

MULTIPLY BY A WHOLE NUMBER

SAVED BY A STRANGER

Johnstown, Pennsylvania

May 31, 1889

The South Fork Creek Dam near Pittsburgh, Pennsylvania, was built in the 1840s to furnish water for a canal between Johnstown and Pittsburgh. When train travel began in 1854, there was no further need for the canal, and the dam was neglected. A group of people from Pittsburgh's wealthy social class bought the lake, built summer resort homes around it, and formed a fishing and hunting club. In 1880, engineers from Cambria Iron Works became concerned about a small break and some rusted pipes in the dam. They sent the lake's owners a report explaining the repairs needed. But the owners did not heed the engineers' warnings and did nothing to repair the dam.

On May 31, 1889, after heavy rain, the South Fork Creek Dam burst, and a sweeping torrent of water rushed down the Allegheny mountainsides above Johnstown, Pennsylvania. The rushing water caught up everything in its current, including people, trees,

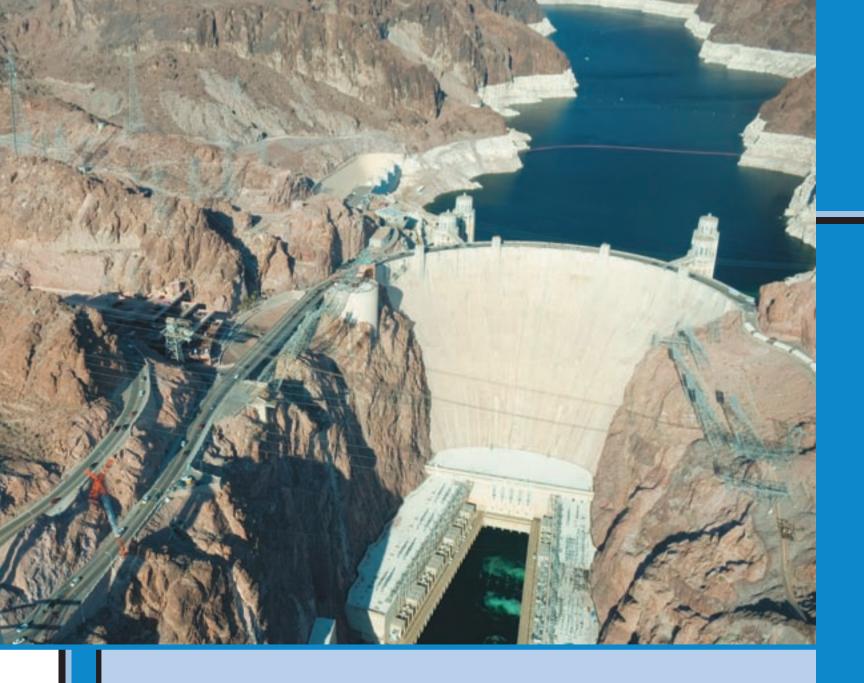


Wreckage was piled up to the third story in one area of town.

animals, and buildings. By the time the current reached the little city in the valley, it was a raging river of debris from the mountain villages through which it had swept. People in Johnstown had to hold onto pieces of houses or whatever they could find to keep from being drowned in the flood.

One of these people was a six-yearold girl named Gertrude Quinn. A workman named Maxwell McAchren saw Gertrude on a mattress being swept along by the flood. Risking his own life, he dove from a large roof into the rushing water and struggled to swim to Gertrude. Finally, he managed to pull himself onto the mattress. Grateful to be rescued, Gertrude clung tightly to Maxwell. As they were swept near the shore, they noticed men on firm ground rescuing people from the floodwaters, but they were still too far from shore to be pulled to safety. Maxwell picked up the little girl and threw her to waiting men, just before the current swept the mattress and him away. He rode on the mattress for over four miles before he was rescued himself. Gertrude never saw Maxwell McAchren again, but she never forgot his kindness to her. Years later, she read of his death in a newspaper and sent red roses to the site of his grave.

Maxwell McAchren gives us a picture of the kind of love Jesus expects when He says, "Thou shalt love thy neighbour as thyself" (Mark 12:31). Maxwell loved his life, but he also loved the life of a little girl he had never met and was determined to do all he could to save her life as well as his own.



When the South Fork Creek Dam broke, 20 million tons of water were released.

Today, the Hoover Dam in Nevada has a reservoir (Lake Mead) with a capacity of 9.2 trillion gallons.

The Hoover Dam is 726.4 feet tall and was built with enough concrete $(4\frac{1}{2}$ million tons) to pave a sidewalk that is 4 feet wide around the earth's equator.

The base of the Hoover Dam is 660 feet thick, the length of two football fields.

Each penstock used to control the flow of water through the Hoover Dam can carry enough water to fill 960,000 twelve-ounce soda cans per second.

Today almost 1,000,000 people tour the Hoover Dam each year.

Multiply by a Whole Number						
Lesson	Topic	Lesson Objectives	Chapter Materials			
12	Multiplication	 Demonstrate an understanding of multiplication and the terms <i>factor</i>, <i>product</i>, and <i>multiple</i> Write a mathematical equation for a word phrase Solve multiplication equations with a multiplication dot, parentheses, or variables Identify prime and composite numbers Identify the Greatest Common Factor (GCF) and the Least Common Multiple (LCM) of a pair of numbers Apply properties of multiplication to numbers and variables: Commutative Property, Associative Property, Identity Property, and Zero Property 	Teacher Manipulatives Packet: • Place Value Kit • Money Kit Student Manipulatives Packet: • Place Value Kit • Money Kit Instructional Aids (Teacher's Toolkit CD): • Cumulative Review Answer Sheet (page IA9) for each student • Sieve of Eratosthenes (page IA10) • Sieve of Eratosthenes (page IA10) for each student • Apply Properties (page IA11) • Powers of 10 (page IA12) • Graph Paper (page IA13) • Graph Paper (page IA13) for each student • Perfect Squares & Square Roots (page IA14) • Perfect Squares & Square Roots (page IA14) • Perfect Squares & Square Roots (page IA14) for each student • Pictures of Multiplication (page IA15) Christian Worldview Shaping (Teacher's Toolkit CD): • Pages 4–6 Other Teaching Aids: • Colored pencils: red and green for each student • A sheet of graph paper for each student • A calculator for each student (optional) Math 6 Tests and Answer Key Optional (Teacher's Toolkit CD): • Fact Review pages • Application pages • Calculator Activities			
13	Multiples of 10	Multiply multiples of 10 Apply the Commutative and Associative Properties of Multiplication to multiply factors that are multiples of 10 Analyze patterns for using mental math to multiply factors that are multiples of 10 Apply the Distributive Property of Multiplication over Addition to multiply by multiples of 10 Identify the applied addition or multiplication property				
14	Exponents	 Develop an understanding of exponents Develop an understanding of squares Write numbers in expanded form with multiplication using exponents (powers of 10) 				
15	1- & 2-Digit Multipliers	 Multiply a whole number by a 1- or 2-digit multiplier Apply the Distributive Property of Multiplication over Addition Estimate the product by rounding to the place of greatest value and by using front-end estimation Solve a multiplication word problem 				
16	Multiply Decimals by a Whole Number	 Multiply a decimal by a 1- or 2-digit multiplier Estimate the product by rounding to the place of greatest value Apply the Distributive Property of Multiplication over Addition Solve decimal word problems, including money problems Solve a multi-step word problem Multiply a decimal by a power of 10 				
17	3-Digit Multipliers	Multiply by a 3-digit multiplier Estimate the product by rounding to the place of greatest value Solve a money multiplication problem Determine the number of partial products Apply strategies to multiply mentally				
18	Squares & Square Roots	 Develop an understanding of finding perfect squares Develop an understanding of finding the square root of a perfect square List the first 20 perfect squares and their square roots Use the Pythagorean Theorem to find the measurement of the hypotenuse of a right triangle 	As you prepare the lessons, you will want to refer to the corresponding Instructional Aids pages located on the Teacher's Toolkit CD. If a page is not specified for the student's or teacher's use in the Chapter Materials list above, you should			
19	Chapter 2 Review	• Review	prepare the page for display.			
20	Chapter 2 Test Cumulative Review	 Identify the addition property applied to an equation Add and subtract whole numbers Solve for the variable in a subtraction equation Identify the standard form of a whole number or a decimal written in expanded form or word form Determine the decimal represented by a point on a number line Solve for the variable in a part-whole model Read and interpret a pictograph 	The Charts and some of the visuals from the Math 4–6 Teacher Manipulatives Packet are located in the Teaching Visuals section of the Teacher's Toolkit CD. Copies of the visuals may be prepared by home educators or by classroom teachers for individual or classroom (group) use.			

A Little Extra Help

Use the following to provide "a little extra help" for the student that is experiencing difficulty with the concepts taught in Chapter 2.

Align columns—To facilitate alignment when solving multiplication problems, allow the student to use graph paper or lined notebook paper turned sideways, or provide copies of the Graph Paper page (page IA13 of the Instructional Aids section of the Teacher's Toolkit CD).

Solve multiplication problems with a 2- or 3-digit multiplier—Provide the student with 1 one square from the Place Value Kit in the Student Manipulatives Packet. Write a problem with a 2-digit multiplier on a sheet of paper. Direct the student to cover the digit in the Tens place of the multiplier with the one square to help him focus only on the first step for solving the problem. After he multiplies by the ones, instruct him to cover the digit in the Ones place of the multiplier and to multiply by the tens.

6 2
394
$\times \square 7$
2,758

Follow the same procedure for problems with a 3-digit multiplier; the student will need 2 one squares. Instruct the student to cover the digits that he is not multiplying by as he solves the problem.

Multiply a decimal by a whole number—Instruct the student to rewrite the multiplication problem without the decimal point in the decimal factor. Direct him to multiply as if both factors are whole numbers and to solve the problem. Next, direct the student to count the number of decimal places in the decimal factor of the original problem and to write the decimal point in the product; remind him that the number of decimal places in the product is the same as the number of decimal places in the decimal factor.

Math Facts

Throughout this chapter, review addition and subtraction facts using Fact Review pages or a Fact Fun activity on the Teacher's Toolkit CD, or you may use flashcards.

Overview 29

Student Text pp. 28-31 Daily Review p. 406a

Objectives

- Demonstrate an understanding of multiplication and the terms *factor, product,* and *multiple*
- Write a mathematical equation for a given picture or word phrase
- Solve multiplication equations with a multiplication dot, parentheses, or variables
- Identify prime and composite numbers
- Identify the Greatest Common Factor (GCF) and the Least Common Multiple (LCM) of a pair of numbers
- Apply properties of multiplication to numbers and variables: Commutative Property, Associative Property, Identity Property, and Zero Property

Teacher Materials

- Place Value Kit: tens and ones
- Sieve of Eratosthenes, page IA10 (CD)
- Apply Properties, page IA11 (CD)

Student Materials

- Sieve of Eratosthenes, page IA10 (CD)
- Colored pencils: red and green

Note

Preview the Fact Review pages, the Application pages, and the Calculator Activities located on the Teacher's Toolkit CD.

Introduce the Lesson

Guide the students in reading aloud the story and facts on pages 28–29 of the Student Text (pages 26–27 of this Teacher's Edition).

Teach for Understanding

Demonstrate an understanding of multiplication

- 1. Display 3 sets of 24 (2 tens and 4 ones).
- ➤ What addition equation can you write for this picture? 24 + 24 + 24 = 72 multiplication equation? 3 × 24 = 72 Write the equations for display.

Display another 3 sets, using tens and ones: a set of 12, a set of 10, and a set of 9.

- ➤ What addition equation can you write for this picture?

 12 + 10 + 9 = 31 multiplication equation? none Why? Elicit that multiplication is used to express a repeated addend.

 Point out that multiplication is referred to as repeated addition.
- 2. Write addend + addend + addend = sum and factor × factor = product below the appropriate equations. Explain that, typically, when multiplication is pictured, the first factor is the multiplier; it tells the number of sets. The second factor is the multiplicand; it tells the number in each set. The product is the total; it is found by multiplying the two factors.
- ➤ What are the factors in 3 × 24 = 72? 3 and 24 Which factor is the multiplier? 3 What does it tell you? There are 3 sets. Which factor is the multiplicand? 24 What does it tell you? There are 24 in each set. What is the product? 72 What does it represent? the total or the answer to the multiplication equation
- 3. Direct the students to write a multiplication equation with the first factor as the multiplier (number of sets) and the second factor as the multiplicand (number in each set) for these statements.

3 plates of 7 cookies $3 \times 7 = 21$

5 rows of 6 chairs $5 \times 6 = 30$

2 dimes in each of 4 pockets $4 \times 2 = 8$

6 chairs at each of 4 tables and 2 more chairs $(4 \times 6) + 2 = 26$ Point out that although the Commutative Property states that the order of the factors does not affect the product, the order is important when representing a picture: the first factor expresses the number of sets, and the second factor expresses the number in each set.

4. Write $8 \times 9 = 72$; $3 \cdot 7 = 21$; and (5)(2) = 10 for display. Explain that a times sign, a dot, or parentheses can be used to indicate the multiplication of 2 factors. Read each equation aloud, using the word *times*.

Write these equations for display. Select students to read the equations aloud and to write the products.

$$8 \cdot 6 = 48$$
 (9)(7) = 63 $12 \cdot 6 = 72$ (5)(11) = 55

5. Write the following equations for display. Explain that when one or both of the factors is a variable, the factors can be written side by side with the whole number preceding the variable. Guide the students in solving these equations if a = 5 and b = 10.

7b = 70 ab = 50 4a = 20 2b - a = 15

Identify prime and composite numbers; identify the GCF and the LCM for a pair of numbers

1. Explain that numbers greater than 1 are classified as *prime* or *composite*. Elicit that numbers with only 2 factors, 1 and the number itself, are prime numbers, and numbers with more than 2 factors are composite numbers. Guide the students in listing the factors of 2, 3, 4, 6, and 12 and in classifying each number as prime or composite.

2: 1, 2; prime 3: 1, 3; prime 4: 1, 2, 4; composite 6: 1, 2, 3, 6; composite 12: 1, 2, 3, 4, 6, 12; composite

Guide the students in identifying common factors and the greatest common factor (GCF) of any pair of the numbers (i.e., 6 and 12: 1, 2, 3, 6; GCF: 6).

2. Explain that *multiples* of a number are found when a number is multiplied by whole numbers (0, 1, 2, 3, 4 . . .). You can find the multiples of a number by *counting by* that number, beginning with zero.

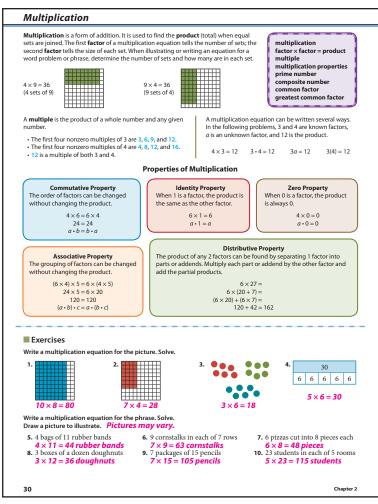
Guide the students in listing the first 12 nonzero multiples of 2, 4, and 6 and in circling in red the common multiples of 2 and 4. 4, 8, 12, 16, 20, 24

2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72

➤ Why do you think every multiple of 4 is common to every other multiple of 2? Elicit that every set of 4 is made up of 2 sets of 2.

Guide the students in circling in green the common multiples of 2 and 6. 6, 12, 18, 24

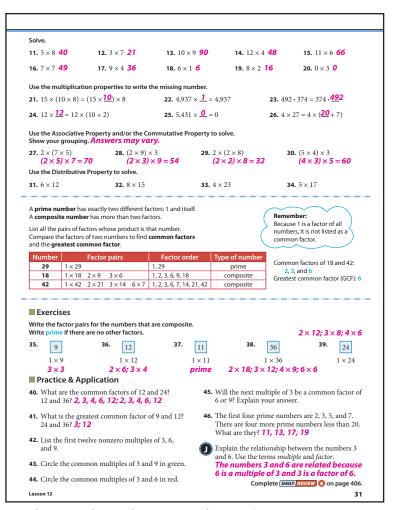
- ➤ Why do you think every multiple of 6 is common to every third multiple of 2? Elicit that every set of 6 is made up of 3 sets of 2.
- ➤ What do you predict about the common multiples of 8 and 2? Why? Elicit that every multiple of 8 is common to every fourth multiple of 2 because every set of 8 is made up of 4 sets of 2.



- 3. Guide the students in identifying the least common multiple (LCM) of 2 and 4 4, 2 and 6 6, 4 and 6 12, and 2, 4, and 6 12. Model the strategy of counting by the largest of the numbers until you come to a multiple that is common to the other number(s). Point out that although 0 is a common multiple of all numbers, it is not used as the LCM.
- 4. Follow a similar procedure to list the multiples of 5 and identify the LCM of 4 and 5. 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60; LCM: 20
- 5. Point out that, although a common multiple of 2 numbers can be found by multiplying the 2 numbers, the product is not always the LCM.
- 6. Display and distribute the Sieve of Eratosthenes page. Explain that Eratosthenes was a mathematician who devised a system for finding prime numbers. Use the directions on Student Text page 259 to guide the students in identifying the prime numbers between 1 and 200.

Apply properties of multiplication

- 1. Lead a discussion to review the Commutative, Associative, and Identity Properties of Addition. (See Lesson 5.) Ask the students which of these properties could also apply to multiplication. Choose students to write a multiplication equation to prove their thinking.
- 2. Display the Apply Properties page. Lead a discussion of the first four properties represented by variables. Emphasize the purpose of these properties as follows: Commutative—order, Associative—grouping, and Identity—restates its name. Guide the students to the conclusion that these properties



do not apply to subtraction or division (e.g., $3-2 \neq 2-3$ and $3 \div 1 \neq 1 \div 3$). Elicit that the Zero Property applies to multiplication.

3. Direct attention to the Distributive Property. Explain that the Distributive Property of Multiplication over Addition states that the product of 2 factors can be found by separating one factor into parts (addends), multiplying each part by the other factor, and adding the partial products.

Write 16×23 for display and elicit ways to separate one of the factors into parts (addends). Demonstrate solving the equations as shown below. Point out that the products are the same (368). *Possible answers:*

$$= (8 + 8) \times 23$$

 $= (8 \times 23) + (8 \times 23)$
 $= 184 + 184$
 $= 368$
 $= 16 \times (20 + 3)$
 $= (16 \times 20) + (16 \times 3)$
 $= 320 + 48$
 $= 368$

- 4. Choose students to tell the value of *n* in problems 1–8 and to identify the property they used to determine the value.
 - 1. n = 45; Commutative Property of Multiplication
 - 2. n = 0; Identity Property of Addition
 - 3. n = 0; Zero Property of Multiplication
 - 4. n = 7; Associative Property of Multiplication
 - 5. n = 21; Associative Property of Addition
 - 6. n = 50; Commutative Property of Multiplication
 - 7. n = 10; Distributive Property
 - 8. n = 11; Distributive Property

Student Text pp. 30–31

Lesson 12 31

Student Text pp. 32-33 Daily Review p. 407b

Objectives

- Multiply multiples of 10
- Apply the Commutative and Associative Properties of Multiplication to multiply factors that are multiples of 10
- Analyze patterns for using mental math to multiply factors that are multiples of 10
- Apply the Distributive Property of Multiplication over Addition to multiply by multiples of 10
- Identify the addition or multiplication property applied in an equation

Teacher Materials

• Place Value Kit

Student Materials

• Place Value Kit

Note

In this lesson, 10; 100; 1,000; 10,000; and 100,000 are referred to as *multiples of 10*. In Lesson 14 the students will learn that these numbers are also *powers of 10*.

Teach for Understanding

Apply multiplication properties and analyze patterns to multiply by a multiple of 10

- 1. Write $2 \times 4 =$ for display. Direct the students to use the ones from their Place Value Kit to picture the equation and solve for the product. Remind the students that the first factor is the *multiplier* (it tells the number of sets), and the
 - second factor is the *multiplicand* (it tells the number in each set). **2 sets of 4 ones; 8** Write the product, 8.

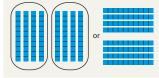


2. Repeat the procedure for 2 × 40

= __ using the tens. 2 sets of 4

tens; 80 Write the product, 80.

Guide the students in writing
below 2 × 40 an expression with
3 factors for this picture, 1 of the



- factors being a multiple of 10. $2 \times (4 \times 10)$.
- 3. Point out that $2 \times (4 \times 10)$ represents the same picture and has the same value as 2×40 . Explain that the parentheses tell you to first make 4 sets of 10. Choose a student to display 4 sets of 10 or 4 tens.
 - Explain that the 2 outside the parentheses tells you that there are 2 sets of the 4 sets of 10. Choose another student to display a second set of 4 tens.
- ➤ Are the pictures the same? yes Has the value changed? no
- ► How could you use the Associative Property to regroup the factors in $2 \times (4 \times 10)$? $(2 \times 4) \times 10$ Write $(2 \times 4) \times 10 =$ __ below $2 \times (4 \times 10) =$ __.
 - Point out that first multiplying the digits that make a fact and then multiplying by a multiple of 10 can make multiplying mentally easier.
- ➤ What is 2×4 ? 8 What is 8×10 ? 80 Write the product.
- 4. Follow a similar procedure for these equations.

(*Note:* Since the student Place Value Kit contains only 1 thousand cube, select a student to use your thousand cubes to picture $2 \times 4,000 =$ for display.)

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2 \times 400 =  __ 2 sets of 4 hundreds; 800; 2 \times (4 \times 100) = (2 \times 4) \times 100

2 \times 4,000 =  __ 2 sets of 4 thousands; 8,000; 2 \times (4 \times 1,000) = (2 \times 4) \times 1,000
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- ▶ What do you notice about the product of these equations when one of the factors is a multiple of 10? Elicit that the product of the fact, 2×4 , is followed by the number of zeros in the multiple of 10.
- 5. Explain that you can mentally multiply by multiples of 10; 100; and 1,000 by first grouping and multiplying the nonzero digits to make a fact, and then annexing the appropriate number of zeros.
- ➤ Write these equations for display and direct the students to solve them mentally.

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4 \times 20 = 80 3 \times 200 = 600 2 \times 3,000 = 6,000 2 \times 50 = 100 4 \times 600 = 2400 6 \times 5,000 = 30,000
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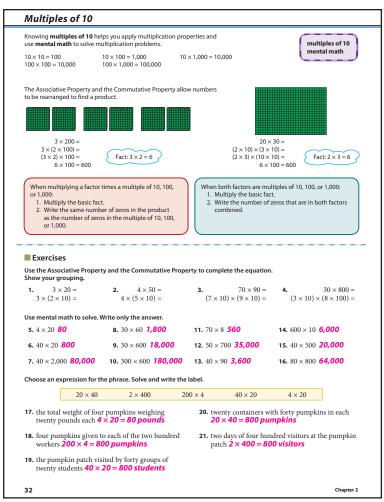
- 6. Use hundreds from the Place Value Kit to display a 20×30 array (2 rows of 3 hundreds) as shown on Student Text page 32.
- ➤ What equation can you write for this array? Why? Elicit 20 × 30 = __; there are 20 rows with 30 squares (ones) in each row. Write the equation for display.
- ➤ How is this equation different from the equations you have already solved? Answers may vary, but elicit that both factors are multiples of 10.
- **What is 20** \times **30? 600** Write the product.
- 7. Turn the array $\frac{1}{4}$ of a turn to the right. Guide the students in writing an equation for the new picture. $30 \times 20 = 600$
- ➤ Did the factors or the product change? no What did change? Elicit the order of the factors and the picture.
- ➤ What property tells you that the order of the factors does not affect the product? Commutative Property of Multiplication
- ➤ What multiplication expressions, using expanded form, can you write for the factors 20 and 30? Elicit 2 × 10 and 3 × 10. Write (2 × 10) × (3 × 10) = below 20 × 30 as shown on Student Text page 32.
- ➤ How can you use the Commutative and Associative Properties (to reorder and regroup the factors) to show the fact and the multiple of 10 that will help you solve this problem mentally? Elicit $(2 \times 3) \times (10 \times 10)$. Write the expression and guide the students in solving it mentally. $6 \times 100 = 600$
- 8. Follow a similar procedure for 30×40 . 1,200
- ➤ What pattern do you notice in the product when both factors are multiples of 10? Elicit that the product of the fact is followed by the total number of zeros in both factors.
- 9. Write for display $10 \times 10 = 100$; $10 \times 100 = 1{,}000$; and $100 \times 1{,}000 = 100{,}000$. Point out the same pattern of zeros in these products.
- 10. Write these equations. Guide the students in solving them mentally and telling how they applied the Commutative and Associative Properties. Possible strategies are provided.

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30 \times 50 = 1,500; (3 \times 5) \times (10 \times 10) = 15 \times 100

40 \times 700 = 28,000; (4 \times 7) \times (10 \times 100) = 28 \times 1,000

600 \times 300 = 180,000; (6 \times 3) \times (100 \times 100) = 18 \times 10,000

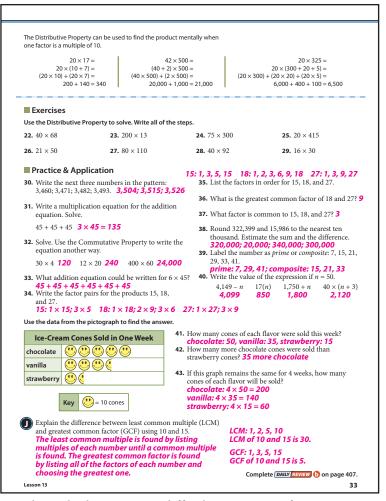
20 \times 9,000 = 180,000; (2 \times 9) \times (10 \times 1,000) = 18 \times 10,000
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Apply the Distributive Property of Multiplication over Addition to multiply by multiples of 10

- 1. Write $2 \times 27 =$ for display.
- ➤ How can you use mental math to find this product? Elicit that you can multiply each part of the expanded form of the 2-digit factor (27) by 2 and add the partial products.
- 2. Write (2)(20 + 7) =__ below $2 \times 27 =$ __. Remind the students of the following: (1) parentheses can be used to indicate multiplication; and (2) the Distributive Property of Multiplication over Addition allows you to separate 1 factor into parts (addends), multiply each part (addend) by the other factor, and add the partial products.
- ► Guide the students in completing the solution. $(2 \times 20) + (2 \times 7) = 40 + 14 = 54$
- 3. Guide the students to the conclusion that the Associative Property of Multiplication is used when only multiplication is used to solve an equation, and the Distributive Property of Multiplication over Addition is used when both multiplication and addition are used to solve a multiplication equation.
- 4. Choose students to demonstrate applying the Distributive Property to 2×27 , using other addends. *Possible answers:* $(2)(17 + 10) = (2 \times 17) + (2 \times 10) = 34 + 20 = 54$; $(2)(15 + 12) = (2 \times 15) + (2 \times 12) = 30 + 24 = 54$
- ➤ Which of these problems do you think is easiest to solve mentally? Why? Answers will vary, but elicit 2 × (20 + 7) because you are multiplying numbers within the basic fact range and multiplying by a multiple of 10.

Point out that the other addends representing 27 are not within the students' memorized basic fact range, making



the multiplication more difficult. Writing one factor in its expanded form limits the multiplying to basic facts and multiples of 10.

5. Direct the students to use the Distributive Property to solve these equations, writing the second factor in expanded form.

$$7 \times 43 = 7 \times (40 + 3) = (7 \times 40) + (7 \times 3) = 280 + 21 = 301$$

 $20 \times 56 = 20 \times (50 + 6) = (20 \times 50) + (20 \times 6) = 1,000 + 120$
 $= 1,120$
 $25 \times 43 = 25 \times (40 + 3) = (25 \times 40) + (25 \times 3) = 1,000 + 75$
 $= 1,075$

Identify the applied property

Write these equations for display. Choose students to identify the property used and to explain their answers.

$$73+92=92+73$$
 Commutative Property of Addition $6+(8+20)=(6+8)+20$ Associative Property of Multiplication $12\times 30=30\times 12$ Commutative Property of Multiplication $5\times (2\times 10)=(5\times 2)\times 10$ Associative Property of Multiplication $3\times 24=3\times (20+4)=(3\times 20)+(3\times 4)$ Distributive Property of Multiplication over Addition

Student Text pp. 32–33

Lesson 13 33

Student Text pp. 34-35 Daily Review p. 407c

Objectives

- Develop an understanding of exponents
- Develop an understanding of squares
- Write numbers in expanded form with multiplication using exponents (powers of 10)

Teacher Materials

- Place Value Kit
- Powers of 10, page IA12 (CD)

Student Materials

• A sheet of graph paper

Teach for Understanding

Develop an understanding of exponents

- 1. Write 7 + 7 + 7 + 7 + 7 + 7 + 7 for display.
- ➤ How many times is 7 repeated as an addend in this addition expression? 7 times What multiplication fact can you write for this expression? Why? 7×7 ; there are 7 sets of 7. Write 7×7 below the addition expression.
- 2. Explain that just as multiplication is a shortened way to show repeated addition, *exponent form* is a shortened way to show *repeated multiplication*.
- ➤ How many times is 7 repeated as a factor in this multiplication expression? 2 times
 - Write 7^2 below 7×7 . Explain that the factor being repeated (7) is the *base*. Write *base* beside the 7.
 - Point to the superscript 2. Explain that the smaller number written to the upper right of the base is called an *exponent*. The exponent indicates the number of times the base is repeated as a factor. Write *exponent* beside the superscript 2. Explain that exponents are read using ordinal number words (i.e., second, third, fourth, and so on). Point to each number as you explain that 7^2 is read *seven to the second power*.
- 3. Write $3 \times 3 \times 3 \times 3 =$ for display. Guide the students in multiplying the factors left to right: $3 \times 3 = 9$, $9 \times 3 = 27$, $27 \times 3 = 81$. Write the product.
- ► How can you write this equation using an exponent? Elicit $3^4 = 81$. Write $3^4 = 81$ below the equation. Point out that another way to find the product is to apply the Associative Property to group the factors. Demonstrate inserting the parentheses: $(3 \times 3) \times (3 \times 3)$.
- ➤ Now what multiplication fact can you write for the equation? $9 \times 9 = 81$ exponent form? $9^2 = 81$
- 4. Follow a similar procedure for these equations. Point out that each repeated multiplication expression is the *factored form* of the number written with an exponent.

$$2 \times 2 \times 2 = 8$$
; $2^3 = 8$ 5×8
 $8 \times 8 = 64$; $8^2 = 64$ 2×8

$$5 \times 5 \times 5 \times 5 = 625; 5^4 = 625$$

 $2 \times 2 \times 2 \times 2 \times 2 = 32; 2^5 = 32$

 $4 \times 4 \times 4 = 64; 4^3 = 64$

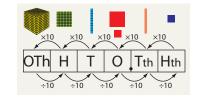
Develop an understanding of squares

1. Distribute the graph paper. Direct the students to draw an array to represent 7×7 (7 rows of 7 units). Then instruct each student to write several other facts with a repeating factor and to draw an array for each fact. Select students to show their arrays and to explain which fact each array represents.

- ➤ What do you notice about arrays when the 2 factors are the same number? A square is formed.
- 2. Direct the students to write each fact in exponent form. Point out that when a square is formed, the base may vary, but the exponent is always 2. Explain that because a square array is formed, 7 to the second power (7²) can also be read *seven* squared.
- 3. Choose students to read aloud the 2 ways that the exponent forms representing their arrays can be read.
- 4. Guide the students in writing the values for 1^2 through 12^2 : $1^2 = 1$; $2^2 = 4$; $3^2 = 9$; $4^2 = 16$; $5^2 = 25$; $6^2 = 36$; $7^2 = 49$; $8^2 = 64$; $9^2 = 81$; $10^2 = 100$; $11^2 = 121$; $12^2 = 144$.

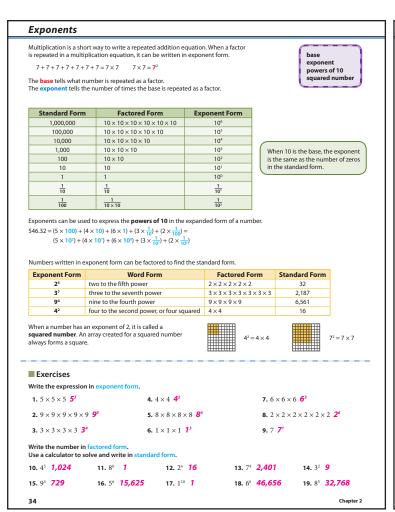
Write the expanded form of a number using exponents

- 1. Draw for display a place value chart for One Thousands through Hundredths. Display a corresponding Place Value Kit piece above each place; allow enough space to draw the arrows and write × 10 and ÷ 10 during the activity. (*Note:* Above the Ones place, display a small red one below a large red one. You may choose to have a student demonstrate the renaming in each step.)
- ➤ How many hundredths are needed to make 1 tenth? 10
 - Draw an arrow above the chart from the Hundredths place to the Tenths place; label it \times 10.

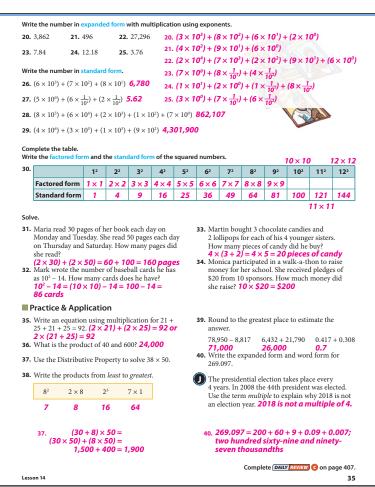


- 2. Follow a similar procedure for each place as you move left on the chart.
- ➤ What do you notice about the value of the place that is just to the left of any place in our base ten place value system?

 The value is 10 times greater than the value of the place to its right.
- 3. Point out that the value of the place that is 2 places to the left of any place is " \times 10 \times 10" greater. Remind the students that 10×10 can be written 10^2 and read *ten to the second power* or *ten squared*.
- > What is the value of 10^2 ? How do you know? 100; 10×10
- ➤ What do you notice about the value of the place that is 2 places to the left of any place in our base ten place value system? The value is 100 times greater than the value of the place that is 2 places to its right.
- ➤ What do you think is the value of the place that is 3 places to the left of any place? The value is 1,000 times greater than the place that is 3 places to its right. Elicit $10 \times 10 \times 10 = 10^3$; it can be read ten to the third power and has a value of 1,000.
- 4. Point to the One Thousands place on the chart.
- ➤ How many hundreds can 1 one thousand be renamed as? 10
- ➤ What part of 1 thousand is 1 hundred? How do you know? Elicit ½ because 1 thousand divided into 10 equal parts is 1 hundred.
 - Draw an arrow below the chart from the One Thousands place to the Hundreds place; label it \div 10.
- 5. Follow a similar procedure for each place as you move right on the chart.
- ➤ What do you notice about the value of the place that is just to the right of any place in our base ten place value system? The value is ½ of the value of the place to its left.



- ➤ What part of 1 hundred is 1 one? How do you know? 1/100; 1 hundred divided into 100 equal parts is 1 one.
- ➤ What part of 1 one is 1 hundredth? How do you know? 1 one divided into 100 equal parts is 1 hundredth.
- ➤ What do you notice about the value of the place that is 2 places to the right of any place? The value is ¹/₁₀₀ of the value of the place that is 2 places to its left.
- 6. Point out that the value of the place that is 2 places to the right of any place is "÷ $10 \div 10$." Explain that when you divide a unit by 10 you have $\frac{1}{10}$ of the original unit, which can be written $\frac{1}{10^1}$ (one tenth to the first power). When you divide the new unit by 10, you have $\frac{1}{100}$ of the original unit ($\frac{1}{10}$ of $\frac{1}{10}$), which can be written $\frac{1}{10^2}$.
- ► What do you think is the value of the place that is 3 places to the right of any place? The value is $\frac{1}{1,000}$ of the value of the place that is 3 places to its left. Elicit $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{10^3}$; it has a value of $\frac{1}{1,000}$.
- 7. Display the Powers of 10 page and review the values written in Standard Form and in Factored Form for each place. Point out that numbers such as 10,000; 1,000; and 100 are *powers of* 10; they can be written using an exponent.
- 8. Direct attention to the values written in Exponent Form.
- ➤ What pattern do you notice between the values written in Exponent Form and in Standard Form? Elicit that when 10 is the base in the exponent form, the standard form is the digit 1 followed by the number of zeros indicated by the exponent.
- > What is the value of 10^5 ? 100,000 $\frac{1}{10^5}$? $\frac{1}{100,000}$ 10^6 ? 1,000,000 $\frac{1}{10^6}$? $\frac{1}{1,000,000}$



- ➤ How would you write one hundred million using 10 as the base and an exponent? 10^8 one hundred millionth? $\frac{1}{10^8}$
 - Point out that any number to the first power is equal to the value of the base (e.g., $10^1 = 10$) and that any number to the zero power is equal to 1 (e.g., $10^0 = 1$).
- 9. Guide the students in writing these and other numbers in expanded form with multiplication, first using powers of 10 and then using exponents.

```
\begin{array}{l} 324,678,009 \; \textbf{(3}\times 100,000,000) + \textbf{(2}\times 10,000,000) + \\ \textbf{(4}\times 1,000,000) + \textbf{(6}\times 100,000) + \textbf{(7}\times 10,000) + \textbf{(8}\times 1,000) \\ + \textbf{(9}\times 1); \textbf{(3}\times \frac{1}{10^{8}}) + \textbf{(2}\times \frac{1}{10^{7}}) + \textbf{(4}\times \frac{1}{10^{6}}) + \textbf{(6}\times \frac{1}{10^{5}}) + \textbf{(7}\times \frac{1}{10^{4}}) + \\ \textbf{(8}\times \frac{1}{10^{3}}) + \textbf{(9}\times \frac{1}{10^{9}}) \\ 5.12 \; \textbf{(5}\times 1) + \textbf{(1}\times \frac{1}{10}) + \textbf{(2}\times \frac{1}{100}); \textbf{(5}\times \frac{1}{10^{9}}) + \textbf{(1}\times \frac{1}{10^{7}}) + \\ \textbf{(2}\times \frac{1}{10^{2}}) \textbf{0} \end{array}
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Student Text pp. 34–35

(Note: Assessment available on Teacher's Toolkit CD.)

Lesson 14 35

Student Text pp. 36-37 Daily Review p. 408d

Objectives

- Multiply a whole number by a 1- or 2-digit multiplier
- Apply the Distributive Property of Multiplication over Addition
- Estimate the product by rounding to the place of greatest value and by using front-end estimation
- Solve a multiplication word problem

Teacher Materials

· Place Value Kit

Teach for Understanding

Multiply by a 1-digit multiplier

- 1. Write $4 \times 307 =$ in vertical form and instruct the students to write it on paper, aligning the digits in the Ones places of the factors. Remind the students that the multiplier (4) tells the number of sets and the multiplicand (307) tells the number in each set.
- ➤ Using the Distributive Property, what equation can you write to solve this problem? Elicit $(4 \times 300) + (4 \times 7) = 1200 + 28 = 1,228$. Write the equation and the solution for display; continue to display the solution.
- 2. Choose a student to use pieces from your Place Value Kit to picture 4×307 . **4 sets of 3 hundreds and 7 ones**
- 3. Demonstrate multiplying the ones (combine the 4 sets of 7 ones [28 ones] and rename them as 2 tens and 8 ones) as you guide the students in multiplying the 7 in the Ones place of the multiplicand by the multiplier 4 for a total of 28 ones; write 8 in the Ones place of the product and write 2 above the Tens place of the multiplicand. (*Note:* Instruct the students to write the answers in the problem as you write them for display.)
- 4. Explain that since there are no tens in the multiplicand to multiply, you add the 2 renamed tens to 0 tens ($[4 \times 0 \text{ tens}] + 2 \text{ tens}$). Write the 2 in the Tens place of the product.
- 5. Demonstrate combining the 4 sets of 3 hundreds (12 hundreds) and renaming them as 1 thousand and 2 hundreds as you guide the students in multiplying the 3 hundreds by 4 in the problem. Point out that since there are no thousands to multiply, you can write 12 hundreds in the product and insert a comma to separate the Thousands period from the Ones period.
- ➤ What do you notice about the 12 hundreds in the product? Elicit that the 12 hundreds rename as 1 one thousand and 2 hundreds; another name for 12 hundreds is 1 thousand, 2 hundred.
- ➤ What does 4 × 307 equal? 1,228
- ▶ How could you estimate the product of 4×307 ? Elicit that you could round 307 to 300 and multiply 4×300 for an estimate of 1,200.
- 6. Point out that estimating is similar to completing only the first step of the Distributive Property (4 \times 300), except that when you estimate, you round to the place of greatest value before multiplying.
 - Remind the students that an estimate is not an exact answer; it can be very close to or far from the actual answer. Point out that the estimate of 1,200 is very close to the exact product 1,228; you only need to add 28 (4 \times 7) to get the exact product.

- 7. Write $6 \times 4{,}492 =$ and $6 \times 4{,}508 =$ horizontally and elicit the estimated products. **24,000** and **30,000**
- 8. Direct the students to find the exact products. Remind them that when multiplying by a single-digit multiplier, the number of digits in the multiplicand does not change the multiplication process. Give guidance as needed. 26,952 and 27,048
- 9. Guide the students in comparing the exact products to the estimated products. Explain that when you multiply a multiplicand in the thousands by a 1-digit multiplier (ones \times thousands), the difference between the estimate and the exact answer can be thousands. Point out that 4,492 rounded down by 492 creates an underestimate of about 3,000 when multiplied by 6 (6 \times 500) and 4,508 rounded up by 492 creates an overestimate of about 3,000 when multiplied by 6 (6 \times 500).
- 10. Follow a similar procedure for these equations.
 - $7 \times 9,786 =$ ___ 68,502 (70,000); 9,786 rounded up by 214 creates an overestimate of about 1,400 (7×200) $8 \times 25,176 =$ ___ 201,408 (240,000); 25,176 rounded up by 4,824 creates an overestimate of about 40,000 ($8 \times 5,000$) $9 \times 104,000 =$ ___ 936,000 (900,000); 104,000 rounded down by 4,000 creates an underestimate of 36,000 ($9 \times 4,000$)
 - ▶ How do you think you can easily find the exact product of $9 \times 104,000$? Elicit that since the multiplicand rounds to 100,000, adding the product of $9 \times 4,000$ to the estimate (900,000) results in the exact answer (936,000).
- 11. Explain that you can also use front-end estimation.
- > Why would you use front-end estimation rather than rounding to the greatest place? to find a more accurate estimate
- ➤ How do you think you would use front-end estimation to estimate a product? Elicit that you multiply the greatest 2 places of the multiplicand by the 1-digit multiplier.
- 12. Direct attention to the three problems in step 10. Point out that you can complete the first 2 steps of the Distributive Property when using front-end estimation. Guide the students in finding the front-end estimate for each problem and comparing it to the rounded estimate.

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7 \times 9,786 = 67,900 (63,000 + 4,900)

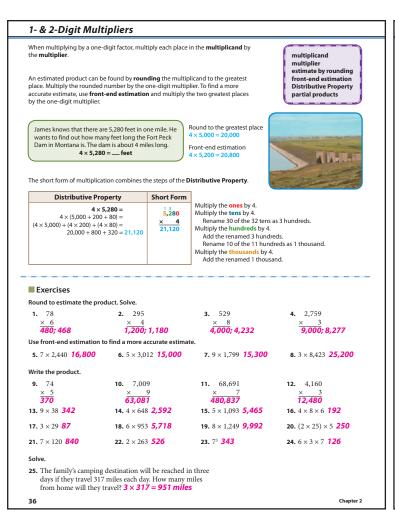
8 \times 25,176 = 200,000 (160,000 + 40,000)

9 \times 104,000 = 900,000; because the value of the second greatest place is zero, the 2 estimates are the same.
```

Multiply by a 2-digit multiplier

An egg carton holds 12 eggs. Each case holds 15 egg cartons. How many eggs are in each case? **180 eggs**

- ➤ What equation is needed to solve this word problem? How do you know? 15 × 12 = __; there are 15 sets of 12 eggs in each case. Write the equation for display.
- ➤ How do you think you can estimate the product? Elicit that you round each factor to the place of greatest value and multiply the rounded factors.
- ➤ What is the estimated product? How do you know? 200; 20 \times 10 = 200 Write the estimate.
- ➤ How can you use the Distributive Property to rewrite 15 × 12? You can separate 1 factor into addends and multiply each addend by the other factor.



Elicit the following equations and write them for display. $(10 + 5) \times 12 = (10 \times 12) + (5 \times 12)$ and $15 \times (10 + 2) = (15 \times 10) + (15 \times 2)$

- Write 15 × 12 and 12 × 15 vertically for display. Remind the students that the Commutative Property states that the order of factors will not affect the product.
 Explain that the bottom factor in a vertical problem is the
 - Explain that the bottom factor in a vertical problem is the multiplier. When you multiply by a multi-digit multiplier, multiply the multiplicand by each digit of the multiplier, beginning with the place of least value (the Ones place in this problem).
- 2. Choose a student to multiply 5×12 , showing the renaming. **60** Remind the students that aligning the digits and crossing out any renamed digits as they are added will help them to solve the problem cor-

rectly.

Write (5×12) beside 60 as shown below. Explain that 60 is a partial product; it is the product of 1 part of the problem and

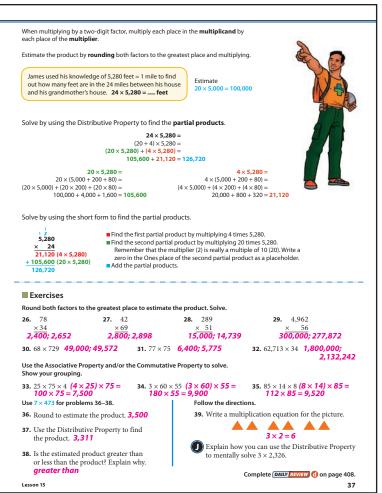
 $\begin{array}{ccc}
 & & & & & \\
12 & & & & 15 \\
 \times 15 & & & \times 12 \\
\hline
60 (5 \times 12) & & & 30 (2 \times 15) \\
\underline{120} (10 \times 12) & & & \underline{150} (10 \times 15) \\
180 & & & & 180
\end{array}$

represents only part of the final product.

- 3. Remind the students that there is a partial product for each digit in a multiplier.
- ➤ When you multiply by a multiple of 10, what digit do you write in the Ones place of the partial product? 0

 Select a student to write a zero in the Ones place of the second partial product and then multiply 1 (ten) × 12. 120

 Write (10 × 12) beside the partial product.



- 4. Choose a student to add the partial products to find the final product (total). 180 Point out that the overestimate of 200 is reasonable because one factor was rounded up slightly more than the other factor was rounded down.
- 5. Follow a similar procedure to multiply 12×15 . Remind the students that they can use the Distributive Property to separate either factor to get the final product: $(10 + 5) \times 12$ or $(10 + 2) \times 15$.
- 6. Guide the students in writing, estimating, and solving a multiplication problem for 40 rows of 39 seats.
- ➤ Since there is a 0 in the Ones place of the multiplier, do you need to multiply the multiplicand by the ones? Why? No; the partial product is 0 (Zero Property of Multiplication) and does not affect the final product (Identity Property of Addition).

Remind the students that they need to write a zero in the Ones place of the product before they multiply by the 4 tens in the multiplier.

39 × 40 1,560 seats

7. Guide the students in writing, estimating, and solving multiplication problems for 96 buses with 45 students on each bus and 61 rows of 78 seats.

$$\begin{array}{c}
45 \\
\times 96 \\
\hline
270 (6 \times 45) \\
+ 4050 (90 \times 45) \\
\hline
4320 students
\end{array}$$

$$\begin{array}{c}
78 \\
\times 61 \\
\hline
78 (1 \times 78) \\
+ 4680 (60 \times 78) \\
\hline
4,758 seats
\end{array}$$

Student Text pp. 36-37

Lesson 15 37

Student Text pp. 38-39 Daily Review p. 408e

Objectives

- Multiply a decimal by a 1- or 2-digit multiplier
- Estimate the product by rounding to the place of greatest value
- Apply the Distributive Property of Multiplication over Addition
- Solve decimal word problems, including money problems
- Solve a multi-step word problem
- Multiply a decimal by a power of 10

Teacher Materials

- · Money Kit
- 5 Student Money Kits
- Place Value Kit

Teach for Understanding

Multiply a decimal by a 1-digit multiplier

Sylvia wants to purchase a headband for 2 of her friends and herself. The headbands cost \$4.26 each. What is the total cost of the headbands? \$12.78

- ➤ What equation can you write to find the total cost of the headbands? $3 \times \$4.26 = c$ Write $3 \times \$4.26 = c$ horizontally and vertically. Point out that the variable c represents the cost.
- ➤ About how much will the headbands cost? Why? \$12.00; $3 \times $4.00 = 12.00
- ➤ Do you think \$12.00 is enough for Sylvia to purchase the headbands? Why? No; elicit that \$12.00 is an underestimate of about \$0.90 because \$4.26 was rounded down by about \$0.30, and $3 \times $0.30 = 0.90 .
- ➤ How could Sylvia adjust her estimate to be sure she has enough money to purchase the headbands? Elicit that she could round \$4.26 up because each headband costs more than \$4.00. (Note: You may choose to explore various ways to round \$4.26 up to the nearest dollar, dime, quarter, and/or half-dollar.)
- 1. Choose a student to show $3 \times \$4.26$ using dollars, dimes, and pennies from your Money Kit. 3 sets of 4 dollars, 2 dimes, and 6 pennies
- Select one student to demonstrate the multiplication by combining the sets of like coins, beginning with the pennies (hundredths), and renaming when necessary; choose a second student to simultaneously demonstrate each step in the problem. 12 dollars, 7 dimes, and 8 pennies; 1278
- ➤ What is needed to show that the product is an amount of money? a dollar sign and a decimal point Choose a student to write the dollar sign and the decimal point. \$12.78 Elicit that the decimal point separates the dollars (ones) from the cents (hundredths) and the dollar sign labels the product as money.
 - Point out that when you multiply by a whole number, there are multiple identical sets of the same units (pennies, dimes, and dollars). Sometimes the units are renamed as larger units, but they are never renamed as smaller units; therefore, the product will be greater than the number being multiplied (the multiplicand).
- 3. Follow a similar procedure with Place Value Kit pieces for the following word problems. Explain that when you multiply a decimal by a whole number, the product will have the same

decimal fraction unit as the multiplicand; tenths multiplied by a whole number results in a product with tenths, and hundredths multiplied by a whole number results in a product with hundredths.

A granola bar weighs 0.87 ounces. How much do 6 granola bars weigh? **5.22 oz**

 $6 \times 0.87 = w$; 6×0.90 oz = 5.40 oz; since 0.87 was rounded up by 0.03 and $6 \times 0.03 = 0.18$, there is an overestimate of 0.18; 6 sets of 8 tenths and 7 hundredths; $6 \times 0.87 = 5.22$ oz.

(*Note:* 0.87 was rounded to the nearest tenth because the Ones place has no value.)

Dad has 4 boards that are 68.75 cm long. When he lays them end to end, what is the combined length of the boards?

 $4 \times 68.75 = b$; 4×70 cm = 280 cm; since 68.75 was rounded up by 1.25 and $4 \times 1.25 = 5.00$, there is an overestimate of 5.00; 4 sets of 6 tens, 8 ones, 7 tenths, and 5 hundredths; $4 \times 68.75 = 275$ cm.

(*Note:* Since the tens and the tenths in the Place Value Kit do not show their proportional difference, you may want to use the Distributive Property to show the repeated sets of each place value: $4 \times 68.75 = 4 \times (60 + 8 + 0.7 + 0.05) = (4 \times 60) + (4 \times 8) + (4 \times 0.7) + (4 \times 0.05) = 240 + 32 + 2.8 + 0.20 = 275$. Then choose a student to solve the vertical problem.)

Multiply a decimal by a 2-digit multiplier

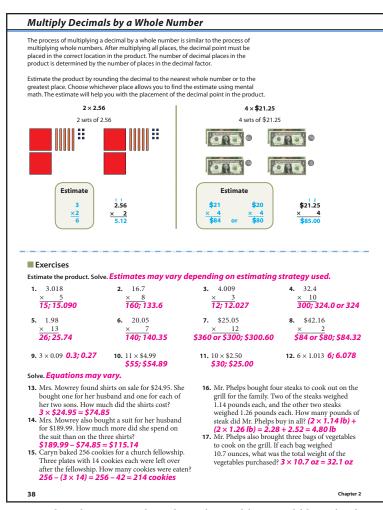
Miss Gable bought 23 cupcakes for \$1.29 each. What was the total cost of the cupcakes? **\$29.67**

1. Elicit the equation for the word problem and demonstrate solving it. Point out that the process for multiplying a decimal by a whole number is similar to multiplying whole numbers; multiply each place of the multiplicand by each place in the multiplier to find the partial products and add the partial products. Remind the students that when you multiply a decimal with hundredths (sets of hundredths), you will have more hundredths. Write the dollar sign and the decimal point.

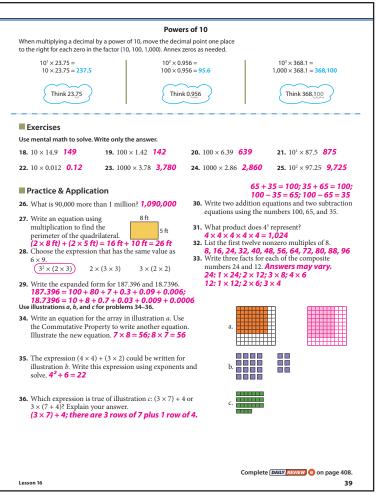
$$\begin{array}{r}
\$1.29 \\
23 \times \$1.29 = c \\
\underline{\times 23} \\
387 (3 \times 129) \\
\underline{+ 2580 (20 \times 129)} \\
\$29 67
\end{array}$$

Miss Gable found decorative pencils for \$0.17 each. If she buys 3 pencils for each of her 10 students, what is the total cost of the pencils? *\$5.10*

- ➤ What equation can you write to find the total cost of the pencils? Accept any combination of the factors: $10 \times 3 \times \$0.17$. Write $10 \times 3 \times \$0.17$ horizontally for display.
- ➤ What does the Commutative Property tell you? The order of the factors does not affect the product. the Associative Property? The grouping of the factors does not affect the product.
- 2. Point out that repeated sets of \$0.17 are found by multiplying. Any equation using the factors 10, 3, and \$0.17 can be used to solve the problem, but the best equation pictures the problem as it is described in the word problem.



- 3. Guide a discussion about how the problem could be solved. Elicit the following explanations and guide the students in writing the equations, using parentheses to identify what they would do first. Find the total number of pencils needed for all 10 students (10 sets of 3) and then multiply the number of pencils needed times the cost of each pencil (10 × 3) × \$0.17; find the cost of 3 pencils and multiply that amount by the number of students: 10 × (3 × \$0.17). Point out that the first factor in each equation tells the number of sets. (Note: You may choose to accept variations in the equations, noting the Commutative or Associative Properties.)
- 4. Guide the students in solving the first step of $(10 \times 3) \times \$0.17$. **30** Demonstrate writing $30 \times \$0.17$ vertically and guide the students in solving it; use the following procedure.
- ➤ Do you need to multiply by the ones? Why? No; the partial product would be 0 and a 0 partial product will not affect the final product.
- 5. Remind the students that they need to write a 0 in the product (below the 0 in 30) before they multiply by the 3 tens. *0510*
- ➤ What is needed in the product? Elicit a decimal point to separate the whole numbers (dollars) from the decimal fraction (cents, part of a dollar) and a dollar sign to label the answer as money. Write the dollar sign and the decimal point. Elicit that the location of the decimal point shows hundredths and that the 0 in the Tens place is not necessary; erase it. \$5.10
- ➤ How can you estimate this problem? Elicit that you can round both factors: 30 sets (pencils) × 20 cents (hundredths) = 600 cents (hundredths).



- ➤ What is the value of 600 cents? \$6.00 Is the answer \$5.10 reasonable? yes Point out that estimating is helpful in checking the placement of the decimal point in the product.
- 6. Follow a similar procedure for $10 \times (3 \times \$0.17)$. $3 \times \$0.17 = \0.51 ; $10 \times \$0.51 = \5.10

Multiply a decimal by a power of 10

- ➤ What do you notice about the decimal point in \$0.51 when you multiplied it by 10? Why? The decimal point moved one place to the right; elicit that each place was multiplied by 10, making the value of each digit 10 times greater and renaming it to the next greater place.
- 1. Choose a student to use the student Money Kits to show $10 \times \$0.51$ 10 sets of 5 dimes and 1 penny and another student to use your Place Value Kit to show 10×0.51 10 sets of 5 tenths and 1 hundredth. Instruct each student to combine his sets, renaming as needed. Point out the 10-times increase in each unit and its being renamed to the next greater unit; e.g., 10×1 penny (1 hundredth) = 10 pennies (10 hundredths) renamed as 1 dime (1 tenth). \$5.10; 5 and 1 tenth = 5.1
- 2. Guide the students in multiplying these decimals by powers of 10. Point out that you move the decimal point one place to the right for each zero in the power of 10, annexing zeros as needed, similar to multiplying a whole number by a power of 10.

$10 \times 0.1 = 1$	$10 \times 0.5 = 5$
$3.42 \times 10 = $ 34.2	\$526.19 × 10 = \$5,261.90
$100 \times 0.1 = 10.0$	$100 \times 0.006 = $ 0.6
$10^2 \times 87.4 = 8,740$	$10^3 \times 9.6 = 9,600$

Student Text pp. 38–39

Lesson 16

Student Text pp. 40-41 Daily Review p. 409f

Objectives

- Multiply by a 3-digit multiplier
- Estimate the product by rounding to the place of greatest value
- Solve a money multiplication problem
- Determine the number of partial products in a multiplication problem
- Apply strategies to multiply mentally

Teacher Materials

• Christian Worldview Shaping, pages 4–6 (CD)

Teach for Understanding

Multiply by a 3-digit multiplier

Timmons Hardware Shop sells a special type of nail for \$1.95 each. Mr. Jones needs 112 of these nails for a construction project. What is the total cost of the nails? \$218.40

- ➤ What equation is needed to solve this word problem? How do you know? $112 \times $1.95 = c$; 112 nails are needed at a cost of \$1.95 each.
- 1. Write the equation for display.
- ➤ How could you estimate the total product? Round both factors to the greatest place and multiply.
- ▶ What is the estimated cost? Why? \$200.00; $100 \times $2.00 = 200.00 Write the estimate for display. Point out that since you rounded down (12) more than you rounded up (0.05), the estimate is an underestimate.
- 2. Write $112 \times \$1.95$ vertically. Explain that the process for multiplying by a 3-digit multiplier is similar to multiplying by a 2-digit multiplier, except that you will have an additional partial product because you have the Hundreds place to multiply by.

Guide the students in solving the equation. Write the factors of each partial product. Point out that you annex 2 zeros in the partial product before multiplying each place in the multiplicand by the digit in the Hundreds place of the multiplier.

$$\$1.95$$
 $\times 112$
 $\hline 390$
 (2×195)
 1950
 (10×195)
 $+ 19500$
 (100×195)
 $\$218.40$
 $(100 + 10 + 2) \times \$1.95$

- 3. Guide the students in using the Distributive Property to write one equation showing the factors for each of the 3 partial products of $112 \times \$1.95$. $(100 \times \$1.95) + (10 \times \$1.95) + (2 \times \$1.95) = \218.40
- 4. Write $203 \times 3{,}567$ in vertical form.
- ➤ How many partial products will you need to write when solving this problem? How do you know? 2; (200 × 3,567) + (3 × 3,567); elicit that you do not need to write the partial product for the 0 in the Tens place of the multiplier because the partial product is 0 (Zero Property of Multiplication) and does not affect the final product (Identity Property of Addition).

5. Guide the students in solving the problem; follow a procedure similar to the one used for the previous word problem. Explain that when you omit a partial product, you need to annex the correct number of zeros in the next partial product.

```
200 \times 4,000 = 800,000; overestimate

3,567

\times 203 2 partial products

10701 (3 × 3,567)

+713400 (200 × 3,567)

724,101 (200 × 3,567) + (3 × 3,567) = 724,101
```

6. Repeat the procedure for 375×427 .

```
400 \times 400 = 160,000; underestimate

427

\times 375 3 partial products

2135 (5 × 427)

29890 (70 × 427)

+ 128100 (300 × 427)

160,125 (300 × 427) + (70 × 427) + (5 × 427) = 160,125
```

7. Christian Worldview Shaping (CD)

Apply strategies to multiply mentally

- 1. Write 7×24 for display. Remind the students that they can use their knowledge of the Distributive Property to multiply mentally.
- ► How could you use the Distributive Property to solve this problem without writing anything on paper? Elicit that you could think of the partial products and add them together: $7 \times 20 = 140$; $7 \times 4 = 28$; 140 + 28 = 168.
- 2. Explain that this strategy is sometimes referred to as *front-end multiplication*. Direct the students to use this strategy to find the products of these problems and to write only the answers on paper.

```
6 \times 53 \ 6 \times 50 = 300; 6 \times 3 = 18; 300 + 18 = 318

4 \times 34 \ 4 \times 30 = 120; 4 \times 4 = 16; 120 + 16 = 136

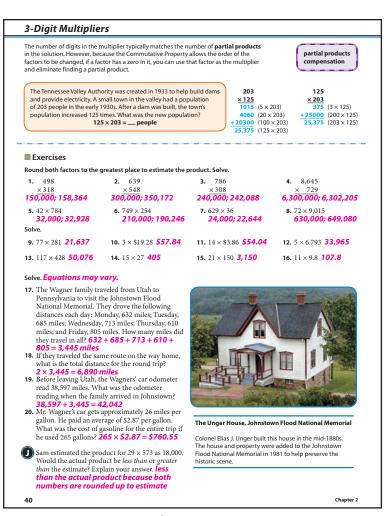
5 \times 48 \ 5 \times 40 = 200; 5 \times 8 = 40; 200 + 40 = 240

7 \times 17 \ 7 \times 10 = 70; 7 \times 7 = 49; 70 + 49 = 119

3 \times 86 \ 3 \times 80 = 240; 3 \times 6 = 18; 240 + 18 = 258

8 \times 64 \ 8 \times 60 = 480; 8 \times 4 = 32; 480 + 32 = 512
```

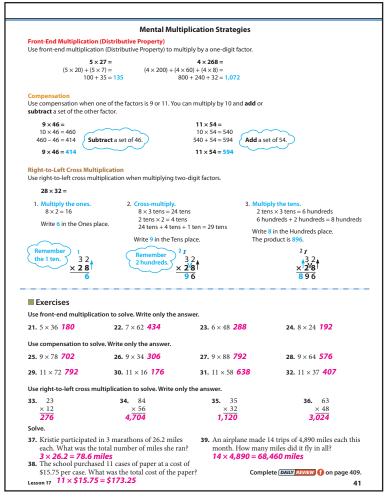
- 3. Write 9×13 for display.
- ▶ How could you use your knowledge of multiplying by a power of 10 to solve this problem without writing anything on paper? Elicit that you could think of the product of 10×13 (130) and subtract 13 from it because 10×13 is one more set of 13 than 9×13 .
- ➤ What is the product of 9×13 ? 117 Write $10 \times 13 = 130$ and 130 - 13 = 117, so $9 \times 13 = 117$ in a think cloud below the equation.
- 4. Follow a similar procedure for 9×27 . $10 \times 27 = 270$ and 270 27 = 243, so $9 \times 27 = 243$
- 5. Write 11×18 for display.
- ➤ How could you solve this problem, using your knowledge of multiplying by a power of 10, without writing anything on paper? Elicit that you could think of the product of 10×18 (180) and add 18 to it because 11×18 is one more set of 18 than 10×18 .
- \blacktriangleright What is the product of 11 \times 18? 198



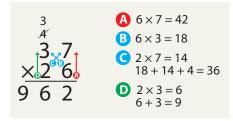
Write $10 \times 18 = 180$ and 180 + 18 = 198, so $11 \times 18 = 198$ in a think cloud below the equation.

- 6. Write 11×43 for display. Choose a student to tell the product and to explain how he determined the answer. $10 \times 43 = 430$ and 430 + 43 = 473, so $11 \times 43 = 473$
 - Explain that this method of mental multiplication is called *compensation*; it is best used when one of the factors is 9 or 11.
- 7. Write 14×12 in vertical form. Explain that there is another strategy for multiplying mentally called *right-to-left cross multiplication*. The title of the strategy gives the steps to follow for finding a product. You begin on the *right* and multiply the ones. Draw an arrow from the 4 to the 2 as shown below (the red A arrow).
- ➤ What is 4 × 2 ones? 8 ones Write 8 in the Ones place of the product.
 - Explain that the next step is to *cross-multiply*. Draw the crossed arrows as shown (the blue B and C arrows).
- ➤ What is 4×1 ten and $1 \text{ ten} \times 2$? 4 tens + 2 tens = 6 tensWrite 6 in the Tens place of the product.
- ➤ What do you think is the next step? Elicit that you need to multiply the tens on the left. Draw an arrow to show the final step (the green D arrow).
- ➤ What is 1 ten × 1 ten? 1 hundred Write 1 in the Hundreds place of the product.





8. Follow a similar procedure for 26×37 . Point out that this problem involves renaming.



9. Write similar problems for display and direct the students to solve them mentally, writing only the answers on paper.

Student Text pp. 40-41

(Note: Assessment available on Teacher's Toolkit CD.)

Lesson 17 41

Student Text pp. 42-43 Daily Review p. 409g

Objectives

- Develop an understanding of finding perfect squares
- Develop an understanding of finding the square root of a perfect square
- List the first 20 perfect squares and their square roots
- Use the Pythagorean Theorem to find the measurement of the hypotenuse of a right triangle

Teacher Materials

- Graph Paper, page IA13 (CD)
- Perfect Squares & Square Roots, page IA14 (CD)

Student Materials

- Graph Paper, page IA13 (CD)
- Perfect Squares & Square Roots, page IA14 (CD)
- A calculator (optional)

Teach for Understanding

Develop an understanding of finding perfect squares

- 1. Display and distribute the Graph Paper page. Direct the students to color a 5×5 array in the first section as shown below.
- ➤ What multiplication expression can you write for this array? 5 × 5

Write the expression below the grid.

- ➤ What is the value of 5×5 ? 25 Write = 25 to complete the equation.
- ► How can you write $5 \times 5 = 25$ using an exponent? $5^2 = 25$ Repeat the procedure with $3 \times 3 = 9$, $3^2 = 9$ and $8 \times 8 = 64$, $8^2 = 64$.
- 2. Underline 25, 9, and 64. Point out that 25, 9, and 64 have something in common. Elicit that each is a product of a number and itself. These products are called *perfect squares*.

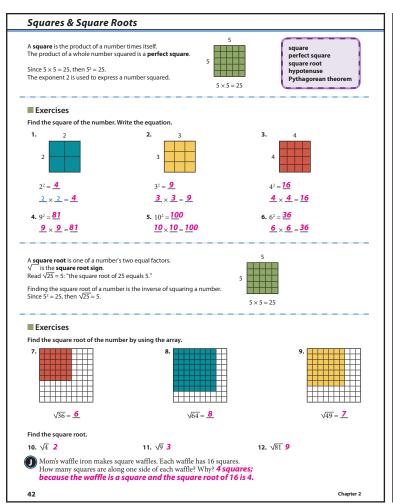
Perfect Squares				
$1 \times 1 = 1^2 =$	1			
$2 \times 2 = 2^2 =$	4			
$3 \times 3 = 3^2 =$	9			
$4 \times 4 = 4^2 =$	16			
$5 \times 5 = 5^2 =$	25			
$6 \times 6 = 6^2 =$	36			
$7 \times 7 = 7^2 =$	49			
$8 \times 8 = 8^2 =$	64			
$9 \times 9 = 9^2 =$	81			
$10 \times 10 = 10^2 =$	100			
$11 \times 11 = 11^2 =$	121			
$12 \times 12 = 12^2 =$	144			
$13 \times 13 = 13^2 =$	169			
$14 \times 14 = 14^2 =$	196			
$15 \times 15 = 15^2 =$	225			
$16 \times 16 = 16^2 =$	256			
$17 \times 17 = 17^2 =$	289			
$18 \times 18 = 18^2 =$	324			
$19 \times 19 = 19^2 =$	361			
$20 \times 20 = 20^2 =$	400			

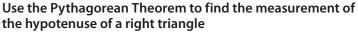
3. Display and distribute the Perfect Squares and Square Roots page. Guide the students in finding the first 20 perfect squares and in completing the Perfect Squares table. You may choose to let them use a calculator for facts that are not memorized.

Develop an understanding of finding the square root of a perfect square

- 1. Remind the students about inverse operations with these examples: 3 + 2 = 5, 5 2 = 3; $3 \times 6 = 18$, $18 \div 6 = 3$.
- 2. Write $\sqrt{25}$ for display. Explain that this symbol written with a number that is a perfect square indicates that you perform the inverse of finding the perfect square. You find the number that was multiplied by itself (squared) to give the number (product) that is inside the symbol.
- ➤ What are the two equal factors that equal 25? 5 and 5 Write = 5 after the expression $\sqrt{25}$. Explain that when you "unsquare" a number, you are finding its *square root*. Explain that this equation is read *the square root of 25 equals 5*.
- 3. Follow a similar procedure to guide the students in determining the square root of each perfect square that was calculated and in writing it in the Square Roots table. You may choose to start in the middle of the chart to discourage students from just writing 1, 2, 3, 4, and so on, as answers without thinking of the number that was multiplied by itself to make the perfect square. Allow the students to refer to the Perfect Squares table to help with facts that are not memorized.

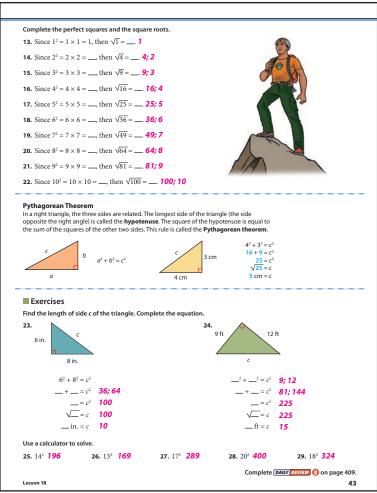
Square Roots		
$\sqrt{1} = 1$		
$\sqrt{4} = 2$		
$\sqrt{9} = 3$		
$\sqrt{16} = 4$		
$\sqrt{25} = 5$		
$\sqrt{36} = 6$		
$\sqrt{49} = 7$		
$\sqrt{64} = 8$		
$\sqrt{81} = 9$		
$\sqrt{100} = 10$		
$\sqrt{121} = 11$		
$\sqrt{144} = 12$		
$\sqrt{169} = 13$		
$\sqrt{196} = 14$		
$\sqrt{225} = 15$		
$\sqrt{256} = 16$		
$\sqrt{289} = 17$		
$\sqrt{324} = 18$		
$\sqrt{361} = 19$		
$\sqrt{400} = 20$		





Instruct the students to locate the information about the Pythagorean Theorem on Student Text page 43. Guide them in reading the information and using the Pythagorean Theorem to find the measurement of the hypotenuse of the right triangles in problems 23 and 24.

Student Text pp. 42-43



Lesson 18 43

Student Text pp. 44-45

Chapter Review

Objectives

- Write a multiplication equation for a given picture
- Apply properties of multiplication: Commutative Property, Associative Property, Identity Property, Zero Property, and Distributive Property
- Use mental math to multiply factors that are multiples of 10
- Estimate the product by rounding to the place of greatest value and by using front-end estimation
- Multiply by a 1-, 2-, or 3-digit multiplier
- Identify the GCF and the LCM of a pair of numbers
- Determine the value of an exponent

Teacher Materials

• Pictures of Multiplication, page IA15 (CD)

Preparation

Write these equations for display. (Do not write the answers or the estimates.)

I.
$$4 \times 30 = 120$$
 $70 \times 80 = 5,600$
 $2 \times 900 = 1,800$ $60 \times 500 = 30,000$
 $10 \times 27,000 = 270,000$ $300 \times 800 = 240,000$
II. $10 \times 2.7 = 27$ $10 \times 8.35 = 83.5$
 $100 \times 4.6 = 460$ $100 \times 0.35 = 35$
III. \$35.17 1.3 8.3 71,249 2,891
 \times 8 \times 20 \times 25 \times 38 \times 356
\$281.36 26 415 569992 17346
(\$320) (20) $+$ 1660 $+$ 2137470 144550
207.5 2,707,462 $+$ 867300
(240) (2,800,000) 1,029,196
(1,200,000)

Note

This lesson reviews the concepts presented in Chapter 2 to prepare the students for the Chapter 2 Test. Student Text pages 44–45 provide the students with an excellent study guide.

Check for Understanding

Write a multiplication equation for a given picture

Display the Pictures of Multiplication transparency. Choose students to write multiplication equations for the pictures and to explain the relationship between the picture and the equation.

1.
$$3 \times 5 = 15$$

2. $2 \times 5 = 10$
3. $6 \times 9 = n$; $n = 54$
4. $3n = 21$; $n = 7$
5. $8 \times 8 = 64$
6. $10 \times 14 = 140$, $10 \times (10 + 4) = 140$, or $(10 \times 10) + (10 \times 4) = 140$

Apply properties of multiplication

Direct the students to write on paper the answers for these statements.

- ➤ Apply the Commutative Property of Multiplication to 8 × 4 = 32. 4 × 8 = 32
- ➤ Explain the Zero Property of Multiplication. When 0 is a factor, the product is 0.
- ➤ Apply the Associative Property of Multiplication to the factors 2, 3, and 5. possible answer: $2 \times (3 \times 5) = (2 \times 3) \times 5 = 30$

- ➤ Apply the Distributive Property of Multiplication over Addition to 3×26 and solve the equation mentally. $3 \times (20 + 6) = (3 \times 20) + (3 \times 6) = 60 + 18 = 78$
- Write a multiplication fact that shows the Identity Property of Multiplication. Accept any multiplication fact with a factor of 1

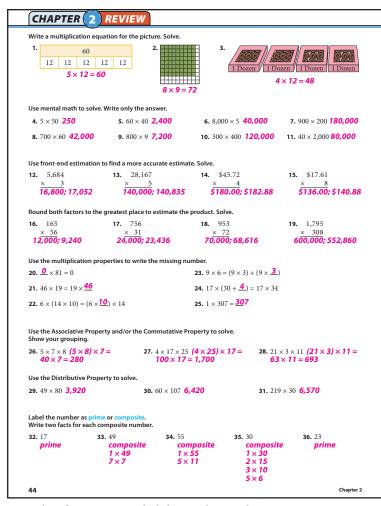
Use mental math to multiply factors that are multiples of 10

- 1. Direct attention to the section I equations written for display. Elicit that you can mentally find the product of factors that are multiples of 10 by multiplying the basic fact, or the non-zero factors, and then annexing the total number of zeros in the factors. Choose students to write the products for display.
- 2. Remind the students that when you multiply a number by 10, the value of every digit in that number becomes 10 times greater and renames to the next greater place, requiring you to write (annex) a zero in the Ones place of the product to show the renaming; e.g., $10 \times 32 = 320$. Point out that the same is true for powers of $10 (100 \times 32 = 3,200; 1,000 \times 32 = 32,000;$ and so on).
- 3. Direct attention to the section II equations.
- ➤ How is multiplying a decimal by 10 similar to multiplying a whole number by 10? Elicit that when multiplying a decimal by 10, the value of each digit in the decimal becomes 10 times greater and renames to the next greater place, requiring you to move the decimal point one place to the right in the product, just as you do when you annex a zero while multiplying a whole number by 10.
- 4. Use the following questions as you guide the students in mentally solving the equations.
- ➤ How can you check to see if you moved the decimal point in the correct direction? Elicit that when multiplying any number by a whole number, the product will be greater than the number being multiplied.
- ➤ When multiplying by a power of 10, what must you do when the decimal being multiplied (multiplicand) has fewer digits to the right of the decimal point than the number of zeros in the multiplier? Why? Elicit that you must annex a zero in the product for each place without a digit to show the renaming of each digit to the next greater place. Elicit that when there is no decimal fraction, a decimal point is not written to the right of the Ones place in a whole number.

Estimate and solve multiplication problems

The Montgomery family spent 14 days visiting historical sites in America. They traveled an average of 234 miles each day. About how many miles did they travel in all? $10 \times 200 = 2,000$ miles

- ➤ How can you find about how many miles the Montgomery family traveled? *estimate*
- 1. Direct the students to estimate the miles traveled by rounding to the greatest place. **2,000 miles**
- ➤ Did the Montgomery family actually travel more or less than 2,000 miles? Why? More; elicit that 2,000 miles is an underestimate because both factors were rounded down.
- 2. Choose a student to demonstrate solving for the exact number of miles traveled. $14 \times 234 = 3,276$ miles Point out that the answer is greater than the underestimate of 2,000 miles. Elicit that when both factors are rounded up, the result is an overestimate; when 1 factor is rounded up more than the



other factor is rounded down, the result is an overestimate; and when 1 factor is rounded down more than the other factor is rounded up, the result is an underestimate.

3. Follow a similar procedure for this word problem.

Mrs. Wilson wants to order 6 books for a ladies Bible study group. Each book costs \$6.35. About how much will 6 books cost? $6 \times $6.00 = 36.00 ; underestimate; $6 \times $6.36 = 38.16

- 4. Direct attention to the section III problems. Direct each student to estimate the answer by rounding to the place of greatest value, to identify the estimate as an underestimate or an overestimate, and to solve for the exact answer.
- 5. Direct the students to use front-end estimation to find a more accurate estimate for the first problem in section III. $8 \times \$35.00 = \280.00

Identify the GCF or the LCM of a pair of numbers

- ➤ What is a factor? a number multiplied to find a product a prime number? a number greater than 1 that has only 2 factors, 1 and the number itself a composite number? a number greater than 1 that has more than 2 factors
- 1. Choose students to list for display the factors of 12 and 30 and to circle the common factors. 1, 2, 3, 6

12: **1, 2, 3, 4, 6, 12**

30: 1, 2, 3, 5, 6, 10, 15, 30

- ➤ What is the GCF of 12 and 30? 6
- ➤ What is a multiple? the product of 2 whole numbers

Complete the table

37.	Standard Form	Factored Form	Exponent Form
	216	6×6×6	6 ³
	625	5 × 5 × 5 × 5	5 ⁴
	1,024	$4 \times 4 \times 4 \times 4 \times 4$	4 ⁵
	1,000,000	10 × 10 × 10 × 10 × 10 × 10	10 ⁶
	1,000	$10 \times 10 \times 10$	10³
	100,000	10×10×10×10×10	10 ⁵

Write the number in standard form

38. $(6 \times 10{,}000) + (3 \times 1{,}000) + (9 \times 10) + (4 \times 1)$ **63,094**

39. $(6 \times 10^3) + (3 \times 10^2) + (9 \times 10^1) + (4 \times 10^0)$ **6,394**

40. $(6 \times 10^2) + (3 \times 10^0) + (9 \times \frac{1}{10^1}) + (4 \times \frac{1}{10^2})$ **603.94**

Solve

Lesson 19

41. List the factors of 18 and 24 in order. 18: 1, 2, 3, 6, 9, 18; 24: 1, 2, 3, 4, 6, 8, 12, 24

42. What is the greatest common factor of 18 and 24? $\boldsymbol{6}$

43. List the multiples of 18 and 24 to find a common multiple. **18: 18, 36, 54, 72; 24: 24, 48, 72 A common multiple of 18 and 24 is 72.**

Solve. **Equations may vary.**

- 44. Mrs. Davidson bought a 10-pound Thanksgiving turkey that was priced at \$0.89 per pound. How much did the turkey cost? 10 x \$0.89 = \$8.90
- 45. Grandma gave \$4.50 to each of her eight grandchildren. How much money did she give in all?

 8 × \$4.50 = \$36.00
- 46. Coach Rees totaled the basketball players' scores for the season. Daniel and Aaron each scored 167 points, and Andrew scored 112 points. What was the total number of points the three boys scored?
- of points the three boys scored?
 (2 × 167) + 112 = 334 + 112 = 446 points
 47. Mr. and Mrs. Calvin have saved \$2,150 each year for
 4 years for their trip to the Holy Land. The trip costs
 \$4,500 per person. How much more money do they
 need for their trip? (2 × \$4,500) (4 × \$2,150) =
 \$9,000 \$8,600 = \$400



Sea of Galilee

2. Instruct the students to list the first 6 nonzero multiples of 12

and 30 and to circle the common multiples. *60* 12: *12*, *24*, *36*, *48*, *60*, *72*

30: 30, 60, 90, 120, 150, 180

➤ What is the LCM of 12 and 30? 60

Remind the students that since 0 is a common multiple of all numbers, it is not considered when determining the LCM.

➤ What other multiple of 30 in the list is also a multiple of 12? How do you know? 120; 10 × 12 = 120

Determine the value of an exponent

Remind the students that repeated factors can be written using exponents. Guide the students in finding the value of these exponents and expressions.

$$\begin{array}{lll} 2^4 = 2 \times 2 \times 2 \times 2 = 16 & 5^3 = 5 \times 5 \times 5 = 125 \\ 4^2 = 4 \times 4 = 16 & 3^2 \times 6 = 3 \times 3 \times 6 = 54 \\ 4 \times 10^2 = 400 & 8 \times \frac{1}{10^3} = 8 \times \frac{1}{10 \times 10 \times 10} = 8 \times \frac{1}{1,000} = \frac{8}{1,000} \text{ or } \end{array}$$

Student Text pp. 44-45

Lesson 19 45

Student Text pp. 46-49

Chapter 2 Test

Cumulative Review

For a list of the skills reviewed in the Cumulative Review, see the Lesson Objectives for Lesson 20 in the Chapter 2 Overview on page 28 of this Teacher's Edition.

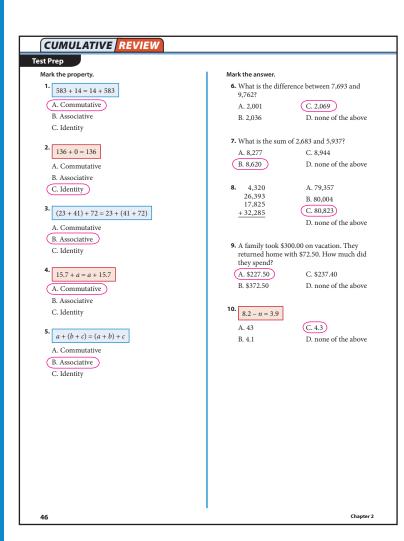
Student Materials

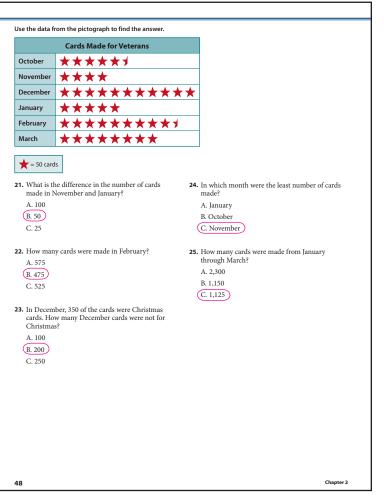
• Cumulative Review Answer Sheet, page IA9 (CD)

Use the Cumulative Review on Student Text pages 46–48 to review previously taught concepts and to determine which students would benefit from your reteaching of the concepts. To prepare the students for the format of achievement tests, instruct them to work on a separate sheet of paper, if necessary, and to mark the answers on the Cumulative Review Answer Sheet.

Use the Exploring Ideas on Student Text page 49 (page 47 of this Teacher's Edition) any time after this chapter.

Mark the answer. **11.** (8 × 10,000) + (6 × 1,000) + (4 × 100) + 180 $(4 \times 10) + (5 \times 1)$ A. 806,445 B. 86,445 A. n = 40B. n = 55C. 864.45 (C. n = 30) **12.** $(8 \times 100) + (2 \times 10) + (3 \times 1) + (6 \times 0.1) + (4 \times 0.01) + (8 \times 0.001)$ A. 823,648 B. 823.68 C. 8.23 A. n = 50B. n = 3013. seven hundred fifty million, four hundred three thousand, eight hundred twelve C. n = 500A. 705,403,812 B. 75,403,812 1.700 300 C. 750,403,812 B. n = 2,000C. n = 1.400Point A 33 A. 6.35 A. n = 91B. 6.365 B. n = 9C. n = 242**15.** Point *B* 9 A. 6.361 B. 6.368 (A. n = 27) C. n = 36Lesson 20 47





HIGH POWER CALCULATIONS

Remember, an exponent tells how many times the base number is multiplied by itself.

The number 10 is multiplied by itself 12 times to equal 1 trillion.

Do you know the word *googol*? A googol is a 1 followed by 100 zeros. In exponential form, a googol is "ten to the hundredth power"—10¹⁰⁰. The mathematician Edward Kasner, who introduced the term *googol*, credited his nine-year-old nephew Milton Sirotta with naming this number.

It can be helpful to use a calculator when working with large numbers in exponential form. Write the following numbers in standard form. Use a calculator to multiply.

Multiply to rename each number in standard form. Use a calculator.

Solve.

$$9^2 - 2^2$$
 77

$$59^2 - 52^2$$
 777

$$559^2 - 552^2$$
 7,777

$$8^2 - 3^2$$
 55

$$58^2 - 53^2$$
 555

