

PERIL FROM ABOVE

New York, New York

July 28, 1945

In Manhattan, New York, the morning of July 28, 1945, dawned foggy with a light drizzling rain. People went about with raincoats and umbrellas, carrying out their normal weekend business. Some grew puzzled when the sound of a plane's engine began to drone not far away. The sound grew louder and louder until it reached an earsplitting roar. Office workers around the city hurried to their windows to find out the cause of the racket. To the terror of drivers and pedestrians, an enormous B-25 bomber was flying low, barely missing the towers of skyscrapers as it soared above Fifth Avenue at a height of 1,000 feet.

The pilot of the aircraft, Lt. Colonel William Smith, a decorated veteran of 100 combat missions, had become disoriented in the fog and must have imagined that he was over the landing strip at his destination, the New Jersey airport. Smith desperately tried to get his plane into an ascent, but it was too late.



Don Molony helps an injured woman down the stairs in the stricken Empire State Building.

At an altitude of 975 feet, the bomber, weighing more than ten tons, crashed into the seventy-eighth and seventy-ninth floors of the Empire State Building, where the fuel tanks exploded, sending flames in all directions. Several workers on the seventy-ninth floor were able to escape the flames and explosions. One of the plane's engines raced through the Empire State Building and exploded on the roof of the Waldorf Building nearby.

Help was soon on the way. The New York Fire Department arrived on the scene. A seventeen-year-old Coast Guardsman named Donald Molony, passing by the building at the time of the accident, quickly ran into a nearby drugstore and ordered first-aid kits and medical supplies. He was able to rescue a woman with severe back injuries who was trapped in an elevator shaft. Then, since the elevator could not be used, he ran up seventy-nine flights of stairs to reach the crash site. Molony helped firemen carry the frightened office workers safely to ground level.

Because it was the weekend, there were only about 1,500 workers in the building. The crash killed fourteen of them and injured twenty-six others. The Empire State Building was repaired within three months' time, and the only visible reminder of the crash is a faint line of black on the skyscraper's limestone surface.



The Empire State Building was constructed in a little over one year, requiring a total of 7,000,000 man-hours. It measures 1,453 feet from the ground to the tip of the broadcast tower. It has 6,514 windows; 73 elevators; and 5 entrances.

The building contains 473 miles of electrical wiring; 70 miles of pipe; 57,000 tons of steel; and 10,000,000 bricks.

The exterior of the building is made of 200,000 cubic feet of Indiana limestone and granite.

The building has 2.85 million square feet of rentable space; has 1,872 steps to the 103rd floor; and sits on 79,288 square feet (approximately two acres). The structure weighs 365,000 tons and has a volume of 37 million cubic feet.

In the Empire State Building Run-Up, an annual tradition since 1978, runners race up 1,576 stairs to the 86th floor. The record time of 9 minutes and 33 seconds was set in 2003.

In 1980 the Empire State Building received its own ZIP code: 10118.

Perimeter & Area

Lesson	Topic	Lesson Objectives	Chapter Materials
98	Perimeter	<ul style="list-style-type: none"> Calculate the perimeter of a polygon using a formula Calculate the unknown length of a side of a polygon Solve an algebraic expression to find the perimeter of a rectangle 	<p>Teaching Visuals (Teacher's Toolkit CD):</p> <ul style="list-style-type: none"> Chart 6: <i>Polygons</i> Chart 7: <i>Center Points, Radii & Diameters</i> Chart 8: <i>Chords & Central Angles</i> Chart 9: <i>Area</i> <p>Instructional Aids (Teacher's Toolkit CD):</p> <ul style="list-style-type: none"> Cumulative Review Answer Sheet (page IA9) for each student Graph Paper (page IA13) Graph Paper (page IA13) for each student Find the Circumference (page IA48) Find the Circumference (page IA48) for each student Area: Rectangles, Squares & Parallelograms (page IA49) Area: Rectangles, Squares & Parallelograms (page IA49) for each student Area: Triangles (page IA50) Area: Triangles (page IA50) for each student Area: Circles (page IA51) Area: Circles (page IA51) for each student Surface Area: Triangular Prism (page IA52) Surface Area: Triangular Prism (page IA52) for each student Surface Area (page IA53) Surface Area (page IA53) for each student Floor Plan Activity (page IA54) Floor Plan Activity (page IA54) for each pair of students Floor Plan Grid (page IA55) for each pair of students Geometry Review I (page IA56) Geometry Review II (page IA57) Nets (page IA58) <p>Christian Worldview Shaping (Teacher's Toolkit CD):</p> <ul style="list-style-type: none"> Pages 28–29 <p>Other Teaching Aids:</p> <ul style="list-style-type: none"> A cereal box (rectangular prism) for every 2 to 3 students and the teacher A 12-inch length of string for each pair of students A cylindrical object, such as a can or mug, for each pair of students A cylinder (potato chip container with top, a can, or similar object) for each student A small oatmeal container (optional) A classroom set of 3-dimensional objects (See Lesson 59.) A sheet of 12×18 construction paper for each pair of students 1–2 sheets of construction paper for each student Two 12×18 sheets of construction paper A calculator for each student A ruler for each student and the teacher Transparent tape for each student and the teacher <p>Math 6 Tests and Answer Key</p> <p>Optional (Teacher's Toolkit CD):</p> <ul style="list-style-type: none"> Fact Review pages Application pages Calculator Activities
99	Circumference	<ul style="list-style-type: none"> Develop an understanding of the relationship between the diameter and the circumference of a circle Calculate the circumference of a circle using a formula Calculate the diameter of a circle given the circumference Relate circumference to real-life situations 	
100	Area of Rectangles, Squares & Parallelograms	<ul style="list-style-type: none"> Calculate the area of rectangles, squares, and parallelograms using a formula Calculate the area of a complex figure Calculate the unknown side (length or width) of a rectangle or a square Relate area to real-life situations 	
101	Area of Triangles	<ul style="list-style-type: none"> Calculate the area of triangles using a formula Calculate the area of a complex figure Calculate the unknown height or base of a triangle Relate area to real-life situations 	
102	Area of Circles	<ul style="list-style-type: none"> Calculate the area of a circle using a formula Estimate the area of a circle Relate area to real-life situations 	
103	Surface Area of Prisms	<ul style="list-style-type: none"> Name the 3-dimensional figure that can be formed from a net Calculate the surface area of rectangular, square, and triangular prisms using formulas Construct a triangular prism Relate surface area to real-life situations 	
104	Surface Area of Cylinders	<ul style="list-style-type: none"> Calculate the surface area of rectangular, square, and triangular prisms using formulas Calculate the surface area of a cylinder using formulas Construct a cylinder net Relate surface area to real-life situations 	
105	Fixed Areas	<ul style="list-style-type: none"> Recognize that perimeter can vary for a fixed area Calculate the area and perimeter of a rectangle Calculate the area of a complex figure Create a basic floor plan from a fixed area Relate geometry to real-life situations 	
106	Chapter 11 Review	<ul style="list-style-type: none"> Review 	
107	Chapter 11 Test Cumulative Review	<ul style="list-style-type: none"> Identify prime and composite numbers Use the guess-and-check strategy to solve problems Identify equivalent expressions Estimate products and sums Identify all the factors of a number Add and multiply decimals Determine the value of a variable Read and interpret a pictograph Read and interpret a Venn Diagram 	

The ruler called for in this chapter and the following chapters is a customary/metric ruler that can be used to measure inches, centimeters, and millimeters.

Before teaching Lesson 103, collect the number of cereal boxes needed for the lesson.

A Little Extra Help

Use the following to provide “a little extra help” for the student that is experiencing difficulty with the concepts taught in Chapter 11.

Identify the net of a 3-dimensional figure—Provide the student with the polyhedrons that were constructed in Lesson 59, or construct new polyhedrons using the faces on page IA38 (Polyhedrons) on the Teacher’s Toolkit CD. Ask the student to select and identify one of the polyhedrons (e.g., a square pyramid). Explain that he can think of the base of the square pyramid as the center of its net with one side of each triangular face connected to each side of the square base. Direct the student to sketch a prediction of what the figure’s net would look like. Then guide him in cutting the square pyramid so that it opens flat to form its net. Guide the student in comparing his sketched net to the net of the actual figure.

Repeat the activity using the other polyhedrons from Lesson 59. Remind the student that prisms have 2 congruent bases.

(*Note:* You may choose to provide the student with objects such as a small cereal box, a Toblerone® chocolate box, or a small oatmeal container rather than use the polyhedrons from Lesson 59.)

Math Facts

Throughout this chapter, review fractions using Fact Review pages on the Teacher’s Toolkit CD. Also review multiplication and division facts using Fact Review pages or a Fact Fun activity on the Teacher’s Toolkit CD, or you may use flashcards.

Objectives

- Calculate the perimeter of a polygon using a formula
- Calculate the unknown length of a side of a polygon
- Solve an algebraic expression to find the perimeter of a rectangle

Teacher Materials

- Chart 6: *Polygons*

Student Materials

- A ruler

Notes

Preview the Fact Review pages, the Application pages, and the Calculator Activities located on the Teacher's Toolkit CD.

To reinforce the student's understanding of finding perimeter and area, encourage him throughout the chapter to first write the appropriate formula, and then to substitute the number of sides and/or the measurement(s) into the formula.

Introduce the Lesson

Guide the students in reading aloud the story and facts on pages 236–37 of the Student Text (pages 234–35 of this Teacher's Edition).

Teach for Understanding

Calculate perimeter using a formula

1. Display the *Polygons* chart. Remind the students that a *polygon* is a closed figure made up of line segments. Choose students to name a polygon and to tell its number of sides. Continue to display the chart.
2. Direct each student to draw a polygon and to write *Park* inside of it. Tell them that the polygon represents an imaginary city park around which they can jog with their dog one time each day.
 - **If 1 centimeter on your drawing represents 1 mile, how can you find the distance that you jog around the entire park each day? Elicit that they can use a ruler to measure the length of each side, and then they can add the lengths together.**
3. Explain that *perimeter* is the distance around a geometric figure. Direct each student to find the perimeter of his park. Elicit that their answers should be in *miles* because each centimeter of the drawing represents one mile. Allow volunteers to tell the distances they can jog around the park each day.
4. Write for display $P = \text{the sum of the lengths of the sides}$. Explain that this formula can be used to find the perimeter of any polygon. The perimeter (represented as P) of any polygon can be found by adding together the lengths of the sides.
5. Direct each student to draw a quadrilateral having four right angles and all four sides measuring 6 centimeters.
 - **What type of quadrilateral did you draw? a square** Draw the square for display; write 6 cm along each side.
 - **Using the formula, what equation can you write to find the perimeter of the square? $P = 6\text{ cm} + 6\text{ cm} + 6\text{ cm} + 6\text{ cm}$** Elicit that the equation can also be written $6\text{ cm} + 6\text{ cm} + 6\text{ cm} + 6\text{ cm} = P$.

(Note: Accept equations with the total perimeter [P] on either side of the equal sign. Label each addend to reinforce that like parts are being combined.)

- **What is the perimeter of the square? 24 cm**
- **What multiplication equation can you write to find the perimeter of the square? Why? $4 \times 6\text{ cm} = 24\text{ cm}$; elicit that since all four sides have the same length, the length of one side can be multiplied by 4 to find the perimeter (the total distance around the square).**

- **Since all the sides of a square are congruent, what multiplication formula can you use for finding the perimeter of a square? Elicit $P = 4 \times s$ or $P = 4s$.** Write the formulas for display.

Choose a student to use the multiplication formula to find the perimeter of a square in which each side measures 7 inches. $P = 4 \times 7\text{ in.}$; $P = 28\text{ in.}$

6. Direct attention to the regular hexagon on the *Polygons* chart.
 - **What formula can you use to find the perimeter of a regular hexagon? Why? $P = 6 \times s$ or $P = 6s$; elicit that a regular hexagon has 6 sides of equal length, so you can multiply the length of one side by 6 to find the perimeter.**

Choose a student to use the multiplication formula to find the perimeter of a regular hexagon in which each side measures 7 centimeters. $P = 6 \times 7\text{ cm}$; $P = 42\text{ cm}$
7. Follow a similar procedure for finding the perimeter of the following polygons.
 - A regular pentagon: 4-inch sides $P = 5s$; $P = 5 \times 4\text{ in.}$; $P = 20\text{ in.}$
 - A regular octagon: 5-inch sides $P = 8s$; $P = 8 \times 5\text{ in.}$; $P = 40\text{ in.}$
8. Direct each student to draw another quadrilateral having four right angles, one pair of congruent opposite sides measuring 6 centimeters, and the other pair of congruent opposite sides measuring 10 centimeters.
 - **What type of quadrilateral did you draw? a rectangle**
9. Draw the rectangle for display and write the measurements (6 cm and 10 cm) along the sides. Elicit two multi-step equations that could be written to find the perimeter of the rectangle and why the equations could be used. Write both equations for display.
 - $P = (2 \times 10\text{ cm}) + (2 \times 6\text{ cm})$; there are 2 congruent lengths and 2 congruent widths, so you can multiply the length by 2 and the width by 2 and then add the products.
 - $P = 2(10\text{ cm} + 6\text{ cm})$; a rectangle has 2 equal sets of length and width measurements, so you can add the length and width and multiply the sum by 2.
- **Why can both of these equations be used to find the perimeter of the rectangle? Elicit that both equations represent the same value. The Distributive Property allows you to write $2(10\text{ cm} + 6\text{ cm})$ as $(2 \times 10\text{ cm}) + (2 \times 6\text{ cm})$.**
10. Choose a student to solve $P = (2 \times 10\text{ cm}) + (2 \times 6\text{ cm})$, and select another student to solve $P = 2(10\text{ cm} + 6\text{ cm})$. **32 cm**
11. Guide the students in writing a formula for each equation: $P = (2 \cdot l) + (2 \cdot w)$ and $P = 2(l + w)$. Write $(2 \cdot l) + (2 \cdot w) = 2(l + w)$ for display.
 - **Is this equation a true mathematical statement? Why? Yes; elicit that both expressions represent the same value and the Distributive Property allows you to write the expression either way.**

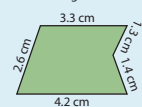
Perimeter

Perimeter is the distance around a geometric figure and is represented by P .

perimeter

Any Polygon

Add the lengths of the sides.



$$P = a + b + c + d + e$$

$$P = 2.6 \text{ cm} + 4.2 \text{ cm} + 1.4 \text{ cm} + 1.3 \text{ cm} + 3.3 \text{ cm}$$

$$P = 12.8 \text{ cm}$$

Regular Polygon

Multiply:
number of sides \cdot length of side



$$P = n \cdot s$$

$$P = 6 \cdot 20 \text{ mm}$$

$$P = 120 \text{ mm}$$

Rectangle

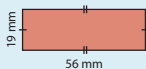
Multiply the length by 2 and the width by 2; add the products.

$$P = (2 \cdot l) + (2 \cdot w)$$

$$P = (2 \cdot 56 \text{ mm}) + (2 \cdot 19 \text{ mm})$$

$$P = 112 \text{ mm} + 38 \text{ mm}$$

$$P = 150 \text{ mm}$$



Multiply the sum of the length and the width by 2.

$$P = 2(l + w)$$

$$P = 2(56 \text{ mm} + 19 \text{ mm})$$

$$P = 2 \cdot 75 \text{ mm}$$

$$P = 150 \text{ mm}$$

Exercises

Measure the length of each side to the nearest centimeter.
Find the perimeter by adding the lengths of the sides.

- $P = n \cdot s$
 $P = 3 \cdot 3$
 $P = 9 \text{ cm}$
- $P = a + b + c + d$
 $P = 2 \frac{1}{2} \text{ cm} + 5 \text{ cm} + 2 \frac{1}{2} \text{ cm} + 1 \text{ cm}$
 $P = 11 \text{ cm}$
- $P = (2 \cdot l) + (2 \cdot w)$
 $P = (2 \cdot 5 \text{ cm}) + (2 \cdot 2 \text{ cm})$
 $P = 10 \text{ cm} + 4 \text{ cm} = 14 \text{ cm}$
- $P = 1 \frac{1}{2} \text{ cm} + 2 \frac{1}{2} \text{ cm} + 1 \text{ cm} + 2 \text{ cm}$
 $P = 7 \text{ cm}$
- $P = 8 \text{ m} + 7 \text{ m} + 2 \text{ m} + 1 \text{ m} + 1.5 \text{ m} + 5 \text{ m}$
 $P = 24.5 \text{ m}$
- $P = 3 \text{ in.} + 2.5 \text{ in.} + 3 \text{ in.} + 2.5 \text{ in.} + 6 \text{ in.} + 5 \text{ in.}$
 $P = 22 \text{ in.}$
- $P = 5 \text{ cm} + 10 \text{ cm} + 6 \text{ cm} + 3 \text{ cm} + 3 \text{ cm} + 2 \text{ cm}$
 $P = 29 \text{ cm}$
- $P = 5 \cdot 5 \text{ cm} = 25 \text{ cm}$
- $P = 3 \cdot 30 \text{ yd} = 90 \text{ yd}$
- $P = 7 \cdot 14 \text{ in.} = 98 \text{ in.}$
- $P = 10 \cdot 8.7 \text{ m} = 87 \text{ m}$

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Chapter 11

Calculate the unknown length of a side of a polygon

- Draw an isosceles triangle for display. Write 5 m along two sides and s along the third side. Write $P = 14 \text{ m}$ beside the triangle.
How can you find the length of the third side of the triangle? Elicit that you can subtract the sum of the lengths of the known sides from the perimeter (14 m); the difference is the length of the third side.
- Write $P = s + s + s$ for display. Point out that s represents each side.
 Choose a student to rewrite the equation using the known measurements. $14 \text{ m} = 5 \text{ m} + 5 \text{ m} + s$
 Write parentheses around $5 \text{ m} + 5 \text{ m}$. Elicit that although the parentheses are not needed to solve the equation, they are helpful in isolating the unknown measurement.
 Select a student to solve the equation. $14 \text{ m} = 10 \text{ m} + s$; $14 \text{ m} - 10 \text{ m} = 10 \text{ m} - 10 \text{ m} + s$; $4 \text{ m} = s$
- Follow a similar procedure to find the measurement of the unknown side of this irregular quadrilateral similar to the one, with a perimeter of 22 cm.
 $22 \text{ cm} = (7 \text{ cm} + 5 \text{ cm} + 6 \text{ cm}) + s$
 $22 \text{ cm} = 18 \text{ cm} + s$
 $22 \text{ cm} - 18 \text{ cm} = 18 \text{ cm} - 18 \text{ cm} + s$; $4 \text{ cm} = s$
- Draw a square for display. Write s along one side.
If the perimeter of the square is 28 inches, what equation can you write to find the unknown length of the side? Solve.
 $28 \text{ in.} = 4 \cdot s$; $28 \text{ in.} \div 4 = 4 \div 4 \cdot s$; $7 \text{ in.} = s$
- Draw a rectangle for display. Write 4 in. as the length and w as the width.

Write an equation to find the perimeter of the rectangle. **Equations may vary.**

12. $P = 67 \text{ mm}$

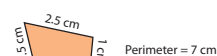
13. $P = 150 \text{ m}$

14. $P = 101 \text{ cm}$

15. $P = 54 \text{ m}$

Find an Unknown Side Measurement Given the Perimeter

Find the sum of the known sides. Subtract the sum from the perimeter to find the measurement of the unknown side.



Perimeter = 7 cm

$$P = s + s + s + s$$

$$7 \text{ cm} = (1.5 \text{ cm} + 2.5 \text{ cm} + 1 \text{ cm}) + s$$

$$7 \text{ cm} = 5 \text{ cm} + s$$

$$7 \text{ cm} - 5 \text{ cm} = 5 \text{ cm} - 5 \text{ cm} + s$$

$$2 \text{ cm} = s$$

The length of the unknown side is 2 cm.

Algebraic Expressions

Find the perimeter of the rectangle if the width is 5 cm.



$$P = (2 \cdot l) + (2 \cdot w)$$

$$P = (2 \cdot 15 \text{ cm}) + (2 \cdot 5 \text{ cm})$$

$$P = 30 \text{ cm} + 10 \text{ cm}$$

$$P = 40 \text{ cm}$$

The length of the rectangle is 3 times the width (3w).
 $l = 3 \cdot 5 \text{ cm} = 15 \text{ cm}$

Exercises

Use the perimeter given to find the length of the unknown side.

- Perimeter = 12 m
 $n = 5 \text{ m}$
- Perimeter = 24 yd
 $n = 2 \text{ yd}$

Solve. **Equations may vary.**

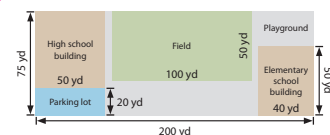
- Dad plans to place molding around the ceiling of the living room. If the length of the living room is 18 feet and the width is 12 feet, how many feet of molding will he use?
 $(2 \cdot 18 \text{ ft}) + (2 \cdot 12 \text{ ft}) = 60 \text{ ft}$

Find the perimeter of the rectangle if the width is 4 cm.

- $P = 24 \text{ cm}$
- $P = 22 \text{ cm}$

Practice & Application Equations may vary.

- What is the perimeter of the field? $P = 300 \text{ yd}$
- What is the perimeter of the parking lot? $P = 140 \text{ yd}$
- What is the perimeter of the high school building? $P = 210 \text{ yd}$
- What is the perimeter of the elementary school building? $P = 180 \text{ yd}$
- What is the perimeter of the entire school property? $P = 550 \text{ yd}$



Complete **DAILY REVIEW** on page 440.

Lesson 98

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If the perimeter of this rectangle is 20 inches, what equation can you write to find the measure of the width? Solve.

$$20 \text{ in.} = (2 \times 4 \text{ in.}) + (2 \times w);$$

$$20 \text{ in.} = 8 \text{ in.} + (2 \times w); 20 \text{ in.} - 8 \text{ in.} = 8 \text{ in.} - 8 \text{ in.} + (2 \times w);$$

$$12 \text{ in.} = 2 \times w; 12 \text{ in.} \div 2 = 2 \div 2 \times w; 6 \text{ in.} = w$$

Solve an algebraic expression to find the perimeter of a rectangle

- Draw a rectangle for display. Write w along the width and $2w$ along the length.

Mr. Alexander plans to put up fencing for a rectangular dog kennel. The length of the kennel will be twice the width. If the width of the kennel will be 10 feet, how much fencing will Mr. Alexander need for the kennel? **60 feet**

- How wide will the kennel be? 10 feet** Write $w = 10 \text{ ft}$.
If the length of the kennel is twice as long as the width, what would the length be? 20 feet ($l = 2 \times 10 \text{ ft}$)
- Write for display $l = 20 \text{ ft}$ and $P = (2 \cdot l) + (2 \cdot w)$. Choose a student to use the length and width measurements to write the equation and solve the problem.
 $P = (2 \cdot 20 \text{ ft}) + (2 \cdot 10 \text{ ft}); P = 40 \text{ ft} + 20 \text{ ft}; P = 60 \text{ ft}$
- Erase $2w$ from the rectangle and write the length $w + 5$.
If the length of the kennel will be 5 feet longer than the width, what will the length be? 15 feet ($l = 10 \text{ ft} + 5 \text{ ft}$)
- Follow a similar procedure to guide the students in finding how much fencing will be needed. $P = (2 \cdot 15 \text{ ft}) + (2 \cdot 10 \text{ ft}); P = 30 \text{ ft} + 20 \text{ ft}; P = 50 \text{ ft}$

Student Text pp. 238–39

Objectives

- Develop an understanding of the relationship between the diameter and the circumference of a circle
- Calculate the circumference of a circle using a formula
- Calculate the diameter of a circle given the circumference
- Relate circumference to real-life situations

Teacher Materials

- Charts 7 & 8: *Center Points, Radii & Diameters; Chords & Central Angles*
- Find the Circumference, page IA48 (CD)
- A ruler

Student Materials

- Find the Circumference, page IA48 (CD)
- A 12-inch length of string for each pair of students
- A cylindrical object, such as a can or mug, for each pair of students (Provide objects varying in size.)
- A ruler
- A calculator

Preparation

Label the cylindrical objects by writing different numbers on pieces of masking tape, placing one label on each object.

Notes

Rather than using cylindrical objects, you may choose to make different sized cylinders by rolling construction paper into a cylinder and taping the sides.

Allow the students to use calculators throughout this chapter so that they can focus on finding the perimeter and area of the various figures rather than on doing lengthy calculations.

Introduce the Lesson

Use the *Center Points, Radii & Diameters* and *Chords & Central Angles* charts to review the parts of a circle.

Teach for Understanding

Develop an understanding of the relationship between the diameter and the circumference of a circle

► **What is the distance around a geometric figure called? the perimeter**

1. Explain that *circumference* is the perimeter or distance around a circle.
2. Display the Find the Circumference page and explain instructions 1–3. Demonstrate finding the circumference: place the string around a cylindrical object to measure the circumference, and then measure that length of string to the nearest millimeter. Explain that measuring around the curved surface anywhere on a cylindrical object will give you the circumference of the circular bases. Demonstrate measuring the object's diameter using a ruler.
3. Distribute the Find the Circumference pages, calculators, cylindrical objects, lengths of string, and rulers. Direct pairs of students to measure the circumference and the diameter of their object to the nearest millimeter and to record the measurements. Instruct the pairs to exchange objects.
4. Repeat the procedure so that each pair measures at least 4 different objects.

5. Direct attention to instruction number 4. Instruct each student to divide the circumference by the diameter for each object he measured and to record his answer.
► **What common relationship do you see between the circumference and diameter of a circle? Elicit that the circumference is a little more than 3 times the diameter.**
6. Explain that the relationship between a circle's circumference and its diameter is a God-ordained constant. The circumference of every circle is a little more than 3 times greater than its diameter.
7. Write $\pi \approx 3.14$ or $\frac{22}{7}$ for display. Explain that mathematicians use a Greek letter, *pi*, to represent the relationship (ratio) of a circle's circumference to its diameter. Pi is a non-repeating, non-terminating decimal. The approximate values 3.14 and $\frac{22}{7}$ are used for calculating the circumference of a circle.

Calculate the circumference of a circle using a formula

► **If the circumference of a circle is about 3.14 or $\frac{22}{7}$ times greater than the diameter of a circle, how could you calculate the circumference of a circle when you know the diameter? Elicit that you can multiply the diameter by 3.14 or $\frac{22}{7}$ to find the circumference.**

1. Direct attention to the formula for finding circumference on the page. Explain that, similar to using a formula to find the perimeter of a polygon, a formula can be used to find the circumference of a circle. Elicit that *C* represents *circumference* and *d* represents *diameter*.
2. Call attention to instruction 5 and direct the students to multiply the diameter by 3.14 to find each circumference. Guide a discussion about how the calculated circumferences compare to the measured circumferences.
3. Direct the students to use their calculators to divide 22 by 7. Point out that since the decimal equivalent of pi, 3.14 . . . , is a repeating decimal, $\frac{22}{7}$ and 3.14 are approximations of pi.
4. Guide the students in calculating the circumference of the circles at the bottom of the page using $\frac{22}{7}$ for pi. Point out that using $\frac{22}{7}$ to find the circumference is especially helpful when working with fractions and whole numbers that are compatible with the denominator 7.

$$C = \frac{22}{7} \times \frac{28}{1} \text{ in.}$$

$$C = 88 \text{ in.}$$

$$C = \frac{22}{7} \times \frac{7}{1} \text{ ft}$$

$$C = 22 \text{ ft}$$

$$C = \frac{22}{7} \times 17\frac{1}{2} \text{ yd}$$

$$C = \frac{22}{7} \times \frac{35}{2}$$

$$C = 55 \text{ yd}$$

5. Draw a circle for display. Draw a radius and write 21 yd as its measure.
► **How do you think you can find the circumference of the circle when given only the measurement of the radius? Elicit that since the radius is half the length of the diameter, you multiply the radius by 2 to find the diameter and then multiply the diameter by pi to find the circumference.**
6. Write $C = 2\pi r$ for display. Guide the students in using the formula to find the circumference of the circle using $\frac{22}{7}$ for pi.
 $C = 2 \times \frac{22}{7} \times \frac{21}{1} \text{ yd}; C = 132 \text{ yd}$ Remind them that the coefficient (a number) precedes the variable.
7. Follow a similar procedure for finding the circumference of circles with the following radii.
radius = 1.4 cm; $\pi \approx 3.14$ $C = 2 \times 3.14 \times 1.4 \text{ cm}; C = 8.792 \text{ cm}$
radius = $\frac{3}{4}$ in.; $\pi \approx \frac{22}{7}$ $C = 2 \times \frac{22}{7} \times \frac{3}{4} \text{ in.}; C = 4\frac{5}{7} \text{ in.}$

Circumference

The **circumference** of a circle is a little more than 3 times its diameter. The ratio $\frac{C}{d}$ has a value of π (pi). Pi is a non-repeating and non-terminating decimal with an approximate value of 3.14 or $\frac{22}{7}$. Use the approximate value of pi to find an unknown circumference or diameter.

circumference
 π (pi) ≈ 3.14
 $C = \pi d$
 $C = 2\pi r$

Find the Circumference Given the Diameter

$$C = \pi d$$

$$C = \pi d$$

$$C = 3.14 \times 10$$

$$C = 31.4 \text{ cm}$$

$$C = \pi d$$

$$C = \frac{22}{7} \times 7$$

$$C = \frac{22}{7} \times \frac{7}{1}$$

$$C = 22 \text{ in.}$$

Find the Circumference Given the Radius

$$C = 2\pi r$$

$$C = 2\pi r$$

$$C = 2 \times 3.14 \times 5$$

$$C = 31.4 \text{ m}$$

$$C = 2\pi r$$

$$C = 2 \times \frac{22}{7} \times 3\frac{1}{2}$$

$$C = 2 \times \frac{22}{7} \times \frac{7}{2}$$

$$C = 22 \text{ in.}$$

Find the Diameter Given the Circumference

$$\text{Since } C = \pi d, \text{ then } \frac{C}{\pi} = d.$$

$$\frac{C}{\pi} = d$$

$$\frac{28.26}{3.14} = d$$

$$d = 9 \text{ cm}$$



Exercises

Find the circumference of the circle using 3.14 for π .

1.



$$C = \pi d$$

$$C = 3.14 \times 6 \text{ cm}$$

$$C = 18.84 \text{ cm}$$

2.



$$C = 2\pi r$$

$$C = 2 \times 3.14 \times 4.5 \text{ m}$$

$$C = 28.26 \text{ m}$$

3.



$$C = 2\pi r$$

$$C = 2 \times 3.14 \times 10 \text{ m}$$

$$C = 62.8 \text{ m}$$

Find the circumference of the circle using $\frac{22}{7}$ for π .

4.



$$C = \pi d$$

$$C = \frac{22}{7} \times 10\frac{1}{2} \text{ in.}$$

$$C = \frac{22}{7} \times \frac{21}{2} \text{ in.}$$

$$C = 33 \text{ in.}$$

5.



$$C = 2\pi r$$

$$C = 2 \times \frac{22}{7} \times 14 \text{ ft}$$

$$C = 88 \text{ ft}$$

6.



$$C = \pi d$$

$$C = \frac{22}{7} \times \frac{7}{8} \text{ yd}$$

$$C = \frac{11}{4} \text{ yd}$$

$$C \approx 2.75 \text{ yd or } 2\frac{3}{4} \text{ yd}$$

Find the diameter using the circumference.

7.

If the circumference is 23.55 cm, then the diameter is $\frac{C}{\pi} = d$.

$$d = \frac{23.55}{3.14}$$

$$d = 7.5 \text{ cm}$$

8.

If the circumference is 47.1 cm, then the diameter is $\frac{C}{\pi} = d$.

$$d = \frac{47.1}{3.14}$$

$$d = 15 \text{ cm}$$

DID YOU KNOW

The Bible indicates that the circumference of a circle is about three times greater than the diameter.

And he made a molten sea, ten cubits from the one brim to the other . . . and a line of thirty cubits did compass it round about.

1 Kings 7:23



Chapter 11

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Find the circumference or the perimeter of the figure. Use 3.14 for π . Round a decimal answer to the nearest hundredth.

9.



$$C = \pi d$$

$$C = 25.12 \text{ yd}$$

10.



$$C = 2\pi r$$

$$C = 10.99 \text{ m}$$

11.



$$C = 2\pi r$$

$$C = 16.956 \text{ cm} \approx 16.96 \text{ cm}$$

12.



$$C = 2\pi r$$

$$C = 5.495 \text{ ft} \approx 5.50 \text{ ft or } 5.5 \text{ ft}$$

13.



$$C = 2\pi r$$

$$C = 81.64 \text{ m}$$

14.



$$P = 6s$$

$$P = 24 \text{ ft}$$

15.



$$P = 2(l + w)$$

$$P = 16 \text{ cm}$$

16.



$$P = \frac{1}{4}(2\pi r + 2r)$$

$$P = 125 \text{ m}$$

Find the perimeter of the unique figure. (Hint: Find the circumference of the whole circle to determine the distance around the half circle. Use 3.14 for π .)

17.



$$P = \frac{1}{2}(C + d)$$

$$P = \frac{1}{2}(3.14 \times 10 + 10)$$

$$P = 15.7 \text{ m}$$

$$30 + 15.7 = 45.7 \text{ m}$$

18.



$$P = \frac{1}{2}(C + d)$$

$$P = \frac{1}{2}(3.14 \times 6 + 6)$$

$$P = 9.42 \text{ m}$$

$$18 + 9.42 = 27.42 \text{ m}$$

Practice & Application Equations may vary.

19.

Molly made a circular braided rug. The radius of the rug is 4 feet. What is the circumference of the rug? $C = 25.12 \text{ ft}$

22.

The Ferris wheel at the Texas State Fair has a diameter of 212 feet. What is the circumference of the Ferris wheel? $C = 665.68 \text{ ft}$

20.

Mom is hemming a circular tablecloth. How many inches will she hem if the diameter of the tablecloth measures 60 inches? $C = 188.4 \text{ in.}$

23.

Noah wants to know how far his bicycle wheel travels when it goes all the way around one time. The wheel is 20 inches across. How far does his bicycle wheel travel in one turn? $C = 62.8 \text{ in.}$

21.

Mr. Byers is placing edging around a rectangular fishpond. The length of the fishpond is 10 feet and the width is 5 feet. If the edging costs \$4.50 per foot, how much will Mr. Byers spend? $P = 30 \text{ ft}; 30 \times \$4.50 = \$135.00$

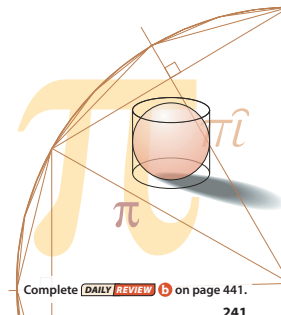
24.

Mrs. Kwan is putting wallpaper border up in the twins' bedroom. The length of the bedroom is 12 feet and the width is 8 feet. Each roll of border is 15 feet. How many rolls will Mrs. Kwan need? $P = 40 \text{ ft}; 40 \div 15 = 2.6 \text{ rolls}; 3 \text{ rolls}$

MEET THE MATHEMATICIAN

Archimedes (287–212 bc) was the greatest mathematician of the ancient world. He lived in Greece over 200 years before Jesus was born. Archimedes developed the first law of hydrostatics and worked out the approximate value of π . Putting mathematics into action, Archimedes was also an inventor. His military inventions included a catapult, a device for dropping heavy objects on ships to sink them, and a machine that picked up ships in the harbor and turned them over! Because of these inventions, the city of Syracuse was able to fend off the Romans for over two years. Archimedes' tomb was engraved with a sphere inside a cylinder to memorialize his great contributions to the study of geometry.

Computers have calculated π to over a trillion decimal places but have not arrived at its exact value. We may never know the exact value of π , but we do know that it is constant for every circle.



Complete **DAILY REVIEW** on page 441.

Lesson 99

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Mrs. Hagan makes and decorates baskets to sell at a craft fair. She made a circular basket that has a diameter of 27 centimeters. Mrs. Hagan has 75 centimeters of ribbon left over from her last basket. Is this enough ribbon to go around the rim of the basket? Why? **No; $C = 3.14 \times 27 \text{ cm}; C = 84.78 \text{ cm}; 84.78 \text{ cm of ribbon are needed.}$**

- **What is the question asking you to determine? whether 75 centimeters of ribbon is enough to go around a basket with a diameter of 27 centimeters**
- **What equation can you use to solve the problem? How do you know? $C = 3.14 \times 27 \text{ cm};$ since the circumference is also the amount of ribbon needed on the basket, multiply pi by the diameter of the basket to find its circumference.**

- Choose a student to write the equation for display and solve it while the other students solve it on paper. **$C = 84.78 \text{ cm}$**
Elicit that the 75 centimeters of ribbon are not enough to go around the rim of the basket.
- Follow a similar procedure for the following word problem. Elicit that it is a multi-step problem; you must combine the circumference of the circular cake and the perimeter of the rectangular cake to find the length of fondant that is needed to decorate both cakes.

Evie is decorating 2 cakes: a circular cake that is 8 inches in diameter and a rectangular cake that is 9 inches \times 13 inches. She needs to place a fondant rope around the bottom of each cake. What is the length of fondant needed for both cakes? **$C = 3.14 \times 8 \text{ in.} = 25.12 \text{ in.};$
 $P = (2 \times 9 \text{ in.}) + (2 \times 13 \text{ in.}) = 44 \text{ in.}; 44 \text{ in.} + 25.12 \text{ in.} = 69.12 \text{ in.}$**

Calculate the diameter of a circle given the circumference

- **How do you think you find the diameter of a circle when given the circumference? Why? Elicit that the product of the diameter and pi equals the circumference $C = \pi d$, so the circumference divided by pi equals the diameter.**

- Elicit the formula for finding the diameter when the circumference is known. $\frac{C}{\pi} = \frac{\pi d}{\pi}; \frac{C}{\pi} = d$
- Guide the students in finding the diameter of circles with these circumferences.
circumference = 37.68 m; $\pi \approx 3.14$ $\frac{37.68 \text{ m}}{3.14} = d; d = 12 \text{ m}$
circumference = 14.13 m; $\pi \approx 3.14$ $\frac{14.13 \text{ m}}{3.14} = d; d = 4.5 \text{ m}$
- Guide the students in solving this word problem. Direct the students to round the answer to the nearest meter.

A circular swimming pool has a circumference of 25 meters. Approximately how far does a swimmer have to swim across the pool to get from one side to the point exactly opposite his starting location? **$25 \text{ m} \div 3.14 = d;$
 $d = 7.96 \text{ m}; 8 \text{ m}$**

Student Text pp. 240–41

Objectives

- Calculate the area of rectangles, squares, and parallelograms using a formula
- Calculate the area of a complex figure
- Calculate the unknown side (length or width) of a rectangle or a square
- Relate area to real-life situations

Teacher Materials

- Chart 9: *Area*
- Area: Rectangles, Squares & Parallelograms, page IA49 (CD)

Student Materials

- Area: Rectangles, Squares & Parallelograms, page IA49 (CD)

Teach for Understanding

Calculate the area of rectangles and squares

1. Display the *Area* chart. Remind students that *area* is the space within a region (figure) and is measured in *square units*. Point out that a small raised 2 is written with the label in the answer to indicate the square units.
2. Display and distribute the Area: Rectangles, Squares & Parallelograms page.
 - **How can you find the number of square units in the rectangle at the top of the page? Elicit that you can count the square units or multiply the length times the width.**
3. Elicit the formula for finding the area of a rectangle. $A = l \times w$
Write the formula for display.
Instruct the students to write the equation to find the area of the rectangle and to write the solution on the blanks.
 $A = 11 \text{ units} \times 5 \text{ units}; A = 55 \text{ units}^2$
Choose a student to count the square units to verify the area.
 - **Can the formula $A = l \times w$ be used to find the area of the square? Why? Yes; a square is a type of rectangle.**
4. Instruct the students to write the equation to find the area of the square and the solution. $A = 9 \text{ units} \times 9 \text{ units}; A = 81 \text{ units}^2$
 - **When finding the area of a square, what do you notice about the equation? Elicit that you multiply a number by itself because the length and width of the square have the same measure.**
 - **How could you rewrite this equation using an exponent? Why? Elicit $A = 9^2$ or $A = (9 \text{ units})^2$; the exponent (2) indicates that 9 units is a factor 2 times in the equation.**
 - **How could you write the formula for finding the area of a square using an exponent? Elicit $A = s^2$.**
5. Guide the students in solving these word problems, using the formulas for finding the area of a rectangle and a square.

Mrs. Barnes's kitchen floor is rectangular, measuring 12 feet long and 9 feet wide. How many square-foot tiles does she need to cover the floor? $A = 12 \text{ ft} \times 9 \text{ ft}; A = 108 \text{ ft}^2$; 108 square-foot tiles

Mr. Hamblin poured a concrete patio in the backyard. The patio is square, measuring 15 feet on each side. What is the area of the patio? $A = 15 \text{ ft} \times 15 \text{ ft}$ or $A = (15 \text{ ft})^2$; $A = 225 \text{ ft}^2$

Calculate the area of a complex figure

- **How could you find the area of the irregular polygon on the page? Possible answers: count the square units; partition the polygon into parts (1 rectangle and 1 square or 2 rectangles), calculate the area of each part, and then add the areas.**
1. Guide the students in drawing a line in the irregular polygon to form an 11×5 rectangle and a 9×9 square. Demonstrate.
 - **What expression can you use to find the area of the left rectangle? $11 \text{ units} \times 5 \text{ units}$** Write the expression for display.
 - **What expression can you use to find the area of the square? $9 \text{ units} \times 9 \text{ units}$ or $(9 \text{ units})^2$** Write either expression beside $11 \text{ units} \times 5 \text{ units}$; leave space between the expressions. Write parentheses around each multiplication expression, a plus sign (+) between them, and $A =$ to the left to show how the entire area is calculated: $A = (11 \text{ units} \times 5 \text{ units}) + (9 \text{ units} \times 9 \text{ units})$ or $A = (11 \text{ units} \times 5 \text{ units}) + (9 \text{ units})^2$. Choose a student to solve the problem. $A = 136 \text{ units}^2$
 2. Guide the students in erasing the line drawn on the figure and then in drawing a new line to show a 20×5 rectangle and a 9×4 rectangle.
 - **What equation could you use to find the total area if you divided the irregular polygon into 2 rectangles? $A = (20 \text{ units} \times 5 \text{ units}) + (9 \text{ units} \times 4 \text{ units}); A = 136 \text{ units}^2$**
 - **Did the area change when you partitioned the complex figure a different way? Why? No; elicit that although a complex figure can be partitioned in different ways, the total area is the same when all the areas are added.**
 3. Guide the students in solving this word problem. Allow them to draw pictures if necessary.

Isaac's dad fenced in the backyard and the adjoining side yard for a play area. The backyard is 25 feet long and 35 feet wide. The side yard is 10 feet long and 15 feet wide. What is the total area of the fenced-in yard?
 $A = (25 \text{ ft} \times 35 \text{ ft}) + (10 \text{ ft} \times 15 \text{ ft}); A = 1,025 \text{ ft}^2$

Calculate the area of parallelograms

1. Direct the students to count the square units in the parallelogram on the page.
 - **How many square units did you count? Accept any reasonable answer, but elicit that it is difficult to determine the exact number of squares because there are so many partial squares.**
2. Direct the students to cut the bottom section from the page and then to cut out the parallelogram. Instruct them to cut the parallelogram into 2 pieces along the dashed vertical line.
 - **What shape can you make if you slide the triangular piece to the opposite side of the parallelogram? a rectangle**
Point out that rectangles and squares have widths that are perpendicular to their lengths, so it is easy to see their length and width. Reshaping the parallelogram formed perpendicular sides, making it easier to see its length and width.
 - **What formula can you use to find the area of the parallelogram now? $A = l \times w$**
 - **What equation and solution can you write for the area of the parallelogram? $A = 9 \text{ units} \times 4 \text{ units}; A = 36 \text{ units}^2$**
3. Draw a parallelogram for display. Explain that it is not necessary to reshape a parallelogram to know its length and width. You can identify the width, or *height*, that is perpendicular to the length, or *base*.

Area of Rectangles, Squares & Parallelograms

Area is the space within a region. The area of a region is the number of square units needed to cover its surface.

Area of a Rectangle
 $A = l \cdot w$
 $A = 8 \cdot 5$
 $A = 40 \text{ units}^2$



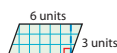
Area of a Square
 $A = l \cdot w \text{ or } A = s^2$
 $A = 6 \times 6 \text{ or } 6^2$
 $A = 36 \text{ units}^2$



Area of a Complex Figure
 $A = l \cdot w$
 $A = b \cdot h$
 $A = l \cdot w \text{ or } A = s^2$
 $A = (l \cdot w) + (b \cdot h)$

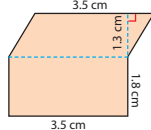
Area of a Parallelogram
 When a parallelogram does not have right angles, its area is still determined by multiplying the length of its base (b) and the length of its height (h). The height is the length of a line segment that is perpendicular to both bases; it is the shortest distance between the bases.

$A = b \cdot h$
 $A = 6 \cdot 3$
 $A = 18 \text{ units}^2$



Area of a Complex Figure
 The area of an irregular polygon is determined by finding the area of each smaller region in the figure and then adding the areas.

$A = (l \cdot w) + (b \cdot h)$
 $A = (3.5 \cdot 1.8) + (3.5 \cdot 1.3)$
 $A = 6.3 + 4.55$
 $A = 10.85 \text{ cm}^2$



Find the Measurement of an Unknown Side Given the Area

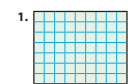
Divide the given area by the measurement of the known side.

$n = A \div s$
 $n = 42 \div 7$
 $n = 6 \text{ in.}$

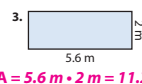


Exercises

Use the formula $l \cdot w$ to find the area of the figure.



$A = 9 \cdot 2 = 18 \text{ units}^2$

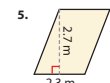


$A = 5.6 \text{ m} \cdot 2 \text{ m} = 11.2 \text{ m}^2$



$A = 12 \text{ yd} \cdot 12 \text{ yd} = 144 \text{ yd}^2$

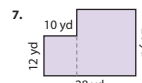
Find the area of the figure.



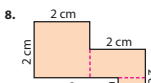
$A = 6.21 \text{ m}^2$



$A = 60 \text{ in.}^2$



$A = 480 \text{ yd}^2$

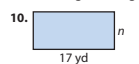


$A = 7 \text{ cm}^2$

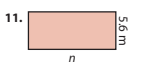
Find the unknown measurement of the figure using the given area.



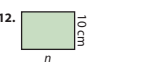
$A = 25 \text{ ft}^2$ 5 ft



$A = 136 \text{ yd}^2$ 8 yd



$A = 70 \text{ m}^2$ 12.5 m

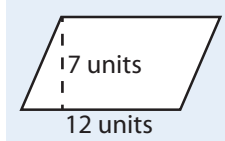


$A = 140 \text{ cm}^2$ 14 cm

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Chapter 11

Write 12 units along the base of the parallelogram. Draw a line segment perpendicular to the bases to represent the height inside the parallelogram and write 7 units beside it. Explain that the height is the shortest distance between both bases (top and bottom edges) and is a line segment that is perpendicular to both bases. You multiply the base (length) times the height (width) to find the area of a parallelogram.



4. Write $A = b \times h$ for display. Elicit that b represents *base* and h represents *height*. Guide the students in using the formula to find the area of the parallelogram. $A = 12 \text{ units} \times 7 \text{ units}$; $A = 84 \text{ units}^2$

5. Follow a similar procedure for parallelograms having these dimensions.
 base = 6 m; height = 3.1 m $A = 6 \text{ m} \times 3.1 \text{ m}$; $A = 18.6 \text{ m}^2$
 base = 5.6 cm; height = 3 cm $A = 5.6 \text{ cm} \times 3 \text{ cm}$; $A = 16.8 \text{ cm}^2$

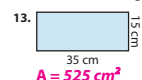
Calculate the unknown side of a rectangle or a square

The men laid 12 square yards of carpet in the bedroom. If the length of the bedroom is 3 yards, what is the width of the room? 4 yd

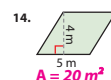
► What information is given in the word problem? Elicit the area of the bedroom is 12 square yards and its length is 3 yards.

1. Choose a student to draw a picture to represent the bedroom. Instruct him to write n along the unknown side. Write $A = l \times w$ below the figure.

Find the area of the figure.



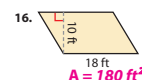
$A = 525 \text{ cm}^2$



$A = 20 \text{ m}^2$



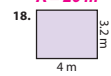
$A = 3.61 \text{ yd}^2$



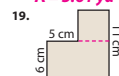
$A = 180 \text{ ft}^2$



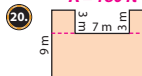
$A = 12 \frac{1}{4} \text{ in.}^2$



$A = 12.8 \text{ m}^2$



$A = 85 \text{ cm}^2$



$A = 96 \text{ m}^2$

Use the dimensions given to find the area.

21. $l = 4 \text{ in.}$
 $w = 8 \text{ in.}$
 $A = 32 \text{ in.}^2$

22. $b = 15 \text{ ft}$
 $h = 24 \text{ ft}$
 $A = 360 \text{ ft}^2$

23. $b = 4.5 \text{ m}$
 $h = 7.5 \text{ m}$
 $A = 33.75 \text{ m}^2$

Use the area given to find the unknown measurement of the rectangle or the square.

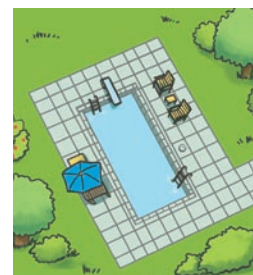
24. $l = 7 \text{ yd}$
 $w = 12 \text{ yd}$
 $A = 84 \text{ yd}^2$

25. $l = 9 \text{ m}$
 $w = 5.5 \text{ m}$
 $A = 49.5 \text{ m}^2$

26. $l = 10 \text{ ft}$
 $w = 10 \text{ ft}$
 $A = 100 \text{ ft}^2$

Practice & Application Equations may vary.

- Luke built a table with a square top. The tabletop measures 30 inches on each side. How many square-inch tiles are needed to cover the whole tabletop? $A = s^2$; $30 \text{ in.} \cdot 30 \text{ in.} = 900 \text{ in.}^2$ tiles
- Owen's family has a rectangular-shaped swimming pool in their backyard. The swimming pool is 30 feet long and 15 feet wide. What is the area of the swimming pool? $A = l \cdot w$; $30 \text{ ft} \cdot 15 \text{ ft} = 450 \text{ ft}^2$
- Owen's backyard has the shape of a rectangle. It is 75 feet long and 50 feet wide. What is the area of the backyard that is not part of the swimming pool? $A = 3,300 \text{ ft}^2$
- Mr. Martinez is placing edging around 2 trees in the front yard. The diameter of each circle will be 3 feet. What is the total amount of edging he needs for these trees? $C = 18.84 \text{ ft}$
- Mrs. Anderson is putting a frame around an octagonal mirror. Each side of the mirror is 12 inches. How much molding will Mrs. Anderson use to frame the mirror? $P = 96 \text{ in.}$
- Explain where to draw a line segment to measure the height of a parallelogram. **perpendicular to the two bases**



Why is area measured in square units and perimeter measured in units? **Perimeter is a 1-dimensional measurement—length (in., cm, ft). Area is a 2-dimensional measurement—length \times width or base \times height (in. \times in. = in.²; cm \times cm = cm²; ft \times ft = ft²).**

Complete **DAILY REVIEW** on page 441.

Lesson 100

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► How can understanding the formula for finding area help you solve the word problem? Elicit that since the product of the length (3 yd) and width (n yd) equals the area (12 yd²), the area divided by the length equals the width: $A = l \times w$; $\frac{A}{l} = \frac{l \times w}{l}$; $\frac{A}{l} = w$

Explain that since multiplication and division are inverse operations, $w = A \div l$ [the inverse of $A = l \times w$] can be used to solve the word problem.

- Select a student to write for display and solve the equation needed to find the width of the bedroom. $w = 12 \text{ yd}^2 \div 3 \text{ yd}$; $w = 4 \text{ yd}$
 Explain that the formula $n = A \div s$ (the unknown side = the area \div the known side) can be used to calculate the unknown side. Write the formula for display.
- Follow a similar procedure for the following problems. Elicit that when you divide the area of a rectangle or a square by the length of one side (length or width), the quotient is the length of the other side (width or length).

The men placed 192 square feet of vinyl flooring in the kitchen. If the width of the kitchen is 12 feet, what is the length of the kitchen? $n = A \div s$; $n = 192 \text{ ft}^2 \div 12 \text{ ft}$; $n = 16 \text{ ft}$

The baby's playpen has an area of 9 square feet. If the length of the playpen is 3 feet, what is the width of the playpen? $n = A \div s$; $n = 9 \text{ ft}^2 \div 3 \text{ ft}$; $n = 3 \text{ ft}$

Student Text pp. 242–43

(Note: Assessment available on Teacher's Toolkit CD.)

Objectives

- Calculate the area of triangles using a formula
- Calculate the area of a complex figure
- Calculate the unknown height or base of a triangle
- Relate area to real-life situations

Teacher Materials

- Area: Triangles, page IA50 (CD)
- A ruler

Student Materials

- Area: Triangles, page IA50 (CD)
- A ruler
- A calculator

Teach for Understanding

Calculate the area of triangles using a formula

- Distribute and display the Area: Triangles page.
 > **What is the formula for finding the area of a rectangle or square?** $A = l \cdot w$ **area of a parallelogram?** $A = b \cdot h$
 Remind the students that rectangles and squares can also be classified as a parallelogram: a quadrilateral whose opposite sides are parallel. Explain that since rectangles and squares are quadrilaterals that have a width that is perpendicular to their length, the width can also be called the height and the length can also be called the base.
 > **Could you use the formula $A = b \cdot h$ to find the area of a square and a rectangle? Why?** *Yes; elicit that a rectangle and a square both have sides perpendicular to each other, so they have a base and a height.*
- Write $A = b \cdot h$ for display and direct the students to write on the page the equation and solution for the area of the rectangle. $A = 12 \text{ units} \times 6 \text{ units}; A = 72 \text{ units}^2$
- Instruct the students to use a ruler and draw a diagonal from one vertex to another to divide the rectangle in half. Demonstrate.
 > **What 2 geometric figures were formed?** *2 triangles*
 > **What fraction of the whole rectangle is each triangle? How do you know?** *$\frac{1}{2}$; elicit that two congruent triangles were formed.*
 > **What is the area of each triangle? How do you know?** *36 units²; elicit that each triangle is half of the rectangle, so the area of each triangle is $\frac{1}{2}$ the area of the rectangle; $\frac{1}{2}$ of 72 units² is 36 units².*
 > **Since every rectangle can be partitioned to form two congruent triangles, what formula can you write to find the area of each triangle? Explain.** *$A = \frac{1}{2}(l \times w)$ or $A = \frac{1}{2}(b \times h)$; elicit that you can find the area of the rectangle and multiply the area by $\frac{1}{2}$ or divide it by 2.*
- Write $A = \frac{1}{2}(b \cdot h)$ for display. Explain that the area of any triangle is $\frac{1}{2}$ the area of its related rectangle (parallelogram). Guide the students in using the formula to write on a separate sheet of paper the equation to find the area of one of the triangles: $A = \frac{1}{2}(12 \times 6)$. Instruct them to solve the equation. $A = \frac{1}{2}(72 \text{ units}^2); A = 36 \text{ units}^2$ Discuss the solution as needed and instruct the students to write on the page the equation and solution for the area of the triangle.
 $A = \frac{1}{2}(12 \text{ units} \times 6 \text{ units}); A = 36 \text{ units}^2$

- Follow a similar procedure to guide the students in finding the area of the square and then the area of the triangles that are formed by drawing a diagonal inside the square.

$$A = 8 \text{ units} \times 8 \text{ units}, A = 64 \text{ units}^2;$$

$$A = \frac{1}{2}(8 \text{ units} \times 8 \text{ units}), A = 32 \text{ units}^2$$

- Direct attention to the parallelogram.

> **Does a parallelogram have perpendicular sides?** *no* **What must you know in order to calculate the area of a parallelogram?** *Answers will vary, but elicit that you must identify the base and the height of the parallelogram.*

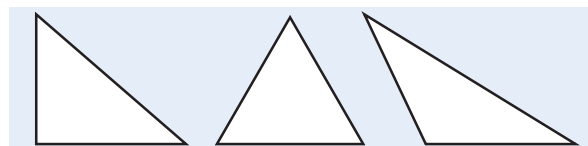
> **What represents the height in a parallelogram?** *the length of a line segment that is perpendicular to both bases*

Follow a procedure similar to the one used for the rectangle and the square as you guide the students in finding the area of the parallelogram and then the area of the triangles that are formed by drawing a diagonal inside the parallelogram.

$$A = 11 \text{ units} \times 5 \text{ units}, A = 55 \text{ units}^2;$$

$$A = \frac{1}{2}(11 \text{ units} \times 5 \text{ units}), A = 27.5 \text{ units}^2$$

- Draw for display a right triangle, an acute triangle, and an obtuse triangle similar to the ones below. Direct the students to draw similar triangles on the back of the Area: Triangles page.



- Instruct the students to measure the base and height of their right triangles to the nearest millimeter and to write each measurement along the line segment. Point out that in a right triangle the perpendicular sides are the base and the height.

> **What formula can you use to find the area of your right triangle?** *$\frac{1}{2}(b \cdot h)$*

Guide the students in writing and solving the equation to find the area of their right triangles. *Answers will vary.*

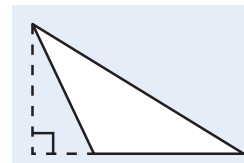
- Direct attention to the acute triangle drawn for display.
 > **How do you think you can identify the height of an acute triangle?** *Elicit that you can draw a line segment from one of the vertices to the opposite side; the line segment must be perpendicular to the opposite side.*

Demonstrate drawing a line segment to represent the height of your acute triangle.

Instruct each student to first draw the line segment representing the height of his acute triangle and then to measure the base and the height to the nearest millimeter and write the measurements. Then direct the students to write the equation to calculate the area of their triangle and solve it.

Answers will vary.

- Explain that the height of an obtuse triangle may be located outside the triangle. Demonstrate drawing the height of the obtuse triangle drawn for display.



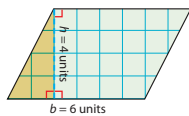
Instruct each student to draw the height of his obtuse triangle, measure the height and the base to the nearest millimeter, and write the measurements. Then direct the students to write the equation to calculate the area of their triangle and solve it. *Answers will vary.*

Area of Triangles

Area of a Parallelogram

$$A = b \cdot h$$

To change a parallelogram to a rectangle, you can remove a triangle from one side of the parallelogram and connect it to the other side.



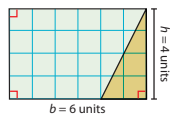
$$A = b \cdot h$$

$$A = 6 \cdot 4$$

$$A = 24 \text{ units}^2$$

Area of a Rectangle

$$A = l \cdot w$$



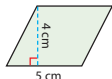
$$A = b \cdot h$$

$$A = l \cdot w$$

$$A = \frac{1}{2}(b \cdot h)$$

Area of a Triangle

A diagonal divides a parallelogram into two congruent triangles. The area of each triangle is $\frac{1}{2}$ of the area of the parallelogram.



$$A = b \cdot h$$

$$A = 5 \cdot 4$$

$$A = 20 \text{ cm}^2$$



$$A = \frac{1}{2}(b \cdot h)$$

$$A = \frac{1}{2}(5 \cdot 4)$$

$$A = \frac{1}{2}(20)$$

$$A = 10 \text{ cm}^2$$

Each triangle has an area of 10 cm².



$$A = l \cdot w$$

$$A = 6 \cdot 4$$

$$A = 24 \text{ cm}^2$$



$$A = \frac{1}{2}(b \cdot h)$$

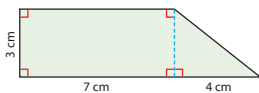
$$A = \frac{1}{2}(6 \cdot 4)$$

$$A = \frac{1}{2}(24)$$

$$A = 12 \text{ cm}^2$$

Each triangle has an area of 12 cm².

Area of a Complex Figure



$$A = (7 \cdot 3) + \frac{1}{2}(4 \cdot 3)$$

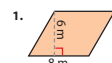
$$A = 21 + \frac{1}{2}(12)$$

$$A = 21 + 6$$

$$A = 27 \text{ cm}^2$$

Exercises

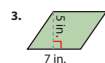
Find the area of the figure.



$$A = 48 \text{ m}^2$$



$$A = 24 \text{ m}^2$$



$$A = 35 \text{ in.}^2$$



$$A = 17.5 \text{ in.}^2$$



$$A = 7.5 \text{ in.}^2$$



$$A = 3.125 \text{ cm}^2$$



$$A = 1 \frac{1}{4} \text{ in.}^2$$

Calculate the area of a complex figure

1. Direct attention to the complex figure on the Area: Triangles page.

► How can you find the area of this irregular polygon?

Possible answers: count the square units; partition the polygon into 1 square and 1 triangle, calculate the two areas, and then add the areas.

2. Direct the students to write and solve an equation to find the area of the square and then to write and solve an equation to find the area of the triangle. $A = 8 \text{ units} \times 8 \text{ units}$, $A = 64 \text{ units}^2$; $A = \frac{1}{2}(8 \text{ units} \times 6 \text{ units})$, $A = 24 \text{ units}^2$

► What equation can you write to find the total area of the polygon? $A = (8 \text{ units} \times 8 \text{ units}) + \frac{1}{2}(8 \text{ units} \times 6 \text{ units})$ Write the equation for display.

Instruct the students to solve the equation and then to write the solution on the page. $A = 88 \text{ units}^2$

Calculate the unknown height or base of a triangle

The sixth-grade classes are making pennants for the game. Each pennant has an area of 45 square inches. If each pennant is 10 inches long, what is the height of each pennant? **9 in.**

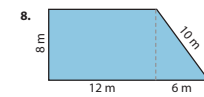
1. Write $A = \frac{1}{2}(b \cdot h)$ for display. Explain that if you know the area of a triangle and the length of its base, you can find the measurement of its height.

► What number can you substitute for A? Why? 45; the area of each pennant is 45 square inches.

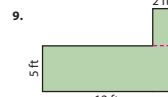
► What number can you substitute for b? Why? 10; each pennant is 10 inches long.

Write $45 = \frac{1}{2}(10h)$ for display.

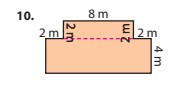
Find the perimeter and the area of the complex figure.



$$P = 48 \text{ m}; A = 120 \text{ m}^2$$



$$P = 42 \text{ ft}; A = 68 \text{ ft}^2$$



$$P = 36 \text{ m}; A = 64 \text{ m}^2$$

Find an Unknown Measurement

Substitute the known measures into the formula and then solve for the unknown.

$$A = \frac{1}{2}(b \cdot h)$$

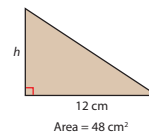
$$48 = \frac{1}{2}(12 \cdot h)$$

$$48 = \frac{1}{2}(12h)$$

$$48 = 6h$$

$$\frac{48}{6} = \frac{6h}{6}$$

$$8 = h$$



$$\text{Area} = 48 \text{ cm}^2$$

$$A = \frac{1}{2}(b \cdot h)$$

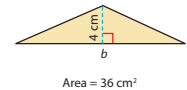
$$36 = \frac{1}{2}(b \cdot 4)$$

$$36 = \frac{1}{2}(4b)$$

$$36 = 2b$$

$$\frac{36}{2} = \frac{2b}{2}$$

$$18 = b$$



$$\text{Area} = 36 \text{ cm}^2$$

Exercises

Find the unknown measurement of the triangle.

$$11. \text{Area} = 30 \text{ ft}^2$$

$$\text{base} = 10 \text{ ft}$$

$$\text{height} = \underline{6 \text{ ft}}$$

$$12. \text{Area} = 15 \text{ m}^2$$

$$\text{base} = \underline{3 \text{ m}}$$

$$\text{height} = 10 \text{ m}$$

$$13. \text{Area} = 24 \text{ yd}^2$$

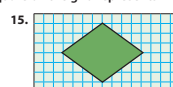
$$\text{base} = 6 \text{ yd}$$

$$\text{height} = \underline{8 \text{ yd}}$$

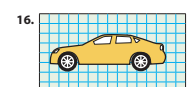
Estimate the area of the figure. Each square on the grid represents 1 cm².



$$42 \text{ cm}^2$$



$$24 \text{ cm}^2$$



$$48 \text{ cm}^2$$

Solve. Round to the nearest whole number if needed. Equations may vary.

17. The "Welcome Home, Soldiers!" banner over the church entrance is a rectangle with a length that is 2 times the width measurement. The width of the banner is 6 feet. What is the area of the banner? $A = 72 \text{ ft}^2$

18. Ani made a circular clay pot with a diameter of 8.6 inches. She wants to glue beads around the rim of the pot. How many beads will she use if she glues 1-inch beads around the pot? **27 beads**

19. Parker is making felt pennants for the school soccer team. Each pennant will be 18 inches long and have a height of 8 inches. How many square inches of felt does he need for each triangular pennant? $A = \frac{1}{2}(b \cdot h)$; $\frac{1}{2}(18 \text{ in.} \cdot 8 \text{ in.}) = 72 \text{ in.}^2$

Lesson 101

20. A builder plans to place ceramic tile in a bathroom that is 10 feet long and 6 feet wide. The tiles are 12 inches by 12 inches. How many tiles will the builder need to cover the entire floor? $A = l \cdot w$; $10 \text{ ft} \cdot 6 \text{ ft} = 60 \text{ ft}^2$; $60 \div 1 = 60 \text{ tiles}$

21. Mr. Jeffrey is placing a small fence around his garden to keep the rabbits out. His square garden is 15 feet on each side. How much fencing does he need to go around the garden? $P = n \cdot s$; $4 \text{ ft} \cdot 15 \text{ ft} = 60 \text{ ft}$

22. On Saturday Colin painted a wall in his bedroom. The wall is 12 feet long and 8 feet high. What is the area of the wall? $A = l \cdot w$; $12 \text{ ft} \cdot 8 \text{ ft} = 96 \text{ ft}^2$

Complete **DAILY REVIEW** on page 442.

► What is $\frac{1}{2}$ of $10h$? $5h$

Write $45 = 5h$ below $45 = \frac{1}{2}(10h)$.

► What is the height of each banner? How do you know?

9 inches; elicit that you divide both sides of the equation by 5 to solve for h: $\frac{45}{5} = \frac{5h}{5}$; $9 = h$; $h = 9 \text{ inches}$.

► Why do you label the answer inches? Elicit that the area is given in square inches and the base is given in inches, so the height is also measured in inches.

Explain that linear units are 1-dimensional units that are used to measure perimeter. Square units are 2-dimensional units that are used to measure area.

2. Follow a similar procedure for the following word problem. Elicit that the Commutative Property of Multiplication allows you to change the order of the factors that represent the base and the height so that the coefficient is written in front of the variable (b): $A = \frac{1}{2}(b \cdot h)$; $6 = \frac{1}{2}(b \cdot 4)$; $6 = \frac{1}{2}(4b)$.

Grandpa is making a triangular table. The table has an area of 6 square feet. If the height of the triangular tabletop is 4 feet, what is the measurement of the base of the tabletop? $6 = \frac{1}{2}(4b)$; $6 = 2b$; $\frac{6}{2} = \frac{2b}{2}$; $3 = b$; base = **3 feet**

3. Direct the students to find the unknown height or base of other triangles using these measurements. Give guidance as needed.

$$\text{Area} = 20 \text{ ft}^2 \quad \text{base} = \underline{4 \text{ ft}} \quad \text{height} = 10 \text{ ft}$$

$$\text{Area} = 25 \text{ ft}^2 \quad \text{base} = 10 \text{ ft} \quad \text{height} = \underline{5 \text{ ft}}$$

$$\text{Area} = 36 \text{ ft}^2 \quad \text{base} = 12 \text{ ft} \quad \text{height} = \underline{6 \text{ ft}}$$

Student Text p. 244–45

Objectives

- Calculate the area of a circle using a formula
- Estimate the area of a circle
- Relate area to real-life situations

Teacher Materials

- Area: Circles, page IA51 (CD)

Student Materials

- Area: Circles, page IA51 (CD)
- A ruler
- A calculator

Teach for Understanding

Calculate the area of a circle using a formula

1. Display and distribute the Area: Circles page. Direct the students to find the approximate area of the top circle by counting the whole and partial square units. **Answers will vary.**
 > **Why is it difficult to find the area of a circle by counting units? A circle contains many partial squares.**
2. Demonstrate each step as you guide the students in determining the formula for finding the area of a circle. (Refer to the figure pictured at the top of page 246 in the Student Text.)
 1. Cut out both circles on the page. Elicit that the circles are congruent. Set aside the circle containing the grid.
 2. Fold the other circle in half along the edge of the shaded area.
 3. Fold the circle in half again: fourths.
 4. Fold the circle in half one more time: eighths.
 5. Unfold the circle and cut out the eighths along the fold lines.
 6. Place the white wedges pointing up in a row, with the bottom edges touching.
 7. Place the shaded wedges between the white wedges, pointing down.

(Note: You may choose to have students glue the wedges onto a sheet of paper and cut out the figure along its outer edges.)

 > **What geometric figure do the wedges resemble when they are arranged like this? Elicit that the wedges form a figure that resembles a parallelogram.**
3. Direct the students to compare the parallelogram-type figure (curved “parallelogram”) to the whole circle. Remind them that both of the circles were congruent.
 > **What formula do you use to find the area of a parallelogram? $A = b \cdot h$** Write the formula for display.
 > **Do you think the height of your curved “parallelogram” is closer in length to the radius or the diameter of the whole circle? the radius**
 Instruct the students to slide the whole circle underneath the “parallelogram” to see that the height is equal to the radius. Direct attention to the base of the “parallelogram.”
 > **How many of the 8 edges of the circle make up the length of the base? 4; elicit that one-half of the edges make up the length of the base.**

Remind the students that the circumference of a circle is the measurement of the edge of a whole circle. Elicit that half of the circumference of the circle makes up the length of the base of the “parallelogram.”

4. Direct attention to $A = b \cdot h$ written for display.
 > **Since the length of the base is $\frac{1}{2}$ of the circumference, what can we substitute for b in the formula? $\frac{1}{2}C$**
 > **Since the height is equal to the measurement of the radius, what can we substitute for h in the formula? r**
5. Write $A = \frac{1}{2}C \cdot r$ for display.
 > **What formula do you use to find the circumference of a circle? md or $2\pi r$**
 Show students that when $2\pi r$ is substituted into the formula for circumference, you have $A = \frac{1}{2}(2\pi r)r$. Explain that this formula can be written in a simpler form. Since $\frac{1}{2} \times 2 = 1$ and the radius multiplied by itself is r -squared, the formula for the area of a circle can be written $A = \pi r^2$.

$$A = (\frac{1}{2}C)(r)$$

$$A = \frac{1}{2}(2\pi r)r$$

$$A = (\frac{1}{2} \cdot \frac{2}{1})\pi(r \cdot r)$$

$$A = \pi r^2$$
6. Direct attention to the whole circle that was cut from the Area: Circles page. Guide the students in using the formula to find the area of the whole circle. **$A = \pi r^2$;
 $A = 3.14 \times (8 \text{ units})^2$; $A = 3.14 \times 64 \text{ units}^2$; $A = 200.96 \text{ units}^2$**
7. Draw for display two circles, each with one of the following radii. Write each measurement along the corresponding radius. Guide the students in using $A = \pi r^2$ to find the area of each circle. Continue to display the circles.

radius = 10 m
 $A = 3.14 \times (10 \text{ m})^2$
 $A = 3.14 \times 100 \text{ m}^2$
 $A = 314 \text{ m}^2$

radius = 4 cm
 $A = 3.14 \times (4 \text{ cm})^2$
 $A = 3.14 \times 16 \text{ cm}^2$
 $A = 50.24 \text{ cm}^2$

 > **How can you determine the radius if you only know the measurement of the diameter? Why? Divide the diameter by 2 or multiply it by $\frac{1}{2}$; the radius is half of the diameter.**
8. Draw for display a circle with a diameter of 10 cm.
 > **What is the length of the radius? How do you know? 5 cm; the radius is $\frac{1}{2}$ of the diameter; $10 \div 2 = 5 \text{ cm}$ or $\frac{1}{2} \times 10 = 5 \text{ cm}$.**
 Guide the students in using the formula to find the area.
 $A = 3.14 \times (5 \text{ cm})^2$; $A = 3.14 \times 25 \text{ cm}^2$; $A = 78.5 \text{ cm}^2$. Continue to display the circle.

Estimate the area of a circle

1. Explain that when an exact answer is not needed, it is helpful to estimate the area of a circle by rounding pi to the nearest whole number.
 > **How could you estimate the area of a circle? Elicit that you can round pi (3.14) to the nearest whole number (3) and then multiply by the radius squared.**
2. Display the whole circle again. Remind the students that the radius is 8 units long and the area is 200.96 units². Choose a student to write for display and solve an equation to estimate the area of the circle. **$A = 3 \times (8 \text{ units})^2$;
 $A = 3 \times 64 \text{ units}^2$; $A = 192 \text{ units}^2$**
3. Guide the students in estimating the area of the circles that were previously drawn for display. Compare the estimated areas to the calculated areas.

radius = 10 m **$A = 300 \text{ m}^2$**
 radius = 4 cm **$A = 48 \text{ cm}^2$**
 diameter = 10 cm **$r = 5 \text{ cm}$; $A = 75 \text{ cm}^2$**

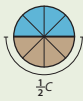
Area of Circles

The **area of a circle** can be discovered by using the formula for the area of a parallelogram ($A = b \cdot h$) and the formula for the circumference of a circle ($C = \pi d$).

area of a circle
 $A = \pi r^2$

Divide a circle into 8 congruent wedges. Shade each half circle.

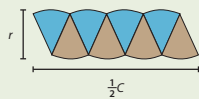
Since $C = \pi d$, then $C = 2\pi r$, or $\frac{1}{2}C = \pi r$.



Arrange the wedges to form a figure that is similar to a parallelogram.

- The base of the figure is half the circumference.
- The height of the figure is the radius of the circle.

$$\begin{aligned} \text{Area of a circle} &= \text{base} \cdot \text{height} \\ &= \left(\frac{1}{2}C\right)(r) \\ &= \frac{1}{2}(2\pi r)r \\ &= \left(\frac{1}{2} \cdot 2\right)\pi(r \cdot r) \\ &= \pi r^2 \end{aligned}$$



The area of a circle is π times the radius squared: $A = \pi r^2$.

Substitute the length of the radius for r and 3.14 for π .

$$\begin{aligned} r &= 7 \text{ ft} \\ A &= \pi r^2 \\ A &= 3.14(7^2) \\ A &= 3.14(49) \\ A &= 153.86 \text{ ft}^2 \end{aligned}$$



Remember that the radius is half the length of the diameter.

$$\begin{aligned} \text{If } d &= 10 \text{ m, then } r = 5 \text{ m.} \\ A &= \pi r^2 \\ A &= 3.14(5^2) \\ A &= 3.14(25) \\ A &= 78.5 \text{ m}^2 \end{aligned}$$



To estimate the area of a circle, round π to 3.

$$\begin{aligned} A &= \pi r^2 \\ A &= 3(4^2) \\ A &= 3(16) \\ A &= 48 \text{ ft}^2 \end{aligned}$$



$$\begin{aligned} A &= \pi r^2 \\ A &= 3(8^2) \\ A &= 3(64) \\ A &= 192 \text{ m}^2 \end{aligned}$$



Exercises

Find the area of the circle. Use 3.14 for π .



$$A = 113.04 \text{ cm}^2$$



$$A = 254.34 \text{ yd}^2$$



$$A = 1,384.74 \text{ in.}^2$$



$$A = 314 \text{ m}^2$$

Estimate the area of the circle. Round π to 3.



$$A = 675 \text{ m}^2$$



$$A = 12 \text{ cm}^2$$



$$A = 108 \text{ ft}^2$$



$$A = 507 \text{ ft}^2$$

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Chapter 11

Write the number that is needed to find the area of the circle.



$$A = 3.14 \cdot \underline{16^2}$$



$$A = 3.14 \cdot \underline{12^2}$$



$$A = 3.14 \cdot \underline{2.5^2}$$



$$A = 3.14 \cdot \underline{4.8^2}$$



$$A = 3.14 \cdot \underline{7.5^2}$$



$$A = 3.14 \cdot \underline{3^2}$$

Find the area and the circumference of the circle.



$$A = 12.56 \text{ in.}^2; C = 12.56 \text{ in.}$$



$$A = 28.26 \text{ ft}^2; C = 18.84 \text{ ft}$$



$$A = 12.56 \text{ yd}^2; C = 12.56 \text{ yd}$$



$$A = 3.14 \text{ yd}^2; C = 6.28 \text{ yd}$$



$$A = 113.04 \text{ cm}^2; C = 37.68 \text{ cm}$$



$$A = 379.94 \text{ m}^2; C = 69.08 \text{ m}$$



$$A = 6.28 \text{ cm}^2$$



$$A = 7.065 \text{ cm}^2$$



$$A = 21.5 \text{ ft}^2$$

Practice & Application Equations may vary.

24. Julia designed a circular flower bed. The radius of the flower bed is 8 feet. What is the area of the flower bed?

$$A = 200.96 \text{ ft}^2$$

25. Mrs. Davenport found a round tablecloth with an area of 11 square feet. Will this tablecloth cover her table that has a diameter of 4 feet? $A = 12.56 \text{ ft}^2$; no

26. Mr. King built a circular sandbox for the playground. The radius of the sandbox is 6 feet. What is the area of the sandbox? $A = 113.04 \text{ ft}^2$



Complete **DAILY REVIEW** on page 442.

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4. Guide the students in estimating and solving the following word problems. While solving the last two problems, remind the students that the Commutative and the Associative Properties of Multiplication allow them to reorder and regroup the factors.

Mrs. Arnold purchased a circular rug to be placed under the kitchen table. The radius of the rug is 3 feet. What is the area of the rug? $A = 3.14(3^2)$; $A = 3.14(9)$; $A = 28.26 \text{ ft}^2$

Mrs. Arnold also purchased a smaller rug to be placed at the doorway in the kitchen. This rug is a half-circle with a radius of 2 feet. What is the area of the rug? $A = \frac{1}{2}(3.14 \times 2^2)$; $A = (\frac{1}{2} \times 2^2) \times 3.14$; $A = (\frac{1}{2} \times 4) \times 3.14$; $A = 2 \times 3.14$; $A = 6.28 \text{ ft}^2$

The school has a circular flower bed around the flag pole. The radius of this circle is 6 feet. The sixth-grade class planted roses in $\frac{1}{4}$ of the flower bed. What is the area of the section that has roses planted by the sixth-grade class?

$$A = \frac{1}{4}(3.14 \times 6^2); A = (\frac{1}{4} \times 6^2) \times 3.14; A = (\frac{1}{4} \times 36) \times 3.14; A = 9 \times 3.14; A = 28.26 \text{ ft}^2$$

Student Text pp. 246–47

Objectives

- Name the 3-dimensional figure that can be formed from a net
- Calculate the surface area of rectangular, square, and triangular prisms using formulas
- Construct a triangular prism
- Relate surface area to real-life situations

Teacher Materials

- Surface Area: Triangular Prism, page IA52 (CD)
- A classroom set of 3-dimensional figures or real-life objects (from Lesson 59)
- A cereal box (rectangular prism)
- A ruler
- Transparent tape

Student Materials

- Surface Area: Triangular Prism, page IA52 (CD)
- A cereal box (rectangular prism) for every 2 to 3 students
- A ruler
- Transparent tape

Teach for Understanding

Name the 3-dimensional figure formed from a net

► **What is the difference between 2-dimensional and 3-dimensional figures? 2-dimensional figures lie in the same plane and have length and width; 3-dimensional figures are in more than one plane and have length, width, and height.**

1. Display your examples of a cone and a cylinder. Remind the students that conical figures are 3-dimensional figures with 1 base, similar to a cone. Cylindrical figures are 3-dimensional figures with 2 congruent bases, similar to a cylinder. The 2 bases may vary in size when compared to other cylindrical figures, but within a figure the 2 bases must be congruent. Display the 3-dimensional objects. Choose students to identify each object, tell the number and shape of the bases, and then classify the object as *cylindrical* or *conical*.
2. Display the cereal box. Elicit that it is a rectangular prism.
 - **How many faces does a rectangular prism have? 6**
 - **What do you notice about the faces of a rectangular prism? Elicit that there are three pairs of congruent rectangular faces.**
 Cut the cereal box along its edges so that it opens flat to form a net with 6 rectangular faces, including 2 rectangular bases. Explain that a *net* is the 2-dimensional pattern of a 3-dimensional figure. The net for a rectangular prism includes 6 rectangles. Point out that shared edges in the rectangular prism figure are common dimensions (length or width) of two faces in the net. (**Note:** Some students may recognize that any pair of the congruent pairs of sides can be chosen as the bases of a rectangular prism.)
3. Direct attention to the nets pictured on Student Text page 248. Elicit which illustrations are pyramid nets and which are prism nets. Point out the polygon base in the net for each pyramid. Also point out that the net for each prism has 2 congruent bases and that all of the side faces are parallelograms (rectangles). Call attention to the pyramids and prisms pictured in the teaching boxes and explain that pyramids and prisms are named for their bases.

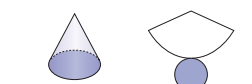
Calculate the surface area of prisms using formulas

1. Arrange the students in groups of 2 or 3 and distribute a cereal box to each group.
 - **How do you find the area of one face on your box? Why? Multiply the face's length times its width; each face is a rectangle and the area of a rectangle is found by multiplying its length times its width.**
 Explain that the *surface area* of a 3-dimensional figure is the sum of the areas of all its surfaces (faces).
 - **How do you think you can find the surface area of your box? Elicit that you must first find the area of each of the 6 faces and then add all of the areas.**
 - **Do you need to measure the length and width of all 6 faces to find the areas? Why? No; elicit that opposite faces in a rectangular prism are congruent, so you only need to measure and find the area of 3 different rectangles (1 in each of the pairs of congruent faces).**
2. Direct each group of students to develop a plan for finding the surface area of their rectangular prism. Allow a student from each group to share the group's ideas. Elicit that there are 3 pairs of congruent faces in any rectangular prism: adding the congruent top and bottom areas, the congruent front and back areas, and the congruent side areas together will give them the surface area of the rectangular prism. Write for display *Surface Area of a rectangular prism = (Area of top and bottom) + (Area of front and back) + (Area of sides)*.
3. Direct each group of students to measure the length and the width of each face of their cereal box to the nearest inch, calculate the area of each face, and add the six areas to find the surface area of the box (rectangular prism). Discuss each group's findings.
 - **Are the length and width dimensions you used to determine the area of each face the same length and width dimensions of the prism? Explain. Yes and no; the top and bottom faces ($l \times w$) are the same length and width measurements of the prism, the side faces are the width and height of the prism ($w \times h$), and the front and back faces are the length and height of the prism ($l \times h$).**
 - **What one mathematical formula could you write to find the surface area of a rectangular prism, using its length, width, and height measurements? Elicit $S = 2(l \times w) + 2(w \times h) + 2(l \times h)$.**
 Write $S = 2(l \times w) + 2(w \times h) + 2(l \times h)$ below the formula that was written in word form. Explain that memorizing this formula is not necessary if you can determine which dimensions of the prism are used as the length and width of the faces.
4. Guide the students in solving the following word problem on paper. Allow them to refer to the formula and to draw a picture if needed. (**Note:** When finding surface area, allow the students to solve one equation or several equations as shown on Student Text page 249.)

Molly is wrapping a birthday present in a rectangular box. The box is 2 feet long, 3 feet wide, and 1 foot high. What is the surface area of the box that Molly needs to cover? $2(2 \text{ ft} \times 3 \text{ ft}) + 2(3 \text{ ft} \times 1 \text{ ft}) + 2(2 \text{ ft} \times 1 \text{ ft}) = 22 \text{ ft}^2$

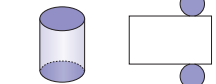
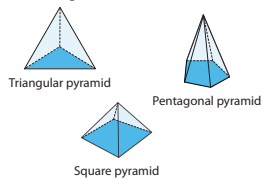
Surface Area of Prisms

A **net** is a flat pattern of 2-dimensional surfaces that can be shaped into a 3-dimensional figure.



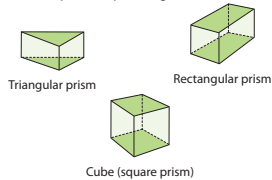
A **cone** has 1 base and an opposite vertex.

A **pyramid** is a cone with a polygon instead of a circle as a base. A pyramid is named for the shape of its base. All other faces of a pyramid are triangles.



A **cylinder** has 2 congruent circular bases.

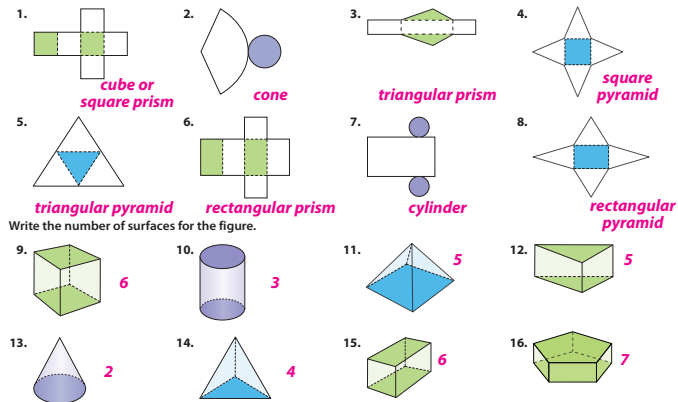
A **prism** is a type of cylinder with 2 congruent polygon bases that are parallel. A prism is named for the shape of its bases. All other faces of a prism are parallelograms.



net
cone
pyramid
cylinder
prism
surface area
rectangular prism
triangular prism

Exercises

Write the name of the figure that the net will make. The bases of the nets are shaded.



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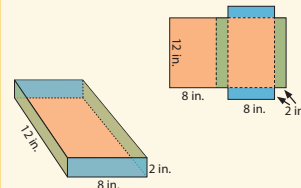
Chapter 11

Use the Given Formula to Find the Surface Area of Prisms

The **surface area** of a 3-dimensional figure is the sum of the areas of all its surfaces.

A **rectangular prism** has 3 sets of congruent faces.

$$S = 2(l \cdot w) + 2(w \cdot h) + 2(l \cdot h)$$



$$\begin{aligned} \text{top and bottom} & 2(12 \cdot 8) = 192 \text{ in.}^2 \\ \text{front and back} & 2(12 \cdot 2) = 48 \text{ in.}^2 \\ \text{sides} & 2(8 \cdot 2) = 32 \text{ in.}^2 \\ \text{Total Surface Area} & = 272 \text{ in.}^2 \end{aligned}$$

A **cube** has 6 congruent faces.

$$S = 6(l \cdot w) \text{ or } S = 6s^2$$



A **triangular prism** has 5 faces.

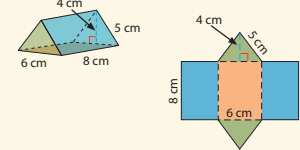
Calculate the area of the 3 rectangular faces.

$$A = l \cdot w$$

Calculate the area of the 2 triangular bases.

$$A = \frac{1}{2}(b \cdot h)$$

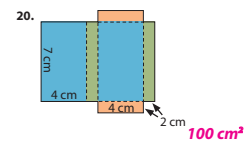
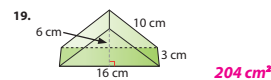
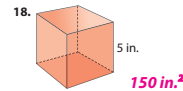
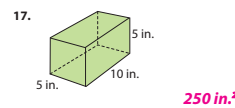
Add the areas of the 5 faces.



$$\begin{aligned} \text{bottom face} & 8 \cdot 6 = 48 \text{ cm}^2 \\ \text{slanted sides} & 2(8 \cdot 5) = 80 \text{ cm}^2 \\ \text{triangular bases} & 2\left(\frac{1}{2}(6 \cdot 4)\right) = 24 \text{ cm}^2 \\ \text{Total Surface Area} & = 152 \text{ cm}^2 \end{aligned}$$

Exercises

Find the surface area of the prism.



J Explain why a cube and a rectangular prism have 6 faces, but a triangular prism has only 5 faces. A rectangle has 4 sides plus the 2 bases, giving the prism 6 faces; a triangle has 3 sides plus the 2 bases, giving the prism 5 faces.

Complete **DAILY REVIEW** on page 443.

Lesson 103

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5. Display your example of a cube. Remind the students that a cube is a square prism.

► **How many faces does a cube have? 6**

► **Do you need to measure the length and width of all 6 faces of a cube to find their areas? Why? No; all the faces of a cube are congruent squares, so you need to measure only one side of one face, find the area of one face, and multiply the area by 6.**

Write for display **Surface Area of a cube** = $6(\text{Area of 1 side})$.

Elicit that "Area of 1 side" is $(l \times w)$ or $(s \times s)$. Write

$$S = 6(l \cdot w) \text{ or } S = 6s^2 \text{ below the formula word form.}$$

6. Choose a student to measure one side of the cube to the nearest inch, centimeter, or millimeter. Direct the students to find the surface area.

7. Instruct the students to solve these word problems. Give guidance as needed.

Mrs. Grant is making a pillow for her granddaughter by covering a foam cube. Each side of the cube is 12 inches. Mrs. Grant needs to calculate the surface area of the foam cube in order to determine how much fabric to purchase. What is the surface area of the foam cube? **$S = 6(12 \text{ in.} \times 12 \text{ in.})$ or $S = 6(12 \text{ in.})^2$; $S = 864 \text{ in.}^2$**

Mrs. Grant purchased 1 yard of fabric that is 45 inches wide. Will she have enough fabric to cover the foam cube for her granddaughter's pillow? **yes; $36 \text{ in.} \times 45 \text{ in.} = 1,620 \text{ in.}^2$**

8. Display and distribute the Surface Area: Triangular Prism page. Remind the students that a net can be used to construct a 3-dimensional figure.

► **What does the net show you about the 3-dimensional figure called a triangular prism? Elicit that the net shows that the triangular prism has 5 faces. Two of the faces are congruent**

triangular bases; they show that the figure is a triangular prism and give the prism its name. The other 3 faces are rectangles; two of the rectangular faces are congruent and the third rectangular face is 1 unit wider than the other two rectangular faces.

► **Do you need to find the area of each base? Why? No; you can find the area of one of the congruent triangular bases and multiply it by 2.**

► **What formula can you use to find the area of one triangular base? $A = \frac{1}{2}(b \cdot h)$ Write the formula for display.**

► **What equation can you use to find the area of both triangular bases? $A = 2[\frac{1}{2}(6 \times 4)]$ Write the equation for display.**

► **Do you need to find the area of each rectangular face? Why? No; you can find the area of one of the congruent faces and multiply it by 2 to find the area of the two congruent faces and then find the area of the other face.**

9. Direct the students to find the area of the 2 triangular bases and the 3 rectangular faces. Give guidance as needed.

$$2 \text{ triangular bases } A = 2[\frac{1}{2}(6 \text{ units} \times 4 \text{ units})];$$

$$A = 2(\frac{1}{2} \times 24 \text{ units}^2); A = 2(12 \text{ units}); A = 24 \text{ units}^2$$

$$2 \text{ rectangular faces } A = 2(l \cdot w); A = 2(5 \text{ units} \times 12 \text{ units});$$

$$A = 2(60 \text{ units}^2); A = 120 \text{ units}^2$$

$$1 \text{ rectangular face } A = l \cdot w; A = 6 \text{ units} \times 12 \text{ units}; A = 72 \text{ units}^2$$

► **How can you find the surface area of the triangular prism? Add the areas of all of the faces.**

► **What is the surface area? $24 + 120 + 72 = 216 \text{ cm}^2$**

10. Instruct the students to cut out the net, fold it along the lines shared by 2 faces, and tape it together.

Student Text pp. 248–49

(Note: Assessment available on Teacher's Toolkit CD.)

Objectives

- Calculate the surface area of rectangular, square, and triangular prisms using formulas
- Calculate the surface area of a cylinder using formulas
- Construct a cylinder net
- Relate surface area to real-life situations

Teacher Materials

- Surface Area, page IA53 (CD)
- A small oatmeal container (or the cylinder from Lesson 103)
- A ruler
- Two 12×18 sheets of construction paper (to cover the surface of the oatmeal container or cylinder)
- Transparent tape

Student Materials

- A cylinder (potato chip container with a top or a can)
- A ruler
- A calculator
- 1–2 sheets of construction paper (to cover the surface of the cylinder)
- Transparent tape

Teach for Understanding

Calculate the surface area of prisms using formulas

- **What is surface area?** *the number of square units it takes to cover the surface of a 3-dimensional figure*
- **How do you find the surface area of a figure?** *Find the area of each of the figure's faces and add all the areas.*

Beth made several scratching toys for her pet cats by covering the surfaces of some wooden blocks with carpet. She made one toy by using a wooden block that had 6 square faces and was 2 feet high. How much carpet did Beth use to cover all the surfaces of the block? **24 ft^2**

- **What is the shape of the wooden block Beth covered with carpet? How do you know?** *A cube or a square prism; each face is a square measuring $2 \text{ ft} \times 2 \text{ ft}$.*

1. Display the Surface Area page. Guide the students in writing the height (2 ft), length (2 ft), and width (2 ft) dimensions along the edges of the cube.

- **How can you find the surface area of a cube?** *Elicit that since all 6 faces are congruent squares, you can first find the area of one square and then multiply the area of one square by 6.*

- **What mathematical formula could you write to help you find the surface area of any cube?** *$S = 6(l \times w)$ or $S = 6(s^2)$*

Write both formulas for display.

Guide the students in finding the surface area of the cube using either formula. Instruct them to write the solution on the page. **$S = 6(2 \text{ ft} \times 2 \text{ ft}); S = 24 \text{ ft}^2$ or $S = 6 \times (2 \text{ ft})^2; S = 24 \text{ ft}^2$**

- **What is the surface area of the cube-shaped scratching toy?** **24 ft^2**

To make the second scratching toy, Beth used a wooden block with 6 rectangular sides. The block has a length of 2 feet, a width of 1 foot, and a height of 1.5 feet. How much carpet did Beth need to cover all the surfaces of this block? **13 ft^2**

- **What is the shape of the wooden block that Beth used to make this scratching toy? How do you know?** *A rectangular prism; elicit that the length, width, and height measurements are all different, so there are 3 pairs of opposite congruent faces that are rectangles.*

2. Guide the students in writing the length, width, and height dimensions along the edges of the rectangular prism.

- **How can you find the surface area of a rectangular prism?** *Elicit that you need to identify each face and its dimensions, find the area of each side or the area of 1 of each pair of congruent sides and multiply the area by 2, then add the areas together to find the surface area.*

- **What formula can help you to find the area of a rectangle?** **$A = l \times w$**

Guide the students in identifying the 6 rectangular faces (3 pairs of congruent faces) and their length and width dimensions. Then use the $l \times w$ formula to guide the students in finding the area of each face.

top and bottom **$A = 2 \text{ ft} \times 1 \text{ ft}; A = 2 \text{ ft}^2$**

front and back **$A = 2 \text{ ft} \times 1.5 \text{ ft}; A = 3 \text{ ft}^2$**

sides **$A = 1 \text{ ft} \times 1.5 \text{ ft}; A = 1.5 \text{ ft}^2$**

- **What equation can you write to find the surface area for the rectangular prism?** **$S = 2(2 \text{ ft}^2) + 2(3 \text{ ft}^2) + 2(1.5 \text{ ft}^2)$; or $S = 2 \text{ ft}^2 + 2 \text{ ft}^2 + 3 \text{ ft}^2 + 3 \text{ ft}^2 + 1.5 \text{ ft}^2 + 1.5 \text{ ft}^2; S = 13 \text{ ft}^2$**

Beth made the third scratching toy from a wooden block with 5 faces. Each of the 3 rectangular sides has a length of 2 feet and a width of 1.5 feet. The 2 triangular faces have a base of 2 feet and a height of 1.75 feet. How much carpet did Beth use to cover all the surfaces of this block? **12.5 ft^2**

- **What is the shape of this scratching toy? How do you know?** *A triangular prism; 3 sides (faces) are rectangles, and the top and bottom faces (bases) are triangles. Elicit that the 3 rectangular sides are congruent and the 2 triangular bases are congruent.*

- **What formula can help you to find the area of a rectangle?** **$A = l \times w$** **area of a triangle?** **$A = \frac{1}{2}(b \times h)$**

3. Guide the students in identifying and writing the length and width dimensions of the 3 congruent rectangular sides (2 ft and 1.5 ft). Use the $l \times w$ formula to find the area of each face. **3 ft^2** Next, guide them in identifying and writing the base and height measurements of the 2 congruent triangles (2 ft and 1.75 ft). Use the formula $\frac{1}{2}(b \times h)$ to find the area of each triangle. **1.75 ft^2**

- **What equation can you write to find the surface area for the triangular prism?** **$S = 3(3 \text{ ft}^2) + 2(1.75 \text{ ft}^2); S = 9 \text{ ft}^2 + 3.5 \text{ ft}^2$ or $S = 3 \text{ ft}^2 + 3 \text{ ft}^2 + 3 \text{ ft}^2 + 1.75 \text{ ft}^2 + 1.75 \text{ ft}^2; S = 12.5 \text{ ft}^2$**

Calculate the surface area of a cylinder using formulas

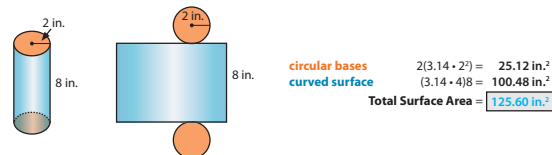
1. Display the oatmeal container.
 - **How do you think you can find the surface area of a cylinder?** *Elicit that you find the sum of the area of all the surfaces.*
 - **How many bases does a cylinder have?** **2** **What is the shape of the base?** *circle*
 - **What formula is used to find the area of a circle?** **$A = \pi r^2$** Write the formula for display.
 - **What information is needed to use this formula?** *Elicit that the measurement of a radius or a diameter is needed.*
2. Trace the circular bases of the oatmeal container onto one sheet of construction paper and cut them out. Choose a

Surface Area of Cylinders

A cylinder has 2 congruent circular bases and 1 curved surface.

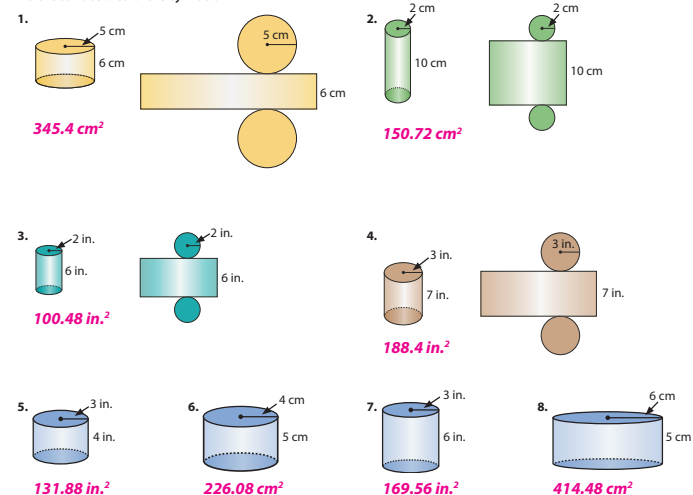
- Calculate the area of one circle using $A = \pi r^2$. Multiply by 2 to find the area of both circular bases.
- Calculate the area of the curved surface. The curved surface is a rectangle when lying flat. Use $A = l \cdot w$. The width of the rectangle is the height of the cylinder. The length of the rectangle is the circumference of the circular bases.

cylinder



Exercises

Find the surface area of the cylinder.



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Chapter 11

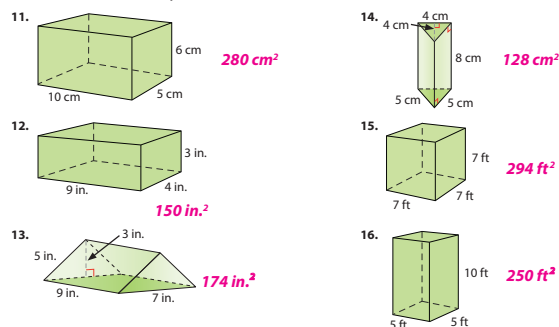
student to measure the circle to the nearest inch to find the diameter.

- **What is the radius? How do you know? Answers will vary based on the size of the circle. The radius is half of the diameter.** Guide the students in using the equation $A = 3.14 \times r^2$ to find the area of one of the circles. Elicit that to find the area of both circular faces, they will need to multiply the expression $3.14 \times r^2$ by 2. Guide them in finding the area of both circular bases using the equation $A = 2(3.14 \times r^2)$. **Answers will vary.**
 - **What kinds of items cover a curved surface similar to the curved surface of a cylinder? possible answers: paper towels, wrapping paper, tape, soup can labels**
 - **What shape do you think the curved surface of the cylinder is when you lay it flat? Answers will vary, but elicit rectangular.**
- Cut the other sheet of construction paper to match the height and distance around the cylinder. Lay it out flat to show the rectangle.
 - **What formula can you use to find the area of a rectangle? $A = l \cdot w$** Write the formula for display.
 - Choose a student to place the cut piece of construction paper on the cylinder without taping it. Tape the top and bottom bases (circles) to the curved surface to make a net. Remove the paper.
 - Display the net and point out that the length of the rectangle is equal to the circumference of the circle. Elicit that you can use the formula for finding the circumference of a circle, πd or $2\pi r$, to find the length of the rectangle. Select a student to calculate the length of the rectangle. **Answers will vary.** Explain that the width of the rectangle is equal to the height of the cylinder. Choose a student to measure the height of the

Write the answer.

- The bases of cylinders in problem 5 and problem 7 are congruent, but the heights are not. How will the nets differ? **The rectangles will be wider (taller).**

Find the surface area of the prism.



Solve. Draw a picture if needed.

- Emma plans to cover a box with colorful adhesive paper. The box is 8 inches long, 6 inches wide, and 5 inches high. Will 225 square inches of adhesive paper cover the box? $2(8 \cdot 6) + 2(8 \cdot 5) + 2(6 \cdot 5) = 236 \text{ in.}^2$; **no**
- Grace made a pillow in the shape of an octagon. Each side is 12 inches long. Will 100 inches of fringe be enough to go around the entire pillow? $8 \cdot 12 = 96 \text{ in.}$; **yes**
- Mr. Watkins painted a dodge ball circle on the playground. The circle has a diameter of 18 feet. What is the circumference of the circle? $3.14 \cdot 18 = 56.52 \text{ ft}$
- Pedro wants to paint 3 walls in his bedroom. His bedroom is 10 feet long and 10 feet wide, and the ceiling is 10 feet high. How much surface area is he going to paint? $3(10 \cdot 10) = 300 \text{ ft}^2$

Why would you measure the perimeter of a rectangle in centimeters and the area of that same rectangle in square centimeters?

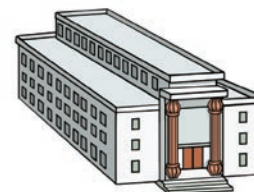
Can a cylinder have circular bases with areas that are different? Why? **No, a cylinder is defined as having 2 congruent circular bases and 1 curved surface.**

DID YOU KNOW

Solomon had the interior of the temple in Jerusalem overlaid with gold. Workers had to figure the area of each surface to be covered in order to prepare the gold overlays.

So Solomon overlaid the house within with pure gold: and he made a partition by the chains of gold before the oracle; and he overlaid it with gold.

1 Kings 6:21



Complete **DAILY REVIEW** on page 443.

Lesson 104

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cylinder to the nearest inch. Guide the students in finding the area of the curved surface (rectangle) by multiplying the calculated length by the width. **Answers will vary.**

Guide the students in adding the areas of the circular bases and the curved surface (rectangle) to find the surface area of the cylinder. **Answers will vary.**

- Distribute the cylinders and construction paper. Direct each student to construct a net for his cylinder and to determine its surface area to the nearest inch or cm. Give guidance as needed.

The final scratching toy Beth made for her cats was 4 feet high. It had 2 circular bases and 1 curved surface. Each circle had a radius of 0.5 of a foot. How much carpet did Beth need to cover all of the surfaces of this toy? **14.13 ft²**

- **What is the shape of this scratching toy? How do you know? A cylinder; it has 2 circular bases and 1 curved surface.**
- Write the dimensions of the cylinder pictured on the Surface Area page and guide the students in finding the surface area. Remind them that the length of the curved surface is the circumference of the cylinder's base, and that the width is the height of the cylinder.
 2 circular bases $A = 2[3.14 \times (0.5 \text{ ft})^2]$; $A = 1.57 \text{ ft}^2$
 1 curved surface $C = 3.14 \times 2(0.5 \text{ ft})$; $C = 3.14 \text{ ft}$
 $A = 3.14 \text{ ft} \times 4 \text{ ft}$; $A = 12.56 \text{ ft}^2$
 total surface area $1.57 \text{ ft}^2 + 12.56 \text{ ft}^2 = 14.13 \text{ ft}^2$

Student Text pp. 250–51

Objectives

- Recognize that perimeter can vary for a fixed area
- Calculate the area and perimeter of a rectangle
- Calculate the area of a complex figure
- Create a basic floor plan from a fixed area
- Relate geometry to real-life situations

Teacher Materials

- Graph Paper, page IA13 (CD)
- Floor Plan Activity, page IA54 (CD)
- Christian Worldview Shaping, pages 28–29 (CD)
- Samples of floor plans

Student Materials

- Graph Paper, page IA13 (CD)
- Floor Plan Activity, page IA54 (CD) for each pair of students
- Floor Plan Grid, page IA55 (CD) for each pair of students
- A ruler
- A calculator
- A sheet of 12×18 construction paper for each pair of students

Notes

Floor plans can be found at online websites, in home decorating magazines, and in brochures provided by builders of local housing developments. You may choose to make a model floor plan using the Floor Plan Grid page to help students better understand the floor plan they will make.

In preparation for the Christian Worldview Shaping in Lesson 112, instruct each student to bring to class a small item for which he will design a package. The item can be new, or something that the student has at home. (For examples of items, see the materials listed for Lesson 112 on the Teacher's Toolkit CD.)

Teach for Understanding

Recognize that perimeter can vary for a fixed area

- **What is perimeter?** *the distance around a geometric figure*
- **What is area?** *the space within a region*

1. Distribute and display the Graph Paper page.
➤ **How many different rectangles do you think you can make with an area of 12 square units?** *Answers will vary.*
2. Direct each student to draw on the graph paper a rectangle with an area of 12 square units. Choose several students to describe their rectangles by giving the dimensions. Draw for display on the Graph Paper page three different rectangles that are congruent to the ones described by the students.
3. Elicit that there are only 3 different factor pairs that make up rectangles with an area of 12 square units: 1 unit \times 12 units or 12 units \times 1 unit; 2 units \times 6 units or 6 units \times 2 units; 3 units \times 4 units or 4 units \times 3 units.
Write $A = 12 \text{ units}^2$ inside each rectangle. Point out that 12 square units can be referred to as a *fixed area*; the area remained the same even though the perimeters changed.
4. Guide the students in calculating the perimeter of each displayed rectangle.
 - 1×12 rectangle $P = (2 \cdot 1) + (2 \cdot 12) = 26 \text{ units}$
 - 2×6 rectangle $P = (2 \cdot 2) + (2 \cdot 6) = 16 \text{ units}$
 - 3×4 rectangle $P = (2 \cdot 3) + (2 \cdot 4) = 14 \text{ units}$

- **What do you notice about the perimeter and the area of the rectangles?** *Elicit that rectangles that are equal in area can have perimeters that are not equal.*

5. Direct each student to find the perimeter of his rectangle and to write the area and perimeter inside the rectangle (e.g. $A = 12 \text{ units}^2$, $P = 26 \text{ units}^2$).
6. Repeat the procedure for rectangles with an area of 24 square units.

$$1 \times 24 \text{ or } 24 \times 1; P = (2 \cdot 1) + (2 \cdot 24) = 50 \text{ units}$$

$$2 \times 12 \text{ or } 12 \times 2; P = (2 \cdot 2) + (2 \cdot 12) = 28 \text{ units}$$

$$3 \times 8 \text{ or } 8 \times 3; P = (2 \cdot 3) + (2 \cdot 8) = 22 \text{ units}$$

$$4 \times 6 \text{ or } 6 \times 4; P = (2 \cdot 4) + (2 \cdot 6) = 20 \text{ units}$$

- **Why can the perimeters be different when rectangles have the same fixed area?** *Elicit that the same number of square units can be arranged in different ways, changing the dimensions which determine the perimeter of a given region.*

Create a basic floor plan from a fixed area

- **Have you ever walked into a friend's house or apartment and noticed that the rooms are not in the same location as in your house?**

1. Display the floor plans. Explain that houses, apartments, offices, and other buildings are designed by architects. There are many ways to design a floor plan for a fixed area. When creating a floor plan, architects also use their knowledge of building codes and the layout of the property where the house or the apartment building will be built. A floor plan is a scaled drawing which is later used in creating blueprints for the builder. Blueprints give more detailed information about the placement of cabinets, electrical outlets, windows, and so on.
2. Arrange the students in pairs. Display the Floor Plan Activity page and distribute the page to each pair of students. Read aloud and explain the requirements of the project.
3. Distribute the Floor Plan Grid page and a sheet of construction paper to each pair of students. Explain that there are 1,500 squares on the page, and each square represents 1 square foot in the house.
Read aloud and discuss the procedures for creating the floor plan. Encourage the students to discuss and sketch possible layouts for their house before cutting apart the Floor Plan Grid. Direct them to cut out the squares for each room or hallway, and to arrange the rooms on the construction paper as they cut out each room. Provide assistance as needed. You may choose to check the floor plans before the students glue the rooms onto the construction paper.

Calculate the area of a complex figure

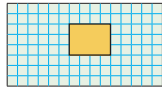
- **What is (or should be) the area of the floor plan of your house? Why?** *1500 ft²; all 1500 squares ("square feet") of the Floor Plan Grid were used to make the floor plans.*
- **How do you calculate the area of a complex figure?** *Elicit that you partition (divide) the complex figure into smaller figures, find the area of each smaller figure, and add all the areas.*
- **How can you calculate the area of your floor plan?** *Elicit that you can find the area of each room or hallway, and then add all the areas.*

Fixed Areas

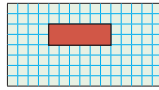
Perimeter is the distance around a geometric figure. **Area** is the space within a region. Geometric figures can have the same area but different perimeters.

Area can be found by counting the number of square units needed to cover the surface. The perimeter of any polygon can be found by adding the lengths of the sides.

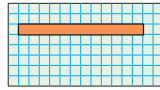
perimeter
 $P = (2 \cdot l) + (2 \cdot w)$
area
 $A = l \cdot w$



$A = 3 \times 4 = 12 \text{ units}^2$
 $P = 4 + 3 + 4 + 3 = 14 \text{ units}$



$A = 2 \times 6 = 12 \text{ units}^2$
 $P = 6 + 2 + 6 + 2 = 16 \text{ units}$

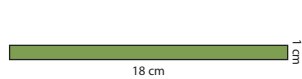


$A = 1 \times 12 = 12 \text{ units}^2$
 $P = 12 + 1 + 12 + 1 = 26 \text{ units}$

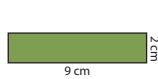
The formula used to calculate the area of a rectangle is length (l) times width (w). $A = l \cdot w$
The formula used to calculate the perimeter of a rectangle is 2 times the length plus 2 times the width. $P = (2 \cdot l) + (2 \cdot w)$

Use factor pairs of 18 as the dimensions of figures with an area of 18 cm^2 .

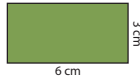
18: 1×18 2×9 3×6



$A = 18 \cdot 1 = 18 \text{ cm}^2$
 $P = (2 \cdot 18) + (2 \cdot 1) = 38 \text{ cm}$



$A = 9 \cdot 2 = 18 \text{ cm}^2$
 $P = (2 \cdot 9) + (2 \cdot 2) = 22 \text{ cm}$



$A = 6 \cdot 3 = 18 \text{ cm}^2$
 $P = (2 \cdot 6) + (2 \cdot 3) = 18 \text{ cm}$

Exercises

Write factor pairs for the given area. Use the factor pairs for the dimension of a figure to find the perimeter. Draw diagrams if needed. **Answers may vary.**

- 6 m^2
- 10 in^2
- 16 ft^2

Write the two perimeter equations for the given area.

4. 16 cm^2	$P = (2 \cdot 6) + (2 \cdot 2)$	$P = (2 \cdot 4) + (2 \cdot 4)$	$P = (2 \cdot 16) + (2 \cdot 16)$
5. 21 cm^2	$P = (2 \cdot 1) + (2 \cdot 21)$	$P = (2 \cdot 11) + (2 \cdot 10)$	$P = (2 \cdot 3) + (2 \cdot 7)$
6. 28 cm^2	$P = (2 \cdot 4) + (2 \cdot 7)$	$P = (2 \cdot 3) + (2 \cdot 7)$	$P = (2 \cdot 14) + (2 \cdot 2)$

- $A = 1 \text{ m} \cdot 6 \text{ m}$
 $P = 2(1 \text{ m}) + 2(6 \text{ m})$
 $P = 14 \text{ m}$

$A = 2 \text{ m} \cdot 3 \text{ m}$
 $P = 2(2 \text{ m}) + 2(3 \text{ m})$
 $P = 10 \text{ m}$
- $A = 1 \text{ in.} \cdot 10 \text{ in.}$
 $P = 2(1 \text{ in.}) + 2(10 \text{ in.})$
 $P = 22 \text{ in.}$

$A = 2 \text{ in.} \cdot 5 \text{ in.}$
 $P = 2(2 \text{ in.}) + 2(5 \text{ in.})$
 $P = 14 \text{ in.}$
- $A = 1 \text{ ft} \cdot 16 \text{ ft}$
 $P = 2(1 \text{ ft}) + 2(16 \text{ ft})$
 $P = 34 \text{ ft}$

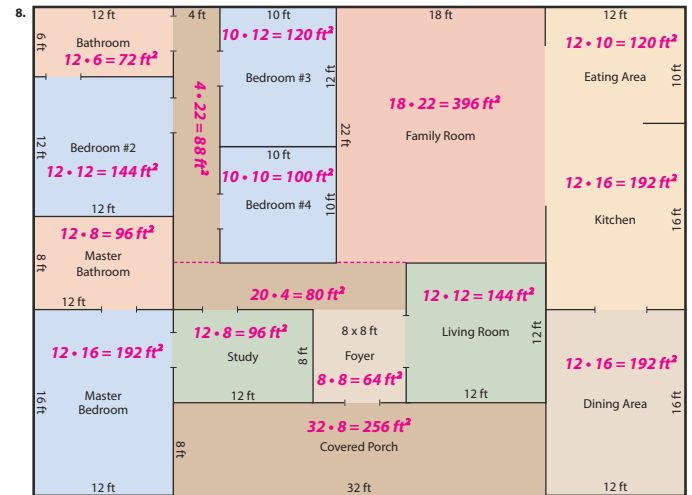
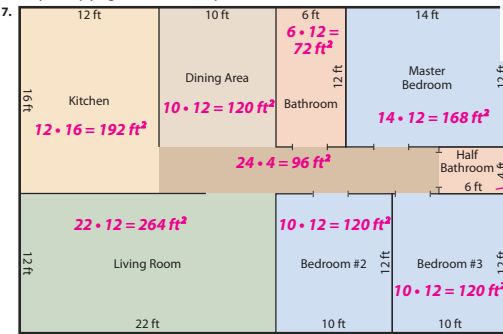
$A = 2 \text{ ft} \cdot 8 \text{ ft}$
 $P = 2(2 \text{ ft}) + 2(8 \text{ ft})$
 $P = 20 \text{ ft}$

$A = 4 \text{ ft} \cdot 4 \text{ ft}$
 $P = 2(4 \text{ ft}) + 2(4 \text{ ft})$
 $P = 16 \text{ ft}$

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Chapter 11

Add the area of each room to find the total area. **Order of addends may vary.**
Check by multiplying $l \times w$ of the floor plan.



Lesson 105

Complete **DAILY REVIEW** on page 444.

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Instruct the students to find the area of each room and hallway on their floor plan, and then find the sum of all these areas. Challenge them to write one equation for the area of their floor plan.

Relate geometry to real-life situations

The following enrichment activities are beneficial in helping the students apply math to real-life situations. The activities may be completed anytime during the school year.

- Instruct the students to calculate the outside perimeter of their floor plan. Guide a discussion about the extra expense that comes from having a larger outside perimeter due to more siding or bricks being needed for the exterior walls.
- Display the students' floor plans. Allow the students in your class or invite the students in other classes to vote for their favorite floor plan. Guide the class in using sidewalk chalk to draw on the parking lot a life-size model of the winning floor plan.
- Instruct each student to measure some or all of the rooms in his home and compare the measurements with the measurements of the corresponding room on his floor plan.
- Place the floor plan on poster board. Cover the floor of each room with carpet or paper tiles. Calculate the cost of all the floor coverings based on prices found online or in local store advertisements.
- Make a 3-dimensional model from the floor plan. Place the floor plan on poster board. Use index cards or cardstock to form walls. Calculate the surface area of the walls. Calculate the cost of painting the walls based on prices found online or in local store advertisements.

Chapter Review

Objectives

- Calculate the perimeter and area of polygons and complex figures
- Calculate the circumference and the area of a circle
- Calculate the unknown side (length or width) of a rectangle or a square
- Name the 3-dimensional figure that can be formed from a net
- Calculate the surface area of rectangular, square, and triangular prisms
- Calculate the surface area of a cylinder
- Relate perimeter, area, and surface area to real-life situations

Teacher Materials

- Geometry Review I, page IA56 (CD)
- Geometry Review II, page IA57 (CD)
- Nets, page IA58 (CD)
- Surface Area (from Lesson 104)

Student Materials

- A calculator

Note

This lesson reviews the concepts presented in Chapter 11 to prepare the students for the Chapter 11 Test. Student Text pages 254–55 provide the students with an excellent study guide.

Check for Understanding

Calculate perimeter, circumference, and area

1. Display the Geometry Review I page and direct attention to the square. Elicit the formulas used to calculate the perimeter and the area of a square. $P = 4 \cdot s$ or $P = s + s + s + s$; $A = s^2$
Review the formulas as needed.

Direct the students to find the perimeter and the area of the square. $P = 4 \cdot 7 \text{ m}$, $P = 28 \text{ m}$; $A = (7 \text{ m})^2$, $A = 49 \text{ m}^2$

2. Follow a similar procedure for all of the figures on both this page and the Geometry Review II page. If additional review of any formula is needed, choose a student to provide different dimensions for the figure and guide the students in calculating the new perimeter or circumference and area.

rectangle $P = (2 \cdot l) + (2 \cdot w)$
 $P = (2 \cdot 9.6 \text{ cm}) + (2 \cdot 4 \text{ cm})$; $P = 27.2 \text{ cm}$
 $A = l \cdot w$
 $A = 9.6 \text{ cm} \cdot 4 \text{ cm}$; $A = 38.4 \text{ cm}^2$

circle $C = 2\pi r$
 $C = 2 \times 3.14 \times 6 \text{ cm}$; $C = 37.68 \text{ cm}$
 $A = \pi r^2$
 $A = 3.14(6 \text{ cm})^2$; $A = 113.04 \text{ cm}^2$

parallelogram $P = s + s + s + s$
 $P = 12 \text{ in.} + 10 \text{ in.} + 12 \text{ in.} + 10 \text{ in.} = 44 \text{ in.}$
 $A = b \cdot h$
 $A = 12 \text{ in.} \cdot 8 \text{ in.}$; $A = 96 \text{ in.}^2$

right triangle $P = s + s + s$
 $P = 3 \text{ ft} + 5 \text{ ft} + 4 \text{ ft}$; $P = 12 \text{ ft}$
 $A = \frac{1}{2}(b \cdot h)$
 $A = \frac{1}{2}(4 \text{ ft} \cdot 3 \text{ ft})$; $A = 6 \text{ ft}^2$

obtuse triangle $P = s + s + s$
 $P = 5 \text{ in.} + 5 \text{ in.} + 8 \text{ in.}$; $P = 18 \text{ in.}$
 $A = \frac{1}{2}(b \cdot h)$
 $A = \frac{1}{2}(8 \text{ in.} \cdot 3 \text{ in.})$; $A = 12 \text{ in.}^2$

(Note: You may choose to guide the students in using more than one equation to find the area of the following figures.)

hexagon $P = n \cdot s$
 $P = 6 \cdot 5 \text{ m}$; $P = 30 \text{ m}$
 $A = (l \cdot w) + 2[\frac{1}{2}(b \cdot h)]$
 $A = (5 \text{ m} \cdot 8 \text{ m}) + 2[\frac{1}{2}(8 \text{ m} \cdot 3 \text{ m})]$; $A = 64 \text{ m}^2$

complex figure $P = s + s + s + s + s$
 $P = 8 \text{ cm} + 5 \text{ cm} + 5 \text{ cm} + 8 \text{ cm} + 12 \text{ cm}$;
 $P = 38 \text{ cm}$
 $A = (b \cdot h) + \frac{1}{2}(b \cdot h)$
 $A = (12 \text{ cm} \cdot 4 \text{ cm}) + \frac{1}{2}(12 \text{ cm} \cdot 4 \text{ cm})$; $A = 72 \text{ cm}^2$

Calculate the unknown side of a rectangle or a square

Mr. Lawrence planted a vegetable garden that has an area of 3,000 square feet. The garden is rectangular and has a width of 100 feet. What is the length of the garden? **30 ft**

- What formula could you use to find the area of the garden if both the length and width were known? $A = l \cdot w$
- How can you find the length of the unknown side since you know the area and the measurement of the other? Divide the area by the known measure; $l = A \div w$ or $n = A \div s$.

1. Direct the students to write an equation for the word problem and solve it. Encourage them to draw pictures if needed.
 $n = 3,000 \text{ ft}^2 \div 100 \text{ ft}$; $n = 30 \text{ ft}$
2. Use the formula $P = 4 \times s$ for the following word problem. Elicit that since you know the perimeter of the square kitchen you can find the length of the unknown side by dividing by 4 because all four sides of the kitchen are equal in length.

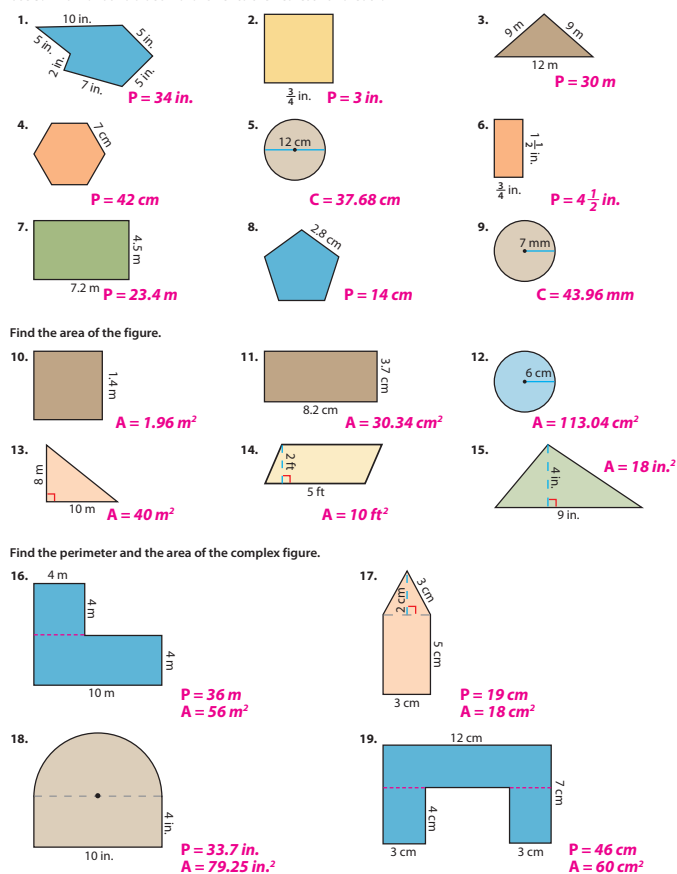
Mrs. Stevens is putting up wallpaper border near the ceiling in her square-shaped kitchen. The perimeter of her kitchen is 28 yards. What is the length of each wall?
 $28 \text{ yd} = 4 \times s$; $28 \text{ yd} \div 4 = 4 \div 4 \times s$; $s = 7 \text{ yd}$

3. Draw a rectangle for display. Write 5 ft along the length and write n along the width.
 ➤ If the perimeter of the rectangle is 22 feet, how can you find the measurement of the unknown width? Elicit that you can subtract the measurement of the two known sides ($2 \cdot 5 \text{ ft}$) from the perimeter (22 ft) and divide the difference (12) by 2.
 Direct the students to find the measurement of the unknown width. $22 \text{ ft} = (2 \cdot 5 \text{ ft}) + (2 \cdot n)$; $22 \text{ ft} - 10 \text{ ft} = 10 \text{ ft} - 10 \text{ ft} + 2n$;
 $12 \text{ ft} = 2n$; $\frac{12 \text{ ft}}{2} = \frac{2n}{2}$; $6 \text{ ft} = n$

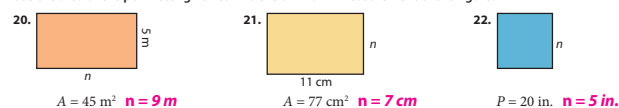
Name the 3-dimensional figure formed from a net

1. Display the Nets page.
 ➤ What is a net? the flat or 2-dimensional pattern of a 3-dimensional figure
 ➤ How many bases do conical figures have? 1 base
 cylindrical figures? 2 bases
2. Choose students to identify each net by name and tell whether it is a conical or a cylindrical figure. Write the name of each figure below the net. Shade the 1 base of each conical net and the 2 congruent bases of each cylindrical net.

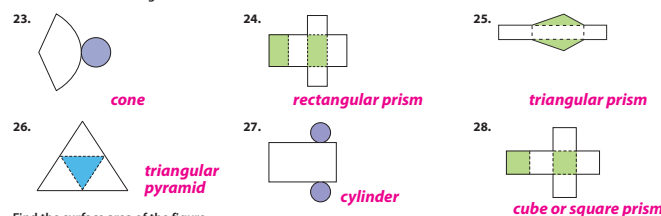
Find the circumference or the perimeter of the figure. Use 3.14 for π . Round a decimal answer to the nearest hundredth.



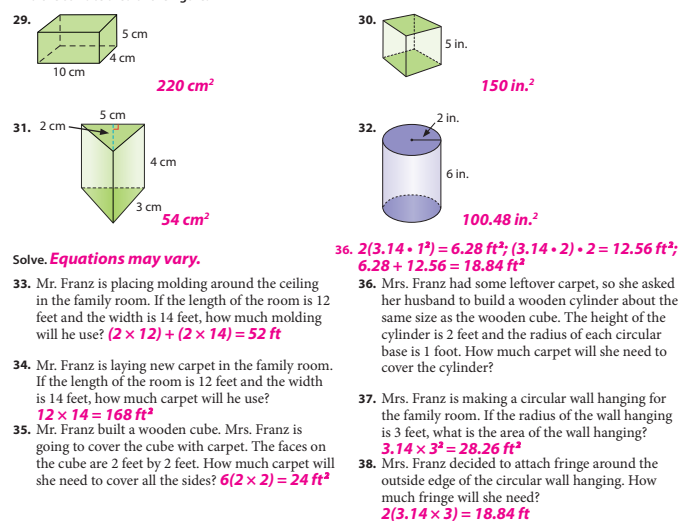
Use the area or the perimeter given to find the unknown measurement of the figure.



Write the name of the figure that the net will make. The bases of the nets are shaded.



Find the surface area of the figure.



1. cylinder; cylindrical figure
2. cone; conical figure
3. triangular prism; cylindrical figure
4. rectangular prism; cylindrical figure
5. square prism or cube; cylindrical figure
6. square pyramid; conical figure
7. triangular pyramid; conical figure
8. rectangular pyramid; conical figure

Calculate the surface area of prisms and a cylinder

1. Display the Surface Area page. Write the following dimensions along the edges of the rectangular prism: length = 4 ft, width = 2 ft, and height = 3 ft.
2. Elicit that you can find the surface area of a rectangular prism by finding the area of 1 face in each pair of congruent faces, multiplying each area by 2, and adding the areas; or you can add the areas of all six faces.
(Note: Allow the students to solve more than one equation to find the surface area of each figure if needed.)
► What formula could you use to find the surface area of a rectangular prism? Elicit $S = 2(l \cdot w) + 2(w \cdot h) + 2(l \cdot h)$. Write the formula for display and review it as needed.
3. Direct the students to find the surface area of the rectangular prism. $S = 2(4 \text{ ft} \times 2 \text{ ft}) + 2(2 \text{ ft} \times 3 \text{ ft}) + 2(4 \text{ ft} \times 3 \text{ ft}) = 52 \text{ ft}^2$ or $S = 8 \text{ ft} + 8 \text{ ft} + 6 \text{ ft} + 6 \text{ ft} + 12 \text{ ft} + 12 \text{ ft} = 52 \text{ ft}^2$

4. Follow a similar procedure for the other figures on the page.
cube—one side = 5 in.
Multiply the area of one face by 6: $S = 6(l \cdot w)$ or $S = 6s^2$;
 $S = 6(5 \text{ in.} \times 5 \text{ in.})$ or $S = 6(5 \text{ in.})^2$; $S = 150 \text{ in.}^2$.
triangular prism—2 rectangular faces: length = 5 ft, width = 3 ft; 1 rectangular face: length = 6 ft, width = 3 ft;
2 triangular bases: base = 6 ft, height = 4 ft
Elicit that since 2 of the rectangular faces are congruent, 1 rectangular face is not congruent, and the triangular bases are congruent, you can multiply the area of one of the 2 congruent rectangular faces by 2, find the area of the non-congruent rectangular face, multiply the area of one of the triangular bases by 2, and add the areas: $S = 2(l \cdot w) + (l \cdot w) + 2[\frac{1}{2}(b \cdot h)]$;
 $S = 2(5 \text{ ft} \times 3 \text{ ft}) + (6 \text{ ft} \times 3 \text{ ft}) + 2[\frac{1}{2}(6 \text{ ft} \times 4 \text{ ft})] = 72 \text{ ft}^2$.
cylinder—radius = 1 ft; height = 3 ft
Elicit that since the circular bases are congruent, you can multiply the area of one of the bases by 2 and add that area to the area of the curved surface. Remind students that the length of the curved surface is the circumference of the cylinder's base, and the width is the height of the cylinder. $S = 2(\pi r^2) + (2\pi r \cdot h)$;
 $S = 2[3.14 \times (1 \text{ ft})^2] + (2 \times 3.14 \times 1 \text{ ft}) 3 \text{ ft}$; $S = 25.12 \text{ ft}^2$
5. If additional review of any formula is needed, choose a student to provide different dimensions for a figure and guide the students in calculating the new surface area.

Student Text pp. 254–55

Chapter 11 Test
Cumulative Review

For a list of the skills reviewed in the Cumulative Review, see the Lesson Objectives for Lesson 107 in the Chapter 11 Overview on page 236 of this Teacher's Edition.

Student Materials

- Cumulative Review Answer Sheet, page IA9 (CD)

Use the Cumulative Review on Student Text pages 256–58 to review previously taught concepts and to determine which students would benefit from your reteaching of the concepts. To prepare the students for the format of achievement tests, instruct them to work on a separate sheet of paper, if necessary, and to mark the answers on the Cumulative Review Answer Sheet.

Use the Exploring Ideas on Student Text page 259 (page 257 of this Teacher's Edition) any time after this chapter.

Use the number cards to find the answer.

0.987

0.087

0.7

11. Mark the numbers from *least to greatest*.

- A. 0.7, 0.087, 0.987
B. 0.087, 0.7, 0.987
C. 0.987, 0.7, 0.087
D. 0.987, 0.087, 0.7

12. Mark the sum of the numbers.

- A. 1.081
B. 1.774
C. 2.557
D. 25.57

13. $\square \times \square =$

- A. 609
B. 6.09
C. 0.609
D. 0.0609

14. $(\square + \square) \times \square$

- A. 0.07
B. 0.7518
C. 7.518
D. 8

15. $\square + n = \square$

- A. 0.9
B. 0.09
C. 0.009
D. 9

Mark the answer.

16. $x > 2$

- A. $x = 16$
B. $x = 2.3$
C. $x = \frac{15}{3}$
D. all of the above

17. $x + 10 - 3 = 29.8$

- A. $x = 19.8$
B. $x = 20.5$
C. $x = 22.8$
D. all of the above

18. $17(n) = 68$

- A. $n = 2$
B. $n = 3$
C. $n = 4$
D. $n = 5$

19. $\frac{n}{8} = 7$

- A. $n = 48$
B. $n = 56$
C. $n = 64$
D. $n = 77$

20. Write an equivalent expression for 8×9 using prime numbers and exponents.

- A. $2^3 \times 3^2$
B. $2^2 \times 3^2$
C. $2^2 \times 3^3$
D. none of the above

CUMULATIVE REVIEW

Test Prep

Mark the answer.

1. What two prime numbers are between 20 and 30?
A. 21 and 23
B. 23 and 29
C. 25 and 27
D. none of the above

2. Two addends have a sum of 30. The second addend is 2 times the amount of the first addend.
A. $14 + 16$
B. $10 + 20$
C. $5 + 25$
D. all of the above

3. $300 = \square$
A. 3×10^2
B. 2×30
C. 30×100
D. none of the above

4. $400 = \square$
A. $1,000 \div 2.5$
B. $8,000 \div 20$
C. $2 \frac{2}{3} \times 150$
D. all of the above

5. $(15 \times 200) \div 60 + 90 = \square$
A. 20
B. 100
C. 140
D. none of the above

6. Estimate the product of 726×398 .

- A. 21,000
B. 28,000
C. 200,000
D. 280,000

7. Use front-end estimation for $189,786 + 346,398$.

- A. 300,000
B. 520,000
C. 600,000
D. 720,000

8. Estimate the inventory of 169,387 nails to the nearest one thousand.

- A. 169,000
B. 170,000
C. 200,000
D. 201,000

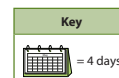
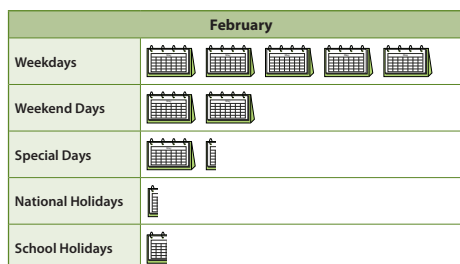
9. Which list shows all the factors of 72?

- A. 8, 9
B. 2, 3, 6, 8, 9
C. 1, 2, 3, 8, 9, 12
D. 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

10. $\frac{3}{4} + \frac{5}{6} = \square$

- A. The estimated sum is 2.
B. The estimated sum is 1.
C. The sum is less than 1.
D. The sum is greater than 2.

Use the data from the pictograph to find the answer.



21. Which two lines of the pictograph give the total number of days in February?
A. weekdays + special days
B. national holidays + weekdays
C. weekdays + weekend days
D. none of the above

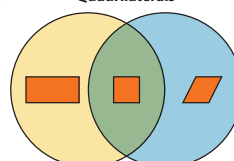
22. How many special days are in February?
A. 3
B. 4
C. 5
D. 6

23. How many more weekdays are there than weekend days?
A. 2 times as many weekdays
B. $2 \frac{1}{2}$ times as many weekdays
C. 15 more weekdays
D. 20 more weekdays

24. If school closes for national holidays and school holidays, how many vacation days will there be?
A. $1 + 2 = 3$ days
B. $4 + 2 = 6$ days
C. $4 + 1 = 5$ days
D. none of the above

Use the Venn diagram to find the true statement.

25. **Quadrilaterals**



- A. A square can be classified as a rectangle and a rhombus.
B. A rectangle is a rhombus.
C. A rhombus is not a quadrilateral.
D. A rectangle, a square, and a rhombus are not related at all.



PRIME NUMBER CALCULATIONS

Find the 45 prime numbers between 1 and 200 using the Sieve of Eratosthenes.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

Step 1: Make a chart with the numbers 1 through 200.

Step 2: Cross out 1.

Step 3: Circle the first six prime numbers. **2, 3, 5, 7, 11, 13**

Step 4: Cross out the multiples of the first six prime numbers.

Step 5: The remaining numbers are prime. List them.

17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199

Christian Goldbach, an eighteenth-century mathematician, devised the theory that every even number greater than 4 can be expressed as the sum of two prime numbers. This theory has never been proved or disproved. Some examples are shown below.

$$10 = 3 + 7$$

$$56 = 43 + 13$$

$$100 = 97 + 3$$

Look at the list of prime numbers you made. Find which two prime numbers greater than 2 can be added together to equal each of the following numbers.

1. 78 **73 + 5; 71 + 7; 67 + 11; 61 + 17; 59 + 19; 47 + 31; or 41 + 37**

2. 116 **113 + 3; 109 + 7; 103 + 13; 97 + 19; 79 + 37; or 73 + 43**

3. 164 **157 + 7; 151 + 13; 127 + 37; 103 + 61; or 97 + 67**

4. 128 **109 + 19; 97 + 31; or 67 + 61**

Every composite number can be illustrated as an array of dots. No prime number greater than 2 can be illustrated this way. Try this idea with several prime and composite numbers.

Number	Array	Explanation	Type of Number
39		This array is 3 equal rows of 13 dots.	composite
7		Equal rows cannot be made with 7 dots.	prime