

1. [(a)]

$$\dim(A) = \text{rank}(A) + \text{null}(A) \quad (1)$$

$$n = m + \text{null}(A) \quad (2)$$

$$\text{null}(A) = n - m \quad (3)$$

(b)

$$\text{null}(A) = \dim(\ker(A)) \quad (4)$$

Then,  $\ker(A)$  can have a basis  $B$  s.t.  $\text{Span}(B) = \ker(A)$ . i.e.

$$\forall \mathbf{v} \in \ker(A), \exists a_1, \dots, a_{n-m} \in \mathbb{F}_2, \quad (5)$$

$$\mathbf{v} = a_1 \mathbf{b}_1 + \dots + a_{n-m} \mathbf{b}_{n-m} (b_i \in B) \quad (6)$$

The answer is  $2^{n-m}$ .

(c)

$$\forall \mathbf{x} \text{ s.t. } \begin{cases} A\mathbf{x} = \mathbf{b} \\ A\mathbf{x}_0 = \mathbf{b} \end{cases} \quad (7)$$

$$A(\mathbf{x} - \mathbf{x}_0) = 0 \quad (8)$$

$$\mathbf{x} - \mathbf{x}_0 \in \ker(A) \quad (9)$$

2. [(a)]

$$foo \quad (10)$$