4

Fraction Theory

HANDS ACROSS THE CHASM

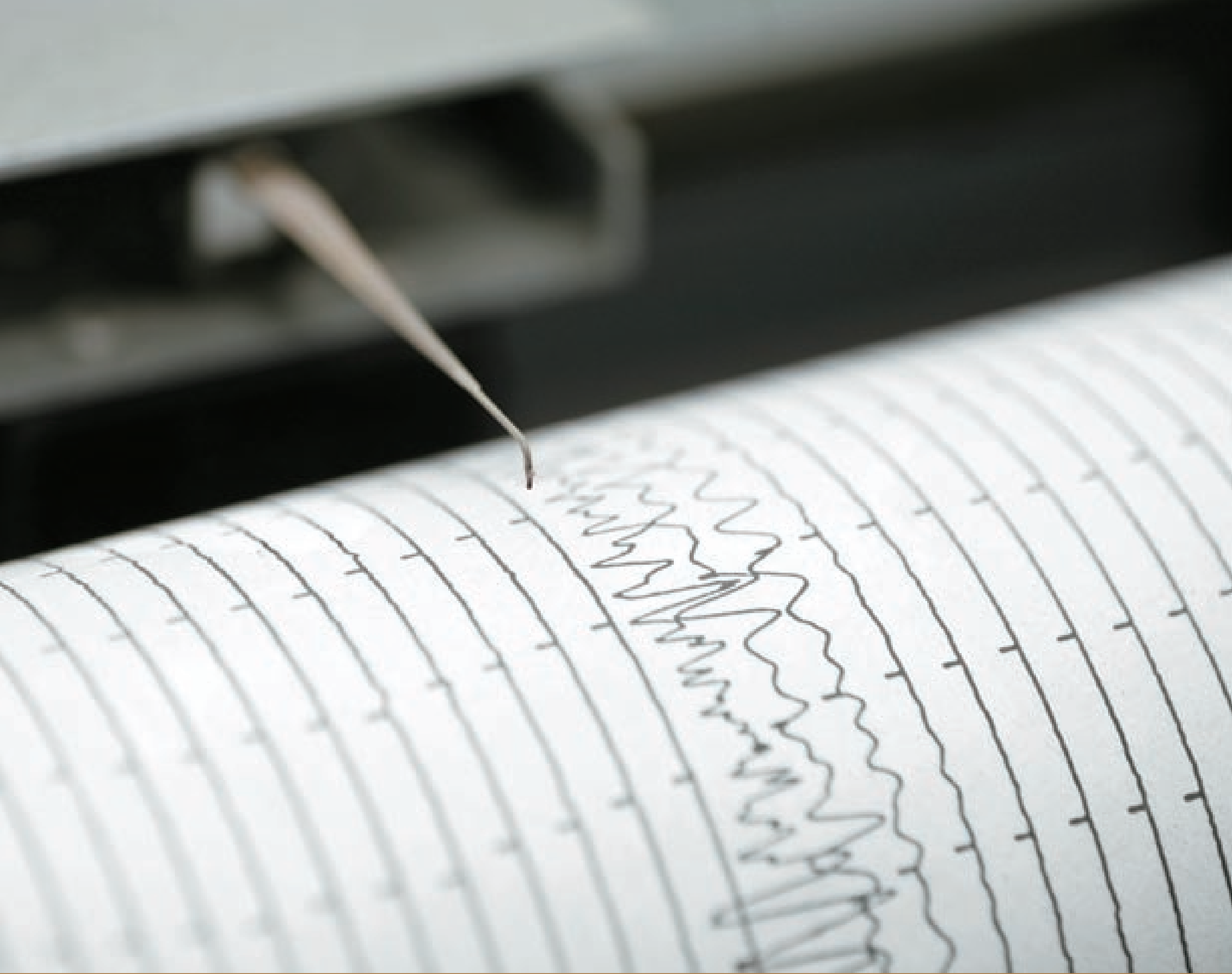
Anchorage, Alaska **March 27, 1964**

On Good Friday, March 27, 1964, people living in Anchorage, Alaska, were eagerly looking forward to the Easter weekend. At 5:30 PM, most people had already left their workplaces and were heading home. But many people lost their homes that same evening before they reached them. At 5:36, the earth's crust shifted beneath Prince William Sound, many miles east of Anchorage, causing an earthquake whose effects were felt across the nation. Anchorage suffered the most widespread destruction. Parts of the town dropped several feet below others, causing buildings and homes to topple from their foundations. Great chasms opened up, and severe landslides carried oceanfront homes into the sea. In other parts of Alaska, the damage was also great. Tsunamis struck the seaport towns of Valdez, Seward, and Kodiak. Giant tidal waves also hit parts of California, Oregon, and Hawaii. In Houston, Texas, the ground rose several inches from the aftershocks caused by the earthquake. Registering at a magnitude of 8.5 on the Richter scale, the earthquake was the strongest ever recorded in the history of North America.

Damage to Fourth Avenue, Anchorage, Alaska, 1964

The Thomas home was situated in a suburb of Anchorage on a bluff overlooking the sea. Tay Thomas and her two young children were in an upstairs bedroom watching television. When the room began to shake, Tay grabbed the children and hurried downstairs. They raced out into the front yard. As the ground trembled violently, they huddled together on the snow-covered lawn. Tay suddenly realized that a large crack was opening up in the ground between her and her daughter Anne. Terror raced through her, and she forgot all fear for her own safety. Her only thought was to keep her children with her. She reached across the chasm as it widened. Grabbing Anne, she pulled her across the chasm and cradled her in her arms. For the next few minutes they hung on while the earth heaved and rocked. When the shaking ceased, they found themselves at sea level, several feet below where their yard had once been. Nearby, only the roof of their house remained. Rocks and rubble lay all around. But they were together and safe. After praying for God's continued protection, they picked their way through the wreckage, enduring the cold until rescue workers spotted them from above and brought them up to higher ground.

While it is not humanly possible to prevent earthquakes, people are endeavoring to build safer buildings that are designed to better withstand earthquakes.



The 1964 Anchorage earthquake produced a tsunami approximately 21 feet high that traveled as far as Japan and New Zealand at a rate of 415 mph.

A Chinese astronomer and mathematician named Chang Heng developed the earliest earthquake recording device in AD 132.

A newer scale for measuring great earthquakes, the Moment Magnitude scale (Mw), recalculated the magnitude of the Anchorage earthquake as 9.2.

Of approximately 1,300,000 detectable seismic disturbances each year, about 100,000 can be felt, about 1,000 cause damage, and only 1 might be recorded as a great earthquake (8.0 or greater).

The Asian earthquake in December 2004 registered 9.0 on the Moment Magnitude scale, and the Haitian earthquake in January 2010 registered 7.0.

On March 11, 2011, a powerful 8.9 earthquake hit northeastern Japan, triggering a tsunami with waves 10 meters high.

Fraction Theory

Lesson	Topic	Lesson Objectives
31	**Greatest Common Factor**	• Determine the greatest common factor (GCF) of two or more numbers by listing the factors, by creating and analyzing a Venn diagram, and by constructing factor trees to find and evaluate the prime factorizations • Apply the GCF to problem-solving situations
32	**Least Common Multiple**	• Determine the GCF of two numbers • Determine the Least Common Multiple (LCM) of two or more numbers by listing the multiples, by creating and analyzing a Venn diagram, and by constructing factor trees to find and evaluate the prime factorizations • Write prime factorizations using exponential notation • Apply the LCM to problem-solving situations
33	**Proper Fractions**	• Demonstrate an understanding of fractions • Write a fraction to name part of a whole, a point on a number line, and part of a set • Draw models of whole shapes, whole sets, and number lines to represent fractions • Identify fractions equivalent to 1 • Complete a fraction model
34	**Improper Fractions & Mixed Numbers**	• Rename a mixed number as an improper fraction • Rename an improper fraction as a whole number or a mixed number • Estimate the value of an improper fraction • Draw a model to solve a word problem
35	**Equivalent Fractions**	• Apply strategies to rename fractions to higher terms • Apply strategies to rename fractions to lower terms and to lowest terms • Use cancellation to rename fractions to lowest terms
36	**Compare & Order Fractions**	• Write an inequality to express unequal relationships • Apply fraction number sense to compare and order fractions • Compare and order unlike fractions by renaming to fractions with a common denominator • Determine equivalent fractions using the LCM • Compare and order mixed numbers and improper fractions
37	**More Comparing Fractions**	• Write an inequality to express unequal relationships • Apply fraction number sense to compare and order fractions • Compare and order fractions by renaming unlike fractions to fractions with a common denominator, by cross-multiplying, and by renaming as decimals
38	**Fractions & Percents**	• Use a fraction model to represent a percent • Write a percent as a fraction in lowest terms • Write a fraction as a percent • Use a circle graph to solve problems • Make a circle graph to communicate data
39	**Chapter 4 Review**	• Review
40	**Chapter 4 Test Cumulative Review**	• Determine the equation represented by a part-whole model or an array • Recognize the expanded form or standard form of an exponent • Apply multiplication properties • Write the expanded form of a number using exponents • Determine the average • Identify all of the factors of a number • Solve word problems • Read and interpret a chart, a stem-and-leaf plot, and a double line graph

Chapter Materials

Teaching Visuals (Teacher's Toolkit CD):
- Chart 2: *Halves, Thirds, Fourths*
- Chart 3: *Sixths, Eighths*
- Chart 4: *Tenths*
- Chart 5: *Part of a Set*

Teacher Manipulatives Packet:
- Fraction Kit

Student Manipulatives Packet:
- Fraction Kit
- Ruler: Inch Ruler (eighths)

Instructional Aids (Teacher's Toolkit CD):
- Cumulative Review Answer Sheet (page IA9) for each student
- Numbers/Facts/Factors (page IA18)
- Venn Diagram: Factors (page IA19)
- Percent Circle (page IA20)

Christian Worldview Shaping (Teacher's Toolkit CD):
- Pages 10–14

Other Teaching Aids:
- A blank sheet of paper for each student and the teacher
- Several blank pages for display, for overlay with Percent Circle (page IA20)

Math 6 Tests and Answer Key

Optional (Teacher's Toolkit CD):
- Fact Review pages
- Application pages
- Calculator Activities

A Little Extra Help

Use the following to provide “a little extra help” for the student that is experiencing difficulty with the concepts taught in Chapter 4.

Determine the least common multiple (LCM) of two numbers—When finding the LCM of two numbers, guide the student in using a calculator to find the first ten nonzero multiples of one number and direct him to write them on paper. Instruct him to repeat the process for the other number. Direct the student to compare the multiples to determine the LCM of the two numbers.

Rename to lowest terms using cancellation—After a student has found the prime factorizations of the numerator and the denominator, he may have difficulty determining fractional names for 1 because the factors are out of order. Instruct him to order the factors from least to greatest and to place like numbers above and below each other. This approach will help him to easily see which numbers can be cancelled because they are equivalent to 1.

$$\frac{3 \times 3 \times 5 \times 2}{2 \times 2 \times 5 \times 3} = \frac{2 \times \quad\;\; 3 \times 3 \times 5}{2 \times 2 \times 3 \times \quad\;\; 5}$$

Math Facts

Throughout this chapter, review addition, subtraction, multiplication, and division facts using Fact Review pages or a Fact Fun activity on the Teacher’s Toolkit CD, or you may use flashcards.

Lesson 31 | **Student Text** pp. 74–77 | **Daily Review** p. 414a

Objectives
- Determine the greatest common factor (GCF) of two or more numbers by listing the factors, by creating and analyzing a Venn diagram, and by constructing factor trees to find and evaluate the prime factorizations
- Apply the GCF to problem-solving situations

Teacher Materials
- Numbers/Facts/Factors, page 18 (CD)
- Venn Diagram: Factors, page 19 (CD)

Notes
Preview the Fact Review pages, the Application pages, and the Calculator Activities located on the Teacher's Toolkit CD.

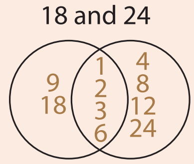
In preparation for dividing by the GCF to rename a fraction to lowest terms, the students will practice various methods for determining the GCF of two or more whole numbers in this lesson.

Introduce the Lesson
Guide the students in reading aloud the story and facts on pages 74–75 of the Student Text (pages 72–73 of this Teacher's Edition).

Teach for Understanding
Determine the greatest common factor (GCF)
➤ **What is a factor?** ***one of the numbers multiplied to find a product***

1. Display the Numbers/Facts/Factors page.
   ➤ **What multiplication fact do you know that has a product of 1?** ***1 × 1***
   Write 1 × 1 in the Facts column to the right of the number 1.
   ➤ **What are the factors of 1?** ***1*** Write 1 in the Factors column.
2. Ask similar questions to complete the table. Model how to examine each counting number as a pair of factors, beginning with 1. Write each pair of factors for the number. Explain that when you come to a factor that has already been listed, you have listed all the factors in the facts (because of the Commutative Property).
3. Remind the students that counting numbers, or natural numbers (i.e., 1, 2, 3, . . .), are either prime or composite. Elicit that a *prime number* is greater than 1 and has exactly 2 factors, 1 and the number itself; a *composite number* is greater than 1 and has more than 2 factors. Point out that 1 is a factor of every number and is neither prime nor composite.
   ➤ **What numbers listed in the table are prime? Why?** ***2, 3, 5, 7, 11; each has only two factors, 1 and itself.***
   Choose students to state facts without 1 as a factor to prove that 4, 6, 8, 9, 10, and 12 are composite. ***Accept any correct facts, allowing students to apply the Commutative Property.***
4. Direct attention to the factors of 8 and 12.
   ➤ **Which factors are common factors of 8 and 12?** ***2 and 4***
   ➤ **Which common factor of 8 and 12 has the greatest value?** ***4***
   Write for display *GCF of 8 and 12: 4*. Remind the students that GCF is the abbreviation for *greatest common factor.* Explain that the greatest common factor (GCF) of 2 numbers is never more than the value of the lesser number because the GCF is a factor of both numbers. Point out that the GCF of any 2 or more numbers is greater than 1 but less than or equal to the number with the least value.
5. Direct attention to the factors of 3 and 9.
   ➤ **What is the GCF of 3 and 9?** ***3*** Write *GCF of 3 and 9: 3* below the previous statement.
   ➤ **Is the GCF of 3 and 9 greater than 1 but less than or equal to the lesser number 3?** ***yes*** **Is 9 a multiple of 3?** ***yes***
   Point out that when the greater number is a multiple of the lesser number, the lesser factor is the GCF of the 2 numbers.
6. Write for display 6, 9, 12 and 4, 8, 12. Elicit the GCF for each set of numbers ***3; 4*** and write *GCF of 6, 9, 12: 3* and *GCF of 4, 8, 12: 4*.
   Explain that since multiplication and division are inverse operations, 2 or more numbers are divisible by any of their common factors, including the GCF. The GCF is the greatest number that any 2 or more numbers are divisible by.
7. Display the Venn Diagram: Factors page. Explain that a Venn diagram pictures the relationship between sets. Common or shared information is placed in the intersecting parts of the circles, while differing information, specific to each set, is placed in the nonintersecting part of the appropriate circle.
   ➤ **What are the factors of 18?** ***1, 2, 3, 6, 9, 18*** **the factors of 24?** ***1, 2, 3, 4, 6, 8, 12, 24*** List the factors in order at the top of the transparency.
   ➤ **What are the common factors of 18 and 24?** ***1, 2, 3, 6*** Circle each pair of common factors.
   ➤ **Where do you write the common factors in a Venn diagram? Why?** ***In the intersecting or overlapping section of the circles; it is shared information.*** Choose a student to write the common factors in the Venn diagram.

Factors of 18 and 24



   ➤ **Where do you write the factors that are not common to 18 and 24?** ***in the nonintersecting section of the appropriate circle*** Choose students to write the uncommon factors in the Venn diagram.
   ➤ **What is the GCF of 18 and 24 as shown in the Venn diagram? How do you know?** ***6; it has the greatest value of the common factors in the intersecting portion of the diagram.***
8. Direct the students to make a Venn diagram to show the relationship between the factors 15 and 25.
   15: ***1, 3, 5, 15*** 25: ***1, 5, 25*** common factors: ***1, 5***
   ➤ **What is the GCF of 15 and 25?** ***5***
9. Direct attention to the factor trees on Student Text page 76. Explain that you can also use *prime factorization* to determine the GCF. The pairs of factors are written below each composite factor in a factor tree until all of the factors are prime, i.e., the prime factorization of the number. Point out that the prime factorization of a number is always the same, regardless of which factors you choose to begin the factor tree.
10. Guide the students in constructing factor trees for 18 ***2 • 3 • 3*** and 24 ***2 • 2 • 2 • 3***.
    Explain that a number's factors can be found by grouping the prime factors into various sets to form the basic facts of the number. Point out that you must use all of the prime factors each time you group them.
    ➤ **How can you group the prime factors of 18 to make a factor pair of 18?** ***Elicit 2 × (3 × 3) and (2 × 3) × 3.***

Greatest Common Factor

When two or more composite numbers have a factor or factors that are the same, each shared factor is called a **common factor**. The **greatest common factor (GCF)** is the greatest factor common to these numbers. The greatest common factor can be found by **listing factors** of each number.

common factor
greatest common factor (GCF)
listing factors
prime factors
factor tree
prime factorization

List the Factors

28: 1 × 28, 2 × 14, 4 × 7 — **28:** 1, 2, 4, 7, 14, 28
40: 1 × 40, 2 × 20, 4 × 10, 5 × 8 — **40:** 1, 2, 4, 5, 8, 10, 20, 40

common factors of **28** and **40:** 1, 2, 4
greatest common factor of **28** and **40:** 4

A composite number can be expressed as the product of a set of **prime factors**. A **factor tree** can be used to find the **prime factorization** of a composite number.

1. Write the number to be factored at the top.
2. Choose any pair of factors.
3. Continue to factor any composite number until all factors are prime.

Every number has only one set of prime factors.

Ways to Factor 40

40	40	40
5 • 8	4 • 10	2 • 20
5 • 2 • 4	2 • 2 • 2 • 5	2 • 2 • 10
5 • 2 • 2 • 2		2 • 2 • 2 • 5

40 = 2 • 2 • 2 • 5

Prime factorization can be used to find the greatest common factor (GCF) of two numbers.

1. List the prime factors of each number in ascending order.
2. Select the factors that are common to both lists.
3. Multiply the common factors.

40	60
4 • 10	6 • 10
2 • 2 • 2 • 5	2 • 3 • 2 • 5

40: 2 • 2 • 2 • 5
60: 2 • 2 • 3 • 5

GCF of 40 and 60: 2 • 2 • 5 = 20

The prime factors of a number can be grouped to show all of the factor pairs or factors of that number.

Prime factors of 40: 40 = 2 • 2 • 2 • 5

Factors of 40: 1, 2, 4, 5, 8, 10, 20, 40

1 × (2 × 2 × 2 × 5) = 1 × 40
2 × (2 × 2 × 5) = 2 × 20
(2 × 2) × (2 × 5) = 4 × 10
(2 × 2 × 2) × 5 = 8 × 5

1 × the prime factorization of a number equals the number.

Exercises

Find the greatest common factor (GCF) by listing the factors of each number.

1. 18 and 24 *GCF: 6*
   18: 1, 2, 3, 6, 9, 18
   24: 1, 2, 3, 4, 6, 8, 12, 24
2. 12 and 20 *GCF: 4*
   12: 1, 2, 3, 4, 6, 12
   20: 1, 2, 4, 5, 10, 20
3. 28 and 42 *GCF: 14*
   28: 1, 2, 4, 7, 14, 28
   42: 1, 2, 3, 6, 7, 14, 21, 42
4. 12 and 21 *GCF: 3*
   12: 1, 2, 3, 4, 6, 12
   21: 1, 3, 7, 21
5. 27 and 54 *GCF: 27*
   27: 1, 3, 9, 27
   54: 1, 2, 3, 6, 9, 18, 27, 54
6. 48 and 72 *GCF: 24*
   6. 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
   72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

Use the numbers in the box to complete the statement.

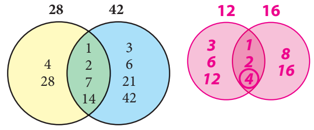
4 and 8
12 and 36
18 and 36

7. The factors 2, 4, and 6 are common factors of ___. *12 and 36*
8. The factors 3, 6, and 9 are common factors of ___. *18 and 36*
9. The numbers 16 and 24 share the common factors ___. *4 and 8*

Write the answer.

10. Use the Venn diagram to list the factors of 28 and 42. Circle the greatest common factor in the list.
    28: 1, 2, 4, 7, 14, 28; 42: 1, 2, 3, 6, 7, 14, 21, 42
11. Create a Venn diagram for the factors of 12 and 16. Circle the greatest common factor in the diagram.
    12: 1, 2, 3, 4, 6, 12
    16: 1, 2, 4, 8, 16

Factors of 28 and 42



Complete the factor tree.

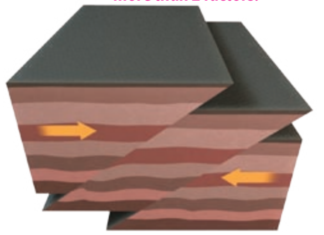
12. 42 → 6 • 7 → *2 • 3 • 7*
13. 36 → 4 • 9 → *2 • 2 • 3 • 3*
14. 24 → 3 • 8 → *3 • 2 • 4* → *3 • 2 • 2 • 2*
15. 18 → 2 • 9 → *2 • 3 • 3*

Write the answer using the completed factor trees above.

16. Group the prime factors to list facts of 24. *2 × 12, 3 × 8, 4 × 6*
17. Group the prime factors to list facts of 36. *2 × 18, 3 × 12, 4 × 9*
18. Find the GCF of 24 and 36. *12; 2 × 2 × 3*
19. Explain the difference between a prime number and a composite number. *19. A prime number is greater than 1 and has only 2 different factors—itself and 1; a composite number is greater than 1 and has more than 2 factors.*

Practice & Application

20. How many partial products does the problem 12 × 18 have? What is the product? *2; 216*
21. Estimate the product of 396 × 420. *160,000*
22. Estimate the quotient of 1,584 ÷ 17. Round the dividend and the divisor to show compatible numbers. *1,600 ÷ 20 = 80*
23. How are the numbers 4 and 16 related?
24. What is the value of the Ten Thousands place when 146 is multiplied by 100? *100 × 146 = 14,600; 10,000*



A tsunami can result from an earthquake that occurs under the ocean. The effect of one tectonic plate shifting under another causes tons of water to move in giant waves.

23. Both are divisible by 4; 4 is a factor of 16; 16 is a multiple of 4; 4 is $\frac{1}{4}$ of 16.

Use the number line to find the answer.

$^-10$ $^-9$ $^-8$ $^-7$ $^-6$ $^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0 1 2 3 4 5 6 7 8 9 10

25. $8 + {}^-5 =$ *3*
26. ${}^-3 + 7 =$ *4*
27. ${}^-2 + {}^-8 =$ *$^-10$*
28. $3 + {}^-4 =$ *$^-1$*

Complete DAILY REVIEW a on page 414.

Write the expressions for display. Point out that first calculating what is in parentheses helps you to see that the Associative Property was used to form the facts 2 × 9 and 6 × 3. Explain that for every number, the Identity Property is applied in the fact 1 × the number's prime factorization. [e.g., 1 × (2 × 3 × 3) = 1 × 18] The factors of 18 are 1, 2, 3, 6, 9, and 18.

11. Follow a similar procedure to identify the factor pairs and factors of 24. *1 × (2 × 2 × 2 × 3) = 1 × 24; 2 × (2 × 2 × 3) = 2 × 12; (2 × 2) × (2 × 3) = 4 × 6; (2 × 2 × 2) × 3 = 8 × 3; 1, 2, 3, 4, 6, 8, 12, 24*
    ➤ **What prime factors are common to 18 and 24?** *2 and 3*
12. Write for display the prime factorizations of 18 and 24. Guide the students in circling the pairs of common factors. Point out that the product of the common prime factors is the GCF of 18 and 24: 2 • 3 = 6.

    18 = 2 × 3 × 3
    24 = 2 × 2 × 2 × 3

13. Direct the students to make factor trees for 28 and 44 and then to multiply the common prime factors to determine the GCF. *28 = 2 • 2 • 7 and 44 = 2 • 2 • 11, GCF of 28 and 44: 4*

Apply the GCF to problem-solving situations

There will be 18 boys and 24 girls going on the school field trip. Each chaperone will monitor a mixed group of boys and girls. The teachers want to distribute both the boys and the girls equally among the groups. What is the greatest number of groups the teachers can make? *6 groups*

➤ **What question do you need to answer?** *What is the greatest number of groups the teachers can make?*
➤ **What are the guidelines for forming each group?** *to distribute both the boys and the girls equally among the groups*

1. Allow the students to work in pairs to find an answer. Discuss their strategies and answers. *Strategies may vary, but should include using factors of 18 and 24 to find that 6 groups can be made.*
   ➤ **How can you find out how many boys and girls will be in each of the 6 groups?** *Elicit that you can divide both 18 and 24 by 6 to find that 3 boys and 4 girls will be in each group.*
2. Explain that since multiplication and division are inverse operations, the greatest number that both 18 and 24 are divisible by is also their greatest common factor. Write *GCF of 18 and 24: 6.*
3. Instruct each student to choose which strategy or strategies he prefers to solve this word problem: examine a list of factors, create a Venn diagram, or use prime factorization. Select students to explain their strategies and tell their answers.

Kaitlyn and Ava baked 36 chocolate chip cookies and 24 peanut butter cookies for the bake sale. If they package both kinds of cookies equally in bags to sell, what is the greatest number of bags the girls can fill? *12 bags; GCF of 24 and 36: 12; 3 chocolate chip cookies and 2 peanut butter cookies in each bag*

Student Text pp. 76–77

Lesson 32 | **Student Text** pp. 78–79 | **Daily Review** p. 414b

Objectives

- Determine the GCF of two numbers
- Determine the Least Common Multiple (LCM) of two or more numbers by listing the multiples, by creating and analyzing a Venn diagram, and by constructing factor trees to find and evaluate the prime factorizations
- Write prime factorizations using exponential notation
- Apply the LCM to problem-solving situations

Note

In preparation for multiplying by the LCM to rename a fraction to higher terms, the students will practice various methods for determining the LCM of two or more whole numbers in this lesson. When adding and subtracting unlike fractions in Chapter 5, the students will determine the least common denominator (LCD) by finding the LCM of the two unlike denominators.

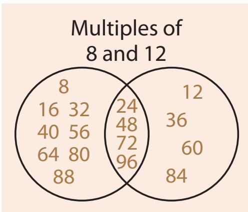
Introduce the Lesson

1. Write *GCF of 24 and 42:* __.
   ➤ **What are the factors of 24? How do you know?** ***1, 2, 3, 4, 6, 8, 12, 24; elicit that a factor of 24 is one of 2 numbers you multiply to find the product 24.*** Write the factors for display.
2. Point out that since multiplication and division are inverse operations, 24 is divisible by each of its factors.
   ➤ **What fact do you know for 42?** ***Elicit 6 × 7.***
   ➤ **Is 3 a factor of 42? How do you know?** ***Yes; elicit that since 6 is a factor of 42 and 6 is 2 sets of 3, then 3 is also a factor of 42. Also, the divisibility rule for 3 applies: the sum of the digits (4 + 2) is divisible by 3 (6 ÷ 3 = 2).***
3. Write $3 \times n = 42$ for display. Point out that you can use your understanding of prime factors and the Commutative and Associative Properties to find the missing factor: $42 = 6 \times 7 = (2 \times 3) \times 7 = 3 \times (2 \times 7) = 3 \times 14 = 42$.
   ➤ **What are the factors of 42?** ***1, 2, 3, 6, 7, 14, 21, 42*** Write the factors.
   ➤ **What are the common factors of 24 and 42?** ***1, 2, 3, 6***
   ➤ **What is the greatest common factor of 24 and 42?** ***6*** Write 6 in the answer blank.

Teach for Understanding

Determine the least common multiple (LCM)

1. Remind the students that zero is a multiple of every number; zero times any number equals zero. The nonzero multiples of a number can be found by counting by that number.
   ➤ **What are the first 10 nonzero multiples of 6?** ***6, 12, 18, 24, 30, 36, 42, 48, 54, 60*** Write 6: and the 10 multiples for display.
   ➤ **What are the first 10 nonzero multiples of 9?** ***9, 18, 27, 36, 45, 54, 63, 72, 81, 90*** Write 9: and the 10 multiples of 9 below the multiples of 6.
   ➤ **What multiples do 6 and 9 have in common?** ***18, 36, 54*** Circle the pairs of common multiples.
   ➤ **What is the least common multiple of 6 and 9?** ***18***
   Write for display *LCM of 6 and 9*: *18*. Remind the students that LCM is the abbreviation for *least common multiple*.
   Explain that since the least common multiple (LCM) is a multiple of 2 numbers, the LCM of any 2 numbers is equal to or greater than the number of greater value.
2. Write 4 and 20, 8 and 9, and 6 and 8 for display. Use the following procedure to guide the students in finding the LCMs after listing the multiples for each pair of numbers.
   ➤ **What is the LCM of 4 and 20?** ***20*** Write *LCM: 20* below the multiples of 4 and 20.
   ➤ **Is the LCM equal to or greater than the greater number (20)?** ***equal to***
   Point out that when the greater number is a multiple of the lesser number, the greater number is the LCM of both numbers.
   ➤ **What is the LCM of 8 and 9?** ***72*** Write *LCM: 72.*
3. Explain that you can find a common multiple of 2 numbers by multiplying the numbers, but a common multiple may not be the LCM.
   ➤ **What is 6 × 8?** ***48***
   ➤ **Is 48 a common multiple of 6 and 8? How do you know?** ***Yes; 6 and 8 are common factors of 48.***
   ➤ **Is 48 the least common multiple of 6 and 8?** ***No; elicit that the LCM of 6 and 8 is 24.*** Write *LCM = 24*.
   Explain that when the greater number is *not* a multiple of the lesser number, the LCM of the 2 numbers is greater than the greater number and less than or equal to the product of the 2 numbers.
4. Write *LCM of 3 and 5*: __. Elicit that since 5 is the greater number and 15 is the product of the 2 numbers, the LCM is greater than 5 and less than or equal to 15. Explain that you can count by the greater number until you come to a number that is also a multiple of the lesser number.
   Choose a student to count by 5s. Direct the class to say "stop" when the student names a multiple of 5 that is also a multiple of 3. ***15*** Write 15 in the blank.
5. Write *LCM of 6 and 10*: __.
   ➤ **What is the range of the possible LCM of 6 and 10? How do you know?** ***11–60; the LCM will be greater than the greater number (10), and less than or equal to the product of the 2 numbers (60).***
   ➤ **Do you need to list multiples greater than 60 to find the LCM? Why?** ***No; elicit that the LCM of 6 and 10 will not be greater than the product of the 2 numbers (60).***
   Direct the students to find the LCM of 6 and 10 by listing the multiples of both numbers. ***30*** Write 30 in the blank.
6. Write *LCM of 8 and 12*:__ for display. Guide the students in creating a Venn diagram for the nonzero multiples of 8 and 12 to find the LCM of 8 and 12. ***24*** (***Note:*** Do not exceed the product 96.) Remind them that the Venn diagram pictures the relationship between 8 and 12. Write 24 in the blank.



7. Guide the students in constructing factor trees for 8 ***2 • 2 • 2*** and 12 ***2 • 2 • 3***. Write the prime factorizations for display using exponential notation: $8 = 2^3$ and $12 = 2^2 \times 3$.
8. Explain that you can use the prime factorizations to calculate the LCM of 2 numbers by multiplying the highest power of each prime factor listed.
   Choose a student to write each of the prime factors of 8 and 12 in ascending order with a multiplication dot between them. ***2 • 3***

Least Common Multiple

There are an infinite number of multiples for any given number. A **multiple** is the product of two whole numbers. A **common multiple** of two numbers can always be found by multiplying the two numbers together.

multiple
common multiple
least common multiple (LCM)
prime factorization with exponents

A common multiple of 6 and 9 is 54 because 6 × 9 = 54.
The following list shows the nonzero multiples of 6 and 9 *less than* 54.
6: 6, 12, 18, 24, 30, 36, 42, 48
9: 9, 18, 27, 36, 45
The common multiples of 6 and 9 are **18** and **36**.
The **least common multiple (LCM)** of 6 and 9 is **18**.

The LCM of any two numbers is *equal to* or *greater than* the greater number.

When the greater number is a multiple of the lesser number, the LCM is *equal to* the greater number.

12: 12, 24, 36, 48, 60, 72
24: 24, 48, 72, 96

24 is a multiple of 12; the LCM of 12 and 24 is 24.

When the greater number is *not* a multiple of the lesser number and the two numbers share a common factor, the LCM is *greater than* the greater number and *less than* the product of the two numbers.

12: 12, 24, 36, 48, 60, 72
18: 18, 36, 54, 72

18 is not a multiple of 12, but 12 and 18 share the common factor 3; the LCM is *greater than* 18 and *less than* the product of 12 × 18 (216); the LCM of 12 and 18 is 36.

Methods for Finding the Least Common Multiple

For smaller numbers or numbers that are easy to count by, list the multiples. (*Hint*: List multiples for the greater number first until you list a number that is also a multiple of the lesser number.)

20: 20, 40, 60
15: 15, 30, 45, 60
LCM of 15 and 20: 60

For numbers that are not easy to list the multiples of, make factor trees. (*Hint*: Write the **prime factorization with exponents** and then multiply the highest power of each prime factor listed.)

24
$4 \cdot 6$
$2 \cdot 2 \cdot 2 \cdot 3$

36
$4 \cdot 9$
$2 \cdot 2 \cdot 3 \cdot 3$

24: $2^3 \cdot 3$
36: $2^2 \cdot 3^2$
LCM of 24 and 36: $2^3 \cdot 3^2 = 72$

Exercises

1. *4: 4, 8, 12 6: 6, 12* 2. *9: 9, 18, 27, 36 12: 12, 24, 36* 3. *6: 6, 12, 18, 24, 30, 36, 42 7: 7, 14, 21, 28, 35, 42*

Find the least common multiple (LCM) by listing multiples. *Number of multiples listed may vary.*

1. 4 and 6 *LCM: 12* **2.** 9 and 12 *LCM: 36* **3.** 6 and 7 *LCM: 42* **4.** 5 and 12 *LCM: 60* **5.** 8 and 10 *LCM: 40* **6.** 3 and 15 *LCM: 15*

Make factor trees and write the prime factorization using exponents.
Find the least common multiple (LCM). *Beginning factors may vary.*

7. 16 and 18 *$2^4 \cdot 3^2 = 16 \cdot 9$; LCM: 144* **8.** 24 and 56 *$2^3 \cdot 3 \cdot 7 = 8 \cdot 21$; LCM: 168* **9.** 14 and 25 *$5^2 \cdot 2 \cdot 7 = 25 \cdot 14$; LCM: 350* **10.** 13 and 21 *$3 \cdot 7 \cdot 13 = 21 \cdot 13$; LCM: 273*

Choose the LCM of the numbers.

11.	3 and 7	7	14	21
12.	12 and 36	36	72	432
13.	4, 8, and 10	20	40	80

4. *5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60 12: 12, 24, 36, 48, 60*
5. *8: 8, 16, 24, 32, 40 10: 10, 20, 30, 40*
6. *3: 3, 6, 9, 12, 15 15: 15*

78 Chapter 4

Write the composite number represented by the prime factorization.

14. $2^2 \cdot 3 \cdot 5$ *60* **15.** $2^2 \cdot 5 \cdot 7$ *140* **16.** 5^2 *25*
17. $2 \cdot 3 \cdot 7$ *42* **18.** $3 \cdot 5$ *15* **19.** $3^2 \cdot 11$ *99*

Make a factor tree for the number. *Beginning factors may vary.*

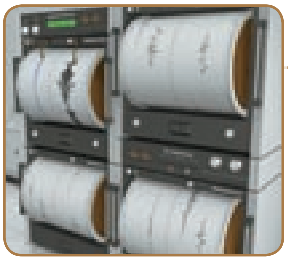
20. 12 **21.** 16 **22.** 56 **23.** 72

Use the factor trees to find the answer. *The factor 1 is optional.*

24. What factors are common to 12 and 16? *2, 4*
25. GCF of 12 and 16: *4* **26.** LCM of 12 and 16: *48*
27. What factors are common to 16 and 56? *2, 4, 8*
28. GCF of 16 and 56: *8* **29.** LCM of 16 and 56: *112*
30. What factors are common to 12 and 72? *2, 3, 4, 6, 12*
31. GCF of 12 and 72: *12* **32.** LCM of 12 and 72: *72*

Practice & Application

33. Write the value of 8 in 708,316,290 in word form. *eight million*
34. Find the product of 23 sets of 896. *20,608*
35. What number does 2×3^3 represent? *54*
36. Write the prime factorization of 210. *$2 \times 3 \times 5 \times 7$*
37. Estimate the difference between 590,600 and 316,900 by rounding each number to its greatest place. *600,000 – 300,000 = 300,000*
38. Find the number of sets that could be made if 5,790 play tickets were bundled into sets of 30. *193 sets*
39. The number 1,530 is divisible by which of the following numbers: 2, 3, 4, 5, 10? *2, 3, 5, 10*
40. Is 391 a prime or a composite number? *prime*
41. Is 16 a multiple or a factor of 32? *16 is a factor of 32.*
J Explain the pattern used in the sequence 21, 42, 63, 84. What would be the next three numbers in the sequence? *The number 21 was added each time to find the next number; 105, 126, 147.*



DID YOU KNOW

A **Richter scale** is used to tell the strength of an earthquake. The scale is a logarithmic scale that measures the strength of earthquakes in powers of 10. An earthquake measuring 3.0 on the Richter scale is 10 times greater than an earthquake measuring 2.0, and one measuring 4.0 is 100 (10 × 10) times greater than one measuring 2.0.

Complete DAILY REVIEW b on page 414.

Lesson 32 79

➤ **How many times is 2 listed in the prime factorization of 8?** *3 times* **of 12?** *2 times* **Which is the higher power?** *3* Write the exponent 3 to the upper right of the prime factor 2: 2^3.

➤ **How many times is 3 listed in the prime factorization of 8?** *0 times* **of 12?** *1 time* **Which is the higher power?** *1* Point out that there is no need to write 1 as an exponent for 3, since any number to the first power is that number.

9. Choose a student to write the prime factorization of $2^3 \times 3$ for display and solve it. *$2 \cdot 2 \cdot 2 \cdot 3 = 24$*

   Point out that just as 8 and 12 are factors of the LCM 24, 2^3 and $(2^2 \times 3)$ are factors of $2^3 \times 3$; therefore, $2^3 \times 3 = 2 \times (2^2 \times 3)$.

10. Follow a similar procedure to find the LCM of 12 and 15 using prime factorization. *12: $2^2 \times 3$; 15: 3×5; LCM: $2^2 \times 3 \times 5 = 60$*

➤ **When might it be more efficient to find the LCM using prime factorizations instead of listing multiples?** *Accept any reasonable answers, but elicit that if you want to find the LCM of large numbers for which you don't know many multiples, it would probably be faster to use the prime-factorization strategy.*

Apply the LCM to problem-solving situations

Every 6 months, Dillon and his parents take a vacation to visit his grandparents. His cousins take a vacation to visit their grandparents every 9 months. This month, the families' vacations overlapped, so they were able to spend time together at their grandparents' house. How many months will it be until both families will again visit their grandparents at the same time? *18 months*

➤ **When will Dillon and his parents visit his grandparents again?** *in 6 months*

➤ **When will Dillon's cousins visit their grandparents again?** *in 9 months*

➤ **What question do you need to answer?** *How long will it be until both families visit again at the same time?*

1. Allow students to work in pairs to find an answer. Discuss their strategies and answers. *Strategies may vary, but should include using multiples of 6 and 9 to find the LCM of 6 and 9.*

➤ **What are the first 6 nonzero multiples of 6?** *6, 12, 18, 24, 30, 36* Write the multiples for display.

➤ **What are the first 6 nonzero multiples of 9?** *9, 18, 27, 36, 45, 54* Write the multiples for display.

➤ **What common multiples of 6 and 9 are listed?** *18 and 36* Circle the common multiples.

➤ **What is the least common multiple of 6 and 9?** *18* Write *LCM of 6 and 9: 18.*

➤ **In how many months will both families visit again at the same time?** *18 months*

2. Follow a similar procedure for visits every 4 months and 6 months *12 months*, visits every 5 months and 6 months *30 months*, visits every 2 months and 5 months *10 months*, visits every 4 months and 12 months *12 months*, and visits every 3 months, 6 months, and 8 months *24 months*.

Student Text pp. 78–79

Lesson 33	Student Text pp. 80–81
	Daily Review p. 415c

Objectives

- Demonstrate an understanding of fractions
- Write a fraction to name part of a whole, a point on a number line, and part of a set
- Draw models of whole shapes, whole sets, and number lines to represent fractions
- Identify fractions equivalent to 1
- Complete a fraction model

Teacher Materials

- Charts 2, 3, and 4: *Halves, Thirds, Fourths; Sixths, Eighths;* and *Tenths*
- Chart 5: *Part of a Set*

Student Materials

- Ruler: Inch Ruler (eighths)

Introduce the Lesson

Explain that fractions are the numbers that are used to name a part or parts of a whole (i.e., part of a whole figure, part of a set, part of a length, or a point on a number line). Remind the students that a fraction also shows the division of 2 whole numbers. Lead a discussion about how fractions are used in everyday life, emphasizing the parts of a whole (e.g., sizes of shoes; measurements of length, weight, and capacity; equal shares; gas mileage).

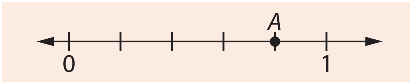
Teach for Understanding

Write a fraction to name part of a whole

Andrew purchased a pizza to share with 2 of his friends. Since 3 people will equally share this pizza, what part of a pizza will each person receive? ***$\frac{1}{3}$ of a pizza***

1. Draw a circle to represent the pizza and draw 3 stick figures to represent Andrew and his 2 friends.
   ➤ **Into how many equal parts do you need to partition or divide the whole pizza?** ***3***
   Partition the circle into 3 equal parts (thirds). Draw lines to show the distribution of 1 part of the pizza to each person.
   ➤ **What part of the pizza will each person receive?** ***1 of the 3 equal parts or $\frac{1}{3}$*** Write $\frac{1}{3}$ for display.
   ➤ **Do you think that $\frac{1}{3}$ of a pizza will be enough for each person to eat? Why?** ***Answers will vary, but elicit that the fraction does not tell the size of the whole pizza. It may be $\frac{1}{3}$ of an extra-large pizza or $\frac{1}{3}$ of a personal pizza.***
2. Draw a rectangle partitioned into thirds; shade 1 third.
   ➤ **What part of the rectangle is shaded?** ***$\frac{1}{3}$***
   Write $\frac{1}{3}$. Point out that the numerator and the denominator are the *terms* of a fraction.
   ➤ **What do the terms of a fraction tell you?** ***The denominator names the equal parts that make up the whole; the numerator tells the number of parts selected.***
3. Explain that the circle and the rectangle are enclosed areas used as fraction models of wholes. Each area is partitioned into the number of equally-sized parts named in the denominator of the fraction, and the numerator tells the number of parts selected or shaded.
4. Display the *Halves, Thirds, Fourths* chart, the *Sixths, Eighths* chart, and the *Tenths* chart to review fraction notation. Point out that in a fraction that is equivalent to 1 both terms of the fraction are the same.
5. Direct the students to draw on paper a picture to represent $\frac{5}{8}$ of an area. Choose students to draw their models for display.
   ➤ **Are all the models the same shape?** ***no*** **What is the same about all the models?** ***Each model is partitioned into 8 equal parts, and 5 of the parts are shaded.***
   Direct the students to have a partner check to see that their figure is divided into eight equal parts and that 5 of the parts are shaded.
6. Repeat the procedure for $\frac{3}{7}$.

Write a fraction to name a point on a number line

1. Draw the following number line for display.
   

   ➤ **What fractional name can you give to the parts of the length shown on the number line? Why?** ***Fifths; the length is partitioned or divided into 5 equal parts.***
   ➤ **What fraction represents point *A*? Why?** ***$\frac{4}{5}$; point A marks the fourth of the 5 equal units in the whole length.*** Choose a student to write the fraction $\frac{4}{5}$ below point *A*.
   ➤ **Is $\frac{4}{5}$ greater than or less than $\frac{1}{2}$? How do you know?** ***Greater than; elicit that $\frac{4}{5}$ is closer to 1 than 0 on the number line.***
   Select a student to trace the length of the number line that $\frac{4}{5}$ represents.
   Lead in counting the fifths from 0 to point *A*: *0 fifths, 1 fifth, 2 fifths, 3 fifths, 4 fifths.*
2. Direct each student to draw on paper a number line with a point that represents $\frac{3}{4}$ and to draw another number line with a point that represents $\frac{7}{10}$.
   ➤ **Are all the number lines with a point representing $\frac{3}{4}$ the same length? Why?** ***No; elicit that the length of the whole is not determined by the fraction and that you did not specify a length for the number line.***
   ➤ **What should be similar for all the number lines?** ***Elicit that each whole unit should be partitioned into the number of parts named by the denominator and the point should be plotted at the number of those parts being selected (indicated by the numerator).*** Allow partners to check each others' number lines.
3. Guide the students in using their inch rulers to draw a number line 2 inches long, labeling each inch (0, 1, 2), and partitioning each whole inch into eighths of an inch. Direct the students to draw a point at $\frac{5}{8}$ inch as you demonstrate.
   ➤ **Why do all of these number lines look the same?** ***Elicit that a specific unit of measurement was given for the number line.***

Write a fraction to name part of a set

1. Display the *Part of a Set* chart and call attention to the first model.
   ➤ **What fraction of the circles is blue?** ***$\frac{1}{2}$*** **red?** ***$\frac{1}{2}$***
   ➤ **What is the whole set for this model?** ***2 circles***
   Direct attention to the second model.
   ➤ **What fraction of this set is green?** ***$\frac{1}{3}$*** **red?** ***$\frac{2}{3}$***
   ➤ **How is this set different from the first set of circles?** ***In the first set, each circle had a value of $\frac{1}{2}$. In this set, each circle has a value of $\frac{1}{3}$.***
2. Follow a similar procedure to analyze the next 2 sets pictured on the chart.

Proper Fractions

Fractions are used to name part of a whole or part of a set.

numerator
denominator
terms
proper fraction

The **numerator** and the **denominator** are the **terms** of a fraction.
In the fraction $\frac{3}{5}$, the terms are 3 and 5.
The fraction $\frac{3}{5}$ is a **proper fraction** because it has a value less than 1.

3 ← **numerator:** tells the number of parts selected
5 ← **denominator:** tells the equal parts of the whole

The fraction $\frac{3}{5}$ tells about the relationship between the parts selected and the whole: the whole has 5 equal parts, and 3 of the parts have been selected.

$\frac{3}{5}$ of the whole is shaded.

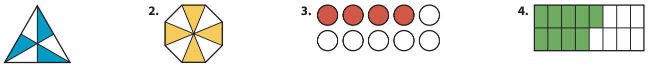
$\frac{3}{5}$ is shown on the number line.

0 1
$\frac{0}{5}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{5}{5}$

$\frac{3}{5}$ of the set is peppermints.

Exercises

Write the fraction for the part that is colored.



1. ***$\frac{3}{6}$ or $\frac{1}{2}$***
2. ***$\frac{4}{8}$ or $\frac{1}{2}$***
3. ***$\frac{4}{10}$ or $\frac{2}{5}$***
4. ***$\frac{9}{16}$***

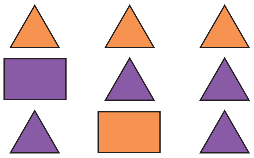
Write the fraction representing the point on the number line.

5. Point *A* ***$\frac{1}{3}$***
6. Point *B* ***$\frac{2}{3}$***
7. Point *C* ***$\frac{1}{5}$***
8. Point *D* ***$\frac{3}{5}$***

Draw a model. ***Models will vary.***

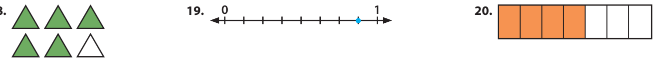
9. a number line to represent $\frac{5}{8}$
10. a circle to represent $\frac{1}{2}$
11. a set of rectangles to represent $\frac{3}{4}$
12. a set of triangles to represent $\frac{2}{7}$

Use the set of figures to find the answer.



13. The triangles are what part of the set? ***$\frac{7}{9}$***
14. The purple triangles are what part of the set? ***$\frac{4}{9}$***
15. What part of the set are the rectangles? ***$\frac{2}{9}$***
16. What part of the set is purple? ***$\frac{5}{9}$***
17. What part of the set is the orange rectangle? ***$\frac{1}{9}$***

Use the picture to write the value of *n*.



18. $\frac{n}{6}$ $n =$ ***5***
19. $\frac{7}{n}$ $n =$ ***8***
20. $\frac{n}{7}$ $n =$ ***4***

Write the fraction to answer the question.
Draw a picture if needed.

21. Three children are sharing 2 cookies. If each child gets the same amount, what fraction of a cookie can each child have? ***$\frac{2}{3}$ of a cookie***
22. Four people are sharing 3 pies so that each person has a fair share. What fraction of a pie will each person receive? ***$\frac{3}{4}$ of a pie***
23. Five people are sharing 1 bottle of juice. What fraction of the bottle of juice will each person receive? ***$\frac{1}{5}$ of a bottle***
24. Twenty students read books about earthquakes for a report. Thirteen of the students read about the Alaskan earthquake of 1964. What fraction of the students read about earthquakes other than the Alaskan earthquake? ***$\frac{7}{20}$ of the students***
25. Six of 11 girls worked on a project about the 1989 San Francisco earthquake. What fraction of the girls worked on the project? ***$\frac{6}{11}$ of the girls***
26. There are 12 boys and 15 girls in the art class. What fraction of the class is girls? ***$\frac{15}{27}$ of the class***
27. There are 206 bones in your body. Each foot has 26 bones. What fraction of the bones in your body are in your feet? ***$\frac{52}{206}$ of the bones***
28. A giraffe sleeps about 2 hours a day. What fraction of the day does a giraffe sleep? ***$\frac{2}{24}$ of the day***

Complete the fraction model. ***Models will vary.***

29. This rectangle represents $\frac{1}{2}$ of a whole. Draw a model of the whole.

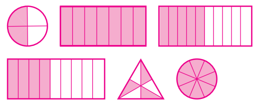
30. This rectangle represents $\frac{2}{3}$ of a whole. Draw a model of the whole.

31. These 4 positive counters represent $\frac{1}{2}$ of a set. Draw a model of the whole set.

32. These 6 negative counters represent $\frac{3}{4}$ of a set. Draw a model of the whole set.

An earthquake with a 7.0 magnitude occurred in Haiti on January 12, 2010. As part of a relief effort, the United States Air Force dropped bottled water and MREs (Meal, Ready-to-Eat) to Haitian victims.

J Draw a picture to represent each fraction: $\frac{2}{4}$, $\frac{7}{7}$, $\frac{5}{10}$, $\frac{4}{9}$, $\frac{3}{6}$, $\frac{8}{8}$. Identify the fractions that are not proper fractions and the fractions that are equivalent to $\frac{1}{2}$. Explain your answer. ***$\frac{7}{7}$ and $\frac{8}{8}$ are not proper fractions. $\frac{2}{4}$, $\frac{5}{10}$, and $\frac{3}{6}$ are equivalent to $\frac{1}{2}$.***

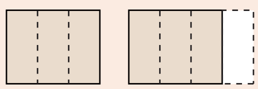


Complete DAILY REVIEW C on page 415.

3. Direct the students to draw on paper a set to represent the fraction $\frac{4}{9}$. Discuss the objects that the students drew in their sets and how 4 parts of the set differ from the other parts.
4. Explain that fractions with a value of less than 1 are *proper fractions*. They have a part-whole meaning and can be modeled using figures, number lines, lengths, and sets.

Complete a fraction model

1. Draw a shaded rectangle for display. Explain that the rectangle represents $\frac{3}{4}$ of a larger rectangle.
   - ➤ **Do all fourths look alike? Why?** ***No; elicit that a fourth can vary in size just as the partitioned figure varies in size.***
   - ➤ **Since this rectangle represents $\frac{3}{4}$ of a larger rectangle, how can you determine the size of $\frac{1}{4}$ of the larger rectangle?** ***Partition this rectangle into 3 equal parts.*** Choose a student to partition the shaded rectangle to show 3 equal parts.
   - ➤ **Since this shaded rectangle is $\frac{3}{4}$ of a larger rectangle, how can you show the size of the larger rectangle?** ***You can draw one more equally-sized part attached to the 3 shaded, partitioned parts, making a total of 4 equal parts in the whole.*** Choose a student to complete the model of the whole.



2. Draw another shaded rectangle for display and tell the students that it represents $\frac{5}{8}$ of a whole rectangle. Direct them to draw the shaded rectangle on paper and complete a model of the whole rectangle. ***Accept correct models (sizes may vary); a rectangle partitioned into 5 equal parts with 3 more equally-sized parts attached.***

3. Follow a similar procedure with a set of 3 shaded triangles that represent $\frac{1}{2}$ of a whole set. ***Correct models will include a whole set of 6 triangles with 3 unshaded.***
4. Repeat the procedure with a set of 4 circles that represent $\frac{2}{3}$ of a whole set. ***Correct models will include a whole set of 6 circles with 2 unshaded.***

Student Text pp. 80–81

Lesson 34 | **Student Text** pp. 82–83 | **Daily Review** p. 415d

Objectives
- Rename a mixed number as an improper fraction
- Rename an improper fraction as a whole number or a mixed number
- Estimate the value of an improper fraction
- Draw a model to solve a word problem

Student Materials
- Fraction Kit: fraction circles

Teach for Understanding

Rename a mixed number as an improper fraction

1. Write $1\frac{5}{8}$ for display.
   ➤ **What is a mixed number?** ***Elicit that a mixed number is a number that is made up of a whole number and a fraction.***
   Write $1 + \frac{5}{8} = 1\frac{5}{8}$ for display. Elicit that a mixed number is the sum of its whole and its fractional part.
2. Distribute the fraction circles from the Fraction Kit. Direct the students to model $1\frac{5}{8}$ by having them place 1 whole and 5 eighths on their desks.
   Draw 2 circles for display. Partition 1 circle into eighths. Shade the 1 whole circle and 5 eighths of the partitioned circle.

> Aunt Kate has some pies to serve for dessert. She has $1\frac{5}{8}$ pies. How many people can Aunt Kate serve dessert to?

➤ **What information must you know to solve the problem?** ***the size of serving for each person***
➤ **If Aunt Kate serves each person $\frac{1}{8}$ of a pie, what does she need to do with the 1 whole pie?** ***Partition the pie into eighths.***
➤ **How many eighths are in 1 whole?** ***8***

3. Direct the students to rename the 1 whole as 8 eighths using their fraction circles as you partition the 1 whole into eighths.
   ➤ **How many people can Aunt Kate serve? How do you know?** ***13 people; there are 13 eighths in $1\frac{5}{8}$.***
   Write $\frac{8}{8} + \frac{5}{8} = \frac{13}{8}$ for display. Elicit that a fraction with the same terms is equal to 1 (e.g., $\frac{8}{8} = 1$, $\frac{5}{5} = 1$, $\frac{n}{n} = 1$). Point out that fractions such as $\frac{8}{8}$ and $\frac{13}{8}$, with values equal to or greater than 1, are *improper fractions*.
   ➤ **How can you quickly recognize an improper fraction?** ***The numerator is equal to or greater than the denominator.***
   Point out that an improper fraction is not in lowest terms. Explain that an improper fraction is in lowest terms (its simplest form), when it is expressed as a mixed number.
4. Direct the students to draw on paper a model for $2\frac{3}{7}$ using whole and partitioned rectangles while a volunteer draws it for display.
   ➤ **How many sevenths are equal to $2\frac{3}{7}$? How do you know?** ***17; each whole is a set of $\frac{7}{7}$ plus the $\frac{3}{7}$.***
5. Guide a discussion in which students explain how they found their answers. Select a student to partition the displayed 2 whole rectangles into sevenths.
   ➤ **What addition equation can you write for the total number of sevenths in $2\frac{3}{7}$?** $\frac{7}{7} + \frac{7}{7} + \frac{3}{7} = \frac{17}{7}$
   Choose a student to write the equation for display.
6. Write $(2 \times \frac{7}{7}) + \frac{3}{7} = \frac{14}{7} + \frac{3}{7} = \frac{17}{7}$ for display. Elicit that multiplication can be used to rename the whole number as sevenths: 2 sets of 7 sevenths $= \frac{14}{7}$.
7. Guide the students in renaming these mixed numbers as improper fractions using addition and multiplication equations. (***Note:*** Allow students who struggle with the renaming to use their Fraction Kits and to draw models until they can rename using addition or multiplication.)
   $2\frac{5}{6} = \frac{6}{6} + \frac{6}{6} + \frac{5}{6} = \frac{17}{6}; (2 \times \frac{6}{6}) + \frac{5}{6} = \frac{12}{6} + \frac{5}{6} = \frac{17}{6}$
   $3\frac{9}{10} = \frac{10}{10} + \frac{10}{10} + \frac{10}{10} + \frac{9}{10} = \frac{39}{10}; (3 \times \frac{10}{10}) + \frac{9}{10} = \frac{30}{10} + \frac{9}{10} = \frac{39}{10}$

Rename an improper fraction as a whole number or a mixed number

1. Write $\frac{12}{3}$ for display. Direct the students to use their fraction circles to model the improper fraction using thirds. Then direct them to group the thirds to show wholes.
   ➤ **How many wholes did you make with the 12 thirds? Why?** ***4; each set of 3 thirds makes 1 whole; there are 4 sets of 3 thirds in 12 thirds.***
   Write the addition equation to show the renaming of $\frac{12}{3}$ as sets of $\frac{3}{3}$: $\frac{12}{3} = \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} = 4$.
2. Follow a similar procedure for $\frac{9}{4}$; use the following questions.
   ➤ **How many wholes did you make with the 9 fourths? Why?** ***2; elicit that each set of 4 fourths makes 1 whole, and there are 2 sets of 4 fourths in 9 fourths with 1 fourth remaining.***
   ➤ **What addition equation shows the renaming of 9 fourths as sets of 4 fourths?** $\frac{9}{4} = \frac{4}{4} + \frac{4}{4} + \frac{1}{4} = 2\frac{1}{4}$
3. Explain to the students that they can estimate the value of an improper fraction by using the multiples of the equivalent fraction of 1. Elicit that for the fraction $\frac{9}{4}$ each set of 4 fourths equals 1 whole, so $\frac{4}{4} = 1$, $\frac{8}{4} = 2$, and $\frac{12}{4} = 3$. Write the 3 equivalents for display.
   ➤ **What 2 numbers is the improper fraction $\frac{9}{4}$ between?** ***2 and 3***
   Elicit that the estimated value of $\frac{9}{4}$ is a mixed number between 2 and 3.
   ➤ **Is $2\frac{1}{4}$ a reasonable answer when renaming $\frac{9}{4}$ as a mixed number? Why?** ***Yes, $2\frac{1}{4}$ is between 2 and 3.***
4. Write $\frac{17}{6}$ for display. Choose students to write the multiples of $\frac{6}{6}$ to find the estimated value of $\frac{17}{6}$. $\frac{6}{6} = 1$, $\frac{12}{6} = 2$, $\frac{18}{6} = 3$
   ➤ **What 2 numbers is $\frac{17}{6}$ between?** ***2 and 3***
5. Direct the students to draw on paper a model for $\frac{17}{6}$ using rectangles while a volunteer draws it for display.
   ➤ **How many parts are in each whole? How do you know?** ***6; the denominator names the parts of the whole.***
   ➤ **How many wholes did you make with 17 sixths? Why?** ***2; there are 2 sets of 6 sixths (12 sixths) in 17 sixths.***
   ➤ **What part of the next whole remains?** ***5 sixths***
   Instruct students to write the addition equation to show the renaming of $\frac{17}{6}$ as a mixed number. $\frac{17}{6} = \frac{6}{6} + \frac{6}{6} + \frac{5}{6} = 2\frac{5}{6}$
6. Write $\frac{34}{2}$ for display and choose a student to read it aloud. ***34 halves***
   ➤ **What operation can be written in fraction form?** ***division***
   Choose a student to read $\frac{34}{2}$ as a division problem. ***34 divided by 2***
   Explain that division is a strategy for renaming an improper fraction as a whole number or a mixed number.
   Direct the students to write $\frac{34}{2}$ in a division frame and solve it. ***17***

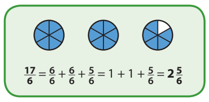
Improper Fractions & Mixed Numbers

improper fraction
mixed number

An **improper fraction** has a value *equal to* or *greater than* 1. $\frac{6}{6}$ $\frac{17}{6}$
An improper fraction is in lowest terms when it is expressed as a **mixed number**.

The improper fraction $\frac{17}{6}$ and the mixed number $2\frac{5}{6}$ represent the same value.

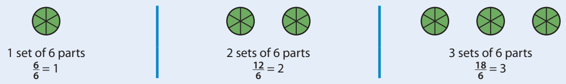
$$\frac{17}{6} = 2\frac{5}{6}$$



$\frac{17}{6} = \frac{6}{6} + \frac{6}{6} + \frac{5}{6} = 1 + 1 + \frac{5}{6} = 2\frac{5}{6}$

A mixed number is the sum of a whole number and a fraction. $2\frac{5}{6} = 2 + \frac{5}{6}$

Knowing that $\frac{6}{6} = 1$ allows you to use multiples of $\frac{6}{6}$ to estimate the value of the improper fraction $\frac{17}{6}$ as a mixed number.

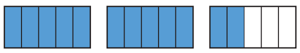


1 set of 6 parts	2 sets of 6 parts	3 sets of 6 parts
$\frac{6}{6} = 1$	$\frac{12}{6} = 2$	$\frac{18}{6} = 3$

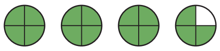
The estimated value of the improper fraction $\frac{17}{6}$ is a mixed number between 2 and 3.
The mixed number $2\frac{5}{6}$ is a reasonable answer.

Exercises

Write the improper fraction for the model.
Write an addition equation to rename the improper fraction as a mixed number.

1. 
$\frac{12}{5}$; $\frac{5}{5} + \frac{5}{5} + \frac{2}{5} = 1 + 1 + \frac{2}{5} = 2\frac{2}{5}$

2. 
$\frac{7}{3}$; $\frac{3}{3} + \frac{3}{3} + \frac{1}{3} = 1 + 1 + \frac{1}{3} = 2\frac{1}{3}$

3. 
$\frac{15}{4}$; $\frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{3}{4} = 1 + 1 + 1 + \frac{3}{4} = 3\frac{3}{4}$

4. 
$\frac{16}{6}$; $\frac{6}{6} + \frac{6}{6} + \frac{4}{6} = 1 + 1 + \frac{4}{6} = 2\frac{4}{6}$

Draw a model. Write the improper fraction as a mixed number.

5. triangles to represent $\frac{7}{2}$ $3\frac{1}{2}$
6. circles to represent $\frac{19}{8}$ $2\frac{3}{8}$
7. hexagons to represent $\frac{11}{6}$ $1\frac{5}{6}$
8. rectangles to represent $\frac{15}{7}$ $2\frac{1}{7}$

Draw a model. Write the mixed number as an improper fraction.

9. squares to represent $1\frac{3}{4}$ $\frac{7}{4}$
10. rectangles to represent $3\frac{4}{5}$ $\frac{19}{5}$
11. hexagons to represent $2\frac{5}{6}$ $\frac{17}{6}$
12. circles to represent $4\frac{1}{3}$ $\frac{13}{3}$

82 Chapter 4

Multiplication can be used to rename mixed numbers as improper fractions, and division can be used to rename improper fractions as mixed numbers.

Rename a mixed number as an improper fraction.

$2\frac{3}{8} =$ ___

1. Multiply to find the number of parts in the wholes. $2 \cdot \frac{8}{8} = \frac{16}{8}$
2. Add the additional parts. $\frac{16}{8} + \frac{3}{8} = \frac{19}{8}$

Rename an improper fraction as a mixed number.

$\frac{19}{8} =$ ___

1. Divide the numerator by the denominator to find the number of wholes.
2. Write the remainder as a fraction to tell how many parts of the next whole there are.

$$\begin{array}{r} 2\frac{3}{8} \\ 8\overline{)19} \\ -16 \\ \hline 3 \end{array}$$

Exercises

Use division to rename an improper fraction as a whole number or a mixed number.
Use multiplication to rename a mixed number as an improper fraction.
Show your work. Draw a picture if needed.

13. $\frac{10}{2}$ 5
14. $\frac{12}{5}$ $2\frac{2}{5}$
15. $3\frac{1}{4}$ $\frac{13}{4}$
16. $4\frac{1}{5}$ $\frac{21}{5}$
17. $1\frac{9}{10}$ $\frac{19}{10}$
18. $\frac{31}{4}$ $7\frac{3}{4}$
19. $\frac{25}{6}$ $4\frac{1}{6}$
20. $5\frac{2}{3}$ $\frac{17}{3}$
21. $\frac{27}{3}$ 9
22. $2\frac{5}{8}$ $\frac{21}{8}$

Write an improper fraction to answer the question.
Draw a picture if needed.

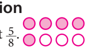
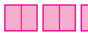
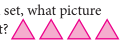
23. Two children are sharing 5 brownies so that each child gets an equal amount. How many brownies will each child get? $\frac{5}{2}$ ***brownies***
24. Four people are sharing 6 personal pan pizzas so that each person has a fair share. How much pizza will each person eat? $\frac{6}{4}$ ***pizzas***

Substitute the value for the variable in the fraction.
Rename improper fractions as a mixed number.

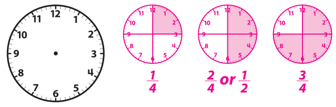
$a = 4$	$b = 7$	$c = 10$	$d = 13$

25. $\frac{a}{b}$ $\frac{4}{7}$
26. $\frac{c}{b}$ $\frac{10}{7} = 1\frac{3}{7}$
27. $\frac{d}{b}$ $\frac{13}{7} = 1\frac{6}{7}$
28. $\frac{a}{d}$ $\frac{4}{13}$
29. $\frac{d}{c}$ $\frac{13}{10} = 1\frac{3}{10}$

Practice & Application

30. Draw a set to represent $\frac{5}{8}$. 
31. Draw rectangles to represent $\frac{6}{2}$. 
32. If ▲▲ represents $\frac{1}{2}$ of a set, what picture represents the whole set? 
33. Write three fractions that represent 1 whole.
Possible answer: $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}$

J Draw three clocks similar to this clock. On each clock draw a horizontal line from 3 to 9 and a vertical line from 12 to 6. Shade the first clock to show "quarter after," the second clock to show "half past," and the third clock to show "quarter to." Write the fraction represented by each clock.



$\frac{1}{4}$ $\frac{2}{4}$ **or** $\frac{1}{2}$ $\frac{3}{4}$

Complete DAILY REVIEW d on page 415.

Lesson 34 83

➤ **Is the quotient of $\frac{34}{2}$ a mixed number or a whole number? Why?** ***Whole number; there is no remainder.***

7. Explain that dividing the numerator of an improper fraction by the denominator tells the number of wholes in the fraction. Any remainder tells how many parts cannot be made into a whole and is written as a fraction. The remainder is the numerator in the fraction of a mixed number, and the divisor is the denominator.
Guide the students in dividing $\frac{9}{4}$ to rename it as a mixed number. $9 \div 4 = 2\frac{1}{4}$
8. Instruct the students to write and solve an addition equation and a division problem to rename these improper fractions as mixed numbers.
(***Note:*** Allow students who struggle with the renaming to use their Fraction Kits and to draw models until they can rename using addition or division.)
$\frac{18}{5}$ $\frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{3}{5} = 3\frac{3}{5}$; $18 \div 5 = 3\frac{3}{5}$
$\frac{28}{10}$ $\frac{10}{10} + \frac{10}{10} + \frac{8}{10} = 2\frac{8}{10}$; $28 \div 10 = 2\frac{8}{10}$
(***Note:*** Some students may recognize that the fraction $\frac{8}{10}$ is not in lowest terms. The emphasis of this lesson is to rename an improper fraction as a mixed number; in Lesson 35 students will apply various strategies for renaming fractions to lowest terms.)
You may choose to have the students use division to rename as mixed numbers the improper fractions that were discussed earlier in this lesson.

Draw a model to solve a word problem

For an afternoon snack, Aunt Kate let her 3 nephews share 5 large cookies. How many cookies did each nephew receive? $\frac{5}{3}$ ***cookies or*** $1\frac{2}{3}$ ***cookies***

1. Direct the students to illustrate the problem to find the answer. Choose students to show their pictures and explain how they found the answer. ***Possible drawing: Draw 5 circles to represent the cookies and 3 stick figures to represent the 3 children. Partition the circles into thirds and draw a line from 1 part of each cookie to a person, continuing until a line has been drawn from each part to a person; you can also draw a line from 1 whole cookie to each person, partition the remaining 2 cookies into thirds, and draw a line from 1 part of each cookie to a person, continuing until a line has been drawn from each part to a person.***
2. Select students to compose similar word problems for the class to solve by drawing pictures.

Student Text pp. 82–83

(*Note*: *Assessment* available on Teacher's Toolkit CD.)

Lesson 35

Student Text pp. 84–85

Daily Review p. 416e

Objectives

- Apply strategies to rename fractions to higher terms
- Apply strategies to rename fractions to lower terms and to lowest terms
- Use cancellation to rename fractions to lowest terms

Teacher Materials

- A blank sheet of paper

Student Materials

- A blank sheet of paper

Introduce the Lesson

Review the divisibility rules on page 502 of the Student Text Handbook.

Teach for Understanding

Rename fractions to higher terms

1. Distribute the blank sheets of paper. Instruct the students to follow each direction after you read it. Demonstrate each step.
   - ➤ **Fold the paper in thirds and shade 1 third. Unfold the paper.**
   - ➤ **What part of the paper is shaded?** $\frac{1}{3}$ Write $\frac{1}{3}$ for display.
   - ➤ **Fold the paper in half in the opposite direction. Unfold the paper.**
   - ➤ **What part of the paper is shaded now?** $\frac{2}{6}$ Write $= \frac{2}{6}$ after $\frac{1}{3}$.
   - ➤ **What do you notice about the numerator and the denominator? Why?** ***Elicit that they increased or doubled because each third was divided into 2 equal parts, making sixths.***
   - ➤ **Refold the paper on the fold lines. When only 1 sixth is showing, fold the paper in half again. Unfold the paper.**
   - ➤ **What part of the paper is shaded now?** $\frac{4}{12}$ Write $= \frac{4}{12}$ after $\frac{2}{6}$.
   - ➤ **Refold the paper on the fold lines. When only 1 twelfth is showing, fold the paper in half again. Unfold the paper.**
   - ➤ **What part of the paper is shaded now?** $\frac{8}{24}$ Write $= \frac{8}{24}$ after $\frac{4}{12}$.
2. Discuss the relationship between $\frac{1}{3}$, $\frac{2}{6}$, $\frac{4}{12}$, and $\frac{8}{24}$. Remind the students that a fraction can be repartitioned without changing its value, renaming the fraction to *higher terms*. Point out that the numerator and the denominator of the new fraction are greater. As the number of parts increases, the size of the parts decreases; however, the fractions are equivalent because they name the same part of a whole.
3. Direct the students to draw on paper a figure that models $\frac{2}{7}$. (***Note:*** Allow the students to draw any shape, but you may want to elicit that a rectangle is easier to repartition.)
   Write $\frac{2}{7} = \frac{n}{14}$ and $\frac{2}{7} = \frac{n}{21}$ for display.
   - ➤ **How can you repartition sevenths to make fourteenths?** ***Repartition each part, dividing the sevenths into 2 equal parts.*** **sevenths to make twenty-firsts?** ***Repartition each part, dividing the sevenths into 3 equal parts.***

   Direct the students to repartition their models into fourteenths or twenty-firsts to show an equivalent fraction in higher terms. Choose students to show their drawings and to tell the equivalent fractions of $\frac{2}{7}$. $\frac{4}{14}$; $\frac{6}{21}$
   - ➤ **What is the Identity Property of Multiplication?** ***When 1 is a factor, the product is the other factor.***
4. Write $a \bullet 1 = a$ and $a \bullet \frac{1}{1} = a$. Guide a discussion about how the Identity Property of Multiplication can be used to rename fractions to higher terms. Elicit that $\frac{2}{7} \bullet 1 = \frac{2}{7}$, $\frac{2}{7} \bullet \frac{2}{2} = \frac{4}{14}$, and $\frac{2}{7} \bullet \frac{3}{3} = \frac{6}{21}$. Remind the students that when renaming fractions the multiplication equations can also be written as $\frac{2 \times 2}{7 \times 2} = \frac{4}{14}$ and $\frac{2 \times 3}{7 \times 3} = \frac{6}{21}$.
   - ➤ **What other fraction names for 1 can be used to find an equivalent fraction in higher terms?** ***Accept any fractions in which the terms are the same.***
5. Instruct the students to write a fraction on paper. Direct them to trade papers with a partner and write a multiplication equation to rename the fraction to higher terms. Instruct partners to check the fractions for equivalency. Choose students to write their equations for display.

Rename fractions to lower terms

1. Draw a set of 8 triangles and shade 4 of them.
   - ➤ **What fraction of the triangles is shaded?** $\frac{4}{8}$

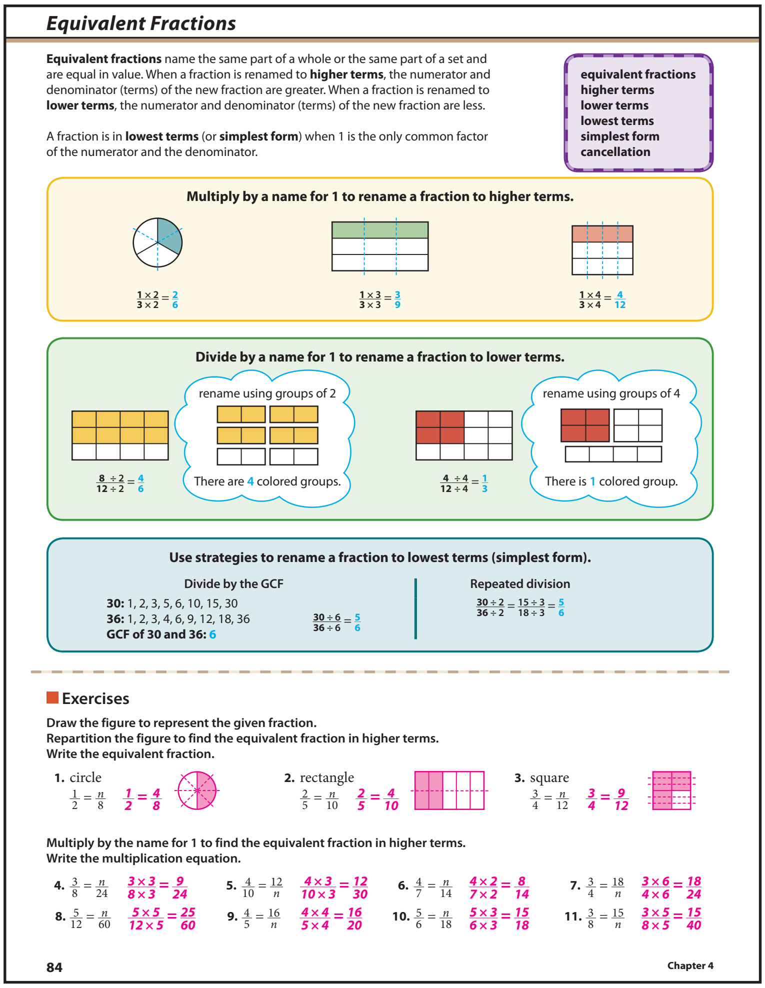
   Choose students to group the set of triangles to find an equivalent fraction and to write the fraction. Discuss the possible groupings and fractions. $\frac{2}{4}$ ***and*** $\frac{1}{2}$
2. Discuss the relationship between $\frac{4}{8}$, $\frac{2}{4}$, and $\frac{1}{2}$. Remind the students that a fraction can be renamed without changing its value. When a fraction is renamed to *lower terms*, the numerator and the denominator of the new fraction decrease, but the fractions are equivalent.
3. Write $a \div 1 = a$ and $a \div \frac{1}{1} = a$ for display.
   - ➤ **What do you know about a number (the dividend) when it is divided by 1?** ***The quotient is that number.***
4. Write $\frac{4}{8} = \frac{n}{4}$ and $\frac{4}{8} = \frac{n}{2}$ for display. Elicit that equivalent fractions of 1 can be used to rename fractions to lower terms. Choose students to write the division equations to rename $\frac{4}{8}$ to lower terms. $\frac{4}{8} \div \frac{2}{2} = \frac{2}{4}$ ***or*** $\frac{4 \div 2}{8 \div 2} = \frac{2}{4}$; $\frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$ ***or*** $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$
   - ➤ **What makes dividing to rename a fraction to lower terms more challenging than multiplying to rename a fraction to higher terms?** ***Elicit that the numerator and denominator must be divided by a common factor; they both must be divisible by the same number.***
5. Direct the students to divide by equivalent fractions of 1 to rename the following fractions to lower terms. Point out that they can use the divisibility rules when analyzing the fraction.

   $\frac{10}{30} = (\div \frac{2}{2})\ \frac{5}{15}; (\div \frac{5}{5})\ \frac{2}{6}; (\div \frac{10}{10})\ \frac{1}{3}$ $\frac{12}{18} = (\div \frac{2}{2})\ \frac{6}{9}; (\div \frac{3}{3})\ \frac{4}{6}; (\div \frac{6}{6})\ \frac{2}{3}$

Rename fractions to lowest terms

1. Write $\frac{30}{36}$ for display.
   - ➤ **How could you divide this fraction to rename it to lower terms?** ***Elicit that dividing the fraction by*** $\frac{2}{2}$, $\frac{3}{3}$, ***and*** $\frac{6}{6}$ ***will rename it to lower terms.***

   Choose students to demonstrate renaming $\frac{30}{36}$ to lower terms. $\frac{30}{36} \div \frac{2}{2} = \frac{15}{18}$; $\frac{30}{36} \div \frac{3}{3} = \frac{10}{12}$; $\frac{30}{36} \div \frac{6}{6} = \frac{5}{6}$
2. Point out that a fraction is in its simplest form when it is in *lowest terms*; 1 is the only common factor of the numerator and the denominator. Elicit that $\frac{5}{6}$ is the simplest form of $\frac{30}{36}$ because 1 is the only common factor of 5 and 6.
3. Explain that repeated division is one of many strategies that can be used to simplify a fraction (rename a fraction to lowest terms). Demonstrate renaming $\frac{30}{36}$ using repeated division: $\frac{30 \div 2}{36 \div 2} = \frac{15 \div 3}{18 \div 3} = \frac{5}{6}$.


Equivalent Fractions

Equivalent fractions name the same part of a whole or the same part of a set and are equal in value. When a fraction is renamed to **higher terms**, the numerator and denominator (terms) of the new fraction are greater. When a fraction is renamed to **lower terms**, the numerator and denominator (terms) of the new fraction are less.

A fraction is in **lowest terms** (or **simplest form**) when 1 is the only common factor of the numerator and the denominator.

equivalent fractions
higher terms
lower terms
lowest terms
simplest form
cancellation

Multiply by a name for 1 to rename a fraction to higher terms.

$\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$ $\frac{1 \times 3}{3 \times 3} = \frac{3}{9}$ $\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$

Divide by a name for 1 to rename a fraction to lower terms.

rename using groups of 2

$\frac{8 \div 2}{12 \div 2} = \frac{4}{6}$

There are 4 colored groups.

rename using groups of 4

$\frac{4 \div 4}{12 \div 4} = \frac{1}{3}$

There is 1 colored group.

Use strategies to rename a fraction to lowest terms (simplest form).

Divide by the GCF

30: 1, 2, 3, 5, 6, 10, 15, 30
36: 1, 2, 3, 4, 6, 9, 12, 18, 36
GCF of 30 and 36: 6

$\frac{30 \div 6}{36 \div 6} = \frac{5}{6}$

Repeated division

$\frac{30 \div 2}{36 \div 2} = \frac{15 \div 3}{18 \div 3} = \frac{5}{6}$

Exercises

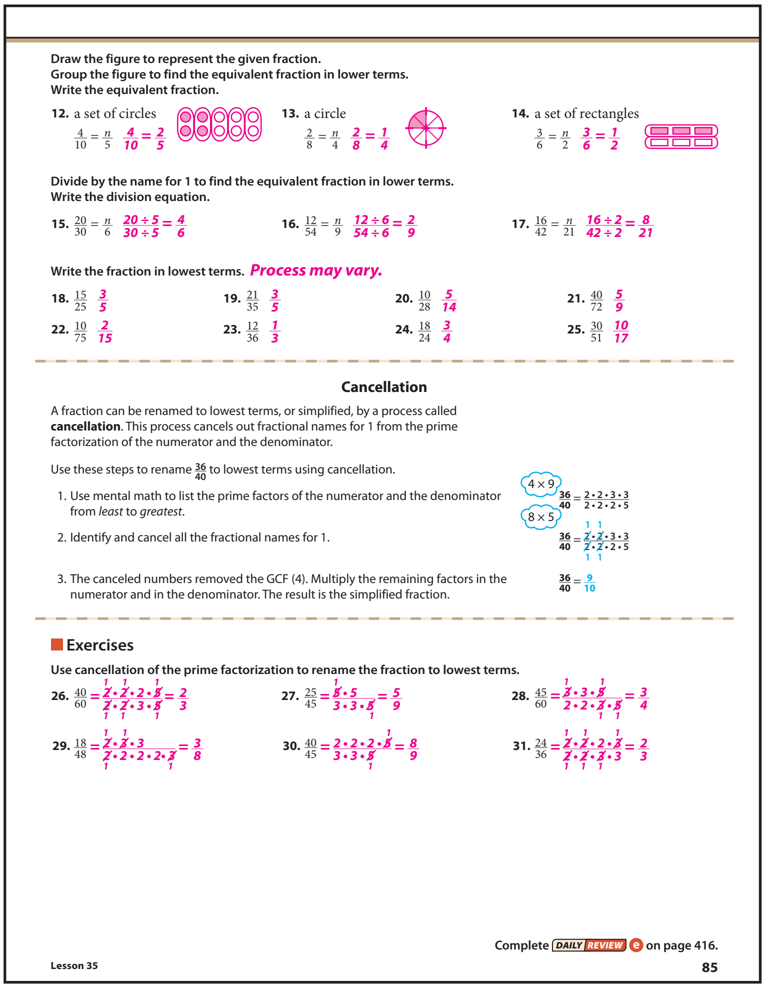
Draw the figure to represent the given fraction.
Repartition the figure to find the equivalent fraction in higher terms.
Write the equivalent fraction.

1. circle $\frac{1}{2} = \frac{n}{8}$ $\frac{1}{2} = \frac{4}{8}$
2. rectangle $\frac{2}{5} = \frac{n}{10}$ $\frac{2}{5} = \frac{4}{10}$
3. square $\frac{3}{4} = \frac{n}{12}$ $\frac{3}{4} = \frac{9}{12}$

Multiply by the name for 1 to find the equivalent fraction in higher terms.
Write the multiplication equation.

4. $\frac{3}{8} = \frac{n}{24}$ $\frac{3 \times 3}{8 \times 3} = \frac{9}{24}$
5. $\frac{4}{10} = \frac{12}{n}$ $\frac{4 \times 3}{10 \times 3} = \frac{12}{30}$
6. $\frac{4}{7} = \frac{n}{14}$ $\frac{4 \times 2}{7 \times 2} = \frac{8}{14}$
7. $\frac{3}{4} = \frac{18}{n}$ $\frac{3 \times 6}{4 \times 6} = \frac{18}{24}$
8. $\frac{5}{12} = \frac{n}{60}$ $\frac{5 \times 5}{12 \times 5} = \frac{25}{60}$
9. $\frac{4}{5} = \frac{16}{n}$ $\frac{4 \times 4}{5 \times 4} = \frac{16}{20}$
10. $\frac{5}{6} = \frac{n}{18}$ $\frac{5 \times 3}{6 \times 3} = \frac{15}{18}$
11. $\frac{3}{8} = \frac{15}{n}$ $\frac{3 \times 5}{8 \times 5} = \frac{15}{40}$

84 Chapter 4


Draw the figure to represent the given fraction.
Group the figure to find the equivalent fraction in lower terms.
Write the equivalent fraction.

12. a set of circles $\frac{4}{10} = \frac{n}{5}$ $\frac{4}{10} = \frac{2}{5}$
13. a circle $\frac{2}{8} = \frac{n}{4}$ $\frac{2}{8} = \frac{1}{4}$
14. a set of rectangles $\frac{3}{6} = \frac{n}{2}$ $\frac{3}{6} = \frac{1}{2}$

Divide by the name for 1 to find the equivalent fraction in lower terms.
Write the division equation.

15. $\frac{20}{30} = \frac{n}{6}$ $\frac{20 \div 5}{30 \div 5} = \frac{4}{6}$
16. $\frac{12}{54} = \frac{n}{9}$ $\frac{12 \div 6}{54 \div 6} = \frac{2}{9}$
17. $\frac{16}{42} = \frac{n}{21}$ $\frac{16 \div 2}{42 \div 2} = \frac{8}{21}$

Write the fraction in lowest terms. *Process may vary.*

18. $\frac{15}{25}$ $\frac{3}{5}$
19. $\frac{21}{35}$ $\frac{3}{5}$
20. $\frac{10}{28}$ $\frac{5}{14}$
21. $\frac{40}{72}$ $\frac{5}{9}$
22. $\frac{10}{75}$ $\frac{2}{15}$
23. $\frac{12}{36}$ $\frac{1}{3}$
24. $\frac{18}{24}$ $\frac{3}{4}$
25. $\frac{30}{51}$ $\frac{10}{17}$

Cancellation

A fraction can be renamed to lowest terms, or simplified, by a process called **cancellation**. This process cancels out fractional names for 1 from the prime factorization of the numerator and the denominator.

Use these steps to rename $\frac{36}{40}$ to lowest terms using cancellation.

1. Use mental math to list the prime factors of the numerator and the denominator from *least* to *greatest*.
2. Identify and cancel all the fractional names for 1.
3. The canceled numbers removed the GCF (4). Multiply the remaining factors in the numerator and in the denominator. The result is the simplified fraction.

4×9 8×5

$\frac{36}{40} = \frac{2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 5}$

$\frac{36}{40} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3 \cdot 3}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2 \cdot 5}$

$\frac{36}{40} = \frac{9}{10}$

Exercises

Use cancellation of the prime factorization to rename the fraction to lowest terms.

26. $\frac{40}{60} = \frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{5}}{\cancel{2} \cdot \cancel{2} \cdot 3 \cdot \cancel{5}} = \frac{2}{3}$
27. $\frac{25}{45} = \frac{\cancel{5} \cdot 5}{3 \cdot 3 \cdot \cancel{5}} = \frac{5}{9}$
28. $\frac{45}{60} = \frac{\cancel{3} \cdot 3 \cdot \cancel{5}}{2 \cdot 2 \cdot \cancel{3} \cdot \cancel{5}} = \frac{3}{4}$
29. $\frac{18}{48} = \frac{\cancel{2} \cdot \cancel{3} \cdot 3}{\cancel{2} \cdot 2 \cdot 2 \cdot 2 \cdot \cancel{3}} = \frac{3}{8}$
30. $\frac{40}{45} = \frac{2 \cdot 2 \cdot 2 \cdot \cancel{5}}{3 \cdot 3 \cdot \cancel{5}} = \frac{8}{9}$
31. $\frac{24}{36} = \frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{3}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot 3} = \frac{2}{3}$

Complete DAILY REVIEW e on page 416.

Lesson 35 85

4. Repeat the procedure for renaming $\frac{48}{132}$: $\frac{48 \div 2}{132 \div 2} = \frac{24 \div 2}{66 \div 2} = \frac{12 \div 3}{33 \div 3} = \frac{4}{11}$.
5. Point out that if you divide a fraction by the greatest common factor (GCF) to find the lowest terms, you have to rename the fraction only once, rather than divide repeatedly, (e.g., $\frac{30 \div 6}{36 \div 6} = \frac{5}{6}$).
6. Write $\frac{48}{132}$ = for display.

   ➤ **What strategies can you use to find the GCF of 48 and 132?** ***List the factors, make Venn diagrams, or use prime factorizations.***

   Tell the students that they should analyze the numbers before choosing the strategy they will use to determine the GCF. Explain that if they do not know many common factors of 48 and 132, using prime factorizations would most likely be the strategy they should choose.
7. Guide the students in constructing factor trees for 48 and 132 to find the prime factorizations. Write $48 = 2 \times 2 \times 2 \times 2 \times 3$ and then write $132 = 2 \times 2 \times 3 \times 11$ below it.

   Remind the students that to calculate the GCF you need to multiply the common factors: 2, 2, 3.

   ➤ **What equation will you use to find the GCF?** ***$2 \times 2 \times 3 = 12$***

   Write $\frac{48 \div 12}{132 \div 12}$ = and direct the students to solve it. $\frac{4}{11}$
8. Direct attention to the prime factorizations of 48 and 132. Write $\frac{48}{132} = \frac{2 \times 2 \times 2 \times 2 \times 3}{2 \times 2 \times \quad 3 \times 11}$.

   ➤ **What do you know about a fraction that has the same number in the numerator and the denominator?** ***The fraction is equal to 1.***

   Draw a rectangle around each $\frac{2}{2}$ and the $\frac{3}{3}$. Explain that since each of these fractions is equal to 1, the Identity Property allows you to cancel them out; there is no need to multiply them.

   Draw a line through the numerator and the denominator in each $\frac{2}{2}$ and the $\frac{3}{3}$ and write a 1 above each canceled numerator and below each canceled denominator. Explain that this process is called *cancellation*.
9. Guide the students in multiplying the remaining factors to find the fraction in lowest terms. Point out that since 1 and 11 are the only factors in the denominator, you do not need to multiply. $\frac{4}{11}$

   Explain that canceling the factors $(2 \times 2 \times 3)$ is similar to dividing by the GCF (12). Multiplying the remaining factors results in lowest terms just as dividing by the GCF does.
10. Follow a similar procedure to guide the students in using cancellation to simplify the following fractions. For the first problem, point out that the lowest terms $\frac{2}{3}$ is what is left after you use cancellation. For the second problem, point out that both factors in the numerator were canceled because they are fractional names for 1; therefore, the numerator in the simplified fraction is 1.

    $\frac{28}{42} = \frac{\cancel{2} \times 2 \times \cancel{7}}{\cancel{2} \times 3 \times \cancel{7}} = \frac{2}{3}$

    $\frac{14}{56} = \frac{\cancel{2} \times \cancel{7}}{\cancel{2} \times 2 \times 2 \times \cancel{7}} = \frac{1}{4}$
11. Instruct the students to simplify the following fractions. Allow them to choose which strategy to use. Discuss their answers, asking students to tell which strategy they chose and their reason for choosing the strategy.

    $\frac{27}{63} = \frac{3}{7}$ $\frac{24}{30} = \frac{4}{5}$

Student Text pp. 84–85

Lesson 36 | **Student Text** pp. 86–87
Daily Review p. 416f

Objectives

- Write an inequality to express unequal relationships
- Apply fraction number sense to compare and order fractions
- Compare and order unlike fractions by renaming to fractions with a common denominator
- Determine equivalent fractions using the LCM
- Compare and order mixed numbers and improper fractions

Teacher Materials

- Fraction Kit: fraction bars

Student Materials

- Fraction Kit: fraction bars

Teach for Understanding

Apply fraction number sense to compare and order fractions

1. Write $\frac{5}{8}$ and $\frac{3}{8}$ for display. Remind the students that $\frac{5}{8}$ and $\frac{3}{8}$ are *like fractions*; they have the same denominator (the parts are the same size).
   ➤ **Which fraction is greater? Why?** ***$\frac{5}{8}$; elicit that when comparing like fractions, the size of the part (the denominator) is the same, so you compare the number of parts selected (the numerator); 5 is greater than 3, so $\frac{5}{8}$ is greater than $\frac{3}{8}$.***
   Explain that an *inequality* is a number sentence in which two expressions are not equal. When comparing expressions, a *greater than* sign (>) or a *less than* sign (<) is used to write an inequality.
   Choose a student to write an inequality expression for $\frac{5}{8}$ and $\frac{3}{8}$. $\frac{5}{8} > \frac{3}{8}$ ***or*** $\frac{3}{8} < \frac{5}{8}$
2. Distribute the fraction bars from the Fraction Kit. Direct the students to display 1 whole fraction bar on their desks. Instruct them to place below the 1 whole a row of fourths that is equal to 1 whole and below that a row of eighths that is equal to 1 whole.
   (***Note:*** Throughout the lesson, use your fraction bars for demonstration.)
   ➤ **Are the eighths the same size as the fourths? Why?** ***No; elicit that the eighths are smaller than the fourths. It takes more eighths (8 of the smaller bars) than fourths (4 of the larger bars) to make 1 whole.***
3. Write $\frac{3}{4}$ and $\frac{3}{8}$ for display. Direct each student to display below his 1 whole a row of 3 fourths and then a row of 3 eighths to determine which is greater. Choose a student to write an inequality. $\frac{3}{8} < \frac{3}{4}$ ***or*** $\frac{3}{4} > \frac{3}{8}$
   ➤ **How can you use your understanding of fractions when comparing fractions with the same numerator?** ***Elicit that when the numerators are the same, there are the same number of selected parts; therefore, you compare the size of the parts (the smaller the denominator, the larger the part). The fraction with the lesser denominator is the greater fraction.*** **fractions with the same denominator?** ***Elicit that when the denominators are the same, the parts are the same size; therefore, you compare the number of selected parts. The fraction with the greater numerator is the greater fraction.***
4. Guide the students in comparing $\frac{5}{9}$ and $\frac{7}{9}$ $\frac{5}{9} < \frac{7}{9}$ and $\frac{4}{5}$ and $\frac{4}{6}$ $\frac{4}{5} > \frac{4}{6}$.
5. Write $\frac{7}{8}$ and $\frac{3}{4}$ for display. Remind the students that $\frac{7}{8}$ and $\frac{3}{4}$ are *unlike fractions:* they do not have the same denominator. Explain that when unlike fractions do not have the same numerator, you can compare each fraction to 1 or $\frac{1}{2}$, using your understanding of the size of the fraction parts.
6. Direct each student to display below his 1 whole a row of 7 eighths and a row of 3 fourths.
   ➤ **What do you know about the relationship between eighths and fourths?** ***Answers may vary, but elicit that eighths are smaller than fourths (2 eighths are needed to make 1 fourth).*** **How many eighths are equivalent to 1 whole?** ***8 eighths*** **How many fourths?** ***4 fourths***
   ➤ **Which fraction is greater, $\frac{7}{8}$ or $\frac{3}{4}$?** $\frac{7}{8}$
   Write $\frac{7}{8} > \frac{3}{4}$ for display. Explain that if each of the fractions being compared is 1 part less than 1 whole, the fraction with the smaller parts will be closer to 1 whole than the fraction with the larger parts; therefore, the fraction with the smaller parts will be the greater fraction: since $\frac{1}{8} < \frac{1}{4}$, then $\frac{7}{8} > \frac{3}{4}$.
7. Follow a similar procedure to compare $\frac{2}{6}$ and $\frac{3}{8}$ by comparing each fraction to $\frac{1}{2}$. Instruct the students to place a 1 half fraction bar below the 1 whole before displaying a row of 2 sixths and a row of 3 eighths. $\frac{3}{8} > \frac{2}{6}$
   Explain that if each of the fractions being compared is 1 part less than $\frac{1}{2}$, the fraction with the smaller parts will be closer to $\frac{1}{2}$ than the fraction with the larger parts; therefore, the fraction with the smaller parts will be the greater fraction: since $\frac{1}{8} < \frac{1}{6}$, then $\frac{3}{8} > \frac{2}{6}$.
8. Repeat the procedure to compare $\frac{4}{6}$ and $\frac{5}{8}$ by comparing each fraction to $\frac{1}{2}$. $\frac{4}{6} > \frac{5}{8}$ Explain that if each of the fractions being compared is 1 part greater than $\frac{1}{2}$, the fraction with the larger parts will be farther from $\frac{1}{2}$ than the fraction with the smaller parts; therefore, the fraction with the larger parts will be the greater fraction.
9. Write $\frac{4}{6}$ and $\frac{3}{8}$ for display. Direct the students to display 4 sixths and 3 eighths below the 1 half fraction bar.
   ➤ **How many sixths are equivalent to 1 half?** ***3 sixths*** **eighths?** ***4 eighths***
   ➤ **How do you know that a fraction is equivalent to $\frac{1}{2}$?** ***Elicit that the numerator is half of the denominator.***
   Choose students to write inequalities, comparing $\frac{4}{6}$ to $\frac{1}{2}$ and $\frac{3}{8}$ to $\frac{1}{2}$. $\frac{4}{6} > \frac{1}{2}$ ***and*** $\frac{3}{8} < \frac{1}{2}$
   ➤ **Since $\frac{4}{6}$ is greater than $\frac{1}{2}$ and $\frac{3}{8}$ is less than $\frac{1}{2}$, how does $\frac{4}{6}$ compare to $\frac{3}{8}$?** $\frac{4}{6} > \frac{3}{8}$ Write $\frac{4}{6} > \frac{3}{8}$.
10. Guide the students in comparing $\frac{4}{10}$ and $\frac{13}{20}$ $\frac{4}{10} < \frac{13}{20}$ and $\frac{3}{4}$ and $\frac{7}{16}$ $\frac{3}{4} > \frac{7}{16}$.
11. Guide the students in comparing these fractions by determining each fraction's relationship to 1 or $\frac{1}{2}$.
    $\frac{6}{7} < \frac{8}{9}$ $\frac{4}{10} > \frac{3}{8}$ $\frac{7}{12} < \frac{4}{6}$
12. Explain that these strategies that are used to compare fractions can also help when ordering fractions. Discuss the strategies used as you guide the students in ordering from least to greatest $\frac{4}{5}$, $\frac{7}{8}$, $\frac{3}{7}$ $\frac{3}{7}$, $\frac{4}{5}$, $\frac{7}{8}$ and $\frac{3}{6}$, $\frac{2}{3}$, $\frac{5}{12}$ $\frac{5}{12}$, $\frac{3}{6}$, $\frac{2}{3}$.

Compare & Order Fractions

Use Fraction Number Sense to Compare

Compare fractions by choosing a strategy that uses your fraction knowledge.

compare fractions and mixed numbers
order fractions and mixed numbers

A. Same Size Parts

Fractions with the same denominators are *like fractions* and have the same size parts. Compare the number of parts selected (numerators).

$\frac{5}{8} > \frac{3}{8}$

5 eighths > 3 eighths

B. Same Number of Parts

Fractions with the same numerator have the same number of selected parts and can be compared by the size of the parts (denominators).

$\frac{2}{6} < \frac{2}{4}$

2 sixths < 2 fourths

C. Greater or Less Than $\frac{1}{2}$

Analyze each fraction's relationship to $\frac{1}{2}$. See if one fraction is *greater than* $\frac{1}{2}$ and one fraction is *less than* $\frac{1}{2}$.

$\frac{7}{8} > \frac{2}{6}$

$\frac{7}{8} > \frac{1}{2}$ $\frac{2}{6} < \frac{1}{2}$

D. Closeness to $\frac{1}{2}$ or 1

Analyze each fraction's closeness to $\frac{1}{2}$ or 1. When both fractions are only one part away from $\frac{1}{2}$ or when both fractions are only one part away from 1, compare the size of the parts.

$\frac{1}{8}$ more to make 1 whole $\frac{7}{8} > \frac{6}{7}$ $\frac{1}{7}$ more to make 1 whole

$\frac{1}{8} < \frac{1}{7}$

Exercises

Practice Strategies A and B.
Write a comparison sentence using > or <.

1. $\frac{4}{12} < \frac{5}{12}$ 2. $\frac{3}{8} < \frac{3}{6}$ 3. $\frac{6}{9} > \frac{6}{10}$
4. $\frac{9}{15} > \frac{8}{15}$ 5. $\frac{3}{10} > \frac{3}{12}$ 6. $\frac{8}{14} < \frac{9}{14}$

Practice Strategy C.
Write a comparison sentence using > or <.

7. $\frac{5}{12} < \frac{1}{2}$ 8. $\frac{7}{10} > \frac{1}{2}$ 9. $\frac{3}{8} < \frac{1}{2}$
10. $\frac{5}{6} > \frac{2}{5}$ 11. $\frac{8}{20} < \frac{9}{11}$ 12. $\frac{9}{16} > \frac{3}{7}$

Practice Strategy D.
Write a comparison sentence using > or <.

13. $\frac{7}{8} > \frac{5}{6}$ 14. $\frac{6}{10} > \frac{7}{12}$ 15. $\frac{4}{10} < \frac{5}{12}$
16. $\frac{2}{3} < \frac{3}{4}$ 17. $\frac{14}{15} < \frac{17}{18}$ 18. $\frac{3}{8} > \frac{2}{6}$

Write a comparison sentence using > or <.

19. $\frac{5}{7} > \frac{5}{8}$ 20. $\frac{3}{5} < \frac{4}{5}$ 21. $\frac{14}{15} > \frac{12}{13}$
22. $\frac{8}{9} > \frac{7}{8}$ 23. $\frac{3}{4} > \frac{3}{8}$ 24. $\frac{7}{16} > \frac{4}{10}$

U.S. military and city employees help remove debris during a cleanup effort at the Misawa Fishing Port in Japan after the 2011 earthquake and tsunami.

86 Chapter 4

Rename Fractions to Compare

Compare fractions by finding a common denominator and renaming the fractions.

If $\frac{3}{4}$ of the purple balloons have polka dots and $\frac{7}{8}$ of the red balloons have polka dots, which color has a larger portion of balloons with polka dots?

Rename One Fraction

When the denominator of one fraction is a multiple of the other fraction, rename only one of the fractions.

$\frac{3}{4} < \frac{7}{8}$

$\frac{6}{8} < \frac{7}{8}$

There are more red balloons with polka dots.

Jared spent $\frac{2}{3}$ of an hour doing chores while Ben spent $\frac{3}{4}$ of an hour. Who spent less time working?

Rename Both Fractions

When the denominators are not related, use the least common multiple (LCM) to rename both fractions.

$\frac{3}{4} > \frac{2}{3}$

LCM of 3 and 4: 12
$\frac{9}{12} > \frac{8}{12}$

Jared spent less time working.

Compare Mixed Numbers

First, compare the whole numbers. $2\frac{2}{3} > 1\frac{4}{9}$

$2 > 1$

If the whole numbers are the same, compare the fractions. $4\frac{3}{8} < 4\frac{5}{6}$

$\frac{3}{8} < \frac{5}{6}$
$\frac{9}{24} < \frac{20}{24}$

Compare a Mixed Number and an Improper Fraction

Rename the mixed number to an improper fraction or rename the improper fraction to a mixed number.

$2\frac{3}{8} < \frac{32}{9}$

$\frac{19}{8} < \frac{32}{9}$ or $2\frac{3}{8} < 3\frac{5}{9}$

Exercises

Write a comparison sentence using > or <. Rename as needed.

25. $\frac{7}{24}$ ○ $\frac{12}{48}$ $\frac{14}{48} > \frac{12}{48}$
26. $\frac{6}{9}$ ○ $\frac{7}{12}$ $\frac{24}{36} > \frac{21}{36}$
27. $\frac{4}{6}$ ○ $\frac{6}{14}$ $\frac{28}{42} > \frac{18}{42}$
28. $\frac{3}{8}$ ○ $\frac{10}{24}$ $\frac{9}{24} < \frac{10}{24}$
29. $\frac{2}{3}$ ○ $\frac{5}{7}$ $\frac{14}{21} < \frac{15}{21}$
30. $\frac{7}{24}$ ○ $\frac{12}{30}$ $\frac{35}{120} < \frac{48}{120}$
31. $2\frac{7}{9}$ ○ $3\frac{1}{3}$ $2\frac{7}{9} < 3\frac{1}{3}$
32. $3\frac{2}{5}$ ○ $3\frac{5}{8}$ $3\frac{16}{40} < 3\frac{25}{40}$
33. $\frac{19}{6}$ ○ $3\frac{5}{6}$ $3\frac{1}{6} < 3\frac{5}{6}$ or $\frac{19}{6} < \frac{23}{6}$
34. $2\frac{11}{12}$ ○ $2\frac{8}{9}$ $2\frac{33}{36} > 2\frac{32}{36}$
35. $4\frac{3}{8}$ ○ $3\frac{4}{8}$ $4\frac{3}{8} > 3\frac{4}{8}$
36. $2\frac{3}{10}$ ○ $\frac{14}{5}$ $2\frac{3}{10} < 2\frac{8}{10}$ or $\frac{23}{10} < \frac{28}{10}$

Write the fractions from *least* to *greatest*.

37. $\frac{2}{3}$ $\frac{7}{8}$ $\frac{5}{6}$ $\frac{2}{3}$ $\frac{5}{6}$ $\frac{7}{8}$
38. $\frac{5}{12}$ $\frac{3}{8}$ $\frac{4}{10}$ $\frac{3}{8}$ $\frac{4}{10}$ $\frac{5}{12}$
39. $\frac{13}{8}$ $2\frac{1}{4}$ $1\frac{3}{5}$ $1\frac{3}{5}$ $\frac{13}{8}$ $2\frac{1}{4}$

Solve.

40. During the class party, $\frac{1}{3}$ of the students bowled, $\frac{4}{15}$ of the students played miniature golf, and $\frac{2}{5}$ of the students rode on go-carts. Which activity was chosen by the most students? *go-carts*

Complete DAILY REVIEW f on page 416.

Lesson 36 87

Compare unlike fractions by renaming

1. Write $\frac{4}{7}$ and $\frac{8}{21}$ for display. Explain that you can compare unlike fractions by renaming them to fractions with a common denominator, using the least common multiple (LCM). Remind the students that the LCM of any 2 numbers is equal to or greater than the number of greater value.
   - ➤ **Are the denominators in $\frac{4}{7}$ and $\frac{8}{21}$ related? How do you know?** ***Yes; 21 is a multiple of 7.***
   - ➤ **What is the LCM of the denominators 7 and 21? Why?** ***21; when the greater number is a multiple of the lesser number, the LCM is the greater number.*** Point out that the LCM can also be referred to as the *least common denominator.*
2. Write for display $\frac{4}{7} = \frac{n}{21}$.
   - ➤ **What fractional name for 1 can you multiply $\frac{4}{7}$ by to rename $\frac{4}{7}$ as twenty-firsts?** ***$\frac{3}{3}$***
3. Choose a student to write and solve an equation to rename $\frac{4}{7}$. *$\frac{4 \times 3}{7 \times 3} = \frac{12}{21}$* Erase n in $\frac{4}{7} = \frac{n}{21}$ and write 12 in its place.
   - ➤ **How does $\frac{4}{7}$ compare with $\frac{8}{21}$? How do you know?** ***$\frac{4}{7} > \frac{8}{21}$ because $\frac{12}{21} > \frac{8}{21}$*** Select a student to write the inequality. Point out that since the denominators are related (i.e., one denominator is a multiple of the other denominator), only one of the fractions needed to be renamed.
4. Follow a similar procedure to guide the students in comparing $\frac{5}{12}$ and $\frac{6}{15}$. Point out that since the denominators are not related, you must find the LCM of the denominators and use the LCM to rename both fractions. Guide the students in finding the LCM of 12 and 15. ***12: $2^2 \cdot 3$, 15: $3 \cdot 5$, LCM: $2^2 \cdot 3 \cdot 5 = 60$; $\frac{5 \times 5}{12 \times 5} = \frac{25}{60}$ and $\frac{6 \times 4}{15 \times 4} = \frac{24}{60}$; $\frac{25}{60} > \frac{24}{60}$; $\frac{5}{12} > \frac{6}{15}$***

Compare mixed numbers and improper fractions

Brittany spent $2\frac{1}{3}$ hours practicing the piano and $2\frac{1}{4}$ hours practicing the flute. Which instrument did she practice the most? *piano*

- ➤ **How could you solve this word problem?** ***Compare $2\frac{1}{4}$ hours and $2\frac{1}{3}$ hours.***
- ➤ **Do you need to use any new strategies to compare mixed numbers? Why?** ***No; elicit that the students have already learned the math concepts needed to compare the mixed numbers. First, you compare the whole numbers. Since the whole numbers are the same, you compare the fractions.***

1. Direct the students to determine which instrument was practiced the most. Choose a student to give the answer. Guide a discussion about how the students compared $\frac{1}{3}$ and $\frac{1}{4}$.
2. Instruct the students to solve the same word problem using $\frac{9}{4}$ hours of piano practice and $1\frac{3}{4}$ hours of flute practice. *piano* Elicit that you can either rename the improper fraction $\frac{9}{4}$ as a mixed number or rename the mixed number $1\frac{3}{4}$ as an improper fraction to solve the problem.
3. Discuss the strategies used as you guide the students in ordering these mixed numbers and improper fractions from least to greatest. $1\frac{4}{5}$, $\frac{11}{7}$, $\frac{12}{10}$ *$\frac{12}{10}$, $\frac{11}{7}$, $1\frac{4}{5}$* and $1\frac{1}{6}$, $2\frac{1}{4}$, $\frac{9}{8}$ *$\frac{9}{8}$, $1\frac{1}{6}$, $2\frac{1}{4}$*.

Student Text pp. 86–87

Lesson 37 | **Student Text** pp. 88–89
Daily Review p. 417g

Objectives

- Write an inequality to express unequal relationships
- Apply fraction number sense to compare and order fractions
- Compare and order fractions by renaming unlike fractions to fractions with a common denominator, by cross-multiplying, and by renaming as decimals

Introduce the Lesson

Review the strategies for comparing and ordering fractions that were taught in Lesson 36; use the information on Student Text pages 86–87.

Teach for Understanding

Apply strategies to compare and order fractions

> Shaun ran $\frac{7}{8}$ of a mile, Bryan ran $\frac{5}{6}$ of a mile, and Caleb ran $\frac{3}{8}$ of a mile. Which boy ran farthest? ***Shaun***

➤ **How could you solve this word problem?** ***Compare the fractions $\frac{7}{8}$, $\frac{5}{6}$, and $\frac{3}{8}$.***

1. Direct the students to compare the fractions and order them from least to greatest. Choose a student to write the ordered fractions for display. ***$\frac{3}{8}$, $\frac{5}{6}$, $\frac{7}{8}$***

   ➤ **Which boy ran farthest?** ***Shaun***

   Discuss the strategies that the students used to determine that $\frac{7}{8}$ is the greatest fraction. If necessary, point out other strategies that could have been used to compare pairs of fractions. Write the corresponding inequalities.

   $\frac{7}{8}$ and $\frac{3}{8}$ ***Same Size Parts, $\frac{7}{8} > \frac{3}{8}$***
   $\frac{5}{6}$ and $\frac{3}{8}$ ***Greater or Less Than $\frac{1}{2}$, $\frac{5}{6} > \frac{3}{8}$***
   $\frac{7}{8}$ and $\frac{5}{6}$ ***Closeness to 1, $\frac{7}{8} > \frac{5}{6}$***

2. Follow a similar procedure for $\frac{8}{14}$, $\frac{7}{12}$, and $\frac{8}{17}$. ***$\frac{8}{17}$, $\frac{8}{14}$, $\frac{7}{12}$***

   $\frac{8}{14}$ and $\frac{7}{12}$ ***Closeness to $\frac{1}{2}$, $\frac{8}{14} < \frac{7}{12}$***
   $\frac{8}{14}$ and $\frac{8}{17}$ ***Same Number of Parts, $\frac{8}{14} > \frac{8}{17}$***
   $\frac{7}{12}$ and $\frac{8}{17}$ ***Greater or Less Than $\frac{1}{2}$, $\frac{7}{12} > \frac{8}{17}$***

   ➤ **Why might the Greater or Less Than $\frac{1}{2}$ strategy be difficult to use when comparing $\frac{7}{12}$ and $\frac{8}{17}$?** ***Elicit that since 17 is an odd number, there is no fraction with a denominator of 17 that is equivalent to $\frac{1}{2}$; $\frac{8}{16} = \frac{1}{2}$, so $\frac{8}{17} < \frac{1}{2}$ and $\frac{9}{17} > \frac{1}{2}$.***

   ➤ **What other strategy could you use to compare $\frac{7}{12}$ and $\frac{8}{17}$?** ***Rename $\frac{7}{12}$ and $\frac{8}{17}$ to equivalent fractions with common denominators.***

3. Write $\frac{7}{12}$ and $\frac{8}{17}$ for display.

   ➤ **Are the denominators in these fractions related? Why?** ***No; neither denominator is a multiple of the other denominator.***

   Elicit that to compare unrelated fractions you need to find the least common multiple to determine the least common denominator.

   ➤ **What strategies can you use to find the LCM of 12 and 17?** ***List multiples, make Venn diagrams, and use prime factorizations.***

4. Guide the students in constructing a factor tree for 12 to find the prime factorization. ***$2 \times 2 \times 3$*** Elicit that you do not need to construct a factor tree for 17 because 17 is a prime number. Write $12 = 2^2 \times 3$ and $17 = 1 \times 17$ for display.

   ➤ **What equation could you use to multiply the highest power of prime numbers from the prime factorizations?** ***$22 \times 3 \times 17$***

   ➤ **What equation could you use to multiply the highest power of prime numbers from the prime factorizations?** ***$2^2 \times 3 \times 17$***

   ➤ **When one of the denominators is a prime number, what strategy do you think you could use to find the LCM?** ***Elicit multiply the denominators, 12×17.***

   Select a student to demonstrate multiplying the denominators and another student to demonstrate multiplying the prime factorizations. ***204***

   ➤ **How would checking for denominators that are prime save time when finding the LCM to rename fractions?** ***Elicit that since the products are the same, it is quicker to multiply the denominators when one of them is prime than to use the prime factorizations.***

5. Write $\frac{7}{12} = \frac{n}{204}$ and $\frac{8}{17} = \frac{n}{204}$ for display. Direct the students to rename the fractions and compare them. ***$\frac{7 \times 17}{12 \times 17} = \frac{119}{204}$ and $\frac{8 \times 12}{17 \times 12} = \frac{96}{204}$; $\frac{119}{204} > \frac{96}{204}$; $\frac{7}{12} > \frac{8}{17}$***

Compare fractions by cross-multiplying

1. Explain that when you rename fractions by multiplying the 2 denominators to find a common multiple, you can use a strategy called *cross multiplication* to determine the numerators that correspond with the common denominator and then compare the fractions.

   ➤ **What did we determine the common denominator (common multiple) of $\frac{7}{12}$ and $\frac{8}{17}$ to be?** ***204***

2. Guide the students in cross-multiplying $\frac{7}{12}$ and $\frac{8}{17}$. Point out that you are finding the numerators that correspond with the common denominator 204 that was calculated by multiplying the denominators 12 and 17.

   $17 \times 7 = 119 \quad \frac{7}{12} \times \frac{8}{17} \quad 12 \times 8 = 96$

3. Compare the process of cross-multiplying to the previous process used to rename $\frac{7}{12}$ and $\frac{8}{17}$ to fractions with the common denominator 204. Point out that since you cross-multiply to find numerators that correspond with a common denominator, you need to be careful not to mix up the numerators, which would result in incorrect comparisons.

4. Guide the students in using cross multiplication to compare these fractions.

   $\frac{4}{10}$ and $\frac{13}{20}$ ***$20 \times 4 = 80$ and $10 \times 13 = 130$; since $80 < 130$, $\frac{4}{10} < \frac{13}{20}$.***
   $\frac{3}{4}$ and $\frac{7}{16}$ ***$16 \times 3 = 48$ and $4 \times 7 = 28$; since $48 > 28$, $\frac{3}{4} > \frac{7}{16}$.***

Compare fractions by renaming as decimals

1. Write $\frac{7}{10}$ and $\frac{67}{100}$ for display. Discuss with the students the strategies that could be used to compare these fractions.

2. Point out that all fractions can be renamed in decimal form and then compared. Discuss renaming fractions in which the denominator is a power of 10 as you guide the students in renaming $\frac{7}{10}$ and $\frac{67}{100}$ as decimals and comparing the decimals. Elicit that you can annex a 0 to rename 7 tenths as 7 hundredths without changing the value of the decimal. ***0.7 and 0.67; $0.7 > 0.67$ or $0.70 > 0.67$***

3. Write $\frac{7}{12}$ and $\frac{8}{17}$ for display.

   ➤ **How can you rename a fraction as a decimal when the denominator is not a power of 10?** ***Elicit that you can divide the numerator by the denominator.***

More Comparing Fractions

Use these strategies to compare and order fractions.
See pages 86–87 for the complete explanation of each strategy.

convert fractions to decimals
cross multiplication

Same Size Parts	$\frac{7}{9} > \frac{6}{9}$	Rename One Fraction	$\frac{2}{8} > \frac{1}{32}$; $\frac{8}{32} > \frac{1}{32}$
Same Number of Parts	$\frac{3}{5} < \frac{3}{4}$		
Greater or Less Than $\frac{1}{2}$	$\frac{5}{6} > \frac{3}{8}$	Rename Both Fractions	$\frac{2}{3} < \frac{5}{7}$; $\frac{14}{21} < \frac{15}{21}$
Closeness to $\frac{1}{2}$ or 1	$\frac{8}{9} > \frac{5}{6}$		

Rename Fractions as Decimals to Compare

Remember that all fractions can be renamed in decimal form. The decimal form can be used to compare fractions.

- If a denominator is a power of 10, simply write the fraction in decimal form.
- If a denominator is not a power of 10, divide the numerator by the denominator. If a fraction does not have an exact decimal equivalent, round to the nearest hundredth or thousandth.

$\frac{7}{10} < \frac{11}{15}$ — $0.7 < 0.73$

$11.000 \div 15 = 0.733$: -105, 50, -45, 50, -45, 5

$1.000 \div 8 = 0.125$: -8, 20, -16, 40, -40, 0

$\frac{1}{8} < \frac{4}{23}$ — $0.13 < 0.17$

$4.000 \div 23 = 0.173$: -23, 170, -161, 90, -69, 21

Exercises

Use any strategy to compare the fractions.
Write a comparison sentence using >, <, or =.

1. $\frac{4}{10} < \frac{5}{12}$
2. $\frac{8}{11} > \frac{3}{5}$
3. $\frac{9}{10} > \frac{9}{13}$
4. $\frac{11}{12} < \frac{15}{16}$
5. $\frac{8}{9} < \frac{13}{14}$
6. $\frac{3}{12} = \frac{12}{48}$
7. $\frac{2}{3} < \frac{3}{4}$
8. $\frac{13}{15} > \frac{8}{10}$

On December 26, 2004, a tsunami hit Thailand.

Rename each fraction as a decimal.
Round to the nearest thousandth if needed.
Write a comparison sentence using >, <, or =.

9. $\frac{7}{10} < \frac{12}{15}$ *(0.7 < 0.8)*
10. $\frac{1}{3} < \frac{9}{23}$ *(0.333 < 0.391)*
11. $\frac{14}{70} = \frac{1}{5}$ *(0.2 = 0.2)*
12. $\frac{3}{4} < \frac{20}{25}$ *(0.75 < 0.8)*
13. $\frac{13}{21} > \frac{30}{49}$ *(0.619 > 0.612)*
14. $\frac{4}{5} > \frac{5}{7}$ *(0.8 > 0.714)*
15. $\frac{12}{19} > \frac{8}{13}$ *(0.632 > 0.615)*
16. $\frac{3}{10} < \frac{4}{11}$ *(0.3 < 0.364)*

88 Chapter 4

Answer the question about the survey.
Explain the processes used for solving.

17. During a recent survey, $\frac{2}{4}$ of the church youth group preferred chocolate milkshakes, $\frac{1}{8}$ preferred vanilla milkshakes, and $\frac{6}{16}$ preferred strawberry milkshakes. What is the favorite milkshake flavor of the youth group? ***chocolate***
18. When asked about their favorite animal for a pet, $\frac{2}{5}$ of the children chose dogs, $\frac{1}{3}$ chose cats, and $\frac{4}{15}$ of the children chose other animals. Did more children choose cats or other animals? ***cats***
19. Survey results indicate that $\frac{1}{6}$ of adults travel to work by bus, $\frac{3}{4}$ travel by car, and $\frac{1}{12}$ walk to work. Which method of transportation is the least common among adults? ***walking***
20. When asked about their favorite subject, $\frac{1}{6}$ of the students preferred science, $\frac{3}{12}$ preferred math, $\frac{1}{3}$ preferred history, and $\frac{1}{4}$ preferred language arts. Which subject was the favorite among the students? ***history***

Cross-Multiply to Compare

Two unlike fractions may be compared by using a process called **cross multiplication**. $\frac{2}{3} \bigcirc \frac{5}{7}$

- Cross-multiply the numerators and denominators. $7 \times 2 = 14$ $\frac{2}{3} \times \frac{5}{7}$ $3 \times 5 = 15$
- Compare the products. $14 < 15$; $\frac{2}{3} < \frac{5}{7}$

This example shows why cross multiplication works. $\frac{2}{3} \bigcirc \frac{5}{7}$

- Find the common denominator by multiplying the two denominators. $3 \times 7 = 21$
- Rename the fractions using the common denominator. $\frac{2 \times 7}{3 \times 7} = \frac{14}{21}$ $\frac{5 \times 3}{7 \times 3} = \frac{15}{21}$; $\frac{14}{21} < \frac{15}{21}$

Notice that the renamed numerators in this example are the same as the products in the cross multiplication above. $\frac{2}{3} < \frac{5}{7}$

Exercises

Write a comparison sentence using >, <, or =.
Use cross multiplication.

21. $\frac{2}{9} > \frac{1}{5}$ *(10 > 9)*
22. $\frac{5}{7} < \frac{8}{11}$ *(55 < 56)*
23. $\frac{3}{5} = \frac{9}{15}$ *(45 = 45)*
24. $\frac{9}{12} < \frac{8}{10}$ *(90 < 96)*
25. $\frac{3}{4} < \frac{7}{8}$ *(24 < 28)*
26. $\frac{6}{9} > \frac{5}{8}$ *(48 > 45)*
27. $\frac{12}{34} < \frac{3}{4}$ *(48 < 102)*
28. $\frac{3}{7} > \frac{10}{28}$ *(84 > 70)*

Complete DAILY REVIEW g on page 417.

Lesson 37 89

Guide students in renaming $\frac{7}{12}$ and $\frac{8}{17}$ as decimals and then comparing the decimals. Remind them that if a fraction does not have an exact decimal equivalent, you can round the decimal to the nearest hundredth or thousandth. ***$7 \div 12 \approx 0.583$ (rounded to nearest thousandth) and $8 \div 17 \approx 0.471$ (rounded to the nearest thousandth); $0.583 > 0.471$, $\frac{7}{12} > \frac{8}{17}$***

Point out that the division problems do not always need to be completed. You can stop dividing as soon as you can identify the greater number: $0.583 > 0.4$, $\frac{7}{12} > \frac{8}{17}$.

4. Guide a discussion to compare the process of renaming the fractions $\frac{7}{12}$ and $\frac{8}{17}$ as decimals to the process of cross-multiplying.
5. Guide the students in comparing these fractions by renaming the fractions as decimals.

   $\frac{8}{13}$ and $\frac{11}{15}$ ***$8 \div 13 \approx 0.615$, $11 \div 15 \approx 0.733$; since $0.6 < 0.7$, $\frac{8}{13} < \frac{11}{15}$***

   $\frac{2}{5}$ and $\frac{26}{65}$ ***$2 \div 5 = 0.4$, $26 \div 65 = 0.4$; since $0.4 = 0.4$, $\frac{2}{5} = \frac{26}{65}$***

Student Text pp. 88–89

(***Note***: *Assessment* available on Teacher's Toolkit CD.)

Lesson 38 | **Student Text** pp. 90–91 | **Daily Review** p. 417h

Objectives

- Use a fraction model to represent a percent
- Write a percent as a fraction in lowest terms
- Write a fraction as a percent
- Use a circle graph to solve problems
- Make a circle graph to communicate data

Teacher Materials

- Percent Circle, page IA20 (CD)
- Christian Worldview Shaping, pages 10–14 (CD)
- Several blank pages for display

Student Materials

- Fraction Kit: fraction circles
- Christian Worldview Shaping, pages 11 and 13 (CD)

Teach for Understanding

Use a fraction model to represent a percent

1. Discuss the uses of percent in everyday life: figuring the tithe for church, a discount at a store, a tip at a restaurant, and so on. Remind the students that *percent* means "by the hundred," "out of a hundred," or "per hundred." Point out that they can use percents to compare parts of wholes that are different sizes since percent uses the common term of 100.

> Brenda and Jessica went shopping. They both purchased items on sale for 50% off the regular price. Brenda spent \$10 for an item with a regular price of \$20. Jessica spent \$25 for an item with a regular price of \$50.

➤ **What fraction of the regular price was the discount Brenda received? Why?** ***$\frac{1}{2}$; 50% of 100 is the same as $\frac{1}{2}$.***

➤ **What fraction of the regular price was the discount Jessica received? Why?** ***$\frac{1}{2}$; 50% of 100 is the same as $\frac{1}{2}$.***

➤ **Why did Jessica pay more than Brenda since both girls received $\frac{1}{2}$ off the whole price?** ***Elicit that $\frac{1}{2}$ of the whole is different because the wholes were different amounts. Jessica paid $\frac{1}{2}$ of \$50, and Brenda paid $\frac{1}{2}$ of \$20.***

2. Distribute the fraction circles from the Fraction Kit to the students and display the Percent Circle page. Point out that fractions are often used to explain percent: 100% of the percent circle is the whole circle and is equal to $\frac{100}{100}$ or 1. Write $\frac{100}{100}$ and 1 in the Ratio and Fraction columns of the table.
3. Place a blank page over the percent circle and shade in $\frac{1}{4}$ of the circle to indicate the part that is 25% of the circle.
   Tell the students to place 1 whole circle on their desks and to use their other fraction pieces to indicate the answers to your questions by placing the corresponding fraction piece(s) on their whole. (Their fraction should look like the fraction (percent) indicated on the page.)

➤ **What fraction is 25% of the circle?** ***Students should place 1 fourth on the whole; $\frac{1}{4}$.*** Write $\frac{25}{100}$ and $\frac{1}{4}$ in the table. Follow a similar procedure for the following.

$50\% = \frac{1}{2}$	$75\% = \frac{3}{4}$	$33\% = \frac{1}{3}$	$66\% = \frac{2}{3}$
$10\% = \frac{1}{10}$	$20\% = \frac{1}{5}$	$40\% = \frac{2}{5}$	

Write a percent as a fraction in lowest terms

➤ **Can all percents be written in fraction form? How?** ***Yes; elicit that since a percent is a ratio in which the quantity is compared to 100, you can write the percent as a fraction with a denominator of 100.***

1. Write 45% for display.

➤ **What is the fraction form for 45%?** ***$\frac{45}{100}$***

➤ **How can $\frac{45}{100}$ be renamed into lowest terms?** ***Possible answers: use the greatest common factor, which is 5, to divide by $\frac{5}{5}$, or determine prime factorizations to use cancellation.***

Direct the students to simplify $\frac{45}{100}$ using the strategy of their choice. Discuss the answer and the strategies the students used. (See Lesson 35 procedures.) ***$\frac{45 \div 5}{100 \div 5}$ or $\frac{3 \times 3 \times \cancel{5}}{2 \times 2 \times 5 \times \cancel{5}} = \frac{9}{20}$***

2. Provide additional practice changing percents to fractions in lowest terms.

$70\% = \frac{70}{100} = \frac{7}{10}$	$60\% = \frac{60}{100} = \frac{3}{5}$
$68\% = \frac{68}{100} = \frac{17}{25}$	$47\% = \frac{47}{100}$

Write a fraction as a percent

1. Write $\frac{3}{5} = \frac{n}{100}$ for display. Guide the students in writing an equation to rename the fraction and in writing the percent. ***$\frac{3 \times 20}{5 \times 20} = \frac{60}{100} = 60\%$***
2. Provide practice renaming fractions and writing the percent using a similar procedure.
   $\frac{9}{10} = \frac{9 \times 10}{10 \times 10} = \frac{90}{100} = 90\%$
   $\frac{3}{25} = \frac{3 \times 4}{25 \times 4} = \frac{12}{100} = 12\%$
3. Point out that if you know the percent equivalents for some fractions, then you can mentally calculate the percent for fractions with the same denominator.

➤ **What percent is $\frac{1}{10}$?** ***10%***

➤ **What percent is $\frac{2}{10}$?** ***20%*** **$\frac{3}{10}$?** ***30%*** **$\frac{7}{10}$?** ***70%***

➤ **What percent is $\frac{1}{5}$?** ***20%***

➤ **What percent is $\frac{2}{5}$** ***40%*** **$\frac{3}{5}$?** ***60%*** **$\frac{4}{5}$?** ***80%***

4. Write $\frac{1}{3} = \frac{n}{100}$ for display. Discuss with students that 3 is not a factor of 100, so there is no form of 1 that can be used to rename the fraction. Point out that another strategy involves using compatible numbers. The fraction $\frac{1}{3}$ can be renamed using a denominator that is close in value to 100 and compatible with 3–99.
   Write $\frac{1}{3} = \frac{n}{99}$ for display. Guide the students in writing an equation to rename the fraction and write the approximate percent. ***$\frac{1 \times 33}{3 \times 33} = \frac{33}{99}$; so $\frac{1}{3} \approx 33\%$***
5. Remind the students that percent can also be expressed as a decimal fraction instead of a common fraction. Renaming a fraction as a decimal is the most common strategy used to find percent when you don't already know the fraction equivalent or when the denominator is not a factor of 100.

➤ **How can you rename a fraction as a decimal?** ***Divide the numerator by the denominator.***

Instruct the students to rename $\frac{1}{3}$ as a decimal by dividing. Guide them in rounding the decimal to the nearest hundredth and renaming it as a percent. Remind the students that to rename a decimal to a percent they can multiply the decimal by 100 by moving the decimal point 2 places to the right, from after the Ones place to after the Hundredths place and replace the term *hundredths* with the term *percent*. ***$1 \div 3 \approx 0.33$; $\frac{1}{3} \approx 33\%$***

6. Write $\frac{5}{6} \approx$ __% for display. Discuss the strategy to use to write the fraction as a percent. ***Divide to rename the fraction as a decimal; round to the nearest hundredth; multiply by 100 by moving the decimal point 2 places to the right to rename the decimal as a percent; 83%.***
7. Follow a similar procedure to provide additional practice in changing fractions to percents.

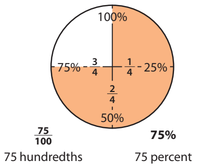
$\frac{3}{7} \approx 0.43 \approx 43\%$	***$\frac{8}{9} \approx 0.89 \approx 89\%$***

Fractions & Percents

A **percent** is a ratio in which a quantity is compared to 100.
A percent can also be written in fraction form.

The fraction $\frac{1}{100}$ can be written as 1%.
The fraction $\frac{3}{4}$ can be renamed as $\frac{75}{100}$ and written as 75%.
A fraction that names 1 whole, such as $\frac{2}{2}$, $\frac{5}{5}$, $\frac{43}{43}$, and $\frac{100}{100}$, is written as 100%.

percent
circle graph



If the denominator of a fraction is a factor of 100, the fraction can easily be renamed as a percent.

$\frac{1}{4} = \frac{25}{100} = 25\%$ $\quad \frac{1}{10} = \frac{10}{100} = 10\%$

$\frac{1}{2} = \frac{50}{100} = 50\%$ $\quad \frac{1}{5} = \frac{20}{100} = 20\%$

$\frac{3}{4} = \frac{75}{100} = 75\%$

Exercises

Write the fraction for the shaded part.
Write the fraction as a percent.

1.  $\frac{3}{3}$ = ***100%***
2.  $\frac{1}{2}$ = ***50%***
3.  $\frac{1}{4}$ = ***25%***
4.  $\frac{1}{5}$ = ***20%***
5.  $\frac{1}{10}$ = ***10%***
6.  $\frac{7}{10}$ = ***70%***

Write the equivalent fraction with a denominator of 100.
Write the new fraction as a percent.

7. $\frac{1}{2} = \frac{50}{100} = 50\%$
8. $\frac{1}{4} = \frac{25}{100} = 25\%$
9. $\frac{3}{10} = \frac{30}{100} = 30\%$
10. $\frac{4}{5} = \frac{80}{100} = 80\%$
11. $\frac{7}{20} = \frac{35}{100} = 35\%$
12. $\frac{6}{25} = \frac{24}{100} = 24\%$

Write the percent as a fraction in lowest terms.

13. $30\% = \frac{30}{100} = \frac{3}{10}$
14. $80\% = \frac{80}{100} = \frac{4}{5}$
15. $15\% = \frac{15}{100} = \frac{3}{20}$
16. $45\% = \frac{45}{100} = \frac{9}{20}$
17. $90\% = \frac{90}{100} = \frac{9}{10}$
18. $25\% = \frac{25}{100} = \frac{1}{4}$

If the denominator of a fraction is *not* a factor of 100, rename the fraction as a decimal and then as a percent.

1. To rename the fraction as a decimal, divide the numerator by the denominator. Round the answer to the nearest hundredth.
2. To rename the decimal as a percent, multiply the decimal by 100 by moving the decimal point 2 places to the right and replace the term *hundredths* with the term *percent*.

$\times 100$
$\frac{2}{9} \approx 0.22 \approx 22\%$
$9\overline{)2}$

$\times 100$
$\frac{5}{7} \approx 0.71 \approx 71\%$
$7\overline{)5}$

Exercises

Write the fraction as a percent by renaming as an equivalent fraction or a decimal.
Round to the nearest hundredth.

19. $\frac{2}{5} = \frac{40}{100} = 40\%$
20. $\frac{1}{3} \approx 0.33 \approx 33\%$
21. $\frac{3}{7} \approx 0.43 \approx 43\%$
22. $\frac{7}{10} = \frac{70}{100} = 70\%$
23. $\frac{5}{8} \approx 0.63 \approx 63\%$
24. $\frac{1}{6} \approx 0.17 \approx 17\%$

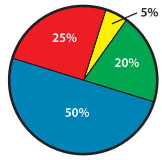
90 Chapter 4

In a **circle graph**, the whole circle represents 100% of the data. The parts of the circle represent parts of the data or a percent of the whole. The parts can also be represented by a fraction.

Exercises

The sixth-grade students attending a state competition participated in a survey about their favorite colors, sports, and subjects. The circle graphs show the results of the survey.

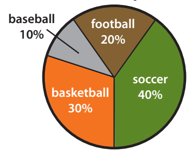
Favorite Color



Use the data from the circle graphs to answer the questions.

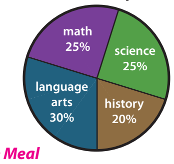
25. Which color do $\frac{1}{2}$ of the students like? ***blue***
26. Which color do $\frac{1}{4}$ of the students like? ***red***
27. What percent of the students like yellow? ***5%***
28. What percent of the students like green or yellow? ***25%***

Favorite Sport



29. What fraction of the students chose football as their favorite sport? $\frac{20}{100} = \frac{1}{5}$
30. What fraction of the students chose soccer as their favorite sport? $\frac{40}{100} = \frac{2}{5}$
31. What percent of the students chose baseball as their favorite sport? ***10%***
32. What percent of the students chose a sport other than baseball? ***100% − 10% = 90%***

Favorite Subject



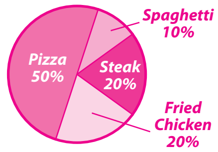
33. What fraction of the students chose math as their favorite subject? $\frac{25}{100} = \frac{1}{4}$
34. What fraction of the students chose language arts as their favorite subject? $\frac{30}{100} = \frac{3}{10}$
35. What percent of the students chose a subject other than science? ***100% − 25% = 75%***
36. What percent of the students chose history or science? ***20% + 25% = 45%***

Draw a circle graph to represent the data in the chart.

37.

Favorite Meal	
Pizza	50%
Spaghetti	10%
Steak	20%
Fried Chicken	20%

Favorite Meal



Complete DAILY REVIEW on page 417.

Lesson 38 91

Use a circle graph to solve problems

1. Refer the students to the first circle graph on Student Text page 91. Explain that in a circle graph, the whole circle indicates 100% of the data. The parts of the circle represent parts of the data or a percent of the whole. This circle graph represents the data collected in a survey.
2. Practice reading the first circle graph using questions similar to these.
   - ➤ **Which color is the least favorite among the sixth graders?** ***yellow***
   - ➤ **What percent of the students like red or blue?** ***75%***
   - ➤ **What percent of the students chose a color other than yellow?** ***95%***
   - ➤ **What fraction of the students chose green?** $\frac{1}{5}$
   - ➤ **What fraction of the students chose yellow?** $\frac{1}{20}$
3. Remind the students that circle graphs are also useful when comparing groups that are different sizes since the wholes do not have to represent the same number. This circle graph showing the sixth-graders' favorite colors could be compared to a circle graph showing the favorite colors of the first graders or of all of the students attending the state competition. The graphs would represent a different number of students; however, the graphs could be analyzed to see if the percents of the color preferences are the same for the younger students or if they are the same for all of the students attending the competition.
4. Take a survey of the class and make a circle graph (optional). Discuss how to make the circle graph. Point out several options if the parts of the whole are not easy to draw, such as finding a common denominator for all the fractions so the circle is easier to partition.
   (*Note:* Students will be taught in Lesson 58 how to use a protractor and the degrees of a circle to make a circle graph.)
5. **Christian Worldview Shaping (CD)**

Student Text pp. 90–91

Lesson 39 **Student Text** pp. 92–93

Chapter Review

Objectives

- Draw models of whole shapes, whole sets, and number lines to represent fractions
- Rename fractions to higher terms and lower terms
- Rename an improper fraction as a mixed number and a mixed number as an improper fraction
- Determine the GCF and the LCM of two numbers
- Rename a fraction to lowest terms (simplify) using the GCF or cancellation
- Write equivalent fractions using the LCM as the least common denominator
- Compare and order fractions and mixed numbers
- Apply fraction concepts to problem-solving situations

Note

This lesson reviews the concepts presented in Chapter 4 to prepare the students for the Chapter 4 Test. Student Text pages 92–93 provide the students with an excellent study guide.

Check for Understanding

Draw models to represent fractions
Rename fractions to higher terms and lower terms

1. Write $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{8}{12}$ for display. Remind the students that fractions name a part or parts of a whole and that you can model fractions using whole shapes, whole sets, and number lines. Direct the students to illustrate the fractions, drawing a different model for each fraction. (See Student Text page 80 for examples.)

➤ **How do you use multiplication or division to rename a fraction to higher terms?** ***multiply the fraction by a fraction form for 1*** **to lower terms?** ***divide the fraction by a fraction form for 1***

2. Direct the students to write an equation to rename $\frac{2}{3}$ to twenty-fourths and an equation to rename $\frac{8}{12}$ to thirds.
   $\frac{2 \times 8}{3 \times 8} = \frac{16}{24}$ ***and*** $\frac{8 \div 4}{12 \div 4} = \frac{2}{3}$
3. Write $\frac{4}{3}$ and $\frac{11}{8}$ for display. Direct the students to draw a model to illustrate each improper fraction and to write the improper fraction as a mixed number. $1\frac{1}{3}$; $1\frac{3}{8}$ (See Student Text page 82 for examples.)

Rename improper fractions and mixed numbers

➤ **How could you rename a mixed number as an improper fraction?** ***Multiply the whole number by the number of parts in the whole and add the fractional parts.*** **an improper fraction as a mixed number?** ***Divide the numerator by the denominator to find the number of wholes and write the remainder as a fraction to show how many parts of the next whole there are.***

Instruct the students to rename these improper fractions and mixed numbers.

$\frac{14}{3} = 4\frac{2}{3}$ $\qquad \frac{55}{9} = 6\frac{1}{9}$ $\qquad \frac{8}{5} = 1\frac{3}{5}$

$3\frac{5}{8} = \frac{29}{8}$ $\qquad 1\frac{7}{12} = \frac{19}{12}$ $\qquad 2\frac{4}{5} = \frac{14}{15}$

Determine the GCF of two numbers

1. Guide the students in constructing factor trees for 18, 28, 30, and 42 and writing the prime factorizations of the numbers. (See Lesson 31.)
   (***Note:*** Continue to display the prime factorizations.)
   $18 = 2 \cdot 3 \cdot 3$ $\qquad 30 = 2 \cdot 3 \cdot 5$
   $28 = 2 \cdot 2 \cdot 7$ $\qquad 42 = 2 \cdot 3 \cdot 7$

➤ **How can you find the GCF of two numbers using prime factorizations?** ***Multiply the prime factors that are common to both numbers.***

2. Instruct the students to use the prime factorizations to find the GCF of 18 and 28 ***2*** and the GCF of 18 and 30 ***6 (2 • 3)***.
3. Choose a student to demonstrate finding the GCF of 8 and 12 by listing the factors and another student to demonstrate finding the GCF by creating a Venn diagram. (See Lesson 31.)
   8: 1, 2, 4, 8
   12: 1, 2, 3, 4, 6, 12 ***GCF: 4***

Simplify a fraction using the GCF or cancellation

1. Remind the students that one way to rename a fraction to lowest terms (simplist form) is to divide the fraction by the GCF. Guide them as they simplify $\frac{18}{30}$ and $\frac{8}{12}$ using the GCF.
   $\frac{18 \div 6}{30 \div 6} = \frac{3}{5}$; $\frac{8 \div 4}{12 \div 4} = \frac{2}{3}$
2. Remind the students that cancellation is another way to simplify fractions. Guide them as they simplify these fractions using cancellation with the prime factorizations. (See Lesson 35.)
   $\frac{18}{28} = \frac{\cancel{2} \times 3 \times 3}{\cancel{2} \times 2 \times 7} = \frac{3 \times 3}{2 \times 7} = \frac{9}{14}$ $\qquad \frac{28}{42} = \frac{\cancel{2} \times 2 \times \cancel{7}}{\cancel{2} \times 3 \times \cancel{7}} = \frac{2}{3}$

Write equivalent fractions using the LCM
Compare fractions

➤ **How can you find the LCM of two numbers using prime factorization?** ***Multiply the highest power of each prime factor listed.***

1. Guide the students in writing the displayed prime factorizations with exponents. $18 = 2 \cdot 3^2$, $30 = 2 \cdot 3 \cdot 5$, $28 = 2^2 \cdot 7$, $42 = 2 \cdot 3$. Use the prime factorizations to find the LCM of 18 and 28 $2^2 \cdot 3^2 \cdot 7 = 252$ and the LCM of 18 and 30 $2 \cdot 3^2 \cdot 5 = 90$. (See Lesson 32.)
2. Choose a student to demonstrate finding the LCM of 8 and 12 by listing the multiples and another student to demonstrate finding the LCM by creating a Venn diagram.
   8: 8, 16, 24, 32, 40, 48
   12: 12, 24, 36, 48, 60, 72 ***LCM: 24***
3. Direct the students to compare these fractions by renaming the fractions using the LCM of the denominators. (See Lesson 36.)
   $\frac{11}{18}$ and $\frac{19}{30}$
   LCM of 18 and 30: 90; $\frac{11 \times 5}{18 \times 5} = \frac{55}{90}$ ***and*** $\frac{19 \times 3}{30 \times 3} = \frac{57}{90}$; $\frac{55}{90} < \frac{57}{90}$, $\frac{11}{18} < \frac{19}{30}$
   $\frac{3}{8}$ and $\frac{5}{12}$
   LCM of 8 and 12: 24; $\frac{3 \times 3}{8 \times 3} = \frac{9}{24}$; $\frac{5 \times 2}{12 \times 2} = \frac{10}{24}$; $\frac{9}{24} < \frac{10}{24}$, $\frac{3}{8} < \frac{5}{12}$

CHAPTER 4 REVIEW

Draw a model.

1. a number line to represent $\frac{5}{6}$ *0* *1*
2. a set to represent $\frac{3}{8}$
3. a figure to represent $\frac{4}{7}$

Draw a model.
Write the improper fraction as a mixed number.

4. triangles to represent $\frac{5}{2}$ $\frac{5}{2} = 2\frac{1}{2}$
5. circles to represent $\frac{7}{4}$ $\frac{7}{4} = 1\frac{3}{4}$
6. rectangles to represent $\frac{9}{5}$ $\frac{9}{5} = 1\frac{4}{5}$

Write the equivalent fraction.

7. $\frac{3}{8} = \frac{n}{24}$ $\frac{3}{8} = \frac{9}{24}$
8. $\frac{4}{7} = \frac{16}{n}$ $\frac{4}{7} = \frac{16}{28}$
9. $\frac{n}{12} = \frac{25}{60}$ $\frac{5}{12} = \frac{25}{60}$
10. $\frac{5}{n} = \frac{15}{18}$ $\frac{5}{6} = \frac{15}{18}$

Rename an improper fraction as a mixed number.
Rename a mixed number as an improper fraction.

11. $\frac{17}{10}$ $1\frac{7}{10}$
12. $\frac{5}{2}$ $2\frac{1}{2}$
13. $\frac{23}{5}$ $4\frac{3}{5}$
14. $4\frac{2}{3}$ $\frac{14}{3}$
15. $3\frac{2}{8}$ $\frac{26}{8}$
16. $1\frac{7}{12}$ $\frac{19}{12}$

Find the greatest common factor using the prime factorization given.

17. 28: 2 • 2 • 7
35: 5 • 7
GCF of 28 and 35: *7*
18. 18: 2 • 3 • 3
27: 3 • 3 • 3
GCF of 18 and 27: *9*
19. 12: 2 • 2 • 3
20: 2 • 2 • 5
GCF of 12 and 20: *4*

Find the least common multiple using the prime factorization given.

20. 28: $2^2 \cdot 7$
35: 5 • 7
LCM of 28 and 35: *140*
21. 18: $2 \cdot 3^2$
27: 3^3
LCM of 18 and 27: *54*
22. 12: $2^2 \cdot 3$
20: $2^2 \cdot 5$
LCM of 12 and 20: *60*

Make a factor tree for the number.
Write the prime factorization using exponents. *Beginning factors may vary.*

23. 24
3 • 8
3 • 2 • 4
3 • 2 • 2 • 2
$24 = 2^3 \cdot 3$

24. 36
4 • 9
2 • 2 • 3 • 3
$36 = 2^2 \cdot 3^2$

25. 40
5 • 8
5 • 2 • 4
5 • 2 • 2 • 2
$40 = 2^3 \cdot 5$

Family Earthquake Survival Kit

92 Chapter 4

Write the fraction in lowest terms. Identify the GCF. *Process and steps used to rename may vary.*

26. $\frac{8}{12}$ *$\frac{2}{3}$; GCF: 4*
27. $\frac{12}{30}$ *$\frac{2}{5}$; GCF: 6*
28. $\frac{28}{35}$ *$\frac{4}{5}$; GCF: 7*
29. $\frac{32}{64}$ *$\frac{1}{2}$; GCF: 32*
30. $\frac{14}{50}$ *$\frac{7}{25}$; GCF: 2*
31. $\frac{18}{27}$ *$\frac{2}{3}$; GCF: 9*

Write a comparison sentence using >, <, or =. *Strategies used to compare may vary.*

32. $\frac{13}{24} > \frac{12}{46}$
33. $3\frac{2}{8} < 3\frac{5}{8}$
34. $\frac{4}{5} > \frac{4}{7}$
35. $2\frac{6}{10} = \frac{13}{5}$
36. $\frac{5}{6} < \frac{13}{14}$
37. $\frac{3}{7} > \frac{2}{5}$

Write the prime factorization of the numerator and the denominator.
Use cancellation to rename the fraction to lowest terms.

38. $\frac{48}{64}$ $\frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 3}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2} = \frac{3}{4}$
39. $\frac{39}{51}$ $\frac{\cancel{3} \cdot 13}{\cancel{3} \cdot 17} = \frac{13}{17}$

Use the strategy given to compare the fractions.
Write a comparison sentence using > or <.

Use the LCM		Rename Fractions as Decimals		Cross-multiply	
40. $\frac{5}{6}$ ○ $\frac{3}{8}$	*$\frac{20}{24} > \frac{9}{24}$*	42. $\frac{3}{8}$ ○ $\frac{4}{5}$	*0.375 < 0.8*	44. $\frac{17}{20}$ ○ $\frac{6}{8}$	*136 > 120*
41. $\frac{4}{9}$ ○ $\frac{7}{15}$	*$\frac{20}{45} < \frac{21}{45}$*	43. $\frac{4}{7}$ ○ $\frac{5}{10}$	*0.571 > 0.5*	45. $\frac{3}{4}$ ○ $\frac{11}{12}$	*36 < 44*

Write the fractions and mixed numbers from *least* to *greatest*.

46. $\frac{3}{3}$ $\frac{5}{6}$ $\frac{7}{8}$ *$\frac{5}{6}$ $\frac{7}{8}$ $\frac{3}{3}$*
47. $1\frac{5}{12}$ $\frac{3}{2}$ $1\frac{3}{8}$ *$1\frac{3}{8}$ $1\frac{5}{12}$ $\frac{3}{2}$*

Solve.

48. The students voted for the food they wanted served at their party. Pizza received $\frac{1}{3}$ of the votes, sub sandwiches received $\frac{4}{15}$ of the votes, and fried chicken received $\frac{2}{5}$ of the votes. What food received the most votes? ***fried chicken***
49. Five people are sharing 3 pizzas so that each person has a fair share. What fraction of a pizza will each person receive? ***$\frac{3}{5}$ of a pizza***
50. Five people are sharing 8 brownies. If they each receive the same amount, how many brownies will each person receive? ***$1\frac{3}{5}$ brownies***

Lesson 39 93

Compare and order fractions and mixed numbers

1. Review the fraction number-sense strategies and the renaming processes shown on Student Text pages 86–87. Guide in comparing these fractions and mixed numbers.

$\frac{7}{8} > \frac{7}{9}$ $\frac{3}{12} < \frac{5}{12}$ $\frac{14}{6} < 2\frac{8}{10}$
$\frac{3}{4} = \frac{6}{8}$ $\frac{9}{10} > \frac{5}{6}$ $1\frac{5}{12} > 1\frac{3}{8}$

Choose students to order these fractions and mixed numbers from least to greatest.

$\frac{4}{6}, \frac{7}{8}, \frac{6}{10}$ *$\frac{6}{10}, \frac{4}{6}, \frac{7}{8}$*
$2\frac{1}{4}, \frac{11}{5}, 1\frac{3}{4}$ *$1\frac{3}{4}, \frac{11}{5}, 2\frac{1}{4}$*

2. Guide the students as they cross-multiply to compare these fractions. (See Lesson 37.)

$\frac{2}{7}$ and $\frac{3}{5}$ *$5 \times 2 = 10$ and $7 \times 3 = 21$; since $10 < 21$, $\frac{2}{7} < \frac{3}{5}$*
$\frac{5}{17}$ and $\frac{4}{13}$ *$13 \times 5 = 65$ and $17 \times 4 = 68$; since $65 < 68$, $\frac{5}{17} < \frac{4}{13}$*

3. Direct the students to compare these fractions by renaming them as decimals. Remind them that if a fraction does not have an exact decimal equivalent, then you can round the decimal to the nearest hundredth or thousandth, or you can stop dividing as soon as you can identify the larger number. (See Lesson 37.)

$\frac{3}{8}$ and $\frac{4}{9}$ *$3 \div 8 = 0.375$ and $4 \div 9 \approx 0.444$; $0.3 < 0.4$, $\frac{3}{8} < \frac{4}{9}$*
$\frac{18}{19}$ and $\frac{6}{10}$ *$18 \div 19 \approx 0.947$ and $6 \div 10 = 0.6$; $0.9 > 0.6$, $\frac{18}{19} > \frac{6}{10}$*

Apply fraction concepts to problem-solving situations

Guide the students in solving these word problems. Discuss the strategies used by the students to find their answers. If necessary, point out other strategies the students could have used.

Seven people will share 4 pizzas so that each person has a fair share. What fraction of a pizza will each person receive? *$\frac{4}{7}$ of a pizza* [BAT: 5b Sharing]

Six people are sharing 9 cookies. If they each receive the same amount, how many cookies will each person receive? *$\frac{9}{6} = 1\frac{1}{2}$ cookies* [BAT: 5b Sharing]

Anna is walking laps on the track. She can walk 1 lap every 8 minutes. Carmen is skating on the track. She can skate 1 lap every 6 minutes. Anna and Carmen started their laps in front of the stadium at the same time. In how many minutes will they both be in front of the stadium at the same time again? *in 24 minutes*

Mom has 45 candy bars and 60 packs of gum for Jessie's birthday party. If she places both items equally into treat bags, what is the greatest number of bags she can fill? *15 bags; 3 candy bars and 4 packs of gum in each bag*

Student Text pp. 92–93

Lesson 40

Student Text pp. 94–97

Chapter 4 Test

Cumulative Review

For a list of the skills reviewed in the Cumulative Review, see the Lesson Objectives for Lesson 40 in the Chapter 4 Overview on page 74 of this Teacher's Edition.

Student Materials

- Cumulative Review Answer Sheet, page IA9 (CD)

Use the Cumulative Review on Student Text pages 94–96 to review previously taught concepts and to determine which students would benefit from your reteaching of the concepts. To prepare the students for the format of achievement tests, instruct them to work on a separate sheet of paper, if necessary, and to mark the answers on the Cumulative Review Answer Sheet.

Read aloud the Career Link on Student Text page 97 and discuss the value of math as it relates to a pediatrician.

CUMULATIVE REVIEW

Test Prep

Mark the answer.

1.

28			
7	7	7	7

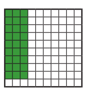
A. 4 + 7
B. 4 × 7
C. 4 + 4 + 4 + 4
D. A and B

2.

30					
5	5	5	5	5	5

A. 6 × 5
B. 6(5)
C. 5 + 5 + 5 + 5 + 5 + 5
D. all of the above

3.



A. 9 • 27
B. 27 ÷ 3
C. 9 × 3
D. B and C

4. 4 dozen eggs

A. 12 ÷ 4
B. 4 × 12
C. 3 × 4
D. A and C

5. 10^2

A. 10 × 2
B. 10 × 10
C. 100
D. B and C

6. $3^2 \times 7$

A. 2 × 3 × 7
B. 3 × 3 × 7
C. 6 × 7
D. A and C

7. 3.94

A. $(3 \times 10^0) + (9 \times \frac{1}{10^1}) + (4 \times \frac{1}{10^2})$
B. $(3 \times 10) + (9 \times \frac{1}{10}) + (4 \times \frac{1}{10})$
C. $(3 \times 10) + (9 \times 10) + (4 \times 100)$
D. all of the above

8. all factors of 36

A. 2, 4, 6, 8, 10, 12
B. 36 ÷ 12
C. 1, 2, 3, 4, 6, 9, 12, 18, 36
D. A and C

9. (3 × 4) × 2

A. 24
B. 3 × (4 × 2)
C. (2 × 4) × 3
D. all of the above

10. Identity Property

A. 57 × 0 = 0
B. 1 × 38 = 38
C. *a* • 1 = *a*
D. B and C

Mark the answer.

11. Which number is not composite?
A. 21
B. 23
C. 25

12. The value of 9 is ninety thousand.
A. 9,348
B. 904,237
C. 295,317

13. Find the quotient of 844 ÷ 2.
A. 420
B. 422
C. 424

14. Which numbers are listed from *least* to *greatest*?
A. 1,672   1,673   1,671   1,674
B. 1.31   1.47   1.5   1.52
C. 0.15   0.7   0.29   0.008

15. Which Roman numeral is 29?
A. XXIX
B. XXVIIII
C. XXXI

16. Jayden bought 7 tickets at $4.95 each. About how much money did he spend?
A. $40
B. $35
C. $30

17. Mrs. Owens made 4 dozen cookies. About how many cookies could each person eat at a reception for 20 people?
A. 1 cookie
B. 2 cookies
C. 3 cookies

18. Audrey's cookie recipe calls for 3 eggs. How many batches of cookies can she make with 1 dozen eggs?
A. 4 batches
B. 3 batches
C. 2 batches

19. Find the average of Mia's science test scores: 95, 89, 90, 86.
A. 100
B. 95
C. 90

5K Run for Kids

Place	Time	Name	Gender
1	21:40	Chris	M
2	22:31	Lucas	M
3	22:35	Kara	F
4	24:40	Brayden	M

Use the data from the chart to find the answer.

20. What was the best time for the 5K run?
A. 21:40
B. 21:31
C. 22:35
D. 24:40

21. How much faster than Kara did Lucas run?
A. 4 minutes
B. 4 seconds
C. 55 seconds
D. 1 minute 25 seconds

Age of Participants in 5K Run for Kids

Stem	Leaf
0	8 8 8 9 9 9
1	1 1 2 2 2 2 3 3

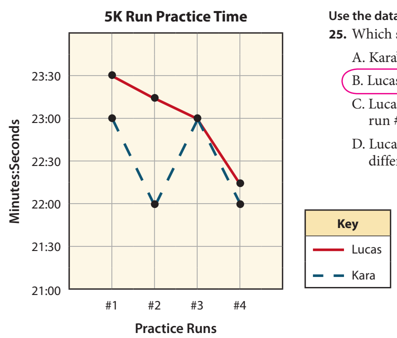
Key 1|3 = 13

Use the data from the stem-and-leaf- plot to find the answer.

22. How many thirteen-year-olds participated in the 5K run?
A. 1
B. 2
C. 8
D. none

23. How many ten-year-olds participated in the 5K run?
A. 1
B. 2
C. 3
D. none

24. What age was represented the most?
A. 11
B. 12
C. 18
D. 19



Use the data from the double line graph to find the answer.

25. Which statement is true according to this graph?
A. Kara's time improved with each practice run.
B. Lucas's time improved with each practice run.
C. Lucas's and Kara's times were the same for practice run #4.
D. Lucas's and Kara's times for practice run #1 had a difference of 1 minute.

CAREER LINK

Pediatrician

A pediatrician is a doctor specially trained to care for children from birth to age eighteen. Pediatricians treat everything from the common cold to childhood injuries. They comfort and care for children and their families.

Simple math is used in the daily work of pediatricians, such as when they take a child's temperature or monitor a child's growth and weight. They also measure and monitor blood pressure and blood sugar, read blood tests, and prescribe medication based on a child's size and weight. If medication is prescribed incorrectly, a child could be harmed or the child's condition could worsen. Prescribing the correct amount and type of medicine is essential to caring for patients.

Pediatricians work closely with parents of infants to ensure that the babies are growing properly. They regularly research the best methods of helping their patients stay healthy. Often they must convert standard measurements of pounds and inches to metric units, since most medications and standards appear in metric terms. Knowing how to read an x-ray, stitch a cut, or give an accurate emergency dose of lifesaving medication are all part of everyday math skills used by pediatricians.

A pediatrician trains for at least eleven years after high school and is required to learn and use math skills, including calculus, probability, and statistics. Knowledge of math and science, as well as a love for children, is essential to being a good doctor. Knowing the intricacies of the human body and the Creator Who designed it helps a pediatrician make wise choices and enjoy a profession of caring for these special patients.