

# Open Source Formal Verification

## Sequential Extended Regular Expressions

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# Sequential Extended Regular Expressions

- PSL offers SEREs
- A powerful way to express complex temporal behaviors
- Example:
  - $\{a;b[*4];c\}$
  - a is active, followed by b active 4 cycles, and then c

# Suffix Implication

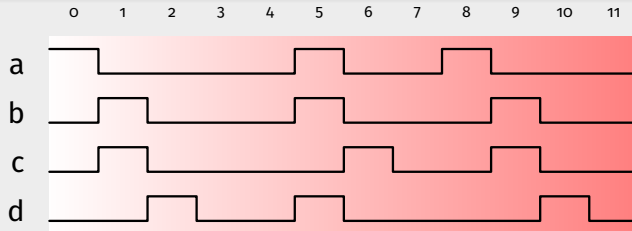
- Two *suffix implication* operators are defined:
  - $| \rightarrow$ 
    - When the left-hand side finishes, then the right-hand side starts
  - $| \Rightarrow$ 
    - When the left-hand side finishes, the right-hand side starts on the next clock cycle

## Example

```
-- Sequence {c;d} is observed when {a;b} finishes
{a;b} |→ {c;d}
-- Sequence {c;d} is observed immediately after {a;b} finishes
{a;b} |⇒ {c;d}
```

# Suffix Implication

## Example



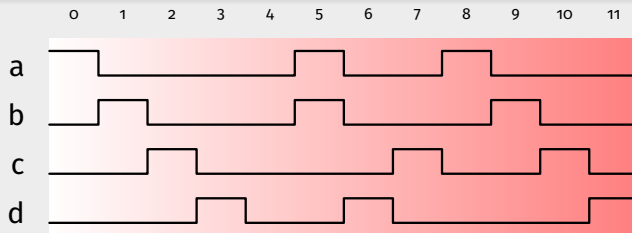
## PSL

```
assert always ({a;b} |-> {c;d});
assert always ({a;b} |=> {c;d});
assert always ({a and b} |-> {d;c});
```



# Suffix Implication

## Example



## PSL

```
assert always ({a;b} |-> {c;d});
```



```
assert always ({a;b} |=> {c;d});
```

# Operator never

- The **never** operator can be used with SEREs to ensure a sequence never happens
- Example:

- Sequence a, b, c shall never be observed

```
assert never {a;b;c};
```

- A read shall never precede a write

```
assert never {rd_o;wr_o};
```

- A signal shall never be asserted on two consecutive cycles

```
assert never {r;r};
```

# Repetition operators

- Repeating a sequence like  $\{a; a; a; a; a\}$  is not ideal as is

Operator	Description
$[*]$	Repeat a number of times between 0 and $\infty$
$[+]$	Repeat a number of times between 1 and $\infty$
$[*n]$	Repeat exactly $n$ times
$[*n \text{ } \color{red}{\text{to}} \text{ } m]$	Repeat a number of times between $n$ and $m$
$[=n]$	$n$ non consecutive repetitions
$[->n]$	$n$ non consecutive goto repetitions



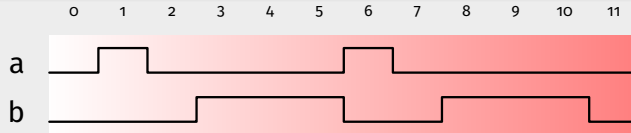
# Repetitions: Examples

Sequence	Equivalent to
$\{a; a; a; a; a\}$	$\{a[*5]\}$
$\{a; b; b; b; c\}$	$\{a; b[*3]; c\}$
$\{a; b; c; b; c; d\}$	$\{a; \{b; c\}[*2]; d\}$

Sequence	Validated by
$\{a; b; b; b; c\}$	$\{a; b[+]; c\}$
$\{a; c\}$	$\{a; b[*]; c\}$
$\{a; b; c; b; c; d\}$	$\{a; b[=2]; d\}$
$\{a; b; c; b; c; d\}$	$\{a; c[->2]; d\}$

# Repetitions

## Exemple



## Assertions

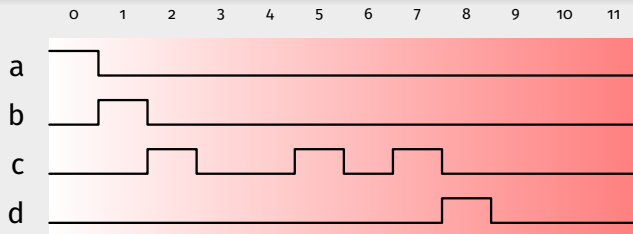
```
assert always (a |-> {[*2];b});
```



```
assert always (a |-> {[*2];b[*3]});
```

# Non consecutive repetitions

## Example



## PSL

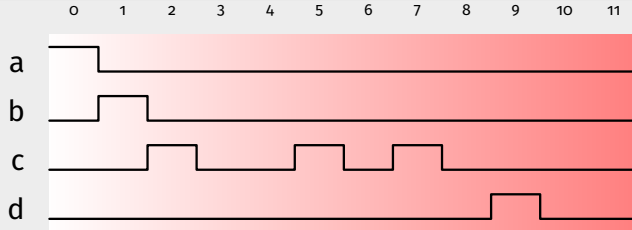
```
assert always ({a} | => {b; c [=3]; d});
```



```
assert always ({a} | => {b; c [->3]; d});
```

# Non consecutive repetitions

## Example



## PSL

```
assert always ({a} | => {b; c [=3]; d});
```



```
assert always ({a} | => {b; c [->3]; d});
```

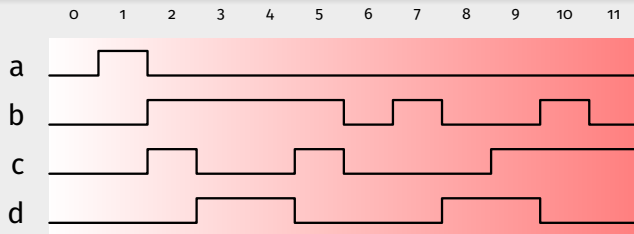
# Sequences Combinations

Function	Description
<code>s1 ; s2</code>	concatenation of two sequences
<code>s1 : s2</code>	fusion of two sequences <sup>1</sup>
<code>s1 &amp; s2</code>	non-length-matching <i>and</i> operator
<code>s1 &amp;&amp; s2</code>	length-matching <i>and</i> operator
<code>s1   s2</code>	<i>Or</i> operator
<code>s1 within s2</code>	s1 shall be observed during the execution of s2

<sup>1</sup>Concatenation, but s2 starts on the last cycle of s1. Not really used

# Non-length-matching *and* operator

## Exemple



## Assertions

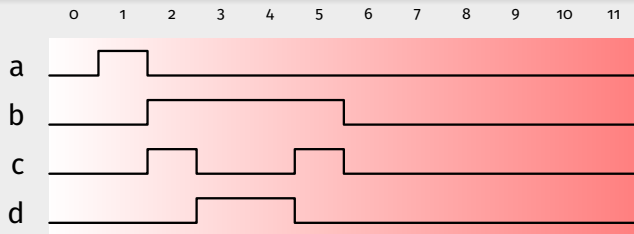
```
assert always {a} | => {{b[*4]} & {c; d}};
```



```
assert always {d[*2]} | => {{c} & {b; not b}};
```

# Length-matching *and* operator

## Exemple



## Assertions

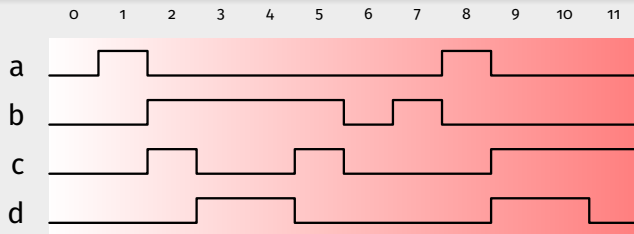
```
assert always {a}|=>{{b[*4]} && {c; d[*2]; c}};
```



```
assert always {a}|=>{{b[*4]} && {c[*1 to 2]; d[*1 to 3]}};
```

# Or operator

## Exemple



## Assertions

```
assert always {a} | => {{b[*4]} | {d; c}};
```

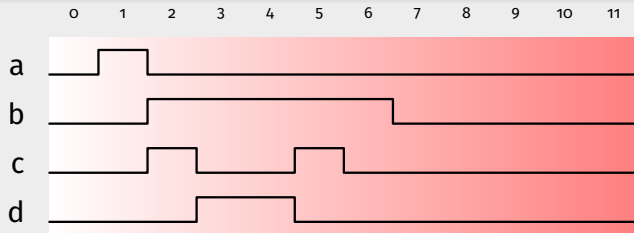


```
assert always {a} | => {[*1]; d; b} | {d[*2]};
```



## s1 within s2

## Example



## PSL

```
assert always ({a} | => {{c;d[*2];c} within {b[*5]}});
```



```
assert always ({a} | => {{d[*2]} within {c;d[*2];c}});
```

# Named sequences

- Like the properties, sequences can be named and then reused
  - Named sequences with parameters are not currently supported
- Particularly useful if a sequence is used in different contexts
  - For instance in an assert and a cover

## PSL

```
sequence seq is {a; not a; a};  
  
assert always ({b} ==> {seq;d[*2];c});  
  
cover seq;
```

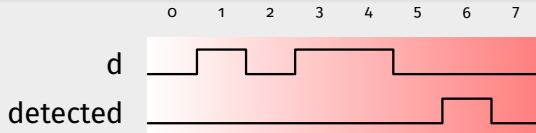
# Detecting the end of a sequence

- Dealing with a signal that shall be asserted at the end of a sequence is tricky
- Examples:
  - A timer triggers after a certain number of clock cycles
  - A sequence detector triggers at the end of the sequence
- Some constructs exist, but are not supported by SBY (`ended()`)
- So, how to solve it?

# Sequence detector

- Let's have a sequence detector that shall assert the signal `detected` when the sequence  $d; d; \bar{d}; \bar{d}$  is observed (asserted at the same cycle as the last `not d`)

## Example: valid chronogram



## A nice assertion

```

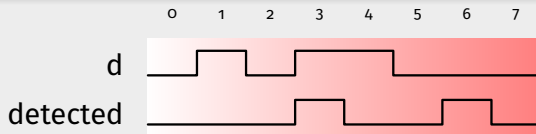
assert always ({d;d;not d;not d} |-> detected);
-- or
assert always ({d[*2]; (not d) [*2]} |-> detected);

```



# Sequence detector

## Invalid chronogram



- What happens then?

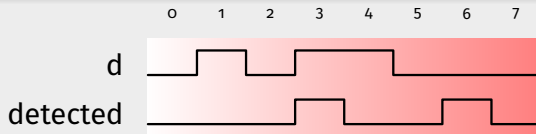
## A nice assertion

```
assert always ({d[*2]; (not d) [*2]} |-> detected);
```

- Is it what we want?
- Well, our assertion does not say when `detected` shall stay low

# Sequence detector

## Invalid chronogram



- Idea : The signal should not be asserted twice before the sequence ends
  - Use `within` and the non consecutive goto repetition
  - Use the length-matching operator and the non consecutive goto repetition

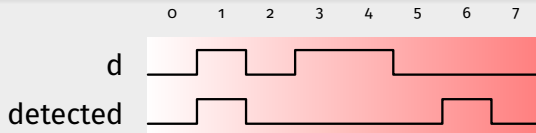
## Assert that the signal does not go high twice within the sequence

```
assert never {{detected[->2]} within {d;d;not d;not d}};
assert never {{detected[->2]} && {d;d;not d;not d}};
```

- What is the issue here?

# Sequence detector

## Invalid chronogram



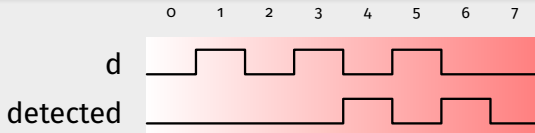
## Assert that the signal does not go high twice within the sequence

```
assert never {{detected[->2]} within {d;d;not d;not d}};
assert never {{detected[->2]} && {d;d;not d;not d}};
```

- Well, maybe not the best idea
- And some issues with sequence overlapping

# Sequence detector

Sequence now d;not d;d;not d : valid chronogram



Assert that the signal does not go high twice within the sequence

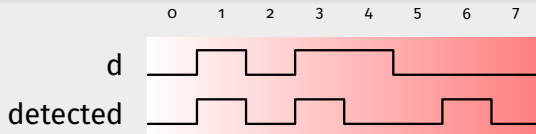
```
assert never {{detected[->2]} within {d;not d;d;not d}};
assert never {{detected[->2]} && {d;not d;d;not d}};
```

- Well, once again, maybe not the best idea



# Sequence detector

## Example



- Idea : reverse the implication
  - Instead of `seq |-> det`, write `det |-> seq` has occurred

Assert that the signal does not go high before the sequence ends

```
assert always (detected |->
  (d = '0' and prev(d) = '0' and prev(d,2) = '1' and prev(d,3) = '1'));
```

- Not easily generalizable. Imagine if the sequence is `{d[*3 to 12]; (not d)[*5 to 15]}`

# Sequence detector: Best option

- Use the following construct, specific to SBY:

```
attribute anyseq : boolean;
signal detected_test: std_logic;
attribute anyseq of detected_test : signal is true; ← the signal is then a free variable
gen_test: assume always ({d[*2]; (not d) [*2]} |-> detected_test);
assert_low: assert always (detected -> detected_test);
assert_high: assert always ({d[*2]; (not d) [*2]} |-> detected);
```

- `gen_test` ensures that `detected_test` is high at the end of the sequence
  - Nothing more
  - So, it could be '0' or '1' at any other time
- `assert_low` only says that if `detected` is high, then `detected_test` **has to be** high
  - Nothing more, so
  - If `detected` was to be high at another time, as `detected_test` could have any value, `assert_low` would fail
- QED

# Sequence detector: Conclusion

- The approach based on the assertion of the sequence detection and the free variable can be applied to any kind of sequence
  - A pure sequence detector
  - Triggering of a timer
  - ...

# LTL - SERE equivalences

- LTL formula can be expressed in SERE-style

LTL	SERE
<code>a until b</code>	<code>{a[*];b}</code>
<code>a and next b</code>	<code>{a;b}</code>
<code>next[i] (a)</code>	<code>{[*i];a}</code>
<code>[next_a[i to j] (b)</code>	<code>{[*i];b[*j-i+1]}</code>
<code>[next_e[i to j] (b)</code>	<code>{[*i to j];b}</code>

# Conclusion

- SEREs are powerful
- But careful with their definition you should be
- Do not be afraid of using coverage to know if a sequence has been observed or not
- Quite often LTL vs SERE is a matter of preference