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CS 3010 Numeric Method and Computing Assignment 2

Ex1 $3x_1+4x_2+3x_3=10$, $x_1+5x_2-x_3=7$, $6x_1+3x_2+7x_3=15$

$$\begin{bmatrix}3&4&3\\1&5&-1\\6&3&7\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}10\\7\\15\end{bmatrix}\quad a=\begin{bmatrix}3&4&3\\1&5&-1\\6&3&7\end{bmatrix}\quad b=\begin{bmatrix}10\\7\\15\end{bmatrix}$$

Forward Elimination

$k=1$ $i=2$. $\text{mult}=\left(\frac{a_{ik}}{a_{kk}}\right)=\frac{1}{3}$. $L_2-\frac{1}{3}L_1\to L_2$

$1-\frac{1}{3}(3)=1-1=0$

$5-\frac{1}{3}(4)=\frac{15}{3}-\frac{4}{3}=\frac{11}{3}$

$-1-\frac{1}{3}(3)=-1-1=-2$

$7-\frac{1}{3}(10)=\frac{21}{3}-\frac{10}{3}=\frac{11}{3}$

$$\begin{bmatrix}3&4&3\\0&\frac{11}{3}&-2\\6&3&7\end{bmatrix}\begin{bmatrix}10\\\frac{11}{3}\\15\end{bmatrix}$$

$k=1$ $i=3$ $\text{mult}=\left(\frac{a_{ik}}{a_{kk}}\right)=\frac{6}{3}=2$ $L_3-2L_1\to L_3$

$6-2(3)=6-6=0$

$3-2(4)=3-8=-5$

$7-2(3)=7-6=1$

$15-2(10)=15-20=-5$

$$\begin{bmatrix}3&4&3\\0&\frac{11}{3}&-2\\0&-5&1\end{bmatrix}\begin{bmatrix}10\\\frac{11}{3}\\-5\end{bmatrix}$$

$k=2$ $i=3$ $\text{mult}=\left(\frac{a_{ik}}{a_{kk}}\right)=\frac{-5}{\frac{11}{3}}=-\frac{15}{11}$ $L_3-\frac{15}{11}L_2\to L_3$

$0-0=0$

$-5-\left(\frac{-15}{11}\right)\left(\frac{11}{3}\right)=-5+5=0$

$1-\left(\frac{-15}{11}\right)(-2)=1-\frac{30}{11}=-\frac{19}{11}$

$-5-\left(\frac{-15}{11}\right)\left(\frac{11}{3}\right)=-5+5=0$

$$\begin{bmatrix}3&4&3\\0&\frac{11}{3}&-2\\0&0&-\frac{19}{11}\end{bmatrix}\begin{bmatrix}10\\\frac{11}{3}\\6\end{bmatrix}$$

Back substitution.

Bottom row $x_n=\frac{b_n}{a_{nn}}$ $x_3=\frac{b_3}{a_{33}}$ $x_3=\frac{0}{-\frac{19}{11}}=0$.

$$x_i=\frac{b_i-\sum_{k=i+1}^{n}x_k a_{ik}}{a_{ii}}$$

$x_2=\frac{\frac{11}{3}-(-2\times0)}{\frac{11}{3}}=\frac{\frac{11}{3}}{\frac{11}{3}}=1$ $x_1=\frac{10-(4\times1+3\times0)}{3}=\frac{10-4}{3}=\frac{6}{3}=2$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}2\\1\\0\end{bmatrix}$$

Ex2 $3x_1+4x_2+3x_3=10$
$x_1+5x_2-1x_3=7$
$6x_1+3x_2+7x_3=15$

$A=\begin{bmatrix}3&4&3\\1&5&-1\\6&3&7\end{bmatrix}$ $b=\begin{bmatrix}10\\7\\15\end{bmatrix}$ scaling = [0,0,0] ind = [1,2,3]

Find max scaling for each row scaling = [4,5,7]

k=1

Scale and pivot.

$\frac{1}{5}<\frac{3}{4}<\frac{6}{7}$ maxInd = 3 swap(row[ind[k]], row[ind[maxInd]]) → ind = [3,2,1]

Forward Elimination.

i=2 mult = $\frac{1}{6}$

$1-\frac{1}{6}(6)=0$
$5-\frac{1}{6}(3)=5-\frac{1}{2}=\frac{9}{2}$
$-1-\frac{1}{6}(7)=-1-\frac{7}{6}=-\frac{13}{6}$
$7-\frac{1}{6}(15)=\frac{27}{6}=\frac{9}{2}$

$\begin{bmatrix}3&4&3\\0&\frac{9}{2}&-\frac{13}{6}\\6&3&7\end{bmatrix}\begin{bmatrix}10\\\frac{9}{2}\\15\end{bmatrix}$ ind = [3,2,1] scaling = [4,5,7]

i=3 mult = $\frac{3}{6}=\frac{1}{2}$

$3-\frac{1}{2}(6)=0$
$4-\frac{1}{2}(3)=\frac{5}{2}$
$3-\frac{1}{2}(7)=\frac{1}{2}$
$10-\frac{1}{2}(15)=\frac{5}{2}$

$\begin{bmatrix}0&\frac{5}{2}&\frac{1}{2}\\0&\frac{9}{2}&-\frac{13}{6}\\6&3&7\end{bmatrix}\begin{bmatrix}\frac{5}{2}\\\frac{9}{2}\\15\end{bmatrix}$ ind = [3,2,1] scaling = [4,5,7]

K=2.

scale and pivot. $\frac{5/2}{4}=\frac{5}{8}$ $\frac{9/2}{5}=\frac{9}{10}$ $\frac{1}{10}<\frac{5}{8}$ no swap. ind = [3,2,1]

Forward elimination.

i=3 multi = $\left(\frac{5}{2}\right)/\left(\frac{9}{2}\right)=\frac{5}{2}\times\frac{2}{9}=\frac{5}{9}$

$0-0=0$
$\frac{5}{2}-\frac{5}{9}\left(\frac{9}{2}\right)=0$
$\frac{1}{2}-\frac{5}{9}\left(-\frac{13}{6}\right)=-\frac{1}{2}+\frac{65}{54}=\frac{38}{54}$
$\frac{5}{2}-\frac{5}{9}\left(\frac{9}{2}\right)=\frac{5}{2}-\frac{5}{2}=0.$

$\begin{bmatrix}0&0&\frac{38}{54}\\0&\frac{9}{2}&-\frac{13}{6}\\6&3&7\end{bmatrix}\begin{bmatrix}0\\\frac{9}{2}\\15\end{bmatrix}$ ind = [3,2,1] scaling [4,5,7]

Back Substitution.

$\frac{38}{54}x_3=0$ $x_3=0$

$x_2=\frac{\frac{9}{2}-\left(-\frac{13}{6}\times 0\right)}{\frac{9}{2}}=\frac{\frac{9}{2}}{\frac{9}{2}}=1$

$x_1=\frac{15-(7\times 0+3\times 1)}{6}=\frac{15-3}{6}=\frac{12}{6}=2.$

$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}2\\1\\0\end{bmatrix}$