RLT: Moody's Analytics ESG Model and Calibration Training

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School of Mathematics

Session 2019

Introduction

Assignment 1 : Risk Neutral Valuation

- Quick view of generating scenario
- Implement the answer of the Assignment 1

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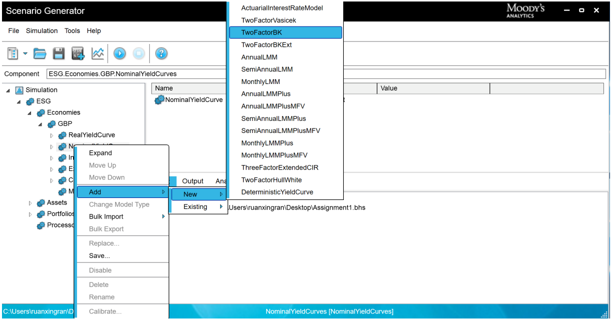
Quick view of generating scenario

GBP-TwoFactorBK Model



Quick view of generating scenario

GBP-TwoFactorBK Model

- The risk-neutral process for the short rate r is :

$$d\log(r) = \alpha_1[\log(m) - \log(r)]dt + \sigma_1 dZ_1$$

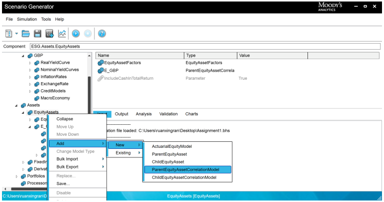
- Where the mean-reversion level m follows the stochastic process :

$$d\log(m) = \alpha_2[\mu - \log(m)]dt + \sigma_2 dZ_2$$

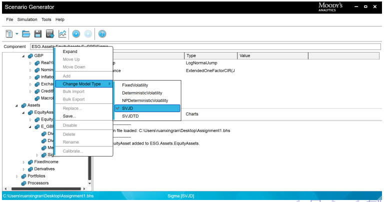
Quick view of generating scenario

Equity Asset-Parant equity correlation model and SVJD

- Using Parant equity correlation model



- Using SVJD model



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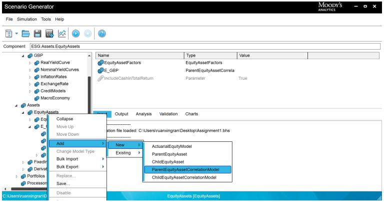

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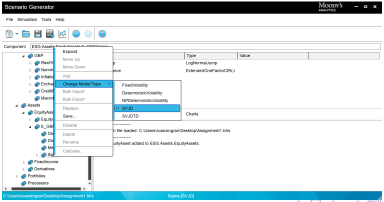

Quick view of generating scenario

Equity Asset-Parant equity correlation model and SVJD

- The log-return of asset $P(t)$ is :

$$\log(\frac{P(t+\Delta t)}{P(t)}) = (r(t) + \mu_p - \pi_p - \frac{1}{2}\sigma_p^2)\Delta t + \sigma_p Z_p(t)\sqrt{\Delta t} + CPP(\Delta t)$$

  where,
  $CPP(\Delta t) = \sum_{f=1}^{F} \beta_f \sum_{i=1}^{N_f(\Delta t)} \log(J_{f,i}) + \sum_{k=1}^{N_s(\Delta t)} \log(J_{s,k})$

- $v_t(volatility^2)$ follows a mean-reverting Stochastic Differential Equation :

$$dv_t = \alpha(\mu - v_t)d_t + \sigma_v\sqrt{v_t}dW_t$$

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Implement the answer of the Assignment 1

The payoff structure of the portfolio

- One zero coupon bond paying 10000 in 10 years
- 10 years European call option on UK Equity with strike price of 7000
- 10 years European put option on UK Equity with strike price of 5000

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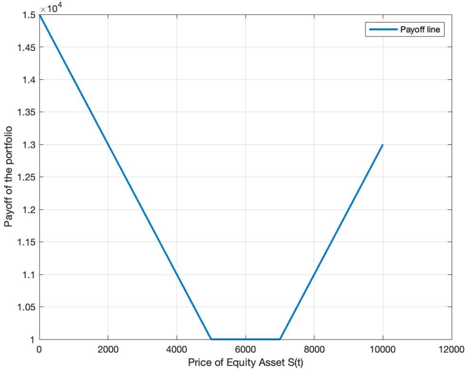
The payoff structure of the portfolio

$$Payoff = 10000 + (5000 - S)_{S \leq 5000} + (S - 7000)_{S \geq 7000}$$



Implement the answer of the Assignment 1
Calculate the market consistent value of the portfolio

- In terms of martingale X_t, it follows :

$$X_0 = E(X_t|F_0)$$

- The market consistent value of portfolio is :

$$P_0 = E(\frac{P_T}{B(T)}|X_0 = 5000) = 12066.67$$

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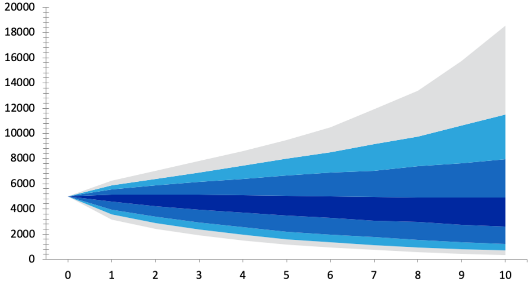
Implement the answer of the Assignment 1

Estimate the 95% confidence interval for the value of the portfolio

- The graph of price of Equity Asset :



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Implement the answer of the Assignment 1

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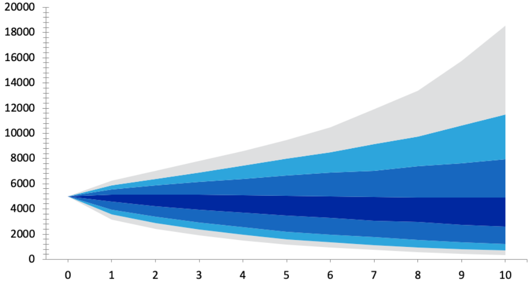

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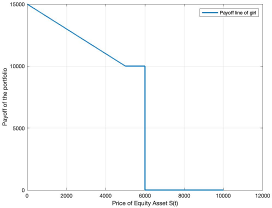
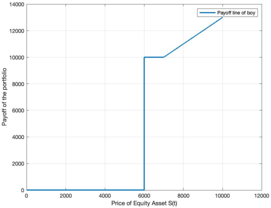
Implement the answer of the Assignment 1

Payoff structure of each child

- The graph payoff structure of each child :





- The market consistent value :
  boy :3989.569.
  girl :8077.096.

Implement the answer of the Assignment 1

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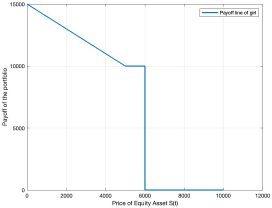

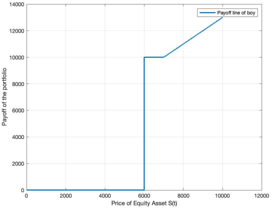

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Weakness of this valuation

- Waste computing resource.
- The result is unfair.

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Assignment 2 : Value at Risk

- Main difference between Real World and Market Consistent
- Value at Risk theory
- Implement the answer of the Assignment 2 part 1

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Real World Model

- Two Economics : GBP and EUR
- Two Equities : E-GBP and E-EUR
- Two Govt Bonds(annually) : GBP-GOV-30ys-0 and EUR-GOV-30ys-0
- Two Risky Bonds : GBP-BBB-30ys-2 (semiannually) and EUR-AAA-30ys-5 (annually)
- Two Portfolios : P-GBP and P-EUR

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Difference between RW and MC

Parent equity asset

- Parent Equity Asset :
  Parent equity assets are addable models. Correlation of these assets is done via the equity asset factor loadings.

Difference between RW and MC

Difference

- Difference between Parent Equity Asset and Parent Equity Asset Correlation Model :
  Depend on whether they are correlated using an equity asset factor model or the correlation matrix.
  The fundamental difference between the two types, is that the former splits risk into systematic risk driven by factor loadings, and specific risk driven by the equity asset volatility model. Whereas the correlated version of the equity assets does not make this distinction, and all risk is driven by the equity asset volatility model.

VaR Theory

- From Financial Risky Theory course, VaR is a measure of the risk of loss for investments, i.e. 0.995 VaR is 0.5 means means that there is a 0.05 probability that the portfolio will increase in value by less than 0.5.
- The definition of Upper-Quantiles(1) :
  For $\alpha \in (0, 1)$, the number $q^{\alpha}(X) = \inf\{x : \alpha < F_X(x)\}$, is called the upper-$\alpha$-quantile of X.
- The definition of VaR(1) :
  For $\alpha \in (0, 1)$, we define the VaR of X, at confidence level 1-α, as : $\text{VaR}^{\alpha}(X) = -q^{\alpha}(X) = -\inf\{x : \alpha < F_X(x)\}$.

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VaR Theory

- In usual case :
  We now consider an investment where at time zero we buy x shares of risky assets and y units of the risk-free asset, and we use $V_{(x;y)}(t)$to denote the value of the portfolio at time t, with $X_{(x;y)}$ to denote the value of the portfolio.
  Thus, VaR is(1) :
  $\mathrm{VaR}^{\alpha}(X_{(x;y)})$=$V_{(x;y)}(0)$-$x\mathrm{e}^{-rT}q^{\alpha}(\mathrm{S(T)})$-$y$
- In my case, I decide to analyse VaR from Macro way, i.e. from the 0.5 percentage of lower bound in Total Return Index plot.

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Implement the answer of the Assignment 2 part 1

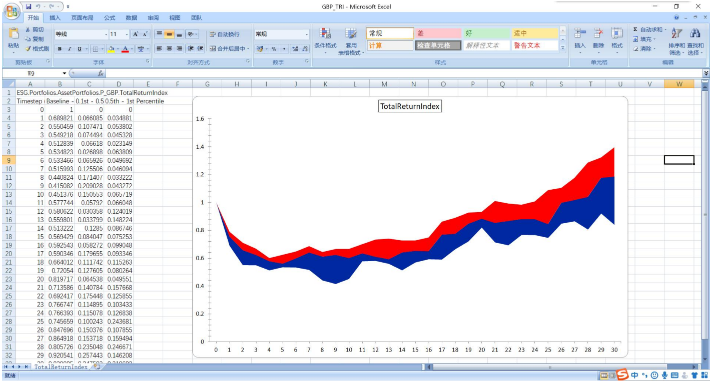
VaR change

- We observe the changes of VaR during these conditions :
  1. Assets' weights in the portfolio : (equity :gov bond :risky bond)
  (based) 0.45 :0.1 :0.45 ; 0.25 :0.5 :0.25 ;
  2. Different random number of seed :
  (based) 1 ; 100 ; 10000 ;
  3. Different numbers of trails :
  100 ; (based) 1000 ; 10000 ;
  4. Different Economic :
  GBP ; EUR.

Implement the answer of the Assignment 2 Part 1

Conclusion

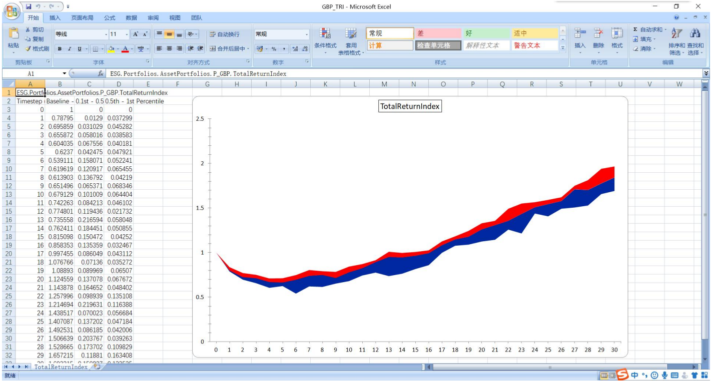
- The graph of 0.45 :0.1 :0.45 is :



Implement the answer of the Assignment 2 Part 1

Conclusion

- Change to 0.25 :0.5 :0.25 is :

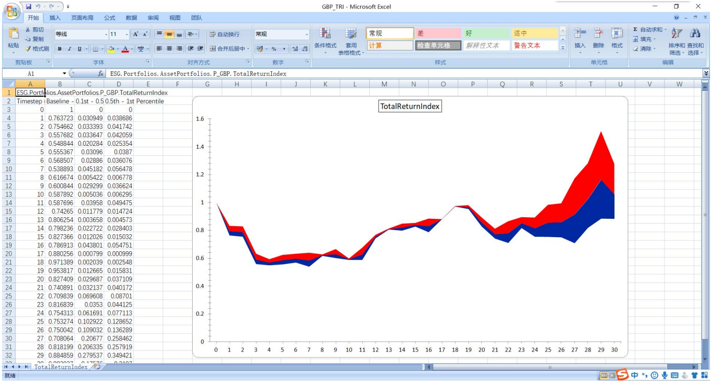


ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex			
Timestep	Baseline - 0.1st - 0.5	0.5th - 1st Percentile	
0	1	0	0
1	0.78795	0.0129	0.037299
2	0.695859	0.031029	0.045282
3	0.655872	0.058016	0.038583
4	0.604035	0.067556	0.040181
5	0.6237	0.042475	0.047921
6	0.539111	0.158071	0.052241
7	0.619619	0.120917	0.065455
8	0.613903	0.136792	0.04219
9	0.651496	0.065371	0.068346
10	0.679129	0.101009	0.064404
11	0.742263	0.084213	0.046102
12	0.774801	0.119436	0.021732
13	0.735558	0.216594	0.058048
14	0.762411	0.184451	0.050855
15	0.815098	0.150472	0.04252
16	0.858353	0.135359	0.032467
17	0.997455	0.086049	0.043112
18	1.076766	0.07136	0.035272
19	1.08893	0.089969	0.06507
20	1.124559	0.137078	0.067672
21	1.143878	0.164652	0.048402
22	1.257996	0.098939	0.135108
23	1.214694	0.219631	0.116388
24	1.438517	0.070023	0.056684
25	1.407087	0.137202	0.047184
26	1.492531	0.086185	0.042006
27	1.506639	0.203767	0.039263
28	1.528665	0.173702	0.109829
29	1.657215	0.11881	0.163408

Implement the answer of the Assignment 2 Part 1

Conclusion

- with 100 trails is :

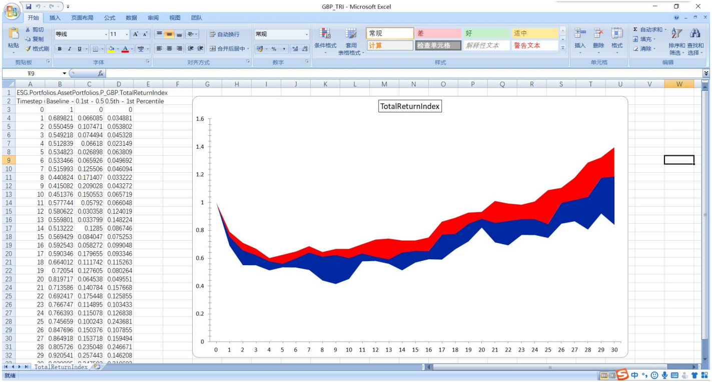


	A	B	C	D
1	ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex			
2	Timestep	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
3	0	1	0	0
4	1	0.763723	0.030949	0.038686
5	2	0.754662	0.033393	0.041742
6	3	0.557682	0.033647	0.042059
7	4	0.548844	0.020284	0.025354
8	5	0.555367	0.03096	0.0387
9	6	0.568507	0.02886	0.036076
10	7	0.538893	0.045182	0.056478
11	8	0.616674	0.005422	0.006778
12	9	0.600844	0.029299	0.036624
13	10	0.587892	0.005036	0.006295
14	11	0.587696	0.03958	0.049475
15	12	0.74265	0.011779	0.014724
16	13	0.806254	0.003658	0.004573
17	14	0.798236	0.022722	0.028403
18	15	0.827366	0.012026	0.015032
19	16	0.786913	0.043801	0.054751
20	17	0.880256	0.000799	0.000999
21	18	0.971389	0.002039	0.002548
22	19	0.953817	0.012665	0.015831
23	20	0.827409	0.029687	0.037109
24	21	0.740891	0.032137	0.040172
25	22	0.709839	0.069608	0.08701
26	23	0.816839	0.0353	0.044125
27	24	0.754313	0.061691	0.077113
28	25	0.753274	0.102922	0.128652
29	26	0.750042	0.109032	0.136289
30	27	0.708064	0.20677	0.258462
31	28	0.818199	0.206335	0.257919
32	29	0.884859	0.279537	0.349421

Implement the answer of the Assignment 2 Part 1

Conclusion

- with 1000 trails is :



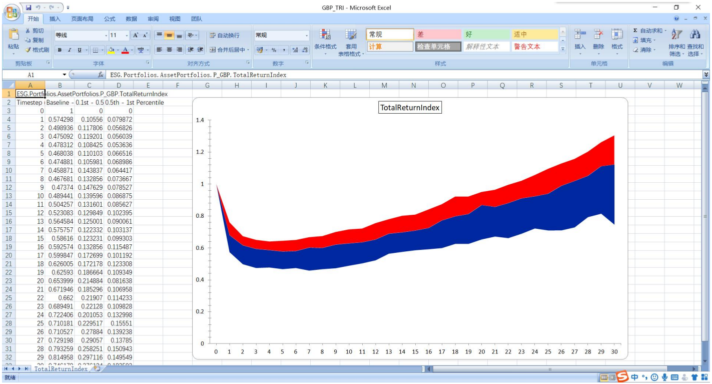
ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex

Timestep	Baseline - 0.1st - 0.5	0.5th - 1st Percentile	
0	1	0	0
1	0.689821	0.066085	0.034881
2	0.550459	0.107471	0.053802
3	0.549218	0.074494	0.045328
4	0.512839	0.06618	0.023149
5	0.534823	0.026898	0.063809
6	0.533466	0.065926	0.049692
7	0.515993	0.125506	0.046094
8	0.440824	0.171407	0.033222
9	0.415082	0.209028	0.043272
10	0.451376	0.150553	0.065719
11	0.577744	0.05792	0.066048
12	0.580622	0.030358	0.124019
13	0.559801	0.033799	0.148224
14	0.513222	0.1285	0.086746
15	0.569429	0.084047	0.075253
16	0.592543	0.058272	0.099048
17	0.590346	0.179655	0.093346
18	0.664012	0.111742	0.115263
19	0.72054	0.127605	0.080264
20	0.819717	0.064538	0.049551
21	0.713586	0.140784	0.157668
22	0.692417	0.175448	0.125855
23	0.766747	0.114895	0.103433
24	0.766393	0.115078	0.126838
25	0.745659	0.100243	0.243681
26	0.847696	0.150376	0.107855
27	0.864918	0.153718	0.159494
28	0.805726	0.235048	0.246671
29	0.920541	0.257443	0.146208

Implement the answer of the Assignment 2 Part 1

Conclusion

- with 10000 trails is :



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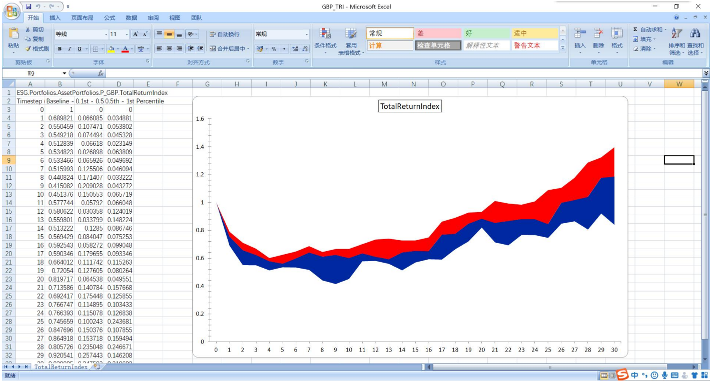
2	Timestep	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
3	0	1	0	0
4	1	0.574298	0.10556	0.079872
5	2	0.498936	0.117806	0.056826
6	3	0.475092	0.119201	0.056039
7	4	0.478312	0.108425	0.053636
8	5	0.468038	0.110103	0.066516
9	6	0.474881	0.105981	0.068986
10	7	0.458871	0.143837	0.064417
11	8	0.467681	0.132856	0.073667
12	9	0.47374	0.147629	0.078527
13	10	0.489441	0.139596	0.086875
14	11	0.504257	0.131601	0.085627
15	12	0.523083	0.129849	0.102395
16	13	0.564584	0.125001	0.090061
17	14	0.575757	0.122332	0.103137
18	15	0.58616	0.123231	0.099303
19	16	0.592574	0.132856	0.115487
20	17	0.599847	0.172699	0.101192
21	18	0.626005	0.172178	0.123308
22	19	0.62593	0.186664	0.109349
23	20	0.653999	0.214884	0.081638
24	21	0.671946	0.185296	0.106958
25	22	0.662	0.21907	0.114233
26	23	0.689491	0.22128	0.109828
27	24	0.722406	0.201053	0.132998
28	25	0.710181	0.229517	0.15551
29	26	0.710527	0.27884	0.139238
30	27	0.729198	0.29057	0.13785
31	28	0.793259	0.258251	0.150943
32	29	0.814958	0.297116	0.149549

Implement the answer of the Assignment 2 Part 1

Conclusion

- with 1 random seed is :

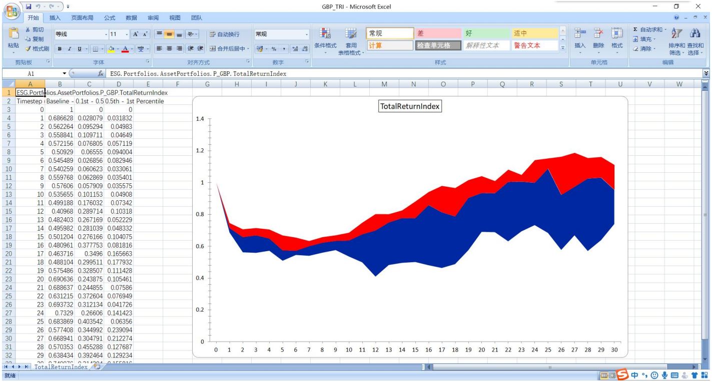


	A	B	C	D
1	ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex			
2	Timestep	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
3	0	1	0	0
4	1	0.689821	0.066085	0.034881
5	2	0.550459	0.107471	0.053802
6	3	0.549218	0.074494	0.045328
7	4	0.512839	0.06618	0.023149
8	5	0.534823	0.026898	0.063809
9	6	0.533466	0.065926	0.049692
10	7	0.515993	0.125506	0.046094
11	8	0.440824	0.171407	0.033222
12	9	0.415082	0.209028	0.043272
13	10	0.451376	0.150553	0.065719
14	11	0.577744	0.05792	0.066048
15	12	0.580622	0.030358	0.124019
16	13	0.559801	0.033799	0.148224
17	14	0.513222	0.1285	0.086746
18	15	0.569429	0.084047	0.075253
19	16	0.592543	0.058272	0.099048
20	17	0.590346	0.179655	0.093346
21	18	0.664012	0.111742	0.115263
22	19	0.72054	0.127605	0.080264
23	20	0.819717	0.064538	0.049551
24	21	0.713586	0.140784	0.157668
25	22	0.692417	0.175448	0.125855
26	23	0.766747	0.114895	0.103433
27	24	0.766393	0.115078	0.126838
28	25	0.745659	0.100243	0.243681
29	26	0.847696	0.150376	0.107855
30	27	0.864918	0.153718	0.159494
31	28	0.805726	0.235048	0.246671
32	29	0.920541	0.257443	0.146208

Implement the answer of the Assignment 2 Part 1

Conclusion

- with 100 random seed is :

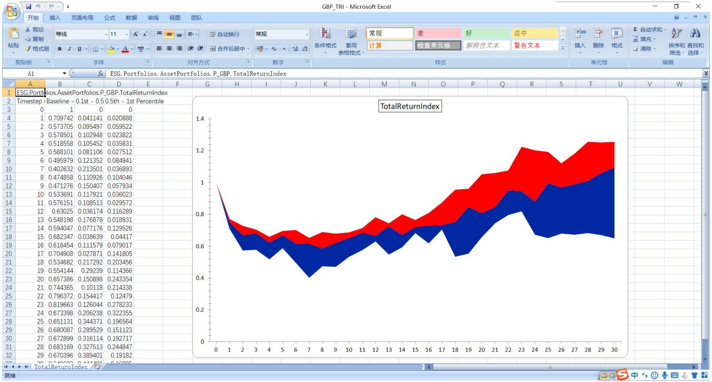


ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex			
Timestep	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
0	1	0	0
1	0.686628	0.028079	0.031832
2	0.562264	0.095294	0.04983
3	0.558841	0.109711	0.04649
4	0.572156	0.076805	0.057119
5	0.50929	0.06555	0.094004
6	0.545489	0.026856	0.082946
7	0.540259	0.060623	0.033061
8	0.559768	0.062869	0.035401
9	0.57606	0.057909	0.035575
10	0.535655	0.101153	0.04908
11	0.499188	0.176032	0.07342
12	0.40968	0.289714	0.10318
13	0.482403	0.267169	0.052229
14	0.495982	0.281039	0.048332
15	0.501204	0.276166	0.104075
16	0.480961	0.377753	0.081816
17	0.463716	0.3496	0.165663
18	0.488104	0.299511	0.177932
19	0.575486	0.328507	0.111428
20	0.690636	0.243875	0.105461
21	0.688637	0.244855	0.07586
22	0.631215	0.372604	0.076949
23	0.693732	0.312134	0.041726
24	0.7329	0.26606	0.141423
25	0.683869	0.403542	0.06356
26	0.577408	0.344992	0.239094
27	0.668941	0.304791	0.212274
28	0.570353	0.455288	0.127687
29	0.638434	0.392464	0.129234

Implement the answer of the Assignment 2 Part 1

Conclusion

- with 10000 random seed is :



	A	B	C	D
1	ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex			
2	Timestep	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
3	0	1	0	0
4	1	0.709742	0.041141	0.020888
5	2	0.573705	0.095497	0.059522
6	3	0.578501	0.102948	0.023822
7	4	0.518558	0.105452	0.035831
8	5	0.588101	0.081106	0.027512
9	6	0.495979	0.121352	0.084941
10	7	0.402632	0.213501	0.036893
11	8	0.474858	0.110926	0.104046
12	9	0.471276	0.150407	0.057934
13	10	0.533691	0.117921	0.036023
14	11	0.576151	0.108513	0.029572
15	12	0.63025	0.036174	0.116289
16	13	0.548198	0.176878	0.018931
17	14	0.594047	0.077176	0.129526
18	15	0.682347	0.038639	0.04417
19	16	0.618454	0.111579	0.079017
20	17	0.704908	0.027871	0.141805
21	18	0.534682	0.217292	0.203456
22	19	0.554144	0.29239	0.114366
23	20	0.657386	0.150898	0.243354
24	21	0.744365	0.10118	0.214338
25	22	0.796372	0.154417	0.12479
26	23	0.819663	0.126044	0.278233
27	24	0.673398	0.206238	0.322355
28	25	0.651131	0.344371	0.196564
29	26	0.680087	0.289529	0.151123
30	27	0.672899	0.316114	0.192717
31	28	0.683169	0.327513	0.244847
32	29	0.670396	0.389401	0.19182

Implement the answer of the Assignment 2 Part 1

Conclusion

- What could we conclude are :
  1. Along with time, the 0.995 VaR changes from negative to positive, which means that if you want this portfolio to earn money, it is better to hold for a long time ;
  2. Changing higher the risk-free asset means you could gain much more, but still you need to hold for a long time ;

Implement the answer of the Assignment 2 Part 1

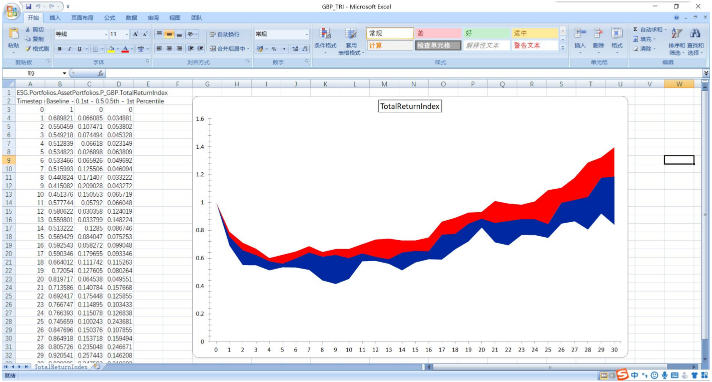
Conclusion

- What could we conclude are :
  3. More trails makes the plot more smoothly, which means during a certain range of value, it could get the point as large as possible, i.e. could simulate more suitable situations ;
  4. More random seeds makes the plot more fluctuating because the seed is an initial number of the pseudorandom number generator which is in fact fully deterministic. Larger numbers somehow returns a sequence of numbers that looks more random.

Implement the answer of the Assignment 2 Part 1

Conclusion

- Similarly, show the EUR plot in the same order :



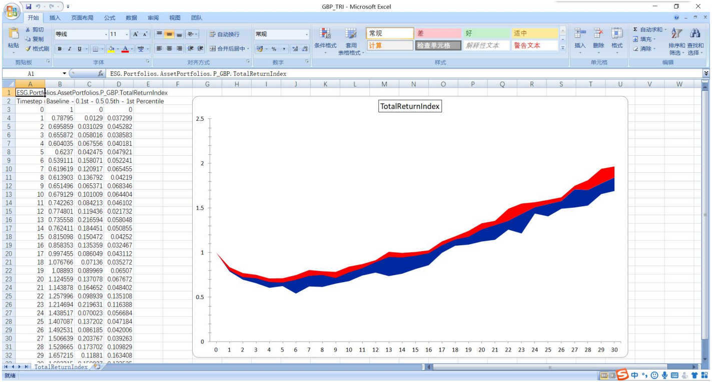
ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex

Timestep	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
0	1	0	0
1	0.689821	0.066085	0.034881
2	0.550459	0.107471	0.053802
3	0.549218	0.074494	0.045328
4	0.512839	0.06618	0.023149
5	0.534823	0.026898	0.063809
6	0.533466	0.065926	0.049692
7	0.515993	0.125506	0.046094
8	0.440824	0.171407	0.033222
9	0.415082	0.209028	0.043272
10	0.451376	0.150553	0.065719
11	0.577744	0.05792	0.066048
12	0.580622	0.030358	0.124019
13	0.559801	0.033799	0.148224
14	0.513222	0.1285	0.086746
15	0.569429	0.084047	0.075253
16	0.592543	0.058272	0.099048
17	0.590346	0.179655	0.093346
18	0.664012	0.111742	0.115263
19	0.72054	0.127605	0.080264
20	0.819717	0.064538	0.049551
21	0.713586	0.140784	0.157668
22	0.692417	0.175448	0.125855
23	0.766747	0.114895	0.103433
24	0.766393	0.115078	0.126838
25	0.745659	0.100243	0.243681
26	0.847696	0.150376	0.107855
27	0.864918	0.153718	0.159494
28	0.805726	0.235048	0.246671
29	0.920541	0.257443	0.146208

Implement the answer of the Assignment 2 Part 1

Conclusion

- Change to 0.25 :0.5 :0.25 is :



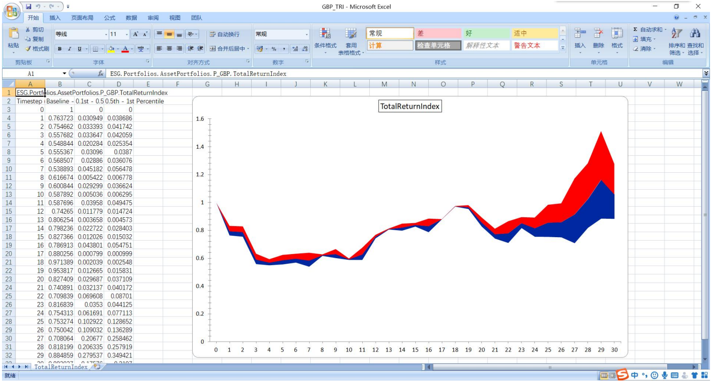
ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex	

Timestep (	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
0	1	0	0
1	0.78795	0.0129	0.037299
2	0.695859	0.031029	0.045282
3	0.655872	0.058016	0.038583
4	0.604035	0.067556	0.040181
5	0.6237	0.042475	0.047921
6	0.539111	0.158071	0.052241
7	0.619619	0.120917	0.065455
8	0.613903	0.136792	0.04219
9	0.651496	0.065371	0.068346
10	0.679129	0.101009	0.064404
11	0.742263	0.084213	0.046102
12	0.774801	0.119436	0.021732
13	0.735558	0.216594	0.058048
14	0.762411	0.184451	0.050855
15	0.815098	0.150472	0.04252
16	0.858353	0.135359	0.032467
17	0.997455	0.086049	0.043112
18	1.076766	0.07136	0.035272
19	1.08893	0.089969	0.06507
20	1.124559	0.137078	0.067672
21	1.143878	0.164652	0.048402
22	1.257996	0.098939	0.135108
23	1.214694	0.219631	0.116388
24	1.438517	0.070023	0.056684
25	1.407087	0.137202	0.047184
26	1.492531	0.086185	0.042006
27	1.506639	0.203767	0.039263
28	1.528665	0.173702	0.109829
29	1.657215	0.11881	0.163408

Implement the answer of the Assignment 2 Part 1

Conclusion

- With 100 trails is :

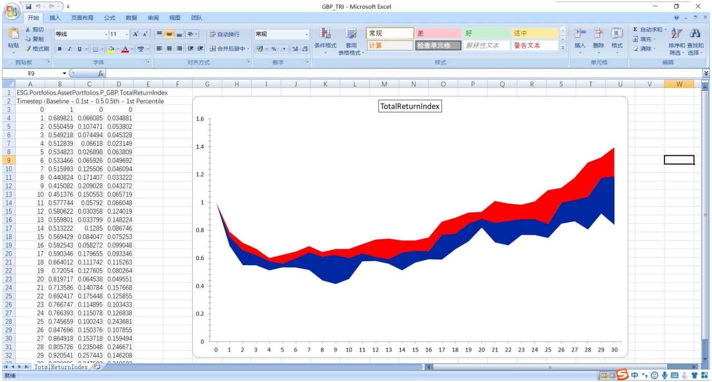


	A	B	C	D
1	ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex			
2	Timestep	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
3	0	1	0	0
4	1	0.763723	0.030949	0.038686
5	2	0.754662	0.033393	0.041742
6	3	0.557682	0.033647	0.042059
7	4	0.548844	0.020284	0.025354
8	5	0.555367	0.03096	0.0387
9	6	0.568507	0.02886	0.036076
10	7	0.538893	0.045182	0.056478
11	8	0.616674	0.005422	0.006778
12	9	0.600844	0.029299	0.036624
13	10	0.587892	0.005036	0.006295
14	11	0.587696	0.03958	0.049475
15	12	0.74265	0.011779	0.014724
16	13	0.806254	0.003658	0.004573
17	14	0.798236	0.022722	0.028403
18	15	0.827366	0.012026	0.015032
19	16	0.786913	0.043801	0.054751
20	17	0.880256	0.000799	0.000999
21	18	0.971389	0.002039	0.002548
22	19	0.953817	0.012665	0.015831
23	20	0.827409	0.029687	0.037109
24	21	0.740891	0.032137	0.040172
25	22	0.709839	0.069608	0.08701
26	23	0.816839	0.0353	0.044125
27	24	0.754313	0.061691	0.077113
28	25	0.753274	0.102922	0.128652
29	26	0.750042	0.109032	0.136289
30	27	0.708064	0.20677	0.258462
31	28	0.818199	0.206335	0.257919
32	29	0.884859	0.279537	0.349421

Implement the answer of the Assignment 2 Part 1

Conclusion

- With 1000 trails is :

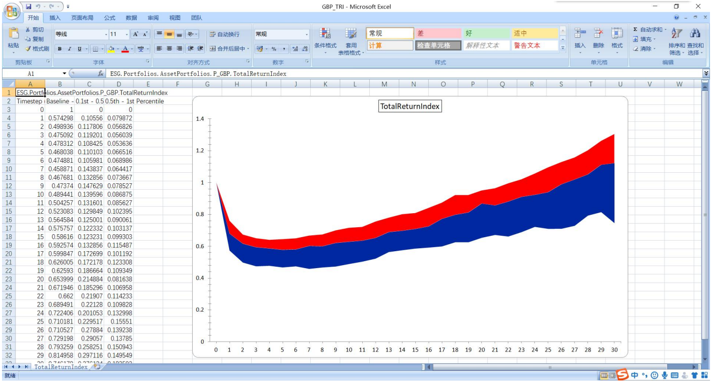


	A	B	C	D
1	ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex			
2	Timestep (	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
3	0	1	0	0
4	1	0.689821	0.066085	0.034881
5	2	0.550459	0.107471	0.053802
6	3	0.549218	0.074494	0.045328
7	4	0.512839	0.06618	0.023149
8	5	0.534823	0.026898	0.063809
9	6	0.533466	0.065926	0.049692
10	7	0.515993	0.125506	0.046094
11	8	0.440824	0.171407	0.033222
12	9	0.415082	0.209028	0.043272
13	10	0.451376	0.150553	0.065719
14	11	0.577744	0.05792	0.066048
15	12	0.580622	0.030358	0.124019
16	13	0.559801	0.033799	0.148224
17	14	0.513222	0.1285	0.086746
18	15	0.569429	0.084047	0.075253
19	16	0.592543	0.058272	0.099048
20	17	0.590346	0.179655	0.093346
21	18	0.664012	0.111742	0.115263
22	19	0.72054	0.127605	0.080264
23	20	0.819717	0.064538	0.049551
24	21	0.713586	0.140784	0.157668
25	22	0.692417	0.175448	0.125855
26	23	0.766747	0.114895	0.103433
27	24	0.766393	0.115078	0.126838
28	25	0.745659	0.100243	0.243681
29	26	0.847696	0.150376	0.107855
30	27	0.864918	0.153718	0.159494
31	28	0.805726	0.235048	0.246671
32	29	0.920541	0.257443	0.146208

Implement the answer of the Assignment 2 Part 1

Conclusion

- With 10000 trails is :

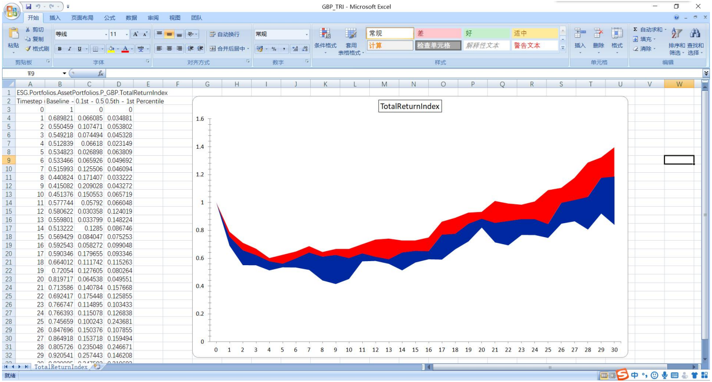


ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex			
Timestep	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
0	1	0	0
1	0.574298	0.10556	0.079872
2	0.498936	0.117806	0.056826
3	0.475092	0.119201	0.056039
4	0.478312	0.108425	0.053636
5	0.468038	0.110103	0.066516
6	0.474881	0.105981	0.068986
7	0.458871	0.143837	0.064417
8	0.467681	0.132856	0.073667
9	0.47374	0.147629	0.078527
10	0.489441	0.139596	0.086875
11	0.504257	0.131601	0.085627
12	0.523083	0.129849	0.102395
13	0.564584	0.125001	0.090061
14	0.575757	0.122332	0.103137
15	0.58616	0.123231	0.099303
16	0.592574	0.132856	0.115487
17	0.599847	0.172699	0.101192
18	0.626005	0.172178	0.123308
19	0.62593	0.186664	0.109349
20	0.653999	0.214884	0.081638
21	0.671946	0.185296	0.106958
22	0.662	0.21907	0.114233
23	0.689491	0.22128	0.109828
24	0.722406	0.201053	0.132998
25	0.710181	0.229517	0.15551
26	0.710527	0.27884	0.139238
27	0.729198	0.29057	0.13785
28	0.793259	0.258251	0.150943
29	0.814958	0.297116	0.149549

Implement the answer of the Assignment 2 Part 1

Conclusion

- With 1 random seed is :

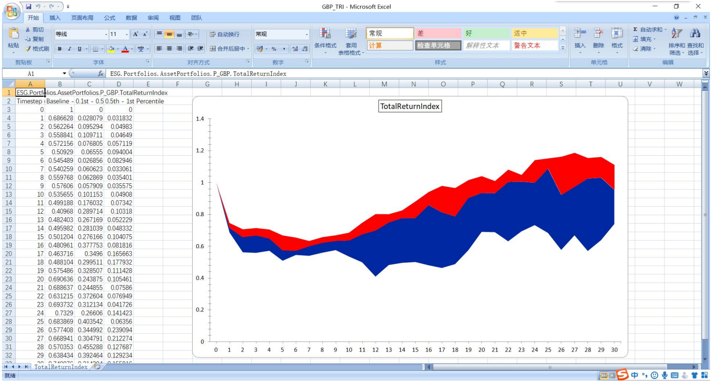


	A	B	C	D
1	ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex			
2	Timestep	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
3	0	1	0	0
4	1	0.689821	0.066085	0.034881
5	2	0.550459	0.107471	0.053802
6	3	0.549218	0.074494	0.045328
7	4	0.512839	0.06618	0.023149
8	5	0.534823	0.026898	0.063809
9	6	0.533466	0.065926	0.049692
10	7	0.515993	0.125506	0.046094
11	8	0.440824	0.171407	0.033222
12	9	0.415082	0.209028	0.043272
13	10	0.451376	0.150553	0.065719
14	11	0.577744	0.05792	0.066048
15	12	0.580622	0.030358	0.124019
16	13	0.559801	0.033799	0.148224
17	14	0.513222	0.1285	0.086746
18	15	0.569429	0.084047	0.075253
19	16	0.592543	0.058272	0.099048
20	17	0.590346	0.179655	0.093346
21	18	0.664012	0.111742	0.115263
22	19	0.72054	0.127605	0.080264
23	20	0.819717	0.064538	0.049551
24	21	0.713586	0.140784	0.157668
25	22	0.692417	0.175448	0.125855
26	23	0.766747	0.114895	0.103433
27	24	0.766393	0.115078	0.126838
28	25	0.745659	0.100243	0.243681
29	26	0.847696	0.150376	0.107855
30	27	0.864918	0.153718	0.159494
31	28	0.805726	0.235048	0.246671
32	29	0.920541	0.257443	0.146208

Implement the answer of the Assignment 2 Part 1

Conclusion

- With 100 random seeds is :

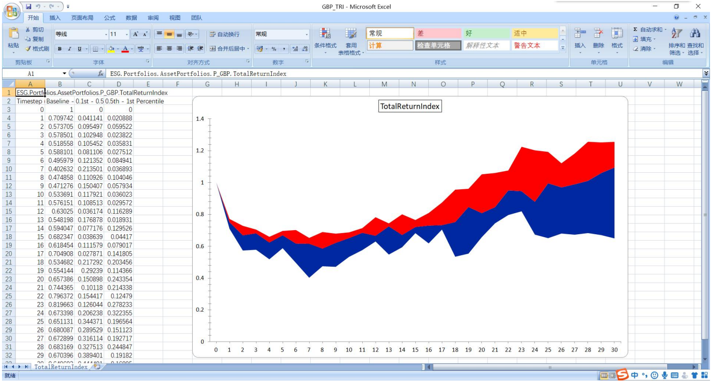


	A	B	C	D
1	ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex			
2	Timestep	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
3	0	1	0	0
4	1	0.686628	0.028079	0.031832
5	2	0.562264	0.095294	0.04983
6	3	0.558841	0.109711	0.04649
7	4	0.572156	0.076805	0.057119
8	5	0.50929	0.06555	0.094004
9	6	0.545489	0.026856	0.082946
10	7	0.540259	0.060623	0.033061
11	8	0.559768	0.062869	0.035401
12	9	0.57606	0.057909	0.035575
13	10	0.535655	0.101153	0.04908
14	11	0.499188	0.176032	0.07342
15	12	0.40968	0.289714	0.10318
16	13	0.482403	0.267169	0.052229
17	14	0.495982	0.281039	0.048332
18	15	0.501204	0.276166	0.104075
19	16	0.480961	0.377753	0.081816
20	17	0.463716	0.3496	0.165663
21	18	0.488104	0.299511	0.177932
22	19	0.575486	0.328507	0.111428
23	20	0.690636	0.243875	0.105461
24	21	0.688637	0.244855	0.07586
25	22	0.631215	0.372604	0.076949
26	23	0.693732	0.312134	0.041726
27	24	0.7329	0.26606	0.141423
28	25	0.683869	0.403542	0.06356
29	26	0.577408	0.344992	0.239094
30	27	0.668941	0.304791	0.212274
31	28	0.570353	0.455288	0.127687
32	29	0.638434	0.392464	0.129234

Implement the answer of the Assignment 2 Part 1

Conclusion

- With 10000 random seeds is :



Implement the answer of the Assignment 2 Part 1

Conclusion

- What could we conclude from EUR and GBP are :
  No marked difference. For each part compared in these two Economies, same trend means these two somehow have the similar market with each other, which could not be difficult to image from history.

Assignment 2 : Correlation Analysis

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- Why we need **CORRELATION** ?
- Correlation in use

  Mr.Pearson

$$\rho = \frac{E(V_1, V_2) - E(V_1 E(V_2)}{SD(V_1)SD(V_2)}$$

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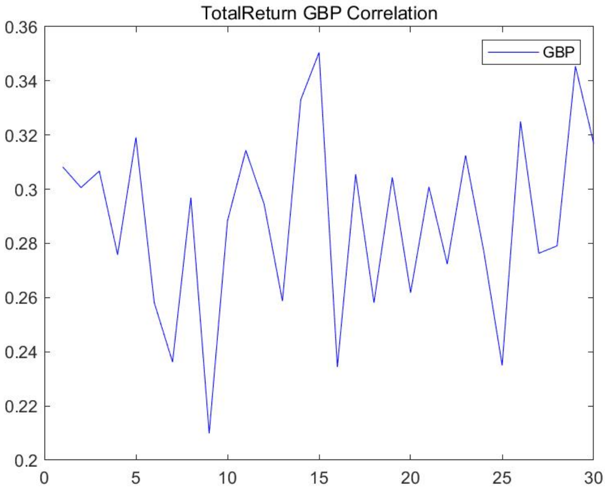

Vary Over Time – GARCH

- Log Return : Stochastic Process
  Equity asset : $\{X_t\}$; Risky Bonds :$\{Y_t\}$ where $t \in \{1, ..., 30\}$
- Reshaped Data : $X_t' = \frac{X_{t+1}-X_t}{X_t}$,$Y_t' = \frac{Y_{t+1}-Y_t}{Y_t}$
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GARCH models can also be used for updating covariance rate estimates and forecasting the future level of covariance rates.For example,the GARCH(1,1) model for updating a covariance rate between X and Y is

- $COV_n = \omega + \alpha X_{n-1} Y_{n-1} + \beta COV_{n-1}$
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- Fit the model with our data, and we got $\alpha and \beta : \alpha = -0.1839; \beta = 34.37.$

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Tail Dependence

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- where p$\in (0,100), donateauser - specifiedpercentage$
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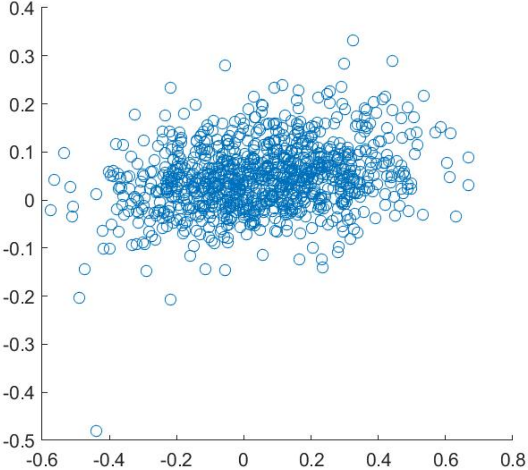

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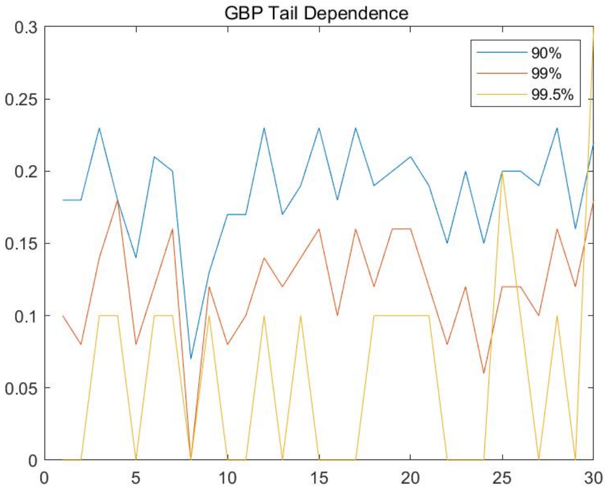
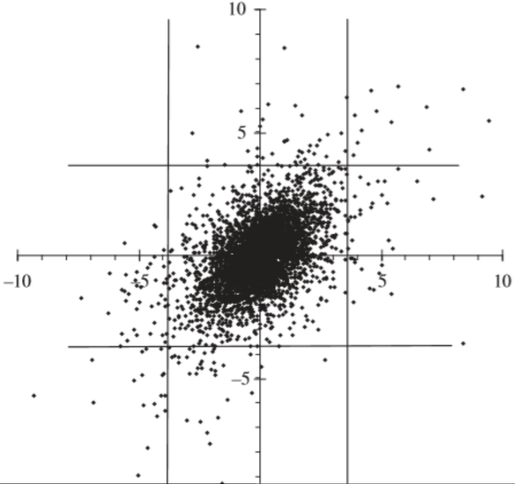


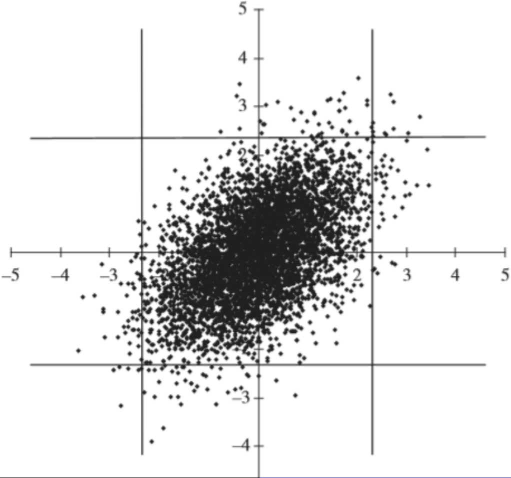
Copula

Student's t-distribution with mark tail dependence



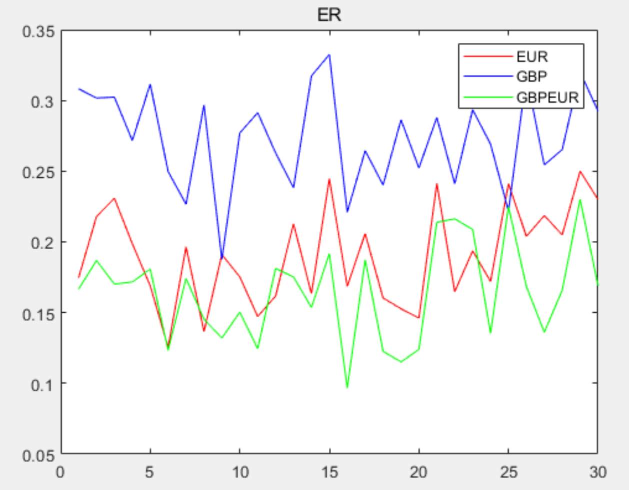
Copula

Gaussian distribution with slight tail dependence



ExcessReturn Anaysis

ExcessReturn Correlation



Conclusion

- We discuss the method to price the portfolio via Monte Carlo Simulation
- We use VaR to measure the risk of portfolio in real world
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Reference

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Thanks

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