CS101 Advanced Engineering Mathematics (I)

工程數學(一)

[Guidelines]

- All the homework in this course will involve solving advanced engineering mathematics problems (differential equations in particular) by hand and computer.
- While discussion in class is allowed, you MUST work independently to generate your own solutions to the problems. You may reference the given Matlab tutorials.
- Matlab (www.mathworks.com) will be used as the computer-aided software (CAS) for solving many problems and plotting solutions.
- For each homework, you must submit a written report. **[Instructions]** are given for solving specific problems, but are *not* required in the written report. However, detail solutions of all the given **[Problems]** are required.

[General Instructions]

To get a good grading on the homework assignments, you are advised to do the following:

- Meet the deadline!! Late homework will **not** be collected. (按時繳交，逾時不候)
- Provide correct answers in details. (詳細推導過程及標明正確答案)
- Prepare your written reports in good quality (使用 Template 並書寫工整).

指導教授：張元翔

Homework Assignment 2

Higher-Order Differential Equations & Systems

Deadline: 1 / 3 / 2019

(期末考前週五下班前繳交至電學 603 研究室)

[Problems 1 ~ 5]

Given the following differential equations, please answer the following: (30%，每題 6 分)

(a) Solve the problem by hand (手寫推導)；

(b) Use Python programming to plot the solution curve. The interval *I* is the given range for the *x*-data in the plots. Similar to the Homework 1, the plots must be carefully ***labeled***, ***titled***, and ***copyright*** for full credits

1. $y'' + 4y = sinx,$
   (For plotting, let I : [0, 2π] and $c_1 = c_2 = 1$)

2. $y'' - 3y' + 2y = e^{2x}$,
   (For plotting, let I : [0, 1] and $c_1 = c_2 = 1$)

3. $x^2y'' - 3xy' + 3y = 0,$
   (For plotting, let I : [0, 1] and $c_1 = c_2 = 1$)

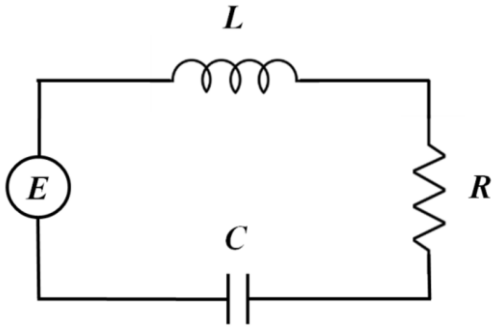
4. $y'' + y = cos^2x,$
   (For plotting, let I : [0, 2π] and $c_1 = c_2 = 1$)

5. $xy'' + 2y' = x, \quad c_1 = c_2 = 1, \quad I : 0 \sim 1$
   (For plotting, let I : [0, 1] and $c_1 = c_2 = 1$)

注意：手寫推導過程與 Python 繪圖必須出現在同一頁面，否則予以扣分。

[Problems 6]

6. Consider the *LRC* Series Circuit, where $L = 1$ H, $R = 10\ \Omega$, $C = 0.04$ F, and $E(t) = 12$ V. Please answer the following: (15%)



(a) Assume the initial charge is 0 and the initial current is 0. Determine the charge $q(t)$ and the current $i(t)$.

(b) Give the two plots for the charge and the current, respectively (for $t = 0 \sim 2$ seconds).

(c) 試用文字敘述所觀察到的電路運作情形。

[Instructions]

Laplace Transform is an integral transform that is often used to solve differential equations. Using Python programming and SymPy package, we are able to find Laplace transform (or Inverse Laplace transform).

[Problems 7 ~ 8]

7. Find the following Laplace transform or Inverse Laplace transform by hand first. Then, use Python programming to verify your results (24%，每題 4 分).

   (a) $\mathcal{L}[t^3 e^t] = ?$

   (b) $\mathcal{L}[t\sin(2t)] = ?$

   (c) $\mathcal{L}[e^{2t}\cos(t)] = ?$

   (d) $\mathcal{L}^{-1}[\frac{1}{s^2+2s+2}] = ?$

   (e) $\mathcal{L}^{-1}[\frac{s}{(s-2)(s-3)(s-6)}] = ?$

   (f) $\mathcal{L}^{-1}[\frac{1}{s^2+1}e^{-\pi s}] = ?$

8. Use Python programming to plot the following functions. The plots must be carefully ***labeled***, ***titled***, and ***copyright*** for full credits. (6%，每題 3 分)

   (a) $f(t) = sin(2t)\ \mathcal{U}(t-\pi)$,
   (For plotting, let I : [0, 4π]

   (b) $f(t) = e^{-t}\,cos(4t)\ \mathcal{U}(t-\pi/2)$,
   (For plotting, let I : [0, 4π]

[Problems 9]

9. Use Python programming to compose a music (or song) and save as a ".wav" file. Please reference the Appendix (附錄) – 弦波。

   Your music must satisfy the following rules:

   (a) Must be a well-known music (or your favorite music) (必須是耳熟能詳的樂曲，可以是歌曲、音樂、電玩音樂等)。
   (b) Duration must be at least 30 seconds (不須是全曲，但至少需長達 30 秒)。
   (c) Filename must be: 學號 + .wav (例如：10727001.wav)。
   (d) You can listen to your music file using Window media player (or software alike).

   Please do the following: (25%)

   (a) Append your Python program in this homework report.
   (b) Upload your music file (wav file) to i-learning.

   Note:

   (a) It's possible to tune your music note as the following function (will sound better), i.e.,

$$x(t) = A \cdot e^{-t} \cos(2\pi ft)$$

   (b) Extra bonus may be given if your music involve chorus (和弦).