8 Divide Fractions

DARING THE WIND

New York, New York **March 11, 1888**

In March of 1888, residents of New York City were enjoying the end of the mildest winter they had experienced in seventeen years. Temperatures had remained in the upper 40s to 50s; spring was a little more than a week away. On the evening of March 11, a cold Arctic air mass, which had been slowly approaching, collided with a warm tropical air mass and headed straight for New York. These two systems created a storm similar to a hurricane. It swept through the city and was known as "The Great Blizzard of 1888" or "The White Hurricane." By the next morning, snow several feet deep covered streets and sidewalks, and more swirled through the air with blinding fury.

One New York resident remarked that the air looked as if people were dumping buckets of flour from every rooftop. Some businesses closed their doors, and others set up cots so that those trapped at work could spend the night. There were reports that the temperature dropped so quickly that sparrows froze on telephone wires right where they had perched!

Snow covers part of West 11th Street after the Blizzard of 1888.

The storm continued its cold, wet downpour for over thirty-six hours so that even the Stock Exchange was closed for three days.

Fifty inches of snow fell in most areas, and wind drifts carried the snow in some areas twenty feet high. Because of the high levels of snow, the elevated trains were paralyzed, leaving nearly 15,000 passengers stranded in the cold.

Sergeant Elias Dunn, chief of the weather observatory, and his workers trudged through the storm to the observatory tower to find out why they were not receiving any weather information. They found their communication equipment completely silent. The anemometer, an important instrument for noting wind direction and velocity, was frozen in a shell of ice. Without the weather instruments, the men would have no way to predict and warn people in the city of more stormy weather forthcoming. People might be caught in unexpected snow and more lives might be lost. Sergeant Dunn asked for a man to climb the twenty-five-foot pole on which the anemometer rested to chop away the ice. It was a risky feat, but Francis Long, his deputy, finally volunteered to attempt it. Winds of seventy-five miles per hour gusted around the ice-covered pole. The other men in the group hung on to the bottom of the pole to try to steady it. But Francis could still feel it shaking as he slowly shinnied up, hatchet in hand. When at last he reached the top, Francis chopped away the ice and replaced broken wiring in the instrument. Soon it was working again.

The surprise blizzard of 1888 caused an estimated twenty-five million dollars in damages from fires alone, and four hundred people lost their lives.

The word *anemometer* is taken from the Greek word *anemos*, which means "wind."

In 1450, Leon Battista Alberti, an Italian architect, invented the first mechanical anemometer. Later, Robert Hooke, an Englishman who received credit for being the inventor of the first anemometer, reinvented it in 1664.

Irishman John Thomas Romney also reinvented the anemometer in 1846 by replacing the four hemispherical cups with plates. This version of the device is still in use today.

Sonic anemometers have no moving parts; rather they use sound waves to measure wind direction and velocity.

Divide Fractions

Lesson	Topic	Lesson Objectives	Chapter Materials
70	**Divide by a Fraction**	• Demonstrate an understanding of dividing a whole number by a fraction and dividing a fraction by a fraction • Divide a mixed number by a fraction • Use a number line to solve a division equation with fractions • Draw a diagram to solve a division equation with fractions • Check a division problem using multiplication • Write and solve an equation for a word problem	**Teacher Manipulatives Packet:** • Fraction Kit **Student Manipulatives Packet:** • Fraction Kit **Instructional Aids (Teacher's Toolkit CD):** • Cumulative Review Answer Sheet (page IA9) for each student • Multi-Step Word Problems (page IA42) **Christian Worldview Shaping (Teacher's Toolkit CD):** • Page 22 **Other Teaching Aids:** • Colored markers or chalk: 3 different colors **Math 6 Tests and Answer Key** **Optional (Teacher's Toolkit CD):** • Fact Review pages • Application pages • Calculator Activities
71	**Divide Fractions**	• Demonstrate an understanding of dividing a fraction to find a quotient with a remainder • Draw a diagram to solve a division equation with fractions • Use a number line to solve a division equation with fractions • Check a division problem using multiplication	
72	**Multiply by the Reciprocal**	• Identify the reciprocal of a fraction • Divide by multiplying by the reciprocal of the divisor • Divide a fraction by a whole number • Draw a diagram to solve a division word problem with fractions • Check a division problem using multiplication • Write related multiplication and division equations for a division problem with fractions	
73	**Mixed Numbers & Reciprocals**	• Demonstrate an understanding of dividing mixed numbers • Divide mixed numbers by multiplying by the reciprocal of the divisor • Check a division problem using multiplication • Draw a diagram to solve a division word problem with a mixed number • Use a number line to solve a division word problem with a mixed number • Solve fraction word problems	
74	**Multi-Step Equations**	• Write and solve a multi-step equation for a multi-step word problem • Demonstrate an understanding of the Order of Operations	
75	**Properties**	• Apply mathematical properties to evaluate an expression with fractions • Use the Order of Operations to evaluate an expression • Substitute a given value for a variable in an expression	
76	**More Multi-Step Word Problems**	• Use the Order of Operations to write and solve a multi-step equation for a multi-step word problem • Determine whether a multi-step problem has too much or too little information	
77	**Chapter 8 Review**	• Review	
78	**Chapter 8 Test Cumulative Review**	• Add integers • Complete a number sequence • Determine an equivalent expression • Evaluate an expression with more than one operation • Read and interpret a double line graph	

A Little Extra Help

Use the following to provide "a little extra help" for the student that is experiencing difficulty with the concepts taught in Chapter 8.

Divide a whole number by a fraction—Prepare four paper strips that are 5, 6, 7, and 8 inches long. Also cut 16 paper strips that are $\frac{1}{2}$ inch long. Write $5 \div \frac{1}{2}$ for display. Direct the student to place $\frac{1}{2}$-inch pieces alongside the 5-inch strip and to count the number of $\frac{1}{2}$-inch pieces. ***10*** Instruct him to solve the problem by first renaming the whole number as an improper fraction with the same denominator as the divisor and then dividing the numerators and the denominators: $5 \div \frac{1}{2} = \frac{10}{2} \div \frac{1}{2} =$ ***10***. Or you may choose to have the student solve the problem by multiplying by the reciprocal of the divisor: $5 \div \frac{1}{2} =$ $\boldsymbol{5 \times 2 = 10}$. If necessary, remind the student that every whole number has a denominator of 1. Direct him to compare the answer with his answer when he used the paper strips. Repeat the procedure for $6 \div \frac{1}{2} =$ ***twelve $\frac{1}{2}$-inch pieces; $\frac{12}{2} \div \frac{1}{2} = 12$; $6 \times 2 = 12$***, $7 \div \frac{1}{2} =$ ***fourteen $\frac{1}{2}$-inch pieces; $\frac{14}{2} \div \frac{1}{2} = 14$; $7 \times 2 = 14$***, and $8 \div \frac{1}{2} =$ ***sixteen $\frac{1}{2}$-inch pieces; $\frac{16}{2} \div \frac{1}{2} = 16$; $8 \times 2 = 16$***.

Divide a mixed number by a fraction—Provide a ruler with sixteenth-inch markings. Write $1\frac{3}{4} \div \frac{1}{16}$ for display and explain to the student that he can use the ruler to solve the equation. Direct him to draw a line $1\frac{3}{4}$ inches long, to make marks on the line to show sixteenth-inch sections, and to count the number of sections. ***28*** Instruct the student to rename the mixed number as an improper fraction and to solve the problem by multiplying by the reciprocal of the divisor. Allow him to write 1 as the denominator of 16 if needed and remind him to use cancellation. $\frac{7}{4} \times$ ***16 = 28*** Direct him to compare the two answers. Repeat the procedure for $1\frac{1}{2} \div \frac{1}{8}$ ***twelve $\frac{1}{8}$-inch sections; $\frac{3}{2} \times 8 = 12$*** and $1\frac{7}{8} \div \frac{5}{8}$ ***three $\frac{5}{8}$-inch sections; $\frac{15}{8} \times \frac{8}{5} = 3$***.

Identify mathematical properties—Make flashcards for each mathematical property. (See Lesson 75.) Write the name of a property on the front of the card and write a fraction example on the back. When the student is shown the name of a property, he should give a fraction example; e.g., Commutative Property of Multiplication $\frac{3}{4} \times \frac{1}{2} = \frac{1}{2} \times \frac{3}{4}$. When the student is shown a fraction example on the back of a card, he should give the name of the property; e.g., $0 + 3\frac{5}{8} = 3\frac{5}{8}$ ***Identity Property of Addition***.

Math Facts

Throughout this chapter, review fractions using Fact Review pages on the Teacher's Toolkit CD. Also, review addition, subtraction, multiplication, and division facts using Fact Review pages or a Fact Fun activity on the Teachers's Toolkit CD, or you may use flashcards.

Lesson 70

Student Text pp. 168–71
Daily Review p. 429a

Objectives
- Demonstrate an understanding of dividing a whole number by a fraction and dividing a fraction by a fraction
- Divide a mixed number by a fraction
- Use a number line to solve a division equation with fractions
- Draw a diagram to solve a division equation with fractions
- Check a division problem using multiplication
- Write and solve an equation for a word problem

Teacher Materials
- Fraction Kit: fraction circles

Student Materials
- Fraction Kit: fraction circles

Notes
The use of fraction circles, number lines, and drawings to picture problems in Lesson 70 and Lesson 71 will aid the student's understanding of dividing fractions and prepare him for dividing fractions by multiplying by the reciprocal in Lesson 72.

Preview the Fact Review pages, the Application pages, and the Calculator Activities located on the Teacher's Toolkit CD.

Introduce the Lesson

Guide the students in reading aloud the story and facts on pages 168–69 of the Student Text (pages 166–67 of this Teacher's Edition).

Teach for Understanding

Demonstrate an understanding of dividing a whole number by a fraction

1. Write 10 ÷ 2 for display. Remind the students that this problem can represent 10 divided into sets of 2.
   Select a student to illustrate the problem. ***Accept any picture showing 10 parts of a whole divided into sets of 2.***

Mrs. Tyler brought 2 pies to the church fellowship dinner. If $\frac{1}{8}$ of a pie is a serving, how many servings will the pies provide? ***16 servings***

➤ **What is the question asking you to find?** ***Elicit that you need to find how many servings of $\frac{1}{8}$ are in 2 pies.***
➤ **How many servings will 2 pies provide? How do you know?** ***16; accept any reasonable explanations based on the students' knowledge of fractions; e.g., since 1 pie divided into eighths is 8 servings, 2 pies will provide 16 servings.***
➤ **What division equation can you write for this word problem?** ***2 ÷ $\frac{1}{8}$ = 16 servings***

2. Write *2 ÷ $\frac{1}{8}$ = 16 servings*. Explain that dividing by a fraction is similar to dividing by a whole number. Elicit that the dividend (2) represents the total, the divisor ($\frac{1}{8}$) represents the number in each set, and the quotient (16) represents the number of sets.
3. Demonstrate solving the equation by first displaying 2 whole fraction circles and then renaming each circle as eighths. Point out that when you divide a whole number by a fraction, the denominator in the divisor tells you the number of parts into which each whole is partitioned.

➤ **How could you check the quotient of the division equation?** ***Elicit that you multiply the quotient and the divisor, which is similar to checking a division problem when the divisor is a whole number.***
Choose a student to demonstrate checking the equation. ***16 × $\frac{1}{8}$ = $\frac{16}{8}$ = 2***

4. Use the following procedure for this word problem.

Mr. Paul bought 4 pizzas for his youth baseball team. If $\frac{1}{4}$ of a pizza is a serving, how many servings will the pizzas provide? ***16 servings***

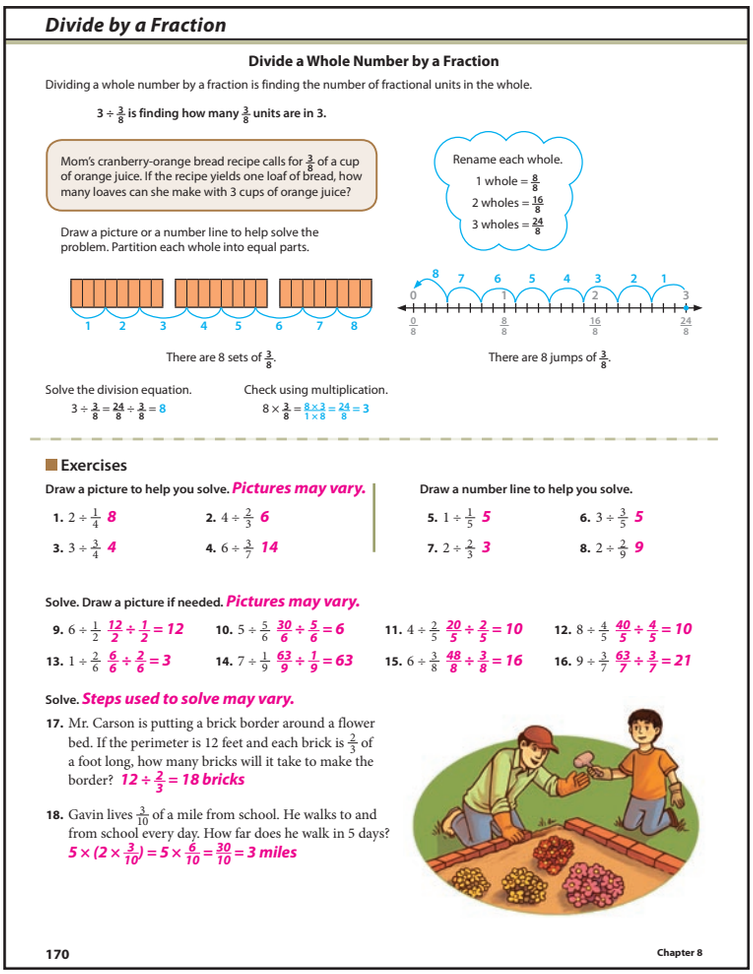
➤ **What is this question asking you to find?** ***Elicit that you need to find how many servings of $\frac{1}{4}$ are in 4 pizzas.***
➤ **What division equation can you write to solve this word problem?** ***4 ÷ $\frac{1}{4}$ = ___*** Write the equation.
➤ **How could you use a number line to solve the equation?** ***Elicit that you could draw a number line from 0 to 4 to represent the 4 pizzas, partition each whole into fourths, and count the number of $\frac{1}{4}$ units.***

5. Select a student to draw the number line. Instruct him to begin at 4 and draw arcs to represent jumps of $\frac{1}{4}$ all the way back to 0. Lead in counting the jumps. (See Student Text page 170 for an example.)
➤ **How many servings will 4 pizzas provide?** ***16 servings***
6. Choose a student to illustrate the solution by drawing a diagram. ***possible drawing: 4 circles partitioned into fourths***
7. Select a student to check the division problem. ***16 × $\frac{1}{4}$ = $\frac{16}{4}$ = 4***
8. Follow a similar procedure for 6 ÷ $\frac{2}{3}$. ***9*** Point out that you need to make groups of 2 thirds to determine the quotient.
9. Distribute the fraction circles and arrange the students in 3 groups. Write 3 ÷ $\frac{3}{8}$ for display. Instruct one group to solve the equation using the fraction circles, another group to solve it by drawing a number line, and the remaining group to draw a picture. ***8*** (***Note:*** Students using the fraction circles will need to share their fraction pieces.)
10. Repeat the procedure for 1 ÷ $\frac{1}{7}$ = ***7*** and 4 ÷ $\frac{4}{5}$ = ***5***. Assign to each group a different method for solving each problem.
➤ **What do you notice about dividing a whole number by a fraction?** ***Elicit that after you rename the whole number as the same fractional parts as the divisor, you divide both the numerators and the denominators to get the quotient, and the quotient is greater than the dividend and the divisor.***
11. Choose students to rewrite 6 ÷ $\frac{2}{3}$ = 9; 1 ÷ $\frac{1}{7}$ = 7; and 4 ÷ $\frac{4}{5}$ = 5 with the renamed dividend and to explain the dividing for each equation. ***$\frac{18}{3}$ ÷ $\frac{2}{3}$ = 9; $\frac{7}{7}$ ÷ $\frac{1}{7}$ = 7; $\frac{20}{5}$ ÷ $\frac{4}{5}$ = 5***

Demonstrate an understanding of dividing a fraction by a fraction

After his birthday party, Austin had $\frac{5}{6}$ of a pizza left. If $\frac{1}{6}$ of a pizza is a serving, how many servings of pizza remained? ***5 servings***

➤ **What is the question asking you to find?** ***Elicit that you need to find how many servings of $\frac{1}{6}$ are in $\frac{5}{6}$.***
➤ **How many servings of $\frac{1}{6}$ are in $\frac{5}{6}$ of a pizza? How do you know?** ***5; elicit that 1 serving of $\frac{1}{6}$ can be given to each of 5 people.***
➤ **What division equation can you write for the word problem?** ***$\frac{5}{6}$ ÷ $\frac{1}{6}$ = 5 servings***



Divide by a Fraction

Divide a Whole Number by a Fraction

Dividing a whole number by a fraction is finding the number of fractional units in the whole.

$3 \div \frac{3}{8}$ is finding how many $\frac{3}{8}$ units are in 3.

Mom's cranberry-orange bread recipe calls for $\frac{3}{8}$ of a cup of orange juice. If the recipe yields one loaf of bread, how many loaves can she make with 3 cups of orange juice?

Rename each whole.
1 whole = $\frac{8}{8}$
2 wholes = $\frac{16}{8}$
3 wholes = $\frac{24}{8}$

Draw a picture or a number line to help solve the problem. Partition each whole into equal parts.

There are 8 sets of $\frac{3}{8}$.

There are 8 jumps of $\frac{3}{8}$.

Solve the division equation.
$3 \div \frac{3}{8} = \frac{24}{8} \div \frac{3}{8} = 8$

Check using multiplication.
$8 \times \frac{3}{8} = \frac{8 \times 3}{1 \times 8} = \frac{24}{8} = 3$

Exercises

Draw a picture to help you solve. *Pictures may vary.*

1. $2 \div \frac{1}{4}$ *8*
2. $4 \div \frac{2}{3}$ *6*
3. $3 \div \frac{3}{4}$ *4*
4. $6 \div \frac{3}{7}$ *14*

Draw a number line to help you solve.

5. $1 \div \frac{1}{5}$ *5*
6. $3 \div \frac{3}{5}$ *5*
7. $2 \div \frac{2}{3}$ *3*
8. $2 \div \frac{2}{9}$ *9*

Solve. Draw a picture if needed. *Pictures may vary.*

9. $6 \div \frac{1}{2}$ $\frac{12}{2} \div \frac{1}{2} = 12$
10. $5 \div \frac{5}{6}$ $\frac{30}{6} \div \frac{5}{6} = 6$
11. $4 \div \frac{2}{5}$ $\frac{20}{5} \div \frac{2}{5} = 10$
12. $8 \div \frac{4}{5}$ $\frac{40}{5} \div \frac{4}{5} = 10$
13. $1 \div \frac{2}{6}$ $\frac{6}{6} \div \frac{2}{6} = 3$
14. $7 \div \frac{1}{9}$ $\frac{63}{9} \div \frac{1}{9} = 63$
15. $6 \div \frac{3}{8}$ $\frac{48}{8} \div \frac{3}{8} = 16$
16. $9 \div \frac{3}{7}$ $\frac{63}{7} \div \frac{3}{7} = 21$

Solve. *Steps used to solve may vary.*

17. Mr. Carson is putting a brick border around a flower bed. If the perimeter is 12 feet and each brick is $\frac{2}{3}$ of a foot long, how many bricks will it take to make the border? $12 \div \frac{2}{3} = 18$ *bricks*

18. Gavin lives $\frac{3}{10}$ of a mile from school. He walks to and from school every day. How far does he walk in 5 days? $5 \times (2 \times \frac{3}{10}) = 5 \times \frac{6}{10} = \frac{30}{10} = 3$ *miles*

Divide a Fraction by a Fraction

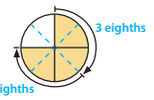
$\frac{3}{4} \div \frac{3}{8}$ is finding how many $\frac{3}{8}$ units are in $\frac{3}{4}$.

Rename the fractions being divided using a common denominator. Divide the numerators. Rename mixed numbers as improper fractions. Then find the common denominator and divide the numerators.

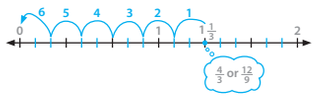
Dividing the common denominators gives a quotient of 1.

A picture or a number line can be drawn to help solve the problem. Repartition the parts to show the common denominator.

$\frac{3}{4} \div \frac{3}{8}$



$1\frac{1}{3} \div \frac{2}{9} = \frac{4}{3} \div \frac{2}{9}$



$\frac{3}{4} \div \frac{3}{8} =$
$\frac{6}{8} \div \frac{3}{8} = \frac{6}{3} = 2$
$\frac{6 \div 3}{8 \div 8} = \frac{6 \div 3}{1} = \frac{2}{1}$

$1\frac{1}{3} \div \frac{2}{9} =$
$\frac{4}{3} \div \frac{2}{9} =$
$\frac{12}{9} \div \frac{2}{9} = \frac{12}{2} = 6$
$\frac{12 \div 2}{9 \div 9} = \frac{12 \div 2}{1} = \frac{6}{1}$

Exercises

Rename fractions using the common denominator.
Write a division equation. Solve. *Pictures may vary.*

19. Draw rectangles to show how many $\frac{3}{4}$-cup servings can be made from $4\frac{1}{2}$ cups of berries.
$4\frac{1}{2} \div \frac{3}{4} = \frac{9}{2} \div \frac{3}{4} = \frac{18}{4} \div \frac{3}{4} = 6$ *servings*

20. Draw a number line to show how many $\frac{9}{16}$-inch blocks can be cut from a $5\frac{5}{8}$-inch block of wood.
$5\frac{5}{8} \div \frac{9}{16} = \frac{45}{8} \div \frac{9}{16} = \frac{90}{16} \div \frac{9}{16} = 10$ *blocks*

Rename fractions using the common denominator.
Draw a picture or a number line if needed to help you solve. *Pictures may vary.*

21. $\frac{2}{3} \div \frac{1}{6}$ $\frac{4}{6} \div \frac{1}{6} = 4$
22. $\frac{4}{5} \div \frac{1}{5}$ *4*
23. $\frac{1}{2} \div \frac{1}{8}$ $\frac{4}{8} \div \frac{1}{8} = 4$
24. $\frac{1}{3} \div \frac{1}{6}$ $\frac{2}{6} \div \frac{1}{6} = 2$
25. $\frac{5}{6} \div \frac{5}{12}$ $\frac{10}{12} \div \frac{5}{12} = 2$
26. $3 \div \frac{1}{2}$ $\frac{6}{2} \div \frac{1}{2} = 6$
27. $\frac{2}{3} \div \frac{1}{12}$ $\frac{8}{12} \div \frac{1}{12} = 8$
28. $\frac{3}{7} \div \frac{1}{7}$ *3*

Rename the mixed number as an improper fraction. Solve.

29. $3\frac{1}{8} \div \frac{5}{8}$
$\frac{25}{8} \div \frac{5}{8} = 5$
30. $1\frac{3}{7} \div \frac{2}{7}$
$\frac{10}{7} \div \frac{2}{7} = 5$
31. $3\frac{1}{3} \div \frac{5}{6}$
$\frac{10}{3} \div \frac{5}{6} = \frac{20}{6} \div \frac{5}{6} = 4$
32. $3\frac{2}{3} \div 1\frac{2}{9}$
$\frac{11}{3} \div \frac{11}{9} = \frac{33}{9} \div \frac{11}{9} = \frac{33}{11} = 3$

Practice & Application

33. $6\frac{5}{7} + 2\frac{1}{2}$ $9\frac{3}{14}$
34. $5\frac{1}{3} - 3\frac{5}{8}$ $1\frac{17}{24}$
35. $4\frac{1}{2} \times 6$ *27*
36. $3\frac{1}{2} + 0.75$ $4\frac{1}{4}$

J Explain why you can use multiplication to check the quotient for $\frac{1}{3} \div \frac{1}{12}$. Solve and check.
Multiplication is the inverse of division.
$\frac{1}{3} \div \frac{1}{12} = \frac{4}{12} \div \frac{1}{12} = 4$; $4 \times \frac{1}{12} = \frac{4}{12} = \frac{1}{3}$

Complete DAILY REVIEW 3 on page 429.

1. Write $\frac{5}{6} \div \frac{1}{6} = 5$ *servings* for display and choose a student to illustrate the equation. ***possible drawing: 1 whole circle partitioned into sixths with 5 sixths shaded***
2. Write $\frac{2}{3} \div \frac{1}{6}$. Direct the students to use the fraction circles to find how many servings of $\frac{1}{6}$ would be left if Austin had $\frac{2}{3}$ of a pizza. ***4 servings*** Select students to explain how they found the answer. ***Explanations may vary, but elicit that $\frac{2}{3}$ needed to be renamed as $\frac{4}{6}$ to equally divide the pizza into servings of $\frac{1}{6}$.***
   Choose a student to write the equation with the renamed dividend. $\frac{4}{6} \div \frac{1}{6} =$ ***4 servings*** Point out that dividing common denominators gives you a denominator of 1 in the quotient. Select one student to draw for display a diagram for the equation and another student to picture the equation on a number line. (See Student Text page 171 for examples.)

Samantha has $\frac{3}{4}$ of a pound of yogurt-covered raisins to put in bags for some of her friends. If she puts $\frac{1}{8}$ of a pound in each bag, how many bags of yogurt-covered raisins will she have? ***6 bags***

➤ **What is the question asking you to find?** ***how many bags of $\frac{1}{8}$ of a pound are in $\frac{3}{4}$***
➤ **What equation can you write to solve the word problem?** $\frac{3}{4} \div \frac{1}{8} =$ ___ Write the equation.

3. Direct the students to use their fraction circles to solve the equation. Choose one student to draw for display a picture for the equation and another student to picture the equation on a number line. ***6 bags*** Complete the equation.
4. Repeat the procedure to find out how many bags of yogurt-covered raisins Samantha will have if she puts $\frac{3}{8}$ of a pound in each bag. ***2 bags***
5. Follow a similar procedure for this word problem.

After serving dessert, Ella has $\frac{2}{3}$ of a cake left. If $\frac{1}{12}$ of the cake is a serving, how many servings can be cut from the leftover cake? $\frac{2}{3} \div \frac{1}{12} =$ ***8 servings*** How many servings could be cut if $\frac{1}{4}$ of the cake were left? $\frac{1}{4} \div \frac{1}{12} =$ ***3 servings***

➤ **What do you notice about dividing related fractions?** ***Elicit that after you rename so that the dividend and the divisor have a common denominator, you divide both the numerators and the denominators to get the quotient, and the quotient is greater than the dividend and the divisor.***

Divide a mixed number by a fraction

1. Write $3\frac{1}{3} \div \frac{2}{3}$ for display.
➤ **How could you solve this division problem?** ***Elicit that you can rename the mixed number $3\frac{1}{3}$ as the improper fraction $\frac{10}{3}$ and then divide.***
2. Select a student to rewrite the problem with the renamed dividend and solve it while the other students solve the problem on paper. $\frac{10}{3} \div \frac{2}{3} =$ ***5***
3. Guide the students in solving these equations.
   $2\frac{1}{3} \div \frac{7}{12} = \frac{7}{3} \div \frac{7}{12} = \frac{28}{12} \div \frac{7}{12} = 4$
   $3\frac{3}{4} \div \frac{3}{8} = \frac{15}{4} \div \frac{3}{8} = \frac{30}{8} \div \frac{3}{8} = 10$
   $4\frac{1}{2} \div \frac{1}{6} = \frac{9}{2} \div \frac{1}{6} = \frac{27}{6} \div \frac{1}{6} = 27$

Student Text pp. 170–71

Lesson 71

Student Text pp. 172–73
Daily Review p. 430b

Objectives
- Demonstrate an understanding of dividing a fraction to find a quotient with a remainder
- Draw a diagram to solve a division equation with fractions
- Use a number line to solve a division equation with fractions
- Check a division problem using multiplication

Teacher Materials
- Fraction Kit
- Christian Worldview Shaping, page 22 (CD)
- Colored markers or chalk: 2 different colors

Student Materials
- Fraction Kit

Introduce the Lesson

Each package of 10 pencils is made on an assembly line. If the assembly line bin contains 1,759 pencils, how many packages can be made? *175 packages*

➤ What equation can you write to solve this word problem? *1,759 ÷ 10 = ___*
➤ How many packages of 10 pencils can be made? *175*
➤ Are there any pencils remaining? *yes* how many? *9*
➤ What fractional part of a package does the remainder represent? How do you know? $\frac{9}{10}$*; elicit that 9 out of the next set of 10 pencils needed to make another package of pencils remain.*

Teach for Understanding

Demonstrate an understanding of dividing by a fraction

1. Distribute the fraction bars.

Sophia has 2 yards of ribbon for decorating picture frames. If she uses $\frac{1}{4}$ of a yard to decorate each frame, how many frames can she decorate? *8 frames*

2. Select a student to write the division equation needed to solve the word problem. *2 ÷ $\frac{1}{4}$ = ___*

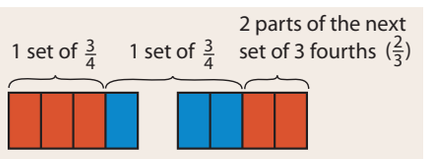
➤ What does the dividend 2 represent? *the total length of the ribbon* What does the divisor $\frac{1}{4}$ represent? *the length of ribbon needed for each frame; the amount in each set*
➤ What is 2 ÷ $\frac{1}{4}$? How do you know? *8; answers will vary, but elicit that when you rename 2 as $\frac{8}{4}$ and divide both the numerators and the denominators, the quotient is 8.* Complete the equation.
➤ If Sophia uses $\frac{3}{4}$ of a yard of ribbon to decorate each picture frame, what equation could you write to find how many frames she could decorate? *2 ÷ $\frac{3}{4}$ = ___*

3. Write 2 ÷ $\frac{3}{4}$ = ___ for display. Direct each student to place 2 whole fraction bars on his desk as you display them. Use the following procedure to guide the students in solving the equation. Demonstrate each step.

➤ What do you do first to solve the equation? *Rename the 2 wholes as 8 fourths.* Instruct the students to rename their wholes as fourths.
➤ What do you do next? *Divide the 8 fourths into sets of 3 fourths.* Direct the students to make sets of 3 fourths.
➤ How many sets of 3 fourths could you make? *2*
Explain that each of the two sets has 3 parts and that "fourths" names or labels the parts.
➤ How many parts (fourths) remain? *2*
➤ How many parts are needed to make the next set? *3*
➤ What fraction represents 2 of the 3 parts needed to make the next set of fourths? $\frac{2}{3}$
Write $\frac{2}{3}$ below the remaining 2 fourths.
➤ What whole number represents the sets of 3 fourths? *2*
➤ What fraction represents the remaining 2 fourths? $\frac{2}{3}$
➤ What does 2 ÷ $\frac{3}{4}$ equal? *2 $\frac{2}{3}$* Complete the equation.

4. Rewrite 2 ÷ $\frac{3}{4}$ = 2 $\frac{2}{3}$ as $\frac{8}{4}$ ÷ $\frac{3}{4}$ = 2 $\frac{2}{3}$. Point out that the divisor $\frac{3}{4}$ is the unit; it tells you to put $\frac{3}{4}$ of the dividend into each set. Explain that since dividing the denominators gives you a denominator of 1 in the quotient and since 1 is the denominator of every whole number, you can solve the equation by dividing the numerators.
Write 8 ÷ 3 in a division frame and demonstrate solving it: 2 $\frac{2}{3}$. Compare the quotient to the quotient in the equation. (See Student Text page 172.)
➤ How many picture frames can Sophia decorate if she uses $\frac{3}{4}$ of a yard of ribbon to decorate each frame? Why? *2 frames; elicit that $\frac{2}{3}$ is less than $\frac{3}{4}$, so Sophia would not have enough ribbon left to decorate another picture frame.*

5. Choose a student to illustrate 2 ÷ $\frac{3}{4}$ = 2 $\frac{2}{3}$ by drawing a diagram. Instruct him to use 2 different colors to shade alternate sets. *possible drawing:*

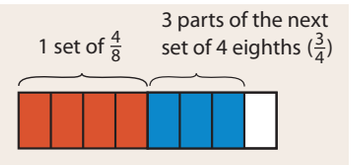


Choose another student to draw a number line to picture the equation.

6. Select a student to demonstrate checking the equation. Point out that since the remainder is represented by the fraction $\frac{2}{3}$, you multiply the mixed number in the quotient and the divisor. *2 $\frac{2}{3}$ × $\frac{3}{4}$ = $\frac{8}{3}$ × $\frac{3}{4}$ = $\frac{24}{12}$ = 2*

7. Write $\frac{7}{8}$ ÷ $\frac{1}{2}$ = ___ for display.

➤ To solve this equation, do you need to rename the dividend or the divisor? *the divisor* What do you rename the divisor as? $\frac{4}{8}$
Write $\frac{7}{8}$ ÷ $\frac{1}{2}$ = ___ and direct the students to use their fraction bars to solve the problem. Choose a student to display the solution. Select a second student to draw a diagram and a third student to picture the solution on a number line.



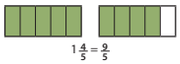
➤ What is $\frac{7}{8}$ ÷ $\frac{1}{2}$? *1 $\frac{3}{4}$*
➤ What does the quotient 1 $\frac{3}{4}$ represent? *1 set of $\frac{1}{2}$ and $\frac{3}{4}$ of the next set of $\frac{1}{2}$*
Choose a student to check the division equation. *1 $\frac{3}{4}$ × $\frac{1}{2}$ = $\frac{7}{8}$*

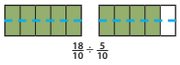
Divide Fractions

When renaming fractions using a common denominator, it is not always possible to divide the numerators evenly. If the numerators are not compatible, the quotient will be a fraction or a mixed number. Drawing a picture can help you solve the equation. Remember to write the final answer in lowest terms.

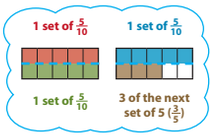
$1\frac{4}{5} \div \frac{1}{2} =$
$\frac{9}{5} \div \frac{1}{2} =$
$\frac{18}{10} \div \frac{5}{10} = \frac{18}{5} = 3\frac{3}{5}$

$5\overline{)18}\ 3\frac{3}{5}$, -15, 3

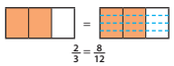




There are $3\frac{3}{5}$ sets of $\frac{1}{2}$ in $1\frac{4}{5}$.



$\frac{2}{3} \div \frac{3}{4} =$
$\frac{8}{12} \div \frac{9}{12} = \frac{8}{9}$



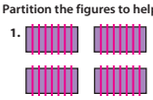
There is $\frac{8}{9}$ of a set.

There are only 8 of the 9 parts needed to make 1 whole set.
There is only part of a set of $\frac{3}{4}$ in $\frac{2}{3}$.

Exercises

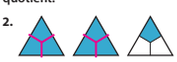
Partition the figures to help you find the quotient.

1.



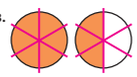
$4 \div \frac{3}{8}$ $\frac{32}{3} = 10\frac{2}{3}$

2.



$2\frac{1}{3} \div \frac{2}{3}$ $\frac{7}{2} = 3\frac{1}{2}$

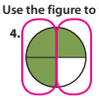
3.



$1\frac{1}{2} \div \frac{5}{6}$ $\frac{9}{5} = 1\frac{4}{5}$

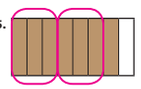
Use the figure to find the quotient.

4.



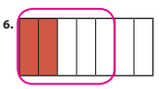
$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \div \frac{2}{4} = \frac{3}{2} = 1\frac{1}{2}$

5.



$\frac{7}{8} \div \frac{3}{8} = \frac{7}{3} = 2\frac{1}{3}$

6.



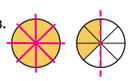
$\frac{2}{7} \div \frac{5}{7} = \frac{2}{5}$

7.

$2\frac{7}{8} \div \frac{3}{4}$

$\frac{23}{8} \div \frac{3}{4} = \frac{23}{8} \div \frac{6}{8} = 3\frac{5}{6}$

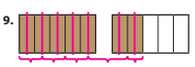
8.



$1\frac{3}{8} \div 1\frac{1}{2}$

$\frac{11}{8} \div \frac{3}{2} = \frac{11}{8} \div \frac{12}{8} = \frac{11}{12}$

9.



$1\frac{2}{5} \div \frac{3}{10}$

$\frac{7}{5} \div \frac{3}{10} = \frac{14}{10} \div \frac{3}{10} = \frac{14}{3} = 4\frac{2}{3}$

Answers on the page are shown using cancellation.

Find the quotient. Multiply to check your answer.
Draw a picture if needed. *Pictures may vary.*

10. $\frac{9}{10} \div \frac{3}{10}$ *3; $3 \times \frac{3}{10} = \frac{9}{10}$*
11. $4\frac{1}{4} \div \frac{3}{4}$ *$5\frac{2}{3}$; $5\frac{2}{3} \times \frac{3}{4} = \frac{17}{3} \times \frac{3}{4} = 4\frac{1}{4}$*
12. $\frac{5}{6} \div \frac{2}{3}$ *$1\frac{1}{4}$; $1\frac{1}{4} \times \frac{2}{3} = \frac{5}{4} \times \frac{2}{3} = \frac{5}{6}$*
13. $\frac{4}{5} \div \frac{4}{5}$ *1; $1 \times \frac{4}{5} = \frac{4}{5}$*
14. $\frac{9}{8} \div \frac{1}{2}$ *$2\frac{1}{4}$; $2\frac{1}{4} \times \frac{1}{2} = \frac{9}{4} \times \frac{1}{2} = \frac{9}{8}$*
15. $\frac{2}{9} \div \frac{5}{9}$ *$\frac{2}{5}$; $\frac{2}{5} \times \frac{5}{9} = \frac{2}{9}$*
16. $6 \div \frac{3}{7}$ *14; $14 \times \frac{3}{7} = 6$*
17. $3\frac{1}{7} \div \frac{4}{7}$ *$5\frac{1}{2}$; $5\frac{1}{2} \times \frac{4}{7} = \frac{11}{2} \times \frac{4}{7} = 3\frac{1}{7}$*
18. $\frac{1}{3} \div \frac{1}{2}$ *$\frac{2}{3}$; $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$*

Solve.

19. One-third of Mia's birthday cake is left. If $\frac{1}{12}$ of the cake is a serving, how many servings are left? *$\frac{1}{3} \div \frac{1}{12} = \frac{4}{12} \div \frac{1}{12} = 4$ servings*
20. Mrs. Korver teaches piano lessons for $3\frac{1}{2}$ hours on Tuesday afternoon. Each lesson is $\frac{1}{2}$ of an hour long. How many students does she teach on Tuesday? *$3\frac{1}{2} \div \frac{1}{2} = \frac{7}{2} \div \frac{1}{2} = \frac{7}{1} = 7$ students*
21. Two-thirds of the pizza was left from the Super Bowl party. If $\frac{1}{6}$ of a pizza is a serving, how many servings will the leftover pizza provide? *$\frac{2}{3} \div \frac{1}{6} = \frac{4}{6} \div \frac{1}{6} = 4$ servings*
22. Jocelyn had 3 yards of fabric. She cut it into pieces that are $\frac{3}{8}$ of a yard long. How many pieces does she have? *$3 \div \frac{3}{8} = \frac{24}{8} \div \frac{3}{8} = 8$ pieces*

Practice & Application

23. How many sets of $\frac{1}{4}$ are in $\frac{1}{2}$? Write a division equation. *$\frac{1}{2} \div \frac{1}{4} = \frac{2}{4} \div \frac{1}{4} = 2$*
24. Write a repeated addition problem for $6 \times \frac{3}{8}$. Solve. *$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{18}{8} = \frac{9}{4} = 2\frac{1}{4}$*
25. Use the Distributive Property to solve $3\frac{1}{2} \times \frac{5}{12}$.
26. Estimate the product of $4\frac{1}{9} \times 17\frac{1}{2}$. *$4 \times 18 = 72$*
27. Write 0.3×0.14 in fraction form. Solve. *$\frac{3}{10} \times \frac{14}{100} = \frac{42}{1000} = \frac{21}{500}$*
28. The paving crew planned to pave $\frac{4}{5}$ of the road on Saturday. By noon on Saturday they had completed $\frac{1}{2}$ of the job. What part of the paving was done by noon? *$\frac{1}{2} \times \frac{4}{5} = \frac{2}{5}$ paved*
29. An equilateral triangle has a perimeter of 29 inches. What is the length of each side? *$\frac{29}{3} = 9\frac{2}{3}$ inches or $29 \div 3 = 9\frac{2}{3}$*
30. Angle F measures 45°. Is the angle acute or obtuse? *acute*
31. How many sides are in a quadrilateral? *4 sides*
32. Draw Circle A. Draw and label radius $\overline{AB}$.
33. On Circle A, draw and label a chord $\overline{BE}$ that is not a diameter.
34.

2			
$\frac{1}{4}$	$\frac{2}{3}$	$\frac{1}{2}$	n

$n = \frac{7}{12}$

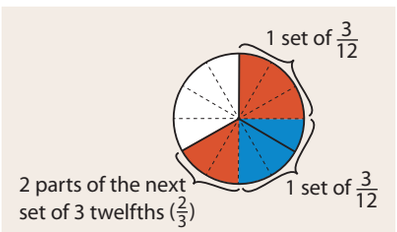
J Find the quotient for $8 \div \frac{4}{5}$. Explain why the quotient is always equal to or larger than the dividend when dividing a whole number by a fraction.

$8 \div \frac{4}{5} = \frac{40}{5} \div \frac{4}{5} = 10$

When dividing a whole number by a fraction, there will be at least 1 fractional part in each whole of the dividend. There will also be some part remaining.

Complete DAILY REVIEW b on page 430.

8. Follow a similar procedure for $\frac{2}{3} \div \frac{1}{4} =$ ___, using the fraction circles. *$\frac{8}{12} \div \frac{3}{12} = 2\frac{2}{3}$; 2 sets of $\frac{3}{12}$ ($\frac{1}{4}$) and $\frac{2}{3}$ of the next set of $\frac{3}{12}$ ($\frac{1}{4}$); $2\frac{2}{3} \times \frac{1}{4} = \frac{8}{12} = \frac{2}{3}$*



9. Follow a similar procedure as you guide the students in solving and checking the following equations; emphasize what the quotient represents. Instruct the students to draw a diagram and a number line for each equation.

$3 \div \frac{5}{7} = \frac{21}{7} \div \frac{5}{7} =$ *$4\frac{1}{5}$; $4\frac{1}{5} \times \frac{5}{7} = 3$*

$\frac{4}{9} \div \frac{4}{9} =$ *1; $1 \times \frac{4}{9} = \frac{4}{9}$*

$\frac{3}{4} \div \frac{2}{5} = \frac{15}{20} \div \frac{8}{20} =$ *$1\frac{7}{8}$; $1\frac{7}{8} \times \frac{2}{5} = \frac{3}{4}$*

$2\frac{4}{9} \div \frac{2}{3} = \frac{22}{9} \div \frac{6}{9} =$ *$3\frac{4}{6} = 3\frac{2}{3}$; $3\frac{2}{3} \times \frac{2}{3} = 2\frac{4}{9}$*

$3\frac{1}{2} \div \frac{4}{5} = \frac{35}{10} \div \frac{8}{10} =$ *$4\frac{3}{8}$; $4\frac{3}{8} \times \frac{4}{5} = 3\frac{1}{2}$*

10. **Christian Worldview Shaping (CD)**

Student Text pp. 172–73

Lesson 72

Student Text pp. 174–75
Daily Review p. 430c

Objectives
- Identify the reciprocal of a fraction
- Divide by multiplying by the reciprocal of the divisor
- Divide a fraction by a whole number
- Draw a diagram to solve a division word problem with fractions
- Check a division problem using multiplication
- Write related multiplication and division equations for a division problem with fractions

Teacher Materials
- Colored markers or chalk: 3 different colors

Teach for Understanding

Identify the reciprocal of a fraction

1. Write $\frac{3}{4} \times \frac{4}{3} =$ ___ and $\frac{5}{6} \times \frac{6}{5} =$ ___ for display. Elicit that the factors in each equation are *reciprocals* because the product of the factors is 1. Point out that you can find the reciprocal of a fraction by *inverting* the numerator and the denominator. Choose students to multiply the numerator and the denominator in the equations to verify that the product of each equation is 1. *$\frac{12}{12} = 1$; $\frac{30}{30} = 1$*
2. Write $8 \times$ ___ $= 1$ for display.

➤ **What fraction completes the equation? Why?** *$\frac{1}{8}$; elicit that a whole number can be expressed as a fraction with a denominator of 1, so the whole number 8 is equal to $\frac{8}{1}$, and its reciprocal is $\frac{1}{8}$.*

3. Guide the students in identifying the reciprocals to complete these equations.

$\frac{1}{4} \times$ ___ $= 1$ *$\frac{4}{1}$ or 4* $\quad\quad$ $\frac{5}{6} \times$ ___ $= 1$ *$\frac{6}{5}$*

$9 \times$ ___ $= 1$ *$\frac{1}{9}$* $\quad\quad$ $\frac{3}{7} \times$ ___ $= 1$ *$\frac{7}{3}$*

Divide by multiplying by the reciprocal of the divisor

Mrs. Olivia made 3 apple cakes for a bake sale at a local retirement center. If $\frac{1}{12}$ of a cake is a serving, how many servings of the apple cake can be sold? *36 servings*

➤ **How many servings of apple cake can be sold? How did you determine the answer?** *36 servings; accept any reasonable explanations.*

➤ **What division equation can you write for the word problem?** *$3 \div \frac{1}{12} = 36$ servings*

1. Write *$3 \div \frac{1}{12} = 36$ servings* for display and choose a student to draw a picture for the equation. *possible drawing: 3 rectangles with each rectangle partitioned into twelfths*

➤ **How many sets of $\frac{1}{12}$ are in 3?** *36*

➤ **Do you think it would be easy to picture the equation for the word problem if Mrs. Olivia had made 3 full-size sheet cakes and $\frac{1}{96}$ of a cake was a serving?**

2. Explain that when dividing fractions, you will not always be able to picture a problem when the divisor is a fraction. Dividing by a whole number would make problems such as $3 \div \frac{1}{96}$ easier to solve.
3. Write $3 \div \frac{1}{12} =$ for display.

➤ **What is the easiest whole number to divide by? Why?** *1; when the divisor is 1, the quotient will be the same as the dividend.*

Ask students to give examples. *Accept any equations with 1 as the divisor; e.g., $\frac{1}{4} \div 1 = \frac{1}{4}$; $5 \div 1 = 5$; $8\frac{5}{6} \div 1 = 8\frac{5}{6}$.*

➤ **How could you change the divisor $\frac{1}{12}$ to 1?** *Elicit that you could multiply $\frac{1}{12}$ by its reciprocal $\frac{12}{1}$.*

4. Write $(\frac{1}{12} \times \frac{12}{1})$ below the $\frac{1}{12}$. Explain that when you change the divisor you must use *compensation* to keep the equation equivalent; i.e., what is done to the divisor must also be done to the dividend.

➤ **Since you multiplied the divisor $\frac{1}{12}$ by $\frac{12}{1}$, what must you do to the dividend 3?** *Multiply 3 by $\frac{12}{1}$.*

Write $(3 \times \frac{12}{1})$ below the 3 and an equal sign (=) at the end of the expression. Remind the students that you can use cancellation to simplify fraction factors before you multiply. Elicit that cancellation can be used in the second part of the equation: $\frac{1}{12} \times \frac{12}{1} = 1$. Guide the students in completing the equation.

$$3 \div \frac{1}{12} =$$
$$(3 \times \frac{12}{1}) \div (\frac{1}{12} \times \frac{12}{1}) =$$
$$(3 \times \frac{12}{1}) \div 1 =$$
$$36 \div 1 = 36$$

➤ **What do you know about dividing by 1?** *The quotient is the same as the dividend.*

5. Explain that since multiplying by the reciprocal makes the divisor equal to 1 and the quotient will be the same as the dividend, it is not necessary to divide by 1. You can multiply the dividend by the reciprocal of the divisor when dividing by a fraction. Because multiplication and division are inverse operations and the reciprocal is the inverse of the divisor, multiplying by the reciprocal of the divisor is the same as dividing by the fraction.
6. Demonstrate solving the equation as shown. Remind the students that $3 \times \frac{12}{1}$ represents 3 sets of 12.

$$3 \div \frac{1}{12} =$$
$$3 \times \frac{12}{1} = 36$$

➤ **What does $3 \div \frac{1}{12}$ equal?** *36* Write $3 \div \frac{1}{12} = 36$ for display.

7. Select a student to demonstrate checking the division equation. *$36 \times \frac{1}{12} = 3$*
8. Explain that the inverse relationship between multiplication and division applies to fractions as well as to whole numbers and that writing related multiplication and division equations can be helpful when checking fraction division problems.

➤ **What related division equation can you write for $3 \div \frac{1}{12} = 36$?** *$3 \div 36 = \frac{1}{12}$* Write the equation.

➤ **What multiplication equation was used to check $3 \div \frac{1}{12} = 36$?** *$36 \times \frac{1}{12} = 3$* Write the equation.

➤ **What related multiplication equation can you write for $36 \times \frac{1}{12} = 3$?** *$\frac{1}{12} \times 36 = 3$* Write the equation.

➤ **What do you notice about the related multiplication equations?** *Elicit that the product of both equations is 3.*

9. Guide the students in multiplying by the reciprocal to find out how many servings of cake could be sold if Mrs. Olivia made 3 full-sized sheet cakes and $\frac{1}{96}$ of a cake was a serving. *$3 \div \frac{1}{96} = 3 \times \frac{96}{1} = 288$ servings*

Multiply by the Reciprocal

Dividing by a fraction is the same as multiplying by the reciprocal of the divisor. The **reciprocal** of the divisor is found by **inverting** the numerator and the denominator.

reciprocal
invert fractions

$\frac{3}{4} \div \frac{3}{4} = 1$ $\quad \frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$

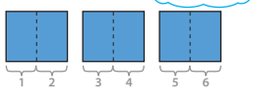
When a fraction is multiplied by its reciprocal, the product is 1.

The example below shows that dividing by a number and multiplying by the reciprocal gives the same result.

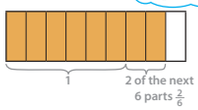
$15 \div 3 = 5$ $\quad 15 \times \frac{1}{3} = \frac{15}{3} = 5$

The reciprocal for 3 is $\frac{1}{3}$.

$3 \div \frac{1}{2}$ How many sets of $\frac{1}{2}$ are in 3?



$\frac{8}{9} \div \frac{2}{3}$ How many sets of $\frac{2}{3}$ are in $\frac{8}{9}$?



Divide	Multiply	Divide	Multiply
$3 \div \frac{1}{2} = \frac{6}{2} \div \frac{1}{2} = 6$	$3 \times \frac{2}{1} = \frac{3 \times 2}{1} = 6$	$\frac{8}{9} \div \frac{6}{9} = \frac{8}{6} = 1\frac{2}{6} = 1\frac{1}{3}$	$\frac{4}{\cancel{8}} \times \frac{\cancel{2}}{\cancel{2}} = \frac{4}{3} = 1\frac{1}{3}$
There are 6 halves in 3.	3 sets of 2 is 6.		

Exercises

Write the reciprocal to complete the equation.

1. $\frac{2}{3} \times \_\_ = 1$ *$\frac{3}{2}$*
2. $6 \times \_\_ = 1$ *$\frac{1}{6}$*
3. $\frac{3}{5} \times \_\_ = 1$ *$\frac{5}{3}$*
4. $12 \times \_\_ = 1$ *$\frac{1}{12}$*

Find the quotient by multiplying by the reciprocal. Write the answer in lowest terms. *Answers are shown using cancellation.*

5. $\frac{6}{8} \div \frac{1}{3}$ *$\frac{6}{8} \times \frac{3}{1} = \frac{18}{8} = \frac{9}{4} = 2\frac{1}{4}$*
6. $\frac{5}{6} \div \frac{1}{2}$ *$\frac{5}{6} \times \frac{2}{1} = \frac{5}{3} = 1\frac{2}{3}$*
7. $\frac{3}{4} \div 3$ *$\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$*
8. $\frac{3}{9} \div \frac{3}{4}$ *$\frac{3}{9} \times \frac{4}{3} = \frac{4}{9}$*
9. $\frac{1}{4} \div \frac{1}{8}$ *$\frac{1}{4} \times \frac{8}{1} = \frac{2}{1} = 2$*
10. $\frac{4}{9} \div 2$ *$\frac{4}{9} \times \frac{1}{2} = \frac{2}{9}$*
11. $\frac{1}{6} \div \frac{1}{3}$ *$\frac{1}{6} \times \frac{3}{1} = \frac{1}{2}$*
12. $4 \div \frac{1}{8}$ *$\frac{4}{1} \times \frac{8}{1} = 32$*
13. $6 \div \frac{2}{3}$ *$\frac{6}{1} \times \frac{3}{2} = \frac{9}{1} = 9$*
14. $7 \div \frac{1}{5}$ *$\frac{7}{1} \times \frac{5}{1} = 35$*
15. $\frac{4}{5} \div \frac{1}{5}$ *$\frac{4}{5} \times \frac{5}{1} = \frac{4}{1} = 4$*
16. $\frac{1}{5} \div \frac{1}{10}$ *$\frac{1}{5} \times \frac{10}{1} = \frac{2}{1} = 2$*
17. $\frac{8}{9} \div 4$ *$\frac{8}{9} \times \frac{1}{4} = \frac{2}{9}$*
18. $\frac{1}{2} \div \frac{1}{2}$ *$\frac{1}{2} \times \frac{2}{1} = \frac{1}{1} = 1$*
19. $\frac{2}{3} \div 5$ *$\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$*
20. $\frac{5}{6} \div \frac{2}{8}$ *$\frac{5}{6} \times \frac{8}{2} = \frac{20}{6} = 3\frac{1}{3}$*

Use the inverse relationship between multiplication and division to write related equations for the given fraction.

$2 \times \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$ ○○○ $2 \times \frac{3}{8} = \frac{3}{4}$ $\quad \frac{3}{8} \times 2 = \frac{3}{4}$ $\quad \frac{3}{4} \div \frac{3}{8} = 2$ $\quad \frac{3}{4} \div 2 = \frac{3}{8}$

Exercises

Use the given equation to write the related multiplication equation. Write two related division equations.

21. $5 \times \frac{2}{15} = \frac{2}{3}$
22. $\frac{2}{5} \times \frac{3}{4} = \frac{3}{10}$
23. $\frac{1}{4} \times \frac{3}{5} = \frac{3}{20}$

Solve. *Answers are shown using cancellation.*

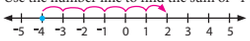
24. Mr. Whitemount bought a 15-pound bag of high-protein dog food for his dog, Bruno. How many feedings can he get from the bag if Bruno eats $\frac{3}{4}$ of a pound per feeding? *$15 \div \frac{3}{4} = \frac{15}{1} \times \frac{4}{3} = \frac{20}{1} = 20$ feedings*

Practice & Application

26. 

What fraction of the counters are negative? *$\frac{4}{7}$*

27. Use the number line to find the sum of $-4 + 6$. *2*



28. Draw a number line to show the quotient of $3 \div \frac{1}{8}$.
29. Draw a picture to show the quotient of $\frac{1}{3} \div \frac{1}{12}$. *Picture may vary.*
30. Solve. Find the difference between the two answers.

$(2 \times 9) + (2 \times 6)$ *30* $\quad 2 \times 9 \times 2 \times 6$ *216*

216 − 30 = 186

31. Use the addition properties to group the addends and solve. *Equations will vary.*

$\frac{3}{8} + \frac{1}{2} + \frac{5}{6} + \frac{5}{8} + \frac{4}{9} = \_\_$ *$2\frac{7}{9}$*

32. Solve the expression that shows that Cole had 4 sets of 24 colored pencils but lost 3 of the pencils.

$(4 \times 24) - 3$ $\quad 4 \times (24 - 3)$

$(4 \times 24) - 3 = 96 - 3 = 93$ colored pencils

25. Mr. Whitemount feeds his dog $1\frac{1}{2}$ pounds of food per day while Bruno is training for search-and-rescue work. How much food does Bruno eat in one week? *$7 \times 1\frac{1}{2} = \frac{7}{1} \times \frac{3}{2} = \frac{21}{2} = 10\frac{1}{2}$ pounds*

$\frac{1}{4} \times 5{,}280 = 1{,}320$ feet

33. One condition of a blizzard is visibility of less than $\frac{1}{4}$ of a mile for more than 3 hours. How many feet is $\frac{1}{4}$ of a mile? (1 mile = 5,280 feet)
34. If the decimal point moves 1 place to the right when a number is multiplied by 10, what is the product of 10×986.39? *9,863.9*

J Complete the table. *Steps used to solve may vary.*

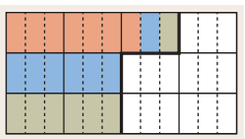
x	$x \div \frac{1}{4}$
1	4
2	*8*
3	*12*
4	*16*
5	*20*
10	*40*

x	$x \div \frac{2}{3}$
1	$1\frac{1}{2}$
2	*3*
3	*$4\frac{1}{2}$*
4	*6*
5	*$7\frac{1}{2}$*
10	*15*

Complete DAILY REVIEW C on page 430.

After the bake sale, Mrs. Olivia had $\frac{7}{12}$ of one apple cake left over. She divided the remaining cake equally among 3 bake-sale volunteers. How many pieces of cake did each volunteer receive? *$2\frac{1}{3}$ pieces*

10. Direct the students to draw a picture to find how many pieces of cake each volunteer received. Choose students to share their drawings. *possible drawing: 1 rectangle partitioned into twelfths with 7 of the twelfths shaded and $2\frac{1}{3}$ of the shaded twelfths placed in each of 3 sets.* Select a student to draw his pictures for display.
    - **How many pieces of cake did each volunteer receive?** *$2\frac{1}{3}$*
    - **How could you picture what fractional part of the cake each volunteer received?** *Elicit that since each of 3 volunteers received an equal part of the cake, each twelfth of the whole cake could be partitioned into 3 equal parts, repartitioning it into thirty-sixths.* Repartition the student's diagram into thirty-sixths.



    - **What fractional part of the whole cake did each volunteer receive?** *$\frac{7}{36}$*
    - **What division equation can you write for the diagram? Why?** *$\frac{7}{12} \div 3 = \frac{7}{36}$; elicit that you are finding what part of $\frac{7}{12}$ can be placed equally into 3 sets.* Write the equation.
11. Choose a student to demonstrate solving $\frac{7}{12} \div 3$ by multiplying by the reciprocal. *$\frac{7}{12} \times \frac{1}{3} = \frac{7}{36}$* Remind the students that the multiplication equation can be read *$\frac{7}{12}$ of $\frac{1}{3}$*. Point out that the product matches the quotient of the division equation written for the diagram.
12. Select a student to check the division equation. *$3 \times \frac{7}{36} = \frac{21}{36} = \frac{7}{12}$*
13. Guide the students in writing the related division and multiplication equations for $\frac{7}{12} \div 3 = \frac{7}{36}$. *$\frac{7}{12} \div \frac{7}{36} = 3$; $3 \times \frac{7}{36} = \frac{7}{12}$; $\frac{7}{36} \times 3 = \frac{7}{12}$*
14. For each of the following problems, direct the students to write the problem on paper. Then guide them in multiplying by the reciprocal to solve the problem, in checking the division problem, and in writing the related division and multiplication equations. Remind them that using cancellation before multiplying will give them a product in lower terms, but they still need to check to make sure that the product is in lowest terms.

    $\frac{1}{3} \div \frac{3}{4} = \frac{1}{3} \times \frac{4}{3} = \frac{4}{9}$; *$\frac{3}{4} \times \frac{4}{9} = \frac{1}{3}$*

    $7 \div \frac{5}{8} =$ *$7 \times \frac{8}{5} = \frac{56}{5} = 11\frac{1}{5}$; $\frac{5}{8} \times 11\frac{1}{5} = \frac{5}{8} \times \frac{56}{5} = 7$*

    $\frac{2}{3} \div 4 = \frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$; *$4 \times \frac{1}{6} = \frac{2}{3}$*

    $\frac{9}{10} \div \frac{3}{5} = \frac{9}{10} \times \frac{5}{3} = \frac{3}{2} = 1\frac{1}{2}$; *$\frac{3}{5} \times 1\frac{1}{2} = \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$*

Student Text pp. 174–75

Lesson 73

Student Text pp. 176–77
Daily Review p. 431d

Objectives

- Demonstrate an understanding of dividing mixed numbers
- Divide mixed numbers by multiplying by the reciprocal of the divisor
- Check a division problem using multiplication
- Draw a diagram to solve a division word problem with a mixed number
- Use a number line to solve a division word problem with a mixed number
- Solve fraction word problems

Teach for Understanding

Demonstrate an understanding of dividing mixed numbers

Maria has $4\frac{1}{2}$ yards of shelf paper. She needs $\frac{3}{4}$ of a yard of the paper to cover the top of each shelf in her kitchen cupboards. How many shelves can she cover? ***6 shelves***

1. Direct the students to draw a picture to solve the word problem. Choose a student to draw his picture for display and explain it. ***Accept any correct illustration and explanation.***

➤ **What equation can you write for the word problem? Why?** ***$4\frac{1}{2} \div \frac{3}{4} = 6$ shelves; elicit that you need to divide $4\frac{1}{2}$ yards (the total) by $\frac{3}{4}$ of a yard (the number in each set) to find that Maria can cover 6 shelves (the number of sets).***

Write $4\frac{1}{2} \div \frac{3}{4} =$ for display.

➤ **What should you do first to solve this problem?** ***Rename the mixed number as an improper fraction.***

➤ **What improper fraction does $4\frac{1}{2}$ rename to?** $\frac{9}{2}$

Write $\frac{9}{2} \div \frac{3}{4} =$ below $4\frac{1}{2} \div \frac{3}{4} =$.

➤ **How could you easily divide these fractions?** ***Elicit that you can multiply $\frac{9}{2}$ by $\frac{4}{3}$, the reciprocal of the divisor $\frac{3}{4}$.***

Choose a student to demonstrate solving the problem. Remind him to use cancellation if possible. $\frac{9}{2} \times \frac{4}{3} =$ ***6 shelves***

➤ **How many shelves can Maria cover?** ***6***

Complete the division problem: $4\frac{1}{2} \div \frac{3}{4} = 6$ *shelves*. Select a student to check the equation. $6 \times \frac{3}{4} = \frac{9}{2} = 4\frac{1}{2}$

2. Follow a similar procedure for this word problem, directing the students to use a number line to find the answer.

Mrs. Riley has 7 yards of ribbon to make bows for Christmas wreaths. She needs $1\frac{1}{4}$ yards of ribbon for each bow. How many bows can she make? ***5 bows; $7 \div 1\frac{1}{4} = 7 \div \frac{5}{4} = 7 \times \frac{4}{5} = \frac{28}{5} = 5\frac{3}{5}$; $5\frac{3}{5} \times 1\frac{1}{4} = \frac{28}{5} \times \frac{5}{4} = 7$***

➤ **What does the fraction in the quotient represent?** ***Elicit that the fraction represents $\frac{3}{5}$ of a yard of ribbon that remains; it is part of the next set of $1\frac{1}{4}$ yards of ribbon needed to make another bow.***

3. Follow a similar procedure for the next word problem. Allow each student to choose whether to draw a picture or to use a number line to find the answer.

Mrs. Jenkins bought $8\frac{1}{3}$ pounds of rice. She placed $1\frac{2}{3}$ pounds of the rice in each food storage container. How many containers did she use to store the rice? ***5 containers; $8\frac{1}{3} \div 1\frac{2}{3} = \frac{25}{3} \div \frac{5}{3} = \frac{25}{3} \times \frac{3}{5} = 5$; $5 \times 1\frac{2}{3} = \frac{25}{3} = 8\frac{1}{3}$***

4. Direct the students to solve the following word problems and to write the equation for the word problem. Choose students to write their solutions for display. Lead a discussion about each answer and the solution.

Mr. Brayden bought Bibles to give as Christmas presents to the children in his Sunday school class. His wife has $4\frac{1}{4}$ yards of fabric to make covers for the Bibles that Mr. Brayden will give to the girls that are in the class. She needs $\frac{5}{8}$ of a yard of fabric to cover each Bible. There are 6 girls in Mr. Brayden's class. Does his wife have enough fabric to make a Bible cover for each girl? How do you know? ***Yes; she can cover 6 Bibles; $4\frac{1}{4} \div \frac{5}{8} = \frac{17}{4} \times \frac{8}{5} = \frac{34}{5} = 6\frac{4}{5}$.*** [BAT: 5b Giving]

Luke picked $3\frac{1}{2}$ bushels of apples. Mason picked $1\frac{1}{4}$ times as many apples as Luke. How many bushels of apples did Mason pick? ***$1\frac{1}{4} \times 3\frac{1}{2} = 4\frac{3}{8}$ bushels*** [BAT: 2e Work]

Tyler has a wooden board that is $10\frac{1}{2}$ feet long. He plans to use the board to make pickets to replace damaged ones in the fence around a flower garden. If each picket will be $1\frac{1}{4}$ feet long, how many pickets can Tyler make? ***8 pickets; $10\frac{1}{2} \div 1\frac{1}{4} = \frac{21}{2} \times \frac{4}{5} = \frac{42}{5} = 8\frac{2}{5}$***

Mixed Numbers & Reciprocals

Rename mixed numbers as improper fractions.
Multiply by the reciprocal of the divisor to solve.

Jordan wants the streamers for his windsock to measure $1\frac{2}{3}$ feet long. He has $8\frac{1}{3}$ feet of nylon fabric. How many streamers can he make?

$8\frac{1}{3} \div 1\frac{2}{3} =$ 5 streamers

$8\frac{1}{3} \div 1\frac{2}{3} =$
$\frac{25}{3} \div \frac{5}{3} =$
$\frac{5}{\cancel{25}_{5}} \times \frac{\cancel{3}^{1}}{\cancel{5}_{1}} = \frac{5}{1} = 5$

Exercises

Rename the mixed number as an improper fraction.
Write the reciprocal.

1. $2\frac{3}{8}$ *$\frac{19}{8}$; $\frac{8}{19}$*
2. $5\frac{3}{4}$ *$\frac{23}{4}$; $\frac{4}{23}$*
3. 26 *$\frac{1}{26}$*
4. $4\frac{1}{3}$ *$\frac{13}{3}$; $\frac{3}{13}$*
5. 2 *$\frac{1}{2}$*
6. $6\frac{5}{6}$ *$\frac{41}{6}$; $\frac{6}{41}$*
7. $3\frac{2}{8}$ *$\frac{26}{8}$; $\frac{8}{26}$*
8. $8\frac{1}{2}$ *$\frac{17}{2}$; $\frac{2}{17}$*
9. $10\frac{3}{10}$ *$\frac{103}{10}$; $\frac{10}{103}$*
10. $6\frac{1}{3}$ *$\frac{19}{3}$; $\frac{3}{19}$*
11. $5\frac{1}{4}$ *$\frac{21}{4}$; $\frac{4}{21}$*
12. $11\frac{5}{9}$ *$\frac{104}{9}$; $\frac{9}{104}$*

Find the quotient by multiplying by the reciprocal.
Write the answer in lowest terms. *Answers are shown using cancellation.*

13. $3\frac{1}{4} \div \frac{1}{2}$ *$\frac{13}{4} \times \frac{2}{1} = \frac{13}{2} = 6\frac{1}{2}$*
14. $9 \div 4\frac{1}{2}$ *$\frac{9}{1} \times \frac{2}{9} = \frac{2}{1} = 2$*
15. $5\frac{1}{2} \div 2\frac{1}{4}$ *$\frac{11}{2} \times \frac{4}{9} = \frac{22}{9} = 2\frac{4}{9}$*
16. $4\frac{2}{5} \div \frac{2}{3}$ *$\frac{22}{5} \times \frac{3}{2} = \frac{33}{5} = 6\frac{3}{5}$*
17. $8 \div 3\frac{3}{4}$ *$\frac{8}{1} \times \frac{4}{15} = \frac{32}{15} = 2\frac{2}{15}$*
18. $1\frac{1}{6} \div 4$ *$\frac{7}{6} \times \frac{1}{4} = \frac{7}{24}$*
19. $6\frac{1}{2} \div \frac{3}{4}$ *$\frac{13}{2} \times \frac{4}{3} = \frac{26}{3} = 8\frac{2}{3}$*
20. $2\frac{3}{4} \div 1\frac{1}{4}$ *$\frac{11}{4} \times \frac{4}{5} = \frac{11}{5} = 2\frac{1}{5}$*
21. $2\frac{3}{5} \div 5$ *$\frac{13}{5} \times \frac{1}{5} = \frac{13}{25}$*
22. $6 \div 3\frac{1}{2}$ *$\frac{6}{1} \times \frac{2}{7} = \frac{12}{7} = 1\frac{5}{7}$*
23. $3\frac{3}{8} \div 1\frac{3}{4}$ *$\frac{27}{8} \times \frac{4}{7} = \frac{27}{14} = 1\frac{13}{14}$*
24. $6\frac{1}{8} \div 3$ *$\frac{49}{8} \times \frac{1}{3} = \frac{49}{24} = 2\frac{1}{24}$*

"No two snowflakes are alike."
—Wilson Bentley

Solve.

25. Ella spends $2\frac{1}{2}$ hours on homework, practicing the flute, and practicing the piano. She spends an equal amount of time on homework and practice. How much time does she spend doing homework?
$2\frac{1}{2} \div 2 = \frac{5}{2} \div \frac{2}{1} = \frac{5}{2} \times \frac{1}{2} = \frac{5}{4} = 1\frac{1}{4}$ hr
26. Use the information from problem 25 to find the amount of time Ella practiced each instrument if she divided her time equally.
$2\frac{1}{2} - 1\frac{1}{4} = 1\frac{1}{4}$ hr for practice;
$1\frac{1}{4} \div 2 = \frac{5}{4} \div \frac{2}{1} = \frac{5}{4} \times \frac{1}{2} = \frac{5}{8}$ hr
27. To help Ella stay organized, Mr. Evans built shelves above her desk. The shelf for homework items measures $3\frac{1}{3}$ feet long. The shelf for her flute and music books measures $2\frac{5}{12}$ feet long. How much longer is the homework shelf than the music shelf?
$3\frac{1}{3} - 2\frac{5}{12} = 3\frac{4}{12} - 2\frac{5}{12} = 2\frac{16}{12} - 2\frac{5}{12} = \frac{11}{12}$ ft
28. Estimate the measurements in problem 27. Explain how Mr. Evans could determine that his 6-foot board would be long enough to build both shelves. *$3 + 2 = 5$ ft; 5 ft < 6 ft; the fractions $\frac{1}{3}$ and $\frac{5}{12}$ are both less than $\frac{1}{2}$, so when added together they would not make another foot.*

Answers are shown using cancellation.
Multiply to check the answer. Write true or false.
Write the correct quotient if the answer is false.

29. $8 \div \frac{2}{3} = 12$ *true*
$\frac{12}{1} \times \frac{2}{3} = \frac{8}{1} = 8$
30. $2\frac{1}{5} \div 3 = \frac{11}{15}$ *true*
$\frac{11}{15} \times \frac{3}{1} = \frac{11}{5} = 2\frac{1}{5}$
31. $8 \div 2\frac{4}{5} = 1\frac{6}{7}$ *false; $2\frac{6}{7}$*
32. $\frac{8}{9} \div \frac{1}{3} = 1\frac{1}{3}$ *false; $2\frac{2}{3}$*
33. $2\frac{5}{8} \div 1\frac{1}{4} = 2\frac{1}{5}$ *false; $2\frac{1}{10}$*
34. $2\frac{1}{2} \div \frac{1}{8} = 20$ *true*
$\frac{20}{1} \times \frac{1}{8} = \frac{5}{2} = 2\frac{1}{2}$

Practice & Application *Answers are shown using cancellation.*

35. Jane read $\frac{1}{3}$ of a 390-page book. How many pages are left to read? *$\frac{1}{3} \times \frac{390}{1} = \frac{130}{1} = 130$;*
$390 - 130 = 260$ pages
36. $3\frac{7}{8} + 4\frac{3}{4} + 2\frac{1}{2}$ *$3\frac{7}{8} + 4\frac{6}{8} + 2\frac{4}{8} = 9\frac{17}{8} = 11\frac{1}{8}$*
37. What is the square root of 100?
$\sqrt{100} =$ ___ *10*
38. How many $\frac{1}{2}$ cups of juice are in $3\frac{3}{4}$ cups?
$3\frac{3}{4} \div \frac{1}{2} = \frac{15}{4} \div \frac{1}{2} = \frac{15}{4} \times \frac{2}{1} = \frac{15}{2} = 7\frac{1}{2}$ cups
39. Draw and label an acute, an obtuse, and a right angle.
40. Draw a picture of a 4×4 pan of brownies. If 5 brownies were eaten, what fraction of the brownies is left? *$\frac{11}{16}$ brownies left;*
$\frac{5}{16}$ were eaten
41. Find the quotient for $3{,}896 \div 42$. Write the remainder in fraction form in lowest terms.
$3{,}896 \div 42 = 92\frac{16}{21}$
42. Write 190,347 in expanded form.
$100{,}000 + 90{,}000 + 300 + 40 + 7$

J Write and solve a word problem for $20 \text{ feet} \div 2\frac{1}{2}$ feet. Use the idea of shelving board for a storage building.

Answers may vary.
Dad has boards that have a total measure of 20 feet in length. How many $2\frac{1}{2}$-foot shelves can he make for the storage building using one board?
$20 \div 2\frac{1}{2} = 20 \times \frac{2}{5}$ ft = 8 shelves

Wilson Bentley was known as the "Snowflake Man." He devoted his life to capturing photomicrographs of snowflakes at his farm in Jericho, Vermont.

Complete DAILY REVIEW 4 on page 431.

Student Text pp. 176–77

(*Note*: *Assessment* available on Teacher's Toolkit CD.)

Lesson 74

Student Text pp. 178–79
Daily Review p. 431e

Objectives
- Write and solve a multi-step equation for a multi-step word problem
- Demonstrate an understanding of the Order of Operations

Notes
Often more than one method can be used to solve multi-step word problems. The equations given in this lesson (including the equations for the problems on the Student Text pages) represent possible solutions; accept other correct solutions. When applicable, encourage the students to write a multi-step equation for a multi-step word problem; accept any order of operations that results in a correct solution. Allow students who have difficulty writing a multi-step equation to write more than one equation.

Throughout this lesson, review the Order of Operations as needed.

Introduce the Lesson

Mrs. Anthony bought $8\frac{3}{4}$ pounds of hamburger. She divided the hamburger into packages of $1\frac{1}{4}$ pounds each. How many packages of hamburger does she have? *7 packages*

➤ **What equation can you write to solve this word problem? Why?** *$8\frac{3}{4} \div 1\frac{1}{4} = \_\_$; elicit that you need to find how many $1\frac{1}{4}$ pounds are in $8\frac{3}{4}$ pounds.*

1. Write $8\frac{3}{4} \div 1\frac{1}{4} = \_\_$ for display and direct the students to solve the equation on paper. Allow the students to draw a diagram or use a number line to help them solve the problem. $\frac{35}{4} \div \frac{5}{4} = \frac{35}{4} \times \frac{4}{5} =$ *7 packages*

➤ **How many packages of hamburger does Mrs. Anthony have?** *7 packages*

Complete the equation and lead a discussion about the solution. Review the dividing of mixed numbers by multiplying by the reciprocal of the divisor as needed.

2. Follow a similar procedure for this word problem.

Ethan walked $1\frac{4}{5}$ miles around the lake. Jeremy only walked $\frac{1}{2}$ of the distance that Ethan walked. How far did Jeremy walk? *$\frac{1}{2} \times 1\frac{4}{5} = \frac{9}{10}$ of a mile; elicit that to find how far Jeremy walked, you need to find $\frac{1}{2}$ of the distance that Ethan walked.*

Teach for Understanding

Solve a multi-step word problem

1. Choose a student to read aloud the sentence at the top of Student Text page 178. Select another student to read the first word problem.

➤ **What do you need to determine before you can find how many of the 75 bags of apples are left to be delivered on Wednesday?** *Answers will vary, but elicit that first you need to determine how many bags of apples were left to be delivered after $\frac{1}{5}$ of the 75 bags were delivered on Monday; then you need to find how many bags of apples were delivered on Tuesday.*

➤ **How many equations do you think are needed to solve the word problem?** *Answers will vary, but elicit that it is possible to find the answer by solving 2 equations.*

➤ **How can you determine how many of the 75 bags of apples needed to be delivered after Mr. Casey and the boys made the deliveries on Monday?** *Elicit that first you need to find $\frac{1}{5}$ of 75 to determine how many bags of apples were delivered on Monday. Then you can subtract the number of bags delivered on Monday from the total number (75 bags of apples) that Mr. Casey and the boys picked.*

2. Point out that the Order of Operations can be used to write one equation to solve this part of the problem.

➤ **What one equation can you write to find how many bags of apples were left to be delivered after Monday's deliveries were made?** *$75 - (\frac{1}{5} \times 75) = \_\_$*

Write the equation for display. Point out that although the equation follows the Order of Operations, writing parentheses around $\frac{1}{5} \times 75$ is a helpful reminder to perform the multiplication before you subtract.

3. Direct the students to solve the equation. Choose a student to write his solution for display, and discuss the solution as needed. *$75 - 15 = 60$*

➤ **Now that you know that 60 bags of apples needed to be delivered after Monday's deliveries were made, how can you determine how many bags of apples were left to be delivered on Wednesday?** *Elicit that you need to find $\frac{1}{3}$ of 60 to determine how many bags of apples were delivered on Tuesday and subtract that amount from the 60 bags of apples that were left to be delivered after Monday's deliveries.*

➤ **Using the Order of Operations, what one equation can you write to find how bags of apples were left to be delivered on Wednesday?** *$60 - (\frac{1}{3} \times 60) = \_\_$*

Elicit that parentheses around $\frac{1}{3} \times 60$ is a reminder to perform the multiplication before you subtract.

4. Direct the students to solve the equation. Choose a student to write his solution for display, and discuss the solution as needed. *$60 - 20 = 40$*

➤ **How many bags of apples did Mr. Casey and the boys have left to deliver on Wednesday?** *40 bags of apples* Write the final answer for display.

➤ **Do you think that one equation could be written for the word problem? Why do you think so?** *Elicit that one equation can be written; the number of bags of apples delivered on Tuesday was subtracted from the number of bags that were left after Monday's deliveries to find the numbers of bags of apples that were left to be delivered on Wednesday.*

➤ **What one final equation can you write for the word problem?** *$75 - (\frac{1}{5} \times 75) - (\frac{1}{3} \times 60) = 40$ bags of apples* Write the equation for display.

➤ **How do you think you can apply the Order of Operations to the equation to correctly solve the word problem?** *Accept any reasonable ideas indicating that first you should solve the expressions in parentheses to determine the number of bags of apples delivered on Monday and the number of bags delivered on Tuesday, and then subtract to find that 40 bags of apples were left to be delivered on Wednesday.*

5. Guide the students in solving the expressions in parentheses first and then subtracting in a left-to-right progression: $75 - 15 - 20 = 60 - 20 = 40$.

Multi-Step Equations

Exercises

Solve. Write a statement to explain the solution. ***Steps used to solve may vary.***

The sixth-grade Sunday school class at Calvary Baptist Church is finding ways to help people in their church and community.

1. Mr. Casey took a group of boys to pick apples for the senior citizens in their church. They picked 75 bags of apples. The boys delivered $\frac{1}{5}$ of the bags on Monday. On Tuesday they delivered $\frac{1}{3}$ of the bags left from Monday. How many bags of apples do they have left to deliver on Wednesday? ***There are 40 bags of apples left to deliver on Wednesday.***
2. Landon picked $2\frac{1}{2}$ boxes of apples. Each box contains $1\frac{1}{2}$ bushels. Drew picked 4 bushels of apples. How many bushels of apples did the two boys pick in all? ***Together the boys picked $7\frac{3}{4}$ bushels of apples.***
3. Mr. Casey and Anthony put up new shelving in Mrs. Bryant's pantry. How many $2\frac{1}{4}$-foot shelves can be cut from two boards that measure $8\frac{7}{8}$ feet each? ***Seven shelves can be built for the pantry.***
4. On Saturday Daniel raked leaves for Mrs. Gulley for $1\frac{3}{4}$ hours. He painted fencing at Mr. Henry's for $2\frac{1}{2}$ hours. How many hours did Daniel work on Saturday? ***Daniel worked $4\frac{1}{4}$ hours on Saturday.***
5. Mrs. Casey and a group of girls made cheer packages for families with illness. Each package had $\frac{1}{2}$ of a pound of mints, $\frac{3}{4}$ of a pound of Crackle Crunch, and $2\frac{7}{8}$ pounds of apples. What is the weight of the food in each cheer package? ***Each cheer package weighs $4\frac{1}{8}$ pounds.***
6. Natalie brought $2\frac{1}{2}$ pounds of mixed nuts for some gift baskets. Leah brought $4\frac{1}{4}$ pounds of mixed nuts. All the nuts were distributed equally among 3 gift baskets. How many pounds of nuts were put in each basket? ***Each gift basket will have $2\frac{1}{4}$ pounds of nuts.***
7. Hannah is making roses out of ribbon to put in each package. She has $10\frac{1}{3}$ yards of red ribbon and $2\frac{2}{3}$ yards of peach ribbon. How many roses can she make if each rose takes $\frac{2}{3}$ of a yard? ***Hannah can make 19 roses.***
8. A family donated sandwich meat for two needy families. There are $5\frac{1}{4}$ pounds of chicken and $6\frac{1}{2}$ pounds of ham. Alexis divided each kind of meat into two equal portions. How much chicken and ham will each family receive? ***Each family will receive $2\frac{5}{8}$ pounds of chicken and $3\frac{1}{4}$ pounds of ham.***

Solve. ***Steps used to solve may vary.***

9. Nicole and Riley invited the 8 new girls in their Sunday school class to a pizza party. If they expect to serve $\frac{3}{8}$ of a pizza to each person at the party, how many pizzas will they need? (Each pizza will be cut into 8 slices.) ***$10 \times \frac{3}{8} = \frac{10}{1} \times \frac{3}{8} = \frac{30}{8} = 3\frac{3}{4} = 4$ pizzas***
10. On Saturday Victoria played her violin for $3\frac{3}{4}$ hours in 5 local nursing homes. How long did she play at each home if she divided her time equally? ***$3\frac{3}{4} \div 5 = \frac{15}{4} \div \frac{5}{1} = \frac{15}{4} \times \frac{1}{5} = \frac{3}{4}$ of an hour***
11. After the snowstorm, Marcus cleared snow from sidewalks in town for $2\frac{1}{2}$ hours on Saturday and $2\frac{1}{2}$ hours on Sunday. He worked an extra $\frac{3}{4}$ of an hour getting wood for Mrs. Carter's wood stove. How much time did Marcus spend this weekend helping others? ***$(2 \times 2\frac{1}{2}) + \frac{3}{4} = (2 \times \frac{5}{2}) + \frac{3}{4} = 5 + \frac{3}{4} = 5\frac{3}{4}$ hr***
12. Jackson shoveled snow from 4 walkways at Mr. Kauffman's home. Each walkway was $4\frac{2}{3}$ yards long. What was the total length of walkways shoveled? ***$4 \times 4\frac{2}{3} = \frac{4}{1} \times \frac{14}{3} = \frac{56}{3} = 18\frac{2}{3}$ yd***

Solve. Write the answer in lowest terms. ***Steps to lowest terms may vary.***

13. $\frac{5}{6} \div \frac{1}{8}$ ***$\frac{5}{6} \times \frac{8}{1} = \frac{20}{3} = 6\frac{2}{3}$***
14. $8\frac{3}{4} \div \frac{1}{3}$ ***$\frac{35}{4} \times \frac{3}{1} = \frac{105}{4} = 26\frac{1}{4}$***
15. $4 \div \frac{4}{5}$ ***$\frac{4}{1} \times \frac{5}{4} = \frac{5}{1} = 5$***
16. $\frac{3}{8} \div \frac{1}{4}$ ***$\frac{3}{8} \times \frac{4}{1} = \frac{3}{2} = 1\frac{1}{2}$***
17. $\frac{3}{4} \div 3$ ***$\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$***
18. $2\frac{2}{5} \div 5$ ***$\frac{12}{5} \times \frac{1}{5} = \frac{12}{25}$***
19. $5 \div \frac{2}{5}$ ***$\frac{5}{1} \times \frac{5}{2} = \frac{25}{2} = 12\frac{1}{2}$***
20. $1\frac{5}{6} \div 3$ ***$\frac{11}{6} \times \frac{1}{3} = \frac{11}{18}$***
21. $7 \div \frac{3}{10}$ ***$\frac{7}{1} \times \frac{10}{3} = \frac{70}{3} = 23\frac{1}{3}$***
22. $6\frac{2}{3} \div 1\frac{5}{8}$ ***$\frac{20}{3} \times \frac{8}{13} = \frac{160}{39} = 4\frac{4}{39}$***
23. $\frac{1}{4} \div \frac{1}{12}$ ***$\frac{1}{4} \times \frac{12}{1} = \frac{3}{1} = 3$***
24. $9 \div 3\frac{1}{2}$ ***$\frac{9}{1} \times \frac{2}{7} = \frac{18}{7} = 2\frac{4}{7}$***
25. $2\frac{1}{2} \div \frac{2}{3}$ ***$\frac{17}{8} \times \frac{3}{2} = \frac{51}{16} = 3\frac{3}{16}$***
26. $1 \div \frac{1}{4}$ ***$\frac{1}{1} \times \frac{4}{1} = \frac{4}{1} = 4$ or $1 \times 4 = 4$***
27. $\frac{2}{3} \div \frac{1}{6}$ ***$\frac{2}{3} \times \frac{6}{1} = \frac{4}{1} = 4$***
28. $\frac{6}{7} \div \frac{1}{2}$ ***$\frac{6}{7} \times \frac{2}{1} = \frac{12}{7} = 1\frac{5}{7}$***
29. $\frac{4}{9} \div \frac{3}{7}$ ***$\frac{4}{9} \times \frac{7}{3} = \frac{28}{27} = 1\frac{1}{27}$***
30. $3\frac{1}{7} \div \frac{1}{3}$ ***$\frac{22}{7} \times \frac{3}{1} = \frac{66}{7} = 9\frac{3}{7}$***

J Write a multi-step word problem using 3 cups of sugar, $1\frac{1}{2}$ cups of sugar, and $1\frac{1}{4}$ cups of sugar. Solve. ***Answers may vary.***

Alyssa and Bryson need $1\frac{1}{2}$ cups of sugar for a cake and $1\frac{1}{4}$ cups of sugar for a batch of cookies. How much sugar will be left from the 3 cups of sugar that they have?
$1\frac{1}{2} + 1\frac{1}{4} = 1\frac{2}{4} + 1\frac{1}{4} = 2\frac{3}{4}$
$3 - 2\frac{3}{4} = 2\frac{4}{4} - 2\frac{3}{4} = \frac{1}{4}$ cup

Complete DAILY REVIEW 6 on page 431.

6. Choose a student to read aloud the second word problem.
   - **How many equations could you write to solve this word problem?** ***Elicit that you can solve the word problem by solving one equation.***
   - **What equation can you write? Why?** ***($2\frac{1}{2} \times 1\frac{1}{2}) + 4 =$ ___; elicit that first you need to multiply $2\frac{1}{2} \times 1\frac{1}{2}$ to find the number of bushels of apples that Landon picked. Then you can add the 4 bushels Drew picked to find the total number of bushels that were picked by both boys.***

   Write the equation for display. Point out that although the equation follows the Order of Operations, writing parentheses around $2\frac{1}{2} \times 1\frac{1}{2}$ is a helpful reminder that the multiplication should be performed first.
7. Direct the students to solve the equation. Discuss the solution as needed. ***$(\frac{5}{2} \times \frac{3}{2}) + 4 = \frac{15}{4} + 4 = 3\frac{3}{4} + 4 = 7\frac{3}{4}$ bushels***
8. You may choose to guide the students in solving two or three more of the word problems before directing them to complete the rest of the problems on Student Text pages 178–79.

   Remind them to use the Order of Operations when using a multi-step equation to solve a word problem. Point out that not all of the word problems are multi-step problems; some of the problems can be solved by using only one operation, and the solution to other problems will require more than one equation.

For some of the word problems, allow students to share other ways they think the word problem could be solved and to explain their reasoning. For example, when solving problem 6, some students may conclude that $\frac{1}{3}$ of the nuts was placed in each basket and that you multiply the total weight by $\frac{1}{3}$ to find how many pounds of nuts were put into each basket rather than divide the total weight by 3.

(***Note:*** You may choose to allow small groups of students to solve the word problems rather than have individual students solve them.)

Student Text pp. 178–79

Lesson 75

Student Text pp. 180–81
Daily Review p. 432f

Objectives
- Apply mathematical properties to evaluate an expression with fractions
- Use the Order of Operations to evaluate an expression
- Substitute a given value for a variable in an expression

Teach for Understanding

Apply properties

1. Write for display the expressions $\frac{1}{4} + \frac{1}{2}$ and $\frac{1}{2} + \frac{1}{4}$ with space between them.
➤ **Are these expressions equal or not equal to one another? How do you know?** ***Equal; elicit that the addends in the expressions are the same and that the Commutative Property of Addition states that the order of the addends can be changed without changing the sum.*** Write an equal sign (=) between the expressions.
➤ **What is the value of each expression?** $\frac{3}{4}$ Write $\frac{3}{4} = \frac{3}{4}$ below the expressions.
2. Follow a similar procedure for $\frac{3}{4} \times \frac{2}{3}$ and $\frac{2}{3} \times \frac{3}{4}$. ***Equal; elicit that the factors in the expressions are the same and that the Commutative Property of Multiplication states that the order of the factors can be changed without changing the product;*** $\frac{1}{2} = \frac{1}{2}$.
3. Write for display the expressions $(\frac{1}{2} + \frac{2}{3}) + \frac{1}{4}$ and $\frac{1}{2} + (\frac{2}{3} + \frac{1}{4})$ with space between them.
➤ **What do the parentheses indicate?** ***Perform the operation inside the parentheses first.***
➤ **What do you notice about the addends in the expressions?** ***The expressions have the same addends in the same order.***
➤ **Do you think that changing the grouping of the addends changes the value of the expression? Why?** ***No; elicit that the Associative Property of Addition states that the grouping of the addends may be changed without changing the sum.***
➤ **Are these addition expressions equal or not equal to one another?** ***equal*** Write an equal sign (=) between the expressions.
Choose a student to simplify each expression to verify the prediction. Guide him in writing the solution below the equation, aligning the equal signs. Explain that when you simplify an expression, you are *evaluating* the expression, or finding its value. $(\frac{6}{12} + \frac{8}{12}) + \frac{3}{12} = \frac{6}{12} + (\frac{8}{12} + \frac{3}{12})$; $\frac{14}{12} + \frac{3}{12} = \frac{6}{12} + \frac{11}{12}$; $\frac{17}{12} = \frac{17}{12}$; $1\frac{5}{12} = 1\frac{5}{12}$
4. Follow a similar procedure for $\frac{1}{4} \times (\frac{4}{5} \times \frac{1}{2})$ and $(\frac{1}{4} \times \frac{4}{5}) \times \frac{1}{2}$. Elicit that the Associative Property of Multiplication states that the grouping of the factors may be changed without changing the product; therefore, the expressions are equal. ***Accept*** $\frac{1}{4} \times \frac{2}{5} = \frac{1}{5} \times \frac{1}{2}$; $\frac{1}{10} = \frac{1}{10}$ ***or*** $\frac{1}{4} \times \frac{4}{10} = \frac{4}{20} \times \frac{1}{2}$; $\frac{4}{40} = \frac{4}{40}$; $\frac{1}{10} = \frac{1}{10}$.
5. Write for display the expressions $2 \times 4\frac{5}{6}$ and $(2 \times 4) + (2 \times \frac{5}{6})$ with space between them.
➤ **Do you think these expressions are equal or not equal to one another? Why?** ***Equal; elicit that 2 sets of 4 and 2 sets of*** $\frac{5}{6}$ ***is the same amount as 2 sets of*** $4\frac{5}{6}$. Write an equal sign (=) between the expressions.
Elicit that the Distributive Property of Multiplication over Addition allows you to determine the product by separating one factor into parts, multiplying each part by the other factor, and adding the partial products.
Choose a student to evaluate each expression to verify the prediction. ***Accept*** $2 \times \frac{29}{6} = 8 + \frac{5}{3}$; $\frac{29}{3} = 8 + 1\frac{2}{3}$; $9\frac{2}{3} = 9\frac{2}{3}$ ***or*** $2 \times \frac{29}{6} = 8 + \frac{10}{6}$; $\frac{58}{6} = 8\frac{10}{6}$; $9\frac{4}{6} = 9\frac{4}{6}$; $9\frac{2}{3} = 9\frac{2}{3}$.
6. Write $\frac{7}{8} + 0 = \frac{7}{8}$ for display.
➤ **What is the identity element for addition?** ***0*** **Which property tells you this?** ***Elicit that the Identity Property of Addition states that when 0 is an addend, the sum is the other addend.***
7. Follow a similar procedure for $1 \times \frac{3}{7} = \frac{3}{7}$. ***1; the Identity Property of Multiplication states that when 1 is a factor, the product is the other factor.***
8. Write $\frac{4}{9} - 0 = \frac{4}{9}$.
➤ **What does the Zero Principle of Subtraction tell you?** ***When 0 is subtracted from a number (the minuend), the answer is that number.***
(***Note:*** There is no Identity Property of Subtraction because $n - 0 = n$, but $0 - n \neq n$.)
9. Write $\frac{3}{4} \times 0 = 0$.
➤ **What does the Zero Property of Multiplication tell you?** ***When 0 is a factor, the product is 0.***

Use the Order of Operations to evaluate an expression

1. Write $\frac{1}{8} + (\frac{3}{8} \times 4) \div 2$ for display.
➤ **What do you do first to simplify the expression?** ***Perform the multiplication inside the parentheses.***
➤ **Do you think that using cancellation when simplifying** $\frac{3}{8} \times 4$ **will be helpful in simplifying the entire expression? Why?** ***Elicit that using cancellation will not be helpful in this instance because you will later be adding eighths.***
➤ **What do you do after you simplify the multiplication in the parentheses? Why?** ***Simplify the division expression; elicit that the Order of Operations tells you to simplify multiplication and division expressions in the order that they appear from left to right before you simplify addition and subtraction expressions.***
Direct the students to evaluate the expression. Remind them that rewriting the expression each time they simplify any part of it will help them to evaluate the expression correctly. Choose a student to explain the process he used to evaluate the expression as he writes his solution for display. Guide the explanation as needed.

$\frac{1}{8} + (\frac{3}{8} \times 4) \div 2$
$= \frac{1}{8} + \frac{12}{8} \div 2$
$= \frac{1}{8} + \frac{6}{8}$
$= \frac{7}{8}$

➤ **Were the parentheses needed for simplifying the expression? Why?** ***Elicit that since the multiplication and division should be performed in the order that they appear from left to right before you perform the addition, the parentheses were not necessary for simplifying the expression.***
2. Repeat the procedure for these equations.

$(2 + 3\frac{1}{8}) - \frac{1}{2} \times \frac{1}{2}$
$= 5\frac{1}{8} - \frac{1}{2} \times \frac{1}{2}$
$= 5\frac{1}{8} - \frac{1}{4}$
$= \frac{39}{8} = 4\frac{7}{8}$

$(2\frac{3}{5} + \frac{11}{15}) \times (4 - \frac{2}{3})$
$= \frac{50}{15} \times \frac{10}{3}$
$= \frac{100}{9} = 11\frac{1}{9}$

Properties

Mathematical Properties

Commutative Property of Addition

$\frac{1}{6}+\frac{2}{6}=\frac{2}{6}+\frac{1}{6}$

$\frac{3}{6}=\frac{3}{6}$

Commutative Property of Multiplication

$\frac{2}{3}\times\frac{2}{5}=\frac{2}{5}\times\frac{2}{3}$

$\frac{4}{15}=\frac{4}{15}$

The order of addends or factors can be changed without changing the sum or product.

Associative Property of Addition

$(\frac{2}{8}+\frac{1}{8})+\frac{4}{8}=\frac{2}{8}+(\frac{1}{8}+\frac{4}{8})$

$\frac{7}{8}=\frac{7}{8}$

Associative Property of Multiplication

$\frac{1}{5}\times(\frac{2}{5}\times\frac{3}{5})=(\frac{1}{5}\times\frac{2}{5})\times\frac{3}{5}$

$\frac{6}{125}=\frac{6}{125}$

The grouping of addends or factors may be changed without changing the sum or product.

Distributive Property of Multiplication over Addition

$2\times1\frac{2}{3}=(2\times1)+(2\times\frac{2}{3})$

$3\frac{1}{3}=3\frac{1}{3}$

The product of any two factors can be found by separating one factor into parts, multiplying each part by the other factor, and adding the partial products.

Identity Property of Addition

$\frac{2}{7}+0=\frac{2}{7}$

When 0 is an addend, the sum is the other addend.

Identity Property of Multiplication

$1\times\frac{4}{9}=\frac{4}{9}$

When 1 is a factor, the product is the other factor.

Zero Principle of Subtraction

$\frac{7}{12}-0=\frac{7}{12}$

When 0 is subtracted from a number (the minuend), the answer is that number.

Zero Property of Multiplication

$\frac{5}{6}\times0=0$

When 0 is a factor, the product is 0.

Exercises

Write the missing number or fraction in the equation.
Write the name of the property illustrated.

1. $\frac{3}{8}\times\_\_=0$ ***0***
   Zero Property of Multiplication
2. $7\times2\frac{3}{5}=(7\times2)+(7\times\_\_)$ ***$\frac{3}{5}$***
   Distributive Property of Multiplication
3. $\frac{2}{9}+\_\_=\frac{1}{3}+\frac{2}{9}$ ***$\frac{1}{3}$***
   Commutative Property of Addition
4. $(\frac{5}{6}+\frac{2}{3})+\_\_=\frac{5}{6}+(\frac{2}{3}+\frac{1}{6})$ ***$\frac{1}{6}$***
   Associative Property of Addition
5. $\frac{9}{10}+0=\_\_$ ***$\frac{9}{10}$***
   Identity Property of Addition
6. $\frac{1}{4}\times(\frac{1}{2}\times\frac{3}{4})=(\frac{1}{4}\times\frac{1}{2})\times\_\_$ ***$\frac{3}{4}$***
   Associative Property of Multiplication
7. $\frac{1}{4}\times\frac{3}{4}=\_\_\times\frac{1}{4}$ ***$\frac{3}{4}$***
   Commutative Property of Multiplication
8. $1\times\_\_=\frac{3}{7}$ ***$\frac{3}{7}$***
   Identity Property of Multiplication
9. $\frac{7}{8}-0=\_\_$ ***$\frac{7}{8}$***
   Zero Principle of Subtraction

Evaluate Expressions

A variable is a letter used to represent a number.

An expression can be evaluated (solved) by substituting a known value for the variable.

$2+(\frac{1}{4}\times a)-1$ if $a=\frac{1}{3}$.

Substitute $\frac{1}{3}$ for a.

$2+(\frac{1}{4}\times\frac{1}{3})-1$

$=2+\frac{1}{12}-1$

$=2\frac{1}{12}-1$

$=1\frac{1}{12}$

Exercises

Evaluate the expression if $a=\frac{1}{4}$. Write the answer in lowest terms. ***Steps used to solve may vary.***

10. $\frac{2}{3}\times a\times\frac{1}{2}$ ***$\frac{1}{12}$***
11. $a\times\frac{4}{9}+\frac{5}{9}$ ***$\frac{2}{3}$***
12. $5+(a\times\frac{1}{5})-a$ ***$4\frac{4}{5}$***
13. $\frac{7}{8}\div a+\frac{3}{4}$ ***$4\frac{1}{4}$***
14. $4-(\frac{3}{4}\div a)\times\frac{1}{2}$ ***$2\frac{1}{2}$***
15. $(a+\frac{1}{5})\times(\frac{3}{4}-a)$ ***$\frac{9}{40}$***

Evaluate the expression.

16. $\frac{2}{3}+\frac{1}{2}\times\frac{3}{4}$ ***$1\frac{1}{24}$***
17. $5-\frac{3}{4}\times\frac{1}{2}$ ***$4\frac{5}{8}$***
18. $\frac{1}{4}+\frac{1}{4}\div\frac{1}{8}$ ***$2\frac{1}{4}$***
19. $(\frac{1}{3}+\frac{1}{5})\div(\frac{5}{8}-\frac{1}{4})$ ***$1\frac{19}{45}$***
20. $(\frac{1}{8}+\frac{2}{3})\times(\frac{1}{2}-\frac{1}{4})$ ***$\frac{19}{96}$***
21. $2\frac{1}{2}\div(\frac{5}{6}-\frac{1}{3})+1$ ***6***
22. $2\frac{3}{4}\div(\frac{6}{8}-\frac{1}{4})+1\frac{1}{2}$ ***7***
23. $1\frac{1}{2}\div(\frac{1}{3}+\frac{5}{6})+2$ ***$3\frac{2}{7}$***
24. $\frac{1}{5}+(\frac{2}{3}\times\frac{2}{3})\div\frac{5}{8}$ ***$\frac{41}{45}$***

Solve. Write the answer in lowest terms. ***Steps used to solve may vary.***

25. $\frac{7}{8}\div\frac{1}{2}$ ***$1\frac{3}{4}$***
26. $3\div\frac{1}{6}$ ***18***
27. $\frac{8}{9}\div2$ ***$\frac{4}{9}$***
28. $3\frac{5}{8}\div\frac{3}{4}$ ***$4\frac{5}{6}$***
29. $\frac{11}{12}\div\frac{1}{6}$ ***$5\frac{1}{2}$***
30. $7\div2\frac{3}{5}$ ***$2\frac{9}{13}$***
31. $5\frac{3}{4}\div2$ ***$2\frac{7}{8}$***
32. $9\frac{2}{3}\div4\frac{3}{8}$ ***$2\frac{22}{105}$***
33. $\frac{5}{9}\div\frac{8}{9}$ ***$\frac{5}{8}$***

Complete DAILY REVIEW on page 432.

3. Follow a similar procedure for the following expressions. Direct the students to substitute the given value for the variable.

$a=\frac{1}{4}$

$5+(\frac{2}{3}\times a)\times2$

$=5+(\frac{2}{3}\times\frac{1}{4})\times2$

$=5+\frac{2}{12}\times2$

$=5+\frac{4}{12}$

$=5\frac{4}{12}=5\frac{1}{3}$

$b=\frac{5}{6}$

$(b\times\frac{1}{3})+(b\times\frac{1}{2})$

$=(\frac{5}{6}\times\frac{1}{3})+(\frac{5}{6}\times\frac{1}{2})$

$=\frac{5}{18}+\frac{5}{12}$

$=\frac{10}{36}+\frac{15}{36}$

$=\frac{25}{36}$

$c=\frac{1}{5}$

$2+(4\frac{1}{2}\div c)\times4$

$=2+(4\frac{1}{2}\div\frac{1}{5})\times4$

$=2+(\frac{9}{2}\times5)\times4$

$=2+\frac{45}{2}\times4$

$=2+\frac{180}{2}$

$=92$

Student Text pp. 180–81

Lesson 76

Student Text pp. 182–83
Daily Review p. 432g

Objectives
- Use the Order of Operations to write and solve a multi-step equation for a multi-step word problem
- Determine whether a multi-step problem has too much or too little information

Teacher Materials
- Multi-Step Word Problems, page IA42 (CD)

Note
Since more than one method often can be used to solve multi-step word problems and since the order in which operations are written within equations may vary, the equations given in this lesson (including the equations for the problems on the Student Text pages) represent possible solutions. Allow students who have difficulty writing a multi-step equation to write more than one equation.

Introduce the Lesson

1. Write the following equations for display. Choose students to identify and explain the property shown in each equation. Allow the students to refer to the properties listed on Student Text page 180 if needed.

   $0.659 + 0 = 0.659$ ***Identity Property of Addition***

   $5 \times 7\frac{3}{5} = (5 \times 7) + (5 \times \frac{3}{5})$ ***Distributive Property of Multiplication over Addition***

   $(27.57 \times 14) \times 3 = 27.57 \times (14 \times 3)$ ***Associative Property of Multiplication***

2. Select students to write and explain equations showing the following properties:

   Associative Property of Addition
   Commutative Property of Addition
   Commutative Property of Multiplication
   Identity Property of Multiplication
   Zero Property of Multiplication

Teach for Understanding

Solve a multi-step word problem

1. Display the Multi-Step Word Problems page and read aloud the first problem. Elicit that all of the information necessary for solving the problem is given, but the price of the purse is not needed.

   ➤ **What do you need to determine first to find the total cost of the shoes? Why?** ***Answers will vary, but elicit that first you need to find how much Jacob will pay for Ella's shoes because only her pair of shoes can be purchased at half price.***

   Instruct the students to solve the problem and to write one equation for their solution. Choose students to write their equation for display and to explain how they found the answer. ***possible equation: $(\frac{1}{2} \times \$46.98) + \$82.69 = \$106.18$***

   (***Note:*** Some students may point out that $\frac{1}{2} \times \$46.98$ is the same as dividing \$46.98 by 2 because 2 and $\frac{1}{2}$ are reciprocals. Accept $(\$46.98 \div 2) + \$82.69 = \$106.18$ as a correct answer.)

2. Write $\$82.69 + \frac{1}{2} \times \$46.98 = \$106.18$ or direct attention to the equation if a student has written it for display.

   ➤ **Are parentheses necessary to correctly solve this equation? Why?** ***Elicit that parentheses are not necessary because the Order of Operations tells you to perform the multiplication before you perform the addition.*** Point out that any multi-step equation for the word problem must be written so that you can first multiply $\frac{1}{2} \times \$46.98$, but the Commutative Property of Addition allows you to show the addition of \$82.69 on either side of the expression. Although parentheses are not necessary, they may be written to indicate the part of the equation that was solved first.

3. Direct the students to determine whether the amount of money that Jacob saved because of the sale would be enough for him to also buy the purse for Ella. Lead a discussion about the solution. ***Yes; elicit that because of the sale, Jacob saved \$23.49, which is more than enough money to buy the purse; $(\$82.69 + \$46.98) - \$106.18 = \$23.49$.***

4. Read aloud the second word problem.

   ➤ **How could you solve this word problem?** ***Elicit that there is not enough information to solve the problem.*** **What other information is needed?** ***The number of trays that the desserts will be placed on.***

   ➤ **How many trays do you think would be needed for the desserts?** ***Accept any reasonable answers.***

   Direct the students to choose one of the given answers (number of trays) and to use that number to solve the problem, writing one equation to show the solution. Choose students to write their equation and to explain their solution. Discuss the similarities and differences between the equations. Point out the use of the Order of Operations and any mathematical properties. ***Equations will vary; possible equation: $(84 + 6 \times 12) \div 5 = 31$ r1; 31 desserts.***

   (***Note:*** You may choose to show the students that brackets may be used to more specifically show the Order of Operations: $[84 + (6 \times 12)] \div 5 = 31\ r1$.)

5. Follow a similar procedure for the third word problem. Elicit that all of the needed information is in the word problem; there is no extra information. ***possible equation: $(75\frac{3}{4} - \frac{2}{3} \times 75\frac{3}{4}) \div 5 = 5\frac{1}{20}$; 5 pounds***

6. Point out that the people mentioned in each of the word problems were ministering to others in different ways. Jacob bought new shoes for missionaries, and Madelyn provided desserts for others in her Sunday school to enjoy. By freely giving the walnuts to his neighbors, Mr. Cameron showed the love of Christ to others. He may also have had the opportunity to share God's plan of salvation with any of the neighbors who do not know the Lord Jesus Christ as their Savior.

   Read aloud I Corinthians 12 and explain that God has given to all who have had their sins forgiven through Christ spiritual gifts with which to serve Him and the church, or the body of Christ. Ask the students to share ways they have served or can serve God in their church, in their family, and in their neighborhood. Challenge them to think of ways they may use math when serving at church.

More Multi-Step Word Problems

Exercises

Solve. Explain what information is needed to solve the problems with *too little information*.

1. A sporting goods store donated 2 new basketball backboards that cost $128.00 each and 8 new basketball shirts that cost $18.75 each to Emmanuel Christian School. What was the value of the donations?
   (2 × $128.00) + (8 × $18.75) = $406.00

2. To raise money for their school, the sixth- and seventh-graders collected 2,136 pounds of paper and aluminum. If the current price for recycled aluminum is $0.90 per pound, how much money will they make on the aluminum products?
   The weight of the aluminum products is needed to solve.

3. In 1972 a record 1,122 inches of snow fell at Mt. Rainier, Washington. During the 1998–1999 winter season, a new snowfall record was set at Mt. Baker. The new record was 18 inches more than the previous record. What is the new snowfall record? *1,122 + 18 = 1,140 in.*

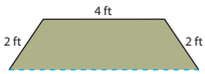
4. Troy weeded 4 rows of vegetables that were each $4\frac{2}{3}$ yards long. Todd weeded 6 rows that were each $2\frac{3}{4}$ yards long. Who weeded more? How much more?
   Troy: $4 \times 4\frac{2}{3} = 4 \times \frac{14}{3} = \frac{56}{3} = 18\frac{2}{3}$
   Todd: $6 \times 2\frac{3}{4} = 6 \times \frac{11}{4} = \frac{66}{4} = 16\frac{1}{2}$
   $18\frac{2}{3} - 16\frac{1}{2} = 2\frac{1}{6}$
   Troy weeded $2\frac{1}{6}$ yards more.

5. The soccer game photographer took 38 photos of the game. He has one photo of each player and some photos of the game. How much will he make if he sells all the photos he took? *The cost of each photo is needed to solve.*

6. Joshua saved $7.25 of his allowance for 6 weeks. How much more money does he need to purchase a 3-game ticket package that costs $89.00 for his favorite baseball team?
   $89.00 – (6 × $7.25) = $45.50

7. Polly's Pet Care ordered 16 pounds of sunflower seeds and $8\frac{1}{2}$ pounds of cracked corn. If these items are divided equally among 4 customers, how many pounds of bird food will each customer receive?
   $(16 \div 4) + (8\frac{1}{2} \div 4) = 4 + 2\frac{1}{8} = 6\frac{1}{8}$ lb

8. The shape below represents $\frac{1}{2}$ of a symmetrical figure (both halves of the figure match when folded on the blue line). Find the perimeter of the figure.



(2 × 2) + (2 × 4) + (2 × 2) = 16 ft

Solve.

9. Kyle's grandparents live 984 miles away. Kyle's family is driving to visit them. They traveled $\frac{1}{3}$ of the distance the first day. How many more miles do they need to travel before they arrive?
   $984 - (\frac{1}{3} \times 984) = 984 - 328 = 656$ miles

10. On day 2 of the trip, Kyle's family decided to travel 1.5 times farther than they did on day 1. How many more miles must they travel to arrive at their destination?
    $656 - (1.5 \times 328) = 656 - 492 = 164$ miles

11. Kyle's family stopped for dinner. They purchased one meal for $6.39, two meals for $4.79 each, and two other meals that were $2.98 each. What was the total cost of the dinner for the family?
    $6.39 + (2 × $4.79) + (2 × $2.98) = $6.39 + $9.58 + $5.96 = $21.93

Evaluate the expression. *Answers are shown using cancellation.*

12. $6 + (4 \times \frac{5}{8}) \div 2$
    $6 + \frac{5}{2} \div 2 = 6 + \frac{5}{2} \times \frac{1}{2} = 6 + \frac{5}{4} = 7\frac{1}{4}$

13. $\frac{2}{3} \times \frac{1}{2} - \frac{1}{8}$
    $\frac{2}{6} - \frac{1}{8} = \frac{8}{24} - \frac{3}{24} = \frac{5}{24}$

14. $\frac{1}{5} \times 2 \times \frac{3}{4}$
    $\frac{2}{5} \times \frac{3}{4} = \frac{3}{10}$

15. $1\frac{1}{3} \times 5 \div \frac{2}{3}$
    $\frac{4}{3} \times 5 \div \frac{2}{3} = \frac{20}{3} \times \frac{3}{2} = \frac{10}{1} = 10$

16. $(\frac{4}{5} + \frac{2}{3}) \times (\frac{7}{8} - \frac{2}{8})$
    $(\frac{12}{15} + \frac{10}{15}) \times \frac{5}{8} = \frac{22}{15} \times \frac{5}{8} = \frac{11}{12}$

17. $(2\frac{4}{9} + 1\frac{2}{3}) \div \frac{1}{2}$
    $(2\frac{4}{9} + 1\frac{6}{9}) \div \frac{1}{2} = 3\frac{10}{9} \times \frac{2}{1} = \frac{37}{9} \times \frac{2}{1} = \frac{74}{9} = 8\frac{2}{9}$

Evaluate the expression if $b = \frac{2}{3}$. *Answers are shown using cancellation.*

18. $b \times \frac{9}{12} + 8$
    $\frac{2}{3} \times \frac{9}{12} + 8 = \frac{3}{6} + 8 = 8\frac{1}{2}$

19. $\frac{8}{9} \div \frac{1}{3} + b$
    $\frac{8}{9} \times \frac{3}{1} + \frac{2}{3} = \frac{8}{3} + \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$

20. $7\frac{6}{7} \div b + 4$
    $7\frac{6}{7} \div \frac{2}{3} + 4 = \frac{55}{7} \times \frac{3}{2} + 4 = \frac{165}{14} + 4 = 11\frac{11}{14} + 4 = 15\frac{11}{14}$

21. $8 - (b \div \frac{1}{6}) \times 1\frac{1}{3}$
    $8 - (\frac{2}{3} \times \frac{6}{1}) \times \frac{4}{3} = 8 - \frac{4}{1} \times \frac{4}{3} = 8 - \frac{16}{3} = 8 - 5\frac{1}{3} = 2\frac{2}{3}$

22. $3\frac{4}{8} + 1\frac{1}{2} \times b$
    $3\frac{4}{8} + 1\frac{1}{2} \times \frac{2}{3} = 3\frac{4}{8} + \frac{3}{2} \times \frac{2}{3} = 3\frac{4}{8} + 1 = 4\frac{4}{8} = 4\frac{1}{2}$

23. $3 + (\frac{5}{6} - b) \times \frac{3}{5}$
    $3 + (\frac{5}{6} - \frac{4}{6}) \times \frac{3}{5} = 3 + \frac{1}{6} \times \frac{3}{5} = 3 + \frac{1}{10} = 3\frac{1}{10}$

Solve. Write the answer in lowest terms. *Answers are shown using cancellation.*

24. $5\frac{6}{10} \div 2\frac{1}{3}$
    $\frac{56}{10} \times \frac{3}{7} = \frac{24}{10} = 2\frac{4}{10} = 2\frac{2}{5}$

25. $2\frac{6}{7} \div 1\frac{1}{2}$
    $\frac{20}{7} \times \frac{2}{3} = \frac{40}{21} = 1\frac{19}{21}$

26. $\frac{9}{10} \div \frac{1}{5}$
    $\frac{9}{10} \times \frac{5}{1} = \frac{9}{2} = 4\frac{1}{2}$

27. $1\frac{2}{6} \div \frac{1}{12}$
    $\frac{8}{6} \times \frac{12}{1} = \frac{16}{1} = 16$

28. $4 \div \frac{1}{2}$
    $\frac{4}{1} \times \frac{2}{1} = \frac{8}{1} = 8$

29. $\frac{1}{15} \div \frac{1}{20}$
    $\frac{1}{15} \times \frac{20}{1} = \frac{4}{3} = 1\frac{1}{3}$

Write two different equations to solve the problem. Explain the two solutions.

J Levi has a board that is $8\frac{1}{2}$ feet long. He wants to cut it into 4-inch pieces. How many pieces can he cut?

$(8\frac{1}{2} \times 12) \div 4 = 25\frac{1}{2}$ pieces; rename feet as inches; divide into 4-inch pieces. $8\frac{1}{2} \div \frac{1}{3} = 25\frac{1}{2}$ pieces; rename 4 inches as part of a foot and then divide. The solution to both equations is the same.

Complete DAILY REVIEW 9 on page 432.

7. Challenge the students to use mental math to answer the question(s) for each of the following word problems as you read them aloud.

> Logan and Daniel bought a large pizza. Logan ate $\frac{5}{8}$ of the pizza, and Daniel ate $\frac{1}{4}$ of it. Who ate the most pizza? *Logan* How much more did he eat? $\frac{3}{8}$ *more of the pizza* Was there any pizza left? *yes* How much was left? $\frac{1}{8}$ *of the pizza*
>
> Ava is $\frac{5}{6}$ of the age of Carter. Carter is 18 years old. How old is Ava? *15 years old*

Student Text pp. 182–83

Lesson 77 **Student Text** pp. 184–85

Chapter Review

Objectives

- Demonstrate an understanding of dividing a fraction by a whole number and dividing a whole number or a fraction by a fraction
- Draw a diagram and use a number line to picture a fraction division equation
- Check a division problem using multiplication
- Divide by multiplying by the reciprocal of the divisor
- Divide mixed numbers
- Apply mathematical properties to fractions
- Use the Order of Operations to evaluate an expression
- Substitute a given value for a variable in an expression
- Solve a multi-step word problem

Notes

Write the names of the mathematical properties for display; include the Zero Principle of Subtraction. (See Student Text page 180.)

This lesson reviews the concepts presented in Chapter 8 to prepare the students for the Chapter 8 Test. Student Text pages 184–85 provide the students with an excellent study guide.

Check for Understanding

Demonstrate an understanding of dividing fractions

After dinner, Mrs. Karl had $\frac{1}{3}$ of a pan of brownies left. If $\frac{1}{12}$ of the pan of brownies is a serving, how many servings were left? *4 servings*

➤ **How could you find how many servings are in the $\frac{1}{3}$ of a pan of brownies? Why?** *Divide $\frac{1}{3}$ by $\frac{1}{12}$; elicit that you need to find how many servings of $\frac{1}{12}$ are in $\frac{1}{3}$.*

1. Write $\frac{1}{3} \div \frac{1}{12} = \_\_$ for display.

➤ **How could you solve this equation using division?** *Elicit that you can rename the dividend $\frac{1}{3}$ as $\frac{4}{12}$ and then divide both the numerators and the denominators.*

➤ **What does $\frac{4}{12} \div \frac{1}{12}$ equal?** *4* Complete the equation: 4 servings.

2. Direct the students to illustrate the equation by drawing a diagram. Select students to draw their diagram for display and explain it. Guide each explanation as needed. *Accept any figure with $\frac{1}{3}$ shaded and the whole figure repartitioned into twelfths, showing $\frac{4}{12}$ in the shaded part.* (See Lesson 70.)
3. Choose a student to picture the equation on a number line.
4. Select another student to demonstrate checking the equation. *$4 \times \frac{1}{12} = \frac{4}{12} = \frac{1}{3}$*
5. Follow a similar procedure for these equations.
   $\frac{4}{5} \div \frac{1}{5} =$ *4; $4 \times \frac{1}{5} = \frac{4}{5}$* $\quad\quad$ $\frac{2}{3} \div \frac{1}{6} =$ *4; $4 \times \frac{1}{6} = \frac{2}{3}$*
   $4 \div \frac{2}{3} =$ *6; $6 \times \frac{2}{3} = 4$*
6. Guide the students in drawing a diagram and in using a number line to solve $\frac{6}{7} \div 3$. $\frac{2}{7}$ (See Lesson 72.)

Divide by multiplying by the reciprocal of the divisor

1. Write these pairs of numbers for display.
   $\frac{1}{2}$ and 2 $\quad$ $\frac{3}{4}$ and $\frac{4}{3}$ $\quad$ $\frac{2}{3}$ and $\frac{3}{2}$ $\quad$ 5 and $\frac{1}{5}$

➤ **What do you notice about these pairs of numbers?** *Elicit that the numbers in each pair are reciprocals because they have a product of 1 when they are multiplied.*

2. Write $\frac{3}{8} \div \frac{1}{2}$ for display.

➤ **What multiplication process can you use to solve $\frac{3}{8} \div \frac{1}{2}$?** *Elicit that you can multiply the dividend $\frac{3}{8}$ by 2 (the reciprocal of $\frac{1}{2}$) to find the quotient.* (See Lesson 72.)

3. Choose a student to rewrite the division problem as a multiplication problem and solve it. Remind him to use cancellation if possible. *Accept $\frac{3}{8} \times 2 = \frac{3}{4}$ or $\frac{3}{8} \times \frac{2}{1} = \frac{3}{4}$.*
   Write $\frac{3}{4}$ as the quotient of the division problem.

➤ **What problem can you use to check the quotient $\frac{3}{4}$?** $\frac{3}{4} \times \frac{1}{2}$
Direct the students to check the division problem. $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

4. Instruct the students to multiply by the reciprocal to solve the following problems on paper. Allow them to write 1 as the denominator of a whole number if needed to help them solve the problem correctly.
   $\frac{6}{7} \div \frac{1}{4} = \frac{6}{7} \times 4 = \frac{24}{7} =$ *$3\frac{3}{7}$* $\quad\quad$ $\frac{1}{6} \div \frac{1}{9} = \frac{1}{6} \times 9 = \frac{3}{2} =$ *$1\frac{1}{2}$*
   $5 \div \frac{3}{8} = 5 \times \frac{8}{3} = \frac{40}{3} =$ *$13\frac{1}{3}$* $\quad\quad$ $\frac{2}{5} \div 5 = \frac{2}{5} \times \frac{1}{5} = \frac{2}{25}$

Divide mixed numbers

James has a rope that is $15\frac{3}{4}$ feet long. If he cuts the rope into lengths of $6\frac{1}{4}$ feet, how many pieces of rope will he have? *3 pieces*

➤ **What equation can you write to solve the word problem? Why?** *$15\frac{3}{4} \div 6\frac{1}{4} = \_\_$; elicit that you need to find how many lengths of $6\frac{1}{4}$ feet are in $15\frac{3}{4}$ feet so you can determine how many pieces of rope James will have.* Write the equation for display.

➤ **How can you solve the equation?** *Rename the mixed numbers as improper fractions and multiply by the reciprocal.*

1. Direct the students to solve the equation. Remind them to use cancellation if possible. $\frac{63}{4} \div \frac{25}{4} = \frac{63}{4} \times \frac{4}{25} =$ *$63 \times \frac{1}{25} = \frac{63}{25} = 2\frac{13}{25}$*

➤ **How many lengths of $6\frac{1}{4}$ feet are in $15\frac{3}{4}$ feet?** *2*

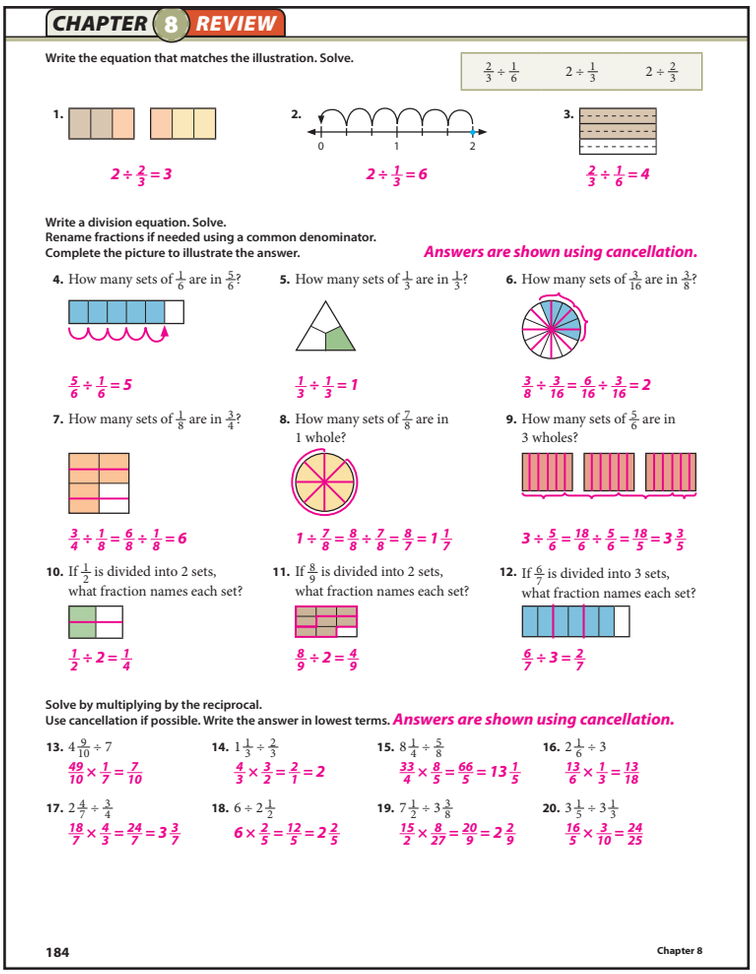
➤ **How many pieces of rope will James have? How do you know?** *3 pieces; elicit that 2 of the pieces of rope will be $6\frac{1}{4}$ feet long and the remainder $\frac{13}{25}$ represents a third piece of rope that will be shorter.*

2. Direct the students to solve the following equations and to check their answers. Choose students to write their solutions for display. Discuss the solutions as needed.
   $3\frac{1}{8} \div \frac{1}{4} =$ *$12\frac{1}{2}$* $\quad\quad$ $2\frac{5}{6} \div 1\frac{1}{4} =$ *$2\frac{4}{15}$*
   $1\frac{1}{5} \div 2 =$ *$\frac{3}{5}$* $\quad\quad$ $8 \div 2\frac{3}{4} =$ *$2\frac{10}{11}$*

Apply mathematical properties to fractions

Write the following equations for display and direct attention to the names of the mathematical properties written for display. For each equation, select students to identify the property represented by the equation, explain the property, and complete the equation.

$\_\_ + \frac{5}{9} = \frac{5}{9}$ *Identity Property of Addition; 0*
$\frac{1}{4} \times (\frac{4}{5} \times \frac{1}{2}) = (\frac{1}{4} \times \_\_) \times \frac{1}{2}$ *Associative Property of Multiplication; $\frac{4}{5}$*
$\frac{2}{3} - \_\_ = \frac{2}{3}$ *Zero Principle of Subtraction, 0*
$2\frac{3}{4} \times \_\_ = 2\frac{3}{4}$ *Identity Property of Multiplication; 1*
$8\frac{1}{3} + 1\frac{1}{4} = 1\frac{1}{4} + \_\_$; *Commutative Property of Addition; $8\frac{1}{3}$*
$2 \times 4\frac{5}{6} = (2 \times \_\_) + (2 \times \_\_)$ *Distributive Property; 4; $\frac{5}{6}$*
$\frac{7}{8} \times \_\_ = 0$ *Zero Property of Multiplication; 0*
$\frac{3}{4} + (\frac{2}{3} + \frac{1}{6}) = (\_\_ + \frac{2}{3}) + \frac{1}{6}$ *Associative Property of Addition; $\frac{3}{4}$*
$\frac{1}{2} \times \frac{2}{5} = \_\_ \times \frac{1}{2}$ *Commutative Property of Multiplication; $\frac{2}{5}$*



CHAPTER 8 REVIEW

Write the equation that matches the illustration. Solve.

$\frac{2}{3} \div \frac{1}{6}$ $2 \div \frac{1}{3}$ $2 \div \frac{2}{3}$

1. $2 \div \frac{2}{3} = 3$

2. $2 \div \frac{1}{3} = 6$

3. $\frac{2}{3} \div \frac{1}{6} = 4$

Write a division equation. Solve.
Rename fractions if needed using a common denominator.
Complete the picture to illustrate the answer. *Answers are shown using cancellation.*

4. How many sets of $\frac{1}{6}$ are in $\frac{5}{6}$? $\frac{5}{6} \div \frac{1}{6} = 5$

5. How many sets of $\frac{1}{3}$ are in $\frac{1}{3}$? $\frac{1}{3} \div \frac{1}{3} = 1$

6. How many sets of $\frac{3}{16}$ are in $\frac{3}{8}$? $\frac{3}{8} \div \frac{3}{16} = \frac{6}{16} \div \frac{3}{16} = 2$

7. How many sets of $\frac{1}{8}$ are in $\frac{3}{4}$? $\frac{3}{4} \div \frac{1}{8} = \frac{6}{8} \div \frac{1}{8} = 6$

8. How many sets of $\frac{7}{8}$ are in 1 whole? $1 \div \frac{7}{8} = \frac{8}{8} \div \frac{7}{8} = \frac{8}{7} = 1\frac{1}{7}$

9. How many sets of $\frac{5}{6}$ are in 3 wholes? $3 \div \frac{5}{6} = \frac{18}{6} \div \frac{5}{6} = \frac{18}{5} = 3\frac{3}{5}$

10. If $\frac{1}{2}$ is divided into 2 sets, what fraction names each set? $\frac{1}{2} \div 2 = \frac{1}{4}$

11. If $\frac{8}{9}$ is divided into 2 sets, what fraction names each set? $\frac{8}{9} \div 2 = \frac{4}{9}$

12. If $\frac{6}{7}$ is divided into 3 sets, what fraction names each set? $\frac{6}{7} \div 3 = \frac{2}{7}$

Solve by multiplying by the reciprocal.
Use cancellation if possible. Write the answer in lowest terms. *Answers are shown using cancellation.*

13. $4\frac{9}{10} \div 7$ $\frac{49}{10} \times \frac{1}{7} = \frac{7}{10}$

14. $1\frac{1}{3} \div \frac{2}{3}$ $\frac{4}{3} \times \frac{3}{2} = \frac{2}{1} = 2$

15. $8\frac{1}{4} \div \frac{5}{8}$ $\frac{33}{4} \times \frac{8}{5} = \frac{66}{5} = 13\frac{1}{5}$

16. $2\frac{1}{6} \div 3$ $\frac{13}{6} \times \frac{1}{3} = \frac{13}{18}$

17. $2\frac{4}{7} \div \frac{3}{4}$ $\frac{18}{7} \times \frac{4}{3} = \frac{24}{7} = 3\frac{3}{7}$

18. $6 \div 2\frac{1}{2}$ $6 \times \frac{2}{5} = \frac{12}{5} = 2\frac{2}{5}$

19. $7\frac{1}{2} \div 3\frac{3}{8}$ $\frac{15}{2} \times \frac{8}{27} = \frac{20}{9} = 2\frac{2}{9}$

20. $3\frac{1}{5} \div 3\frac{1}{3}$ $\frac{16}{5} \times \frac{3}{10} = \frac{24}{25}$

Write the reciprocal.

21. $5\frac{3}{5}$ $\frac{5}{28}$ 22. 2 $\frac{1}{2}$ 23. $8\frac{1}{8}$ $\frac{8}{65}$ 24. $\frac{2}{3}$ $\frac{3}{2}$ 25. 11 $\frac{1}{11}$

Multiply to check the answer. Write true or false.
Write the correct quotient if the answer is false. *Answers are shown using cancellation.*

26. $6 \div \frac{5}{6} = 6\frac{2}{3}$ *false; $7\frac{1}{5}$*
$\frac{20}{3} \times \frac{5}{6} = \frac{50}{9} = 5\frac{5}{9}$
$6 \times \frac{6}{5} = \frac{36}{5} = 7\frac{1}{5}$

27. $\frac{7}{8} \div \frac{1}{6} = 5\frac{1}{4}$ *true*
$\frac{21}{4} \times \frac{1}{6} = \frac{7}{8}$

28. $3\frac{1}{8} \div 2\frac{1}{4} = 1\frac{1}{9}$ *false; $1\frac{7}{18}$*
$\frac{10}{9} \times \frac{9}{4} = \frac{5}{2} = 2\frac{1}{2}$
$\frac{25}{8} \times \frac{4}{9} = \frac{25}{18} = 1\frac{7}{18}$

Solve.

29. Mrs. Stanley has a square flower garden that is $8\frac{1}{2}$ feet on each side. She has $10\frac{3}{4}$ feet of landscape edging. How much more edging does she need for the perimeter of her garden?
$(4 \times 8\frac{1}{2}) - 10\frac{3}{4} = (4 \times \frac{17}{2}) - 10\frac{3}{4} =$
$34 - 10\frac{3}{4} = 33\frac{4}{4} - 10\frac{3}{4} = 23\frac{1}{4}$ ft

30. Mr. Martin has 2 packages of hamburger in the freezer. One package is $2\frac{3}{4}$ pounds and the other is $1\frac{3}{4}$ pounds. He needs $1\frac{1}{2}$ pounds of hamburger for each pot of chili he makes. How many pots of chili can be made with the hamburger he has?
$(2\frac{3}{4} + 1\frac{3}{4}) \div 1\frac{1}{2} = 4\frac{1}{2} \div 1\frac{1}{2} = \frac{9}{2} \div \frac{3}{2} =$
$\frac{9}{2} \times \frac{2}{3} = 3$ pots

Answers are shown using cancellation.
Evaluate the expression.

31. $2\frac{3}{4} \div (\frac{1}{8} + \frac{2}{8})$ $\frac{11}{4} \times \frac{8}{3} = \frac{22}{3} = 7\frac{1}{3}$

32. $6 - \frac{3}{9} \times \frac{1}{3}$ $6 - \frac{1}{9} = 5\frac{9}{9} - \frac{1}{9} = 5\frac{8}{9}$

33. $(\frac{1}{3} - \frac{1}{6}) \times (\frac{2}{12} + \frac{1}{12})$ $(\frac{2}{6} - \frac{1}{6}) \times \frac{3}{12} = \frac{1}{6} \times \frac{3}{12} = \frac{1}{24}$

Evaluate the expression if $a = \frac{1}{3}$. *Answers are shown using cancellation.*

34. $\frac{1}{5} \times a \times 2$ $\frac{1}{5} \times \frac{1}{3} \times \frac{2}{1} = \frac{2}{15}$

35. $\frac{3}{4} \div a \times 3$ $\frac{3}{4} \div \frac{1}{3} \times \frac{3}{1} =$
$\frac{3}{4} \times \frac{3}{1} \times \frac{3}{1} = \frac{27}{4} = 6\frac{3}{4}$

36. $6 \times (3 - a)$ $6 \times (3 - \frac{1}{3}) =$
$6 \times 2\frac{2}{3} = 6 \times \frac{8}{3} = 16$

Write the missing number or fraction that illustrates a mathematical property.
Write the property.

37. $\frac{2}{8} \times (\frac{3}{8} \times \frac{1}{8}) = (\_\_ \times \frac{3}{8}) \times \frac{1}{8}$ $\frac{2}{8}$
Associative Property

38. $\frac{3}{4} \times \_\_ = 0$ *0*
Zero Property of Multiplication

39. $\frac{2}{5} \times \frac{4}{5} = \_\_ \times \frac{2}{5}$ $\frac{4}{5}$
Commutative Property

40. $3 \times 1\frac{1}{6} = (\_\_ \times 1) + (3 \times \_\_)$ *3; $\frac{1}{6}$*
Distributive Property

41. $\frac{1}{3} \times \_\_ = \frac{1}{3}$ *1*
Identity Property of Multiplication

Associative Property
Commutative Property
Distributive Property
Identity Property of Multiplication
Zero Property of Multiplication

Use the Order of Operations to evaluate an expression

Direct the students to use the given values and the Order of Operations to simplify these expressions. Choose students to write their solution for display and explain it. Review the Order of Operations as needed.

$a = \frac{5}{6}$
$(\frac{2}{3} \times a) \div \frac{1}{2}$
$= (\frac{2}{3} \times \frac{5}{6}) \div \frac{1}{2}$
$= \frac{5}{9} \div \frac{1}{2}$
$= \frac{10}{9} = 1\frac{1}{9}$

$b = \frac{3}{4}$
$= (b \div \frac{1}{2}) - (4 \times \frac{1}{3})$
$= (\frac{3}{4} \div \frac{1}{2}) - (4 \times \frac{1}{3})$
$= (\frac{3}{4} \times 2) - (4 \times \frac{1}{3})$
$= \frac{3}{2} - \frac{4}{3}$
$= \frac{9}{6} - \frac{8}{6}$
$= \frac{1}{6}$

Solve a multi-step word problem

Jacob read $\frac{2}{3}$ of a book that was 315 pages. Kevin read $\frac{1}{2}$ of a book that was 414 pages. Who read the most pages? How many more pages did he read? *Jacob; 3 pages more*

- How could you find the answers to the questions in the word problem? *Answers will vary, but elicit that first you need to determine the number of pages each of the boys read and then subtract to find the difference in the number of pages.*
- What operation would you use to determine the number of pages read by each boy? Why? *Multiplication; you need to find how many pages are in a fraction (a part) of the total number of pages.*
- Who do you think read the most pages? *Answers will vary.*
- Do you think one equation could be used to solve this multi-step problem? Why? *Answers will vary, but elicit that more than one equation is needed because you do not know who read the most pages; therefore, you do not know which expression would be the minuend in a multi-step equation and which expression would be the subtrahend.*

Direct the students to solve the problem. Discuss the solution as needed. Challenge the students to write a multi-step equation to show how they solved the word problem.
$(\frac{2}{3} \times 315) - (\frac{1}{2} \times 414) = 3$ pages

Student Text pp. 184–85

Lesson 78

Student Text pp. 186–89

Chapter 8 Test

Cumulative Review

For a list of the skills reviewed in the Cumulative Review, see the Lesson Objectives for Lesson 78 in the Chapter 8 Overview on page 168 of this Teacher's Edition.

Student Materials

- Cumulative Review Answer Sheet, page IA9 (CD)

Use the Cumulative Review on Student Text pages 186–88 to review previously taught concepts and to determine which students would benefit from your reteaching of the concepts. To prepare the students for the format of achievement tests, instruct them to work on a separate sheet of paper, if necessary, and to mark the answers on the Cumulative Review Answer Sheet.

Read aloud the Career Link on Student Text page 189 (page 187 of this Teacher's Edition) and discuss the value of math as it relates to an environmental analyst.

Mark the answer.

11.

48			
12	12	12	12

A. 48 − 12
B. 4 × 12
C. 4 + 12

12. 6^3
A. 6 • 6 • 6
B. 6 × 3
C. 3 • 6

13. $\frac{1}{5} = \_\_$
A. 5 × 1
B. 1 ÷ 5
C. 5 ÷ 1

14. $\frac{3}{8} = \frac{n}{24}$
A. $n = 6$
B. $n = 9$
C. $n = 12$

15. $\frac{5}{2} = \_\_$
A. $5\frac{1}{2}$
B. $2\frac{1}{2}$
C. $1\frac{1}{2}$

16. $7 + 8 \times 3$
A. 29
B. 30
C. 31

17. $2 \times 3 + 16 \div 4$
A. 5
B. 10
C. 15

18. $4 + 6 \times (3 + 2)$
A. 34
B. 36
C. 38

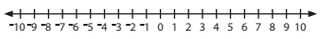
19. $9 \times 4 - 2^2$
A. 52
B. 42
C. 32

20. $27 - 4 \times 3 \div 6$
A. 25
B. 20
C. 15

CUMULATIVE REVIEW

Test Prep

Use the number line to find the answer.



1. $-5 + 2 = \_\_$
A. −1
B. −2
C. −3

2. $-4 + -1 = \_\_$
A. 0
B. −5
C. 5

3. $8 + -7 = \_\_$
A. 1
B. −1
C. 15

4. $-3 + 4 = \_\_$
A. 0
B. 1
C. 2

5. $5 + -5 = \_\_$
A. 10
B. 1
C. 0

Complete the sequence.

6. 2, 4, 6, __
A. 12
B. 10
C. 8

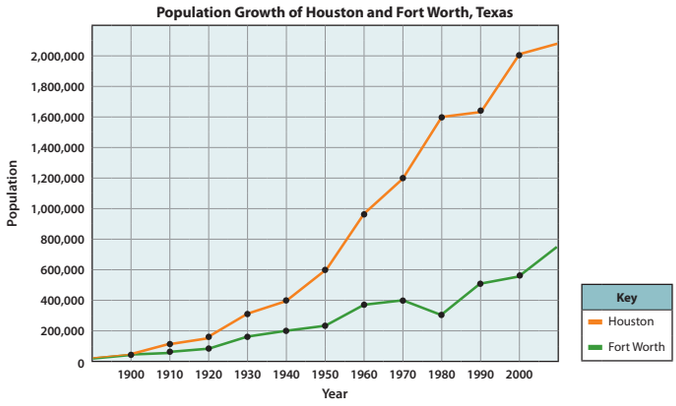
7. 3, 9, 27, 81, __
A. 243
B. 162
C. 84

8. −5, −3, −1, 1, __
A. 5
B. 3
C. 1

9. $\frac{1}{2}$, 1, $1\frac{1}{2}$, __
A. $\frac{3}{4}$
B. $2\frac{1}{2}$
C. 2

10. 0.25, 0.50, 0.75, __
A. 100
B. 10
C. 1

Use the data from the line graph to find the answer.



21. What was Houston's population in 1950?
A. 500,000
B. 600,000
C. 700,000

22. How much greater was the population of Houston than that of Fort Worth in 1970?
A. 400,000
B. 600,000
C. 800,000

23. Which city's population decreased from 1970 to 1980?
A. Houston
B. Fort Worth
C. both cities

24. Which city grew at a faster rate?
A. Houston
B. Fort Worth
C. both cities

25. The lines for Houston and Fort Worth show
A. a decline in population.
B. no change in population.
C. a steady growth in population.

CAREER LINK

Environmental Analyst

God has given man the ability to design and build many wonderful inventions. Modern travel and machinery are made possible by the sacrifices and hard work of many people. Knowing how to regulate and maintain these businesses in such a way that keeps people and our environment safe is the job of an environmental analyst. He collects, studies, and analyzes data so that businesses and the government know how to work together.

An environmental analyst collects and uses data every day to make suggestions and provide solutions to problems that affect environmental safety. His working knowledge of science and math is critical to doing his job effectively. He spends time with business owners reviewing procedures and production of their businesses to help them work within government regulations. He must maintain a comprehensive knowledge of policies and legislation, as well as study the geography and unique qualities of land and water for the businesses for which he works.

Gathering scientific and mathematical information is part of an analyst's everyday work. Using the results of his research, he creates proposals to help companies establish good procedures for doing business in their community. That means keeping the people, land, water, and even animals within that area safe. Because of the need to communicate with business owners and officials, he must develop professional writing and communication skills.

You might be wondering what caring for the environment has to do with being a Christian. God tells us in Genesis 1 that He has created our world and has given us the responsibility to use and care for the earth in order to help our fellow man and to serve Him. Being proficient in math and science skills enables the environmental analyst to use those gifts not only to help his employers but also to help protect the world that God has created.