# HYPERION 2021

## TEAM VEGABONDS

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## Contents

## Abstract

Dark matter accounts for about 85% of the matter of the Universe. However, much about its behaviour and constitution remains unknown. This makes it difficult to evaluate metrics such as the density and distribution of dark matter across the galactic system. Here, we explore the dark matter density in the Milky Way by calculating the change in the radial velocity of stars over time, accounting for the respective stellar neighbourhood and instrumentation noise. This change in velocity gives the stellar acceleration averaged over a period of 10 years, which may then be used to retrieve the density profile of Dark Matter in the galaxy.
Also underlined here are the possible sources of error that might arise while calculating velocities using Doppler shift. Emphasis is also laid on the numerical error ranges in the density. Note that this is only an approximate method since contribution due to the Galactic disk has been neglected. However, it still holds reasonably accurately based on experimental evidence, as we would realise later.


# 1 Gauss's Law of Gravitation

### Statement

The gravitational flux through any closed surface is proportional to the mass enclosed within the surface.

### Derivation

The Gauss's Law for gravity can be derived from Newton's Law of gravitation which is:

$$\boldsymbol{g(r)} = \frac{-GM\widehat{r}}{r^2} \tag{1}$$

where $\widehat{r}$ is the radial unit vector, and r is the radius. M is the mass of the particle, assumed to be a point mass.
By applying superposition principle we can calculate the contribution to $g(r)$ associated with the mass at s where it is calculated by Newton's law:

$$\boldsymbol{g}(r) = -G\int \rho(s)\frac{r-s}{|r-s|^3}d^3s \tag{2}$$

Taking divergence on both sides with respect to r

$$\boldsymbol{\nabla}\boldsymbol{\cdot}(\frac{r}{|r|^3}) = 4\pi\delta(r)$$

**$\delta(\mathbf{r})$ is the Dirac delta function**

$$\boldsymbol{\nabla} \cdot g(r) = -4\pi G \int \rho(s)\delta(r-s)d^3s$$

By the unique property of the Dirac-Delta function we get:

$$\boxed{\boldsymbol{\nabla \cdot g(r)} = -4\pi G\rho(r)}$$

# 2 Expressing local DM density

Consider $M_r$ to be the mass of the dark matter encapsulated in a sphere of radius $r$

$$M_r = \int_0^r 4\pi x^2 \rho_x dx$$

$$\frac{\partial M_r}{\partial r} = 4\pi r^2 \rho_r$$

$$a_r = -\frac{GM_r}{r^2}$$

$$\frac{\partial a_r}{\partial r} = 2\frac{GM_r}{r^3} - \frac{G}{r^2}\frac{\partial M}{\partial r}$$

$$= 2\frac{GM_r}{r^3} - \frac{G\rho_r}{r^2}.4\pi r^2$$

$$\rho_r = \frac{1}{4\pi G}(\frac{2}{r^2}(\frac{GM}{r}) - \frac{\partial a_r}{\partial r})$$

$$\rho_r = \frac{1}{4\pi G}(2[\frac{v}{r}]^2 - \frac{\partial a_r}{\partial r})$$

By definition

$$v/r = A - B$$

$$\boxed{\rho_r = \frac{1}{4\pi G}(2(A-B)^2 - \frac{\partial a_r}{\partial r})}$$

The $A-B$ term denotes the **angular velocity** of the galactic plane at a certain radius r.

Taking into consideration the **differential rotation** of galaxies, and the known fact that the Oort constants are indicative of the **shear forces**, this term's decreasing nature lies in coherence with the observation that dark matter density does indeed reduce as we move away from the galactic nucleus.

# 3 Relative Uncertainty

The relative uncertainty in the Dark Matter density can be ascertained by basing the equation on standard error and uncertainty models.
Taking the respective partial derivatives on either side, and dividing by the original equation, we simplify the equation to reach the following expression.

$$\frac{\partial \rho_{DM}}{\rho_{DM}} \approx \frac{\partial a'_r}{a_r} \frac{(A-B)^2}{2\pi G \rho_{DM}} \tag{3}$$

$\partial \rho_{DM}$ is the uncertainty in the DM Density $\partial$a'$_r$ is the uncertainty in $\frac{\partial a_r}{\partial r}$

It is noteworthy that the relative uncertainty in the DM density is magnified compared to the relative uncertainty in $\frac{\partial a_r}{\partial r}$ because certain terms are cancelled in the equation we derived previously.

On substituting the values of A $= 15.3 \pm 0.4 kms^{-1}kpc$ and B $= -11.9 \pm 0.4 kms^{-1}kpc$, along with other constants:

$$\approx 2.6 \frac{\partial a'_r}{a'_r} \tag{4}$$

# 4 Observation: Problems and solutions

We expect the observations to be cross-galactic, this might lead to several issues while making observations for collecting radial velocity data . Some of them are discussed below.

- **Aberration**
  Aberration is a phenomenon which produces an **apparent motion** of celestial objects about their true positions, dependent on the velocity of the observer. Thus, when we are measuring from the earth, aberration accounts for some irregular observations.
  In order to receive light from the star the instruments on earth have to be offset slightly in the direction of earth's motion.
  **Extremely stable calibrations** of the spectrograph can lead to measurement of more smaller values. Specialised laser frequency combs, **Astrocombs** can be used.

- **Reddening and Interstellar Extinction**
  The scattering and absorption of light by the dust grains (which are usually of size comparable to the wavelength of blue light) along the line makes the object appear **redder and dimmer** than they really are. These effects are called reddening and interstellar extinction. This in particular leads to misleading results in the the analysis of the data of the radial velocities. For observations made from the Earth, even the atmosphere can lead to extinction and reddening.

Usage of a **global mean extinction curve** and performing an error analysis is the most common solution. Applying the appropriate corrections to the photometric observations might lead to more approximate results.

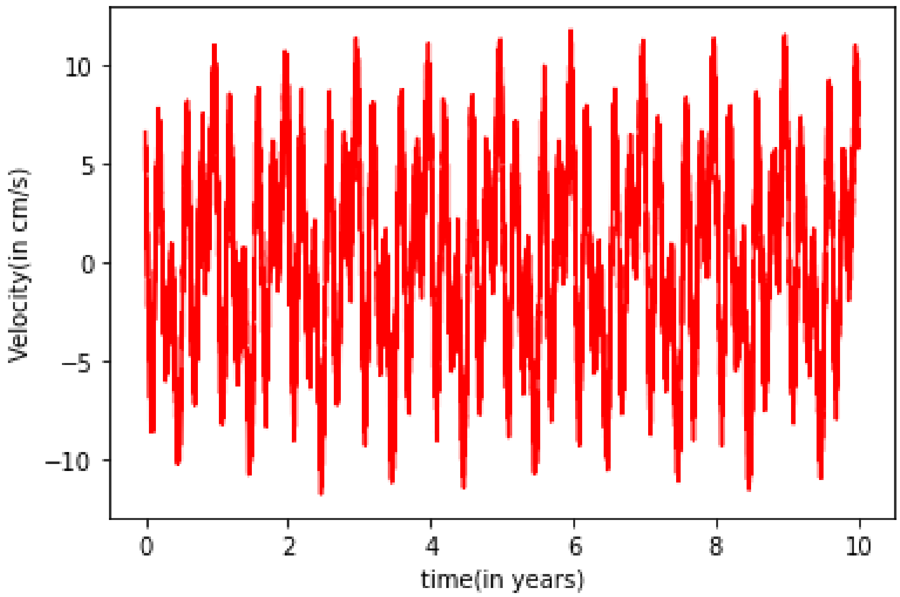
- **Gravitational Lensing**
  Gravitational lensing effect occurs when huge amount of matter say a cluster of galaxies creates a gravitational field that distorts and magnifies light from distant galaxies behind it, in the same line of sight.Thus, the relative transverse velocity of a lens with respect to a source star in gravitational lensing results in a wavelength shift in the light rays passing by a lens. thus interfering with the measurements.

  High precision spectrographs with an accuracy of detecting this wavelength shift can be used to measure this effect to detect the radial velocities with higher accuracy.

- **Convective Blueshift**
  Stellar convection which produces line shifts can impact the wavelengths measured. Magnetic activity also induces radial velocity variations with major component being the hindrance in the convective Blueshift due to **Plages**. A possible solution for this correction can be by using good radial velocity noise levels which are combined with a good instrumental stability and **realistic granulation noise**.

## 5 Radial Velocity due to DM and GC

The given raw data is plotted below.

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It is easy to infer both visually and intuitively that the plot is a combination of a linear and a periodic sinusoidal function. The former arising due to the Galactic center and the Dark Matter halo, while the latter due to the star's neighbourhood, most likely exoplanets or companion stars.
Using a linear + sin model will also allow us to smooth-en out instrumentation noise, and it can be assumed to be perfectly random, and hence modelled as a Gaussian distribution centred around 0. Thus, a best-fit straight line can nullify the random noise. We try to find an appropriate best fit of the type mentioned above, using SciPy.

```
def fit_func(x, a, b, c, d):
    z=[]
    for e in x:
        z.append(float(a*e+b)+float(c*(np.sin(d*e))))
    return z
```

Using curve_fit functionality available in SciPy, we derive the parameters to the function written above.

```
with open('vel_data.csv','r') as csvfile:
        lines = csv.reader(csvfile, delimiter=',')
```

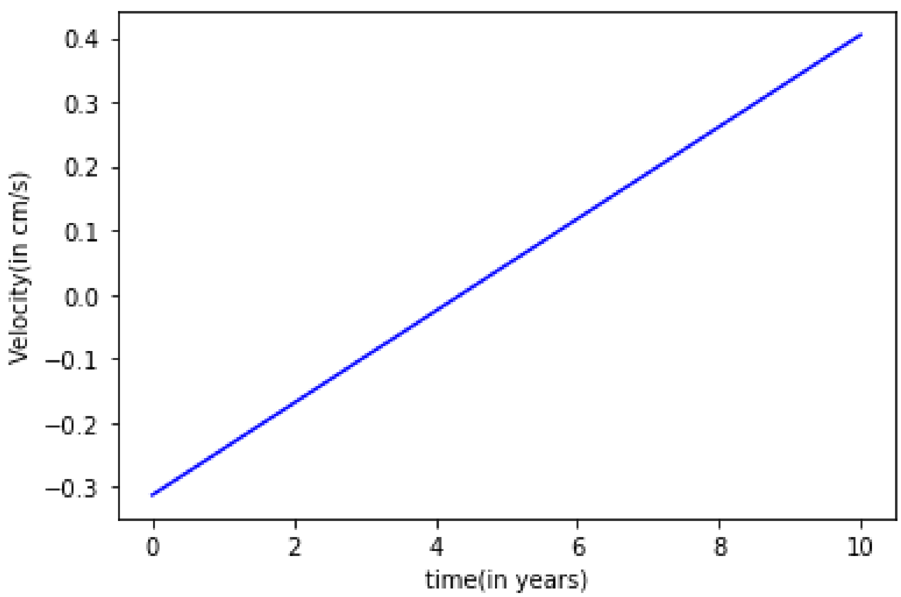
```
        for row in lines:
                x.append(float(row[0]))
                y.append(float(row[1]))

params = curve_fit(fit_func, x, y)

[a, b, c, d] = params[0]
z=[]
for e in x:
    z.append(float(a*e+b))

plt.plot(x, y, color='r')
plt.plot(x, z, color='b')
```

The final plots obtained are displayed below. The slope of the graph evaluates to $0.0717 cm s^{-1} yr^{-1}$.
This is the radial acceleration of the star averaged over the given time duration.



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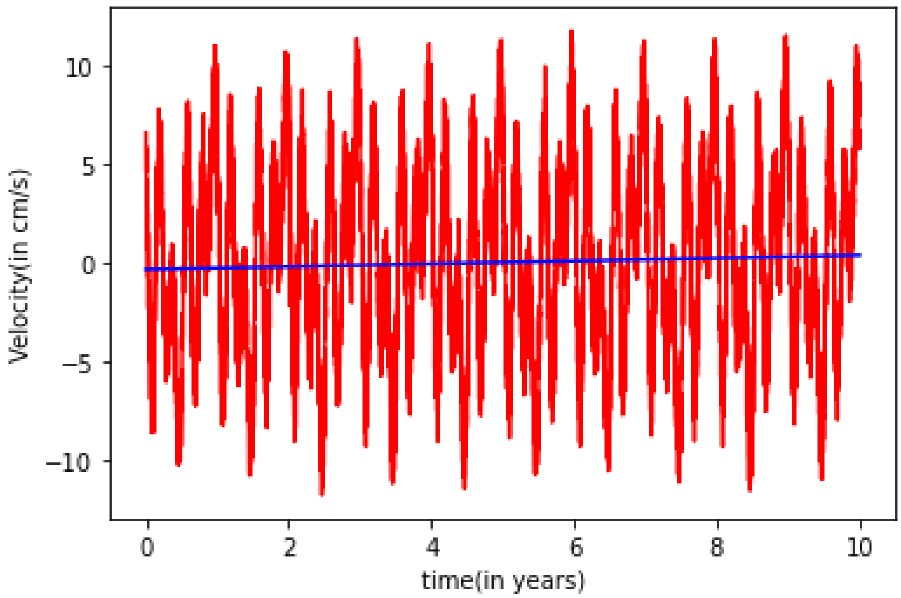
Radial velocity due to DM and GC vs Time

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Both graphs, superimposed for visualisation

### The Sinusoidal Part

As observed from the galactic centre, we can observe two components of stellar velocity – the radially outward part and a sinusoidal part.
The sine curve is indicative of the periodic motion of the star about the barycentre of its neighbourhood. Since the orbital plane of the star is deemed to be co-incidental with the galactic plane, the component of orbital velocity in the direction of the star's radial coordinate will obey a sinusoidal period.

## 6 Calculating Average Acceleration

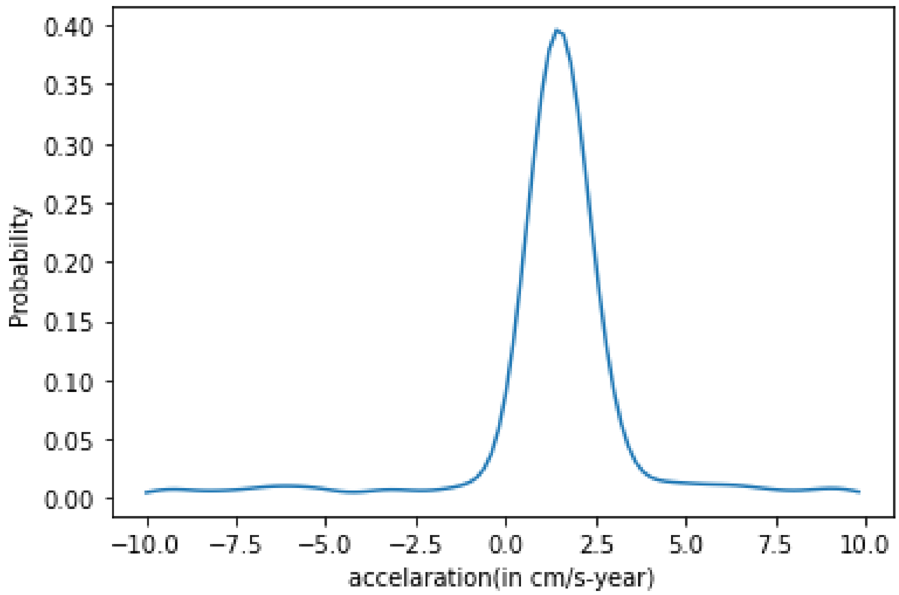
Using SciPy's Gaussian functionality and and the Linspace function available in NumPy, the Gaussian fit for the given data comes out as below.
Only the relevant snippets are included for reference.

```
from scipy.stats.kde import gaussian_kde
from numpy import linspace
```

```
with open('acc_data.csv','r') as csvfile:
        lines = csv.reader(csvfile, delimiter=',')
        for row in lines:
                x.append(float(row[0]))
                y.append(float(row[1]))
```

```
a=np.array(y)
kde = gaussian_kde(a)
dist_space = linspace( min(a), max(a), 100 )
plt.plot( dist_space, kde(dist_space))
```

The mean acceleration comes out to be 1.3242 $cms^{-1}yr^{-1}$

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# 7 Calculating Dark Matter Density

$A - B$, is $9 \times 10^{-16} s^{-1}$ Taking forward the assumption of a uniform density profile, we claim:

$$\frac{\partial a_r}{\partial r} = \frac{\bar{a_r}}{\bar{r}}$$

i.e, equating the slope of their respective curve to the ratio of the means.
This gives us:

$$\frac{\partial a_r}{\partial r} = 5 \times 10^{-31} s^{-2}$$

Now,

$$\rho_r = \frac{1}{4\pi G}(2(A-B)^2 - \frac{\partial a_r}{\partial r})$$

$$\rho_r = \frac{1}{4\pi G}(1.1 \times 10^{-30})$$

$$\rho_r \approx 5 \times 10^{-22} kgm^{-3}$$

Which is reasonably close to the experimentally deduced value of $7.2 \times 10^{-22} kgm^{-3}$.

## 8 Conclusion

The result obtained at the end of this exercise can be deemed to be reasonable since, experimental observation vindicate our theoretical findings.
However, the error margins need to be kept in mind at all times, thus reiterating the importance of the numerical analysis.
The closeness of the results also proves that the Dark Matter halo model is reasonably accurate. We can improve upon it by taking into account the oblate shape of the halo, but that is beyond the scope of this text.

## References


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