

Exponential family

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Exponential family

A pdf or pmf is said to belong to an exponential family if it can be written in the form

$$f(x|\theta) = h(x) \exp \left(\sum_{i=1}^k w_i(\theta) t_i(x) - A(\theta) \right), \quad x \in R$$

with $h(\cdot), w_i(\cdot), t(\cdot), A(\cdot)$ being real-valued functions.

- $h(x)$ often contains information about the support (or the range of X) through the use of indicator function.
- $A(\theta)$ is the normalizing constant to make f a valid pdf or pmf.

Examples

Examples

Natural parameters

The exponential family can be parameterized into the **canonical form**

$$\begin{aligned} f(x|\theta) &= h(x) \exp \left(\sum_{i=1}^k \eta_i t_i(x) - A^*(\boldsymbol{\eta}) \right) \\ &= h(x) \exp \left(\boldsymbol{\eta}^\top T(x) - A^*(\boldsymbol{\eta}) \right), \quad x \in R \end{aligned}$$

where $\boldsymbol{\eta} = (\eta_1, \dots, \eta_k)$ is the set of **natural parameters**, and $T(x) = (t_1(x), \dots, t_k(x))^\top$ is the set of **sufficient statistics** (more on it later).

Examples

Examples

Properties

- $E \{T(X)\} = \frac{\partial A^*}{\partial \boldsymbol{\eta}^\top}$
- $\text{Var} \{T(X)\} = \frac{\partial A^*}{\partial \boldsymbol{\eta} \partial \boldsymbol{\eta}^\top}$.
- $\log M_{T_X}(s) = A^*(s + \boldsymbol{\eta}) - A^*(\boldsymbol{\eta})$

Examples