# Math 269B Sample Final Exam

[**1**] True or False

(a) ________ If a numerical method is unconditionally stable, then one can use any size timestep to get an acceptable numerical approximation.

(b) ________ The timestep restrictions associated with explicit methods for the diffusion equation are usually more severe than the timestep restrictions associated with transport equations.

(c) ________ If a tri-diagonal matrix is strictly diagonally dominant, then there exists an LU factorization.

(d) ________ The LU factorization of a tri-diagonal matrix requires O(M$^2$) operations, where M is the dimension of the matrix.

(e) ________ The relation between the timestep and the mesh size required to satisfy a CFL condition will be more restrictive than any relation required to establish stability.

(f) ________ A consistent method can satisfy a CFL condition and yet not converge to the solution of the differential equation.

(g) ________ Von Neumann stability analysis can be applied to multi-step schemes.

(h) ________ All finite difference methods for linear constant coefficient PDE's can be obtained by applying an ODE method to the equations arising from a semi-discretization of the PDE that is continuous in time and discrete in space.

[2] Consider the semi-implicit finite difference method

$$v_m^{n+1} = v_m^n - k\,a\left(\frac{v_m^n - v_{m-1}^n}{h}\right) + k\,b\,\frac{v_{m+1}^{n+1} - 2\,v_m^{n+1} + v_{m-1}^{n+1}}{h^2}$$

used to create an approximate solution to the equation $v_t + a\,v_x = b\,v_{xx}$, $a$ and $b > 0$, for $x \in [0,\,1]$, $t \in [0,\,\mathrm{T}]$, and v periodic.

[2][a] Give a derivation of the leading term of an expression for the local truncation error of the method.

[2][b] Derive a stability estimate that can be used to establish an error bound for this method.

[2][c] Using your results from parts [a] and [b] give a derivation of an error bound for the numerical solution.

[3] Consider the equation $v_t + a\, v_x = b\, v_{xx}$, $b > 0$, for $x \in [0,\ 1]$, $t \in [0,\ \mathrm{T}]$, with initial condition $v(0, x) = v_0(x)$ and boundary conditions $v(t, 0) = 1$ and $v_x(t, 1) = 2$.

[a] If the interval [0,1] is decomposed into 5 panels, what is the dimension of the vector used to store the unknown values of the solution at a given time level?

[b] Give, in matrix/vector form, the finite difference method that arises when one uses $\mathrm{D}_+\mathrm{D}_-$ to approximate $v_{xx}$, $\mathrm{D}_0$ to approximate $v_x$, and backward Euler is used for the timestepping. (Since there are only a small number of unknowns – please give all entries of the resulting matrices and vectors)

[4] Consider the equation

$$\begin{aligned} u_t &= b_1\, u_{xx} + b_2\, u_{yy} + c\, u \\ u(x, y, 0) &= f(x, y) \end{aligned}$$

to be solved for $0 \le x \le 1$, $0 \le y \le 1$, $t > 0$ and $u$ periodic in $x$ and $y$, $b_1$, $b_2 > 0$ and $c < 0$.

**(a)** Give a convergent method that is second order accurate in space, second order accurate in time, is unconditionally stable, and only requires the solution of tri-diagonal systems of equations to advance the solution one timestep.

**(b)** Prove that your scheme is unconditionally stable.