Algorithm Template Library

ytz123

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Contents

1 图论

1.1 线段树维护树直径

```
/*LCA 用 ST 表，总复杂度 O(nlog)*/
/* 每次询问删去两条边后，剩下 3 棵树的最大直径长度 */
const int N = 2e5 + 5;
int T, n, m;
int len, head[N], ST[20][N];
struct edge{int u, v, w;}ee[N];
int cnt, fa[N], log_2[N], st[N], en[N], dfn[N], dis[N], dep[N], pos[N];
struct edges{int to, next, cost;}e[N];
void add(int u, int v, int w) {
    e[++ len] = (edges){v, head[u], w}, head[u] = len;
    e[++ len] = (edges){u, head[v], w}, head[v] = len;
}
void dfs1(int u) {
    st[u] = ++ cnt, dfn[cnt] = u;
    for (int v, i = head[u]; i; i = e[i].next) {
        v = e[i].to;
        if (v == fa[u]) continue;
        fa[v] = u, dep[v] = dep[u] + 1;
        dis[v] = dis[u] + e[i].cost, dfs1(v);
    }
    en[u] = cnt;
}
void dfs2(int u) {
    dfn[++ cnt] = u, pos[u] = cnt;
    for (int v, i = head[u]; i; i = e[i].next) {
        v = e[i].to;
        if (v == fa[u]) continue;
        dfs2(v), dfn[++ cnt] = u;
    }
}
int mmin(int x, int y) {
    if (dep[x] < dep[y]) return x;
    return y;
}
int lca(int u, int v) {
    static int w;
    if (pos[u] > pos[v]) swap(u, v);
    w = log_2[pos[v] - pos[u] + 1];
    return mmin(ST[w][pos[u]], ST[w][pos[v] - (1 << w) + 1]);
}
int dist(int u, int v) {
    int Lca = lca(u, v);
    return dis[u] + dis[v] - dis[Lca] * 2;
}
void build() {
    for (int i = 1; i <= cnt; i ++)
        ST[0][i] = dfn[i];
    for (int i = 1; i < 20; i ++)
        for (int j = 1; j <= cnt; j ++)
            if (j + (1 << (i - 1)) > cnt) ST[i][j] = ST[i - 1][j];
```

```
            else ST[i][j] = mmin(ST[i - 1][j], ST[i - 1][j + (1 << (i - 1))]);
}
int M;
struct node {
    int l, r, dis;
}tr[N << 1];
void update(int o, int o1, int o2) {
    static int d; static node tmp;
    if (tr[o1].dis == -1) {tr[o] = tr[o2]; return;}
    if (tr[o2].dis == -1) {tr[o] = tr[o1]; return;}
    if (tr[o1].dis > tr[o2].dis) tmp = tr[o1];
    else tmp = tr[o2];
    d = dist(tr[o1].l, tr[o2].l);
    if (d > tmp.dis) tmp.l = tr[o1].l, tmp.r = tr[o2].l, tmp.dis = d;
    d = dist(tr[o1].l, tr[o2].r);
    if (d > tmp.dis) tmp.l = tr[o1].l, tmp.r = tr[o2].r, tmp.dis = d;
    d = dist(tr[o1].r, tr[o2].l);
    if (d > tmp.dis) tmp.l = tr[o1].r, tmp.r = tr[o2].l, tmp.dis = d;
    d = dist(tr[o1].r, tr[o2].r);
    if (d > tmp.dis) tmp.l = tr[o1].r, tmp.r = tr[o2].r, tmp.dis = d;
    tr[o] = tmp;
}
void ask(int s, int t) {
    if (s > t) return;
    for (s += M - 1, t += M + 1; s ^ t ^ 1; s >>= 1, t >>= 1) {
        if (~s&1) update(0, 0, s ^ 1);
        if ( t&1) update(0, 0, t ^ 1);
    }
}
int main() {
    ios::sync_with_stdio(false);
    int u, v, w, ans; log_2[1] = 0;
    for (int i = 2; i <= 200000; i ++)
        if (i == 1 << (log_2[i - 1] + 1))
            log_2[i] = log_2[i - 1] + 1;
        else log_2[i] = log_2[i - 1];
    for (cin >> T; T --; ) {
        cin >> n >> m, cnt = len = 0;
        for (int i = 1; i <= n; i ++)
            head[i] = 0;
        for (int i = 1; i < n; i ++) {
            cin >> ee[i].u >> ee[i].v >> ee[i].w;
            add(ee[i].u, ee[i].v, ee[i].w);
        }
        dfs1(1);
        for (M = 1; M < n + 2; M <<= 1);
        for (int i = 1; i <= n; i ++)
            tr[i + M].l = tr[i + M].r = dfn[i], tr[i + M].dis = 0;
        for (int i = n + M + 1; i <= (M << 1) + 1; i ++)
            tr[i].dis = -1;
        cnt = 0, dfs2(1), build();
        for (int i = M; i; i --)
            update(i, i << 1, i << 1 | 1);
        for (int i = 1; i < n; i ++)
```

```
            if (dep[ee[i].u] > dep[ee[i].v])
                swap(ee[i].u, ee[i].v);
        for (int u, v, i = 1; i <= m; i ++) {
            cin >> u >> v, ans = 0;
            u = ee[u].v, v = ee[v].v, w = lca(u, v);
            if (w == u || w == v) {
                if (w != u) swap(u, v);
                tr[0].dis = -1, ask(1, st[u] - 1), ask(en[u] + 1, n), ans = max(ans,
                ↪  tr[0].dis);
                tr[0].dis = -1, ask(st[u], st[v] - 1), ask(en[v] + 1, en[u]), ans =
                ↪  max(ans, tr[0].dis);
                tr[0].dis = -1, ask(st[v], en[v]), ans = max(ans, tr[0].dis);
            }
            else {
                if (st[u] > st[v]) swap(u, v);
                tr[0].dis = -1, ask(1, st[u] - 1), ask(en[u] + 1, st[v] - 1), ask(en[v] +
                ↪  1, n), ans = max(ans, tr[0].dis);
                tr[0].dis = -1, ask(st[u], en[u]), ans = max(ans, tr[0].dis);
                tr[0].dis = -1, ask(st[v], en[v]), ans = max(ans, tr[0].dis);
            }
            printf("%d\n", ans);
        }
    }
    return 0;
}
```

1.2 有向图判断两个点能否到达

```
/* 原题:n 个点的有向图, k 对点 (u,v) 满足 u 可达 v
 * p 对点 (u,v) 满足 u 不可达 v, 问这个图是否存在
 * 解法: 按照 p 对可达点直接连边构图, 然后考虑验证
 * a[i][j]=0/1 表示 i 是否可达 j, 我们对 j 分块, bitset 加速
 * 时间 O(n*m/32) 空间 O(n*n/blk) blk 随便取
 */
const int N = 1e5 + 2;
const int BLK = 5000;
int n, k, p;
int d[N];
vector <int> e[N], f[N], ck[N];
int top, sta[N], in[N];
int cnt, dfn[N], low[N], vis[N];
int sum, bel[N];
bitset <BLK> a[N];
queue <int> q;
int topo[N];
void tarjan(int u) {
        vis[u] = in[u] = 1;
        sta[++ top] = u, dfn[u] = low[u] = ++ cnt;
        for (int v : e[u])
                if (!vis[v]) {
                        tarjan(v);
                        low[u] = min(low[v], low[u]);
                }
                else if (in[v])
```

```
                        low[u] = min(low[v], low[u]);
        if (low[u] == dfn[u]) {
                sum ++; int i;
                while (1) {
                        i = sta[top --];
                        in[i] = 0, bel[i] = sum;
                        if (i == u) break;
                }
        }
}
int main() {
        cin >> n >> k;
        for (int u, v, i = 1; i <= k; i ++) {
                cin >> u >> v;
                e[u].push_back(v);
        }
        cin >> p;
        for (int u, v, i = 1; i <= p; i ++) {
                cin >> u >> v;
                f[u].push_back(v);
        }
        for (int i = 1; i <= n; i ++)
                if (!vis[i])
                        tarjan(i);
        for (int i = 1; i <= n; i ++) {
                for (int j : f[i]) {
                        if (bel[i] == bel[j]) return puts("NO"), 0;
                        ck[bel[i]].push_back(bel[j]);//check
                }
                f[i].clear();
        }
        for (int i = 1; i <= n; i ++)
                for (int j : e[i]) {
                        if (bel[i] == bel[j]) continue;
                        f[bel[i]].push_back(bel[j]);
                        d[bel[j]] ++;
                }
        cnt = 0;
        for (int i = 1; i <= sum; i ++)
                if (!d[i])
                        q.push(i);
        while (!q.empty()) {
                int now = q.front(); q.pop();
                topo[++ cnt] = now;
                for (int j : f[now]) {
                        d[j] --;
                        if (d[j] == 0) q.push(j);
                }
        }
        for (int i = 1, t = (sum + BLK - 1) / BLK; i <= t; i ++) {
                for (int j = sum; j; j --) {
                        int u = topo[j];
                        a[u].reset();
                        if (BLK * (i - 1) < u && u <= BLK * i)
```

```
                                a[u][u - BLK * (i - 1) - 1] = 1;
                        for (int v : f[u])
                                a[u] |= a[v];
                }
                for (int j = 1; j <= sum; j ++)
                        for (int v : ck[j])
                                if (BLK * (i - 1) < v && v <= BLK * i &&
                                        a[j][v -  BLK * (i - 1) - 1] == 1) {
                                        puts("NO");
                                        return 0;
                                }
        }
        printf("YES\n%d\n", k);
        for (int i = 1; i <= n; i ++)
                for (int j : e[i])
                        printf("%d %d\n", i, j);
        return 0;
}
```

1.3 spfa 费用流

```
const int N = 600, M = 800000, inf = 0x3f3f3f3f;
int s, t, ans, len, maxflow;
int T, n, m, K, W;
int head[N], incf[N], path[N], pre[N], vis[N], d[N];
struct edge{int to, next, cap, cost;}e[M];
struct video {int s, t, w, op; }a[N];
void add(int u, int v, int w, int c) {
        e[++ len] = (edge){v, head[u], w, c}, head[u] = len;
        e[++ len] = (edge){u, head[v], 0, -c}, head[v] = len;
}
bool spfa() {
        deque <int> q;
        q.push_back(s), incf[s] = inf;
        for (int i = 1; i <= t; i ++) d[i] = inf;
        d[s] = 0;
        while (!q.empty()) {
                int x = q.front();
                q.pop_front(), vis[x] = 0;
                for (int i = head[x]; i; i = e[i].next) {
                        if (e[i].cap && d[e[i].to] > d[x] + e[i].cost) {
                                d[e[i].to] = d[x] + e[i].cost;
                                pre[e[i].to] = x, path[e[i].to] = i;
                                incf[e[i].to] = min(incf[x], e[i].cap);
                                if (!vis[e[i].to]) {
                                        vis[e[i].to] = 1;
                                        if (q.empty() || d[e[i].to] < d[q.front()])
                                        ↪  q.push_front(e[i].to);
                                        else q.push_back(e[i].to);
                                }
                        }
                }
        }
        maxflow += incf[t];
```

```
        if (d[t] == inf) return 0;
        for (int i = t; i != s; i = pre[i]) {
                e[path[i]].cap -= incf[t];
                e[path[i] ^ 1].cap += incf[t];
        }
        return ans += incf[t] * d[t], 1;
}
int main() {
        /*build graph*/
        while(spfa());
}
```

1.4 dsu

```
/*DSU
 * 用途:O(nlogn) 解决无修改的子树询问问题，需要保证操作支持删除
 * 解决方法：对于每个节点，先对所有轻儿子，dfs 下去求一遍，再消除影响
 *         然后再 dfs 自己的重儿子，然后不消除影响，再加上所有轻儿子
 *         就得到当前节点为根的子树的答案了
 */
int n, c[N];
int cnt[N], maxCnt;
int siz[N], son[N];
vector <int> e[N];
ll ans[N], sum[N];
void dfs1(int u, int fr) {
        siz[u] = 1;
        for (int v : e[u]) {
                if (v == fr) continue;
                dfs1(v, u);
                siz[u] += siz[v];
                if (siz[v] > siz[son[u]]) son[u] = v;
        }
}
void update(int x, int y) {
        sum[cnt[x]] -= x;
        cnt[x] += y;
        sum[cnt[x]] += x;
        if (cnt[x] > maxCnt) maxCnt = cnt[x];
        if (sum[maxCnt] == 0) maxCnt --;
}
void dfs3(int u, int fr, int val) {
        update(c[u], val);
        for (int v : e[u]) {
                if (v == fr) continue;
                dfs3(v, u, val);
        }
}
void dfs2(int u, int fr) {
        for (int v : e[u]) {
                if (v == fr || v == son[u]) continue;
                dfs2(v, u), dfs3(v, u, -1);
        }
        if (son[u]) dfs2(son[u], u);
```

```
        for (int v : e[u]) {
                if (v == fr || v == son[u]) continue;
                dfs3(v, u, 1);
        }
        update(c[u], 1);
        ans[u] = sum[maxCnt];
}
int main() {
        ios::sync_with_stdio(false);
        cin >> n;
        for (int i = 1; i <= n; i ++)
                cin >> c[i];
        for (int u, v, i = 1; i < n; i ++) {
                cin >> u >> v;
                e[u].push_back(v);
                e[v].push_back(u);
        }
        dfs1(1, 1), dfs2(1, 1);
        for (int i = 1; i <= n; i ++)
                cout << ans[i] << ' ';
        return 0;
}
```

1.5 长链剖分

```
/* 长链剖分，选择深度最大的儿子作为重儿子，用于合并以深度为下标的信息
 * 像 dsu 一样，直接继承重儿子信息，然后按深度暴力合并其他儿子信息
 * 时间复杂度考虑每个节点作为轻儿子里的节点被合并只会有一次，所以 O(n)
 * 另一种用法，可以 O(nlogn) 预处理后,O(1) 找到 k 级祖先
 * example problem: 给个树，第 i 个点有两个权值 ai 和 bi
 * 现在求一条长度为 m 的路径，使得 Σai/Σbi 最小
 * 防爆栈 trick: 像重链剖分一样改成 bfs
 */
int n, m;
double k, a[N], b[N];
int len[N], son[N];
vector <int> e[N];
double tmp[N], *ptr, *f[N], temp[N];
void dfs(int u, int fr) {
    for (int v : e[u]) {
        if (v == fr) continue;
        dfs(v, u);
        if (len[v] > len[son[u]]) son[u] = v;
    }
    len[u] = len[son[u]] + 1;
}
inline double F(int x, int y) {return y >= len[x] ? 0 : f[x][y];}
bool solve(int u, int fr) {
    /* 为实现 O(1) 继承，采用 f[u]-f[v] 来保存 u-fa[v] 路径上的最小权值和 (dep[v]>dep[u])
     * 即自底向上累加
     */
    if (son[u]) {
        f[son[u]] = f[u] + 1;
        if (solve(son[u], u)) return 1;
```

```
        f[u][0] = val[u] + f[u][1];
        if (len[u] >= m && f[u][0] - F(u, m) <= 0) return 1;
        for (int v : e[u]) {
            if (v == son[u] || v == fr) continue;
            f[v] = ptr, ptr += len[v];
            if (solve(v, u)) return 1;
            for (int j = 1; j <= len[v] && j <= m; j ++) {
                if (len[u] + j < m) continue;
                if (f[v][0] - F(v, j) + f[u][0] - F(u, m - j) <= 0) return 1;
            }
            temp[0] = val[u];
            for (int j = 1; j <= len[v]; j ++)
                temp[j] = val[u] + min(f[u][1] - F(u, j + 1), f[v][0] - F(v, j));
            if (len[v] + 1 == len[u]) f[u][0] = temp[len[v]];
            for (int j = 1; j <= len[v]; j ++)
                f[u][j] = f[u][0] - temp[j - 1];
            if (len[v] + 1 != len[u]) f[u][len[v] + 1] = f[u][0] - temp[len[v]];
        }
    }
    else {
        f[u][0] = val[u];
        if (m == 1 && f[u][0] <= 0) return 1;
    }
    return 0;
}
bool judge(double mid) {
    f[1] = ptr = tmp, ptr += len[1], k = mid;
    return solve(1, 1);
}
int main() {
    cin >> n >> m;
    for (int i = 1; i <= n; i ++) cin >> a[i];
    for (int i = 1; i <= n; i ++) cin >> b[i];
    if (m == -1) {
        double ans = 1e9;
        for (int i = 1; i <= n; i ++)
            ans = min(ans, a[i] / b[i]);
        printf("%.2f\n", ans);
        return 0;
    }
    for (int u, v, i = 1; i < n; i ++) {
        cin >> u >> v;
        e[u].push_back(v);
        e[v].push_back(u);
    }
    dfs(1, 1);
    int flag = 0;
    double l = 0, r = 2e5, mid, ans;
    for (int i = 0; i < 50; i ++) {
        mid = (l + r) / 2;
        if (judge(mid)) r = mid - eps, flag = 1, ans = mid;
        else l = mid + eps;
    }
    if (flag) printf("%.2f\n", ans);
```

```
    else puts("-1");
    return 0;
}
```

1.6 LCT

```
const int N = 3e5 + 5;
int n, m;
int fa[N], ch[N][2], rev[N], val[N], sum[N];
int sta[N], top;
bool isroot(int x) {
    return ch[fa[x]][0] != x && ch[fa[x]][1] != x;
}
void reverse(int x) {
    if (!x) return;
    swap(ch[x][0], ch[x][1]);
    rev[x] ^= 1;
}
void pushdown(int x) {
    if (!rev[x]) return;
    reverse(ch[x][0]);
    reverse(ch[x][1]);
    rev[x] = 0;
}
void pushup(int x) {
    sum[x] = sum[ch[x][0]] ^ sum[ch[x][1]] ^ val[x];
}
void rot(int x) {
    int y = fa[x], z = fa[y], d = ch[y][1] == x, c = ch[x][!d];
    fa[x] = z; if (!isroot(y)) ch[z][ch[z][1] == y] = x;
    ch[y][d] = c; if (c) fa[c] = y;
    fa[y] = x, ch[x][!d] = y;
    pushup(y), pushup(x);
}
void splay(int x) {
    int u = x, top = 0, y, z;
    while (!isroot(u)) sta[++ top] = u, u = fa[u];
    sta[++ top] = u;
    while (top) pushdown(sta[top --]);
    while (!isroot(x)) {
        y = fa[x], z = fa[y];
        if (!isroot(y)) {
            if ((ch[z][0] == y) ^ (ch[y][0] == x)) rot(x);
            else rot(y);
        }
        rot(x);
    }
}
void access(int x) {//把 x 到根的路径拎出来
    for (int y = 0; x != 0; y = x, x = fa[x]) {
        splay(x), ch[x][1] = y, pushup(x);
    }
}
void makeroot(int x) {//令 x 成为这棵树的根
```

```
    access(x), splay(x), reverse(x);
}
int findroot(int x) {//找根
    access(x), splay(x);
    while (ch[x][0]) pushdown(x), x = ch[x][0];
    splay(x);//把根转到顶保证复杂度
    return x;
}
void split(int x, int y) {//拉出 x-y 的路径
    makeroot(x);
    access(y), splay(y);
    //y 存了这条路径的信息
}
void link(int x, int y) {
    makeroot(x);
    if (findroot(y) != x) fa[x] = y;
}
void cut(int x, int y) {
    makeroot(x);
    if (findroot(y) == x && fa[y] == x && ch[x][1] == y) {
        fa[y] = ch[x][1] = 0;
        pushup(x);
    }
}
int main() {
    ios::sync_with_stdio(false);
    cin >> n >> m;
    for (int i = 1; i <= n; i ++)
        cin >> val[i];
    for (int op, x, y; m --; ) {
        cin >> op >> x >> y;
        switch(op) {
            case 0:split(x, y);printf("%d\n", sum[y]);break;
            case 1:link(x, y);break;
            case 2:cut(x, y);break;
            case 3:splay(x);val[x] = y;pushup(x);break;
            //单点更新完记得 pushup
        }
    }
}
```

1.7 hungary

```
/* 二分图最大匹配，时间复杂度 O(nm)
 * 例题:n1 个男，n2 个女，m 个配对关系
 * 输出最大配对数，然后对于每个男输出配对的女
 */
int n1, n2, m;
vector <int> e[N];
int vis[N], pre[N], ans, tim;
bool dfs(int u) {
        if (vis[u] == tim) return 0;
        vis[u] = tim;
        for (int v : e[u])
```

```
                if (!pre[v] || dfs(pre[v]))
                        return pre[v] = u, 1;
        return 0;
}
int main() {
        scanf("%d %d %d", &n1, &n2, &m);
        for (int u, v, i = 1; i <= m; i ++) {
                scanf("%d %d", &u, &v);
                e[v].push_back(u);
                // 要输出左侧点连接的右侧点，连边时就由右边点向左边连边
                // 连边 (u->v)，输出的 pre[v] 就是右边的点了
        }
        for (tim = 1; tim <= n2; tim ++)
                if (dfs(tim)) ans ++;
        printf("%d\n", ans);
        for (int i = 1; i <= n1; i ++)
                printf("%d%c", pre[i], i == n1 ? '\n' : ' ');
        return 0;
}
```

1.8 dinic 最大流

```
const int N = 20000;
const int M = 500000;
const int inf = 0x3f3f3f3f;
int n, m;
int s, t, len = 1;
int to[M], cap[M], nex[M];
int g[N], p[N], q[N], d[N];
void add(int x, int y, int v) {
        to[++ len] = y, cap[len] = v, nex[len] = g[x], g[x] = len;
        to[++ len] = x, cap[len] = 0, nex[len] = g[y], g[y] = len;
}
bool bfs() {
        int l = 1, r = 1, x, i;
        memset (d, 0, sizeof d);
        d[s] = 1, q[1] = s;
        while (l <= r) {
                x = q[l ++];
                for (i = g[x]; i; i = nex[i])
                        if (cap[i] && !d[to[i]])
                                d[to[i]] = d[x] + 1, q[++ r] = to[i];
        }
        return d[t];
}
int dfs(int x, int y) {
        if (x == t || y == 0) return y;
        int flow = 0;
        for (int &i = p[x]; i; i = nex[i]) {
                if (!cap[i] || d[to[i]] != d[x] + 1) continue;
                int f = dfs(to[i], min(y, cap[i]));
                flow += f, y -= f;
                cap[i] -= f, cap[i ^ 1] += f;
                if (!y) break;
```

```
        }
        return flow;
}
int  dinic() {
        int maxflow = 0;
        while (bfs()) {
                memcpy(p, g, sizeof g);
                maxflow += dfs(s, inf);
        }
        return maxflow;
}
```

1.9 DAG 删去无用边

```
/* 无用边定义：对于边 (u,v) 如果存在从 u 到 v 不经过该边的另一条路径，则称该边无用
 * 时间复杂度:O(n^3) */
bool f[N][N];//i 是否可达 j
vector <int> e[N];
int main() {
        rep(i, 1, n)
                for (int j : e[i]) {
                        rep (k, 1, n)
                                if (i != k && j != k && f[i][k] && f[k][j])
                                        no_use_edge;
                }
}
```

1.10 树分治

```
const int N = 1e5 + 5;
vector <int> e[N];
int n, a[N];
int root, _left, vis[N];
int siz[N], maxv[N];
void find_root(int u, int fr) {
        siz[u] = 1, maxv[u] = 0;
        for (int v : e[u]) {
                if (v == fr || vis[v]) continue;
                find_root(v, u);
                siz[u] += siz[v];
                maxv[u] = max(maxv[u], siz[v]);
        }
        maxv[u] = max(maxv[u], _left - siz[u]);
        if (!root || maxv[u] < maxv[root])
                root = u;
}
void dfs(int u, int fr) {
        siz[u] = 1;
        for (int v : e[u]) {
                if (v == fr || vis[v]) continue;
                find_root(v, u);
                siz[u] += siz[v];
        }
}
```

```
void solve(int u, int w) {
        dfs(u, u);//update siz[]
        a[u] = w, vis[u] = 1;
        for (int v : e[u]) {
                if (vis[v]) continue;
                _left = siz[v];
                root = 0;
                find_root(v, v);
                solve(root, w + 1);
        }
}
int main() {
    ios::sync_with_stdio(false);
    cin >> n;
    for (int u, v, i = 1; i < n; i ++) {
            cin >> u >> v;
            e[u].push_back(v);
            e[v].push_back(u);
    }
    _left = n, root = 0, find_root(1, 1);
    solve(root, 0);
    for (int i = 1; i <= n; i ++)
            printf("%c ", 'A' + a[i]);
    return 0;
}
```

2 数据结构

2.1 splay

```
#define mid (l + r >> 1)
int n, m, a, b, len, tot;
struct node {
        bool rev; //翻转标记
        int v, siz;
        node *c[2];
        node():rev(0),v(0),siz(0),c{NULL, NULL}{}
        node *init(int x);
        void pushdown();
        void mata() {siz = c[0] -> siz + c[1] -> siz + 1;}
        int cmp(int k) {return k<=c[0]->siz?0:(k==c[0]->siz+1?-1:1);}
        void print();
}pool[N], *null = new node();
node *node::init(int x){rev=0, v=x, siz=1, c[0]=c[1]=null;return this;}
void node::pushdown() {
        if (!rev) return;
        if (c[0] != null) c[0] -> rev ^= 1;
        if (c[1] != null) c[1] -> rev ^= 1;
        swap(c[0], c[1]), rev = 0;
}
void node::print() {
        pushdown();
        if (c[0] != null) c[0] -> print();
        if (1 <= v && v <= n) printf("%d ", v);
        if (c[1] != null) c[1] -> print();
}
node *build(int l, int r) {//初始序列为 1-n
        if (l == r) return pool[tot ++].init(r);
        node *tmp = pool[tot ++].init(mid);
        if (l < mid) tmp -> c[0] = build(l, mid - 1);
        if (mid < r) tmp -> c[1] = build(mid + 1, r);
        tmp -> mata(); return tmp;
}
void rot(node *&o, int k) {//把 k 儿子提上来
        o -> pushdown(); node *tmp = o -> c[k];
        tmp -> pushdown(); o -> c[k] = tmp -> c[!k];
        tmp -> c[!k] -> pushdown(); tmp -> c[!k] = o;
        o -> mata(), tmp -> mata(), o = tmp;
}
void splay(node *&o, int k) {//把以 o 为根的 splayTree 中 rk 为 k 的点提到根
        int k1 = o -> cmp(k); o -> pushdown();
        if (k1 == -1) return; o -> c[k1] -> pushdown();
        if (k1) k -= o -> c[0] -> siz + 1;
        int k2 = o -> c[k1] -> cmp(k);
        if (~k2) {//k2 != -1
                if (k2) k -= o -> c[k1] -> c[0] -> siz + 1;
                o -> c[k1] -> c[k2] -> pushdown();
                splay(o -> c[k1] -> c[k2], k);
                if (k2 == k1) rot(o, k1);
                else rot(o -> c[k1], k2);
```

```
        }
        rot(o, k1);
}
int main() {
        scanf("%d %d", &n, &m);
        node *root = build(0, n + 1); //方便边界左右处理各多开一个
        for (; m --; ) {
                scanf("%d %d", &a, &b), a ++, b ++, len = b - a + 1;
                splay(root, a - 1), splay(root -> c[1], len + 1);
                root -> c[1] -> c[0] -> rev ^= 1, root -> c[1] -> c[0] -> pushdown();
        }
        root -> print(); return 0;
}
```

2.2 treap

```
/* 容易实现的预开内存池 treap, 每次 head 清空即可
如果初始要插入 n 个 1, 可改为类似 splay 的 O(n)build 写法
*/
const int poolSize = 5e5 + 10;
struct node {
        node *c[2];
        int v, r, siz;
        void update();
        void init(int x);
};
node *null = new node(), *root = null;
void node::update() {
        siz = c[0] -> siz + c[1] -> siz + 1;
}
void node::init(int x) {
        v = x, r = rand(), siz = 1;
        c[0] = c[1] = null;
}
node nodesPool[poolSize];
int head;//每次 head=0 清空
node *newnode(int x) {
        node *res = &nodesPool[head ++];
        res -> init(x);
        return res;
}
void rot(node *&o, int d) {
        node *tmp = o -> c[!d];
        o -> c[!d] = tmp -> c[d], tmp -> c[d] = o;
        o -> update(), tmp -> update(), o = tmp;
}
void insert(node *&o, int x) {
        if (o == null) {
                o = newnode(x);
                return;
        }
        int d = x > o -> v ? 0 : 1;
        insert(o -> c[d], x);
        if (o -> c[d] -> r < o -> r) rot(o, !d);
```

```
        o -> update();
}
void del(node *&o, int x) {
        if (x == o -> v) {
                if (o -> c[0] == null) {o = o -> c[1]; return;}
                if (o -> c[1] == null) {o = o -> c[0]; return;}
                int d = o -> c[0] -> r < o -> c[1] -> r ? 1 : 0;
                rot(o, d), del(o -> c[d], x);
        }
        else del(o -> c[x <= o -> v], x);
        o -> update();
}
void build(node *&o, int l, int r) {
        o = newnode(1);
        if (l == r) return;
        int mid = l + r >> 1;
        if (l < mid) build(o -> c[0], l, mid - 1);
        if (o -> c[0] != null && o -> c[0] -> r < o -> r) swap(o -> c[0] -> r, o -> r);
        if (mid < r) build(o -> c[1], mid + 1, r);
        if (o -> c[1] != null && o -> c[1] -> r < o -> r) swap(o -> c[1] -> r, o -> r);
        o -> update();
}
```

2.3 主席树

```
const int N = 130000;
const int M = N * 20;
//主席树节点数，可以直接稳妥选择 N*(5+logN)
struct {
    int siz, l, r;
    ll sum, val;
}tr[M];
#define l(x) tr[x].l
#define r(x) tr[x].r
#define s(x) tr[x].sum
#define v(x) tr[x].val
#define sz(x) tr[x].siz
#define mid (l + r >> 1)
int tot, root[N];
int build(int l, int r) {
    int x = ++ tot;
    if (l < r) {
        l(x) = build(l, mid);
        r(x) = build(mid + 1, r);
    }
    return x;
}
int change(int o, int l, int r, int p, int y) {
    //在 p 的位置插入一个 y
    int x = ++ tot;
    s(x) = s(o) + y, sz(x) = sz(o) + 1;
    l(x) = l(o), r(x) = r(o), v(x) = y;
    if (l < r) {
        if (p <= mid) l(x) = change(l(o), l, mid, p, y);
```

```
        else r(x) = change(r(o), mid + 1, r, p, y);
    }
    return x;
}
ll ask(int o1, int o2, int l, int r, int k) {
    //求 (l,r] 区间前 k 小的数之和, 有 o1=root[l],o2=root[r]
    if (l == r) return v(o2) * k;
    int lsz = sz(l(o2)) - sz(l(o1));
    if (lsz == k) return s(l(o2)) - s(l(o1));
    if (lsz < k) return s(l(o2)) - s(l(o1)) + ask(r(o1), r(o2), mid + 1, r, k - lsz);
    return ask(l(o1), l(o2), l, mid, k);
}
```

2.4 吉老师线段树

```
/* 区间每个数变为 min(a[i],t) + 区间最大 + 区间和 O(nlog)*/
const int N = (1 << 20) + 5;
#define lc (o << 1)
#define rc (lc | 1)
struct node {
        int max1, max2, cnt;
        ll sum;
}tr[N << 1];
int T, n, m;
int op, x, y, t;
int a[N];
void pushup(int o) {
        if (tr[lc].max1 == tr[rc].max1) {
                tr[o].max1 = tr[lc].max1;
                tr[o].cnt = tr[lc].cnt + tr[rc].cnt;
                tr[o].max2 = max(tr[lc].max2, tr[rc].max2);
        }
        else {
                if (tr[lc].max1 > tr[rc].max1) {
                        tr[o] = tr[lc];
                        tr[o].max2 = max(tr[o].max2, tr[rc].max1);
                }
                else {
                        tr[o] = tr[rc];
                        tr[o].max2 = max(tr[o].max2, tr[lc].max1);
                }
        }
        tr[o].sum = tr[lc].sum + tr[rc].sum;
}
void pushdown(int o) {
        if (tr[o].max1 < tr[lc].max1) {
                tr[lc].sum += 1ll * (tr[o].max1 - tr[lc].max1) * tr[lc].cnt;
                tr[lc].max1 = tr[o].max1;
        }
        if (tr[o].max1 < tr[rc].max1) {
                tr[rc].sum += 1ll * (tr[o].max1 - tr[rc].max1) * tr[rc].cnt;
                tr[rc].max1 = tr[o].max1;
        }
}
```

```
void build(int o, int l, int r) {
        if (l == r) {
                tr[o].max1 = tr[o].sum = a[r];
                tr[o].cnt = 1, tr[o].max2 = 0;
                return;
        }
        int mid = l + r >> 1;
        build(lc, l, mid);
        build(rc, mid + 1, r);
        pushup(o);
}
void update(int o, int l, int r, int s, int t, int v) {
        if (v >= tr[o].max1) return;
        pushdown(o);
        if (s <= l && r <= t) {
                if (v > tr[o].max2) {
                        tr[o].sum += 1ll * (v - tr[o].max1) * tr[o].cnt;
                        tr[o].max1 = v;
                        return;
                }
        }
        int mid = l + r >> 1;
        if (s <= mid) update(lc, l, mid, s, t, v);
        if (t >  mid) update(rc, mid + 1, r, s, t, v);
        pushup(o);
}
ll ask_max(int o, int l, int r, int s, int t) {
        if (s <= l && r <= t) return tr[o].max1;
        pushdown(o);
        int mid = l + r >> 1;
        ll res = 0;
        if (s <= mid) res = max(res, ask_max(lc, l, mid, s, t));
        if (t >  mid) res = max(res, ask_max(rc, mid + 1, r, s, t));
        return res;
}
ll ask_sum(int o, int l, int r, int s, int t) {
        if (s <= l && r <= t) return tr[o].sum;
        pushdown(o);
        int mid = l + r >> 1;
        ll res = 0;
        if (s <= mid) res += ask_sum(lc, l, mid, s, t);
        if (t >  mid) res += ask_sum(rc, mid + 1, r, s, t);
        return res;
}
int main() {
        ios::sync_with_stdio(false);
        for (cin >> T; T --; ) {
                cin >> n >> m;
                for (int i = 1; i <= n; i ++)
                        cin >> a[i];
                build(1, 1, n);
                while (m --) {
                        cin >> op;
                        if (op == 0) {
```

```
                        cin >> x >> y >> t;
                        update(1, 1, n, x, y, t);
                    }
                    else if (op == 1) {
                        cin >> x >> y;
                        cout << ask_max(1, 1, n, x, y) << '\n';
                    }
                    else {
                        cin >> x >> y;
                        cout << ask_sum(1, 1, n, x, y) << '\n';
                    }
                }
            }
            return 0;
}
```

2.5 KDTree

2.5.1 3 维 KDtree

```
/*O(n*n^(1-1/k)),k 为维度 */
const int N = 1e5 + 5;
const int Mod = 1e9 + 7;
int nowD, ans, x[3], y[3];
int n, m, a[N], b[N], c[N], d[N];
struct node {
    int Max[3], Min[3], d[3];
    int val, maxv;
    node *c[2];
    node() {
        c[0] = c[1] = NULL;
        val = maxv = 0;
    }
    void pushup();
    bool operator < (const node &a) const {
        return d[nowD] < a.d[nowD];
    }
}Null, nodes[N];
node *root = &Null;
inline void node::pushup() {
    if (c[0] != &Null) {
        if (c[0] -> Max[1] > Max[1]) Max[1] = c[0] -> Max[1];
        if (c[0] -> Max[2] > Max[2]) Max[2] = c[0] -> Max[2];
        if (c[0] -> Min[0] < Min[0]) Min[0] = c[0] -> Min[0];
        if (c[0] -> Min[2] < Min[2]) Min[2] = c[0] -> Min[2];
        if (c[0] -> maxv > maxv) maxv = c[0] -> maxv;
    }
    if (c[1] != &Null) {
        if (c[1] -> Max[1] > Max[1]) Max[1] = c[1] -> Max[1];
        if (c[1] -> Max[2] > Max[2]) Max[2] = c[1] -> Max[2];
        if (c[1] -> Min[0] < Min[0]) Min[0] = c[1] -> Min[0];
        if (c[1] -> Min[2] < Min[2]) Min[2] = c[1] -> Min[2];
        if (c[1] -> maxv > maxv) maxv = c[1] -> maxv;
    }
}
```

```
inline node *build(int l, int r) {
    int mid = l + r >> 1; nowD = rand() % 3;
    nth_element(nodes + l, nodes + mid, nodes + r + 1);
    node *res = &nodes[mid];
    if (l != mid) res -> c[0] = build(l, mid - 1);
    else res -> c[0] = &Null;
    if (r != mid) res -> c[1] = build(mid + 1, r);
    else res -> c[1] = &Null;
    res -> pushup();
    return res;
}
inline int calc(node *o) {
    if (y[0] < o -> Min[0] || x[1] > o -> Max[1] || x[2] > o -> Max[2] || y[2] < o ->
    ↪  Min[2]) return -1;
    return o -> maxv;
}
inline void query(node *o) {
    if (o -> val > ans && y[0] >= o -> d[0] && x[1] <= o -> d[1] && x[2] <= o -> d[2] &&
    ↪  y[2] >= o -> d[2]) ans = o -> val;
    int dl, dr;
    if (o -> c[0] != &Null) dl = calc(o -> c[0]);
    else dl = -1;
    if (o -> c[1] != &Null) dr = calc(o -> c[1]);
    else dr = -1;
    if (dl > dr) {
        if (dl > ans) query(o -> c[0]);
        if (dr > ans) query(o -> c[1]);
    } else {
        if (dr > ans) query(o -> c[1]);
        if (dl > ans) query(o -> c[0]);
    }
}
int main() {
    ios::sync_with_stdio(false);
    cin >> n >> m;
    for (int i = 1; i <= n; i ++) {
        cin >> a[i];
        b[i] = d[a[i]];
        d[a[i]] = i;
    }
    for (int i = 1; i <= n; i ++) d[i] = n + 1;
    for (int i = n; i; i --) {
        c[i] = d[a[i]];
        d[a[i]] = i;
    }
    for (int i = 1; i <= n; i ++) {
        nodes[i].Min[0] = nodes[i].d[0] = b[i];
        nodes[i].Max[1] = nodes[i].d[1] = c[i];
        nodes[i].Max[2] = nodes[i].Min[2] = nodes[i].d[2] = i;
        nodes[i].val = nodes[i].maxv = a[i];
    }
    root = build(1, n);
    for (int l, r; m --; ) {
        cin >> l >> r;
```

```
        l = (l + ans) % n + 1;
        r = (r + ans) % n + 1;
        if (l > r) swap(l, r);
        y[0] = l - 1;
        x[1] = r + 1;
        x[2] = l, y[2] = r;
        ans = 0, query(root);
        cout << ans << endl;
    }
    cout << endl;
    return 0;
}
```

2.5.2 KDtree 二维空间区间覆盖单点查询

```
/* 类似线段树 */
const int N = 1e5 + 5;
const int Mod = 1e9 + 7;
int nowD, x[2], y[2], z;
struct node {
    int Max[2], Min[2], d[2];
    int val, lazy;
    node *c[2];
    node() {
        c[0] = c[1] = NULL;
    }
    void pushup();
    void pushdown();
    bool operator < (const node &a) const {
        return d[nowD] < a.d[nowD];
    }
}Null, nodes[N];
node *root = &Null;
inline void node::pushup() {
    if (c[0] != &Null) {
        if (c[0] -> Max[0] > Max[0]) Max[0] = c[0] -> Max[0];
        if (c[0] -> Max[1] > Max[1]) Max[1] = c[0] -> Max[1];
        if (c[0] -> Min[0] < Min[0]) Min[0] = c[0] -> Min[0];
        if (c[0] -> Min[1] < Min[1]) Min[1] = c[0] -> Min[1];
    }
    if (c[1] != &Null) {
        if (c[1] -> Max[0] > Max[0]) Max[0] = c[1] -> Max[0];
        if (c[1] -> Max[1] > Max[1]) Max[1] = c[1] -> Max[1];
        if (c[1] -> Min[0] < Min[0]) Min[0] = c[1] -> Min[0];
        if (c[1] -> Min[1] < Min[1]) Min[1] = c[1] -> Min[1];
    }
}
inline void node::pushdown() {
    if (c[0] != &Null) c[0] -> val = c[0] -> lazy = lazy;
    if (c[1] != &Null) c[1] -> val = c[1] -> lazy = lazy;
    lazy = -1;
}
inline node *build(int l, int r, int D) {
    int mid = l + r >> 1; nowD = D;
```

```
    nth_element(nodes + l, nodes + mid, nodes + r + 1);
    node *res = &nodes[mid];
    if (l != mid) res -> c[0] = build(l, mid - 1, !D);
    else res -> c[0] = &Null;
    if (r != mid) res -> c[1] = build(mid + 1, r, !D);
    else res -> c[1] = &Null;
    res -> pushup();
    return res;
}
inline int query(node *o) {
    if (o == &Null) return -1;
    if (o -> lazy != -1) o -> pushdown();
    if (x[0] > o -> Max[0] || y[0] > o -> Max[1] || x[0] < o -> Min[0] || y[0] < o ->
    ↪  Min[1]) return -1;
    if (x[0] == o -> d[0]) return o -> val;
    return max(query(o -> c[0]), query(o -> c[1]));
}
inline void modify(node *o) {
    if (o == &Null) return;
    if (o -> lazy != -1) o -> pushdown();
    if (x[0] > o -> Max[0] || y[0] > o -> Max[1] || x[1] < o -> Min[0] || y[1] < o ->
    ↪  Min[1]) return;
    if (x[0] <= o -> Min[0] && y[0] <= o -> Min[1] && x[1] >= o -> Max[0] && y[1] >= o ->
    ↪  Max[1]) {
        o -> val = o -> lazy = z;
        return;
    }
    if (x[0] <= o -> d[0] && y[0] <= o -> d[1] && x[1] >= o -> d[0] && y[1] >= o -> d[1])
    ↪  o -> val = z;
    modify(o -> c[0]), modify(o -> c[1]);
}
int n, m, k, a[N], c[N], d[N];
int cnt, st[N], en[N], dfn[N], dep[N];
vector <int> e[N];
void dfs(int u) {
    st[u] = ++ cnt, dfn[cnt] = u;
    for (int v : e[u])
        dep[v] = dep[u] + 1, dfs(v);
    en[u] = cnt;
}
int main() {
    ios::sync_with_stdio(false);
    int T, ans;
    for (cin >> T; T --; ) {
        cin >> n >> m >> k, ans = cnt = 0;
        for (int i = 1; i <= n; i ++)
            e[i].clear();
        for (int u, i = 2; i <= n; i ++) {
            cin >> u;
            e[u].push_back(i);
        }
        dfs(1);
        for (int i = 1; i <= n; i ++) {
            nodes[i].Min[0] = nodes[i].Max[0] = nodes[i].d[0] = i;
```

```
            nodes[i].Min[1] = nodes[i].Max[1] = nodes[i].d[1] = dep[dfn[i]];
            nodes[i].val = 1, nodes[i].lazy = -1;
        }
        root = build(1, n, 0);
        for (int u, v, w, i = 1; i <= k; i ++) {
            cin >> u >> v >> w;
            if (w == 0) {
                x[0] = st[u], y[0] = dep[u];
                ans = (ans + 1ll * i * query(root) % Mod) % Mod;
            } else {
                x[0] = st[u], x[1] = en[u];
                y[0] = dep[u], y[1] = dep[u] + v;
                z = w, modify(root);
            }
        }
        cout << ans << endl;
    }
    return 0;
}
```

2.5.3 KDtree 二维空间单点修改区间查询

```
/*
 * 调整重构系数可以影响常数
 * 询问多就让系数接近 0.70-0.75, 询问少就让系数在 0.8-0.90
 */
const int inf = 1e9;
int n, m, tot, nowD;
struct node {
    int Max[2], Min[2], d[2];
    int sum, siz, val;
    node *c[2];
    node() {
        Max[0] = Max[1] = -inf;
        Min[0] = Min[1] = inf;
        sum = val = siz = 0;
        c[0] = c[1] = NULL;
        d[0] = d[1] = 0;
    }
    void update();
}Null, nodes[200010], *temp[200010];
node *root = &Null;
inline void node::update() {
    siz = c[0] -> siz + c[1] -> siz + 1;
    sum = c[0] -> sum + c[1] -> sum + val;
    if (c[0] != &Null) {
        if (c[0] -> Max[0] > Max[0]) Max[0] = c[0] -> Max[0];
        if (c[0] -> Max[1] > Max[1]) Max[1] = c[0] -> Max[1];
        if (c[0] -> Min[0] < Min[0]) Min[0] = c[0] -> Min[0];
        if (c[0] -> Min[1] < Min[1]) Min[1] = c[0] -> Min[1];
    }
    if (c[1] != &Null) {
        if (c[1] -> Max[0] > Max[0]) Max[0] = c[1] -> Max[0];
        if (c[1] -> Max[1] > Max[1]) Max[1] = c[1] -> Max[1];
```

```
        if (c[1] -> Min[0] < Min[0]) Min[0] = c[1] -> Min[0];
        if (c[1] -> Min[1] < Min[1]) Min[1] = c[1] -> Min[1];
    }
}
inline bool cmp(const node *a, const node *b) {
    return a -> d[nowD] < b -> d[nowD];
}
inline void traverse(node *o) {
    if (o == &Null) return;
    temp[++ tot] = o;
    traverse(o -> c[0]);
    traverse(o -> c[1]);
}
inline node *build(int l, int r, int D) {
    int mid = l + r >> 1; nowD = D;
    nth_element(temp + l, temp + mid, temp + r + 1, cmp);
    node *res = temp[mid];
    res -> Max[0] = res -> Min[0] = res -> d[0];
    res -> Max[1] = res -> Min[1] = res -> d[1];
    if (l != mid) res -> c[0] = build(l, mid - 1, !D);
    else res -> c[0] = &Null;
    if (r != mid) res -> c[1] = build(mid + 1, r, !D);
    else res -> c[1] = &Null;
    res -> update();
    return res;
}
int x, y, a, b, tmpD;
node **tmp;
inline void rebuild(node *&o, int D) {
    tot = 0;
    traverse(o);
    o = build(1, tot, D);
}
inline void insert(node *&o, node *p, int D) {
    if (o == &Null) {o = p; return;}
    if (p -> Max[0] > o -> Max[0]) o -> Max[0] = p -> Max[0];
    if (p -> Max[1] > o -> Max[1]) o -> Max[1] = p -> Max[1];
    if (p -> Min[0] < o -> Min[0]) o -> Min[0] = p -> Min[0];
    if (p -> Min[1] < o -> Min[1]) o -> Min[1] = p -> Min[1];
    o -> siz ++, o -> sum += p -> sum;
    insert(o -> c[p -> c[D] >= o -> c[D]], p, !D);
    if (max(o -> c[0] -> siz, o -> c[1] -> siz) > int(o -> siz * 0.75 + 0.5)) tmpD = D,
    ↪  tmp = &o;
}
inline int query(node *o) {
    if (o == &Null) return 0;
    if (x > o -> Max[0] || y > o -> Max[1] || a < o -> Min[0] || b < o -> Min[1]) return
    ↪  0;
    if (x <= o -> Min[0] && y <= o -> Min[1] && a >= o -> Max[0] && b >= o -> Max[1])
    ↪  return o -> sum;
    return (x <= o -> d[0] && y <= o -> d[1] && a >= o -> d[0] && b >= o -> d[1] ? o ->
    ↪  val : 0)
        + query(o -> c[1]) + query(o -> c[0]);
}
```

```
int main() {
    ios::sync_with_stdio(false);
    cin >> m;
    node *ttt = &Null;
    for (int t, ans = 0; ; ) {
        cin >> t;
        if (t == 3) break;
        if (t == 1) {
            cin >> x >> y >> a;
            x ^= ans, y ^= ans, n ++;
            nodes[n].sum = nodes[n].val = a ^ ans, nodes[n].siz = 1;
            nodes[n].Max[0] = nodes[n].Min[0] = nodes[n].d[0] = x;
            nodes[n].Max[1] = nodes[n].Min[1] = nodes[n].d[1] = y;
            nodes[n].c[0] = nodes[n].c[1] = &Null;
            tmp = &(ttt), insert(root, &nodes[n], 0);
            if (*tmp != &Null) rebuild(*tmp, tmpD);
        } else {
            cin >> x >> y >> a >> b;
            x ^= ans, y ^= ans, a ^= ans, b ^= ans;
            if (x > a) swap(x, a);
            if (y > b) swap(y, b);
            ans = query(root);
            printf("%d\n", ans);
        }
    }
    return 0;
}
```

2.5.4 KDtree 找最近点

```
/*
 * 为了维持树的平衡，可以一开始把所有点都读进来 build
 * 然后打 flag 标记该点是否被激活
 */
const int N = 5e5 + 5;
const int inf = 1 << 30;
int n, m;
int ql, qr, ans, tot, nowD;
//nowD = rand() & 1 ?
struct Node {
    int d[2];
    bool operator < (const Node &a) const {
        if (d[nowD] == a.d[nowD]) return d[!nowD] < a.d[!nowD];
        return d[nowD] < a.d[nowD];
    }
}pot[N];
struct node {
    int min[2], max[2], d[2];
    node *c[2];
    node() {
        min[0] = min[1] = max[0] = max[1] = d[0] = d[1] = 0;
        c[0] = c[1] = NULL;
    }
    node(int x, int y);
```

```
    void update();

}t[N], Null, *root;
node::node(int x, int y) {
    min[0] = max[0] = d[0] = x;
    min[1] = max[1] = d[1] = y;
    c[0] = c[1] = &Null;
}
inline void node::update() {
    if (c[0] != &Null) {
        if (c[0] -> max[0] > max[0]) max[0] = c[0] -> max[0];
        if (c[0] -> max[1] > max[1]) max[1] = c[0] -> max[1];
        if (c[0] -> min[0] < min[0]) min[0] = c[0] -> min[0];
        if (c[0] -> min[1] < min[1]) min[1] = c[0] -> min[1];
    }
    if (c[1] != &Null) {
        if (c[1] -> max[0] > max[0]) max[0] = c[1] -> max[0];
        if (c[1] -> max[1] > max[1]) max[1] = c[1] -> max[1];
        if (c[1] -> min[0] < min[0]) min[0] = c[1] -> min[0];
        if (c[1] -> min[1] < min[1]) min[1] = c[1] -> min[1];
    }
}
inline void build(node *&o, int l, int r, int D) {
    int mid = l + r >> 1;
    nowD = D;
    nth_element(pot + l, pot + mid, pot + r + 1);
    o = new node(pot[mid].d[0], pot[mid].d[1]);
    if (l != mid) build(o -> c[0], l, mid - 1, !D);
    if (r != mid) build(o -> c[1], mid + 1, r, !D);
    o -> update();
}
inline void insert(node *o) {
    node *p = root;
    int D = 0;
    while (1) {
        if (o -> max[0] > p -> max[0]) p -> max[0] = o -> max[0];
        if (o -> max[1] > p -> max[1]) p -> max[1] = o -> max[1];
        if (o -> min[0] < p -> min[0]) p -> min[0] = o -> min[0];
        if (o -> min[1] < p -> min[1]) p -> min[1] = o -> min[1];
        if (o -> d[D] >= p -> d[D]) {
            if (p -> c[1] == &Null) {
                p -> c[1] = o;
                return;
            } else p = p -> c[1];
        } else {
            if (p -> c[0] == &Null) {
                p -> c[0] = o;
                return;
            } else p = p -> c[0];
        }
        D ^= 1;
    }
}
inline int dist(node *o) {
```

```
    int dis = 0;
    if (ql < o -> min[0]) dis += o -> min[0] - ql;
    if (ql > o -> max[0]) dis += ql - o -> max[0];
    if (qr < o -> min[1]) dis += o -> min[1] - qr;
    if (qr > o -> max[1]) dis += qr - o -> max[1];
    return dis;
}
inline void query(node *o) {
    int dl, dr, d0;
    d0 = abs(o -> d[0] - ql) + abs(o -> d[1] - qr);
    if (d0 < ans) ans = d0;
    if (o -> c[0] != &Null) dl = dist(o -> c[0]);
    else dl = inf;
    if (o -> c[1] != &Null) dr = dist(o -> c[1]);
    else dr = inf;
    if (dl < dr) {
        if (dl < ans) query(o -> c[0]);
        if (dr < ans) query(o -> c[1]);
    } else {
        if (dr < ans) query(o -> c[1]);
        if (dl < ans) query(o -> c[0]);
    }
}
int main() {
    ios::sync_with_stdio(false);
    cin >> n >> m;
    for (int i = 1; i <= n; i ++)
        cin >> pot[i].d[0] >> pot[i].d[1];
    build(root, 1, n, 0);

    for (int x, y, z; m --; ) {
        cin >> x >> y >> z;
        if (x == 1) {
            t[tot].max[0] = t[tot].min[0] = t[tot].d[0] = y;
            t[tot].max[1] = t[tot].min[1] = t[tot].d[1] = z;
            t[tot].c[0] = t[tot].c[1] = &Null;
            insert(&t[tot ++]);
        } else {
            ans = inf, ql = y, qr = z;
            query(root), printf("%d\n", ans);
        }
    }
    return 0;
}
```

3 构造

3.1 若干排列使所有数对都出现一次

```
/*n/2 个排列使得所有数对 (i,j) 且 i<j 都出现一次
 *n 为奇数则首尾相连，偶数不连
 */
typedef vector<int> vi;
void get_even(int n, vi ans[]) {
        vi a(n);
        for (int i = 0; i < n; i ++)
                a[i] = i + 1;
        for (int i = 1; i <= n / 2; i ++) {
                ans[i].resize(n + 1);
                for (int j = 0; j < n / 2; j ++)
                        ans[i][j * 2] = a[j], ans[i][j * 2 + 1] = a[n - 1 - j];
                int t = a[n - 1];
                for (int j = n - 1; j > 0; j --)
                        a[j] = a[j - 1];
                a[0] = t;
        }
}
void get_odd(int n, vi ans[]) {
        get_even(n - 1, ans);
        for (int i = 1; i <= n / 2; i ++) {
                for (int j = n - 1; j > 0; j --)
                        ans[i][j] = ans[i][j - 1] + 1;
                ans[i][0] = 1;
        }
}
int main() {
        vi ans[2019];
        int n; cin >> n;
        if (n & 1) get_odd(n, ans);
        else get_even(n, ans);
        return 0;
}
```

3.2 rec-free

```
/* 不存在四个 1 构成一个矩形，并使得 1 尽量多，输出 01 矩阵 */
const int N = 200, n = 150, M = 13;//M 为质数，N>M*M>n
int a[N][N], b[N][N], c[N][N];
void make() {
        for (int i = 1; i <= M; i ++)
        for (int j = 1; j <= M; j ++)
            a[i][j + 1] = M * (j - 1) + i;
    for (int i = 1; i <= M; i ++)
        for (int j = 1; j <= M; j ++)
            c[i][a[i][j]] = 1;
    for (int k = 1; k < M; k ++) {
        memcpy(b, a, sizeof b);
        for (int i = 1; i <= M; i ++)
            for (int j = 1; j <= M; j ++)
```

```
                a[i][j] = (b[i + j - 1 - ((i + j - 1) > M ? M : 0)][j]);
        for (int i = 1; i <= M; i ++)
            for (int j = 1; j <= M; j ++)
                c[k * M + i][a[i][j]] = 1;
    }
}
```

3.3 点边均整数多边形

```
/* 构造满足以下条件的简单多边形（可以凹但边不能相交）
 * 有 k 个点 k 条边，所有边都为整数，所有点的坐标都为整数
 * 不存在与坐标轴平行的边
 */
typedef pair<int, int> P;
P operator * (P a, int b){return P(a.first * b, a.second * b);}
P operator + (P a, P b) {return P(a.first + b.first, a.second + b.second);}
int main() {
        int n; cin >> n;
        if (n == 3) return printf("0 0\n4 3\n-20 21\n"), 0;
        int m = n / 2 - 1;
        vector<P> v;
        v.push_back(P(0, 0));
        v.push_back(P(4, -3) * m);
        v.push_back(P(4, 0) * (2 * m));
        int lim = (n & 1)? n - 1: n;
        for (int i = 4; i <= lim; ++ i) {
                P last = v.back();
                if ((i % 2) == 0) v.push_back(last + P(-4, 3));
                else v.push_back(last + P(-4, -3));
        }
        if (n & 1) v.push_back(P(-20, 48));
        for (P p: v) printf("%d %d\n", p.first, p.second);
}
```

4 计算几何

4.1 最小矩形覆盖含凸包和旋转卡壳

```
/* 最小矩形覆盖，保留六位小数，逆时针输出四个顶点坐标 */
namespace minRectCover {
        const int N = 1e5 + 5;
        const double eps = 1e-8;
        struct point{
                double x, y;
                point(){}
                point(double x, double y):x(x), y(y){}
                bool operator < (const point &a) const {
                        return fabs(y - a.y) < eps ? x < a.x : y < a.y;}
                point operator - (const point &a) const {
                        return point(x - a.x, y - a.y);}
                point operator + (const point &a) const {
                        return point(x + a.x, y + a.y);}
                point operator / (const double &a) const {
                        return point(x / a, y / a);}

                point operator * (const double &a) const {
                        return point(x * a, y * a);}
                double operator / (const point &a) const { // .
                        return x * a.x + y * a.y;}
                double operator * (const point &a) const { // X
                        return x * a.y - y * a.x;}

        }p[N], q[N], rc[4];
        double sqr(double x) {return x * x;}
        double abs(point a) {return sqrt(a / a);}
        int sgn(double x) {return fabs(x) < eps ? 0 : (x < 0 ? -1 : 1);}
        point vertical(point a, point b) {//与 ab 向量垂直的向量
                return point(a.x + a.y - b.y, a.y - a.x + b.x) - a;}
        point vec(point a){return a / abs(a);}
        void convexhull(int n, point *hull, int &top) {//如果要计算周长需要特判 n==2
                for (int i = 1; i < n; i ++)
                        if (p[i] < p[0])
                                swap(p[i], p[0]);
                sort (p + 1, p + n, [&](point a, point b){
                        double t = (a - p[0]) * (b - p[0]);
                        if (fabs(t) < eps) return sgn(abs(p[0] - a) - abs(p[0] - b)) < 0;
                        return t > 0;
                });
                int cnt = 0;//去重
                for (int i = 1; i < n; i ++)
                        if (sgn(p[i].x - p[cnt].x) != 0 || sgn(p[i].y - p[cnt].y) != 0)
                                p[++ cnt] = p[i];
                n = cnt + 1;
                hull[top = 1] = p[0];
                for (int i = 1; i < n; i ++) {
                        while (top > 1 &&
                                (hull[top] - hull[top - 1]) * (p[i] - hull[top]) < eps)
                                ↪  top --;
```

```
                hull[++ top] = p[i];
            }
            hull[0] = hull[top];
        }
        void main() {
            int n;
            scanf("%d", &n);
            for (int i = 0; i < n; i ++)
                scanf("%lf %lf", &p[i].x, &p[i].y);
            convexhull(n, q, n);
            double ans = 1e20;
            int l = 1, r = 1, t = 1;
            double L, R, D, H;
            for (int i = 0; i < n; i ++) {
                D = abs(q[i] - q[i + 1]);
                while (sgn((q[i + 1] - q[i]) * (q[t + 1] - q[i]) -
                    (q[i + 1] - q[i]) * (q[t] - q[i])) > -1) t = (t + 1) % n;
                while (sgn((q[i + 1] - q[i]) / (q[r + 1] - q[i]) - (q[i + 1] -
                ↪  q[i]) / (q[r] - q[i])) > -1) r = (r + 1) % n;
                if (i == 0) l = r;
                while (sgn((q[i + 1] - q[i]) / (q[l + 1] - q[i]) - (q[i + 1] -
                ↪  q[i]) / (q[l] - q[i])) <  1) l = (l + 1) % n;
                L = fabs((q[i + 1] - q[i]) / (q[l] - q[i]) / D);
                R = fabs((q[i + 1] - q[i]) / (q[r] - q[i]) / D);
                H = fabs((q[i + 1] - q[i]) * (q[t] - q[i]) / D);
                double tmp = (R + L) * H;
                if (tmp < ans) {
                    ans = tmp;
                    rc[0] = q[i] + (q[i + 1] - q[i]) * (R / D);//右下
                    rc[1] = rc[0] + vec(vertical(q[i], q[i + 1])) * H;//右上
                    rc[2] = rc[1] - (rc[0] - q[i]) * ((R + L) / abs(q[i] -
                    ↪  rc[0]));//左上
                    rc[3] = rc[2] - (rc[1] - rc[0]);
                }
            }
            printf("%.6f\n", ans);
            int fir = 0;
            for (int i = 1; i < 4; i ++)
                if (rc[i] < rc[fir])
                    fir = i;
            for (int i = 0; i < 4; i ++)
                printf("%.6f %.6f\n", rc[(fir + i) % 4].x, rc[(fir + i) % 4].y);
        }
}
```

5 数论

5.1 CRT

```
void exgcd(ll a, ll b, ll &d, ll &x, ll &y) {
        if (!b) {
                d = a, x = 1, y = 0;
                return;
        }
        exgcd(b, a % b, d, y, x);
        y -= a / b * x;
}
ll crt(ll *m, ll *a, int n) {//n 个式子: y = mx + a, 下标从 1 开始
        ll A = a[1], M = m[1], d, x, y, m2;
        for (int i = 2; i <= n; i ++) {// k1 * m1 - k2 * m2 = a2 - a1
                exgcd(M, m[i], d, x, y);
                if ((a[i] - A) % d) return -1;
                m2 = M / d * m[i];
                x = (a[i] - A) / d * x % m[i];
                A = (A + x * M % m2) % m2;
                if (A < 0) A += m2;//保证 A>=0
                M = m2;
        }
        return A;//y = Mx + A
}
```

6 字符串

6.1 KMP

```
int nxt[N];
void kmp(int n, char *a, int m, char *b) {
        //长度为 m 的 b 中找 a, 下标从 0 开始, 得到的是匹配成功的末尾位置
        static int i, j, cnt, tmp[2]; cnt = 0;
        for (nxt[0] = j = -1, i = 1; i < n; nxt[i ++] = j) {
                while (~j && a[j + 1] != a[i]) j = nxt[j];
                if (a[j + 1] == a[i]) j ++;
        }
        for (j = -1, i = 0; i < m; i ++) {
                while (~j && a[j + 1] != b[i]) j = nxt[j];
                if (a[j + 1] == b[i]) j ++;
                if (j == n - 1) {
                        printf("%d ", i);
                        j = nxt[j];
                }
        }
}
```

7 其他

7.1 数字哈希

```
namespace my_hash {
        const int N = (1 << 19) - 1;//散列大小，一定要取 2^k-1，不超内存的情况下，N越大碰撞越少
        struct E {
                int v;
                E *nxt;
        }*g[N + 1], pool[N], *cur = pool, *p;
        int vis[N + 1], T;
        void ins(int v) {
                int u = v & N;
                if (vis[u] < T) vis[u] = T, g[u] = NULL;
                for (p = g[u]; p; p = p -> nxt) if (p -> v == v) return;
                p = cur ++; p -> v = v; p -> nxt = g[u]; g[u] = p;
        }
        int ask(int v) {
                int u = v & N;
                if (vis[u] < T) return 0;
                for (p = g[u]; p; p = p -> nxt) if (p -> v == v) return 1;
                return 0;
        }
        void init() {T ++, cur = pool;}//应对多组数据使用
}
```

7.2 海岛分金币

7.2.1 海岛分金币 1

```
/*
非朴素模型，有额外条件：
每个人做决定时如果有多种方案可以使自己获得最大收益
那么他会让决策顺序靠前的人获得的收益尽可能的大！
solution:
贪心模拟
*/
#define v first
#define id second
typedef pair<int, int> pr;
const int N = 1010;
int a[N][N];
pr b[N];
int n, m;
int main() {
    cin >> n >> m;
    a[1][1] = m;
    for (int i = 2; i <= n; i ++) {
        for (int j = 1; j < i; j ++)
            b[j] = pr(a[i - 1][j], j);
        sort (b + 1, b + i, [&](pr x, pr y){return x.v != y.v ? (x.v < y.v) : (x.id > y.id);});
        //按照是否容易满足来排序，因为容易满足的人消耗掉的金币比较少，也就使得当前的人获利最大
        int s = m, nd = (i - 1) / 2;
```

```
        for (int j = 1; j < i && nd; j ++) {
            nd --;
            s -= (a[i][b[j].id] = a[i - 1][b[j].id] + 1);
        }
        if (s < 0) {
            for (int j = 1; j < i; j ++)
                a[i][j] = a[i - 1][j];
            a[i][i] = -1;
        }
        else {
            a[i][i] = s;
        }
    }
    for (int i = n; i; i --)
        printf("%d ", a[n][i]);
    return 0;
}
```

7.2.2 海岛分金币 2

```
/*
海盗分金币朴素模型：
n 个海盗分 m 个金币，依次做决策，如果不少于半数的人同意则方案通过，否则当前做决策的人会被淘汰
↪ (收益视为-1)，由下一人做出决策
如果一个海盗有多种方案均为最大收益，那么他会希望淘汰的人越多越好
求出第 x 个做决策的海盗的最大可能受益和最小可能收益
*/
struct node {
    int min_v, max_v;
    node():min_v(0), max_v(0) {}
    node(int min_v, int max_v):min_v(min_v), max_v(max_v) {}
};
node ask(int n, int m, int x) {//n 个人分 m 个金币，第 x 个做决策的人最少/最多分到多少个金币
    int y = n + 1 - x;
    if (n >= (m + 2) * 2) {
        int a = (m + 1) * 2, b = 2, c = 4;
        //前 a 个为 [0,1]，后 b 个为 [0,0]，将持续 c 个
        while (a + b + c <= n) {
            a += b;
            b *= 2;
            c *= 2;
        }
        if (y <= a) return node(0, 1);
        else if (y <= a + b) return node(0, 0);
        else return node(-1, -1);
    }
    else if (n == m * 2 + 3) {
        if (x == 1) return node(-1, -1);
        else if (y <= m * 2 && y % 2 == 1 || x == 2) return node(0, 0);
        else return node(0, 1);
    }
    else if (n == m * 2 + 2) {
        if (y <= m * 2 && y % 2 == 1 || x == 1) return node(0, 0);
        else return node(0, 1);
```

```
    }
    else if (n == m * 2 + 1) {
        if (y <= m * 2 && y % 2 == 1) return node(1, 1);
        else return node(0, 0);
    }
    else {
        if (x & 1) {
            if (x != 1) return node(1, 1);
            else return node(m - (n - 1) / 2, m - (n - 1) / 2);
        }
        else return node(0, 0);
    }
}

int main() {
    ios::sync_with_stdio(false);
    int x, n, m, k; node y;
    cin >> n >> m >> k;
    while (k --) {
        cin >> x;
        y = ask(n, m, x);
        printf("%d %d\n", y.min_v, y.max_v);
    }
    return 0;
}

/*
m = 5

1  5
2  0 5
3  1 0 4
4  0 1 0 4
5  1 0 1 0 3
6  0 1 0 1 0 3
7  1 0 1 0 1 0 2
8  0 1 0 1 0 1 0 2
9  1 0 1 0 1 0 1 0 1
10 0 1 0 1 0 1 0 1 0 1
11 1 0 1 0 1 0 1 0 1 0 0
12 0 _ 0 _ 0 _ 0 _ 0 _ _ 0
13 0 _ 0 _ 0 _ 0 _ 0 _ _ 0 -1
14 _ _ _ _ _ _ _ _ _ _ _ _ 0 0
15 _ _ _ _ _ _ _ _ _ _ _ _ 0 0 -1
16 _ _ _ _ _ _ _ _ _ _ _ _ 0 0 -1 -1
17 _ _ _ _ _ _ _ _ _ _ _ _ 0 0 -1 -1 -1
18 _ _ _ _ _ _ _ _ _ _ _ _ _ 0 0 0 0
19 _ _ _ _ _ _ _ _ _ _ _ _ _ 0 0 0 0 -1
20 _ _ _ _ _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1
21 _ _ _ _ _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1 -1
22 _ _ _ _ _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1 -1 -1
23 _ _ _ _ _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1 -1 -1 -1
24 _ _ _ _ _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1 -1 -1 -1 -1
25 _ _ _ _ _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1 -1 -1 -1 -1 -1
```

```
26 _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ 0 0 0 0 0 0 0 0
*/
```

7.3 根号枚举

```
for (int i = 1, last; i <= n; i = last + 1) {
        last = n / (n / i);
        //当前枚举区间为 [i, last]
}
```

7.4 读入输出外挂

```
namespace IO {//only for int!!!
    static const int SIZE = 1 << 20;
    inline int get_char() {
        static char *S, *T = S, buf[SIZE];
        if (S == T) {
            T = fread(buf, 1, SIZE, stdin) + (S = buf);
            if (S == T) return -1;
        }
        return *S ++;
    }
    inline void in(int &x) {//for int
        static int ch;
        while (ch = get_char(), ch < 48);x = ch ^ 48;
        while (ch = get_char(), ch > 47) x = x * 10 + (ch ^ 48);
    }
    char buffer[SIZE];
    char *s = buffer;
    void flush() {//最后需要 flush!!
        fwrite(buffer, 1, s - buffer, stdout);
        s = buffer;
        fflush(stdout);
    }
    inline void print(const char ch) {
        if(s - buffer > SIZE - 2) flush();
        *s++ = ch;
    }
    inline void print(char *str) {//for string
        while(*str != 0)
            print(char(*str ++));
    }
    inline void print(int x) {
        static char buf[25];
        static char *p = buf;
        if (x <  0) print('-'), x = -x;
        if (x == 0) print('0');
        while(x) *(++ p) = x % 10, x /= 10;
        while(p != buf) print(char(*(p --) ^ 48));
    }
};
```

7.5 给定小数化成分数

```
# 本题答案的分母不超过 1e9, 给定小数的小数点位为 18 位
# 单次 O(log2n)
inf, inff = 10 ** 9, 10 ** 18
for i in range(int(input())):
    n = int(input()[2:])
    if n == 0: print('0 1')
    else:
        lp, lq, rp, rq = 0, 1, 1, 1
        while max(lq, rq) <= inf:
            mp, mq = lp + rp, lq + rq
            if mp * inff <= mq * n:
                l, r, mid, cnt = 1, (inf - lq) // rq + 1, -1, -1
                while l <= r:
                    mid = l + r >> 1
                    if (lp + rp * mid) * inff <= (lq + rq * mid) * n:
                        cnt, l = mid, mid + 1
                    else:
                        r = mid - 1
                lp, lq = lp + rp * cnt, lq + rq * cnt
            else:
                l, r, mid, cnt = 1, (inf - rq) // lq + 1, -1, -1
                    while l <= r:
                                mid = l + r >> 1
                        if (rp + lp * mid) * inff > (rq + lq * mid) * n:
                            cnt, l = mid, mid + 1
                        else:
                            r = mid - 1
            rp, rq = rp + lp * cnt, rq + lq * cnt
        if lq <= inf: print(lp, lq)
        else: print(rp, rq)
```