CS101 Advanced Engineering Mathematics (I)

工程數學(一)

[Guidelines]

- All the homework in this course will involve solving advanced engineering mathematics problems (differential equations in particular) by hand and computer.
- While discussion in class is allowed, you MUST work independently to generate your own solutions to the problems.
- Python programming will be used for solving problems and plotting solutions. You should reference the Python Tutorial (Python 教學投影片與解說).
- For each homework, you must submit a written report.

[General Instructions]

To get a good grading on the homework assignments, you are advised to do the following:

- Do not copy other classmate's works! (請遵守學術倫理，嚴禁抄襲)
- Provide correct answers in details. (詳細推導過程及標明正確答案)
- Prepare your written reports in good quality (使用 Template 檔並書寫工整).
- Meet the deadline!! Late homework will **not** be collected. (按時繳交，逾時不候)

指導教授：張元翔

Homework Assignment 1

Review of Calculus & First-Order Differential Equations

Deadline: 11 / 8 / 2019 (星期五)

(期中考當週週五下班前繳交至電學 603 計算機視覺研究室)

【Instructions】

The concept of Taylor series is very important in the study of Calculus and Engineering Mathematics. Using Python, it's possible to compare two functions in one plot using the command:

>>> plt.plot (x, y1, '-', x, y2, '--')

given data points in x-coordinate. Please reference the Python Tutorial (Python 教學投影片與解說) for details.

【Problems】

1. The Taylor series for the function $y = x\sin(x)$ at $x = 0$ is given by:

$$f(x) = x\sin x = x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!} + \frac{x^{10}}{9!} - \cdots$$

(assuming $-\pi \le x \le \pi$). Please do the following (20%)

(a) Derive the Taylor series by hand. Then, use the package <SymPy> to verify your results. Demonstrate your Python source codes and the results.

(b) Plot both the functions $x\sin x$ and $x^2 - \frac{x^4}{3!}$ in one plot.

(c) Plot both the functions $x\sin x$ and $x^2 - \frac{x^4}{3!} + \frac{x^6}{5!}$ in one plot.

(d) Plot both the functions $x\sin x$ and $x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!}$ in one plot.

(e) Compare the results and discuss your findings (請用中文解釋).

Note: The figures must be carefully ***labeled***, ***titled***, and with your own ***copyright*** for full credits.

【**Instructions**】

Calculus is useful for solving optimization problems. In this homework assignment, our goal is to learn the ***method of least squares***, also known as ***Linear Regression*** (線性迴歸).

【**Problems**】

2. 給定一組資料點 (x_i, y_i), $i = 1\ldots n$，**最小平方法** (Method of Least Squares) 的目的是找一直線 $y = ax + b$，使得每一點至直線的垂直距離總和(又稱為**平方誤差和** Sum of Square Errors) 可以達到最小值：

$$\varepsilon = \sum_{i=1}^{n}\left[y_i - ax_i - b\right]^2$$

試回答下列問題：

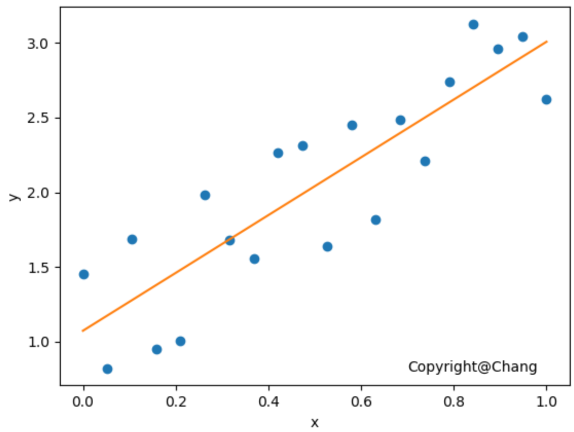
(a) 試利用最小平方法求得最佳之直線方程式 (試手寫推導) (10%)

【**提示**】 分別**設微分為 0**，即 $\dfrac{\partial\varepsilon}{\partial a} = 0$ 與 $\dfrac{\partial\varepsilon}{\partial b} = 0$，得到的結果會是下列型態:

$$\begin{bmatrix} \sum\_\_ & \sum\_\_ \\ \sum\_\_ & \sum\_\_ \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum\_\_ \\ \sum\_\_ \end{bmatrix}$$

(b) 給定一組資料點 (2, 1)、(3, 2)、(4, 3)、(5, 2)，試利用最小平方法求得最佳之直線方程式 (5%)

(c) 使用 Python 程式產生 20 個亂數點，亂數為一次多項式且 $a = 2$ 與 $b = 1$，同時加入介於–0.5 ~ 0.5 的均勻雜訊。試設計 Python 程式求最佳直線方程式，列印 a 與 b 的值並繪圖 (範例如下) (15%)



Note: The figures must be carefully ***labeled***, ***titled***, and with your own ***copyright*** for full credits.

【提示】

```
import numpy as np
import np.random as random
a = 2
b = 1
x = np.linspace(0, 1, n)
noise = (random.rand(n) - 0.5)
y = a * x + b + noise
…
```

【Instructions】

Direction fields are particularly useful for solving first-order differential equations when analytic solutions can't be found. To plot a direction field for a first-order differential equation with Python programming, the equation must be in **normal form**. A first-order differential equation is in normal form if it is expressed as:

$$\frac{dy}{dx} = f(x, y)$$

Please reference the Python Tutorial (Python 教學投影片與解說) for details.

【Problems】

3. Following the aforementioned instructions, use the Python programming to obtain the direction field for each of the following differential equations (the interval I is given for (x, y) coordinates accordingly). Attach the figures in your written report. The figures must be carefully ***labeled***, ***titled***, and with your own ***copyright*** for full credits. (20%，每題 5 分)

   (a) $\frac{dy}{dx} = x + y$; I: [-5 : 0.5 : 5, -5 : 0.5 : 5]

   (b) $\frac{dy}{dx} = x - y$; I: [-5 : 0.5 : 5, -5 : 0.5 : 5]

   (c) $\frac{dy}{dx} = xy$; I: [-5 : 0.5 : 5, -5 : 0.5 : 5]

   (d) $\frac{dy}{dx} = \sin x \cos y$; I: [-5 : 0.5 : 5, -5 : 0.5 : 5]

   Note: The figures must be carefully ***labeled***, ***titled***, and with your own ***copyright*** for full credits.

【**Instructions**】

In the field of differential equations, *Initial Value Problems* (IVPs) are problems with specific initial conditions. In general, special solutions (or functions) can be found at a given point in the solution domain. Here, we will learn to derive the special solutions by hand, and generate plots for the solutions by Python programming.

【**Problems**】

4. Solve the following initial value problems, and plot the solution curves. The interval I is given for the x-data in the plots. The figures must be carefully ***labeled***, ***titled***, and with your own ***copyright*** for full credits.

   注意：均先用手寫推導解題，再用 Python 程式畫圖，每一題須附手寫推導過程、程式碼與繪圖 (30%，每題 6 分)

   (a) $\dfrac{dy}{dx} = (y-1)^2$, $y(1) = 0$, $I: [0, 1]$

   (b) $\dfrac{dy}{dx} = x\sqrt{1-y^2}$, $y(0) = 0$, $I: [0, 2\pi]$

   (c) $x\dfrac{dy}{dx} - y = x^2 \sin x$, $y(2\pi) = 0$, $I: [0, 8\pi]$

   (d) $y' + (\tan x)y = \cos^2 x$, $y(0) = 1$, $I: [0, 4\pi]$

   (e) $\dfrac{dy}{dx} = (x+y+1)^2$, $y(0) = -1$, $I: [0, 4\pi]$

   Note: The figures must be carefully ***labeled***, ***titled***, and with your own ***copyright*** for full credits.