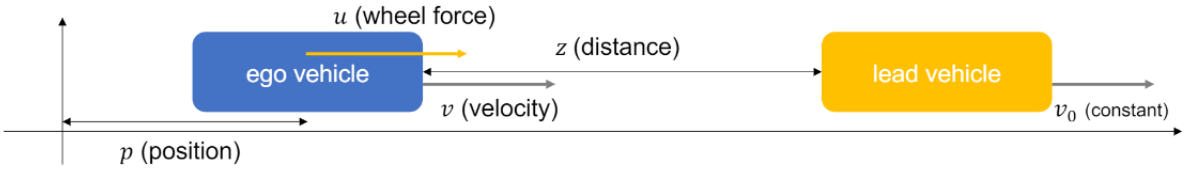
QP Testcase

Problem Formulation

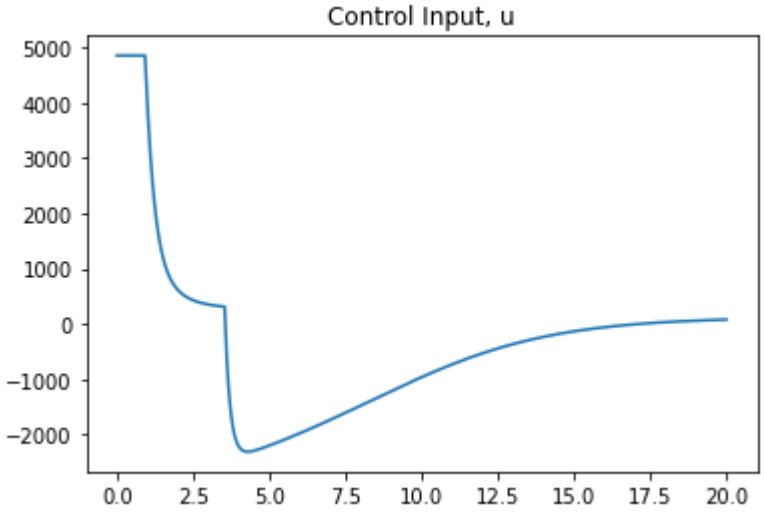


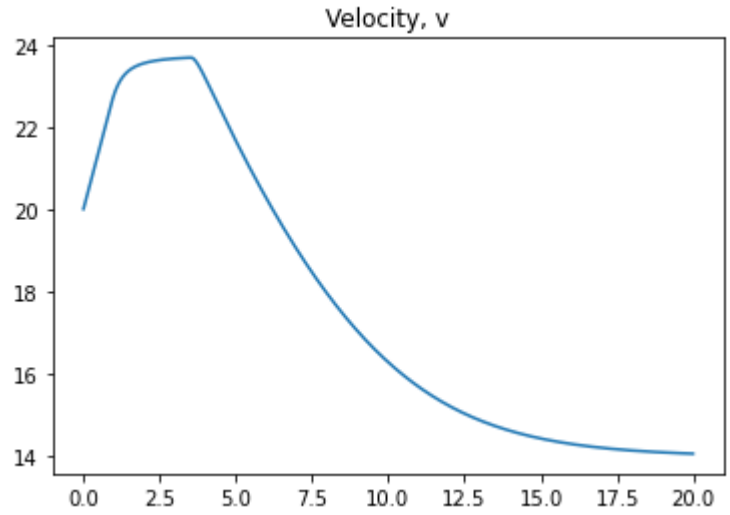
- State $x = [p\ v\ z]^T \in \mathbb{R}^3$ Control Input $u \in \mathbb{R}^1$
- Dynamics $\dot{x} = \begin{bmatrix} v \\ -\frac{1}{m}F_r(v) \\ v_0 - v \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \\ 0 \end{bmatrix} u$ $\dot{x} = f(x) + g(x)u$, where $F_r(v) = f_0 + f_1 v + f_2 v^2$ is the resistance
- Input Constraints $-mc_d g \leq u \leq mc_a g$
- Stability objective $v \longrightarrow v_d$ (v_d : desired velocity)
- Safety objective $z \geq T_h v$ (T_h : lookahead time)
- CLF $V(x) = (v - v_d)^2$ Constraint $(v - v_d)\left\{\frac{2}{m}(u - F_r) + \lambda(v - v_d)\right\} \leq \delta$
- CBF $h(x) = z - T_h v - \frac{\frac{1}{2}(v - v_0)^2}{c_d g}$ Constraint $\frac{1}{m}(T_h + \frac{v - v_0}{c_d g})(F_r(v) - u) + \gamma\left(v_0 - v + z - T_h v - \frac{(v - v_0)^2}{2c_d g}\right) \geq 0$ (Feasible Guaranteed)
- Final QP $argmin(u, \delta)$ $\frac{1}{2}\begin{bmatrix} u \\ \delta \end{bmatrix}^T \begin{bmatrix} \frac{4}{m^2} & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} u \\ \delta \end{bmatrix} + \begin{bmatrix} -\frac{2F_r}{m^2} & 0 \end{bmatrix} \begin{bmatrix} u \\ \delta \end{bmatrix}$

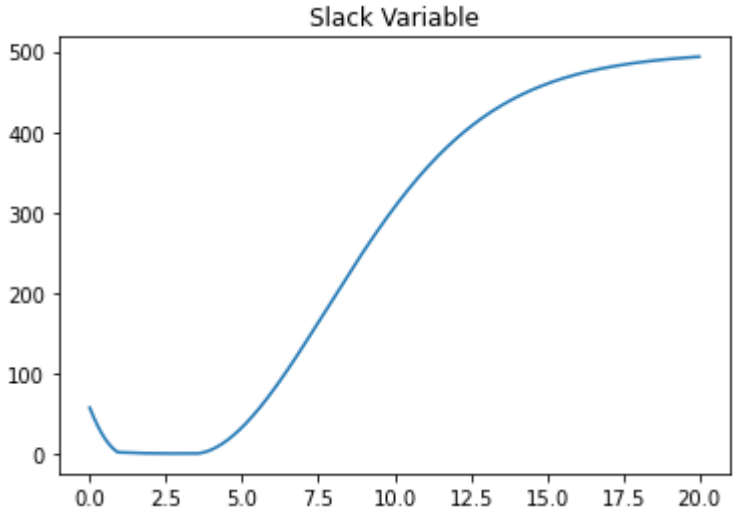
Performance on Different Solvers

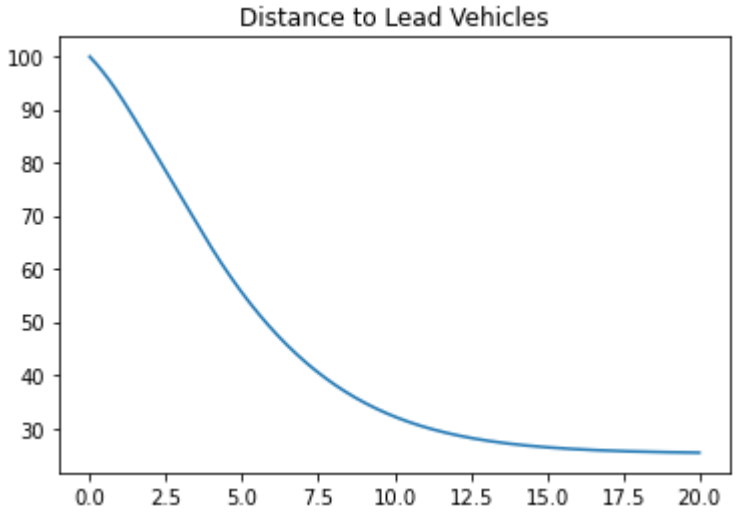
CVXOPT

Results:





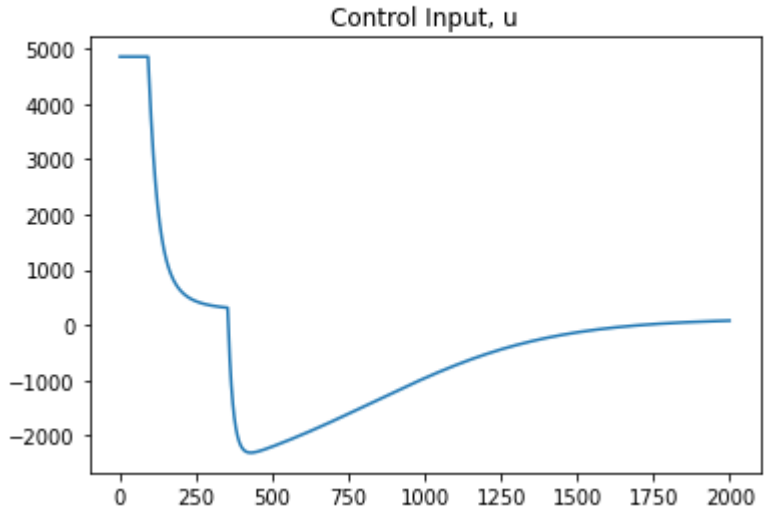




Executing time: 21.39902138710022 (2000 Iteration)

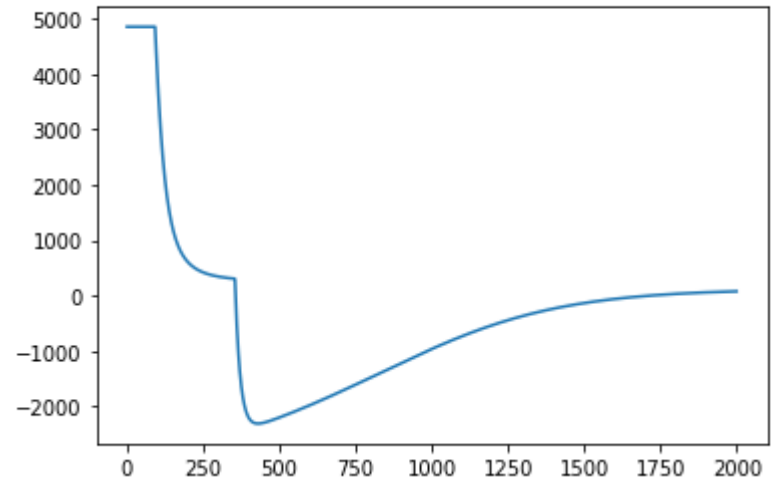
Quadprog

```
Executing time: 0.07271766662597656
Text(0.5, 1.0, 'Control Input, u')
```



CVXPY

```
Executing time: 25.958006858825684
```



Gurobi

```
Executing time: 135.90692472457886
```