Spatial Modulation: Optimal Detection and Performance Analysis

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***Abstract*—In this letter, we derive the optimal detector for the so-called spatial modulation (SM) system introduced by Mesleh et al. in [1]. The new detector performs significantly better than the original (~ 4 dB gain), and we support our results by deriving a closed form expression for the average bit error probability. As well, we show that SM with the optimal detector achieves performance gains ($\sim 1.5-3$ dB) over popular multiple antenna systems, making it an excellent candidate for future wireless communication standards.**

***Index Terms*—Antenna modulation, maximum likelihood detection, MIMO, spatial modulation.**

I. Introduction

USING multiple antennas in wireless communications allows unprecedented improvements over current systems. Large spectral efficiency is obtained by using transmission techniques designed for multiple input multiple output (MIMO) systems, such as the vertical Bell Laboratories layered space-time (V-BLAST) architecture [2]. Due to inter-channel interference (ICI) caused by coupling multiple symbols in time and space, maximum likelihood (ML) detection increases exponentially in complexity with the number of transmit antennas. Consequently, avoiding ICI greatly reduces receiver complexity, and contributes in attaining performance gains.

The so-called spatial modulation (SM), introduced by Mesleh et al. in [1], [3], is an effective means to remove ICI and the need for precise time synchronization amongst antennas. SM is a pragmatic approach for transmitting information, where the modulator uses well known amplitude/phase modulation (APM) techniques such as phase shift keying (PSK) and quadrature amplitude modulation (QAM), but also employs the antenna index to convey information. Only one antenna remains active during transmission so that ICI is avoided, and inter-antenna synchronization (IAS) is no longer needed as in the case of V-BLAST, where all antennas transmit at the same time. However, the detector in [1] is sub-optimal, and hinders the full performance gains achievable by SM.

Contribution: The contribution of this paper is twofold. We first derive the optimal detector for SM, improving over the sub-optimal detection rules suggested in [1]. Thanks to our development, SM compares favorably to other transmission schemes, such as APM with maximum ratio combining (MRC), and the well known V-BLAST.

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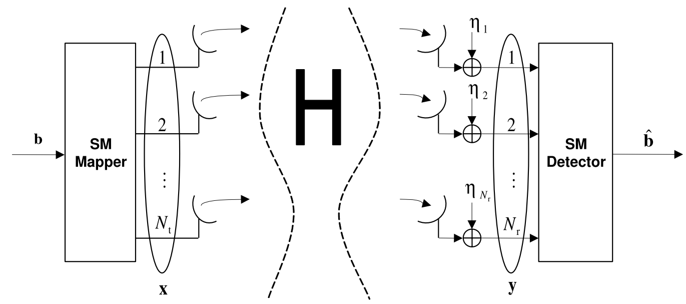

Fig. 1. Spatial modulation system model.

Second, in order to support our results, we analyze the performance of the SM system, and derive a closed form expression for the bit error probability when real constellations are used.

Organization: Section II introduces the basic SM system model, the mapper, and the original detector. In Section III, we derive the optimal detector, and provide a comprehensive look at its complexity. An in depth analysis of SM's performance is then provided in Section IV. Section V presents some simulation results, and we conclude the paper in Section VI.

II. Spatial Modulation

A. SM Transmission

The general system model is shown in Fig. 1, which consists of a MIMO wireless link with N_t transmit and N_r receive antennas.[1] A random sequence of independent bits $\mathbf{b}$ enters the SM mapper, which groups $m = \log_2(MN_\text{t})$ bits and maps them to a constellation vector $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_{N_\text{t}} \end{bmatrix}^T$, where we assume a power constraint of unity (i.e. $E_\mathbf{x}\left[\mathbf{x}^H\mathbf{x}\right] = 1$). In SM, only one antenna remains active during transmission and hence, only one of the x_j in $\mathbf{x}$ is nonzero. The signal is transmitted over an $N_\text{r} \times N_\text{t}$ wireless channel $\mathbf{H}$, and experiences an $N_\text{r} - \dim$ additive white Gaussian (AWGN) noise $\eta = \begin{bmatrix} \eta_1 & \eta_2 & \cdots & \eta_{N_\text{r}} \end{bmatrix}^T$. The received signal is given by $\mathbf{y} = \sqrt{\rho}\mathbf{H}\mathbf{x} + \eta$, where ρ is the average signal to noise ratio (SNR) at each receive antenna, and $\mathbf{H}$ and η have independent and identically distributed (iid) entries according to $\mathcal{CN}(0,1)$.

As mentioned earlier, SM exploits the antenna index as an additional means to transmit information. The antenna

[1] The following notations are used throughout the paper. Italicized symbols denote scalar values, while bold lower/upper case symbols denote vectors/matrices. We use $(\cdot)^T$ for transpose, $(\cdot)^H$ for conjugate transpose, and $\binom{\cdot}{\cdot}$ for the binomial coefficient. We use $|\cdot|$ for absolute value of a scalar, and $\|\cdot\|_\text{F}$ for the Frobenius norm of a vector/matrix. We use $\mathcal{CN}(\mu, \sigma^2)$ for the complex Gaussian distribution of a random variable, having independent Gaussian distributed $\mathcal{N}(\mu, \frac{1}{2}\sigma^2)$ real and imaginary parts with mean μ and variance $\frac{\sigma^2}{2}$. We use $P(\cdot)$ for the probability of an event, $p_\mathbf{Y}(\cdot)$ for the probability density function (PDF) of a random variable $\mathbf{Y}$, and $E_\mathbf{x}[\cdot]$ for the statistical expectation with respect to $\mathbf{x}$. We use $\text{Re}\{\cdot\}$ for the real part of a complex variable, and $\mathcal{X}$ represents a constellation of size M.

combined with the symbol index make up the SM mapper, which outputs a constellation vector of the following form:

$$\mathbf{x}_{jq} \triangleq \begin{bmatrix} 0 & 0 & \cdots & \underset{j^{\text{th}}\text{ position}}{\underset{\uparrow}{x_q}} & 0 & \cdots & 0 \end{bmatrix}^T,$$

where j represents the activated antenna, and x_q is the q^{th} symbol from the M-ary constellation $\mathcal{X}$. Hence, only the j^{th} antenna remains active during symbol transmission. Figure 2 in [1] illustrates an example of the mapper and is omitted here due to space constraints. For example, in 3 bits/s/Hz transmission with $N_{\text{t}} = 4$ antennas, the information bits are mapped to a ± 1 binary PSK (BPSK) symbol, and transmitted on one of the four available antennas. The output of the channel when x_q is transmitted from the j^{th} antenna is expressed as

$$\mathbf{y} = \sqrt{\rho}\mathbf{h}_j x_q + \eta, \tag{1}$$

where $\mathbf{h}_j$ denotes the j^{th} column of $\mathbf{H}$.

B. SM Detection (Sub-Optimal)

In [1], a sub-optimal detection rule based on MRC is given by

$$\begin{aligned} \hat{\jmath} &= \arg\max_j z_j & (2)\\ \hat{q} &= \mathcal{D}(z_{\hat{\jmath}}), & (3) \end{aligned}$$

where $z_j = \frac{\left|\mathbf{h}_j^H \mathbf{y}\right|}{\|\mathbf{h}_j\|_{\text{F}}^2}$, $\hat{\jmath}$ and $\hat{q}$ represent the estimated antenna and symbol index, respectively, and $\mathcal{D}$ is the constellation demodulator function. Since the mapping is one to one, the demapper obtains an estimate of the transmitted bits by taking $\hat{\jmath}$ and $\hat{q}$ as inputs.

We note that the simulation results of [1] could not be reproduced using the *conventional* channel assumptions of Section II. The reason for this can be seen (in the noiseless case) by substituting (1) for $\mathbf{y}$ in (2). Therefore, z_j reduces to $\frac{\sqrt{\rho}\left|\mathbf{h}_k^H \mathbf{h}_j x_q\right|}{\|\mathbf{h}_k\|_{\text{F}}^2}$ and, in order to detect the correct antenna index (i.e. $k = j$), we require $\frac{\left|\mathbf{h}_k^H \mathbf{h}_j\right|}{\|\mathbf{h}_k\|_{\text{F}}^2} < 1$. By invoking Cauchy's inequality to the left hand side, we find that $\|\mathbf{h}_j\|_{\text{F}} \le \|\mathbf{h}_k\|_{\text{F}}$ is a necessary condition for antenna detection without errors, which should materialize in the absence of noise. One way to ensure this condition is by normalizing the channel prior to transmission (i.e. $\|\mathbf{h}_j\|_{\text{F}}^2 = c$ for all j, where c is a constant). We refer to these channels as *constrained*.

We now derive the optimal detector for SM, where the need for constrained channels is not necessary.

III. Optimal Detection

A. SM Detection (Optimal)

Since the channel inputs are assumed equally likely, the optimal detector is based on the ML principle:

$$\begin{aligned} [\hat{\jmath}_{\text{ML}}, \hat{q}_{\text{ML}}] &= \arg\max_{j,q} p_{\mathbf{Y}}(\mathbf{y} \mid \mathbf{x}_{jq}, \mathbf{H}) \\ &= \arg\min_{j,q} \sqrt{\rho}\,\|\mathbf{g}_{jq}\|_{\text{F}}^2 - 2\text{Re}\left\{\mathbf{y}^H \mathbf{g}_{jq}\right\}, \quad (4) \end{aligned}$$

where $\mathbf{g}_{jq} = \mathbf{h}_j x_q$, $1 \le j \le N_{\text{t}}$, $1 \le q \le M$, and $p_{\mathbf{Y}}(\mathbf{y} \mid \mathbf{x}_{jq}, \mathbf{H}) = \pi^{-N_{\text{r}}} \exp\left(-\left\|\mathbf{y} - \sqrt{\rho}\mathbf{H}\mathbf{x}_{jq}\right\|_{\text{F}}^2\right)$ is the PDF of $\mathbf{y}$, conditioned on $\mathbf{x}_{jq}$ and $\mathbf{H}$. It can be seen that optimal detection requires a joint detection of the antenna indices and symbols, as opposed to the scheme outlined in Section II-B, where the problem is decoupled.

B. Complexity

To compare the complexity of the optimal detector with the one given in Section II-B, we use the number of multiplications required in the detection process. The number of additions can be shown to have a similar value for both detectors. The complexity of Mesleh's SM detector is obtained from [1] as[2] $\delta_{\text{SM}} = 2N_{\text{r}}N_{\text{t}} + N_{\text{t}} + f(M)$, where the last term depends on the type of demodulation assumed.

Similar to [1], we analyze (4) to obtain the complexity of optimal SM detection. It can be shown that the first and second terms in (4) result in $N_{\text{r}}N_{\text{t}} + M$ and $N_{\text{r}}N_{\text{t}} + N_{\text{t}}M$ multiplications, respectively (for the first term, we simplified the computation by splitting $\|\mathbf{h}_j x_q\|_{\text{F}}^2 = \|\mathbf{h}_j\|_{\text{F}}^2 |x_q|^2$). Hence, the total computational complexity for optimal SM detection is given by $\delta_{\text{SM,opt}} = 2N_{\text{r}}N_{\text{t}} + N_{\text{t}}M + M$.

We observe that the complexity involved between both detectors is comparable. These results motivate the use of our optimal SM detector in order to fully exploit SM's advantages.

IV. Performance Analysis

The performance of the SM system (with the optimal detector) is derived using the well known union bounding technique [4, p. 261-262]. The average bit error rate (BER) in SM is bounded as

$$\begin{aligned} P_{\text{e,bit}} &\le E_{\mathbf{x}}\left[\sum_{\hat{\jmath},\hat{q}} N(q,\hat{q}) P(\mathbf{x}_{jq} \to \mathbf{x}_{\hat{\jmath}\hat{q}})\right] \\ &= \sum_{j=1}^{N_{\text{t}}}\sum_{q=1}^{M}\sum_{\hat{\jmath}=1}^{N_{\text{t}}}\sum_{\hat{q}=1}^{M} \frac{N(q,\hat{q}) P(\mathbf{x}_{jq} \to \mathbf{x}_{\hat{\jmath}\hat{q}})}{N_{\text{t}}M}, \quad (5) \end{aligned}$$

where $N(q,\hat{q})$ is the number of bits in error between the symbol x_q and $x_{\hat{q}}$, and $P(\mathbf{x}_{jq} \to \mathbf{x}_{\hat{\jmath}\hat{q}})$ denotes the pairwise error probability (PEP) of deciding on the constellation vector $\mathbf{x}_{\hat{\jmath}\hat{q}}$ given that $\mathbf{x}_{jq}$ is transmitted. By using (4), the PEP conditioned on $\mathbf{H}$ is given by

$$P(\mathbf{x}_{jq} \to \mathbf{x}_{\hat{\jmath}\hat{q}} \mid \mathbf{H}) = P(d_{jq} > d_{\hat{\jmath}\hat{q}} \mid \mathbf{H}) = Q\left(\sqrt{\kappa}\right),$$

where $d_{jq} = \left(\sqrt{\rho}\,\|\mathbf{g}_{jq}\|_{\text{F}}^2 - 2\text{Re}\left\{\mathbf{y}^H \mathbf{g}_{jq}\right\}\right)$, and $Q(x) = \int_x^\infty \frac{1}{2\pi} e^{-\frac{t^2}{2}} dt$. We define κ as

$$\kappa \triangleq \tfrac{\rho}{2}\,\|\mathbf{g}_{jq} - \mathbf{g}_{\hat{\jmath}\hat{q}}\|_{\text{F}}^2 = \sum_{n=1}^{N_{\text{r}}} |A(n) + iB(n)|^2, \tag{6}$$

where $i = \sqrt{-1}$ and

$$\begin{aligned} A(n) &= \sqrt{\tfrac{\rho}{2}}\left(h_{nj}^R x_q^R - h_{nj}^I x_q^I - h_{n\hat{\jmath}}^R x_{\hat{q}}^R + h_{n\hat{\jmath}}^I x_{\hat{q}}^I\right) \\ B(n) &= \sqrt{\tfrac{\rho}{2}}\left(h_{nj}^R x_q^I + h_{nj}^I x_q^R - h_{n\hat{\jmath}}^R x_{\hat{q}}^I - h_{n\hat{\jmath}}^I x_{\hat{q}}^R\right). \end{aligned}$$

[2] In [1], the complex multiplications involved with $\|\mathbf{h}_j\|_{\text{F}}^2$ and $|\cdot|$ are ignored, and results in a slightly different δ_{SM} .

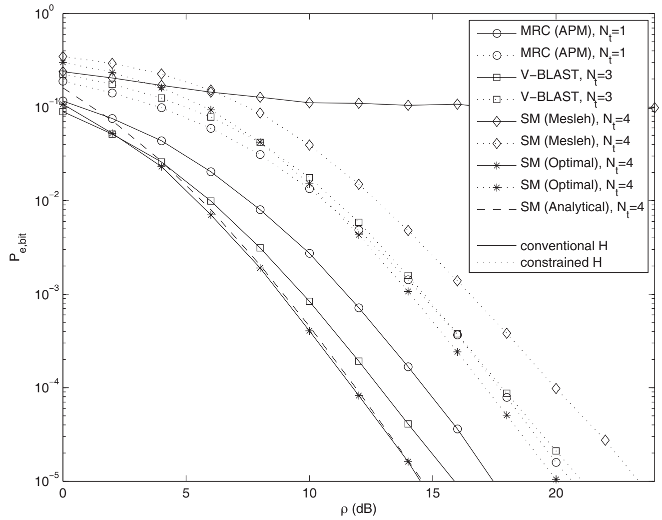

Fig. 2. BER performance of spatial modulation versus SNR, for $m = 3$ bits/s/Hz transmission ($N_\mathrm{r} = 4$).

The superscript R and I denote the real and imaginary part, respectively, and h_{nj} is the element of $\mathbf{H}$ in the n^th row, and j^th column. The distribution of the random variable κ in (6) is not easily obtained since $A\left(n\right)$ and $B\left(n\right)$ are not, in general, independent. In this case, the performance can be evaluated numerically. However, for symbols x drawn from a real constellation $\mathcal{X}$, this independence is satisfied and (6) reduces to $\kappa = \sum_{n=1}^{2N_\mathrm{r}} \alpha_n^2$ where $\alpha_n \sim \mathcal{N}\left(0, \sigma_\alpha^2\right)$ with $\sigma_\alpha^2 = \frac{\rho\left(|x_q|^2 + |x_{\hat{q}}|^2\right)}{4}$. Hence, κ is a chi-squared random variable with $2N_\mathrm{r}$ degrees of freedom and PDF $p_\kappa\left(v\right)$ given in [4, p. 41]. The PEP can then be formulated as $P\left(\mathbf{x}_{jq} \rightarrow \mathbf{x}_{\hat{j}\hat{q}}\right) = \int_{v=0}^{\infty} Q\left(\sqrt{v}\right) p_\kappa\left(v\right) dv$, which has a closed form expression given in [5, Eq. (64)]. Thus,

$$P\left(\mathbf{x}_{jq} \rightarrow \mathbf{x}_{\hat{j}\hat{q}}\right) = \mu_\alpha^{N_\mathrm{r}} \sum_{k=0}^{N_\mathrm{r}-1} \binom{N_\mathrm{r} - 1 + k}{k} \left[1 - \mu_\alpha\right]^k . \quad (7)$$

where $\mu_\alpha = \frac{1}{2}\left(1 - \sqrt{\frac{\sigma_\alpha^2}{1+\sigma_\alpha^2}}\right)$. Plugging (7) into (5), we obtain

$$P_\text{e,bit} \leq \sum_{q=1}^{M} \sum_{\hat{q}=1}^{M} \frac{N_\mathrm{t} N(q,\hat{q}) \mu_\alpha^{N_\mathrm{r}} \sum_{k=0}^{N_\mathrm{r}-1} \binom{N_\mathrm{r}-1+k}{k} \left[1 - \mu_\alpha\right]^k}{M}. \quad (8)$$

V. Simulation Results

In this section, we present some examples to compare the optimal SM detector over the original detection scheme [1]. We perform Monte Carlo simulations for 10^6 channel realizations and plot the average BER performance versus ρ (the average SNR per receive antenna). We target $m = 3$ bits/s/Hz transmission with $N_\mathrm{r} = 4$ antennas. Figure 2 illustrates the simulation results for both constrained (dotted line) and conventional (solid line) channel assumptions (see Section II-B).

For reference, we use two different transmission setups. The first one is APM, 8-QAM transmission with $N_\mathrm{t} = 1$ (single antenna transmission) and $M = 8$. The second is V-BLAST with BPSK modulation, $N_\mathrm{t} = 3$, and ordered successive interference cancellation (OSIC) with the minimum mean squared error (MMSE) receiver [6]. SM with BPSK and $N_\mathrm{t} = 4$ antennas is shown for both sub-optimal [1] and optimal receivers (derived in Section III-A). We also plot the $P_\text{e,bit}$ bound of (8) for SM using BPSK modulation, where $N_\mathrm{t} = 4, M = 2, \sigma_\alpha^2 = \frac{\rho}{2}$ and $N\left(q,\hat{q}\right) = \begin{cases} 0, & q = \hat{q} \\ 1, & q \neq \hat{q} \end{cases}$.

Let us first consider the case of constrained channels (dotted lines). As shown, the optimal SM detector gains 4 dB at $P_\text{e,bit} = 10^{-5}$ over the sub-optimal detector of [1]. Also, (minor) gains are also evident over MRC and V-BLAST, which is not the case with sub-optimal detection. For the conventional channel model (solid line), it is shown that optimal SM provides performance improvements of 3 dB over APM, and 1 dB over V-BLAST (at $P_\text{e,bit} = 10^{-5}$). As well, we notice that the derived BER bounds are relatively tight, and support our simulation results.

VI. Conclusion

In this letter, we derive the optimal SM detector for which, significant performance gains are observed over the detector in [1]. To support our results, we also derive a closed form expression bounding the average BER of SM when real constellations are used. The simulation results indicate that SM with optimal detection outperforms V-BLAST and APM transmission, which makes it a promising candidate for low complexity transmission techniques.

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