13 Ratios, Proportions & Percents

JOURNEY THROUGH ASH

Cougar, Washington **May 18, 1980**

In the late 1970s, people in the beautiful resort region surrounding Mount Saint Helens had reason to be uneasy. Geologists were carefully watching this mountain located in eastern Washington. Although dormant for more than a century, the snowcapped volcano was due for another eruption. On March 20, 1980, a series of small tremors began to shake the earth. A few days later, a small eruption of ash and steam blew out of the peak. However, scientists were still concerned about a large, expanding bulge on the mountain's north slope. They advised the governor to set up a danger zone around the volcano and to order everyone to evacuate.

On the morning of May 18, the north slope exploded sideways. Huge pieces of the mountain slid into Spirit Lake and the Toutle River below. A vertical explosion sent ash and smoke billowing thousands of feet into the air. The blast destroyed much more than the danger zone. Ash and mud covered the landscape for hundreds of miles. Homes, vegetation, people, and animals were buried. Those who managed to escape clustered together in public shelters that had been set up outside of the danger area.

Plants grow again around Mount Saint Helens.

Bruce Nelson and Sue Ruff were part of a group of six people who had been camping about sixteen miles north of the volcano. As they prepared breakfast at their campfire the morning of May 18, hot ash, mud, and stones began to rain down on them. They fell into a hole where several trees had been uprooted. It was so hot in the hole that they could hear their own hair sizzle. When the downpour stopped, they dug themselves out with their hands, burning their fingers on the hot ash. They called to their friends but found only two of them, both seriously injured by trees that had fallen on them. After sheltering the two injured young men, Bruce and Sue decided to go for help. They waded with burning feet through the sea of hot ash, coughing from the dust in the air and stumbling over fallen trees. Wandering for three hours, they finally were able to flag down a rescue helicopter. They directed the pilot back to their campsite and helped him rescue their two injured friends.

Although losses from the volcano were great, the landscape has gradually returned to normal again. The ash actually enriched the soil and helped vegetation grow back more quickly. The area surrounding Mount Saint Helens is now a 100,000-acre national monument.

The Mount Saint Helens volcanic eruption was the first eruption in the forty-eight contiguous states of the United States to result in any deaths.

A 360º view of the crater left by the eruption as well as a live webcam of Mount Saint Helens is available on the Internet.

Ninety percent of the species of plants that were destroyed around Mount Saint Helens had grown back after only three years.

Many survival stories have been written by those who lived through the Mount Saint Helens volcanic eruption.

The U.S. Geological Survey monitors volcanic activity and analyzes data to determine the possibility of an eruption. Their website provides more information on observatories and monitoring volcanoes.

Hot volcanic lava can reach temperatures greater than 2,000ºF.

Ratios, Proportions & Percents

Lesson	Topic	Lesson Objectives	Chapter Materials
114	**Ratios & Rates**	• Write a ratio in three forms: word form, ratio form, fraction form • Write ratios to describe part-to-part, part-to-whole, and whole-to-part comparisons • Find equivalent ratios • Determine the unit rate • Find an equivalent ratio using the unit rate • Use ratios to represent real-life situations and to solve problems	**Teacher Manipulatives Packet:** • Shapes Kit • Place Value Kit **Student Manipulatives Packet:** • Black and red counters **Instructional Aids (Teacher's Toolkit CD):** • Cumulative Review Answer Sheet (page IA9) for each student • Graph Paper (page IA13) • Graph Paper (page IA13) for each student • Parent Letter (page IA63), a half page for each student • Pictured Ratios (page IA64) • Ratio Tables (page IA65) • Missing Measurements (page IA66) • Percent (page IA67) • Percent Models: Find the Part (page IA68) • Percent Models: Find the Whole (page IA69) • Circle Graph: Elements in the Earth's Crust (page IA70) **Christian Worldview Shaping (Teacher's Toolkit CD):** • Pages 31–32 **Other Teaching Aids:** • A map • Samples of floor plans • Modeling clay • A calculator for each student • A map for each group of students (optional) • A ruler for each student and the teacher • A straight edge for each student and the teacher (optional) **Math 6 Tests and Answer Key** **Optional (Teacher's Toolkit CD):** • Fact Review pages • Application pages • Calculator Activities
115	**Ratio Tables**	• Complete a ratio table • Find equivalent ratios • Make a ratio table • Solve problems using ratio tables • Use ratios to represent real-life situations and to solve problems	
116	**Solving Proportions**	• Develop an understanding of proportions using models • Determine whether two ratios are proportional • Solve for a missing term in a proportion • Use ratios to represent real-life situations and to solve problems	
117	**Similar Figures**	• Develop an understanding of proportions in similar figures • Solve for a missing term in a proportion • Find the unknown measure in similar figures using proportions • Use indirect measurement to find the unknown measure in similar objects • Use ratios to represent real-life situations and to solve problems	
118	**Scale**	• Find actual measurements using a scale and a scale drawing, map, or model • Determine the unknown measure on a scale drawing given the scale and the actual measurement • Solve word problems using ratios	
119	**Percent**	• Develop an understanding of percent using models • Express percents as ratios, decimals, and fractions in lowest terms • Express decimals and fractions as percents • Compare percents to decimals and fractions using >, <, or = • Solve percent word problems using proportions	
120	**Finding Percent of a Number**	• Find a percent of a number using an equation, a model, and a proportion • Solve percent word problems	
121	**Finding the Unknown Whole**	• Find the unknown whole in a percent problem using a model, an equation, and a proportion • Solve percent word problems	
122	**Speed, Distance & Time**	• Calculate the distance given the rate of speed and the time, the rate of speed given the distance and the time, and the time given the distance and the rate of speed • Rename to calculate distance, rate of speed, or time • Find an equivalent rate using a proportion	
123	**Chapter 13 Review**	• Review	
124	**Chapter 13 Test Cumulative Review**	• Read and interpret a circle graph • Solve word problems • Simplify square roots, exponents, and expressions • Solve for a missing term in a proportion • Calculate the area of a complex figure • Calculate the circumference of a circle • Calculate the volume of a cylinder • Identify the equation for the area of a rectangular prism and a triangle	

Send home the Parent Letter, page IA63, to inform parents of the items needed for Chapter 14.

A Little Extra Help

Use the following to provide "a little extra help" for the student that is experiencing difficulty with the concepts taught in Chapter 13.

Write ratios—Direct the student to read Matthew 14:17 from his Bible. Remind him that a ratio compares two quantities and can be written using the word *to* (word form), using a colon (:) to represent the word *to* (ratio form), and as a fraction (fraction form). Ask him to identify the objects that are mentioned in the verse. ***5 loaves, 2 fish*** Instruct the student to first write the ratio of loaves to fish in word form, and then to write it in ratio form and fraction form. ***5 to 2, 5:2,*** $\frac{5}{2}$ Direct him to read aloud each form of the ratio. If necessary, allow the student to label the terms in each ratio form (e.g., 5 loaves:2 fish) and to read aloud each ratio form.

Repeat the procedure using the following references: Luke 17:11–19 ***1 thankful leper to 10 healed lepers—1 to 10, 1:10,*** $\frac{1}{10}$ and Job 1:1–2 ***7 sons to 3 daughters—7 to 3, 7:3,*** $\frac{7}{3}$. You may choose to further examine Job's substance before and after his trials as shown in Job 1:3 and Job 42:10–12.

Find equivalent ratios by multiplying and dividing—Write $\frac{7}{5} = \frac{n}{50}$ for display. Ask the student to identify the relationship between the second terms of the ratios. ***50 is 10 times greater than 5 or 5 × 10 = 50*** Draw an arrow from 5 to 50 and write × 10 below it. (See page 279 of the Student Text for an example.) Remind the student that to find an equivalent ratio, the operation that is performed on one term must also be performed on the other term. Ask him to tell what operation needs to be performed to find the unknown term in the second ratio. ***multiplication*** Direct the student to draw an arrow from the 7 to the *n* and write × 10 above it. Then instruct him to erase the *n* and complete the ratio. ***70***

Follow a similar procedure for $\frac{20}{15} = \frac{4}{n}$. ***4 is*** $\frac{1}{5}$ ***of 20 or 20 ÷ 5 = 4; division; 15 ÷ 5 = 3***

Continue the activity as needed using the following, or similar, problems.

$\frac{60}{6} = \frac{30}{3}$ $\quad \frac{5}{2} = \frac{40}{16}$ $\quad \frac{8}{100} = \frac{2}{25}$

$\frac{3}{20} = \frac{18}{120}$ $\quad \frac{60}{25} = \frac{12}{5}$ $\quad \frac{28}{4} = \frac{14}{2}$

Math Facts

Throughout this chapter, review fractions using Fact Review pages on the Teacher's Toolkit CD. Also, review multiplication and division facts using Fact Review pages or a Fact Fun activity on the Teacher's Toolkit CD, or you may use flashcards.

Lesson 114 | **Student Text** pp. 276–79 | **Daily Review** p. 446a

Objectives

- Write a ratio in three forms: word form, ratio form, fraction form
- Write ratios to describe part-to-part, part-to-whole, and whole-to-part comparisons
- Find equivalent ratios
- Determine the unit rate
- Find an equivalent ratio using the unit rate
- Use ratios to represent real-life situations and to solve problems

Teacher Materials

- Shapes Kit

Note

Preview the Fact Review pages, the Application pages, and the Calculator Activities located on the Teacher's Toolkit CD.

Introduce the Lesson

Guide the students in reading aloud the story and facts on pages 276–77 of the Student Text (pages 274–75 of this Teacher's Edition).

Teach for Understanding

Write ratios in 3 forms to describe comparisons

1. Display 4 rhombi and 6 trapezoids and write *rhombi to trapezoids* for display.
   ➤ **How many rhombi are displayed?** ***4*** **trapezoids?** ***6***
   ➤ **If you were to substitute the number of each figure for its name in the written statement, how would it read?** ***4 to 6***
   Write *ratio* for display and write the comparison *4 to 6* below it. Explain that a ratio is a comparison of two quantities and is read using the word *to*. This ratio compares the number of rhombi *to* the number of trapezoids.
   Explain that a ratio can be written in *word form*, *ratio form*, or *fraction form*. The ratio *4 to 6* is written in word form because the word *to* is written. Write *word form* beside *4 to 6*.
2. Write *4:6* for display and label it *ratio form*. Explain that the ratio form is written with a colon.
3. Write $\frac{4}{6}$ for display and label it *fraction form*. Explain that a ratio can also be written in fraction form.
4. Point out that all 3 forms represent the same ratio. Similar to the terms of a fraction, the numbers of a ratio are referred to as terms: the first number (4) is referred to as the *first term* and the second number (6) is referred to as the *second term*.
   ➤ **What ratio compares the number of rhombi to the number of quadrilaterals displayed?** ***4 to 10***
   Choose students to write for display the ratio 4 to 10 in the three forms. ***4:10, 4 to 10,*** $\frac{4}{10}$
5. Explain that ratios can describe different comparisons. This ratio compares part of the set (rhombi) to the whole set (quadrilaterals). Write *part-to-whole* for display.
   ➤ **Is the ratio of rhombi to trapezoids a part-to-whole comparison? Why?** ***No; elicit that it compares one part of the set to another part of the set.*** Write *part-to-part* for display.
   ➤ **What ratio compares the number of quadrilaterals to the number of rhombi?** ***10 to 4***
   Choose a student to write the ratio in the three forms. ***10 to 4, 10:4,*** $\frac{10}{4}$
   ➤ **What comparison does this ratio describe?** ***the whole set to a part of the set*** Write *whole-to-part* for display.
6. Follow a similar procedure using other quadrilaterals.

Find equivalent ratios

1. Display 4 blue squares and 6 blue rectangles.
   ➤ **What part-to-part ratio compares the number of squares to the number of rectangles?** ***4 to 6***
   Choose a student to write the ratio in the three forms, naming each form as he writes the ratio. ***4 to 6, word form; 4:6, ratio form;*** $\frac{4}{6}$***, fraction form***
2. Select a student to display a second set (row) of 4 green squares and 6 green rectangles. Write for display *4 to 6 is like 8 to ___, 4:6 is like 8:___*, and $\frac{4}{6}$ *is like* $\frac{8}{n}$. Remind the students that a fraction is a part-to-whole ratio.
   ➤ **What number would complete each of these ratios to make the statement true?** ***12*** Write 12 to complete each ratio.
3. Write for display *4 to 6 is like 12 to ___, 4:6 is like 12:___*, and $\frac{4}{6}$ *is* $\frac{12}{n}$ and *4 to 6 is like ___ to 24, 4:6 is like ___:24*, and $\frac{4}{6}$ *is like* $\frac{n}{24}$. Choose students to display repeating sets of 4 squares and 6 rectangles to picture and complete the equivalent ratios. ***12 to 18; 16 to 24***
   ➤ **How do you think you can find an equivalent ratio in higher terms?** ***Elicit that you can multiply each term by the same nonzero number*** **in lower terms?** ***Elicit that you can divide each term by the same nonzero divisor.***
   ➤ **Do you think the ratio 4:6 is written in lowest terms? Why?** ***No; elicit that 4 and 6 have a common factor of 2.***
   Choose a student to write an equivalent ratio in lower terms. ***2:3*** Divide the original set of 4 squares to 6 rectangles into two equal sets to show repeating sets of 2 squares to 3 rectangles.
   ➤ **Is the ratio 2:3 written in lowest terms? How do you know?** ***Yes; elicit that 2 and 3 have no common factor other than 1.***
4. Follow a similar procedure for a whole-to-part comparison (quadrilaterals to rectangles). ***10 to 6, 10:6,*** $\frac{10}{6}$***; possible answers: higher terms—20 to 12, 30:18,*** $\frac{100}{60}$***; lower terms—5:3***
   ➤ **What does the ratio 4 to 10 represent?** ***the number of squares to the number of quadrilaterals in the set***
   ➤ **What type of comparison does the ratio 4 to 10 describe for this set of quadrilaterals?** ***part-to-whole***
5. Guide the students in completing these equivalent ratios.
   10 to 7 = ___ to 49 ***70***   5:12 = ___:60 ***25***
   $\frac{15}{45} = \frac{3}{n}$ ***9*** $\frac{16}{4} = \frac{8}{n}$ ***2***
6. Point out that when you multiply each term of a ratio by the same factor or divide each term of a ratio by the same divisor, you are multiplying or dividing the entire ratio by a value of 1 (e.g., $\times \frac{4}{4}$ or $\div \frac{3}{3}$). Elicit from the students that the Identity Property of Multiplication states that when 1 is a factor, the product is the other factor. When a ratio written in fraction form is multiplied by a fraction name for 1, the same ratio is expressed in higher terms.

Determine the unit rate

1. Write *rate* for display. Explain that a *rate* is a special ratio that compares two quantities having different units.
   Write the following statement for display and choose a student to tell what units are being compared. Select another student to write for display the rate in word form, ratio form, and fraction form.

Ratios & Rates

A **ratio** is a mathematical comparison of two quantities. The **terms** represent the items being compared. A ratio is read using the word *to*.

ratio
terms
equivalent ratios
rate
unit rate

There are 4 girls to every 6 boys that play in the county soccer league. The ratio of girls to boys can be written three ways: 4 to 6, 4:6, or $\frac{4}{6}$.

Quantities can be compared differently; therefore, the order of the terms must match the order of the quantities being compared in the written statement.

Quantities	Ratio	Word Form	Ratio Form	Fraction Form
part to part	girls to boys	4 to 6	4:6	$\frac{4}{6}$
part to whole	girls to players	4 to 10	4:10	$\frac{4}{10}$
whole to part	players to girls	10 to 4	10:4	$\frac{10}{4}$

Equivalent ratios can be found by multiplying or dividing both terms of the ratio by the same nonzero number (a form of 1). It is similar to renaming a fraction into higher or lower terms. To simplify a ratio, rename it to lowest terms.

4 to 6 = 8 to 12 (×2, ×2) 4 to 6 = 2 to 3 (÷2, ÷2) 4:10 = 12:30 (×3, ×3) 4:10 = 2:5 (÷2, ÷2) $\frac{10}{4} = \frac{40}{16}$ (×4, ×4) $\frac{10}{4} = \frac{5}{2}$ (÷2, ÷2)

Exercises

Use the data from the chart to write the ratio.

Number of Volcanoes Per State	
State	Volcanoes
Washington	5
Oregon	5
California	3

1. volcanoes in Washington to volcanoes in California (ratio form) ***5:3***
2. volcanoes in California to volcanoes in Oregon (word form) ***3 to 5***
3. volcanoes in Washington to total volcanoes (fraction form) $\frac{5}{13}$
4. volcanoes in Washington and Oregon to total volcanoes (ratio form) ***10:13***

Answers will vary.
Write an equivalent ratio in higher terms.

5. 3 to 4 ***6 to 8*** 6. 4:5 ***12:15*** 7. $\frac{7}{15}$ $\frac{28}{60}$

Answers will vary.
Write an equivalent ratio in lower terms.

8. 12 to 20 ***3 to 5*** 9. 27:45 ***3:5*** 10. $\frac{150}{10}$ $\frac{15}{1}$

Find the missing term that completes the equivalent ratio.

11. $\frac{6}{9} = \frac{n}{3}$ ***n = 2*** 12. $\frac{5}{8} = \frac{n}{64}$ ***n = 40*** 13. $\frac{3}{4} = \frac{n}{100}$ ***n = 75*** 14. $\frac{21}{30} = \frac{7}{n}$ ***n = 10*** 15. $\frac{5}{6} = \frac{30}{n}$ ***n = 36*** 16. $\frac{24}{42} = \frac{4}{n}$ ***n = 7***

Write the ratio as a fraction in lowest terms.

17. 6 parakeets to 36 dogs $\frac{6}{36} = \frac{1}{6}$
18. 18 right-handed students to 4 left-handed students $\frac{18}{4} = \frac{9}{2}$
19. 2 cups of sugar to 10 cups of water $\frac{2}{10} = \frac{1}{5}$
20. 15 red candies to 10 green candies $\frac{15}{10} = \frac{3}{2}$

Write a ratio in fraction form.

The school's soccer team won 7 games and lost 5 games during the fall soccer season.

21. wins to games played $\frac{7}{12}$
22. wins to losses $\frac{7}{5}$
23. games played to losses $\frac{12}{5}$

278 Chapter 13

A **rate** is a special ratio comparing two quantities having different measuring units. The **unit rate** tells how many of a quantity there are *per* one unit of another quantity.

Gasoline is purchased by a *per-gallon rate*.	\$3.00 per 1 gallon = \$3/gal
A babysitter is paid by a *per-hour rate*.	\$6.50 per 1 hour = \$6.50/hr
Calories are reported in a *per-serving rate*.	180 calories per 1 serving = 180 calories/serving
Speed is calculated as *miles per hour*.	60 miles per 1 hour = 60 mph or 60 mi/hr

To find the unit rate, rename the ratio using a denominator of 1.

Mom spent \$11.00 for 4 packages of cookies. What is the unit rate (cost per package)?

$\frac{\text{cost}}{\text{pkg}} = \frac{11}{4} = \frac{2.75}{1}$ (÷4, ÷4) The cookies cost \$2.75 per package.

Multiply the terms of the unit rate to find an equivalent ratio.

Each package contains 12 cookies. How many cookies are in 4 packages?

$\frac{\text{cookies}}{\text{packages}} = \frac{12}{1} = \frac{48}{4}$ (×4, ×4) There are 48 cookies in 4 packages.

What distance will a car travel in 3 hours at an average speed of 50 miles per hour?

$\frac{\text{miles}}{\text{hour}} = \frac{50}{1} = \frac{150}{3}$ (×3, ×3) The car will travel 150 miles in 3 hours.

Exercises

Find the unit rate.

24. Kevin drove 480 miles on 16 gallons of gasoline. ***30 mi/gal***
25. Elizabeth earned \$48 in 8 hours. ***\$6/hr***
26. Mr. Monroe drove 2,250 miles in 3 days. ***750 mi/day***
27. Juliet read 30 pages in 60 minutes. ***0.5 page/min or*** $\frac{1}{2}$ ***page/min***
28. Mom paid \$3.16 for 4 pounds of apples. ***\$0.79/lb***
29. The office assistant can type 165 words in 3 minutes. ***55 words/min***

Use the unit rate to find the answer.

30. 5 gallons at \$3.15/gallon ***\$15.75***
31. 4 hours at \$7/hr ***\$28***
32. 6 hours at 60 mph ***360 mi***
33. 9 days at 230 mi/day ***2,070 mi***
34. 0.5 hour at 7 km/hr ***3.5 km***
35. 3.5 hours at \$10/hr ***\$35***

Solve.

36. Eva is traveling by train. She has traveled 180 miles in 2 hours. At this rate, her trip will take 6 hours. How far will she travel in all? ***540 miles***
37. Madison is traveling by plane. She has traveled 420 miles in 2 hours. At this rate, her trip will take 4 hours. How far will she travel in all? ***840 miles***
38. Bethany drove 300 miles on 12 gallons of gasoline. At this rate, how many gallons of gasoline will she need to travel 1,000 miles? ***40 gallons***

Complete DAILY REVIEW a on page 446.

Lesson 114 279

A car traveled 405 miles using 15 gallons of gasoline. ***miles to gallons; 405 to 15, 405:15,*** $\frac{405}{15}$

➤ **If a car went 405 miles using 15 gallons of gasoline, how can you find out how many miles it would go using 1 gallon of gasoline?** ***Elicit that you can find a ratio that is equivalent to*** $\frac{\text{405 miles}}{\text{15 gallons}}$ ***with "1 gallon" as the second term.***

➤ **What equivalent ratios could you write to find how many miles the car would travel using 1 gallon of gasoline?** ***Elicit*** $\frac{\text{405 miles}}{\text{15 gallons}} = \frac{m}{\text{1 gallon}}$.

2. Write $\frac{405\ mi}{15\ gal} = \frac{m}{1\ gal}$ for display. Point out that a unit rate tells how many of a quantity there are per 1 unit of another quantity. The second term of a unit rate is always 1.

➤ **What relationship do you notice between these equivalent ratios?** ***Elicit that 15 gallons is divided by 15 to get the unit rate of 1 gallon.***

➤ **How could you find the first term in the unit rate?** ***Answers may vary, but elicit that since the second term in*** $\frac{\text{405 mi}}{\text{15 gal}}$ ***is divided by 15 to get 1 gal; the same operation must be performed on the first term; 405 ÷ 15 = n.*** Point out that when you divide both terms by the same number, you are dividing by a name for 1 to find an equivalent ratio ($\frac{405}{15} \div \frac{15}{15} = \frac{m}{1}$).

3. Direct the students to divide 405 miles by 15 gallons to find m (the number of miles per gallon). ***m = 27 miles*** Complete the solution and write the final answer: $\frac{405\ mi}{15\ gal} = \frac{m}{1\ gal} = \frac{27\ mi}{1\ gal}$ or 27 mi/gal. Explain that it is not necessary to write the 1 in the second term of a unit rate.

4. Follow a similar procedure for these statements.

Lisa earned \$42 in 7 hours. ***dollars to hours; 42 to 7, 42:7,*** $\frac{42}{7}$; $\frac{\$42}{7\ hr} = \frac{d}{1\ hr} = \frac{6\ dollars}{1\ hr}$ ***or \$6/hr***

There are 141 calories in 10 potato chips. ***calories to potato chips; 141 to 10, 141:10,*** $\frac{141}{10}$; $\frac{141\ calories}{10\ chips} = \frac{c}{1\ chip} = \frac{14.1\ calories}{1\ chip}$ ***or 14.1 calories/chip***

5. Follow a similar procedure as you guide the students in finding an equivalent ratio using the unit rates in the following word problems. Elicit that both terms in the unit rate must be multiplied by the same number to find an equivalent ratio.

Alex's go-cart can travel at a maximum speed of 15 miles per hour. If he drives on a go-cart track at this speed, how many miles will he have driven in 0.5 hours? $\frac{15\ miles}{hour}$; $\frac{15\ mi}{1\ hr} = \frac{m}{0.5\ hr} = \frac{7.5}{0.5}$; ***7.5 miles***

Carrots are on sale for \$0.79 per pound. Anna is purchasing 3.75 pounds of carrots. How much will she pay for the carrots? $\frac{\$0.79}{pound}$; $\frac{\$0.79}{1\ lb} = \frac{d}{3.75\ lb} = \frac{\$2.96}{3.75}$; ***\$2.96***

Student Text pp. 278–79

Lesson 115 **Student Text** pp. 280–81
Daily Review p. 447b

Objectives
- Complete a ratio table
- Find equivalent ratios
- Make a ratio table
- Solve problems using ratio tables
- Use ratios to represent real-life situations and to solve problems

Teacher Materials
- Pictured Ratios, page IA64 (CD)
- Ratio Tables, page IA65 (CD)

Teach for Understanding

Complete a ratio table

1. Display the Pictured Ratios page. Explain that each of the 6 pictures shows a relationship. A *unit rate* is shown when one of the pictured objects is being compared with a quantity of another object.
   ➤ What two objects are being compared in each picture? *Elicit horse trailers to horses or horses to horse trailers.*
   ➤ Which of these pictures shows a unit rate? *Elicit the picture showing 3 horses to 1 horse trailer.*
   Point out that a *unit rate* for the number of horse trailers to 1 horse is not shown.
2. Draw for display a ratio table with the first two ratio entries: $\frac{\text{horses}}{\text{trailers}}$ and $\frac{3}{1}$. Explain that a *ratio table* is a method for organizing information to show the comparison of two quantities or equivalent ratios. Unit rates are written at the beginning of each row, and the table is used to extend the pattern between the terms (a 3 to 1 ratio).

horses	3	6	9	12	15
trailers	1	2	3	4	5

   ➤ Which of the other pictures show the same 3:1 ratio of horses to horse trailers? How do you know? *Elicit the picture showing 6 horses to 2 horse trailers, and the picture showing 9 horses to 3 horse trailers. Each of the quantities shown in the pictures can be divided to show a ratio of 3 horses to 1 trailer.* Write the next two ratios in the table: $\frac{6}{2}$ and $\frac{9}{3}$.
3. Write 4 and 5 as the last two trailer entries. Choose students to complete the table and explain their reasoning: $\frac{12}{4}$; $\frac{15}{5}$. Point out that the Identity Property of Multiplication (multiplying by a name for 1) can be applied to the unit rate so that each term of a ratio is multiplied by the same number to find an equivalent ratio (e.g., $\frac{6}{2} \times \frac{2}{2} = \frac{12}{4}$ and $\frac{3}{1} \times \frac{5}{5} = \frac{15}{5}$). Explain that every ratio in the table, when written in lowest terms, is equal to the unit rate.
   ➤ What horse to trailer ratio can you write for picture number 2 on the Pictured Ratios page? $\frac{\text{8 horses}}{\text{2 trailers}}$
   ➤ What is the unit rate for this ratio? $\frac{\text{4 horses}}{\text{1 trailer}}$
   ➤ What is the unit rate of horses to trailers shown in picture number 5? How do you know? $\frac{\text{4 horses}}{\text{1 trailer}}$; $\frac{6}{1.5} \div \frac{1.5}{1.5} = \frac{4}{1}$ in picture number 6? $\frac{\text{4 horses}}{\text{1 trailer}}$; $\frac{12}{3} \div \frac{3}{3} = \frac{4}{1}$
4. Display only the first table on the Ratio Tables page. Guide students in completing the table: $\frac{12}{3}$, $\frac{16}{4}$.
5. Guide the students in using their knowledge of unit rates and equivalent ratios to complete the next two tables: $\frac{200}{2}$, $\frac{300}{3}$, $\frac{400}{4}$, $\frac{500}{5}$; and $\frac{32}{2}$, $\frac{64}{4}$, $\frac{128}{8}$, $\frac{256}{16}$.

➤ Would the ratio $\frac{600}{6}$ be in the same ratio table as $\frac{300}{3}$? How do you know? *Yes; elicit that when both are renamed to lowest terms, they share the same $\frac{100}{1}$ unit rate; therefore, they are equivalent ratios.*
➤ Would the ratio $\frac{850}{8}$ go in the same ratio table as $\frac{300}{3}$? Why? *No; $\frac{850}{8}$ does not rename to a $\frac{100}{1}$ unit rate.*

Make a ratio table

1. Guide the students in drawing a ratio table to show the relationship of *Inches* to *Yards*.
   ➤ What unit rate compares the number of inches to 1 yard? *36:1* Instruct the students to write the unit rate as the first ratio in the table.
2. Direct the students to write ratios in their tables as you ask the following questions.
   ➤ How many inches are in 2 yards? *72 inches, $\frac{72}{2}$* 3 yards? *108 inches, $\frac{108}{3}$* 4 yards? *144 inches, $\frac{144}{4}$*

Solve problems using ratio tables

Patrick paid \$0.90 sales tax for a \$12 purchase. He wants to find out how much sales tax he would have to pay for a purchase of \$42.00. *\$3.15*

➤ What is being compared in the word problem? *the amount of sales tax to the amount of a purchase; \$0.90 to \$12*

1. Display the $\frac{\text{Tax}}{\text{Purchase}}$ ratio table and write $\frac{\$0.90}{\$12}$ as the first entry. Explain that ratio tables can be used to find a specific ratio such as the amount of sales tax you need to pay when purchasing an item.
   ➤ What equivalent ratio can be made by multiplying each term of this first ratio by 2? $\frac{\$1.80}{\$24}$ by 3? $\frac{\$2.70}{\$36}$ by 4? $\frac{\$3.60}{\$48}$ Write the ratios in the table.
   ➤ What property did you apply when renaming the ratio $\frac{\$0.90}{\$12}$ to higher terms? How do you know? *Identity Property of Multiplication; each ratio was multiplied by a name for 1.*
   ➤ Since the purchase price of \$42 is \$6 more than \$36, how do you think you can find an equivalent ratio with \$6 as its second term using the ratios in this table? *Elicit that you can divide the second term of any of the equivalent ratios by 6 to find the common divisor needed for dividing both the first and second term. (e.g., Since \$12 ÷ 6 = 2, you know that \$12 ÷ 2 = \$6; therefore, you can divide $\frac{\$0.90}{\$12}$ by $\frac{2}{2}$ to find an equivalent ratio of $\frac{\$0.45}{\$6}$.)*
   Write the ratio $\frac{\$0.45}{\$6}$ in the table.
   (***Note:*** A common error made by students is to add \$6 to both terms in the previous ratio of $\frac{\$2.70}{\$36}$. Adding \$6 to each term results in the ratio $\frac{\$8.70}{\$42}$, which is not equivalent to the other ratios in the table; \$8.70 is not halfway between \$2.70 and \$3.60 like the second term, \$42, is halfway between \$36 and \$48. Point out that adding \$6 to each term of a ratio means you are adding a value of $\frac{6}{6}$ to the entire ratio. Adding 1 is not the same as multiplying by 1. Adding 1 changes the value of the original ratio, so $\frac{\$0.90}{\$12} \neq \frac{\$8.70}{\$42}$.)
   ➤ How can you use the ratio $\frac{\$0.45}{\$6}$ to help you find how much sales tax will be on a purchase of \$42? *Elicit that since \$36 + \$6 = \$42, you can add the \$0.45 of tax for a \$6 purchase to \$2.70, or since \$48 − \$6 = \$42, you can subtract \$0.45 from \$3.60.*
   Choose a student to write and solve the addition equation to find the amount of tax Patrick would need to pay for a \$42 purchase. Select another student to write and solve the subtraction equation. *\$2.70 + \$0.45 = \$3.15 or \$3.60 − \$0.45 = \$3.15* Write $\frac{\$3.15}{\$42}$ in the table.

Ratio Tables

A **ratio table** is a method for organizing equivalent ratios. The table extends the pattern between the terms by multiplying or dividing both terms of the ratio by the same nonzero number (a name for 1).

ratio table

		×2	×3	×4	×5
nickels	5	10	15	20	25
quarters	1	2	3	4	5

$\frac{5}{1} = \frac{10}{2} = \frac{15}{3} = \frac{20}{4} = \frac{25}{5}$

		÷2	÷4	÷8
pennies	80	40	20	10
dimes	8	4	2	1

$\frac{80}{8} = \frac{40}{4} = \frac{20}{2} = \frac{10}{1}$

Exercises

Complete the ratio table. Use the ratios to answer the question.

1. What is the unit rate of inches to feet? ***12 : 1***
2. How many inches are in 3 feet? ***36 in.***
3. How many inches are in 4 feet? ***48 in.***
4. Sixty inches make up how many feet? ***5 ft***

inches	12	24			60
feet	1	2	3	4	

5. How many quarts can be made from 20 cups? ***5 qt***
6. How many cups are in 2.5 quarts? ***10 c***
7. What is the unit rate of cups to quarts? ***4 : 1***

cups	40	20		
quarts	10		2.5	1

Complete the ratio table.

8.

cm	2.54	***5.08***	10.16	20.32	***40.64***
in.	1	2	4	8	16

9.

km	1.61	***3.22***	4.83	***6.44***	***8.05***
mi	1	2	3	4	5

Use the unit rate 5,280 ft/mi to make a ratio table. Answer the question.

10. How many feet are in 2 miles? ***2 × 5,280 ft = 10,560 ft***
11. How many feet are in 5 miles? ***5 × 5,280 ft = 26,400 ft***
12. How many miles are 21,120 feet? ***21,120 ft ÷ 5,280 ft = 4 mi***

Write yes if the ratio could be in a ratio table with $\frac{5}{7}$. Write no if the ratio could *not* be in a ratio table with $\frac{5}{7}$.

13. $\frac{25}{35}$ ***yes*** 14. $\frac{50}{60}$ ***no*** 15. $\frac{60}{84}$ ***yes***

Write yes if the ratio could be in a ratio table with $\frac{3}{4}$. Write no if the ratio could *not* be in a ratio table with $\frac{3}{4}$.

16. $\frac{27}{32}$ ***no*** 17. $\frac{33}{44}$ ***yes*** 18. $\frac{42}{56}$ ***yes***

Write the ratio that could *not* be in a ratio table with the given ratio.

19. 5 to 10 $\frac{30}{60}$ $\frac{45}{90}$ $\frac{25}{100}$

20. 18 : 12 6 : 4 72 : 36 36 : 24

280 Chapter 13

Make a ratio table to solve problems. Extend the pattern by making equivalent ratios, or use combinations of the ratios.

A gallon of paint can cover a wall area of about 350 ft². How much area can 6 gallons of paint cover? How much area can 15 gallons cover?

$\frac{350\text{ ft}^2}{1\text{ gal}} \times \frac{6}{6} = \frac{2{,}100\text{ ft}^2}{6\text{ gal}}$

feet²	350	700	1,050	1,400	1,750	2,100
gallon	1	2	3	4	5	6

Use the ratios in the table to find the area for 15 gallons.
15 : *n*
15 gal : 5,250 ft²

15 = 6 + 4 + 5
15 : 2,100 + 1,400 + 1,750

The sales tax on a purchase of \$20 is \$1.30. What will the tax be on a purchase of \$70?

70 is not a multiple of 20 and would not extend this pattern.
70 = 60 + 10 or 70 = 80 − 10

tax	\$1.30	\$2.60	\$3.90	\$5.20	\$0.65	\$4.55
purchase	\$20	\$40	\$60	\$80	\$10	\$70

If \$20 : \$1.30, then \$10 : \$0.65.
\$70 : *n*
\$70 : \$4.55

70 = 60 + 10
70 : \$3.90 + \$0.65

Exercises

Use the ratio table to answer the question. ***Steps to solve may vary.***

A group of students is visiting the history museum. The table shows the price of admission that different groups will pay.

21. How much will a group of 20 students pay? ***\$130.00***
22. How much will a group of 24 students pay? ***\$156.00***
23. How much will a group of 27 students pay? ***\$175.50***

students	3	5	6	12
admission	\$19.50	\$32.50	\$39.00	\$78.00

The gasoline tank in Rachel's car holds 15 gallons. Her car can travel 420 miles on a tank of gas.

24. How far can the car travel on 40 gallons? ***1,120 mi***
25. How far can the car travel on 50 gallons? ***1,400 mi***
26. How far can the car travel on 4 gallons? ***112 mi***

gallons	15	20	24	30
miles	420	560	672	840

There is less gravity on the moon than on the earth. A person weighing 120 pounds on the earth would weigh approximately 20 pounds on the moon.

27. About how much would a 200-pound person weigh on the moon? ***34 lb***
28. About how much would a 160-pound person weigh on the moon? ***27 lb***

earth	120	100	60	40
moon	20	17	10	7

J Graph the earth and moon ratio table on a coordinate plane. Use the moon as the *x*-coordinate and the earth as the *y*-coordinate. Draw a line to connect the points. Locate your weight on the graph and find your approximate weight on the moon.

Complete DAILY REVIEW b on page 447.

Lesson 115 281

2. Display the $\frac{\text{Earth Weight}}{\text{Mars Weight}}$ ratio table. Explain that there is less gravity on Mars than on Earth. If an object weighs 100 pounds on Earth, it would weigh approximately 40 pounds on Mars.
(***Note:*** The $\frac{\text{Earth Weight}}{\text{Mars Weight}}$ ratio of $\frac{1}{0.377}$ has been rounded to $\frac{1}{0.4}$ for this lesson.)
3. Guide the students in determining approximately how much people would weigh on Mars if they weighed the following numbers of pounds on Earth. For each weight, ask a student to tell how he would find how much the person would weigh on Mars and to explain his reasoning. Discuss each method as needed.

   160 pounds ***Since the first terms in the ratios $\frac{100}{40}$ and $\frac{60}{24}$ added together equal 160, you can add the second terms; 64 lb.***

   70 pounds ***Possible answers: since the first terms in the ratios $\frac{40}{16}$, $\frac{20}{8}$, and $\frac{10}{4}$ added together equal 70, you can add the second terms; or you can multiply both terms of $\frac{10}{4}$ by 7; 28 lb.***

   115 pounds ***Possible answer: you can divide both terms of $\frac{10}{4}$ by 2 to find that 5 pounds on Earth would be about 2 pounds on Mars ($\frac{5}{2}$). Then since the first terms in the ratios $\frac{100}{40}$, $\frac{10}{4}$, and $\frac{5}{2}$ added together equal 115, you can add the second terms; 46 lb.***

Student Text pp. 280–81

Lesson 116 | **Student Text** pp. 282–83 | **Daily Review** p. 447c

Objectives

- Develop an understanding of proportions using models
- Determine whether two ratios are proportional
- Solve for a missing term in a proportion
- Use ratios to represent real-life situations and to solve problems

Teacher Materials

- Christian Worldview Shaping, pages 31–32 (CD)

Student Materials

- Black and red counters

Teach for Understanding

Develop an understanding of proportions using models

1. Distribute the counters. Direct each student to place 4 black counters in one row, and then to place 12 red counters in another row below the black counters.

➤ **What part-to-part ratio compares the black counters to the red counters?** ***4 to 12***

Write the ratio for display: $\frac{\text{4 black counters}}{\text{12 red counters}} = \frac{4}{12}$.

2. Direct the students to arrange the black and the red counters into 2 equal groups.

➤ **What is the ratio of the black and red counters in each group?** ***2 to 6*** Write the ratio for display: $\frac{\text{2 black counters}}{\text{6 red counters}} = \frac{2}{6}$.

3. Repeat the procedure for 4 equal groups of red and black counters. Write the ratio: $\frac{\text{1 black counter}}{\text{3 red counters}} = \frac{1}{3}$.

4. Explain that the ratios $\frac{2}{6}$ and $\frac{1}{3}$ are equivalent to $\frac{4}{12}$. Write for display $\frac{4}{12} = \frac{2}{6}$ and the term *proportion*. Explain that a *proportion* is an equation stating that two ratios are equivalent. Ratios are proportional only if they are equivalent. This proportion is read *4 is to 12 like 2 is to 6*. Lead the students in reading aloud the proportion.

Determine whether two ratios are proportional

1. Write $\frac{4}{8}$ __ $\frac{1}{2}$ for display.

➤ **Are these ratios equivalent? How do you know?** ***Yes; answers may vary, but elicit relationships that exist between the ratios such as 4 is one-half of 8 and 1 is one-half of 2 and 4 ÷ 4 = 1 and 8 ÷ 4 = 2.***

2. Write $\frac{a}{b} = \frac{c}{d}$. Explain that every proportion has vertical, horizontal, and diagonal relationships between its terms. Use the following activity to guide the students to the conclusion that if any one of these relationships is equivalent, then the ratios are proportional.

   Vertical: *a* is to *b* like *c* is to *d*; 4 is to 8 like 1 is to 2
   Elicit the relationship between the first and second terms of each ratio to determine if the ratios are equivalent:
   4 is one-half of 8, just as 1 is one-half of 2; $4 \div 8 = \frac{1}{2}$ and $1 \div 2 = \frac{1}{2}$. Rename one or both ratios to common lowest terms or higher terms: $\frac{1}{2} = \frac{1}{2}$ or $\frac{4}{8} = \frac{4}{8}$.

   Horizontal: *a* is to *c* like *b* is to *d*; 4 is to 1 like 8 is to 2
   Elicit the relationship between the first terms of the two ratios and between the second terms of the two ratios to determine if the ratios are equivalent: 4 is four times (× 4) greater than 1, just as 8 is four times (× 4) greater than 2; 4 ÷ 4 = 1 and 8 ÷ 4 = 2.

   Diagonal: (cross multiplication) $d \times a = b \times c$ or $da = bc$; $2 \times 4 = 8 \times 1$.

   Guide the students in cross-multiplying diagonal terms to write a possible equation to determine if the ratios are equivalent: 2×4 __ 8×1; $8 = 8$.

3. Write = to complete the proportion: $\frac{4}{8} = \frac{1}{2}$.

4. Follow a similar procedure for $\frac{6}{9}$ __ $\frac{3}{4}$. $\frac{6}{9} \neq \frac{3}{4}$ Guide the students to the conclusion that if any one of the relationships (vertical, horizontal, or diagonal) is not equivalent, the ratios are not proportional.

   Vertical: 6 is two-thirds of 9, and 3 is three-fourths of 4; $\frac{6}{9} = \frac{24}{36}$ and $\frac{3}{4} = \frac{27}{36}$; $\frac{24}{36} \neq \frac{27}{36}$.

   Horizontal: 6 is two times (× 2) greater than 3, but 9 is more than two times greater than 4; 6 ÷ 3 = 2, but 9 ÷ 4 = 2.25.

   Diagonal: 4×6 __ 9×3; $24 \neq 27$.

➤ **To what can you relate the process of determining whether ratios are equivalent?** ***Elicit that it is similar to determining whether fractions are equivalent.***

Remind the students that all three types of ratios (part-to-whole, part-to-part, and whole-to-part) can be written in fraction form, and that if ratios are equivalent, they are proportional.

5. Explain that using strategies to solve proportions can be useful in real-life situations such as finding the best price when making a purchase.

> Balloons come in different sized packages. A large package contains 15 balloons and costs \$3. A smaller package contains 6 balloons and costs \$2. Are the prices equivalent? Why? ***no;*** $\frac{\text{15 balloons}}{\$3} \neq \frac{\text{6 balloons}}{\$2}$

➤ **What is being compared?** ***the prices of balloons*** Write $\frac{\text{balloons}}{\text{price}}$ for display.

➤ **What ratio can you write for the larger package of balloons?** ***15 to \$3*** **the smaller package?** ***6 to \$2*** Write $\frac{15}{\$3}$ __ $\frac{6}{\$2}$.

➤ **Are the costs of the packages the same? How do you know?** ***No; possible answers: the ratios renamed to lowest terms are different unit rates (*** $\frac{\text{5 balloons}}{\$1} \neq \frac{\text{3 balloons}}{\$1}$***); the ratios renamed to common second terms are different (*** $\frac{30}{6} \neq \frac{18}{6}$***); when you cross-multiply 2 × 15 and 3 × 6, the products are different (30 ≠ 18).***

Write ≠ to complete the problem.

➤ **Which package of balloons is the better deal? Why?** ***The larger package of balloons; you get 5 balloons for \$1 in the larger package rather than just 3 balloons for \$1 in the smaller package.***

➤ **How else do you think you can compare the prices of the packages of balloons?** ***Elicit that you can find the price of 1 balloon in each package.***

➤ **How do you think you can you find the price per balloon (the unit rate) for the packages?** ***Elicit that you can compare the ratio*** $\frac{\$3}{\text{15 balloons}}$ ***to*** $\frac{\$2}{\text{6 balloons}}$***; divide each first and second term by a common factor to rename the second term as 1 balloon, or divide the first term of the ratio by the second term.***

6. Write $\frac{\$3}{15}$ and $\frac{\$2}{6}$ for display. Direct the students to find the price per balloon in each package. $\frac{3}{15} \div \frac{15}{15} = \frac{\$0.20}{\text{1 balloon}}$ ***and*** $\frac{2}{6} \div \frac{6}{6} \approx \frac{\$0.33}{\text{1 balloon}}$

➤ **Are the prices of the packages of balloons equivalent? Why?** ***No; the price per balloon in the larger package is less than the price per balloon in the smaller package.*** Write $\frac{\$3}{\text{15 balloons}} \neq \frac{\$2}{\text{6 balloons}}$.

➤ **Are the ratios $\frac{3}{15}$ and $\frac{2}{6}$ proportional? Why?** ***No; they are not equivalent.***

Solving Proportions

A **proportion** is an equation stating that two ratios are equivalent. Ratios are proportional when they are equivalent. The terms can be compared vertically, horizontally, or diagonally to test for equivalency.

proportion

Use Number Sense

$\frac{1}{2} \bigcirc \frac{3}{6}$

This proportion can be read "1 is to 2 like 3 is to 6." Compare the relationship of the numerator to the denominator of each ratio.

1:2 1 is *one-half* of 2.
3:6 3 is *one-half* of 6.

$\frac{1}{2}$ is **proportional** to $\frac{3}{6}$.

$\frac{1}{2} = \frac{3}{6}$

These ratios are equivalent and form a proportion.

Test for Equivalent Ratios

The same operation must be performed on both terms.

$\frac{1}{2} \overset{\times 3}{=} \frac{3}{6}$ (× 3)

$\frac{1}{2}$ is **proportional** to $\frac{3}{6}$.

$\frac{2}{3} \overset{\times 4}{\neq} \frac{8}{10}$ (× 4)

$\frac{2}{3}$ is **not proportional** to $\frac{8}{10}$.

Use Cross Multiplication

Cross-multiply to compare the numerators of like fractions.

$\frac{1}{2} \bigcirc \frac{3}{6}$

$6 \times 1 = 6$ $2 \times 3 = 6$

$\frac{1}{2} = \frac{3}{6}$

$\frac{2}{3} \bigcirc \frac{8}{10}$

$10 \times 2 = 20$ $3 \times 8 = 24$

$\frac{2}{3} \neq \frac{8}{10}$

Compare Lowest Terms

$\frac{1}{2} = \frac{3}{6}$ $\frac{3}{6} = \frac{1}{2}$

$\frac{2}{3} \neq \frac{8}{10}$ $\frac{8}{10} = \frac{4}{5}$

Equivalent ratios will have the same lowest terms.

Use Division

$\frac{2}{3} \bigcirc \frac{8}{10}$

$2 \div 3 \bigcirc 8 \div 10$

$0.\overline{6} \neq 0.8$

$\frac{2}{3}$ is **not proportional** to $\frac{8}{10}$.

Exercises

Write the ratios in lowest terms to compare. Write a fraction comparison using = or ≠.

1. $\frac{12}{14}$ and $\frac{36}{42}$ ***$\frac{6}{7} = \frac{6}{7}$***
2. $\frac{9}{27}$ and $\frac{7}{21}$ ***$\frac{1}{3} = \frac{1}{3}$***
3. $\frac{22}{54}$ and $\frac{20}{26}$ ***$\frac{11}{27} \neq \frac{10}{13}$***
4. $\frac{9}{15}$ and $\frac{6}{10}$ ***$\frac{3}{5} = \frac{3}{5}$***

Write the operation that was performed on both terms.

5. $\frac{9}{14} = \frac{27}{42}$ ***$\times \frac{3}{3}$***
6. $\frac{32}{80} = \frac{4}{10}$ ***$\div \frac{8}{8}$***
7. $\frac{9}{11} = \frac{81}{99}$ ***$\times \frac{9}{9}$***
8. $\frac{7}{12} = \frac{56}{96}$ ***$\times \frac{8}{8}$***

Use cross multiplication to prove that the ratios are proportional.

9. $\frac{15}{20} = \frac{6}{8}$ ***120 = 120***
10. $\frac{2}{3} = \frac{12}{18}$ ***36 = 36***
11. $\frac{30}{12} = \frac{20}{8}$ ***240 = 240***
12. $\frac{5}{7} = \frac{40}{56}$ ***280 = 280***

Use division to find whether the ratios are proportional. Write a decimal comparison using = or ≠.

13. $\frac{2}{4}$ and $\frac{15}{30}$ ***0.5 = 0.5***
14. $\frac{3}{8}$ and $\frac{4}{16}$ ***0.375 ≠ 0.25***
15. $\frac{2}{3}$ and $\frac{3}{5}$ ***$0.\overline{6} \neq 0.6$***
16. $\frac{9}{72}$ and $\frac{6}{48}$ ***0.125 = 0.125***

Solve Proportions to Find Equal Ratios

Compare Unit Rates

Isabella earned \$11 for 2 hours of babysitting. Felicia earned \$14 for 3 hours of babysitting. Were the girls paid the same hourly rate?

unit rate = $\frac{\text{pay}}{1\text{ hour}}$

Isabella

$\frac{\$11}{2\text{ hr}} = \frac{n}{1\text{ hr}}$

$n = \$11 \div 2$

$n = \$5.50$

Felicia

$\frac{\$14}{3\text{ hr}} = \frac{n}{1\text{ hr}}$

$n = \$14 \div 3$

$n = \$4.67$

Isabella earned more per hour than Felicia.

Find the Equivalent Fraction

The coffee shop sells 4 doughnuts for \$2. At this price, how much will 8 doughnuts cost?

$\frac{\text{doughnuts}}{\text{cost}}$ $\frac{4}{2} = \frac{8}{c}$ (× 2)

$4 \times 2 = 8$
$2 \times 2 = c$
$c = 4$

8 doughnuts will cost \$4.

This strategy works well when the terms are related.

Cross-Multiply

The coffee shop sells 3 muffins for \$6. At this price, how much will 8 muffins cost?

$\frac{\text{muffins}}{\text{cost}}$

$\frac{3}{6} = \frac{8}{c}$

$c \times 3 = 3c$ $6 \times 8 = 48$

$3c = 48$

$\frac{3c}{3} = \frac{48}{3}$

$c = 16$

8 muffins will cost \$16.

Exercises

Steps to solve may vary.

Solve the proportion.

17. $\frac{a}{24} = \frac{9}{36}$ ***a = 6***
18. $\frac{14}{1} = \frac{b}{2}$ ***b = 28***
19. $\frac{4}{16} = \frac{c}{12}$ ***c = 3***
20. $\frac{33}{6} = \frac{d}{10}$ ***d = 55***
21. $\frac{h}{2} = \frac{120}{80}$ ***h = 3***
22. $\frac{6}{m} = \frac{18}{42}$ ***m = 14***

Write a possible proportion for the situation. Write yes if the prices are equivalent. Write no if one price is a better buy.

23. 4 pounds of apples for \$6 or 10 pounds of apples for \$15 ***yes***
24. 12 eggs for \$2 or 18 eggs for \$3 ***yes***
25. 5 bottles of soda for \$7 or 6 bottles of soda for \$8 ***no***
26. A 12 oz can of corn for 55¢ or a 20 oz can of corn for 95¢ ***no***
27. 1 quart of milk for \$1 or 1 gallon of milk for \$4 ***yes***
28. A 5 oz candy bar for 50¢ or an 8 oz candy bar for 75¢ ***no***

Solve the proportion to find an equivalent ratio.

29. Gabriel shoveled snow from 2 driveways in 3 hours. At this rate, how long will it take him to shovel 5 driveways? ***7.5 hours***
30. During the race, Driver 1 traveled 480 miles in 3 hours. At this rate, how far will Driver 1 travel in 5 hours? ***800 miles***
31. Two pizzas cost \$15. At this rate, how much would 7 pizzas cost? ***\$52.50***
32. Factory workers can produce 25 items in 30 hours. How many hours will it take them to produce 60 items at this rate? ***72 hours***
33. During the election for class president, Colton received 3 votes for every vote cast for Bryce. Bryce received 5 votes. How many votes did Colton receive? ***15 votes***
34. Three servings of yogurt are 24 ounces. How many ounces are in 4 servings? ***32 ounces***
35. A survey of middle school students revealed that 36 out of 120 students have green eyes. How many students with green eyes would you expect to find out of 10 of these students? ***3 students***

Complete DAILY REVIEW C on page 447.

7. Follow a similar procedure for this word problem.

> Last Saturday, Mark earned \$20 for working 4 hours. This Saturday, Mark earned \$30 for working 6 hours. Did Mark receive the same hourly rate on each Saturday? ***Yes; $\frac{\$20}{4\text{ hr}} = \frac{\$30}{6\text{ hr}}$; $\frac{\$5}{1\text{ hr}} = \frac{\$5}{1\text{ hr}}$; $\frac{20}{4}$ is proportional to $\frac{30}{6}$.***

Solve for a missing term in a proportion

> A cookie recipe calls for 2 cups of chocolate chips to make 48 cookies. How many cups of chocolate chips are needed to make 120 cookies? ***5 cups***

- ➤ **What is being compared?** ***cups of chocolate chips to number of cookies*** Write $\frac{\textit{cups}}{\textit{cookies}}$.
- ➤ **What ratios can you write to compare the cups of chocolate chips to the number of cookies?** ***2 cups to 48 cookies and c to 120 cookies*** Write $\frac{2\textit{ cups}}{48\textit{ cookies}} = \frac{c}{120\textit{ cookies}}$.

1. Point out that when comparing ratios, the terms must show the same comparison; therefore, the order of terms is important.
2. Direct the students to solve for the missing term in the proportion using the algorithm of their choice. ***c = 5*** Discuss the methods that were used to solve the problem.
   - ➤ **What does c = 5 represent?** ***5 cups of chocolate chips are needed to make 120 cookies*** Write the label *cups* after the 5. Erase the *c* in the proportion and write *5 cups*.

> In a survey, 4 out of every 10 people chose pizza as their favorite food. If 30 people were surveyed, how many people could you predict would choose pizza? ***12 people***

- ➤ **What is being compared?** ***the number of people who chose pizza as their favorite food and the number of people surveyed*** Write $\frac{\textit{people choosing pizzas}}{\textit{people surveyed}}$ for display.
- ➤ **What ratios can you write to compare the number of people who chose pizza to the number of people surveyed?** ***4 to 10 and p to 30*** Write $\frac{4}{10} = \frac{p}{30}$.

3. Direct the students to solve for the missing term in the proportion. ***p = 12***
   - ➤ **What does p = 12 represent?** ***12 people are likely to choose pizza if 30 people are surveyed*** Write the label *people*. Erase the *p* in the proportion and write *12 people*.
4. Follow a similar procedure for the following problems. (***Note:*** Labeling the terms in the ratios will help the students remember what the missing term represents.)

> Mom purchased 12 bottles of soft drink for \$15. At this rate, how much will 14 bottles of soft drink cost? ***$\frac{\text{bottles}}{\text{cost}}$; $\frac{12}{15} = \frac{14}{c}$; c = \$17.50***
>
> During an election, Jared received 2 votes for every 3 votes that Kevin received. Kevin received 42 votes. How many votes did Jared receive? ***$\frac{\text{votes for Jared}}{\text{votes for Kevin}}$; $\frac{2}{3} = \frac{v}{42}$; v = 28 votes***
>
> If a farmer can raise 10 goats on 3 acres of pasture, how many acres of pasture would he need to raise 25 goats? ***$\frac{\text{goats}}{\text{acres}}$; $\frac{10}{3} = \frac{25}{g}$; g = 7.5 acres***

5. **Christian Worldview Shaping (CD)**

Student Text pp. 282–83

Lesson 117 | **Student Text** pp. 284–85 | **Daily Review** p. 448d

Objectives

- Develop an understanding of proportions in similar figures
- Solve for a missing term in a proportion
- Find the unknown measure in similar figures using proportions
- Use indirect measurement to find the unknown measure in similar objects
- Use ratios to represent real-life situations and to solve problems

Teacher Materials

- Graph Paper, page IA13 (CD)
- Missing Measurements, page IA66 (CD)
- A ruler or straight edge

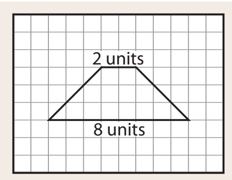
Student Materials

- Graph Paper, page IA13 (CD)
- A ruler or straight edge

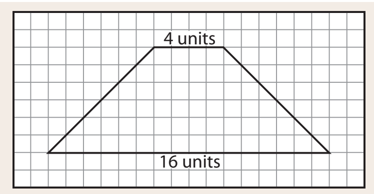
Teach for Understanding

Develop an understanding of proportions in similar figures

1. Display and distribute the Graph Paper page.
   ➤ **What are similar figures?** ***Elicit that similar figures are geometric figures with the same shape, but not necessarily the same size.***
2. Draw on the displayed page a trapezoid with its dimensions as shown. Direct the students to draw a similar trapezoid on their Graph Paper using the same number of units.
   ➤ **Compare your trapezoid to a classmate's trapezoid. Are they identical?** ***yes*** **What word is used to describe two figures that are identical in shape and size?** ***Elicit the term*** **congruent.**



3. Explain that a copier produces a *congruent* figure when it exactly copies a figure, and it produces a *similar* figure when it enlarges or reduces a figure. To produce a similar figure, the copier lengthens or shortens each side of the figure by the same amount without changing the angles of the figure.
4. Direct the students to draw a trapezoid that is similar to their first trapezoid, doubling the length of each side, and then to write the dimensions. Demonstrate on the displayed page.
   ➤ **What ratio describes the top:top relationship between the small trapezoid and large trapezoid?** ***2:4*** **the bottom:bottom relationship?** ***8:16*** Write $\frac{2}{4}$ __ $\frac{8}{16}$ for display.



   ➤ **Are these ratios proportional? How do you know?** ***Yes; accept that the ratios are equivalent because of any of the following possible reasons: the second term in each ratio is twice the first term ($2 \times 2 = 4$ and $8 \times 2 = 16$); both terms of the second ratio are 4 times greater than the terms in the first ratio ($\frac{2}{4} \times \frac{4}{4} = \frac{8}{16}$); when the terms of the ratios are cross-multiplied, the products are the same ($16 \times 2 = 32$ and $4 \times 8 = 32$).*** Write = to complete the proportion.

   Elicit that corresponding angles in similar figures and congruent figures are congruent (have the same measure), and corresponding side lengths in congruent figures are congruent (equal).

   ➤ **What do you think is true about corresponding side lengths of similar figures?** ***Elicit that corresponding side lengths are proportional.***
5. Explain that the proportion $\frac{2}{4} = \frac{8}{16}$ was written using ratios that compare measures *between* the corresponding sides of similar figures. A proportion can also be made using ratios that compare measures of sides *within* figures.
   ➤ **What ratio describes the top:bottom relationship in the small trapezoid?** ***2:8*** **the top:bottom relationship in the large trapezoid?** ***4:16*** Write $\frac{2}{8}$ __ $\frac{4}{16}$.
   ➤ **Are these ratios proportional? How do you know?** ***Yes; accept that the ratios are equivalent because of any of the following possible reasons: the second term in each ratio is 4 times greater than the first term ($2 \times 4 = 8$ and $4 \times 4 = 16$); both terms of the second ratio are 2 times greater than the terms in the first ratio ($\frac{2}{8} \times \frac{2}{2} = \frac{4}{16}$); when the terms of the ratios are cross-multiplied, the products are the same ($16 \times 2 = 32$ and $8 \times 4 = 32$).*** Write = to complete the proportion.

Find the unknown measure in similar figures

1. Display the Missing Measurements page. Remind the students that if they know three terms in a proportion they can find the value of the missing term. Since sides of similar figures are proportional, writing a proportion can help them find an unknown side measure.
2. Write $\frac{\text{length A}}{\text{length B}} = \frac{\text{width A}}{\text{width B}}$ for display. Point out that the ratios in the proportion compare corresponding sides *between* Figure A and Figure B.
   ➤ **What ratios can be written between the corresponding sides of the figures?** ***$\frac{7m}{14m}$ and $\frac{3m}{n}$*** Write $\frac{7m}{14m} = \frac{3m}{n}$ for display.
   ➤ **How can you find the value of *n*?** ***Possible answers: 7 is one half of 14, so $7 \times 2 = 14$ and $3 \times 2 = 6$; cross-multiply: $7n = 14 \times 3$, $\frac{7n}{7} = \frac{42}{7}$, $n = 6$.*** Choose students to demonstrate solving the proportion and to explain the algorithms they used.
   ➤ **What does $n = 6$ represent?** ***the width of Figure B, 6 meters.***
3. Write $\frac{\text{length A}}{\text{width A}} = \frac{\text{length B}}{\text{width B}}$ for display. Explain that a proportion can also be written using ratios for measures *within* each figure.
   ➤ **What ratios can be written within each figure?** ***$\frac{7m}{3m}$ and $\frac{14m}{n}$*** Write $\frac{7m}{3m} = \frac{14m}{n}$.
   ➤ **How can you find the value of *n* using what you know about equivalent fractions?** ***Elicit that you can multiply $\frac{7}{3}$ by a name for 1 ($\frac{2}{2}$).*** Select a student to demonstrate solving the proportion. $\frac{7}{3} \times \frac{2}{2} = \frac{14}{6}$
   ➤ **What does $n = 6$ represent?** ***the width of Figure B, 6 meters***
   ➤ **Does it matter which method you use to solve a proportion?** ***no***
   ➤ **What might make you choose one method over another method?** ***Possible answers: the method that seems easiest based on the ratios; it is easier to find equivalent fractions or ratios when the terms are related.***
   (***Note:*** Allow students to use the terms *equivalent ratios* and *equivalent fractions* interchangeably.)
4. Point out that in the first proportion, $\frac{\text{length A}}{\text{length B}} = \frac{\text{width A}}{\text{width B}}$, the ratios each showed an $\frac{A}{B}$ comparison, and in the second proportion, $\frac{\text{length A}}{\text{width A}} = \frac{\text{length B}}{\text{width B}}$, the ratios each showed a $\frac{\text{length}}{\text{width}}$ comparison. Write $\frac{\text{length A}}{\text{width A}} \neq \frac{\text{width B}}{\text{length B}}$ for display.
   ➤ **Why are these ratios not proportional?** ***Elicit that the ratios are not showing the same comparison of figure to figure or length to width. The terms of the ratios within a proportion must make the same comparison.*** Point out that the order of the

Similar Figures

Similar figures are geometric figures with the same shape but not necessarily the same size. Figures are similar when the corresponding angle measurements are equal *and* the ratios of corresponding side lengths are proportionate.

similar figures
indirect measurement

Finding the Unknown Measure

Solve a proportion to find an unknown measure in similar figures.

Ratios Between Figures

Write a ratio for the corresponding sides of the two figures.

$6 \cdot 10 \quad \frac{10}{4} = \frac{n}{6} \quad 4 \cdot n$

$60 = 4n$

$\frac{60}{4} = \frac{\cancel{4}n}{\cancel{4}}$

$15\text{ cm} = n$

Ratios Within a Figure

Write a ratio for the sides *within* the same figure.

$6 \cdot 10 \quad \frac{10}{n} = \frac{4}{6} \quad n \cdot 4$

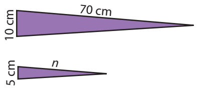
$60 = 4n$

$\frac{60}{4} = \frac{\cancel{4}n}{\cancel{4}}$

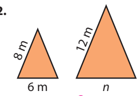
$15\text{ cm} = n$

Exercises

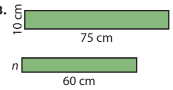
Write a proportion to find the unknown measure for the pair of similar figures. *Steps to solve may vary.*

1. 

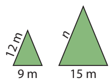
n = 35 cm

2. 

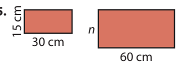
n = 9 m

3. 

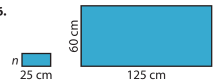
n = 8 cm

4. 

n = 20 m

5. 

n = 30 cm

6. 

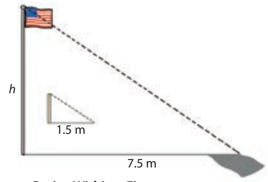
n = 12 cm

Solve. *Steps to solve may vary.*

7. Sydney enlarged a picture to be 3 times larger than the original. The original picture was 2 inches long by 4 inches wide. The enlarged picture has a length of 6 inches. What is the width? $\frac{2}{4} = \frac{6}{n}$; *n = 12 in.*
8. The school photographs given in Ami's package are not similar in size. The larger photographs are 9 inches × 12 inches and 8 inches × 10 inches. If the smaller of these photos remained 10 inches wide, what would the length need to be so the photos would be similar? $\frac{9}{12} = \frac{n}{10}$; *n = 7.5 in.*
9. If 3 balloons cost 48¢, how much will 20 balloons cost? $\frac{\$0.48}{3\text{ balloons}} = \frac{n}{20\text{ balloons}}$; *n = $3.20*
10. A salmon swam 126 miles in 4.5 hours. At this rate, how far could it travel in 8 hours? $\frac{126\text{ mi}}{4.5\text{ hr}} = \frac{n}{8\text{ hr}}$; *n = 224 miles*
11. The office has two sizes of envelopes. One size is 9 cm × 16.5 cm. The other size is 10.5 cm × 24 cm. Are these envelopes similar? *no;* $\frac{9}{16.5} \neq \frac{10.5}{24}$
12. A family wants to build a swimming pool that is similar in size to the standard Olympic pool. An Olympic pool is 50 meters long by 25 meters wide. If their pool will be 20 meters long, how wide will it be? $\frac{50}{25} = \frac{20}{n}$; *n = 10 m*

Indirect measurement uses similar objects to find the measurement of an object difficult to measure. Solve a proportion to find the unknown measurement.

The sixth-grade class wanted to know the height of the flagpole in the schoolyard. The teacher taught them how to determine the height of the flagpole without measuring it using the length of the flagpole's shadow and the length of a meter stick's shadow.



Ratios Between Figures

$\frac{\text{flagpole } h}{\text{meter } h} = \frac{\text{flagpole } s}{\text{meter } s}$

$1.5 \cdot h \quad \frac{h}{1} = \frac{7.5}{1.5} \quad 1 \cdot 7.5$

$1.5h = 7.5$

$\frac{\cancel{1.5}h}{\cancel{1.5}} = \frac{7.5}{1.5}$

$h = 5\text{ m}$

Ratios Within a Figure

$\frac{\text{flagpole } h}{\text{flagpole } s} = \frac{\text{meter } h}{\text{meter } s}$

$1.5 \cdot h \quad \frac{h}{7.5} = \frac{1}{1.5} \quad 7.5 \cdot 1$

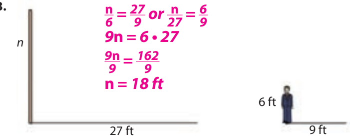
$1.5h = 7.5$

$\frac{\cancel{1.5}h}{\cancel{1.5}} = \frac{7.5}{1.5}$

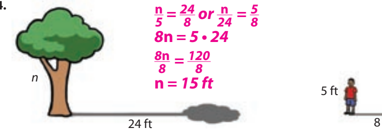
$h = 5\text{ m}$

Exercises

Write and solve a proportion to find the unknown height. *Steps to solve may vary.*

13. 

$\frac{n}{6} = \frac{27}{9}$ *or* $\frac{n}{27} = \frac{6}{9}$
$9n = 6 \cdot 27$
$\frac{9n}{9} = \frac{162}{9}$
n = 18 ft

14. 

$\frac{n}{5} = \frac{24}{8}$ *or* $\frac{n}{24} = \frac{5}{8}$
$8n = 5 \cdot 24$
$\frac{8n}{8} = \frac{120}{8}$
n = 15 ft

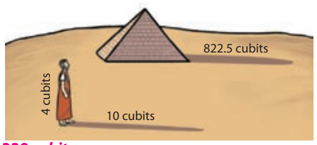
15. $\frac{n}{5} = \frac{5}{2}$
$2n = 5 \cdot 5$
$\frac{2n}{2} = \frac{25}{2}$
n = 12.5 m

16. $\frac{n}{10} = \frac{1}{4}$
$\frac{10}{4} = \frac{4n}{4}$
n = 2.5 m

15. A barn casts a shadow that is 5 meters long. A house is 5 meters high and casts a shadow that is 2 meters long. How high is the barn?
16. A meter stick casts a shadow that is 4 meters long. A tree casts a shadow that is 10 meters long. How tall is the tree?

MEET THE MATHEMATICIAN

The Greek mathematician **Thales**, who lived about 600 years before Jesus was born, devised the system for finding the height of something that cannot be measured. He discovered the height of the Egyptian pyramid Cheops using his height, the length of his shadow, and the length of the pyramid's shadow. Solve a proportion to find the height of Cheops. $\frac{4}{10} = \frac{n}{822.5}$; *n = 329 cubits*



Complete DAILY REVIEW d on page 448.

terms of each ratio within a proportion must be the same to make the same comparison.

5. Follow a similar procedure to find the unknown measurement in Figure D and Figure F. *Proportions may vary.*
   Figure D: $\frac{8\text{ m}}{10\text{ m}} = \frac{12\text{ m}}{n}$ *and* $\frac{8\text{ m}}{12\text{ m}} = \frac{10\text{ m}}{n}$, *n = 15 m*
   Figure F: $\frac{4\text{ cm}}{10\text{ cm}} = \frac{5\text{ cm}}{n}$ *and* $\frac{5\text{ cm}}{4\text{ cm}} = \frac{n}{10\text{ cm}}$, *n = 12.5 cm*

Use indirect measurement to find the unknown measure

Lee is 5 feet tall. He wondered how his height compared to the height of the tree in his front yard. Lee's father told him that they could find out the height of the tree without actually measuring it.

On a sunny day, Lee and his father went outside and measured the length of the tree's shadow and the length of Lee's shadow. The tree's shadow was 20 feet, and Lee's shadow was 8 feet. What is the height of the tree? *12.5 feet*

1. Explain that *indirect measurement* is the method of using similar figures and a proportion to find a measurement too difficult to measure directly, such as the height of a building or a tree. Remind the students that sometimes it is helpful to draw a picture of the situation. Draw for display a stick figure diagram of a boy, a tree, and the shadows of each, similar to the ones pictured on Student Text page 285.
   ➤ **What information is given?** *Lee is 5 feet tall, and his shadow is 8 feet long. The tree's shadow is 20 feet long.* Write the measurements.
   ➤ **How do you think you can solve the word problem?** *Answers will vary, but elicit that you can write ratios to describe the relationship of Lee's height and the length of his shadow and the relationship of the tree's height and the length of its shadow, and then write a proportion to compare the ratios within the figures;* $\frac{\text{Lee h}}{\text{Lee s}} = \frac{\text{tree h}}{\text{tree s}}$.
2. Write $\frac{\text{Lee h}}{\text{Lee s}} = \frac{\text{tree h}}{\text{tree s}}$. Direct the students to write a proportion and solve it. $\frac{5\text{ ft}}{8\text{ ft}} = \frac{n}{20\text{ ft}}$; *n = 12.5 ft*
3. Choose students to demonstrate solving the proportion and to explain the algorithms they used. *Possible answers: cross-multiply or find an equivalent ratio (e.g., Since 20 is $2\frac{1}{2}$ sets of 8, $2\frac{1}{2}$ sets of 5 equals 12.5).*
   ➤ **What ratios can you write to compare the measures between the figures?** *Elicit ratios that compare Lee's height to the tree's height and Lee's shadow to the tree's shadow;* $\frac{\text{Lee h}}{\text{tree h}} = \frac{\text{Lee s}}{\text{tree s}}$.
4. Direct the students to write the proportion and solve it. $\frac{5\text{ ft}}{n} = \frac{8\text{ ft}}{20\text{ ft}}$; *n = 12.5 ft*
5. Follow a similar procedure for these word problems.

A meter stick casts a shadow that is 3 meters long. A house casts a shadow that is 24 meters long. How tall is the house? *8 meters*

A 6-foot-tall man casts a shadow that is 15 feet long. A flagpole casts a shadow that is 35 feet long. How tall is the flagpole? *14 ft*

(*Note:* Students frequently make errors because they do not write proportions using corresponding ratios. You may choose to write for display an incorrect proportion and direct the students to identify the error; e.g., $\frac{1}{3} = \frac{24}{n}$, $\frac{\text{stick h}}{\text{stick s}} = \frac{\text{house s}}{\text{house h}}$.)

Student Text pp. 284–85

Lesson 118

Student Text pp. 286–87

Daily Review p. 448e

Objectives

- Find actual measurements using a scale and a scale drawing, map, or model
- Determine the unknown measure on a scale drawing given the scale and the actual measurement
- Solve word problems using ratios

Teacher Materials

- A map
- Samples of floor plans
- Modeling clay
- A ruler
- A Bible

Student Materials

- A ruler
- A calculator
- A map for each group of students (optional)

Preparation

Form a piece of modeling clay into a ball that has a diameter of approximately 2 inches and a circumference of approximately $6\frac{1}{4}$ inches.

Measure the length of a room that is familiar to the students (e.g., the classroom, the lunchroom, the library). Prepare for display a scale drawing of the room using a scale of $\frac{1 \text{ in.}}{5 \text{ ft}}$. (Do not write the actual room dimensions on the drawing.)

Notes

Floor plans can be found on websites, in home decorating magazines, and in brochures provided by builders of local housing developments.

Allow the students to use calculators to solve proportion and percent problems throughout the remainder of this chapter.

Teach for Understanding

Find actual measurements using a scale and a model

1. Display the ball of modeling clay and explain that it is a model representing the earth. The model has a diameter of 2 inches, but the actual diameter of the earth is about 8,000 miles.

➤ **What ratio compares the size of the model of the earth to the actual size of the earth?** ***2 in. to 8,000 mi***

2. Write *2 in.:8,000 mi* and $\frac{2 \text{ in.}}{8{,}000 \text{ mi}}$ for display. Explain that this ratio is a *scale,* a ratio of measurements that compares the size of a model, a drawing, or a map to the size of the actual object. Write $\frac{\text{model measurement}}{\text{actual measurement}}$ for display.
3. Explain that the diameter of the sun is about 865,000 miles.

➤ **If you used the same scale, 2 inches to represent every 8,000 miles, to make a model of the sun, how could you find out how large to make your model?** ***Possible answers: write and solve a proportion; find an equivalent ratio.***

4. Write for display $\frac{2 \text{ in.}}{8{,}000 \text{ mi}} = \frac{\text{Sun model measurement}}{\text{Sun actual measurement}}$ for display. Direct the students to write a proportion using ratios that compare the diameter measurement of the model with the actual diameter measurement and then solve the proportion to find the diameter of the model of the sun. Discuss the answer.

$$\frac{2}{8{,}000} = \frac{n}{865{,}000}$$

$$1{,}730{,}000 = 8{,}000n$$

$$\frac{1{,}730{,}000}{8{,}000} = \frac{\cancel{8{,}000}n}{\cancel{8{,}000}}$$

$$216.25 = n$$

➤ **What is the diameter of a model of the sun when a scale of 2 inches for every 8,000 miles is used?** ***216.25 inches or $216\frac{1}{4}$ inches***

➤ **Do you think a model of the sun that is made using this scale would fit in the classroom?** ***Answers may vary.***

5. Explain that a model sphere with a diameter of $216\frac{1}{4}$ inches equals a diameter of approximately 18 feet. Discuss the size of the classroom compared to the size of the model of the sun. Guide the students to the conclusion that the model of the sun would not fit in an average-size classroom.

   Display the model of the earth. Point out that the sun is much larger than the earth; God made it just the right size and distance away from the earth in order to give the right amount of sunlight and heat needed for our planet. Read aloud Colossians 1:15–18. Remind the students that it is Jesus Christ Who created and now sustains all of creation. [Bible Promise: I. God as Master]

Find actual measurements using a scale and a scale drawing

1. Explain that architects and engineers use scale drawings or blueprints when designing buildings.
2. Write for display *drawing measurement:actual measurement = 1 in.:5 ft* and display the prepared scale drawing of the selected room. Explain that 1 inch on the drawing represents 5 feet of the actual room measurement. Choose a student to measure the length of the room on the scale drawing and to write the measurement along the length.
3. Direct students to find the actual measurement of the room's length by solving a proportion that includes the scale and a ratio that compares the drawing length to the actual length. ***Answers will vary according to room dimensions.***
4. Repeat the procedure to find the actual width of the room.

Find actual measurements using a scale and a map

1. Display a map and explain that maps are a type of scale drawing. A map is created based on a scale that compares the map to the actual distance. A city map might have a scale of 1 inch to represent 3 miles, but a state map might have a scale of 2 inches to represent 60 miles. A map of a country might have a scale of 3 inches to represent 400 miles.
2. Choose a student to locate the scale on the map and write it for display. Select another student to measure the distance between two locations on the map and write it for display. Direct the students to find the actual distance between the locations by solving a proportion using the map scale. ***Answers will vary.***
3. Repeat the procedure using several other pairs of locations on the map.

   (***Note:*** You may choose to distribute maps to groups of students and direct them to locate the scale on their maps. Instruct them to measure the distance between two locations on the map, and then find the actual distance between the places by solving a proportion using their map scale. Direct the students to exchange maps. Repeat the activity.)

Determine the unknown measure on a scale drawing given the scale and the actual measurement

1. Explain that house plans and maps are examples of scale drawings that are *smaller* than the actual object. In a science

Scale

A **scale** is a ratio of measurements that compares the size of a drawing, a map, or a model with the size of the actual object. A **scale drawing** has dimensions proportionate to the size of the actual object. Maps and house plans are drawn smaller than the actual object. Illustrations of microscopic objects are drawn larger than the actual object. A **scale model** is a three-dimensional model that is proportionate to the size of the actual object.

scale
scale drawing
scale model

Actual Measurement

The distance between two cities on a map is 2.5 centimeters. Given a map scale of 1 cm : 100 km, what is the actual distance?

$$\frac{\text{map distance (cm)}}{\text{actual distance (km)}} = \frac{1}{100} = \frac{2.5}{n}$$

n = **250 km**

Drawing or Model Measurement

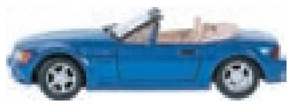
The length of a car is 156 inches. Find the length of a model car using the scale 1 in. : 52 in.

$$\frac{\text{model length}}{\text{actual length}} = \frac{1}{52} = \frac{n}{156}$$

156 = 52*n*

3 in. = *n*

1 cm : 100 km

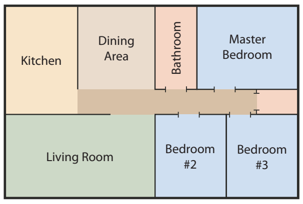


1 in. : 52 in.

Exercises

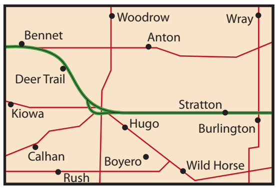
Write a proportion using the scale 1 in. : 12 ft to find the answer.

1. If the length of the master bedroom is 1.2 inches, what is the actual length? ***14.4 ft***
2. If the outside wall from the kitchen to the master bedroom is 3.5 inches, what is the actual length? ***42 ft***
3. If the outside wall from the kitchen to the living room is 2.3 inches, what is the actual length? ***27.6 ft***
4. If the bathroom has a length of 0.5 inch and a width of 1 inch, what are the actual dimensions? ***6 ft long and 12 ft wide***



Measure the *road distance* between cities to the nearest $\frac{1}{2}$ cm. Write a proportion using the scale 1 cm : 32 km to find the approximate distance.

5. Wray to Burlington $\frac{1}{32} = \frac{3}{n}$; ***n = 96 km***
6. Deer Trail to Wild Horse $\frac{1}{32} = \frac{4.5}{n}$; ***n = 144 km***
7. Kiowa to Burlington $\frac{1}{32} = \frac{7}{n}$; ***n = 224 km***
8. Calhan to Stratton $\frac{1}{32} = \frac{5.5}{n}$; ***n = 176 km***
9. Rush to Wild Horse $\frac{1}{32} = \frac{3.5}{n}$; ***n = 112 km***
10. Deer Trail to Burlington $\frac{1}{32} = \frac{6}{n}$; ***n = 192 km***
11. Calhan to Wild Horse $\frac{1}{32} = \frac{5}{n}$; ***n = 160 km***

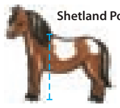


286 Chapter 13

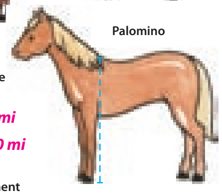
Write a proportion using the scale factor 1 cm : 35 cm to find the answer.

12. The height of the model Shetland pony is 2.6 cm. What is the actual height of this pony? $\frac{1}{35} = \frac{2.6}{n}$; ***n = 35 • 2.6; n = 91 cm***
13. The height of the model Hackney pony is 3.6 cm. What is the actual height of this pony? $\frac{1}{35} = \frac{3.6}{n}$; ***n = 35 • 3.6; n = 126 cm***
14. The height of a Palomino is 168 cm. What should the height of the model Palomino be? $\frac{1}{35} = \frac{n}{168}$; $\frac{168}{35} = \frac{35n}{35}$; ***n = 4.8 cm***

Hackney Pony



Shetland Pony



Palomino

Write a proportion using the map scale 1 in. : 150 mi to find the actual distance represented by the measurement.

15. 4 in. ***600 mi***
16. 9 in. ***1,350 mi***
17. 3.6 in. ***540 mi***
18. 0.6 in. ***90 mi***
19. 5.8 in. ***870 mi***
20. 13 in. ***1,950 mi***

Write a proportion using the map scale 1 in. : 16 mi to find the map measurement equal to the actual distance.

21. 80 mi ***5 in.***
22. 160 mi ***10 in.***
23. 120 mi ***7.5 in.***
24. 48 mi ***3 in.***
25. 8 mi ***0.5 in.***
26. 67.2 mi ***4.2 in.***

Practice & Application

27. A playground is 72 feet wide. How wide would a scale drawing of the playground be in which 3 inches represents 12 feet?
28. A scale drawing of a plant cell is 2.4 cm in length. What is the actual length of the cell if the drawing uses a scale of 8 cm : 1 mm?
29. A model car is 12 inches long. How long is the actual car if it was made with a scale factor of 1 in. : 16 in.? $\frac{1}{16} = \frac{12}{n}$; ***n = 16 • 12; n = 192 in.***
30. A dollhouse was built with a scale factor of 1 in. : 12 in. The dollhouse is 48 inches long. What is the actual length of the house it represents? $\frac{1}{12} = \frac{48}{n}$; ***n = 12 • 48; n = 576 in.***
31. The length of a highway is 225 miles long. How long would the highway be on a map with a scale of 2 in. : 75 mi?
32. A model train was made with a scale factor of 1 in. : 96 in. The model railroad car is 5.5 inches long. What is the actual length of the railroad car? $\frac{1}{96} = \frac{5.5}{n}$; ***n = 96 • 5.5; n = 528 in.***

J A square microchip has an actual measure of 0.8 mm per side. Make a scale drawing of the microchip using the given scale.

scale = 1 cm : 0.1 mm ***8 cm each side***
scale = 5 mm : 0.2 mm ***20 mm each side***

$\frac{1}{0.1} = \frac{n}{0.8}$
$\frac{0.8}{0.1} = \frac{0.1n}{0.1}$
n = 8 cm
$\frac{5}{0.2} = \frac{n}{0.8}$
0.8 • 5 = 0.2n
$\frac{4}{0.2} = \frac{0.2n}{0.2}$
n = 20 mm

27. $\frac{3}{12} = \frac{n}{72}$
72 • 3 = 12n
$\frac{216}{12} = \frac{12n}{12}$
n = 18 in.

31. $\frac{2}{75} = \frac{n}{225}$
225 • 2 = 75n
$\frac{450}{75} = \frac{75n}{75}$
n = 6 in.

28. $\frac{8}{1} = \frac{2.4}{n}$
$\frac{8n}{8} = \frac{2.4}{8}$
n = 0.3 mm

Complete DAILY REVIEW e on page 448.

Lesson 118 287

book, illustrations of microscopic cells and other objects are examples of scale drawings that are *larger* than the actual object.

➤ **If you were to make a scale drawing, what information would you need to know?** ***Elicit that you need to know the actual measurements of the object and the scale for the drawing.***

2. Explain to the students that they will make scale drawings of their desktops using a scale of 1 inch for every 6 inches of the actual desk. Write *model measurement:actual measurement = 1 in.:6 in.* for display. Direct each student to measure the length and width of his desktop.

➤ **What proportion could you write to find the length for your scale drawing?** $\frac{1 \text{ in.}}{6 \text{ in.}} = \frac{n}{\text{actual length}}$ **the width for your scale drawing?** $\frac{1 \text{ in.}}{6 \text{ in.}} = \frac{n}{\text{actual width}}$

3. Guide the students in solving the proportions to find the length and width for the drawings. ***Answers will vary based on desk sizes.*** Then guide them in making the scale drawings of their desktops, using their rulers to draw the correct length and width measurements. Point out that the scale tells them that their model drawing will be $\frac{1}{6}$ of the length and width of the actual desktop.

➤ **How would the model drawing of your desktop change if the scale was 1 in.:10 in.?** ***Elicit that the scale drawing would be smaller, $\frac{1}{10}$ of the length and width of the actual desktop rather than $\frac{1}{6}$ of the length and width.*** **if the scale was 1 in.:2 in.?** ***Elicit that the scale drawing would be larger, $\frac{1}{2}$ of the length and width of the actual desktop rather than $\frac{1}{6}$ of the length and width.*** **if the scale was 2 in.:1 in.?** ***Elicit that the length and width measurements of the scale drawing would be twice the length and width measurements of the actual desk; the scale drawing would be larger than the desk.***

4. Guide the students in determining the length and width of the scale drawing using the scale 2 in.:1 in. and using the actual length and width measurements of their desks. $\frac{2 \text{ in.}}{1 \text{ in.}} = \frac{n}{\text{actual length}}$; $\frac{2 \text{ in.}}{1 \text{ in.}} = \frac{n}{\text{actual width}}$

5. Guide the students in using proportions to solve the following word problems.

The length of a car is 258 inches. How long would a model of this car be if it were built using a scale of 1 in.:43 in.? $\frac{1 \text{ in.}}{43 \text{ in.}} = \frac{n}{258 \text{ in.}}$; ***n = 6 in.***

The floor plan of a house has a scale of 1 in.:6 ft. If the actual living room is 24 feet long and 18 feet wide, what are the length and width dimensions on the floor plan? $\frac{1 \text{ in.}}{6 \text{ ft}} = \frac{l}{24 \text{ ft}}$; ***l = 4 in.;*** $\frac{1 \text{ in.}}{6 \text{ ft}} = \frac{w}{18 \text{ ft}}$; ***w = 3 in.***

The distance between 2 cities is 75 miles. If the map scale is 1 in.:15 mi, what is the map measurement? $\frac{1 \text{ in.}}{15 \text{ mi}} = \frac{n}{75 \text{ mi}}$; ***n = 5 in.***

A plant cell is 0.5 mm in length. How long is the drawing of the cell if the scale is 5 cm:1 mm? $\frac{5 \text{ cm}}{1 \text{ mm}} = \frac{l}{0.5 \text{ mm}}$; $\frac{50 \text{ mm}}{1 \text{ mm}} = \frac{l}{0.5 \text{ mm}}$; ***l = 25 mm or 2.5 cm***

Student Text pp. 286–87

(*Note*: *Assessment* available on Teacher's Toolkit CD.)

Lesson 119 | **Student Text** pp. 288–89 | **Daily Review** p. 449f

Objectives

- Develop an understanding of percent using models
- Express percents as ratios, decimals, and fractions in lowest terms
- Express decimals and fractions as percents
- Compare percents to decimals and fractions using >, <, or =
- Solve percent word problems using proportions

Teacher Materials

- Place Value Kit
- Percent, page IA67 (CD)

Student Materials

- A calculator

Teach for Understanding

Express percents as ratios, decimals, and fractions in lowest terms

1. Write *percent* and a percent sign (%) for display. Explain that *percent* is a part-to-whole ratio in which a part is compared to a whole that is made up of 100 equal parts. The term *percent* and the percent sign (%) mean "out of 100" or "per 100."
2. Display a large red one from the Place Value Kit.
   ➤ **How many shaded squares are there?** ***1***
   Display a large purple hundredths square (partitioned into 100 hundredths). Elicit that the red *1 whole* is equivalent to the purple *100 hundredths*.
   ➤ **What fraction tells the part of this whole that is shaded purple?** $\frac{100}{100}$
   ➤ **How can you write the value of 1 whole or $\frac{100}{100}$ in ratio form?** ***1:1 or 100:100*** **in decimal form?** ***1 or 1.00*** Write $1 = \frac{100}{100} =$ 100:100 = 1.00 for display.
   ➤ **Since percent means "out of 100" or "per 100," how can you write the value in percent form?** ***100%***
   Write = 100% after the other equivalencies. Point out that anything greater than 100% indicates more than one whole.
   ➤ **Is it possible to have 110% of the displayed square? Why?** ***Answers may vary, but elicit that 110% = $\frac{110}{100}$ = $1\frac{1}{10}$ or 1 whole square and $\frac{1}{10}$ of a second square.*** Display a large red one and an orange tenth.
   ➤ **What would 200% of the square look like?** ***Elicit that $\frac{200}{100}$ = 2 whole squares.*** Display 2 large red ones.
3. Display 1 orange tenth on a white hundredths mat.
   ➤ **How many of the 100 squares are shaded?** ***10***
   Write *10 per 100* for display.
   ➤ **How can you write a value of 10 per 100 in ratio form?** ***10:100*** **percent form?** ***10%*** **decimal form?** ***0.10*** **fraction form?** $\frac{10}{100}$
   Write the equivalencies beside *10 per 100*: = 10:100 = 10% = 0.10 = $\frac{10}{100}$.
   ➤ **What is $\frac{10}{100}$ written in lowest terms?** $\frac{1}{10}$ Guide the students to the conclusion that the other forms can also be written in lowest terms: 10:100 5 1:10, 0.10 5 0.1.
4. Repeat the procedure, displaying 4 orange tenths on the white hundredths mat. ***40 per 100 = 40:100 = 40% = 0.40 = $\frac{40}{100}$; lowest terms: $\frac{40}{100} = \frac{2}{5}$; 40:100 = 2:5; 0.40 = 0.4***
   Point out that although the decimal 0.4 has the same value as $\frac{2}{5}$ (2 ÷ 5 = 0.4), a decimal cannot be written as fifths because decimals are a base ten system.
5. Follow a similar procedure for these values, displaying the appropriate number of tenths on the hundredths mat.
   25 per 100 = ***25:100; 25%; 0.25; $\frac{25}{100}$***
   lowest terms: ***$\frac{25}{100} = \frac{1}{4}$; 25:100 = 1:4***
   4 per 100 = ***4:100 = 4% = 0.04 = $\frac{4}{100}$***
   lowest terms: ***$\frac{4}{100} = \frac{1}{25}$; 4:100 = 1:25***
6. Direct attention to the number line on the Percent page.
   ➤ **Which fractional parts does this number line show? How do you know?** ***Elicit that the number line shows tenths and hundredths; the larger marks partition the length of the 1 whole into 10 equal parts, and the smaller lines partition it into 100 equal parts.***
   ➤ **How can you write the value of point *A* as a fraction? Why?** ***$\frac{1}{10}$ or $\frac{10}{100}$*** **a decimal?** ***0.1 or 0.10*** **a percent?** ***10%; accept all reasonable explanations.***
7. Follow a similar procedure for the other points. *B* ***$\frac{25}{100}$; 0.25; 25%*** *C* ***$\frac{4}{10}$ or $\frac{40}{100}$; 0.4 or 0.40; 40%*** *D* ***$\frac{58}{100}$; 0.58; 58%*** *E* ***$\frac{8}{10}$ or $\frac{80}{100}$; 0.8 or 0.80; 80%***
   ➤ **What percent does the 0 represent?** ***0%*** **the 1?** ***100%***
8. Write for display *A* = *40%*, *B* = *90%*, *C* = *35%*. Instruct the students to draw a number line, partition it into tenths, and graph the three points.
9. Direct attention to the first rectangular model.
   ➤ **What percent of the whole rectangle does each part represent? Why?** ***10%; since there are 10 parts in the rectangle, each part is $\frac{1}{10}$ or 10% of the rectangle.***
   ➤ **How many parts of the rectangle are shaded?** ***$2\frac{1}{2}$***
   ➤ **What percent of the whole rectangle do the $2\frac{1}{2}$ shaded parts represent? How do you know?** ***25%; elicit that each of the 2 completely shaded parts is 10% of the rectangle and the half-shaded part is 5%; 10% + 10% + 5% = 25%.***
   ➤ **Since the shading is halfway to 50%, what percent is half of 50%?** ***25%***
10. Choose students to shade the last rectangle to show 50%, 51%, 49%, 26%, and 98%, removing the shading before showing the next percent.

Express decimals and fractions as percents

1. Draw a circle for display and shade one half of it.
   ➤ **What fraction represents the shaded part of the circle?** $\frac{1}{2}$
   Write $\frac{1}{2}$ for display.
   ➤ **How can you write the fraction $\frac{1}{2}$ as a percent? Why?** ***50%; accept all reasonable explanations, but guide them to the conclusion that since percent is a "per 100" ratio, you can rename $\frac{1}{2}$ as an equivalent fraction with 100 as its second term ($\frac{1}{2} \times \frac{50}{50} = \frac{50}{100}$), which is the fraction form of 50%.***
   Elicit that the terms of the fraction can also divide to find the decimal equivalent, which can be renamed as a percent: $\frac{1}{2}$ = 1 ÷ 2 = 0.5 = 0.50 = 50%. Point out that 5 tenths must be renamed as 50 hundredths to determine the percent. Choose a student to write = 50% after the $\frac{1}{2}$.
2. Follow a similar procedure for $\frac{3}{4}$ of a circle: $\frac{3}{4} \times \frac{25}{25} = \frac{75}{100}$ = 75% or $\frac{3}{4}$ = 3 ÷ 4 = 0.75 = 75%.
3. Follow a similar procedure to guide the students in finding the percent for $\frac{1}{3}$ and $\frac{2}{3}$ of a circle by dividing the terms of each fraction. ***1 ÷ 3 = $0.\overline{3} \approx$ 33%; 2 ÷ 3 = $0.\overline{6} \approx$ 67%***

Point out that when the second term of a ratio is related to 100, as with halves and fourths, it is easy to rename the ratio as an equivalent fraction. However, whether the terms are

Percent

Percent is a ratio in which a quantity is compared to 100. The symbol for percent is **%**. Percent means "out of 100," "per 100," or "÷ 100."

percent (%)

1 whole	32 out of a hundred	Divide by 100
$100\% = \frac{100}{100} = 1.0$	$\frac{32}{100}$ fraction, 0.32 decimal, 32% percent	$50\% = \frac{50}{100} = 50 \div 100 = 0.5$

This number line shows equivalent fractions, decimals, and percents.

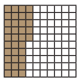
0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\frac{0}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$
0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
$\frac{0}{100}$	$\frac{10}{100}$	$\frac{20}{100}$	$\frac{30}{100}$	$\frac{40}{100}$	$\frac{50}{100}$	$\frac{60}{100}$	$\frac{70}{100}$	$\frac{80}{100}$	$\frac{90}{100}$	$\frac{100}{100}$

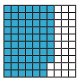
Exercises

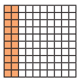
Draw a number line. Divide the number line into ten equal parts. Graph a point on the number line to represent the percent.

1. 40% **2.** 70% **3.** 85%

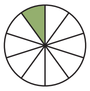
Write the decimal for the shaded part of the grid. Write the decimal as a percent.

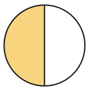
4.  *0.35 = 35%*

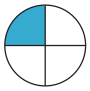
5.  *0.68 = 68%*

6.  *0.2 = 20%*

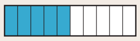
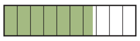
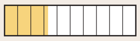
Write the fraction for the shaded part of the circle. Write the percent of the whole.

7.  *$\frac{1}{10}$ = 10%*

8.  *$\frac{1}{2}$ = 50%*

9.  *$\frac{1}{4}$ = 25%*

Estimate the percent shaded of the rectangle.

10.		10%	25%	50%	75%
11.		2%	36%	68%	95%
12.		33%	50%	75%	95%

Draw a rectangle. Shade the rectangle to represent an approximation of the percent.

13. 25% **14.** 48% **15.** 90%

288 Chapter 13

Change a Decimal to a Percent	Change a Percent to a Decimal
Use mental math: move the decimal point two places to the right when multiplying by 100. $0.71 \times 100 = 71\%$; $0.2 \times 100 = 20\%$	Use mental math: move the decimal point two places to the left when dividing by 100. $60\% = \frac{60}{100} = 60 \div 100 = 0.6$; $4\% = \frac{4}{100} = 4 \div 100 = 0.04$

Change a Fraction to a Percent	Change a Percent to a Fraction
1. Divide. Round the answer to the nearest hundredth. 2. Change the decimal to a percent. $\frac{3}{5} = 3 \div 5 = 0.6 = 60\%$	1. Rename the percent as a fraction with 100 as the denominator. 2. Simplify the fraction by renaming to lowest terms. $40\% = \frac{40}{100} = \frac{2}{5}$

Exercises

Write the percent in decimal form. Write the decimal in percent form.

16. 52% *0.52* **17.** 89% *0.89* **18.** 2% *0.02*

19. 0.07 *7%* **20.** 0.54 *54%* **21.** 23% *0.23*

Write the percent as a fraction in lowest terms.
Write the fraction as a decimal. Write the decimal as a percent. Round to the nearest percent.

22. 30% *$\frac{30}{100} = \frac{3}{10}$* **23.** 80% *$\frac{80}{100} = \frac{4}{5}$* **24.** 15% *$\frac{15}{100} = \frac{3}{20}$*

25. $\frac{2}{5}$ *$2 \div 5 = 0.4 = 40\%$* **26.** $\frac{3}{4}$ *$3 \div 4 = 0.75 = 75\%$* **27.** $\frac{3}{8}$ *$3 \div 8 = 0.375 \approx 38\%$*

Write a comparison sentence using >, <, or =. *Steps to solve may vary.*

28. $\frac{5}{6}$ > 60% **29.** 0.73 = 73% **30.** $\frac{3}{10}$ > 3% **31.** 0.4 > 4%

32. 0.48 > 5% **33.** $\frac{2}{3}$ < 85% **34.** 0.7 < 75% **35.** 0.8 = 80%

Use the data from the circle graph to find the answer.

36. What fraction of those surveyed chose cats as their favorite pet? *$\frac{1}{4}$*

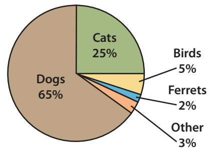
37. What fraction chose an animal other than cats as their favorite pet? *$\frac{3}{4}$*

38. What percent chose dogs as their favorite pet? *65%*

39. What percent chose a pet other than dogs? *35%*

40. What percent represents the whole graph? *100%*

Favorite Pet



Solve.

41. The art teacher took 50 students on a field trip to the museum. Thirty of the students bought a post card while there. What percent of the students bought a post card? *$\frac{30}{50} = \frac{60}{100} = 60\%$*

42. Dillon spelled 24 words correctly out of 25 on a spelling test. What percent of the words did he spell correctly? *$\frac{24}{25} = \frac{96}{100} = 96\%$*

Complete DAILY REVIEW f on page 449.

Lesson 119 289

related or not, the first term of a ratio can always be divided by its second term to find a decimal (e.g., $\frac{1}{3} = 1 \div 3 = 0.\overline{3} \approx$ 33%). If the terms do not divide equally, the ratio and the percent are approximately equal.

Compare percents to decimals and fractions

1. Write 0.6, 0.14, and 0.08 for display. Direct the students to write each decimal as a percent. Choose students to share their answers and explain their reasoning. ***60%, 14%, 8%***
   ➤ **What do you notice about the decimal point when a decimal is renamed as a percent?** ***The decimal point moves 2 places to the right.*** Elicit that a percent can be found by multiplying a decimal by 100 to move the decimal point two places to the right.
   Select students to demonstrate multiplying 0.6, 0.14, and 0.08 by 100 to find the percents. ***0.6 × 100 = 60%, 0.14 × 100 = 14%, 0.08 × 100 = 8%***
2. Follow a similar procedure to guide the students in writing 43%, 30%, and 2% as decimals. ***0.43, 0.3, 0.02; the decimal point moves 2 places to the left.*** Elicit that a decimal value can be found by dividing a percent by 100 to move the decimal point two places to the left. ***43% = $\frac{43}{100}$ = 43 ÷ 100 = 0.43; 30% = $\frac{30}{100}$ = 30 ÷ 100 = 0.3; 2% = $\frac{2}{100}$ = 2 ÷ 100 = 0.02***
3. Write $\frac{3}{5}$, $\frac{7}{10}$, and $\frac{2}{7}$ for display. Direct the students to write each fraction as a decimal and as a percent. Choose students to write their answers for display and to explain their reasoning. ***$\frac{3}{5}$ = 0.6 = 60%; $\frac{7}{10}$ = 0.7 = 70%; $\frac{2}{7}$ ≈ 0.29 ≈ 29%***
4. Write for display $\frac{7}{10}$ __ 7%. Instruct each student to decide whether >, =, or < completes the statement. Select a student to complete the comparison and explain his answer. ***$\frac{7}{10}$ > 7%***
5. Follow a similar procedure for the following comparisons. Review finding equivalent fractions, decimals, and percents as needed.
   $\frac{2}{5}$ < 50%   0.75 = $\frac{6}{8}$ $\frac{1}{3}$ > 25%

Solve percent word problems

Guide the students in writing proportions to solve the following word problems. Explain that to find a percent using a proportion, the second term in the ratio with the unknown value must be 100 because the second term in the known ratio is equivalent to 100%. (e.g., the 15 points scored by Carmen's team is equal to 100%.)

(*Note:* Some students may realize that the word problems can also be solved by writing a ratio to compare the information given in the word problem and dividing the terms. This will rename the ratio as a decimal that can be renamed as a percent.)

Carmen scored 3 of her team's 15 points during a volleyball game. What percent of her team's points did she score? ***$\frac{3}{15} = \frac{n}{100}$; 20%***

Samuel paid the sale price of $35 for a $50 game. What percent of the original cost did Samuel pay? ***$\frac{35}{50} = \frac{n}{100}$; 70%***

Dad left a tip of $5 for the server after purchasing a meal for $25. What percent of the cost of the meal did he leave for a tip? ***$\frac{5}{25} = \frac{n}{100}$; 20%***

On a math test, Tony answered 40 out of 45 questions correctly. What percent of the test questions did he answer correctly? ***$\frac{40}{45} = \frac{n}{100}$; approximately 89%***

Student Text pp. 288–89

Lesson 120 | **Student Text** pp. 290–91
Daily Review p. 449g

Objectives
- Find a percent of a number using an equation, a model, and a proportion
- Solve percent word problems

Teacher Materials
- Percent Models: Find the Part, page IA68 (CD)

Student Materials
- A calculator

Teach for Understanding

Find a percent of a number using an equation

> If 50% of the 20 students in Miss Gardner's class are boys, how many students are boys? *10 boys*

1. Elicit that 50% of the whole class of 20 students are boys, and that 100% of the class is equal to 20. To solve the problem you must find a number that is a part (50%) of the whole class.
   ➤ **What does "percent" mean?** ***out of 100 or per 100***
   ➤ **What do you think 50% of 20 is? Why?** ***10; possible answers: 50 is half of 100, and 10 is half of 20 or since 50% = $\frac{50}{100}$ = $\frac{1}{2}$, 50% of a number is equal to $\frac{1}{2}$ of that number; $\frac{1}{2}$ of 20 = 10.*** Point out that their reasoning indicated the statement *50 is to 100 like __ is to 20.*
   Elicit that the word *of* in the word problem means "multiply" and the word *are* means "equals." Write 50% × 20 = __ for display.
   ➤ **How can you write 50% as a numerical value without the percent sign?** ***$\frac{50}{100}$ or 0.50***
2. Write *n% of a number* = $\frac{n}{100}$ × *the number* and write $\frac{50}{100}$ × 20 = __ below the formula. Explain that you can find the part that is the percent of a number by writing the percent as a fraction and multiplying it by the whole amount.
3. Choose a student to solve the equation. Point out that you can use cancellation or write the percent as a fraction in lowest terms before multiplying. ***$\frac{50}{100}$ × 20 = $\frac{50}{5}$ = 10 or $\frac{1}{2}$ × 20 = 10***
   ➤ **How many students are boys?** ***10***
4. Follow a similar procedure to find the percent of these numbers.
   30% of 40 ***$\frac{30}{100}$ × 40 = 12***
   70% of 50 ***$\frac{70}{100}$ × 50 = 35***
   25% of 60 ***$\frac{25}{100}$ × 60 = 15 or $\frac{1}{4}$ × 60 = 15***

> Pauline has saved 36% of the money she needs for a new computer that costs $500. How much money has she saved for the computer? *$180*

   ➤ **How much money is Pauline trying to save?** ***$500***
   ➤ **What part of the money has Pauline saved?** ***36%***
   ➤ **How can you use the formula *n% of a number* = $\frac{n}{100}$ × *the number* to solve this word problem?** ***Elicit that you can substitute 36% for the percent and 500 for the whole; then multiply to find the number that is equal to 36% of $500.***
5. Write for display *36% of $500 is __.*
   ➤ **What is the decimal form for 36%?** ***0.36***
   Choose a student to use the formula and the decimal form of 36% to solve the problem. ***0.36 × 500 = $180***
6. Follow a similar procedure for these problems.
   40% of 75 ***0.40 × 75 = 30***
   8% of 35 ***0.08 × 35 = 2.8***

Find a percent of a number using a model

➤ **What is 10% of 30?** ***3*** **of 70?** ***7*** **of 25?** ***2.5*** **How did you determine your answers?** ***Accept all reasonable explanations, but elicit that since finding 10% of a number is the same as finding $\frac{1}{10}$ of a number (the whole), you can divide the number by 10, moving the decimal point one place to the left.***
➤ **How can you mentally find 10% of a number?** ***Elicit that you can divide the number by 10 by mentally moving the decimal point one place to the left.***

1. Display the first word problem and model on the Percent Models: Find the Part page. Read aloud the word problem and explain that this type of model is helpful for picturing how to find the percent of a number.
   ➤ **What percent does the entire model represent?** ***100%***
   ➤ **What whole amount is 100% in this problem?** ***$60***
   ➤ **How much money does each part of the model represent? How do you know?** ***$6.00; elicit that each part of the model is 10% of the whole and $6 is 10% of $60.***
   ➤ **What part of the money did Brad give to the church?** ***Elicit 10% of $60, $\frac{1}{10}$ of $60, $6.***
   Shade 10% of the model and write $6 above the 10% line.
   ➤ **Since 10% of 60 is 6, how can you find the amount of money Brad saved?** ***Elicit that he saved 40%, and 40% is 4 times more than 10%, so 4 × $6 = $24 or $6 + $6 + $6 + $6 = $24.***
   Shade to the 40% line of the model and write $24 above the 40% line. Point out that if you know 10% or $\frac{1}{10}$ of a price, you can find 20%, 30%, 40%, and so on.
   ➤ **How can you find the total amount that Brad gave to the church and saved?** ***possible answers: 10% + 40% ($6 + $24) or 50% = 5 × 10% (5 × $6)***
   ➤ **How much money did Brad give to the church and save?** ***$30***
2. Direct attention to the next word problem.
   ➤ **What is the whole amount or 100% in this word problem?** ***$75*** Write $75 above the 100% line of the model.
   ➤ **What part of the $75 did Shannon save?** ***60%*** Write 60% below the model and shade the 60% .
   ➤ **What is 10% of $75? How do you know?** ***$7.50; elicit that since 10% is $\frac{1}{10}$ of the whole, you can mentally divide $75 by 10, moving the decimal point in $75 one place to the left.*** Write $7.50 above the 10% line.
   ➤ **How can you find 60% of $75, using mental math?** ***6 × 10% = 6 × $7.50 = $45 [possible mental solution: (6 × $7) + (6 × $0.50) = $45]*** Write $45 above the 60% section.
3. Direct the students to draw a model to solve the third word problem. Choose a student to complete the model on the displayed page. Select another student to write the equations he used to solve the problem and to explain his solutions. Discuss the solutions.
   discount solution: ***10% of $180 = $18; 30% = 3 × 10% = 3 × $18 = $54***
   sale price solution: ***$180 − $54 = $126***
   Point out that this model is best used when a multiple of 10 is the percent. However, the model could be partitioned into more parts to solve problems in which the percent is not a multiple of 10: 20 sections of 5% or 100 sections of 1%.

Finding Percent of a Number

Percents are used to find the discount of a sale item, the sales tax on a purchase, or an appropriate tip for the cost of a meal at a restaurant. 100% represents all of a given number. Less than 100% of a number represents part of that number. Use the formula to find the percentage of a number. **n% of a number = $\frac{n}{100}$ × the number**

Rename the percent as a fraction or a decimal to solve an equation.

Elise wanted a sweater that cost $50. She saved $27 towards the purchase and waited for a sale. The first sale was 25% off the original price. Does Elise have enough money to buy the sweater with the discount of 25%?

What is 25% of $50?

$\frac{25}{100} \times 50 = \_\_$

$\frac{25}{\cancel{100}_2} \times \frac{\cancel{50}^1}{1} = \frac{25}{2} = 12\frac{1}{2}$ discount = $12.50

Sale Price: $50.00 – $12.50 = $37.50

Elise does *not* have enough money to buy the sweater with a discount of 25%.

Near the end of the season, the sale increased to 60% off the original price. Does Elise have enough money to buy the sweater with the discount of 60%?

What is 60% of $50?

$0.60 \times 50 = \_\_$

$0.60 \times 50 = 30.00$ discount = $30.00

Sale Price: $50.00 – $30.00 = $20.00

Elise has enough money to buy the sweater with a discount of 60%.

Make a Model

Garrett wanted basketball shoes that cost $125. The first sale price was 30% off. Then the shoes went on sale for 60% off. What was the discount of the shoes during each sale?

0 30% $125

10%	10%	10%	10%	10%	10%	10%	10%	10%	10%

Each section represents 10% of 125.

10% of 125 = $12.50

30%: 3 × $12.50 = $37.50

30% discount = $37.50

60%: 6 × $12.50 = $75.00

60% discount = $75.00

Exercises

Cancellation steps may vary.

Find the percent of the number. Solve by writing a fraction for the percent.

1. 50% of 78 ***39***
2. 30% of 80 ***24***
3. 40% of 200 ***80***
4. 25% of 48 ***12***
5. 60% of 25 ***15***
6. 75% of 52 ***39***
7. 20% of 85 ***17***
8. 33% of 100 ***33***
9. 10% of 250 ***25***
10. 70% of 15 ***$10\frac{1}{2}$***

Find the percent of the number. Use a decimal for the percent if needed.

11. 15% of 80 ***12***
12. 35% of 120 ***42***
13. 24% of 400 ***96***
14. 5% of 64 ***3.2***
15. 100% of 25 ***25***
16. 18% of 65 ***11.7***
17. 52% of 65 ***33.8***
18. 39% of 200 ***78***
19. 99% of 50 ***49.5***
20. 45% of 20 ***9***

Make a model to find the percent of the number.

21.	10% of 70	20% of 70	80% of 70	***7; 14; 56***
22.	10% of 50	40% of 50	90% of 50	***5; 20; 45***
23.	10% of 85	30% of 85	70% of 85	***8.5; 25.5; 59.5***

Problems involving percents can be solved by setting up a proportion. An unknown in a proportion can be found by cross-multiplying or by finding the equivalent ratios. $\frac{\text{part}}{100} = \frac{\text{part}}{\text{whole}}$

A survey showed that 6 of 23 sixth-grade students preferred Gooey Cluster candy bars over Nutty Crunch. What percent of students preferred Gooey Cluster?

$\frac{n}{100} = \frac{6 \text{ (part liking Gooey Cluster)}}{23 \text{ (whole class)}}$

Cross-Multiply

1. Write the proportion.
2. Multiply to find the cross products.
3. Solve the equation.

$\frac{n}{100} = \frac{6}{23}$

$23 \cdot n = 100 \cdot 6$

$23n = 600$

$\frac{\cancel{23}^1 n}{\cancel{23}_1} = \frac{600}{23}$

$n = 26.09$

About 26% of the sixth-grade class preferred Gooey Cluster bars.

The survey was given to 25 fifth-graders. Eight students preferred Gooey Cluster. What percent of the class preferred Gooey Cluster?

$\frac{n}{100} = \frac{8 \text{ (part liking Gooey Cluster)}}{25 \text{ (whole class)}}$

Equivalent Ratios

1. Write the proportion.
2. Multiply or divide by a form of 1.
3. Solve the equation.

$\frac{n}{100} \overset{\div 4}{=} \frac{8}{25}$

$n \div 4 = 8$

$\frac{n}{\cancel{4}_1} \cdot \cancel{4}^1 = 8 \cdot 4$

$n = 32$

32% of the fifth-grade class preferred Gooey Cluster bars.

Exercises

Method used to solve may vary.

Write a proportion to find the percent of the number.

24. 55% of 80 ***$\frac{55}{100} = \frac{n}{80}$; n = 44***
25. 60% of 95 ***$\frac{60}{100} = \frac{n}{95}$; n = 57***
26. 80% of 60 ***$\frac{80}{100} = \frac{n}{60}$; n = 48***
27. 48% of 20 ***$\frac{48}{100} = \frac{n}{20}$; n = 9.6***
28. 2% of 100 ***$\frac{2}{100} = \frac{n}{100}$; n = 2***
29. 35% of 50 ***$\frac{35}{100} = \frac{n}{50}$; n = 17.5***
30. 16% of 45 ***$\frac{16}{100} = \frac{n}{45}$; n = 7.2***
31. 25% of 25 ***$\frac{25}{100} = \frac{n}{25}$; n = 6.25***
32. 10% of 300 ***$\frac{10}{100} = \frac{n}{300}$; n = 30***

Practice & Application

33. Miles spends 5% of his day practicing basketball. How many hours does he practice each day? ***1.2 hours***
34. Miles made 35% of his 40 free-throw shots during the basketball season. How many free-throw shots did he make? ***14 shots***
35. Mr. Callahan bought a cordless screwdriver during a $\frac{1}{4}$-off sale. The original price was $64. What was the discount? What was the sale price? ***$16; $48***
36. If sales tax is 8%, how much tax would Evelyn pay on a purchase of $37.50? ***$3.00***
37. The cost of dinner at a restaurant was $60. If Charles gave a 20% tip to the server, how much was the tip? ***$12***
38. Maggie baby-sat and made $80 during the week. She plans to give 10% to the church on Sunday and to save 40%. How much will she give to the church and how much will she save? ***$8; $32***
39. Alicia deposited $150 in a simple savings account that earns 2% each year. If she does not deposit or withdraw any money, how much interest will it earn in one year? ***$3***
40. On a survey, 65% of the respondents said they prefer dogs over cats as pets. If 200 people completed the survey, how many people prefer dogs? ***130 people***
41. Manuel is 80% of the height of his father. If his father is 6 feet tall, how tall is Manuel? ***4.8 feet***

Complete DAILY REVIEW g on page 449.

Find a percent of a number using a proportion

During the flu season, 26% of the 250 students were absent from school. How many students were absent? ***65 students***

1. Explain that another method of finding the percent of a number is using a proportion. Write: $\frac{part}{100} = \frac{part}{whole}$ for display.
   ➤ **What is the fraction form for 26%?** ***$\frac{26}{100}$***
   ➤ **What is the whole or 100% of the students?** ***250***
2. Choose a student to write a proportion using the information. $\frac{26}{100} = \frac{s}{250}$
   Direct the students to solve the proportion to find the number of students that were absent. ***6,500 = 100s; $\frac{6{,}500}{100} = \frac{100s}{100}$; s = 65***
   ➤ **How many students were absent?** ***65 students***
3. Follow a similar procedure to guide the students in solving 75% of 96 and 52% of 25.
   $\frac{75}{100} = \frac{n}{96}$; 7,200 = 100n; $\frac{7{,}200}{100} = \frac{100n}{100}$; n = 72
   $\frac{52}{100} = \frac{n}{25}$; 1,300 = 100n; $\frac{1{,}300}{100} = \frac{100n}{100}$; n = 13
4. Explain that you can also find 52% of 25 by renaming $\frac{52}{100}$ as an equivalent ratio since the second terms, 100 and 25, are related. Guide the students in using this method to solve the problem.

$$\frac{52}{100} \overset{\div 4}{=} \frac{n}{25}$$

5. Direct the students to solve the following word problems; give guidance as needed. Encourage them to use the method of their choice: writing an equation, using a model, or using a proportion. Discuss the answers and the methods that were used.

Cheri purchased a book during a 25% off sale. The original price was $13.50. What was the amount of discount? What was the sale price? ***$3.38; $10.12***

If the sales tax is 7%, how much tax would Clint pay on his purchase of $43.25? ***$3.03***

Lisa is 85% of the height of her older sister. If her older sister is 60 inches tall, how tall is Lisa? ***51 inches***

Student Text pp. 290–91

Lesson 121 | **Student Text** pp. 292–93 | **Daily Review** p. 450h

Objectives

- Find the unknown whole in a percent problem using a model, an equation, and a proportion
- Solve percent word problems

Teacher Materials

- Percent Models: Find the Whole, page IA69 (CD)

Student Materials

- A calculator

Teach for Understanding

Find the unknown whole using a model

1. Display the Percent Models: Find the Whole page. Point out that a percent model can be made using any figure that can easily be partitioned into 10 equal parts.
   - ➤ **What does each of the 10 equal sections in a percent model represent?** ***$\frac{1}{10}$ or 10% of the 100% in the whole model***
   - ➤ **What is the question in the first word problem asking you to find?** ***The whole amount or the total amount Dori earned during the week.***
   - ➤ **What information is given?** ***Elicit that $120 is 60% of the whole amount that Dori earned.*** Point out 60% and $120 on the model.
   - ➤ **What do you think the *n* on the model represents?** ***The whole amount that Dori earned during the week.***
2. Shade 60% of the model. Remind the students that the shaded sections are the part that equals the percent of the whole.
   - ➤ **How many equal parts make up 60%?** ***6***
   - ➤ **How can you find the value of 10%? Why?** ***Divide 120 by 6; elicit that since 60% is made up of 6 parts, each of which is 10%, dividing 120 by 6 will give you the value of each 10% section of the model.***
   - ➤ **What is the value of 10%? How do you know?** ***$20; $120 ÷ 6 = $20*** Write 10% below its line in the model and $20 above the line.
   - ➤ **Since 10% of the unknown whole is $20, what equation can you write to find 100%? Why?** ***10 × $20 = __; elicit that 10 sections of 10% make up 100%, so multiply the value of 10% ($20) by 10 to find the whole.***
   - ➤ **What does 10 × $20 equal?** ***$200*** Write $200 above the 100% line of the model.
   - ➤ **What whole amount did Dori earn?** ***$200***
3. Direct attention to the second word problem.
   - ➤ **What information is given in this word problem?** ***Elicit that $14 is 40% of the whole (the original price of the game).*** Write 40% below its line and $14 above the line. Shade 40% of the model.
   - ➤ **How many equal parts make up 40%?** ***4***
   - ➤ **What is the value of 10%? How do you know?** ***$3.50; $14 (40%) divided by 4 tells you the value of each 10% section; $14 ÷ 4 = $3.50.*** Write 10% below its line and $3.50 above the line.
   - ➤ **Since 10% of the unknown whole is $3.50, how can you find 100%?** ***Multiply $3.50 by 10.*** Elicit that when you multiply a number by 10, the decimal point moves one place to the right.
   - ➤ **What was the original price of Eric's game?** ***$35.00*** Write $35 above the 100% line of the model.
4. Direct the students to draw a model to solve the third word problem. Choose a student to complete the model on the displayed page. Select another student to write and explain the equations he used to solve the problem. Discuss the solution. ***$72 ÷ 3 = $24; 10 × $24 = $240***

   Remind the students that this model is best used when a multiple of 10 is the percent. However, the model could be partitioned into more parts: 20 sections of 5% or 100 sections of 1%.
   - ➤ **How could you use the model to find 85% of $240?** ***Possible answers: partition the whole model into twice as many parts so that each secttion is $\frac{1}{20}$ or 5% of the whole and multiply the value of 5% by 17; find 80% of $240 ($192) and 90% of $240 ($216), divide the difference by 2 (24 ÷ 2 = 12), and add the quotient to $192 (80% of $240); $204.***

Find the unknown whole using an equation

1. Write *percent × whole = part* for display and explain that you can use this formula to find the whole or 100%.

> Brian counted 8 trucks in the parking lot. If 20% of the vehicles in the parking lot are trucks, how many vehicles are in the parking lot? ***40 vehicles***

   - ➤ **What information are you given in the word problem?** ***8 trucks is 20% of the total.***
   - ➤ **What equation can you write in words to solve the problem? Why?** ***20% of v is 8; 20% of the unknown number of vehicles is trucks (8).*** Write for display *20% of v is 8.*
   - ➤ **How can you write 20% as a numerical value without the percent sign?** ***$\frac{20}{100}$ or 0.20***
2. Direct the students to rewrite an equation using mathematical symbols and an equivalent numerical value for 20%. ***$0.20 \times v = 8$ or $\frac{20}{100} \times v = 8$*** Choose a student to write both equations for display.
   - ➤ **How can you find the value of *v*?** ***Divide each side of the equation by 20 hundredths to isolate the v.***
   - ➤ **Which equation do you think is easier to solve? Why?** ***Possible answer: the equation that has the percent expressed as a decimal; when dividing by a fraction, you multiply by the reciprocal to solve the problem.***
3. Instruct the students to find the total number of vehicles by solving one of the equations. Discuss the answer and then choose a student to complete both of the equations written for display. ***$\frac{0.20v}{0.20} = \frac{8}{0.20}$; v = 40 vehicles***
4. Follow a similar procedure for these problems.

   70% of what number is 42? ***60***

   45% of what number is 18? ***40***

   21 is 14% of what number? ***150***

Find the unknown whole using a proportion

1. Write $\frac{part}{100} = \frac{part}{whole}$ for display. Explain that a proportion can also be used to find the unknown whole in a percent problem. Since a percent is a part-to-whole comparison with 100 as the whole, it can be compared to another part-to-whole comparison.

> A church gave $240 to a missionary family. This amount is 30% of the church's weekly missions budget. What is the weekly missions budget of the church? ***$800***

Finding the Unknown Whole

Simon spent $24 in the sporting goods store. This is 30% of his birthday money. How much birthday money did Simon have to begin with?

Write an Equation

Substitute known information into the formula to find an unknown.

percent × whole = part

$30\% \times n = \$24$

$0.3n = \$24$

$\frac{\cancel{0.3}n}{\cancel{0.3}} = \$24 \div 0.3$

$n = \$80$

whole = part ÷ percent

$n = \$24 \div 30\%$

$n = \$24 \div 0.3$

$n = \$80$

The original amount was $80.

Write a Proportion

Find the unknown whole by solving a proportion. Cross-multiply or find the equivalent ratio to solve.

$\frac{\textbf{part}}{\textbf{100}} = \frac{\textbf{part}}{\textbf{whole}}$

$\frac{30}{100} = \frac{\$24}{n}$

$30n = \$2{,}400$

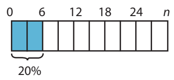
$\frac{\cancel{30}n}{\cancel{30}} = \frac{\$2{,}400}{30}$

$n = \$80$

The original amount was $80.

Make a Model

Allison scored 20% of the Blue Jays' points during the basketball game. If she scored 6 points, what was her team's score?



Each section represents 3 points.

20% of the Blue Jays' score = 6 points

100% = 5 × 20%

5 × 6 points = 30 points

100% of the Blue Jays' score = 30 points

Exercises

Write an equation to find the unknown whole.

1. 15% of what number is 12? ***n = 80***

2. 20% of what number is 50? ***n = 250***

3. 60% of what number is 15? ***n = 25***

4. 75% of what number is 9? ***n = 12***

5. 16 is 25% of what number? ***n = 64***

6. 14 is 35% of what number? ***n = 40***

Write a proportion to find the unknown whole. ***Proportions may vary.***

7. 35% of what number is 42? ***120***

8. 60% of what number is 24? ***40***

9. 52% of what number is 78? ***150***

10. 3% of what number is 6? ***200***

11. 7 is 14% of what number? ***50***

12. 36 is 45% of what number? ***80***

Make a model to find the unknown whole.

13. 40% of what number is 8? ***20***

14. 80% of what number is 56? ***70***

15. 50% of what number is 75? ***150***

16. 70% of what number is 84? ***120***

17. 117 is 90% of what number? ***130***

18. 15 is 25% of what number? ***60***

Find the percent of the number. ***Method used to solve may vary.***

19. 10% of 75 ***7.5***

20. 20% of 75 ***15***

21. 40% of 75 ***30***

22. 5% of 60 ***3***

23. 40% of 9 ***3.6***

24. 18% of 50 ***9***

25. 40% of 120 ***48***

26. 63% of 200 ***126***

27. 96% of 40 ***38.4***

Write the fraction as a percent.

28. $\frac{3}{4}$ ***75%***

29. $\frac{3}{50}$ ***6%***

30. $\frac{7}{10}$ ***70%***

31. $\frac{1}{2}$ ***50%***

32. $\frac{2}{5}$ ***40%***

33. $\frac{4}{25}$ ***16%***

Practice & Application

34. Caden mowed 4 lawns in 6 hours. At this rate, how long will it take him to mow 5 lawns? ***7.5 hr***

35. During the race, Car 14 traveled 240 miles in 2 hours. At this rate, what distance will the car travel in 5 hours? ***600 mi***

36. Aubrey bought a skirt during a $\frac{1}{3}$-off sale. The original price was $45. What was the discount? What was the sale price? ***$15; $30***

37. If sales tax is 8%, how much tax would be charged on a purchase of $6.50? ***$0.52***

38. Catherine spends 35% of her day at work. How many hours does she spend at work each day? ***8.4 hr***

39. All items in the sports department were on sale for 30% off. Weston received a discount of $12 on the basketball he purchased. What was the original price of the basketball? ***$40***

40. Nicholas scored 20% of his team's goals during a soccer game. He scored 2 goals. How many goals did his team score? ***10 goals***

41. During the election for class representative, Stella received 2 votes for every vote cast for her opponent. Since her opponent received 8 votes, how many votes did Stella receive? ***16 votes***

42. A house casts a shadow 28 feet long. A 6-foot man casts a shadow that is 8 feet long. How high is the house? ***21 ft***

43. A meter stick casts a shadow that is 3 meters long. A tree casts a shadow that is 12 meters long. How tall is the tree? ***4 m***

44. Brad got 29 answers correct out of 33 problems on a math test. About what percent of the problems did he get right? ***88%***

45. Caiti wanted a pair of shoes that cost $65. She watched for them to go on sale. At first, the sale price was 20% off the original price. Caiti bought the shoes when they went on sale for 45% off. What was the discount of the shoes during each sale? What did she pay for the shoes? ***$13.00; $29.25; $35.75***

MEET THE MATHEMATICIAN

John Napier (1550–1617) was a Scotsman who fervently defended the great Protestant reformer John Knox. As a Christian, he was deeply involved in political and religious struggles and turned to mathematics for relaxation. Although he hoped to be remembered for a commentary he wrote on the book of Revelation, he is chiefly remembered as the man who invented "Napier's bones," a multiplication table which was made from bone or ivory. This was one of the earliest forms of calculators used to do multiplication and division! John Napier also introduced the idea of a point to separate the whole number part of a number from its fraction part. This point is the decimal point.

Complete DAILY REVIEW on page 450.

➤ **What does the word *this* refer to in the word problem?** ***Elicit that it refers to the amount that was given to the missionary family; $240.***

➤ **What is the fraction form for 30%?** $\frac{30}{100}$

➤ **What part-to-whole relationship will help you find the weekly missions budget amount?** ***$240 is a part of the weekly missions budget.***

2. Choose a student to write the part-to-whole ratios to make a proportion. $\frac{30}{100} = \frac{\$240}{n}$

➤ **How can you solve for the missing term in the proportion?** ***Find an equivalent ratio or cross-multiply.***

Direct the students to solve the proportion. ***$800*** Discuss the answer and the method students used to solve the proportion. Elicit that $800 represents the whole or the church's weekly missions budget.

3. Follow a similar procedure for these problems.

   45% of what number is 18? ***40***

   15% of what number is 24? ***160***

   45 is 90% of what number? ***50***

Solve percent word problems

Guide the students in solving the following word problems. For each problem, elicit whether they need to find the part or the whole and encourage them to use the method of their choice: using an equation, a model, or a proportion. Discuss the answers and the methods that the students used.

Daniel scored 30% of his team's points during a basketball game. Daniel scored 6 points. What was the team's score? ***20 points***

At the Christian school, there are 15 boys in the 6th grade. If 60% of the students in the 6th grade are boys, how many students are in the 6th grade? ***25 students***

A $350 recliner is on sale for 45% off the original price. What is the discount? ***$157.50*** What is the sale price? ***$192.50***

If sales tax is 8%, how much tax would be charged for a purchase of $21.95? ***$1.76***

Student Text pp. 292–93

(***Note***: *Assessment* available on Teacher's Toolkit CD.)

Lesson 122

Student Text pp. 294–95

Daily Review p. 450i

Objectives

- Calculate the distance given the rate of speed and the time, the rate of speed given the distance and the time, and the time given the distance and the rate of speed
- Rename to calculate distance, rate of speed, or time
- Find an equivalent rate using a proportion

Student Materials

- A calculator

Teach for Understanding

Calculate distance, rate of speed, or time

1. Write r (*rate of speed*) = $\frac{d\ (distance)}{t\ (time)}$ for display. Remind the students that speed is a *rate*, a special ratio that compares two quantities having different measuring units. A rate of speed is expressed as a *distance* measurement divided by a *time* measurement and is written using the word "per" such as "miles *per* hour." If a rate is expressed using *miles per hour*, the unit that was used to measure the distance was *miles* and the unit that was used to measure the time was *hours*.
   Elicit that since speed is a rate, a proportion can be used to find equivalent rates of speed when you know the distance or the time traveled.
2. Elicit from the students that their knowing the formula $r = \frac{d}{t}$ helps them to write a proportion for finding the unit rate of speed for the following word problems. Instruct them to solve for the rate by finding an equivalent ratio or by cross-multiplying. (See Lesson 116.)

> Alexa walked 5 miles in 2 hours. What was her average speed per hour? $\frac{n \text{ mi}}{1 \text{ hr}} = \frac{5 \text{ mi}}{2 \text{ hr}}$; *n = 2.5; r = 2.5 miles per hour*
>
> The ranger traveled 78.6 miles through the park in 3 hours on his 4-wheeler. What was his average speed per hour? $\frac{n \text{ mi}}{1 \text{ hr}} = \frac{78.6 \text{ mi}}{3 \text{ hr}}$; *n = 26.2; r = 26.2 miles per hour*

3. Read aloud this word problem.

> In 1852, Henri Giffard built a steam airship that flew at a speed of 5 miles per hour. The airship traveled 17 miles. About how many hours did it take to travel that distance? *t = 3.4 hours*

➤ **Could you use a proportion to find the amount of time it took the airship to travel 17 miles? Why?** *Yes; elicit that since you know 3 of the 4 numbers in the proportion (speed = 5 miles per 1 hour, and distance = 17 miles), you can solve for the unknown time.*

➤ **Using the formula $r = \frac{d}{t}$, what proportion can you write to find the unknown time?** $\frac{5 \text{ mi}}{1 \text{ hr}} = \frac{17 \text{ mi}}{t}$

Direct the students to use cross multiplication to solve for the unknown time. *5t = 17; t = 3.4 hr*

4. Guide the students in using a proportion to find the unknown time using the following information.
   $r = \frac{65\text{mi}}{\text{hr}}$; d = 195 mi $\frac{65 \text{ mi}}{1 \text{ hr}} = \frac{195 \text{ mi}}{t}$; *65t = 195; t = 3 hr*

> Mrs. Collins walked at a speed of 4 miles per hour. If she maintained the same speed, how far did she walk in 3 hours? *d = 12 miles*

➤ **What information is given in this word problem?** *The rate of speed is $\frac{4 \text{ mi}}{\text{hr}}$, and the time is 3 hours.*

➤ **What information is not given? How do you know?** *The distance; the question is asking how far she walked.*

5. Direct the students to write a proportion to find the unknown distance. Choose a student to give the answer and explain the solution. $\frac{4 \text{ mi}}{1 \text{ hr}} = \frac{d}{3 \text{ hr}}$; *d = 12; accept explanations of finding an equivalent ratio or cross-multiplying.*

➤ **What measuring unit is used to find the distance in this problem? How do you know?** *Miles; the rate of speed was measured in miles per hour.* Write the answer: *distance* = *12 miles*.

6. Follow a similar procedure to guide the students in solving these problems.

> Juanita traveled 528 miles by train in 6 hours. What was the average speed of the train? *r* = $\frac{88 \text{ mi}}{\text{hr}}$
>
> Ashley and her friend biked 15 miles at an average speed of 6 miles per hour. About how long did it take to travel that distance? *t = 2.5 hours*
>
> Robert drove his car an average speed of 40 miles per hour. Jason drove his car an average speed of 50 miles per hour. How many more hours did it take Robert to travel 200 miles? *Robert: t = 5 hours; Jason: t = 4 hours; 5 − 4 = 1 hour*

Rename to calculate distance, rate of speed, or time

> A cheetah ran at a speed of 60 miles per hour for 10 minutes. How far did the cheetah travel? *d = 10 miles*

➤ **What information is given in this word problem?** *r* = $\frac{60 \text{ mi}}{\text{hr}}$; *t = 10 min*

➤ **What measuring units are used to measure the rate of speed in the problem?** *miles per hour*

➤ **What measuring unit is used to measure time?** *minutes*

1. Explain that a proportion compares *like* units. When the units of measure in a proportion are not alike, you need to rename a unit of measure so that you can compare like units before you begin to solve the problem.

➤ **What two units are related? How are they related?** *Minutes and hours; elicit that there are 60 minutes in 1 hour.*

Write $\frac{60 \text{ mi}}{1\text{hr}} = \frac{60 \text{ mi}}{60 \text{ min}}$ for display.

2. Direct the students to use the renamed rate of speed and the formula $r = \frac{d}{t}$ to find the unknown distance. Choose a student to tell the answer and explain his solution. *Solutions will vary;* $\frac{60}{60} = \frac{d}{10}$; *d = 10.*

➤ **What measuring unit is used to measure distance in this problem? How do you know?** *Miles; the rate of speed was measured in miles per minute.* Write the answer: *d* = *10 miles*.

3. Follow a similar procedure for the following word problem. Elicit that you need to find the rate or speed of a train, which is typically measured in miles per hour, and that 1 day can be renamed as 24 hours.

> The Fuller family went on a cross-country trip by train. They traveled 2,040 miles in 1 day. How fast did the train travel? *r* = $\frac{2{,}040 \text{ mi}}{24 \text{ hr}}$; *r* = $\frac{85 \text{ mi}}{\text{hr}}$

Speed, Distance & Time

Speed is a **rate**, a special ratio which compares distance to time and is usually written as a unit: miles per hour, kilometers per hour, or meters per second. A known rate of speed can be used to find an unknown distance or time traveled at the same rate of speed. Make a proportion to find the unknown distance or time.

speed
rate
distance
time

r (rate of speed) is how *fast* **d (distance)** is how *far* **t (time)** is how *long*

How many hours will it take to travel a distance of 1,500 miles at a speed of 100 miles per hour?

$\frac{\text{distance}}{\text{time}} = \frac{100 \text{ mi}}{1 \text{ hr}} \overset{\times 15}{=} \frac{1{,}500 \text{ mi}}{n \text{ hr}}$

$n = 15$ hr

It will take 15 hours to travel 1,500 miles.

Or cross-multiply

$100n = 1{,}500$

$\frac{\cancel{100}n}{\cancel{100}} = \frac{1{,}500}{\cancel{100}}$ ($n = 15$)

Exercises *Steps to solve may vary.*

Find the distance traveled.

1. How many miles can you walk in 2 hours if your average speed is 4 mi/hr? ***8 mi***
2. How many kilometers can you drive in 5 hours if your car's average speed is 52 km/hr? ***260 km***
3. How many feet can you run in 12 seconds at a speed of 7 ft/sec? ***84 ft***

Find the time traveled.

4. How many hours would it take to walk 5 miles at 3 mi/hr? ***1.67 hr***
5. How many hours would it take to drive 200 miles at 50 mi/hr? ***4 hr***
6. How many hours would it take to fly 165 miles at 330 mi/hr? ***0.5 hr***

Find the average speed (unit rate).

7. What is the average speed if a car traveled 224 kilometers in 3.5 hours? ***64 km/hr***
8. What is the average speed if a car traveled 140 miles in 4 hours? ***35 mi/hr***
9. What is the average speed if a train traveled 270 miles in 3 hours? ***90 mi/hr***

Solve.

10. Anderson's airplane trip was $2\frac{1}{4}$ hours long. If the plane averaged 350 mi/hr, how far did Anderson travel? ***787.5 mi***
11. Justin is flying 1,190 miles in a cross-country flight. If the plane averages 340 mi/hr, how long will the flight take? ***3.5 hr***
12. Sean and his friend Tristan biked 24 miles at an average speed of 12 miles per hour. How many hours did they bike? ***2 hr***
13. The average speed of a truck was 50 mi/hr, and the average speed of a car was 60 mi/hr. How many fewer hours did it take the car than the truck to travel 600 miles? ***2 hr***
14. During a race, Lorie's horse averaged 30 mi/hr and Sadie's horse averaged 25 mi/hr. About how many fewer minutes did it take Lorie's horse than Sadie's horse to go 5 miles? ***2 min***

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Rename to Make a Proportion

A proportion compares *like* units. Use equivalent times and distances to rename when units are *not* alike.

Caitlyn's grandmother lives 10 miles from her house. If Caitlyn travels at an average speed of 60 mi/hr, how many minutes will it take her to travel to Grandmother's house?

$r = 60$ mi/hr
$d = 10$ mi
$t = \_\_$

Minutes are needed to measure the time it takes to travel to Grandmother's house, but the known rate of speed is per hour.

1 hour = 60 minutes

$\frac{60 \text{ mi}}{1 \text{ hr}} = \frac{60 \text{ mi}}{60 \text{ min}}$

Write a proportion using the renamed units of speed.

$\frac{\text{distance}}{\text{time}} = \frac{\text{mi}}{\text{min}} = \frac{60}{60} = \frac{10}{n}$; $n = 10$ minutes

If Caitlyn travels at a speed (rate) of 60 mi/hr (or 60 mi/60 min), she will travel 10 miles in 10 minutes.

The packaging from Jeffrey's rocket boasts an average rocket speed of 195 ft/sec. If his rocket flies upward for 8.2 seconds, how many yards high will it fly?

$r = 195$ ft/sec
$d = \_\_$
$t = 8.2$ sec

A distance in yards is needed to measure the flight, but the known rate of speed is feet per second.

1 yard = 3 ft

$\frac{195 \text{ ft}}{1 \text{ sec}} = \frac{65 \text{ yd}}{1 \text{ sec}}$

Write a proportion using the renamed units of speed.

$\frac{\text{distance}}{\text{time}} = \frac{\text{yd}}{\text{sec}} = \frac{65}{1} = \frac{n}{8.2 \text{ sec}}$; $n = 533$ yd

If the rocket travels at a speed (rate) of 195 ft/sec (or 65 yd/sec), it will travel 533 yards in 8.2 seconds.

Exercises

Find the rate (r), distance (d), or time (t).

15. $r = 50$ mi/hr; $d = \_\_$ ***50 mi***; $t = 60$ min
16. $r = 5$ ft/min; $d = 20$ yd; $t = \_\_$ ***12 min***
17. $r = \frac{1}{2}$ mi/hr; $d = \_\_$ ***$2\frac{1}{2}$ mi***; $t = 5$ hr
18. $r = \_\_$ mi/min ***0.1 mi/min***; $d = 0.5$ mi; $t = 5$ min
19. $r = 250$ m/min; $d = 3$ km; $t = \_\_$ ***12 min***
20. $r = \_\_$ mi/hr ***45 mi/hr***; $d = 540$ mi; $t = \frac{1}{2}$ day
21. $r = 21$ ft/sec; $d = \_\_$ ***315 ft***; $t = 15$ sec
22. $r = \_\_$ mi/hr ***25 mi/hr***; $d = 600$ mi; $t = 1$ day
23. $r = 40$ ft/sec; $d = 1$ mi; $t = \_\_$ ***132 sec***
24. $r = 70$ km/hr; $d = \_\_$ ***35 km***; $t = 30$ min
25. Robert's remote-control car can travel at a rate of 66 feet per minute. How far could the car travel in 15 minutes? At this rate, how long would it take the car to travel 1 mile? $\frac{66 \text{ ft}}{1 \text{ min}} = \frac{d}{15 \text{ min}}$; ***d = 990 ft;*** $\frac{66 \text{ ft}}{1 \text{ min}} = \frac{5{,}280 \text{ ft}}{t}$; ***t = 80 min***

Complete DAILY REVIEW on page 450.

Lesson 122 295

4. Direct the students to find the unknown measurement for each of the following groups of information. Guide the students in renaming units of measure as needed. (***Note:*** For the following problems and the problems on Student Text page 295, allow the students to refer to the equivalency charts on pages 499–500 of the Student Text Handbook.)

$r = \frac{12 \text{ km}}{\text{min}}$ $r = \frac{20 \text{ ft}}{\text{min}}$
$t = 5$ min $t = 90$ sec
$d =$ ***60 km*** $d =$ ***30 ft***

$d = 208$ km $d = 60$ ft
$t = 3.2$ hr $t = 90$ sec
$r = \frac{65 \text{ km}}{\text{hr}}$ $r = \frac{40 \text{ ft}}{\text{min}}$

$r = \frac{6 \text{ km}}{\text{hr}}$ $r = \frac{15 \text{ ft}}{\text{min}}$
$d = 3$ km $d = 20$ yd
$t =$ ***0.5 hr*** $t =$ ***4 min***

Student Text pp. 294–95

Lesson 123 **Student Text** pp. 296–97

Chapter Review

Objectives

- Find equivalent ratios
- Determine whether two ratios are proportional
- Determine the unit rate
- Find the unknown measure in similar figures using proportions
- Use an indirect measurement to find the unknown measure in similar objects
- Find actual measurements using a scale
- Determine the unknown measure on a scale drawing given the scale and the actual measurement
- Express percents as ratios, decimals, and fractions
- Express fractions and ratios as percents
- Find a percent of a number or the unknown whole

Teacher Materials

- Circle Graph: Elements in the Earth's Crust, page IA70 (CD)

Student Materials

- A calculator

Note

This lesson reviews the concepts presented in Chapter 13 to prepare the students for the Chapter 13 Test. Student Text pages 296–97 provide the students with an excellent study guide.

Check for Understanding

Find equivalent ratios; determine the unit rate

➤ **What is a ratio?** ***a comparison of two quantities***

1. Tell the students that there are 240 mL in 1 cup. Choose a student to write the ratio that compares the number of milliliters to 1 cup. ***240 mL:1 c***
2. Draw a ratio table similar to the ones used in Lesson 115. Write *milliliters* and the first two entries, 240 and 480, in the top row. Write *cups* and the entries 1, 2, 3, 4, and 6 in the bottom row. Elicit that all the ratios in a ratio table are equivalent or proportional.

   ➤ **How can you complete this table?** ***Multiply the number of cups by 240 or write and solve a proportion.*** Choose students to write the number of milliliters in 3 cups ***720*** and 4 cups ***960***.

   ➤ **What other ways can you find the number of milliliters in 6 cups?** ***Possible answers: double the number of milliliters in 3 cups or add the number of milliliters in 2 cups with the number of milliliters in 4 cups.*** Complete the table. ***1440***
3. Write the ratios $\frac{25}{36}$ and $\frac{60}{72}$ for display.

   ➤ **How can you find out if these ratios could be in a ratio table with $\frac{5}{6}$?** ***Possible answer: determine if each ratio is equivalent to $\frac{5}{6}$ by cross-multiplying and comparing the products.*** Choose students to write possible proportions and cross-multiply to determine if the ratios are equivalent to $\frac{5}{6}$. $\frac{5}{6} \neq \frac{25}{36}$ ***and*** $\frac{5}{6} = \frac{60}{72}$
4. Instruct the students to solve the following word problem. Elicit that the second term in a unit rate is 1. The unit rate can be found by dividing both terms of a ratio by a name for 1. Discuss the solution as needed. (See Lesson 114.)

> Mom used 20 gallons of gas to travel 500 miles. How many miles per gallon did her vehicle get? $\frac{25\text{ mi}}{\text{gal}}$

5. Follow a similar procedure for the following information.
   14 gal of gas cost \$47.60 $\frac{3.40}{\text{gal}}$ 3 lb of grapes cost \$7.05 $\frac{\$2.35}{\text{lb}}$
6. Follow a similar procedure for the following problems. Elicit that both terms of the unit rate can be multiplied by a name for 1 to find an equivalent ratio. Remind the students that speed is a rate, a special ratio that compares distance to time.
   3 min at $\frac{12\text{ yd}}{\text{min}}$ ***36 yd*** 5 days at $\frac{17\text{ mi}}{\text{day}}$ ***85 mi*** 2.7 lb at $\frac{\$1.25}{\text{lb}}$ ***\$3.38***

Determine whether two ratios are proportional

➤ **When are two ratios proportional?** ***When they are equivalent.***

➤ **How can you determine whether two ratios are proportional?** ***Elicit the various strategies that can be used to compare the terms of the ratios vertically, horizontally, or diagonally.***

Review the strategies as needed. (See Lesson 116.) Direct the students to write ratios for the information in each of the following word problems and to determine whether the ratios are proportional. Select students to answer the questions and to write a mathematical statement that supports their answer. Instruct each student to explain the strategy he used to determine whether the statement is a proportion.

> Alyssa earned \$15 for working 3 hours, and Mackenzie earned \$24 for working 5 hours. Did they receive the same hourly rate? ***no;*** $\frac{15}{3} \neq \frac{24}{5}$
>
> Aiden bought a 16-ounce drink for \$1.92. Dylan bought a 12-ounce drink for \$1.44. Were the prices per ounce equivalent? ***yes;*** $\frac{16}{\$1.92} = \frac{12}{\$1.44}$

Find the unknown measure in similar figures; use indirect measurement

1. Draw for display a small rectangle and write *40 cm* and *60 cm* along its sides. Also draw a similar rectangle and write *100 cm* along its shorter side and *n* along its longer side. Elicit that the rectangles are similar figures; they are the same shape, but are not the same size.

   ➤ **What proportion can you write to find the unknown measurement using ratios *between* these similar figures?** $\frac{40\text{ cm}}{100\text{ cm}} = \frac{60\text{ cm}}{n}$ ***within*** **the figures?** $\frac{40\text{ cm}}{60\text{ cm}} = \frac{100\text{ cm}}{n}$
2. Write both proportions for display and instruct the students to solve them. Compare the answers and allow students to explain the method they used to solve the proportions.
   n = 150 cm
3. Guide the students in solving the following word problem. Elicit that the method of indirect measurement uses similar objects and a proportion to find the unknown measure of an object that is difficult to measure.

> A building casts a shadow that is 3.6 meters long. A sign that is 2 meters tall casts a shadow that is 1.2 meters long. What is the height of the building? ***6 meters;*** $\frac{h}{2\text{ m}} = \frac{3.6\text{ m}}{1.2\text{ m}}$, ***h = 6 m; or*** $\frac{2\text{ m}}{1.2\text{ m}} = \frac{h}{3.6\text{ m}}$, ***h = 6 m***

Find actual measurements using a scale
Determine the unknown measure on a scale drawing

➤ **What is a scale?** ***a ratio of measurements that compares the size of a drawing or model to the size of the actual object***

➤ **What are examples of scale drawings and models used in everyday life?** ***possible answers: maps, floor plans, horses, doll houses, cars, trains, airplanes***

CHAPTER 13 REVIEW

Complete the ratio table.

1.

centimeters	2.54	5.08	*7.62*	10.16	*12.7*
inches	1	2	3	4	5

Write yes if the ratio could be in a ratio table with $\frac{4}{7}$.
Write no if the ratio could *not* be in a ratio table with $\frac{4}{7}$.

2. $\frac{16}{28}$ ***yes***
3. $\frac{48}{80}$ ***no***
4. $\frac{56}{98}$ ***yes***

Find the unit rate.

5. Reese used 17 gallons of gas to drive 510 miles. ***30 mi/gal***
6. Jonah earned $56.00 in 8 hours. ***$7.00/hr***
7. Allie read 30 pages in 20 minutes. ***1.5 pg/min***
8. Sarah bought 5 pounds of apples for $4.25. ***$0.85/lb***

Find the distance traveled.

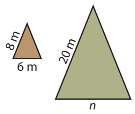
9. 4 minutes at 15 yd/min ***60 yd***
10. 3.5 hours at 50 mi/hr ***175 mi***
11. 18 seconds at 20 ft/sec ***360 ft***
12. 6 days at 21 mi/day ***126 mi***

Write a possible proportion for the situation.
Write yes if the prices are equivalent. Write no if one price is a better buy. ***Steps to solve may vary.***

13. 8 ears of corn for $1 or 15 ears of corn for $2 $\frac{\$1}{8} \neq \frac{\$2}{15}$; ***no***
14. 4 pencils for $2 or 12 pencils for $6 $\frac{\$2}{4} = \frac{\$6}{12}$; ***yes***
15. 22 oz drink for $1.50 or 28 oz drink for $1.75 $\frac{\$1.50}{22} \neq \frac{\$1.75}{28}$; ***no***
16. 12 eggs for $2 or 18 eggs for $3 $\frac{\$2}{12} = \frac{\$3}{18}$; ***yes***

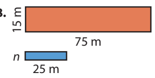
Write a proportion to find the unknown measure for the pair of similar figures. ***Proportions may vary.***

17.



$\frac{8}{6} = \frac{20}{n}$
$\frac{8n}{8} = \frac{120}{8}$
$n = 15$ m

18.



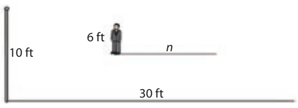
$\frac{15}{n} = \frac{75}{25}$
$\frac{375}{75} = \frac{75n}{75}$
$n = 5$ m

Use a map scale of 1 in.:75 mi to find the actual distance represented by the measurement.

19. 4 in. $\frac{1 \text{ in.}}{75 \text{ mi}} = \frac{4 \text{ in.}}{d}$; $d = 4 \cdot 75$ mi; $d = 300$ mi
20. 2.6 in. $\frac{1 \text{ in.}}{75 \text{ mi}} = \frac{2.6 \text{ in.}}{d}$; $d = 2.6 \cdot 75$ mi; $d = 195$ mi

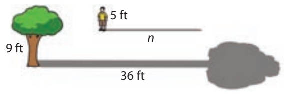
Write a proportion to find the unknown height.

21.



$\frac{6}{10} = \frac{n}{30}$; $30 \cdot 6 = 10n$; $\frac{180}{10} = \frac{10n}{10}$; $n = 18$ ft

22.



$\frac{5}{9} = \frac{n}{36}$; $36 \cdot 5 = 9n$; $\frac{180}{9} = \frac{9n}{9}$; $n = 20$ ft

Method used to solve may vary.

Write the percent in decimal form. Write the decimal in percent form.

23. 64% ***0.64***
24. 4% ***0.04***
25. 0.09 ***9%***
26. 0.83 ***83%***

Write the percent as a fraction in lowest terms. Write the fraction as a percent.

27. 40% $\frac{2}{5}$
28. 10% $\frac{1}{10}$
29. $\frac{3}{5}$ ***60%***
30. $\frac{5}{10}$ ***50%***

Find the percent of the number.

31. 30% of 70 $0.30 \cdot 70 = 21$
32. 42% of 75 $0.42 \cdot 75 = 31.5$
33. 25% of 64 $0.25 \cdot 64 = 16$
34. 60% of 30 $0.60 \cdot 30 = 18$

Find the unknown whole.

35. 5% of what number is 3? ***60***
36. 40% of what number is 32? ***80***
37. 25% of what number is 5? ***20***

Solve. ***Steps to solve may vary.***

38. During a survey, it was discovered that 84 out of 124 students have brown eyes. About how many students with brown eyes would you expect to find out of 10 of these students? ***7 students***
39. Factory workers can produce 18 items in 15 hours. How many hours will it take them to produce 12 items at this rate? ***10 hr***
40. A playground is 144 feet wide. How wide would a scale drawing of the playground be in which 2 inches represents 12 feet? ***24 in.***
41. Jenna made $200 during the week. She plans to give 10% to the church and to put 40% in her savings account. How much will she give to the church and how much will she save? ***$20; $80***
42. Alicia deposited $500 in a simple savings account that earns 3% each year. If she does not deposit or withdraw any money, how much interest will she earn in one year? ***$15***
43. On a survey, 85% of the respondents said there are 2 vehicles in their households. If 200 people completed the survey, how many people own 2 vehicles? ***170 people***
44. Sienna enlarged a picture to be 5 times larger than the original. The original picture was 2 inches long by 4 inches wide. The enlarged picture has a length of 10 inches. What is the width? ***20 in.***
45. Zane answered 42 out of 50 questions correctly on a science test. What percent of the questions did he answer correctly? ***84%***
46. Erin bought a purse during a 25%-off sale. The original price was $45. What was the discount? What was the sale price? ***$11.25; $33.75***
47. If sales tax is 7%, how much tax would be charged on a purchase of $11? ***$0.77***
48. All items in the store were marked 30% off. Tucker received a discount of $21 on a soccer ball he bought. What was the original price of the ball? ***$70***
49. Elliot scored 60% of his free-throw attempts. If he made 3 free throws during the game, how many attempts did he make? ***5 attempts***

Guide the students in solving the following word problems. Review the solutions as needed. (See Lesson 118.)

> A map has a scale of 1 in.:25 mi. If the distance between 2 cities is 4.5 inches, what is the actual distance?
> $\frac{1 \text{ in.}}{25 \text{ mi}} = \frac{4.5 \text{ in.}}{n}$; $n = 112.5$ ***mi***
>
> The distance between 2 cities is 90 miles. If the map scale is 1 in.:15 mi, what is the map measurement?
> $\frac{1 \text{ in.}}{15 \text{ mi}} = \frac{n}{90 \text{ mi}}$; $n = 6$ ***in.***

Express percents as ratios, fractions, and decimals
Express fractions and ratios as percents

➤ What is the definition of a percent? ***a ratio in which a quantity (part) is compared to 100 (whole)***

1. Display the Circle Graph page. Explain that the table and graph show the major elements that are found in the Earth's crust. Complete the table as students give the ratio, fraction, and decimal for each percent (e.g., 28%: $\frac{28}{100}$; $\frac{7}{25}$; *0.28*).
2. Elicit the process for writing a ratio as a percent. (See Lesson 119.) Instruct the students to solve these word problems.

> On a math test, Anthony answered 46 out of 50 questions correctly. What percent of the questions did he answer correctly? $\frac{46}{50} = 0.92$ (or $\frac{92}{100}$) $= 92\%$
>
> During the summer, 4 out of 12 students attend summer school. What percent of the students attend summer school? $\frac{4}{12} = 0.3 \approx 33\%$

Find a percent of a number or the unknown whole

> During basketball season Jacob made 25% of his 32 three-point shots. How many three-point shots did he make? ***8 shots***

➤ How can you solve this word problem? Why? ***Find 25% of 32 (n% of a number = $\frac{n}{100}$ × the number); the answer represents a part or a percent of the total (32) three-point shots that Jacob attempted.*** Elicit that the percent can be renamed as a fraction or a decimal.

1. Direct the students to solve the problem. Discuss the solutions. (See Lesson 120.) ***Equations will vary.***
2. Follow a similar procedure for the following word problem. Elicit that since the answer represents the whole (100% of the shots that John attempted), the formula *percent* × *part* = *whole* can be used to write an equation or a proportion. (See Lesson 121.) ***Equations will vary;*** $\frac{30}{100} = \frac{6}{s}$, $s = 20$.

> John made 30% of his three-point shots during the basketball season. If John made 6 three-point shots that season, what was the total number of three-point shots attempted by John? ***20 shots***

3. Direct the students to solve these percent problems using the method of their choice.

2% of 500 = ***10***	18% of 25 = ***4.5***
70% of n is 35 $n = 50$	36 is 45% of n $n = 80$

Student Text pp. 296–97

Lesson 124

Student Text pp. 298–301

Chapter 13 Test

Cumulative Review

For a list of the skills reviewed in the Cumulative Review, see the Lesson Objectives for Lesson 124 in the Chapter 13 Overview on page 276 of this Teacher's Edition.

Student Materials

- Cumulative Review Answer Sheet, page IA9 (CD)

Use the Cumulative Review on Student Text pages 298–300 to review previously taught concepts and to determine which students would benefit from your reteaching of the concepts. To prepare the students for the format of achievement tests, instruct them to work on a separate sheet of paper, if necessary, and to mark the answers on the Cumulative Review Answer Sheet.

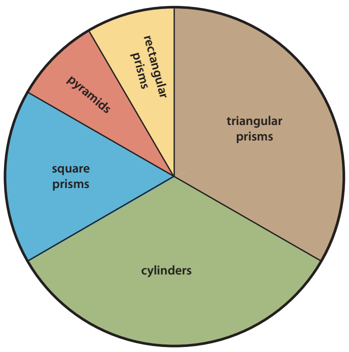
Read aloud the Career Link on Student Text page 301 (page 299 of this Teacher's Edition) and discuss the value of math as it relates to an electrician.

CUMULATIVE REVIEW

Test Prep

Use the data from the circle graph to find the answer.

The circle graph represents examples of three-dimensional figures found at home.



1. What categories are least represented?
   A. pyramids and triangular prisms
   B. cylinders and rectangular prisms
   C. square prisms and pyramids
   D. pyramids and rectangular prisms

2. Cylinders represent what part of the graph?
   A. $\frac{1}{3}$ C. $\frac{1}{6}$
   B. $\frac{1}{2}$ D. $\frac{1}{9}$

3. Square prisms represent what part of the graph?
   A. $\frac{1}{3}$ C. $\frac{1}{6}$
   B. $\frac{1}{2}$ D. $\frac{1}{9}$

4. What part of the graph is made up of cylinders and square prisms?
   A. $\frac{1}{2}$ C. $\frac{3}{4}$
   B. $\frac{2}{3}$ D. $\frac{5}{6}$

5. What type of figure is as equally represented as cylinders?
   A. pyramids
   B. triangular prisms
   C. square prisms
   D. rectangular prisms

Mark the answer.

6. A roll of quarters has a value of $10.00. Ana has $16\frac{3}{4}$ rolls of quarters. How much money does she have?
   A. $165.75 C. $170.25
   B. $167.50 D. $175

7. Dad bought grass seed for the lawn. Each bag covers 1,000 square feet. How many bags did he buy if the yard is 120 feet × 60 feet and the house takes up about $\frac{1}{2}$ of the area?
   A. 3 bags C. 6 bags
   B. 4 bags D. 8 bags

8. Mariah collected 1 dozen eggs on Monday and twice as many on Tuesday. How many eggs did she collect in all?
   A. 2 dozen C. 36 eggs
   B. 30 eggs D. $1\frac{1}{2}$ dozen

9. Jude learned 15 Bible verses for the Bible quiz team. Dylan learned 3 times as many verses as Jude. How many verses did the two boys learn altogether?
   A. 15 + 15 + 15 = 45 verses
   B. 15 + (15 + 3) = 33 verses
   C. 15 + (3 • 15) = 60 verses
   D. none of the above

10. Working together, it takes Jace and Spencer $2\frac{1}{2}$ hours to mow and trim the Allens' lawn. It takes them $3\frac{3}{4}$ hours to mow and trim the Reas' lawn. How many hours will it take them on Saturday to mow and trim both lawns?
    A. 5 hr C. $5\frac{1}{2}$ hr
    B. $5\frac{1}{4}$ hr D. $6\frac{1}{4}$ hr

11. $\sqrt{121}$
    A. 10 C. 12
    B. 11 D. 13

12. 5^4
    A. 125 C. 500
    B. 200 D. 625

13. $3 + 5 \times 8 - 2$
    A. 23 C. 41
    B. 32 D. 43

14. $^-2 + 6$
    A. $^-4$ C. 4
    B. 0 D. 8

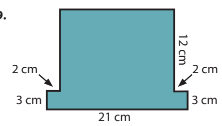
15. 31×15
    A. 375 C. 455
    B. 405 D. 465

16. $590 \div 14$
    A. $42.\overline{1}$ C. 42.14
    B. 42.04 D. 42.4

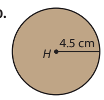
17. $600 \div 0.25$
    A. 2.4 C. 240
    B. 24 D. 2,400

18. $\frac{3}{11} = \frac{n}{44}$
    A. $n = 12$ C. $n = 18$
    B. $n = 15$ D. $n = 33$

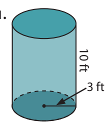
Mark the answer.

19.


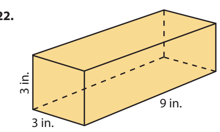
Area is found using the formula $l \bullet w$. What is the area of the figure?
A. 267 cm²
B. 303 cm²
C. 258 cm²
D. 315 cm²

20.


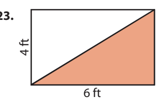
The circumference of a circle is found using the formula πd. What is the circumference of circle H?
A. 63.59 cm
B. 14.13 cm
C. 28.26 cm
D. 7.07 cm

21.


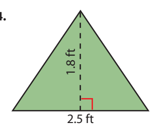
The formula for the volume of a cylinder is $(\pi r^2) \times h$. What is the volume of this cylinder?
A. 28.26 ft³
B. 94.2 ft³
C. 124.2 ft³
D. 282.6 ft³

22.


Which equation shows the volume of the rectangular prism?
A. 3 × 9 = 27 in.²
B. (3 • 9) + (3 • 3) = 36 in.²
C. 2(3 • 3) + 2(3 • 9) = 72 in.²
D. 3 × 3 × 9 = 81 in.³

23.


Which equation can be used to find the area of the shaded part?
A. 4 ft × 6 ft = 24 ft²
B. $\frac{1}{2}$(6 ft × 4 ft) = 12 ft²
C. (2 • 4 ft) + (2 • 6 ft) = 20 ft²
D. $\frac{1}{3}$(4 ft + 6 ft) = 3.3 ft²

24.


Which equation can be used to find the area of the triangle?
A. $\frac{1}{2}$(2.5 × 1.8) = 2.25 ft²
B. 2.5 × 1.8 = 4.5 ft²
C. 2(2.5 × 1.8) = 9 ft²
D. (2 • 2.5) + (2 • 1.8) = 8.6 ft²

CAREER LINK

Electrician

Think of some things in your house that run on electricity: an oven, a refrigerator, a washer, a dryer, a microwave, and a computer. When a house is built, an electrician must be able to read blueprints and plan for the correct voltage for the house and the number and kinds of outlets for each room.

An electrician can install, maintain, and repair many types of electrical systems. He uses math every day to calculate electrical loads, voltage, power, and amps. On a daily basis, he must be able to solve problems to find an unknown quantity, making algebra and problem solving important to his work. He must be able to work with whole numbers, decimals, fractions, percentages, and geometry. He needs to be able to read and chart data. He must know how to use standard and metric measurements and how to convert them.

Every appliance uses a certain amount of electricity. An electrician must know how to calculate and convert information. He uses that information to find the correct box and the length, size, and type of wires to handle the electrical load of the building. An electrician must make sure that his customers have a safe home or business.

An electrician must know and understand Ohm's Law, which shows the relationship between power, voltage, current, and resistance. For example, a stove uses electricity to produce heat. If an electric range uses more power, such as for turning the elements on high, more current is needed. Mathematical formulas help the electrician know how to install the correct size of wire to carry an electrical load.

A Christian electrician has many opportunities to help other people. He can inspect homes to be sure that the wiring is safe. He can also show people low-cost ways to use their appliances and heating and cooling systems. An electrician can be a great asset to the ministry of a church. He can repair electrical problems and help build a new building. He can also help missionaries by meeting their needs.