{-}2 + {}^{-}2 + {}^{-}2$

- **What integer represents the points scored each turn?** *$^{-}2$*
- **How many times (turns) was this same score repeated? How do you know?** *5; each player had 5 turns during the game.*
- **What multiplication equation can you write for this problem? Why?** *$5 \times {}^{-}2 = \_\_$; there were 5 turns of a loss of 2 points ($^{-}10$ points).* Write $5 \times {}^{-}2 = \_\_$.

  Direct one of the students in each pair to show $5 \times {}^{-}2$ on the mat while the other student shows it on the number line. *Place 5 sets of 2 counters on the negative side of the mat for a total of $^{-}10$; begin at 0 and move 2 units (points) to the left 5 times repeatedly, ending at $^{-}10$.*
- **What was Katelyn's score after her fifth turn?** *$^{-}10$ points*

  Complete the equation.

2. Follow a similar procedure to solve these word problems.

> Allen lost 3 points on each of his first 3 turns of the game. On each of his last 2 turns he gained 5 points. What was his score at the end of 5 turns? *$^{-}3 + {}^{-}3 + {}^{-}3 + 5 + 5 = {}^{-}9 + 10 = 1$; $(3 \times {}^{-}3) + (2 \times 5) = {}^{-}9 + 10 = 1$*
>
> The value of stock in the Hancock Steel Company gained a total of 15 points in 5 days. What was the average change each day? *$15 \div 5 = 3$ points per day*

Student Text pp. 388–89

Lesson 162

Student Text pp. 390–91
Daily Review p. 466h

Objectives
- Add, subtract, multiply, and divide integers
- Subtract integers by adding the opposite
- Apply the Order of Operations to integers

Teacher Materials
- Number Line
- Positive & Negative Number Line (from Lesson 155)
- Algebra Mat (from Lesson 156)
- Order of Operations, page IA107 (CD)
- Plastic counters: opaque (black) and transparent red

Student Materials
- Algebra Mat Kit
- Order of Operations, page IA107 (CD)

Note
In this lesson the red (negative) and black (positive) counters may be used apart from the mat.

Teach for Understanding

Add and subtract integers

1. Write 5 + 4 = __ and $^{-}5 + {}^{-}4$ = __ for display. Display the Algebra Mat and distribute the Algebra Mat Kits.

➤ **What is true about the sum when all the addends are positive integers? Why?** ***Elicit that the sum will be a greater positive integer; when adding only positive integers, you get more positives.***

➤ **What is true about the sum when all the addends are negative integers? Why?** ***Elicit that the sum will be a lesser negative integer; when adding only negative integers, you get more negatives.***

Direct each student to write his predicted answer to each equation and then to show the equation using the mat and counters: 5 + 4 = ***9; place 5 positive counters, place 4 more positive counters, and then combine the counters for a total of 9 positive counters*** and $^{-}5 + {}^{-}4 = {}^{-}9$***; place 5 negative counters, place 4 more negative counters, and then combine the counters for a total of 9 negative counters***.

2. Write 5 − 4 = __ and 4 − 5 = __ for display.

➤ **What is true about the difference of two positive integers? Why?** ***Elicit that the difference is a positive integer if the minuend has the greater value because you are taking away less positives than you have; the difference will be a negative integer if the subtrahend has the greater value because you are taking away more positives than you have.***

Direct each student to write his predicted answer to each equation and then to show the equation using counters: 5 − 4 = ***1; place 5 positive counters and remove 4 positive counters, leaving a difference of 1 positive counter*** and 4 − 5 = $^{-}1$***; place 4 positive counters, since there is 1 less positive counter than the 5 needed to be taken away, add 1 positive counter and 1 negative counter (1 + $^{-}1$ = 0), and remove the 5 positive counters, leaving a difference of 1 negative counter.***

3. Repeat the procedure with the following problems: 7 − 5 = ***2; place 7 positive counters and remove 5 positive counters, leaving a difference of 2 positive counters*** and 5 − 7 = $^{-}2$***; place 5 positive counters since there are 2 less positive counters than the 7 needed to be taken away, add 1 positive counter and 1 negative counter (2 + $^{-}2$ = 0), and remove the 7 positive counters, leaving a difference of 2 negative counters***.

4. Write 4 − 2 = __ and 4 + $^{-}2$ = __ for display. Arrange students in pairs. Direct each pair of students to write their predicted answers to each equation. One student will picture one equation using counters, while the other student pictures the other equation. (***Note:*** Guide the students in aligning positive and negative counters in a 1-to-1 correspondence so the canceling out can be seen.) ***4 − 2 = 2; place 4 positive counters and remove 2 positive counters, leaving a difference of 2 positive counters. 4 + $^{-}2$ = 2; place 4 positive counters, place 2 negative counters, and combine the counters to show $^{-}2$ + 2 = 0, with a difference of 2 positive counters.***

Elicit that since 4 − 2 = 2 and 4 + $^{-}2$ = 2, then 4 – 2 = 4 + $^{-}2$. Write 4 – 2 = 4 + $^{-}2$ for display.

➤ **What conclusion can you reach about subtracting an integer and adding its opposite?** ***The answers are the same.***

➤ **Why do you think the sum reflects the difference in the absolute value of the integers when the addends are positive and negative integers?** ***Elicit that subtracting an integer is the same as adding its opposite.***

5. Repeat a similar procedure to guide students in predicting and solving these pairs of problems: $^{-}9 + 4 = {}^{-}5$ and $9 + {}^{-}4 = 5$; $^{-}2 + 7 = 5$ and $2 + {}^{-}7 = {}^{-}5$; $^{-}9 - {}^{-}5 = {}^{-}4$ and $^{-}5 - {}^{-}9 = 4$.

6. Display the Positive & Negative Number Line.

➤ **When using a number line, how is adding two negative numbers different from adding two positive numbers?** ***Elicit that when adding two positive numbers, both moves are in a positive direction (to the right), but when adding two negative numbers, both moves are in a negative direction (to the left).***

Choose students to demonstrate solving 5 + 4 = ***9*** and $^{-}2 + {}^{-}6 = {}^{-}8$ on the number line.

➤ **When using a number line, how is adding $^{-}3 + 7$ different from $3 + {}^{-}7$?** ***Elicit that you begin at 0 and move in a negative direction or in a positive direction as each addend indicates. For these problems, you are moving in opposite directions for each addend.***

Choose students to demonstrate solving $^{-}3 + 7 = 4$ and $3 + {}^{-}7 = {}^{-}4$ on the Number Line.

Multiply and divide integers

1. Write the equation $6 \times {}^{-}2$ = __ for display. Direct the students to demonstrate finding the product using counters or a number line. ***6 sets of 2 negative counters; Begin at 0 and draw an arrow to the left to $^{-}2, {}^{-}4, {}^{-}6, {}^{-}8, {}^{-}10,$ and $^{-}12$.***

➤ **What repeated addition equation does this represent?** $^{-}2 + {}^{-}2 + {}^{-}2 + {}^{-}2 + {}^{-}2 + {}^{-}2 = {}^{-}12$

➤ Write the repeated addition equation and complete the equation $6 \times {}^{-}2 = {}^{-}12$.

➤ **What is the inverse operation of multiplication?** ***division***

➤ **What division equations can you write from this multiplication equation?** $^{-}12 \div {}^{-}2 = 6$ **and** $^{-}12 \div 6 = {}^{-}2$ Write the equations.

2. Direct students to show finding the quotient using counters or a number line. Remind the students that you use repeated addition to multiply integers and repeated subtraction to divide integers. ***Place 12 negative counters ($^{-}12$) and divide into sets of 2 negative counters ($^{-}2$); begin at $^{-}12$ and subtract $^{-}2$ until you reach 0.***

➤ **What does the equation $^{-}12 \div {}^{-}2 = 6$ tell you?** ***Elicit that there are 6 sets of $^{-}2$ in $^{-}12$.***

Mixed Review

Order of Operations

1. Do all operations inside the parentheses first.
2. Do all multiplication and division in order from left to right.
3. Do all addition and subtraction in order from left to right.

$2(^-5 - 4) =$ $2 \times ^-9 = ^-18$ $\quad ^-9 + ^-8 \div 4 =$ $^-9 + ^-2 = ^-11$

Exercises

Solve. Follow the Order of Operations.

1. $^-7 + ^-8 - 3$ *$^-18$*
2. $5 - (6 + ^-5)$ *4*
3. $2(^-7 + 5)$ *$2(^-2) = ^-4$*
4. $^-8 \div 2 - 3$ *$^-4 - 3 = ^-7$*
5. $^-4 - 5 + ^-3$ *$^-12$*
6. $8 + (2 - 7)$ *3*
7. $4 + ^-6 \div 3$ *$4 + ^-2 = 2$*
8. $6 \div (6 - 9)$ *$6 \div ^-3 = ^-2$*
9. $^-1 + 1 - 1$ *$^-1$*
10. $9 - 7 + ^-3$ *$^-1$*
11. $8(^-1 + ^-3)$ *$8(^-4) = ^-32$*
12. $^-5 - 2 \times 2$ *$^-5 - 4 = ^-9$*

Add.

13. $^-8 + 3$ *$^-5$*
14. $1 + ^-5$ *$^-4$*
15. $^-7 + ^-3$ *$^-10$*
16. $^-2 + 7$ *5*
17. $6 + 8$ *14*
18. $^-4 + 7$ *3*
19. $2 + ^-2$ *0*
20. $14 + ^-3$ *11*
21. $^-10 + 9$ *$^-1$*
22. $13 + ^-2$ *11*
23. $^-15 + 9$ *$^-6$*
24. $^-6 + 6$ *0*

Subtract.

25. $3 - 8$ *$^-5$*
26. $^-4 - 10$ *$^-14$*
27. $^-7 - ^-2$ *$^-5$*
28. $5 - ^-4$ *9*
29. $10 - ^-2$ *12*
30. $^-6 - ^-4$ *$^-2$*
31. $^-2 - 5$ *$^-7$*
32. $3 - 9$ *$^-6$*
33. $5 - 7$ *$^-2$*
34. $^-4 - ^-4$ *0*
35. $2 - 3$ *$^-1$*
36. $6 - 5$ *1*

Multiply.

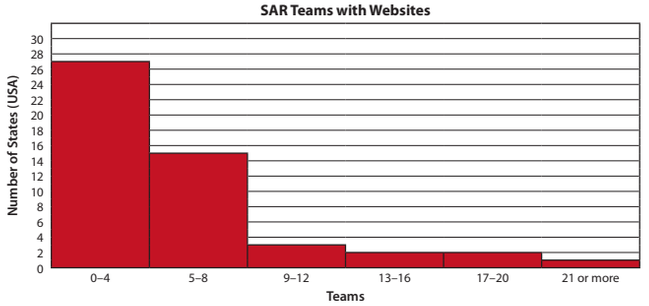
37. $6 \times ^-5$ *$^-30$*
38. $^-8 \times ^-7$ *56*
39. $^-4 \times 3$ *$^-12$*
40. $^-1 \times 1$ *$^-1$*
41. $2 \times ^-3$ *$^-6$*
42. $^-4 \times ^-5$ *20*
43. $^-3 \times 6$ *$^-18$*
44. $^-1 \times ^-9$ *9*
45. $7 \times ^-2$ *$^-14$*

Divide.

46. $^-15 \div 3$ *$^-5$*
47. $4 \div ^-2$ *$^-2$*
48. $^-12 \div ^-6$ *2*
49. $6 \div ^-3$ *$^-2$*
50. $^-10 \div ^-5$ *2*
51. $16 \div 4$ *4*
52. $^-4 \div ^-1$ *4*
53. $18 \div ^-9$ *$^-2$*
54. $39 \div 3$ *13*

Practice & Application

As of 2011, there were more than 1,500 search-and-rescue (SAR) teams in the United States. The histogram shows the number of teams that have their own website by state.



55. How many states list more than 20 search-and-rescue teams? *1 state*
56. What interval shows the mode? *0–4*
57. Pennsylvania has 14 teams listed. In which interval is Pennsylvania? *13–16*
58. How many states have 13 or more search-and-rescue teams listed? *5 states*
59. How many states have 12 or fewer search-and-rescue teams listed? *45 states*
60. What percentage of states is in the 0–4 interval? *$\frac{27}{50} = \frac{54}{100} = 54\%$*
61. Hannah is training for a 5K race. Her finish time has decreased 30 seconds each week for 6 weeks. What is the total change in her finish time at the end of the 6 weeks? *$6 \times ^-30 = ^-180$ sec $= ^-3$ min*
62. The weatherman predicted that the temperature would rise to 12°F. If the temperature is $^-3$°F, how many degrees must it rise to make the prediction true? *$^-3 + n = 12°$, $^-3 - ^-3 + n = 12° - ^-3°$, $n = 15°$*
63. The youth group filled 15 trash bags with leaves while helping Mrs. Slocum with yard work. When they carried the bags to the street to be picked up, 2 of the bags broke, spilling the leaves. How many full bags of leaves did they still have? *$15 + ^-2 = 13$ bags or $15 - 2 = 13$ bags*

Complete DAILY REVIEW on page 466.

3. Write $^-5 \times 2 =$ ___ for display. Elicit that the Commutative Property can help you solve a problem with a negative multiplier: $^-5 \times 2 = 2 \times ^-5$. *$^-10$*
4. Write $^-2 \times ^-3 =$ ___ for display.

➤ **What do you know about 2 and $^-2$?** ***Elicit that they are opposites.***

➤ **What is $2 \times ^-3$ equal to?** *$^-6$*

➤ **If 2 sets of $^-3 = ^-6$, what do you think the opposite of 2 sets ($^-2$ sets) of $^-3$ would be?** ***6; the opposite of $^-6$***

Write 6 in the answer blank of the problem.

➤ **What rules apply to both multiplication and division?** ***Elicit that multiplication and division with like signs result in a positive answer and multiplication and division with unlike signs result in a negative answer.***

5. Guide students in applying their knowledge of fact families and inverse operations to solve these division problems.

$^-9 \div 3 =$ *$^-3$* $\qquad 8 \div ^-2 =$ *$^-4$*

Apply the Order of Operations to integers

Display the Order of Operations page and review the explanation at the top of the page. Guide the students in following the Order of Operations to complete the problems. Remind them that if a step does not apply to an expression, you proceed to the next step. (***Note:*** If a student struggles with performing the operations in the correct order, direct him to underline the operation that he is about to apply.) ***Answers: $^-9$, $^-5$, $^-18$, $^-8$, $^-5$, $^-7$, 4, 7, 16, 41***

Student Text pp. 390–91

Lesson 163

Student Text pp. 392–93
Daily Review p. 467i

Objectives
- Graph points on a four-quadrant coordinate plane
- Write ordered pairs to identify points on a four-quadrant coordinate plane

Teacher Materials
- Coordinate Plane, page IA24 (CD)

Student Materials
- Coordinate Planes, page IA25 (CD)

Preparation
Write these terms for display: *coordinate plane, quadrants, x-axis, y-axis, origin (0, 0), ordered pair (x, y), x-coordinate, y-coordinate.*

Teach for Understanding

Graph points on a four-quadrant coordinate plane

1. Display the Coordinate Plane and distribute the Coordinate Planes page. Remind the students that a *coordinate plane* is formed by two number lines intersecting at right angles and that the axes divide the coordinate plane into the four quadrants.
2. Write the Roman numerals I–IV to label the quadrants and instruct the students to label their coordinate plane; begin with the top right quadrant and proceed in a counterclockwise direction.
3. Elicit that the horizontal number line is the *x-axis* and the vertical number line is the *y-axis*. Remind students that the point where the *x*-axis and *y*-axis intersect is called the *origin*.
4. Elicit that an *ordered pair* describes the location of every point on a coordinate plane. The *x-coordinate* (first coordinate) tells the distance of the point along the *x*-axis—how far to move to the right or left from the origin. The *y-coordinate* (second coordinate) tells the distance of the point along the *y*-axis—how far to move up or down from the origin.
5. Remind the students that the location of the origin is described by the ordered pair (0, 0). To find the location of any point, you begin at the origin (0,0) and move the number of units right or left as specified by the *x*-coordinate; then move the number of units up or down as specified by the *y*-coordinate (*x*, *y*).

➤ **Where is the location of the point described by the ordered pair (2, 3)?** ***Elicit that it is 2 units right and 3 units up from the origin (0, 0) in Quadrant I.***

6. Direct attention to the first coordinate plane (top left) on the Coordinate Planes page. Instruct the students to graph the point on this graph and to label it point *A* as you demonstrate.

➤ **Where is the point described by the ordered pair located (−2, 3)?** ***Elicit that it is 2 units left and 3 units up from the origin (0, 0) in Quadrant II.*** Graph and label point *B* (−2, 3).

➤ **Where is the point described by the ordered pair located (−2, −3)?** ***Elicit that it is 2 units left and 3 units down from the origin (0, 0) in Quadrant III.*** Graph and label point *C* (−2, −3).

➤ **Where is the point described by the ordered pair located (2, −3)?** ***Elicit that it is 2 units right and 3 units down from the origin (0, 0) in Quadrant IV.*** Graph and label point *D* (2, −3).

7. Direct the students to graph points *E* (4, 6), *F* (4, −6), *G* (−4, −6) and *H* (−4, 6) on the same coordinate plane as points *A*, *B*, *C*, and *D*.

➤ **What is true about the *x*- and *y*-coordinates of an ordered pair whose point is located in Quadrant I?** ***Elicit that both coordinates are positive integers.*** Write (+, +) below the Roman numeral I in Quadrant I.

➤ **What is true about the *x*- and *y*-coordinates of an ordered pair whose point is located in Quadrant II?** ***Elicit that the x-coordinate is a negative integer, and the y-coordinate is a positive integer.*** Write (–, +) below the Roman numeral II in Quadrant II.

➤ **What is true about the *x*- and *y*-coordinates of an ordered pair whose point is located in Quadrant III?** ***Elicit that the x-coordinate is a negative integer, and the y-coordinate is a negative integer.*** Write (–, –) below the Roman numeral III in Quadrant III.

➤ **What is true about the *x*- and *y*-coordinates of an ordered pair whose point is located in Quadrant IV?** ***Elicit that the x-coordinate is a positive integer, and the y-coordinate is a negative integer.*** Write (+, –) below the Roman numeral IV in Quadrant IV.

➤ **In which of the four quadrants could a point with a positive *x*-coordinate possibly be located?** ***Quadrants I and IV*** **a point with a negative *x*-coordinate?** ***Quadrants II and III***

➤ **In which of the four quadrants could a point with a positive *y*-coordinate possibly be located?** ***Quadrants I and II*** **a point with a negative *y*-coordinate?** ***Quadrants III and IV***

8. Follow a similar procedure to graph points *J* (0, 3) and *K* (0, −3) on the top right coordinate plane.

➤ **What do the ordered pairs of points *J* and *K* have in common, and how does it affect their location?** ***Elicit that they both have an x-coordinate of 0 causing them to be located somewhere along the y-axis; their y-coordinate are opposite integers with the same absolute value of 3, causing them to be 3 units from the origin in opposite directions.***

➤ **What do you think is true of the *x*- and *y*-coordinates of the ordered pair of a point whose location is somewhere along the *x*-axis?** ***Elicit that the x-coordinate could be any integer, but the y-coordinate must be 0.***

Elicit ordered pairs with 0 as their *y*-coordinate and guide the students in graphing them.

Write ordered pairs to identify points on a four-quadrant coordinate graph

1. Direct the students to graph 8 points on the bottom left coordinate plane and to label them *A*–*H*. Encourage them to vary the location of the points among all four quadrants and the axes.
2. Next, direct the students to write each letter (*A*–*H*) and the ordered pair that indicates its location on the backside of their page.
3. Direct them to trade papers with a partner. Then tell them to write the ordered pair beside each point's letter name on the graph. After writing an ordered pair for each point, each partner can compare his ordered pairs with those written on the back of the page.

(***Note:*** You may also choose to have students identify the quadrant number or axis name of the location.)

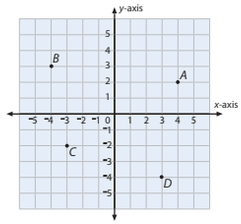
Coordinate Plane

An **ordered pair** is used to describe the location of a point on a **coordinate plane** in relation to the **origin** (0, 0). Each ordered pair is located in a quadrant of the graph or on the ***x*-axis** or ***y*-axis**.

ordered pair
coordinate plane
origin
***x*-axis**
***y*-axis**

Quadrant II contains points with a negative *x*-coordinate and a positive *y*-coordinate.

B = (−4, 3)



Quadrant I contains points with positive *x*- and *y*-coordinates.

A = (4, 2)

Quadrant III contains points with negative *x*- and *y*-coordinates.

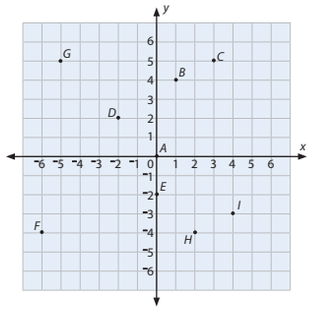
C = (−3, −2)

Quadrant IV contains points with a positive *x*-coordinate and a negative *y*-coordinate.

D = (3, −4)

Exercises

Write the ordered pair for each point on the coordinate plane.
Answer the questions.



1. *A* ***(0, 0)***
2. *B* ***(1, 4)***
3. *C* ***(3, 5)***
4. *D* ***(−2, 2)***
5. *E* ***(0, −2)***
6. *F* ***(−6, −4)***
7. *G* ***(−5, 5)***
8. *H* ***(2, −4)***
9. *I* ***(4, −3)***

10. How many points are in quadrant I? ***2***
11. How many points are in quadrant II? ***2***
12. What point is located in quadrant III? ***F***
13. What points are located in quadrant IV? ***H and I***
14. Where is point *E* located? ***on the y-axis***
15. Name the point located at the origin of the coordinate plane. ***Point A***

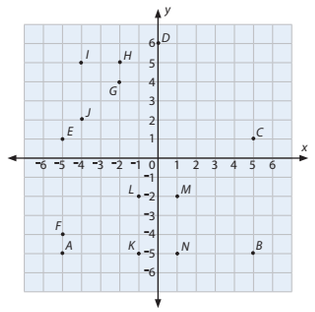
Graph each ordered pair on a coordinate plane.
Connect the points in order to form a figure.

16. (1, 1) → (5, −3) → (3, 3) → (0, 0) → (−3, −3)
17. (3, −3) → (−3, 3) → (−5, −3) → (−1, 1)

Graph each ordered pair on a coordinate plane.
Connect the points in order. Connect the last point to the first.
Identify the quadrilateral.

18. (1, −2) → (5, −2) → (5, −5) → (1, −5) ***rectangle***
19. (−1, 0) → (−5, 0) → (−5, 3) → (−3, 3) ***trapezoid***
20. (0, 0) → (2, 2) → (5, 2) → (3, 0) ***parallelogram***
21. (−6, 6) → (6, 6) → (6, −6) → (−6, −6) ***square***

Write the ordered pair for each point on the coordinate plane.



22. *A* ***(−5, −5)***
23. *B* ***(5, −5)***
24. *C* ***(5, 1)***
25. *D* ***(0, 6)***
26. *E* ***(−5, 1)***
27. *F* ***(−5, −4)***
28. *G* ***(−2, 4)***
29. *H* ***(−2, 5)***
30. *I* ***(−4, 5)***
31. *J* ***(−4, 2)***

Write the letter of the point named by the ordered pair.

32. (−1, −5) **K**
33. (1, −2) **M**
34. (−1, −2) **L**
35. (1, −5) **N**

Connect the points in order.
Identify the shape.

36. On a 4-quadrant coordinate plane, plot the points for problems 22–35.
A → *B* → *C* → *D* → *E* → *F* → *A*
G → *H* → *I* → *J*
K → *L* → *M* → *N*

37. Name the shape that is made. ***a house***

J What is always true about the location of a point whose first coordinate is 0? ***The point is on the y-axis.***

Complete DAILY REVIEW on page 467.

4. Write the ordered pairs (4, 0), (0, 8), and (0, −4) for display. Ask the students to write a prediction of the shape that would be formed if these 3 points were graphed and connected in order. Direct them to graph the points on the remaining coordinate plane to check their prediction. ***equilateral triangle***

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(*Note*: *Assessment* available on Teacher's Toolkit CD.)

Lesson 164 **Student Text** pp. 394–95

Chapter Review

Objectives

- Compare and order integers
- Add and subtract integers
- Multiply and divide integers
- Apply the Order of Operations to integers
- Write an equation for a word problem

Teacher Materials

- Positive & Negative Number Line (from Lesson 155)
- Algebra Mat (from Lesson 156)
- Plastic counters: opaque (black) and transparent red

Note

This lesson reviews the concepts presented in Chapter 17 to prepare the students for the Chapter 17 Test. Student Text pages 394–95 provide the students with an excellent study guide.

Check for Understanding

Compare and order integers

1. Display the Positive & Negative Number Line page.

➤ **What number is in the center of the number line? *0* What kind of numbers are to the right of zero? *positive numbers* What kind of numbers are to the left of zero? *negative numbers***

Remind the students that zero is neither negative nor positive, but it separates the negative and positive numbers.

➤ **What name is given to the set of whole numbers (0, 1, 2, 3 . . .) and their opposites? *Integers*** Write integers for display.

➤ **What integer is three units to the right of 0? *3* What integer is three units to the left of 0? *−3***

➤ **What is the absolute value of 5? How do you know? *5; elicit that the absolute value of a number is its distance from zero on the number line.***

➤ **What other integer has the same absolute value as 5? Why? *−5; elicit that −5 is 5 units from zero on the number line.***

2. Write $|5| = 5$ and $|-5| = 5$ for display. Remind the students that these symbols indicate the absolute value of a number and are read "the absolute value of 5 is 5" and "the absolute value of negative 5 is 5." Absolute value of a number is always a positive number since distance is expressed as a positive value.

➤ **What types of integers always have the same absolute value? *opposites***

➤ **In what direction are you moving on a number line if the values of the numbers are increasing? *in a positive direction; elicit right on a horizontal number line and up on a vertical number line.* In what direction are you moving on a number line if the values of the numbers are decreasing? *in a negative direction; elicit left on a horizontal number line and down on a vertical number line***

3. Write the following number comparisons for display. Direct the students to copy and complete the number sentences using > or <.

$-1 > -2$ $\quad 6 > -6$ $\quad -1 < 0$ $\quad 0 < 5$

4. Write the following sets of numbers for display and direct the students to order each set from least to greatest on his own paper.

8, −1, −8, 3 $\quad$ ***−8, −1, 3, 8***

−9, 5, −4, −1 $\quad$ ***−9, −4, −1, 5***

Add and subtract integers

1. Write the following equations for display. Direct the students to copy and complete the equations. Choose students to solve each problem using the Algebra Mat and a number line. (Refer to Lessons 156–158.)

$-4 + 5 =$ ***1*** $\quad -3 + -5 =$ ***−8*** $\quad 3 + 2 =$ ***5***

$3 - 7 =$ ***−4*** $\quad 9 - -6 =$ ***15*** $\quad -8 - -2 =$ ***−6***

$3 - 5 =$ ***−2*** $\quad -9 - -2 =$ ***−7*** $\quad -6 + 2 =$ ***−4***

$-7 - 3 =$ ***−10*** $\quad 6 - -5 =$ ***11*** $\quad -9 - 6 =$ ***−15***

2. Remind the students that when you subtract a number it is the same as adding its opposite. Write and solve an addition equation for each of the subtraction problems. (Refer to Lesson 158.)

3 + −7 = −4 $\quad$ ***9 + 6 = 15*** $\quad$ ***−8 + 2 = 6***

3 + −5 = −2 $\quad$ ***−9 + 2 = −7*** $\quad$ ***−9 + −6 = −15***

−7 + −3 = −10 $\quad$ ***6 + 5 = 11***

3. Write the following comparisons for display and choose students to write > or <.

$-1°F < 1°F$ $\quad -6 + -3 > -1 - 10$

$-4 - 2 < 1 + 2$ $\quad 8° C > 2° C$

Multiply and divide integers

1. Write the following problems for display. Direct the students to draw a number line and use arrows to show the following multiplication and division problems. Choose students to solve each problem using the Algebra Mat and a number line.

$4 \times -3 =$ ***−12*** $\quad -12 \div -3 =$ ***4***

$7 \times -2 =$ ***−14*** $\quad -14 \div -2 =$ ***7***

2. Write the following problems for display. Elicit that negative sets cannot be pictured. Direct the students to solve the problems using the Commutative Property or by thinking of the negative multiplier as resulting in an answer opposite to that of a positive multiplier.

$-3 \times 2 =$ ***−6*** $\quad -2 \times 4 =$ ***−8*** $\quad -4 \times -2 =$ ***8***

Elicit the related division equations.

$-6 \div 2 =$ ***−3*** $\quad -8 \div 4 =$ ***−2*** $\quad 8 \div -2 =$ ***−4***

➤ **What pattern do the integer signs follow in both multiplication and division? *Elicit that multiplication and division with like signs result in a positive answer, and multiplication and division with unlike signs result in a negative answer.***

3. Write the following comparisons for display and choose students to write > or <.

$-7 \times 2 < -3 \times -2$ $\quad -6 \div -2 > -16 \div 2$

$-10 \div 2 > -3 \times 2$ $\quad -3 \times 2 < -3 \div -1$

4. Elicit the Order of Operations from the students. ***parentheses, exponents, multiplication and division (left to right), addition and subtraction (left to right)***

5. Write the following equations for display. Guide the students in applying the Order of Operations to solve the equations.

$10 \div -2 + -5 =$ ***−10*** $\quad 4 + -6 \times 2 =$ ***−8***

$5(2 - 3) =$ ***−5*** $\quad -3 \times 4 - 5 =$ ***−17***

CHAPTER 17 REVIEW

Complete the sentence.

1. The set of whole numbers and their opposites is called ___. ***integers***
2. The distance a number is from zero is that number's ___. ***absolute value***
3. The numbers 4 and $^-4$ are the same distance from ___. ***zero***

zero
integers
absolute value

Write the integer.

4. The thermometer shows the temperature as 13 degrees below zero. ***$^-13$***
5. Every four years February has an extra day. ***1***
6. The distance to school is 25 miles. ***25***
7. Three inches of rain fell in the month of June. ***3***
8. Part of the city of New Orleans is 7 feet below sea level. ***$^-7$***
9. David's miniature golf score was 4 strokes under par. ***$^-4$***

Write a comparison sentence using > or <.

10. $^-2$ **>** $^-27$
11. $^-16$ **<** $^-14$
12. 3 **>** $^-6$
13. 6 **>** $^-6$
14. 8 **>** $^-9$
15. $^-2$ **<** $^-1$
16. 0 **>** $^-1$
17. $^-3$ **<** 1
18. 2 **<** 3

Write the integers from *least* to *greatest*.

19. $^-6$ 3 $^-8$ 1 — ***$^-8$ $^-6$ 1 3***
20. 16 $^-17$ $^-13$ 12 — ***$^-17$ $^-13$ 12 16***
21. $^-1$ 5 2 $^-5$ — ***$^-5$ $^-1$ 2 5***
22. $^-4$ $^-1$ $^-2$ 1 — ***$^-4$ $^-2$ $^-1$ 1***

Add.

23. $^-7 + 9$ ***2***
24. $^-8 + 5$ ***$^-3$***
25. $^-4 + ^-7$ ***$^-11$***
26. $^-6 + ^-4$ ***$^-10$***
27. $^-12 + ^-1$ ***$^-13$***
28. $7 + ^-4$ ***3***
29. $^-3 + 3$ ***0***
30. $0 + ^-7$ ***$^-7$***
31. $8 + ^-9$ ***$^-1$***

Subtract.

32. $6 - ^-3$ ***9***
33. $^-12 - 8$ ***$^-20$***
34. $0 - ^-6$ ***6***
35. $5 - 13$ ***$^-8$***
36. $^-4 - ^-1$ ***$^-3$***
37. $8 - 17$ ***$^-9$***

Solve.

38. $6 \times ^-2$ ***$^-12$***
39. $^-3 \times ^-2$ ***6***
40. $12 \div ^-6$ ***$^-2$***
41. $^-16 \div ^-4$ ***4***
42. $27 \div ^-9$ ***$^-3$***
43. $^-3 \times 7$ ***$^-21$***
44. $^-21 \div ^-3$ ***7***
45. $^-3 \times ^-8$ ***24***
46. $24 \div ^-8$ ***$^-3$***

Write a comparison sentence using > or <.

47. $3 + ^-5$ **<** $1 - 1$ ***$^-2 < 0$***
48. $0 - ^-3$ **>** $2 + ^-2$ ***$3 > 0$***
49. $^-6 + 3$ **<** $4 - 5$ ***$^-3 < ^-1$***
50. $4 - 7$ **>** $2 \times ^-3$ ***$^-3 > ^-6$***
51. $^-2 - ^-1$ **<** $^-4 + 6$ ***$^-1 < 2$***
52. $4 + 2$ **>** $^-6 \div 1$ ***$6 > ^-6$***

Use the Order of Operations to solve.

53. $3(^-5 + ^-1)$ ***$^-18$***
54. $^-4 \times 3 - 6$ ***$^-18$***
55. $2 + ^-12 \div 2$ ***$^-4$***

A sign marks sea level about two-thirds of the way up a cliff in Badwater Basin, California.

Write an equation for a word problem

Direct the students to write an equation for the following word problems. Choose students to explain to the class the procedure they used to solve the problem.

At 6:00 PM the temperature was $^-8$°F. By midnight the temperature had dropped 6 degrees. What was the temperature then? ***$^-8$°F − 6°F = $^-14$°F***

Arden's checking account balance was $5.00. He wrote a check for $7.00. Then he hurried to the bank to deposit $12.00. What is his account balance now? ***$5.00 − $7.00 + $12.00 = $10.00 or $5.00 + $^-$7.00 + $12.00 = $10.00***

Jonathan's game score was 12. Madison's score was $^-8$. What is the difference between the two game scores? ***12 − $^-8$ = 20 points***

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Lesson 165

Student Text pp. 396–99

Chapter 17 Test

Cumulative Review

For a list of the skills reviewed in the Cumulative Review, see the Lesson Objectives for Lesson 165 in the Chapter 17 Overview on page 374 of this Teacher's Edition.

Student Materials

- Cumulative Review Answer Sheet, page IA9 (CD)

Use the Cumulative Review on Student Text pages 396–98 to review previously taught concepts and to determine which students would benefit from your reteaching of the concepts. To prepare the students for the format of achievement tests, instruct them to work on a separate sheet of paper, if necessary, and to mark the answers on the Cumulative Review Answer Sheet.

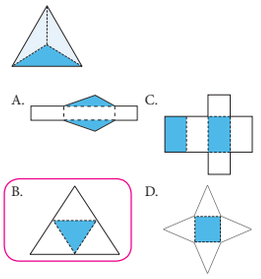
Read aloud the Career Link on Student Text page 399 (page 397 of this Teacher's Edition) and discuss the value of math as it relates to a physical education teacher.

Mark the answer.

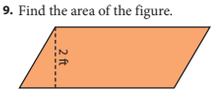
7. Which polygon has 7 sides and 7 angles?

A. pentagon
C. heptagon
B. hexagon
D. quadrilateral

8. Match the figure to its net.

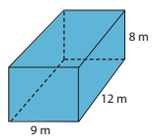


9. Find the area of the figure.



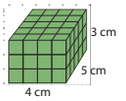
A. 7 ft
C. 10 ft²
B. 10 ft³
D. 12 ft

10. Find the surface area.



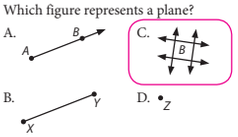
A. 204 m³
C. 864 m³
B. 552 m²
D. 900 m²

11. Find the volume.

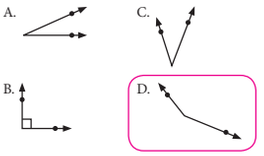


A. 20 cm³
C. 35 cm³
B. 60 cm³
D. 70 cm³

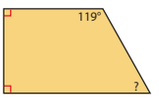
12. Which figure represents a plane?



13. Which angle is obtuse?



14. What is the measure of the unknown angle?



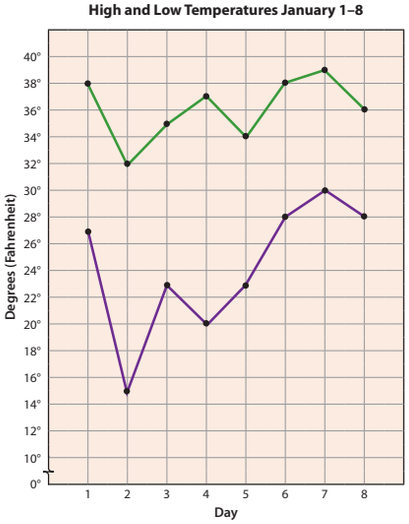
A. 21°
C. 50°
B. 39°
D. 61°

CUMULATIVE REVIEW

Test Prep

Use the data from the double line graph to find the answer.

High and Low Temperatures January 1–8



1. What is the range in the temperature on the graph?

A. 24°
C. 30°
B. 27°
D. 38°

2. What is the range of high temperatures?

A. 5°
C. 7°
B. 6°
D. 8°

3. What is the average high temperature?

A. 35.75°F
C. 36.5°F
B. 36.125°F
D. 37°F

4. What day shows the least difference between the high and low temperatures?

A. Day 1
C. Day 4
B. Day 2
D. Day 8

5. What is the mode for the high temperature?

A. 32°F
C. 38°F
B. 34°F
D. 40°F

6. What was the change in low temperature from Day 1 to Day 2?

A. −12°
C. 10°
B. 12°
D. −10°

Mark the answer.

15. Use rounding to estimate.

$$\begin{array}{r} 28.36 \\ -\ 12.52 \\ \hline \end{array}$$

A. 10
C. 30
B. 20
D. 40

16. Round to the nearest cent.

$26\overline{)\$86.00}$

A. $2.07
C. $3.08
B. $3.07
D. $3.31

17.

$$\begin{array}{r} 4 \text{ pt } 3 \text{ c} \\ -\ 3 \text{ pt } 1 \text{ c} \\ \hline \end{array}$$

A. 1 pt 7 c
C. 2 pt 5 c
B. 1 pt 2 c
D. 4 pt

18. Solve for n.

$n \times 132 = 528$

A. $n = 1$
C. $n = 3$
B. $n = 2$
D. $n = 4$

19. Estimate the answer.

$\frac{3}{8} + \frac{5}{6} \approx$ ___

A. $1\frac{1}{2}$
C. $2\frac{1}{2}$
B. 2
D. 3

20. $3\frac{1}{8} - \frac{3}{4} =$ ___

A. $1\frac{5}{8}$
C. $2\frac{1}{4}$
B. $2\frac{3}{8}$
D. $3\frac{1}{4}$

21. $4\frac{1}{2} \times 2\frac{1}{6} =$ ___

A. $8\frac{1}{6}$
C. $9\frac{1}{4}$
B. $8\frac{1}{2}$
D. $9\frac{3}{4}$

22. $2\frac{3}{4} \div \frac{1}{2} =$ ___

A. $1\frac{3}{8}$
C. $5\frac{1}{2}$
B. $3\frac{1}{8}$
D. 6

23. Devin has 60 minutes to do his chores. If he has worked for 36 minutes, what part of his chore time has passed?

A. $\frac{2}{5}$
C. $\frac{3}{5}$
B. $\frac{4}{5}$
D. $\frac{5}{5}$

24. What is three hundred one and six thousand, four hundred two ten thousandths in standard form?

A. 301.6402
C. 3,001.6042
B. 310.6042
D. 301.642

25. A photographer took student pictures for three schools. He took 125 pictures at the first school, 230 at the second, and 560 at the third. Which expression could be used to determine the number of pictures taken at the three schools?

A. 560 − (230 + 125)
C. (125 + 230) × 560
B. 125 + 230 + 560
D. (230 − 125) + 560

CAREER LINK

Physical Education Teacher

Do you know what your target heart rate is? Your target heart rate is based on your age and heartbeats per minute. If your heart rate is too fast, you may hurt yourself when exercising. If your heart rate does not increase enough, you may not be getting the full benefit of your exercise. To calculate a target heart rate, subtract your age from 220 beats per minute. If you are twelve years old, your maximum safe target rate is 208 beats per minute. Doctors recommend that you stay within 55%–90% of the maximum rate that is safe for exercise. A physical education teacher uses target heart rate ranges to help his students achieve a safe workout without placing them in danger.

Counting, multiplying, and dividing are math skills a physical education teacher uses every day while collecting equipment to correlate with the number of students in his class. He uses time and measurement skills when measuring for events such as the distance you kick or throw a ball and the time it takes you to run around a track. Math skills are also used for student assessment. The teacher must carefully record daily participation, skill tests, fitness exams, and written tests in order to calculate an accurate grade for each student.

Many schools and individuals use the President's Challenge Physical Fitness Program. An award is given to individuals who meet the guidelines for running a mile and doing pushups, sit-ups, and sit-and-reaches. Time limits and number of repetitions are established according to the age of the student. Math can be used by both teachers and students as a motivational tool to achieve faster times and more repetitions while performing drills.

The Bible teaches that our bodies are marvelous gifts from God; we are responsible for using them in a way that pleases Him (1 Corinthians 6:19–20). Part of pleasing God is making sure that we keep our bodies physically fit. A physical education teacher plays an important role in God's world. He shows people how to properly value one of God's most precious gifts, the gift of life.