CST8233: Quiz# 1

This Quiz Assets Your Knowledge in Floating Point Representation

1. Which is 0.0000675 meter in correct scientific notation?

   a) 6.75×10^{-5} meter
   b) $6.75 \times 10^{+3}$ meter
   c) $0.675 \times 10^{+2}$ meter
   d) 0.675×10^{-2} meter

the answer is A

2. Convert the decimal number 6.75 to binary.
   a) 0110.1110
   b) 0111.1100
   c) 0110.0110
   d) 0110.110

   The answer is D

3. Convert the binary number 1010.1100 to decimal.

a) 12.65
b) 10.875
c) 10.55
d) 10.75

   The ans. Is D

4. A de-normalized number has

a) exponent: 0, mantissa: 0
b) exponent: 0, mantissa: not 0
c) exponent: not 0, mantissa: 0
d) exponent: not 0, mantissa: not 0

answer is b

5. Suppose we have a 7-bit computer that uses IEEE floating-point arithmetic where a floating point number has 1 sign bit, 3 exponent bits, and 3 fraction bits. The number -3.875 in the 7-bit format is

    i. 1 101010
    ii. 0 100111
    iii. 1 100110
    iv. 1 100111

The answer is D

6. A machine **based on the IEEE 754 floating-point standard** stores floating point numbers in 7-bit word. The first bit is used for the sign of the number, the next three for the biased exponent and the next three for the magnitude of the mantissa. The number (0010111)represented in base-10 is:

    (A) 0.375
    (B) 1.5
    (C) 0.9375
    (D) 3.5

    Answer is c

7. A machine **based on the IEEE 754 floating-point standard** stores floating point numbers in 7-bit word. The first bit is used for the sign of the number, the next three for the biased exponent and the next three for the magnitude of the mantissa. The number (0000101)represented in base-10 is:

    (A) 0.475
    (B) 1.5
    (C) 0.15625
    (D) 0.2635

   Answer is c

8. . For the float number 128.0 the exponent in memory based **on the IEEE 754 floating-point standard single precision** is

    (A) 1011 1010
    (B) 1011 1000

(C) 1001 1000

(D) 1000 0110

Answer is D

9. For the bit pattern `1 1000 0001 011 0000 0000 0000 0000 0000` **based on the IEEE 754 floating-point standard single precision** is

(A) 5.9

(B) -5.5

(C) 7.5

(D) -3.5

Answer b

True/False

The IEEE single precision floating-point representation of the $(-3.875)_{10}$ is

1 00100000 01101000000000000000000

False

CST8233: quiz# 2

This Quiz assets your knowledge in Interpolation

1. Polynomials are the most commonly used functions for interpolation because they are easy to
   (A) evaluate
   (B) differentiate
   (C) integrate
   (D) evaluate, differentiate and integrate

   the answer is D

2. The Lagrange polynomial that passes through the 3 data points is given by

x	15	18	22
y	24	37	25

$$f_2(x) = L_0(x)(24) + L_1(x)(37) + L_2(x)(25)$$

The value of $L_1(x)$ at $x = 16$ is most nearly
   (A) –0.071430
   (B) 0.50000
   (C) 0.57143
   (D) 4.3333

   The answer is B

3. Given the following data :.

x	0	16	19	23	24
y	233	246	378	255	342

Using Lagrange Interpolation to estimate the value at x= 22, what the best two data points would you choose for interpolation?
   (E) 23, 24
   (F) 16, 19
   (G) 0, 16
   (H) 19, 24

The answer is A

4. Find the Lagrange polynomial which interpolates the points (0, 1), (1, 0), (3, 4).

Answer: p(x) = 1/3 (x − 1)(x − 3) + 0 + 2/3 x(x − 1)

5- Given the following points: (1,1), (4,2), and (9,3), Find the quadratic polynomial P2 using Newton's divided difference formula.

Answer: P2(x)=1+1/3(x−1)−1/60(x−1)(x−4)

CST8233: Quiz#3

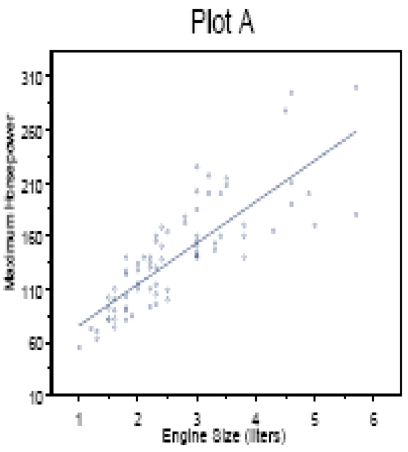
This Quiz assets your knowledge in Regression

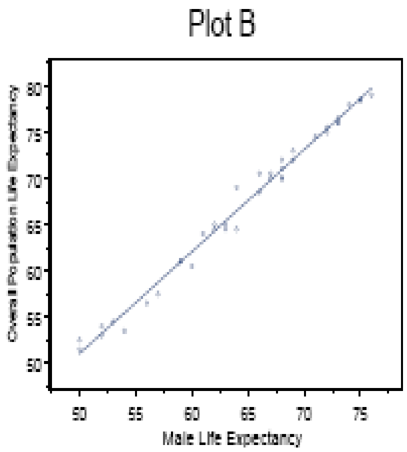
1- A regression line always passes through the point

a) $(0,\ 0)$.
b) $(0,\ \bar{y})$.
c) $(\bar{x},\ 0)$.
d) $(\bar{x},\ \bar{y})$.

The answer is d

2- Which least-squares regression line would have a smaller sum of squared errors (SSE)?





a) The line in Plot A.
b) The line in Plot B.
c) It would be the same for both plots.

The answer is b

3. given the regression equation represent the relation between weight and height:
$\hat{y}$ = 9.9994+0.1332x
If a person increased his/her weight by 10 pounds, by how much (in inches) would one expect to see their height increase?

a) 0.1332
b) 11.332
c) 99.994
d) 1.332 + 9.9994

the answer is b

4. The following fact is not true about standard normal distribution curve:

(A) the mean $\mu = 0$ and $\sigma = 1$
(B) can be used with any data distribution
(C) the area under the curve is 1
(D) can be used to find the probability of a random variable is less than a constant

the answer is b

4. A regression line using 25 observations produced SSR = 118.68 and SSE = 56.32. The standard error of estimate was:
(A) 2.11
(B) 2.44
(C) 1.56
(D) None of these choices.

the answer is c

5. A scatter diagram includes the following data points:

X	3	2	5	4	5
Y	8	6	12	10	14

Two regression models are proposed: (1) $\hat{y} = 1.2 + 2.5x$, and (2) $\hat{y} = 0 + 4.0x$. Using the least squares method, which of these regression models provides the better fit to the data? Why?

(A) The first regression model
(B) The second regression model
(C) Both models are fit the data
(D) None of these choices.

The answer is A

Name : --

ID:---

CST8233: Quiz# 4

This Quiz assets your knowledge in Maclurin and Taylor Series

1. **The difference between a Maclaurin series and a Taylor series is**

a. A Maclaurin series uses derivatives of x and a Taylor series uses powers of x

b. A Maclaurin series uses powers of x and a Taylor series uses powers of the derivatives of x

c. A Maclaurin series uses derivatives at x=0 and a Taylor series uses derivatives at x = a

d. A Maclaurin series uses derivatives at x=a and a Taylor series uses derivatives at x = 0

e. there is no difference between a Maclaurin series and a Taylor series

the answer is c

2. Find the first two nonzero terms of the Maclaurin expansion of the function.
$f(x) = xe^{-x^2}$

a. $x - \frac{1}{2}x^2 + ...$

b. $x + \frac{1}{6}x^3 + ...$

c. $x + x^3 + ...$

d. $x - x^3 + ...$

the answer is D

3. **Find four Terms of Maclaurin Expansion of the function f(x) = x sin 4x**

a. $4x - \frac{32}{3}x^4 + \frac{512}{15}x^6 - \frac{4096}{315}x^8 + ...$

b. $4x - 8x^3 + \frac{32}{3}x^5 - \frac{4096}{45}x^7 + ...$

c. $4x^2 - \frac{32}{3}x^4 + \frac{128}{15}x^6 - \frac{1024}{315}x^8 + ...$

d. $4x^2 + \frac{32}{3}x^4 + \frac{128}{15}x^6 + \frac{1024}{315}x^8 + ...$

the answer is C

4- **Find the Maclaurin polynomial of degree 6 for $\frac{1}{1+4x^2}$**

$$\approx (1 - 4x^2 + 16x^4 - 64x^6)$$

5. **Use a first 3 nonzero term of the series to evaluate the following limits:**

$$\lim_{x \to 0} \frac{x^2\, e^x}{cos(x)-1}$$

$$\approx \frac{1}{3}$$

CST8233: Quiz#5

This Quiz assets your knowledge in Numerical Differentiation

1. Using the forward divided difference approximation with a step size of 0.2, the derivative of the function at $x = 2$ is given as

x	1.8	2.0	2.2	2.4	2.6
$f(x)$	6.0496	7.3890	9.0250	11.023	13.464

(A) 6.697
(B) 7.389
(C) 7.438
(D) 8.180

2. A student finds the numerical value of $f'(x) = 20.220$ at $x = 3$ using a step size of 0.2. Which of the following methods did the student use to conduct the differentiation if $f(x)$ is given in the table below?

x	2.6	2.8	3.0	3.2	3.4	3.6
$f(x)$	$e^{2.6}$	$e^{2.8}$	e^{3}	$e^{3.2}$	$e^{3.4}$	$e^{3.6}$

(A) Backward divided difference
(B) Calculus, that is, exact
(C) Central divided difference
(D) Forward divided difference

3. The recommender method for numerical differentiation is:

(A) backward divided difference
(B) Calculus, that is, exact
(C) Central divided difference
(D) Forward divided difference

4. The given table lists the absolute errors from three Finite difference approximations of the derivative $f'(x)$ (labelled A, B, C). The errors are computed for a sequence of decreasing values of the grid spacing h. Which of the statements below regarding the order of accuracy for the three formulas is TRUE?

h	Formula A	Formula B	Formula C
0.500000	5.764e-01	6.670e-02	9.329e-03
0.250000	3.096e-01	1.683e-02	3.565e-04
0.125000	1.596e-01	4.218e-03	5.181e-05
0.062500	8.093e-02	1.055e-03	4.116e-06
0.031250	4.074e-02	2.638e-04	2.836e-07
0.015625	2.044 e-02	6.595e-05	1.853e-08
0.007812	1.024 e-02	1.649e-05	1.183e-09
0.003906	5.122e-03	4.122e-06	7.513e-11

(A) Formulas A and B are order 1, Formula C is order 2.
(B) Formula A is order 1, Formula B is order 2, Formula C is order 3.
(C) Formula A is order 1, Formula B is order 2, Formula C is order 4.
(D) All formulas have the same order of accuracy, but different error constants.

5. Why we need to approximate the derivative?

CST8233: Quiz#6

This Quiz assets your knowledge in ODE

Use Euler's method to find y-values of the solution for the given values of x and Δx, if the curve of the solution passes through the given point.

1) $\frac{dy}{dx} = 1 + \frac{y}{x}$; x = 3 to x = 4.5; Δx = 0.5; (3, 0)

A)

x	3	3.5	4	4.5
y	0	1.0000	2.1429	6.8214

B)

x	3	3.5	4	4.5
y	0	0.5000	1.0714	1.7054

C)

x	3	3.5	4	4.5
y	0	0.7500	1.2857	2.0464

D)

x	3	3.5	4	4.5
y	0	1.0000	1.6071	3.4107

Answer: B

2. To solve the ordinary differential equation

$$3\frac{dy}{dx} + xy^2 = \sin x,\ y(0) = 5$$

you need to rewrite the equation as

(A) $\frac{dy}{dx} = \sin x - xy^2,\ y(0) = 5$

(B) $\frac{dy}{dx} = \frac{1}{3}\left(\sin x - xy^2\right),\ y(0) = 5$

(C) $\frac{dy}{dx} = \frac{1}{3}\left(-\cos x - \frac{xy^3}{3}\right),\ y(0) = 5$

(D) $\frac{dy}{dx} = \frac{1}{3}\sin x,\ y(0) = 5$

<u>The answer is B</u>

3. Given

$$3\frac{dy}{dx} + 5y^2 = \sin x,\ y(0.3) = 5$$

and using a step size of $h = 0.3$, the value of $y(0.9)$ using the Runge-Kutta 2nd order method is most nearly

(A) –4297.4
(B) –4936.7
(C) -0.21336×10^{14}
(D) -0.24489×10^{14}

The answer is A

First rewrite your equation to be

$$\frac{dy}{dx}=\frac{1}{3}(sinx-5y^2)$$

first step compute y1
use your equations :
k1=f(x0,y0)=f(0.3,5)=-41.5682

k2=

$$=\frac{1}{3}\left(\sin(0.6)-5(-7.4704)^2\right.$$

$$=\frac{1}{3}(0.56464-279.04)$$

K2= −92.824
Y1=y(0.6) =5+(1/2(-*4* 1.5682)+1/2(−92.824)*0.3
= -15.159

Step2:
Now use in your equation y1= -15.159 and x1 =0.6

K1= -382.80
K2= -28166
Y2= -4297.4

Hybrid 2. Bit field representation of Floating point Numbers

1. Find the big-endian hexadecimal representation of the fields in memory that represent the following base 10 float numbers: -0.75, 2.25, 256.25, -0.0625.
[Answers: -0.75 = $BF400000_{16}$, 2.25 = 40100000_{16}, 256.25 = 43802000_{16}, -0.0625 = $BD800000_{16}$]

2. Find the base 10 float numbers that are represented by the following hex fields in memory: C0A80000, 410A0000, 44802000, 3EC00000.
[Answers: $C0A80000_{16}$ = -5.25_{10}, $410A0000_{16}$ = 8.625_{10}, 44802000_{16} = 1025_{10}, $3EC00000_{16}$ = 0.375_{10}]

Hybrid 5

Numerical Differentiation/Integration

<u>Example 1</u>

The following data is used in question 1:

x	0	1	2	3	4	5
y	5.68	6.75	7.32	7.35	6.88	6.24

1.a. Calculate the 1st order derivatives

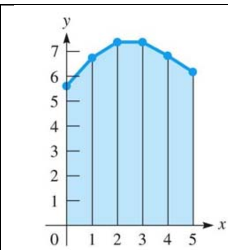
x	0	1	2	3	4	5
y	5.68	6.75	7.32	7.35	6.88	6.24
dy/dx	-	0.82	0.3	-0.22	-0.555	-

1.b. Calculate the 2nd order derivatives

x	0	1	2	3	4	5
y	5.68	6.75	7.32	7.35	6.88	6.24
d^2y/dx^2	-	-0.5	-0.54	-0.5	-0.17	-

1.c. Use the trapezoidal method to calculate the integral of the data between x = 0 and x = 4.

Plotted, the data looks like this, with a step size Δx = 1.



Trapezoid integral = 1.0 (5.68/2 + 6.75 + 7.32 + 7.35 + 6.88/2) = 27.70

1.c. Use Simpson's 1/3 rule to calculate the integral of the data between x = 0 and x = 4.
Integral by Simpson's rule = 1/3(5.68 + 4(6.75) + 2(7.32) + 4(7.35) + 6.88)

= 1.0/3(5.68 + 27.0 + 14.64+ 29.4 + 6.88) = 27.87

<u>Example 2</u>
Calculate the integral of the following data using both the trapezoidal rule and Simpson's 1/3 rule

x	2	4	6	8	10	12	14
y	0.670	2.34	4.56	3.67	3.56	4.78	6.87

The step size is Δx = 2.

Answer. Using the trapezoidal rule
Integral = 2(0.670/2 + 2.34 + 4.56 + 3.67 + 3.56 + 4.78 + 6.87/2) = 45.36

Answer. Using Simpson's 1/3 rule
Integral = 2/3(0.67 +4*2.34 + 2*4.56 + 4*3.67 + 2*3.56 + 4*4.78 + 6.87)

= 2/3(0.67 + 9.36 + 9.12 + 14.68 + 7.12 + 19.12 + 6.87) = 44.63

Example 3 – Approximating an integral of a mathematical function

(If you can't integrate a function analytically, you can evaluate the function at regular steps and then integrate it numerically.)

Using both the trapezoidal method and Simpson's 1/3 method, evaluate the integral of the function

f(x) = $x\sqrt{(x+1)}$

over the range x = 2.0 to x = 3.0 in steps of 0.1

(the exact integral = 4.6954 to 4 decimal places.)

Answer

By evaluating the function, the data for the integral is

x	2.0	2.1	2.2	2.3	2.4
f(x)	3.4641016	3.6974315	3.9354796	4.1781575	4.4253813

x	2.5	2.6	2.7	2.8	2.9	3.0
f(x)	4.6770717	4.9331532	5.1935537	5.4582048	5.7270411	6.0

a. Trapezoidal

Integral = $\frac{0.1}{2}[3.464\,101\,6 + 2(3.697\,431\,5) + \cdots + 2(5.727\,041\,1) + 6.000\,000\,0]$

= 4.6958 to 4 decimal places

b. Simpson 1/3

$$\int_2^3 x\sqrt{x+1}\,dx = \frac{0.1}{3}\Big[3.464\,101\,6 + 4(3.697\,431\,5) + 2(3.935\,479\,6) + 4(4.178\,157\,5) + 2(4.425\,381\,3) + 4(4.677\,071\,7) + 2(4.933\,153\,2) + 4(5.193\,553\,7) + 2(5.458\,204\,8) + 4(5.727\,041\,1) + 6.000\,000\,0\Big]$$

$$= \frac{0.1}{3}(140.86156) = 4.695\,385\,4$$

= 4.6954 to 4 decimal places

which agrees with the exact value to this number of decimal places

Example 4

The velocity v (kilometres per hour) of a car was recorded at 1 minute intervals as shown:

t(min)	0	1	2	3	4	5	6	7	8	9	10
V(km/h)	60	62	65	69	72	74	76	77	77	75	76

4.a. Calculate the acceleration of the car in kilometres/hour/hour

Answer. The acceleration is the 1st derivative of the velocity.

The step size between measurements Δx = 1 minute = 1/60 hour

t(min)	0	1	2	3	4	5	6	7	8	9	10
V(km/h)	60	62	65	69	72	74	76	77	77	75	76

Accn(km/h/h)	-	150	210	210	150	120	90	30	-60	-30	-

4.b. Calculate the distance travelled over the 0 to 10 minutes using the trapezoidal method
Answer
Distance = 1/60(60/2 + 62 + 65 + 69 + 72 + 74 + 76 + 77 + 77 + 75 + 76/2) = 11.917 km

4.c. Calculate the distance travelled over the 0 to 10 minutes using Simpson's 1/3 method.
Answer
Distance = (1/60)/3(60 + 4(62) + 2(65) + 4(69) + 2(72) + 4(74) + 2(76) + 4(77) + 2(77) + 4(75) + 76)
= 11.911

Hybrid 3 - STATISTICS

In Exercises below, use the following data.

An airline's records showed that the percent of on-time flights each day for a 20-day period was as follows:

72, 75, 76, 70, 77, 73, 80, 75, 82, 85, 77, 78, 74, 86, 72, 77, 67, 78, 69, 80

3. Determine the mean.

5. Construct a frequency distribution table with five classes and a lowest class limit of 67.

7. Draw a histogram for the data in Exercise 5.

In Exercises 11–16, use the following data: An important property of oil is its coefficient of viscosity, which gives a measure of how well it flows. In order to determine the viscosity of a certain motor oil, a refinery took samples from 12 different storage tanks and tested them at 50°C. *The results (in pascal-seconds) were 0.24, 0.28, 0.29, 0.26, 0.27, 0.26, 0.25, 0.27, 0.28, 0.26, 0.26, 0.25*.

11. Find the mean.

13. Find the standard deviation.

In Exercises 31–36, use the following data: Police radar on a city street recorded the speeds of 110 cars in a 65 km/h *zone. The following table shows the class marks of the speeds recorded and the number of cars in each class*.

Speed (km/h)	40	45	50	55	60	65	70	75	80	85
No. cars	3	4	4	5	8	22	48	10	4	2

31. Find the mean.

33. Find the standard deviation.

In Exercises 47–50, use the following data: After analysing data for a long period of time, it was determined that samples of500 readings of an organic pollutant for an area are distributed normally. For this pollutant, $\mu = 2.20\ \mu\text{g/m}^3$ *and* $\sigma = 0.50\ \mu\text{g/m}^3$.

47. In a sample, how many readings are expected to be between 1.50 μg/m^3 and 2.50 μg/m^3?

49. In a sample, how many readings are expected to be above 1.00 mg/m^3?

51. In a certain experiment, the resistance *R* of a certain resistor was measured as a function of the temperature *T*. The data found are shown in the following table. Find the least-squares line, expressing *R* as a function of *T*.

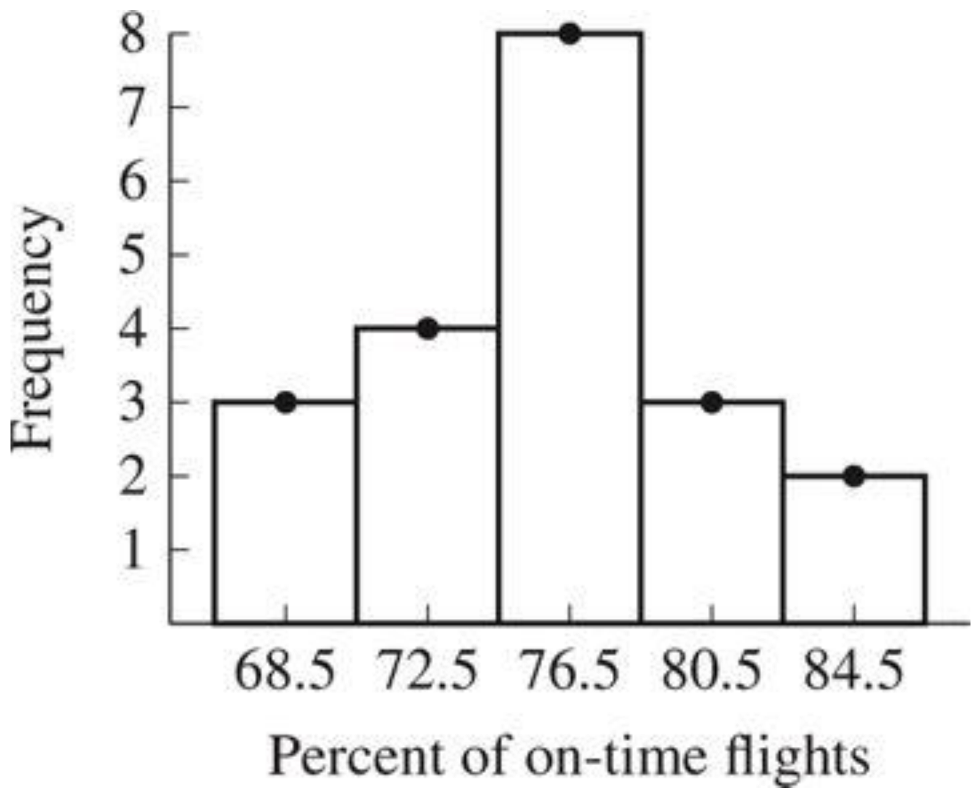
T (°c)	0.0	20.0	40.0	60.0	80.0	100
R (Ω)	25.0	26.8	28.9	31.2	32.8	34.7

ANSWERS

3. 76.2

5.

class	*f*
67–70	3
71–74	4
75–78	8
79–82	3
83–86	2



7.

9.

Class	f
<71	3
<75	7
<79	15
<83	18
<87	20

11. 0.264 Pa · s

13. 0.014 Pa · s

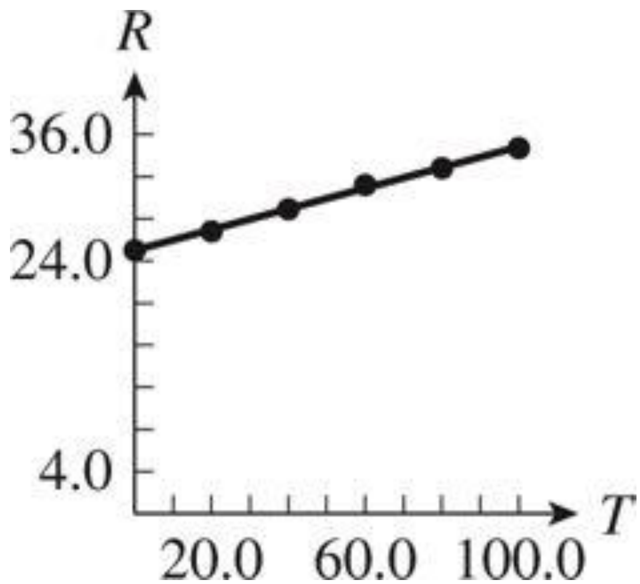
31. 66.2 km/h

33. 9.0 km/h

47. 322

49. 495

51. $R = 0.0983T + 25.0$



Hybrid 4

Solving Ordinary Differential equations using Euler's Method

Answers included

Use Euler's method to find y-values of the solution for the given values of x and Δx, if the curve of the solution passes through the given point.

1.

$$\frac{dy}{dx} = x + 1;$$

$x = 0$ to $x = 1$; $\Delta x = 0.2$; $(0, 1)$

Answer:

x	0.0	0.2	0.4	0.6	0.8	1.0
y	1.00	1.20	1.44	1.72	2.04	2.40

2.

$$\frac{dy}{dx} = y(0.4x + 1);$$

$x = -0.2$ to $x = 0.3$; $\Delta x = 0.1$; $(-0.2, 2)$

Answer:

x	−0.2	−0.1	0.0	0.1	0.2	0.3
y	2.0000	2.1840	2.3937	2.6330	2.9069	3.2208

3. The differential equation of Exercise 1 with $\Delta x = 0.1$

Answer:

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
y	1.00	1.10	1.21	1.33	1.46	1.60	1.75	1.91

x	0.8	0.9	1.0
y	2.08	2.26	2.45

Hybrid 2 Maclaurin (Taylor) Series

Exercise 1

Using the basic definition of a Maclaurin series find the first three nonzero terms of the following functions.

#1 f(x) = e^x	answer: $1 + x + \frac{1}{2}x^2 + \ldots$
#2. f(x) = cos(x)	answer: $1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \ldots$
#3 $f(x) = e^{-2x}$	answer: $1 - 2x + 2x^2 - \ldots$
#4 $f(x) = \cos 4\pi x$	answer: $1 - 8\pi^2 x^2 + \frac{32}{3}\pi^4 x^4 - \ldots$

Exercise 2

In the following, find the first four nonzero terms of the Maclaurin expansions of the given functions by using the following standard results:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

#1 $f(x) = e^{3x}$	answer: $1 + 3x + \frac{9}{2}x^2 + \frac{9}{2}x^3 + \ldots$
#2 $f(x) = \sin \frac{1}{2}x$	answer: $\frac{x}{2} - \frac{x^3}{2^3 3!} + \frac{x^5}{2^5 5!} - \frac{x^7}{2^7 7!} + \ldots$
#3 $f(x) = x \cos 4x$	answer: $x - 8x^3 + \frac{32}{3}x^5 - \frac{256}{45}x^7 + \ldots$

Exercise 3.

Calculate the value of each of the given functions. Use the indicated number of terms of the appropriate Maclaurin series. Compare with the value found directly on a calculator.

#1 $e^{0.2}$ (3)	answer: 1.22, 1.221 402 8
#2 sin 0.1 (2)	answer: 0.099 833 3, 0.099 833 4
#3 e (7)	answer: 2.718 055 6, 2.718 281 8
#4 $\cos \pi^{\circ}$ (2)	answer: 0.998 496 77, 0.998 497 15

Exercise 4.
Calculate the value of the following function using a Taylor series, taking a = 1.0, using all the terms up to and including x^3

#1 $e^{1.2}$	answer: 3.32

Exercise 5.

#1a. Derive the Maclaurin series expansion for the function $f(x) = (e^x + e^{-x})/2$ for the first three non-zero terms	*Answer.* $e^x = 1 + x + x^2/2 + x^3/6 + x^4/24 + ...$ $e^{-x} = 1 - x + x^2/2 - x^3/6 + x^4/24 + ...$ So $(e^x + e^{-x})/2 = 1 + x^2/2 + x^4/24 +$
#1b. Write a numerical expression for the estimated % fractional (relative) error in your series from 1**a** at x = 1.0 when only two terms are used.	*Answer.* At x = 1 with two terms f(x) = 1 + 1/2! = 3/2 error is ~first truncated term = 1/4! = 1/24 so the % fractional error = 100 x 1/24 x 2/3 = 2.8%
#2a. Derive the Maclaurin series expansion for the function $f(x) = (e^x - e^{-x})/2$ for the first three non-zero terms.	*Answer.* $e^x = 1 + x + x^2/2 + x^3/6 + x^4/24 + x^5/120 + ...$ $e^{-x} = 1 - x + x^2/2 - x^3/6 + x^4/24 - x^5/120 + ...$ So $(e^x - e^{-x})/2 = x + x^3/6 + x^5/120 + ...$
#2b. Write a numerical expression for the estimated % fractional (relative) error in your series from **2a** at x = 1.0 when only two terms are used.	*Answer.* At x = 1, f(x) = 1 + 1/6 = 7/6 error is ~first truncated term = 1/120 so the % fractional error = 100 x 1/120 x 6/7 = 100/140 = 0.71%

Exercise 6

#1a. Derive the Maclaurin series expansion for the function f(x) = xcos(4x) for the first three nonzero terms	*Answer.* *This is an example from exercise 2:* $f(x) = x(1 - (4x)^2/2! + (4x)^4/4!) = x - 4^2x^3/2! + 4^4x^5/4! = x - 8x^3 + 32x^5/3$
#1b. Write down a numerical expression for an estimate of the % relative series error in your series from 1a at x = 0.1 when the series is truncated after the second term (the third term and higher are omitted from the series)	*Answer.* *At x = 0.1 for two terms* $f(x) = 0.1 - 8*(0.1)^3 = 0.1 - 0.008 = 0.092$ *%relative error from 3rd term* $= 100*(32*10^{-5}/3)/0.092 = 0.116\%$

Exercise 7

#1a. Derive the Maclaurin series expansion for the function `f(x) = e`^{x}`cos(x)` up to and including the term in x^4.	*Answer.* $e^x\cos x = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots\right)$ By multiplying the series on the right, we have the following result, considering through the x^4 terms in the product. $1\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}\right)$ $x\left(1 - \frac{x^2}{2!}\right)$ $\frac{x^2}{2!}\left(1 - \frac{x^2}{2!}\right)$ $\left(\frac{x^3}{3!} + \frac{x^4}{4!}\right)(1)$ $e^x\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} + x - \frac{x^3}{2} + \frac{x^2}{2} - \frac{x^4}{4} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots$ $= 1 + x - \frac{1}{3}x^3 - \frac{1}{6}x^4 + \cdots$
#1b. Write down a numerical expression for the estimated % fractional error in your series from 1a at x = 1 when the series contains only up to and including the term in x^3	*Answer.* At x = 1, f(x) = 1 + 1 - 1/3 = 5/3 The error is approximated as the first neglected term = $-x^4/6 = -1/6$ so the % fractional error = -(1/6)/(5/3)*100 = (-1/10)*100 = -10%

Example 1.10.1 Consider the initial-value problem

$$y' = y - x, \quad y(0) = \tfrac{1}{2}.$$

Use Euler's method with (a) $h = 0.1$ and (b) $h = 0.05$ to obtain an approximation to $y(1)$. Given that the exact solution to the initial-value problem is

$$y(x) = x + 1 - \tfrac{1}{2}e^x,$$

compare the errors in the two approximations to $y(1)$.

Solution: In this problem we have

$$f(x, y) = y - x, \qquad x_0 = 0, \qquad y_0 = \tfrac{1}{2}.$$

(a) Setting $h = 0.1$ in (1.10.2) yields

$$y_{n+1} = y_n + 0.1(y_n - x_n).$$

Hence,

$$y_1 = y_0 + 0.1(y_0 - x_0) = 0.5 + 0.1(0.5 - 0) = 0.55,$$
$$y_2 = y_1 + 0.1(y_1 - x_1) = 0.55 + 0.1(0.55 - 0.1) = 0.595.$$

Continuing in this manner, we generate the approximations listed in Table 1.10.1, where we have rounded the calculations to six decimal places.

n	x_n	y_n	Exact Solution	Absolute Error
1	0.1	0.55	0.547414	0.002585
2	0.2	0.595	0.589299	0.005701
3	0.3	0.6345	0.625070	0.009430
4	0.4	0.66795	0.654088	0.013862
5	0.5	0.694745	0.675639	0.019106
6	0.6	0.714219	0.688941	0.025278
7	0.7	0.725641	0.693124	0.032518
8	0.8	0.728205	0.687229	0.040976
9	0.9	0.721026	0.670198	0.050828
10	1.0	0.703129	0.640859	0.062270

2- Use Euler's method to find the solution to the differential equation $dy/dx=3x+4y$ at $x=1$ with the initial condition $y(0)=0$ and step size $h=0.25$.

We continue using Euler's method until $x=1$. The results of Euler's method are in the table below.

n	xn	yn
0	0	0
1	0.25	0
2	0.5	0.1875
3	0.75	0.75
4	1	2.0625