

Algorithm Template Library

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1 注意事项

1.1 需要想到的方法

分块方法的应用：

图的所有点，按照度数与 $\text{sqrt}(m)$ 的关系分为大点小点

分块的实现：

实现上一般不需要 `block` 结构体，分块的元素其实还是在同一个数组里的

数论：

威尔逊定理：

$(n-2)! \% n$ n 为素数为 1，否则 n 为 4 是答案为 2，其他为 0

费马小定理：

$x^{(p-1)\%p}=1$ 必须满足 p 为质数且 ap 互质

鸽巢原理：

$x^k \% p$ 的循环节不会超过 p

计数问题：

计算多种情况的答案之和，转变为枚举某个量，计算这个量对多少种情况做贡献

`random_shuffle`：

对于验证比较耗时，但是理论上满足条件的比较少情况

为了防止被构造数据卡掉，可以随机一下（前提是顺序不影响意义），另外还要注意能去重就去重

判定问题：

随机赋值：某些判定可以考虑随机赋值 `val, val` 的运算看情况可以考虑异或 `or` 乘法

判定两个数相等的一种思路：取若干个质数，模数都相等就默认两个数相同，要保证质数乘积 $> \text{MAX_X}$

快速幂：

快速幂不只有 2 进制，一般 2 进制最快

使用 k 进制快速幂实际复杂度是 $k \log(k, n)$

当指数是个很长的数字时，根据给定数字的进制选择快速幂的进制才是缀吼的

1.2 需要注意的问题

通用：

检查代码觉得没有问题时，再多读两遍题，是否有漏掉的东西

分块的块大小：

具体问题具体分析！

分析修改和查询的次数以及复杂度，再决定块大小！

多组数据的初始化：

网络流 `len = 1`

主席树 `tot = 1`

变量打错

不要交错题

读入是否有强制在线的加密

错误率较高的简单题，考虑各种可能的特殊情况，以及对特殊情况是否会引起一般情况的错误

2 图论

2.1 线段树维护树直径

```

/*LCA 用 ST 表, 总复杂度  $O(n \log)$ */
/* 每次询问删去两条边后, 剩下 3 棵树的直径长度 */
const int N = 2e5 + 5;
int T, n, m;
int len, head[N], ST[20][N];
struct edge{int u, v, w;}ee[N];
int cnt, fa[N], log_2[N], st[N], en[N], dfn[N], dis[N], dep[N], pos[N];
struct edges{int to, next, cost;}e[N];
void add(int u, int v, int w) {
    e[++ len] = (edges){v, head[u], w}, head[u] = len;
    e[++ len] = (edges){u, head[v], w}, head[v] = len;
}
void dfs1(int u) {
    st[u] = ++ cnt, dfn[cnt] = u;
    for (int v, i = head[u]; i; i = e[i].next) {
        v = e[i].to;
        if (v == fa[u]) continue;
        fa[v] = u, dep[v] = dep[u] + 1;
        dis[v] = dis[u] + e[i].cost, dfs1(v);
    }
    en[u] = cnt;
}
void dfs2(int u) {
    dfn[++ cnt] = u, pos[u] = cnt;
    for (int v, i = head[u]; i; i = e[i].next) {
        v = e[i].to;
        if (v == fa[u]) continue;
        dfs2(v), dfn[++ cnt] = u;
    }
}
int mmin(int x, int y) {
    if (dep[x] < dep[y]) return x;
    return y;
}
int lca(int u, int v) {
    static int w;
    if (pos[u] > pos[v]) swap(u, v);
    w = log_2[pos[v] - pos[u] + 1];
    return mmin(ST[w][pos[u]], ST[w][pos[v] - (1 << w) + 1]);
}
int dist(int u, int v) {
    int Lca = lca(u, v);
    return dis[u] + dis[v] - dis[Lca] * 2;
}
void build() {
    for (int i = 1; i <= cnt; i++)
        ST[0][i] = dfn[i];
    for (int i = 1; i < 20; i++)
        for (int j = 1; j <= cnt; j++)
            if (j + (1 << (i - 1)) > cnt) ST[i][j] = ST[i - 1][j];
}

```

```

        else ST[i][j] = mmmin(ST[i - 1][j], ST[i - 1][j + (1 << (i - 1))]);
    }
    int M;
    struct node {
        int l, r, dis;
    } tr[N << 1];
    void update(int o, int o1, int o2) {
        static int d; static node tmp;
        if (tr[o1].dis == -1) {tr[o] = tr[o2]; return;}
        if (tr[o2].dis == -1) {tr[o] = tr[o1]; return;}
        if (tr[o1].dis > tr[o2].dis) tmp = tr[o1];
        else tmp = tr[o2];
        d = dist(tr[o1].l, tr[o2].l);
        if (d > tmp.dis) tmp.l = tr[o1].l, tmp.r = tr[o2].l, tmp.dis = d;
        d = dist(tr[o1].l, tr[o2].r);
        if (d > tmp.dis) tmp.l = tr[o1].l, tmp.r = tr[o2].r, tmp.dis = d;
        d = dist(tr[o1].r, tr[o2].l);
        if (d > tmp.dis) tmp.l = tr[o1].r, tmp.r = tr[o2].l, tmp.dis = d;
        d = dist(tr[o1].r, tr[o2].r);
        if (d > tmp.dis) tmp.l = tr[o1].r, tmp.r = tr[o2].r, tmp.dis = d;
        tr[o] = tmp;
    }
    void ask(int s, int t) {
        if (s > t) return;
        for (s += M - 1, t += M + 1; s ^ t ^ 1; s >>= 1, t >>= 1) {
            if (~s&1) update(0, 0, s ^ 1);
            if (t&1) update(0, 0, t ^ 1);
        }
    }
    int main() {
        ios::sync_with_stdio(false);
        int u, v, w, ans; log_2[1] = 0;
        for (int i = 2; i <= 200000; i++)
            if (i == 1 << (log_2[i - 1] + 1))
                log_2[i] = log_2[i - 1] + 1;
            else log_2[i] = log_2[i - 1];
        for (cin >> T; T--; ) {
            cin >> n >> m, cnt = len = 0;
            for (int i = 1; i <= n; i++)
                head[i] = 0;
            for (int i = 1; i < n; i++) {
                cin >> ee[i].u >> ee[i].v >> ee[i].w;
                add(ee[i].u, ee[i].v, ee[i].w);
            }
            dfs1(1);
            for (M = 1; M < n + 2; M <<= 1);
            for (int i = 1; i <= n; i++)
                tr[i + M].l = tr[i + M].r = dfn[i], tr[i + M].dis = 0;
            for (int i = n + M + 1; i <= (M << 1) + 1; i++)
                tr[i].dis = -1;
            cnt = 0, dfs2(1), build();
            for (int i = M; i; i--)
                update(i, i << 1, i << 1 | 1);
            for (int i = 1; i < n; i++)

```

```

        if (dep[ee[i].u] > dep[ee[i].v])
            swap(ee[i].u, ee[i].v);
    for (int u, v, i = 1; i <= m; i++) {
        cin >> u >> v, ans = 0;
        u = ee[u].v, v = ee[v].v, w = lca(u, v);
        if (w == u || w == v) {
            if (w != u) swap(u, v);
            tr[0].dis = -1, ask(1, st[u] - 1), ask(en[u] + 1, n), ans = max(ans,
                ↪ tr[0].dis);
            tr[0].dis = -1, ask(st[u], st[v] - 1), ask(en[v] + 1, en[u]), ans =
                ↪ max(ans, tr[0].dis);
            tr[0].dis = -1, ask(st[v], en[v]), ans = max(ans, tr[0].dis);
        }
        else {
            if (st[u] > st[v]) swap(u, v);
            tr[0].dis = -1, ask(1, st[u] - 1), ask(en[u] + 1, st[v] - 1), ask(en[v] +
                ↪ 1, n), ans = max(ans, tr[0].dis);
            tr[0].dis = -1, ask(st[u], en[u]), ans = max(ans, tr[0].dis);
            tr[0].dis = -1, ask(st[v], en[v]), ans = max(ans, tr[0].dis);
        }
        printf("%d\n", ans);
    }
}
return 0;
}

```

2.2 有向图判断两个点能否到达

```

/* 原题:  $n$  个点的有向图,  $k$  对点  $(u, v)$  满足  $u$  可达  $v$ 
*  $p$  对点  $(u, v)$  满足  $u$  不可达  $v$ , 问这个图是否存在
* 解法: 按照  $p$  对可达点直接连边构图, 然后考虑验证
*  $a[i][j]=0/1$  表示  $i$  是否可达  $j$ , 我们对  $j$  分块, bitset 加速
* 时间  $O(n*m/32)$  空间  $O(n*n/blk)$   $blk$  随便取
*/
const int N = 1e5 + 2;
const int BLK = 5000;
int n, k, p;
int d[N];
vector<int> e[N], f[N], ck[N];
int top, sta[N], in[N];
int cnt, dfn[N], low[N], vis[N];
int sum, bel[N];
bitset<BLK> a[N];
queue<int> q;
int topo[N];
void tarjan(int u) {
    vis[u] = in[u] = 1;
    sta[++top] = u, dfn[u] = low[u] = ++cnt;
    for (int v : e[u])
        if (!vis[v]) {
            tarjan(v);
            low[u] = min(low[v], low[u]);
        }
    else if (in[v])

```

```
        low[u] = min(low[v], low[u]);
    if (low[u] == dfn[u]) {
        sum++; int i;
        while (1) {
            i = sta[top--];
            in[i] = 0, bel[i] = sum;
            if (i == u) break;
        }
    }
}

int main() {
    cin >> n >> k;
    for (int u, v, i = 1; i <= k; i++) {
        cin >> u >> v;
        e[u].push_back(v);
    }
    cin >> p;
    for (int u, v, i = 1; i <= p; i++) {
        cin >> u >> v;
        f[u].push_back(v);
    }
    for (int i = 1; i <= n; i++)
        if (!vis[i])
            tarjan(i);
    for (int i = 1; i <= n; i++) {
        for (int j : f[i]) {
            if (bel[i] == bel[j]) return puts("NO"), 0;
            ck[bel[i]].push_back(bel[j]); //check
        }
        f[i].clear();
    }
    for (int i = 1; i <= n; i++)
        for (int j : e[i]) {
            if (bel[i] == bel[j]) continue;
            f[bel[i]].push_back(bel[j]);
            d[bel[j]]++;
        }
    cnt = 0;
    for (int i = 1; i <= sum; i++)
        if (!d[i])
            q.push(i);
    while (!q.empty()) {
        int now = q.front(); q.pop();
        topo[++ cnt] = now;
        for (int j : f[now]) {
            d[j]--;
            if (d[j] == 0) q.push(j);
        }
    }
    for (int i = 1, t = (sum + BLK - 1) / BLK; i <= t; i++) {
        for (int j = sum; j; j--) {
            int u = topo[j];
            a[u].reset();
            if (BLK * (i - 1) < u && u <= BLK * i)
```



```

        a[u][u - BLK * (i - 1) - 1] = 1;
    for (int v : f[u])
        a[u] |= a[v];
}
for (int j = 1; j <= sum; j++)
    for (int v : ck[j])
        if (BLK * (i - 1) < v && v <= BLK * i &&
            a[j][v - BLK * (i - 1) - 1] == 1) {
            puts("NO");
            return 0;
        }
}
printf("YES\n%d\n", k);
for (int i = 1; i <= n; i++)
    for (int j : e[i])
        printf("%d %d\n", i, j);
return 0;
}

```

2.3 spfa 费用流

```

const int N = 600, M = 800000, inf = 0x3f3f3f3f;
int s, t, ans, len, maxflow;
int T, n, m, K, W;
int head[N], incf[N], path[N], pre[N], vis[N], d[N];
struct edge {int to, next, cap, cost;} e[M];
struct video {int s, t, w, op;} a[N];
void add(int u, int v, int w, int c) {
    e[++len] = (edge){v, head[u], w, c}, head[u] = len;
    e[++len] = (edge){u, head[v], 0, -c}, head[v] = len;
}
bool spfa() {
    deque<int> q;
    q.push_back(s), incf[s] = inf;
    for (int i = 1; i <= t; i++) d[i] = inf;
    d[s] = 0;
    while (!q.empty()) {
        int x = q.front();
        q.pop_front(), vis[x] = 0;
        for (int i = head[x]; i; i = e[i].next) {
            if (e[i].cap && d[e[i].to] > d[x] + e[i].cost) {
                d[e[i].to] = d[x] + e[i].cost;
                pre[e[i].to] = x, path[e[i].to] = i;
                incf[e[i].to] = min(incf[x], e[i].cap);
                if (!vis[e[i].to]) {
                    vis[e[i].to] = 1;
                    if (q.empty() || d[e[i].to] < d[q.front()])
                        q.push_front(e[i].to);
                    else q.push_back(e[i].to);
                }
            }
        }
    }
}
maxflow += incf[t];

```

```

    if (d[t] == inf) return 0;
    for (int i = t; i != s; i = pre[i]) {
        e[path[i]].cap -= incf[t];
        e[path[i] ^ 1].cap += incf[t];
    }
    return ans += incf[t] * d[t], 1;
}
int main() {
    /*build graph*/
    while(spfa());
}

```

2.4 dsu

```

/*DSU
* 用途:  $O(n \log n)$  解决无修改的子树询问问题, 需要保证操作支持删除
* 解决方法: 对于每个节点, 先对所有轻儿子, dfs 下去求一遍, 再消除影响
*           然后再 dfs 自己的重儿子, 然后不消除影响, 再加上所有轻儿子
*           就得到当前节点为根的子树的答案了
*/
int n, c[N];
int cnt[N], maxCnt;
int siz[N], son[N];
vector<int> e[N];
ll ans[N], sum[N];
void dfs1(int u, int fr) {
    siz[u] = 1;
    for (int v : e[u]) {
        if (v == fr) continue;
        dfs1(v, u);
        siz[u] += siz[v];
        if (siz[v] > siz[son[u]]) son[u] = v;
    }
}
void update(int x, int y) {
    sum[cnt[x]] -= x;
    cnt[x] += y;
    sum[cnt[x]] += x;
    if (cnt[x] > maxCnt) maxCnt = cnt[x];
    if (sum[maxCnt] == 0) maxCnt--;
}
void dfs3(int u, int fr, int val) {
    update(c[u], val);
    for (int v : e[u]) {
        if (v == fr) continue;
        dfs3(v, u, val);
    }
}
void dfs2(int u, int fr) {
    for (int v : e[u]) {
        if (v == fr || v == son[u]) continue;
        dfs2(v, u), dfs3(v, u, -1);
    }
    if (son[u]) dfs2(son[u], u);
}

```

```

    for (int v : e[u]) {
        if (v == fr || v == son[u]) continue;
        dfs3(v, u, 1);
    }
    update(c[u], 1);
    ans[u] = sum[maxCnt];
}
int main() {
    ios::sync_with_stdio(false);
    cin >> n;
    for (int i = 1; i <= n; i++)
        cin >> c[i];
    for (int u, v, i = 1; i < n; i++) {
        cin >> u >> v;
        e[u].push_back(v);
        e[v].push_back(u);
    }
    dfs1(1, 1), dfs2(1, 1);
    for (int i = 1; i <= n; i++)
        cout << ans[i] << ' ';
    return 0;
}

```

2.5 长链剖分

/* 长链剖分，选择深度最大的儿子作为重儿子，用于合并以深度为下标的信息
 * 像 *dsu* 一样，直接继承重儿子信息，然后按深度暴力合并其他儿子信息
 * 时间复杂度考虑每个节点作为轻儿子中的节点被合并只会有一次，所以 $O(n)$
 * 另一种用法，可以 $O(n \log n)$ 预处理后， $O(1)$ 找到 k 级祖先
 * *example problem*: 给个树，第 i 个点有两个权值 a_i 和 b_i
 * 现在求一条长度为 m 的路径，使得 $\sum a_i / \sum b_i$ 最小
 * 防爆栈 *trick*: 像重链剖分一样改成 *bfs*
 */

```

int n, m;
double k, a[N], b[N];
int len[N], son[N];
vector<int> e[N];
double tmp[N], *ptr, *f[N], temp[N];
void dfs(int u, int fr) {
    for (int v : e[u]) {
        if (v == fr) continue;
        dfs(v, u);
        if (len[v] > len[son[u]]) son[u] = v;
    }
    len[u] = len[son[u]] + 1;
}
inline double F(int x, int y) {return y >= len[x] ? 0 : f[x][y];}
bool solve(int u, int fr) {
    /* 为实现  $O(1)$  继承，采用  $f[u]-f[v]$  来保存  $u-fa[v]$  路径上的最小权值和 ( $dep[v]>dep[u]$ )
    * 即自底向上累加
    */
    if (son[u]) {
        f[son[u]] = f[u] + 1;
        if (solve(son[u], u)) return 1;
    }
}

```

```

    f[u][0] = val[u] + f[u][1];
    if (len[u] >= m && f[u][0] - F(u, m) <= 0) return 1;
    for (int v : e[u]) {
        if (v == son[u] || v == fr) continue;
        f[v] = ptr, ptr += len[v];
        if (solve(v, u)) return 1;
        for (int j = 1; j <= len[v] && j <= m; j++) {
            if (len[u] + j < m) continue;
            if (f[v][0] - F(v, j) + f[u][0] - F(u, m - j) <= 0) return 1;
        }
        temp[0] = val[u];
        for (int j = 1; j <= len[v]; j++)
            temp[j] = val[u] + min(f[u][1] - F(u, j + 1), f[v][0] - F(v, j));
        if (len[v] + 1 == len[u]) f[u][0] = temp[len[v]];
        for (int j = 1; j <= len[v]; j++)
            f[u][j] = f[u][0] - temp[j - 1];
        if (len[v] + 1 != len[u]) f[u][len[v] + 1] = f[u][0] - temp[len[v]];
    }
}
else {
    f[u][0] = val[u];
    if (m == 1 && f[u][0] <= 0) return 1;
}
return 0;
}

bool judge(double mid) {
    f[1] = ptr = tmp, ptr += len[1], k = mid;
    return solve(1, 1);
}

int main() {
    cin >> n >> m;
    for (int i = 1; i <= n; i++) cin >> a[i];
    for (int i = 1; i <= n; i++) cin >> b[i];
    if (m == -1) {
        double ans = 1e9;
        for (int i = 1; i <= n; i++)
            ans = min(ans, a[i] / b[i]);
        printf("%.2f\n", ans);
        return 0;
    }
    for (int u, v, i = 1; i < n; i++) {
        cin >> u >> v;
        e[u].push_back(v);
        e[v].push_back(u);
    }
    dfs(1, 1);
    int flag = 0;
    double l = 0, r = 2e5, mid, ans;
    for (int i = 0; i < 50; i++) {
        mid = (l + r) / 2;
        if (judge(mid)) r = mid - eps, flag = 1, ans = mid;
        else l = mid + eps;
    }
    if (flag) printf("%.2f\n", ans);
}

```

```

    else puts("-1");
    return 0;
}

```

2.6 LCT

```

const int N = 3e5 + 5;
int n, m;
int fa[N], ch[N][2], rev[N], val[N], sum[N];
int sta[N], top;
bool isroot(int x) {
    return ch[fa[x]][0] != x && ch[fa[x]][1] != x;
}
void reverse(int x) {
    if (!x) return;
    swap(ch[x][0], ch[x][1]);
    rev[x] ^= 1;
}
void pushdown(int x) {
    if (!rev[x]) return;
    reverse(ch[x][0]);
    reverse(ch[x][1]);
    rev[x] = 0;
}
void pushup(int x) {
    sum[x] = sum[ch[x][0]] ^ sum[ch[x][1]] ^ val[x];
}
void rot(int x) {
    int y = fa[x], z = fa[y], d = ch[y][1] == x, c = ch[x][!d];
    fa[x] = z; if (!isroot(y)) ch[z][ch[z][1] == y] = x;
    ch[y][d] = c; if (c) fa[c] = y;
    fa[y] = x, ch[x][!d] = y;
    pushup(y), pushup(x);
}
void splay(int x) {
    int u = x, top = 0, y, z;
    while (!isroot(u)) sta[++top] = u, u = fa[u];
    sta[++top] = u;
    while (top) pushdown(sta[top--]);
    while (!isroot(x)) {
        y = fa[x], z = fa[y];
        if (!isroot(y)) {
            if ((ch[z][0] == y) ^ (ch[y][0] == x)) rot(x);
            else rot(y);
        }
        rot(x);
    }
}
void access(int x) { //把 x 到根的路径拎出来
    for (int y = 0; x != 0; y = x, x = fa[x]) {
        splay(x), ch[x][1] = y, pushup(x);
    }
}
void makeroot(int x) { //令 x 成为这棵树的根

```

```

    access(x), splay(x), reverse(x);
}
int findroot(int x) { //找根
    access(x), splay(x);
    while (ch[x][0]) pushdown(x), x = ch[x][0];
    splay(x); //把根转到顶保证复杂度
    return x;
}
void split(int x, int y) { //拉出 x-y 的路径
    makeroot(x);
    access(y), splay(y);
    //y 存了这条路径的信息
}
void link(int x, int y) {
    makeroot(x);
    if (findroot(y) != x) fa[x] = y;
}
void cut(int x, int y) {
    makeroot(x);
    if (findroot(y) == x && fa[y] == x && ch[x][1] == y) {
        fa[y] = ch[x][1] = 0;
        pushup(x);
    }
}
int main() {
    ios::sync_with_stdio(false);
    cin >> n >> m;
    for (int i = 1; i <= n; i++)
        cin >> val[i];
    for (int op, x, y; m--; ) {
        cin >> op >> x >> y;
        switch(op) {
            case 0: split(x, y); printf("%d\n", sum[y]); break;
            case 1: link(x, y); break;
            case 2: cut(x, y); break;
            case 3: splay(x); val[x] = y; pushup(x); break;
            //单点更新完记得 pushup
        }
    }
}

```

2.7 hungary

```

/* 二分图最大匹配, 时间复杂度  $O(nm)$ 
 * 例题:  $n1$  个男,  $n2$  个女,  $m$  个配对关系
 * 输出最大配对数, 然后对于每个男输出配对的女
 */
int n1, n2, m;
vector<int> e[N];
int vis[N], pre[N], ans, tim;
bool dfs(int u) {
    if (vis[u] == tim) return 0;
    vis[u] = tim;
    for (int v : e[u])

```

```

        if (!pre[v] || dfs(pre[v]))
            return pre[v] = u, 1;
    return 0;
}
int main() {
    scanf("%d %d %d", &n1, &n2, &m);
    for (int u, v, i = 1; i <= m; i++) {
        scanf("%d %d", &u, &v);
        e[v].push_back(u);
        // 要输出左侧点连接的右侧点, 连边时就由右边点向左边连边
        // 连边 (u->v), 输出的 pre[v] 就是右边的点了
    }
    for (tim = 1; tim <= n2; tim++)
        if (dfs(tim)) ans++;
    printf("%d\n", ans);
    for (int i = 1; i <= n1; i++)
        printf("%d%c", pre[i], i == n1 ? '\n' : ' ');
    return 0;
}

```

2.8 dinic 最大流

```

const int N = 20000;
const int M = 500000;
const int inf = 0x3f3f3f3f;
int n, m;
int s, t, len = 1;
int to[M], cap[M], nex[M];
int g[N], p[N], q[N], d[N];
void add(int x, int y, int v) {
    to[++len] = y, cap[len] = v, nex[len] = g[x], g[x] = len;
    to[++len] = x, cap[len] = 0, nex[len] = g[y], g[y] = len;
}
bool bfs() {
    int l = 1, r = 1, x, i;
    memset(d, 0, sizeof d);
    d[s] = 1, q[1] = s;
    while (l <= r) {
        x = q[l++];
        for (i = g[x]; i; i = nex[i])
            if (cap[i] && !d[to[i]])
                d[to[i]] = d[x] + 1, q[++r] = to[i];
    }
    return d[t];
}
int dfs(int x, int y) {
    if (x == t || y == 0) return y;
    int flow = 0;
    for (int &i = p[x]; i; i = nex[i]) {
        if (!cap[i] || d[to[i]] != d[x] + 1) continue;
        int f = dfs(to[i], min(y, cap[i]));
        flow += f, y -= f;
        cap[i] -= f, cap[i ^ 1] += f;
        if (!y) break;
    }
    return flow;
}

```

```

    }
    return flow;
}
int dinic() {
    int maxflow = 0;
    while (bfs()) {
        memcpy(p, g, sizeof g);
        maxflow += dfs(s, inf);
    }
    return maxflow;
}

```

2.9 DAG 删去无用边

/* 无用边定义：对于边 (u, v) 如果存在从 u 到 v 不经过该边的另一条路径，则称该边无用

* 时间复杂度： $O(n^3)$ */

```

bool f[N][N]; // i 是否可达 j
vector<int> e[N];
int main() {
    rep(i, 1, n)
        for (int j : e[i]) {
            rep(k, 1, n)
                if (i != k && j != k && f[i][k] && f[k][j])
                    no_use_edge;
        }
}

```

2.10 树分治

```

const int N = 1e5 + 5;
vector<int> e[N];
int n, a[N];
int root, _left, vis[N];
int siz[N], maxv[N];
void find_root(int u, int fr) {
    siz[u] = 1, maxv[u] = 0;
    for (int v : e[u]) {
        if (v == fr || vis[v]) continue;
        find_root(v, u);
        siz[u] += siz[v];
        maxv[u] = max(maxv[u], siz[v]);
    }
    maxv[u] = max(maxv[u], _left - siz[u]);
    if (!root || maxv[u] < maxv[root])
        root = u;
}
void dfs(int u, int fr) {
    siz[u] = 1;
    for (int v : e[u]) {
        if (v == fr || vis[v]) continue;
        find_root(v, u);
        siz[u] += siz[v];
    }
}

```



```

void solve(int u, int w) {
    dfs(u, u); //update siz[]
    a[u] = w, vis[u] = 1;
    for (int v : e[u]) {
        if (vis[v]) continue;
        _left = siz[v];
        root = 0;
        find_root(v, v);
        solve(root, w + 1);
    }
}

int main() {
    ios::sync_with_stdio(false);
    cin >> n;
    for (int u, v, i = 1; i < n; i++) {
        cin >> u >> v;
        e[u].push_back(v);
        e[v].push_back(u);
    }
    _left = n, root = 0, find_root(1, 1);
    solve(root, 0);
    for (int i = 1; i <= n; i++)
        printf("%c ", 'A' + a[i]);
    return 0;
}

```

2.11 支配树

2.11.1 DAG 支配树 (含倍增 LCA)

```

/* 对于一个 dag, 假设每个联通块中都只有一个点出度为 0
 * 那么对于一个联通块, 假设这个出度为 0 的点为 s
 * 可以处理出这个块中所有点到 s 的必经点
 * rt 为新建的虚根, 把所有联通块整合在一棵树里
 */
int fa[N][19];
vector<int> e[N], f[N], E[N];
int du[N], dep[N];
int lca(int x, int y) {
    if (dep[x] < dep[y]) swap(x, y); // dep[x] > dep[y]
    for (int j = 18; j >= 0; j--)
        if (dep[fa[x][j]] >= dep[y])
            x = fa[x][j];
    if (x == y) return x;
    for (int j = 18; j >= 0; j--)
        if (fa[x][j] != fa[y][j])
            x = fa[x][j], y = fa[y][j];
    return fa[x][0];
}

void topo() {
    static int q[N * 2], l, r; l = 1, r = 0;
    for (int i = 1; i <= n; i++)
        if (du[i] == 0) {
            q[++r] = i;

```

```

        E[rt].push_back(i); //支配树的边
        fa[i][0] = rt;
        dep[i] = 1;
    }
    while (l <= r) {
        int u = q[l ++];
        for (int v : e[u]) {
            du[v] --;
            if (du[v] == 0) {
                int las = -1;
                for (int w : f[v]) {
                    if (las == -1) las = w;
                    else las = lca(las, w);
                }
                E[las].push_back(v);
                fa[v][0] = las;
                dep[v] = dep[las] + 1;
                for (int j = 1; j <= 18; j++)
                    fa[v][j] = fa[fa[v][j - 1]][j - 1];
                q[++ r] = v;
            }
        }
    }
}

int main() {
    int cas, q;
    for (scanf("%d", &cas); cas --; ) {
        scanf("%d %d", &n, &m);
        rt = n + 1;
        for (int i = 0; i <= rt; i++) {
            e[i].clear(); f[i].clear();
            E[i].clear(); du[i] = dep[i] = 0;
            for (int j = 0; j <= 18; j++)
                fa[i][j] = 0;
        }
        for (int u, v, i = 1; i <= m; i++) {
            scanf("%d %d", &u, &v);
            e[v].push_back(u); //反向边
            f[u].push_back(v); //正向边
            du[u] ++;
        }
        topo();
    }
    return 0;
}

```

3 数据结构

3.1 莫队

3.1.1 带修改莫队

```

/* 原问题：给定长度为  $n$  的数列，询问操作会询问某段区间内
 * 有多少个子区间的异或和不为 0，修改操作会交换两个相邻位置的数字
 * 转化：原数列做前缀异或和，询问变成区间内多少对不一样的数字
 * 修改变为单点修改，直接带修改莫队来做即可
 *
 * 带修改莫队复杂度分析：
 * 块大小  $B$  设为  $N^{2/3}$ ，故有  $(N^{1/3})$  块，修改操作  $O(1)$  完成
 * 在时间轴上滚动的总复杂度：
 * 左右端点所在块不动时，时间单调增加， $O((N^{1/3})^2) * O(N)$ 
 * 左右端点所在块有一个变化，时间可能回退  $O(N)$ ，同上
 * 无关时间的复杂度：
 * 左端点所在块不变，右端点单增， $O(N^{1/3}) * O(N)$ 
 * 左端点变化，右端点最多回退  $O(N)$ ， $O(N^{1/3}) * O(N)$ 
 * 综上，总复杂度  $O(N^{2/3})$ 
 */
const int N = 1e5 + 5;
const int M = (1 << 20) + 2019;
const int B = 2155;
int n, m, pl, pr, cur, a[N], b[N], d[N], cnt[M];
int idxC, idxQ, tim[N], pos[N], val[N], pre[N];
/*pos[i] 第 i 次修改的位置
 *pre[i] 第 i 次修改前，pos[i] 的位置的值
 *val[i] 第 i 次修改要将这个位置的值改为 val[i]
 */
ll res, ans[N];
#define bel(x) (((x) - 1) / B + 1)
struct query {
    int id, tim, l, r;
    bool operator < (const query &b) const {
        if(bel(l) != bel(b.l)) return l < b.l;
        if(bel(r) != bel(b.r)) return r < b.r;
        return id < b.id;
    }
} q[N];
void add(int p){
    res += cnt[b[p]];
    cnt[b[p]]++;
}
void del(int p){
    cnt[b[p]]--;
    res -= cnt[b[p]];
}
void modify(int cur, int dir = 1){
    if(pos[cur] >= pl && pos[cur] <= pr) del(pos[cur]);
    b[pos[cur]] = dir == 1 ? val[cur] : pre[cur];
    if(pos[cur] >= pl && pos[cur] <= pr) add(pos[cur]);
}
void change(int now){

```

```

while(cur < idxC && tim[cur + 1] <= now) modify(++ cur);
while(cur > 0 && tim[cur] > now) modify(cur --, -1);
}
int main(){
    int op, x, y;
    while (scanf("%d %d", &n, &m) != EOF) {
        for (int i = 1; i <= n; i++) {
            scanf("%d", &a[i]);
            b[i] = b[i - 1] ^ a[i];
            d[i] = a[i];
        }
        idxQ = idxC = 0;
        for (int i = 1; i <= m; i++) {
            scanf("%d", &op);
            if (op == 1) {
                idxQ++;
                q[idxQ].id = idxQ;
                q[idxQ].tim = i;
                scanf("%d %d", &q[idxQ].l, &q[idxQ].r);
                q[idxQ].l--;
                ans[idxQ] = 1ll * (q[idxQ].r - q[idxQ].l)
                    * (q[idxQ].r - q[idxQ].l + 1) / 2;
            }
            else {
                tim[++ idxC] = i;
                scanf("%d", &x);
                pre[idxC] = b[x];
                b[x] ^= a[x], b[x] ^= a[x + 1];
                swap(a[x], a[x + 1]);
                pos[idxC] = x;
                val[idxC] = b[x];
            }
        }
        //因为要获取 pre[i] 的值, 所以读入时数组要相应改变, 处理完后再恢复为初始值
        //或者这里把 cur 设为最后时刻也可以哦
        for (int i = 1; i <= n; i++)
            b[i] = b[i - 1] ^ d[i];
        pl = 1, pr = 0; //保证初始时 [pl, pr] 是个空区间即可
        cur = res = 0;
        sort(q + 1, q + idxQ + 1);
        for(int i = 1; i <= idxQ; i++){
            change(q[i].tim);
            while(pl > q[i].l) add(-- pl);
            while(pr < q[i].r) add(++ pr);
            while(pl < q[i].l) del(pl ++);
            while(pr > q[i].r) del(pr --);
            ans[q[i].id] -= res;
        }
        for(int i = 1; i <= idxQ; i++)
            printf("%lld\n", ans[i]);
        for (int i = pl; i <= pr; i++)
            cnt[b[i]]--;
    }
    return 0;
}

```

```
}
```

3.2 splay

```
#define mid (l + r >> 1)
int n, m, a, b, len, tot;
struct node {
    bool rev; //翻转标记
    int v, siz;
    node *c[2];
    node():rev(0),v(0),siz(0),c{NULL, NULL}{}
    node *init(int x);
    void pushdown();
    void mata() {siz = c[0] -> siz + c[1] -> siz + 1;}
    int cmp(int k) {return k<=c[0]->siz?0:(k==c[0]->siz+1?-1:1);}
    void print();
}pool[N], *null = new node();
node *node::init(int x){rev=0, v=x, siz=1, c[0]=c[1]=null;return this;}
void node::pushdown() {
    if (!rev) return;
    if (c[0] != null) c[0] -> rev ^= 1;
    if (c[1] != null) c[1] -> rev ^= 1;
    swap(c[0], c[1]), rev = 0;
}
void node::print() {
    pushdown();
    if (c[0] != null) c[0] -> print();
    if (1 <= v && v <= n) printf("%d ", v);
    if (c[1] != null) c[1] -> print();
}
node *build(int l, int r) {//初始序列为 1-n
    if (l == r) return pool[tot++].init(r);
    node *tmp = pool[tot++].init(mid);
    if (l < mid) tmp -> c[0] = build(l, mid - 1);
    if (mid < r) tmp -> c[1] = build(mid + 1, r);
    tmp -> mata(); return tmp;
}
void rot(node *&o, int k) {//把 k 儿子提上来
    o -> pushdown(); node *tmp = o -> c[k];
    tmp -> pushdown(); o -> c[k] = tmp -> c[!k];
    tmp -> c[!k] -> pushdown(); tmp -> c[!k] = o;
    o -> mata(), tmp -> mata(), o = tmp;
}
void splay(node *&o, int k) {//把以 o 为根的 splayTree 中 rk 为 k 的点提到根
    int k1 = o -> cmp(k); o -> pushdown();
    if (k1 == -1) return; o -> c[k1] -> pushdown();
    if (k1) k -= o -> c[0] -> siz + 1;
    int k2 = o -> c[k1] -> cmp(k);
    if (~k2) {//k2 != -1
        if (k2) k -= o -> c[k1] -> c[0] -> siz + 1;
        o -> c[k1] -> c[k2] -> pushdown();
        splay(o -> c[k1] -> c[k2], k);
        if (k2 == k1) rot(o, k1);
        else rot(o -> c[k1], k2);
    }
```

```

    }
    rot(o, k1);
}
int main() {
    scanf("%d %d", &n, &m);
    node *root = build(0, n + 1); //方便边界左右处理各多开一个
    for (; m --; ) {
        scanf("%d %d", &a, &b), a ++, b ++, len = b - a + 1;
        splay(root, a - 1), splay(root -> c[1], len + 1);
        root -> c[1] -> c[0] -> rev ^= 1, root -> c[1] -> c[0] -> pushdown();
    }
    root -> print(); return 0;
}

```

3.3 treap

```

/*
容易实现的预开内存池 treap, 每次 head 清空即可
如果初始要插入  $n$  个 1, 可改为类似 splay 的  $O(n)build$  写法
poolSize 是单组数据的最大节点数, 对于单组数据有很多插入和删除
导致使用的节点很多的数据, 无法使用
*/
const int poolSize = 5e5 + 10;
struct node {
    node *c[2];
    int v, r, siz;
    void update();
    void init(int x);
};
node *null = new node(), *root = null;
void node::update() {
    siz = c[0] -> siz + c[1] -> siz + 1;
}
void node::init(int x) {
    v = x, r = rand(), siz = 1;
    c[0] = c[1] = null;
}
node nodesPool[poolSize];
int head; //每次 head=0 清空
node *newnode(int x) {
    node *res = &nodesPool[head ++];
    res -> init(x);
    return res;
}
void rot(node *&o, int d) {
    node *tmp = o -> c[!d];
    o -> c[!d] = tmp -> c[d], tmp -> c[d] = o;
    o -> update(), tmp -> update(), o = tmp;
}
void insert(node *&o, int x) {
    if (o == null) {
        o = newnode(x);
        return;
    }
}

```

```

    int d = x > o -> v ? 0 : 1;
    insert(o -> c[d], x);
    if (o -> c[d] -> r < o -> r) rot(o, !d);
    o -> update();
}

void del(node *&o, int x) {
    if (x == o -> v) {
        if (o -> c[0] == null) {o = o -> c[1]; return;}
        if (o -> c[1] == null) {o = o -> c[0]; return;}
        int d = o -> c[0] -> r < o -> c[1] -> r ? 1 : 0;
        rot(o, d), del(o -> c[d], x);
    }
    else del(o -> c[x <= o -> v], x);
    o -> update();
}

void build(node *&o, int l, int r) {
    o = newnode(1);
    if (l == r) return;
    int mid = l + r >> 1;
    if (l < mid) build(o -> c[0], l, mid - 1);
    if (o -> c[0] != null && o -> c[0] -> r < o -> r) swap(o -> c[0] -> r, o -> r);
    if (mid < r) build(o -> c[1], mid + 1, r);
    if (o -> c[1] != null && o -> c[1] -> r < o -> r) swap(o -> c[1] -> r, o -> r);
    o -> update();
}

```

3.4 主席树

```

const int N = 130000;
const int M = N * 20;
//主席树节点数, 可以直接稳妥选择  $N*(5+\log N)$ 
struct {
    int siz, l, r;
    ll sum, val;
}tr[M];
#define l(x) tr[x].l
#define r(x) tr[x].r
#define s(x) tr[x].sum
#define v(x) tr[x].val
#define sz(x) tr[x].siz
#define mid (l + r >> 1)
int tot, root[N];
int build(int l, int r) {
    int x = ++tot;
    s(x) = sz(x) = 0;
    if (l < r) {
        l(x) = build(l, mid);
        r(x) = build(mid + 1, r);
    }
    return x;
}

int change(int o, int l, int r, int p, int y) {
    //在  $p$  的位置插入一个  $y$ 
    int x = ++tot;

```

```

    s(x) = s(o) + y, sz(x) = sz(o) + 1;
    l(x) = l(o), r(x) = r(o), v(x) = y;
    if (l < r) {
        if (p <= mid) l(x) = change(l(o), l, mid, p, y);
        else r(x) = change(r(o), mid + 1, r, p, y);
    }
    return x;
}
ll ask(int o1, int o2, int l, int r, int k) {
    //求 (l,r] 区间前 k 小的数之和, 有 o1=root[l],o2=root[r]
    if (l == r) return v(o2) * k;
    int lsz = sz(l(o2)) - sz(l(o1));
    if (lsz == k) return s(l(o2)) - s(l(o1));
    if (lsz < k) return s(l(o2)) - s(l(o1)) + ask(r(o1), r(o2), mid + 1, r, k - lsz);
    return ask(l(o1), l(o2), l, mid, k);
}

```

3.5 吉老师线段树

```

/* 区间每个数变为 min(a[i],t) + 区间最大 + 区间和 O(nlog)* */
const int N = (1 << 20) + 5;
#define lc (o << 1)
#define rc (lc | 1)
struct node {
    int max1, max2, cnt;
    ll sum;
}tr[N << 1];
int T, n, m;
int op, x, y, t;
int a[N];
void pushup(int o) {
    if (tr[lc].max1 == tr[rc].max1) {
        tr[o].max1 = tr[lc].max1;
        tr[o].cnt = tr[lc].cnt + tr[rc].cnt;
        tr[o].max2 = max(tr[lc].max2, tr[rc].max2);
    }
    else {
        if (tr[lc].max1 > tr[rc].max1) {
            tr[o] = tr[lc];
            tr[o].max2 = max(tr[o].max2, tr[rc].max1);
        }
        else {
            tr[o] = tr[rc];
            tr[o].max2 = max(tr[o].max2, tr[lc].max1);
        }
    }
    tr[o].sum = tr[lc].sum + tr[rc].sum;
}
void pushdown(int o) {
    if (tr[o].max1 < tr[lc].max1) {
        tr[lc].sum += 1ll * (tr[o].max1 - tr[lc].max1) * tr[lc].cnt;
        tr[lc].max1 = tr[o].max1;
    }
    if (tr[o].max1 < tr[rc].max1) {

```



```
        tr[rc].sum += 1ll * (tr[o].max1 - tr[rc].max1) * tr[rc].cnt;
        tr[rc].max1 = tr[o].max1;
    }
}

void build(int o, int l, int r) {
    if (l == r) {
        tr[o].max1 = tr[o].sum = a[r];
        tr[o].cnt = 1, tr[o].max2 = 0;
        return;
    }
    int mid = l + r >> 1;
    build(lc, l, mid);
    build(rc, mid + 1, r);
    pushup(o);
}

void update(int o, int l, int r, int s, int t, int v) {
    if (v >= tr[o].max1) return;
    pushdown(o);
    if (s <= l && r <= t) {
        if (v > tr[o].max2) {
            tr[o].sum += 1ll * (v - tr[o].max1) * tr[o].cnt;
            tr[o].max1 = v;
            return;
        }
    }
    int mid = l + r >> 1;
    if (s <= mid) update(lc, l, mid, s, t, v);
    if (t > mid) update(rc, mid + 1, r, s, t, v);
    pushup(o);
}

ll ask_max(int o, int l, int r, int s, int t) {
    if (s <= l && r <= t) return tr[o].max1;
    pushdown(o);
    int mid = l + r >> 1;
    ll res = 0;
    if (s <= mid) res = max(res, ask_max(lc, l, mid, s, t));
    if (t > mid) res = max(res, ask_max(rc, mid + 1, r, s, t));
    return res;
}

ll ask_sum(int o, int l, int r, int s, int t) {
    if (s <= l && r <= t) return tr[o].sum;
    pushdown(o);
    int mid = l + r >> 1;
    ll res = 0;
    if (s <= mid) res += ask_sum(lc, l, mid, s, t);
    if (t > mid) res += ask_sum(rc, mid + 1, r, s, t);
    return res;
}

int main() {
    ios::sync_with_stdio(false);
    for (cin >> T; T--; ) {
        cin >> n >> m;
        for (int i = 1; i <= n; i++)
            cin >> a[i];
```

```

        build(1, 1, n);
        while (m --) {
            cin >> op;
            if (op == 0) {
                cin >> x >> y >> t;
                update(1, 1, n, x, y, t);
            }
            else if (op == 1) {
                cin >> x >> y;
                cout << ask_max(1, 1, n, x, y) << '\n';
            }
            else {
                cin >> x >> y;
                cout << ask_sum(1, 1, n, x, y) << '\n';
            }
        }
    }
    return 0;
}

```

3.6 KDTree

3.6.1 3 维 KDtree

```

/*O(n*n^(1-1/k)), k 为维度 */
const int N = 1e5 + 5;
const int Mod = 1e9 + 7;
int nowD, ans, x[3], y[3];
int n, m, a[N], b[N], c[N], d[N];
struct node {
    int Max[3], Min[3], d[3];
    int val, maxv;
    node *c[2];
    node() {
        c[0] = c[1] = NULL;
        val = maxv = 0;
    }
    void pushup();
    bool operator < (const node &a) const {
        return d[nowD] < a.d[nowD];
    }
} Null, nodes[N];
node *root = &Null;
inline void node::pushup() {
    if (c[0] != &Null) {
        if (c[0] -> Max[1] > Max[1]) Max[1] = c[0] -> Max[1];
        if (c[0] -> Max[2] > Max[2]) Max[2] = c[0] -> Max[2];
        if (c[0] -> Min[0] < Min[0]) Min[0] = c[0] -> Min[0];
        if (c[0] -> Min[2] < Min[2]) Min[2] = c[0] -> Min[2];
        if (c[0] -> maxv > maxv) maxv = c[0] -> maxv;
    }
    if (c[1] != &Null) {
        if (c[1] -> Max[1] > Max[1]) Max[1] = c[1] -> Max[1];
        if (c[1] -> Max[2] > Max[2]) Max[2] = c[1] -> Max[2];
        if (c[1] -> Min[0] < Min[0]) Min[0] = c[1] -> Min[0];
    }
}

```

```
        if (c[1] -> Min[2] < Min[2]) Min[2] = c[1] -> Min[2];
        if (c[1] -> maxv > maxv) maxv = c[1] -> maxv;
    }
}

inline node *build(int l, int r) {
    int mid = l + r >> 1; nowD = rand() % 3;
    nth_element(nodes + l, nodes + mid, nodes + r + 1);
    node *res = &nodes[mid];
    if (l != mid) res -> c[0] = build(l, mid - 1);
    else res -> c[0] = &Null;
    if (r != mid) res -> c[1] = build(mid + 1, r);
    else res -> c[1] = &Null;
    res -> pushup();
    return res;
}

inline int calc(node *o) {
    if (y[0] < o -> Min[0] || x[1] > o -> Max[1] || x[2] > o -> Max[2] || y[2] < o ->
        Min[2]) return -1;
    return o -> maxv;
}

inline void query(node *o) {
    if (o -> val > ans && y[0] >= o -> d[0] && x[1] <= o -> d[1] && x[2] <= o -> d[2] &&
        y[2] >= o -> d[2]) ans = o -> val;
    int dl, dr;
    if (o -> c[0] != &Null) dl = calc(o -> c[0]);
    else dl = -1;
    if (o -> c[1] != &Null) dr = calc(o -> c[1]);
    else dr = -1;
    if (dl > dr) {
        if (dl > ans) query(o -> c[0]);
        if (dr > ans) query(o -> c[1]);
    } else {
        if (dr > ans) query(o -> c[1]);
        if (dl > ans) query(o -> c[0]);
    }
}

int main() {
    ios::sync_with_stdio(false);
    cin >> n >> m;
    for (int i = 1; i <= n; i++) {
        cin >> a[i];
        b[i] = d[a[i]];
        d[a[i]] = i;
    }
    for (int i = 1; i <= n; i++) d[i] = n + 1;
    for (int i = n; i; i--) {
        c[i] = d[a[i]];
        d[a[i]] = i;
    }
    for (int i = 1; i <= n; i++) {
        nodes[i].Min[0] = nodes[i].d[0] = b[i];
        nodes[i].Max[1] = nodes[i].d[1] = c[i];
        nodes[i].Max[2] = nodes[i].Min[2] = nodes[i].d[2] = i;
        nodes[i].val = nodes[i].maxv = a[i];
    }
}
```

```

    }
    root = build(1, n);
    for (int l, r; m --; ) {
        cin >> l >> r;
        l = (l + ans) % n + 1;
        r = (r + ans) % n + 1;
        if (l > r) swap(l, r);
        y[0] = l - 1;
        x[1] = r + 1;
        x[2] = l, y[2] = r;
        ans = 0, query(root);
        cout << ans << endl;
    }
    cout << endl;
    return 0;
}

```

3.6.2 KDtree 二维空间区间覆盖单点查询

```

/* 类似线段树 */
const int N = 1e5 + 5;
const int Mod = 1e9 + 7;
int nowD, x[2], y[2], z;
struct node {
    int Max[2], Min[2], d[2];
    int val, lazy;
    node *c[2];
    node() {
        c[0] = c[1] = NULL;
    }
    void pushup();
    void pushdown();
    bool operator < (const node &a) const {
        return d[nowD] < a.d[nowD];
    }
} Null, nodes[N];
node *root = &Null;
inline void node::pushup() {
    if (c[0] != &Null) {
        if (c[0] -> Max[0] > Max[0]) Max[0] = c[0] -> Max[0];
        if (c[0] -> Max[1] > Max[1]) Max[1] = c[0] -> Max[1];
        if (c[0] -> Min[0] < Min[0]) Min[0] = c[0] -> Min[0];
        if (c[0] -> Min[1] < Min[1]) Min[1] = c[0] -> Min[1];
    }
    if (c[1] != &Null) {
        if (c[1] -> Max[0] > Max[0]) Max[0] = c[1] -> Max[0];
        if (c[1] -> Max[1] > Max[1]) Max[1] = c[1] -> Max[1];
        if (c[1] -> Min[0] < Min[0]) Min[0] = c[1] -> Min[0];
        if (c[1] -> Min[1] < Min[1]) Min[1] = c[1] -> Min[1];
    }
}
inline void node::pushdown() {
    if (c[0] != &Null) c[0] -> val = c[0] -> lazy = lazy;
    if (c[1] != &Null) c[1] -> val = c[1] -> lazy = lazy;
}

```

```

    lazy = -1;
}
inline node *build(int l, int r, int D) {
    int mid = l + r >> 1; nowD = D;
    nth_element(nodes + l, nodes + mid, nodes + r + 1);
    node *res = &nodes[mid];
    if (l != mid) res -> c[0] = build(l, mid - 1, !D);
    else res -> c[0] = &Null;
    if (r != mid) res -> c[1] = build(mid + 1, r, !D);
    else res -> c[1] = &Null;
    res -> pushup();
    return res;
}
inline int query(node *o) {
    if (o == &Null) return -1;
    if (o -> lazy != -1) o -> pushdown();
    if (x[o] > o -> Max[0] || y[o] > o -> Max[1] || x[o] < o -> Min[0] || y[o] < o ->
        Min[1]) return -1;
    if (x[o] == o -> d[0]) return o -> val;
    return max(query(o -> c[0]), query(o -> c[1]));
}
inline void modify(node *o) {
    if (o == &Null) return;
    if (o -> lazy != -1) o -> pushdown();
    if (x[o] > o -> Max[0] || y[o] > o -> Max[1] || x[o] < o -> Min[0] || y[o] < o ->
        Min[1]) return;
    if (x[o] <= o -> Min[0] && y[o] <= o -> Min[1] && x[o] >= o -> Max[0] && y[o] >= o ->
        Max[1]) {
        o -> val = o -> lazy = z;
        return;
    }
    if (x[o] <= o -> d[0] && y[o] <= o -> d[1] && x[o] >= o -> d[0] && y[o] >= o -> d[1])
        o -> val = z;
    modify(o -> c[0]), modify(o -> c[1]);
}
int n, m, k, a[N], c[N], d[N];
int cnt, st[N], en[N], dfn[N], dep[N];
vector<int> e[N];
void dfs(int u) {
    st[u] = ++ cnt, dfn[cnt] = u;
    for (int v : e[u])
        dep[v] = dep[u] + 1, dfs(v);
    en[u] = cnt;
}
int main() {
    ios::sync_with_stdio(false);
    int T, ans;
    for (cin >> T; T --; ) {
        cin >> n >> m >> k, ans = cnt = 0;
        for (int i = 1; i <= n; i++)
            e[i].clear();
        for (int u, i = 2; i <= n; i++) {
            cin >> u;
            e[u].push_back(i);
        }
    }
}

```

```

    }
    dfs(1);
    for (int i = 1; i <= n; i++) {
        nodes[i].Min[0] = nodes[i].Max[0] = nodes[i].d[0] = i;
        nodes[i].Min[1] = nodes[i].Max[1] = nodes[i].d[1] = dep[dfn[i]];
        nodes[i].val = 1, nodes[i].lazy = -1;
    }
    root = build(1, n, 0);
    for (int u, v, w, i = 1; i <= k; i++) {
        cin >> u >> v >> w;
        if (w == 0) {
            x[0] = st[u], y[0] = dep[u];
            ans = (ans + 1ll * i * query(root) % Mod) % Mod;
        } else {
            x[0] = st[u], x[1] = en[u];
            y[0] = dep[u], y[1] = dep[u] + v;
            z = w, modify(root);
        }
    }
    cout << ans << endl;
}
return 0;
}

```

3.6.3 KDtree 二维空间单点修改区间查询

```

/*
 * 调整重构系数可以影响常数
 * 询问多就让系数接近 0.70-0.75, 询问少就让系数在 0.8-0.90
 */
const int inf = 1e9;
int n, m, tot, nowD;
struct node {
    int Max[2], Min[2], d[2];
    int sum, siz, val;
    node *c[2];
    node() {
        Max[0] = Max[1] = -inf;
        Min[0] = Min[1] = inf;
        sum = val = siz = 0;
        c[0] = c[1] = NULL;
        d[0] = d[1] = 0;
    }
    void update();
} Null, nodes[200010], *temp[200010];
node *root = &Null;
inline void node::update() {
    siz = c[0] -> siz + c[1] -> siz + 1;
    sum = c[0] -> sum + c[1] -> sum + val;
    if (c[0] != &Null) {
        if (c[0] -> Max[0] > Max[0]) Max[0] = c[0] -> Max[0];
        if (c[0] -> Max[1] > Max[1]) Max[1] = c[0] -> Max[1];
        if (c[0] -> Min[0] < Min[0]) Min[0] = c[0] -> Min[0];
        if (c[0] -> Min[1] < Min[1]) Min[1] = c[0] -> Min[1];
    }
}

```

```

    }
    if (c[1] != &Null) {
        if (c[1] -> Max[0] > Max[0]) Max[0] = c[1] -> Max[0];
        if (c[1] -> Max[1] > Max[1]) Max[1] = c[1] -> Max[1];
        if (c[1] -> Min[0] < Min[0]) Min[0] = c[1] -> Min[0];
        if (c[1] -> Min[1] < Min[1]) Min[1] = c[1] -> Min[1];
    }
}

inline bool cmp(const node *a, const node *b) {
    return a -> d[nowD] < b -> d[nowD];
}

inline void traverse(node *o) {
    if (o == &Null) return;
    temp[++ tot] = o;
    traverse(o -> c[0]);
    traverse(o -> c[1]);
}

inline node *build(int l, int r, int D) {
    int mid = l + r >> 1; nowD = D;
    nth_element(temp + l, temp + mid, temp + r + 1, cmp);
    node *res = temp[mid];
    res -> Max[0] = res -> Min[0] = res -> d[0];
    res -> Max[1] = res -> Min[1] = res -> d[1];
    if (l != mid) res -> c[0] = build(l, mid - 1, !D);
    else res -> c[0] = &Null;
    if (r != mid) res -> c[1] = build(mid + 1, r, !D);
    else res -> c[1] = &Null;
    res -> update();
    return res;
}

int x, y, a, b, tmpD;
node **tmp;
inline void rebuild(node *&o, int D) {
    tot = 0;
    traverse(o);
    o = build(1, tot, D);
}

inline void insert(node *&o, node *p, int D) {
    if (o == &Null) {o = p; return;}
    if (p -> Max[0] > o -> Max[0]) o -> Max[0] = p -> Max[0];
    if (p -> Max[1] > o -> Max[1]) o -> Max[1] = p -> Max[1];
    if (p -> Min[0] < o -> Min[0]) o -> Min[0] = p -> Min[0];
    if (p -> Min[1] < o -> Min[1]) o -> Min[1] = p -> Min[1];
    o -> siz ++, o -> sum += p -> sum;
    insert(o -> c[p -> c[D] >= o -> c[D]], p, !D);
    if (max(o -> c[0] -> siz, o -> c[1] -> siz) > int(o -> siz * 0.75 + 0.5)) tmpD = D,
        ↪ tmp = &o;
}

inline int query(node *o) {
    if (o == &Null) return 0;
    if (x > o -> Max[0] || y > o -> Max[1] || a < o -> Min[0] || b < o -> Min[1]) return
        ↪ 0;
    if (x <= o -> Min[0] && y <= o -> Min[1] && a >= o -> Max[0] && b >= o -> Max[1])
        ↪ return o -> sum;
}

```

```

    return (x <= o -> d[0] && y <= o -> d[1] && a >= o -> d[0] && b >= o -> d[1] ? o ->
        ↪ val : 0)
        + query(o -> c[1]) + query(o -> c[0]);
}
int main() {
    ios::sync_with_stdio(false);
    cin >> m;
    node *ttt = &Null;
    for (int t, ans = 0; ; ) {
        cin >> t;
        if (t == 3) break;
        if (t == 1) {
            cin >> x >> y >> a;
            x ^= ans, y ^= ans, n++;
            nodes[n].sum = nodes[n].val = a ^ ans, nodes[n].siz = 1;
            nodes[n].Max[0] = nodes[n].Min[0] = nodes[n].d[0] = x;
            nodes[n].Max[1] = nodes[n].Min[1] = nodes[n].d[1] = y;
            nodes[n].c[0] = nodes[n].c[1] = &Null;
            tmp = &(ttt), insert(root, &nodes[n], 0);
            if (*tmp != &Null) rebuild(*tmp, tmpD);
        } else {
            cin >> x >> y >> a >> b;
            x ^= ans, y ^= ans, a ^= ans, b ^= ans;
            if (x > a) swap(x, a);
            if (y > b) swap(y, b);
            ans = query(root);
            printf("%d\n", ans);
        }
    }
    return 0;
}

```

3.6.4 KDtree 找最近点

```

/*
 * 为了维持树的平衡，可以一开始把所有点都读进来 build
 * 然后打 flag 标记该点是否被激活
 */
const int N = 5e5 + 5;
const int inf = 1 << 30;
int n, m;
int ql, qr, ans, tot, nowD;
//nowD = rand() % 1 ?
struct Node {
    int d[2];
    bool operator < (const Node &a) const {
        if (d[nowD] == a.d[nowD]) return d[!nowD] < a.d[!nowD];
        return d[nowD] < a.d[nowD];
    }
}pot[N];
struct node {
    int min[2], max[2], d[2];
    node *c[2];
    node() {

```



```
    min[0] = min[1] = max[0] = max[1] = d[0] = d[1] = 0;
    c[0] = c[1] = NULL;
}
node(int x, int y);
void update();

}t[N], Null, *root;
node::node(int x, int y) {
    min[0] = max[0] = d[0] = x;
    min[1] = max[1] = d[1] = y;
    c[0] = c[1] = &Null;
}
inline void node::update() {
    if (c[0] != &Null) {
        if (c[0] -> max[0] > max[0]) max[0] = c[0] -> max[0];
        if (c[0] -> max[1] > max[1]) max[1] = c[0] -> max[1];
        if (c[0] -> min[0] < min[0]) min[0] = c[0] -> min[0];
        if (c[0] -> min[1] < min[1]) min[1] = c[0] -> min[1];
    }
    if (c[1] != &Null) {
        if (c[1] -> max[0] > max[0]) max[0] = c[1] -> max[0];
        if (c[1] -> max[1] > max[1]) max[1] = c[1] -> max[1];
        if (c[1] -> min[0] < min[0]) min[0] = c[1] -> min[0];
        if (c[1] -> min[1] < min[1]) min[1] = c[1] -> min[1];
    }
}
inline void build(node *&o, int l, int r, int D) {
    int mid = l + r >> 1;
    nowD = D;
    nth_element(pot + l, pot + mid, pot + r + 1);
    o = new node(pot[mid].d[0], pot[mid].d[1]);
    if (l != mid) build(o -> c[0], l, mid - 1, !D);
    if (r != mid) build(o -> c[1], mid + 1, r, !D);
    o -> update();
}
inline void insert(node *o) {
    node *p = root;
    int D = 0;
    while (1) {
        if (o -> max[0] > p -> max[0]) p -> max[0] = o -> max[0];
        if (o -> max[1] > p -> max[1]) p -> max[1] = o -> max[1];
        if (o -> min[0] < p -> min[0]) p -> min[0] = o -> min[0];
        if (o -> min[1] < p -> min[1]) p -> min[1] = o -> min[1];
        if (o -> d[D] >= p -> d[D]) {
            if (p -> c[1] == &Null) {
                p -> c[1] = o;
                return;
            } else p = p -> c[1];
        } else {
            if (p -> c[0] == &Null) {
                p -> c[0] = o;
                return;
            } else p = p -> c[0];
        }
    }
}
```

```
        D ^= 1;
    }
}

inline int dist(node *o) {
    int dis = 0;
    if (ql < o -> min[0]) dis += o -> min[0] - ql;
    if (ql > o -> max[0]) dis += ql - o -> max[0];
    if (qr < o -> min[1]) dis += o -> min[1] - qr;
    if (qr > o -> max[1]) dis += qr - o -> max[1];
    return dis;
}

inline void query(node *o) {
    int dl, dr, d0;
    d0 = abs(o -> d[0] - ql) + abs(o -> d[1] - qr);
    if (d0 < ans) ans = d0;
    if (o -> c[0] != &Null) dl = dist(o -> c[0]);
    else dl = inf;
    if (o -> c[1] != &Null) dr = dist(o -> c[1]);
    else dr = inf;
    if (dl < dr) {
        if (dl < ans) query(o -> c[0]);
        if (dr < ans) query(o -> c[1]);
    } else {
        if (dr < ans) query(o -> c[1]);
        if (dl < ans) query(o -> c[0]);
    }
}

int main() {
    ios::sync_with_stdio(false);
    cin >> n >> m;
    for (int i = 1; i <= n; i++)
        cin >> pot[i].d[0] >> pot[i].d[1];
    build(root, 1, n, 0);

    for (int x, y, z; m--; ) {
        cin >> x >> y >> z;
        if (x == 1) {
            t[tot].max[0] = t[tot].min[0] = t[tot].d[0] = y;
            t[tot].max[1] = t[tot].min[1] = t[tot].d[1] = z;
            t[tot].c[0] = t[tot].c[1] = &Null;
            insert(&t[tot++]);
        } else {
            ans = inf, ql = y, qr = z;
            query(root), printf("%d\n", ans);
        }
    }
    return 0;
}
```

4 构造

4.1 若干排列使所有数对都出现一次

```

/*n/2 个排列使得所有数对 (i,j) 且 i<j 都出现一次
 *n 为奇数则首尾相连, 偶数不连
 */
typedef vector<int> vi;
void get_even(int n, vi ans[]) {
    vi a(n);
    for (int i = 0; i < n; i++)
        a[i] = i + 1;
    for (int i = 1; i <= n / 2; i++) {
        ans[i].resize(n + 1);
        for (int j = 0; j < n / 2; j++)
            ans[i][j * 2] = a[j], ans[i][j * 2 + 1] = a[n - 1 - j];
        int t = a[n - 1];
        for (int j = n - 1; j > 0; j--)
            a[j] = a[j - 1];
        a[0] = t;
    }
}
void get_odd(int n, vi ans[]) {
    get_even(n - 1, ans);
    for (int i = 1; i <= n / 2; i++) {
        for (int j = n - 1; j > 0; j--)
            ans[i][j] = ans[i][j - 1] + 1;
        ans[i][0] = 1;
    }
}
int main() {
    vi ans[2019];
    int n; cin >> n;
    if (n & 1) get_odd(n, ans);
    else get_even(n, ans);
    return 0;
}

```

4.2 rec-free

```

/* 不存在四个 1 构成一个矩形, 并使得 1 尽量多, 输出 01 矩阵 */
const int N = 200, n = 150, M = 13; //M 为质数, N>M*M>n
int a[N][N], b[N][N], c[N][N];
void make() {
    for (int i = 1; i <= M; i++)
        for (int j = 1; j <= M; j++)
            a[i][j + 1] = M * (j - 1) + i;
    for (int i = 1; i <= M; i++)
        for (int j = 1; j <= M; j++)
            c[i][a[i][j]] = 1;
    for (int k = 1; k < M; k++) {
        memcpy(b, a, sizeof b);
        for (int i = 1; i <= M; i++)
            for (int j = 1; j <= M; j++)

```

```

        a[i][j] = (b[i + j - 1 - ((i + j - 1) > M ? M : 0)][j]);
    for (int i = 1; i <= M; i++)
        for (int j = 1; j <= M; j++)
            c[k * M + i][a[i][j]] = 1;
}
}

```

4.3 点边均整数多边形

/* 构造满足以下条件的简单多边形 (可以凹但边不能相交)

* 有 k 个点 k 条边, 所有边都为整数, 所有点的坐标都为整数

* 不存在与坐标轴平行的边

*/

```

typedef pair<int, int> P;
P operator * (P a, int b){return P(a.first * b, a.second * b);}
P operator + (P a, P b) {return P(a.first + b.first, a.second + b.second);}
int main() {
    int n; cin >> n;
    if (n == 3) return printf("0 0\n4 3\n-20 21\n"), 0;
    int m = n / 2 - 1;
    vector<P> v;
    v.push_back(P(0, 0));
    v.push_back(P(4, -3) * m);
    v.push_back(P(4, 0) * (2 * m));
    int lim = (n & 1) ? n - 1 : n;
    for (int i = 4; i <= lim; ++i) {
        P last = v.back();
        if ((i % 2) == 0) v.push_back(last + P(-4, 3));
        else v.push_back(last + P(-4, -3));
    }
    if (n & 1) v.push_back(P(-20, 48));
    for (P p: v) printf("%d %d\n", p.first, p.second);
}

```

5 计算几何

5.1 最小矩形覆盖含凸包和旋转卡壳

/* 最小矩形覆盖，保留六位小数，逆时针输出四个顶点坐标 */

```
namespace minRectCover {
    const int N = 1e5 + 5;
    const double eps = 1e-8;
    struct point{
        double x, y;
        point(){}
        point(double x, double y):x(x), y(y){}
        bool operator < (const point &a) const {
            return fabs(y - a.y) < eps ? x < a.x : y < a.y;}
        point operator - (const point &a) const {
            return point(x - a.x, y - a.y);}
        point operator + (const point &a) const {
            return point(x + a.x, y + a.y);}
        point operator / (const double &a) const {
            return point(x / a, y / a);}

        point operator * (const double &a) const {
            return point(x * a, y * a);}
        double operator / (const point &a) const { // .
            return x * a.x + y * a.y;}
        double operator * (const point &a) const { // X
            return x * a.y - y * a.x;}

    }p[N], q[N], rc[4];
    double sqr(double x) {return x * x;}
    double abs(point a) {return sqrt(a / a);}
    int sgn(double x) {return fabs(x) < eps ? 0 : (x < 0 ? -1 : 1);}
    point vertical(point a, point b) {//与 ab 向量垂直的向量
        return point(a.x + a.y - b.y, a.y - a.x + b.x) - a;}
    point vec(point a){return a / abs(a);}
    void convexhull(int n, point *hull, int &top) {//如果要计算周长需要特判 n==2
        for (int i = 1; i < n; i++)
            if (p[i] < p[0])
                swap(p[i], p[0]);
        sort(p + 1, p + n, [&](point a, point b){
            double t = (a - p[0]) * (b - p[0]);
            if (fabs(t) < eps) return sgn(abs(p[0] - a) - abs(p[0] - b)) < 0;
            return t > 0;
        });
        int cnt = 0; //去重
        for (int i = 1; i < n; i++)
            if (sgn(p[i].x - p[cnt].x) != 0 || sgn(p[i].y - p[cnt].y) != 0)
                p[++cnt] = p[i];
        n = cnt + 1;
        hull[top = 1] = p[0];
        for (int i = 1; i < n; i++) {
            while (top > 1 && (hull[top] - hull[top - 1])
                * (p[i] - hull[top]) < eps) top--;
            hull[++top] = p[i];
        }
    }
}
```

```

    }
    hull[0] = hull[top];
}
void main() {
    int n;
    scanf("%d", &n);
    for (int i = 0; i < n; i++)
        scanf("%lf %lf", &p[i].x, &p[i].y);
    convexhull(n, q, n);
    double ans = 1e20;
    int l = 1, r = 1, t = 1;
    double L, R, D, H;
    for (int i = 0; i < n; i++) {
        D = abs(q[i] - q[i + 1]);
        while (sgn((q[i + 1] - q[i]) * (q[t + 1] - q[i]) -
            (q[i + 1] - q[i]) * (q[t] - q[i])) > -1) t = (t + 1) % n;
        while (sgn((q[i + 1] - q[i]) / (q[r + 1] - q[i]) - (q[i + 1]
            - q[i]) / (q[r] - q[i])) > -1) r = (r + 1) % n;
        if (i == 0) l = r;
        while (sgn((q[i + 1] - q[i]) / (q[l + 1] - q[i]) - (q[i + 1]
            - q[i]) / (q[l] - q[i])) < 1) l = (l + 1) % n;
        L = fabs((q[i + 1] - q[i]) / (q[l] - q[i]) / D);
        R = fabs((q[i + 1] - q[i]) / (q[r] - q[i]) / D);
        H = fabs((q[i + 1] - q[i]) * (q[t] - q[i]) / D);
        double tmp = (R + L) * H;
        if (tmp < ans) {
            ans = tmp;
            rc[0] = q[i] + (q[i + 1] - q[i]) * (R / D); //右下
            rc[1] = rc[0] + vec(vertical(q[i], q[i + 1])) * H; //右上
            rc[2] = rc[1] - (rc[0] - q[i]) * ((R + L)
                / abs(q[i] - rc[0])); //左上
            rc[3] = rc[2] - (rc[1] - rc[0]);
        }
    }
    printf("%.6f\n", ans);
    int fir = 0;
    for (int i = 1; i < 4; i++)
        if (rc[i] < rc[fir])
            fir = i;
    for (int i = 0; i < 4; i++)
        printf("%.6f %.6f\n", rc[(fir + i) % 4].x, rc[(fir + i) % 4].y);
}
}

```

6 数论

6.1 BSGS

```

/*bsgs 简单原理: 求  $y^x \equiv z \pmod{p}$ 
 * 令  $x=km+b$ ,  $m$  取  $\sqrt{p}$ 
 * 先把  $b$  的  $m$  个取值存起来, 然后枚举  $k$  即可
 * 拓展  $\gcd(y, p) \neq 1$ 
 * 先找到最小的  $k$  使得  $\gcd(y^k, p) = \gcd(y^{k+1}, p)$ 
 * 然后约去这个  $\gcd$  就得到互质了, 最后再加上  $k$  就行
 */
//限制  $\gcd(y, p) = 1$ 
ll bsgs(ll y, ll z, ll p){ //  $y^x \equiv z \pmod{p}$ 
    static map<ll, int> mp; //可替换为 hash
    z %= p; mp.clear();
    if (z == 1) return 0;
    ll m = sqrt(p) + 1, s = z;
    for (int i = 0; i < m; i++) {
        mp[s] = i;
        s = s * y % p;
    }
    ll yy = qpow(y, m, p); s = 1;
    for (int i = 1; i <= m + 1; i++) {
        s = s * yy % p;
        if (mp.find(s) != mp.end())
            return i * m - mp[s];
    }
    return -1;
}
//拓展 bsgs, 不再限制  $\gcd(y, p) = 1$ 
ll exbsgs(ll y, ll z, ll p){
    static map<ll, int> mp; //可替换为 hash
    z %= p; mp.clear();
    if (z == 1) return 0;
    ll cnt = 0, t = 1;
    for (ll d = __gcd(y, p); d != 1; d = __gcd(y, p)){
        if (z % d) return -1;
        cnt++, z /= d, p /= d, t = 1LL * t * y / d % p;
        if (z == t) return cnt;
    }
    ll s = z, m = sqrt(p) + 1;
    for (int i = 0; i < m; i++){
        mp[s] = i;
        s = s * y % p;
    }
    ll x = qpow(y, m, p); s = t;
    for (int i = 1; i <= m; i++)
        if (mp.count(s = s * x % p))
            return i * m - mp[s] + cnt;
    return -1;
}

```

6.2 CRT

```

void exgcd(ll a, ll b, ll &d, ll &x, ll &y) {
    if (!b) {
        d = a, x = 1, y = 0;
        return;
    }
    exgcd(b, a % b, d, y, x);
    y -= a / b * x;
}

ll crt(ll *m, ll *a, int n) { // n 个式子:  $y = mx + a$ , 下标从 1 开始
    ll A = a[1], M = m[1], d, x, y, m2;
    for (int i = 2; i <= n; i++) { //  $k_1 * m_1 - k_2 * m_2 = a_2 - a_1$ 
        exgcd(M, m[i], d, x, y);
        if ((a[i] - A) % d) return -1;
        m2 = M / d * m[i];
        x = (a[i] - A) / d * x % m[i];
        A = (A + x * M % m2) % m2;
        if (A < 0) A += m2; // 保证  $A \geq 0$ 
        M = m2;
    }
    return A; //  $y = Mx + A$ 
}

```

6.3 蔡勒公式

```

/* 0-6 对应周日-周六 */
int calc(int yy, int m, int d) {
    if (m < 3) m += 12, yy--;
    int c = yy / 100, y = yy % 100;
    int w = (c / 4 - c * 2 + y + y / 4 + (13 * (m + 1)) / 5 + d - 1) % 7;
    return (w + 7) % 7;
}

```

6.4 素数判定 + 大整数质因数分解

```

/* miller_rabin 可以判定 ll 以内数字是否为素数
 * 时间复杂度  $O(T \log n)$ , 错误率  $(1/4)^T$ ,  $T$  是测试组数
 *
 * pollard_rho 算法,  $O(n^{1/4})$  实现大整数的质因数分解
 */
namespace PollardRho {
    const int T = 20; // 测试次数
    ll qmul(ll a, ll b, ll p) {
        ll c = 0;
        for (a %= p; b > 0; b >= 1) {
            if (b & 1) c += a;
            if (c >= p) c -= p;
            a <<= 1;
            if (a >= p) a -= p;
        }
        return c;
    }
    ll qpow(ll x, ll k, ll p) {
        ll res = 1;

```



```

    for (x %= p; k > 0; k >>= 1, x = qmul(x, x, p))
        if (k & 1) res = qmul(res, x, p);
    return res;
}
bool check(ll a, ll n, ll x, ll t) {
    ll res = qpow(a, x, n), last = res;
    for (int i = 1; i <= t; i++) {
        res = qmul(res, res, n);
        if (res == 1 && last != 1 && last != n - 1) return 1;
        last = res;
    }
    if (res != 1) return 1;
    return 0;
}
//素数判定函数 (ret = 0) -> prime
bool millerRabin(ll n) {
    if (n < 2) return 1;
    ll x = n - 1, t = 0;
    while (!(x & 1)) x >>= 1, t++;
    bool flag = 1;
    if (t >= 1 && (x & 1)) {
        for (int k = 0; k < T; k++) {
            ll a = rand() % (n - 1) + 1;
            if (check(a, n, x, t)) {
                flag = 1;
                break;
            }
        }
        flag = 0;
    }
    if (!flag || n == 2) return 0;
    return 1;
}
ll pollardRho(ll x, ll c) {
    ll i = 1, x0 = rand() % x, y = x0, k = 2;
    while (1) {
        i++;
        x0 = qmul(x0, x0, x) + c % x;
        ll d = abs(__gcd(y - x0, x));
        if (d != 1 && d != x) return d;
        if (y == x0) return x;
        if (i == k) y = x0, k <<= 1;
    }
}
void findFac(ll n, ll *f) {
    if (!millerRabin(n)) {
        f[++f[0]] = n;
        return;
    }
    ll p = n;
    while (p >= n) p = pollardRho(p, rand() % (n - 1) + 1);
    findFac(p, f), findFac(n / p, f);
}
//质因数分解函数, 因子放在 f 数组, 有重复且无序

```

```
void getFac(ll n, ll *f) {
    f[0] = 0;
    if (n <= 1) return;
    findFac(n, f);
}
}
int main() {
    srand(time(NULL));
}
```

7 字符串

7.1 KMP

```
int nxt[N];
void kmp(int n, char *a, int m, char *b) {
    //长度为 m 的 b 中找 a, 下标从 0 开始, 得到的是匹配成功的末尾位置
    static int i, j, cnt, tmp[2]; cnt = 0;
    for (nxt[0] = j = -1, i = 1; i < n; nxt[i++] = j) {
        while (~j && a[j + 1] != a[i]) j = nxt[j];
        if (a[j + 1] == a[i]) j++;
    }
    for (j = -1, i = 0; i < m; i++) {
        while (~j && a[j + 1] != b[i]) j = nxt[j];
        if (a[j + 1] == b[i]) j++;
        if (j == n - 1) {
            printf("%d ", i);
            j = nxt[j];
        }
    }
}
```

8 其他

8.1 十进制快速幂

// $x_i = x_{i-1} * a + x_{i-2} * b$, 求 x_n , n 贼大那种

```
const int K = 2;
ll mod;
struct matrix {
    ll c[K][K];
    void clear() {memset (c, 0, sizeof c);}
    matrix operator * (const matrix&a) const {
        matrix res; res.clear();
        for (int k = 0; k < K; k++)
            for (int i = 0; i < K; i++)
                for (int j = 0; j < K; j++)
                    res.c[i][j] = (res.c[i][j]
                        + c[i][k] * a.c[k][j] % mod) % mod;
        return res;
    }

    matrix operator ^ (const char *s) const {
        matrix res, x = *this, y = x; res.clear();
        for (int i = 0; i < K; i++) res.c[i][i] = 1;
        int len = strlen(s + 1);
        for (int i = len; i >= 1; i--) {
            for (int j = 1; j < 10; j++) {
                if (s[i] == '0' + j) res = res * y;
                y = y * x;
            }
            x = y;
        } return res;
    }
}a, b;
char s[N];
int main(){
    scanf("%lld %lld %lld %lld %s %lld",
        &a.c[0][0], &a.c[0][1],
        &b.c[1][1], &b.c[0][1],
        s + 1, &mod);
    b.c[1][0] = 1;
    printf("%lld\n", (a * (b ^ s)).c[0][0]);
    return 0;
}
```

8.2 数字哈希

```
namespace my_hash {
    const int N = (1 << 19) - 1; //散列大小, 一定要取  $2^k - 1$ , 不超内存的情况下, N越大碰撞
    ↪ 越少
    struct E {
        int v;
        E *nxt;
    }*g[N + 1], pool[N], *cur = pool, *p;
    int vis[N + 1], T;
```

```

void ins(int v) {
    int u = v & N;
    if (vis[u] < T) vis[u] = T, g[u] = NULL;
    for (p = g[u]; p; p = p -> nxt) if (p -> v == v) return;
    p = cur ++; p -> v = v; p -> nxt = g[u]; g[u] = p;
}
int ask(int v) {
    int u = v & N;
    if (vis[u] < T) return 0;
    for (p = g[u]; p; p = p -> nxt) if (p -> v == v) return 1;
    return 0;
}
void init() {T ++, cur = pool;}//应对多组数据使用
}

```

8.3 海岛分金币

8.3.1 海岛分金币 1

```

/*
非朴素模型，有额外条件：
每个人做决定时如果有多种方案可以使自己获得最大收益
那么他会让决策顺序靠前的人获得的收益尽可能的大！
solution:
贪心模拟
*/
#define v first
#define id second
typedef pair<int, int> pr;
const int N = 1010;
int a[N][N];
pr b[N];
int n, m;
int main() {
    cin >> n >> m;
    a[1][1] = m;
    for (int i = 2; i <= n; i++) {
        for (int j = 1; j < i; j++)
            b[j] = pr(a[i - 1][j], j);
        sort(b + 1, b + i, [&](pr x, pr y){return x.v != y.v ? (x.v < y.v) : (x.id >
            y.id);});
        //按照是否容易满足来排序，因为容易满足的人消耗掉的金币比较少，也就使得当前的人获利最大
        int s = m, nd = (i - 1) / 2;
        for (int j = 1; j < i && nd; j++) {
            nd--;
            s -= (a[i][b[j].id] = a[i - 1][b[j].id] + 1);
        }
        if (s < 0) {
            for (int j = 1; j < i; j++)
                a[i][j] = a[i - 1][j];
            a[i][i] = -1;
        }
        else {
            a[i][i] = s;
        }
    }
}

```

```

    }
}
for (int i = n; i; i --)
    printf("%d ", a[n][i]);
return 0;
}

```

8.3.2 海岛分金币 2

/*

海盗分金币朴素模型:

n 个海盗分 m 个金币, 依次做决策, 如果不少于半数的人同意则方案通过, 否则当前做决策的人会被淘汰
 \hookrightarrow (收益视为 -1), 由下一人做出决策

如果一个海盗有多种方案均为最大收益, 那么他会希望淘汰的人越多越好

求出第 x 个做决策的海盗的最大可能受益和最小可能收益

*/

```

struct node {
    int min_v, max_v;
    node():min_v(0), max_v(0) {}
    node(int min_v, int max_v):min_v(min_v), max_v(max_v) {}
};

node ask(int n, int m, int x) { // n 个人分 m 个金币, 第 x 个做决策的人最少/最多分到多少个金币
    int y = n + 1 - x;
    if (n >= (m + 2) * 2) {
        int a = (m + 1) * 2, b = 2, c = 4;
        // 前 a 个为 [0, 1], 后 b 个为 [0, 0], 将持续 c 个
        while (a + b + c <= n) {
            a += b;
            b *= 2;
            c *= 2;
        }
        if (y <= a) return node(0, 1);
        else if (y <= a + b) return node(0, 0);
        else return node(-1, -1);
    }
    else if (n == m * 2 + 3) {
        if (x == 1) return node(-1, -1);
        else if (y <= m * 2 && y % 2 == 1 || x == 2) return node(0, 0);
        else return node(0, 1);
    }
    else if (n == m * 2 + 2) {
        if (y <= m * 2 && y % 2 == 1 || x == 1) return node(0, 0);
        else return node(0, 1);
    }
    else if (n == m * 2 + 1) {
        if (y <= m * 2 && y % 2 == 1) return node(1, 1);
        else return node(0, 0);
    }
    else {
        if (x & 1) {
            if (x != 1) return node(1, 1);
            else return node(m - (n - 1) / 2, m - (n - 1) / 2);
        }
        else return node(0, 0);
    }
}

```

```

    }
}

int main() {
    ios::sync_with_stdio(false);
    int x, n, m, k; node y;
    cin >> n >> m >> k;
    while (k --) {
        cin >> x;
        y = ask(n, m, x);
        printf("%d %d\n", y.min_v, y.max_v);
    }
    return 0;
}

/*
m = 5

1 5
2 0 5
3 1 0 4
4 0 1 0 4
5 1 0 1 0 3
6 0 1 0 1 0 3
7 1 0 1 0 1 0 2
8 0 1 0 1 0 1 0 2
9 1 0 1 0 1 0 1 0 1
10 0 1 0 1 0 1 0 1 0 1
11 1 0 1 0 1 0 1 0 1 0 0
12 0 _ 0 _ 0 _ 0 _ 0 _ 0
13 0 _ 0 _ 0 _ 0 _ 0 _ 0 -1
14 _ _ _ _ _ _ _ _ _ 0 0
15 _ _ _ _ _ _ _ _ _ 0 0 -1
16 _ _ _ _ _ _ _ _ _ 0 0 -1 -1
17 _ _ _ _ _ _ _ _ _ 0 0 -1 -1 -1
18 _ _ _ _ _ _ _ _ _ 0 0 0 0
19 _ _ _ _ _ _ _ _ _ 0 0 0 0 -1
20 _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1
21 _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1 -1
22 _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1 -1 -1
23 _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1 -1 -1 -1
24 _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1 -1 -1 -1 -1
25 _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1 -1 -1 -1 -1 -1
26 _ _ _ _ _ _ _ _ _ 0 0 0 0 0 0 0 0 0
*/

```

8.4 根号枚举

```

for (int i = 1, last; i <= n; i = last + 1) {
    last = n / (n / i);
    //当前枚举区间为 [i, last]
}

```

8.5 读入输出外挂

```

namespace IO {//only for int!!!
    static const int SIZE = 1 << 20;
    inline int get_char() {
        static char *S, *T = S, buf[SIZE];
        if (S == T) {
            T = fread(buf, 1, SIZE, stdin) + (S = buf);
            if (S == T) return -1;
        }
        return *S ++;
    }
    inline void in(int &x) {//for int
        static int ch;
        while (ch = get_char(), ch < 48); x = ch ^ 48;
        while (ch = get_char(), ch > 47) x = x * 10 + (ch ^ 48);
    }
    char buffer[SIZE];
    char *s = buffer;
    void flush() {//最后需要 flush!!
        fwrite(buffer, 1, s - buffer, stdout);
        s = buffer;
        fflush(stdout);
    }
    inline void print(const char ch) {
        if(s - buffer > SIZE - 2) flush();
        *s++ = ch;
    }
    inline void print(char *str) {//for string
        while(*str != 0)
            print(char(*str ++));
    }
    inline void print(int x) {
        static char buf[25];
        static char *p = buf;
        if (x < 0) print('-'), x = -x;
        if (x == 0) print('0');
        while(x) *(++ p) = x % 10, x /= 10;
        while(p != buf) print(char(*(p --) ^ 48));
    }
};

```

8.6 给定小数化成分数

本题答案的分母不超过 $1e9$, 给定小数的小数点位为 18 位
 # 单次 $O(\log 2n)$

```

inf, inff = 10 ** 9, 10 ** 18
for i in range(int(input())):
    n = int(input()[2:])
    if n == 0: print('0 1')
    else:
        lp, lq, rp, rq = 0, 1, 1, 1
        while max(lq, rq) <= inf:
            mp, mq = lp + rp, lq + rq
            if mp * inff <= mq * n:

```



```
l, r, mid, cnt = 1, (inf - lq) // rq + 1, -1, -1
while l <= r:
    mid = l + r >> 1
    if (lp + rp * mid) * inff <= (lq + rq * mid) * n:
        cnt, l = mid, mid + 1
    else:
        r = mid - 1
lp, lq = lp + rp * cnt, lq + rq * cnt
else:
    l, r, mid, cnt = 1, (inf - rq) // lq + 1, -1, -1
    while l <= r:
        mid = l + r >> 1
        if (rp + lp * mid) * inff > (rq + lq * mid) * n:
            cnt, l = mid, mid + 1
        else:
            r = mid - 1
    rp, rq = rp + lp * cnt, rq + lq * cnt
if lq <= inf: print(lp, lq)
else: print(rp, rq)
```