DB - HW7

7.1

Suppose that we decompose the schema R = (A, B, C, D, E) into (A, B, C) (A, D, E).
Show that this decomposition is a lossless decomposition if the following set F of functional dependencies holds:

$$
\begin{gathered}
A \longrightarrow BC \\
CD \longrightarrow E \\
B \longrightarrow D \\
E \longrightarrow A
\end{gathered}
$$

Answer:

$$
\begin{gathered}
Let:\ R_1 \ = \ (A, B, C),\ R_2 \ = \ (A, D, E). \\
\Longrightarrow \begin{cases} R_1 \cap R_2 = A \\ A\ is\ a\ candidate\ key \end{cases} \Longrightarrow R_1 \cap R_2 \longrightarrow R_1 \\
\therefore Decomposition\ \{(A, B, C), (A, D, E)\}\ \ is\ a\ lossless - join\ decomposition.
\end{gathered}
$$

7.13

Show that the decomposition in Exercise 7.1 is not a dependency-preserving decomposition.

Answer:

$$1.\ Let\ F_1 = restriction\ of\ F\ to\ (A, B, C)$$

$$\begin{cases} A \to A \\ A \to B \\ A \to C \\ A \to AB \\ A \to AC \\ A \to BC \\ A \to ABC \\ AB \to A \\ AB \to B \\ AB \to C \\ AB \to AB \\ AB \to AC \\ AB \to BC \\ AB \to ABC \\ AC/BC\ldots \\ B \to B \\ C \to C \end{cases}$$

$$2.\ Let\ F_2 = restriction\ of\ F\ to\ (A, D, E)$$

$$\begin{cases} A \to A \\ A \to D \\ A \to E \\ A \to AD \\ A \to AE \\ A \to DE \\ A \to ADE \\ D \to D \\ E/AD/AE/DE/ADE\ldots \end{cases}$$

$$\because (F_1 \cup F_2)^+\ does\ not\ contain\ B \to D$$

$$\therefore the\ decomposition\ in\ Exercise\ 7.1\ is\ not\ a\ dependency-preserving\ decomposition.$$

7.21

Give a lossless decomposition into BCNF of schema *R* of Exercise 7.1.

Answer:

$$\because B \to D\ is\ nontrivial\ \wedge\ B\ is\ not\ a\ key$$

$$\therefore R\ is\ not\ in BCNF$$

$$By\ using\ following\ algorithm,\ we\ could\ find\ a\ lossless\ decomposition\ in\ BCNF:$$

$$F = \begin{cases} A \longrightarrow BC \\ CD \longrightarrow E \\ B \longrightarrow D \\ E \longrightarrow A \end{cases} \implies F^+$$

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while(there's a schema Ri in result that is not in BCNF)
{
     let a -> b be a notrtivial functional dependency holds on Ri
     if(a->Ri is not in F+ AND a & b has no intersecton)
     {
         result = (result-Ri)+(a,b)+(Ri-b);
     }
}
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$$We\ would\ have\ R^1 = \{(A, B, C, E), (B, D)\},\ which\ is\ a\ lossless\ decomposition\ in\ BCNF$$

7.22

Give a lossless, dependency-preserving decomposition into 3NF of schema *R* of exercise 7.1.

Answer:

$$\because \mathit{The\ given\ functional\ dependenies} \begin{cases} A \longrightarrow BC \\ CD \longrightarrow E \\ B \longrightarrow D \\ E \longrightarrow A \end{cases} \mathit{is\ a\ canonical\ cover.}$$

$$\mathit{For\ each\ functional\ dependency\ } FD_i : \alpha \to \beta,\ \mathit{let\ } R_i = \alpha\beta.$$

$$\mathit{We\ would\ have} \begin{cases} R_1 = (A, B, C) \\ R_2 = (C, D, E) \\ R_3 = (B, D) \\ R_4 = (E, A) \end{cases} \mathit{inside\ which\ } (A, B, C) \subset \mathit{candidate\ key\ } (B, C).$$

$$\mathit{Let}\ \ R^1 = \{(A, B, C), (C, D, E), (B, D), (E, A)\}$$
$$\Longrightarrow R^1\ \mathit{is\ a\ } 3NF,\ \mathit{lossless},\ \mathit{dependency-preserving\ decomposition}.$$

7.29

Show that the following decomposition of the schema *R* of Exercise 7.1 is not a lossless decomposition:

$$(A, B, C)$$
$$(C, D, E).$$

Hint: **Give an example of relation r(R) such that :**

$$\Pi_{A,B,C}(r) \bowtie \Pi_{C,D,E}(r) \neq r$$

Answer:

Assume there is an example of relation r(R) like this:

A	B	C	D	E
a_1	b_1	c_1	d_1	e_1
a_2	b_2	c_2	d_2	e_2

$$\mathit{Let}:\ R_1\ =\ (A, B, C),\ R_2\ =\ (A, D, E).$$
$$\mathit{Then}\ \ \Pi_{R_1}(r) \bowtie \Pi_{R_2}(r)\ \ \mathit{would\ be\ look\ like}:$$

A	B	C	D	E
a_1	b_1	c_1	d_1	e_1
a_1	b_1	c_1	d_2	e_2
a_2	b_2	c_2	d_1	e_1
a_2	b_2	c_2	d_2	e_2

$$\mathit{which\ obviously\ meets\ the\ condition\ that}: \Pi_{A,B,C}(r) \bowtie \Pi_{C,D,E}(r) \neq r$$
$$\therefore\ \ \mathit{Decomposition}\ \{(A, B, C), (C, D, E)\}\ \ \mathit{is\ not\ a\ lossless\ decomposition.}$$