

# Algorithm Template Library

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# 1 图论

## 1.1 线段树维护树直径

```

/*LCA 用 ST 表, 总复杂度  $O(n \log)$ */
/* 每次询问删去两条边后, 剩下 3 棵树的直径长度 */
const int N = 2e5 + 5;
int T, n, m;
int len, head[N], ST[20][N];
struct edge{int u, v, w;}ee[N];
int cnt, fa[N], log_2[N], st[N], en[N], dfn[N], dis[N], dep[N], pos[N];
struct edges{int to, next, cost;}e[N];
void add(int u, int v, int w) {
    e[++ len] = (edges){v, head[u], w}, head[u] = len;
    e[++ len] = (edges){u, head[v], w}, head[v] = len;
}
void dfs1(int u) {
    st[u] = ++ cnt, dfn[cnt] = u;
    for (int v, i = head[u]; i; i = e[i].next) {
        v = e[i].to;
        if (v == fa[u]) continue;
        fa[v] = u, dep[v] = dep[u] + 1;
        dis[v] = dis[u] + e[i].cost, dfs1(v);
    }
    en[u] = cnt;
}
void dfs2(int u) {
    dfn[++ cnt] = u, pos[u] = cnt;
    for (int v, i = head[u]; i; i = e[i].next) {
        v = e[i].to;
        if (v == fa[u]) continue;
        dfs2(v), dfn[++ cnt] = u;
    }
}
int mmin(int x, int y) {
    if (dep[x] < dep[y]) return x;
    return y;
}
int lca(int u, int v) {
    static int w;
    if (pos[u] > pos[v]) swap(u, v);
    w = log_2[pos[v] - pos[u] + 1];
    return mmin(ST[w][pos[u]], ST[w][pos[v] - (1 << w) + 1]);
}
int dist(int u, int v) {
    int Lca = lca(u, v);
    return dis[u] + dis[v] - dis[Lca] * 2;
}
void build() {
    for (int i = 1; i <= cnt; i++)
        ST[0][i] = dfn[i];
    for (int i = 1; i < 20; i++)
        for (int j = 1; j <= cnt; j++)
            if (j + (1 << (i - 1)) > cnt) ST[i][j] = ST[i - 1][j];
}

```

```

        else ST[i][j] = mmmin(ST[i - 1][j], ST[i - 1][j + (1 << (i - 1))]);
    }
    int M;
    struct node {
        int l, r, dis;
    } tr[N << 1];
    void update(int o, int o1, int o2) {
        static int d; static node tmp;
        if (tr[o1].dis == -1) {tr[o] = tr[o2]; return;}
        if (tr[o2].dis == -1) {tr[o] = tr[o1]; return;}
        if (tr[o1].dis > tr[o2].dis) tmp = tr[o1];
        else tmp = tr[o2];
        d = dist(tr[o1].l, tr[o2].l);
        if (d > tmp.dis) tmp.l = tr[o1].l, tmp.r = tr[o2].l, tmp.dis = d;
        d = dist(tr[o1].l, tr[o2].r);
        if (d > tmp.dis) tmp.l = tr[o1].l, tmp.r = tr[o2].r, tmp.dis = d;
        d = dist(tr[o1].r, tr[o2].l);
        if (d > tmp.dis) tmp.l = tr[o1].r, tmp.r = tr[o2].l, tmp.dis = d;
        d = dist(tr[o1].r, tr[o2].r);
        if (d > tmp.dis) tmp.l = tr[o1].r, tmp.r = tr[o2].r, tmp.dis = d;
        tr[o] = tmp;
    }
    void ask(int s, int t) {
        if (s > t) return;
        for (s += M - 1, t += M + 1; s ^ t ^ 1; s >>= 1, t >>= 1) {
            if (~s&1) update(0, 0, s ^ 1);
            if (t&1) update(0, 0, t ^ 1);
        }
    }
    int main() {
        ios::sync_with_stdio(false);
        int u, v, w, ans; log_2[1] = 0;
        for (int i = 2; i <= 200000; i++)
            if (i == 1 << (log_2[i - 1] + 1))
                log_2[i] = log_2[i - 1] + 1;
            else log_2[i] = log_2[i - 1];
        for (cin >> T; T--; ) {
            cin >> n >> m, cnt = len = 0;
            for (int i = 1; i <= n; i++)
                head[i] = 0;
            for (int i = 1; i < n; i++) {
                cin >> ee[i].u >> ee[i].v >> ee[i].w;
                add(ee[i].u, ee[i].v, ee[i].w);
            }
            dfs1(1);
            for (M = 1; M < n + 2; M <<= 1);
            for (int i = 1; i <= n; i++)
                tr[i + M].l = tr[i + M].r = dfn[i], tr[i + M].dis = 0;
            for (int i = n + M + 1; i <= (M << 1) + 1; i++)
                tr[i].dis = -1;
            cnt = 0, dfs2(1), build();
            for (int i = M; i; i--)
                update(i, i << 1, i << 1 | 1);
            for (int i = 1; i < n; i++)

```

```

        if (dep[ee[i].u] > dep[ee[i].v])
            swap(ee[i].u, ee[i].v);
    for (int u, v, i = 1; i <= m; i++) {
        cin >> u >> v, ans = 0;
        u = ee[u].v, v = ee[v].v, w = lca(u, v);
        if (w == u || w == v) {
            if (w != u) swap(u, v);
            tr[0].dis = -1, ask(1, st[u] - 1), ask(en[u] + 1, n), ans = max(ans,
                ↪ tr[0].dis);
            tr[0].dis = -1, ask(st[u], st[v] - 1), ask(en[v] + 1, en[u]), ans =
                ↪ max(ans, tr[0].dis);
            tr[0].dis = -1, ask(st[v], en[v]), ans = max(ans, tr[0].dis);
        }
        else {
            if (st[u] > st[v]) swap(u, v);
            tr[0].dis = -1, ask(1, st[u] - 1), ask(en[u] + 1, st[v] - 1), ask(en[v] +
                ↪ 1, n), ans = max(ans, tr[0].dis);
            tr[0].dis = -1, ask(st[u], en[u]), ans = max(ans, tr[0].dis);
            tr[0].dis = -1, ask(st[v], en[v]), ans = max(ans, tr[0].dis);
        }
        printf("%d\n", ans);
    }
}
return 0;
}

```

## 1.2 有向图判断两个点能否到达

/\* 原题: $n$  个点的有向图,  $k$  对点  $(u, v)$  满足  $u$  可达  $v$   
 \*  $p$  对点  $(u, v)$  满足  $u$  不可达  $v$ , 问这个图是否存在  
 \* 解法: 按照  $p$  对可达点直接连边构图, 然后考虑验证  
 \*  $a[i][j]=0/1$  表示  $i$  是否可达  $j$ , 我们对  $j$  分块, *bitset* 加速  
 \* 时间  $O(n*m/32)$  空间  $O(n*n/blk)$   $blk$  随便取  
 \*/

```

const int N = 1e5 + 2;
const int BLK = 5000;
int n, k, p;
int d[N];
vector<int> e[N], f[N], ck[N];
int top, sta[N], in[N];
int cnt, dfn[N], low[N], vis[N];
int sum, bel[N];
bitset<BLK> a[N];
queue<int> q;
int topo[N];
void tarjan(int u) {
    vis[u] = in[u] = 1;
    sta[++top] = u, dfn[u] = low[u] = ++cnt;
    for (int v : e[u])
        if (!vis[v]) {
            tarjan(v);
            low[u] = min(low[v], low[u]);
        }
    else if (in[v])

```

```
        low[u] = min(low[v], low[u]);
    if (low[u] == dfn[u]) {
        sum++; int i;
        while (1) {
            i = sta[top--];
            in[i] = 0, bel[i] = sum;
            if (i == u) break;
        }
    }
}

int main() {
    cin >> n >> k;
    for (int u, v, i = 1; i <= k; i++) {
        cin >> u >> v;
        e[u].push_back(v);
    }
    cin >> p;
    for (int u, v, i = 1; i <= p; i++) {
        cin >> u >> v;
        f[u].push_back(v);
    }
    for (int i = 1; i <= n; i++)
        if (!vis[i])
            tarjan(i);
    for (int i = 1; i <= n; i++) {
        for (int j : f[i]) {
            if (bel[i] == bel[j]) return puts("NO"), 0;
            ck[bel[i]].push_back(bel[j]); //check
        }
        f[i].clear();
    }
    for (int i = 1; i <= n; i++)
        for (int j : e[i]) {
            if (bel[i] == bel[j]) continue;
            f[bel[i]].push_back(bel[j]);
            d[bel[j]]++;
        }
    cnt = 0;
    for (int i = 1; i <= sum; i++)
        if (!d[i])
            q.push(i);
    while (!q.empty()) {
        int now = q.front(); q.pop();
        topo[++ cnt] = now;
        for (int j : f[now]) {
            d[j]--;
            if (d[j] == 0) q.push(j);
        }
    }
    for (int i = 1, t = (sum + BLK - 1) / BLK; i <= t; i++) {
        for (int j = sum; j; j--) {
            int u = topo[j];
            a[u].reset();
            if (BLK * (i - 1) < u && u <= BLK * i)
```

```

        a[u][u - BLK * (i - 1) - 1] = 1;
    for (int v : f[u])
        a[u] |= a[v];
}
for (int j = 1; j <= sum; j++)
    for (int v : ck[j])
        if (BLK * (i - 1) < v && v <= BLK * i &&
            a[j][v - BLK * (i - 1) - 1] == 1) {
            puts("NO");
            return 0;
        }
}
printf("YES\n%d\n", k);
for (int i = 1; i <= n; i++)
    for (int j : e[i])
        printf("%d %d\n", i, j);
return 0;
}

```

### 1.3 spfa 费用流

```

const int N = 600, M = 800000, inf = 0x3f3f3f3f;
int s, t, ans, len, maxflow;
int T, n, m, K, W;
int head[N], incf[N], path[N], pre[N], vis[N], d[N];
struct edge {int to, next, cap, cost;} e[M];
struct video {int s, t, w, op;} a[N];
void add(int u, int v, int w, int c) {
    e[++len] = (edge){v, head[u], w, c}, head[u] = len;
    e[++len] = (edge){u, head[v], 0, -c}, head[v] = len;
}
bool spfa() {
    deque<int> q;
    q.push_back(s), incf[s] = inf;
    for (int i = 1; i <= t; i++) d[i] = inf;
    d[s] = 0;
    while (!q.empty()) {
        int x = q.front();
        q.pop_front(), vis[x] = 0;
        for (int i = head[x]; i; i = e[i].next) {
            if (e[i].cap && d[e[i].to] > d[x] + e[i].cost) {
                d[e[i].to] = d[x] + e[i].cost;
                pre[e[i].to] = x, path[e[i].to] = i;
                incf[e[i].to] = min(incf[x], e[i].cap);
                if (!vis[e[i].to]) {
                    vis[e[i].to] = 1;
                    if (q.empty() || d[e[i].to] < d[q.front()])
                        q.push_front(e[i].to);
                    else q.push_back(e[i].to);
                }
            }
        }
    }
}
maxflow += incf[t];

```

```

    if (d[t] == inf) return 0;
    for (int i = t; i != s; i = pre[i]) {
        e[path[i]].cap -= incf[t];
        e[path[i] ^ 1].cap += incf[t];
    }
    return ans += incf[t] * d[t], 1;
}
int main() {
    /*build graph*/
    while(spfa());
}

```

## 1.4 dsu

```

/*DSU
* 用途:  $O(n \log n)$  解决无修改的子树询问问题, 需要保证操作支持删除
* 解决方法: 对于每个节点, 先对所有轻儿子, dfs 下去求一遍, 再消除影响
*           然后再 dfs 自己的重儿子, 然后不消除影响, 再加上所有轻儿子
*           就得到当前节点为根的子树的答案了
*/
int n, c[N];
int cnt[N], maxCnt;
int siz[N], son[N];
vector<int> e[N];
ll ans[N], sum[N];
void dfs1(int u, int fr) {
    siz[u] = 1;
    for (int v : e[u]) {
        if (v == fr) continue;
        dfs1(v, u);
        siz[u] += siz[v];
        if (siz[v] > siz[son[u]]) son[u] = v;
    }
}
void update(int x, int y) {
    sum[cnt[x]] -= x;
    cnt[x] += y;
    sum[cnt[x]] += x;
    if (cnt[x] > maxCnt) maxCnt = cnt[x];
    if (sum[maxCnt] == 0) maxCnt--;
}
void dfs3(int u, int fr, int val) {
    update(c[u], val);
    for (int v : e[u]) {
        if (v == fr) continue;
        dfs3(v, u, val);
    }
}
void dfs2(int u, int fr) {
    for (int v : e[u]) {
        if (v == fr || v == son[u]) continue;
        dfs2(v, u), dfs3(v, u, -1);
    }
    if (son[u]) dfs2(son[u], u);
}

```



```

    for (int v : e[u]) {
        if (v == fr || v == son[u]) continue;
        dfs3(v, u, 1);
    }
    update(c[u], 1);
    ans[u] = sum[maxCnt];
}
int main() {
    ios::sync_with_stdio(false);
    cin >> n;
    for (int i = 1; i <= n; i++)
        cin >> c[i];
    for (int u, v, i = 1; i < n; i++) {
        cin >> u >> v;
        e[u].push_back(v);
        e[v].push_back(u);
    }
    dfs1(1, 1), dfs2(1, 1);
    for (int i = 1; i <= n; i++)
        cout << ans[i] << ' ';
    return 0;
}

```

## 1.5 长链剖分

/\* 长链剖分，选择深度最大的儿子作为重儿子，用于合并以深度为下标的信息  
 \* 像 *dsu* 一样，直接继承重儿子信息，然后按深度暴力合并其他儿子信息  
 \* 时间复杂度考虑每个节点作为轻儿子中的节点被合并只会有一次，所以  $O(n)$   
 \* 另一种用法，可以  $O(n \log n)$  预处理后， $O(1)$  找到  $k$  级祖先  
 \* *example problem*: 给个树，第  $i$  个点有两个权值  $a_i$  和  $b_i$   
 \* 现在求一条长度为  $m$  的路径，使得  $\sum a_i / \sum b_i$  最小  
 \* 防爆栈 *trick*: 像重链剖分一样改成 *bfs*  
 \*/

```

int n, m;
double k, a[N], b[N];
int len[N], son[N];
vector<int> e[N];
double tmp[N], *ptr, *f[N], temp[N];
void dfs(int u, int fr) {
    for (int v : e[u]) {
        if (v == fr) continue;
        dfs(v, u);
        if (len[v] > len[son[u]]) son[u] = v;
    }
    len[u] = len[son[u]] + 1;
}
inline double F(int x, int y) {return y >= len[x] ? 0 : f[x][y];}
bool solve(int u, int fr) {
    /* 为实现  $O(1)$  继承，采用  $f[u]-f[v]$  来保存  $u-fa[v]$  路径上的最小权值和 ( $dep[v]>dep[u]$ )
    * 即自底向上累加
    */
    if (son[u]) {
        f[son[u]] = f[u] + 1;
        if (solve(son[u], u)) return 1;
    }
}

```

```

    f[u][0] = val[u] + f[u][1];
    if (len[u] >= m && f[u][0] - F(u, m) <= 0) return 1;
    for (int v : e[u]) {
        if (v == son[u] || v == fr) continue;
        f[v] = ptr, ptr += len[v];
        if (solve(v, u)) return 1;
        for (int j = 1; j <= len[v] && j <= m; j++) {
            if (len[u] + j < m) continue;
            if (f[v][0] - F(v, j) + f[u][0] - F(u, m - j) <= 0) return 1;
        }
        temp[0] = val[u];
        for (int j = 1; j <= len[v]; j++)
            temp[j] = val[u] + min(f[u][1] - F(u, j + 1), f[v][0] - F(v, j));
        if (len[v] + 1 == len[u]) f[u][0] = temp[len[v]];
        for (int j = 1; j <= len[v]; j++)
            f[u][j] = f[u][0] - temp[j - 1];
        if (len[v] + 1 != len[u]) f[u][len[v] + 1] = f[u][0] - temp[len[v]];
    }
}
else {
    f[u][0] = val[u];
    if (m == 1 && f[u][0] <= 0) return 1;
}
return 0;
}

bool judge(double mid) {
    f[1] = ptr = tmp, ptr += len[1], k = mid;
    return solve(1, 1);
}

int main() {
    cin >> n >> m;
    for (int i = 1; i <= n; i++) cin >> a[i];
    for (int i = 1; i <= n; i++) cin >> b[i];
    if (m == -1) {
        double ans = 1e9;
        for (int i = 1; i <= n; i++)
            ans = min(ans, a[i] / b[i]);
        printf("%.2f\n", ans);
        return 0;
    }
    for (int u, v, i = 1; i < n; i++) {
        cin >> u >> v;
        e[u].push_back(v);
        e[v].push_back(u);
    }
    dfs(1, 1);
    int flag = 0;
    double l = 0, r = 2e5, mid, ans;
    for (int i = 0; i < 50; i++) {
        mid = (l + r) / 2;
        if (judge(mid)) r = mid - eps, flag = 1, ans = mid;
        else l = mid + eps;
    }
    if (flag) printf("%.2f\n", ans);
}

```

```

    else puts("-1");
    return 0;
}

```

## 1.6 LCT

```

const int N = 3e5 + 5;
int n, m;
int fa[N], ch[N][2], rev[N], val[N], sum[N];
int sta[N], top;
bool isroot(int x) {
    return ch[fa[x]][0] != x && ch[fa[x]][1] != x;
}
void reverse(int x) {
    if (!x) return;
    swap(ch[x][0], ch[x][1]);
    rev[x] ^= 1;
}
void pushdown(int x) {
    if (!rev[x]) return;
    reverse(ch[x][0]);
    reverse(ch[x][1]);
    rev[x] = 0;
}
void pushup(int x) {
    sum[x] = sum[ch[x][0]] ^ sum[ch[x][1]] ^ val[x];
}
void rot(int x) {
    int y = fa[x], z = fa[y], d = ch[y][1] == x, c = ch[x][!d];
    fa[x] = z; if (!isroot(y)) ch[z][ch[z][1] == y] = x;
    ch[y][d] = c; if (c) fa[c] = y;
    fa[y] = x, ch[x][!d] = y;
    pushup(y), pushup(x);
}
void splay(int x) {
    int u = x, top = 0, y, z;
    while (!isroot(u)) sta[++top] = u, u = fa[u];
    sta[++top] = u;
    while (top) pushdown(sta[top--]);
    while (!isroot(x)) {
        y = fa[x], z = fa[y];
        if (!isroot(y)) {
            if ((ch[z][0] == y) ^ (ch[y][0] == x)) rot(x);
            else rot(y);
        }
        rot(x);
    }
}
void access(int x) { //把 x 到根的路径拎出来
    for (int y = 0; x != 0; y = x, x = fa[x]) {
        splay(x), ch[x][1] = y, pushup(x);
    }
}
void makeroot(int x) { //令 x 成为这棵树的根

```

```

    access(x), splay(x), reverse(x);
}
int findroot(int x) { //找根
    access(x), splay(x);
    while (ch[x][0]) pushdown(x), x = ch[x][0];
    splay(x); //把根转到顶保证复杂度
    return x;
}
void split(int x, int y) { //拉出 x-y 的路径
    makeroot(x);
    access(y), splay(y);
    //y 存了这条路径的信息
}
void link(int x, int y) {
    makeroot(x);
    if (findroot(y) != x) fa[x] = y;
}
void cut(int x, int y) {
    makeroot(x);
    if (findroot(y) == x && fa[y] == x && ch[x][1] == y) {
        fa[y] = ch[x][1] = 0;
        pushup(x);
    }
}
int main() {
    ios::sync_with_stdio(false);
    cin >> n >> m;
    for (int i = 1; i <= n; i++)
        cin >> val[i];
    for (int op, x, y; m--; ) {
        cin >> op >> x >> y;
        switch(op) {
            case 0: split(x, y); printf("%d\n", sum[y]); break;
            case 1: link(x, y); break;
            case 2: cut(x, y); break;
            case 3: splay(x); val[x] = y; pushup(x); break;
            //单点更新完记得 pushup
        }
    }
}

```

## 1.7 hungary

```

/* 二分图最大匹配, 时间复杂度  $O(nm)$ 
 * 例题:  $n1$  个男,  $n2$  个女,  $m$  个配对关系
 * 输出最大配对数, 然后对于每个男输出配对的女
 */
int n1, n2, m;
vector<int> e[N];
int vis[N], pre[N], ans, tim;
bool dfs(int u) {
    if (vis[u] == tim) return 0;
    vis[u] = tim;
    for (int v : e[u])

```

```

        if (!pre[v] || dfs(pre[v]))
            return pre[v] = u, 1;
    return 0;
}
int main() {
    scanf("%d %d %d", &n1, &n2, &m);
    for (int u, v, i = 1; i <= m; i++) {
        scanf("%d %d", &u, &v);
        e[v].push_back(u);
        // 要输出左侧点连接的右侧点, 连边时就由右边点向左边连边
        // 连边 (u->v), 输出的 pre[v] 就是右边的点了
    }
    for (tim = 1; tim <= n2; tim++)
        if (dfs(tim)) ans++;
    printf("%d\n", ans);
    for (int i = 1; i <= n1; i++)
        printf("%d%c", pre[i], i == n1 ? '\n' : ' ');
    return 0;
}

```

## 1.8 dinic 最大流

```

const int N = 20000;
const int M = 500000;
const int inf = 0x3f3f3f3f;
int n, m;
int s, t, len = 1;
int to[M], cap[M], nex[M];
int g[N], p[N], q[N], d[N];
void add(int x, int y, int v) {
    to[++len] = y, cap[len] = v, nex[len] = g[x], g[x] = len;
    to[++len] = x, cap[len] = 0, nex[len] = g[y], g[y] = len;
}
bool bfs() {
    int l = 1, r = 1, x, i;
    memset(d, 0, sizeof d);
    d[s] = 1, q[1] = s;
    while (l <= r) {
        x = q[l++];
        for (i = g[x]; i; i = nex[i])
            if (cap[i] && !d[to[i]])
                d[to[i]] = d[x] + 1, q[++r] = to[i];
    }
    return d[t];
}
int dfs(int x, int y) {
    if (x == t || y == 0) return y;
    int flow = 0;
    for (int &i = p[x]; i; i = nex[i]) {
        if (!cap[i] || d[to[i]] != d[x] + 1) continue;
        int f = dfs(to[i], min(y, cap[i]));
        flow += f, y -= f;
        cap[i] -= f, cap[i ^ 1] += f;
        if (!y) break;
    }
    return flow;
}

```

```

    }
    return flow;
}
int dinic() {
    int maxflow = 0;
    while (bfs()) {
        memcpy(p, g, sizeof g);
        maxflow += dfs(s, inf);
    }
    return maxflow;
}

```

## 1.9 DAG 删去无用边

/\* 无用边定义：对于边  $(u, v)$  如果存在从  $u$  到  $v$  不经过该边的另一条路径，则称该边无用

\* 时间复杂度： $O(n^3)$  \*/

```

bool f[N][N]; // i 是否可达 j
vector<int> e[N];
int main() {
    rep(i, 1, n)
        for (int j : e[i]) {
            rep(k, 1, n)
                if (i != k && j != k && f[i][k] && f[k][j])
                    no_use_edge;
        }
}

```

## 1.10 树分治

```

const int N = 1e5 + 5;
vector<int> e[N];
int n, a[N];
int root, _left, vis[N];
int siz[N], maxv[N];
void find_root(int u, int fr) {
    siz[u] = 1, maxv[u] = 0;
    for (int v : e[u]) {
        if (v == fr || vis[v]) continue;
        find_root(v, u);
        siz[u] += siz[v];
        maxv[u] = max(maxv[u], siz[v]);
    }
    maxv[u] = max(maxv[u], _left - siz[u]);
    if (!root || maxv[u] < maxv[root])
        root = u;
}
void dfs(int u, int fr) {
    siz[u] = 1;
    for (int v : e[u]) {
        if (v == fr || vis[v]) continue;
        find_root(v, u);
        siz[u] += siz[v];
    }
}

```

```
void solve(int u, int w) {
    dfs(u, u); //update siz[]
    a[u] = w, vis[u] = 1;
    for (int v : e[u]) {
        if (vis[v]) continue;
        _left = siz[v];
        root = 0;
        find_root(v, v);
        solve(root, w + 1);
    }
}

int main() {
    ios::sync_with_stdio(false);
    cin >> n;
    for (int u, v, i = 1; i < n; i++) {
        cin >> u >> v;
        e[u].push_back(v);
        e[v].push_back(u);
    }
    _left = n, root = 0, find_root(1, 1);
    solve(root, 0);
    for (int i = 1; i <= n; i++)
        printf("%c ", 'A' + a[i]);
    return 0;
}
```

## 2 数据结构

### 2.1 splay

```
#define mid (l + r >> 1)
int n, m, a, b, len, tot;
struct node {
    bool rev; //翻转标记
    int v, siz;
    node *c[2];
    node():rev(0),v(0),siz(0),c{NULL, NULL}{}
    node *init(int x);
    void pushdown();
    void mata() {siz = c[0] -> siz + c[1] -> siz + 1;}
    int cmp(int k) {return k<=c[0]->siz?0:(k==c[0]->siz+1?-1:1);}
    void print();
}pool[N], *null = new node();
node *node::init(int x){rev=0, v=x, siz=1, c[0]=c[1]=null;return this;}
void node::pushdown() {
    if (!rev) return;
    if (c[0] != null) c[0] -> rev ^= 1;
    if (c[1] != null) c[1] -> rev ^= 1;
    swap(c[0], c[1]), rev = 0;
}
void node::print() {
    pushdown();
    if (c[0] != null) c[0] -> print();
    if (1 <= v && v <= n) printf("%d ", v);
    if (c[1] != null) c[1] -> print();
}
node *build(int l, int r) {//初始序列为 1-n
    if (l == r) return pool[tot++].init(r);
    node *tmp = pool[tot++].init(mid);
    if (l < mid) tmp -> c[0] = build(l, mid - 1);
    if (mid < r) tmp -> c[1] = build(mid + 1, r);
    tmp -> mata(); return tmp;
}
void rot(node *&o, int k) {//把 k 儿子提上来
    o -> pushdown(); node *tmp = o -> c[k];
    tmp -> pushdown(); o -> c[k] = tmp -> c[!k];
    tmp -> c[!k] -> pushdown(); tmp -> c[!k] = o;
    o -> mata(), tmp -> mata(), o = tmp;
}
void splay(node *&o, int k) {//把以 o 为根的 splayTree 中 rk 为 k 的点提到根
    int k1 = o -> cmp(k); o -> pushdown();
    if (k1 == -1) return; o -> c[k1] -> pushdown();
    if (k1) k -= o -> c[0] -> siz + 1;
    int k2 = o -> c[k1] -> cmp(k);
    if (~k2) {//k2 != -1
        if (k2) k -= o -> c[k1] -> c[0] -> siz + 1;
        o -> c[k1] -> c[k2] -> pushdown();
        splay(o -> c[k1] -> c[k2], k);
        if (k2 == k1) rot(o, k1);
        else rot(o -> c[k1], k2);
    }
```



```

    }
    rot(o, k1);
}
int main() {
    scanf("%d %d", &n, &m);
    node *root = build(0, n + 1); //方便边界左右处理各多开一个
    for (; m --; ) {
        scanf("%d %d", &a, &b), a ++, b ++, len = b - a + 1;
        splay(root, a - 1), splay(root -> c[1], len + 1);
        root -> c[1] -> c[0] -> rev ^= 1, root -> c[1] -> c[0] -> pushdown();
    }
    root -> print(); return 0;
}

```

## 2.2 treap

/\* 容易实现的预开内存池 *treap*, 每次 *head* 清空即可  
 如果初始要插入  $n$  个 1, 可改为类似 *splay* 的  $O(n)build$  写法  
 \*/

```

const int poolSize = 5e5 + 10;
struct node {
    node *c[2];
    int v, r, siz;
    void update();
    void init(int x);
};
node *null = new node(), *root = null;
void node::update() {
    siz = c[0] -> siz + c[1] -> siz + 1;
}
void node::init(int x) {
    v = x, r = rand(), siz = 1;
    c[0] = c[1] = null;
}
node nodesPool[poolSize];
int head; //每次 head=0 清空
node *newnode(int x) {
    node *res = &nodesPool[head ++];
    res -> init(x);
    return res;
}
void rot(node *&o, int d) {
    node *tmp = o -> c[!d];
    o -> c[!d] = tmp -> c[d], tmp -> c[d] = o;
    o -> update(), tmp -> update(), o = tmp;
}
void insert(node *&o, int x) {
    if (o == null) {
        o = newnode(x);
        return;
    }
    int d = x > o -> v ? 0 : 1;
    insert(o -> c[d], x);
    if (o -> c[d] -> r < o -> r) rot(o, !d);
}

```

```

        o -> update();
    }
    void del(node *&o, int x) {
        if (x == o -> v) {
            if (o -> c[0] == null) {o = o -> c[1]; return;}
            if (o -> c[1] == null) {o = o -> c[0]; return;}
            int d = o -> c[0] -> r < o -> c[1] -> r ? 1 : 0;
            rot(o, d), del(o -> c[d], x);
        }
        else del(o -> c[x <= o -> v], x);
        o -> update();
    }
    void build(node *&o, int l, int r) {
        o = newnode(1);
        if (l == r) return;
        int mid = l + r >> 1;
        if (l < mid) build(o -> c[0], l, mid - 1);
        if (o -> c[0] != null && o -> c[0] -> r < o -> r) swap(o -> c[0] -> r, o -> r);
        if (mid < r) build(o -> c[1], mid + 1, r);
        if (o -> c[1] != null && o -> c[1] -> r < o -> r) swap(o -> c[1] -> r, o -> r);
        o -> update();
    }
}

```

## 2.3 主席树

```

const int N = 130000;
const int M = N * 20;
//主席树节点数, 可以直接稳妥选择  $N*(5+\log N)$ 
struct {
    int siz, l, r;
    ll sum, val;
}tr[M];
#define l(x) tr[x].l
#define r(x) tr[x].r
#define s(x) tr[x].sum
#define v(x) tr[x].val
#define sz(x) tr[x].siz
#define mid (l + r >> 1)
int tot, root[N];
int build(int l, int r) {
    int x = ++tot;
    if (l < r) {
        l(x) = build(l, mid);
        r(x) = build(mid + 1, r);
    }
    return x;
}
int change(int o, int l, int r, int p, int y) {
    //在  $p$  的位置插入一个  $y$ 
    int x = ++tot;
    s(x) = s(o) + y, sz(x) = sz(o) + 1;
    l(x) = l(o), r(x) = r(o), v(x) = y;
    if (l < r) {
        if (p <= mid) l(x) = change(l(o), l, mid, p, y);
    }
}

```

```

        else r(x) = change(r(o), mid + 1, r, p, y);
    }
    return x;
}
ll ask(int o1, int o2, int l, int r, int k) {
    //求 (l,r] 区间前 k 小的数之和, 有 o1=root[l], o2=root[r]
    if (l == r) return v(o2) * k;
    int lsz = sz(l(o2)) - sz(l(o1));
    if (lsz == k) return s(l(o2)) - s(l(o1));
    if (lsz < k) return s(l(o2)) - s(l(o1)) + ask(r(o1), r(o2), mid + 1, r, k - lsz);
    return ask(l(o1), l(o2), l, mid, k);
}

```

## 2.4 吉老师线段树

```

/* 区间每个数变为 min(a[i],t) + 区间最大 + 区间和 O(nlog)* */
const int N = (1 << 20) + 5;
#define lc (o << 1)
#define rc (lc | 1)
struct node {
    int max1, max2, cnt;
    ll sum;
} tr[N << 1];
int T, n, m;
int op, x, y, t;
int a[N];
void pushup(int o) {
    if (tr[lc].max1 == tr[rc].max1) {
        tr[o].max1 = tr[lc].max1;
        tr[o].cnt = tr[lc].cnt + tr[rc].cnt;
        tr[o].max2 = max(tr[lc].max2, tr[rc].max2);
    }
    else {
        if (tr[lc].max1 > tr[rc].max1) {
            tr[o] = tr[lc];
            tr[o].max2 = max(tr[o].max2, tr[rc].max1);
        }
        else {
            tr[o] = tr[rc];
            tr[o].max2 = max(tr[o].max2, tr[lc].max1);
        }
    }
    tr[o].sum = tr[lc].sum + tr[rc].sum;
}
void pushdown(int o) {
    if (tr[o].max1 < tr[lc].max1) {
        tr[lc].sum += 1ll * (tr[o].max1 - tr[lc].max1) * tr[lc].cnt;
        tr[lc].max1 = tr[o].max1;
    }
    if (tr[o].max1 < tr[rc].max1) {
        tr[rc].sum += 1ll * (tr[o].max1 - tr[rc].max1) * tr[rc].cnt;
        tr[rc].max1 = tr[o].max1;
    }
}

```

```
void build(int o, int l, int r) {
    if (l == r) {
        tr[o].max1 = tr[o].sum = a[r];
        tr[o].cnt = 1, tr[o].max2 = 0;
        return;
    }
    int mid = l + r >> 1;
    build(lc, l, mid);
    build(rc, mid + 1, r);
    pushup(o);
}

void update(int o, int l, int r, int s, int t, int v) {
    if (v >= tr[o].max1) return;
    pushdown(o);
    if (s <= l && r <= t) {
        if (v > tr[o].max2) {
            tr[o].sum += 1ll * (v - tr[o].max1) * tr[o].cnt;
            tr[o].max1 = v;
            return;
        }
    }
    int mid = l + r >> 1;
    if (s <= mid) update(lc, l, mid, s, t, v);
    if (t > mid) update(rc, mid + 1, r, s, t, v);
    pushup(o);
}

ll ask_max(int o, int l, int r, int s, int t) {
    if (s <= l && r <= t) return tr[o].max1;
    pushdown(o);
    int mid = l + r >> 1;
    ll res = 0;
    if (s <= mid) res = max(res, ask_max(lc, l, mid, s, t));
    if (t > mid) res = max(res, ask_max(rc, mid + 1, r, s, t));
    return res;
}

ll ask_sum(int o, int l, int r, int s, int t) {
    if (s <= l && r <= t) return tr[o].sum;
    pushdown(o);
    int mid = l + r >> 1;
    ll res = 0;
    if (s <= mid) res += ask_sum(lc, l, mid, s, t);
    if (t > mid) res += ask_sum(rc, mid + 1, r, s, t);
    return res;
}

int main() {
    ios::sync_with_stdio(false);
    for (cin >> T; T--; ) {
        cin >> n >> m;
        for (int i = 1; i <= n; i++)
            cin >> a[i];
        build(1, 1, n);
        while (m--) {
            cin >> op;
            if (op == 0) {
```

```

        cin >> x >> y >> t;
        update(1, 1, n, x, y, t);
    }
    else if (op == 1) {
        cin >> x >> y;
        cout << ask_max(1, 1, n, x, y) << '\n';
    }
    else {
        cin >> x >> y;
        cout << ask_sum(1, 1, n, x, y) << '\n';
    }
}
}
return 0;
}

```

## 2.5 KDTree

### 2.5.1 3 维 KDtree

```

/*O(n*n^(1-1/k)), k 为维度 */
const int N = 1e5 + 5;
const int Mod = 1e9 + 7;
int nowD, ans, x[3], y[3];
int n, m, a[N], b[N], c[N], d[N];
struct node {
    int Max[3], Min[3], d[3];
    int val, maxv;
    node *c[2];
    node() {
        c[0] = c[1] = NULL;
        val = maxv = 0;
    }
    void pushup();
    bool operator < (const node &a) const {
        return d[nowD] < a.d[nowD];
    }
} Null, nodes[N];
node *root = &Null;
inline void node::pushup() {
    if (c[0] != &Null) {
        if (c[0] -> Max[1] > Max[1]) Max[1] = c[0] -> Max[1];
        if (c[0] -> Max[2] > Max[2]) Max[2] = c[0] -> Max[2];
        if (c[0] -> Min[0] < Min[0]) Min[0] = c[0] -> Min[0];
        if (c[0] -> Min[2] < Min[2]) Min[2] = c[0] -> Min[2];
        if (c[0] -> maxv > maxv) maxv = c[0] -> maxv;
    }
    if (c[1] != &Null) {
        if (c[1] -> Max[1] > Max[1]) Max[1] = c[1] -> Max[1];
        if (c[1] -> Max[2] > Max[2]) Max[2] = c[1] -> Max[2];
        if (c[1] -> Min[0] < Min[0]) Min[0] = c[1] -> Min[0];
        if (c[1] -> Min[2] < Min[2]) Min[2] = c[1] -> Min[2];
        if (c[1] -> maxv > maxv) maxv = c[1] -> maxv;
    }
}
}

```

```
inline node *build(int l, int r) {
    int mid = l + r >> 1; nowD = rand() % 3;
    nth_element(nodes + l, nodes + mid, nodes + r + 1);
    node *res = &nodes[mid];
    if (l != mid) res -> c[0] = build(l, mid - 1);
    else res -> c[0] = &Null;
    if (r != mid) res -> c[1] = build(mid + 1, r);
    else res -> c[1] = &Null;
    res -> pushup();
    return res;
}

inline int calc(node *o) {
    if (y[0] < o -> Min[0] || x[1] > o -> Max[1] || x[2] > o -> Max[2] || y[2] < o ->
        Min[2]) return -1;
    return o -> maxv;
}

inline void query(node *o) {
    if (o -> val > ans && y[0] >= o -> d[0] && x[1] <= o -> d[1] && x[2] <= o -> d[2] &&
        y[2] >= o -> d[2]) ans = o -> val;
    int dl, dr;
    if (o -> c[0] != &Null) dl = calc(o -> c[0]);
    else dl = -1;
    if (o -> c[1] != &Null) dr = calc(o -> c[1]);
    else dr = -1;
    if (dl > dr) {
        if (dl > ans) query(o -> c[0]);
        if (dr > ans) query(o -> c[1]);
    } else {
        if (dr > ans) query(o -> c[1]);
        if (dl > ans) query(o -> c[0]);
    }
}

int main() {
    ios::sync_with_stdio(false);
    cin >> n >> m;
    for (int i = 1; i <= n; i++) {
        cin >> a[i];
        b[i] = d[a[i]];
        d[a[i]] = i;
    }
    for (int i = 1; i <= n; i++) d[i] = n + 1;
    for (int i = n; i; i--) {
        c[i] = d[a[i]];
        d[a[i]] = i;
    }
    for (int i = 1; i <= n; i++) {
        nodes[i].Min[0] = nodes[i].d[0] = b[i];
        nodes[i].Max[1] = nodes[i].d[1] = c[i];
        nodes[i].Max[2] = nodes[i].Min[2] = nodes[i].d[2] = i;
        nodes[i].val = nodes[i].maxv = a[i];
    }
    root = build(1, n);
    for (int l, r; m--; ) {
        cin >> l >> r;
```

```

        l = (l + ans) % n + 1;
        r = (r + ans) % n + 1;
        if (l > r) swap(l, r);
        y[0] = l - 1;
        x[1] = r + 1;
        x[2] = l, y[2] = r;
        ans = 0, query(root);
        cout << ans << endl;
    }
    cout << endl;
    return 0;
}

```

### 2.5.2 KDtree 二维空间区间覆盖单点查询

```

/* 类似线段树 */
const int N = 1e5 + 5;
const int Mod = 1e9 + 7;
int nowD, x[2], y[2], z;
struct node {
    int Max[2], Min[2], d[2];
    int val, lazy;
    node *c[2];
    node() {
        c[0] = c[1] = NULL;
    }
    void pushup();
    void pushdown();
    bool operator < (const node &a) const {
        return d[nowD] < a.d[nowD];
    }
} Null, nodes[N];
node *root = &Null;
inline void node::pushup() {
    if (c[0] != &Null) {
        if (c[0] -> Max[0] > Max[0]) Max[0] = c[0] -> Max[0];
        if (c[0] -> Max[1] > Max[1]) Max[1] = c[0] -> Max[1];
        if (c[0] -> Min[0] < Min[0]) Min[0] = c[0] -> Min[0];
        if (c[0] -> Min[1] < Min[1]) Min[1] = c[0] -> Min[1];
    }
    if (c[1] != &Null) {
        if (c[1] -> Max[0] > Max[0]) Max[0] = c[1] -> Max[0];
        if (c[1] -> Max[1] > Max[1]) Max[1] = c[1] -> Max[1];
        if (c[1] -> Min[0] < Min[0]) Min[0] = c[1] -> Min[0];
        if (c[1] -> Min[1] < Min[1]) Min[1] = c[1] -> Min[1];
    }
}
inline void node::pushdown() {
    if (c[0] != &Null) c[0] -> val = c[0] -> lazy = lazy;
    if (c[1] != &Null) c[1] -> val = c[1] -> lazy = lazy;
    lazy = -1;
}
inline node *build(int l, int r, int D) {
    int mid = l + r >> 1; nowD = D;

```

```

    nth_element(nodes + l, nodes + mid, nodes + r + 1);
    node *res = &nodes[mid];
    if (l != mid) res -> c[0] = build(l, mid - 1, !D);
    else res -> c[0] = &Null;
    if (r != mid) res -> c[1] = build(mid + 1, r, !D);
    else res -> c[1] = &Null;
    res -> pushup();
    return res;
}

inline int query(node *o) {
    if (o == &Null) return -1;
    if (o -> lazy != -1) o -> pushdown();
    if (x[o] > o -> Max[0] || y[o] > o -> Max[1] || x[o] < o -> Min[0] || y[o] < o ->
        Min[1]) return -1;
    if (x[o] == o -> d[0]) return o -> val;
    return max(query(o -> c[0]), query(o -> c[1]));
}

inline void modify(node *o) {
    if (o == &Null) return;
    if (o -> lazy != -1) o -> pushdown();
    if (x[o] > o -> Max[0] || y[o] > o -> Max[1] || x[o] < o -> Min[0] || y[o] < o ->
        Min[1]) return;
    if (x[o] <= o -> Min[0] && y[o] <= o -> Min[1] && x[o] >= o -> Max[0] && y[o] >= o ->
        Max[1]) {
        o -> val = o -> lazy = z;
        return;
    }
    if (x[o] <= o -> d[0] && y[o] <= o -> d[1] && x[o] >= o -> d[0] && y[o] >= o -> d[1])
        o -> val = z;
    modify(o -> c[0]), modify(o -> c[1]);
}

int n, m, k, a[N], c[N], d[N];
int cnt, st[N], en[N], dfn[N], dep[N];
vector<int> e[N];
void dfs(int u) {
    st[u] = ++ cnt, dfn[cnt] = u;
    for (int v : e[u])
        dep[v] = dep[u] + 1, dfs(v);
    en[u] = cnt;
}

int main() {
    ios::sync_with_stdio(false);
    int T, ans;
    for (cin >> T; T --; ) {
        cin >> n >> m >> k, ans = cnt = 0;
        for (int i = 1; i <= n; i++)
            e[i].clear();
        for (int u, i = 2; i <= n; i++) {
            cin >> u;
            e[u].push_back(i);
        }
        dfs(1);
        for (int i = 1; i <= n; i++) {
            nodes[i].Min[0] = nodes[i].Max[0] = nodes[i].d[0] = i;

```



```

        nodes[i].Min[1] = nodes[i].Max[1] = nodes[i].d[1] = dep[dfn[i]];
        nodes[i].val = 1, nodes[i].lazy = -1;
    }
    root = build(1, n, 0);
    for (int u, v, w, i = 1; i <= k; i++) {
        cin >> u >> v >> w;
        if (w == 0) {
            x[0] = st[u], y[0] = dep[u];
            ans = (ans + 1ll * i * query(root) % Mod) % Mod;
        } else {
            x[0] = st[u], x[1] = en[u];
            y[0] = dep[u], y[1] = dep[u] + v;
            z = w, modify(root);
        }
    }
    cout << ans << endl;
}
return 0;
}

```

### 2.5.3 KDtree 二维空间单点修改区间查询

```

/*
 * 调整重构系数可以影响常数
 * 询问多就让系数接近 0.70-0.75, 询问少就让系数在 0.8-0.90
 */
const int inf = 1e9;
int n, m, tot, nowD;
struct node {
    int Max[2], Min[2], d[2];
    int sum, siz, val;
    node *c[2];
    node() {
        Max[0] = Max[1] = -inf;
        Min[0] = Min[1] = inf;
        sum = val = siz = 0;
        c[0] = c[1] = NULL;
        d[0] = d[1] = 0;
    }
    void update();
} Null, nodes[200010], *temp[200010];
node *root = &Null;
inline void node::update() {
    siz = c[0] -> siz + c[1] -> siz + 1;
    sum = c[0] -> sum + c[1] -> sum + val;
    if (c[0] != &Null) {
        if (c[0] -> Max[0] > Max[0]) Max[0] = c[0] -> Max[0];
        if (c[0] -> Max[1] > Max[1]) Max[1] = c[0] -> Max[1];
        if (c[0] -> Min[0] < Min[0]) Min[0] = c[0] -> Min[0];
        if (c[0] -> Min[1] < Min[1]) Min[1] = c[0] -> Min[1];
    }
    if (c[1] != &Null) {
        if (c[1] -> Max[0] > Max[0]) Max[0] = c[1] -> Max[0];
        if (c[1] -> Max[1] > Max[1]) Max[1] = c[1] -> Max[1];
    }
}

```

```

        if (c[1] -> Min[0] < Min[0]) Min[0] = c[1] -> Min[0];
        if (c[1] -> Min[1] < Min[1]) Min[1] = c[1] -> Min[1];
    }
}

inline bool cmp(const node *a, const node *b) {
    return a -> d[nowD] < b -> d[nowD];
}

inline void traverse(node *o) {
    if (o == &Null) return;
    temp[++ tot] = o;
    traverse(o -> c[0]);
    traverse(o -> c[1]);
}

inline node *build(int l, int r, int D) {
    int mid = l + r >> 1; nowD = D;
    nth_element(temp + l, temp + mid, temp + r + 1, cmp);
    node *res = temp[mid];
    res -> Max[0] = res -> Min[0] = res -> d[0];
    res -> Max[1] = res -> Min[1] = res -> d[1];
    if (l != mid) res -> c[0] = build(l, mid - 1, !D);
    else res -> c[0] = &Null;
    if (r != mid) res -> c[1] = build(mid + 1, r, !D);
    else res -> c[1] = &Null;
    res -> update();
    return res;
}

int x, y, a, b, tmpD;
node **tmp;
inline void rebuild(node *&o, int D) {
    tot = 0;
    traverse(o);
    o = build(1, tot, D);
}

inline void insert(node *&o, node *p, int D) {
    if (o == &Null) {o = p; return;}
    if (p -> Max[0] > o -> Max[0]) o -> Max[0] = p -> Max[0];
    if (p -> Max[1] > o -> Max[1]) o -> Max[1] = p -> Max[1];
    if (p -> Min[0] < o -> Min[0]) o -> Min[0] = p -> Min[0];
    if (p -> Min[1] < o -> Min[1]) o -> Min[1] = p -> Min[1];
    o -> siz ++, o -> sum += p -> sum;
    insert(o -> c[p -> c[D] >= o -> c[D]], p, !D);
    if (max(o -> c[0] -> siz, o -> c[1] -> siz) > int(o -> siz * 0.75 + 0.5)) tmpD = D,
        ↪ tmp = &o;
}

inline int query(node *o) {
    if (o == &Null) return 0;
    if (x > o -> Max[0] || y > o -> Max[1] || a < o -> Min[0] || b < o -> Min[1]) return
        ↪ 0;
    if (x <= o -> Min[0] && y <= o -> Min[1] && a >= o -> Max[0] && b >= o -> Max[1])
        ↪ return o -> sum;
    return (x <= o -> d[0] && y <= o -> d[1] && a >= o -> d[0] && b >= o -> d[1] ? o ->
        ↪ val : 0)
        + query(o -> c[1]) + query(o -> c[0]);
}

```

```

int main() {
    ios::sync_with_stdio(false);
    cin >> m;
    node *ttt = &Null;
    for (int t, ans = 0; ; ) {
        cin >> t;
        if (t == 3) break;
        if (t == 1) {
            cin >> x >> y >> a;
            x ^= ans, y ^= ans, n++;
            nodes[n].sum = nodes[n].val = a ^ ans, nodes[n].siz = 1;
            nodes[n].Max[0] = nodes[n].Min[0] = nodes[n].d[0] = x;
            nodes[n].Max[1] = nodes[n].Min[1] = nodes[n].d[1] = y;
            nodes[n].c[0] = nodes[n].c[1] = &Null;
            tmp = &(ttt), insert(root, &nodes[n], 0);
            if (*tmp != &Null) rebuild(*tmp, tmpD);
        } else {
            cin >> x >> y >> a >> b;
            x ^= ans, y ^= ans, a ^= ans, b ^= ans;
            if (x > a) swap(x, a);
            if (y > b) swap(y, b);
            ans = query(root);
            printf("%d\n", ans);
        }
    }
    return 0;
}

```

#### 2.5.4 KDtree 找最近点

```

/*
 * 为了维持树的平衡，可以一开始把所有点都读进来 build
 * 然后打 flag 标记该点是否被激活
 */
const int N = 5e5 + 5;
const int inf = 1 << 30;
int n, m;
int ql, qr, ans, tot, nowD;
//nowD = rand() % 1 ?
struct Node {
    int d[2];
    bool operator < (const Node &a) const {
        if (d[nowD] == a.d[nowD]) return d[!nowD] < a.d[!nowD];
        return d[nowD] < a.d[nowD];
    }
}pot[N];
struct node {
    int min[2], max[2], d[2];
    node *c[2];
    node() {
        min[0] = min[1] = max[0] = max[1] = d[0] = d[1] = 0;
        c[0] = c[1] = NULL;
    }
    node(int x, int y);
}

```

```

    void update();

}t[N], Null, *root;
node::node(int x, int y) {
    min[0] = max[0] = d[0] = x;
    min[1] = max[1] = d[1] = y;
    c[0] = c[1] = &Null;
}
inline void node::update() {
    if (c[0] != &Null) {
        if (c[0] -> max[0] > max[0]) max[0] = c[0] -> max[0];
        if (c[0] -> max[1] > max[1]) max[1] = c[0] -> max[1];
        if (c[0] -> min[0] < min[0]) min[0] = c[0] -> min[0];
        if (c[0] -> min[1] < min[1]) min[1] = c[0] -> min[1];
    }
    if (c[1] != &Null) {
        if (c[1] -> max[0] > max[0]) max[0] = c[1] -> max[0];
        if (c[1] -> max[1] > max[1]) max[1] = c[1] -> max[1];
        if (c[1] -> min[0] < min[0]) min[0] = c[1] -> min[0];
        if (c[1] -> min[1] < min[1]) min[1] = c[1] -> min[1];
    }
}
inline void build(node *&o, int l, int r, int D) {
    int mid = l + r >> 1;
    nowD = D;
    nth_element(pot + l, pot + mid, pot + r + 1);
    o = new node(pot[mid].d[0], pot[mid].d[1]);
    if (l != mid) build(o -> c[0], l, mid - 1, !D);
    if (r != mid) build(o -> c[1], mid + 1, r, !D);
    o -> update();
}
inline void insert(node *o) {
    node *p = root;
    int D = 0;
    while (1) {
        if (o -> max[0] > p -> max[0]) p -> max[0] = o -> max[0];
        if (o -> max[1] > p -> max[1]) p -> max[1] = o -> max[1];
        if (o -> min[0] < p -> min[0]) p -> min[0] = o -> min[0];
        if (o -> min[1] < p -> min[1]) p -> min[1] = o -> min[1];
        if (o -> d[D] >= p -> d[D]) {
            if (p -> c[1] == &Null) {
                p -> c[1] = o;
                return;
            } else p = p -> c[1];
        } else {
            if (p -> c[0] == &Null) {
                p -> c[0] = o;
                return;
            } else p = p -> c[0];
        }
        D ^= 1;
    }
}
inline int dist(node *o) {

```

```
int dis = 0;
if (ql < o -> min[0]) dis += o -> min[0] - ql;
if (ql > o -> max[0]) dis += ql - o -> max[0];
if (qr < o -> min[1]) dis += o -> min[1] - qr;
if (qr > o -> max[1]) dis += qr - o -> max[1];
return dis;
}

inline void query(node *o) {
    int dl, dr, d0;
    d0 = abs(o -> d[0] - ql) + abs(o -> d[1] - qr);
    if (d0 < ans) ans = d0;
    if (o -> c[0] != &Null) dl = dist(o -> c[0]);
    else dl = inf;
    if (o -> c[1] != &Null) dr = dist(o -> c[1]);
    else dr = inf;
    if (dl < dr) {
        if (dl < ans) query(o -> c[0]);
        if (dr < ans) query(o -> c[1]);
    } else {
        if (dr < ans) query(o -> c[1]);
        if (dl < ans) query(o -> c[0]);
    }
}

int main() {
    ios::sync_with_stdio(false);
    cin >> n >> m;
    for (int i = 1; i <= n; i++)
        cin >> pot[i].d[0] >> pot[i].d[1];
    build(root, 1, n, 0);

    for (int x, y, z; m--; ) {
        cin >> x >> y >> z;
        if (x == 1) {
            t[tot].max[0] = t[tot].min[0] = t[tot].d[0] = y;
            t[tot].max[1] = t[tot].min[1] = t[tot].d[1] = z;
            t[tot].c[0] = t[tot].c[1] = &Null;
            insert(&t[tot++]);
        } else {
            ans = inf, ql = y, qr = z;
            query(root), printf("%d\n", ans);
        }
    }
    return 0;
}
```

### 3 构造

#### 3.1 若干排列使所有数对都出现一次

```

/*n/2 个排列使得所有数对 (i,j) 且 i<j 都出现一次
 *n 为奇数则首尾相连, 偶数不连
 */
typedef vector<int> vi;
void get_even(int n, vi ans[]) {
    vi a(n);
    for (int i = 0; i < n; i++)
        a[i] = i + 1;
    for (int i = 1; i <= n / 2; i++) {
        ans[i].resize(n + 1);
        for (int j = 0; j < n / 2; j++)
            ans[i][j * 2] = a[j], ans[i][j * 2 + 1] = a[n - 1 - j];
        int t = a[n - 1];
        for (int j = n - 1; j > 0; j--)
            a[j] = a[j - 1];
        a[0] = t;
    }
}
void get_odd(int n, vi ans[]) {
    get_even(n - 1, ans);
    for (int i = 1; i <= n / 2; i++) {
        for (int j = n - 1; j > 0; j--)
            ans[i][j] = ans[i][j - 1] + 1;
        ans[i][0] = 1;
    }
}
int main() {
    vi ans[2019];
    int n; cin >> n;
    if (n & 1) get_odd(n, ans);
    else get_even(n, ans);
    return 0;
}

```

#### 3.2 rec-free

```

/* 不存在四个 1 构成一个矩形, 并使得 1 尽量多, 输出 01 矩阵 */
const int N = 200, n = 150, M = 13; //M 为质数, N>M*M>n
int a[N][N], b[N][N], c[N][N];
void make() {
    for (int i = 1; i <= M; i++)
        for (int j = 1; j <= M; j++)
            a[i][j + 1] = M * (j - 1) + i;
    for (int i = 1; i <= M; i++)
        for (int j = 1; j <= M; j++)
            c[i][a[i][j]] = 1;
    for (int k = 1; k < M; k++) {
        memcpy(b, a, sizeof b);
        for (int i = 1; i <= M; i++)
            for (int j = 1; j <= M; j++)

```

```

        a[i][j] = (b[i + j - 1 - ((i + j - 1) > M ? M : 0)][j]);
    for (int i = 1; i <= M; i++)
        for (int j = 1; j <= M; j++)
            c[k * M + i][a[i][j]] = 1;
}
}

```

### 3.3 点边均整数多边形

```

/* 构造满足以下条件的简单多边形 (可以凹但边不能相交)
 * 有  $k$  个点  $k$  条边, 所有边都为整数, 所有点的坐标都为整数
 * 不存在与坐标轴平行的边
 */
typedef pair<int, int> P;
P operator * (P a, int b){return P(a.first * b, a.second * b);}
P operator + (P a, P b) {return P(a.first + b.first, a.second + b.second);}
int main() {
    int n; cin >> n;
    if (n == 3) return printf("0 0\n4 3\n-20 21\n"), 0;
    int m = n / 2 - 1;
    vector<P> v;
    v.push_back(P(0, 0));
    v.push_back(P(4, -3) * m);
    v.push_back(P(4, 0) * (2 * m));
    int lim = (n & 1) ? n - 1 : n;
    for (int i = 4; i <= lim; ++i) {
        P last = v.back();
        if ((i % 2) == 0) v.push_back(last + P(-4, 3));
        else v.push_back(last + P(-4, -3));
    }
    if (n & 1) v.push_back(P(-20, 48));
    for (P p: v) printf("%d %d\n", p.first, p.second);
}

```

## 4 计算几何

### 4.1 最小矩形覆盖含凸包和旋转卡壳

/\* 最小矩形覆盖，保留六位小数，逆时针输出四个顶点坐标 \*/

```
namespace minRectCover {
    const int N = 1e5 + 5;
    const double eps = 1e-8;
    struct point{
        double x, y;
        point(){}
        point(double x, double y):x(x), y(y){}
        bool operator < (const point &a) const {
            return fabs(y - a.y) < eps ? x < a.x : y < a.y;}
        point operator - (const point &a) const {
            return point(x - a.x, y - a.y);}
        point operator + (const point &a) const {
            return point(x + a.x, y + a.y);}
        point operator / (const double &a) const {
            return point(x / a, y / a);}

        point operator * (const double &a) const {
            return point(x * a, y * a);}
        double operator / (const point &a) const { // .
            return x * a.x + y * a.y;}
        double operator * (const point &a) const { // X
            return x * a.y - y * a.x;}

    }p[N], q[N], rc[4];
    double sqr(double x) {return x * x;}
    double abs(point a) {return sqrt(a / a);}
    int sgn(double x) {return fabs(x) < eps ? 0 : (x < 0 ? -1 : 1);}
    point vertical(point a, point b) {//与 ab 向量垂直的向量
        return point(a.x + a.y - b.y, a.y - a.x + b.x) - a;}
    point vec(point a){return a / abs(a);}
    void convexhull(int n, point *hull, int &top) {//如果要计算周长需要特判 n==2
        for (int i = 1; i < n; i++)
            if (p[i] < p[0])
                swap(p[i], p[0]);
        sort(p + 1, p + n, [&](point a, point b){
            double t = (a - p[0]) * (b - p[0]);
            if (fabs(t) < eps) return sgn(abs(p[0] - a) - abs(p[0] - b)) < 0;
            return t > 0;
        });
        int cnt = 0; //去重
        for (int i = 1; i < n; i++)
            if (sgn(p[i].x - p[cnt].x) != 0 || sgn(p[i].y - p[cnt].y) != 0)
                p[++cnt] = p[i];
        n = cnt + 1;
        hull[top = 1] = p[0];
        for (int i = 1; i < n; i++) {
            while (top > 1 && (hull[top] - hull[top - 1])
                * (p[i] - hull[top]) < eps) top--;
            hull[++top] = p[i];
        }
    }
}
```



```

    }
    hull[0] = hull[top];
}
void main() {
    int n;
    scanf("%d", &n);
    for (int i = 0; i < n; i++)
        scanf("%lf %lf", &p[i].x, &p[i].y);
    convexhull(n, q, n);
    double ans = 1e20;
    int l = 1, r = 1, t = 1;
    double L, R, D, H;
    for (int i = 0; i < n; i++) {
        D = abs(q[i] - q[i + 1]);
        while (sgn((q[i + 1] - q[i]) * (q[t + 1] - q[i]) -
            (q[i + 1] - q[i]) * (q[t] - q[i])) > -1) t = (t + 1) % n;
        while (sgn((q[i + 1] - q[i]) / (q[r + 1] - q[i]) - (q[i + 1]
            - q[i]) / (q[r] - q[i])) > -1) r = (r + 1) % n;
        if (i == 0) l = r;
        while (sgn((q[i + 1] - q[i]) / (q[l + 1] - q[i]) - (q[i + 1]
            - q[i]) / (q[l] - q[i])) < 1) l = (l + 1) % n;
        L = fabs((q[i + 1] - q[i]) / (q[l] - q[i]) / D);
        R = fabs((q[i + 1] - q[i]) / (q[r] - q[i]) / D);
        H = fabs((q[i + 1] - q[i]) * (q[t] - q[i]) / D);
        double tmp = (R + L) * H;
        if (tmp < ans) {
            ans = tmp;
            rc[0] = q[i] + (q[i + 1] - q[i]) * (R / D); //右下
            rc[1] = rc[0] + vec(vertical(q[i], q[i + 1])) * H; //右上
            rc[2] = rc[1] - (rc[0] - q[i]) * ((R + L)
                / abs(q[i] - rc[0])); //左上
            rc[3] = rc[2] - (rc[1] - rc[0]);
        }
    }
    printf("%.6f\n", ans);
    int fir = 0;
    for (int i = 1; i < 4; i++)
        if (rc[i] < rc[fir])
            fir = i;
    for (int i = 0; i < 4; i++)
        printf("%.6f %.6f\n", rc[(fir + i) % 4].x, rc[(fir + i) % 4].y);
}
}

```

## 5 数论

### 5.1 CRT

```
void exgcd(ll a, ll b, ll &d, ll &x, ll &y) {
    if (!b) {
        d = a, x = 1, y = 0;
        return;
    }
    exgcd(b, a % b, d, y, x);
    y -= a / b * x;
}

ll crt(ll *m, ll *a, int n) { // n 个式子:  $y = mx + a$ , 下标从 1 开始
    ll A = a[1], M = m[1], d, x, y, m2;
    for (int i = 2; i <= n; i++) { //  $k_1 * m_1 - k_2 * m_2 = a_2 - a_1$ 
        exgcd(M, m[i], d, x, y);
        if ((a[i] - A) % d) return -1;
        m2 = M / d * m[i];
        x = (a[i] - A) / d * x % m[i];
        A = (A + x * M % m2) % m2;
        if (A < 0) A += m2; // 保证  $A \geq 0$ 
        M = m2;
    }
    return A; //  $y = Mx + A$ 
}
```

## 6 字符串

### 6.1 KMP

```
int nxt[N];
void kmp(int n, char *a, int m, char *b) {
    //长度为 m 的 b 中找 a, 下标从 0 开始, 得到的是匹配成功的末尾位置
    static int i, j, cnt, tmp[2]; cnt = 0;
    for (nxt[0] = j = -1, i = 1; i < n; nxt[i++] = j) {
        while (~j && a[j + 1] != a[i]) j = nxt[j];
        if (a[j + 1] == a[i]) j++;
    }
    for (j = -1, i = 0; i < m; i++) {
        while (~j && a[j + 1] != b[i]) j = nxt[j];
        if (a[j + 1] == b[i]) j++;
        if (j == n - 1) {
            printf("%d ", i);
            j = nxt[j];
        }
    }
}
```

## 7 其他

### 7.1 数字哈希

```
namespace my_hash {
    const int N = (1 << 19) - 1; // 散列大小, 一定要取  $2^k-1$ , 不超内存的情况下, N越大碰撞
    ↪ 越少
    struct E {
        int v;
        E *nxt;
    } *g[N + 1], pool[N], *cur = pool, *p;
    int vis[N + 1], T;
    void ins(int v) {
        int u = v & N;
        if (vis[u] < T) vis[u] = T, g[u] = NULL;
        for (p = g[u]; p; p = p -> nxt) if (p -> v == v) return;
        p = cur++; p -> v = v; p -> nxt = g[u]; g[u] = p;
    }
    int ask(int v) {
        int u = v & N;
        if (vis[u] < T) return 0;
        for (p = g[u]; p; p = p -> nxt) if (p -> v == v) return 1;
        return 0;
    }
    void init() {T++; cur = pool;} // 应对多组数据使用
}
```

### 7.2 海岛分金币

#### 7.2.1 海岛分金币 1

```
/*
非朴素模型, 有额外条件:
每个人做决定时如果有多种方案可以使自己获得最大收益
那么他会让决策顺序靠前的人获得的收益尽可能的大!
solution:
贪心模拟
*/
#define v first
#define id second
typedef pair<int, int> pr;
const int N = 1010;
int a[N][N];
pr b[N];
int n, m;
int main() {
    cin >> n >> m;
    a[1][1] = m;
    for (int i = 2; i <= n; i++) {
        for (int j = 1; j < i; j++)
            b[j] = pr(a[i - 1][j], j);
        sort(b + 1, b + i, [&](pr x, pr y){return x.v != y.v ? (x.v < y.v) : (x.id >
            ↪ y.id)});
        // 按照是否容易满足来排序, 因为容易满足的人消耗掉的金币比较少, 也就使得当前的人获利最大
        int s = m, nd = (i - 1) / 2;
```

```

    for (int j = 1; j < i && nd; j++) {
        nd--;
        s -= (a[i][b[j].id] = a[i - 1][b[j].id] + 1);
    }
    if (s < 0) {
        for (int j = 1; j < i; j++)
            a[i][j] = a[i - 1][j];
        a[i][i] = -1;
    }
    else {
        a[i][i] = s;
    }
}
for (int i = n; i; i--)
    printf("%d ", a[n][i]);
return 0;
}

```

### 7.2.2 海岛分金币 2

/\*

海盗分金币朴素模型:

$n$  个海盗分  $m$  个金币, 依次做决策, 如果不少于半数的人同意则方案通过, 否则当前做决策的人会被淘汰

→ (收益视为  $-1$ ), 由下一人做出决策

如果一个海盗有多种方案均为最大收益, 那么他会希望淘汰的人越多越好

求出第  $x$  个做决策的海盗的最大可能受益和最小可能收益

\*/

```

struct node {
    int min_v, max_v;
    node():min_v(0), max_v(0) {}
    node(int min_v, int max_v):min_v(min_v), max_v(max_v) {}
};

node ask(int n, int m, int x) { // n 个人分 m 个金币, 第 x 个做决策的人最少/最多分到多少个金币
    int y = n + 1 - x;
    if (n >= (m + 2) * 2) {
        int a = (m + 1) * 2, b = 2, c = 4;
        // 前 a 个为 [0, 1], 后 b 个为 [0, 0], 将持续 c 个
        while (a + b + c <= n) {
            a += b;
            b *= 2;
            c *= 2;
        }
        if (y <= a) return node(0, 1);
        else if (y <= a + b) return node(0, 0);
        else return node(-1, -1);
    }
    else if (n == m * 2 + 3) {
        if (x == 1) return node(-1, -1);
        else if (y <= m * 2 && y % 2 == 1 || x == 2) return node(0, 0);
        else return node(0, 1);
    }
    else if (n == m * 2 + 2) {
        if (y <= m * 2 && y % 2 == 1 || x == 1) return node(0, 0);
        else return node(0, 1);
    }
}

```

```

    }
    else if (n == m * 2 + 1) {
        if (y <= m * 2 && y % 2 == 1) return node(1, 1);
        else return node(0, 0);
    }
    else {
        if (x & 1) {
            if (x != 1) return node(1, 1);
            else return node(m - (n - 1) / 2, m - (n - 1) / 2);
        }
        else return node(0, 0);
    }
}

int main() {
    ios::sync_with_stdio(false);
    int x, n, m, k; node y;
    cin >> n >> m >> k;
    while (k --) {
        cin >> x;
        y = ask(n, m, x);
        printf("%d %d\n", y.min_v, y.max_v);
    }
    return 0;
}

```

```

/*
m = 5

```

```

1 5
2 0 5
3 1 0 4
4 0 1 0 4
5 1 0 1 0 3
6 0 1 0 1 0 3
7 1 0 1 0 1 0 2
8 0 1 0 1 0 1 0 2
9 1 0 1 0 1 0 1 0 1
10 0 1 0 1 0 1 0 1 0 1
11 1 0 1 0 1 0 1 0 1 0 0
12 0 _ 0 _ 0 _ 0 _ 0 _ 0
13 0 _ 0 _ 0 _ 0 _ 0 _ 0 -1
14 _ _ _ _ _ _ _ _ _ 0 0
15 _ _ _ _ _ _ _ _ _ 0 0 -1
16 _ _ _ _ _ _ _ _ _ 0 0 -1 -1
17 _ _ _ _ _ _ _ _ _ 0 0 -1 -1 -1
18 _ _ _ _ _ _ _ _ _ 0 0 0 0
19 _ _ _ _ _ _ _ _ _ 0 0 0 0 -1
20 _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1
21 _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1 -1
22 _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1 -1 -1
23 _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1 -1 -1 -1
24 _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1 -1 -1 -1 -1
25 _ _ _ _ _ _ _ _ _ 0 0 0 0 -1 -1 -1 -1 -1 -1 -1

```

```
26 - - - - - 0 0 0 0 0 0 0 0
*/
```

### 7.3 根号枚举

```
for (int i = 1, last; i <= n; i = last + 1) {
    last = n / (n / i);
    //当前枚举区间为 [i, last]
}
```

### 7.4 读入输出外挂

```
namespace IO {//only for int!!!
    static const int SIZE = 1 << 20;
    inline int get_char() {
        static char *S, *T = S, buf[SIZE];
        if (S == T) {
            T = fread(buf, 1, SIZE, stdin) + (S = buf);
            if (S == T) return -1;
        }
        return *S ++;
    }
    inline void in(int &x) {//for int
        static int ch;
        while (ch = get_char(), ch < 48); x = ch ^ 48;
        while (ch = get_char(), ch > 47) x = x * 10 + (ch ^ 48);
    }
    char buffer[SIZE];
    char *s = buffer;
    void flush() {//最后需要 flush!!
        fwrite(buffer, 1, s - buffer, stdout);
        s = buffer;
        fflush(stdout);
    }
    inline void print(const char ch) {
        if (s - buffer > SIZE - 2) flush();
        *s++ = ch;
    }
    inline void print(char *str) {//for string
        while(*str != 0)
            print(char(*str ++));
    }
    inline void print(int x) {
        static char buf[25];
        static char *p = buf;
        if (x < 0) print('-'), x = -x;
        if (x == 0) print('0');
        while(x) *(++ p) = x % 10, x /= 10;
        while(p != buf) print(char(*(p --) ^ 48));
    }
};
```

## 7.5 给定小数化成分数

# 本题答案的分母不超过  $1e9$ , 给定小数的小数点位为 18 位

# 单次  $O(\log 2n)$

```

inf, inff = 10 ** 9, 10 ** 18
for i in range(int(input())):
    n = int(input()[2:])
    if n == 0: print('0 1')
    else:
        lp, lq, rp, rq = 0, 1, 1, 1
        while max(lq, rq) <= inf:
            mp, mq = lp + rp, lq + rq
            if mp * inff <= mq * n:
                l, r, mid, cnt = 1, (inf - lq) // rq + 1, -1, -1
                while l <= r:
                    mid = l + r >> 1
                    if (lp + rp * mid) * inff <= (lq + rq * mid) * n:
                        cnt, l = mid, mid + 1
                    else:
                        r = mid - 1
                lp, lq = lp + rp * cnt, lq + rq * cnt
            else:
                l, r, mid, cnt = 1, (inf - rq) // lq + 1, -1, -1
                while l <= r:
                    mid = l + r >> 1
                    if (rp + lp * mid) * inff > (rq + lq * mid) * n:
                        cnt, l = mid, mid + 1
                    else:
                        r = mid - 1
                rp, rq = rp + lp * cnt, rq + lq * cnt
        if lq <= inf: print(lp, lq)
        else: print(rp, rq)

```