16

Probability

WHEN THE SERPENT STUNG

Charleston, South Carolina **April 25, 1956**

On April 25, 1956, fourteen-year-old Harold Murray of Charleston, South Carolina, was visiting his friend Raymond Beal. The sunny weather drew the boys outdoors to play in Raymond's backyard. A woman's scream suddenly disrupted the calm afternoon, and Harold felt a chill run down his back. He and Raymond stared at each other for a moment and then turned and ran in the direction from which the scream had come. They stopped in horror at the sight of a large snake coiling itself slowly around the foot of Mrs. Rakoske, Raymond's neighbor. As the creature uncoiled itself and slithered into the grass, Harold's Boy Scout training flashed into his mind. That snake was a poisonous copperhead!

This "Scouts in Action" story appeared in *Boys' Life* magazine, November 1956.

In one quick movement, Harold grabbed a shovel lying nearby and brought it down sharply on the snake, killing it. Mrs. Rakoske began to sob. Harold dropped to his knees to look at her foot. Two tiny red puncture marks stood out against the white of her skin. Harold straightened and grabbed Raymond's arm. "Go find Mr. Rakoske," he ordered. "We've got to get her to the hospital right away!" Speaking calmly to Mrs. Rakoske, Harold tied his handkerchief tightly around her leg halfway between her ankle and her knee. He knew this would stop the flow of blood from the poisoned foot to the rest of her body. Then he hesitated. He had learned in Boy Scouts that the next thing to do was to cut around the bite and suck the poison out. But he had a cavity in one of his teeth, and he knew that the poison could possibly go into his own bloodstream through the tooth. He paused only a moment longer. Then he made up his mind. Swiftly he cut an X over the puncture marks with his Scout knife. As the cut bled, he sucked out the blood and poison and spat it out. Again and again Harold repeated this step until Raymond arrived with Mr. Rakoske, who rushed his wife to the hospital.

The doctor at the hospital examined Mrs. Rakoske with pleasant surprise. When he found out the rescue had been performed by a fourteen-year-old Boy Scout, he was stunned for a moment. Then he congratulated Harold for doing such a good job. Harold's quick thinking and action had saved Mrs. Rakoske's life.

Copperheads usually grow to be about 2.5 feet long, but occasionally some are as long as 4 feet.

Young copperheads have bright yellow tails, which are often used to attract prey.

Copperheads are known as pit vipers because they have two heat-sensing pits between their eyes and nostrils.

The copperhead bite is more venomous than that of any other type of snake. The venom is toxic and can cause painful damage to tissue, but it is often not fatal.

The state of North Carolina has more reported copperhead snakebites than any other state in the United States.

Most snakebites can be avoided by not trying to handle the snake. If you see a snake, back away from it.

In the event of a snakebite, the Center for Disease Control recommends that you call 911 to get immediate medical attention, position the person so that the bite is below the level of the heart, calm the patient, and cover the area with a clean, dry covering.

Probability

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• Write probability as a fraction, a decimal, and a percent
• Find the theoretical probability of an event and its complement</td><td rowspan="7">Instructional Aids (Teacher's Toolkit CD):
• Cumulative Review Answer Sheet (page IA9) for each student
• Probability (page IA93)
• Probability (page IA93) for each student
• Theoretical Probability (page IA94)
• Sample Spaces (page IA95)
• Tree Diagram (page IA96)
• Tree Diagram (page IA96) for each student
• Multiplication Counting Principle (page IA97)
• Experimental Probability (page IA98)
• Experimental Probability (page IA98) for each student
• Spinning Penny Experiment (page IA99) for each pair of students (optional)
• Fair or Unfair Games (page IA100)
• Fair or Unfair Games (page IA100) for each group of students (optional)
• Probability Spinner (page IA101)
• Probability Spinner (page IA101) for each student
Christian Worldview Shaping (Teacher's Toolkit CD):
• Page 36
Other Teaching Aids:
• Colored markers: red and blue
• Colored pencils or crayons: red and blue for each student
• A calculator (optional) for each student
• A paper clip for each student and the teacher
• A penny for every two students (optional)
• 6 Unifix Cubes: 4 orange, 1 brown, 1 yellow
• An opaque bag
• Two 1–6 number cubes
• Two 1–6 number cubes for each group of students (optional)
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• Find an equivalent fraction or ratio
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A Little Extra Help

Use the following to provide "a little extra help" for the student that is experiencing difficulty with the concepts taught in Chapter 16.

Predict the results for a large sample of data using probability—A student who is having difficulty using probability to predict the results for a large sample of data may find it easier to write and solve a missing term proportion to predict the results. Draw for display the "Favorite Number From 0 to 5" chart and elicit that the tallies indicate that a total of 20 people were surveyed. Guide the students in determining the probability of each number being chosen as a favorite.

Favorite Number From 0 to 5		
0	||	$P(0) = \frac{2}{20} = \frac{1}{10}$
1	𝍸 |||	$P(1) = \frac{8}{20} = \frac{2}{5}$
2	|||	$P(2) = \frac{3}{20}$
3		$P(3) = \frac{0}{20} = 0$
4	||	$P(4) = \frac{2}{20} = \frac{1}{10}$
5	𝍸	$P(5) = \frac{5}{20} = \frac{1}{4}$

Remind the student that the probability of the sample of 20 people surveyed can be used to predict the results if a larger sample of 100 people were surveyed. (e.g., $P(0) = \frac{1}{10}$; therefore, $\frac{1}{10} \times 100 =$ the number of people out of 100 that are expected to choose 0 as their favorite number.). Direct the student to write and solve a proportion that has a missing term to find the expected number of people out of 100 that would choose 0 as their favorite number. $\frac{1}{10} = \frac{n}{100}$, $n = 10$ Elicit that since 1 out of 10 people chose 0 as their favorite number when 20 people were surveyed, you could predict that 10 people would choose 0 as their favorite number if 100 people were surveyed. Follow a similar procedure for each of the other numbers.

Mental Math

Throughout this chapter, select problems from the list of mental math problems provided on pages IA108–IA110 of the Teacher's Toolkit CD.

Lesson 148

Student Text pp. 356–59
Daily Review p. 460a

Objectives
- Develop an understanding of probability
- Write probability as a fraction, a decimal, and a percent
- Find the theoretical probability of an event and its complement

Teacher Materials
- Probability, page IA93 (CD)
- Theoretical Probability, page IA94 (CD)
- Christian Worldview Shaping, page 36 (CD)
- Colored markers: red and blue

Student Materials
- Probability, page IA93 (CD)
- Colored pencils or crayons: red and blue
- A calculator (optional)

Notes
Permitting the students to use their calculators, as needed, throughout this chapter will allow them to focus on the concepts that are being taught.

Preview the Fact Review pages, the Application pages, and the Calculator Activities located on the Teacher's Toolkit CD.

Introduce the Lesson

Guide the students in reading aloud the story and facts on pages 356–57 of the Student Text (pages 354–55 of this Teacher's Edition).

Teach for Understanding

Develop an understanding of probability

1. Write for display: *It will rain today.*
   ➤ **Is it possible or impossible that it will rain today? Why?** ***Possible; it has rained other days.***
   ➤ **Since it is possible, is it unlikely or likely it will rain today? Why?** ***possible answers: likely since it rains often or unlikely since it seldom rains***
2. Repeat the procedure using these statements.
   It will snow today. ***Possible; unlikely or likely is dependent on the time of year and climate.***
   The chair is alive. ***Impossible; elicit that it is certain that chairs are not alive; they do not eat or breathe.***
   Most people in this room are left-handed. ***possible; unlikely because most people are right-handed***
   ➤ **Where might it be likely that most people are left-handed?** ***Elicit in a setting where there are many left-handed people.***
   Most people in this room have brown eyes. ***Possible; unlikely or likely is dependent on the physical characteristics of your students.***
   God rules supreme. ***Certain; elicit that it is impossible for God not to rule supreme because of who He is.***
3. Display and distribute the Probability page. Explain that probability is the likelihood (or uncertainty) that an event will occur.
4. Direct attention to the probability scale as represented on the number line on the page. Point out that probability can be expressed as a fraction or as a decimal between 0 and 1. It can also be expressed as a percent between 0% and 100%. The closer a probability is to 1, the more likely the event will happen. A probability of 1, or 100%, means it is certain that the event will occur. A probability of 0, or 0%, means it is impossible for the event to occur. A probability of $\frac{1}{2}$ means an event will occur 1 out of 2 times or approximately 50% of the time.
5. Explain that the circles below the probability scale represent spinners. Ask the following question as you guide the students in using their red and blue colored pencils (or crayons) to color the first two spinners so that the probability of landing on blue matches the probability above it on the scale. Demonstrate on the displayed page.
   ➤ **If the probability of landing on blue is 0, what part of the spinner will be blue?** ***none or 0%***
   ➤ **Since there are only two options, red or blue, what part of the spinner will you color red?** ***all 4 sections ($\frac{4}{4}$) or 100%***
   ➤ **If the probability of landing on blue is 25%, what part of the spinner will be blue? How do you know?** ***$\frac{1}{4}$; 25% = $\frac{1}{100}$ or $\frac{1}{4}$*** **What part of the spinner will be red? How do you know?** ***$\frac{3}{4}$; elicit that whatever is not blue must be red (the only other color).***
6. Follow a similar procedure for the 3 remaining spinners. ***$\frac{1}{2}$ blue and $\frac{1}{2}$ red; $\frac{3}{4}$ blue and $\frac{1}{4}$ red; $\frac{4}{4}$ blue***

Find the theoretical probability of an event and its complement

1. Display the Theoretical Probability page. Explain that *theoretical probability* is calculated when all of the possible outcomes are known and are equally likely to occur.
   ➤ **What is the ratio of 1-tiles to all the tiles in the bag?** ***3 to 6***
   ➤ **What fraction describes the probability of drawing a 1-tile from the bag?** ***$\frac{3}{6}$*** **What is the probability in lowest terms?** ***$\frac{1}{2}$***
   Point out that $P(1)$ on the table represents the probability of drawing a 1-tile from the bag. Write $\frac{1}{2}$ in the fraction column. Elicit that the probability can be read *one out of two.*
   ➤ **What decimal is equivalent to $\frac{1}{2}$?** ***0.5*** **What percent is equivalent to $\frac{1}{2}$?** ***50%*** Write 0.5 and 50% to complete the first row of the table.
2. Explain that since theoretical probability assumes that all outcomes are equally likely to occur, P(event) can be expressed as a part to whole ratio; the number of favorable (desired) outcomes divided by the number of possible outcomes.
3. Write the following statements for display: *It will rain today* and *It will not rain today.* Explain that these events are *complementary events*; only one or the other can occur. Point out that $P(1)$ and $P(not\ 1)$ are complementary events; they cannot happen simultaneously.
   ➤ **What is the ratio of non-1-tiles to the tiles in the bag?** ***3 to 6***
   ➤ **What fraction describes the probability of drawing a non-1-tile from the bag?** ***$\frac{3}{6}$*** **What is the probability in lowest terms?** ***$\frac{1}{2}$***
   ➤ **What is the decimal equivalent to $\frac{1}{2}$?** ***0.5*** **the percent equivalent to $\frac{1}{2}$?** ***50%***
   Write all three forms of the probability in the table.
4. Direct attention to the equation: $P(\text{event}) + P(not\ \text{event}) = 1$ or 100%. Explain that the sum of the probability of complementary events will always equal 1 or 100%. When you know the probability of an event, you can subtract the known propability from 1 or 100% to find the probability of its complement.

Theoretical Probability

Probability is the likelihood that an event will occur. **Theoretical probability** is found when the total possible outcomes of an event are known and all outcomes are equally likely to occur. Probability is written as a ratio or a percent.

probability
theoretical probability
complementary events

What is the probability of drawing a blue marble from each bag?

$P(\text{event}) = \dfrac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

impossible	unlikely	equally likely	likely	certain
$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{4}$
0	0.25	0.5	0.75	1
0%	25%	50%	75%	100%

0 out of 4	1 out of 4	2 out of 4	3 out of 4	4 out of 4

Complementary events are two events that could happen, but both events cannot happen at the same time. The sum of the two events must equal 1 or 100%.

The complement of it snowing today is *not* snowing today.

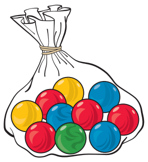
25% + *P*(*not* snow) = 100%
25% + 75% = 100%

Think 25% + ___ = 100%

When there is a 25% chance that it will snow, there is a 75% chance that it will *not* snow.

Exercises

A marble is drawn from the pictured bag.
Write the probability of the event as a fraction and as a percent.



1. P(green) ***$\frac{1}{10}$; 10%***
2. P(red) ***$\frac{4}{10}$; 40%***
3. P(either red or blue) ***$\frac{7}{10}$; 70%***
4. P(purple) ***$\frac{0}{10}$; 0%***
5. P(blue) ***$\frac{3}{10}$; 30%***
6. P(yellow) ***$\frac{2}{10}$; 20%***
7. P(*not* red) ***$\frac{6}{10}$; 60%***
8. P(*not* purple) ***$\frac{10}{10}$; 100%***

Use the spinner to find the probability of the event.
Write it as a fraction in lowest terms.

9. P(blue) ***$\frac{4}{6} = \frac{2}{3}$***
10. P(even) ***$\frac{3}{6} = \frac{1}{2}$***
11. P(green and odd) ***$\frac{1}{6}$***
12. P(blue and even) ***$\frac{2}{6} = \frac{1}{3}$***
13. P(green) ***$\frac{2}{6} = \frac{1}{3}$***
14. P(odd) ***$\frac{3}{6} = \frac{1}{2}$***
15. P(green and even) ***$\frac{1}{6}$***
16. P(blue and odd) ***$\frac{2}{6} = \frac{1}{3}$***

Use the spinner above to find the probability of the event and its complement.
Write both as a percent. Round to the nearest whole percent.

17. P(blue) and P(*not* blue) ***67%; 33%***
18. P(green) and P(*not* green) ***33%; 67%***
19. P(odd) and P(*not* odd) ***50%; 50%***
20. P(5) and P(*not* 5) ***17%; 83%***

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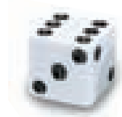
A 1–6 number cube is rolled once. Write the probability of the event as a fraction in lowest terms and as a percent. Round to the nearest whole percent.



21. P(3) ***$\frac{1}{6}$; 17%***
22. P(2 or 5) ***$\frac{1}{3}$; 33%***
23. P(greater than 1) ***$\frac{5}{6}$; 83%***
24. P(4) ***$\frac{1}{6}$; 17%***
25. P(odd) ***$\frac{1}{2}$; 50%***
26. P(multiple of 2) ***$\frac{1}{2}$; 50%***
27. P(7) ***0; 0%***
28. P(less than 5) ***$\frac{2}{3}$; 67%***
29. P(*not* 6) ***$\frac{5}{6}$; 83%***
30. P(composite) ***$\frac{1}{3}$; 33%***

The probability of event A is given. Find the complement, *P*(*not* A).

31. P(A) = 30% **P(not *A*) = 70%**
32. P(A) = 45% **P(not *A*) = 55%**
33. P(A) = $\frac{1}{4}$ **P(not *A*) = $\frac{3}{4}$**
34. P(A) = $\frac{2}{5}$ **P(not *A*) = $\frac{3}{5}$**
35. P(A) = 40% **P(not *A*) = 60%**
36. P(A) = 29% **P(not *A*) = 71%**

Write certain, equally likely, or impossible.
Each choice will be used only once.

37. If a 1–6 number cube is rolled one time, what word or phrase best describes the event?
    a) roll a 3 ***equally likely***
    b) roll a number 1–6 ***certain***
    c) roll a number greater than 6 ***impossible***

Write the probability as a fraction and as a percent.
Round to the nearest whole percent.

38. Eighty people attended the Sunday morning service at Regency Bible Church. Twenty people sang in the choir. What is the probability that a person attending is a choir member?
    $\frac{20}{80} = \frac{1}{4}$; $\frac{1}{4}$ = 25%
39. You are given 5 choices for a multiple-choice test question. If you do not know the answer, what is the probability of guessing the correct answer?
    $\frac{1}{5}$ = 20%

J The Heritage Christian School soccer team has a record of 8 wins and 3 losses. Victory Christian School has a record of 6 wins and 2 losses. Which team is more likely to win its next game?
Heritage: $\frac{8}{11} \approx$ 73%
Victory: $\frac{6}{8}$ = 75%
Victory Christian School is more likely to win.

Soccer Records

	HCS	VCS
Wins	8	6
Losses	3	2
Games Played	11	8

Complete DAILY REVIEW a on page 460.

Lesson 148 359

Elicit the related equations specific to the probability of drawing the 1-tile from the bag. **P(*1*) + P(not *1*) = *1*; *1* − P(not *1*) = P(*1*); P(not *1*) + P(*1*) = *100%*; *100%* − P(*1*) = P(not *1*)**

5. Follow a similar procedure to complete the table. Elicit that since probabilities can be written as a percent, the decimal should be rounded to the nearest hundredth when dividing the number of favorable outcomes by the number of possible outcomes. The percent (probability) is an approximation.
   P(2) $\frac{1}{3}$, ***0.33, 33%*** P(*not* 2) $\frac{2}{3}$, ***0.67, 67%***
   P(3) $\frac{1}{6}$, ***0.17, 17%*** P(*not* 3) $\frac{5}{6}$, ***0.83, 83%***
   P(odd number) $\frac{2}{3}$, ***0.67, 67%*** P(*not* odd number) $\frac{1}{3}$, ***0.33, 33%***
6. Draw for display a bag containing the following number squares: two 1-tiles, three 2-tiles, and four 3-tiles. Repeat the activity.
   P(1) $\frac{2}{9}$, ***0.22, 22%;*** P(*not* 1) $\frac{7}{9}$, ***0.78, 78%;***
   P(2) $\frac{1}{3}$, ***0.33, 33%;*** P(*not* 2) $\frac{2}{3}$, ***0.67, 67%;***
   P(3) $\frac{4}{9}$, ***0.44, 44%;*** P(*not* 3) $\frac{5}{9}$, ***0.56, 56%***
7. Display the Probability page again. Elicit the following complement [P(*not* blue)] for each spinner.
   P(blue) = 0; P(*not* blue) = ***1 or 100%*** Elicit that when it is impossible to spin blue, it is certain that you will spin (*not* blue).
   P(blue) = $\frac{1}{4}$; P(*not* blue) = ***$\frac{3}{4}$ or 75%*** Elicit that when it is unlikely that you will spin blue, it is likely that you will spin red (*not* blue).
   P(blue) = $\frac{1}{2}$; P(*not* blue) = ***$\frac{1}{2}$ or 50%*** Elicit that it is equally likely that you will spin blue or red (*not* blue).
   P(blue) = $\frac{3}{4}$; P(*not* blue) = ***$\frac{1}{4}$ or 25%*** Elicit that when it is likely that you will spin blue, it is unlikely that you will spin red (*not* blue).
   P(blue) = 1; P(*not* blue) = ***0 or 0%*** Elicit that when it is certain that you will spin blue, it is impossible that you will spin red (*not* blue).
8. Guide students in determining the probability for problem 38 on Student Text page 359 using the following questions.
   ➤ **How many people attended the Sunday morning service at Regency Bible Church? *80***
   ➤ **How many people sang in the choir? *20***
   ➤ **What is the probability that a person attending is a choir member? *Elicit 20 out of 80; $\frac{20}{80}$ or $\frac{1}{4}$.***
   ➤ **What is the probability that a person attending is not a choir member? *Elicit 60 out of 80; $\frac{60}{80}$ or $\frac{3}{4}$.***
9. **Christian Worldview Shaping (CD)**

Student Text pp. 358–59

Lesson 149 | **Student Text** pp. 360–61 | **Daily Review** p. 461b

Objectives
- Find the sample space for and the probability of an event
- Make a tree diagram to list the sample space for an event
- Determine the number of possible outcomes using the Multiplication Counting Principal
- Find the theoretical probability of an event

Teacher Materials
- Sample Spaces, page IA95 (CD)
- Tree Diagram, page IA96 (CD)
- Counting Principle, page IA97 (CD)

Student Materials
- Tree Diagram, page IA96 (CD)
- A calculator (optional)

Teach for Understanding

Find the sample space for and the probability of an event

1. Display the Sample Spaces page.
   - ➤ **If you were to make 1 spin using this spinner, what are the possible outcomes?** ***A, B, or C***
2. Explain that the *sample space* for an event is the set of all possible outcomes for the event. The sample space can be listed using brackets (set notation). Point out that the sample space {A, B, C} in the table lists the 3 possible outcomes for 1 spin of this spinner.
   - ➤ **If you desired to spin an A with 1 spin of this spinner, what fraction represents the probability of spinning an A? Why?** ***$\frac{1}{3}$, 0.33, or 33%; elicit that the ratio of favorable (desired) outcomes (A) to possible outcomes (A, B, C) is 1 to 3.***

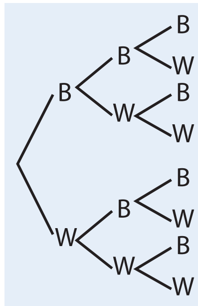
   Write $P(A) = \frac{1}{3}$ for display. Elicit that the probability can be read *1 out of 3*.
   - ➤ **How can you write *P*(A) as a decimal?** ***0.33*** **a percent?** ***33%***
3. Follow a similar procedure for *P*(B) ***$\frac{1}{3}$, 0.33, 33%***; *P*(C) ***$\frac{1}{3}$, 0.33, 33%***; and *P*(*not* A) ***$\frac{2}{3}$, 0.67, 67%***.
4. Explain that if you were to spin the spinner 2 times, you could land on A for spin 1 and land on A again for spin 2, so one possible outcome for 2 spins would be AA. Elicit the sample space for 2 spins and list it on the displayed page. ***{AA, AB, AC, BA, BB, BC, CA, CB, CC}***
   - ➤ **How many possible outcomes are there for 2 spins?** ***9*** Write 9 in the table.
5. Elicit the following probabilities for 2 spins on the spinner. Choose students to calculate the decimal form to the nearest hundredth. *P*(AA) ***$\frac{1}{9}$, 0.11, 11%***; *P*(AC) ***$\frac{1}{9}$, 0.11, 11%***; *P*(at least one A) ***$\frac{5}{9}$, 0.56, 56%***

Make a tree diagram to list the sample space for an event

1. Point out that AAA on the Sample Spaces page is one possible outcome if you were to spin the spinner 3 times. Elicit several other outcomes.
2. Display the Tree Diagram page. Explain that a *tree diagram* can be used to organize all the possible outcomes and to list the sample space of an event. You begin the tree diagram with the possible outcomes for the first event or 1 spin. Point out that the number of possible outcomes for 1 spin (3) is recorded below the tree.
   - ➤ **If you spin A on the first spin, what are the possible outcomes for the second spin?** ***A, B, or C***
3. Trace each dotted line to draw a branch from A to A, A to B, and A to C.
   (***Note:*** Throughout the activity, instruct the students to draw and label the branches after you draw and label them for display.)
   - ➤ **If you spin B on the first spin, what are the possible outcomes for the second spin?** ***A, B, or C***

   Draw 3 similar branches from B. Label the branches A, B, C.
   - ➤ **If you spin C on the first spin, what are the possible outcomes for the second spin?** ***A, B, or C***

   Draw 3 branches from C and label them A, B, C. Model reading each branch of the tree to find each possible outcome in the sample space for 2 spins and list them for display: {*AA*, *AB*, *AC*, *BA*, *BB*, *BC*, *CA*, *CB*, *CC*}.
   - ➤ **How many possible outcomes are there for 2 spins?** ***9*** Write 9 in the table below the second spin.
4. Follow a similar procedure to extend the tree diagram to show the third spin. Lead the students in reading aloud each branch of the tree diagram and guide them in listing all of the possible outcomes to create the sample space for 3 spins: {*AAA*, *AAB*, *AAC*, *ABA*, *ABB*, *ABC*, *ACA*, *ACB*, *ACC*, *BAA*, *BAB*, *BAC*, *BBA*, *BBB*, *BBC*, *BCA*, *BCB*, *BCC*, *CAA*, *CAB*, *CAC*, *CBA*, *CBB*, *CBC*, *CCA*, *CCB*, *CCC*}.
   - ➤ **How many possible outcomes are there for 3 spins?** ***27*** Write 27 in the table.
5. Elicit the following probabilities for 3 spins: *P*(AAA) ***$\frac{1}{27}$, 0.04, or 4%***; *P*(CCC) ***$\frac{1}{27}$, 0.04, or 4%***; *P*(at least 2 Bs) ***$\frac{7}{27}$, 0.26, or 26%***.
6. Direct the students to draw a circular spinner on the back of their worksheet and to shade half of the spinner. Then guide them in drawing a tree diagram and identifying the sample spaces (black or white) for 3 spins of the spinner.

   1 spin ***{B,W}***
   2 spins ***{BB, BW, WB, WW}***
   3 spins ***{BBB, BBW, BWB, BWW, WBB, WBW, WWB, WWW}***

Determine the number of possible outcomes using the Multiplication Counting Principle

1. Display the first word problem on the Counting Principle page with the tree diagram covered. Instruct the students to think of the different combinations of peanut butter sandwiches if one other spread and one type of bread were chosen. Elicit several possible combinations.
   - ➤ **What can help you list the sample space for this word problem?** ***Elicit a tree diagram.***
2. Uncover the tree diagram. Point out the 1 choice of peanut butter, the 3 choices of spread (G, S, H) and the 2 choices of bread (W, M).
   - ➤ **How many possible combinations of peanut butter (P) and a spread are there?** ***3***

   Lead the students in reading each tree branch to list the possible combinations of peanut butter and a spread: PG, PS, PH.
3. Follow a similar procedure to list the possible combinations of peanut butter, a spread, and a type of bread. ***6 possible combinations***
   - ➤ **What is the probability of having a peanut butter sandwich with jelly? Why?** ***$\frac{4}{6} = \frac{2}{3}$, 0.67, or 67%; Elicit that there are 4 sandwiches that contain jelly out of 6 possible combinations (outcomes).*** Point out that 2 of the 3 spreads are jellies, and 4 of

Sample Spaces

Knowing the number of possible outcomes is necessary when calculating probability. The **sample space** for an event is the set of all possible outcomes. A **tree diagram** is an organized way to show all the possible outcomes.

sample space
tree diagram
Multiplication Counting Principle

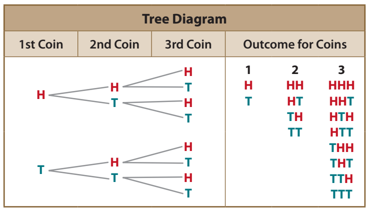
$P(\text{event}) = \dfrac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

The sample spaces show the outcomes of flipping 1, 2, and 3 coins. Each coin has a head (H) and a tail (T).

flipping 1 coin: 2 possible outcomes
{H, T}

flipping 2 coins: 4 possible outcomes
{HH, HT, TH, TT}

flipping 3 coins: 8 possible outcomes
{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}



Exercises

Students may or may not use simplest fraction form.

Use the sample space above to write the probability of the event as a fraction and as a percent.

1. 1 coin, P(heads) ***$\frac{1}{2}$; 50%***

2. 2 coins, P(at least one head) ***$\frac{3}{4}$; 75%***

3. 2 coins, P(*not* tails) ***$\frac{1}{4}$; 25%***

4. 3 coins, P(at least one tail) ***$\frac{7}{8}$; 88%***

5. 3 coins, P(at least 2 tails) ***$\frac{4}{8} = \frac{1}{2}$; 50%***

6. 3 coins, P(heads and tails) ***$\frac{6}{8} = \frac{3}{4}$; 75%***

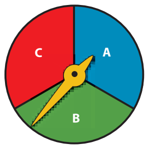
Use the spinner to find the answer.

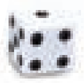
7. List the sample space for spinning the spinner two times. ***{rr, rb, rg, br, bb, bg, gr, gb, gg}***

8. Use the sample space to find the probability of landing on the same color both times. ***$\frac{3}{9} = \frac{1}{3}$ or 33%***

9. What is the probability of landing on a different color both times? ***$\frac{6}{9} = \frac{2}{3}$ or 67%***

Use the spinner and/or the number cube to find the answer.





10. List the sample space for one spin of the spinner. ***{A, B, C}***

11. List the sample space for one roll of the number cube. ***{1, 2, 3, 4, 5, 6}***

12. List the sample space for one spin of the spinner *or* one roll of the number cube. ***{A, B, C, 1, 2, 3, 4, 5, 6}***

13. List the sample space for one spin of the spinner *and* one roll of the number cube. ***{A1, A2, A3, A4, A5, A6, B1, B2, B3, B4, B5, B6, C1, C2, C3, C4, C5, C6}***

14. Write the probability (as a fraction and as a percent) for rolling a 6 after any letter for one spin on the spinner *and* one roll of the number cube. ***$\frac{3}{18} = \frac{1}{6}$; 17%***

360 Chapter 16

Use the **Multiplication Counting Principle** to find the number of outcomes when choices are given.

Jaden has 5 different-colored shirts and 3 different-colored pants. How many different outfits can he make?

5 shirt choices × 3 pant choices
$5 \times 3 =$ 15 outfit choices

Clark's Catering offers 3 meats, 4 vegetables, and 2 desserts. How many different meal combinations can be made?

3 meat choices × 4 vegetable choices × 2 dessert choices
$3 \times 4 \times 2 =$ 24 meal combinations

Exercises

Use the Multiplication Counting Principle to find the number of possible outcomes.

15. Lorenzo is purchasing a new shirt. The store has short-sleeve and long-sleeve shirts in 8 different colors. How many different shirts does Lorenzo have to choose from? ***16 shirt choices***

16. Morgan is ordering an egg sandwich for breakfast. Her choices are biscuit or croissant; sausage, bacon, or ham; and with cheese or without cheese. How many sandwich choices does Morgan have? ***12 sandwich choices***

17. Julian bought a combination lock with 4 dials. Each dial contains the numbers 1 to 9. How many possible combinations can be made for a lock like Julian's? ***6,561 combinations***

18. Sasha is redecorating her bedroom. From the store's showroom, she can choose one of 4 beds, one of 2 nightstands, and one of 3 desks. How many possible bedroom sets can Sasha choose from? ***24 bedroom sets***

Find the number of possible outcomes and list the sample space. Write the probability as a fraction in lowest terms and as a percent.

19. Options on a new car are a standard or an automatic transmission in a 2- or 4-door model. Find P(automatic transmission, 4-door). ***$2 \times 2 = 4$ combinations; $\frac{1}{4}$; 25%***

20. You may choose one kind of ice-cream cone (sugar or regular), one scoop of ice cream (chocolate or vanilla), and one topping (sprinkles, peanuts, or chocolate chips). Find P(cone with chocolate ice-cream). ***$2 \times 2 \times 3 = 12$ combinations; $\frac{1}{2}$; 50%***

21. White and black cars come with a red, tan, or black interior. Find P(white or black car, black interior). ***$2 \times 3 = 6$ combinations; $\frac{1}{3}$; 33%***

22. Carson packed 3 shirts (red, orange, green), 2 pants (blue, khaki), and 2 sweatshirts (solid, print). If Carson wears a shirt, a pair of pants and a sweatshirt, how many different combinations can he make? Find P(red shirt, blue pants, solid sweatshirt). ***$3 \times 2 \times 2 = 12$ combinations; $\frac{1}{12}$; 8%***

Practice & Application

Katelyn has enough money to buy a small pizza with one topping. Thick and thin crusts cost the same price.

Bochi's Pizzeria		
Sizes	**Crusts**	**Toppings**
small medium large extra-large	thick thin	mushrooms olives spinach pepperoni ham extra cheese

23. Make a tree diagram of Katelyn's choices.

24. List a sample space of her choices.

25. Find P(small pizza, thick crust).

26. Find P(small pizza, thick crust, pepperoni).

Complete DAILY REVIEW b on page 461.

Lesson 149 361

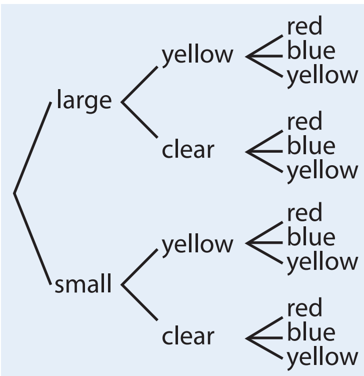
the 6 possible sandwiches have jelly. Elicit the complement $P(not\ \text{jelly}) = \frac{2}{6}$ or $\frac{1}{3}$.

4. Guide the students in comparing the number of branches from each choice to the number of possible outcomes. Elicit that multiplying the number of choices and the number of branches from those choices gives you the number of possible outcomes. (e.g., 1 peanut butter choice × 3 spread choices = 3 possible combinations of peanut butter and a spread and 1 peanut butter choice × 3 spread choices × 2 bread choices = 6 possible combinations of peanut butter, a spread, and a type of bread)

5. Explain that the process of multiplying the number of choices and the number of branches from those choices to find the total number of possible outcomes is the *Multiplication Counting Principle*.

6. Read aloud the second word problem.

➤ **What multiplication equation can you write to show the total number of fruit basket combinations Mrs. Parker can make?** ***Elicit 2 × 2 × 3 = 12; 2 basket sizes × 2 cellophane choices × 3 bow choices = 12 combinations.***



7. Guide the students in making a tree diagram to list the sample space. Demonstrate.

➤ **What is the probability of Mrs. Parker making a fruit basket with a red bow?** ***$\frac{4}{12} = \frac{1}{3}$, 0.33, or 33%*** Elicit that 2 basket sizes × 2 cellophane choices × 1 bow choice = 4 combinations (outcomes) with a red bow.

8. Display the Sample Spaces page again. Guide students in applying the Multiplication Counting Principle to write a multiplication equation for finding the number of possible outcomes for 2 spins. ***3 × 3 = 9*** Compare the product to the number of possible outcomes listed in the sample space for 2 spins.

Repeat the procedure for 3 spins. ***3 × 3 × 3 = 27***

9. Guide the students in using the Multiplication Counting Principle to answer the following questions.

➤ **How many different outfits can you make from 7 shirts and 3 pairs of pants?** ***7 × 3 = 21 outfits***

➤ **How many different lunch combinations of a drink, a burger, and fries can you make from 5 types of drink, 4 types of burgers, and 2 types of fries?** ***5 × 4 × 2 = 40 lunch combinations***

➤ **How many possible outcomes are there if you roll a 1–6 number cube two times?** ***6 × 6 = 36 possible outcomes***

Student Text pp. 360–61

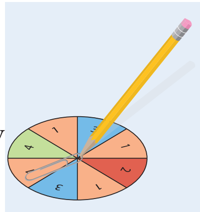
Lesson 150

Student Text pp. 362–63
Daily Review p. 461c

Objectives
- Find the theoretical probability of an event
- Predict the results of an experiment using the theoretical probability of an event
- Conduct a probability experiment
- Find the experimental probability of an event
- Create a line plot for the results of an experiment

Teacher Materials
- Experimental Probability, page IA98 (CD)
- A paper clip

Student Materials
- Experimental Probability, page IA98 (CD)
- Spinning Penny Experiment, page IA99 (CD) for each pair of students (optional)
- A paper clip
- A calculator (optional)
- A penny for every two students (optional)



Note
A paper clip or bobby pin can be used as the pointer for a spinner. Place the paper clip (or bobby pin) so that one end is in the center of the spinner. In order to spin the paper clip, place a pencil point inside the end of the paper clip so that the pencil point is touching the center of the spinner, holding the paper clip in place.

Teach for Understanding

Find the theoretical probability of an event; predict the results of an experiment

➤ **What is probability?** ***Elicit the likelihood that an event will occur.***

1. Display and distribute the Experimental Probability page and the paper clips. Direct attention to the spinner and remind students that the *theoretical probability* of an event is a ratio of the number of favorable (desired) outcomes divided by the number of possible outcomes that are equally likely to occur.

➤ **If you were to make 1 spin using this spinner, what are the possible outcomes?** ***1, 2, 3, or 4***

➤ **What fraction represents the probability of spinning 1? Why?** ***$\frac{4}{8}$ or $\frac{1}{2}$; elicit that the ratios express the number of favorable outcomes (1) to the number of possible outcomes.*** Elicit that $\frac{4}{8}$ and $\frac{1}{2}$ are equivalent ratios.

2. Direct the students to complete the first column of the table by writing the theoretical probability for each event as a fraction in lowest terms. $P(1) = \frac{1}{2}$, $P(2) = \frac{1}{8}$, $P(3) = \frac{1}{4}$, $P(4) = \frac{1}{8}$

➤ **If you were to make 8 spins using this spinner, what do you predict is the number of times you would land on 1? Why?** ***4; elicit that since the probability of landing on 1 is $\frac{1}{2}$, it is likely that 1 out of every 2 spins (one-half of the spins) would result in 1; 4 is one-half of 8 spins.***

➤ **What mathematical expression can you write for one-half of 8?** ***$\frac{1}{2} \times 8$***

Choose a student to write the expression for display and solve it while the other students solve it on paper. ***$\frac{1}{2} \times 8 = \frac{8}{2} = 4$***

➤ **Since $P(1)$ is $\frac{1}{2}$, what proportion could you write to find the expected number out of 12 spins that would result in 1?** ***$\frac{1}{2} = \frac{n}{12}$; n = 6 spins***

3. Explain that probability can be used to predict the expected results of an experiment.

➤ **What equation can you write to predict the number of times you would land on 1 if you made 16 spins of the spinner?** ***$\frac{1}{2} \times 16 = 8$ or $\frac{1}{2} = \frac{n}{16}$*** Write $P(1) = 8$ in the Expected Results column.

➤ **What do you notice about the probability of landing on 1 for 1 spin ($\frac{1}{2}$), 8 spins ($\frac{4}{8}$), and 16 spins ($\frac{8}{16}$)?** ***Elicit that they are equivalent fractions or ratios.***

4. Direct the students to predict the expected results (probability) of each event if the spinner is spun 16 times and to write their predictions on the page. ***$P(2) = \frac{1}{8} \times 16 = 2$, $P(3) = \frac{1}{4} \times 16 = 4$, $P(4) = \frac{1}{8} \times 16 = 2$***

Conduct a probability experiment; find the experimental probability of an event

1. Explain that *experimental probability* is the probability found during an experiment. Demonstrate spinning the paper clip on the spinner and recording the result by drawing a tally in the Actual Results column of the table for the appropriate letter. Allow the students to make several practice spins using their paper clips.
2. Instruct each student to spin his paper clip 1 time and to record the result on the table. Direct the students to continue the experiment for a total of 16 spins.
(***Note:*** While the students are conducting the experiment, you may choose to complete the experiment and record the results which will be used to demonstrate recording the data or a line plot later in the lesson.)

➤ **Are the actual results the same as your expected results?** ***Answers will vary.***

3. Point out that there are a number of variables (reasons) that affect the actual results of an experiment so that the actual results are not exactly the same as the expected results. In this experiment, students may not have spun their paper clip the same way, held their pencil at the same angle, begun with the pointer on the same space, and so on.
4. Explain that experimental probability can be found by taking the actual number of favorable outcomes of the event (the frequency) and dividing it by the total number of trials (16). Demonstrate calculating the experimental probability for 1.
5. Direct each student to use his results to calculate the experimental probability for each number and then to record it on his table.
6. Guide each student in comparing the theoretical probability to his experimental probability. Lead a discussion about the differences and similarities. Explain that if accurate spinners were used and more trials were performed, the experimental probability would be closer to the theoretical probability.

Create a line plot for the results of an experiment

1. Direct attention to the line at the bottom of the page and remind the students that a line plot is used to record data. Demonstrate counting the tallies for 1 to find the frequency for 1 and drawing the appropriate number of Xs above the number 1 to correspond with the frequency for 1.

➤ **How many Xs are needed to complete the line plot? Why?** ***16; there were 16 trials.***

Experimental Probability

Experimental probability is found using data collected from an experiment or a survey. The experimental probability of an event is the number of observed occurrences of an event in relation to the total number of trials (or people surveyed).

experimental probability

$P(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{total number of trials}}$

Theoretical probability tells the outcome of an experiment if all the given outcomes are equally likely to occur. The actual experimental results may not be the same as the expected (or predicted) results.

Exercises

Write the theoretical probability of the event when rolling a 1–6 number cube.

1. $P(5)$ *$\frac{1}{6}$ or 17%*
2. $P(3)$ *$\frac{1}{6}$ or 17%*
3. $P(6)$ *$\frac{1}{6}$ or 17%*

Use the tally chart to find the experimental probability of the event. Write it as a fraction in lowest terms and as a percent. Answer the question.

Number Cube Rolls						
Number	1	2	3	4	5	6
Results	II	III	~~IIII~~	IIII	~~IIII~~	I

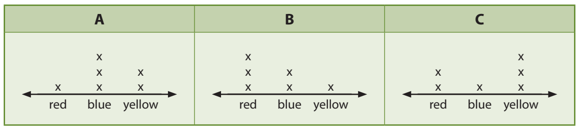
4. $P(1)$ *$\frac{2}{20} = \frac{1}{10}$; 10%*
5. $P(2)$ *$\frac{3}{20}$; 15%*
6. $P(3)$ *$\frac{5}{20} = \frac{1}{4}$; 25%*
7. $P(4)$ *$\frac{4}{20} = \frac{1}{5}$; 20%*
8. $P(5)$ *$\frac{5}{20} = \frac{1}{4}$; 25%*
9. $P(6)$ *$\frac{1}{20}$; 5%*
10. How would variables such as an unbalanced cube, a toss from a different angle, or the placement of each number on the cube before the toss cause a difference in the theoretical and experimental probability for the same experiment? *Any variable would most likely change the outcome of experimental probability. The theoretical probability would most likely stay the same.*

Use the spinner to find the probability of the event. Write it as a fraction in lowest terms and as a percent. Round to the nearest whole percent.

11. $P(\text{red})$ *$\frac{3}{6} = \frac{1}{2}$; 50%*
12. $P(\text{yellow})$ *$\frac{1}{6}$; 17%*
13. $P(\text{red and blue})$ *0%; Spinner cannot land on 2 colors.*
14. $P(\text{blue})$ *$\frac{2}{6} = \frac{1}{3}$; 33%*
15. $P(\textit{not}\text{ blue})$ *$\frac{4}{6} = \frac{2}{3}$; 67%*
16. $P(\textit{not}\text{ red})$ *$\frac{3}{6} = \frac{1}{2}$; 50%*

Use your findings in problems 11–13 to complete the following.

17. Which line plot would you expect best represents the results of an experiment that consisted of spinning the spinner 6 times? Why? *B; $P(r) = \frac{3}{6} = \frac{1}{2}$; $P(b) = \frac{2}{6} = \frac{1}{3}$; $P(y) = \frac{1}{6}$*



362 Chapter 16

Create a spinner similar to the model given.
Write the theoretical probability for the expected results.
Conduct the experiment and complete the table.

18. *Spinner Experiment:* Spin a paper clip on the spinner 12 times.

Theoretical Probability	Number of Trials	Expected Results	Actual Results	Experimental Probability
$P(\text{red}) = \frac{1}{2}$	12	$\frac{1}{2} = \frac{6}{12}$		
$P(\text{blue}) = \frac{1}{3}$	12	$\frac{1}{3} = \frac{4}{12}$		
$P(\text{yellow}) = \frac{1}{6}$	12	$\frac{1}{6} = \frac{2}{12}$		

Use the survey results to find the probability of the event. Write it as a fraction in lowest terms and as a percent. Round to the nearest whole percent.

Notebook Survey	
Color	Results
blue	6
red	3
green	4
mixed	6
other	1

19. $P(\text{red notebook})$ *$\frac{3}{20}$; 15%*
20. $P(\text{green notebook})$ *$\frac{4}{20} = \frac{1}{5}$; 20%*
21. $P(\text{red or blue notebook})$ *$\frac{9}{20}$; 45%*
22. $P(\text{mixed colors})$ *$\frac{6}{20} = \frac{3}{10}$; 30%*

Practice & Application

23. Five crayons (red, blue, green, yellow, and orange) were placed in a bag. Find $P(\text{orange})$. *$\frac{1}{5}$ or 20%*
24. Make a list or a tree diagram to determine the sample space for flipping the same coin 3 times. Find $P(\text{at least 2 heads})$. *{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}; $\frac{4}{8} = \frac{1}{2}$ or 50%*
25. Suppose you roll a 1–6 number cube 2 times. Use the Multiplication Counting Principle to find the number of possible outcomes. Find $P(\text{both rolls result in the same number})$. *6 × 6 = 36 combinations; $\frac{6}{36}$ or 17%*
26. Mrs. Larson surveyed her class and found that 13 out of 25 students have brown eyes. Find $P(\textit{not}\text{ brown eyes})$. *$\frac{12}{25}$ or 48%*
27. Mr. Hernandez surveyed his students to find their favorite type of book. He found that 37% of the class prefers historical fiction. What is $P(\text{complement})$? *P(not prefer historical fiction) = 63%*
28. Meteorologists provide weather forecasts expressed in percents. What does the meteorologist mean by a 50% chance of rain on Monday? *There is a 1 in 2 chance that it will rain on Monday.*
29. What percentage would a meteorologist say if $P(\text{snowstorm on Tuesday}) = \frac{3}{5}$? *There is a 60% chance of a snowstorm on Tuesday.*
30. Write the value of n if $\frac{10}{7} = \frac{n}{21}$. *n = 30*
31. Find the product of 15 × 16 × 20. *4,800*
32. How long will it take to travel 330 miles at a speed of 60 mph? *5.5 hr, $5\frac{1}{2}$ hr, or 5 hr 30 min*

J Roll a 1–6 number cube 18 times. Draw a line plot showing the frequencies of rolling 1, 2, 3, 4, 5, and 6 during the experiment. *Answers will vary.*

1 2 3 4 5 6

Number showing on number cube

Complete DAILY REVIEW C on page 461.

Lesson 150 363

2. Direct the students to complete the line plot using their results. Then guide a discussion about the varying line plots.
   - ➤ **Which numbers have a similar frequency? Why do you think this is true?** ***Answers may vary, but the numbers that will most likely have a similar frequency are 2 and 4; elicit that 2 and 4 have the same theoretical probability, a 1 out of 8 probability of being landed on.***
   - ➤ **Which number has the highest frequency? Why?** ***Answers may vary, but will most likely be 1; it has the greatest theoretical probability, a 4 out of 8 probability of being landed on.***

   (***Note:*** The following activity is optional. You may choose to assign the experiment to be completed at home or use the following procedure for the students to complete the experiment during class.)
3. Arrange the students in pairs and distribute the Spinning Penny Experiment page. Direct each pair of students to conduct the experiment.
   - ➤ **What variables exist when conducting the Spinning Penny Experiment?** ***Possible answers: students may not have spun their pennies at the same speed or on the same type of surface, held their finger at the same place, and so on.***
4. Allow students to tell their experimental probabilities for the Spinning Penny Experiment. Guide the students to the conclusion that an expected result or probability for an experiment is similar to a scientific hypothesis for a science experiment or an estimate for a math problem. The expected results are educated guesses.

Student Text pp. 362–63

Point out that the *number of outcomes in the event* in the teaching on Student Text page 362 refers to the favorable (desired) outcomes rather than total outcomes in the event. Also, explain that when the question asks for the probability of an event, the students are to give the theoretical (equally likely) probability.

Lesson 151 | **Student Text** pp. 364–65 | **Daily Review** p. 462d

Objectives
- Determine whether a game is fair or unfair using probability
- Conduct a probability experiment
- Find the experimental probability and the theoretical probability of an event
- List the sample space for an event
- Make predictions using probability

Teacher Materials
- Fair or Unfair Games, page IA100 (CD)
- 6 Unifix® Cubes in an opaque bag: 4 orange, 1 brown, 1 yellow
- Two 1–6 number cubes

Student Materials
- Fair or Unfair Games, page IA100 (CD) for each pair or group of students (optional)
- Two 1–6 number cubes for each pair or group of students (optional)

Note
Six marbles (3 different colors) or 6 squares of construction paper may be substituted for the 6 Unifix Cubes.

Teach for Understanding

Use probability to determine whether a game is fair or unfair

1. Display the bag of Unifix Cubes. Tell the students that the bag contains orange, brown, and yellow cubes. Choose 3 students (A, B, and C) to play "Draw a Cube." Explain that if an orange cube is drawn, Student A receives 1 point; if a brown cube is drawn, Student B receives 1 point; and if a yellow cube is drawn, Student C receives 1 point. The first person to receive 5 points wins the game. Shake the bag, allow Student A to draw a cube, and direct the appropriate student to record his point by writing a tally. Return the cube to the bag and shake the bag. Allow Student B to draw a cube and direct the appropriate student to record his point. Repeat the procedure, allowing Student C to draw a cube. Continue the activity until a winner has been declared. Show the students the 6 cubes.
   - ➤ **Is this game fair or unfair? Why?** ***Elicit that this game is unfair; Student A had more opportunities to receive points since there were more orange cubes than brown and yellow cubes.***
2. Elicit the probabilities of drawing an orange cube, a brown cube, or a yellow cube from the bag. **P(*orange*) = $\frac{4}{6}$ = $\frac{2}{3}$, P(*brown*) = $\frac{1}{6}$, P(*yellow*) = $\frac{1}{6}$**
   - ➤ **What would you do to make the game fair?** ***Elicit that you would place the same number of each color cube in the bag.***
3. Explain that in a fair game all the players are equally likely to win; no player has an advantage. Remove 3 of the orange cubes and return the other 3 cubes to the bag.
4. Guide the students in finding the theoretical probabilities of drawing an orange cube, a brown cube, or a yellow cube in a fair game. **P(*orange*) = $\frac{1}{3}$, P(*brown*) = $\frac{1}{3}$, P(*yellow*) = $\frac{1}{3}$**

Conduct a probability experiment; find the experimental probability of an event

1. Display the Fair or Unfair Games page and read aloud the procedure for the "Two-Cube Sum" game. Instruct each student to choose the number from 2 to 12 that they think will be the winning number, write the number on a small piece of paper, and exchange the paper with a classmate.
   - ➤ **Why do you think there is not a column for 1 or 13 in the table?** ***Elicit that the smallest number on a cube is 1 and the largest number is 6. The smallest sum will be 2 (1 + 1 = 2), and the largest sum will be 12 (6 + 6 = 12). The sample space for this event is {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}.***
2. Allow the students to take turns rolling the cubes and giving the sum of the two numbers. Record each result by drawing a tally in the table on the displayed page. Continue the activity until a number has a frequency of 10 tallies.
   You may choose to allow pairs or small groups of students to play the game. Provide a Fair or Unfair Games page and 2 number cubes for each group.
   - ➤ **What was the winning number?** ***Answers will vary.***
3. Direct the students to find the experimental probability of the number with the highest frequency and the number with the lowest frequency. Instruct them to write the probability as a fraction in lowest terms, a decimal, and a percent. (***Note:*** If pairs or groups of students played the game, each group should calculate the experimental probability using its data.)
   - ➤ **How can you determine whether this game is fair or unfair?** ***Elicit that you can determine whether all the sums are equally likely to occur by listing the possible pairs of addend combinations for each sum.***
   - ➤ **How many possible outcomes are there for 1 cube?** ***6***
   - ➤ **If you apply the Multiplication Counting Principle, what multiplication equation can you write to find the number of possible outcomes for 2 cubes?** ***6 × 6 = 36***
4. Elicit the possible addend combinations for each sum and list them for display.
   - 2 = ***1 + 1***
   - 3 = ***1 + 2, 2 + 1***
   - 4 = ***1 + 3, 3 + 1, 2 + 2***
   - 5 = ***1 + 4, 4 + 1, 2 + 3, 3 + 2***
   - 6 = ***1 + 5, 5 + 1, 2 + 4, 4 + 2, 3 + 3***
   - 7 = ***1 + 6, 6 + 1, 2 + 5, 5 + 2, 3 + 4, 4+ 3***
   - 8 = ***2 + 6, 6 + 2, 3 + 5, 5 + 3, 4 + 4***
   - 9 = ***3 + 6, 6 + 3, 4 + 5, 5 + 4***
   - 10 = ***4 + 6, 6 + 4, 5 + 5***
   - 11 = ***5 + 6, 6 + 5***
   - 12 = ***6 + 6***
   - ➤ **Is this game fair or unfair? Why?** ***Unfair; all of the sums are not equally likely to occur.***
   - ➤ **Which number has the most possible combinations?** ***7 (6 combinations)*** **the least possible combinations?** ***2 and 12 (1 combination each)***
5. Elicit the theoretical probability of 2 rolled number cubes having a sum of 2–12. **P(*2 or 12*) = $\frac{1}{36}$, *0.03, 3%;* P(*3 or 11*) = $\frac{2}{36}$ = $\frac{1}{18}$, *0.06, 6%;* P(*4 or 10*) = $\frac{3}{36}$ = $\frac{1}{12}$, *0.08, 8%;* P(*5 or 9*) = $\frac{4}{36}$ = $\frac{1}{9}$, *0.11, 11%;* P(*6 or 8*) = $\frac{5}{36}$, *0.14, 14%;* P(*7*) $\frac{6}{36}$ = $\frac{1}{6}$, *0.17, 17%***
6. Instruct the students to use the list of possible combinations to find *P*(odd sum) and *P*(*not* odd), expressing the probability as a fraction in lowest terms. Remind them that *P*(*not* odd) is the complement of *P*(odd sum). **P(*odd sum*) = $\frac{18}{36}$ = $\frac{1}{2}$, P(*not odd*) = $\frac{18}{36}$ = $\frac{1}{2}$**

Fair or Unfair?

In a **fair game**, all players are *equally likely* to win. No player has an unfair advantage. Compare the probabilities of winning for each player to determine if a game is fair.

fair game

Number Cube Roll

1. Roll a 1–6 number cube.
2. If the number is even, Player A wins. If the number is odd, Player B wins.

Sample space: {1, 2, 3, 4, 5, 6}

$P(\text{even}) = \frac{3}{6} = \frac{1}{2}$ or 50%

$P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$ or 50%

Since P(even) = P(odd), this game is fair for the two players.

Three-Coin Toss

1. Toss 3 coins.
2. If the coins match, Player A wins. If the coins do not match, Player B wins.

Sample space: {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

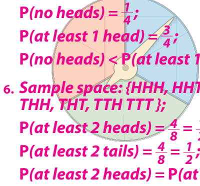
$P(\text{match}) = \frac{2}{8} = \frac{1}{4}$ or 25%

$P(\text{no match}) = \frac{6}{8} = \frac{3}{4}$ or 75%

Since P(match) < P(no match), this game is unfair. The players are not equally likely to win. Player B has an unfair advantage.

Exercises

Use the spinner to find the answer.
Write the probability as a fraction in lowest terms.



1. Three players choose a color on the spinner. If the spinner lands on red, Player A wins. If it lands on blue, Player B wins. If it lands on green, Player C wins. Find P(red), P(blue), and P(green). Is this game fair or unfair? ***P(blue) = $\frac{1}{3}$; P(red) = $\frac{1}{3}$; P(green) = $\frac{1}{3}$; fair***

2. The three players change colors. This time Player A chooses blue, and Player B chooses red. Player C does not want green again, so he chooses *not* green. Find P(blue), P(red), and P(*not* green). Is this game fair or unfair? ***P(blue) = $\frac{1}{3}$; P(red) = $\frac{1}{3}$; P(not green) = $\frac{2}{3}$; unfair***

List the sample space to find the answer.
Write the probability as a fraction in lowest terms.

3. A bag contains 1 red marble and 1 blue marble. One marble is drawn from the bag. If the marble is red, Player A wins. If the marble is blue, Player B wins. Find P(drawing a red marble) and P(drawing a blue marble). Is this game fair or unfair? Explain. ***Sample Space: {r, b}; P(red) = $\frac{1}{2}$; P(blue) = $\frac{1}{2}$; P(red) = P(blue); fair***

4. A 1–6 number cube is rolled once. Player A wins if the number is less than 4. Player B wins if the number is greater than 4. Find P(less than 4) and P(greater than 4). Is this game fair or unfair? ***Sample Space: {1, 2, 3, 4, 5, 6};***
P(less than 4) = $\frac{3}{6} = \frac{1}{2}$;
P(greater than 4) = $\frac{2}{6} = \frac{1}{3}$;
P(less than 4) > P(greater than 4); unfair

5. Cory and Edward have to decide who will empty the dishwasher. Edward suggests that they toss 2 coins. If no heads come up, Edward will empty the dishwasher. If at least 1 head comes up, Cory will empty the dishwasher. Find P(*no* heads) and P(at least 1 head) for tossing 2 coins. Is Edward's proposal fair or unfair?

6. Cory suggests that they toss 3 coins. If at least 2 heads come up, Edward will empty the dishwasher. If at least 2 tails come up, Cory will empty the dishwasher. Find P(at least 2 heads) and P(at least 2 tails) for tossing 3 coins. Is Cory's proposal fair or unfair?

3. Explanation: The game is fair because each player has a 50% chance of winning.

5. Sample space: {HH, HT, TH, TT};
P(no heads) = $\frac{1}{4}$;
P(at least 1 head) = $\frac{3}{4}$;
P(no heads) < P(at least 1 head); unfair

6. Sample space: {HHH, HHT, HTH HTT, THH, THT, TTH TTT };
P(at least 2 heads) = $\frac{4}{8} = \frac{1}{2}$;
P(at least 2 tails) = $\frac{4}{8} = \frac{1}{2}$;
P(at least 2 heads) = P(at least 2 tails); fair

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The probability of event A is given. Find the complement, *P*(*not* A).

7. $P(A) = \frac{6}{10}$ ***P(not A) = $\frac{4}{10}$*** **8.** $P(A) = \frac{2}{3}$ ***P(not A) = $\frac{1}{3}$*** **9.** $P(A) = \frac{3}{8}$ ***P(not A) = $\frac{5}{8}$***

10. P(A) = 35% ***P(not A) = 65%*** **11.** P(A) = 85% ***P(not A) = 15%*** **12.** P(A) = 37% ***P(not A) = 63%***

A 1–6 number cube is rolled once. Write the probability of the event as a fraction in lowest terms and as a percent. Round to the nearest whole percent.

13. P(6) ***$\frac{1}{6}$; 17%*** **14.** P(9) ***0; 0%*** **15.** P(odd) ***$\frac{1}{2}$; 50%***

16. P(a number other than 4) ***$\frac{5}{6}$; 83%*** **17.** P(a multiple of 3) ***$\frac{1}{3}$; 33%*** **18.** P(a prime number) ***$\frac{2}{3}$; 67%***

Solve.

19. At Peggy's Pancake House, you can order regular or whole-wheat pancakes with a choice of blueberry, strawberry, or maple syrup. How many different choices of pancakes with syrup do you have? ***2 pancakes × 3 syrups = 6 choices***

20. The combination lock on the family's storage building has 3 dials. Each dial contains the numbers 1 to 9. How many possible combinations are there for this lock? ***9 digits × 9 digits × 9 digits = 729 combinations***

21. Tessa attempted 15 free throws and made 9 of them. What is the probability that she will be successful and make the next throw? ***$\frac{9}{15} = \frac{3}{5}$ or 60%***

22. You are given 4 choices for a multiple-choice question. If you do not know the answer, what is the probability of guessing the correct answer? ***$\frac{1}{4}$ or 25%***

23. You are answering a true or false question. If you do not know the answer, what is the probability of guessing the correct answer? ***$\frac{1}{2}$ or 50%***

Use the survey results to write the probability as a decimal and as a percent.

Scientists use the probability of an event to make predictions. A survey using a sample of 100 sixth-graders can be used to predict the genetic traits of other sixth-graders.

Genetic Survey Results (sample = 100 students)

Trait	Dimples	Straight Hair	Attached Earlobes	Widow's Peak
Yes	40	50	30	50
No	60	50	70	50

24. P(dimples) ***0.4; 40%***

25. P(straight hair) ***0.5; 50%***

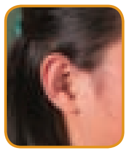
26. P(attached earlobes) ***0.3; 30%***

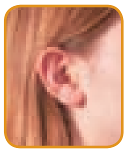
27. P(widow's peak) ***0.5; 50%***

J Use the information given for the sample of 100 sixth-graders to predict the number of students that will and will *not* have a genetic trait. Make a chart to show your predictions for a sample of 200 students.


widow's peak


no widow's peak


attached earlobe


detached earlobe

Complete DAILY REVIEW d on page 462.

Lesson 151 365

➤ **Would a game of rolling an odd sum or a *not* odd sum be fair or unfair? Why?** ***Fair; the probability of rolling an odd sum is equal to the probability of rolling a*** **not** ***odd sum.***

➤ **What is the probability of rolling an even sum using 2 cubes? How do you know?** ***$\frac{18}{36}$ or $\frac{1}{2}$; elicit that*** **P(even)** ***is the same as*** **P(not odd).**

7. Guide the students in playing "Two-Cube Even or Odd Sum" to determine the experimental probability of even and odd sums.

Make predictions using probability

1. Explain that meteorologists look at past conditions and use probability to forecast weather; insurance companies use probabilities to decide the cost of insuring a car, a house, or a person's life; and testing companies look at the probability of students guessing correct answers when writing standardized tests.

> Jack likes to eat Cinnamon Rings cereal for breakfast every morning. His mother wants him to have more variety, so he eats it every 3 days. Jack made a spinner and divided it into 3 equal sections hoping that his mother would let him use the spinner to decide his breakfast. He labeled the sections Cinnamon Rings, Oatmeal, and Eggs. Jack hopes that he designed the spinner so that it lands on Cinnamon Rings the most often.

➤ **Will this spinner provide Jack with more chances to eat Cinnamon Rings than the other breakfast items? Why?** ***No; elicit that all of the outcomes are equally likely to occur;*** **P(CR)** $= \frac{1}{3}$, **P(O)** $= \frac{1}{3}$, ***and*** **P(E)** $= \frac{1}{3}$.

2. Instruct the students to draw a spinner that will give Jack twice the opportunity to eat Cinnamon Rings as each of the other breakfast items. ***Possible drawing: a spinner divided into fourths; 2 fourths labeled Cinnamon Rings, 1 fourth labeled Oatmeal, and 1 fourth labeled Eggs.***
3. Survey 5 sixth-grade students to determine their favorite pets. Guide all of your students in recording the responses in a frequency table and in identifying the sample space [e.g., cat (2), dog (2), horse (1)]. Direct the students to calculate the probability of each type of pet being chosen as a favorite (e.g., $P(\text{cat}) = \frac{2}{5}$, $P(\text{dog}) = \frac{2}{5}$, $P(\text{horse}) = \frac{1}{5}$).
4. Explain that the probability calculated using the data gathered from a small sample of people can be used to make predictions about a larger population of the same people. Scientists use probability when studying genetic traits. Pollsters use probability when predicting the winner in an election.
5. Direct the students to use the survey data to predict the expected results for 100 students. Remind them that this prediction is theoretical (an educated guess). Then guide the students in conducting an actual survey of 100 students' favorite pets using the same animal choices. Guide them to the conclusion that as the number of students surveyed increases, the actual results (experimental probability) will be closer to the predicted results (theoretical probability).

Student Text pp. 364–65

(*Note*: *Assessment* available on Teacher's Toolkit CD.)

Lesson 152

Student Text pp. 366–67
Daily Review p. 462e

Objectives
- Differentiate between independent and dependent compound events
- Find the probability of compound events using a formula

Teacher Materials
- 6 Unifix Cubes: 4 orange, 1 brown, 1 yellow (from Lesson 151)
- An opaque bag (from Lesson 151)
- A 1–6 number cube (from Lesson 151)

Student Materials
- A calculator

Teach for Understanding

Differentiate between independent and dependent compound events

1. Explain that a *compound event* involves 2 or more simple events. Spinning a spinner more than once, drawing more than one item from a bag, tossing 2 coins, or tossing 1 coin 3 times are examples of compound events. A compound event can be an *independent event* or a *dependent event*.
   Compound events are independent when the sample space (possible outcomes) of one event remains the same regardless of the outcome of a previous event. If a coin lands on heads for the first toss, it does not affect the outcome of how the coin will land on a second or third toss. The color on which a spinner lands on spin 1 does not affect where the spinner will land on spin 2.
   Compound events are dependent when the outcome of the first event affects the sample space (possible outcomes) of the second event.
2. Display 3 of the cubes (1 orange, 1 brown, and 1 yellow) and then place them in the bag.
   ➤ **What is the probability of drawing the orange cube from the bag?** ***$\frac{1}{3}$ or 33%*** **the brown cube?** ***$\frac{1}{3}$ or 33%*** **the yellow cube?** ***$\frac{1}{3}$ or 33%***
   Choose a student to draw 1 cube from the bag without returning it to the bag.
   ➤ **How many cubes remain in the bag?** ***2***
   ➤ **What is the probability of the same color cube being drawn again? Why?** ***$\frac{0}{2}$ or 0%; since the cube was not returned, there are no more cubes of that color.***
   ➤ **What is the probability of choosing one or the other of the colored cubes? Why?** ***$\frac{1}{2}$ or 50%; since there are only 2 cubes remaining in the bag, the probability has changed.***
3. Follow a similar procedure for drawing 1 of the 2 remaining cubes from the bag. Point out that these events are dependent because the sample space (possible outcomes) was affected by what was drawn during the first event.
4. Return the cubes to the bag. Elicit that the probability of drawing any of the colors is back to $\frac{1}{3}$ or 33%. Repeat the experiment.
5. Return the cubes to the bag again. Choose a student to draw 1 cube from the bag, show the cube to the other students, and then return the cube to the bag.
   ➤ **What is the probability of choosing each color cube on a second draw?** ***P(orange) = $\frac{1}{3}$ or 33%; P(brown) = $\frac{1}{3}$ or 33%; P(yellow) = $\frac{1}{3}$ or 33%***
   Choose another student to draw 1 cube, show the cube to the other students, and then return the cube to the bag.
   ➤ **Why did the probability remain the same for both drawings?** ***Elicit that the bag still contained 3 cubes, 1 cube of each color. The events are independent because the outcome of the first draw did not affect the outcome of the second draw.***
6. Direct the students to determine whether the following descriptions of compound events are independent or dependent. If the events are dependent, instruct a student to explain what changed.
   - Roll a number cube 3 times. ***independent***
   - Draw names from a container to form different math teams. ***Dependent; you would not return the name to possibly be drawn again.***
   - Flip a coin 2 times. ***independent***
   - Predict whether it will rain or not rain during the next 2 days. ***independent***
   - Choose 3 tiles from a bag without returning the tile after each draw. ***Dependent; there is one less tile to choose from each time.***

Find the probability of compound events

1. Display the 1–6 number cube.
   ➤ **What is the probability of rolling 5? Why?** ***$\frac{1}{6}$; 1 out of 6 sides will produce the favorable outcome.***
   ➤ **What is the probability of rolling 3?** ***$\frac{1}{6}$***
   ➤ **How can you find the probability of rolling a 5 on the first roll and a 3 on the second roll?** ***List the sample space (all possible outcomes) or make a tree diagram.***
2. Demonstrate making a tree diagram to find $P(5 \text{ and } 3) = \frac{1}{36}$. (***Note:*** The word "and" can be substituted for the comma in compound event notation.)
3. Explain that the probability of compound events can be found by multiplying the probability of the individual events. Write $P(\text{A and B}) = P(\text{A}) \times P(\text{B})$ for display.
   ➤ **What equation can you use to find *P*(5 and 3)?** ***$P(5) \times P(3) = \_\_$***
   Choose a student to write the equation for display and solve it. $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$ Write $P(\text{A and B}) = \frac{1}{36}$.
   ➤ **Are these events independent or dependent? How do you know?** ***Independent; the outcome of the first event does not affect the outcome of the second event.***
4. Follow a similar procedure for the following independent events.
   toss a coin 2 times: $P(\text{heads, heads}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
   toss a coin 3 times: $P(\text{tails, heads, tails}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
   roll a 1–6 number cube 3 times $P(1, 2, 3) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$
5. Display 3 of the cubes (1 orange, 1 brown, and 1 yellow) and then place them in the bag.
   ➤ **What is the probability of drawing the brown cube from the bag?** ***$\frac{1}{3}$*** **the orange cube?** ***$\frac{1}{3}$*** **the yellow cube?** ***$\frac{1}{3}$***
   ➤ **If you were to select the brown cube on the first draw and not return it to the bag, what is the probability of drawing the yellow cube?** ***$\frac{1}{2}$***

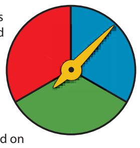
Independent & Dependent Events

A **compound event** involves two or more simple events; they can be independent or dependent. An **independent event** occurs when the sample space of one event remains the same regardless of the outcome of a previous event. A **dependent event** occurs when the outcome of one event affects the sample space of a later event. You can find the probability of a compound event by multiplying the individual theoretical probabilities.

compound event
independent event
dependent event

Independent Event

Alberto spins the spinner. He lands on a red section. He spins a second time and lands on blue.



Sample space for each spin:
{r, b, g}
$P(\text{red}) = \frac{1}{3}$ $P(\text{blue}) = \frac{1}{3}$

Find the probability of spinning red on the first spin and blue on the second spin.

$\boldsymbol{P(\text{A, B}) = P(\text{A}) \times P(\text{B})}$
Sample space for 2 spins: {rr, rb, rg, bb, br, bg, gg, gr, gb}
$P(\text{red, blue}) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Dependent Event

Rosalie draws a marble from the bag. She keeps the red marble and passes the bag to Jolene to draw a marble. Jolene chooses a blue marble.

Sample space for the first draw: {r, b, g}
$P(\text{red}) = \frac{1}{3}$

Sample space for the second draw: {b, g}
$P(\text{blue after 1 red marble drawn}) = \frac{1}{2}$

Find the probability of drawing a red marble followed by a blue marble.

$\boldsymbol{P(\text{A, B}) = P(\text{A}) \times P(\text{B after A})}$
$P(\text{red, blue}) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

Exercises

Write independent or dependent to describe the event.

1. roll a 5 and then roll a 6 on a 1–6 number cube ***independent***
2. choose a soft drink and then choose a sandwich from a menu ***independent***
3. select a marble from a bag and return it; then select a second marble ***independent***
4. select a marble from a bag and keep it; then select a second marble ***dependent***
5. draw names from a container to form soccer teams ***dependent***
6. flip a coin 3 times ***independent***

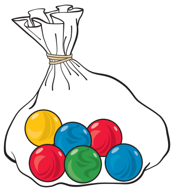
Write an equation to find the probability of the event as a fraction in lowest terms.

A marble is drawn from the bag and then replaced before the next selection.

7. What is the sample space of each draw from the bag? ***{r, r, b, b, y, g}***
8. $P(\text{red, yellow})$ $\frac{2}{6} \times \frac{1}{6} = \frac{2}{36} = \frac{1}{18}$
9. $P(\text{yellow, green})$ $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$
10. $P(\text{red, } \textit{not} \text{ yellow})$ $\frac{2}{6} \times \frac{5}{6} = \frac{10}{36} = \frac{5}{18}$
11. $P(\text{red, blue})$ $\frac{2}{6} \times \frac{2}{6} = \frac{4}{36} = \frac{1}{9}$
12. $P(\text{yellow, blue})$ $\frac{1}{6} \times \frac{2}{6} = \frac{2}{36} = \frac{1}{18}$
13. $P(\text{green, } \textit{not} \text{ blue})$ $\frac{1}{6} \times \frac{4}{6} = \frac{4}{36} = \frac{1}{9}$

A marble is drawn from the bag but is *not* replaced after each selection. List the sample space for each individual event.

14. $P(\text{red, yellow})$ $\frac{1}{15}$
15. $P(\text{yellow, green})$ $\frac{1}{30}$
16. $P(\text{red, } \textit{not} \text{ yellow})$ $\frac{4}{15}$
17. $P(\text{red, blue})$ $\frac{2}{15}$
18. $P(\text{yellow, blue})$ $\frac{1}{15}$
19. $P(\text{green, } \textit{not} \text{ blue})$ $\frac{1}{10}$



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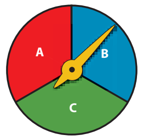
Spin the spinner and/or roll the 1–6 number cube for the event.
Write an equation to find the probability of the event as a fraction.

20. $P(\text{A}, 2)$ $\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$
21. $P(\text{A, B, C})$ $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$
22. $P(\text{B}, 3, 4)$ $\frac{1}{3} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{108}$
23. $P(\text{B, C}, 5)$ $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{6} = \frac{1}{54}$
24. Were the compound events above dependent or independent? ***independent***



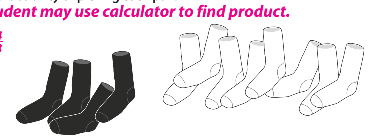
Write an equation to find the probability of the following dependent events.
Write the probability as a fraction in lowest terms. ***Student may use calculator to find product.***

The 20 students in class were each assigned a number, 1–20. The numbers were written on slips of paper and placed in a container. The teacher drew numbers to determine which students would be grouped together for a science project. She did *not* return any numbers to the container.

25. If the numbers are *not* returned to the container, is each drawn number dependent or independent? ***dependent***
26. What will happen to the number of outcomes in the sample space for each drawing? ***26. The sample space will decrease by one name at the end of each individual event.***
27. $P(1, 4)$ $\frac{1}{20} \times \frac{1}{19} = \frac{1}{380}$
28. $P(3, 17)$ $\frac{1}{18} \times \frac{1}{17} = \frac{1}{306}$
29. $P(2, 6, 10)$ $\frac{1}{16} \times \frac{1}{15} \times \frac{1}{14} = \frac{1}{3{,}360}$
30. $P(16, 18, 20)$ $\frac{1}{13} \times \frac{1}{12} \times \frac{1}{11} = \frac{1}{1{,}716}$
31. $P(11, 12, 13, 14)$ $\frac{1}{10} \times \frac{1}{9} \times \frac{1}{8} \times \frac{1}{7} = \frac{1}{5{,}040}$
32. $P(5, 7, 8, 9)$ $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{360}$
33. What numbers are left in the container? ***15, 19***

If you select 2 socks at random, find the probability of picking each pair.
Write it as a fraction in lowest terms. ***Student may use calculator to find product.***

34. $P(\text{white, white})$ $\frac{8}{12} \times \frac{7}{11} = \frac{56}{132} = \frac{14}{33}$
35. $P(\text{white, black})$ $\frac{8}{12} \times \frac{4}{11} = \frac{8}{33}$
36. $P(\text{black, black})$ $\frac{4}{12} \times \frac{3}{11} = \frac{4}{44} = \frac{1}{11}$
37. $P(\text{black, white})$ $\frac{4}{12} \times \frac{8}{11} = \frac{8}{33}$



During the last 2 years in June, it rained 12 out of 60 days.
Find the probability of the independent event.
Write it as a fraction in lowest terms.

38. $P(\text{1 rain day})$ $\frac{12}{60} = \frac{1}{5}$
39. $P(\text{1 rain day, 1 rain day})$ $\frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$
40. $P(\text{1 rain day, 1 no-rain day})$ $\frac{1}{5} \times \frac{4}{5} = \frac{4}{25}$
41. $P(\text{1 no-rain day})$ $\frac{48}{60} = \frac{4}{5}$
42. $P(\text{1 no-rain day, 1 no-rain day})$ $\frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$
43. $P(\text{1 no-rain day, 1 rain day})$ $\frac{4}{5} \times \frac{1}{5} = \frac{4}{25}$

Complete DAILY REVIEW e on page 462.

Lesson 152 367

6. Write $P(\text{A, B}) = P(\text{A}) \times P(\text{B after A})$. Point out that the comma represents the word *and*. Explain that when a cube is not returned to the bag, the choices in the bag change, which also changes the probability of future (subsequent) draws.
   - ➤ **Are these events independent or dependent? How do you know?** ***Dependent; the outcome of the first event affects the possible outcomes of the second event.***
   - ➤ **What equation can you use to find P(brown, yellow) if you are *not* returning the cube to the bag after each draw?** ***P(brown, yellow) = P(brown) × P(yellow after brown);*** $\frac{1}{3} \times \frac{1}{2} = \_\_$. Choose a student to write the equation and solve it. $\frac{1}{6}$ Write $P(\text{brown, yellow}) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$.
7. Follow a similar procedure using all 6 of the cubes.
   - ➤ **What is the probability of drawing an orange cube from the bag?** $\frac{4}{6}$ ***or*** $\frac{2}{3}$
   - ➤ **If you were to select an orange cube on the first draw and not return the cube to the bag, what is the probability of drawing a yellow cube?** $\frac{1}{5}$
   - ➤ **What equation can you use to find P(orange, yellow) when not returning the drawn cube to the bag?** $\frac{2}{3} \times \frac{1}{5} = \_\_$ Choose a student to write the equation and solve it. $\frac{2}{15}$
8. Repeat the procedure to find $P(\text{yellow, orange, brown})$ $\frac{1}{6} \times \frac{4}{5} \times \frac{1}{4} = \frac{4}{120} = \frac{1}{30}$
9. Direct each student to write his name on a slip of paper. Remove the cubes from the bag and place the slips of paper in the bag.
   - ➤ **What is the probability of drawing a specific student's name from the bag?** $\frac{1}{n}$ ***(n = number of students in the class)***
10. Write $P(\text{student 1, student 2})$ for display using 2 students' names.
    - ➤ **What equation do you think you could use to find the probability of drawing two specific students to be partners on a field trip?** ***Guide students to the equation*** $\frac{1}{n} \times \frac{1}{n-1}$ ***(where n = number of students in the class).*** Choose a student to write the equation for display and solve it using a calculator, while the other students solve it on paper.
11. Follow a similar procedure to find the probability of drawing the names of 4 specific students that are on the same team. $\frac{1}{n} \times \frac{1}{n-1} \times \frac{1}{n-2} \times \frac{1}{n-3}$

Student Text pp. 366–67

You may choose to allow the students to work in pairs or small groups to complete the Student Text pages.

Lesson 153 **Student Text** pp. 368–69

Chapter Review

Objectives
- Predict the results of an experiment using the theoretical probability of an event
- Conduct a probability experiment to determine the experimental probability of an event
- Determine whether a game is fair or unfair using probability
- Determine the theoretical probability of an event
- Find the complement of the probability of an event
- Make a tree diagram to list the sample space for an event
- Find the number of possible outcomes using the Multiplication Counting Principle
- Make predictions using probability

Teacher Materials
- Probability Spinner, page IA101 (CD)
- A paper clip

Student Materials
- Probability Spinner, page IA101 (CD) for each student
- A paper clip
- A calculator (optional)

Note

This lesson reviews the concepts presented in Chapter 16 to prepare the students for the Chapter 16 Test. Student Text pages 368–69 provide the students with an excellent study guide.

Teach for Understanding

Predict and determine the experimental probability of an event; determine whether a game is fair or unfair

1. Display and distribute the Probability Spinner page and the paper clips. Point out that the spinner on the page will be used for both activities: "Spin 1, 2, or 3" and "Even or Odd Spin."
   - ➤ **What is the set of all possible outcomes for an event called?** ***the sample space***
   - ➤ **What is the sample space for 1 spin of this spinner?** ***{1, 2, 3}***
2. Instruct the students to determine the theoretical probability of spinning each number and to write the probability as a fraction in lowest terms to complete the first column of the table: $P(1) = \frac{1}{6}$, $P(2) = \frac{1}{2}$, $P(3) = \frac{1}{3}$. Then direct them to use the theoretical probability to predict the expected results of each event for 12 spins. $P(1) = \frac{1}{6} \times 12 = 2$, $P(2) = \frac{1}{2} \times 12 = 6$, $P(3) = \frac{1}{3} \times 12 = 4$ or $\frac{1}{6} = \frac{n}{12}$; $\frac{1}{2} = \frac{n}{6}$; $\frac{1}{3} = \frac{n}{12}$
3. Instruct the students to spin the paper clip 12 times and to record each result by drawing a tally in the Actual Results column of the table.
   - ➤ **Are your actual results the same as your expected results? Why?** ***Answers will most likely be that the actual results differed from the expected results; elicit that every experiment has variables.***
   - ➤ **How can you use the actual results to calculate the experimental probability?** ***Divide the actual results (the number of favorable results or the frequency) for each event by the number of trials (12).***
4. Direct each student to calculate the experimental probability of spinning each number and then to record it on his table.

   Lead a discussion about the differences and similarities of the probabilities.
   - ➤ **If 3 players were each to pick a different number on the spinner, and the player whose number was spun most often won, would "Spin 1, 2, or 3" be a fair or an unfair game? Why?** ***Unfair; the probabilities of spinning each of the numbers are not equally likely:*** $P(1) = \frac{1}{6}$; $P(2) = \frac{3}{6} = \frac{1}{2}$; $P(3) = \frac{2}{6} = \frac{1}{3}$.
5. Follow a similar procedure to guide the students in completing the "Even or Odd Spin" table: $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$, $P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$; each expected result is 10; the actual results and the experimental probabilities will vary.
   - ➤ **If 1 player was to pick odd numbers and another player was to pick even numbers, and the player whose numbers were spun most often won, would "Even or Odd Spin" be a fair or an unfair game? Why?** ***Fair; the probabilities of spinning an even or an odd number are equally likely:*** $P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$; $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$.

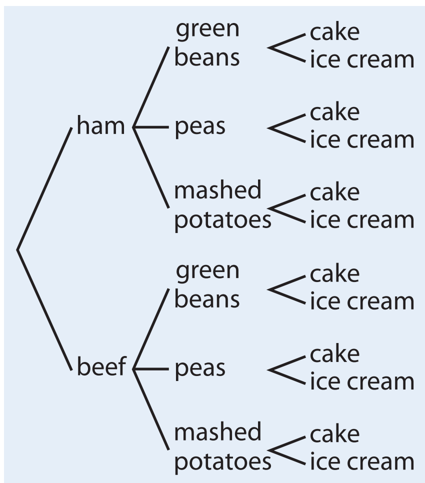
Find the complement of the probability of an event

Direct attention to the spinner on the page.
- ➤ **What is the complement of spinning 2 on the spinner? How do you know?** ***Not spinning 2; elicit that complementary events cannot occur at the same time; either 2 will be spun or 2 will not be spun.***
- ➤ **If $P(2)$ is $\frac{1}{2}$, what is $P(not\ 2)$? How do you know?** $\frac{1}{2}$; $P(2) + P(\text{not } 2) = 1$, so $1 - P(2) = P(\text{not } 2)$; $1 - \frac{1}{2} = \frac{1}{2}$
- ➤ **If $P(3)$ is $\frac{1}{3}$, what is $P(not\ 3)$?** $\frac{2}{3}$
- ➤ **If $P(1)$ is $\frac{1}{6}$, what is $P(not\ 1)$?** $\frac{5}{6}$
- ➤ **If $P(4)$ for rolling a 1–6 number cube is 17%, what is $P(not\ 4)$? How do you know?** ***83%; 100% − 17% = 83%***

Make a tree diagram to list the sample space; find the number of possible outcomes using the Multiplication Counting Principle

> For dinner some students can choose either ham or beef for their meat; green beans, peas, or mashed potatoes for their vegetable; and cake or ice cream for dessert. How many different combinations of meals do the students have to choose from?

1. Elicit the number of choices that the students have for each part of their dinner: meat ***2***, vegetable ***3***, and dessert ***2***.
   - ➤ **What equation can you write to find the number of possible combinations of a meat, a vegetable, and a dessert?** $2 \times 3 \times 2 =$ __
   - ➤ **How many possible combinations are there?** ***12***
   - ➤ **What can help you to list the sample space when choices are given?** ***Make a tree diagram.***
2. Guide the students in making a tree diagram and using it to list the sample space. ***{hgc, hgi, hpc, hpi, hmc, hmi, bgc, bgi, bpc, bpi, bmc, bmi}***



CHAPTER 16 REVIEW

A marble is drawn from the pictured bag.
Write the probability of the event as a fraction and as a percent.

1. P(green) $\frac{1}{10}$; 10%
2. P(red) $\frac{4}{10}$; 40%
3. P(either red or blue) $\frac{7}{10}$; 70%
4. P(blue) $\frac{3}{10}$; 30%
5. P(yellow) $\frac{2}{10}$; 20%
6. P(*not* green) $\frac{9}{10}$; 90%

Use the spinner to find the probability of the event.
Write it as a fraction in lowest terms and as a percent.

7. P(blue) $\frac{1}{3}$; 33%
8. P(green) $\frac{1}{6}$; 17%
9. P(red and odd) $\frac{1}{3}$; 33%
10. P(*not* blue) $\frac{2}{3}$; 67%
11. P(red) $\frac{1}{2}$; 50%
12. P(odd) $\frac{1}{2}$; 50%
13. P(red or blue) $\frac{5}{6}$; 83%
14. P(multiple of 2) $\frac{1}{2}$; 50%

The probability of event A is given. Find the complement, *P(not A)*.

15. P(A) = $\frac{1}{4}$ $\frac{3}{4}$
16. P(A) = $\frac{1}{5}$ $\frac{4}{5}$
17. P(A) = 20% 80%

Make a list or a tree diagram to find the possible outcomes.
Write the probability as a fraction in lowest terms.

18. Moriah has a spinner with 2 equal sections labeled A and B and another spinner with 3 equal sections labeled 1, 2, and 3. If she spins both spinners, what is the probability of the outcome being B and an odd number? $\frac{1}{3}$
19. Reuben has a spinner with 3 equal sections labeled A, B, and C and another spinner with 3 equal sections labeled 1, 2, and 3. If he spins both spinners, what is the probability of the outcome being A and an odd number? $\frac{2}{9}$
20. Luis has a spinner with 2 equal sections labeled A and B and a number cube labeled 1–6. If he rolls the number cube and spins the spinner, what is the probability of the outcome being A and an even number? $\frac{1}{4}$
21. A spinner has 2 equal sections of red (*r*) and blue (*b*). If you spin the spinner 2 times, what is the probability of landing on blue exactly 2 times? $\frac{1}{4}$

Make a list or a tree diagram to find the sample space for the game.
Write the probability as a fraction in lowest terms. Determine whether the game is fair or unfair.

22. A spinner has 2 equal sections of yellow (*y*) and green (*g*). Spin the spinner 2 times. Player A wins if the spinner lands on yellow 2 times. Player B wins if it lands on green at least 1 time. What is the probability of both spins being yellow? What is the probability of at least one spin being green? Is this game fair or unfair?
P(*yellow and yellow*) = $\frac{1}{4}$
P(*at least 1 green*) = $\frac{3}{4}$; *unfair*
23. A 1–6 number cube is rolled once. Player A wins if the number is less than 4. Player B wins if the number is greater than 3. What is the probability of rolling a number less than 4? What is the probability of rolling a number greater than 3? Is this game fair or unfair?
P(*is less than 4*) = $\frac{1}{2}$
P(*is greater than 3*) = $\frac{1}{2}$; *fair*

368 Chapter 16

Use the Multiplication Counting Principle to find the number of possible outcomes.

24. Damien is buying an ice-cream sundae. He has a choice of ice cream (vanilla, chocolate, or strawberry), a choice of syrup (chocolate, caramel, or strawberry), and a choice of one topping (nuts, chocolate chips, gummy bears, or whipped cream). How many different combinations does Damien have to choose from?
3 ice creams × 3 syrups × 4 toppings = 36 combinations
25. Elisa is buying a milkshake. She has a choice of ice cream (vanilla or chocolate) and a choice of add-ins (candies, cookie crumbles, mint pieces, chocolate chips, or caramel). How many different combinations does Elisa have to choose from?
2 ice creams × 5 add-ins = 10 combinations

Use the menu to find the answer.

Pasta Palace		
Pasta	Type	Sauce
Spaghetti	Egg	Classic Italian
Linguini	Wheat	Alfredo
Fettuccine	Spinach	Pesto

26. At the Pasta Palace, you can choose the shape and type of pasta, as well as the sauce. How many different combinations do the customers have to choose from?
3 pastas × 3 types × 3 sauces = 27 combinations
27. What is the probability that a customer will order pasta made with spinach? $\frac{9}{27} = \frac{1}{3}$ *or 33%*
28. What is the probability that a customer will order pasta with the Classic Italian sauce? $\frac{9}{27} = \frac{1}{3}$ *or 33%*
29. What is the probability that a customer will order egg fettuccine with Alfredo sauce? $\frac{1}{27}$ *or 4%*

Write the probability as a percent.
Round to the nearest whole percent.

30. Mateo scored a field goal in 6 out of 10 games. What is the probability that he will score a field goal in today's game? $\frac{6}{10}$ = 60%
31. There are 11 boys and 14 girls in the school choir. What would be the probability of drawing a girl's name from a container having the names of all the choir members? $\frac{14}{25}$ = 56%
32. Mrs. Reed surveyed her class and found that 6 out of 20 students wear glasses. Find the complement of students *not* wearing glasses.
32. P(*glasses*) = $\frac{6}{20}$
P(*not glasses*) = $\frac{14}{20}$ = 70%
33. Lisa bought a bag of beads to make jewelry. All the beads are the same size and shape. There are 3 silver beads, 7 gold beads, and 5 white beads. What is the probability that the first bead Lisa pulls out of the bag will *not* be gold? $\frac{8}{15}$ ≈ 53%

Use the survey results to find the probability of favorite books.
Write the probability as a fraction and as a percent.

Survey Results (sample = 20 people)	
Book Type	Results
Mystery	8
Historical Fiction	6
Fantasy	4
Nonfiction	2

34. P(mystery) $\frac{8}{20}$ or $\frac{2}{5}$; 40%
35. P(historical fiction) $\frac{6}{20}$ or $\frac{3}{10}$; 30%
36. P(fantasy) $\frac{4}{20}$ or $\frac{1}{5}$; 20%
37. P(nonfiction) $\frac{2}{20}$ or $\frac{1}{10}$; 10%

Use the survey results to predict the type of book chosen in a group of 100 people.

38. How many people would choose mysteries as their favorite type of book? $\frac{8}{20} = \frac{n}{100}$; *n* = 40 *people*
39. How many people would choose historical fiction? $\frac{6}{20} = \frac{n}{100}$; *n* = 30 *people*
40. How many people would choose nonfiction? $\frac{2}{20} = \frac{n}{100}$; *n* = 10 *people*

Lesson 153 369

➤ **What is the probability of a student choosing a meal with ice cream for dessert? Why?** **P(*ice cream*) = $\frac{6}{12} = \frac{1}{2}$, 0.5, 50%;** ***6 out of the 12 possible meal combinations (outcomes) include ice cream for dessert.***

➤ **What is the probability of a student choosing ham, peas, and cake for dinner?** **P(*ham, peas, cake*) = $\frac{1}{12}$, 0.08, 8%**

Make predictions using probability

Samuel is hoping his parents will let him use a spinner to determine what time he must to go to bed at night. He divided the spinner into 3 equal sections and labeled the sections 9:00, 10:00, and 11:00. If his parents agree to use the spinner, will it give Samuel more opportunities to stay up until 11:00?

1. Direct the students to draw the spinner on paper and determine the probability of each outcome.
P(*9:00*) = $\frac{1}{3}$, P(*10:00*) = $\frac{1}{3}$, P(*11:00*) = $\frac{1}{3}$

➤ **Will this spinner provide Samuel with a greater opportunity to stay up until 11:00 than 9:00 or 10:00? Why?** ***No; all the outcomes are equal.***

2. Instruct the students to draw a different spinner that includes the same times (9:00, 10:00, 11:00) but will give Samuel a greater opportunity to stay up until 11:00. ***Spinners will vary, but the size of the space for landing on 11:00 must indicate a greater probability than the probability of landing on 9:00 or 10:00.***

3. Write *red* ||, *yellow* |, *blue* ||||, and *green* ||| for display. Explain that 10 students were asked to identify which of these four colors is their favorite. The tallies indicate the favorite color choices of the 10 students that were surveyed.

➤ **What is the sample space for the survey?** ***red, yellow, blue, green***

4. Remind students that the probability calculated using the data gathered from a small sample of people can be used to make predictions about a larger population of the same people. Instruct the students to use the data from the survey to predict the expected results if 100 students were surveyed: red $\frac{1}{5} \times 100 = 20$; yellow $\frac{1}{10} \times 100 = 10$; blue $\frac{2}{5} \times 100 = 40$; green $\frac{3}{10} \times 100 = 30$.

5. Direct the students to use the data from the survey results of 10 students to predict the expected results for each color if 40 students were surveyed: red $\frac{1}{5} \times 40 = 8$; yellow $\frac{1}{10} \times 40 = 4$; blue $\frac{2}{5} \times 40 = 16$; green $\frac{3}{10} \times 40 = 12$.

Student Text pp. 368–69

Lesson
154 Student Text pp. 370–73

Chapter 16 Test

Cumulative Review

For a list of the skills reviewed in the Cumulative Review, see the Lesson Objectives for Lesson 154 in the Chapter 16 Overview on page 356 of this Teacher's Edition.

Student Materials

- Cumulative Review Answer Sheet, page IA9 (CD)

Use the Cumulative Review on Student Text pages 370–72 to review previously taught concepts and to determine which students would benefit from your reteaching of the concepts. To prepare the students for the format of achievement tests, instruct them to work on a separate sheet of paper, if necessary, and to mark the answers on the Cumulative Review Answer Sheet.

Read aloud the Career Link on Student Text page 373 (page 371 of this Teacher's Edition) and discuss the value of math as it relates to a firefighter.

CUMULATIVE REVIEW

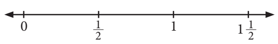
Test Prep

Mark the answer.

1. What is 10^6 in standard form?
   A. 10,000 C. 1,000,000
   B. 100,000 D. none of the above

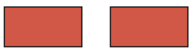
2. Round 67.39788 to the nearest ten thousandth.
   A. 67.3979 C. 67.3988
   B. 67.3980 D. none of the above

3. The fraction $\frac{13}{20}$ is closest to what point on the number line?



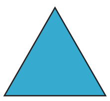
   A. 0 C. 1
   B. $\frac{1}{2}$ D. $1\frac{1}{2}$

4. Which best describes the figures?



   A. obtuse angles C. congruent figures
   B. similar figures D. all of the above

5. What two words identify the type of triangle?
   A. acute, equilateral
   B. right, isosceles
   C. scalene, obtuse
   D. none of the above



6. How many pieces of ribbon 12.5 cm long can Jaclyn cut from a piece 145 cm long?
   A. 11 pieces C. 12 pieces
   B. 13 pieces D. 14 pieces

7. Leah paid $25.95 for a new coat. If she gave the cashier forty dollars, what was her change?
   A. $12.05 C. $13.05
   B. $14.05 D. $11.05

8. Grandfather's house is $104\frac{7}{10}$ km from Finn's house. If Finn's family drove $60\frac{1}{2}$ km before lunch, how many more kilometers will they need to drive after lunch to reach Grandfather's house?
   A. $42\frac{1}{2}$ km C. $43\frac{1}{10}$ km
   B. $44\frac{1}{5}$ km D. $44\frac{4}{5}$ km

9. How many hours are in 420 minutes?
   A. 6 hours C. 7 hours
   B. 8 hours D. 9 hours

10. What is the mean temperature for the park for the given months?

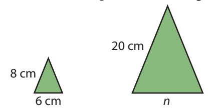
Monthly Average High Temperature for Bryce Canyon National Park

May	June	July	August	September
63°F	75°F	80°F	77°F	72°F

   A. 72.8°F C. 77.3°F
   B. 73.4°F D. 78°F

Mark the answer.

11. Use a proportion to find the unknown measure for the pair of similar figures.



   A. n = 15 cm C. n = 24 cm
   B. n = 32 cm D. none of the above

12. Rename $\frac{32}{48}$ in lowest terms.
   A. $\frac{1}{3}$ C. $\frac{2}{5}$
   B. $\frac{3}{7}$ D. $\frac{2}{3}$

13. What is the least common multiple of 14 and 21?
   A. 35 C. 42
   B. 49 D. 60

14. $\frac{9}{10} \div 3$
   A. $\frac{1}{10}$ C. $\frac{3}{5}$
   B. $\frac{3}{10}$ D. $\frac{1}{2}$

15. Estimate the sum.
   $1\frac{6}{14} + 2\frac{7}{12}$
   A. 1 C. 4
   B. 2 D. 5

16. Find an equivalent ratio for $\frac{11}{12}$.
   A. $\frac{30}{36}$ C. $\frac{55}{56}$
   B. $\frac{44}{48}$ D. $\frac{90}{100}$

17. $25 - 7 \times 2 + 44$
   A. 35 C. 50
   B. 40 D. 55

18. $34\overline{)6,908}$
   A. $203\frac{6}{34}$ C. $213\frac{3}{34}$
   B. $203\frac{3}{17}$ D. $230\frac{1}{34}$

19. 100×365.93
   A. 36,593 C. 3.5993
   B. 0.35993 D. none of the above

20. 85×6.051 g
   A. 51435 g C. 514.005 g
   B. 514.335 g D. 5143.35 g

Use the data from the stem-and-leaf plot to find the answer.

Ages of the First 44 United States Presidents at Inauguration

Stem	Leaf
4	2 3 6 6 7 7 8 9 9
5	0 1 1 1 1 1 2 2 4 4 4 4 4 5 5 5 5 6 6 6 7 7 7 7 8
6	0 1 1 1 2 4 4 5 8 9

Key 4|2 = 42

21. What is the age of the youngest president?
   A. 40 C. 49
   B. 42 D. 50

22. What age is the median?
   A. 52 C. $54\frac{1}{2}$
   B. 54 D. 55

23. What is the range of ages?
   A. 20 C. 30
   B. 27 D. 32

24. What ages show the mode?
   A. 51 and 54 C. 51 and 61
   B. 56 and 57 D. 54 and 57

25. The oldest president served 8 years as president. How old was he at the end of his term?
   A. 66 C. 75
   B. 68 D. 77

CAREER LINK

Firefighter

When you see a fire engine racing down the street to help at an accident or a fire, do you ever think about math? Probably not, but firefighters do. Knowing how to get the right amount of water flow from their trucks and hydrants is critical to minimizing the damage of a fire. Certain size hoses allow different amounts of water flow per gallon per minute. A firefighter knows how to calculate how much water flow he needs in order to select the correct hose based on the color code of a hydrant or the pumper truck he uses. The flow of water per minute varies by the color of the top of the hydrant. The firefighters at the other end of the hose need a sufficient amount of water to spray at the proper velocity; otherwise, they may find themselves unprotected from the flames. They are first in the line of danger and exposure to the heat.

A firefighter must be able to calculate the amount of friction loss that occurs in the water flow through the hoses. For every hundred feet of hose, five pounds of friction loss occurs. If the firefighter must travel up or down stairs to put out a fire, he needs to account for a gain or a loss of pounds per square inch of water pressure. Sometimes firefighters may use multiple hoses at a pumper or hydrant. Each hose has different friction loss depending on its length and diameter. The firefighter must know instinctively which to choose and how to adjust the water flow.

Saving lives, homes, and buildings is an important way to show love to members of a community. Man is made in the image of God, and a firefighter works very hard to glorify God by protecting other "image-bearers." He uses his God-given talents and abilities to encourage and comfort those who are hurt or distressed during a traumatic event in their lives.