1. (a)

$$dim(A) = rank(A) + null(A) \tag{1}$$
$$n = m + null(A) \tag{2}$$
$$null(A) = n - m \tag{3}$$

(b)

$$null(A) = dim(ker(A)) \tag{4}$$

Then, $ker(A)$ can have a basis B s.t. $Span(B) = ker(A)$. i.e.

$$\forall \mathbf{v} \in ker(A), \exists a_1, ..., a_{n-m} \in \mathbb{F}_2, \tag{5}$$
$$\mathbf{v} = a_1\mathbf{b}_1 + \cdots + a_{n-m}\mathbf{b}_{n-m}(b_i \in B) \tag{6}$$

$\therefore$ The answer is 2^{n-m}.

(c)

$$\forall \mathbf{x} \text{s.t.} \begin{cases} A\mathbf{x} = \mathbf{b} \\ A\mathbf{x_0} = \mathbf{b} \end{cases} \tag{7}$$
$$\therefore \;\; A(\mathbf{x} - \mathbf{x}_0) = 0 \tag{8}$$
$$\mathbf{x} - \mathbf{x}_0 \in ker(A) \tag{9}$$

Then, choosing each element of $\mathbf{x}$ carefully (1 or 0), $\mathbf{x} - \mathbf{x}_0$ can be any element of $\mathbb{F}_2^n$.

$$\therefore \;\; \{\mathbf{x} - \mathbf{x}_0 | A\mathbf{x} = b\} = ker(A) \tag{10}$$

$\therefore$ $\mathbf{x} - \mathbf{x}_0$ has 2^{n-m} solutions.
$\therefore$ $\mathbf{x}$ has 2^{n-m} solutions.

2.

$$f(c\mathbf{v} + (-c)\mathbf{v}) \geq \min\{f(\mathbf{v}), f(\mathbf{v})\} \tag{11}$$
$$\therefore \;\; f(\mathbf{0}_V) \geq f(\mathbf{v}) \tag{12}$$