

# Algorithm Template Library

ytz123

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## 1 图论

### 1.1 线段树维护树直径

```
#include <bits/stdc++.h>

using namespace std;

const int N = 2e5 + 5;

int T, n, m;

int len, head[N], ST[20][N];

struct edge{int u, v, w;}ee[N];

int cnt, fa[N], log_2[N], st[N], en[N], dfn[N], dis[N],
    ↪ dep[N], pos[N];

struct edges{int to, next, cost;}e[N];

void add(int u, int v, int w) {
    e[++ len] = (edges){v, head[u], w}, head[u] = len;
    e[++ len] = (edges){u, head[v], w}, head[v] = len;
}

void dfs1(int u) {
    st[u] = ++ cnt, dfn[cnt] = u;
    for (int v, i = head[u]; i; i = e[i].next) {
        v = e[i].to;
        if (v == fa[u]) continue;
        fa[v] = u, dep[v] = dep[u] + 1;
        dis[v] = dis[u] + e[i].cost, dfs1(v);
    }
    en[u] = cnt;
}

void dfs2(int u) {
    dfn[++ cnt] = u, pos[u] = cnt;
    for (int v, i = head[u]; i; i = e[i].next) {
        v = e[i].to;
        if (v == fa[u]) continue;
        dfs2(v), dfn[++ cnt] = u;
    }
}
```

```

}

int mmin(int x, int y) {
    if (dep[x] < dep[y]) return x;
    return y;
}

int lca(int u, int v) {
    static int w;
    if (pos[u] > pos[v]) swap(u, v);
    w = log_2[pos[v] - pos[u] + 1];
    return mmin(ST[w][pos[u]], ST[w][pos[v] - (1 << w) + 1]);
}

int dist(int u, int v) {
    int Lca = lca(u, v);
    return dis[u] + dis[v] - dis[Lca] * 2;
}

void build() {
    for (int i = 1; i <= cnt; i++)
        ST[0][i] = dfn[i];
    for (int i = 1; i < 20; i++)
        for (int j = 1; j <= cnt; j++)
            if (j + (1 << (i - 1)) > cnt) ST[i][j] = ST[i - 1][j]
            ↪ 1][j];
            else ST[i][j] = mmin(ST[i - 1][j], ST[i - 1][j +
            ↪ (1 << (i - 1))]);
}

int M;

struct node {
    int l, r, dis;
}tr[N << 1];

void update(int o, int o1, int o2) {
    static int d; static node tmp;
    if (tr[o1].dis == -1) {tr[o] = tr[o2]; return;}
    if (tr[o2].dis == -1) {tr[o] = tr[o1]; return;}
    if (tr[o1].dis > tr[o2].dis) tmp = tr[o1];
    else tmp = tr[o2];
    d = dist(tr[o1].l, tr[o2].l);

```

```

    if (d > tmp.dis) tmp.l = tr[o1].l, tmp.r = tr[o2].l,
        ↪ tmp.dis = d;
    d = dist(tr[o1].l, tr[o2].r);
    if (d > tmp.dis) tmp.l = tr[o1].l, tmp.r = tr[o2].r,
        ↪ tmp.dis = d;
    d = dist(tr[o1].r, tr[o2].l);
    if (d > tmp.dis) tmp.l = tr[o1].r, tmp.r = tr[o2].l,
        ↪ tmp.dis = d;
    d = dist(tr[o1].r, tr[o2].r);
    if (d > tmp.dis) tmp.l = tr[o1].r, tmp.r = tr[o2].r,
        ↪ tmp.dis = d;
    tr[o] = tmp;
}

void ask(int s, int t) {
    if (s > t) return;
    for (s += M - 1, t += M + 1; s ^ t ^ 1; s >>= 1, t >>= 1)
        ↪ {
            if (~s&1) update(0, 0, s ^ 1);
            if (t&1) update(0, 0, t ^ 1);
        }
}

int main() {
    ios::sync_with_stdio(false);
    int u, v, w, ans; log_2[1] = 0;
    for (int i = 2; i <= 2000000; i++)
        if (i == 1 << (log_2[i - 1] + 1))
            log_2[i] = log_2[i - 1] + 1;
        else log_2[i] = log_2[i - 1];
    for (cin >> T; T--;) {
        cin >> n >> m, cnt = len = 0;
        for (int i = 1; i <= n; i++)
            head[i] = 0;
        for (int i = 1; i < n; i++) {
            cin >> ee[i].u >> ee[i].v >> ee[i].w;
            add(ee[i].u, ee[i].v, ee[i].w);
        }
        dfs1(1);
        for (M = 1; M < n + 2; M <= 1);
        for (int i = 1; i <= n; i++)
            tr[i + M].l = tr[i + M].r = dfn[i], tr[i + M].dis
                ↪ = 0;
        for (int i = n + M + 1; i <= (M << 1) + 1; i++)

```

```

        tr[i].dis = -1;
    cnt = 0, dfs2(1), build();
    for (int i = M; i; i --)
        update(i, i << 1, i << 1 | 1);
    for (int i = 1; i < n; i ++){
        if (dep[ee[i].u] > dep[ee[i].v])
            swap(ee[i].u, ee[i].v);
    }
    for (int u, v, i = 1; i <= m; i ++){
        cin >> u >> v, ans = 0;
        u = ee[u].v, v = ee[v].v, w = lca(u, v);
        if (w == u || w == v){
            if (w != u) swap(u, v);
            tr[0].dis = -1, ask(1, st[u] - 1), ask(en[u] +
                ↪ 1, n), ans = max(ans, tr[0].dis);
            tr[0].dis = -1, ask(st[u], st[v] - 1),
                ↪ ask(en[v] + 1, en[u]), ans = max(ans,
                ↪ tr[0].dis);
            tr[0].dis = -1, ask(st[v], en[v]), ans =
                ↪ max(ans, tr[0].dis);
        }
        else {
            if (st[u] > st[v]) swap(u, v);
            tr[0].dis = -1, ask(1, st[u] - 1), ask(en[u] +
                ↪ 1, st[v] - 1), ask(en[v] + 1, n), ans =
                ↪ max(ans, tr[0].dis);
            tr[0].dis = -1, ask(st[u], en[u]), ans =
                ↪ max(ans, tr[0].dis);
            tr[0].dis = -1, ask(st[v], en[v]), ans =
                ↪ max(ans, tr[0].dis);
        }
        printf("%d\n", ans);
    }
}
return 0;
}

```

## 1.2 有向图判断两个点能否到达

```

/* 时间  $O(n*m/32)$  空间  $O(n*n/blk)$  blk 随便取 */
#include <bits/stdc++.h>

using namespace std;

const int N = 1e5 + 2;

```

```

const int BLK = 5000;

int n, k, p;

int d[N];

vector<int> e[N], f[N], ck[N];

int top, sta[N], in[N];

int cnt, dfn[N], low[N], vis[N];

int sum, bel[N];

bitset<BLK> a[N];

queue<int> q;

int topo[N];

void tarjan(int u) {
    vis[u] = in[u] = 1;
    sta[++top] = u, dfn[u] = low[u] = ++cnt;
    for (int v : e[u])
        if (!vis[v]) {
            tarjan(v);
            low[u] = min(low[v], low[u]);
        }
        else if (in[v])
            low[u] = min(low[v], low[u]);
    if (low[u] == dfn[u]) {
        sum++; int i;
        while (1) {
            i = sta[top--];
            in[i] = 0, bel[i] = sum;
            if (i == u) break;
        }
    }
}

int main() {
    freopen("input.txt", "r", stdin);
    freopen("output.txt", "w", stdout);
}

```

```

ios::sync_with_stdio(false);
cin >> n >> k;
for (int u, v, i = 1; i <= k; i++) {
    cin >> u >> v;
    e[u].push_back(v);
}
cin >> p;
for (int u, v, i = 1; i <= p; i++) {
    cin >> u >> v;
    f[u].push_back(v);
}

for (int i = 1; i <= n; i++)
    if (!vis[i])
        tarjan(i);
for (int i = 1; i <= n; i++) {
    for (int j : f[i]) {
        if (bel[i] == bel[j]) return
            ↪ puts("NO"), 0;
        ck[bel[i]].push_back(bel[j]);
    }
    f[i].clear();
}
for (int i = 1; i <= n; i++)
    for (int j : e[i]) {
        if (bel[i] == bel[j]) continue;
        f[bel[i]].push_back(bel[j]);
        d[bel[j]]++;
    }

cnt = 0;
for (int i = 1; i <= sum; i++)
    if (!d[i])
        q.push(i);
while (!q.empty()) {
    int now = q.front(); q.pop();
    topo[++ cnt] = now;
    for (int j : f[now]) {
        d[j]--;
        if (d[j] == 0) q.push(j);
    }
}

```



```

    for (int i = 1, t = (sum + BLK - 1) / BLK; i <= t; i
        ↪ ++ ) {
        for (int j = sum; j; j --) {
            int u = topo[j];
            a[u].reset();
            if (BLK * (i - 1) < u && u <= BLK * i)
                a[u][u - BLK * (i - 1) - 1] =
                    ↪ 1;
            for (int v : f[u])
                a[u] |= a[v];
        }
        for (int j = 1; j <= sum; j ++ )
            for (int v : ck[j])
                if (BLK * (i - 1) < v && v <=
                    ↪ BLK * i && a[j][v - BLK *
                    ↪ (i - 1) - 1] == 1) {
                    puts("NO");
                    return 0;
                }
    }

    printf("YES\n%d\n", k);
    for (int i = 1; i <= n; i ++ )
        for (int j : e[i])
            printf("%d %d\n", i, j);

    return 0;
}

```

### 1.3 spfa 费用流

```

const int N = 600, M = 800000, inf = 0x3f3f3f3f;

int s, t, ans, len, maxflow;

int T, n, m, K, W;

int head[N], incf[N], path[N], pre[N], vis[N], d[N];

struct edge{
    int to, next, cap, cost;
}e[M];

struct video {z
    int s, t, w, op;
}

```

```

}a[N];

void add(int u, int v, int w, int c) {
    e[++ len] = (edge){v, head[u], w, c}, head[u] = len;
    e[++ len] = (edge){u, head[v], 0, -c}, head[v] = len;
}

bool spfa() {
    deque<int> q;
    q.push_back(s), incf[s] = inf;
    for (int i = 1; i <= t; i++) d[i] = inf;
    d[s] = 0;
    while (!q.empty()) {
        int x = q.front();
        q.pop_front(), vis[x] = 0;
        for (int i = head[x]; i; i = e[i].next) {
            if (e[i].cap && d[e[i].to] > d[x] +
                ↪ e[i].cost) {
                d[e[i].to] = d[x] + e[i].cost;
                pre[e[i].to] = x,
                ↪ path[e[i].to] = i;
                incf[e[i].to] = min(incf[x],
                ↪ e[i].cap);
                if (!vis[e[i].to]) {
                    vis[e[i].to] = 1;
                    if (q.empty() ||
                        ↪ d[e[i].to] <
                        ↪ d[q.front()])
                        ↪ q.push_front(e[i].to);
                    else
                        ↪ q.push_back(e[i].to);
                }
            }
        }
    }
    maxflow += incf[t];
    if (d[t] == inf) return 0;
    for (int i = t; i != s; i = pre[i]) {
        e[path[i]].cap -= incf[t];
        e[path[i] ^ 1].cap += incf[t];
    }
    return ans += incf[t] * d[t], 1;
}

```

```
int main() {  
    /*build graph*/  
    while(spfa());  
}
```

## 1.4 dinic 最大流

```
#include <bits/stdc++.h>  
  
using namespace std;  
  
const int N = 20000;  
  
const int M = 500000;  
  
const int inf = 0x3f3f3f3f;  
  
int n, m;  
  
int s, t, len = 1;  
  
int to[M], cap[M], nex[M];  
  
int g[N], p[N], q[N], d[N];  
  
void add(int x, int y, int v) {  
    to[++ len] = y, cap[len] = v, nex[len] = g[x], g[x] =  
        ↪ len;  
    to[++ len] = x, cap[len] = 0, nex[len] = g[y], g[y] =  
        ↪ len;  
}  
  
bool bfs() {  
    int l = 1, r = 1, x, i;  
    memset(d, 0, sizeof d);  
    d[s] = 1, q[1] = s;  
    while (l <= r) {  
        x = q[l ++];  
        for (i = g[x]; i; i = nex[i])  
            if (cap[i] && !d[to[i]])  
                d[to[i]] = d[x] + 1, q[++ r] =  
                    ↪ to[i];  
    }  
    return d[t];  
}
```

```
}

int dfs(int x, int y) {
    if (x == t || y == 0) return y;
    int flow = 0;
    for (int &i = p[x]; i; i = nex[i]) {
        if (!cap[i] || d[to[i]] != d[x] + 1) continue;
        int f = dfs(to[i], min(y, cap[i]));
        flow += f, y -= f;
        cap[i] -= f, cap[i ^ 1] += f;
        if (!y) break;
    }
    return flow;
}

int dinic() {
    int maxflow = 0;
    while (bfs()) {
        memcpy(p, g, sizeof g);
        maxflow += dfs(s, inf);
    }
    return maxflow;
}

int main() {
    ios::sync_with_stdio(false);

    return 0;
}
```

## 1.5 DAG 删去无用边

```
/*
无用边定义：对于边  $(u, v)$  如果存在从  $u$  到  $v$  不经过该边的另一条路
 $\hookrightarrow$  径，则称该边无用
时间复杂度： $O(n^3)$ 
*/
bool f[N][N]; //  $i$  是否可达  $j$ 

vector<int> e[N];

int main() {
    rep(i, 1, n)
        for (int j : e[i]) {
```

```
        for (k, 1, n)
            if (i != k && j != k &&
                ⇨ f[i][k] && f[k][j])
                no_use_edge;
    }
}
```

## 1.6 树分治

```
#include <bits/stdc++.h>

using namespace std;

typedef long long ll;

const int N = 1e5 + 5;

vector <int> e[N];

int n, a[N];

int root, _left, vis[N];

int siz[N], maxv[N];

void find_root(int u, int fr) {
    siz[u] = 1, maxv[u] = 0;
    for (int v : e[u]) {
        if (v == fr || vis[v]) continue;
        find_root(v, u);
        siz[u] += siz[v];
        maxv[u] = max(maxv[u], siz[v]);
    }
    maxv[u] = max(maxv[u], _left - siz[u]);
    if (!root || maxv[u] < maxv[root])
        root = u;
}

void dfs(int u, int fr) {
    siz[u] = 1;
    for (int v : e[u]) {
        if (v == fr || vis[v]) continue;
        find_root(v, u);
        siz[u] += siz[v];
    }
}
```

```

    }
}

void solve(int u, int w) {
    dfs(u, u); //update siz[]
    a[u] = w, vis[u] = 1;
    for (int v : e[u]) {
        if (vis[v]) continue;
        _left = siz[v];
        root = 0;
        find_root(v, v);
        solve(root, w + 1);
    }
}

int main() {
    ios::sync_with_stdio(false);
    cin >> n;
    for (int u, v, i = 1; i < n; i++) {
        cin >> u >> v;
        e[u].push_back(v);
        e[v].push_back(u);
    }
    _left = n, root = 0, find_root(1, 1);
    solve(root, 0);
    for (int i = 1; i <= n; i++)
        printf("%c ", 'A' + a[i]);
    return 0;
}

```

## 2 数据结构

### 2.1 treap

/\* 容易实现的预开内存池 *treap*,  
 ↳ 每次 *head* 清空即可  
 如果初始要插入  $n$  个 1, 可改为类  
 ↳ 似 *splay* 的  $O(n)build$  写法  
 \*/

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
const int poolSize = 5e5 + 10;
```

```
struct node {
    node *c[2];
    int v, r, siz;
    void update();
    void init(int x);
};
```

```
node *null = new node(), *root;
↳ = null;
```

```
void node::update() {
    siz = c[0] -> siz +
    ↳ c[1] -> siz + 1;
}
```

```
void node::init(int x) {
    v = x, r = rand(), siz
    ↳ = 1;
    c[0] = c[1] = null;
}
```

```
node nodesPool[poolSize];
```

```
int head; //每次 head=0 清空
```

```
node *newnode(int x) {
    node *res =
    ↳ &nodesPool[head
    ↳ ++];
}
```

```
res -> init(x);
return res;
}

void rot(node *&o, int d) {
    node *tmp = o ->
    ↳ c[!d];
    o -> c[!d] = tmp ->
    ↳ c[d], tmp -> c[d]
    ↳ = o;
    o -> update(), tmp ->
    ↳ update(), o = tmp;
}
```

```
void insert(node *&o, int x) {
    if (o == null) {
        o =
        ↳ newnode(x);
        return;
    }
    int d = x > o -> v ? 0
    ↳ : 1;
    insert(o -> c[d], x);
    if (o -> c[d] -> r < o
    ↳ -> r) rot(o, !d);
    o -> update();
}
```

```
void del(node *&o, int x) {
    if (x == o -> v) {
        if (o -> c[0]
        ↳ == null)
        ↳ {o = o ->
        ↳ c[1];
        ↳ return;}
        if (o -> c[1]
        ↳ == null)
        ↳ {o = o ->
        ↳ c[0];
        ↳ return;}
    }
```

```

        int d = o ->
        ↪ c[0] -> r
        ↪ < o ->
        ↪ c[1] -> r
        ↪ ? 1 : 0;
        rot(o, d),
        ↪ del(o ->
        ↪ c[d], x);
    }
    else del(o -> c[x <= o
    ↪ -> v], x);
    o -> update();
}

void build(node *&o, int l,
    ↪ int r) {
    o = newnode(1);
    if (l == r) return;
    int mid = l + r >> 1;
    if (l < mid) build(o
    ↪ -> c[0], l, mid -
    ↪ 1);
    if (o -> c[0] != null
    ↪ && o -> c[0] -> r
    ↪ < o -> r) swap(o
    ↪ -> c[0] -> r, o ->
    ↪ r);
    if (mid < r) build(o
    ↪ -> c[1], mid + 1,
    ↪ r);
    if (o -> c[1] != null
    ↪ && o -> c[1] -> r
    ↪ < o -> r) swap(o
    ↪ -> c[1] -> r, o ->
    ↪ r);
    o -> update();
}

struct node {
    int Max[3], Min[3], d[3];
    int val, maxv;
    node *c[2];

    node() {
        c[0] = c[1] = NULL;
        val = maxv = 0;
    }

    void pushup();

    bool operator < (const
    ↪ node &a) const {
        return d[nowD] <
        ↪ a.d[nowD];
    }
} Null, nodes[N];

node *root = &Null;

inline void node::pushup() {
    if (c[0] != &Null) {
        if (c[0] -> Max[1] >
        ↪ Max[1]) Max[1] =
        ↪ c[0] -> Max[1];
        if (c[0] -> Max[2] >
        ↪ Max[2]) Max[2] =
        ↪ c[0] -> Max[2];
    }
}

/*
 * O(n * n^(1-1/k)), k 为维度

```

## 2.2 KDTree

### 2.2.1 3 维 KDtree



```

        if (c[0] -> Min[0] <
            ↪ Min[0]) Min[0] =
            ↪ c[0] -> Min[0];
        if (c[0] -> Min[2] <
            ↪ Min[2]) Min[2] =
            ↪ c[0] -> Min[2];
        if (c[0] -> maxv >
            ↪ maxv) maxv = c[0]
            ↪ -> maxv;
    }
    if (c[1] != &Null) {
        if (c[1] -> Max[1] >
            ↪ Max[1]) Max[1] =
            ↪ c[1] -> Max[1];
        if (c[1] -> Max[2] >
            ↪ Max[2]) Max[2] =
            ↪ c[1] -> Max[2];
        if (c[1] -> Min[0] <
            ↪ Min[0]) Min[0] =
            ↪ c[1] -> Min[0];
        if (c[1] -> Min[2] <
            ↪ Min[2]) Min[2] =
            ↪ c[1] -> Min[2];
        if (c[1] -> maxv >
            ↪ maxv) maxv = c[1]
            ↪ -> maxv;
    }
}

inline node *build(int l, int
    ↪ r) {
    int mid = l + r >> 1; nowD
    ↪ = rand() % 3;
    nth_element(nodes + l,
    ↪ nodes + mid, nodes + r
    ↪ + 1);
    node *res = &nodes[mid];
    if (l != mid) res -> c[0]
    ↪ = build(l, mid - 1);
    else res -> c[0] = &Null;
    if (r != mid) res -> c[1]
    ↪ = build(mid + 1, r);
    else res -> c[1] = &Null;
    res -> pushup();
}

return res;
}

inline int calc(node *o) {
    if (y[0] < o -> Min[0] ||
    ↪ x[1] > o -> Max[1] ||
    ↪ x[2] > o -> Max[2] ||
    ↪ y[2] < o -> Min[2])
    ↪ return -1;
    return o -> maxv;
}

inline void query(node *o) {
    if (o -> val > ans && y[0]
    ↪ >= o -> d[0] && x[1]
    ↪ <= o -> d[1] && x[2]
    ↪ <= o -> d[2] && y[2]
    ↪ >= o -> d[2]) ans = o
    ↪ -> val;
    int dl, dr;
    if (o -> c[0] != &Null) dl
    ↪ = calc(o -> c[0]);
    else dl = -1;
    if (o -> c[1] != &Null) dr
    ↪ = calc(o -> c[1]);
    else dr = -1;
    if (dl > dr) {
        if (dl > ans) query(o
        ↪ -> c[0]);
        if (dr > ans) query(o
        ↪ -> c[1]);
    } else {
        if (dr > ans) query(o
        ↪ -> c[1]);
        if (dl > ans) query(o
        ↪ -> c[0]);
    }
}

int main() {
    ↪ ios::sync_with_stdio(false);
    cin >> n >> m;
}

```

```

for (int i = 1; i <= n; i
↪ ++ ) {
    cin >> a[i];
    b[i] = d[a[i]];
    d[a[i]] = i;
}
for (int i = 1; i <= n; i
↪ ++ ) d[i] = n + 1;
for (int i = n; i; i --) {
    c[i] = d[a[i]];
    d[a[i]] = i;
}
for (int i = 1; i <= n; i
↪ ++ ) {
    nodes[i].Min[0] =
↪ nodes[i].d[0] =
↪ b[i];
    nodes[i].Max[1] =
↪ nodes[i].d[1] =
↪ c[i];
    nodes[i].Max[2] =
↪ nodes[i].Min[2] =
↪ nodes[i].d[2] = i;
    nodes[i].val =
↪ nodes[i].maxv =
↪ a[i];
}
root = build(1, n);
for (int l, r; m --; ) {
    cin >> l >> r;
    l = (l + ans) % n + 1;
    r = (r + ans) % n + 1;
    if (l > r) swap(l, r);
    y[0] = l - 1;
    x[1] = r + 1;
    x[2] = l, y[2] = r;
    ans = 0, query(root);
    cout << ans << endl;
}
cout << endl;
return 0;
}

```

**2.2.2 KDtree 二维空间区间覆盖  
单点查询**

```

/* 类似线段树 */
#include <bits/stdc++.h>

using namespace std;

const int N = 1e5 + 5;

const int Mod = 1e9 + 7;

int nowD, x[2], y[2], z;

struct node {
    int Max[2], Min[2], d[2];
    int val, lazy;
    node *c[2];

    node() {
        c[0] = c[1] = NULL;
    }

    void pushup();

    void pushdown();

    bool operator < (const
↪ node &a) const {
        return d[nowD] <
↪ a.d[nowD];
    }
} Null, nodes[N];

node *root = &Null;

inline void node::pushup() {
    if (c[0] != &Null) {
        if (c[0] -> Max[0] >
↪ Max[0]) Max[0] =
↪ c[0] -> Max[0];
        if (c[0] -> Max[1] >
↪ Max[1]) Max[1] =
↪ c[0] -> Max[1];
    }
}

```

```

        if (c[0] -> Min[0] <
            ↪ Min[0]) Min[0] =
            ↪ c[0] -> Min[0];
        if (c[0] -> Min[1] <
            ↪ Min[1]) Min[1] =
            ↪ c[0] -> Min[1];
    }
    if (c[1] != &Null) {
        if (c[1] -> Max[0] >
            ↪ Max[0]) Max[0] =
            ↪ c[1] -> Max[0];
        if (c[1] -> Max[1] >
            ↪ Max[1]) Max[1] =
            ↪ c[1] -> Max[1];
        if (c[1] -> Min[0] <
            ↪ Min[0]) Min[0] =
            ↪ c[1] -> Min[0];
        if (c[1] -> Min[1] <
            ↪ Min[1]) Min[1] =
            ↪ c[1] -> Min[1];
    }
}

inline void node::pushdown() {
    if (c[0] != &Null) c[0] ->
        ↪ val = c[0] -> lazy =
        ↪ lazy;
    if (c[1] != &Null) c[1] ->
        ↪ val = c[1] -> lazy =
        ↪ lazy;
    lazy = -1;
}

inline node *build(int l, int
    ↪ r, int D) {
    int mid = l + r >> 1; nowD
        ↪ = D;
    nth_element(nodes + l,
        ↪ nodes + mid, nodes + r
        ↪ + 1);
    node *res = &nodes[mid];
    if (l != mid) res -> c[0]
        ↪ = build(l, mid - 1,
        ↪ !D);
    else res -> c[0] = &Null;
    if (r != mid) res -> c[1]
        ↪ = build(mid + 1, r,
        ↪ !D);
    else res -> c[1] = &Null;
    res -> pushup();
    return res;
}

inline int query(node *o) {
    if (o == &Null) return -1;
    if (o -> lazy != -1) o ->
        ↪ pushdown();
    if (x[0] > o -> Max[0] ||
        ↪ y[0] > o -> Max[1] ||
        ↪ x[0] < o -> Min[0] ||
        ↪ y[0] < o -> Min[1])
        ↪ return -1;
    if (x[0] == o -> d[0])
        ↪ return o -> val;
    return max(query(o ->
        ↪ c[0]), query(o ->
        ↪ c[1]));
}

inline void modify(node *o) {
    if (o == &Null) return;
    if (o -> lazy != -1) o ->
        ↪ pushdown();
    if (x[0] > o -> Max[0] ||
        ↪ y[0] > o -> Max[1] ||
        ↪ x[1] < o -> Min[0] ||
        ↪ y[1] < o -> Min[1])
        ↪ return;
    if (x[0] <= o -> Min[0] &&
        ↪ y[0] <= o -> Min[1] &&
        ↪ x[1] >= o -> Max[0] &&
        ↪ y[1] >= o -> Max[1]) {
        o -> val = o -> lazy =
            ↪ z;
        return;
    }
}

```

```

    if (x[0] <= o -> d[0] &&
        ↪ y[0] <= o -> d[1] &&
        ↪ x[1] >= o -> d[0] &&
        ↪ y[1] >= o -> d[1]) o
        ↪ -> val = z;
    modify(o -> c[0]),
        ↪ modify(o -> c[1]);
}

int n, m, k, a[N], c[N], d[N];

int cnt, st[N], en[N], dfn[N],
    ↪ dep[N];

vector <int> e[N];

void dfs(int u) {
    st[u] = ++ cnt, dfn[cnt] =
        ↪ u;
    for (int v : e[u])
        dep[v] = dep[u] + 1,
            ↪ dfs(v);
    en[u] = cnt;
}

int main() {
    ↪ ios::sync_with_stdio(false);
    int T, ans;
    for (cin >> T; T --; ) {
        cin >> n >> m >> k,
            ↪ ans = cnt = 0;
        for (int i = 1; i <=
            ↪ n; i++)
            e[i].clear();
        for (int u, i = 2; i
            ↪ <= n; i++) {
            cin >> u;
            e[u].push_back(i);
        }
        dfs(1);
        for (int i = 1; i <=
            ↪ n; i++) {
nodes[i].Min[0] =
    ↪ nodes[i].Max[0]
    ↪ =
    ↪ nodes[i].d[0]
    ↪ = i;
nodes[i].Min[1] =
    ↪ nodes[i].Max[1]
    ↪ =
    ↪ nodes[i].d[1]
    ↪ = dep[dfn[i]];
nodes[i].val = 1,
    ↪ nodes[i].lazy
    ↪ = -1;
}
root = build(1, n, 0);
for (int u, v, w, i =
    ↪ 1; i <= k; i++) {
    cin >> u >> v >>
        ↪ w;
    if (w == 0) {
        x[0] = st[u],
            ↪ y[0] =
            ↪ dep[u];
        ans = (ans +
            ↪ 1ll * i *
            ↪ query(root)
            ↪ % Mod) %
            ↪ Mod;
    } else {
        x[0] = st[u],
            ↪ x[1] =
            ↪ en[u];
        y[0] = dep[u],
            ↪ y[1] =
            ↪ dep[u] +
            ↪ v;
        z = w,
            ↪ modify(root);
    }
}
cout << ans << endl;
}
return 0;
}

```

### 2.2.3 KDtree 二维空间单点修改 区间查询

```

/*
 * 调整重构系数可以影响常数
 * 询问多就让系数接近
 ↪ 0.70-0.75, 询问少就让系数在
 ↪ 0.8-0.90
 */
#include <bits/stdc++.h>

using namespace std;

const int inf = 1e9;

int n, m, tot, nowD;

struct node {
    int Max[2], Min[2], d[2];
    int sum, siz, val;
    node *c[2];

    node() {
        Max[0] = Max[1] =
            ↪ -inf;
        Min[0] = Min[1] = inf;
        sum = val = siz = 0;
        c[0] = c[1] = NULL;
        d[0] = d[1] = 0;
    }

    void update();
} Null, nodes[200010],
↪ *temp[200010];

node *root = &Null;

inline void node::update() {
    siz = c[0] -> siz + c[1]
    ↪ -> siz + 1;
    sum = c[0] -> sum + c[1]
    ↪ -> sum + val;
    if (c[0] != &Null) {

```

```

        if (c[0] -> Max[0] >
            ↪ Max[0]) Max[0] =
            ↪ c[0] -> Max[0];
        if (c[0] -> Max[1] >
            ↪ Max[1]) Max[1] =
            ↪ c[0] -> Max[1];
        if (c[0] -> Min[0] <
            ↪ Min[0]) Min[0] =
            ↪ c[0] -> Min[0];
        if (c[0] -> Min[1] <
            ↪ Min[1]) Min[1] =
            ↪ c[0] -> Min[1];
    }
    if (c[1] != &Null) {
        if (c[1] -> Max[0] >
            ↪ Max[0]) Max[0] =
            ↪ c[1] -> Max[0];
        if (c[1] -> Max[1] >
            ↪ Max[1]) Max[1] =
            ↪ c[1] -> Max[1];
        if (c[1] -> Min[0] <
            ↪ Min[0]) Min[0] =
            ↪ c[1] -> Min[0];
        if (c[1] -> Min[1] <
            ↪ Min[1]) Min[1] =
            ↪ c[1] -> Min[1];
    }
}

inline bool cmp(const node *a,
    ↪ const node *b) {
    return a -> d[nowD] < b ->
    ↪ d[nowD];
}

inline void traverse(node *o)
    ↪ {
    if (o == &Null) return;
    temp[++ tot] = o;
    traverse(o -> c[0]);
    traverse(o -> c[1]);
}

```

```

inline node *build(int l, int
↪ r, int D) {
    int mid = l + r >> 1; nowD
    ↪ = D;
    nth_element(temp + l, temp
    ↪ + mid, temp + r + 1,
    ↪ cmp);
    node *res = temp[mid];
    res -> Max[0] = res ->
    ↪ Min[0] = res -> d[0];
    res -> Max[1] = res ->
    ↪ Min[1] = res -> d[1];
    if (l != mid) res -> c[0]
    ↪ = build(l, mid - 1,
    ↪ !D);
    else res -> c[0] = &Null;
    if (r != mid) res -> c[1]
    ↪ = build(mid + 1, r,
    ↪ !D);
    else res -> c[1] = &Null;
    res -> update();
    return res;
}

int x, y, a, b, tmpD;

node **tmp;

inline void rebuild(node *&o,
↪ int D) {
    tot = 0;
    traverse(o);
    o = build(1, tot, D);
}

inline void insert(node *&o,
↪ node *p, int D) {
    if (o == &Null) {o = p;
    ↪ return;}
    if (p -> Max[0] > o ->
    ↪ Max[0]) o -> Max[0] =
    ↪ p -> Max[0];
    if (p -> Min[0] < o ->
    ↪ Min[0]) o -> Min[0] =
    ↪ p -> Min[0];
    if (p -> Min[1] < o ->
    ↪ Min[1]) o -> Min[1] =
    ↪ p -> Min[1];
    o -> siz ++, o -> sum += p
    ↪ -> sum;
    insert(o -> c[p -> c[D] >=
    ↪ o -> c[D]], p, !D);
    if (max(o -> c[0] -> siz,
    ↪ o -> c[1] -> siz) >
    ↪ int(o -> siz * 0.75 +
    ↪ 0.5)) tmpD = D, tmp =
    ↪ &o;
}

inline int query(node *o) {
    if (o == &Null) return 0;
    if (x > o -> Max[0] || y >
    ↪ o -> Max[1] || a < o
    ↪ -> Min[0] || b < o ->
    ↪ Min[1]) return 0;
    if (x <= o -> Min[0] && y
    ↪ <= o -> Min[1] && a >=
    ↪ o -> Max[0] && b >= o
    ↪ -> Max[1]) return o ->
    ↪ sum;
    return (x <= o -> d[0] &&
    ↪ y <= o -> d[1] && a >=
    ↪ o -> d[0] && b >= o ->
    ↪ d[1] ? o -> val : 0)
    + query(o -> c[1]) +
    ↪ query(o -> c[0]);
}

int main() {
    ↪ ios::sync_with_stdio(false);
    cin >> m;
    node *ttt = &Null;

```

```

for (int t, ans = 0; ; ) {
    cin >> t;
    if (t == 3) break;
    if (t == 1) {
        cin >> x >> y >>
        a;
        x ^= ans, y ^=
        ans, n ++;
        nodes[n].sum =
        nodes[n].val =
        a ^ ans,
        nodes[n].siz =
        1;
        nodes[n].Max[0] =
        nodes[n].Min[0]
        =
        nodes[n].d[0]
        = x;
        nodes[n].Max[1] =
        nodes[n].Min[1]
        =
        nodes[n].d[1]
        = y;
        nodes[n].c[0] =
        nodes[n].c[1]
        = &Null;
        tmp = &(ttt),
        insert(root,
        &nodes[n], 0);
        if (*tmp != &Null)
            rebuild(*tmp,
            tmpD);
    } else {
        cin >> x >> y >> a
        >> b;
        x ^= ans, y ^=
        ans, a ^= ans,
        b ^= ans;
        if (x > a) swap(x,
        a);
        if (y > b) swap(y,
        b);
        ans = query(root);
    }
    printf("%d\n",
    ans);
}
return 0;
}
}

2.2.4 KDtree 找最近点
/*
 * 为了维持树的平衡, 可以一开始
 * 把所有点都读进来 build
 * 然后打 flag 标记该点是否被激
 * 活
 */
#include <bits/stdc++.h>
using namespace std;
const int N = 5e5 + 5;
const int inf = 1 << 30;
int n, m;
int ql, qr, ans, tot, nowD;
//nowD = rand() % 1 ?
struct Node {
    int d[2];
    bool operator < (const
    Node &a) const {
        if (d[nowD] ==
        a.d[nowD]) return
        d[!nowD] <
        a.d[!nowD];
        return d[nowD] <
        a.d[nowD];
    }
}pot[N];
struct node {
    int min[2], max[2], d[2];
    node *c[2];

```

```

node() {
    min[0] = min[1] =
        ↪ max[0] = max[1] =
        ↪ d[0] = d[1] = 0;
    c[0] = c[1] = NULL;
}

node(int x, int y);

void update();

}t[N], Null, *root;

node::node(int x, int y) {
    min[0] = max[0] = d[0] =
        ↪ x;
    min[1] = max[1] = d[1] =
        ↪ y;
    c[0] = c[1] = &Null;
}

inline void node::update() {
    if (c[0] != &Null) {
        if (c[0] -> max[0] >
            ↪ max[0]) max[0] =
            ↪ c[0] -> max[0];
        if (c[0] -> max[1] >
            ↪ max[1]) max[1] =
            ↪ c[0] -> max[1];
        if (c[0] -> min[0] <
            ↪ min[0]) min[0] =
            ↪ c[0] -> min[0];
        if (c[0] -> min[1] <
            ↪ min[1]) min[1] =
            ↪ c[0] -> min[1];
    }
    if (c[1] != &Null) {
        if (c[1] -> max[0] >
            ↪ max[0]) max[0] =
            ↪ c[1] -> max[0];
        if (c[1] -> max[1] >
            ↪ max[1]) max[1] =
            ↪ c[1] -> max[1];
        if (c[1] -> min[0] <
            ↪ min[0]) min[0] =
            ↪ c[1] -> min[0];
        if (c[1] -> min[1] <
            ↪ min[1]) min[1] =
            ↪ c[1] -> min[1];
    }
}

inline void build(node *&o,
    ↪ int l, int r, int D) {
    int mid = l + r >> 1;
    nowD = D;
    nth_element(pot + l, pot +
        ↪ mid, pot + r + 1);
    o = new
        ↪ node(pot[mid].d[0],
        ↪ pot[mid].d[1]);

    if (l != mid) build(o ->
        ↪ c[0], l, mid - 1, !D);
    if (r != mid) build(o ->
        ↪ c[1], mid + 1, r, !D);
    o -> update();
}

inline void insert(node *o) {
    node *p = root;
    int D = 0;
    while (1) {
        if (o -> max[0] > p ->
            ↪ max[0]) p ->
            ↪ max[0] = o ->
            ↪ max[0];
        if (o -> max[1] > p ->
            ↪ max[1]) p ->
            ↪ max[1] = o ->
            ↪ max[1];
    }
}
    
```



```

    if (o -> min[0] < p ->
        ↪ min[0]) p ->
        ↪ min[0] = o ->
        ↪ min[0];
    if (o -> min[1] < p ->
        ↪ min[1]) p ->
        ↪ min[1] = o ->
        ↪ min[1];

    if (o -> d[D] >= p ->
        ↪ d[D]) {
        if (p -> c[1] ==
            ↪ &Null) {
            p -> c[1] = o;
            return;
        } else p = p ->
            ↪ c[1];
    } else {
        if (p -> c[0] ==
            ↪ &Null) {
            p -> c[0] = o;
            return;
        } else p = p ->
            ↪ c[0];
    }
    D ^= 1;
}

inline int dist(node *o) {
    int dis = 0;
    if (ql < o -> min[0]) dis
        ↪ += o -> min[0] - ql;
    if (ql > o -> max[0]) dis
        ↪ += ql - o -> max[0];
    if (qr < o -> min[1]) dis
        ↪ += o -> min[1] - qr;
    if (qr > o -> max[1]) dis
        ↪ += qr - o -> max[1];
    return dis;
}

inline void query(node *o) {
    int dl, dr, d0;
    d0 = abs(o -> d[0] - ql) +
        ↪ abs(o -> d[1] - qr);
    if (d0 < ans) ans = d0;
    if (o -> c[0] != &Null) dl
        ↪ = dist(o -> c[0]);
    else dl = inf;
    if (o -> c[1] != &Null) dr
        ↪ = dist(o -> c[1]);
    else dr = inf;

    if (dl < dr) {
        if (dl < ans) query(o
            ↪ -> c[0]);
        if (dr < ans) query(o
            ↪ -> c[1]);
    } else {
        if (dr < ans) query(o
            ↪ -> c[1]);
        if (dl < ans) query(o
            ↪ -> c[0]);
    }
}

int main() {
    ↪ ios::sync_with_stdio(false);
    cin >> n >> m;
    for (int i = 1; i <= n; i
        ↪ ++ )
        cin >> pot[i].d[0] >>
            ↪ pot[i].d[1];
    build(root, 1, n, 0);

    for (int x, y, z; m --; )
        ↪ {
            cin >> x >> y >> z;
            if (x == 1) {
                t[tot].max[0] =
                    ↪ t[tot].min[0]
                    ↪ = t[tot].d[0]
                    ↪ = y;
            }
        }
}

```

```

        t[tot].max[1] =
        ↪ t[tot].min[1]
        ↪ = t[tot].d[1]
        ↪ = z;
        t[tot].c[0] =
        ↪ t[tot].c[1] =
        ↪ &Null;
        insert(&t[tot
        ↪ ++]);
    } else {
        ans = inf, ql = y,
        ↪ qr = z;
        query(root),
        ↪ printf("%d\n",
        ↪ ans);
    }
}
return 0;
}

```

### 3 构造

#### 3.1 若干排列使所有数对都出现一次

/\* $n/2$  个排列使得所有数对  $(i, j)$   
 $\rightarrow$  且  $i < j$  都出现一次  
 $*n$  为奇数则首尾相连, 偶数不连  
 $*/$

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
typedef vector<int> vi;
```

```
void get_even(int n, vi ans[])  
{
```

```
    vi a(n);
```

```
    for (int i = 0; i < n;  
         $\rightarrow$  i++)
```

```
        a[i] = i + 1;
```

```
    for (int i = 1; i <= n  
         $\rightarrow$  / 2; i++) {
```

```
        ans[i].resize(n  
             $\rightarrow$  + 1);
```

```
        for (int j =  
             $\rightarrow$  0; j < n /  
             $\rightarrow$  2; j++)
```

```
            ans[i][j
```

```
                 $\rightarrow$  *
```

```
                 $\rightarrow$  2]
```

```
                 $\rightarrow$  =
```

```
                 $\rightarrow$  a[j],
```

```
                 $\rightarrow$  ans[i][j
```

```
                 $\rightarrow$  *
```

```
                 $\rightarrow$  2
```

```
                 $\rightarrow$  +
```

```
                 $\rightarrow$  1]
```

```
                 $\rightarrow$  =
```

```
                 $\rightarrow$  a[n
```

```
                 $\rightarrow$  -
```

```
                 $\rightarrow$  1
```

```
                 $\rightarrow$  -
```

```
                 $\rightarrow$  j];
```

```
int t = a[n -  
     $\rightarrow$  1];
```

```
for (int j = n  
     $\rightarrow$  - 1; j >  
     $\rightarrow$  0; j --)
```

```
    a[j] =
```

```
         $\rightarrow$  a[j
```

```
         $\rightarrow$  -
```

```
         $\rightarrow$  1];
```

```
a[0] = t;
```

```
}
```

```
void get_odd(int n, vi ans[])
```

```
{
```

```
    get_even(n - 1, ans);
```

```
    for (int i = 1; i <= n  
         $\rightarrow$  / 2; i++) {
```

```
        for (int j = n
```

```
             $\rightarrow$  - 1; j >
```

```
             $\rightarrow$  0; j --)
```

```
            ans[i][j]
```

```
                 $\rightarrow$  =
```

```
                 $\rightarrow$  ans[i][j
```

```
                 $\rightarrow$  -
```

```
                 $\rightarrow$  1]
```

```
                 $\rightarrow$  +
```

```
                 $\rightarrow$  1;
```

```
        ans[i][0] = 1;
```

```
}
```

```
int main() {
```

```
    vi ans[2019];
```

```
    int n; cin >> n;
```

```
    if (n & 1) get_odd(n,
```

```
         $\rightarrow$  ans);
```

```
    else get_even(n,
```

```
         $\rightarrow$  ans);
```

```
    return 0;
```

### 3.2 rec-free

```

/* 不存在四个 1 构成一个矩形,
   ↳ 并使得 1 尽量多, 输出 01 矩
   ↳ 阵 */
#include <bits/stdc++.h>

using namespace std;

const int N = 200, n = 150, M
↳ = 13;
//M 为质数, N>M*M>n
int a[N][N], b[N][N], c[N][N];

void make() {
    for (int i = 1; i <=
↳ M; i++)
        for (int j = 1; j <=
↳ M; j++)
            a[i][j + 1] = M *
↳ (j - 1) + i;
    for (int i = 1; i <= M; i
↳ ++)
        for (int j = 1; j <=
↳ M; j++)
            c[i][a[i][j]] = 1;
    for (int k = 1; k < M; k
↳ ++) {
        memcpy(b, a, sizeof
↳ b);
        for (int i = 1; i <=
↳ M; i++)
            for (int j = 1; j
↳ <= M; j++)
                a[i][j] = (b[i
↳ + j - 1 -
↳ ((i + j -
↳ 1) > M ? M
↳ : 0)][j]);
        for (int i = 1; i <=
↳ M; i++)
            for (int j = 1; j
↳ <= M; j++)
                c[k * M +
↳ i][a[i][j]]
↳ = 1;
    }

int main() {
    make();
    return 0;
}

```

## 4 计算几何

### 4.1 最小矩形覆盖含凸包和旋转卡壳

```

/*
 * 最小矩形覆盖，保留六位小数，
 * 逆时针输出四个顶点坐标
 */
#include <bits/stdc++.h>

using namespace std;

namespace minRectCover {

    const int N = 1e5 + 5;
    const double eps =
        ↪ 1e-8;

    struct point{
        double x, y;
        point(){}
        point(double
            ↪ x, double
            ↪ y):x(x),
            ↪ y(y){}

        bool operator
            ↪ < (const
            ↪ point &a)
            ↪ const {

```

```

        return
            ↪ fabs(y
            ↪ -
            ↪ a.y)
            ↪ <
            ↪ eps
            ↪ ?
            ↪ x
            ↪ <
            ↪ a.x
            ↪ :
            ↪ y
            ↪ <
            ↪ a.y;
    }

    point operator
        ↪ - (const
        ↪ point &a)
        ↪ const {
            return
                ↪ point(x
                ↪ -
                ↪ a.x,
                ↪ y
                ↪ -
                ↪ a.y);
        }

    point operator
        ↪ + (const
        ↪ point &a)
        ↪ const {
            return
                ↪ point(x
                ↪ +
                ↪ a.x,
                ↪ y
                ↪ +
                ↪ a.y);
        }
}

```

```

point operator
↳ / (const
↳ double &a)
↳ const {
    return
    ↳ point(x
    ↳ /
    ↳ a,
    ↳ y
    ↳ /
    ↳ a);
}

point operator
↳ * (const
↳ double &a)
↳ const {
    return
    ↳ point(x
    ↳ *
    ↳ a,
    ↳ y
    ↳ *
    ↳ a);
}

double
↳ operator /
↳ (const
↳ point &a)
↳ const { //
↳ .
    return
    ↳ x
    ↳ *
    ↳ a.x
    ↳ +
    ↳ y
    ↳ *
    ↳ a.y;
}

double
↳ operator *
↳ (const
↳ point &a)
↳ const { //
↳ X
    return
    ↳ x
    ↳ *
    ↳ a.y
    ↳ -
    ↳ y
    ↳ *
    ↳ a.x;
}

}p[N], q[N], rc[4];

double sqr(double x)
↳ {return x * x;}
double abs(point a)
↳ {return sqrt(a /
↳ a);}
int sgn(double x)
↳ {return fabs(x) <
↳ eps ? 0 : (x < 0 ?
↳ -1 : 1);}
point vertical(point
↳ a, point b)
↳ {return point(a.x
↳ + a.y - b.y, a.y -
↳ a.x + b.x) -
↳ a;}//与 ab 向量垂
↳ 直的向量
point vec(point
↳ a){return a /
↳ abs(a);}

void convexhull(int n,
↳ point *hull, int
↳ &top) {//如果要计
↳ 算周长需要特判 n==2

```

```

for (int i =
    ↪ 1; i < n;
    ↪ i++)
    if
        ↪ (p[i]
        ↪ <
        ↪ p[0])
            swap(p[i],
                ↪ p[0]);
sort (p + 1, p
    ↪ + n,
    ↪ [&](point
    ↪ a, point
    ↪ b){
        double
            ↪ t
            ↪ =
            ↪ (a
            ↪ -
            ↪ p[0])
            ↪ *
            ↪ (b
            ↪ -
            ↪ p[0]);
        if
            ↪ (fabs(t)
            ↪ <
            ↪ eps)
            ↪ return
            ↪ sgn(abs(p[0]
            ↪ -
            ↪ a)
            ↪ -
            ↪ abs(p[0]
            ↪ -
            ↪ b))
            ↪ <
            ↪ 0;
        return
            ↪ t
            ↪ >
            ↪ 0;
    });

int cnt =
    ↪ 0; //去重
for (int i =
    ↪ 1; i < n;
    ↪ i++)
    if
        ↪ (sgn(p[i].x
        ↪ -
        ↪ p[cnt].x)
        ↪ !=
        ↪ 0
        ↪ ||
        ↪ sgn(p[i].y
        ↪ -
        ↪ p[cnt].y)
        ↪ !=
        ↪ 0)
        ↪ p[++]
        ↪ cnt]
        ↪ =
        ↪ p[i];
n = cnt + 1;

hull[top = 1]
    ↪ = p[0];
for (int i =
    ↪ 1; i < n;
    ↪ i++) {

```

```

        while
            ↪ (top
            ↪ >
            ↪ 1
            ↪ &&
            ↪ (hull[top]
            ↪ -
            ↪ hull[top
            ↪ -
            ↪ 1])
            ↪ *
            ↪ (p[i]
            ↪ -
            ↪ hull[top])
            ↪ <
            ↪ eps)
            ↪ top
            ↪ --;
        hull[++
            ↪ top]
            ↪ =
            ↪ p[i];
    }
    hull[0] =
        ↪ hull[top];
}

void main() {
    int n;
    scanf("%d",
        ↪ &n);
    for (int i =
        ↪ 0; i < n;
        ↪ i ++)
        scanf("%lf
            ↪ %lf",
            ↪ &p[i].x,
            ↪ &p[i].y);

    convexhull(n,
        ↪ q, n);

    double ans =
        ↪ 1e20;

```

```

    int l = 1, r =
        ↪ 1, t = 1;
    double L, R,
        ↪ D, H;
    for (int i =
        ↪ 0; i < n;
        ↪ i ++) {
        D =
            ↪ abs(q[i]
            ↪ -
            ↪ q[i
            ↪ +
            ↪ 1]);
        while
            ↪ (sgn((q[i
            ↪ +
            ↪ 1]
            ↪ -
            ↪ q[i]
            ↪ *
            ↪ (q[t
            ↪ +
            ↪ 1]
            ↪ -
            ↪ q[i]
            ↪ -
            ↪ (q[i
            ↪ +
            ↪ 1]
            ↪ -
            ↪ q[i]
            ↪ *
            ↪ (q[t
            ↪ -
            ↪ q[i]))
            ↪ >
            ↪ -1)
            ↪ t
            ↪ =
            ↪ (t
            ↪ +
            ↪ 1)
            ↪ %
            ↪ n;

```



<b>while</b>	<b>while</b>
↪ (sgn((q[i	↪ (sgn((q[i
↪ +	↪ +
↪ 1]	↪ 1]
↪ -	↪ -
↪ q[i])	↪ q[i])
↪ /	↪ /
↪ (q[r	↪ (q[l
↪ +	↪ +
↪ 1]	↪ 1]
↪ -	↪ -
↪ q[i])	↪ q[i])
↪ -	↪ -
↪ (q[i	↪ (q[i
↪ +	↪ +
↪ 1]	↪ 1]
↪ -	↪ -
↪ q[i])	↪ q[i])
↪ /	↪ /
↪ (q[r]	↪ (q[l]
↪ -	↪ -
↪ q[i]))	↪ q[i]))
↪ >	↪ <
↪ -1)	↪ 1)
↪ r	↪ l
↪ =	↪ =
↪ (r	↪ (l
↪ +	↪ +
↪ 1)	↪ 1)
↪ %	↪ %
↪ n;	↪ n;
<b>if</b> (i	L =
↪ ==	↪ fabs((q[i
↪ 0)	↪ +
↪ 1	↪ 1]
↪ =	↪ -
↪ r;	↪ q[i])
	↪ /
	↪ (q[l]
	↪ -
	↪ q[i])
	↪ /
	↪ D);

```

R =
↪ fabs((q[i
↪ +
↪ 1]
↪ -
↪ q[i])
↪ /
↪ (q[r]
↪ -
↪ q[i])
↪ /
↪ D);
H =
↪ fabs((q[i
↪ +
↪ 1]
↪ -
↪ q[i])
↪ *
↪ (q[t]
↪ -
↪ q[i])
↪ /
↪ D);
double
↪ tmp
↪ =
↪ (R
↪ +
↪ L)
↪ *
↪ H;
if
↪ (tmp
↪ <
↪ ans)
↪ {
↪     ans
↪     ↪ =
↪     ↪ tmp;

rc[0]
↪ =
↪ q[i]
↪ +
↪ (q[i
↪ +
↪ 1]
↪ -
↪ q[i])
↪ *
↪ (R
↪ /
↪ D); //右
↪ 下
rc[1]
↪ =
↪ rc[0]
↪ +
↪ vec(vertical(q[
↪ q[i
↪ +
↪ 1]))
↪ *
↪ H; //右
↪ 上
rc[2]
↪ =
↪ rc[1]
↪ -
↪ (rc[0]
↪ -
↪ q[i])
↪ *
↪ ((R
↪ +
↪ L)
↪ /
↪ abs(q[i]
↪ -
↪ rc[0])); //左
↪ 上

```

```

        rc[3]
        ↪ =
        ↪ rc[2]
        ↪ -
        ↪ (rc[1]
        ↪ -
        ↪ rc[0]);
    }
}

printf("%.6f\n",
    ↪ ans);
int fir = 0;
for (int i =
    ↪ 1; i < 4;
    ↪ i ++)
    if
        ↪ (rc[i]
        ↪ <
        ↪ rc[fir])
            fir
                ↪ =
                ↪ i;
for (int i =
    ↪ 0; i < 4;
    ↪ i ++)
    printf("%.6f
        ↪ %.6f\n",
        ↪ rc[(fir
        ↪ +
        ↪ i)
        ↪ %
        ↪ 4].x,
        ↪ rc[(fir
        ↪ +
        ↪ i)
        ↪ %
        ↪ 4].y);
}

}

int main() {
    minRectCover::main();
    return 0;
}

```

## 5 其他

### 5.1 数字哈希

```
namespace my_hash {
    const int N = (1 <<
        ↪ 19) - 1; //散列大小,
        ↪ 一定要取  $2^k-1$ , 不
        ↪ 超内存的情况下, N
        ↪ 越大碰撞越少
```

```
struct E {
    int v;
    E *nxt;
}*g[N + 1], pool[N],
    ↪ *cur = pool, *p;
```

```
int vis[N + 1], T;
```

```
void ins(int v) {
    int u = v & N;
    if (vis[u] <
        ↪ T) vis[u]
        ↪ = T, g[u]
        ↪ = NULL;
    for (p = g[u];
        ↪ p; p = p
        ↪ -> nxt) if
        ↪ (p -> v ==
        ↪ v) return;
    p = cur ++; p
    ↪ -> v = v;
    ↪ p -> nxt =
    ↪ g[u]; g[u]
    ↪ = p;
}
```

```
int ask(int v) {
    int u = v & N;
    if (vis[u] <
        ↪ T) return
        ↪ 0;
```

```
for (p = g[u];
    ↪ p; p = p
    ↪ -> nxt) if
    ↪ (p -> v ==
    ↪ v) return
    ↪ 1;
return 0;
```

```
}
```

```
void init() {T ++, cur
    ↪ = pool;} //应对多组
    ↪ 数据使用
```

```
}
```

### 5.2 海岛分金币

#### 5.2.1 海岛分金币 1

/\*  
非朴素模型, 有额外条件:  
每个人做决定时如果有多种方案可  
 ↪ 以使自己获得最大收益  
那么他会让决策顺序靠前的人获得  
 ↪ 的收益尽可能的大!

*solution:*

贪心模拟

\*/

```
#include <bits/stdc++.h>
```

```
#define v first
```

```
#define id second
```

```
using namespace std;
```

```
typedef pair<int, int> pr;
```

```
const int N = 1010;
```

```
int a[N][N];
```

```
pr b[N];
```

```
int n, m;
```

```

int main() {
    cin >> n >> m;
    a[1][1] = m;
    for (int i = 2; i <= n; i++) {
        for (int j = 1; j < i; j++) {
            b[j] = pr(a[i - 1][j], j);
            sort(b + 1, b + i, [&](pr x, pr y){return x.v != y.v ? (x.v < y.v) : (x.id > y.id)});
            //按照是否容易满足来排序, 因为容易满足的人消耗掉的金币比较少, 也就使得当前的人获利最大
            int s = m, nd = (i - 1) / 2;
            for (int j = 1; j < i; j++) {
                nd--;
                s -= (a[i][b[j].id] - a[i - 1][b[j].id] + 1);
            }
            if (s < 0) {
                for (int j = 1; j < i; j++) {
                    a[i][j] = a[i - 1][j];
                    a[i][i] = -1;
                }
            }
            else {
                a[i][i] = s;
            }
        }
    }
    for (int i = n; i; i--)
        printf("%d ", a[n][i]);
    return 0;
}
    
```

## 5.2.2 海岛分金币 2

```

/*
海盗分金币朴素模型:
n 个海盗分 m 个金币, 依次做决策,
如果不少于半数的人同意则方案通过, 否则当前做决策的人会被淘汰 (收益视为 -1), 由下一人做出决策
如果一个海盗有多种方案均为最大收益, 那么他会希望淘汰的人越多越好
求出第 x 个做决策的海盗的最大可能受益和最小可能收益
*/
#include <bits/stdc++.h>

using namespace std;

struct node {
    int min_v, max_v;
    node():min_v(0), max_v(0) {}
    node(int min_v, int max_v):min_v(min_v), max_v(max_v) {}
};

node ask(int n, int m, int x) {
    //n 个人分 m 个金币, 第 x 个做决策的人最少/最多分到多少个金币
    int y = n + 1 - x;
    if (n >= (m + 2) * 2) {
        int a = (m + 1) * 2, b = 2, c = 4;
        //前 a 个为 [0, 1], 后 b 个为 [0, 0], 将持续 c 个
    }
    }
    
```

```

while (a + b + c <= n)
    ↪ {
        a += b;
        b *= 2;
        c *= 2;
    }
if (y <= a) return
    ↪ node(0, 1);
else if (y <= a + b)
    ↪ return node(0, 0);
else return node(-1,
    ↪ -1);
}
else if (n == m * 2 + 3) {
    if (x == 1) return
        ↪ node(-1, -1);
    else if (y <= m * 2 &&
        ↪ y % 2 == 1 || x ==
        ↪ 2) return node(0,
        ↪ 0);
    else return node(0,
        ↪ 1);
}
else if (n == m * 2 + 2) {
    if (y <= m * 2 && y %
        ↪ 2 == 1 || x == 1)
        ↪ return node(0, 0);
    else return node(0,
        ↪ 1);
}
else if (n == m * 2 + 1) {
    if (y <= m * 2 && y %
        ↪ 2 == 1) return
        ↪ node(1, 1);
    else return node(0,
        ↪ 0);
}
else {
    if (x & 1) {
        if (x != 1) return
            ↪ node(1, 1);
    }
}

else return node(m
    ↪ - (n - 1) / 2,
    ↪ m - (n - 1) /
    ↪ 2);
}
else return node(0,
    ↪ 0);
}

int main() {
    ↪ ios::sync_with_stdio(false);
    int x, n, m, k; node y;
    cin >> n >> m >> k;
    while (k --) {
        cin >> x;
        y = ask(n, m, x);
        printf("%d %d\n",
            ↪ y.min_v, y.max_v);
    }
    return 0;
}

/*
m = 5
1 5
2 0 5
3 1 0 4
4 0 1 0 4
5 1 0 1 0 3
6 0 1 0 1 0 3
7 1 0 1 0 1 0 2
8 0 1 0 1 0 1 0 2
9 1 0 1 0 1 0 1 0 1
10 0 1 0 1 0 1 0 1 0 1
11 1 0 1 0 1 0 1 0 1 0 0
12 0 _ 0 _ 0 _ 0 _ 0 _ 0
13 0 _ 0 _ 0 _ 0 _ 0 _ 0 -1
14 _ _ _ _ _ _ _ _ _ 0 0
15 _ _ _ _ _ _ _ _ _ 0 0
    ↪ -1

```

```

16 _ _ _ _ _ _ _ _ _ _ 0 0
   ↪ -1 -1
17 _ _ _ _ _ _ _ _ _ _ 0 0
   ↪ -1 -1 -1
18 _ _ _ _ _ _ _ _ _ _
   ↪ 0 0 0 0
19 _ _ _ _ _ _ _ _ _ _
   ↪ 0 0 0 0 -1
20 _ _ _ _ _ _ _ _ _ _
   ↪ 0 0 0 0 -1 -1
21 _ _ _ _ _ _ _ _ _ _
   ↪ 0 0 0 0 -1 -1 -1
22 _ _ _ _ _ _ _ _ _ _
   ↪ 0 0 0 0 -1 -1 -1 -1
23 _ _ _ _ _ _ _ _ _ _
   ↪ 0 0 0 0 -1 -1 -1 -1 -1
24 _ _ _ _ _ _ _ _ _ _
   ↪ 0 0 0 0 -1 -1 -1 -1 -1 -1
25 _ _ _ _ _ _ _ _ _ _
   ↪ 0 0 0 0 -1 -1 -1 -1 -1 -1
   ↪ -1
26 _ _ _ _ _ _ _ _ _ _
   ↪ _ _ _ _ 0 0 0 0 0 0 0 0
*/

```

### 5.3 根号枚举

```

for (int i = 1, last; i <= n;
     ↪ i = last + 1) {
    last = n / (n / i);
    //当前枚举区间为 [i,
    ↪ last]
}

```

### 5.4 读入输出外挂

```

namespace IO {//only for
  ↪ int!!!
  static const int SIZE = 1
  ↪ << 20;

  inline int get_char() {
    static char *S, *T =
    ↪ S, buf[SIZE];

```

```

    if (S == T) {
      T = fread(buf, 1,
        ↪ SIZE, stdin) +
        ↪ (S = buf);
      if (S == T) return
        ↪ -1;
    }
    return *S++;
}

```

```

inline void in(int &x)
  ↪ {//for int
  static int ch;
  while (ch =
    ↪ get_char(), ch <
    ↪ 48); x = ch ^ 48;
  while (ch =
    ↪ get_char(), ch >
    ↪ 47) x = x * 10 +
    ↪ (ch ^ 48);
}

```

```

char buffer[SIZE];
char *s = buffer;

```

```

void flush() {//最后需要
  ↪ flush!!
  fwrite(buffer, 1, s -
    ↪ buffer, stdout);
  s = buffer;
  fflush(stdout);
}

```

```

inline void print(const
  ↪ char ch) {
  if (s - buffer > SIZE -
    ↪ 2) flush();
  *s++ = ch;
}

```

```

inline void print(char
  ↪ *str) {//for string
  while(*str != 0)

```

```

        print(char(*str
            ↪ ++));
    }

    inline void print(int x) {
        static char buf[25];
        static char *p = buf;
        if (x < 0)
            ↪ print('-', x =
            ↪ -x;
        if (x == 0)
            ↪ print('0');
        while(x) *(++ p) = x %
            ↪ 10, x /= 10;
        while(p != buf)
            ↪ print(char(*(p --)
            ↪ ^ 48));
    }
};

```

```

        lp, lq = lp + rp * cnt, lq
            ↪ + rq * cnt
    else:
        l, r, mid, cnt = 1, (inf -
            ↪ rq) // lq + 1, -1, -1
        while l <= r:
            mid = l + r >> 1
            if (rp + lp * mid) * inff
                ↪ > (rq + lq * mid) *
                ↪ n:
                cnt, l = mid, mid + 1
            else:
                r = mid - 1
        rp, rq = rp + lp * cnt, rq
            ↪ + lq * cnt
    if lq <= inf: print(lp, lq)
    else: print(rp, rq)

```

### 5.5 给定小数化成分数

```

# 本题答案的分母不超过 1e9, 给
↪ 定小数的小数点位为 18 位
inf, inff = 10 ** 9, 10 ** 18
for i in range(int(input())):
    n = int(input()[2:])
    if n == 0: print('0 1')
    else:
        lp, lq, rp, rq = 0, 1, 1, 1
        while max(lq, rq) <= inf:
            mp, mq = lp + rp, lq + rq
            if mp * inff <= mq * n:
                l, r, mid, cnt = 1, (inf -
                    ↪ lq) // rq + 1, -1, -1
                while l <= r:
                    mid = l + r >> 1
                    if (lp + rp * mid) * inff
                        ↪ <= (lq + rq * mid) *
                        ↪ n:
                        cnt, l = mid, mid + 1
                    else:
                        r = mid - 1

```