

Introduction to Statistical Machine Learning

Homework 5

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1.

$$\begin{aligned}
 p(y=1|\mathbf{x}) &= \frac{p(\mathbf{x}|y=1)p(y=1)}{p(\mathbf{x})} \\
 &= \frac{p(y=1)}{p(\mathbf{x})} N(\mu_1, \Sigma_1) \\
 &= \frac{p(y=1)}{p(\mathbf{x})} \prod_{j=1}^d N(\mu_{1,j}, \sigma_j^2) \\
 &= \frac{p(y=1)}{p(\mathbf{x})} \prod_{j=1}^d \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{(x_j - \mu_{1,j})^2}{2\sigma_j^2}\right) \\
 &= \frac{p(y=1) \prod_{j=1}^d \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{(x_j - \mu_{1,j})^2}{2\sigma_j^2}\right)}{p(y=0) \prod_{j=1}^d \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{(x_j - \mu_{0,j})^2}{2\sigma_j^2}\right) + p(y=1) \prod_{j=1}^d \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{(x_j - \mu_{1,j})^2}{2\sigma_j^2}\right)} \\
 &= \frac{1}{1 + \frac{p(y=0) \prod_{j=1}^d \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{(x_j - \mu_{0,j})^2}{2\sigma_j^2}\right)}{p(y=1) \prod_{j=1}^d \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{(x_j - \mu_{1,j})^2}{2\sigma_j^2}\right)}} \\
 &= \frac{1}{1 + \frac{p(y=0)}{p(y=1)} \prod_{j=1}^d \exp((x_j - \mu_{1,j})^2 - (x_j - \mu_{0,j})^2)} \\
 &= \frac{1}{1 + \exp\left(\log \frac{p(y=0)}{p(y=1)} + \sum_{j=1}^d (\mu_{1,j}^2 - \mu_{0,j}^2) + 2 \sum_{j=1}^d (\mu_{0,j} - \mu_{1,j}) x_j\right)}
 \end{aligned}$$

\therefore The posterior $p(y=1|x)$ resulting from Gaussian generative model has the same form as the posterior in logistic regression model with the bias and weight below.

$$\begin{cases} w_0 = \sum_{j=1}^d (\mu_{1,j}^2 - \mu_{0,j}^2) \\ \mathbf{w} = 2(\mu_0 - \mu_1) \end{cases}$$

2. Yes, they produce the same classifier because it's a convex optimization problem.
3. Please, see a Jupyter notebook file submitted together.

References

- [1] Christopher M. Bishop, Pattern Recognition and Machine Learning
- [2] Discussion with Tomoki Tsujimura and Bowen Shi