THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Tutorial Week 3

STAT3023: Statistical Inference — Semester 2, 2023

1. Let $\mathbf{X} = (X_1, \ldots, X_n)$ follow a multinomial distribution with m trials and cell probabilities $\mathbf{p} = (p_1, \ldots, p_n)$ such that $\sum_{i=1}^n p_i = 1$.

(a) Show that $\text{Cov}(X_i, X_j) = -mp_ip_j$.

(b) Derive the marginal distribution of (X_1, X_2).

(c) Derive the conditional distribution $(X_3, \ldots X_n | X_1 = x_1, X_2 = x_2)$.

2. (Probably integral transform) Let X be a continuous random variable with a strictly increasing cdf $F_X(x)$. Define the random variable $Y = F_X(X)$. Show that Y is uniformly distributed on $(0, 1)$.

3. Let X be a standard exponential random variable Exp(1), and define Y to be the integer part of $X + 1$, that is

$$Y = i + 1 \text{ if and only if } i \leq X < i + 1,\ i = 0, 1, 2, \ldots$$

Identify the distribution of Y.

4. (Problem 7.30 from Freund's) Consider the two random variables (X, Y) with the joint density

$$f_{X,Y}(x, y) = \begin{cases} 12xy(1-y) & 0 < x < 1,\ 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Let $Z = XY^2$. Find the joint probability density of (Y, Z) and then integrate Y out to find the probability density of Z.

5. Let U and V be independent χ^2_m and χ^2_n random variables. Define

$$X = \frac{U/m}{V/n}$$

This random variable X is said to follow an F-distribution with m and n degrees of freedom and we write, $X \sim F_{m,n}$.

(a) Derive the density of X.

(b) Let T denote the t-distribution with n degrees of freedom. Show that $T^2 \sim F_{1,n}$.