THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Tutorial Week 1

STAT3023: Statistical Inference — Semester 2, 2023

1. Exercises from the textbook (Freund's, 8th ed.) 4.36, 4.37, 4.40.

(a) Explain why there can be no random variable for which $M_X(t) = \frac{t}{1-t}$.

(b) Show that if a random variable has the probability density function

$$f(x) = \frac{1}{2}e^{-|x|} \quad \text{for } -\infty < x < \infty,$$

its moment-generating function is given by

$$M_X(t) = \frac{1}{1-t^2}.$$

(c) Given the moment-generating function $M_X(t) = e^{3t+8t^2}$, find the moment-generating function of the random variable $Z = \frac{1}{4}(X-3)$, and use it to determine the mean and the variance of Z.

2. (Based on 6.12) Find the moment generating function of $X \sim \text{Gamma}(\alpha, \beta)$. Recall the density function is

$$f_X(x) = \begin{cases} \frac{e^{-x/\beta}x^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

3. (Based on 7.43) If n independent random variables have the same gamma distribution with the parameters α and β, find the moment generating function of their sum and identify its distribution.

4. A continuous (positive) random variable X is said to have a standard log normal distribution if $Y = \log(X) \sim N(0,1)$.

(a) Using the fact that $M_Y(t) = \exp\left(\frac{t^2}{2}\right)$, derive $E(X^r)$ for any $r = 1, 2, \ldots$

(b) It can be proved that (the proof will be covered in Week 3), the pdf of X is given by

$$f_X(x) = \frac{1}{x\sqrt{2\pi}} \exp\left(-\frac{\{\log(x)\}^2}{2}\right), \quad x > 0.$$

Prove that the moment generating function of X does not exist (in the way it is defined in the lecture). Specifically, prove that for any positive t, the expectation $E\{\exp(tX)\}$ is not finite.

5. Let $X_i,\ i = 1, 2, \ldots, n$, be a sequence of independent Rademacher random variables, i.e., $P(X_i = 1) = P(X_i = -1) = 0.5$. Let $S = \sum_{i=1}^n X_i$ for $i = 1, 2, \ldots, n$. Using the Chernoff bound, prove that for any $x \in (0, 1)$, we have

$$P(S \geq nx) \leq \exp\left\{-nF(x)\right\},$$

where

$$F(x) = \frac{1}{2}(1 - x)\log(1 - x) + \frac{1}{2}(1 + x)\log(1 + x).$$