OMF assignment 1

CMF Zihao Song

1. Mean-Variance Model

e) the efficient frontier:

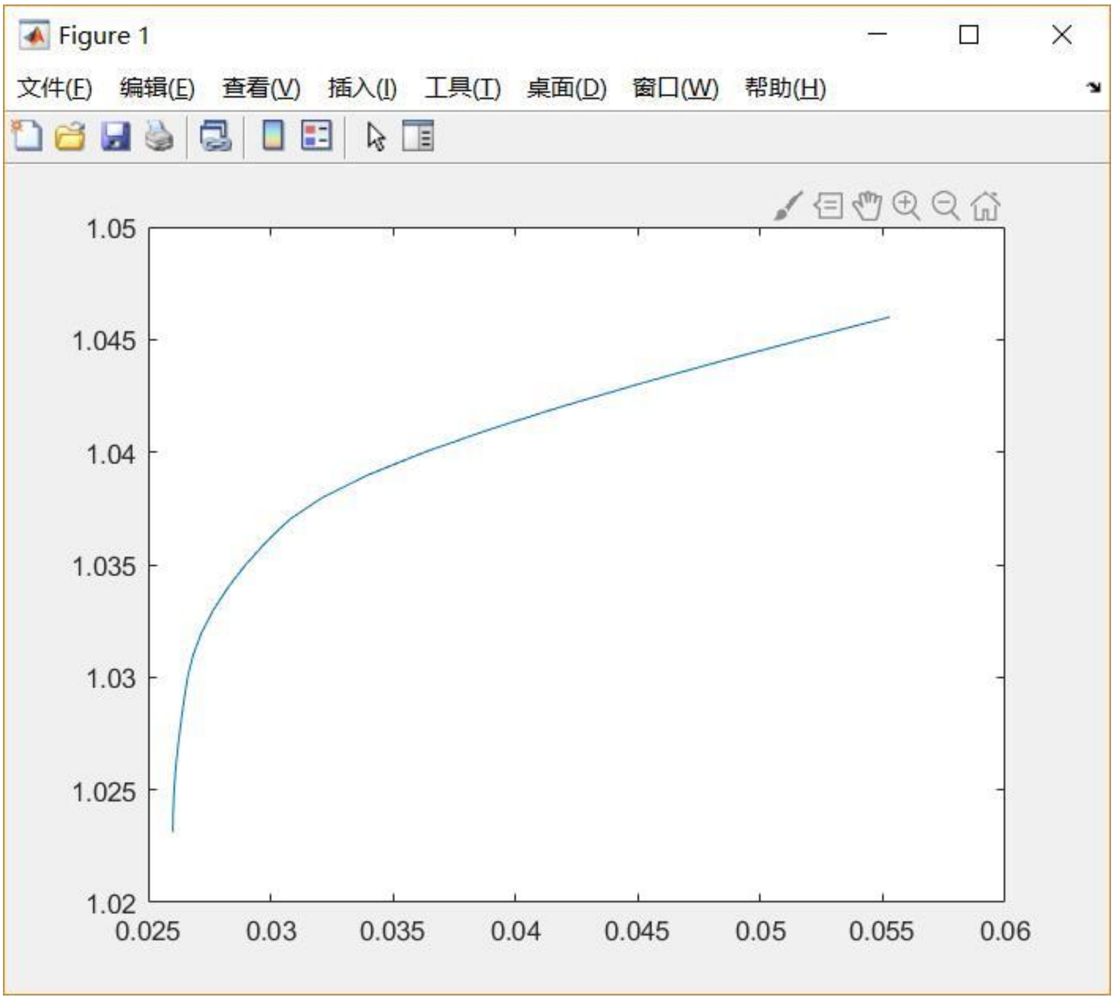

Figure 1. the efficient frontier

the asset weights as a function of R:

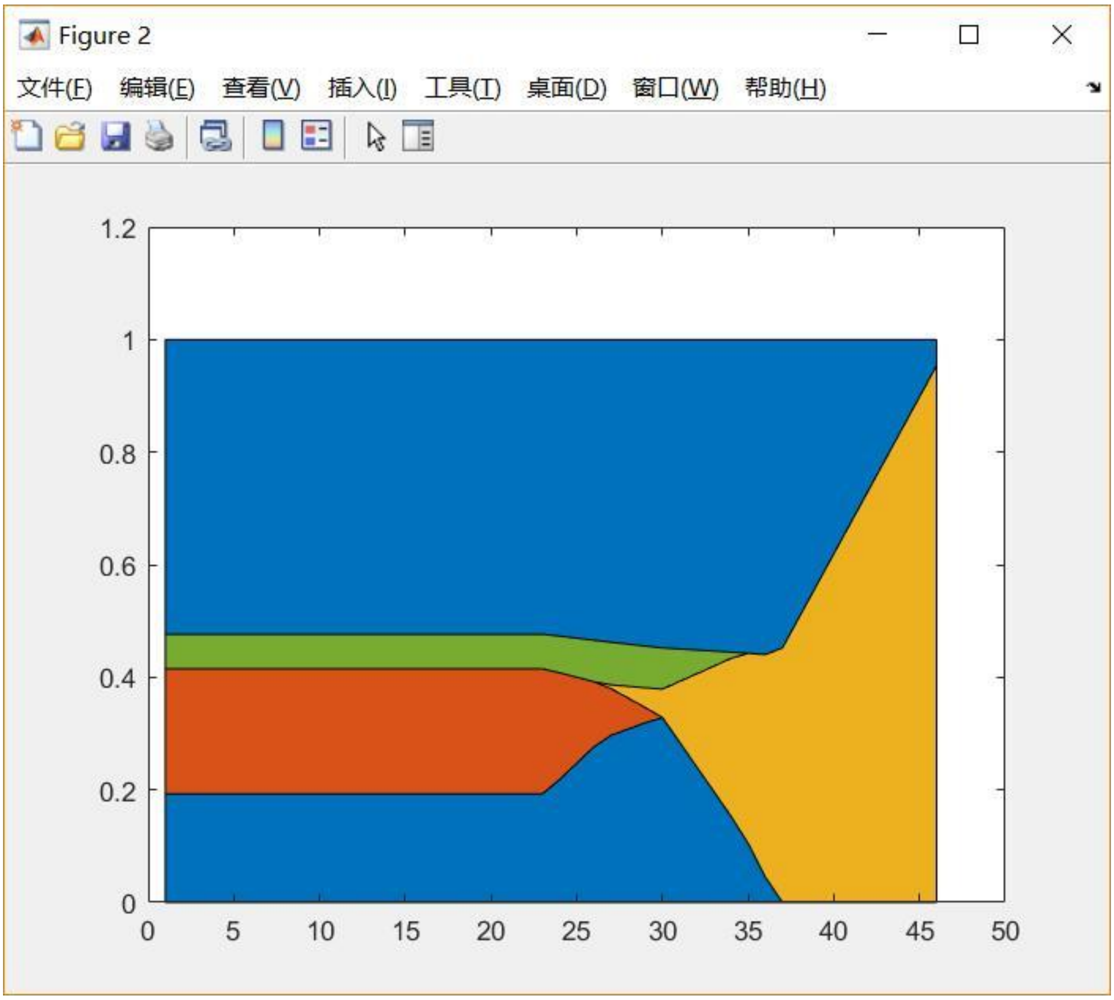

Figure 2. the asset weights as a function of R

f)

1.Interpret the results:

For the plot, the y-axis represents different levels of target returns while x-axis uses the variance of returns to represent the lowest levels of risk corresponding to the target returns. The return value changes from appx. 1.023 to 1.046, and the risk changes from 0.026 to 0.056. It is a positive correlation concave function with lower ratio when the risk becomes larger. For the meaning, it shows the lowest risk achievable for a given (target) return.

For the area, every line shows the column of xx’ we get in model which reflect the asset weights the model chooses and every area or color shows different portfolios when various target returns are required. Different colors represent different assets, from the figure 2, the trend for the percentages of different assets is shown by the area of each color.

2.Are they surprising?

Definitely not, they could be explained by the data from model.

3. Why does the model invest in the assets that it has chosen? How can you explain the changes of the portfolio as R changes?

Actually, I viewed the xx’ (the first 3 rows and the last 4rows) model gets, the geometric mean and the cov (pictures in the same order):

ans =

0.1928	0.2222	0.0000	0.0000	0.0614	0.0000	0.0000	0.5236
0.1928	0.2222	0.0000	0.0000	0.0614	0.0000	0.0000	0.5236
0.1928	0.2223	0.0000	0.0000	0.0613	0.0000	0.0000	0.5237

Figure 3. xx' (the first 3 rows)

0.0000	0.0000	0.7867	0.0000	0.0000	0.0000	0.0000	0.2133
0.0000	0.0000	0.8425	0.0000	0.0000	0.0000	0.0000	0.1575
0.0000	0.0000	0.8982	0.0000	0.0000	0.0000	0.0000	0.1018
0.0000	0.0000	0.9540	0.0000	0.0000	0.0000	0.0000	0.0460

Figure 4. xx' (the last 4rows)

变量 - GM

xx | x | GM

1x8 double

	1	2	3	4	5	6	7	8	9
1	1.0294	1.0023	1.0468	0.9868	1.0296	0.9743	0.9467	1.0289	
2									
3									

Figure 5. the geometric mean

变量 - COVK

xx | x | GM | COVK

8x8 double

	1	2	3	4	5	6	7	8
1	0.0020	0.0016	0.0021	0.0019	0.0019	0.0021	0.0019	-3.4042e-...
2	0.0016	0.0019	0.0020	0.0020	0.0017	0.0021	0.0019	-2.9235e-...
3	0.0021	0.0020	0.0034	0.0028	0.0025	0.0027	0.0022	-4.6784e-...
4	0.0019	0.0020	0.0028	0.0028	0.0023	0.0024	0.0022	-4.6895e-...
5	0.0019	0.0017	0.0025	0.0023	0.0039	0.0027	0.0022	-5.8747e-...
6	0.0021	0.0021	0.0027	0.0024	0.0027	0.0044	0.0033	9.3124e-05
7	0.0019	0.0019	0.0022	0.0022	0.0022	0.0033	0.0043	1.4457e-04
8	-3.4042e-...	-2.9235e-...	-4.6784e-...	-4.6895e-...	-5.8747e-...	9.3124e-05	1.4457e-04	0.0016
9								
10								

Figure 6. the monthly cov

As shown in pictures, firstly discuss the return against risk, it is concave because low risk makes low profit and high risk gives high profit and the sensitivities of risk becomes lower and lower. Secondly because we need to get return in low risk at the beginning, the model does not choose the asset in high cov, like 3 and does not choose the assets which cannot make profit, like 4, 6, 7. Thus, in area, at the beginning, there are 4 lines. When R (risk) goes up, we need the asset which gives us more profit with high risk, like 3, and reduce the assets with low profit and low risk, like 1, 2, 5, 8. Thus, in area, there are two lines at last. At last, when we consider who exists all the time, it is asset 8, even though its mean return is not very high among all the assets. This is because the covariances of the 8th asset with other assets are all negative, adding the 8th asset in the portfolio can decrease the total risk.

2. Risk-adjusted Return model

b) the efficient frontier:

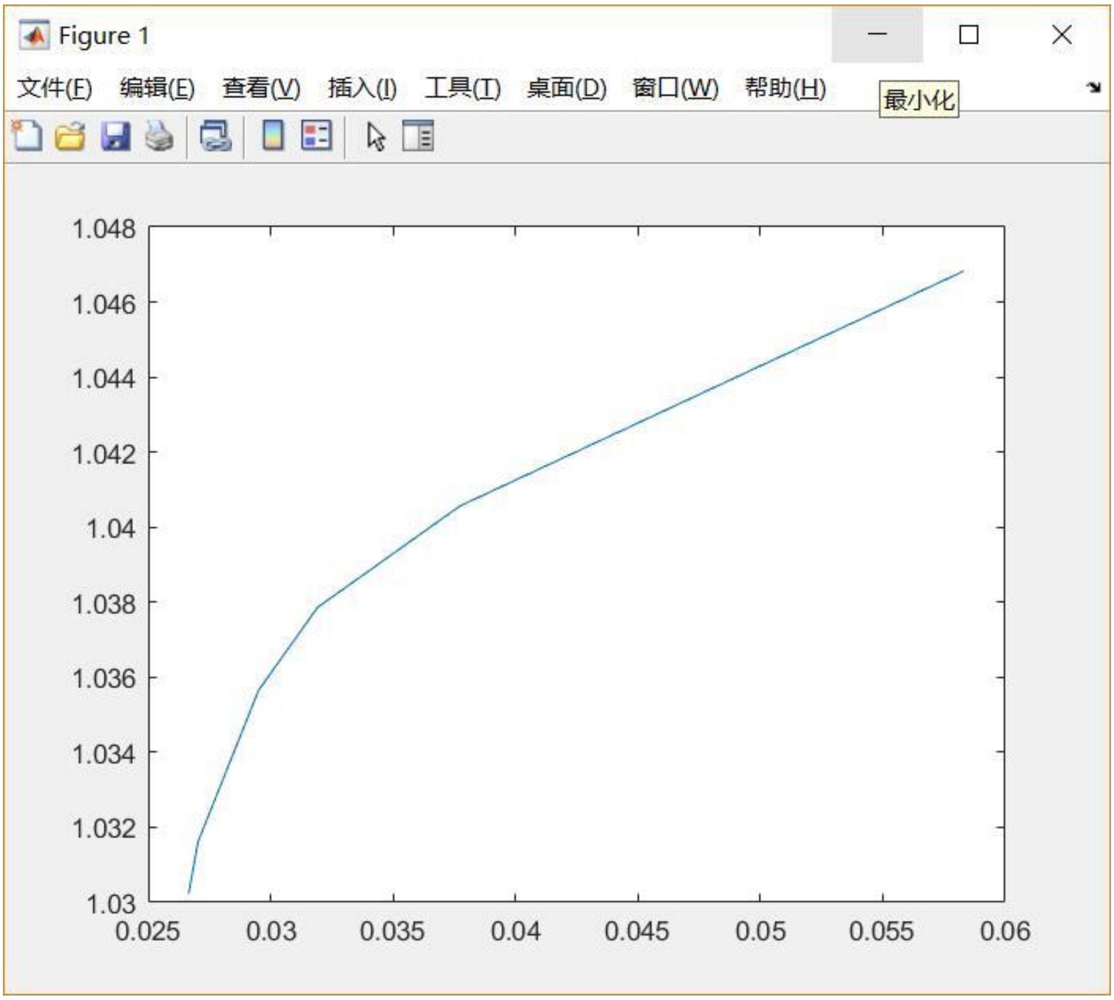

Figure 7. the efficient frontier

the asset weights as a function of R:

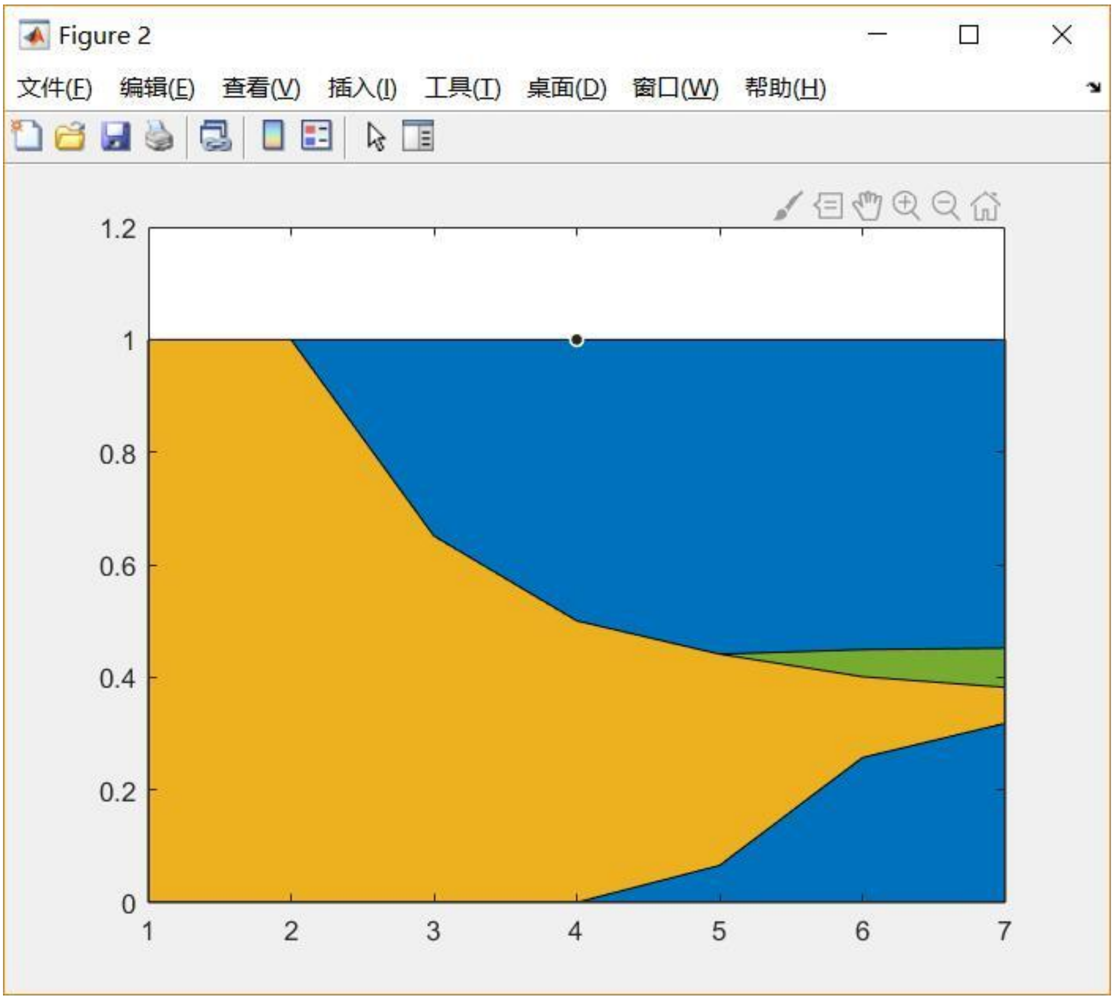

Figure 8. the asset weights as a function of R

Do the plots for parts 1(e) and 2(b) efficient frontiers and asset allocations coincide? Discuss!

Compare with 1e), firstly we discuss the same stuffs, there are 2 same in plot, one is the range of x and y axis, which means the similar level of return and risk (obviously because the same model with same assets) and another is the trend, i.e. the concave function. Secondly, let's discuss the different parts, checking the xx' the geometric mean and the cov (pictures in the same order):

变量 - GM

xx x GM

1x8 double

	1	2	3	4	5	6	7	8	9
1	1.0294	1.0023	1.0468	0.9868	1.0296	0.9743	0.9467	1.0289	
2									
3									

Figure 5. the geometric mean

变量 - COVK

xx | x | GM | COVK

8x8 double

	1	2	3	4	5	6	7	8
1	0.0020	0.0016	0.0021	0.0019	0.0019	0.0021	0.0019	-3.4042e-...
2	0.0016	0.0019	0.0020	0.0020	0.0017	0.0021	0.0019	-2.9235e-...
3	0.0021	0.0020	0.0034	0.0028	0.0025	0.0027	0.0022	-4.6784e-...
4	0.0019	0.0020	0.0028	0.0028	0.0023	0.0024	0.0022	-4.6895e-...
5	0.0019	0.0017	0.0025	0.0023	0.0039	0.0027	0.0022	-5.8747e-...
6	0.0021	0.0021	0.0027	0.0024	0.0027	0.0044	0.0033	9.3124e-05
7	0.0019	0.0019	0.0022	0.0022	0.0022	0.0033	0.0043	1.4457e-04
8	-3.4042e-...	-2.9235e-...	-4.6784e-...	-4.6895e-...	-5.8747e-...	9.3124e-05	1.4457e-04	0.0016
9								
10								

Figure 6. the monthly cov

```
>> xx'

ans =

    0.0000    0.0000    1.0000    0.0000    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    1.0000    0.0000    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.6510    0.0000    0.0000    0.0000    0.0000    0.3490
    0.0000    0.0000    0.5003    0.0000    0.0000    0.0000    0.0000    0.4997
    0.0656    0.0000    0.3754    0.0000    0.0000    0.0000    0.0000    0.5589
    0.2573    0.0000    0.1437    0.0000    0.0480    0.0000    0.0000    0.5509
    0.3182    0.0000    0.0639    0.0000    0.0696    0.0000    0.0000    0.5483
```

Figure 9. xx'

For the plot, it does not quite coincide with the one in Task 1 (e) and there is an obvious difference: the slope of this concave function. With different values of lambda, the sensitivity of the model to risk would change. The higher of the value of lambda, the more sensitive the model is. Thus, in the beginning, when lam is small, if we need min risk but high return, the model choose 100% asset 1, which causes the asset allocations having less sensitivity of risk (but in 1e, we choose four assets which causes having more sensitivity); and when lam becomes larger, this model becomes having more sensitivity on risk, thus, it chooses to decline asset 3, but at the same time, we need higher return and higher risk, i.e. choosing 1, 5, 8; Also, it has more sensitivity than 1e), so the slope is bigger than 1e) in this part. On conclusion, in Task 2, at first, when the lambda is small, the risk is less sensitive to the risk than in Task 1, resulting to a same level of change of return causing different changes in risk for these two plots. The one in Task 2 increases more with the returns increasing. But later, the value of lambda increases slower than the sensitivity, Task 2 becomes more sensitive than the one in Task 1, i.e. when return increases, the risk in Task 2 increases less than Task 1.

For the area, it intuitively shows the opinion we get above, with the value for lambda increases, the effect of risk on the objective function is increasing, in this case, the proportion of those assets with higher risk is decreasing, and the proportion of those with lower risk is increasing, i.e.

yellow rep. asset 3, model changes from 1 asset to 4 assets when r becomes larger and also the asset 2 is chosen in model due to the less risk.

3. Different estimates for the covariance matrix

a) the efficient frontier:

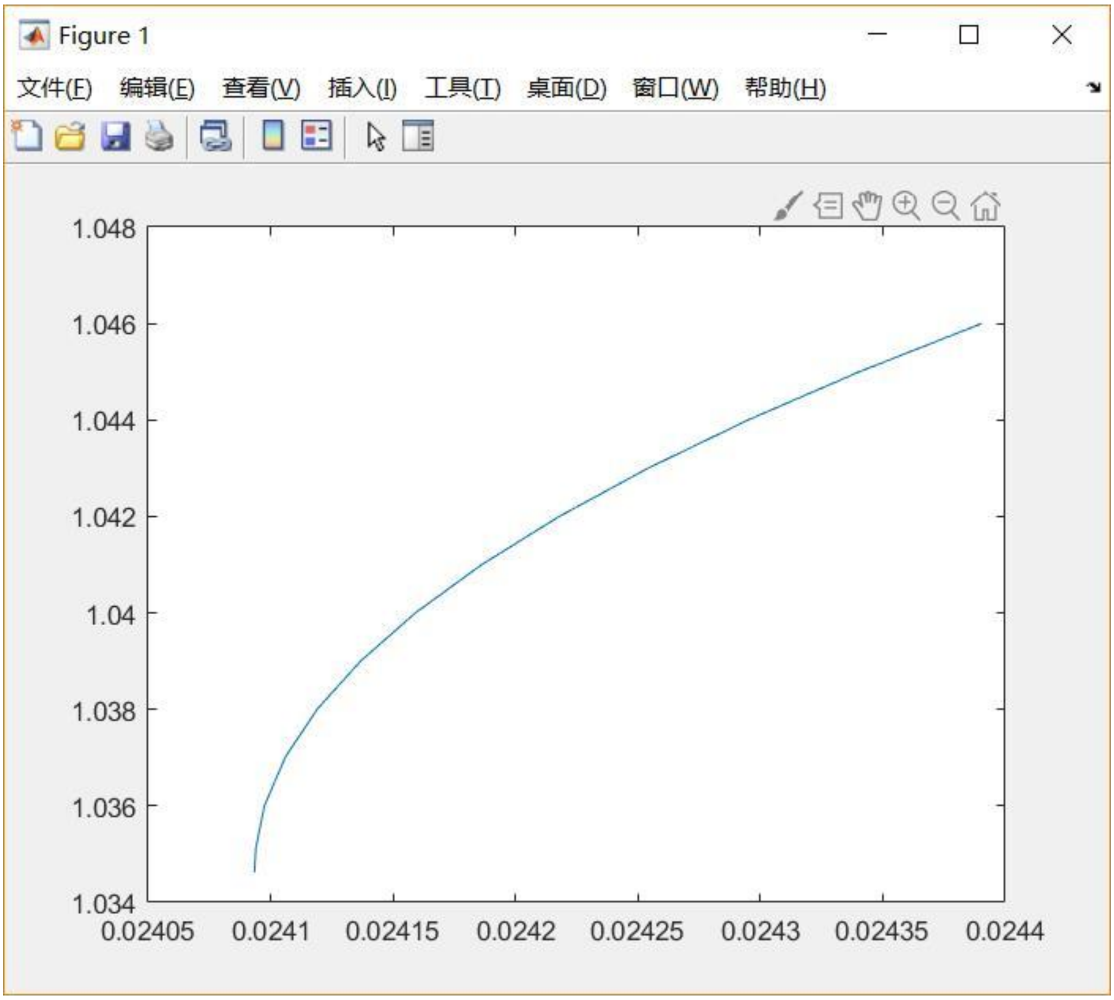

Figure 10. the efficient frontier

b) the efficient frontier:

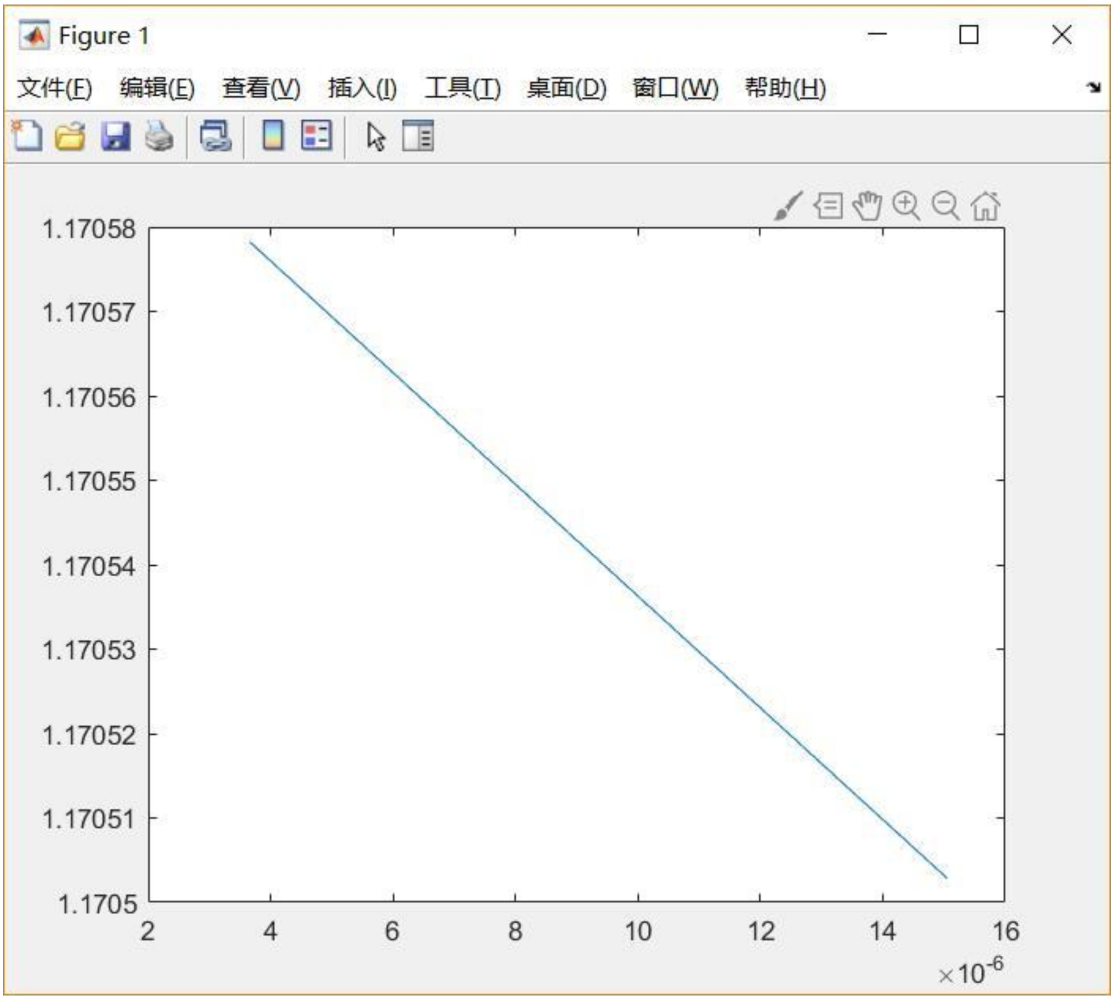

Figure 11. the efficient frontier

c)

You should observe a strange behavior in the second case. Can you comment on why this is the case?

Actually, I check the xx’ (the first several rows and the last several rows) in b), the yearly cov, the geometric mean as well (pictures in the same order):

```
>> xx'

ans =

   -1.3447   -1.8489    2.0651    0.0479    1.4501   -1.7344    0.4555    1.9093
   -1.3447   -1.8489    2.0651    0.0479    1.4501   -1.7344    0.4555    1.9093
   -1.3447   -1.8489    2.0651    0.0479    1.4501   -1.7344    0.4555    1.9093
   -1.3447   -1.8489    2.0651    0.0479    1.4501   -1.7344    0.4555    1.9093
   -1.3447   -1.8489    2.0651    0.0479    1.4501   -1.7344    0.4555    1.9093
   -1.3447   -1.8489    2.0651    0.0479    1.4501   -1.7344    0.4555    1.9093
   -1.3447   -1.8489    2.0651    0.0479    1.4501   -1.7344    0.4555    1.9093
   -1.3447   -1.8489    2.0651    0.0479    1.4501   -1.7344    0.4555    1.9093
   -1.3447   -1.8489    2.0651    0.0479    1.4501   -1.7344    0.4555    1.9093
   -1.3447   -1.8489    2.0651    0.0479    1.4501   -1.7344    0.4555    1.9093
```

Figure 12. xx' (the first several rows)

-1.3446	-1.8486	2.0649	0.0480	1.4500	-1.7343	0.4555	1.9092
-1.3446	-1.8485	2.0648	0.0480	1.4500	-1.7343	0.4555	1.9092
-1.3446	-1.8485	2.0648	0.0480	1.4500	-1.7343	0.4555	1.9091
-1.3446	-1.8484	2.0647	0.0480	1.4499	-1.7342	0.4554	1.9091
-1.3446	-1.8483	2.0647	0.0481	1.4499	-1.7342	0.4554	1.9091
-1.3446	-1.8482	2.0646	0.0481	1.4499	-1.7342	0.4554	1.9090
-1.3446	-1.8481	2.0645	0.0481	1.4498	-1.7342	0.4554	1.9090
-1.3445	-1.8481	2.0645	0.0481	1.4498	-1.7342	0.4554	1.9090
-1.3445	-1.8480	2.0644	0.0481	1.4498	-1.7341	0.4554	1.9089
-1.3445	-1.8479	2.0644	0.0482	1.4497	-1.7341	0.4554	1.9089
-1.3445	-1.8478	2.0643	0.0482	1.4497	-1.7341	0.4554	1.9089
-1.3445	-1.8477	2.0642	0.0482	1.4497	-1.7341	0.4553	1.9088
-1.3445	-1.8476	2.0642	0.0482	1.4496	-1.7341	0.4553	1.9088
-1.3445	-1.8475	2.0641	0.0483	1.4496	-1.7340	0.4553	1.9087

Figure 13. xx' (the last several rows)

变量 - COV

COV

8x8 double

	1	2	3	4	5	6	7	8
1	0.0372	0.0307	0.0414	0.0357	0.0402	0.0489	0.0405	0.0146
2	0.0307	0.0274	0.0360	0.0314	0.0340	0.0463	0.0399	0.0152
3	0.0414	0.0360	0.0519	0.0452	0.0515	0.0587	0.0430	0.0106
4	0.0357	0.0314	0.0452	0.0425	0.0496	0.0531	0.0417	0.0062
5	0.0402	0.0340	0.0515	0.0496	0.0651	0.0543	0.0371	-0.0048
6	0.0489	0.0463	0.0587	0.0531	0.0543	0.0844	0.0780	0.0314
7	0.0405	0.0399	0.0430	0.0417	0.0371	0.0780	0.0905	0.0407
8	0.0146	0.0152	0.0106	0.0062	-0.0048	0.0314	0.0407	0.0357
9								
10								

Figure 14. the yearly cov

变量 - GM

xx x GM

1x8 double

	1	2	3	4	5	6	7	8	9
1	1.0294	1.0023	1.0468	0.9868	1.0296	0.9743	0.9467	1.0289	
2									
3									

Figure 5. the geometric mean

Firstly, we figure out that in plot b), with extremely small risk and higher return than other plots, the line is strictly negative. Secondly, for reasons, Using the hedging method can effectively reduce the total risk of the portfolios, like buying one of those have positive correlation and sell another, or buy both when the correlation is negative. Thus, in xx’, we find asset 1, 2, 6 are negatively purchase and asset 3, 4, 5, 7, 8 are positively purchase with 3 and 8 are much higher than the other assets. For much higher return, I think it is because asset 3 and 8 have high return and also the model choose almost 200% of them. For the extremely low risk, checking the cov matrix, the model probably uses the negative assets to cancel out the positive assets especially the asset 3, 5, 8 which have been purchased much. Thus, it maybe leaves asset 4 and 7 with not too large number and high risk. At last, let's comment for the negative correlation line function. Obviously, in cov matrix, asset 5 and 8 have negative cov, and also among the other 6 assets, the model chooses 3 positively purchasing and 3 negatively purchasing with the same decreasing trend. Thus, when risk going up, all the pair assets have negative correlation, which is shown as a negative correlation line.

4. Mean-absolute deviation model

b) the efficient frontier:

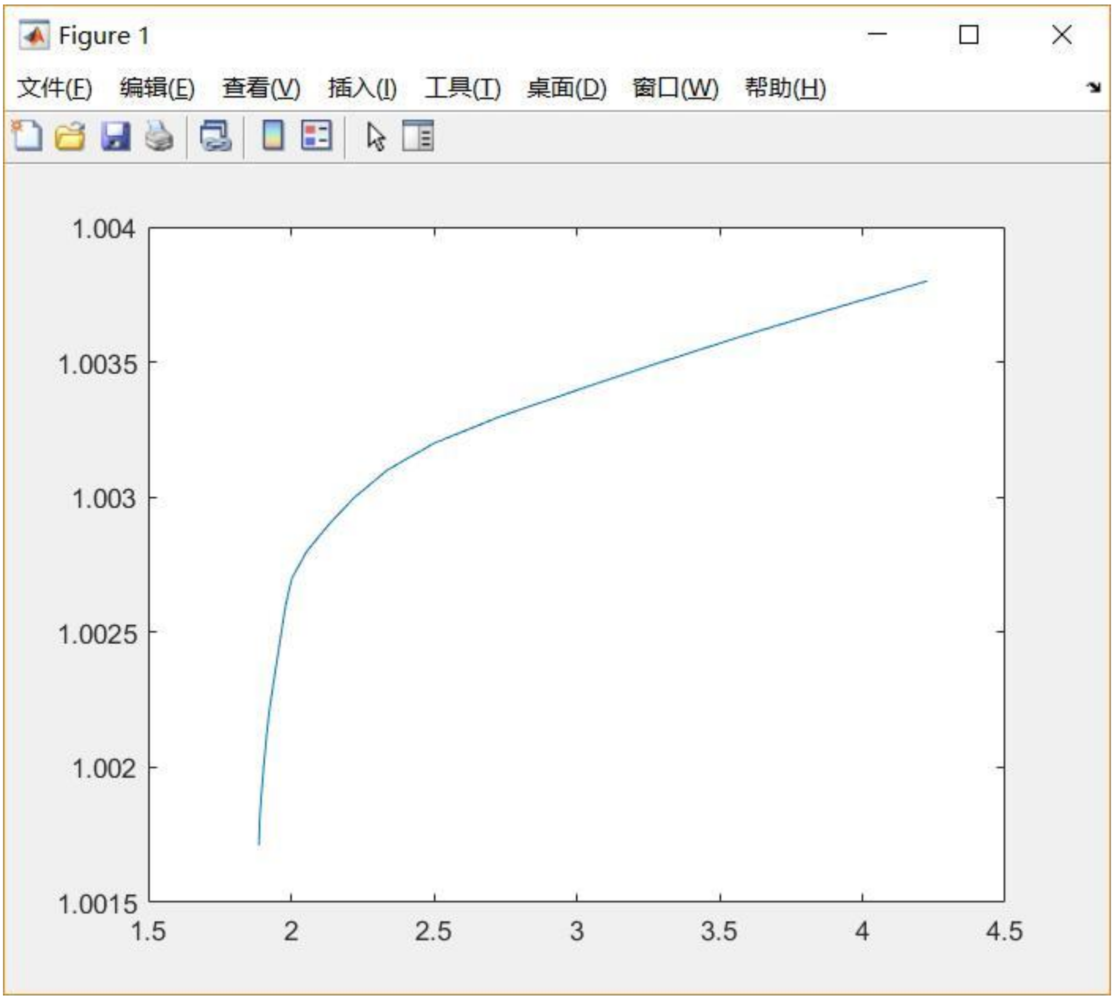

Figure 15. the efficient frontier

the asset weights as a function of R:

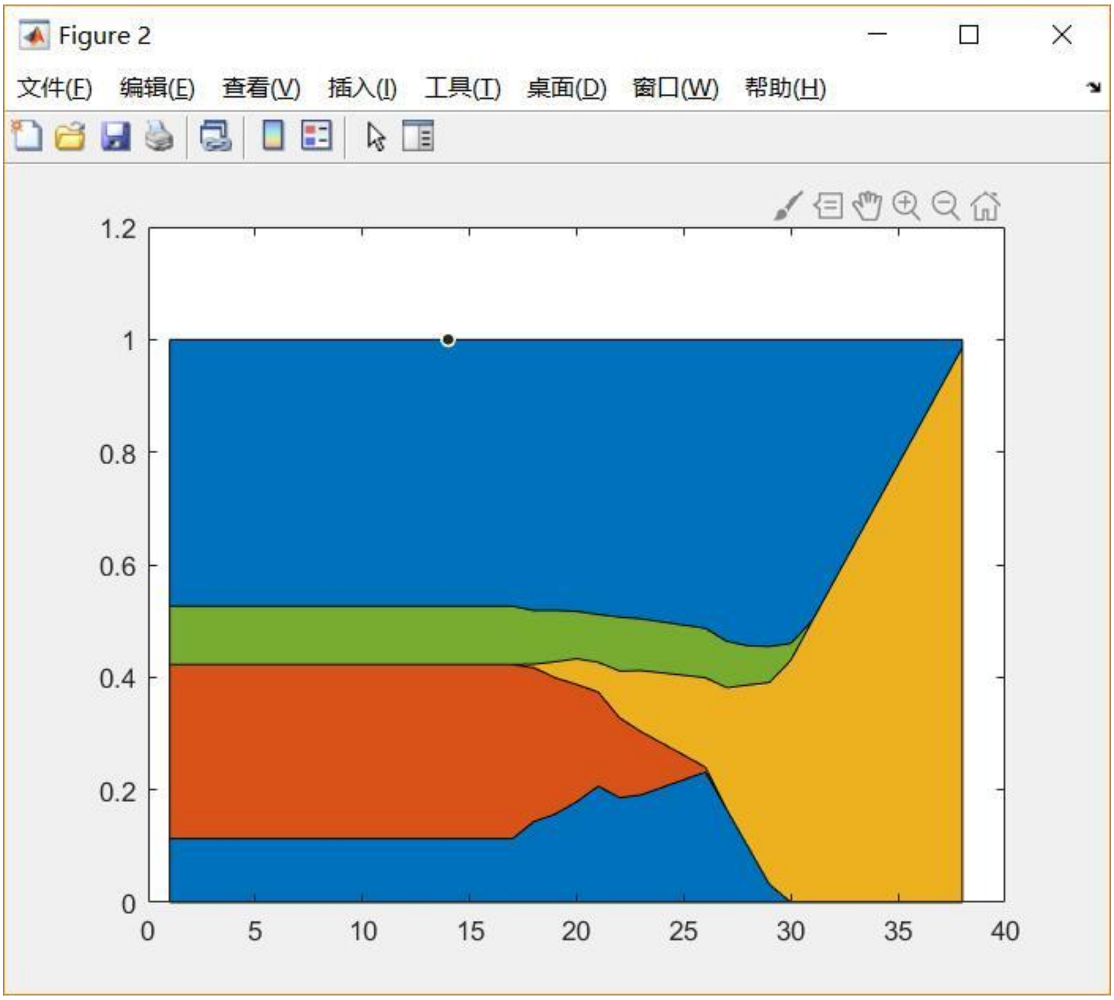

Figure 16. the asset weights as a function of R